I. BARYON-ANTIBARYON PHASE TRANSITION AT HIGH TEMPERATURE

II. INCLUSIVE VIRTUAL PHOTON-HADRON REACTIONS IN THE PARTON MODEL

Thesis by

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In Partial Fulfillment of the Requirements

For the Degree of

Doctor of Philosophy

California Institute of Technology

Pasadena, California

1973

(Submitted May 21, 1973)

ACKNOWLEDGEMENTS

I am indebted to Professor S. C. Frautschi for suggesting the study presented in Part I of this thesis and for guidance at all stages of the work. Many thanks go to Professor R. P. Feynman for much constructive criticism on Part II of this thesis.

I have received very valuable support from the Latin American Scholarship Program of American Universities during my years as a graduate student.

ABSTRACT

PART I

Present experimental data on nucleon-antinucleon scattering allow a study of the possibility of a phase transition in a nucleon-antinucleon gas at high temperature. Estimates can be made of the general behavior of the elastic phase shifts without resorting to theoretical derivation. A phase transition which separates nucleons from antinucleons is found at about 280 MeV in the approximation of the second virial coefficient to the free energy of the gas.

Part II

The parton model is used to derive scaling laws for the hadrons observed in deep inelastic electron-nucleon scattering which lie in the fragmentation region of the virtual photon. Scaling relations are obtained in the Bjorken and Regge regions. It is proposed that the distribution functions become independent of both q^2 and ν where the Bjorken and Regge regions overlap. The quark density functions are discussed in the limit $x \rightarrow 1$ for the nucleon octet and the pseudoscalar mesons. Under certain plausible assumptions it is found that only one or two quarks of the six types of quarks and antiquarks have an appreciable density function in the limit $x \rightarrow 1$. This has implications for the quark fragmentation functions near the large momentum boundary of their fragmentation region. These results are used to propose a method of measuring the proton and neutron quark density functions for all x by making measurements on inclusively produced hadrons in electroproduction only. Implications are also discussed for the hadrons produced in electron-positron annihilation.

TABLE OF CONTENTS

PART I. BARYON-ANTIBARYON PHASE TRANSITION AT HIGH TEMPERATURE

PART II. INCLUSIVE VIRTUAL PHOTON-HADRON REACTIONS IN THE PARTON MODEL

PART I

I. INTRODUCTION

Harrison[1] has suggested that if baryon-antibaryon inhomogeneities existed in the early universe, several problems of galaxy formation could be solved. Harrison's suggestion and the earlier conjecture[2] of a charge symmetrical boundary condition between baryons and antibaryons has led to the proposal[3] for mechanisms to separate baryons and antibaryons at high temperature. Statistical fluctuations in the baryon number density are not adequate to explain the present baryon density in the universe. Dynamical mechanisms are therefore required to separate baryons and antibaryons if a symmetrical boundary condition is assumed. Apart from the necessity of finding a separation mechanism, symmetrical models must explain various observational data such as the present ratio of the number of photons to baryons and the absence of any appreciable mixture of matter and antimatter[4] in interstellar gas.

A model has been proposed by Omnes[3] which gives baryon antibaryon separation in the blackbody radiation at a temperature of 350 MeV. The system under consideration is a gas of pions, nucleons and antinucleons at constant volume and temperature. To obtain the equilibrium configuration, the free energy is minimized with respect to variations in the numbers of nucleons and antinucleons. The free energy is expanded in powers of the numbers of nucleons and antinucleons. It is found in minimizing the free energy that, if the second virial coefficient has a large enough positive value (corresponding to an effective repulsion between nucleons and antinucleons) separation is possible.

An effective repulsion between nucleons and antinucleons arises in the Omnes model from the assumption of validity of Levinson's theorem, and considering that the corresponding bound states of the $N\overline{N}$ system $(\pi, \rho, \ldots)$ are an independent component of the radiation. The approximation is made that only S waves are important with Levinson's theorem holding for scattering states with the quantum numbers of the π, η, ρ and ω mesons. The $N\overline{N}$ phase shifts therefore fall from π to 0 as momentum goes from 0 to ∞.

To understand how a falling phase shift causes repulsion it is sufficient to look at the modification of the number of states in a range of momentum due to the interaction. The asymptotic wave function in spherical coordinates is proportional to

$$\sin\,[pr + \delta + \ell\pi/2].$$

We assume the particle is contained in a spherical volume of radius R. The condition that the wave function vanishes at the boundary gives

$$pR + \delta + \ell\pi/2 = n\pi.$$

The number of states dn in the range of momentum dp is given by

$$\frac{dn}{dp} = \frac{R}{\pi} + \frac{1}{\pi}\,\frac{d\delta}{dp} \quad .$$

We therefore find that if $d\delta/dp$ is negative, the number of states in the range dp is reduced below that in the absence of interactions.

We find a falling phase shift, for example, in a system in which there is one bound state and Levinson's theorem holds. In this case the phase space which is excluded from the scattering states has gone into the formation of the bound state as pointed out by Omnes. The presence of the bound state must ordinarily be taken into account in the calculation of the second virial coefficient for a gas of such particles. The second virial coefficient for a gas of particles interacting through an attractive potential is in fact negative. This coefficient consists of two terms, one negative due to the bound state and the other positive depending on $d\delta/dp$. At high temperatures the two terms approach opposite values giving zero for the second virial coefficient.

In Omnes' model the bound states of the nucleon-antinucleon system are assumed to be the π, η, ρ, and ω mesons. The phase shifts are taken to be monotonically decreasing from π to 0 in a range of momentum of the order of the ω mass. Since the π, η, ρ, and ω are considered to be independent components of the radiation they are <u>not</u> included in the calculation of the second virial coefficient B. As a result a positive value of B is obtained corresponding to an effective repulsion. Separation is possible if there is a large enough number of nucleons and antinucleons interacting with momenta of a few hundred MeV. In the blackbody radiation the density of particles is a rapidly increasing function of temperature. Increasing the temperature eventually produces a density of nucleons and antinucleons large enough that it becomes more profitable (for lowering the free energy) to have different numbers of nucleons and antinucleons. For these statements to have any relevance it is necessary, of course, that the separation temperature occur within the range in which the original assumptions are valid.

It is the purpose of this thesis to point out that the present experimental data on low energy nucleon antinucleon scattering are adequate to make estimates of the general behavior of the elastic phase shifts without resorting to theoretical derivations such as the one made by Omnes. Every known model of the nucleon-antinucleon interaction which makes an attempt to fit the data contains an absorptive potential(5) that causes some of the real phase shifts to attain negative values of the order of $-\pi/2$ at 600 MeV center of mass momentum; whereas the phase shifts that take positive values are small. We take the simplest model of the nucleon antinucleon interaction which consists of a purely absorptive potential. This simple model gives good fits to the low energy total inelastic and differential cross sections.(5) We find that the phase shifts in this model fall fast enough that separation is again possible at 280 MeV. If we had used any of the more sophisticated models of the nucleon antinucleon interaction which include spin dependent interactions, the answer would not be changed in any essential way. In all these models the phase shifts fall fast enough to give a second virial coefficient large enough to cause separation in the blackbody radiation at around

300 MeV. The analysis that leads to this conclusion, however, disregards the effect of the inelastic channels. Since insufficient data exist on these channels we can only give a theoretical argument which suggests that their effect is small. Our conclusion that nucleon-antinucleon separation occurs does not, unfortunately, rely only on the data.

II. FORMALISM

Thermodynamic quantities are calculated for the high temperature radiation assuming thermal equilibrium. The system considered is a gas of pions, nucleons and antinucleons at constant volume and temperature which can exchange particles with the surroundings. The various particle densities and the configuration of this system will be such as to minimize the free energy.

The contribution to the free energy coming from the interaction among the various particles is expanded in a power series in the densities of nucleons and antinucleons N/V and $\bar{N}/V$. Only terms up to quadratic order are kept in this expansion. The term linear in N and $\bar{N}$ is due to the pion-nucleon interaction. The term of order $N\bar{N}$ is due to the nucleon-antinucleon interaction. Terms of order N^2 and $\bar{N}^2$ are due to nucleon-nucleon interactions. If the effects of Fermi statistics are taken in the first approximation, they provide additional terms in the free energy proportional to N^2 and $\bar{N}^2$.

Bouchiat[6] has analyzed the modifications to the free energy of the nucleon gas due to the presence of pions. At temperatures around 200 MeV, the approximation is made that the nucleon-pion interaction occurs only in the Δ state. The zero width limit is taken for the Δ resonance. With these simplifying assumptions the baryon gas consists of nucleons and Δ resonances; the Δ being considered as an excited state of the nucleon. The free energy of the baryons is found to be

$$F_1 = -NT \log \frac{eN^o}{N} - \bar{N}T \log \frac{eN^o}{\bar{N}} \tag{II.1}$$

where

$$\frac{N^o}{V} = 4\left(\frac{M_N T}{2\pi}\right)^{3/2} \exp\left(-\frac{M_N}{T}\right) + 16\left(\frac{M_\Delta T}{2\pi}\right)^{3/2} \exp\left(-\frac{M_\Delta}{T}\right). \tag{II.2}$$

M_N and M_Δ are the masses of the nucleon and Δ resonance respectively. V is the volume of the gas. $\hbar$, c and k have been placed equal to one in this expression and throughout this thesis. In the absence of other interactions the equilibrium state will be that with a density of nucleons equal to N^o/V. The presence of pions permits a larger density of nucleons at equilibrium given by the second term on the right-hand side of Eq. (II.2).

The contribution to the free energy due to the nucleon-antinucleon interaction is given by

$$F_2 = 2TBN\bar{N}/V \tag{II.3}$$

where B is determined by the Beth-Uhlenbeck formula:

$$B = -8\left(\frac{\pi}{M_N T}\right)^{3/2} \Sigma\, g_n \exp\left(\frac{E_n}{T}\right)$$

$$-8\left(\frac{\pi}{M_N T}\right)^{3/2} \sum_{I\,J} \frac{(2I+1)(2J+1)}{16\pi} \int_0^\infty \frac{d\delta^{IJ}}{dp} \exp\left(-\frac{p^2}{M_N T}\right) dp, \tag{II.4}$$

p is the center of mass momentum of the nucleon, E_n is the binding energy of the bound state of the system and g_n is its degeneracy.

The nucleon-nucleon interaction contribution to the free energy is

$$F_3 = TB'(N^2 + \bar{N}^2)/V \qquad \text{(II.5)}$$

where B' is again given by an expression of the form (II.4) with the sum over J and I subject to those states allowed by the exclusion principle.

Corrections due to Fermi statistics for the nucleons and antinucleons can be taken into account in first approximation by adding an appropriate term to the virial coefficient in F_3. This correction is given by[7]

$$F_4 = TB''(N^2 + \bar{N}^2)/V \qquad \text{(II.6)}$$

where

$$B'' = \frac{1}{8}\left(\frac{\pi}{M_N T}\right)^{3/2} \qquad \text{(II.7)}$$

The part of the free energy of the gas which depends on the density of nucleons and antinucleons is therefore given by

$$F = -NT\log\frac{eN^o}{N} - \bar{N}T\log\frac{eN^o}{\bar{N}} + 2TB\frac{N\bar{N}}{V} + T(B'+B'')(N^2+\bar{N}^2)/V \qquad \text{(II.8)}$$

The free energy F is minimized with respect to the numbers of nucleons and antinucleons. It is found that for a positive and greater than $\exp\left(\frac{a+b}{a-b}\right)+b$ the minimum of free energy is achieved for a state with unequal numbers of nucleons and antinucleons; where $a = 2BN_o/V$,

$b = 2(B'+B'')N_o/V$. For negative a, which corresponds to an effective attraction between nucleons and antinucleons, the minimum of F is at $N = \bar{N}$; the same holds when $a = b$ or when $a < \exp(\frac{a+b}{a-b})+b$. In the case where all nucleon-nucleon interactions are neglected or cancel out, the condition for separation becomes simply $2BN_o/V > e$.

The temperature dependence of N_o/V is dominated by the exponential factors in (II.2) at the temperatures under consideration. For this reason the critical temperature for separation is not sensitive to the value of B. To achieve a separation temperature of a few hundred MeV it is only necessary for B to have a value of the order of 1 $(\text{fermi})^3$.

III. THE OMNES MODEL

The main assumptions in the Omnes model which give separation are: a) Nucleon-antinucleon interactions occur mainly in $\ell = 0$ states and the corresponding bound states which have the quantum numbers of the π, η, ω and ρ mesons are an independent component of the radiation, b) Levinson's theorem holds for scattering in states of the corresponding quantum numbers.

Assumption a) allows Omnes to drop the first term on the right-hand side of (II.4) which is the contribution of the N-$\overline{\text{N}}$ bound states to the second virial coefficient. Levinson's theorem states that the phase shifts fall from π to 0 as p goes from 0 to ∞. If the phase shifts fall to zero in a sufficiently small range of momentum, a sufficiently large positive value of B is obtained.

