# ANALYSIS OF AN INCLINED THRUST AXIS AS APPLIED TO A RAMJET PROPELLED AIRCRAFT

Thesis by

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## SUMMARY

The low values of lift-drag ratio attained by supersonic wing configurations provide the opportunity for the utilization of an inclined thrust axis. The exhaust jet of a ramjet propelled aircraft is inclined in order to use some of the jet force to supply additional lift. This has the effect of augmenting the lift by the relatively large sine component of the jet force whereas the thrust in the flight direction is reduced only by the smaller change in the cosine component.

It was found that this principle offers a substantial decrease in fuel consumption over that of a normal ramjet for probable values of lift-drag ratios above Mach number 1.5. Almost all of the possible decrease can be obtained with jet inclination angles of $15^{\circ}$ or less.

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# INTRODUCTION

This investigation has its origin in the low lift-drag ratios attainable in the supersonic flight range. It was reasoned that the thrust axis could be inclined at an angle and thus the lift of the wing would be augmented by the relatively large sine component of the jet force. The forward speed would be decreased only in proportion to the smaller change in the cosine component of the jet force. An inclined thrust axis ramjet would then require less wing area and consequently have less wing drag than a normal ramjet operating at the same flight velocity. The purpose of this analysis is to investigate the relation between lift-drag ratio, angle of thrust inclination, and other applicable parameters, and to establish limits of probable gain by using an inclined thrust axis in the supersonic flight range. The ramjet power plant was selected for comparison purposes because it offers the most practicable propulsion system in the critical range of lift-drag ratios (Mach No. 1.5 - 4).

The analysis involves only elementary mathematics and is primarily concerned with the decrease in fuel consumption obtainable by comparing an inclined thrust axis ramjet with a normal ramjet. The comparison of the two ramjets is made for three different assumptions of weight conditions in level flight and two conditions in climb. The results are obtained in the form of relatively concise analytical expressions which are plotted for various weight conditions.

There are technical aspects of an inclined thrust axis system that are not discussed herein as the main objective of this paper is only to show the theoretical feasibility of such a system. However, the results indicate that the maximum benefits in fuel consumption can be secured with relatively small jet inclination angles, and this fact would be of engineering importance. It is believed that the parameters involved in this analysis can be adapted to any practical configuration of a ramjet aircraft.

The abbreviation, I.T.A., will be used throughout this paper to mean inclined thrust axis. In formulas the subscript, $I$, will be used to denote a quantity pertaining to the inclined thrust axis ramjet, whereas the subscript, $N$, will denote a quantity pertaining to the normal ramjet with a conventional straight thrust axis.

# I. NOTATION AND SYMBOLS

| | | |
|---|---|---|
| $A$ | - | Cross sectional area of ramjet |
| $C_D$ | - | Drag Coefficient |
| $C_L$ | - | Lift Coefficient |
| $C_F$ | - | Thrust Coefficient |
| $C$ | - | Ramjet exit velocity |
| $D$ | - | Drag |
| $F$ | - | Thrust |
| $L$ | - | Lift |
| $L/D$ | - | Lift Drag Ratio of the Wing |
| $M$ | - | Mach Number |
| $S$ | - | Wing Area |
| $V$ | - | Flight velocity |
| $W$ | - | Weight |
| $f/a$ | - | Fuel-air ratio |
| $g$ | - | Acceleration of gravity |
| $m$ | - | Mass flow of air |
| $m_f$ | - | Mass flow of fuel |
| $q$ | - | Dynamic pressure |
| $\rho$ | - | Air density |
| $\mu$ | - | Ratio of mass flow of combustion products to mass of entering air |
| $\theta$ | - | Angle of climb |

| | | |
|---|---|---|
| $\phi$ | - | Angle of inclination of thrust axis |
| $\alpha$ | - | Proportionality Constant $\alpha = \frac{D_b}{mc}$ |
| $\epsilon$ | - | Proportionality Constant $\epsilon = \frac{W_b}{mc}$ |
| $\sigma$ | - | Proportionality Constant $\sigma = \frac{W_w}{S_w}$ |
| $\beta$ | - | Dimensionless Parameter $\beta = \frac{L}{D}\left(1 - \frac{\sigma}{q C_L}\right)$ |

Subscripts

| | | |
|---|---|---|
| I | - | Ramjet with inclined thrust axis |
| N | - | Ramjet with normal straight thrust axis |
| w | - | Pertaining to wing only and not entire airplane |
| eng. | - | Engine |
| b | - | Body of aircraft |

## II. SUPERSONIC LIFT-DRAG RATIOS

The application of the inclined thrust axis has a relative importance based on the low values of lift-drag ratios obtained in the supersonic flight range. Before discussing the inclined thrust axis principle, it is appropriate to consider the magnitude and variation of lift-drag ratio for different wing configurations.

It is a well established fact that the lift-drag ratios of all types of wings tend to decrease with increasing Mach number in the supersonic flight range. By using radically swept wings and high aspect ratios, lift-drag values of ten or greater can be maintained up to a Mach number of 1.4. However, as Mach number increases beyond this point the gain due to sweepback diminishes and the maximum lift-drag ratios steadily decrease. At Mach number four the lift-drag ratio for any wing configuration of practical thickness appears to be limited to about five.

Reference 2 gives a good condensation of theoretical aerodynamic characteristics for various supersonic wing planforms. It is noted that while there is some spread in lift-drag ratios for different wing configurations below Mach number 1.5, above this speed there is a tendency for the maximum ratios to converge very closely. At Mach number four the lift-drag ratios for all wing planforms with practical thickness ratios are about the same. Values of lift-drag ratio for a delta wing with 60° sweepback as given in reference 2 are listed below for two thickness ratios.

| | | M = 1.5 | M = 2 | M = 3 | M = 4 |
|---|---|---|---|---|---|
| $\tau$ = .06 | $\frac{L}{D}$ | 6.6 | 6.0 | 5.5 | 5.2 |
| $\tau$ = .08 | $\frac{L}{D}$ | 5.2 | 4.7 | 4.4 | 4.2 |

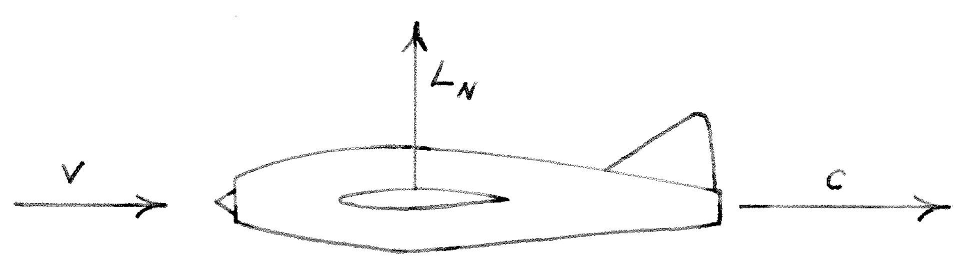
$$\text{where} \quad \tau = \frac{\text{Wing Thickness}}{\text{Wing Chord}}$$

The above data are based on the assumption of a double wedge wing section. A thickness ratio of .02 would increase the values of lift-drag ratio at $\tau$ = .06 by the order of 20% to 30%. A thickness ratio of .07 represents a good figure for present wing construction using a main spar type of structural support.

## III. BASIC PERFORMANCE CHARACTERISTICS OF I.T.A. AND NORMAL RAMJETS

Inclining the exhaust jet of a ramjet aircraft will modify the usual expressions for thrust and lift. The thrust will be reduced by a factor proportional to the cosine of the inclination angle. The total lift of the I.T.A. ramjet will consist of the lift of the wing plus an additional lift component proportional to the sine of the jet inclination angle. The drag of the I.T.A. ramjet will be considered equal to that of the normal ramjet, (i.e., inclining the thrust axis will not change the drag). The force of the inclined jet is assumed in all cases to act through the center of gravity of the I.T.A. aircraft. The two configurations are shown schematically below with the applicable expressions for thrust and lift in each case.

### A. Normal Ramjet




The thrust of the ramjet is obtained by computing the change in momentum between the inlet and exhaust stations. Considering the inlet and exhaust pressures equal, the thrust is given as

$$F_N = (m + m_f)\,c - mV$$

or $$F_N = m(\mu c - V)$$

where $$\mu = 1 + \frac{m_f}{m}$$

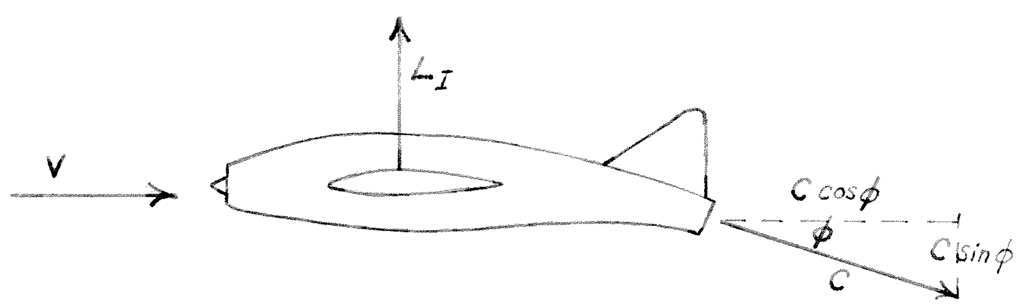
The quantity $\mu$ is slightly greater than unity since the fuel-air ratio of a ramjet is of the order of .05. In this treatment $\mu$ shall be taken equal to unity since it has a very small effect and this assumption will simplify the analysis. The thrust then becomes

$$F_N = m(c - V) \qquad \text{Eq. (1)}$$

The lift of a normal ramjet aircraft is simply the lift of the wing.

$$L_N = \frac{1}{2}\rho V^2 C_L S_N \qquad \text{Eq. (2)}$$

B. Inclined Thrust Axis Ramjet.




The change in momentum between inlet and exhaust stations of the I.T.A. ramjet gives a force which can be resolved into two components. The component in the flight direction is the thrust, which is

$$F_I = m(C\cos\phi - V) \qquad \text{Eq. (3)}$$

The force component perpendicular to the flight direction is an additional lift force.

$$\Delta L = m\,C\,\sin\phi$$

The total lift of the I.T.A. ramjet will consist of the lift of the wing plus the additional lift of the inclined jet force.

$$L_I = \frac{1}{2}\rho V^2 C_L S_I + m\,C\,\sin\phi \qquad \text{Eq. (4)}$$

C. <u>Difference in Fuel Consumption.</u>

Two ramjets are considered to be operating with the same fuel-air ratio but with different mass flows ($m_N$ and $m_I$). It is easy to show that for this case the fractional difference in fuel flow is equal to the fractional difference in mass flow.

