THE SCATTERING OF ELECTRONS IN GASES

THESIS BY

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SUMMARY OF RESULTS

It is found

(1) That the angular distribution function of electrons scattered in mercury is not a monotonic function of the angle, but has a minimum whose angular position is an inverse cotangent function of the energy of the electrons.

(2) That the angular distribution function of electrons scattered in air is a monotonic function of the angle within the range investigated.

(3) That measurements of small angle scattering in mercury are in qualitative agreement with the work of other observers.

B I B L I O G R A P H Y

1. K. T. Compton and F. L. Mohler, Bul. Nat. Research Council, Vol. 9.
2. K. T. Compton and Irving Langmuir, Reviews of Mod. Physics, Vol. 2.
3. C. Ramsauer, Ann. d. Physik, 64, 513.
4. E. Brüche, Ann. d. Physik, 4, 387; C. Ramsauer u. R. Kollath, Ann. d. Physik, 4, 91; T. J. Jones, Phys. Rev., 32, 459; R. B. Brode, Pro. Roy. Soc., 125, 134.
5. E. G. Dymond, Phys. Rev., 29, 433.
6. G. P. Harnwell, Pro. Nat. Acad. of Sciences, 14, 564.
7. G. P. Harnwell, Phys. Rev., 34, 661; Dymond and Watson, Pro. Roy. Soc., 122, 571.
8. J. M. Pearson, Journ. Optical Soc., 19, 371.
9. Whiddington, Nature, 124, 758.
10. P. L. Copeland, Phys. Rev., 35, 982.
11. F. L. Arnot, Pro. Roy. Soc., 125, 660.
12. I. Langmuir and H. A. Jones, Phys. Rev., 31, 357.

THE SCATTERING OF ELECTRONS IN GASES

I

THE PROBLEM

In the studies of the atom, as a structure made up of nuclei and electrons, a great deal of information has been obtained from experiments dealing with energy relations. Modern spectroscopy has revealed an enormous complexity of "excited states" to which the atom may be raised, and the quantum theory has given a very satisfactory description of the phenomena. The problem has also been attacked by observing the energy losses suffered by electrons in their interaction with atoms and molecules. A complete account of the experiments in this field is given by Compton and Mohler,(1) and by Compton and Langmuir.(2)

Another method of attack has been to observe the effective cross-sectional area presented to a moving electron by an atom. This was done most accurately by a method due to Ramsauer(3) which has since been refined and specialized by others.(4) The results are expressed by a quantity α which is the area in sq. cm. presented to a moving electron by one c.c. of the gas under consideration at a pressure of one mm. of mercury. The reciprocal of α is thus the mean free path of the electron in the gas.

If a homogeneous beam of electrons passes through a gas we may say that an electron has collided with an atom if it suffers an appreciable change in energy or momentum.

We shall call those collisions in which the energy does not change appreciably "elastic" collisions and all others "inelastic". By an appreciable change in energy is meant a change which is comparable to the lowest resonance potential of the atom, and a corresponding change in momentum will be defined by the geometry of the apparatus in any particular case.

In any theory professing to give a complete picture of the atom as an electrical structure it will be necessary to know the field of force in its neighborhood. We can hope to do this only by investigating these momentum transfers in detail. Because of the enormous disparity in masses, it is obvious that the electron alone need be followed. This problem of determining the angular distribution of electrons scattered by atoms has only recently claimed attention. The early experiments of Dymond (5) and Harnwell (6) were in disagreement, but some of the difficulties of the problem were made clear. Both observers have since reported results which seem to be in qualitative agreement with the present theories. (7) A series of experiments was begun at this Institute by Mr. John Pearson and the author in an attempt to improve the technique and to measure the scattering at large angles. An account of the first results obtained was given by Mr. Pearson in his Doctor's Thesis at this Institute (1930). It is with the further development of the experiment and the results obtained that this thesis will deal.

II

THE APPARATUS

A detailed account of the design and construction of the apparatus has been given by Mr. Pearson, but for completeness the essential points of construction and further modifications are given here.

FIRST: The scattering chamber should be free from electric and magnetic fields. The early experiments already referred to [(5),(6)] showed that this was imperative. It was decided to construct the whole apparatus of brass and two pairs of square Helmholtz coils, 150 cm. on a side, were made to neutralize the earth's magnetic field. Storage batteries furnished the current for these and a potentiometer measured very accurately the current which neutralized the vertical component. The chamber was a cylinder 5 inches in diameter and 4½ inches deep. A vertical cross-sectional plan through the slits S_1 and S_2, which were in optical alignment, is shown in Figure 1. The pumps and pressure gauge were connected to the chamber by pyrex glass tubes at two diametrically opposite openings not shown in the figure.

SECOND: The source of electrons was an 8 mil tungsten wire bent in the form of a narrow hairpin about as long as the slits and heated with A.C. through an insulating transformer. The filament was spot-welded to leads which led through the glass stopper, Q, as shown. Later filaments were firmly fastened to these leads by twisting tightly and

