Elimination of Parity Doubled States
from Regge Amplitudes

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ABSTRACT

The common belief that fermions lying on linear Regge trajectories must have opposite-parity partners is shown to be false. The mechanism by which these experimentally nonexistant states are eliminated from the theory depends on the presence of fixed Regge cuts in fermion exchange amplitudes. Thus it is predicted that fermion Regge trajectories are always accompanied by fixed Regge cuts. More generally, if particles may be classified as composites of spin-1/2 (fermion) quarks, fixed cuts are expected to be present in boson exchange amplitudes as well. This result is demonstrated in the framework of the Van Hove model and a few further experimental consequences are discussed.

CONTENTS

PART I

INTRODUCTION

In Regge pole theory there has been a long standing difficulty in the treatment of half-integral spin particles (fermions). Theoretical arguments utilizing the experimental fact that trajectory functions are approximately linear in the $(\text{energy})^2$ predict that fermion states should occur in approximately degenerate parity doublets. There is, however, no convincing example of a parity partner for any of the known fermion states.

The principal result of this thesis is the demonstration that linear fermion trajectories need not have degenerate opposite parity partners. In order that this be the case it is necessary that each fermion trajectory be accompanied by a fixed Regge cut of a specific type. Using the Van Hove model we show how such cuts arise in a natural way. We conclude that fixed cuts should be present in all fermion exchange processes.

In the quark model particles are classified as composites of spin 1/2 (fermion) quarks. In this model the exchange of any particle is equivalent to the exchange of a set of quarks. If in Regge theory any fermion exchange is accompanied by a fixed Regge cut, the exchange of a set of fermion quarks should also require the presence of a cut. This is indeed the case, and again the cuts play the role of eliminating extraneous states. Were the quarks allowed to be parity doublets, three-quark states would be parity doublets and quark-antiquark states would be simple doublets (that is, pairs of states of the same parity).

In Part II we demonstrate the existence of fixed cuts in fermion exchange reactions. Properties of the cuts are discussed and a few experimental correlations are noted. Part III elaborates on several points raised in Part II. We review the experimental situation regarding fermion parity doublets, and give a detailed discussion of the fixed cuts which are present in the solution of the Dirac equation with a Coulomb potential.

Part IV discusses the origin of fixed cuts in the quark model. First we show that the quark model implies the vertex symmetry $SU(6)_W$; then we show how a Regge amplitude constructed from $SU(6)_W$-symmetric vertices must contain fixed Regge cuts. A detailed discussion of the experimental consequences of these cuts is given along with a comparison with qualitative features in vector meson exchange data.

PART II

FERMION REGGEIZATION WITHOUT PARITY DOUBLING

Gribov[1] showed that every fermion Regge trajectory $\alpha^{+}(W)$ must be accompanied by a MacDowell symmetric[2] trajectory $\alpha^{-}(W) = \alpha^{+}(-W)$ of the opposite parity. If (as is indicated by experiment for N_α and Δ_δ) a trajectory is linear in $u = W^2$, its MacDowell twin will be degenerate with it. Hence it has always seemed puzzling that no parity partners of the N and $\Delta(1238)$ have been found. Attempts to find an analytic form in which states on the MacDowell twin are systematically suppressed have not been successful.[3] We deduce the appropriate analytic form from a model containing only resonances of positive parity lying on a linear trajectory. The partial-wave amplitudes are found to have a fixed Regge cut, and the negative-parity (MacDowell twin) trajectory lies on an unphysical sheet of the J plane at positive energies. The idea of a fixed Regge cut is not new; it is present in the solution of the Dirac equation with a Coulomb potential.[4] In the present problem it is, of course, possible to have parity doubling and no Regge cut; but lacking any a priori reason for parity doubling, we anticipate in general the presence of a fixed Regge cut in fermion-exchange amplitudes.

We will illustrate the origin of the fixed cut in the Van Hove model.[5] The amplitude in this model is the sum of Feynman diagrams for the exchange of all resonances along a given trajectory. Clearly, this amplitude satisfies the usual analyticity requirements and contains only the resonances of the input trajectory. In πN scattering, the Feynman diagram for the exchange of a natural-parity ($J^P = 1/2^+, 3/2^-, 5/2^+, \ldots$) fermion resonance of spin $J = \ell + 1/2$ and mass $m(\ell)$ in the u channel[6,7] is

$$\bar{u}_2 \mathcal{M}(J)\, u_1 = \bar{u}_2\, \gamma_5\, g^2(\ell)\, p'_{\mu_1} \ldots p'_{\mu_\ell}\, T^J_{\mu_1 \ldots \mu_\ell ; \nu_1 \ldots \nu_\ell}\, p_{\nu_1} \ldots p_{\nu_\ell}\, \gamma_5\, u_1$$

$$= \bar{u}_2\, \frac{g^2(\ell)\, p^{2\ell}\, P'_{\ell+1}(z_u)}{u - m^2(\ell)} \left[\frac{\not{k}}{m(\ell)} - 1\right] u_1 + O\left(z_u^{\ell-1}\right) , \quad (1)$$

where $T^J_{\mu,\nu}$ is the propagator for a spin-J fermion. We Reggeize by summing a sequence of resonances and transforming the sum into an integral a la Sommerfeld and Watson[8]:

$$\mathcal{M} = \sum_J \mathcal{M}(J) \simeq \frac{i}{2} \int d\ell\, \frac{g^2(\ell)\, p^{2\ell}\, P'_{\ell+1}(-z_u)}{(u - m^2(\ell)) \sin \pi\ell} \left[\frac{\not{k}}{m(\ell)} - 1\right] . \quad (2)$$

All terms but those contributing to the leading power of the asymptotic expansion of $\mathcal{M}(u,z_u)$ as $z_u \to \infty$ have been dropped.

If we take $m^2(\ell) = (\ell - \alpha_o)/\alpha'$ and assume for convenience that $g^2(\ell)$ is analytic in ℓ,[9] we can open the contour in the ℓ plane and obtain a contribution from the pole at $m^2(\ell) = u$ and the cut with branch point at $\ell = \alpha_o$ (see Fig. 1). This gives

$$\mathcal{M}(u,z_u) = \frac{\pi\, g^2(\alpha(u))\, p^{2\alpha(u)}\, P'_{\alpha(u)+1}(-z_u)\, \alpha'}{\sin \pi\, \alpha(u)} \left[\frac{\not{k} - W}{W}\right] \quad (3)$$

$$- \not{k} \int_{-\infty}^{\alpha_o} d\ell\, \frac{g^2(\ell)\, p^{2\ell}\, P'_{\ell+1}(-z_u)}{\sqrt{-m^2(\ell)}\, (u - m^2(\ell)) \sin\pi\ell} ,$$

where $$\alpha(u) = \alpha_o + \alpha' u \quad . \tag{4}$$

In the limit $z_u \to \infty$,

$$\mathcal{M}(u,z_u) \to \frac{\pi\, g^2(\alpha(u))\, c(\alpha(u))\, s^{\alpha(u)}\, \alpha'}{\sin \pi\, \alpha(u)} \left[\frac{\not{k}-W}{W}\right]$$

$$-\not{k} \int_{-\infty}^{\alpha_o} d\ell \; \frac{g^2(\ell)\; c(\ell)\; s^\ell}{\sqrt{-m^2(\ell)}\;(u - m^2(\ell))\; \sin \pi\ell} \tag{5}$$

where $$c(\ell) = \frac{\Gamma(2\ell+2)}{4^\ell\;(\Gamma(\ell+1))^2} \quad . \tag{6}$$

The first term of (5) has the form of a Regge pole contribution while the second term has the form of a Regge cut. The singularity at u=0 in the residue of the pole term is cancelled by the cut term, so that $\mathcal{M}$ is analytic in u (see Eq. (11)).

The principal features of our solution can be seen in the partial-wave amplitudes $f^{\mp}_{J\pm1/2}(W)$,[2] which can be read off directly from (1). We find that

$$f^{\mp}_{J\pm1/2} = -\frac{E\mp M}{8\pi W}\left(\frac{\sqrt{\alpha' u} \pm \sqrt{\ell-\alpha_o}}{\ell - \alpha_o - \alpha' u}\right)\frac{\alpha'}{\sqrt{\ell-\alpha_o}}\; p^{2\ell}\, g^2(\ell) \quad , \tag{7}$$

where $\ell = J - 1/2$. There is a moving pole at $\ell = \alpha(u)$ and a fixed cut at $\ell = \alpha_o$. Note that the moving pole in $f^{+}_{J-1/2}$ is on the physical sheet of the ℓ plane only for $W < 0$. As we move from $W < 0$ to $W > 0$, the pole at $\ell = \alpha(W^2)$ moves through the fixed cut onto the second sheet of the ℓ plane as shown in Fig. 2; this explains why there are no negative-parity resonances in our model.

The cut $(\ell-\alpha_o)^{-1/2}$ in (7) is due to the presence of odd powers of $m(\ell)$ in (1). We can obtain a solution with no Regge cut only if we include negative-parity states along with the positive-parity ones. This would correspond the the usual solution; but it clearly involves the ad hoc assumption that the negative-parity states exist.