Omnes takes for the N-$\overline{\text{N}}$ phase shifts in all the $\ell = 0$ states:

$$\delta = \begin{cases} \pi(1 - p^2/p_o^2) & p \leq p_o \\ 0 & p > p_o \end{cases} \qquad \text{(III.1)}$$

The value Omnes used for p_o ($\sim M_\omega/2$) leads to a violation of the Wigner bound ($d\delta/dp >$ - Range of forces) provided we take the range of the forces to be 1.4 fermi. This can be corrected by taking a suitable value for p_o. The smallest possible value of p_o consistent with the Wigner bound is p_o = 885 MeV corresponding to $(d\delta/dp)$minimum = -1.4 fermi. The results of calculations using p_o = 885 MeV give a value of the critical temperature for separation of

T_c = 378 MeV. Fermi statistics taken in the approximation of the second virial coefficient raise the critical temperature to 381 MeV; this justifies the additional approximation made by Omnes that the effect of Fermi statistics is small.

IV. ABSORPTIVE MODEL FOR THE N-$\bar{N}$ INTERACTION

Phillips[5] has discussed various models of the N-$\bar{N}$ interaction in the region of a few hundred MeV. The simplest model which gives reasonable fits to the data is the pure absorptive model. The interaction is due to a pure imaginary Woods-Saxon potential

$$W = -iW_o[1 - A \exp Dr]^{-1} \quad . \qquad (IV.1)$$

Good fits to the differential, total and inelastic cross sections are obtained with the parameters $A = 1$, $D = 3(\text{fermi})^{-1}$ and $W_o = 3.3$ GeV.

We have calculated the scattering phase shifts due to the potential given in (IV.1); the real parts are shown in Table 1 for S, P and D waves. The phase shifts of all partial waves, except S waves, show the same qualitiative features. Re δ is small and positive near threshold; it becomes negative at a value of momentum which is higher for higher partial waves. Re δ for S waves is always negative. The significant result as far as the problem of separation is concerned, is that S and P phase shifts fall through an angle of the order of $\pi/2$ when p varies from 0 to 600 MeV.

TABLE I

E (lab.)	Re δ_0	Re δ_1	Re δ_2
10 MeV	-7.2°	$+2.1^\circ$	$+0.4$
20	-27.7°	$+0.9^\circ$	$+0.7^\circ$
60	-49.2°	-6.0°	$+1.8^\circ$
100	-62.5°	-12.5°	$+0.06^\circ$
300	-93.6°	-32.0°	-5.8°
500	-103.8°	-39.9°	-9.7°

Phase shifts due to the potential of equation (IV.1)

Nucleon antinucleon separation is again possible in this purely absorptive model of N-$\bar{N}$ interactions. We get a positive value for B of the order of 1 (fermi)^3 at temperatures of a few hundred MeV due to the falling phase shifts. Numerical calculations give a value of 280 MeV for the critical temperature for separation. Including the effect of Fermi statistics to the approximation of the second virial coefficient raises T_c to 283 MeV.

Although we have used the simplest theoretical model of the nucleon-nucleon interactions, it is important to note that the feature which gives negative phase shifts is present in other more sophisticated models. All models discussed by Phillips[(5)] which make an attempt to fit the total and differential cross sections contain an absorptive core. In particular, Bryan and Phillips[8] take the model of nucleon-nucleon interactions of Bryan and Scott[9] consisting of various one boson exchanges, change the sign of negative G parity exchanges and add an imaginary Woods-Saxon potential. They state that due to the absorptive core negative real parts are obtained for the low partial amplitudes; all spin and isospin dependence is contained in the one boson exchange terms. The absorptive potential gives the short range interaction, while the long range interactions are mainly contributed by the exchange terms. Another example is the model of Ball and Chew.[10] Ball and Chew take the nucleon-nucleon model of Signell and Marshak;[11] they adapt it to the nucleon-antinucleon system by changing the sign of the one-pion exchange term and adding an absorptive core. They give a table for their theoretical phase shifts at 140 MeV laboratory energy. The same features of the absorption only model are again noticed: low partial waves have sizeable negative values for the real parts of the phase shifts, high partial waves have positive but small values for their phase shifts.

V. FROISSART PHASE SHIFTS

It is necessary to justify our use of the real parts of the phase shifts in the Beth-Uhlenbeck formula (II.4). To do this, we re-examine the validity of the formula in the presence of inelastic channels. According to Dashen, Ma and Bernstein,[12] the general expression for the second virial in as S wave is

$$b_2 = - \frac{1}{2^{3/2}} \frac{1}{4\pi i} \int_0^\infty dE \; e^{-E/T} < N\bar{N}| S^{-1} \overleftrightarrow{\frac{\partial}{\partial E}} S \; |N\bar{N} > \qquad (V.1)$$

where S is now a matrix and $\overleftrightarrow{\frac{\partial}{\partial E}} = \overrightarrow{\frac{\partial}{\partial E}} - \overleftarrow{\frac{\partial}{\partial E}}$. The one channel case gives $S = \exp(2i\delta)$

$$S^{-1} \overleftrightarrow{\frac{\partial}{\partial E}} S = 4i \frac{d\delta}{dE}$$

which leads to the Beth-Uhlenbeck formula. However, in the multi-channel case

$$< n|S|i > = \eta^{in} \exp(2i\delta^{in})$$

$$< n|S|i >^\dagger = \eta^{in} \exp(-2i\delta^{in})$$

η, δ are real and depend on E. Putting a complete set of states $|n >< n|$ and working out the derivatives, one finds

$$< N\bar{N}|S^{-1} \overleftrightarrow{\frac{\partial}{\partial E}} S|N\bar{N} > = 4i \sum_n |\eta^{in}|^2 \frac{d\delta^{in}}{dE}$$

One recovers the Beth-Uhlenbeck formula only by ignoring all inelastic channels and setting $\eta^{\text{elastic}} = 1$. We may argue, however, that the effect of the inelastic channels is small. Define

$$S = \Sigma\, S_F$$

where S_F is a diagonal matrix satisfying elastic unitarity in each channel, it follows that Σ is also unitary. S_F is

$$S_F = \exp(2i\delta_F)$$

where

$$\delta_F = \delta - \delta_\alpha$$

$$\delta_\alpha = \frac{q}{\pi} \int_0^\infty \frac{\mathrm{Im}\ \delta\ (\nu')d\nu'}{q'(\nu' - \nu - i\epsilon)} \qquad \text{(V.2)}$$

which is defined so that $\mathrm{Im}\ \delta_\alpha = \mathrm{Im}\ \delta$ leaving δ_F purely real; δ is the physical phase shift and δ_F are called the Froissart phase shifts.[13] Now write

$$\langle n|S_F|i\rangle = \exp(2i\ \delta_{Fii})\ \text{(diagonal)}$$

$$\langle n|\Sigma|i\rangle = \eta^{in} \exp(2i\alpha_{in})$$

and we find

$$< N\bar{N}|S^{-1} \frac{\overleftrightarrow{\partial}}{\partial E} S|N\bar{N} > = 4i \frac{d\delta_F^{N\bar{N}\to N\bar{N}}}{dE} +$$

$$+ 4i \sum_n |\eta^{N\bar{N}\to n}|^2 \frac{d\alpha^{N\bar{N}\to n}}{dE} \qquad (V.3)$$

Omnes[14] argues that the second term on the right-hand side of the above equation vanishes in some versions of the Veneziano model. Alternatively we may argue that due to the large number of channels involved in our problem the individual terms in the sum in (V.3) may cancel each other leaving a term which is small compared to the contribution of the elastic channel. This is, however, not conclusive and only suggests that the contribution of the inelastic channels is small. If this is the case, we can neglect the second term in equation (V.3) and recover the Beth-Uhlenbeck formula with δ replaced by δ_F.

The results we have derived using δ (physical) in (II.4) are nevertheless not essentially changed by using δ_F. This is due to the behavior of Im δ in the region of a few hundred MeV. The principal part of the integral in (V.2) gives Re δ_α, which in the non-relativistic limit becomes

$$\text{Re}\ \delta_\alpha = \frac{E^{\frac{1}{2}}}{\pi} P \int_0^\infty \frac{\text{Im}\ \delta\ dE'}{(E')^{\frac{1}{2}}(E'-E)} \qquad (V.4)$$

where E is the laboratory energy of the nucleon. In the absorptive model Im δ is very closely equal to $C\ E^{\frac{1}{2}}$ where C is a constant. The result of this is that the integral in (V.4) is positive and has a monotonic dependence on energy which is weaker than the factor $E^{\frac{1}{2}}$ multiplying it. It follows that $\delta_F = \mathrm{Re}\delta - \mathrm{Re}\delta_\alpha$ will show the same negative derivative behavior with energy characteristic of $\mathrm{Re}\delta$. If we can neglect the second term of (V.3) our conclusions of previous sections on baryon-antibaryon separation are not changed.

The important question which remains unsettled is that of the size of the contribution of the inelastic channels to the second virial coefficient. If the contribution is positive, it will not change our results; if it is negative, its size will determine the degree to which our results will be modified. The inelastic contributions could reduce the elastic contributions by as much as 70 percent without changing our results qualitatively; but separation will not occur if the two contributions is negative.

VI. COMMENTS

From causality arguments it is possible to establish that $d\delta/dp >$ -Range of the forces. This limit on how fast the phase shifts can fall places a lower bound on the critical temperature for separation. We assume the radiation is at a temperature where only S and P waves are important in N-$\bar{N}$ scattering. If we let δ fall linearly as fast as possible ($d\delta/dp = -1.4$ fermi) through as many multiples of π as we wish, we find that T_c cannot be lower than 247 MeV. The answer justifies the assumption that only S and P waves are important.

A resonance in the N-N system could provide an attractive force among nucleons and among antinucleons. This attraction would lower the critical temperature for separation in the presence of a mechanism which separates nucleons from antinucleons. Phase shift analyses have been performed for nucleon-nucleon scattering up to energies of a few hundred MeV. No resonances have been observed; this permits us to estimate a limit on the effect of a possible resonance near the energy limits of the phase shift analyses. The change in the critical temperature is found not to be significant. For definiteness, if we assume a zero width resonance in the $^1D_2$ state at 450 MeV center of mass momentum, the critical temperature for separation is not lowered by more than 50 MeV in either the Omnes or the absorptive model. Resonances in higher angular momentum states would enter with a larger statistical weight but they would be expected to occur at a higher energy which would make their effect small.

Pion exchange affects the scattering in high partial waves. High angular momentum phase shifts contribute with high statistical weight to the second virial coefficient but the individual phase shifts are small. Numerically, it is found that the pion exchange phase shifts are small and contribute little to B in spite of their high statistical weights.

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PART II

I. INTRODUCTION

The parton model was first proposed by Feynman[1,2] to suggest regularities in multiparticle high energy collisions. It was suggested that the invariant single particle differential cross section $Ed\sigma/d^3p$, for the reaction hadron + hadron → hadron + anything, would become a function of the ratio $p_z/P_{c.m.}$ of the final hadron longitudinal momentum to the initial momentum in the center of mass at very high energies. This prediction was later confirmed by experiment.[3] It was suggested further that the distribution of hadrons in the reaction just mentioned would be constant for small values of $p_z/P_{c.m.}$. This prediction also seems to be confirmed by experiment.[4]

Although the parton model has been successful in making predictions for hadronic interactions, it has been much more useful in understanding the electromagnetic and weak interactions of hadrons. Experiments at the Stanford Linear Accelerator Center[5] found scaling in the structure functions which describe the inelastic scattering of electrons from nucleons, as predicted by Bjorken.[6] It was noticed shortly after that the parton model provided a simple explanation of the Bjorken scaling phenomenon. Other regularities in the data could also be explained in terms of the parton model. If it is assumed that partons are quarks, further results can be derived such as relations between electromagnetic and weak structure functions. These relations have not yet been tested experimentally.

The "Light cone algebra" proposed by Fritzsch and Gell-Mann[7] is a specification of the behavior of current commutators when the currents operate on space-time points which are on each other's light cone. This suggestion was abstracted from a parton model of quarks and includes as its consequences those predictions of the quark-parton model which are believed most likely to be true experimentally.

It is of interest to propose tests of those features of the parton model which go beyond light cone algebra. Also of interest is to extend the parton model further and propose tests to determine how far we can reasonably trust the model. It is the purpose of Part II of this thesis to examine the experimental consequences of these additional features of the parton model and to add assumptions to extend its range of applicability. These additional assumptions are suggested by pre-existing properties of the model and some experimental facts.