Fuel flow $= m_f = \frac{f}{a} \times m$

Difference in fuel flow $= m_{f_N} - m_{f_I} = \frac{f}{a}(m_N - m_I)$

The fractional difference in fuel flow then becomes

$$\frac{m_{f_N} - m_{f_I}}{m_{f_N}} = \frac{m_N - m_I}{m_N} \qquad \text{Eq. (5)}$$

This relationship is the basis of all fuel consumption comparisons made in this paper.

## IV. SIMPLIFIED LEVEL FLIGHT COMPARISON OF I.T.A. RAMJET AND NORMAL RAMJET

This section is a comparison of the fuel consumption of an I.T.A. ramjet with that of a normal ramjet for the case in which both aircraft have the same gross weight. This weight assumption is unfavorable to the I.T.A. ramjet which will require less wing area and, consequently, less wing weight. However, even under the assumption of equal gross weights, the I.T.A. ramjet will show a decrease in fuel consumption. This decrease results from the fact that the I.T.A. ramjet will have less drag because of the smaller wing area and hence will require less thrust for a given flight velocity.

The purpose of this simplified analysis is to obtain an expression for the decrease in fuel consumption of an I.T.A. ramjet, based only upon the reduction in drag resulting from the smaller required wing area. The effect of weight corrections will be discussed later. The following specific assumptions will apply to this case.

Assumptions:

1. The two configurations have the same flight and jet exhaust velocities but different mass flows. This condition could be satisfied by using a different engine or the same engine with different inlet and nozzle areas.

2. The two configurations have the same wing section, but I.T.A. type will have less wing area as part of the lift is obtained from the vertical component of the jet force.

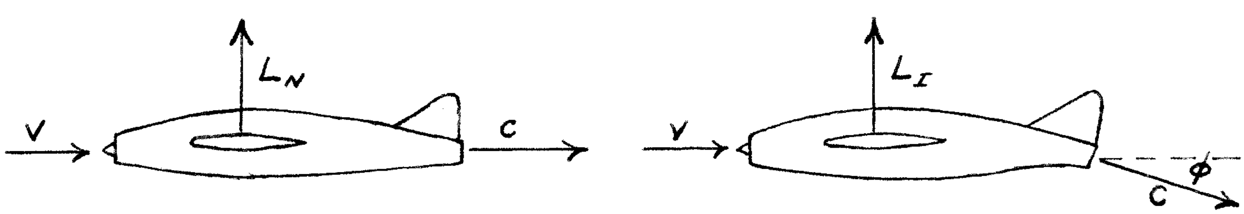
3. Wing and body interference effects will be neglected. The fuselage body will be assumed to contribute no lift due to aerodynamic shape.

4. The two configurations will have the same fuel-air ratio, but the weight of fuel added will be neglected.

5. The drag of the aircraft body will be assumed the same for both configurations.

6. The two configurations will have the same gross weight. The I.T.A. ramjet will require less wing area and consequently less wing weight, but it is assumed in this case that the weight difference will be made up by a greater payload for the I.T.A. ramjet.

Normal　　　　　　　　　　Inclined Thrust Axis




Thrust = $F_N = m_N (C - V)$　　　　$F_I = m_I (C \cos\phi - V)$

Lift = $L_N = \frac{1}{2}\rho V^2 C_L S_N$　　　　$L_I = \frac{1}{2}\rho V^2 C_L S_I + m_I C \sin\phi$

A. <u>Reduction in Required Wing Area for I.T.A. Ramjet.</u>

For level flight the lift of each configuration is

$$L_N = W_N = \frac{1}{2}\rho V^2 C_L S_N$$

$$L_I = W_I = \frac{1}{2}\rho V^2 C_L S_I + m_I c \sin\phi$$

Since the gross weights are equal, subtracting the two expressions for lift gives

$$\frac{1}{2}\rho V^2 C_L \left(S_N - S_I\right) = m_I c \sin\phi$$

Let $\Delta S$ denote the decrease in required wing area.

$$\Delta S = S_N - S_I = \frac{m_I c \sin\phi}{\frac{\rho V^2}{2} C_L} \qquad \text{Eq. (6)}$$

This can be expressed as a fractional decrease of the normal ram-jet wing area.

$$\frac{\Delta S}{S_N} = \frac{m_I c \sin\phi}{W_N} = \frac{m_I c \sin\phi}{W_I} \qquad \text{Eq. (7)}$$

$$\frac{\Delta S}{S_N} = \frac{F_I \sin\phi}{W_I \left(\cos\phi - \frac{V}{c}\right)} \qquad \text{Eq. (8)}$$

Equation (8) shows that the reduction in wing area increases as the thrust-weight ratio and jet inclination angle increase, and as the velocity ratio approaches unity.

B. <u>Reduction in Drag of the I.T.A. Ramjet Due to the Decrease in Required Wing Area.</u>

Since the body drag of the two configurations is assumed equal, the reduction in total drag will be simply the difference in the drag of the wings. The drag of a wing expressed in coefficient form is

$$D_W = \frac{1}{2} \rho V^2 S C_D$$

Let the difference in total drag be denoted by $\Delta D$.

$$\Delta D = \frac{1}{2} \rho V^2 C_D (S_N - S_I) = \frac{1}{2} \rho V^2 C_D \Delta S$$

Using the expression for $\Delta S$ from Equation (6), the reduction in drag is

$$\Delta D = \frac{m_I c \sin \phi}{C_L / C_D} = \frac{m_I c \sin \phi}{L/D} \qquad \text{Eq. (9)}$$

where $L/D$ is the lift-drag ratio of the wing alone.

C. <u>Decrease in Fuel Consumption of I.T.A. Ramjet Due to Decrease in Drag.</u>

Since thrust equals drag for level flight, the thrust of the I.T.A. ramjet can be reduced from that required by a normal ramjet in an amount equal to the difference in drag. The obvious method of reducing the thrust and keeping the flight and exhaust velocities equal is to reduce the mass flow. The difference in fuel consumption will then be given by the difference in mass flow of the two configurations if they are operated at the same fuel-air ratio.

The thrust required by an I.T.A. ramjet expressed in terms of thrust required by a normal ramjet of the same weight is

$$F_I = F_N - \Delta D$$

$$F_I = m_N (c - V) - \frac{m_I c \sin \phi}{L/D} \qquad \text{Eq. (10)}$$

When this expression is equated to Equation (3), it gives a relation between the two mass flows.

$$m_I\left(C\cos\phi - V\right) = m_N\left(C - V\right) - \frac{m_I\, C\,\sin\phi}{L/D}$$

The ratio of the mass flows is

$$\frac{m_I}{m_N} = \frac{C - V}{C\left(\frac{\sin\phi}{L/D} + \cos\phi\right) - V} \qquad \text{Eq. (11)}$$

Subtracting both sides of Equation (11) from unity gives the fractional reduction in mass flow which can be expressed in non-dimensional form.

$$\frac{m_N - m_I}{m_N} = \frac{\frac{\sin\phi}{L/D} + \cos\phi - 1}{\frac{\sin\phi}{L/D} + \cos\phi - V/C} \qquad \text{Eq. (12)}$$

From Equation (5), the fractional difference in fuel flow is equal to the fractional difference in mass flow for the same fuel-air ratio. Equation (13) gives the fractional decrease in fuel consumption which results from the decreased wing drag of the I.T.A. ramjet.

$$\frac{m_{f_N} - m_{f_I}}{m_{f_N}} = \frac{L/D\left(\cos\phi - 1\right) + \sin\phi}{L/D\left(\cos\phi - V/C\right) + \sin\phi} \qquad \text{Eq. (13)}$$

## V. LEVEL FLIGHT COMPARISON OF I.T.A. AND NORMAL RAMJETS WITH WEIGHT CORRECTIONS INCLUDED.

The I.T.A. ramjet will have less total weight than a normal ramjet for the same performance (equal flight speed) in level flight. This weight decrease results mainly from the reduced wing area required by the I.T.A. ramjet. Also, there will be the possibility of using a smaller engine with the I.T.A. configuration since the mass flow required is less than that for a normal ramjet. A smaller engine will give a reduction in body weight and also a decrease of body drag.

