AIRPLANE TAKEOFF PERFORMANCE

Thesis by

Victor J. Martin

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SUMMARY

In this paper a general equation for calculating the takeoff performance of landplanes is developed in terms of the usual airplane parameters.

The analysis enables the ground effect, wind and atmospheric conditions, altitude, and the use of flaps to be taken into account. A simple and rapid method is presented for the determination of the best flap angle to be used for takeoff.

Variation in takeoff performance due to changes in any of the airplane or engine-propeller characteristics is readily determined through the introduction of a new takeoff acceleration parameter.

A method for calculating the propeller speed, engine power, and thrust variation for both fixed-pitch and constant-speed propellers is included, and a new significance is given to the thrust versus square of velocity curve for takeoff computations.

Examples are included to illustrate the use of the analysis in solving the various takeoff problems.

INTRODUCTION

Recent rapid advances in the design of high speed airplanes with high wing and power loadings have made the problem of takeoff performance one of major importance.

Propellers designed to give maximum efficiency at the normal cruising or high speeds of modern airplanes are as a result of such selection usually very inefficient in the takeoff region, even for constant-speed or two-pitch arrangements. Power loadings are kept as high as possible to increase the range and payload, resulting in lower thrust-weight ratios. High speed performance necessitates high wing loadings to reduce parasite drag, with the result that takeoff speeds are increased. These and other factors tend to increase the takeoff distances to an extent such that they become critical in design, often determining the limiting values for power and wing loadings, and propeller diameters.

Among the various means used to improve this condition are the use of constant-speed propellers to obtain the maximum possible engine power during the takeoff run, and the use of flaps to decrease the takeoff speed.

Due to the critical nature of the takeoff problem in modern airplane design, there was felt to be a definite need for some rapid and accurate method of calculating the takeoff performance, and of readily determining the influence on takeoff of the various airplane and engine-propeller characteristics, to aid the designer in weighing the relative importance of different compromises in meeting the takeoff requirements.

The present analysis was started with the foregoing in view.

I. DEVELOPMENT OF THE GENERAL TAKEOFF EQUATION

Neglecting the tail lift and drag, which is separately accounted for by a correction to the initial thrust, the equations of motion of the airplane during takeoff become:

$$T - D - \mu N = \frac{W}{g} a_x \tag{1.0}$$

$$L + N - W = 0 \tag{1.1}$$

where

T = propeller thrust

D = air drag

μ = coefficient of rolling friction

N = normal ground force

W = gross weight of airplane

g = acceleration due to gravity

a_x = horizontal acceleration

L = lift of wing

Eliminating N between equations (1.0) and (1.1) and putting the resulting equation into dimensionless form,

$$\frac{T}{W} - \frac{D}{W} - \mu\left(1 - \frac{L}{W}\right) = \frac{a_x}{g} \tag{1.2}$$

The drag may be split up into two parts in accordance with the Prandtl wing theory,

$$\frac{D}{W} = \frac{D_p}{W} + \frac{D_i}{W} \tag{1.3}$$

where

D_p = parasite drag = qf

f = equivalent parasite area (See reference 1)

ρ = mass density of air

V = velocity

$q = \frac{1}{2}\rho V^2$

D_i = induced drag = $L^2/\pi q b_1^2$

b_1 = effective span including ground effect

$b_1^2 = e_1 k b^2$

e_1 = airplane efficiency factor for takeoff calculations

k = Munk's span factor

b = largest individual span of wing cellule

The total drag may then be written,

$$\frac{D}{W} = \frac{q}{l_p} + \left(\frac{L}{W}\right)^2 \frac{l_{s_1}}{\pi q} \tag{1.4}$$

where

$l_p = W/f$ = parasite loading

$l_{s_1} = W/b_1^2$ = effective span loading for takeoff

Substituting equation (1.4) into equation (1.2),

$$\frac{T}{W} - \left[\frac{q}{l_p} + \left(\frac{L}{W}\right)^2 \frac{l_{s_1}}{\pi q}\right] - \mu\left(1 - \frac{L}{W}\right) = \frac{a_x}{g} \tag{1.5}$$

The variation of thrust with velocity must now be determined. Customary practice is to assume a linear variation of thrust with the speed or with the square of the speed. Examination of a number of modern arrangements having constant-speed propellers revealed the fact that the thrust variation can be closely approximated by assuming a linear variation with the square of the velocity, so this type of variation has been used in the analysis. The particular significance of this choice of thrust variation will be discussed later.