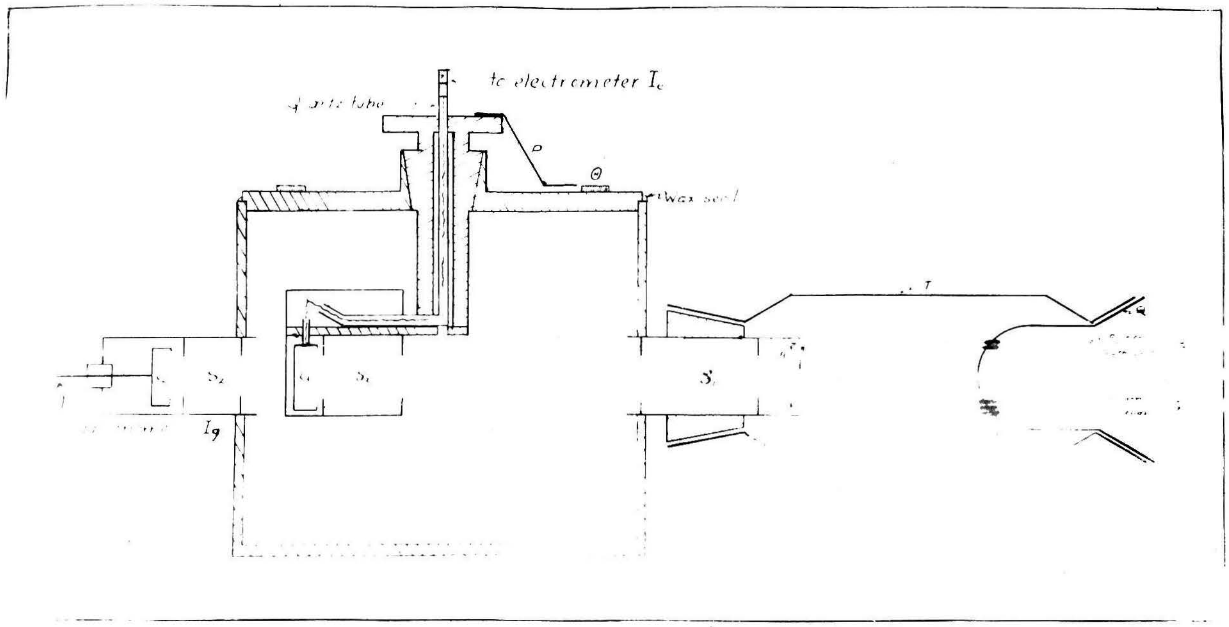

to electrometer I_c
wax seal
S_2
I_g
S_1

Fig. 1.

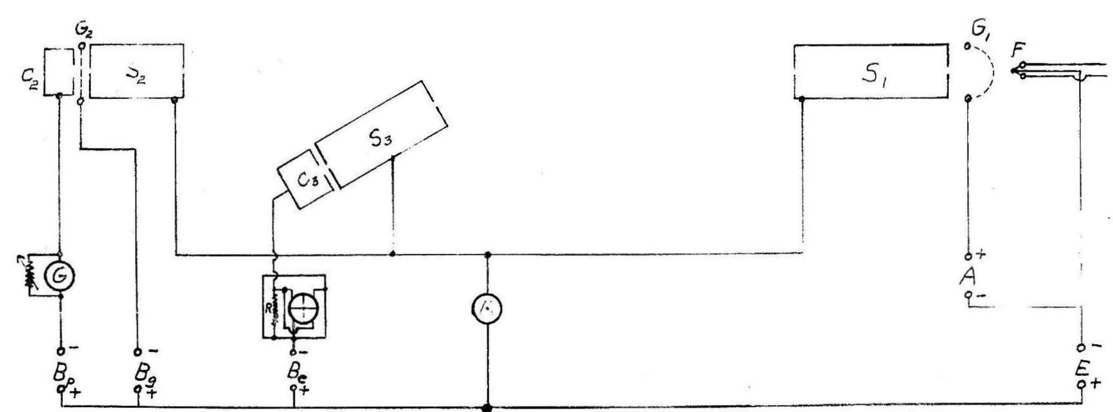

G_2
C_2
S_2
S_3
C_3
S_1
G_1
F
G
A
B_p
B_g
B_e
E

Fig. 2.

seemed to work just as well. The emission was controlled by manual operation of resistances in the filament heating circuit. The loop in the filament was supported by a nickel wire which served as the cathode lead. The inhomogeneity because of the "I R" drop in the filament was thus cut down and in most cases the spread because of this effect was less than 3 volts. An accelerating grid, A, of fine mesh nickel gauze bent in the form of a semi-circle with its axis parallel to the slits and convex toward the filament was part of the electron "gun" and served to focus a large part of the emission on the slits and thence to the chamber as a narrow beam. The stopper, Q, fitted into the glass tube, T, which in turn fastened to the chamber as shown. The various ground glass joints were made vacuum tight with picien wax and those near the hot filament were water cooled. The dimensions in the electron gun were made small so that arc discharges would not occur. Sputtered tungsten on the glass near the electrode leads sometimes caused insulation troubles but it was found possible to burn this out by applying high voltages.

<u>THIRD</u>: In order to interpret the results as single scattering it is necessary that the mean free path of the electron shall be larger than the dimensions of the chamber. According to the kinetic theory this is given by $4\sqrt{2}$ times the mean free path of the gas atoms. Direct measurements of electronic mean free paths have given values in most cases larger than this,[2] so it can safely be taken to calculate an

upper limit to the pressures which can be used. This limit is approximately 10^{-3} mm. for the gases used. In all the scattering experiments undertaken, this condition was amply satisfied.

FOURTH: In the results of this experiment first reported, an anomalous loss of energy of the electrons in the main beam was described. In order to investigate this point more closely, an auxiliary grid of nickel wire was mounted between the fixed Faraday cage and the last slit. This grid, which could be maintained at any desired potential, is shown in Fig. 2, but through an error was left out of Fig. 1. Results showed that this formed a fairly good equipotential surface.

FIFTH: A schematic diagram of the electrical connections is shown in Fig. 2. The filament, F, was maintained at a negative potential, E, with respect to ground and the accelerating grid at a positive potential, A, with respect to, F. All the other potentials were with respect to ground. The milliammeter, M, measured the current to the slits and chamber. G_2, is the retarding grid and its potential will be called, B_g. The potentials, E, A, and B_g, were obtained from potentiometers fed by a D.C. motor generator set and batteries supplied, B_p, and B_e. The galvanometer, G, had a sensitivity of 8×10^{-9} amp. per cm. and was provided with shunts in order to read larger currents. A Dolazalek electrometer shunted with a large resistance, R, measured the current to the rotating collector C_3. It was regularly operated at a sensitivity of 2000 mm.

per volt. An analysis of the theory of the instrument and a method of obtaining greater sensitiveness is given by Pearson.[8] The resistance, R, was a narrow line of india ink drawn on drawing paper, boiled in paraffin, and sealed in a glass tube. After many trials one was made which had a constant resistance of 2×10^{11} ohms. Another resistance (2×10^{9} ohms) was used to read larger currents. Thus it was possible to measure currents as small as 10^{-14} amp. It is interesting that these high resistances should be as constant as they were found to be. Even at high voltages (2000 volts) the resistance decreased less than $\frac{1}{4}$ of its value at 1 volt. The resistance of the insulation about the lead to collector, C_3, was measured and found to be of the order of 10^{15} ohms at 200 volts and so a correction to the electrometer readings because of this was not large enough to be considered.