Unless $g^2(\alpha_o) = 0$, the partial-wave amplitudes (7) have infinities for $\ell \to \alpha_o$, in violation of unitarity. Of course, our model has zero-width resonances, so it is clearly not unitary. We wish to demonstrate that there exists a smooth limit from the unitarized theory to the zero-width limit, and that this limit should be useful in parametrizing experimental data. The procedure for unitarizing the model has been discussed by Sugar and Sullivan,[11] who find that certain fixed poles are converted into moving poles in the process.

In our case[12] the unitarized partial-wave amplitudes have the form

$$f^{\mp}_{J\pm1/2} = \frac{E\pm M}{8\pi W} \frac{p^{2\ell} g^2(\ell)}{\left[m(\ell)-g^2(\ell)a(u)\right] W - \left[m^2(\ell)+g^2(\ell)b(u)\right]} , \tag{8}$$

where a(u) and b(u) are functions with the proper right-hand cuts in u. The amplitudes $f^{\mp}_{J\pm1/2}$ have a fixed cut at $\ell = \alpha_o$ and only moving poles. In particular if $g^2(\ell) = c$, a constant, $f^{\mp}_{J\pm1/2}$ have two moving poles,[13] with trajectories $\alpha_{1,2}$ [14] given by

$$2m(\alpha_{1,2}(u)) = W \pm\sqrt{W^2 -4cWa(u) -4cb(u)} . \tag{9}$$

In the limit $a,b \to 0$, $\alpha_1 \to \alpha(u)$ (the input positive-parity trajectory) and $\alpha_2 \to \alpha_o$ (a fixed pole). Thus we can interpret the factor $(\ell - \alpha_o)^{-1/2}$ in (7) as the coincidence of a fixed cut, $(\ell - \alpha_o)^{1/2}$, and a fixed pole, $(\ell - \alpha_o)^{-1}$. When the model is unitarized, the fixed pole becomes a moving pole just as in Ref. 11. Although the pole at $\ell = \alpha_2(u)$ does contribute to the asymptotic scattering amplitude, as long as a and b are small the trajectory $\alpha_2(u)$ will never rise high enough to produce any physical resonances.

Unless a(u) and b(u) are small, the unitarized trajectory will deviate from the linear form (4). Since experiment indicates approximately linear trajectories, we conclude that a and b may be neglected and data may be parametrized using (5).

For small negative u, we make the approximation

$$\frac{g^2(\alpha(u))\ c(\alpha(u))\ \alpha'}{\sin \pi\alpha(u)} \simeq G_o + G_1 u \quad . \tag{10}$$

Then (5) becomes

$$M \simeq \pi \left(\frac{G_o}{W} + G_1 W\right) s^{\alpha_o + \alpha' u} (\not{k} - W)$$

$$- \pi \left[\left(\frac{G_o}{W} + G_1 W\right) s^{\alpha' u} \left(1 - \mathrm{erf}\left(\sqrt{\alpha' u \ln s}\right)\right) - \frac{G_1}{\sqrt{\alpha' \pi \ln s}}\right] s^{\alpha_o} \not{k} \quad . \tag{11}$$

The first term is clearly a Regge pole, and the second is a fixed Regge cut, since

$$\mathrm{erf}\ (x) \xrightarrow[|x| \to \infty]{} 1 - \frac{1}{\sqrt{\pi}} e^{-x^2} \left[\frac{1}{x} - \frac{1}{2x^3} + \ldots\right] \qquad |\arg x| < 3\pi/4 \quad . \tag{12}$$

We can see explicitly how the singularity in the pole residue is canceled by the cut. The remaining part of the cut term has no $\sqrt{u}$ singularity because erfx is odd in x. Signature may be incorporated in our formulas by the modification

$$\mathcal{M}(u,z_u) \rightarrow \frac{1}{2}\left[\mathcal{M}(u,z_u) + \tau\,\mathcal{M}(u,-z_u)\right].$$

With this modification, the pole term in (11) will acquire the usual signature factor and the cut will have a complicated varying phase.

The strongest experimental support of our work lies in the absence of parity partners to known fermion resonances. Also our conclusion that the partial-wave amplitudes contain a fixed Regge cut does not clash with experiment. Note that by an appropriate choice of G_o and G_1, the ratio of the cut contribution to that of the pole can be chosen arbitrarily for a given range of s. If the pole contribution is dominant, we expect to see typical Regge shrinkage and dips where the trajectory passes through wrong-signature nonsense points. When the cut dominates, there will be no shrinkage and no wrong-signature nonsense dips. Nucleon-exchange data support this correlation. In π^+p backward scattering, the data show Regge shrinkage and a marked dip at $u = -0.2$. In backward π^o photoproduction, there is no shrinkage and no dip at $u = -0.2$.

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2. S.W. MacDowell, Phys. Rev. 116, 774 (1959).
3. V. Barger and D. Cline, Phys. Letters 26B, 85 (1967), suggest that only the lowest state on each MacDowell twin is suppressed. This ad hoc assumption has also been made in Veneziano models of πN scattering, which have failed to fit backward scattering data even qualitatively. See, e.g., S. Chu and B.R. Desai, University of California at Riverside Report No. UCR-34P107-96 (to be published).
4. The fixed cut gives the trajectory functions a two-sheeted J-plane structure. In an analytic continuation from $E>0$ to $E<0$, the Regge poles move from one sheet of the J plane to the other. See V. Singh, Phys. Rev. 127, 632 (1962).
5. L. Van Hove, Phys. Letters 24B, 183 (1967); R.P. Feynman, unpublished.
6. The total momentum in the u channel is denoted by k_μ, and p_μ and p_μ' are the initial and final nucleon momenta, respectively. The energy, momentum, and scattering angle in the u-channel center of mass are W $(=\sqrt{u})$, p, and z_u.
7. The factor $\left[\not{\ell} - m(\ell)\right]/m(\ell)$ corresponds to a normalization $\bar{u}_\ell u_\ell = 2$, independent of ℓ.

8. Properly speaking, we should work with Legendre functions of the second kind to make an analytic continuation to the left of $\mathrm{Re}\,\ell = -1/2$.

9. In general, we expect $g^2(\ell)$ to be a meromorphic function of $m(\ell)$. Even powers of $m(\ell)$ yield a fixed cut in the spin-flip amplitude, while odd powers give a fixed cut in the nonflip amplitude. Nucleon exchange in backward π^+p scattering may be qualitatively fitted with only even powers of $m(\ell)$ in $g^2(\ell)$.

10. The contribution of the Feynman diagram for the exchange of an unnatural parity resonance has the form of (1) with $m(\ell)$ replaced by $-m(\ell)$. $\mathcal{M}(m(\ell)) + \mathcal{M}(-m(\ell))$ has no odd powers of $m(\ell)$ and hence no cut in ℓ.

11. R.L. Sugar and J.D. Sullivan, Phys. Rev. 166, 1515 (1968).

12. See D.L. Steele, thesis, University of Illinois, 1969 (unpublished).

13. If $g^2(\alpha_o) = 0$, there would be no infinity in (7) and no auxiliary pole $\alpha_2(u)$. However, the Regge-pole contribution to the nonflip part of $\mathcal{M}$ at $u = 0$ would be of order $b(u)$, which by assumption is small. Nucleon-exchange data in backward π^+p scattering show no indication of any dip at $u = 0$. (See also Ref.9.)

14. Note that α_1 and α_2 collide at $u = 0$ and become complex for $u < 0$. If there is only one moving pole (see Ref.13), then α does not become complex. Also note that $\alpha_{1,2}(0)$ no longer coincide with the branch point α_o.

FIGURE CAPTIONS

Fig. 1: Dashed line-initial contour; solid line-opened contour.

Fig. 2: Pole trajectory for f^+. Dashed lines show path on second sheet; solid lines refer to the principal sheet.