If T_o is the initial thrust, and T_T the thrust at take-off speed, the variation will be:

$$\frac{T}{W} = \frac{T_o}{W} - \left(\frac{T_o - T_T}{W}\right)\left(\frac{V}{V_T}\right)^2 = \frac{T_o}{W} - a\left(\frac{V}{V_T}\right)^2 \qquad (1.6)$$

where V_T = takeoff speed

Equation (1.6) is now substituted in equation (1.5),

$$\frac{T_o}{W} - a\left(\frac{V}{V_T}\right)^2 - \left[\frac{q}{\ell_p} + \left(\frac{L}{W}\right)^2 \frac{\ell s_1}{\pi q}\right] - \mu\left(1 - \frac{L}{W}\right) = \frac{a_x}{g} \qquad (1.7)$$

The only remaining variable to be evaluated is the lift, which will vary with the speed as well as with the technique of takeoff, which will in turn depend on the type of field; i.e., the coefficient of friction, μ.

For fields with a high μ, the lift should be kept high to reduce the frictional force, μN; while if μ is low, the lift should be kept small to reduce the induced drag.

Clearly there is some optimum lift or C_L at which the total resistance during takeoff is a minimum. This is readily found from the total resistance,

$$R = qf + \frac{L^2}{\pi q b_1^2} + \mu(W - L) \qquad (1.8)$$

At any time or velocity, the dynamic pressure, q, is independent of the lift, so for the minimum total resistance equation (1.8) is differentiated with respect to L, with q constant, and equated to zero,

$$\frac{dR}{dL} = \frac{2L}{\pi q b_1^2} - \mu = 0 \tag{1.9}$$

Putting $L = qSC_L$, where S = wing area,

$$\frac{2qSC_L}{\pi q b_1^2} = \mu \tag{1.10}$$

from which is determined the optimum C_L for takeoff,

$$C_{L_0} = \frac{\pi}{2}\frac{b_1^2}{S}\mu = \frac{\pi}{2} AR_1 \mu \tag{1.11}$$

where AR_1 is the effective aspect ratio near the ground.

Since the optimum C_L is independent of velocity, it is held constant during the takeoff run. When the airplane has accelerated to the takeoff speed corresponding to the maximum lift coefficient, it is pulled into the attitude of maximum lift and flown off. It is evident that this technique will result in the shortest possible takeoff run.

The optimum lift becomes,

$$\frac{L}{W} = \frac{\pi}{2}\mu q \frac{b_1^2}{W} = \frac{\pi \mu q}{2 l_{s_1}} \tag{1.12}$$

In some cases the optimum C_L cannot be held during takeoff due to C_{L_m} limitations or to flaps, so a factor ϵ will be included in the lift term, where

$$\epsilon = C_L/C_{L_0}$$

The lift becomes,

$$\frac{L}{W} = \frac{\pi \mu q}{2 l_{s_1}}\epsilon \tag{1.13}$$

This value for the lift is now substituted into equation

Note: This result has been given by Hartman in reference 9.

(1.7), and after cancelling and rearranging terms,

$$\left\{\frac{a_x}{g} + \left(\frac{\rho}{2 l_p} - \frac{\pi \mu^2 \rho (2\epsilon - \epsilon^2)}{8 l_{s_1}}\right) V^2 + a\left(\frac{V}{V_T}\right)^2 - \left(\frac{T_0}{W} - \mu\right) = 0\right\} \quad (1.14)$$

Since the acceleration is independent of a constant wind velocity, and the drag and thrust depend on the relative velocity of the wind with respect to the plane, equation (1.14) may be regarded as the equation of motion of the airplane relative to the air. If we now define:

V = velocity of plane with respect to ground

V_a = velocity of plane with respect to the air

V_w = velocity of headwind with respect to ground

V_T = takeoff velocity of plane

Then $V_a = V + V_w$ (1.15)

Putting $a_x = \frac{dV_a}{dt}$, equation (1.14) becomes,

$$\left\{\frac{1}{g}\frac{dV_a}{dt} + \left(\frac{\rho}{2 l_p} - \frac{\pi \mu^2 \rho (2\epsilon - \epsilon^2)}{8 l_{s_1}}\right) V_a^2 + a\left(\frac{V_a}{V_T}\right)^2 - \left(\frac{T_0}{W} - \mu\right) = 0\right\} \quad (1.16)$$

This may be written,

$$\frac{dV_a}{dt} + A V_a^2 - B = 0 \quad (1.17)$$

where $A = g\left(\frac{\rho}{2 l_p} - \frac{\pi \mu^2 \rho (2\epsilon - \epsilon^2)}{8 l_{s_1}} + \frac{a}{V_T^2}\right)$

$B = g\left(\frac{T_0}{W} - \mu\right)$

$$\frac{dV_a}{\left(\frac{B}{A} - V_a^2\right)} = A\,dt \quad (1.18)$$

Integrating,

$$\left[\sqrt{\frac{A}{B}} \tanh^{-1} V_a \sqrt{\frac{A}{B}} = At + C_1\right] \quad (1.19)$$

At $t = 0$, $V_a = V_W$,

$$\therefore \quad C_1 = \sqrt{\frac{A}{B}} \tanh^{-1} V_W \sqrt{\frac{A}{B}}$$

$$t = \frac{1}{\sqrt{AB}} \left[\tanh^{-1} V_a \sqrt{\frac{A}{B}} - \tanh^{-1} V_W \sqrt{\frac{A}{B}} \right] \tag{1.20}$$

Equation (1.20) gives the time for the plane to reach a velocity V_a, relative to the wind. For takeoff time, $V_a = V_T$,

$$t = \frac{1}{\sqrt{AB}} \left[\tanh^{-1} V_T \sqrt{\frac{A}{B}} - \tanh^{-1} V_W \sqrt{\frac{A}{B}} \right] \tag{1.21}$$

Equation (1.21) gives the takeoff time with any wind velocity. Putting $V_a = \frac{dSa}{dt}$ into equation (1.21) and rearranging terms,

$$\sqrt{\frac{A}{B}}\, dS_a = \tanh \left[\sqrt{AB}\, t + \tanh^{-1} V_W \sqrt{\frac{A}{B}} \right] dt \tag{1.22}$$

Integrating,

$$\sqrt{\frac{A}{B}}\, S_a = \frac{1}{\sqrt{AB}} \log \cosh \left[\sqrt{AB}\, t + \tanh^{-1} V_W \sqrt{\frac{A}{B}} \right] + C_2 \tag{1.23}$$

At $t = 0$, $S_a = 0$,

$$\therefore \quad C_2 = -\frac{1}{\sqrt{AB}} \log \cosh \left[\tanh^{-1} V_W \sqrt{\frac{A}{B}} \right]$$

$$S_a = \frac{1}{A} \log \frac{\cosh \left[\sqrt{AB}\, t + \tanh^{-1} V_W \sqrt{\frac{A}{B}} \right]}{\cosh \left[\tanh^{-1} V_W \sqrt{\frac{A}{B}} \right]} \tag{1.24}$$

Substituting the time for takeoff into equation (1.24),

$$S_a = \frac{1}{A} \log \frac{\cosh \left[\tanh^{-1} V_T \sqrt{\frac{A}{B}} \right]}{\cosh \left[\tanh^{-1} V_W \sqrt{\frac{A}{B}} \right]} \tag{1.25}$$

S_a, however, is not the takeoff distance, but the distance the airplane has travelled with respect to the air. If S is the takeoff distance, and t is the takeoff time,

$$S = \int_0^t V dt = \int_0^t (V_a - V_w) dt = \int_0^t V_a dt - V_w \int_0^t dt$$