All the slits were 13 mm. long and 1 mm. wide except S_2, which were .8 mm. wide. The slits, S_1, were $1\frac{1}{4}$" apart, S_2 were $\frac{3}{4}$", and S_3 1". The rotating collector was mounted in a ground metal joint in the cover, which was sealed to the chamber with picien wax. The index, P, attached to the rotator gave the angular position on the engraved scale (N).

A two stage mercury pump backed by an oil pump was used to evacuate the apparatus. Liquid air traps isolated the pumps and McLeod gauge from the chamber. With the pumps in operation, the pressure was lower than could be read on the gauge; i.e., less than 10^{-6} cm. of Hg. Under favorable running conditions, the system would hold a vacuum of

$5x10^{-5}$ cm. for a couple of days without pumping, showing that it was quite tight for a metal apparatus. Because the chamber was constructed of brass, it was impossible to bake out, but it was found that after several days of pumping and intermittent bombardment with an intense electron beam, the vacuum conditions were very good.

III

PROCEDURE

The value of the current necessary to neutralize the horizontal component of the earth's magnetic field had been determined with a magnetic pendulum before the apparatus was mounted in position. It was found that comparatively large variations in this current had no effect on I_g, the current to the galvanometer, so this adjustment was never critical. The current in these coils was set at this value and then, with the electron gun furnishing a steady current to the slits, the current in the other pair of coils was varied until the galvanometer deflection was a maximum. Experiments showed that this current was slightly different when B_g and B_p were zero, than when the potentials were adjusted so that only full speed electrons could reach the collector. This difference is due to the formation of slow speed secondaries at or near the slits. The voltage, A, was then varied in order to get as intense a beam as possible and the reading of, M, was kept constant during a run.

With $(I_g)_{max.}$ for full speed electrons found in this way, runs were taken observing, I_g, as a function of B_ρ, or B_g, as will be described later. Also, I_g, as a function of the vertical magnetic field was investigated. This gave the "shape" of the main beam. In general this was not symmetrical about the value $(I_g)_{max.}$ when a new filament was being tried out, and so it was necessary to again line up the filament with the slits. This was a slow process, but when it was done carefully, the readings at positive and negative angles checked very well. Fortunately for the experiment a good filament would last some time.

Mercury vapor was the most convenient gas to investigate and so most of the work was done with it. A mixture of ice and water was placed about the "liquid air trap" near the chamber which contained a small quantity of liquid mercury. The temperature of this bath determined the pressure of mercury vapor in the chamber when equilibrium was reached, as long as the bath was below room temperature. The other liquid air trap next to the McLeod gauge was connected to the chamber by a long narrow glass tube and consequently had little to do with the pressure in the chamber. Liquid air was usually left on this trap as it didn't seem to make a difference whether it was cold or not. The ice-water mixture was vigorously agitated by a stream of air bubbling through it and the temperature was checked with a thermometer. The International Critical Tables gives the value 1.8×10^{-4} mm. for the pressure of mercury vapor at 0° C.

which was used in the calculations.

Experiments were attempted with hydrogen but it was found impossible to keep the pressure constant long enough to take a run because of some kind of a "getter" action removing the gas. Electrolytic hydrogen was introduced into the vacuum tight apparatus through a liquid air trap and allowed to come to equilibrium. Then the filament was turned on and observations started. It was noticed that the pressure steadily diminished as long as the electron gun was operating. In one instance the pressure fell from 2.7×10^{-3} cm. to 1.2×10^{-3} cm. in half an hour and then remained constant for over three hours after the filament was turned off. This phenomenon was observed several times and so work with this gas was given up. With air in the apparatus at similar pressures the phenomenon was not noticed.

In making scattering measurements the current to the rotating collector, I_e, was observed as a function of the angular position θ. Experiments showed that no current reached the collector C_3 in vacuum when θ was greater than 15°. This absence of scattering at larger angles was used as a criterion for good vacuum conditions. Because of the inhomogeneity in the main beam it was considered practical to measure elastic scattering only, and so the value of B_e was always adjusted to collect the full speed electrons. At angles greater than 30° simultaneous readings of I_e and $(I_g)_{max.}$ were made. At times the main beam would vary considerably due to variable slit scattering or filament variations, but the ratio I_e/I_g for a given angle and pressure was remarkably constant. In general two readings at each setting were made and if these checked to within 5% they were accepted. At large angles it was also necessary to correct for the positive ions which diffused into the collector. This was conveniently done by raising the bias B_e about 10 volts above E, and observing the current to the collector. For very large angles the positive ion current was often greater than the electron current.

Fluctuations in the line voltage were particularly bothersome during the day, sometimes making it impossible to operate. However, experiments were conducted in the early morning when conditions were usually very constant so that one man could operate the apparatus. More weight was attached to curves taken under the conditions in interpreting the results.

IV

RESULTS

1. ENERGY OF ELECTRONS: Typical curves showing I_g as a function of B_p before the introduction of the retarding grid are shown. The initial drop at small values of the retarding field was due to the exclusion of slow speed secondaries. When the mercury was present this group also included positive ions which were attracted to the collector by the negative field. The subsequent rise in the curve in the case of gas was due, probably, to a decrease in ionization as the electrons

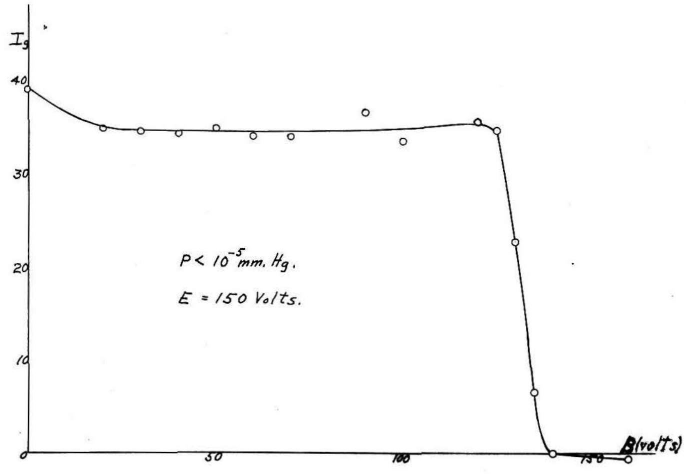

I_g
40
30
20
10
0
50
100
150
B(volts)
$P < 10^{-5}$ mm. Hg.
E = 150 Volts.