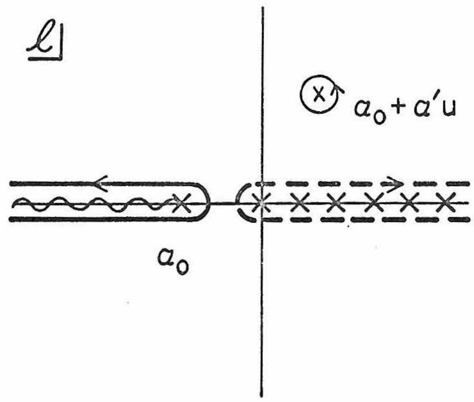

Fig. 1

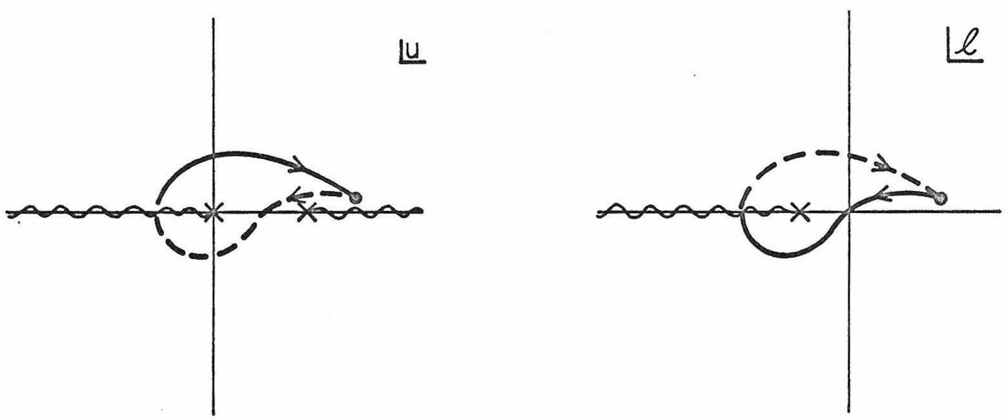

Fig. 2

PART III

1. Experimental Evidence on Parity Doubling

Experimental evidence strongly indicates that there are no fermion parity doublets. One way of seeing this is to note the success of the conventional SU(6) quark model[1] in classifying known fermion states. In Table 1 (page 16) we list the low-lying quark model states and indicate experimental candidates[2] for each state. Only 1 of the 17 states in the chart has no experimental candidate. Even more striking is the fact that of all the known S = 0 resonances[2] with masses under 1950 MeV, only 1 state is not accomodated in Table 1. And this state, the $P_{11}(1780)$, fits nicely into the picture as part of another 56, $L = 0^+$, the second daughter of the $P_{11}(939)$. Thus if we accept the SU(6) classification, we can find no evidence for parity doubled fermion states.

Ignoring the success of the SU(6) classification scheme, we may ask if any candidates exist for a fermion parity doublet. Barger and Cline[3] suggested several possibilities, but new data on spins and parities have eliminated all but one of their candidates. This is the pair of octets of spin-parity $5/2^+$ and $5/2^-$. Although the masses of these octets make them possible candidates for a parity doublet, other properties make their status dubious. In particular the F/D ratios for coupling to pseudoscalar mesons and baryons are 1.2 for the $5/2^+$ octet and -0.2 for the $5/2^-$ octet.[4] If these were really parity doublets, the F/D ratios would be identical. Furthermore the difference in F/D ratios is no mystery; it is correctly predicted by duality.[4]

2. Fixed Cuts in Potential Scattering Theory

As was noted in Part II (page 3) fixed Regge Cuts do occur in some potential theory problems. Here we will examine V. Singh's solution[5] to the Dirac equation with a Coulomb potential. His expression for the scattering matrix is

$$S(E,J,L=J-1/2)=\frac{(J+1/2+m\rho)\ \Gamma\left(\sqrt{(J+1/2)^2-e^4}-E\rho\right)}{\Gamma\left(\sqrt{(J+1/2)^2-e^4}+1+E\rho\right)}\ e^{i\pi\sigma}\ , \qquad (1)$$

where

$$\rho=\frac{e^2}{\sqrt{m^2-E^2}}\ ,$$

$$\sigma=J+1/2-\sqrt{(J+1/2)^2-e^4}\ .$$

It is clear that S has a cut in J running from $J=-1/2-e^2$ to $J=-1/2+e^2$. The Regge trajectories are given by the location of the zeroes of the Γ function in the numerator of (1). These occur at

$$\sqrt{(J+1/2)^2-e^4}=E\rho-n \qquad n=0,1,2,\ldots \ . \qquad (2)$$

The nth trajectory will move onto the second sheet of the J plane at the value of E where

$$E\rho-n=0\ ,$$

that is,

$$E = \frac{nm}{\sqrt{n^2 + e^4}} \quad .$$

In Fig. 1 we plot the real part of the leading two trajectory functions for $|E| < m$. The $\mathrm{Re}J < -1/2$ branch of the leading $(n = 0)$ trajectory is absent on account of the factor $(J + 1/2 + m\rho)$ in Eq. (1).

We see that as ReE is decreased, trajectories move through the cut in the J-plane and onto the second sheet, exactly as was discussed in Part II. Note, however, that the cuts in the potential theory problem have nothing to do with MacDowell twins, since the potential problem has no MacDowell symmetry. Here the physical reason for the disappearance of the poles from the principal sheet is that if the fermion sees an attractive potential, its antiparticle (with opposite charge) will see a repulsive potential.

TABLE 1. Nucleon resonances.

SU(6) Class	(SU(3), quark spin)	Resonances	J^P
$\underset{\sim}{56}$, L = 0^+	($\underset{\sim}{8}$, 1/2)	$P_{11}(939)$	$1/2^+$
	($\underset{\sim}{10}$, 3/2)	$P_{33}(1236)$	$3/2^+$
$\underset{\sim}{70}$, L = 1^-	($\underset{\sim}{8}$, 1/2)	$S_{11}(1535)$	$1/2^-$
		$D_{13}(1520)$	$3/2^-$
	($\underset{\sim}{10}$, 1/2)	$S_{31}(1650)$	$1/2^-$
		$D_{33}(1670)$	$3/2^-$
	($\underset{\sim}{8}$, 3/2)	$S_{11}(1700)$	$1/2^-$
		$D_{13}(1700)$	$3/2^-$
		$D_{15}(1670)$	$5/2^-$
	($\underset{\sim}{1}$, 1/2)		
$\underset{\sim}{56}$, L = 0^+	($\underset{\sim}{8}$, 1/2)	$P_{11}(1470)$	$1/2^+$
	($\underset{\sim}{10}$, 3/2)	$P_{33}(1690?)$	$3/2^+$
$\underset{\sim}{56}$, L = 2^+	($\underset{\sim}{8}$, 1/2)	$P_{13}(1860)$	$3/2^+$
		$F_{15}(1688)$	$5/2^+$
	($\underset{\sim}{10}$, 3/2)	$P_{31}(1910)$	$1/2^+$
		P_{33}	$3/2^+$
		$F_{35}(1890)$	$5/2^+$
		$F_{37}(1950)$	$7/2^+$

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3. V. Barger, D. Cline, Phys. Letters 26B, 85 (1967).
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5. V. Singh, Phys. Rev. 127, 632 (1962).

FIGURE CAPTION

Fig. 1: Regge trajectories for Dirac equation with a Coulomb Field. Solid lines indicate trajectory on principal sheet of the J plane; dashed lines, trajectory on second sheet of the J plane.