II. DESCRIPTION OF THE PARTON MODEL

In the parton model, the proton or any hadron is seen as composed of various field constituents called partons. The existence of a wave function giving the amplitudes for different numbers of partons with various momenta in a frame in which the proton has a large momentum P is assumed. That is, there is an amplitude $\Psi_1(p_1)$ to find one parton of momentum p_1 in the proton, an amplitude $\Psi_2(p_1,p_2)$ to find two partons with momenta p_1, p_2 in the proton, etc. The wave functions for different P have the property that the probability to find a parton of longitudinal momentum xP and fixed transverse momentum becomes a function of the number x only, as $P \to \infty$. This probability is unconditional in the sense that the number of partons which carry the remaining momentum $(1 - x)P$ is not restricted in any way. The statement can be made slightly more general; the probability to find a finite number of partons with longitudinal momenta x_1P, x_2P, ... x_nP and fixed transverse momenta becomes a function of the x's only as $P \to \infty$ if all $x_i > 0$ and $\Sigma\, x_i < 1$. This scaling assumption is made for the probability, not for the amplitude. The amplitude depends on the momenta of all the partons; the number of partons increases with P and there are always partons of finite momentum. This prevents the amplitude from satisfying a simple scaling property.

The transverse momenta of the partons are assumed to be limited independently of their longitudinal momenta. This assumption is suggested by the fact that in high energy hadron-hadron collisions the outgoing particles have limited transverse momenta with an average of 0.3 GeV independently of the energy of the collision. This observation is one of the main experimental inputs of the parton model; the scaling in longitudinal momentum is believed to be related to it.

In contrast to the scaling property of the fast partons, there is a fixed distribution of finite momentum partons which becomes independent of P as $P \to \infty$. That is, at any large momentum P there are finite momentum partons whose distribution remains fixed. These are called wee partons. This assumption is suggested by the approximate constancy of total cross sections. In a high energy collision of hadrons, it is assumed that there are interactions only between partons whose relative momentum is less than some finite value. A constant number of wee partons is required to have a constant cross section.

Continuity is assumed between the wee parton distribution and the finite x scaling distribution. This can be achieved only if the distribution is of the form dx/x for small x and dp_z/E for wee partons. The rapidity variable $y = \frac{1}{2} \ell n[(E+p_z)/(E-p_z)]$ is more appropriate in this discussion. The distribution of partons in a hadron of momentum P is illustrated in Figure II.1a. At a larger momentum P' we have the same distribution of low momentum partons and the same distribution of fast partons (fixed x). The only way to join these distributions in a frame-independent way is to assume a flat distribution in rapidity; as illustrated in Figure II.1b. An important consequence of this continuity is that the average number of partons increases logarithmically with the momentum of the hadrons. The rapidity variable has the property that a fixed distance from the maximum $y_{max} = \ell n(2P/M)$ corresponds to a fixed x, and a fixed distance from y = 0 corresponds to a fixed momentum.

A more detailed description of the parton model may be found in Ref. 8. We have only presented those features of the model on which we shall rely in the following sections.

An important phenomenon which can be understood in terms of parton model ideas is the scaling of the structure functions which describe the inelastic scattering of electrons from nucleons $e + N \rightarrow e +$ anything. The differential cross section for this process (the kinematics are given at the end of Section III.A)is described by two structure functions W_1 and W_2 which in general can be functions of the variables q^2 and ν. q^2 is the square of the virtual photon four-momentum q_μ, and $\nu = P \cdot q/M$; P_μ is the four-momentum of the nucleon and M is its mass. The parton model predicts that the virtual photon interacts with a parton whose longitudinal momentum is a fraction $x = -q^2/2M\nu$ of the momentum of the proton in a frame in which the momentum of the proton is large. As a consequence of this, the structure functions become functions of x only in the Bjorken limit ($-q^2$ and $\nu \rightarrow \infty$, $-q^2/2M\nu$ finite). If it is assumed that charged partons have spin 1/2 both structure functions depend on a single function of x through the relations $\nu W_2 = x\, f(x)$ and $2MW_1 = f(x)$; an assumption which seems to be supported by experiment.[5] The function

$f(x)$ has a simple interpretation in the parton model. $f(x)$ is the probability to find a parton with a fraction x of the momentum of the nucleon, weighted by the square of its charge. It has been assumed in the derivation that the strong interactions before and after the virtual photon interaction can only move the parton a finite distance off its mass shell.

If we assume that partons are quarks, the function $f(x)$ is given in terms of the probabilities to find the various quarks with momentum fraction x. For example, in the case of inelastic scattering from protons we have

$$f^p(x) = \frac{4}{9}(u(x) + \bar{u}(x)) + \frac{1}{9}(d(x)+\bar{d}(x)) + \frac{1}{9}(s(x) + \bar{s}(x)) \qquad (II.1)$$

where $u(x)dx$ is the probability to find a u quark with momentum fraction in the range x to x + dx in the proton. The other functions are defined similarly and are denoted by the names of the quarks; u and $\bar{u}$ for up and anti-up quarks, d and $\bar{d}$ for down and anti-down quarks; s and $\bar{s}$ for strange and anti-strange quarks. The functions $u(x)$, $\bar{u}(x)$ ..., etc., are called the quark density functions. From isospin invariance the neutron scaling function is obtained from (II.1) replacing $u \leftrightarrow d$ and $\bar{u} \leftrightarrow \bar{d}$:

$$f^n(x) = \frac{4}{9}(d(x) + \bar{d}(x)) + \frac{1}{9}(u(x) + \bar{u}(x)) + \frac{1}{9}(s(x) + \bar{s}(x)) \quad . \qquad (II.2)$$

The quark density functions have the same meaning as above. The number of u quarks in the neutron is the same as the number of d quarks in the proton.

III. SCALING IN INCLUSIVE ELECTROPRODUCTION OF HADRONS

A. Kinematics

The process we wish to describe is the inclusive electroproduction of a hadron

$$e + N \to e' + h + \text{Anything} \tag{III.1}$$

illustrated in Figure III.1. The four-momenta of the incoming and outgoing electrons e and e' are p_μ and $p_\mu{}'$ respectively. The four-momentum of the nucleon N is P_μ and that of the hadron h is h_μ. The possibility of production of any number of other hadrons beside h is not restricted in the reaction; a state of this set is denoted by n and its total four-momentum by $(p_n)_\mu$.

In the laboratory frame in which N is at rest we have $P_\mu = (M, \vec{0})$, $p_\mu = (E, \vec{p})$ and $p'_\mu = (E', \vec{p}')$.

The Bjorken limit of the deep inelastic region is defined by

$$\begin{aligned} q^2 &= (p' - p)^2 \to -\infty \\ M\nu &= P\cdot q \to \infty \\ x &= -q^2/2M\nu \quad \text{fixed} \end{aligned} \tag{III.2}$$

This defines one of the limits of the virtual photon variables with which we shall be concerned. Another region of interest is the Regge region in which q^2 is fixed and $\nu \to \infty$.

The variables used to describe the outgoing hadron h are not uniform in the literature. We shall present the kinematics in terms of a set of variables commonly used[9] and later define variables more natural to the model we shall use. Since we take the spin average and azimuthal average with respect to the direction of the virtual photon, only two variables are needed to describe h; a convenient pair is:

$$\begin{aligned} \epsilon &= h \cdot P \\ \kappa &= h \cdot q \end{aligned} \qquad \text{(III.3)}$$

In the Bjorken limit, the target fragmentation region is defined by

$$\begin{aligned} &\kappa \rightarrow \infty \\ &\epsilon \quad \text{fixed} \\ &u = \kappa/M\nu \quad \text{fixed} \end{aligned} \qquad \text{(III.4)}$$

In a frame in which the nucleon N has a large momentum (e.g. the center of mass) this corresponds to a hadron with a longitudinal momentum given by a fraction $u/(1 - x)$ of the momentum of N.

In the Regge limit, the target fragmentation region is again defined by (III.4). The longitudinal momentum of h is given by a fraction u of the momentum of N in a frame in which N is fast-moving.

The current fragmentation region is defined in the Bjorken limit by

$$\begin{aligned}
&\epsilon \to \infty \\
&\kappa \to \infty \\
&u = \kappa/M\nu \quad \text{fixed} \\
&\epsilon/M\nu \quad \text{fixed}
\end{aligned} \tag{III.5}$$

The longitudinal momentum of h is a fraction $z = -u/x$ of the momentum of the virtual photon in the laboratory frame.

In the Regge region, the current fragmentation limit is no longer given by conditions (III.5). We want h to have a longitudinal momentum given by a fraction z of the momentum of the virtual photon in the center of mass frame. This condition is given by

$$\begin{aligned}
&\epsilon \to \infty \\
&\kappa \quad \text{fixed} \quad .
\end{aligned} \tag{III.6}$$

The fraction z is again given by $-u/x$; u and x go to zero at the same rate.

The differential cross section for the inclusive electroproduction process illustrated in Figure III.1 is given by

$$d\sigma = \left\{ \sum_{\text{av.}} |M|^2 \right\} \frac{1}{2E2E'2M2E_h} \quad \frac{d^3p'}{(2\pi)^3} \quad \frac{d^3h}{(2\pi)^3} \tag{III.7}$$

This can be written in terms of the leptonic tensor

$$\ell_{\mu\nu} = p_\mu p'_\nu + p_\nu p'_\mu - p\cdot p'\delta_{\mu\nu}$$

and the tensor $\tilde{W}_{\mu\nu}$ which summarizes the unknown structure of the coupling of the virtual photon to the hadrons in question. Equation (III.7) becomes:

$$\frac{d\sigma}{d\nu dq^2 d\epsilon d\kappa} = \ell_{\mu\nu} \tilde{W}_{\mu\nu} \left(\frac{4\pi\alpha}{q^2}\right)^2 \frac{2\pi M\nu}{E} \tag{III.8}$$

where

$$\tilde{W}_{\mu\nu} = \int d^4h\theta(h_o)\delta(h^2 - m_n^2)\delta(h\cdot P - \epsilon)\delta(h\cdot q - \kappa) \; x$$
$$x \sum_n \langle N|j_\mu(0)|n,h\rangle \langle n,h|j_\nu(0)|N\rangle \; 2\pi\delta^4(p - p' + P - h - p_n) \; . \tag{III.9}$$

Since we are taking the spin and azimuthal average there are only two structure functions, denoted by $\tilde{W}_1$ and $\tilde{W}_2$ which in general are functions of q^2, ν, κ and ϵ. $\tilde{W}_\mu$ is given in terms of $\tilde{W}_1$ and $\tilde{W}_2$ by:

$$\tilde{W}_{\mu\nu} = \frac{1}{M^2}\left(P_\mu - \frac{P\cdot q}{q^2} q_\mu\right)\left(P_\nu - \frac{P\cdot q}{q^2} q_\nu\right) W_2(q^2, \nu, \kappa, \epsilon) +$$
$$- \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W_1(q^2, \nu, \kappa, \epsilon) \quad . \tag{III.10}$$

In terms of the structure functions the differential cross section for reaction III.1 is given by:

$$\frac{d\sigma}{d\nu dq^2 d\epsilon d\kappa} = \frac{4\pi\alpha^2}{q^2} \frac{E'}{E} \left\{\cos^2\frac{\theta}{2} \tilde{W}_2 + 2\sin^2\frac{\theta}{2}\tilde{W}_1\right\} \tag{III.11}$$

where θ is the angle between p and p' in the laboratory. The variables $z = -u/x$ and the transverse momentum h_T of the hadron h are more appropriate to the parton model in the current fragmentation region. In terms of these the differential cross section is:

$$\frac{d\sigma}{d\nu dq^2 dz dh_T^2} = \frac{4\pi\alpha^2}{q^2}\frac{E'}{E}\frac{M\nu}{2z}\left\{\cos^2\frac{\theta}{2}\tilde{W}_2 + 2\sin^2\frac{\theta}{2}\tilde{W}_1\right\} \qquad \text{(III.12)}$$

We present briefly, for reference purposes, the kinematics of the inelastic process $e + N \rightarrow e' +$ Anything where no hadrons are observed in the final state. We use the same notation as above for the momenta of the incoming and outgoing electrons, the virtual photon and the nucleon N. We define the hadronic tensor

$$W_{\mu\nu}(q^2, \nu) = \frac{1}{2\pi}\int d^4x\, e^{iq\cdot x} < N|j_\mu(x)\, j_\nu(0)|N > \quad . \qquad \text{(III.13)}$$

By relativity and gauge invariance this tensor can be written as (average over spins is taken)

$$W_{\mu\nu}(q^2,\nu) = \frac{1}{M^2}\left(P_\mu - \frac{P\cdot q}{q^2}q_\mu\right)\left(P_\nu - \frac{P\cdot q}{q^2}q_\nu\right) W_2(q^2,\nu) +$$

$$- \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W_1(q^2,\nu) \quad . \qquad \text{(III.14)}$$

The differential cross section is given in terms of the structure functions W_1 and W_2 as

$$\frac{d\sigma}{d\nu dq^2} = \frac{4\pi\alpha^2}{q^2} \frac{E'}{E} \left\{ \cos^2\frac{\theta}{2} W_2(q^2,\nu) + 2\sin^2\frac{\theta}{2} W_1(q^2,\nu) \right\}$$

(III.15)

B. The Bjorken Limit

It is convenient, when we analyze process (III.1) in terms of the parton model, to look at the reaction in the Breit frame (see Figure III.2). In this frame the virtual photon four-momentum has a space component only. The four-momenta of the virtual photon and nucleon are

$$q_\mu = (0,\ 0,\ 0,\ -2Px)$$

$$P_\mu = (\sqrt{P^2+M^2},\ 0,\ 0,\ P)$$

with invariants

$$q^2 = -4P^2 x^2$$

$$P\cdot q = M\nu = 2P^2 x \ . \qquad \text{(III.16)}$$

Figure III.2a illustrates the parton distribution in the nucleon and the virtual photon before the reaction. The photon interacts with a parton of momentum xP and reverses its motion, producing the parton distribution illustrated in Figure III.2b. This parton distribution after the reaction has the property that if we let x be fixed and take $P \to \infty$ (Bjorken limit), the momentum distribution of the partons scales with P. That is, the probability to find a parton with a fixed ratio of its longitudinal momentum to the momentum P remains constant as $P \to \infty$. In particular, moving to the left in Figure III.2b there is only one parton whose momentum scales with P, the scale factor given by -x.