Fig. 3.

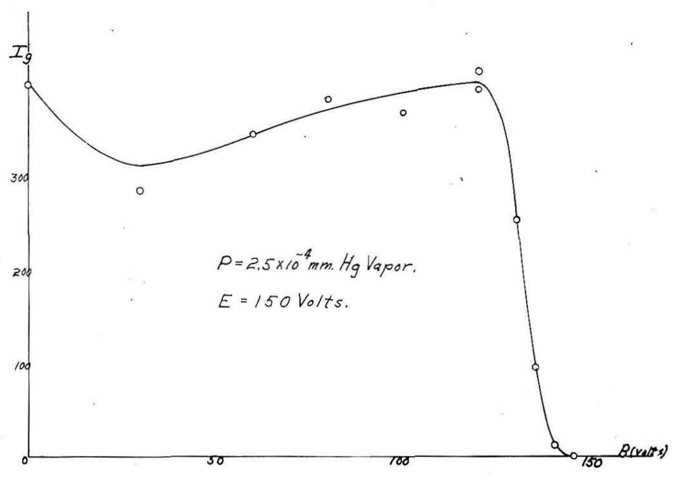

I_g
300
200
100
0
50
100
150
B(volts)
$P = 2.5 \times 10^{-4}$ mm. Hg Vapor.
E = 150 Volts.

Fig. 4.

were slowed up. This part of the curve was very unsteady when gas was present, while in vacuum the curve was quite flat. The most interesting feature is the early drop in the curves which was repeatedly found regardless of the value of E. According to these curves the electrons have lost some 25 volts of their original energy which is much too large to be due to contact e.m.f.'s, etc. A similar effect has been observed by Whiddington[9] in gases but the present phenomenon seems to be more complicated in that it is present even with good vacuum conditions. However, when the grid G_2 was used to retard the electrons and $B_p = 0$, curves similar to the upper curve in Fig. 5 were obtained. When gas was present, the middle of the curves was always lower than either side as in Fig. 4. Two grids of different mesh wire screen were tried but the curves remained essentially the same. These facts make it difficult to explain the phenomenon on the basis

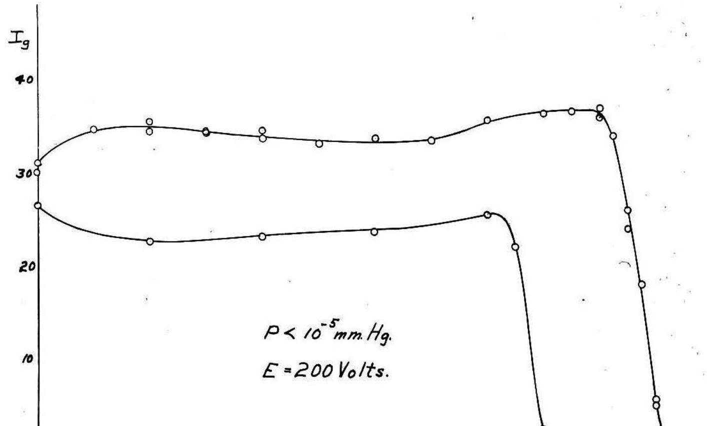


Fig. 5.

of electrostatic interpenetration of fields and as the effect seemed independent of current densities, ordinary space charge does not help. Further, it was found that B_p could be any value between zero and about 30 volts positive with respect to B_g and the retardation curves would not drop off below the bias E. But when B_p was made less positive with respect to B_g the drop off occured below E and when $B_p = B_g$ curves similar to the lower curve in Fig. 5 were obtained. It should be noticed that in this case there is supposedly no field between collector and grid but, nevertheless, a curve similar to Fig. 3 was found. Some direct experiments by Copeland (10) on "Secondary Electrons from Contaminated Surfaces" have shown the effect of coating the collector, in this case a tungsten wire, with an oil film. This produced an insulating layer which very efficiently reflected the incident electrons when there was no field at the surface of the collector tending to prevent the charges from leaving. If such a film were present on the surface of the collector used in our experiments, which is quite possible as brass contaminates very easily, it is easy to see how the collector at 140 volts can reflect 150 volt electrons. The incident electrons will collect on the surface of the film until the potential due to space charge becomes sufficiently negative with respect to the plate so electrons can pass to the plate. But, as the plate is made more and more negative, a point is finally reached where the surface of the film is at a potential E and the current starts to drop off. However, the collector is not yet at the potential E and so curves like Figures 3 and 4 were obtained. Further experiments with other collectors are being conducted to test this point more carefully. It seems at least in view of this evidence that the speed of the electrons in the main beam may safely be taken as given by E.

2. FOCUSING EFFECT: When I_g was observed as a function of the vertical magnetic field, an interesting focusing effect was observed. Figure 6 shows a typical curve. The low broad peak represents the observations in vacuum and the sharp peak the corresponding measurements in mercury. The "half width" is indicated in both cases. To obtain the resultant magnetic field in gauss, multiply the difference in abscissa of any point with the abscissa for $(I_g)_{max.}$ by 1.10. This effect has been observed in cathode ray oscillograph tubes and has been qualitatively explained as due to positive ions of low mobility, compared to electron speeds in the electron beam. The effect varied with current density, pressure, and velocity of electrons but was not studied in detail. It is important in the present scattering experiments as giving a measure of the number of positive ions present in the scattering volume.