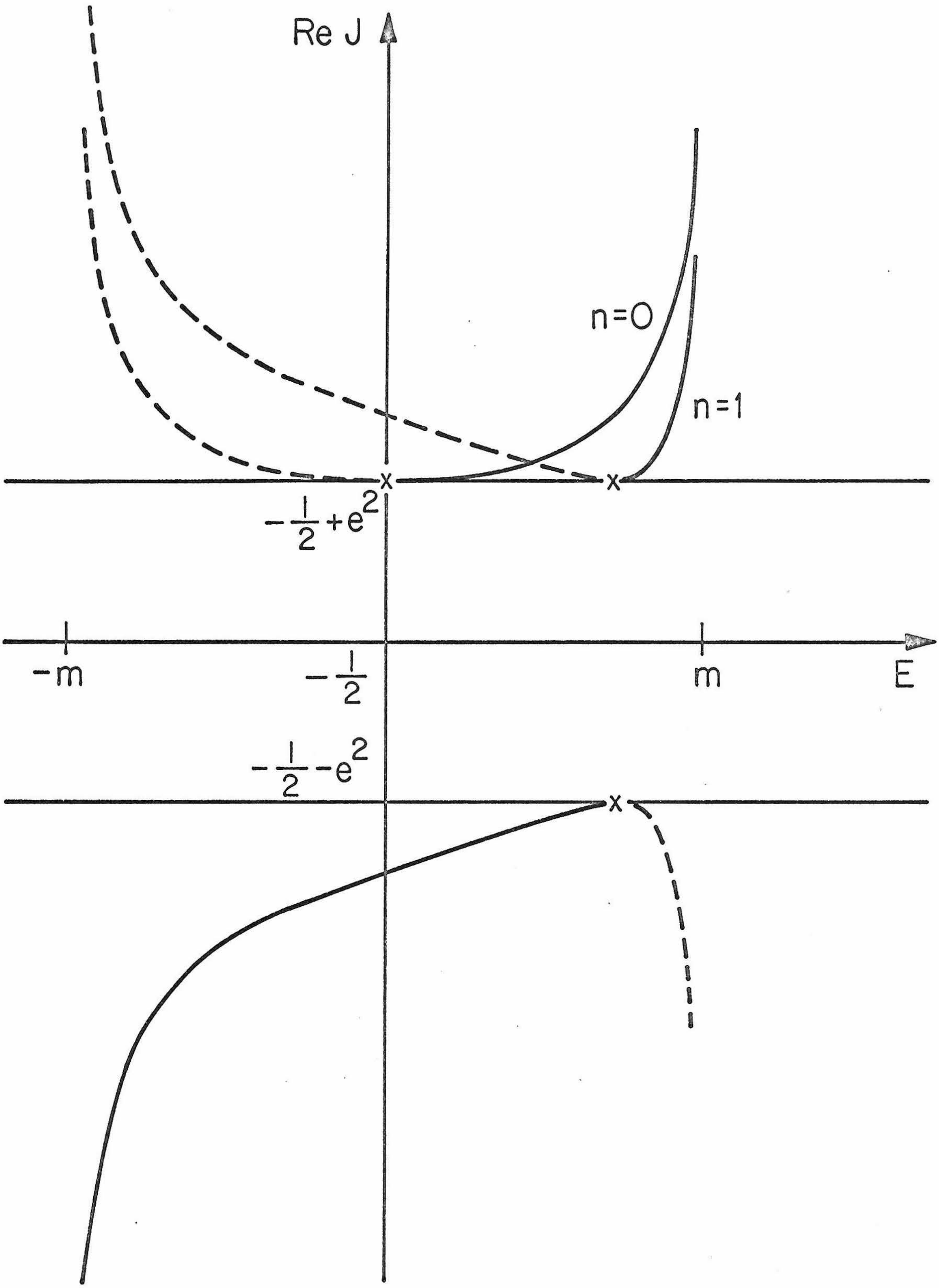

Fig. 1

PART IV

REGGE AMPLITUDE ARISING FROM $SU(6)_W$ VERTICES

1. Introduction

The SU(3) symmetry of the quark model has been extremely useful in classifying strongly interacting particles and in predicting the relative strengths of their couplings. Spin has been incorporated in the model to give a successful classification of hadron states under SU(6).[1] The most natural way of treating spin at three particle vertices yields the symmetry $SU(6)_W$.[2,3] While this symmetry correctly describes many vertices,[3] there has previously been no successful application to scattering[4] amplitudes.

In this thesis we determine the form of the Regge amplitude which results from assuming $SU(6)_W$ as a vertex symmetry. Knowledge of vertices involving a spin J resonance enables us to construct the Feynman amplitude for the exchange of the resonance. Then, using the Van Hove model,[5] we can express the Regge amplitude as a formal sum (on J) of such resonance exchanges. Hence in this model, the form of the Regge amplitude is determined by the assumption of $SU(6)_W$ symmetric vertices. We find that Regge poles are, in general, accompanied by fixed Regge cuts, with branch points at the zero energy intercept of the trajectory. These fixed cuts are similar to those suggested in Part II of this thesis for fermion exchange amplitudes as a consequence of the absence of parity doubled fermion states.[6] The fixed cuts found here for meson (quark-antiquark) exchange amplitudes may be viewed as a consequence of the absence of parity doubled quarks.[7]

The presence of significant fixed cut terms has important experimental consequences. The shrinkage characteristic of a Regge trajectory with normal slope will be absent in those amplitudes which have large fixed cuts, and there will be no dips at wrong-signature nonsense points along the trajectory. Given the magnitudes of pole and cut terms in some reaction, we are able to predict the magnitudes of these terms for a whole class of SU(6) related reactions. Applying our approach to a set of vector meson exchange processes, we find that the numerical importance of cut terms--as indicated by the presence or absence of wrong-signature nonsense dips in differential cross sections--is in accord with our predictions.

An outline of Part IV of this thesis is as follows. In Section 2 we show that $SU(6)_W$ is the natural vertex symmetry arising from the quark model and remind the reader how to calculate $SU(6)_W$ vertices. Construction of a Regge amplitude from the $SU(6)_W$ vertices is carried out in Section 3. Some consequences of our approach are given in Section 4, with particular attention to the question of wrong-signature nonsense dips. A discussion of our work is given in Section 5 with some suggestions for further research on this problem.

2. The Quark Model and $SU(6)_W$

The classification of baryons as qqq composites and mesons as $q\bar{q}$ composites implies that any SU(3) invariant vertex may be pictorially represented by quark graphs. (See Fig. 1.) These graphs are drawn according to the following rules: (a) Each quark or antiquark is represented by a directed line. (b) A baryon or antibaryon is represented by three lines running in the same direction. (c) A meson is represented by two lines running in opposite directions. Zweig[8] suggested an additional rule: (d) The quark and antiquark lines of a single meson should not be connected. (This rule accounts for the absence of the decay $\phi \to \rho\pi$ and the weak coupling of the ϕ to nucleons.) Note that the rules (a) - (d) are precisely those used by Harari[9] and Rosner[10] to construct their duality diagrams.

We wish to incorporate spin into the quark graph picture in the simplest possible fashion. We choose a Lorentz frame in which the particle momenta are collinear along the z axis. In such a frame, we assume that the spins of quarks a, b, and d (in Fig. 1) are unchanged in the reaction. How, then, must the spins of the annihilating quarks c and e be related? The parity of a $q\bar{q}$ pair is $-(-1)^L$, so if parity is to be conserved, q_c and $\bar{q}_e$ must annihilate in an odd angular momentum state. Then angular momentum conservation requires that L = 1 and that the quark spins form a triplet. If, furthermore, the transverse motions of the annihilating quarks may be neglected, then the quark momenta lie along the z axis, so $L_z = 0$ and hence $S_z = 0$.

Thus we see that the $q\bar{q}$ state annihilates with the quark spins in a triplet state, $S = 1$, $S_z = 0$. This is exactly the result given by the collinear symmetry $SU(2)_W$.[2] Taking into account the SU(3) quantum numbers of the quarks, we obtain $SU(6)_W$ as the natural vertex symmetry of the quark model.[11] Note that the derivation above is independent of what collinear frame we choose, since $SU(2)_W$ states are invariant under boosts along the z axis.

Choosing some collinear frame, it is easy to calculate the $SU(6)_W$ symmetric vertex functions. Let each quark be represented by a pair of indices $(\alpha, \underline{a})$, where α specifies its SU(3) nature and $\underline{a}$ gives its spin orientation along the z axis. In a collinear frame, the meson-baryon vertex (Fig. 1) has the form

$$\bar{B}_{(\alpha a)(\beta b)(\gamma c)}\, B^{(\alpha a)(\beta b)(\delta d)}\, M^{(\gamma e)}{}_{(\delta d)}\, D_{ec} \quad . \tag{1}$$

The matrix

$$D = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{2}$$

in (1) specifies that q_c and $\bar{q}_e$ annihilate in a spin state $S = 1$, $S_z = 0$.

Let us recall the form of the SU(6) wave functions[12] for the $\underset{\sim}{56}$ baryons, $B^{(\alpha a)(\beta b)(\gamma c)}$, and the $\underset{\sim}{35}$ mesons, $M^{(\alpha a)}{}_{(\beta b)}$.

$$B^{(\alpha a)(\beta b)(\gamma c)m} = \frac{1}{3}\left[\varepsilon^{\alpha\beta\sigma} B^{\gamma}{}_{\sigma}\, C_{ab}\, \chi_c^{(m)} + \varepsilon^{\beta\gamma\sigma} B^{\alpha}{}_{\sigma}\, C_{bc}\, \chi_a^{(m)} + \varepsilon^{\gamma\alpha\sigma} B^{\beta}{}_{\sigma}\, C_{ca}\, \chi_b^{(m)}\right] + D^{\alpha\beta\gamma} (\sigma^i C)_{ab}\, \xi_{ci}^{(m)} \quad . \tag{3}$$

The superscript $m = a+b+c$ gives the spin projection of the baryon. χ is a two-component spinor

$$\chi^{(+1/2)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad ; \quad \chi^{(-1/2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{4}$$

and $\xi^{(m)}{}_a$ is a vector-spinor,

$$\xi^{(m)}{}_a = \sum_{\sigma\rho} (\tfrac{1}{2}\,\sigma,\ 1\rho \mid \tfrac{3}{2}\, m)\ \chi_a^{(\sigma)}\ \varepsilon^{(\rho)} \quad , \tag{5}$$

where

$$\varepsilon^{\pm 1} = \frac{1}{\sqrt{2}} (\mp 1,\ -i,\ 0) \quad ,$$

$$\varepsilon^{o} = (0,\ 0,\ 1) \quad . \tag{6}$$

The matrix C is given by

$$C = i\, \sigma_y = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad . \tag{7}$$

$B^{\alpha}{}_{\beta}$ and $D^{\alpha\beta\gamma}$ are the SU(3) matrices for the octet and decuplet:

$$B^{\alpha}{}_{\beta} = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^o + \frac{1}{\sqrt{6}}\Lambda^o & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^o + \frac{1}{\sqrt{6}}\Lambda^o & n \\ -\Xi^- & \Xi^o & -\frac{2}{\sqrt{6}}\Lambda^o \end{pmatrix}, \tag{8}$$

$$
\begin{aligned}
D^{111} &= \Delta^{++} & D^{112} &= \frac{1}{\sqrt{3}}\,\Delta^{+} \\
D^{113} &= \frac{1}{\sqrt{3}}\,\Sigma^{*+} & D^{122} &= \frac{1}{\sqrt{3}}\,\Delta^{o} \\
D^{123} &= \frac{1}{\sqrt{6}}\,\Sigma^{*o} & D^{133} &= \frac{1}{\sqrt{3}}\,\Xi^{o} \\
D^{222} &= \Delta^{-} & D^{223} &= \frac{1}{\sqrt{3}}\,\Sigma^{*-} \\
D^{233} &= \frac{1}{\sqrt{3}}\,\Xi^{-} & D^{333} &= \Omega^{-} \quad .
\end{aligned}
\tag{9}
$$