So finally,

$$\left[S = S_a - V_w t \right] \tag{1.26}$$

The takeoff distance is then,

$$S = \frac{1}{A}\left[\text{LOG} \frac{\text{COSH TANH}^{-1} V_T \sqrt{\frac{A}{B}}}{\text{COSH TANH}^{-1} V_W \sqrt{\frac{A}{B}}} - V_W \sqrt{\frac{A}{B}} \left(\text{TANH}^{-1} V_T \sqrt{\frac{A}{B}} - \text{TANH}^{-1} V_W \sqrt{\frac{A}{B}} \right) \right] \tag{1.27}$$

Using the relations:

$$\text{COSH TANH}^{-1} X = \frac{1}{\sqrt{1 - X^2}} \qquad \text{AND} \qquad \text{LOG} \sqrt{X} = \frac{1}{2} \text{LOG } X,$$

the takeoff distance may be written,

$$S = \frac{1}{2A}\left[\text{LOG} \frac{\left(1 - \frac{A}{B} V_W^2\right)}{\left(1 - \frac{A}{B} V_T^2\right)} - 2 V_W \sqrt{\frac{A}{B}} \left(\text{TANH}^{-1} V_T \sqrt{\frac{A}{B}} - \text{TANH}^{-1} V_W \sqrt{\frac{A}{B}} \right) \right] \tag{1.28}$$

For zero wind the takeoff distance reduces to,

$$S = \frac{1}{2A} \text{ LOG} \frac{1}{\left(1 - \frac{A}{B} V_T^2\right)} \tag{1.29}$$

In the present form equation (1.29) becomes indeterminate when A is zero, and the equation as a whole does not give a clear idea as to the importance of the various terms, so write

$$S = \frac{1}{2A} \times \frac{B}{V_T^2} \times \frac{V_T^2}{B} \text{ LOG} \frac{1}{\left(1 - \frac{A}{B} V_T^2\right)} = \left[\frac{V_T^2}{2B} \times \frac{1}{\frac{A}{B} V_T^2} \text{ LOG} \frac{1}{\left(1 - \frac{A}{B} V_T^2\right)} \right] \tag{1.30}$$

Defining $\lambda_a = \frac{A}{B} V_T^2$, the takeoff distance may be written,

$$S = \frac{V_T^2}{2B}\left[\frac{1}{\lambda_a} \text{LOG} \frac{1}{(1-\lambda_a)}\right] = \frac{V_T^2}{2B}\Phi(\lambda_a) \tag{1.31}$$

For zero wind, the takeoff time is,

$$t = \frac{1}{\sqrt{AB}} \text{TANH}^{-1} V_T\sqrt{\frac{A}{B}} = \frac{V_T}{B} \times \frac{1}{V_T\sqrt{\frac{A}{B}}} \text{TANH}^{-1} V_T\sqrt{\frac{A}{B}} \tag{1.32}$$

$$t = \frac{V_T}{B}\left[\frac{\text{TANH}^{-1}\sqrt{\lambda_a}}{\sqrt{\lambda_a}}\right] = \frac{V_T}{B}\psi(\lambda_a) \tag{1.33}$$

Substituting into λ_a the values for the constants A and B,

$$\lambda_a = \frac{g\left(\frac{\rho}{2 l_p} - \frac{\pi\mu^2\rho(2\epsilon-\epsilon^2)}{8 l_{s_1}} + \frac{T_0 - T_T}{W V_T^2}\right)}{g\left(\frac{T_0}{W} - \mu\right)} V_T^2 \tag{1.34}$$

Putting,

$$\rho V_T^2 = \frac{2W}{C_{L_m} S}$$

$$l_p = \frac{W}{f}$$

$$l_{s_1} = \frac{W}{b_1^2}$$

$$\lambda_a = \frac{\left(\frac{f/S}{C_{L_m}} - \frac{(2\epsilon-\epsilon^2)}{C_{L_m}} \times \frac{\pi\mu^2 b_1^2}{4S} + \frac{T_0 - T_T}{W}\right)}{\left(\frac{T_0}{W} - \mu\right)} \tag{1.35}$$