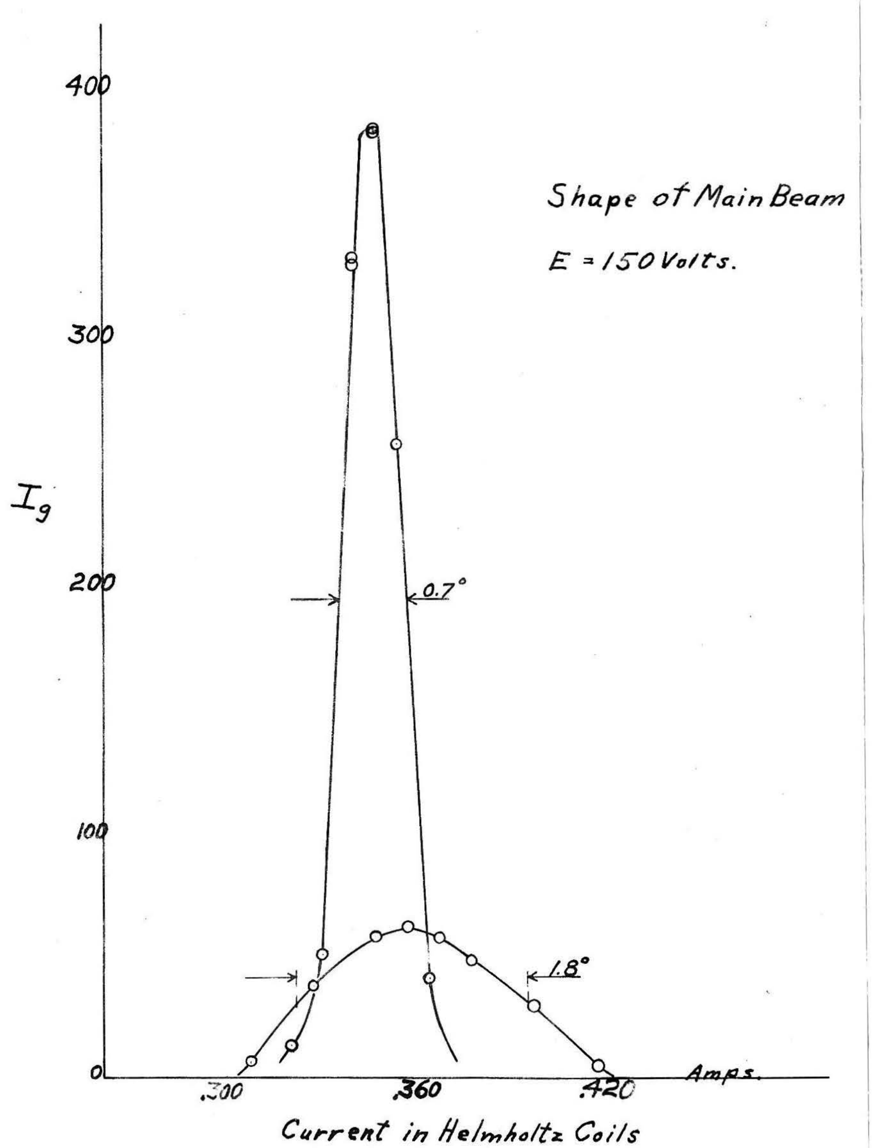

Shape of Main Beam
E = 150 Volts.
400
300
200
100
0
I_g
0.7°
1.8°
.300
.360
.420
Amps.
Current in Helmholtz Coils

Fig. 6.

3. <u>SCATTERING CURVES</u>: The equation:

$$\frac{1}{K} f(\theta) = \frac{I_e}{I_g} \frac{\sin\theta}{p} \qquad (1)$$

was used in interpreting the results (appendix I). I_e is the scattered current reaching the collector C_3 , I_g the main beam, θ the angle of setting and p the pressure. $f(\theta)$ is the probability per unit solid angle of an electron being scattered through an angle θ in going 1 cm. through the gas considered, at a pressure of 1 mm. of mercury. The curves show $\frac{1}{K} f(\theta)$ plotted in arbitrary units against θ in degrees. Because of the very small currents scattered at large angles, the values of M and A were usually adjusted to give a maximum current. However, it was possible to work with lower current densities. Results in a typical case are shown in Figure 7. The smooth curve is drawn through the crosses, which were taken when the focusing effect was very pronounced. The value of $(I_g)_{max.}$ with gas was 5 times the corresponding value in vacuum. Several months later, using a new filament, the curve marked with circles was taken when conditions were adjusted so that this focusing effect was much smaller. $(I_g)_{max.}$ was in this case but 10% greater in gas than in vacuum. In plotting this curve the "K" in the above equation had to be adjusted by a small factor to bring the curves together. This "K" which includes the slit scattering and some of the geometry of the chamber was always found to vary from day to day and sometimes during the progress of a run. However, qualitative results were all that were hoped for and it can be seen that

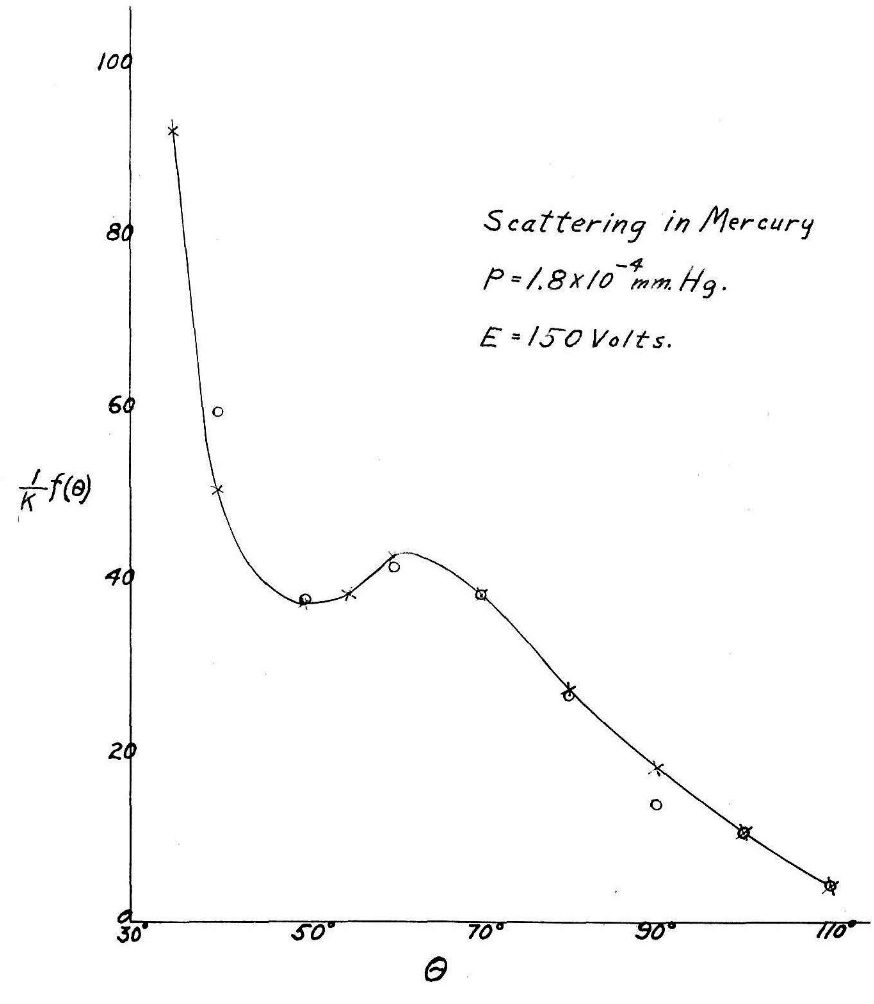

Scattering in Mercury
P = 1.8 x 10^-4 mm. Hg.
E = 150 Volts.
100
80
60
40
20
0
1/K f(θ)
30°
50°
70°
90°
110°
θ

