Appendix A

DAFALIAS-MANZARI MODEL

Here we provide a brief description of the Dafalias-Manzari constitutive model [18] in triaxial stress space. Interested readers may refer to the original work to get a complete description. The model primarily utilizes concepts of kinematic hardening, critical state soil mechanics, and the effect of soil fabric on dilatancy; this enables it to capture both monotonic and cyclic response of soils under different loading conditions.

We denote σ_1 as the major principal effective stress, $\sigma_2 = \sigma_3$ as the minor principal effective stress, and ϵ_1, $\epsilon_2 = \epsilon_3$ the corresponding principal strains. We define pressure $p = (1/3)(\sigma_1 + 2\sigma_3)$, deviatoric stress $q = \sigma_1 - \sigma_3$, volumetric strain $\epsilon_v = \epsilon_1 + 2\epsilon_3$, and deviatoric strain $\epsilon_q = (2/3)(\epsilon_1 - \epsilon_3)$. We use superscripts e and p to denote the elastic and plastic parts of strain, respectively, and $\dot{\square}$ to denote increment in $\square$. With the notation outlined, the *i*ncremental stress-strain relations are:

$$\dot{\epsilon}_q^e = \frac{\dot{q}}{3G}; \quad \dot{\epsilon}_v^e = \frac{\dot{p}}{K} \tag{A.1}$$

$$\dot{\epsilon}_q^p = \frac{\dot{\eta}}{H}; \quad \dot{\epsilon}_v^p = \beta|\dot{\epsilon}_q^p| \tag{A.2}$$

where G is the elastic shear modulus, K is the elastic bulk modulus, H is the plastic hardening modulus associated with the increment in stress ratio $\dot{\eta}$, and β is dilatancy. Note that $\eta = q/p$ is the stress ratio. The model assumes a *h*ypo-elastic response, where the evolution of elastic moduli G and K is given by:

$$G = G_0 \frac{(2.97 - e)^2}{1 + e} \sqrt{\frac{p}{p_{at}}}; \quad K = \frac{2(1 + \nu)}{3(1 - 2\nu)} G \tag{A.3}$$

where G_0 is a constant, ν is Poisson ratio, and p_{at} is the atmospheric pressure. The yield surface f is proposed to be:

$$f = |\eta - \alpha| - m = 0 \tag{A.4}$$

This defines a wedge in effective stress space, with α as the back stress, and m as a constant defining the width of the wedge such that it has an opening of $2mp$ for a given value of p. The evolution of α is governed by a kinematic hardening law:

$$\dot{\alpha} = H\dot{\epsilon}_q^p \tag{A.5}$$

where H is the hardening modulus given by:

$$H = h(M^b - \eta) \text{ with } h = \frac{G_0 h_0 (1 - c_h e)}{|\eta - \eta_{in}|} \sqrt{\frac{p_{at}}{p}} \tag{A.6}$$

where h is a positive function, M^b is the bounding stress ratio, and h_0 and c_h are scalar parameters. η_{in} is the value of η at the initiation of a loading process. To calculate $\dot{\epsilon}_v^p$, we need the dilatancy β that is given by:

$$\beta = A_d(M^d - \eta) \tag{A.7}$$

where M^d is the dilatancy stress ratio. A_d is a positive function given by:

$$A_d = A_0(1 + \langle sz \rangle) \text{ with } \dot{z} = -c_z \langle -\dot{\epsilon}_v^p \rangle (s z_{\max} + z) \tag{A.8}$$

where A_0 is a constant and $s = \pm 1$ according to $\eta = \alpha \pm m$. $\langle\rangle$ are Macaulay brackets and $z_{\max}$ is the maximum possible of state parameter z, which has an initial value of 0.

The model complies with critical state mechanics by postulating evolution laws for M^b and M^d:

$$M^b = M \exp(-n^b \psi) \text{ and } M^d = M \exp(-n^d \psi) \tag{A.9}$$

where n^b and n^d are positive constant. M is the critical state stress ratio and $\psi = e - e_c$ is the state parameter as defined by Been and Jefferies [7]. e is the current void ratio and e_c is the critical void ratio. e_c is obtained according to the relationship proposed by Li and Wang [51]:

$$e_c = e_{c0} - \lambda_c (p_c / p_{at})^{\xi} \tag{A.10}$$

where p_c is the pressure at critical state, e_{c0} is the void ratio at $p_c = 0$, and λ_c and ξ are constants.