Remembering that the optimum $C_{Lo} = \frac{\pi}{2} AR_1 \mu$

Corresponding $C_{Di_o} = \frac{C_{Lo}^2}{\pi AR_1} = \frac{\pi}{4} AR_1 \mu^2$ (1.36)

Also $f/S = C_{Dp}$

Then,

$$\lambda a = \frac{\left(\frac{C_{Dp} - C_{Di_o}(2\epsilon - \epsilon^2)}{C_{Lm}} + \frac{T_o - T_T}{W}\right)}{\left(\frac{T_o}{W} - \mu\right)} \tag{1.37}$$

When the airplane is taking off at the optimum C_{Lo}, λa becomes,

$$\lambda a = \frac{\left(\frac{C_{Dp} - C_{Di_o}}{C_{Lm}} + \frac{T_o - T_T}{W}\right)}{\left(\frac{T_o}{W} - \mu\right)} \tag{1.38}$$

For V_T in miles per hour, the takeoff distance is,

$$S = \frac{V_T^2}{29.9\left(\frac{T_o}{W} - \mu\right)} \Phi(\lambda a) \tag{1.39}$$

Once λa has been calculated, $\Phi(\lambda a)$ may be found in Fig.6.

For V_T in miles per hour, the takeoff time is,

$$t = \frac{V_T}{21.9\left(\frac{T_o}{W} - \mu\right)} \psi(\lambda a) \tag{1.40}$$

The function $\psi(\lambda a)$ is also plotted in Fig.6.

For calculations without the use of Fig.6, the following series are useful:

$$\Phi(\lambda a) = \left(1 + \frac{\lambda a}{2} + \frac{\lambda a^2}{3} + \cdots\right) \quad (1.41)$$

$$\psi(\lambda a) = \left(1 + \frac{\lambda a}{3} + \frac{\lambda a^2}{5} + \cdots\right) \quad (1.42)$$

Significance of the parameter λa

When λ_a approaches the value 1.0, it is seen from Fig.6 or equations (1.41) and (142) that both $\Phi(\lambda_a)$ and $\psi(\lambda_a)$ become infinite, and consequently the takeoff distance and time become infinite. Setting $\lambda_a = 1.0$, and reducing, the following expression results,

$$T_T = \mu\,(W-L) + D \quad (1.43)$$

The term μ(W-L) is recognized as the ground friction force, so it is seen that when $\lambda_a = 1.0$, the thrust at takeoff speed is just equal to the total ground plus air resistance, and the plane could never reach takeoff speed.

It is evident then that λ_a is the ratio of the change in net accelerating force during the takeoff run, to the initial net accelerating force.

II EFFECT OF WIND ON TAKEOFF PERFORMANCE

A. Takeoff distance

Using equations (1.28) and (1.29), the ratio of takeoff distance with wind to that without wind is found,

$$\frac{S_W}{S_0} = 1 - \frac{\log\left[1-\lambda_a\left(\frac{V_W}{V_T}\right)^2\right]}{\log\left[1-\lambda_a\right]} + \frac{2\frac{V_W}{V_T}\sqrt{\lambda_a}}{\log(1-\lambda_a)}\left[\tanh^{-1}\sqrt{\lambda_a} - \tanh^{-1}\left(\frac{V_W}{V_T}\sqrt{\lambda_a}\right)\right] \quad (2.0)$$

where

S_W = takeoff distance with headwind V_W

S_o = takeoff distance in zero wind

Equation (2.0) shows the ratio $\frac{S_W}{S_o}$ to be a function only of λ_a and the ratio $\frac{V_W}{V_T}$, so for a given λ_a, there is a definite variation with wind velocity.