We now bring in an assumption used in predicting scaling in hadronic interactions.(2) This assumption states that the final hadrons resulting from a parton distribution which scales in longitudinal momentum will also scale. Applying this assumption to our case above, we conclude that if we ask for a right moving outgoing hadron with longitudinal momentum which is a fixed fraction of the momentum $(1 - x)P$ or for a left moving hadron with longitudinal momentum which is a fixed fraction of the momentum $-xP$, the probability of finding such a hadron goes to a constant as $P \to \infty$. These hadrons are in the fragmentation region of the target and current respectively. We shall be concerned only with the current fragmentation region here. A consequence of the assumption above is that the structure functions defined previously satisfy the scaling relations

$$\left(\frac{M\nu}{2z}\right)\nu\, \tilde{W}_2^h\,(q^2, \nu, z, h_T^2) = x\, f^h\,(x, z, h_T^2) \qquad \text{a)}$$

$$(III.17)$$

$$\left(\frac{M\nu}{2z}\right)2M\, \tilde{W}_1^h\,(q^2, \nu, z, h_T^2) = f^h(x, z, h_T^2) \qquad \text{b)}$$

in the Bjorken limit. The fact that we have the same function f on the right-hand side of equations (III.17) is a consequence of the assumption that all charged partons have spin 1/2, an assumption we make throughout this thesis. The variable z defined in Section III A on kinematics is the ratio of the longitudinal momentum of the hadron h to the momentum -xP of the left moving hadron. The scaling relations (III.17) state that the structure functions $\nu^2\tilde{W}_2$ and $\nu\tilde{W}_1$ depend on q^2 and ν only through their ratio. The transverse momentum distribution of the hadron h will very likely be limited the same way it is in hadron-hadron reactions; this is in fact important for the longitudinal scaling to be valid.

The scaling assumption made in deriving relations (III.17) has as a consequence the concept of limiting fragmentation of a parton. It is suggested that if we have a single parton moving with large momentum, the hadrons resulting from the fragmentation of such parton will show longitudinal scaling. That is, if we ask for the probability to find a hadron with fixed transverse momentum and fixed fraction of its longitudinal momentum to the momentum p of the parton, that probability goes to a constant as $p \to \infty$. This idea was used by Drell and Yan,[10] and Berman, Bjorken and Kogut[11] in deriving scaling relations similar to (III.17). If we have a finite number of different types of partons (labeled by α), for every hadron h there will be a distribution function $D^h_\alpha(z, k_T^2)$ which gives the probability for the fragmentation of a parton of type α into a hadron h with longitudinal momentum fraction z and transverse momentum k_T, and any other hadrons. The function f in (III.17) will be a superposition over the index α of the probability that the virtual photon hits a parton of type α times the distribution D^h_α. The transverse momentum h_T of the outgoing hadron will, however, not be given by the transverse momentum distribution in $D_\alpha^h(z, k_T^2)$ because the parton α has some initial distribution in transverse momentum. What we have is a convolution of the initial transverse momentum distribution of the partons with the distribution in transverse momentum of the parton fragmentation. We have a superposition of factorized terms for the scaling function $f^h(x, z, h_T^2)$ only when we integrate over h_T^2; we denote the integrated function by $f^h(x, z)$:

$$f^h(x,z) = \sum_\alpha Q_\alpha^2\, \alpha(x)\, D_\alpha^h(z) \qquad \text{(III.18)}$$

In this expression $\alpha(x)$ is the density of partons of type α at that value of x integrated over transverse momentum; $D_\alpha^h(z)$ is the function $D_\alpha^h(z, k_T^2)$ integrated over k_T^2. Q_α is the charge of the parton of type α, measured in units of the electron charge.

C. The Regge Limit

We now let q^2 be fixed but large enough for the parton model to be applicable. We again have a distribution of partons before and after the interaction with the virtual photon as illustrated in Figure III.2, in the Breit frame. When q^2 is fixed, the momentum of the left moving parton after the interaction is also fixed. As $\nu \to \infty$ $(P \to \infty)$ the distribution of right moving partons scales as indicated in Section II. The fast partons (those with fixed x) scale in longitudinal momentum with P. The partons of finite momentum have a fixed distribution independent of P as $P \to \infty$. This fixed distribution property of the slow partons allows us to derive a scaling relation for the production of hadrons in the current fragmentation limit. We have a fixed momentum left moving parton and a fixed distribution of low momentum partons of the original nucleon. The distribution of partons out to any fixed range in rapidity in the Breit frame remains fixed as $P \to \infty$; this is illustrated in Figure III.3. From the assumption that interactions between partons occur only within a finite range in rapidity we conclude that, for a given q^2, the distribution of hadrons of low momentum in the Breit frame approaches a constant as $\nu \to \infty$. As a consequence of this the structure functions for inclusive electroproduction of hadrons satisfy the scaling relations

$$\left(\frac{M\nu}{2z}\right) \nu\tilde{W}_2^h\,(q^2, \nu, z, h_T^{\;2}) = g^h(q^2, z, h_T^{\;2}) \qquad a)$$

$$\left(\frac{M\nu}{2z}\right) 2M\tilde{W}_1^h\,(q^2, \nu, z, h_T^{\;2}) = \frac{1}{x} g^h(q^2, z, h_T^{\;2}) \qquad b) \qquad \text{(III.19)}$$

in the Regge limit and in the fragmentation region of the current. The derivation of the scaling relations (III.19) does not involve the assumption of limiting fragmentation of the parton.

The scaling relations in the Bjorken limit (III.17) and in the Regge limit (III.19) must agree where their ranges of applicability overlap. This occurs when x is small in (III.17) and when $-q^2$ is large in (III.19). The two pairs of formulas can only agree if the shape of the distribution becomes independent of both $-q^2$ and ν:

$$\left(\frac{M\nu}{2z}\right) \nu\tilde{W}_2^h (q^2, \nu, z, h_T^2) = g^h(z, h_T^2) \qquad a)$$

$$\left(\frac{M\nu}{2z}\right) 2M\tilde{W}_1^h (q^2, \nu, z, h_T^2) = \frac{1}{x} g^h(z, h_T^2) \qquad b) \qquad \text{(III.20)}$$

We conclude that in the Bjorken limit, for small x, the distribution of hadrons produced in the current fragmentation region approaches a fixed shape.

These last scaling relations (III.20), if integrated over transverse momentum, can also be derived from (III.18) and the assumed behavior of the parton density functions $\alpha(x)$ for small x. For small x, $\alpha(x) = \gamma_\alpha / x$ where γ_α is a constant, as required by the continuity between the fast parton distribution and the low momentum parton distribution. Substituting this into (III.18) we find

$$f^h(x,z) = \frac{1}{x} \sum_\alpha Q_\alpha \gamma_\alpha D_\alpha^h (z) \qquad \text{(III.21)}$$

for small x. The conclusion again is that the distribution of hadrons in the current fragmentation region approaches a fixed shape (as a function of z) for small x in the Bjorken limit. We see that the property which is responsible for the continuity between slow and fast momentum parton distributions is also responsible for the continuity between the inclusively produced hadron distributions in the Bjorken and Regge limits.

It is worth noting that none of the scaling relations derived in this section depend on the nature of the partons, such as assuming quarks or any other type of constituents. The spin 1/2 assumption is, however, necessary to obtain the relation $\nu\tilde{W}_2/2M\tilde{W}_1 = x$.

A light cone analysis has been made by Ellis,[12] Stack,[13] and Fritzsch and Minkowski[14] of the reaction $e + N \rightarrow e + h + \text{Anything}$. They propose that the structure functions scale in the Bjorken limit in the target fragmentation limit. The current fragmentation region we have discussed is, however, not accessible to the light cone analysis since it cannot be shown that the light cone dominates.

IV. THE QUARK DENSITY FUNCTIONS IN THE LIMIT $x \to 1$

In this section we shall give an argument for a plausible behavior of the quark density functions for x near 1. To motivate our assumptions we present an argument given by Feynman[8] for the behavior of the deep inelastic structure function $f(x)$ for x near 1. Let us ask for the probability that a proton of large momentum P has one parton carrying all the momentum P except for a finite amount (e.g. 1 GeV). This probability will be shown to fall with an inverse power of P. There are only a finite number of low momentum partons; all the partons of finite x, except for $x = 1 - 1\ \text{GeV}/P$, are excluded in the configuration we require. In particular, all the partons in the dx/x region of the distribution are excluded. Since the presence of the dx/x partons is essentially independent of the other partons, the probability that they are not present is proportional to $e^{-c\bar{n}}$ where c is a constant and $\bar{n}$ is the average number of excluded partons. This independence is due to the finite range of interaction in the rapidity variable. The average number $\bar{n}$ is given by

$$\bar{n} = a \int_{1/P}^{x_1} dx/x = a(\ell n x_1 - \ell n \tfrac{1}{P}) \tag{IV.1}$$

where a is a constant and x_1 is a fixed upper limit. It follows that the probability that the proton has one parton carrying almost all of its momentum falls as an inverse power of P, $e^{-c\bar{n}} \propto 1/P^{\gamma}$ where γ is a constant.

The configuration discussed above is important in that the proton must be in such a configuration to scatter elastically from a virtual photon in the Breit frame, as illustrated in Figure IV.1. If the virtual photon interacts with a parton of $x \neq 1$, the intermediate state has a mass of order P^2 and high energy interactions between partons are required for any appreciable amplitude that the final proton is in such a state. This leads to the conclusion that both the electric and magnetic form factors of the proton must fall with the same inverse power of $(-q^2)^{\frac{1}{2}}$ for large $(-q^2)^{\frac{1}{2}}$, the power is given by the constant γ mentioned above.

The probability that the proton has a parton at $x = 1 - (1\ \mathrm{GeV})/P$ is also proportional to

$$\int_{1-1/P}^{1} f(x)dx$$

which we argued must behave like $1/P^{\gamma}$. $f(x)$ must therefore have the form

$$f(x) \propto (1 - x)^{\gamma-1} \quad \text{for } x \to 1. \tag{IV.2}$$

From elastic form factor measurements γ is near 4 so γ-1 is near 3, a conclusion which is consistent with measurements on the deep inelastic structure functions. This relation between the power fall-off of the elastic form factors and the behavior of the inelastic structure functions near x = 1 was first derived by Drell and Yan.[15]

We now turn to the special case of the quark parton model. The structure function $f^p(x)$ for the proton is given by

$$f^p(x) = \frac{4}{9}(u(x) + \bar{u}(x)) + \frac{1}{9}(d(x)+\bar{d}(x)) + \frac{1}{9}(s(x)+\bar{s}(x)) \qquad (IV.3)$$

where $u(x)$, $\bar{u}(x)$, ..., etc., are the quark density functions defined in Section II. In terms of the same density functions, the neutron structure function $f^n(x)$ is given by

$$f^n(x) = \frac{4}{9}(d(x)+\bar{d}(x)) + \frac{1}{9}(u(x)+\bar{u}(x)) + \frac{1}{9}(s(x)+\bar{s}(x)). \qquad (IV.4)$$

We shall discuss the behavior of the quark density functions when $x \to 1$. We ask for the the probability that there is any one type of quark carrying all the momentum of the proton except for a finite amount. This configuration excludes any other type of quark in the dx/x region. From this it is possible to conclude, the same way as was done for $f(x)$, that each of the quark density functions must behave as a power of $(1 - x)$ for x near 1:

$$\begin{aligned} u(x) &\propto (1 - x)^{\gamma(u)-1} & \bar{u}(x) &\propto (1 - x)^{\gamma(\bar{u})-1} \\ d(x) &\propto (1 - x)^{\gamma(d)-1} & \bar{d}(x) &\propto (1 - x)^{\gamma(\bar{d})-1} \qquad (IV.5) \\ s(x) &\propto (1 - x)^{\gamma(s)-1} & \bar{s}(x) &\propto (1 - x)^{\gamma(\bar{s})-1} . \end{aligned}$$

The powers $\gamma(u)$, $\gamma(d)$, ..., etc., may in principle be all different;

in fact, it is possible to argue guided by experiment that $\gamma(u)$ is the smallest power for the proton.