The meson wave function is

$$
M^{(\alpha a)}{}_{(\beta b)}{}^{m} = P^{\alpha}{}_{\beta}\, C_{ab} + V^{\alpha}{}_{\beta}\, \varepsilon_{i}^{(m)}\, (\sigma^{i}\, C)_{ab} \quad , \tag{10}
$$

where

$$
P^{\alpha}{}_{\beta} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^{o} + \frac{1}{\sqrt{6}}\eta^{o} & \pi^{+} & K^{+} \\ \pi^{-} & -\frac{1}{\sqrt{2}}\pi^{o} + \frac{1}{\sqrt{6}}\eta^{o} & K^{o} \\ K^{-} & \overline{K}^{o} & -\frac{2}{\sqrt{6}}\eta^{o} \end{pmatrix} \tag{11}
$$

and

$$
V^{\alpha}{}_{\beta} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\omega^{o} + \rho^{o}) & \rho^{+} & K^{*+} \\ \rho^{-} & \frac{1}{\sqrt{2}}(\omega^{o} - \rho^{o}) & K^{*o} \\ K^{*-} & \overline{K}^{*o} & \phi \end{pmatrix} . \tag{12}
$$

The superscript m specifies polarization for the vector meson nonet.

Using (3) and (10) in (1), we obtain the vertex functions given in Table 1. The 56-56-35 vertices are all determined to within a single constant factor by the $SU(6)_W$ symmetry. Similarly, the 35-35-35 vertices may be computed from the coupling

$$M^{1(\alpha a)}{}_{(\beta b)} \; M^{2(\beta b)}{}_{(\gamma c)} \; M^{3(\gamma d)}{}_{(\alpha a)} \; D_{cd} \quad . \tag{13}$$

The results are summarized in Table 2.

Using the information in Tables 1 and 2, it is a straightforward matter to calculate the invariant vertex functions which, in a collinear frame, reduce to those given in the Tables. These invariant vertex functions are listed in Table 3.[13] Common mass factors have been absorbed in the constants c and d to give the entries a simple form.

The extension of the couplings in Table 3 to vertices involving Regge recurrences of the listed states is easily made. For example, the $SU(6)_W$ symmetric coupling of two pseudoscalar mesons to a V recurrence (quark spin 1) of spin $J = L + 1$ will be

$$-d(J)M_3(J)\varepsilon_{3\nu\mu_1\ldots\mu_L}(p_1+p_2)_\nu(p_1+p_2)_{\mu_1}\cdots(p_1+p_2)_{\mu_L}\left[\langle P_1P_2V\rangle - \langle P_1VP_2\rangle\right]$$

(ε denotes the polarization of the V recurrence.) The higher spin indices simply couple to appropriate momentum factors. In general, couplings for excited states with a given quark spin assignment may be constructed by decomposing the spin of the states into quark spin and orbital angular momentum and coupling the quark spin according to $SU(6)_W$. This will give a unique result whenever two of the states have no orbital excitation. In other cases there will be more than a single coupling for each class of SU(6)-related reactions.

Note the presence in Table 3 of factors involving the masses M_1, M_2, M_3. These factors have a simple kinematic origin. Some of these, e.g., $(M_1 + M_2)^2 - M_3^2$, arise because the $SU(6)_W$ symmetry relates vertices involving different angular momenta. Other factors, e.g., M_3 in the $P_1\ P_2\ V$ vertex, arise because the symmetry relates vertices involving vector mesons of different helicities. The polarization vector for a vector meson of zero helicity is proportional to $1/M_V$.[14] Hence, within a class of vertices related by $SU(6)_W$, those vertices involving a zero helicity vector meson will contain an extra factor of M_V relative to those not involving a zero helicity vector meson. This is the only source of odd powers of meson mass; the angular momentum factors will always contain factors of $(\text{meson mass})^2$.

3. Construction of a Regge Amplitude

The presence of extra "kinematic" factors in the vertex functions has important consequences when we construct a Regge amplitude. In general, we find the presence of fixed Regge cuts with branch points coinciding with the zero energy intercepts of the Regge trajectories.

This phenomenon has been discussed previously for fermion exchange processes; here we find cuts for boson or fermion exchange processes and predict the relative strengths of the cuts in different processes.

The Van Hove model[15] expresses a Regge amplitude as a formal sum of Feynman diagrams for the exchange of all resonances along a given trajectory.[16] Consider, for example, the reaction

$$\pi^+ + \pi^- \to \pi^o + \omega^o \quad ,$$

mediated by ρ exchange (see Fig. 2). The coupling at a $\omega\rho^*\pi$ vertex is

$$-i\sqrt{2}\; d(J)\varepsilon^{\alpha\mu_1\gamma\delta}\; \varepsilon^*_{(\omega)\alpha}\; P_{(\rho^*)\gamma}\; P_{(\pi)\delta}\; P_{(\pi)\mu_2}\cdots P_{(\pi)\mu_J}\; 2^J \varepsilon_{(\rho^*)\mu_1\mu_2\cdots\mu_J} \tag{14}$$

for a ρ^* of spin J. The $\pi\pi\rho^*$ coupling is

$$-\sqrt{2}\; d(J)m_{\rho^*}(J)P_{(\pi)\mu_1}\; P_{(\pi)\mu_2}\cdots P_{(\pi)\mu_J}\; 2^J\; \varepsilon_{(\rho^*)\mu_1\mu_2\cdots\mu_J} \quad . \tag{15}$$

The Feynman diagram for the exchange of a ρ^* of a spin J is therefore

$$\mathcal{M}(J)=2id^2(J)m(J)4^J$$

$$\times\left(\frac{P^+_{\mu_1}\cdots P^+_{\mu_J}\; T^{(J)}_{\mu_1\cdots\mu_J;\nu_1\cdots\nu_J}\; P^-_\delta\; P^-_{\nu_2}\cdots P^-_{\nu_J}\; Q_\gamma\; \varepsilon^*_{(\omega)\alpha}\; \varepsilon^{\alpha\nu_1\gamma\delta}}{t - m^2\;(J)}\right)$$

$$= \frac{2id^2(J)m(J)4J}{t - m^2(J)} \varepsilon^*_{(\omega)\alpha} Q_\gamma p^-_\delta \varepsilon^{\alpha\beta\gamma\delta} \frac{1}{J} \frac{\partial}{\partial p^-_\beta} \mathcal{P}_J(p^+, p^-) \quad , \quad (16)$$

where $T^{(s)}_{\mu;\nu}/(t - m^2(J))$ is the Feynman propagator for a particle of spin J,

$$\mathcal{P}_J(p^+, p^-) = 4^J(|p^+||p^-|)^J \; P_J\left(\frac{p^+ \cdot p^-}{|p^+||p^-|}\right) \quad , \quad (17)$$

and $t = Q^2$. We assume that $m_\pi = m_\omega$ to simplify the kinematics. Summing over J and transforming the sum into a contour integral gives

$$\mathcal{M} = \sum_J \mathcal{M}(J) = \frac{1}{2} Z_\beta \int_C dJ \; \frac{d^2(J)m(J)}{t - m^2(J)} \frac{1}{J} \frac{\partial}{\partial p^-_\beta} \frac{\mathcal{P}_J(p^+, -p^-)}{\sin \pi J} \quad , \quad (18)$$

using the abbreviation

$$Z_\beta = 2i \, \varepsilon^*_{(\omega)\alpha} Q_\gamma p^-_\delta \; \varepsilon^{\alpha\beta\gamma\delta} \quad . \quad (19)$$

The contour C is indicated in Fig. 3. If we assume that the m^2 is a linear function of J,

$$m^2(J) = \frac{J - \alpha_o}{\alpha'} \quad , \quad (20)$$

and that $d^2(J)$ is analytic, then we can open the contour in the J plane obtaining contributions from the pole at $m^2(J) = t$ and the cut with branch point $J = \alpha_o$. This gives

$$\mathcal{M} = \frac{\pi\, d^2(\alpha)\sqrt{t}\ \ \alpha'}{\sin\ \pi\alpha}\ Z_\beta\ \frac{1}{J}\ \frac{\partial}{\partial p_\beta^-}\ \mathcal{P}_J(p^+, -p^-)$$

$$- Z_\beta \int_{-\infty}^{\alpha_o} dJ\ \frac{d^2(J)\sqrt{-m^2(J)}}{(t - m^2(J))\ \sin\ \pi J}\ \frac{1}{J}\ \frac{\partial}{\partial p_\beta^-}\ \mathcal{P}_J(p^+, -p^-) \qquad (21)$$

where $\alpha(t) = \alpha_o + \alpha' t$. (22)