This comparison is similar to the simplified case except that the condition of equal gross weights is no longer imposed. It is assumed that the I.T.A. ramjet will have less total weight. This weight decrease will consist of a reduction in wing weight proportional to the wing area and a decrease in body weight which will be proportional to the mass flow. It is also assumed that there will be a decrease in body drag of the I.T.A. ramjet which will be proportional to the mass flow. This comparison will have the effect of superimposing the above weight corrections upon the results obtained in Part IV.

Assumptions:

1. The two configurations have the same flight and jet

exhaust velocities, but different mass flows.

2. The two configurations have the same wing section, but I.T.A. type will require less wing area due to the additional lift of the inclined jet force.

3. Wing and body interference effects, and lift due to body shape will be neglected.

4. The two configurations will have the same fuel-air ratio, but the weight of fuel added will be neglected.

5. The reduction in required wing area of the I.T.A. ramjet will result in a proportional decrease in wing weight (i.e., wing weight is proportional to wing area).

$$W_{W_N} - W_{W_I} = \sigma \left( S_N - S_I \right) \qquad \text{where} \quad \sigma = \frac{W_W}{S_W}$$

6. The reduction in required mass flow will give a decrease in body weight which will be proportional to the mass flow. This assumption represents the ideal case in which the body consists of ramjet engine alone. The actual case would have this assumption as an upper limit.

$$W_{b_N} - W_{b_I} = \epsilon \left( m_N - m_I \right) \qquad \text{where} \quad \epsilon = \frac{W_b}{m}$$

7. The reduction in drag of the body of the I.T.A. ramjet will be proportional to the decrease in mass flow. This is a condition to be expected, but the assumption is made chiefly to simplify

the analysis and, as shown later, the effect of this assumption is small.

$$D_{b_N} - D_{b_I} = \alpha\,(m_N - m_I) \qquad \text{where} \quad \alpha = \frac{D_b}{m}$$

It should be noted that assumptions 5. to 7. may be suppressed by taking the proportionality constants $\sigma$, $\epsilon$ or $\alpha$ equal to zero. In the event that all three proportionality constants are assumed zero, the assumptions reduce to those of the former simplified case.

The lift force equals the weight of an airplane in level flight. The difference in lift of the two wings is obtained by subtracting the equations for lift that apply in each case.

$$L_N = W_N$$

$$L_I = W_I - m_I\, C \sin\phi$$

$$L_N - L_I = m_I\, C \sin\phi + W_N - W_I$$

Since the total weight consists of the wing weight and body weight, the total decrease in weight of I.T.A. ramjet is

$$W_N - W_I = (W_{W_N} - W_{W_I}) + (W_{b_N} - W_{b_I})$$

By assumptions 5. and 6., the weight decrease can be expressed as

$$W_N - W_I = \sigma\,(S_N - S_I) + \epsilon\,(m_N - m_I) \qquad \text{Eq. (14)}$$

This relation substituted in the expression for difference in wing lift gives

$$L_N - L_I = m_I c \sin\phi + \frac{\sigma(L_N - L_I)}{\frac{\rho V^2}{2} C_L} + \epsilon(m_N - m_I)$$

where $\rho$, $V$ and $C_L$ are constants for both configurations.

$$L_N - L_I = \frac{m_I c \sin\phi + \epsilon(m_N - m_I)}{\left(1 - \frac{2\sigma}{\rho V^2 C_L}\right)}$$

The difference in drag of the two wings is obtained by multiplying both sides by the reciprocal of the lift-drag ratio.

$$D_{W_N} - D_{W_I} = \frac{m_I c \sin\phi + \epsilon(m_N - m_I)}{L/D\left(1 - \frac{2\sigma}{\rho V^2 C_L}\right)} \qquad \text{Eq. (15)}$$

The other condition for level flight is that thrust equals drag. The difference in thrust of the two configurations will be equal to the difference in drag.

$$F_N = m_N(c - V) = D_N$$
$$F_I = m_I(c\cos\phi - V) = D_I$$

$$m_N(c - V) - m_I(c\cos\phi - V) = (D_{W_N} - D_{W_I}) + (D_{b_N} - D_{b_I})$$

Assumption 3. gives

$$D_{b_N} - D_{b_I} = \alpha(m_N - m_I)$$

The difference in drag of the two wings becomes

$$D_{W_N} - D_{W_I} = m_N(c - V - \alpha) - m_I(c\cos\phi - V - \alpha) \qquad \text{Eq. (16)}$$

By equating Equation (15) and Equation (16), a relationship between the two mass flows is obtained.

$$m_N(c - V - \alpha) - m_I(c\cos\phi - V - \alpha) = \frac{m_I c \sin\phi + \varepsilon(m_N - m_I)}{\frac{L}{D}\left(1 - \frac{2\sigma}{\rho V^2 C_L}\right)}$$

The ratio of the mass flows becomes

$$\frac{m_I}{m_N} = \frac{\beta(c - V - \alpha) - \varepsilon}{\beta(c\cos\phi - V - \alpha) + c\sin\phi - \varepsilon} \qquad \text{Eq. (17)}$$

where $\beta = \frac{L}{D}\left(1 - \frac{2\sigma}{\rho V^2 C_L}\right)$

Dividing numerator and denominator by the exhuast velocity will give the mass flow ratio in terms of dimensionless parameters.

$$\frac{m_I}{m_N} = \frac{\beta(1 - V/c - \alpha/c) - \varepsilon/c}{\beta(\cos\phi - V/c - \alpha/c) + \sin\phi - \varepsilon/c} \qquad \text{Eq. (18)}$$

Subtracting both sides from unity gives the reduction in fuel flow expressed as a fractional decrease.

$$\frac{m_N - m_I}{m_N} = \frac{\beta(\cos\phi - 1) + \sin\phi}{\beta(\cos\phi - V/c - \alpha/c) + \sin\phi - \varepsilon/c} \qquad \text{(Eq. (19)}$$

Equation (19) represents the level flight decrease in fuel consumption of an I.T.A. ramjet. This expression compensates for the weight and drag effects of a smaller body, in addition to the reduced wing area corrections, It is directly evident that Equation (19) reduces to Equation (12) when $\alpha/c$, $\varepsilon/c$ and $\sigma$ are taken equal to zero.

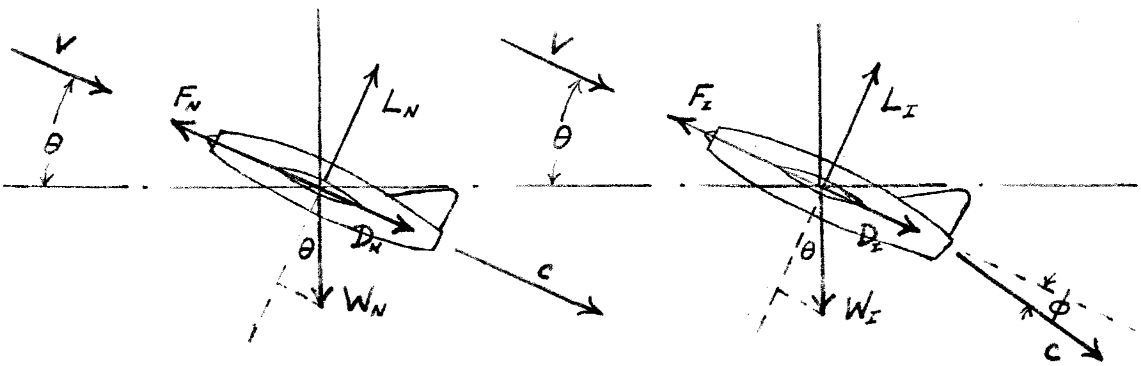
# VI. COMPARISON OF I.T.A. AND NORMAL RAMJETS FOR THE GENERAL CASE OF AIRCRAFT IN CLIMB

For the high speeds considered, the time for a ramjet aircraft to climb to operating ceiling would be only a matter of a few minutes. However, since the fuel consumption at these high speeds is large, the time to climb would constitute an appreciable part of the flight duration of a supersonic ramjet.

It was believed the I.T.A. ramjet would show a greater decrease in fuel consumption in climb than in level flight, and the calculations in this section show this is true. The greater decrease in fuel consumption in climb over level flight results from the influence of the angle of climb upon the reduced weight of the I.T.A. ramjet. It will be shown in this section that if the weights of the two configurations are assumed equal, then the simplified expression (Equation 12 which was derived for level flight) will also hold for climb. The assumptions for this case are the same as those of Part V for the aircraft in level flight, but here both ramjets are assumed to be operating at a small angle of climb.

The two ramjet configurations are shown below with the equations for thrust and lift that apply in each case.