Fig. 7.

the two curves are almost identical. All of the scattering curves taken in mercury at large angles appeared as if they would be quite monotonic except for a definite minimum, as if the scattered current at a particular angle was entirely missing. Because of the spread of electron velocities and the finite resolving power of the slits this appears as a rather broad minimum whose location is a definite function of the speed of the electrons. Curves were taken for $E = 100$ to $E = 200$ volts and the summary of the results is given in Fig. 8. It was impossible to work with slower electrons because the slit scattering increased very fast for these and practically no current reached the collectors. Above 200 volts the minimum appears in the steep part of the curves and so is smoothed out. An interesting correlation is seen when cotangent $\Theta_{min.}$ is plotted against the energy of

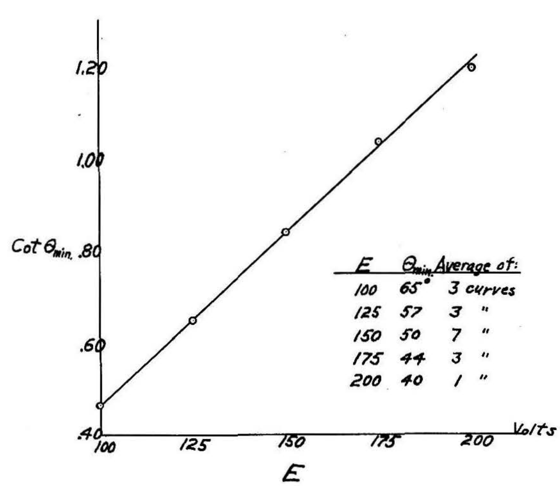
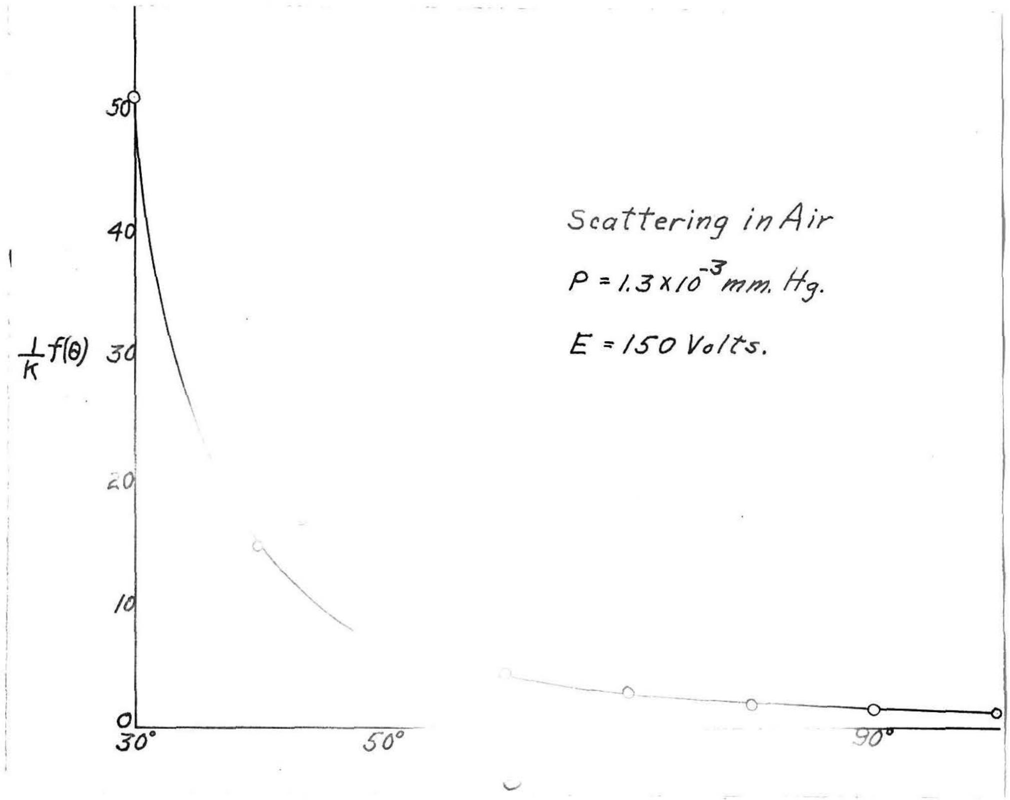


Fig. 8.

the electrons in volts. This is something like what would be expected if the scattering followed an inverse square law of force. The agreement should not be emphasized too much, however, as it may be only fortuitous.