In the configuration we are studying, aside from a specific linear combination of quarks of momentum nearly equal to the momentum of the proton, there is also a low momentum "core" of quarks. The overall quantum numbers of the core depend on the quantum numbers of the large momentum quark. Since it is very likely that the probability for the presence of a low momentum core will depend on its quantum numbers, we make the following non-degeneracy assumption: The value of the power fall-off with momentum of the probability that one quark, or linear combination of quarks, carries almost all the momentum of the proton will be different for different values of the quantum numbers of the core. A consequence of this assumption is that the ratio $f^n(x)/f^p(x)$ of the neutron and proton structure functions can only take a discrete set of values depending on the quantum numbers of the core as $x \to 1$. We first eliminate the possibility that a strange quark or anit-quark dominates near $x = 1$. The likelihood that a strange or negative baryon number quark dominates near $x = 1$ in a non-strange and positive baryon number object is considered to be very low. We only consider either u or d quarks, or a linear combination, to have the lowest power γ. There are only two possibilities for the nucleon, the core may have isospin 0 or 1; we call the corresponding powers γ_0 and γ_1. In the case of isospin 0 core the u quark only dominates near $x = 1$ in the proton and the d quark in the neutron. For isospin 1 core there is a linear combination of u and d which gives the correct quantum numbers of the proton and neutron. If $\gamma_1 < \gamma_0$, that is, isospin 1 core dominates then $f^n(x)/f^p(x) \to 3/2$ when $x \to 1$, a value which is almost certainly excluded by experiment. Experimentally,[16] the ratio $f^n(x)/f^p(x)$ falls approximately linearly from $0.79 \pm .06$ at $x = 0.2$ to $0.38 \pm .06$ at $x = 0.8$. If $\gamma_0 < \gamma_1$ then $f^n(x)/f^p(x) \to 1/4$ as $x \to 1$, a value which is consistent with the data. We therefore conclude that it is the u quark which dominates near $x = 1$ in the proton and the d quark in the neutron. The value 1/4 is the minimum the ratio $f^n(x)/f^p(x)$ can take for any value of x consistent with the quark model. From equations (IV.3) and (IV.4) and the positivity of the quark density functions it follows that $1/4 \leq f^n(x)/f^p(x) \leq 4$.

The above arguments given by Feynman for the behavior of the structure functions as $x \to 1$ motivate the assumptions made in the rest of this section.

As an extension of these ideas, we discuss what the behavior of the electromagnetic deep inelastic structure function would be near $x = 1$ for the other particles in the nucleon octet if these were available experimentally. The reason for making this discussion is that it will bear on the behavior of the fragmentation of a quark into these particles, a quantity which is measurable. After having excluded the possibility that an antiquark or a strange quark dominates near $x = 1$ in the nucleon, we were left with two alternatives. The core could have either isospin 0 or 1. This is precisely what we would get if we viewed a large momentum nucleon near $x = 1$ as built of a core which is in a $\overline{3}$ representation of SU(3) and a quark which is a member of a 3; or as built of a core which is in a 6 representation of SU(3) and a quark, respectively. We shall extend this to all the baryons in the nucleon octet by assuming that near $x = 1$ they are built either as (3 quark $\otimes$ $\overline{3}$ core) or (3 quarks $\otimes$ 6 core). These decompositions are shown in Tables IV.1 and IV.2. In both tables the core is represented by $\phi(I, I_3, S)$, where the labels are the total isospin, third component of isospin and strangeness respectively. SU(3) multiplets other than $\overline{3}$ or 6 for the core can give an octet when combined with a quark, but these are not accessible from a combination of two quarks. At least one quark-antiquark pair would be required in addition to two quarks in the core. It is more difficult to put a larger number of quarks in a packet of low momentum, so we consider only the $\overline{3}$ and 6 case. This also excludes a strange quark or anti-quark from dominating near $x = 1$ in the nucleon.

We have argued from non-degeneracy and the experimental data on $f^n(x)/f^p(x)$ that $\phi^{\overline{3}}(0, 0, 0)$ dominates over $\phi^6(1, I_3, 0)$ in the case of the nucleon; that is, the amplitude that the proton looks like $u\phi^3(0, 0, 0)$ falls with a lower inverse power of the momentum P when $x = 1 - 1$ GeV/P than the amplitude for it to look like

$$\sqrt{\frac{1}{3}}\, u\phi^6(1, 0, 0) - \sqrt{\frac{2}{3}}\, d\phi^6(1, 1, 0).$$

We shall carry over the non-degeneracy assumption to the other baryons and add the following: Larger units of strangeness in the core imply a larger inverse power fall-off of the amplitude. That is, we assume that a core like $\phi^{\overline{3}}(I, I_3, \pm |S|)$ dominates over $\phi^{\overline{3}}(I', I_3', \pm (|S|+1))$ and similarly for a ϕ^6 core. This assumption is suggested by the observation that, in all SU(3) multiplets of particles, larger units of strangeness always involve higher mass. In our case we require a core of quarks of low relative momentum and we assume it is harder to have a core with higher strangeness. We do not make this comparison between members of the different core multiplets ($\phi^{\overline{3}}$ and ϕ^6), but we have deduced that $\phi^{\overline{3}}(0, 0, 0)$ dominates over $\phi^6(1, I_3, 0)$ from the nucleon data. From this and our strangeness ordering assumption we conclude that $\phi^{\overline{3}}(0, 0, 0)$ dominates over any ϕ^6 member irrespective of its strangeness.

Before we continue discussing the nucleon octet we wish to make a general assumption regarding the dependence of the inverse power fall-off γ of the amplitude for $x = 1 - 1$ GeV/P on the quantum numbers of the fast quark. Since the amplitude must depend on the difficulty of having the core quarks in a low momentum packet we have argued that γ must depend on the quantum numbers of the core. As the momentum P of the hadron gets large, so does the relative momentum of the core and the fast quark. It is possible that in this limit any interactions between the core and the fast quark become independent of the quantum numbers of the quark. This is intended to apply only in the following situation: Let us suppose we have two hadrons, h_1 and h_2, of the same SU(3) multiplet and let c_1 and c_2 be the cores which dominate as $x \to 1$ and $P \to \infty$. If it so happens that c_1 and c_2 have the same quantum numbers (except possibly for I_3) and are members of the same multiplet, we assume that $\gamma_1 = \gamma_2$ regardless of the quantum numbers of the fast quark. This is a statement of a limit in which we believe SU(3) will be exact. It is only in this limit that we make an SU(3) invariance assumption for the structure functions.

Returning to the nucleon octet, we now consider the Λ. We notice from Table IV.1 that in the 3 quark $\otimes$ 3 core decomposition the Λ has a component of the form s $\phi^{\overline{3}}(0, 0, 0)$. We have concluded that $\phi^{\overline{3}}(0, 0, 0)$ dominates over any ϕ^6, it also dominates over $\phi^{\overline{3}}(\frac{1}{2}, I_3, -1)$ because of our strangeness ordering assumption. We conclude that it is the s quark which dominates near x = 1 in the Λ particle. The core happens to be identical to that which occurs in the nucleon; we therefore conclude further that the power γ_Λ is the same as γ_N for proton and neutron.

For the Σ triplet we cannot say which type of core dominates since we are not sure how to compare $\phi^{\overline{3}}(\frac{1}{2}, I_3, -1)$ of Table IV.1 with $\phi^6(1, I_3, 0)$ of Table IV.2. We can conclude, however, that γ_Σ is necessarily larger than γ_N since we either must have a core of the form $\phi^6(1, I_3, 0)$ or have a unit of strangeness in the core.

We have the same type of difficulty with the Ξ as we had for the Σ since we do not know how to compare $\phi^{\overline{3}}(\frac{1}{2}, I_3, -1)$ with $\phi^6(\frac{1}{2}, I_3, -1)$. We can conclude, however, that γ_Ξ is larger than or equal to γ_Σ.

Summarizing our results for the nucleon octet we have found that the electromagnetic deep inelastic structure function behaves like $f(x) \propto (1 - x)^{\gamma-1}$ as $x \to 1$. The probability that a hadron of large momentum P has a quark carrying all the momentum P except for a finite amount is proportional to $P^{-\gamma}$. The powers γ for the nucleons, Λ, Σ and Ξ are ordered as follows: $\gamma_N = \gamma_\Lambda < \gamma_\Sigma \leq \gamma_\Xi$. The u quark dominates near x = 1 for the proton, the d for the neutron and the s for the Λ.

We next analyze the pseudoscalar mesons. Here there are no data, as is available for the nucleons, to suggest which quarks dominate near x = 1. We have, however, some clues. As we have noted, there is evidence for the belief that the u quark dominates near x = 1 in the proton and the d quark in the neutron. These are precisely the quarks which occur as constituents of the nucleons in the low energy three-quark model of the baryons. Using this as a guide we shall assume that it is the quark and (or) antiquark which occur in the low energy quark model description of each meson that carry almost all the momentum of the meson as $x \to 1$. This means that we have two possibilities: At large momentum near x = 1 the mesons have the form (3 quark $\otimes$ $\overline{3}$ core) or ($\overline{3}$ antiquark $\otimes$ 3 core). Another reason for making this choice is that if we allow the possibility for a quark which does not occur in the low energy quark model to dominate near x = 1, then the average number of quarks and antiquarks in the core must be at least three; whereas with the choice above it could be as low as one making it easier for the core quarks to be in a low momentum packet.

We shall discuss only the structure functions of the pions and kaons since these will be the only mesons of practical interest when we turn to quark fragmentation in the next section. The structure function of the π^+ is given in terms of its quark density functions as

$$f^{\pi^+}(x) = \frac{4}{9}(u_{\pi^+}(x) + \bar{u}_{\pi^+}(x)) + \frac{1}{9}(d_{\pi^+}(x) + \bar{d}_{\pi^+}(x)) +$$

$$+ \frac{1}{9}(s_{\pi^+}(x) + \bar{s}_{\pi^+}(x)), \qquad \text{(IV.6)}$$

with similar expressions for π^0, π^- and the kaons.

From charge conjugation and isospin invariance, relations are found between the quark density functions of the mesons (the variable x has been omitted):

$$\begin{aligned}
& u_{\pi^+} = d_{\pi^-} = \bar{d}_{\pi^+} = \bar{u}_{\pi^-} && \text{a)} \\
& d_{\pi^+} = u_{\pi^-} = \bar{u}_{\pi^+} = \bar{d}_{\pi^-} && \text{b)} \\
& s_{\pi^+} = s_{\pi^-} = \bar{s}_{\pi^+} = \bar{s}_{\pi^-} = s_{\pi^0} = \bar{s}_{\pi^0} && \text{c)} \\
& u_{\pi^0} = d_{\pi^0} = \bar{d}_{\pi^0} = \bar{u}_{\pi^0} = \frac{1}{2}(u_{\pi^+} + u_{\pi^-}) && \text{d)} \\
& u_{K^+} = d_{K^0} = \bar{d}_{\bar{K}^0} = \bar{u}_{K^-} && \text{e)} \qquad \text{(IV.7)} \\
& d_{K^+} = u_{K^0} = \bar{u}_{\bar{K}^0} = \bar{d}_{K^-} && \text{f)} \\
& s_{K^+} = s_{K^0} = \bar{s}_{\bar{K}^0} = \bar{s}_{K^-} && \text{g)} \\
& u_{K^-} = d_{\bar{K}^0} = \bar{d}_{K^0} = \bar{u}_{K^+} && \text{h)} \\
& d_{K^-} = u_{\bar{K}^0} = \bar{u}_{K^0} = \bar{d}_{K^+} && \text{i)} \\
& s_{K^-} = s_{\bar{K}^0} = \bar{s}_{K^0} = \bar{s}_{K^+} && \text{j)}
\end{aligned}$$

These relations hold for all x and in particular as $x \to 1$. Each quark density function will behave like $(1-x)^{\gamma-1}$ as $x \to 1$. According to our assumption, the quarks which will have the lowest powers for their density functions as $x \to 1$ will be:

$u, \bar{d}$	for π^+
$u, \bar{u}, d, \bar{d},$	for π^0
$\bar{u}, d$	for π^-
$u, \bar{s}$	for K^+
$d, \bar{s}$	for K^0
$\bar{u}, s$	for K^-
$\bar{d}, s$	for $\bar{K}^0$.