Normal Ramjet — Inclined Thrust Axis Ramjet




$$F_N = m_N (c - V)$$
$$L_N = \tfrac{1}{2} \rho V^2 C_L S_N$$

$$F_I = m_I (c \cos\phi - V)$$
$$L_{W_I} = \tfrac{1}{2} \rho V^2 C_L S_I$$

The equilibrium of forces perpendicular to the flight direction gives equations from which the difference in wing lift can be obtained.

$$L_N = W_N \cos\theta$$
$$L_I + m_I c \sin\phi = W_I \cos\theta$$

$$L_N - L_I = (W_N - W_I)\cos\theta + m_I c \sin\phi \qquad \text{Eq. (20)}$$

The equi librium of forces in the flight direction gives equations from which the difference in drag can be obtained.

$$F_N = D_N + W_N \sin\theta$$
$$F_I = D_I + W_I \sin\theta$$

$$D_N - D_I = F_N - F_I - (W_N - W_I)\sin\theta$$

$$D_N - D_I = m_N(c - V) - m_I(c\cos\phi - V) - (W_N - W_I)\sin\theta \qquad \text{Eq. (21)}$$

The difference in total weight can be expressed as

$$W_N - W_I = \sigma(S_N - S_I) + \varepsilon(m_N - m_I)$$

but $$S_N - S_I = \frac{L_N - L_I}{q\,C_L} \quad , \quad q = \tfrac{1}{2}\rho V^2$$

so that

$$W_N - W_I = \frac{\sigma}{q C_L}\left(L_N - L_I\right) + \varepsilon\left(m_N - m_I\right) \qquad \text{Eq. (22)}$$

Equations (20) and (22) can be solved simultaneously to give Equations (23) and (24)

$$W_N - W_I = \frac{\dfrac{\sigma m_I C \sin\phi}{q C_L} + \varepsilon(m_N - m_I}{\left(1 - \dfrac{\sigma}{q C_L}\cos\theta\right)} \qquad \text{Eq. (23)}$$

$$L_N - L_I = \frac{\varepsilon \cos\theta\,(m_N - m_I) + m_I C \sin\phi}{\left(1 - \dfrac{\sigma}{q C_L}\cos\theta\right)} \qquad \text{Eq. (24)}$$

Multiplying both sides of Equation (24) by reciprocal of the lift-drag ratio will give an expression for the difference in drag of the two wings.

$$D_{W_N} - D_{W_I} = \frac{\varepsilon\cos\theta\,(m_N - m_I) + m_I C \sin\phi}{L/D\left(1 - \dfrac{\sigma}{q C_L}\cos\theta\right)} \qquad \text{Eq. (25)}$$

From assumption 4. the difference in drag of the wings is

$$D_{W_N} - D_{W_I} = D_N - D_I - \alpha\left(m_N - m_I\right)$$

Substitute Equation (25) into left side and Equation (21) into right side of the above expression

$$\frac{\epsilon \cos\theta\,(m_N - m_I) + m_I\, c \sin\phi}{\frac{L}{D}\left(1 - \frac{\sigma}{q\, C_L}\cos\theta\right)} = m_N(c - v) - m_I\,(c\cos\phi - v - \alpha) - \sin\theta\,(W_N - W_I)$$

Introducing Equation (23) into the term containing the difference in weight and letting

$$B = \left(1 - \frac{\sigma}{q\, C_L}\cos\theta\right)$$

gives

$$\epsilon \cos\theta\,(m_N - m_I) + m_I\, c\sin\phi = B\left(\frac{L}{D}\right) m_N\,(c - v - \alpha) - B\left(\frac{L}{D}\right) m_I\,(c\cos\phi - v - \alpha) - \frac{m_I\,\sigma \sin\theta\; c \sin\phi}{q\, C_L}\left(\frac{L}{D}\right) - \frac{L}{D}\sin\theta\, m_N\,\epsilon + \frac{L}{D}\sin\theta\, m_I\,\epsilon$$

The ratio of the mass flows becomes

$$\frac{m_I}{m_N} = \frac{\frac{L}{D}\left[B(c - v - \alpha) - \epsilon\sin\theta\right] - \epsilon\cos\theta}{\frac{L}{D}\left[B(c\cos\phi - v - \alpha) + \sin\theta\left(\frac{\sigma c}{q\, C_L}\sin\phi - \epsilon\right)\right] - \epsilon\cos\theta + c\sin\phi} \quad \text{Eq. (26)}$$

In non-dimensional form the mass flow ratio for the general case is

$$\frac{m_I}{m_N} = \frac{\frac{L}{D}\left[B\left(1 - \frac{v}{c} - \frac{\alpha}{c}\right) - \frac{\epsilon}{c}\sin\theta\right] - \frac{\epsilon}{c}\cos\theta}{\frac{L}{D}\left[B\left(\cos\phi - \frac{v}{c} - \frac{\alpha}{c}\right) + \sin\theta\left(\frac{\sigma}{q\, C_L}\sin\phi - \frac{\epsilon}{c}\right)\right] - \frac{\epsilon}{c}\cos\theta + \sin\phi}$$

The fractional decrease in mass flow for the general case is given by Equation (27).

$$\frac{m_N - m_I}{m_N} = \frac{\frac{L}{D}\left[B(\cos\phi - 1) + \frac{\sigma}{q\, C_L}\sin\theta\right] + \sin\phi}{\frac{L}{D}\left[B\left(\cos\phi - \frac{v}{c} - \frac{\alpha}{c}\right) + \sin\theta\left(\frac{\sigma}{q\, C_L}\sin\phi - \frac{\epsilon}{c}\right)\right] - \frac{\epsilon}{c}\cos\theta + \sin\phi} \quad \text{Eq. (27)}$$

where $\theta$ = angle of climb

$\phi$ = jet inclination angle

$B = \left(1 - \frac{\sigma}{q C_L} \cos\theta\right)$

By inspection it can be seen that Equation (27) reduces to Equation (19) if the angle of climb is taken as zero. Furthermore Equation (27) will reduce to the simplified case of Equation (12) if the parameters $\frac{\alpha}{c}$, $\epsilon/c$, and $\nu/c$ are all taken equal to zero.

# VII. EVALUATION OF DIMENSIONLESS PARAMETERS

The decrease in fuel consumption of an I.T.A. ramjet, as expressed by Equations (19) and (27), has been plotted for a wide range of possible conditions and is presented graphically in Figs. 1 to 8. Excluding lift-drag ratios which were covered in Part II, there are four other basic dimensionless parameters ($V/c$, $\beta$, $\alpha/c$ and $\epsilon/c$) which enter the above equations. A general range of values of each parameter had to be established in order to present significant data. The purpose of this section is to indicate the methods used in evaluating the dimensionless parameters.

## A. Parameter $V/c$

Values of the ratio of flight velocity to jet exit velocity can be determined from available ramjet performance calculations such as reference (4). The usual value of this parameter is near .5, and it is considered that a range of values from .3 to .7 would cover the practical limits. High values of this velocity ratio are advantageous for decreasing the fuel consumption of an I.T.A. ramjet. This occurs because, for constant thrust, the mass flow increases much faster than the exhaust velocity decreases, and the product of these two quantities will become infinite when the velocity ratio approaches unity.

For constant thrust the exhaust velocity of a ramjet engine can be controlled by varying the mass flow or, with a given mass flow, by varying the nozzle exit area. It would not be practical to obtain a high velocity ratio by increasing the mass flow as this would lead to a larger engine size with increased body drag and weight. But, on the other hand, low values of the velocity ratio are desirable for maximum thrust with a fixed mass flow. Consequently, the selection of exhaust velocity for a particular I.T.A. ramjet design would present a compromise problem.

B. <u>Parameter</u> $\beta = \frac{L}{D}\left(1 - \frac{\sigma}{q C_L}\right)$

The term $\frac{\sigma}{q C_L}$ can be expressed as the ratio of the weight of the wing to the lift of the wing.

$$\frac{\sigma}{q C_L} = \frac{W_W}{q C_L S_W} = \frac{W_W}{L_W}$$

For a normal ramjet, wing lift is equal to the gross weight, so that $\beta$ becomes simply a function of lift-drag ratio and the ratio of wing weight to gross weight.

$$\beta = \frac{L}{D}\left(1 - \frac{\sigma}{q C_L}\right) = \frac{L}{D}\left(1 - \frac{W_W}{W}\right)$$

From Part II, the maximum variation in lift-drag ratio is 10 to 4 for supersonic wings at Mach numbers of 1.5 to 4, respectively. It is estimated that the variation in ratio of wing weight to total weight would range from 25% to 50% for supersonic ramjets. On this basis the limits for $\beta$ are:

$$\beta_{max.} = 10\left(1 - \frac{1}{4}\right) = 7.5$$

$$\beta_{min.} = 4\left(1 - \frac{1}{2}\right) = 2.0$$

The parameter B which enters the climb equations is evaluated in the same manner as $\beta$ .

$$B = \left(1 - \frac{W_w}{W} \cos \theta\right)$$

C. <u>Parameter</u> $\frac{\alpha}{c} = \frac{D_b}{mc}$

If it is assumed that the drag of the body will vary from 20% to 40% of the total drag, then for level flight the body drag will equal the same percentage of thrust. The product of mass flow and exhaust velocity can be evaluated from the thrust formula in terms of thrust and velocity ratio.

$$F = m(c - v) \qquad mc = \frac{F}{1 - v/c}$$

For $v/c = .3 \qquad mc = 1.43\,F$

$v/c = .7 \qquad mc = 3.33\,F$

$$\left(\frac{\alpha}{c}\right)_{max.} = \frac{.4}{1.43} = .28$$

$$\left(\frac{\alpha}{c}\right)_{min} = \frac{.2}{3.33} = .06$$

D. <u>Parameter</u> $\frac{\epsilon}{c} = \frac{W_b}{mc}$

Fig. V-6d of reference 3 gives curves of net thrust per unit engine weight for various flight speeds and altitudes. These values are tabulated below for specific conditions.

| Sea Level | | 50,000 ft. | | 100,000 ft. |
|---|---|---|---|---|
| M = 2 | M = 4 | M = 2 | M = 4 | M = 3 |
| 18 | 8 | 7 | 7 | 2 |

If it is assumed that the engine weight is a definite fraction of body weight (50% to 75%), then $\varepsilon/c$ can be expressed as follows:

$$\frac{\varepsilon}{c} = \frac{W_b}{mc} = \frac{W_{eng}}{F} \times \frac{W_b}{W_{eng}} \times \left(1 - \frac{v}{c}\right)$$

$$\left(\frac{\varepsilon}{c}\right)_{max} = \frac{1}{2} \times 2 \times .7 = .7$$

$$\left(\frac{\varepsilon}{c}\right)_{min} = \frac{1}{18} \times \frac{4}{3} \times .3 = .022$$

E. Values of Parameters Used for Weight Correction Curves.

$v/c$ = .5 This is a conservative value of this parameter as the usual values are slightly higher than .5.