Defining $\frac{A}{B}V_W^2 = \lambda_W$, equation (2.0) may be written,

$$\frac{S_W}{S_o} = 1 - 2\frac{V_W}{V_T}\left[\frac{\psi(\lambda a)}{\Phi(\lambda a)}\right] + \left(\frac{V_W}{V_T}\right)^2\left[2\,\frac{\psi(\lambda_W)}{\Phi(\lambda a)} - \frac{\Phi(\lambda_W)}{\Phi(\lambda a)}\right] \tag{2.1}$$

Using this equation and Fig.6, curves of the variation of takeoff distance with wind velocity for various values of λ_a have been plotted in Fig.7.

When λ_a is small, that is if the net accelerating force is nearly constant during the takeoff run, $\Phi(\lambda_a) = \psi(\lambda_a) = 1.0$, and $\Phi(\lambda_W) = \psi(\lambda_W) = 1.0$, and equation (2.1) reduces to,

$$\frac{S_W}{S_o} = 1 - 2\frac{V_W}{V_T} + \left(\frac{V_W}{V_T}\right)^2 = \left(1 - \frac{V_W}{V_T}\right)^2 \tag{2.2}$$

The range of values of λ_a for which this simple expression may be used with sufficient accuracy can be seen from Fig.7, since equation (2.2) is exact for $\lambda_a = 0$.

B. Takeoff time

Using equations (1.21) and (1.32), the ratio of takeoff time with wind to that with zero wind is,

$$\frac{t_W}{t_o} = 1 - \frac{\tanh^{-1}\frac{V_W}{V_T}\sqrt{\lambda a}}{\tanh^{-1}\sqrt{\lambda a}} \tag{2.3}$$

where

t_w = takeoff time with headwind V_w

t_o = takeoff time with zero wind

The takeoff time variation with wind is also a function of only λ_a and $\frac{V_w}{V_T}$, and curves of the variation of takeoff time with wind velocity for various values of λ_a have been plotted in Fig.8 using Fig.6 and the following form of equation (2.3):

$$\frac{t_w}{t_o} = 1 - \frac{V_w}{V_T}\left[\frac{\psi(\lambda_w)}{\psi(\lambda_a)}\right] \tag{2.4}$$

For λ_a small, this reduces to,

$$\frac{t_w}{t_o} = \left(1 - \frac{V_w}{V_T}\right) \tag{2.5}$$

III EFFECT OF TAIL LOAD ON TAKEOFF PERFORMANCE

In developing the general takeoff equation, the tail lift and drag were neglected. Since, however, the tail must lift a load of the order 0.1W near the start of the takeoff run, it was thought advisable to investigate the induced drag of the tail caused by this load, especially as the aspect ratio of the tail is usually small. The parasite drag of the tail was assumed to be accounted for in the total parasite drag of the airplane and was therefore neglected in this part of the analysis.

Assuming a mean center of gravity position of .33 chord back from the wheel center, and a mean length from the center of gravity position to the center of pressure on the tail surface of three times the chord, the tail load necessary to hold the tail off the ground during the takeoff run was calculated for a number of airplanes having various takeoff speeds.

The velocity of the slipstream was calculated using the momentum theory as described in reference 3.

The induced drag of the tail was then calculated,

$$\frac{D_{i_t}}{W} = \frac{L_t^2}{\pi q_t b_{e_t}^2} \tag{3.0}$$

where

D_{i_t} = induced drag of tail

L_t = lift of tail

V_t = average velocity of air over tail

$q_t = \frac{1}{2}\rho V_t^2$

b_{e_t} = effective span of tail

A mean efficiency factor for the tail surface was taken as 0.8, from which,

$$b_{e_t}^2 = 0.8 b_t^2 \tag{3.1}$$

b_t = tail span

For multi-engined planes, V_t was assumed to be the slipstream velocity, i.e., the propeller wakes were considered to completely cover the tail surface. For single-engined planes, V_t was taken as the average velocity of the slipstream and the surrounding air over the tail based on the tail area included in the propeller diameter, using a normally shaped tail surface.

The tail drag variation with velocity was calculated for a number of different types of airplanes.

The assumption of a linear variation of tail drag with q, from the value calculated at the start of the takeoff run, to zero at takeoff speed was found to be a close approximation. Thus the tail drag could be taken into account by applying a correction to the initial thrust.