In the limit $s = (p^+ + p^-)^2 \to \infty$,

$$\mathcal{M} \to \frac{\pi\, d^2(\alpha(t))\ b(\alpha(t))\ \alpha'\ s^{\alpha(t)-1}}{\sin\ \pi\alpha(t)}\ Z_\beta\ p_\beta^+\sqrt{t}$$

$$- Z_\beta\ p_\beta^+ \int_{-\infty}^{\alpha_o} dJ\ \frac{d^2(J)\sqrt{-m^2(J)}\ b(J)\ s^{J-1}}{(t - m^2(J))\ \sin\ \pi J} \qquad (23)$$

where

$$b(J) = \frac{\Gamma(2J + 1)}{(\Gamma(J+1))^2} \quad . \qquad (24)$$

It is clear that (23) consists of a moving Regge pole term and a fixed Regge cut. Nonsense couplings along the trajectories are presumably eliminated in the usual way by zeros of $d^2(J)$ at $J=0, -1, -2, \ldots$. Therefore, for negative t, we can approximate

$$\frac{d^2(\alpha(t))\ b(\alpha(t))\ \alpha'}{\sin\ \pi\alpha(t)} \doteq d_o \quad . \qquad (25)$$

Then (23) becomes

$$\mathfrak{M} \simeq \pi\, d_o\, Z_\beta\, p_\beta^{\,+} \left[\sqrt{t}\ \mathrm{erf}\left(\sqrt{\alpha' t\ \ln s}\right)\ s^{\alpha(t)-1} + \frac{1}{\sqrt{\alpha'\pi\ \ln s}}\ S^{\alpha_o - 1}\right]$$

$$\simeq \pi\, d_o\, Z_\beta\, p_\beta^{\,+} \left[\sqrt{t}\ s^{\alpha(t)-1} + \frac{S^{\alpha_o-1}}{2t\sqrt{\pi}\ (\alpha'\ \ln s)^{3/2}}\ \left(1+\mathcal{O}\left(\frac{1}{\alpha' t\ \ln s}\right)\right)\right] . \tag{26}$$

In Eq. (18) it is clear that the fixed cut arises from the presence of an odd power of $m(J)$ and the assumption that $d^2(J)$ is even in $m(J)$, i.e., analytic in J. If, for example, $d^2(J) = m(J)\ d_1(J)$, with $d_1(J)$ analytic, then the $\pi\pi \to \pi\omega$ amplitude would have no fixed cuts. In this case, however, there would be fixed cuts in the amplitudes $\pi\pi \to \pi\pi$ and $\pi\omega \to \pi\omega$. Hence the presence of fixed cuts in some amplitudes is inescapable.

4. Experimental Consequences

The presence of fixed Regge cuts has three important experimental consequences. (1) Asymptotic behavior. At sufficiently high energies, the Regge cut term gives an energy falloff independent of t.[17] (2) Polarization. When signature is incorporated in the model by the replacement

$$\mathfrak{M}(t, z_t) \to \frac{1}{2}\left[\mathfrak{M}(t, z_t) + \tau\,\mathfrak{M}(t, -z_t)\right] \quad , \tag{27}$$

Regge pole terms will acquire the usual Regge phase, but cut terms will have some complicated varying phase. Thus there can be interference between the pole and cut terms, and exchange of a single Regge trajectory will be able to give non-zero polarization. (3) Wrong-signature nonsense dips. The contribution from a single Regge pole term vanishes when the trajectory passes through a nonsense value of the wrong-signature. The cut term does not vanish, however, so an amplitude with a significant cut term will show no dip at wrong-signature nonsense points.[18]

A qualitative discussion of the third point is easy to make. We will restrict our attention to processes involving vector meson exchange. In Section 2 we gave a simple criterion for determining the presence of odd powers of M_V in vector meson vertices. The argument required examining the vertex in a collinear frame. Note that the t-channel center-of-mass frame is collinear for both vertices, so we can apply the argument of Section 2 directly to t-channel helicity amplitudes. The factors of $m(J)$ which will appear in t-channel helicity amplitudes involving various ρ^* or ω^* vertices are given in Table 4. Assuming the absence of a cut term in some particular helicity amplitude, this Table allows us to predict the presence or absence of cut terms in other helicity amplitudes. In general, a reaction will have no cuts in some amplitudes and cuts in others. We must pay attention to the relative magnitudes of the different amplitudes in order to assess the importance of cut terms in any given process.

In Table 5 we tabulate the magnitudes of the helicity amplitudes and the factors of $m(J)$ which lead to Regge cuts for a number of reactions[19] involving the vertices of Table 4. The relative magnitudes of the contribution of a spin J V^* exchange to the various t-channel helicity amplitudes are given by

$$f_{\lambda\mu}(t=m^2(J),s) = V_1^{(\lambda)}\, V_2^{(\mu)}\, \varepsilon_1^{(\lambda)^*} \cdot \varepsilon_2^{(\mu)}\, G(J) \; . \qquad (28)$$

$V_1^{(\lambda)}$ and $V_2^{(\mu)}$ are the $SU(6)_W$ vertex coefficients from Tables 1 and 2, and $\varepsilon_1^{(\mu)}$ and $\varepsilon_2^{(\mu)}$ specify the orientation of the quark spin of the V^* at vertices 1 and 2 respectively in the t-channel center-of-mass. Equation (28) is obvious for $J = 1$ and is valid for arbitrary J because the higher spin indices always couple to additional momentum factors at each vertex. In the limit $s \to \infty$,

$$|\varepsilon_1^{(\lambda)} \cdot \varepsilon_2^{(\mu)}| \to |\varepsilon_1^{(0)*} \cdot \varepsilon_2^{(0)}| / (\sqrt{2})^{|\lambda| + |\mu|} \quad .$$

In Table 5, then, we tabulate simply

$$|V_1^{(\lambda)}\, V_2^{(\mu)}\, c(J)\, d(J)/(\sqrt{2})^{|\lambda| + |\mu|}| \quad .$$

For convenience, we have defined

$$g(J) = \frac{c(J)\; d(J)}{36\; m(J)} \quad .$$

Genuine kinematic factors (kinematics of $\pi N \to \pi N$ are assumed for all reactions listed) are tabulated as $\sqrt{t}$ or t. The factors of $m(J)$ induced by $SU(6)_W$ are determined from Table 4 and tabulated in the appropriate helicity amplitudes.

The qualitative features of the data for the reaction $\pi^- p \to \pi^0 n$ indicate that the reaction is dominated by a Regge pole in the t-channel helicity 1 amplitude. Therefore, the function $g(J)$ in Table 5 must be approximately even in $m(J)$, i.e., analytic in J, so that the helicity 1 amplitude is purely a Regge pole term while the helicity 0 amplitude contains a Regge cut as well. This cut arises from the presence in the amplitude of a factor $m(J)$ and is referred to in Table 5 as a weak cut. As can be seen in (26), the contribution to the scattering amplitude of a weak cut $(m(J)\ g(J))$ at $t = 0$ is suppressed by a factor $(\pi\,\alpha'\,\ln s)^{-1/2}$ relative to a pole term $(g(J))$. A cut arising from the presence of a factor $1/m(J)$ is referred to as a strong cut. The magnitude of a strong cut $(g(J)/m(J))$ is larger than that of a weak cut by a factor of $2\,\alpha'\,\ln s$.

Now in Table 5 we see that the helicity 1 amplitude in $\pi^- p \to \omega n$ contains a strong cut with a numerical coefficient larger than that for the pole term in the helicity 0 amplitude. Therefore, in this reaction, the cut effects should be appreciable and we expect no wrong-signature nonsense dip. Proceeding in this manner, we may make the other predictions given in the last column of Table 5. These predictions agree with experiment for all the reactions listed.[20]

5. Discussion

The qualitative discussion above should be largely unaffected by the manner in which the SU(6) symmetry of our theory is broken. Symmetry breaking will alter the SU(3) factors and numerical coefficients in Table 3, but will not affect the mass factors, which arise solely from kinematic considerations. In Table 5 it is apparent that the question of dip or no dip depends primarily on these mass factors. In a quantitative fit of differential cross sections and polarization phenomena, symmetry breaking effects will be important and a more detailed theory will be necessary.

Aside from symmetry breaking, an important question concerns the relation of our work to duality. Since both schemes are based on identical quark graphs, it seems likely that they may be fused in a unified approach. After completing our work on the manner in which $SU(6)_W$ leads to fixed Regge cuts, we learned that K. Bardacki and M.B. Halpern[21] have in fact constructed a dual amplitude containing fixed cuts and have proposed that this amplitude be utilized in the quark model. S. Ellis[22] has also investigated this problem, which we expect to open a fruitful area of new research.