$\beta$ = 3.33 This value was determined by using $L/D$ = 5 and a wing-to-total-weight ratio of one-third. The value of $L/D$ = 5 is reasonable for Mach numbers of two to four.

$\alpha/c$ = .1 This is a conservative value based on $v/c$ = .5 and body drag equal to 20% of the total drag.

$\varepsilon/c$ = .4 This value would correspond to high altitude and $v/c$ = .5.

## VIII. DISCUSSION AND RESULTS

The method of analysis yields concise and relatively simple analytical expressions for the decrease in fuel consumption. The decrease amounts to the reduction in fuel weight per unit distance since the decrease in mass flow of fuel is taken at a given flight velocity. General results indicate that the I.T.A. ramjet will give appreciable fuel savings in the supersonic range where lift-drag ratios of ten (10) or less are encountered.

Figs. 1 and 2 are a plot of Equation (19) in which the parameters $\alpha/c$ and $\epsilon/c$ are taken equal to zero. These graphs show the level flight decrease in fuel consumption of an I.T.A. ramjet considering only the decreased weight and drag of the smaller required wing area. Figs. 1 and 2 may be considered to represent the minimum fuel consumption decrease, since the possible reduction in engine size is not taken into account. In Fig. 1 the effect of varying $\beta$ is shown to have a large effect on the general shape of the curves. For $V/c$ = .5 the decrease in fuel consumption of an I.T.A. ramjet varies from 3% at $\beta$ = 6 to over 20% at $\beta$ = 2. Fig. 1 also shows that the jet inclination angle required for maximum decrease in fuel consumption is dependent upon $\beta$, and this angle increases as $\beta$ decreases. In Fig. 2 the effect of varying the velocity ratio with constant $\beta$ ($\beta$ = 3.33) is shown to give a decrease of 6 - 17% in the fuel consumption of an I.T.A. ramjet for values of $V/c$ = .3 to .8.

If the lift-drag ratio is substituted for the parameter $\beta$ , Fig. 1 will then represent a plot of Equation (12), since $\beta$ is equal to the lift-drag ratio when $\sigma = 0$. In this case Fig. 1 will show the reduction in fuel consumption due only to decreased drag of the smaller wing necessary for the I.T.A. ramjet. For a lift-drag ratio of four and velocity ratio of .5, there is a decrease of 5.5% resulting solely from the lower drag of I.T.A. ramjet wing.

Figs. 3 to 6 are plotted from Equation (19) and show the effect of reducing the size of the aircraft body in addition to the decrease in wing weight and drag. These figures are plots in which the dimensionless parameters are varied in order to show the influence of each upon the decrease in fuel consumption. Fig. 3 gives the effect of a change in $V/c$ or $\alpha/c$ for constant values of $\beta$ and $\epsilon/c$ . Since $V/c$ and $\alpha/c$ occur together in the equations, these two quantities could be combined into a single parameter. A change in $\alpha/c$ is equivalent to a change in $V/c$ of the same amount. Fig. 3 shows that for the assumed values of the other parameters ( $\beta = 3.33$, $\frac{\epsilon}{c} = .4$, $\frac{\alpha}{c} = .1$) a maximum fuel decrease of 35% is obtained by the I.T.A. ramjet at a $V/c$ ratio of .7.

Fig. 4 gives the effect of a change in $\epsilon/c$ with the other parameters held constant. However, as shown in Section VII, the parameters $\alpha/c$ and $\epsilon/c$ are evaluated in terms of $V/c$ . In Fig. 5 the parameter $\epsilon/c$ is varied with $V/c$ , but $\alpha/c$ is held at a conservative small value. Comparing Fig. 5 with Fig. 2 which includes only the wing weight correction, it will be noted that the additional assumptions of a decrease in body size and drag double the maximum decrease in fuel consumption. Fig. 5 is representative of the upper limit of the maximum decrease in fuel consumption, whereas Fig. 2 is representative of the lower limit. Depending mainly upon the body configuration, the actual ramjet aircraft will be somewhere between these two limits. With $\beta$ = 3.33 and $\alpha/C$ = .1, Fig. 5 shows a range of decrease in fuel consumption of 9% at $V/C$ = .3 and 25% at $V/C$ = .7.

The basic effect of $\beta$ upon the shape of the curves is demonstrated again in Fig. 6, in which $\beta$ is varied while the other parameters are held constant. Fig. 6 shows almost twice the decrease in fuel consumption over that of Fig. 2 for high values of $\beta$, but this effect diminishes for lower values of $\beta$.

Figs. 7 and 8 indicate that the decrease in fuel consumption of an I.T.A. ramjet is aided appreciably by angle of climb. This condition occurs as a result of the decreased wing and engine weight of the I.T.A. ramjet. Equation (27) for climb reduces to the simplified level flight case when the weight reductions are assumed zero. Fig. 7, which includes only the wing weight correction, shows a maximum decrease in I.T.A. ramjet fuel consumption of

13% at a climb angle of 15°. This compares with a value of 8% under the same conditions in level flight. In climb the effect of a decrease in body weight and drag is much greater than the decrease of wing weight and drag. Fig. 8, which includes the body corrections, shows a maximum decrease of 35% for a climb angle of 15°. This is almost three times the level flight value for the same conditions.

## IX. CONCLUSIONS

From the foregoing analysis the following conclusions are evident regarding the application of the inclined thrust axis principle to a ramjet aircraft:

(1) As a result of the low values of lift-drag ratio for supersonic wing configurations, an I.T.A. ramjet will give an appreciable reduction in fuel consumption for Mach numbers greater than 1.5.

(2) For a fixed thrust inclination angle, the maximum decrease in fuel consumption will be primarily a function of lift-drag ratio and the ratio of flight velocity to jet exhaust velocity. For a velocity ratio of .5 the decrease will vary from 3% at a lift-drag ratio of ten, to 22% at a lift-drag ratio of four.

(3) The decrease in fuel consumption increases with higher values of the ratio of flight velocity to exhaust velocity. A velocity ratio of .7 and a lift-drag ratio of four give a 30% reduction in fuel consumption.

(4) The maximum decrease in fuel consumption can be obtained with thrust inclination angles of less than 30°. Since the curves showing fuel decrease rise sharply, nearly all of the possible decrease can be secured with inclination angles of 15° or less.

(5) When the possible decrease in aircraft weight is considered, the effective reduction in fuel consumption increases considerably for small angles of climb.

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2. SNOW, R. M. and BONNEY, E.A.: Aerodynamic Characteristics of Wings at Supersonic Speeds; Bumblebee Report No. 55, March 1947.

3. CLEVELAND LABORATORY STAFF: Performance and Ranges of Application of Various Types of Aircraft - Propulsion Systems. NACA TN No. 1349, August 1947.

4. NO. AMERICAN AVIATION CORP.: Determination of Ramjet Performance Characteristics at Supersonic Flight Speeds.. No. Am. Report No. NA46-631, 1946.