Relations (IV.7) a), d), e) and j) allow only three of these powers to be independent. $u_{\pi^+}(x)$, $\bar{d}_{\pi^+}(x)$, $u_{\pi^0}(x)$, $\bar{u}_{\pi^0}(x)$, $d_{\pi^0}(x)$, $\bar{d}_{\pi^0}(x)$, $d_{\pi^-}(x)$, $\bar{u}_{\pi^-}(x)$, have the power behavior $(1-x)^{\gamma_1-1}$ as $x \to 1$; $u_{K^+}(x)$, $d_{K^0}(x)$, $\bar{d}_{\bar{K}^0}(x)$, $\bar{u}_{K^-}(x)$, have the power behavior $(1-x)^{\gamma_2-1}$ as $x \to 1$ and $\bar{s}_{K^+}(x)$, $\bar{s}_{K^0}(x)$, $s_{\bar{K}^0}(x)$, $s_{K^-}(x)$ have the power behavior $(1-x)^{\gamma_3-1}$ as $x \to 1$.

In the K^+ it can be either the u of the $\bar{s}$ which dominate near $x = 1$. When the u dominates there must be one strangeness unit in the core as opposed to zero when the $\bar{s}$ dominates. From this we conclude that γ_2 is larger than γ_3. Furthermore, when the $\bar{s}$ dominates in the K^+ the core is identical to that in the π^+ when the $\bar{d}$ dominates, from which we conclude that $\gamma_1 = \gamma_3$; we use the notation $\gamma_M = \gamma_1 = \gamma_3$.

To summarize, the quark density functions with the lowest power of $(1 - x)$ as $x \to 1$ for the pions and kaons are $\bar{d}_{\pi^+}(x)$, $\bar{s}_{K^+}(x)$ and those related by charge conjugation and isospin invariance, these are:

$$\begin{aligned} &u_{\pi^+}(x) = d_{\pi^-}(x) = \bar{d}_{\pi^+}(x) = \bar{u}_{\pi^-}(x) \propto (1-x)^{\gamma_M - 1} && \text{a)} \\ &u_{\pi^0}(x)\ ,\ d_{\pi^0}(x) = \bar{d}_{\pi^0}(x) = \bar{u}_{\pi^0}(x) \propto (1-x)^{\gamma_M - 1} && \text{b)} \qquad \text{(IV.8)} \\ &\bar{s}_{K^+}(x) = \bar{s}_{K^0}(x) = s_{K^0}(x) = s_{K^-}(x) \propto (1-x)^{\gamma_M - 1}\ , && \text{c)} \end{aligned}$$

It follows immediately from the Drell-Yan relation and our conclusions above that the form factors (both electric and magnetic) of the proton, neutron and Λ fall with the same inverse power of $\sqrt{-q^2}$, and the power is given by γ_N. In the case of the pseudoscalar mesons, the Drell-Yan relation must be modified as pointed out by Ravndal.(17) This is because the meson must interact with the longitudinal virtual photon to scatter elastically, whereas its constituent charged partons have spin 1/2 and prefer to scatter (with a factor of $-q^2$ higher) from the transverse virtual photon. For this reason the pion form factor falls off like $(1/\sqrt{-q^2})^{\gamma_M + 1}$ rather than with the power γ_M as would be the case for spin 1/2 particles. From (IV.8) we have that the form factors of both pion and kaon must fall with the same power of $\sqrt{-q^2}$.

V. THE QUARK FRAGMENTATION FUNCTIONS IN THE LIMIT $z \to 1$

In Section III, $D_\alpha^h(z, k_T^2)dzdk_T^2$ was defined as the probability for the fragmentation of a parton of type α into a hadron h (and any other hadrons) with longitudinal momentum fraction z and transverse momentum k_T in a range $dzdk_T^2$. We shall not say anything about the transverse momentum distribution except that it will have a small and fixed average (~0.3 GeV) typical of hadron-hadron collisions. We shall henceforth only consider the integrated function $D_\alpha^h(z)$.

We are interested in the behavior of the function $D_\alpha^h(z)$ in the limit $z \to 1$ in the special case of the quark parton model. As we shall argue later (see Section VI), the fragmentation function $D_\alpha^h(z)dz$ behaves like dz/z for small z and cuts off to dp_z/E for low momentum. This implies that the multiplicity of hadrons resulting from the fragmentation of a quark increases logarithmically with the momentum of the quark. From this, in an entirely analogous way to the analysis of the deep inelastic structure functions, it can be shown that $D_\alpha^h(z)$ behaves like $(1 - z)^{\gamma-1}$ for $z \to 1$ and that the probability for the fragmentation of a quark α of momentum P into a hadron h which carries all the momentum of α except for a finite amount has the momentum dependence $P^{-\gamma}$.

When a quark of momentum P fragments into a hadron h of momentum P-δ (δsmall) it must project into that part of the amplitude for the hadron to have a quark of the same type carrying almost all its momentum. We therefore conclude that the power γ which appears in the quark density function of type α for the hadron h $\alpha_h(x) \propto (1-x)^{\gamma-1}$ as $x \to 1$ is the same as the power γ which appears in the fragmentation function of quark α into the hadron h $D_\alpha^h(z) \propto (1-z)^{\gamma-1}$ as $z \to 1$. All our previous results as to which quarks dominate near x = 1 in the nucleon octet and the pseudoscalar mesons can now be translated into statements as to which of these particles are produced more copiously from the fragmentation of quarks near z = 1. For the proton, neutron and Λ, we have the following results:

$$D_u^p(z) \propto (1-z)^{\gamma_N-1} \qquad a)$$

$$D_d^n(z) \propto (1-z)^{\gamma_N-1} \qquad b) \qquad (V.1)$$

$$D_s^\Lambda(z) \propto (1-z)^{\gamma_N-1} \qquad c) .$$

We do not know which quarks dominate in the case of the Σ and Ξ, but we have concluded that regardless of which quarks dominate, their powers γ_Σ and γ_Ξ must be greater than γ_N and $\gamma_\Sigma \leq \gamma_\Xi$. All quark fragmentation functions other than those in (V.1) for the production of p, n, and Λ behave like powers of $(1 - z)$ as $z \to 1$ which are larger than γ_N -1. From this we conclude that the ratio of protons to any other particle in the nucleon octet produced from the fragmentation of a u quark diverges as an inverse power of (1- z) as $z \to 1$. The same statement can be made regarding neutrons produced from d quarks and

Λ's produced from s quarks.

The quark fragmentation functions for the pseudoscalar mesons are subject to relations similar to (IV.7) from charge conjugation and isospin invariance. This and our results as to which quarks dominate near x = 1 in the quark density functions can be translated into statements on the limit as $z \to 1$ of the quark fragmentation functions; those with the lowest power of (1 - z) as $z \to 1$ are:

$$D_u^{\pi^+}(z) = D_d^{\pi^-}(z) = D_{\bar{d}}^{\pi^+}(z) = D_{\bar{u}}^{\pi^-}(z) \propto (1-z)^{\gamma_M - 1} \qquad a)$$

$$D_u^{\pi^0}(z) = D_d^{\pi^0}(z) = D_{\bar{d}}^{\pi^0}(z) = D_{\bar{u}}^{\pi^0}(z) \propto (1-z)^{\gamma_M - 1} \qquad b) \qquad (V.2)$$

$$D_{\bar{s}}^{K^+}(z) = D_{\bar{s}}^{K^0}(z) = D_s^{\bar{K}^0}(z) = D_s^{K^-}(z) \propto (1-z)^{\gamma_M - 1} \qquad c) \quad .$$

The power γ_M is the same as that appearing in (IV.8). All other quark fragmentation functions into pions or kaons not appearing in (V.2) behave like $(1 - z)^{\gamma - 1}$ as $z \to 1$ with $\gamma > \gamma_M$.

An interesting application of the above results lies in the possibility of measuring the quark density functions of the proton and neutron for all x from inclusive electroproduction experiments in the Bjorken limit. As discussed in Section III, the function

$$f_p^h(x,z) = \frac{4}{9}\, u(x)D_u^h(z) + \frac{4}{9}\, \bar{u}(x)D_{\bar{u}}^h(z) + \frac{1}{9}\, d(x)D_d^h(z) +$$

$$+ \frac{1}{9}\, \bar{d}(x)D_{\bar{d}}^h(z) + \frac{1}{9}\, s(x)D_s^h(z) + \frac{1}{9}\, \bar{s}(x)D_{\bar{s}}^h(z) \qquad (V.3)$$

is measurable. The subindex p on $f_p^h(x,z)$ indicates that the target is the proton. When the target is the neutron

$$f_n^h(x,z) = \frac{4}{9}\, d(x)D_u^h(z) + \frac{4}{9}\, \bar{d}(x)D_{\bar{u}}^h(z) + \frac{1}{9}\, u(x)D_d^h(z) +$$

$$+ \frac{1}{9}\, \bar{u}(x)D_{\bar{d}}^h(z) + \frac{1}{9}\, s(x)D_s^h(z) + \frac{1}{9}\, \bar{s}(x)D_{\bar{s}}^h(z) \; . \qquad \text{(V.4)}$$

We have defined the quark density functions $u(x)$, $d(x)$, ..., etc., without subindices when they refer to the proton. By isospin invariance, the density of u quarks in the neutron is equal to the density of d quarks in the proton. The first term on the right-hand side of (V.4) is therefore $\frac{4}{9}\, d(x)D_u^h(z)$; it is the product of the probability that a u quark in the neutron interacts with a virtual photon $(\frac{4}{9}\, d(x))$ and the probability of fragmentation of the u quark into the hadron h. As we go to the limit $z \rightarrow 1$ there are only one or two terms contributing to (V.3) or (V.4) for certain hadrons h; this permits us to extract the quark density functions.

Possibly the easiest way to obtain the nucleon quark density functions using this method is to measure only charged pions and kaons. Using the proton as a target and going to the limit $z \rightarrow 1$, we have:

$$
\begin{aligned}
f_p^{\pi^+}(x,z) &= \tfrac{4}{9}\, u(x)\beta(1-z)^{\gamma_M-1} + \tfrac{1}{9}\, \bar{d}(x)\beta(1-z)^{\gamma_M-1} && \text{a)}\\
f_p^{\pi^-}(x,z) &= \tfrac{4}{9}\, \bar{u}(x)\beta(1-z)^{\gamma_M-1} + \tfrac{1}{9}\, d(x)\beta(1-z)^{\gamma_M-1} && \text{b)}\\
f_p^{K^+}(x,z) &= \tfrac{1}{9}\, \bar{s}(x)\delta(1-z)^{\gamma_M-1} && \text{c)}\\
f_p^{K^-}(x,z) &= \tfrac{1}{9}\, s(x)\delta(1-z)^{\gamma_M-1} && \text{d)}
\end{aligned}
\qquad (V.5)
$$

Only u and $\bar{d}$ contribute to π^+, $\bar{u}$ and d to π^-, $\bar{s}$ to K^+ and s to K^-. β and δ are unknown constants. We know the shape of $D_\alpha^h(z)$ as $z \to 1$ but not the absolute normalization; there are relations, however, from charge conjugation and isospin invariance which reduce the number of constants. The functions on the left-hand side of (V.5) all behave like $(1-z)^{\gamma_M-1}$, the z dependence can therefore be factored out. Defining $f_p^{\pi,K}(x) = f_p^{\pi,K}(x,z)/(1-z)^{\gamma_M-1}$ we have the simpler relations

$$
\begin{aligned}
f_p^{\pi^+}(x) &= \tfrac{4}{9}\,\beta\, u(x) + \tfrac{1}{9}\,\beta\, \bar{d}(x) && \text{a)}\\
f_p^{\pi^-}(x) &= \tfrac{4}{9}\,\beta\, \bar{u}(x) + \tfrac{1}{9}\,\beta\, d(x) && \text{b)}\\
f_p^{K^+}(x) &= \tfrac{1}{9}\,\delta\, \bar{s}(x) && \text{c)}\\
f_p^{K^-}(x) &= \tfrac{1}{9}\,\delta\, s(x) && \text{d)}\ .
\end{aligned}
\qquad (V.6)
$$

We see that we can determine the functions $\bar{s}(x)$ up to a multiplicative constant from making measurements of K^+ and K^- on a proton target only.