TABLE 1

Baryon-Baryon-Meson Vertices

Vertex	Spins†	Value*
BBP	1/2, 1/2, 0	$\frac{1}{18}\left[5\langle\bar{B}PB\rangle + \langle\bar{B}BP\rangle\right]$
BBV	1/2, -1/2, 1	$\frac{1}{9\sqrt{2}}\left[5\langle\bar{B}VB\rangle + \langle\bar{B}BV\rangle - \langle\bar{B}B\rangle\langle V\rangle\right]$
	1/2, 1/2, 0	$\frac{1}{6}\left[\langle\bar{B}VB\rangle - \langle\bar{B}BV\rangle + \langle\bar{B}B\rangle\langle V\rangle\right]$
DBP	1/2, 1/2, 0	$-\frac{1}{3}\sqrt{\frac{2}{3}}\left[\bar{D}_{\alpha\beta\gamma}\,\varepsilon^{\beta\delta\tau}\,B^{\alpha}{}_{\tau}\,P^{\gamma}{}_{\delta}\right]$
DBV	3/2, 1/2, 1	$\frac{1}{3}\left[\bar{D}_{\alpha\beta\gamma}\,\varepsilon^{\beta\delta\tau}\,B^{\alpha}{}_{\tau}\,V^{\gamma}{}_{\delta}\right]$
	1/2, 1/2, 0	0
	1/2, -1/2, 1	$\frac{1}{3\sqrt{3}}\left[\bar{D}_{\alpha\beta\gamma}\,\varepsilon^{\beta\delta\tau}\,B^{\alpha}{}_{\tau}\,V^{\gamma}{}_{\delta}\right]$
DDP	3/2, 3/2, 0	$\frac{1}{2}\left[\bar{D}_{\alpha\beta\gamma}\,D^{\alpha\beta\delta}\,P^{\gamma}{}_{\delta}\right]$
	1/2, 1/2, 0	$\frac{1}{6}\left[\bar{D}_{\alpha\beta\gamma}\,D^{\alpha\beta\delta}\,P^{\gamma}{}_{\delta}\right]$
DDV	3/2, 3/2, 0	$\frac{1}{2}\left[\bar{D}_{\alpha\beta\gamma}\,D^{\alpha\beta\delta}\,V^{\gamma}{}_{\delta}\right]$
	3/2, 1/2, 1	$\frac{1}{\sqrt{6}}\left[\bar{D}_{\alpha\beta\gamma}\,D^{\alpha\beta\delta}\,V^{\gamma}{}_{\delta}\right]$
	1/2, 1/2, 0	$\frac{1}{2}\left[\bar{D}_{\alpha\beta\gamma}\,D^{\alpha\beta\delta}\,V^{\gamma}{}_{\delta}\right]$
	1/2, -1/2, 1	$\frac{\sqrt{2}}{3}\left[\bar{D}_{\alpha\beta\gamma}\,D^{\alpha\beta\delta}\,V^{\gamma}{}_{\delta}\right]$
	1/2, 3/2, -1	$\frac{1}{\sqrt{6}}\left[\bar{D}_{\alpha\beta\gamma}\,D^{\alpha\beta\delta}\,V^{\gamma}{}_{\delta}\right]$

†Particle 1 is outgoing; particles 2 and 3, incoming.

* $\bar{B}PB$ denotes $\bar{B}^{\alpha}{}_{\beta}\,P^{\beta}{}_{\gamma}\,B^{\gamma}{}_{\alpha}$.

TABLE 2

Meson-Meson-Meson Vertices

Vertex	Spins	Value
P_1P_2V	0, 0, 0	$\frac{1}{2}\left[\langle P_1P_2V\rangle - \langle P_1VP_2\rangle\right]$
V_1V_2P	1, 1, 0	$\frac{1}{2}\left[\langle V_1V_2P\rangle + \langle V_1PV_2\rangle\right]$
$V_1V_2V_3$	0, 0, 0	$\frac{1}{2}\left[\langle V_1V_2V_3\rangle - \langle V_3V_2V_1\rangle\right]$
	1, 1, 0	$\frac{1}{2}\left[\langle V_1V_2V_3\rangle - \langle V_3V_2V_1\rangle\right]$
	1, 0, 1	$\frac{1}{2}\left[\langle V_1V_2V_3\rangle - \langle V_3V_2V_1\rangle\right]$
	0, 1, -1	$\frac{1}{2}\left[\langle V_1V_2V_3\rangle - \langle V_3V_2V_1\rangle\right]$

TABLE 3

Invariant Vertex Functions

Vertex	Function*
BBP	$\frac{c}{9}\left[(M_1+M_2)^2 - M_3^2\right] (\bar{u}_1\ \gamma_5\ u_2) \left[5\ \langle\bar{B}PB\rangle + \langle\bar{B}BP\rangle\right]$
BBV	$\frac{c}{9}\left\{\left[\varepsilon_3 \cdot (p_1+p_2)(\bar{u}_1\ u_2)\right]\left[(M_1+M_2)(5\ \langle\bar{B}VB\rangle + \langle\bar{B}BV\rangle - \langle\bar{B}B\rangle\langle V\rangle) - 3M_3(\langle\bar{B}VB\rangle - \langle\bar{B}BV\rangle + \langle\bar{B}B\rangle\langle V\rangle)\right]\right.$
	$\left. -(\bar{u}_1\ \not{\varepsilon}_3\ u_2)\left[(M_1+M_2)^2 - M_3^2\right]\left[5\ \langle\bar{B}VB\rangle + \langle\bar{B}BV\rangle - \langle\bar{B}B\rangle\langle V\rangle\right]\right\}$
DBP	$+\frac{4c}{3}\ M_1\ p_{2\mu}\ (\bar{u}_{1\mu}\ u_2)\left[D_{\alpha\beta\gamma}\ \varepsilon^{\beta\delta\tau}\ B^{\alpha}{}_{\tau}\ P^{\gamma}{}_{\delta}\right]$
DBV	$-\frac{2c}{3}\left\{\left[(M_1+M_2)^2 - M_3^2\right]\varepsilon_{3\mu}\ (\bar{u}_{1\mu}\ \gamma_5\ u_2) - p_{1\mu}\ (\bar{u}_{1\mu}\ \gamma_5\ u_2)\ \varepsilon_3 \cdot (p_1+p_2)\right.$
	$\left. +\ 2\ M_1\ p_{2\mu}\ (\bar{u}_{1\mu}\ \not{\varepsilon}_3\ \gamma_5\ u_2)\right\}\left[\bar{D}_{\alpha\beta\gamma}\ \varepsilon^{\beta\delta\tau}\ B^{\alpha}{}_{\tau}\ V^{\gamma}{}_{\delta}\right]$
DDP	$c\left\{2\ p_{2\mu}\ (\bar{u}_{1\mu}\ \gamma_5\ u_{2\nu})\ p_{1\nu} - \left[(M_1+M_2)^2 - M_3^2\right](\bar{u}_{1\mu}\ \gamma_5\ u_{2\mu})\right\}\bar{D}_{\alpha\beta\gamma}\ D^{\alpha\beta\delta}\ P^{\gamma}{}_{\delta}$
DDV	$c\left\{\left[(M_1+M_2)^2 - M_3^2\right](\bar{u}_{1\mu}\ \not{\varepsilon}_3\ u_{2\mu}) - \left(M_1+M_2-M_3\right)(\bar{u}_{1\mu}\ u_{2\mu})\ \varepsilon_3 \cdot (p_1+p_2) - 2\ p_{2\mu}\ (\bar{u}_{1\mu}\ \not{\varepsilon}_3\ u_{2\nu})p_{1\nu}\right.$
	$\left. +\ \frac{2\ p_{2\mu}\ (u_{1\mu}\ u_{2\nu})\ p_{1\nu}\ \varepsilon_3 \cdot (p_1+p_2)}{M_1+M_2+M_3}\right\}\left[\bar{D}_{\alpha\beta\gamma}\ D^{\alpha\beta\delta}\ V^{\gamma}{}_{\delta}\right]$
P_1P_2V	$-\ d\ M_3\ \varepsilon_3 \cdot (p_1+p_2)\left[\langle P_1P_2V\rangle - \langle P_1VP_2\rangle\right]$

TABLE 3 (cont.)

Vertex	Function*
V_1V_2P	$-\, i\, d\, \varepsilon^{\mu\nu\rho\sigma}\, p_{3\mu}\, (p_1+p_2)_\nu\, \varepsilon_{2\rho}\, \varepsilon_{1\sigma}^{*} \left[\langle V_1V_2P\rangle + \langle V_1PV_2\rangle\right]$
$V_1V_2V_3$	$d\left[M_3\, (\varepsilon_1^{*}\cdot\varepsilon_2)(\varepsilon_3\cdot(p_1+p_2)) - M_2\, (\varepsilon_1^{*}\cdot\varepsilon_3)(\varepsilon_2\cdot(p_3+p_1)) - M_1\, (\varepsilon_2\cdot\varepsilon_3)(\varepsilon_1^{*}\cdot(p_2-p_3)) + \frac{\varepsilon_1^{*}\cdot(p_2-p_3)\ \varepsilon_2\cdot(p_3+p_1)\ \varepsilon_3\cdot(p_1+p_2)}{2(M_1+M_2+M_3)}\right]\left[\langle V_1V_2V_3\rangle - \langle V_3V_2V_1\rangle\right]$

*M_i and p_i refer to the mass and momentum of the i-th particle, $i = 1, 2, 3$. Particle 1 is outgoing; particles 2 and 3 incoming.

TABLE 4

Odd powers of m(J) in vector meson vertices.