# FIG. 1

## PERCENTAGE DECREASE IN FUEL CONSUMPTION

COMPARISON OF I.T.A. AND NORMAL RAMJETS
FOR CONDITION OF LEVEL FLIGHT WITH
WING WEIGHT CORRECTION ONLY

$V/C = .5 \quad \alpha/C = 0 \quad e/C = 0$

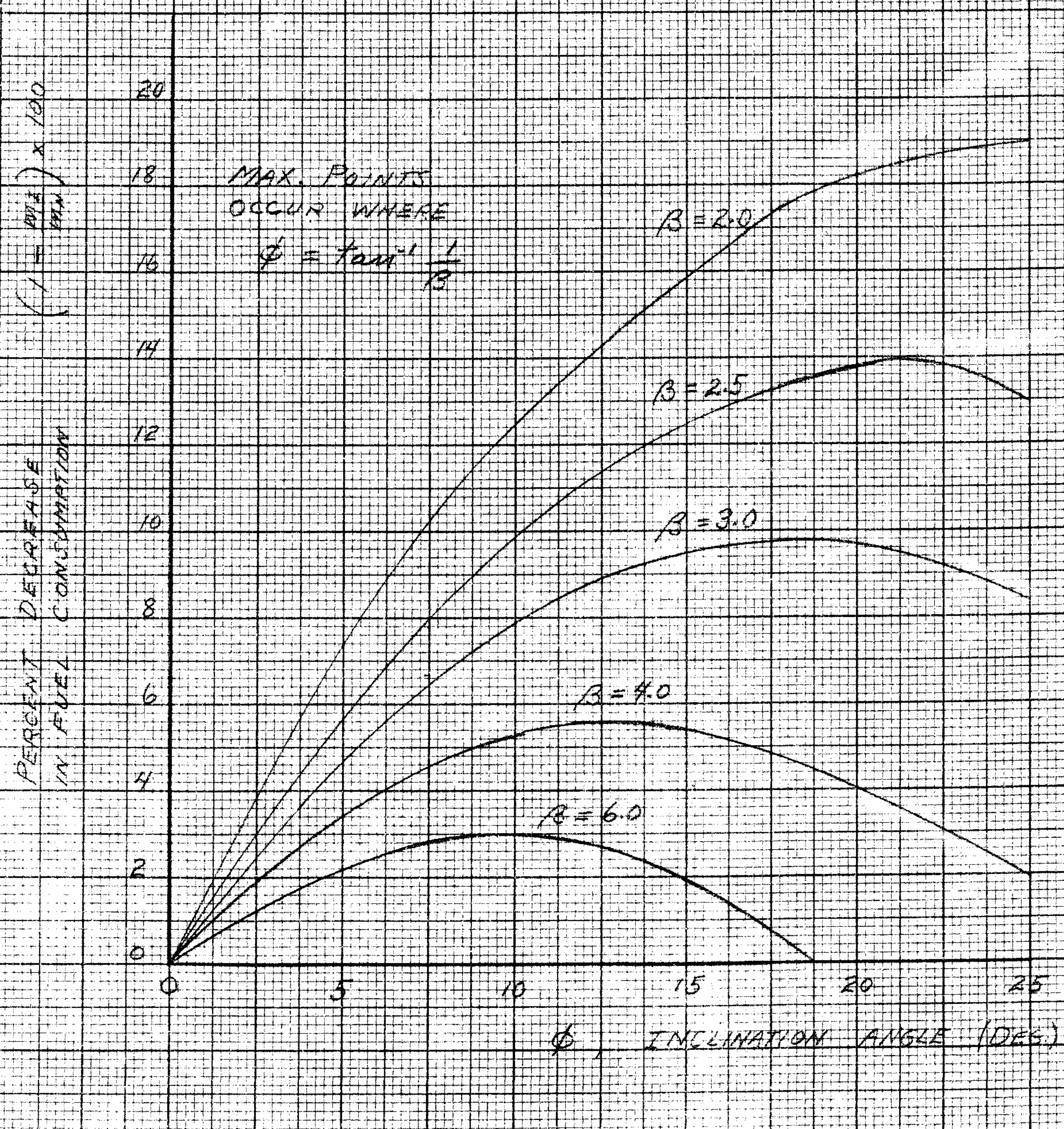
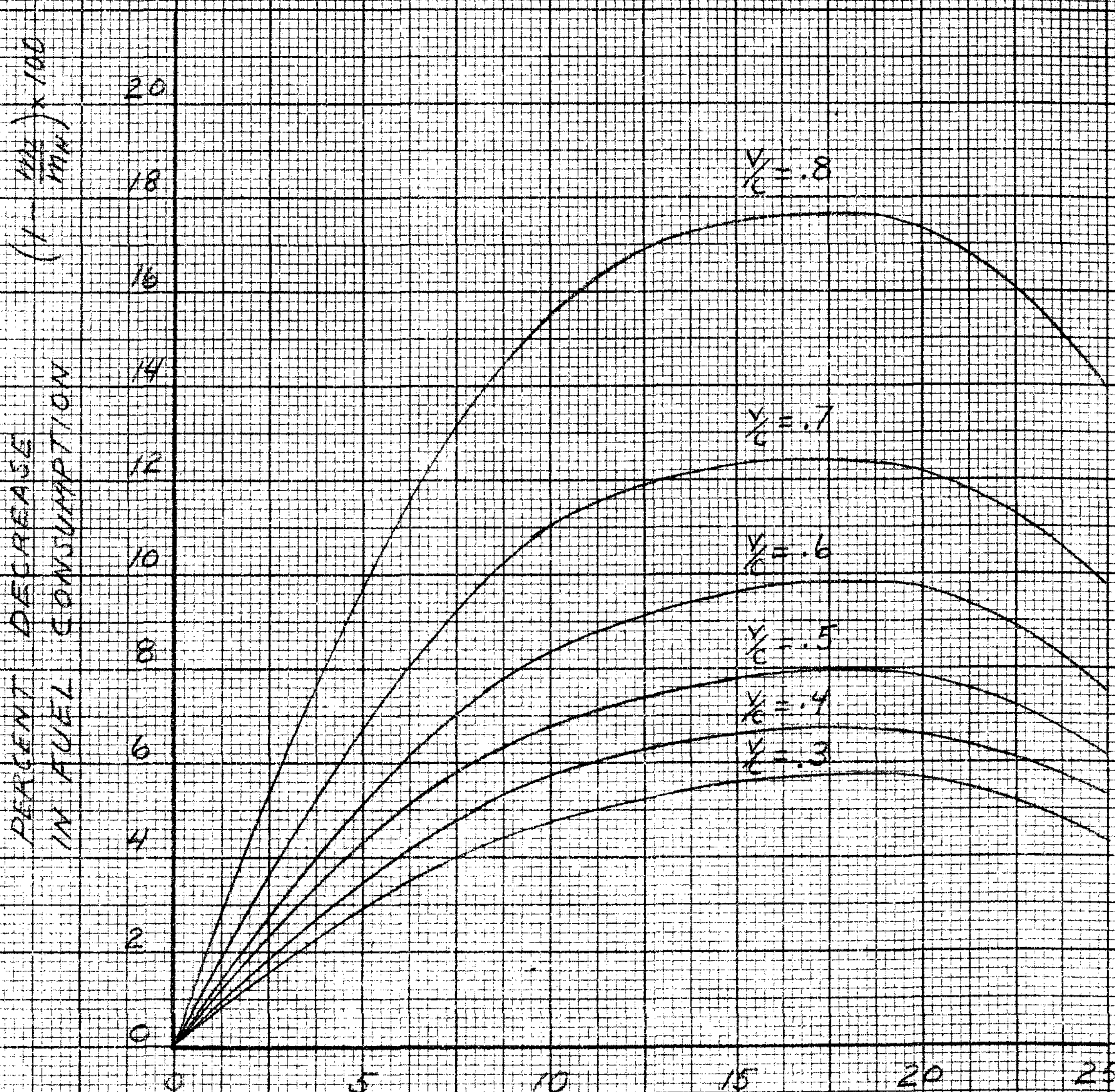


FIG. 2

PERCENTAGE DECREASE IN FUEL CONSUMPTION

COMPARISON OF I.T.A AND NORMAL RAMJETS FOR CONDITION OF LEVEL FLIGHT WITH WING WEIGHT CORRECTION ONLY

$\beta = 3.33 \quad \frac{a}{c} = 0 \quad \frac{b}{c} = 0$



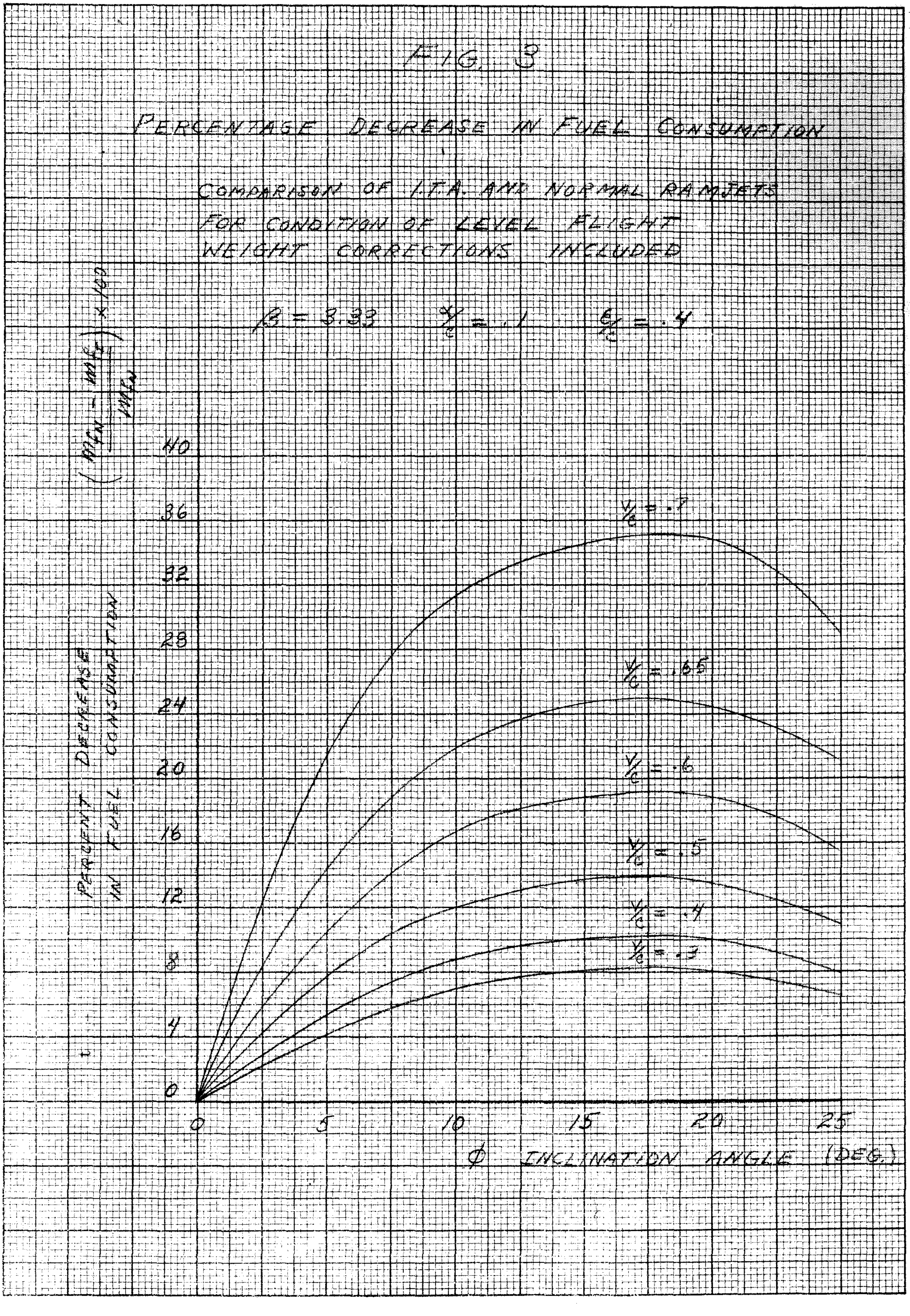


FIG. 3
PERCENTAGE DECREASE IN FUEL CONSUMPTION
COMPARISON OF I.T.A. AND NORMAL RAMJETS
FOR CONDITION OF LEVEL FLIGHT
WEIGHT CORRECTIONS INCLUDED
$\beta = 3.33$ $\frac{a}{c} = .1$ $\frac{b}{c} = .4$
PERCENT DECREASE IN FUEL CONSUMPTION $\left(\frac{\dot{m}_{f_N} - \dot{m}_{f_I}}{\dot{m}_{f_N}}\right) \times 100$
$\frac{V}{c} = .7$
$\frac{V}{c} = .65$
$\frac{V}{c} = .6$
$\frac{V}{c} = .5$
$\frac{V}{c} = .4$
$\frac{V}{c} = .3$
0
4
8
12
16
20
24
28
32
36
40
0
5
10
15
20
25
$\phi$ INCLINATION ANGLE (DEG.)