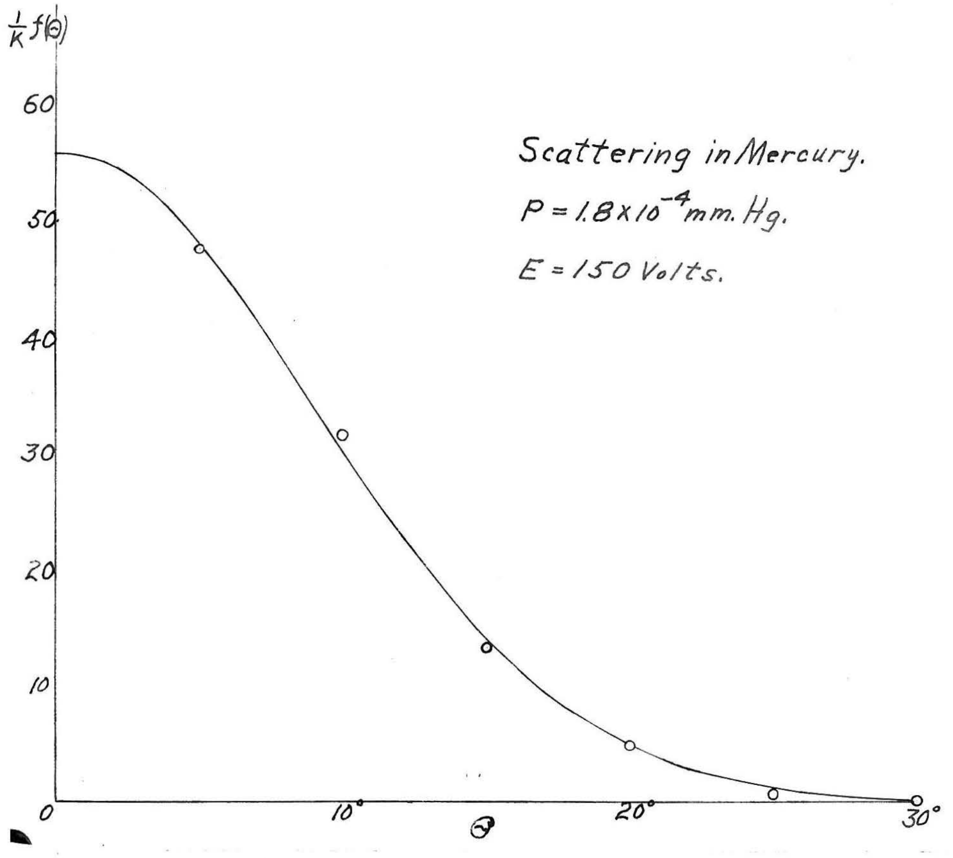
Scattering measurements at large angles were taken with air as the scattering gas and Fig. 9 gives the results for $E = 150$ volts. It was necessary to use slightly higher pressures to obtain large enough scattered currents to measure but the condition for single scattering was still satisfied. In contrast to the curves for mercury, this one shows a simple monotonic decrease as θ increases. This curve can be shown to agree very closely with an inverse square law of force but not enough curves were taken with air to make this very certain, as it was thought that the scattering due to a mixture of gases would be hard to interpret. Very little focusing effect was found with air.

Toward the completion of the work the large resistance R (Fig. 2) developed an e.m.f. and as it usually took several months for a new resistance to become constant, it was decided to try to measure the scattering at small angles in mercury with this apparatus. The procedure was the same as for large angles except that low filament emissions were used. It was found that the value of $(I_g)_{max.}$ did not vary appreciably under these conditions. The main beam was measured at the beginning of the run and checked at the end. The value of $\frac{1}{K} f(\theta)$ was computed as before and the results are shown by the circles



Scattering in Air
P = 1.3 x 10^-3 mm. Hg.
E = 150 Volts.
1/K f(θ)
50
40
30
20
10
0
30°
50°
90°

Fig. 9.



$\frac{1}{K}f(\theta)$
60
50
40
30
20
10
0
10°
20°
30°
θ
Scattering in Mercury.
$P = 1.8 \times 10^{-4}$ mm. Hg.
E = 150 Volts.

Fig. 10.

in Fig. 10. Recently Langmuir[(2)] has shown that practically all the measurements of the scattering function for electrons at small angles can be expressed by the empirical equation:

$$f(\theta) = f(\theta)_{\theta=0}\, e^{-\frac{\theta^2}{\theta_0^2}} \qquad (2)$$

where θ_0 is the "mean square angle of deflection". Using the data obtained in this experiment, the constants in this equation can be obtained graphically by plotting log $f(\theta)$ against θ^2. When this is done and $f(\theta)$ plotted, the smooth curve shown in Fig. 10 is obtained. The experimental points fit this equation quite well. Arnot[(11)] has recently reported direct measurements of the angular scattering of 82 volt electrons in mercury. Langmuir has shown that these results are consistent with equation (2) where $\theta_0 = 11.3°$. A value of $\theta_0 = 12.7°$ was found in the present experiments for 150 volt electrons. Langmuir and Jones[(12)] have measured the small angle scattering in mercury by an indirect method and they find $\theta_0 = 10°$, approximately, for 150 volt electrons. Their method does not distinguish sharply between loss of energy and loss of forward momentum, however. It is impossible to calculate the value of $f(\theta)$ from the present experiments because of the uncertainty in "K" (equation 1). As was mentioned, "K" would vary from day to day as much as a factor 5. All that can be said is that the value of the total probability of scattering α as computed from this data is in agreement with the value found by other observers in order of magnitude. The calculation is shown in Appendix II.

It has been objected that since the scattering volume increases rapidly at small angles, there is an uncertainty in the interpretation of the results as the main beam does not remain constant throughout the scattering volume. However, at the low pressures used in this experiment, this is not a serious objection because it is easily computed that the change in the main beam when $\theta = 5^\circ$ is less than 1% in crossing the scattering volume. A much more serious objection presents itself because of the length of the slits. At these angles, an electron may enter the collector having been deflected as little as θ or as much as ψ where $\cos\psi = \cos\theta \frac{d_3}{\sqrt{d_3^2 + h^2}}$ "h" is the length of the slit and "d " its distance from the scattering volume. When $\theta = 5^\circ$, $\psi = 18^\circ$ Which is quite an appreciable difference. Further work may justify a correction for this effect.

V

CONCLUSIONS

It is found

(1) That the angular distribution function of electrons scattered in mercury is not a monotonic function of the angle, but has a minimum whose angular position is an inverse cotangent function of the energy of the electrons.

(2) That the angular distribution function of electrons scattered in air is a monotonic function of the angle within the range investigated.

(3) That measurements of small angle scattering in mercury are in qualitative agreement with the work of other observers.