Vertex	V^* Helicity	m(J) Factors
$NN\rho^*$	±1	c(J)
	0	m(J) c(J)
$\Delta N\rho^*$	±1 (a)	c(J)
$NN\omega^*$	±1	c(J)
	0	m(J) c(J)
$\pi\pi\rho^*$	0 (b)	m(J) d(J)
$\pi\rho\omega^*$	±1 (b)	d(J)
$\pi\omega\rho^*$	±1 (b)	d(J)
$\eta\rho\rho^*$	±1 (b)	d(J)

(a) Only helicities allowed by $SU(6)_W$.

(b) Only helicities allowed by parity and angular momentum.

TABLE 5

Wrong signature nonsense dips for vector meson exchange.[19]

Reaction	Exchange	Helicities	Relative Magnitude	Cut?	Dip?
$\pi^- p \to \pi^0 n$	ρ	1, 0	$5\sqrt{2}\sqrt{t}\ g(J)$	none	yes
		0, 0	$3\sqrt{2}\ m(J)\ g(J)$	weak	
$\pi^- p \to \omega n$	ρ	1, 1	$5\ t\ g(J)/m(J)$	strong	no
		0, 1	$3\sqrt{t}\ g(J)$	none	
$\pi^+ p \to \pi^0 \Delta^{++}$	ρ	1(3/2,1/2), 0	$6\sqrt{t}\ g(J)$	none	yes
		1(1/2,-1/2), 0	$2\sqrt{3}\sqrt{t}\ g(J)$	none	
$\pi^+ p \to \omega \Delta^{++}$	ρ	1(3/2,1/2), 1	$3\sqrt{2}\ t\ g(J)/m(J)$	strong	no
		1(1/2,-1/2), 1	$\sqrt{6}\ t\ g(J)/m(J)$	strong	
$\pi^0 p \to \rho^0 p$	ω	1, 1	$(3/\sqrt{2})\ t\ g(J)/m(J)$	strong	yes[a]
		0, 1	$(9/\sqrt{2})\sqrt{t}\ g(J)$	none	
$\rho^0 p \to \pi^0 p$	ω	1, 1	$(3/\sqrt{2})\ t\ g(J)/m(J)$	strong	yes[b]
		0, 1	$(9/\sqrt{2})\sqrt{t}\ g(J)$	none	
$\rho^0 p \to \eta p$	ρ	1, 1	$(5/\sqrt{6})\ t\ g(J)/m(J)$	strong	no[c]
		0, 1	$(3/\sqrt{6})\sqrt{t}\ g(J)$	none	
$\omega p \to \pi^+ n$	ρ	1, 1	$5\ t\ g(J)/m(J)$	strong	no[d]
		0, 1	$3\sqrt{t}\ g(J)$	none	

TABLE 5 (cont.)

a)
$$\frac{d\sigma}{dt}\,(\pi^{+}p\to\rho^{+}p)+\frac{d\sigma}{dt}\,(\pi^{-}p\to\rho^{-}p)-\frac{d\sigma}{dt}\,(\pi^{-}p\to\rho^{o}n)$$

b)
$$\frac{d\sigma}{dt}\,(\gamma p\to\pi^{o}p)$$

c)
$$\frac{d\sigma}{dt}\,(\gamma p\to\eta p)$$

d)
$$\frac{d\sigma}{dt}\,(\gamma p\to\pi^{+}n)-\frac{d\sigma}{dt}\,(\gamma n\to\pi^{-}p)$$

REFERENCES

1. See, for example, rapporteur talks of B. French and A. Donnachie in Proceedings of 14th International Conference on High Energy Physics (CERN, Geneva, 1968).
2. $SU(6)_W$ was first proposed by H.J. Lipkin and S. Meshkov, Phys. Rev. Letters 14, 670 (1965).
3. A good review of $SU(6)_W$ is given in H. Harari's lectures in Lectures in Theoretical Physics (University of Colorado Press, Boulder, Colorado, 1965), Vol. VIII-B.
4. $SU(6)_W$ is, of course, meaningless for non-collinear processes. If we admit the idea of Regge pole dominance and factorization, we can see why it does not work for collinear processes either. This is because $SU(6)_W$ relates spin nonflip amplitudes to flip-flip amplitudes, and the latter vanish as a consequence of factorization.
5. L. Van Hove, Physics Letters 24B, 183 (1967); R.P. Feynman, Caltech lecture, unpublished (1967).
6. R. Carlitz and M. Kislinger, Phys. Rev. Letters 24, 186 (1970).
7. See R. Delbourgo and H. Rashid, Phys. Rev. 176, 2074 (1968). They give a model with $SU(6)_W$ symmetric vertices and no Regge cuts, but find it necessary to use parity-doubled quarks to eliminate $\sqrt{t}$ singularities. In our model, these singularities are cancelled by the cut terms.
8. G. Zweig, CERN report Th-402 (1964), unpublished.

9. H. Harari, Phys. Rev. Letters 22, 562 (1969).

10. J. Rosner, Phys. Rev. Letters 22, 689 (1969).

11. Normal SU(6) is clearly not an appropriate symmetry for the vertices, for it requires a $q\bar{q}$ pair to annihilate in a singlet spin state--contrary to angular momentum and parity conservation.

12. See B.W. Lee's lectures in Particle Symmetries (Gordon and Breach, New York, 1966).

13. We have calculated the entries in Table 3 by selecting a complete set of invariant vertex functions, evaluating these functions in a collinear frame, equating these values to those in Tables 1 and 2, and solving the resulting set of linear equations. An alternate method would be to construct relativistic wave functions and write down a covariant version of (1). See B. Sakita and K.C. Wali, Phys. Rev. 139, B1355 (1965).

14. For a meson with four-momentum $(E; 0, 0, q)$, $E^2 - q^2 = M_V{}^2$, the normalized polarization vector is $\varepsilon^o = (q; 0, 0, E)/M_V$.

15. Details of the Van Hove construction are discussed by R. Blankenbecler and R.L. Sugar, Phys. Rev. 168, 1597 (1968). See also Ref. 6.

16. There is an ambiguity in the Van Hove model of Reggeization corresponding to the ambiguity in the behavior of the propagator and vertex functions off the mass shell. We eliminate this ambiguity by choosing to Reggeize t-channel helicity amplitudes and including in the Van Hove sum (16) terms whose only t dependence is in the denominator $t - m(J)^2$.

17. This may not occur until quite high energies. From Eq. (26) we see that the leading term in the asymptotic expansion of the cut dominates only when $\alpha' \, t \, \ell n \, s >> 1$.

18. Note that our approach to wrong-signature nonsense dips is compatible with the argument that the exchange of a pair of exchange degenerate trajectories will produce no dip.

19. Photoproduction processes are included in Table 5 assuming vector dominance. This is an unambiguous procedure for the reactions listed because the VVP vertex is automatically gauge invariant. This will not be the case in general.

20. A summary of the data and an empirical rule for the presence of wrong-signature nonsense dips in vector exchange processes is given in a review talk on photoproduction by H. Harari at the Fourth International Symposium on Electron and Photon Interactions at High Energies (Liverpool, England; September 1969), unpublished.

21. K. Bardacki and M.B. Halpern, "Dual Models Without Parity Doubling", University of California at Berkeley preprint, to be published.

22. S. Ellis, to be published.

FIGURE CAPTIONS

Fig. 1: Quark graph for meson-baryon vertex.

Fig. 2: Kinematics for $\pi\pi \to \pi\omega$.

Fig. 3: Dashed line - initial contour; solid line - opened contour.

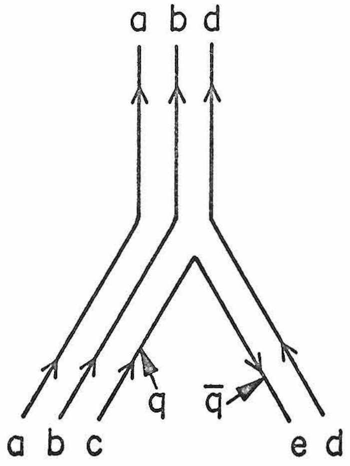

Fig. 1

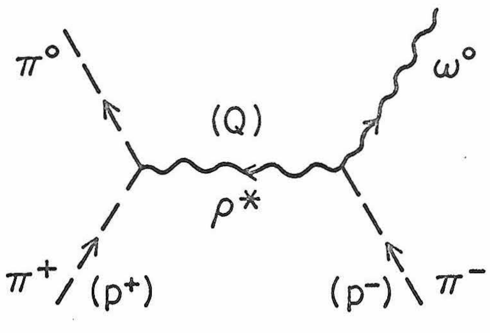

Fig. 2

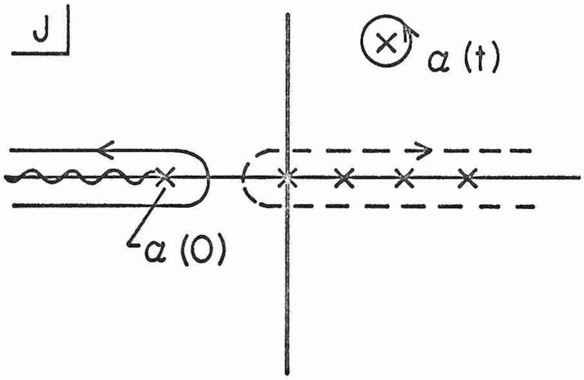

Fig. 3