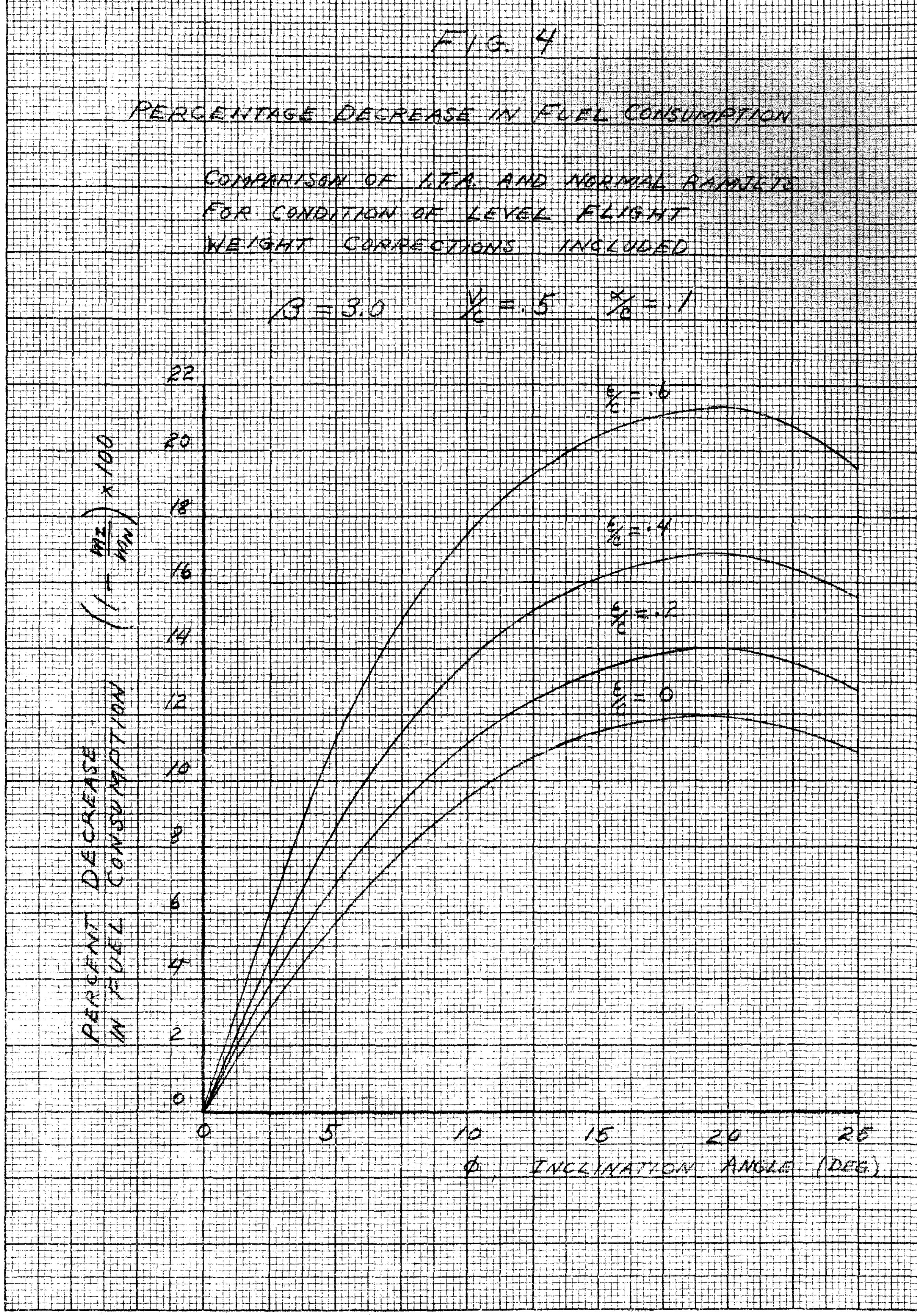


FIG. 4
PERCENTAGE DECREASE IN FUEL CONSUMPTION
COMPARISON OF I.T.A. AND NORMAL RAMJETS
FOR CONDITION OF LEVEL FLIGHT
WEIGHT CORRECTIONS INCLUDED
$\beta = 3.0$ $V/c = .5$ $x/c = .1$
$\epsilon/c = .6$
$\epsilon/c = .4$
$\epsilon/c = .2$
$\epsilon/c = 0$
PERCENT DECREASE IN FUEL CONSUMPTION $\left(1 - \frac{m_I}{m_N}\right) \times 100$
0 2 4 6 8 10 12 14 16 18 20 22
0 5 10 15 20 25
$\phi$ INCLINATION ANGLE (DEG)