During the course of the experiment the following phenomena were observed:

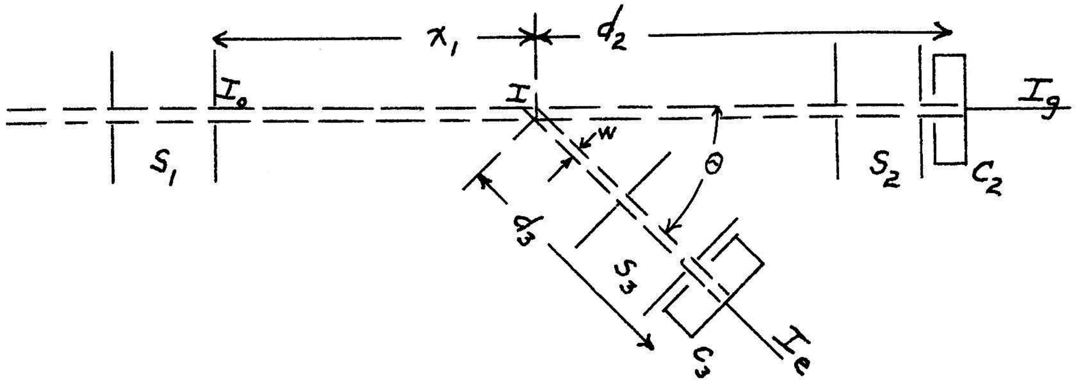
1. The apparent loss of energy of the electrons in the main beam as measured by retarding potentials.

2. Focusing effect.

3. The "getter" action which prevented work with hydrogen.

In conclusion the author would like to express his appreciation and indebtedness to Professor R. A. Millikan for his interest, advice and encouragement in the work, to Mr. Selby Skinner for his patient assistance in taking readings and to Mr. Julius Pearson and Mr. William Clancy for the technical assistance which they so kindly contributed.

APPENDIX I



Let a current I_0 pass through the first slits into the chamber and denote by I_g the current collected by the fixed Faraday cage and by I_e that collected by the rotating cage set at an angle θ. Let the current in the main beam at the scattering volume be I and the total current scattered in this volume I_s. The currents are diminished by scattering in passing through the gas according to the well-known relation $I = I_0 e^{-\alpha p l}$ where "I" is the current in the beam after traversing a distance "l" in a gas at pressure "p" when the absorption coefficient is "α". Let $(1-k_2)$ be the fraction of the current scattered by slits S_2 and $(1-k_3)$ the fraction scattered by S_3. With these notations the following relations hold:

$$I = I_0 e^{-\alpha p x_1} \tag{1}$$

$$I_s = -\frac{dI}{dx_1}\delta x_1 = \alpha p I_0 e^{-\alpha p x_1}\delta x_1 \tag{2}$$

$$I_s f(\theta)\delta\Omega\, k_3 e^{-\alpha p d_3} = I_e \tag{3}$$

$$I_0 e^{-\alpha p (x_1 + d_2)} k_2 = I_g \tag{4}$$

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interchange

$f(\Theta)$ is the fraction of the total scattered current that is scattered in the direction Θ per unit solid angle and $\delta\Omega$ is the solid angle subtended at the scattering volume by the collector. By using equations (2), (3), and (4) we can express I_s in terms of the known quantity I_g and solve for $f(\Theta)$.

$$I_e = \frac{k_3}{k_2} \alpha p I_g e^{\alpha p(d_2 - d_3)} \delta x_1 \delta\Omega$$

$$f(\Theta) = \frac{k_2}{k_3} \frac{I_e}{I_g} \frac{e^{-\alpha p(d_2 - d_3)}}{\alpha p \, \delta x_1 \delta\Omega} \qquad (5)$$

This equation contains correcting factors for finite solid angle $(\delta\Omega)$, scattering volume (δx_1), density of gas (p), and for difference in path length $(d_2 - d_3)$. Substituting $\delta x_1 = \frac{W}{\sin\Theta}$ where "W" is the width of slits S_3 we have:

$$f(\Theta) = K \frac{I_e}{I_g} \frac{\sin\Theta}{p}$$

$$\text{where: } K = C \frac{e^{-\alpha p(d_2 - d_3)}}{\alpha} = \frac{k_2}{w \, \delta\Omega \, k_3} \frac{e^{-\alpha p(d_2 - d_3)}}{\alpha} \qquad (6)$$

It is obvious from the definition of $f(\Theta)$ that:

$$\int_{\Omega} f(\Theta) \, d\Omega = \alpha$$

$$\text{and so: } \frac{C}{\alpha} e^{-\alpha p(d_2 - d_3)} \int_0^{\pi} \left(\frac{I_e}{I_g} \frac{\sin\Theta}{p} \right) \sin\Theta \, d\Theta = \frac{\alpha}{2\pi} \qquad (7)$$

This can be solved for α provided the constant "C" is known.

APPENDIX II

If the scattering of electrons through small angles is properly described by the equation,[2]

$$f(\Theta) = f(\Theta)_{\theta=0}\, e^{-\frac{\Theta^2}{\Theta_0^2}} \qquad (1)$$

where Θ_0 is the mean square angle of deflection and $f(\Theta)$ is the probability that an electron will be deflected through an angle Θ per unit solid angle in going a unit distance in the gas at 1 mm. of mercury pressure, we may calculate the total scattering produced by the gas.

$$\alpha = \int_{\Omega} f(\Theta)\, d\Omega = 2\pi \int_0^{\pi} f(\Theta) \sin\Theta\, d\Theta \qquad (2)$$

Combining (1) and (2) and noticing that most of the contribution to the integral comes from small angles because of the exponential term we may write:

$$\alpha = 2\pi f(\Theta)_{\theta=0} \int_0^{\pi} \sin\theta\, e^{-\frac{\Theta^2}{\Theta_0^2}} d\Theta \doteq 2\pi f(\Theta)_{\theta=0} \int_0^{\infty} e^{-\frac{\Theta^2}{\Theta_0^2}} \Theta\, d\theta$$

and so approximately: $\alpha = \pi \Theta_0^2 f(\Theta)_{\theta=0}$

But $\frac{1}{K} f(\Theta)$ was measured. Let this quantity at $\Theta = 0$ be denoted by $\mathcal{U}$.

Then:

$$\alpha = \pi \Theta_0^2 K \mathcal{U}.$$

From the graph (Fig. 10)

$$\mathcal{U} = 55.6$$

If we assume $k_2 = k_3$ in equation (6) Appendix I the value of K becomes $61.3 \frac{e^{-\alpha p(d_2 - d_3)}}{\alpha}$

The exponential term is practically unity so we have:

$$\alpha^2 = (61.3)(55.6)(\pi)(.22)^2 \cong 540$$

$$\therefore \alpha = 23.3 \frac{cm^2}{cm^3}.$$

This result agrees with the value obtained by Brode[4] $27 \frac{cm^2}{cm^3}$ in order of magnitude at least.