To measure the other quark density functions a neutron target must be used. Defining analogous quantities $f_n^{\pi,K}(x) = f_n^{\pi,K}(x,z)/(1-z)^{\gamma_M - 1}$ for the neutron, we have the relations

$$
\begin{aligned}
f_n^{\pi^+}(x) &= \frac{4}{9}\beta\, d(x) + \frac{1}{9}\beta\, \bar{u}(x) && \text{a)}\\
f_n^{\pi^-}(x) &= \frac{4}{9}\beta\, \bar{d}(x) + \frac{1}{9}\beta\, u(x) && \text{b)}\\
f_n^{K^+}(x) &= \frac{1}{9}\delta\, \bar{s}(x) && \text{c)}\\
f_n^{K^-}(x) &= \frac{1}{9}\delta\, s(x) && \text{d)}
\end{aligned}
\qquad (\text{V.7})
$$

Equations (V.5) a), b) and (V.7) a), b) can easily be solved to obtain $u(x)$, $\bar{u}(x)$, $d(x)$ and $\bar{d}(x)$:

$$
\begin{aligned}
u(x) &= \frac{5}{12\beta}\left[\, f_p^{\pi^+}(x) - \frac{1}{4} f_n^{\pi^-}(x) \,\right] && \text{a)}\\
\bar{u}(x) &= \frac{5}{12\beta}\left[\, f_p^{\pi^-}(x) - \frac{1}{4} f_n^{\pi^+}(x) \,\right] && \text{b)}\\
d(x) &= \frac{5}{12\beta}\left[\, f_n^{\pi^+}(x) - \frac{1}{4} f_p^{\pi^-}(x) \,\right] && \text{c)}\\
\bar{d}(x) &= \frac{5}{12\beta}\left[\, f_n^{\pi^-}(x) - \frac{1}{4} f_p^{\pi^+}(x) \,\right] && \text{d)}
\end{aligned}
\qquad (\text{V.8})
$$

Hence, these quark density functions can be measured up to the same multiplicative constant. A way to obtain the absolute normalization is to substitute into the scaling function of the proton

$$f(x) = \frac{4}{9}\,(u(x) + \bar{u}(x)) + \frac{1}{9}\,(d(x) + \bar{d}(x)) + \frac{1}{9}\,(s(x) + \bar{s}(x))$$

which has been measured.

Another way to obtain the nucleon quark density functions is to measure p, n, Λ and their antiparticles near $z = 1$. This is impractical experimentally and is only mentioned briefly; much higher energies are required than in the case of the mesons. Since protons near $z = 1$ are produced only from u quarks, neutrons from d quarks, Λ's from s quarks and their antiparticles from the corresponding antiquarks, we have only one term on the right-hand side of (V.3) as $z \to 1$. Using only the proton as a target we have

$$\begin{aligned}
f_p^p(x,z) &= \frac{4}{9}\,u(x)\zeta(1-z)^{\gamma_N-1} && \text{a)}\\
f_p^n(x,z) &= \frac{1}{9}\,d(x)\zeta(1-z)^{\gamma_N-1} && \text{b)}\\
f_p^\Lambda(x,z) &= \frac{1}{9}\,s(x)\eta(1-z)^{\gamma_N-1} && \text{c)}\\
f_p^{\bar{p}}(x,z) &= \frac{4}{9}\,\bar{u}(x)\zeta(1-z)^{\gamma_N-1} && \text{d)}\\
f_p^{\bar{n}}(x,z) &= \frac{1}{9}\,\bar{d}(x)\zeta(1-z)^{\gamma_N-1} && \text{e)}\\
f_p^{\bar{\Lambda}}(x,z) &= \frac{1}{9}\,\bar{s}(x)\eta(1-z)^{\gamma_N-1} && \text{f)}
\end{aligned} \qquad \text{(V.9)}$$

The quark density functions can be obtained up to the two unknown constants ζ and η. These constants are, as before, determined by substituting into the scaling function of the proton.

VI. ELECTRON-POSITRON INCLUSIVE PRODUCTION OF HADRONS

We wish to describe the process

$$e^- + e^+ \to h + \text{Anything} \tag{VI.1}$$

in which one final hadron is observed in the collision of an electron and a positron where an unrestricted number of other hadrons may be produced. To lowest order in the electromagnetic coupling constant the process is described by one photon exchange as illustrated in Figure VI.1. We define

$$\overline{W}_{\mu\nu}(q^2,\nu)=\frac{1}{2\pi}\int d^4x\; e^{iq\cdot x} \sum_n < 0|J_\mu(x)|h,n > < h,n|J_\nu(0)|0 > \tag{VI.2}$$

There are two structure functions as indeep inelastic scattering:

$$\overline{W}_{\mu\nu}(q^2,\nu) = -\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\overline{W}_1(q^2,\nu) +$$

$$+ \frac{1}{M^2}\left(h_\mu - \frac{h\cdot q}{q^2}\, q_\mu\right)\left(h_\nu - \frac{h\cdot q}{q^2}\, q_\nu\right)\overline{W}_2(q^2,\nu) \tag{VI.3}$$

the four-momentum of the hadron h is h_μ, q_μ is the four-momentum of the virtual photon. The invariants used are q^2 and ν where $M\nu = h\cdot q$, M is the mass of h.

The differential cross section is given by

$$\frac{d^2\sigma}{dE d\cos\theta} = \frac{4\pi\alpha^2}{(q^2)^2} \frac{M^2\nu}{(q^2)^{\frac{1}{2}}} \left(1 - \frac{q^2}{\nu^2}\right)^{\frac{1}{2}} \times$$

$$\times \left[2\overline{W}_1(q^2,\nu) + \frac{2M\nu}{q^2}\left(1 - \frac{q^2}{\nu^2}\right) \frac{\nu\overline{W}_2(q^2,\nu)}{2M} \sin^2\theta\right] \tag{VI.4}$$

E is the energy of h in the center of mass and θ is the angle of the momentum of h with respect to the direction of the incident particles in the same frame.

We define the Bjorken limit, in analogy to the case in electroproduction, as the limit in which $q^2 \to \infty$, $\nu \to \infty$ with their ratio fixed. The invariant $2M\nu/q^2$ is the ratio of the energy of h to the energy of the incident electron in the center of mass; this ratio is also the ratio of the momenta of h and e^- in the center of mass in the Bjorken limit.

Drell, Levy and Yan,(18) using a parton model they proposed,(19) in which the elementary fields are pions, nucleons and antinucleons with a transverse momentum cut-off, showed that in the Bjorken limit the structure functions $\overline{W}_1$ and $\nu\overline{W}_2$ become a function of the ratio $2M\nu/q^2$ only. Light cone analyses have been made with similar results.(14,20,20) In our view of the parton model, this scaling relation is a result of the assumption of limiting parton fragmentation. The time-like virtual photon produces a parton and an antiparton moving in opposite directions in the center of mass. These partons fragment into hadrons producing two jets of particles of low

(~0.3 GeV) transverse momentum relative to the direction of motion of the partons. The probability of the production of a hadron with a fixed ratio of its longitudinal momentum to the momentum of the parton or antiparton goes to a constant as $q^2 \to \infty$. If charged partons have spin 1/2 and we have limiting fragmentation, the following relations for the structure functions can be derived in the Bjorken limit:

$$2M\overline{W}_1^h\,(q^2,\nu) = \frac{G^h(z)}{z} \qquad \text{a)}$$

$$\nu\overline{W}_2^h\,(q^2,\nu) = -\,\frac{G^h(z)}{z^2} \qquad \text{b)} \qquad \text{(VI.5)}$$

where $z = 2M\nu/q^2$. The function $G^h(z)$ is given in terms of the parton fragmentation functions $D_\alpha^h(z)$, defined in Section III, by

$$G^h(z) = \sum_\alpha Q_\alpha^2\, D_\alpha^2\,(z) \quad . \qquad \text{(VI.6)}$$

The sum runs over partons and antipartons of all types; Q_α is the charge of the parton of type α measured in units of the electron charge.

More than scaling of the structure functions is implied by the fragmentation model. If electron-positron annihilation experiments fail to see hadron jets at sufficiently high energy, the model would be invalidated.

The fragmentation functions $D_\alpha^h(z)$ are the same as those which appear in (III.3) for the inclusive structure function in electroproduction. In the quark parton model the $D_\alpha^h(z)$ could in principle be measured in neutrino and antineutrino deep inelastic inclusive electroproduction of hadrons. These could be used to predict the outcome of other experiments (as $G^h(z)$ above), but the low weak coupling constant makes this impractical.

We may obtain various interesting results applying what we have concluded in Section V about the behavior of the quark fragmentation functions in the limit $z \to 1$. Relations (V.2) imply that the ratio of the number of π^+'s to the number of K^+'s produced at any given angle in reaction (VI.1) goes to a constant independent of z as $z \to 1$, that is

$$\frac{G^{\pi^+}(z)}{G^{K^+}(z)} \to \text{constant} \qquad z \to 1 \ . \tag{VI.7}$$

This ratio could in principle be a function of z everywhere. From (V.2) the same relation holds for any type of pion or kaon with possibly different constants. If the numbers β and δ in (V.5) were known from inclusive electroproduction these constants could be obtained in terms of the charges of the quarks. From (V.1) and (VI.6) we conclude that the ratio of protons to neutrons and the ratio of protons to Λ's produced in reaction (VI.1) go to a constant as $z \to 1$. From isospin invariance $D_u^p(z) = D_d^n(z)$; this implies that the constant in the ratio of protons to neutrons is 4 near $z = 1$:

$$\frac{G^p(z)}{G^n(z)} \to 4 \qquad \text{as } z \to 1. \tag{VI.8}$$

Protons are produced only from u quarks and neutrons from d quarks near $z = 1$, since $u\,\bar{u}$ pairs are produced four times as often as $d\,\bar{d}$ pairs from time-like photons we have the relation above. We cannot obtain the constant for the ratio of protons to Λ's since we do not know the relative normalizations in (V.2) a) and (V.2) c).

As a last point we shall discuss the behavior of the quark fragmentation functions $D_\alpha^h(z)$ when z is small. We shall rely on an argument given by Feynman[1] in suggesting that the hadrons produced in a hadron-hadron interaction have a distribution dp_z/E for small values of p_z/P; p_z and E are the z component of momentum and energy of the inclusively produced outgoing hadron, and P is the incident momentum in the center of mass. The incident particles are moving along the z axis. The cross section for exclusive reactions in which a quantum number coupled to hadrons must be exchanged are known to fall as an inverse power of the center of mass energy. An example of such a reaction is $\pi^- + p \rightarrow \pi^0 + n$, in which the third component of isospin changes rapidly from -3/2 units moving in the direction of the π^- to + 1/2 unit moving in the direction of the π^0 (when the π^0 goes in the forward direction). It is argued that the cross section for this reaction must fall with energy because as energy increases the probability of changing the isospin current without radiating other hadrons decreases. Becuase of the rapid change in the isospin currents, the radiated hadrons have a sharp distribution in coordinate space in the z direction. By Fourier transform, the energy is uniform in p_z; if the energy is distributed in fixed ratios among the different types of particles then each kind of particle will have a distribution dp_z/E if p_z/P is small. An entirely analogous argument can be made in the case of electron-positron annihilation into hadrons. In the quark parton model the virtual photon creates a quark-antiquark pair which fragments into hadrons. When a $u\,\bar{u}$ or $d\,\bar{d}$ pair is created, the isospin current changes from zero to one unit; when an $s\,\bar{s}$ pair is created, it is the strangeness current that changes. We shall assume that this rapid change in currents which are coupled to hadrons produces a distribution of the form dp_z/E argued for above. This implies a distribution dz/z for the quark fragmentation function $D_\alpha^h(z)dz$ for small z This argument applies also to any parton model in which the partons carry quantum numbers coupled to hadrons.