FIG 5

PERCENTAGE DECREASE IN FUEL CONSUMPTION

COMPARISON OF I.T.A. AND NORMAL RAMJETS
FOR CONDITION OF LEVEL FLIGHT
WEIGHT CORRECTIONS INCLUDED

$\delta/c$ VARIED WITH $V/c$

$\beta = 3.33 \qquad \alpha/c = .1$

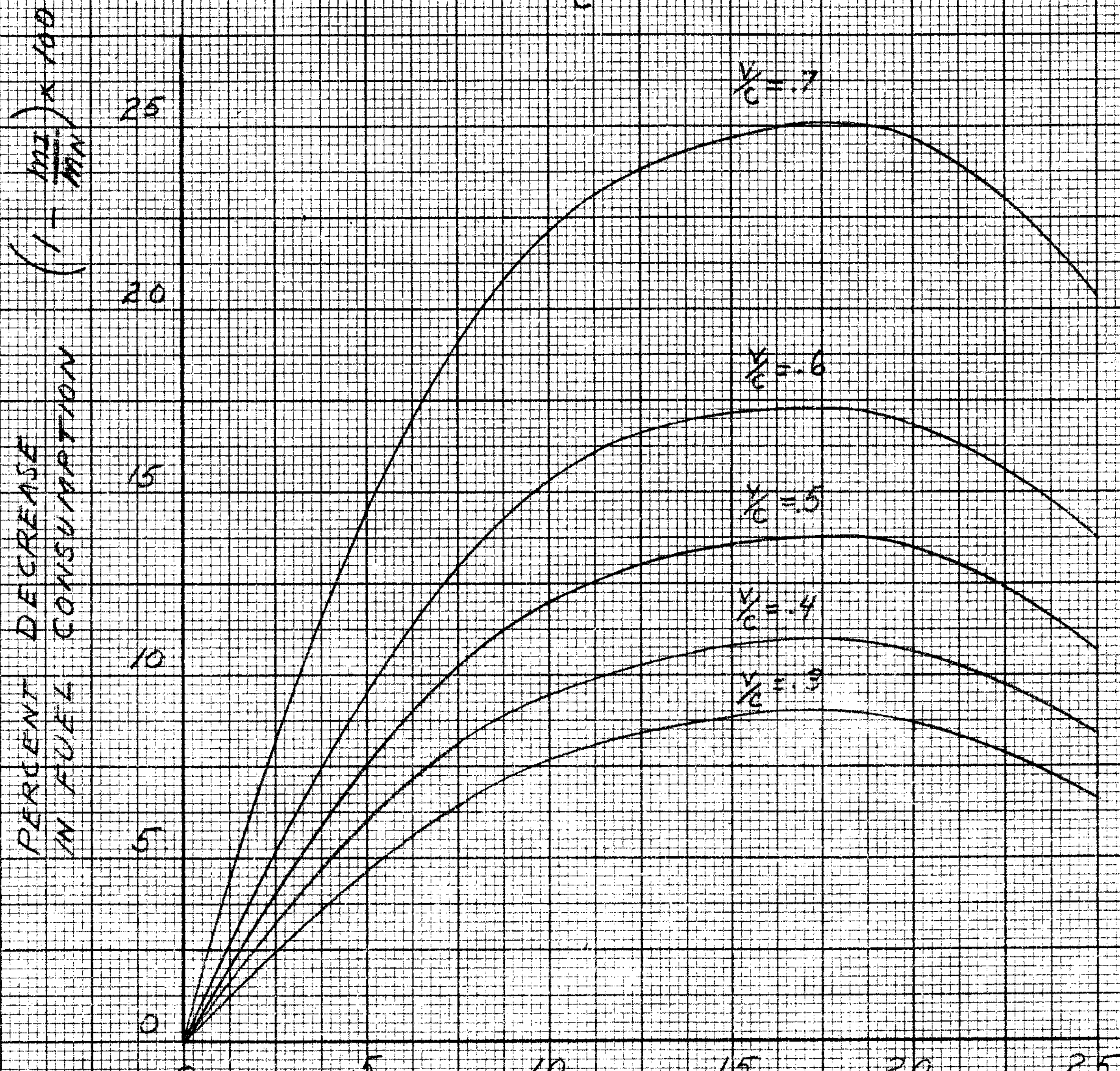


FIG. 6

PERCENTAGE DECREASE IN FUEL CONSUMPTION

COMPARISON OF I.T.A. AND NORMAL RAMJETS
FOR CONDITION OF LEVEL FLIGHT
WEIGHT CORRECTIONS INCLUDED

$\frac{V}{C} = .5 \quad \frac{\alpha}{C} = .1 \quad \frac{E}{C} = .4$

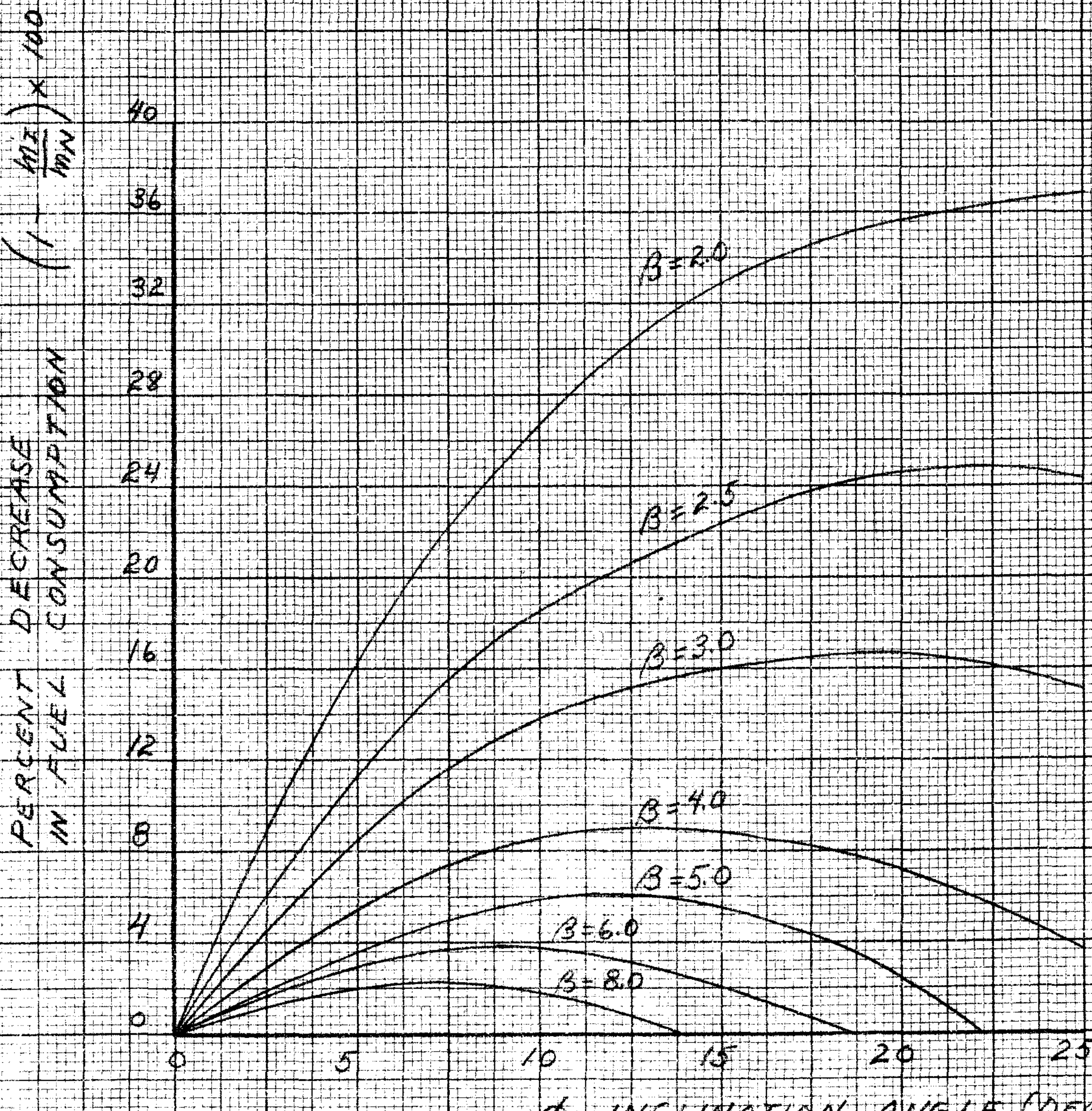


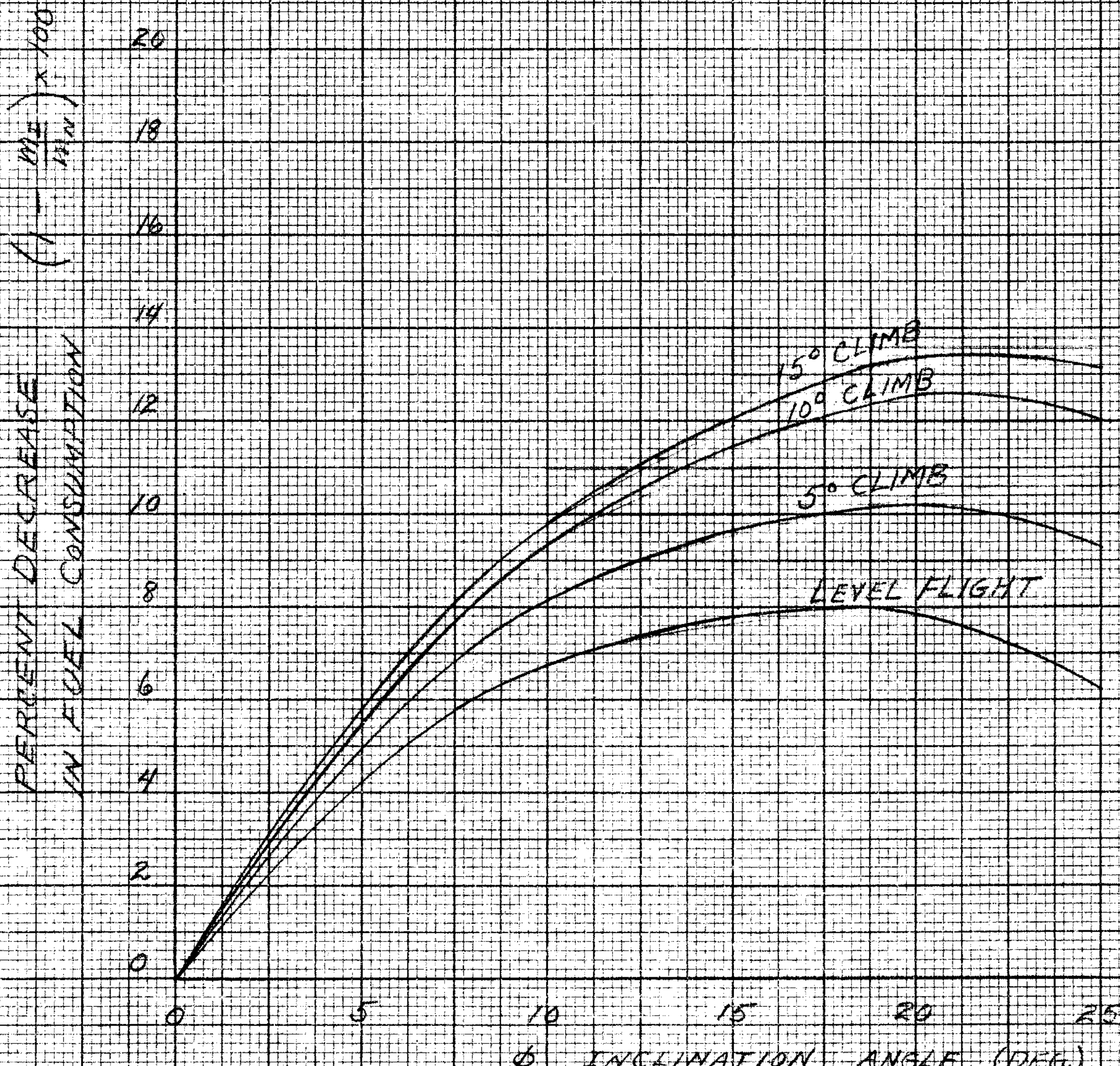
FIG. 7

PERCENTAGE DECREASE IN FUEL CONSUMPTION

COMPARISON OF I.T.A. AND NORMAL RAMJETS
IN CLIMB WITH WING WEIGHT
CORRECTION ONLY

$y/c = .5 \quad x/c = 0 \quad \epsilon/c = 0$

$L/D = 5 \quad \frac{\sigma}{q C_L} = 1/3$



# FIG. 8

## PERCENTAGE DECREASE IN FUEL CONSUMPTION

COMPARISON OF I.T.A. AND NORMAL RAMJETS IN CLIMB WITH WEIGHT CORRECTIONS INCLUDED

$V/c = .5 \quad \alpha/c = .1 \quad b/c = .4 \quad L/D = 5 \quad \frac{\sigma}{q c_L} = \frac{1}{3}$