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FIGURES

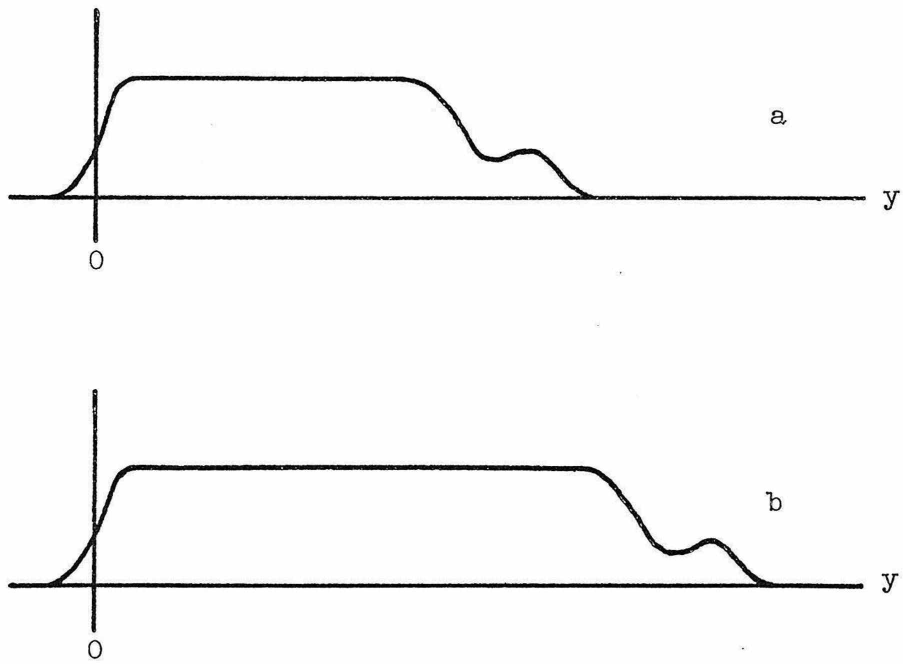

Figure II.1 a) The distribution of partons in rapidity for a hadron of momentum P. b) The same distribution for momentum P' greater than P. The point y = 0 corresponds to a parton at rest in the frame of the observer.

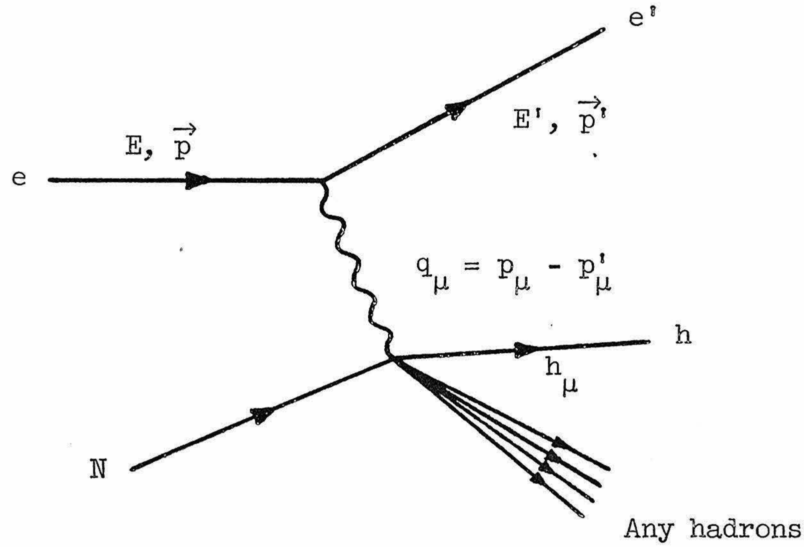

Figure III.1 The process $e + N \to e' + h +$ Anything via one photon exchange.

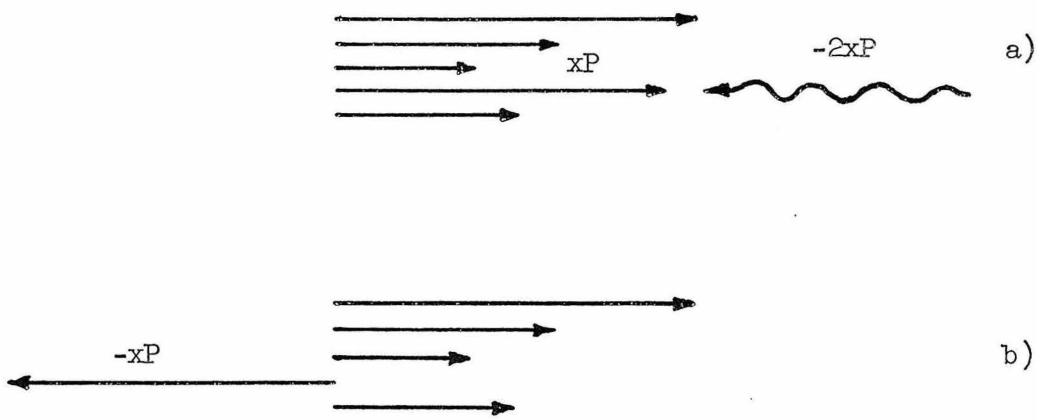

Figure III.2 Parton distributions: a) before interaction with virtual photon; b) after interaction.

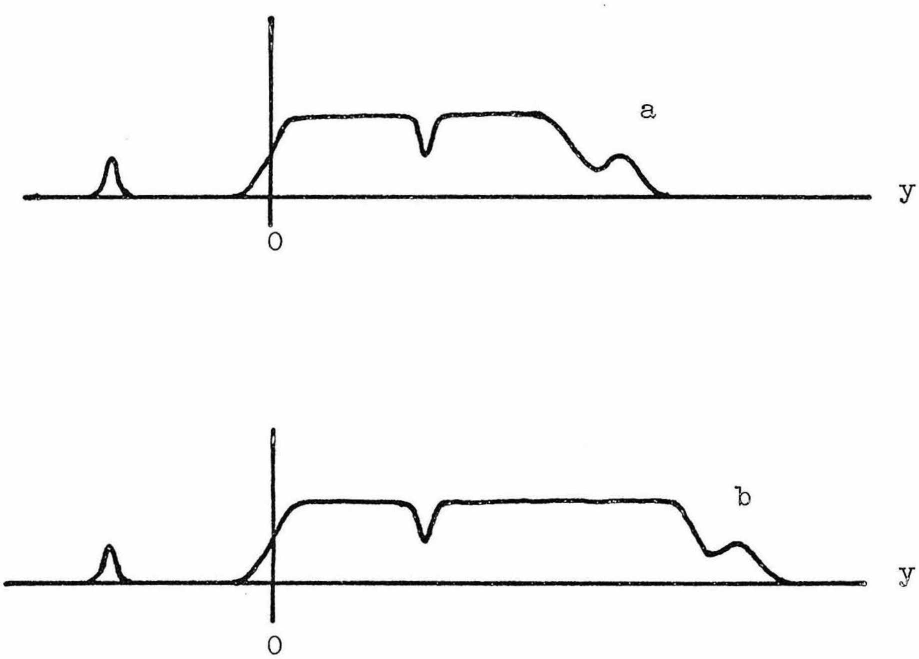

Figure III.3 Parton distributions in the rapidity variable y after the collision of a fixed q^2 virtual photon with ν higher in b) than in a). The parton distribution in the neighborhood of $y = 0$ remains fixed as $\nu \to \infty$.

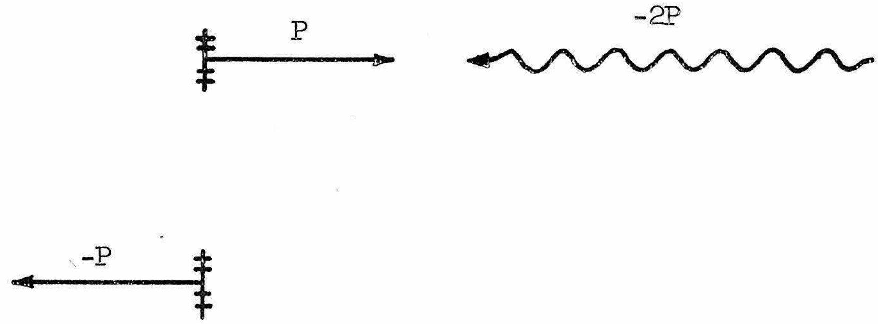

Figure IV.1 Elastic scattering of a proton from a virtual photon. In the parton model the proton must be in a configuration where one parton is carrying almost all its momentum P with a low momentum core, before and after the reaction.

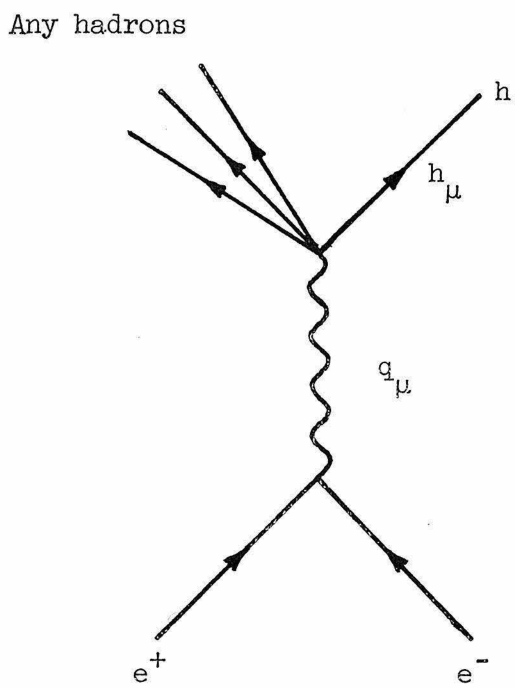

Figure VI.1 One photon exchange diagram for the process $e^+ + e^- \rightarrow h + \text{Anything}$.

TABLES

$$p = u\,\phi^{\bar{3}}(0,\ 0,\ 0)$$

$$n = d\,\phi^{\bar{3}}(0,\ 0,\ 0)$$

$$\Lambda = \sqrt{\tfrac{1}{6}}\,[u\,\phi^{\bar{3}}(\tfrac{1}{2},\ -\tfrac{1}{2},\ -1) - d\,\phi^{\bar{3}}(\tfrac{1}{2},\ \tfrac{1}{2},\ -1)] - \sqrt{\tfrac{2}{3}}\,s\,\phi^{\bar{3}}(0,\ 0,\ 0)$$

$$\Sigma^{+} = u\,\phi^{\bar{3}}(\tfrac{1}{2},\ \tfrac{1}{2},\ -1)$$

$$\Sigma^{0} = \sqrt{\tfrac{1}{2}}\,[u\,\phi^{\bar{3}}(\tfrac{1}{2},\ -\tfrac{1}{2},\ -1) + d\,\phi^{\bar{3}}(\tfrac{1}{2},\ \tfrac{1}{2},\ -1)]$$

$$\Sigma^{-} = d\,\phi^{\bar{3}}(\tfrac{1}{2},\ -\tfrac{1}{2},\ -1)$$

$$\Xi^{0} = s\,\phi^{\bar{3}}(\tfrac{1}{2},\ \tfrac{1}{2},\ -1)$$

$$\Xi^{-} = s\,\phi^{\bar{3}}(\tfrac{1}{2},\ -\tfrac{1}{2},\ -1)$$

Table IV.1. Decomposition of the nucleon octet at large momentum and $x \to 1$ as a quark and a core $\phi^{\bar{3}}(I,\ I_3,\ S)$ which is a member of a $\bar{3}$ representation of SU(3).

$$p = \sqrt{\tfrac{1}{3}}\, u\, \phi^6(1, 0, 0) - \sqrt{\tfrac{2}{3}}\, d\, \phi^6(1, 1, 0)$$

$$n = -\sqrt{\tfrac{1}{3}}\, d\, \phi^6(1, 0, 0) + \sqrt{\tfrac{2}{3}}\, u\, \phi^6(1, -1, 0)$$

$$\Lambda = \sqrt{\tfrac{1}{2}}\, u\, \phi^6(\tfrac{1}{2}, -\tfrac{1}{2}, -1) - \sqrt{\tfrac{1}{2}}\, d\, \phi^6(\tfrac{1}{2}, \tfrac{1}{2}, -1)$$

$$\Sigma^+ = \sqrt{\tfrac{1}{3}}\, u\, \phi^6(\tfrac{1}{2}, \tfrac{1}{2}, -1) - \sqrt{\tfrac{2}{3}}\, s\, \phi^6(1, 1, 0)$$

$$\Sigma^0 = \sqrt{\tfrac{1}{6}}\, [u\, \phi^6(\tfrac{1}{2}, -\tfrac{1}{2}, -1) + d\, \phi^6(\tfrac{1}{2}, \tfrac{1}{2}, -1)] - \sqrt{\tfrac{2}{3}}\, s\, \phi^6(1, 0, 0)$$

$$\Sigma^- = -\sqrt{\tfrac{1}{3}}\, d\, \phi^6(\tfrac{1}{2}, -\tfrac{1}{2}, -1) + \sqrt{\tfrac{2}{3}}\, s\, \phi^6(1, -1, 0)$$

$$\Xi^0 = \sqrt{\tfrac{2}{3}}\, u\, \phi^6(0, 0, -2) - \sqrt{\tfrac{1}{3}}\, s\, \phi^6(\tfrac{1}{2}, \tfrac{1}{2}, -1)$$

$$\Xi^- = -\sqrt{\tfrac{2}{3}}\, d\, \phi^6(0, 0, -2) + \sqrt{\tfrac{1}{3}}\, s\, \phi^6(\tfrac{1}{2}, -\tfrac{1}{2}, -1)$$

Table IV.2. Decomposition of the nucleon octet at large momentum and $x \to 1$ as a quark and a core $\phi^6(I, I_3, S)$ which is a member of a 6 representation of SU(3).