The correction to be applied may be found from Fig.4 and Fig.5 which were constructed from the following approximate formulas:

For single-engined airplanes,

$$\Delta \frac{T_0}{W} = - \frac{.005}{\frac{T_0}{W}\left(1.4 - 4\frac{D}{b_t}\right)^2} \tag{3.2}$$

For multi-engined airplanes,

$$\Delta \frac{T_0}{W} = - \frac{.005\,n}{\frac{T_0}{W}} \left(\frac{D}{b_t}\right)^2 \tag{3.3}$$

where

$\Delta\, T_0/W$ = correction to be applied to initial thrust

D = propeller diameter

b_t = tail span

n = number of engines

T_0/W = total initial thrust-weight ratio

IV DETERMINATION OF FACTORS IN TAKEOFF FORMULAS

(A) Stalling speed

The normal power-off stalling speed may be estimated in the manner described in either reference 1 or reference 4. The stalling speed used in the analysis is for the particular altitude for which the takeoff is to be calculated. The stalling speed at altitude may be found from the sea level stalling speed by the relation,

$$V_s = \frac{V_{so}}{\sqrt{\sigma}} \qquad (4.0)$$

where

V_s = stalling speed at altitude

V_{so} = sea level stalling speed

In case the airplane is attempting to pass a takeoff requirement, it may be desirable to take off at the power-on stalling speed, which in many cases will considerably reduce the takeoff distance. In reference 5, takeoff measurements show that it is possible to realize this reduction in takeoff speed due to power.

At the takeoff speed, the weight of the airplane is effectively reduced by the amount $(T_T \sin \theta)$, where θ is the angle of the thrust axis with the horizontal, with the airplane in the attitude of maximum lift. Since the stalling speed is proportional to the square root of the weight,

$$\frac{V_P}{V_T} = \sqrt{\left(1 - \frac{T_T}{W} \sin \theta\right)} \qquad (4.1)$$

Conservatively neglecting the small change in λ_a due to the apparently increased C_{L_m}, the ratio of takeoff distance corresponding to the power-on takeoff speed to the normal takeoff distance is,

$$\frac{S_P}{S_O} = \left(\frac{V_P}{V_T}\right)^2 \qquad (4.2)$$

$\frac{V_P}{V_T}$ has been plotted in Fig.9.

(B) Parasite and induced drags

If wind-tunnel lift and drag data are available, C_D is plotted against C_L^2, and the best fitting straight extrapolated to $C_L = 0$, where the value of C_{D_p} to be used is found.

If the slope of the straight line is $m = dC_D/dC_L^2$, the airplane efficiency factor, e, neglecting ground effect, becomes,

$$e = \frac{1}{m \pi AR} \tag{4.3}$$

where $AR = b^2/S$ = geometrical aspect ratio

In case no wind-tunnel data is available, C_{D_p} and e may be estimated as described in reference 1.

Effect of ground on induced drag

Using the efficiency factor found from equation (4.3), the total drag coefficient, neglecting ground effect, is

$$C_D = C_{D_p} + \frac{C_L^2}{\pi AR\, e} \tag{4.4}$$

For the purpose of introducing the effect of the ground, equation (4.10) may be written,

$$C_D = C_{D_p} + \frac{C_L^2}{\pi AR} + \frac{C_L^2}{\pi AR}\left(\frac{1-e}{e}\right) \tag{4.5}$$

The term $\frac{C_L^2}{\pi AR}\left(\frac{1-e}{e}\right)$ depends on the increase in parasite drag of the wing and fuselage with angle of attack, so the only term affected by the presence of the ground is $\frac{C_L^2}{\pi AR}$, the actual induced drag of the ground.

If γ is the ratio of the induced drag near the ground to the induced drag in the air, the drag coefficient in the presence of the ground becomes,

$$C_D = C_{D_p} + \frac{C_L^2}{\pi AR}\gamma + \frac{C_L^2}{\pi AR}\left(\frac{1-e}{e}\right) = C_{D_p} + \frac{C_L^2}{\pi AR}\left(\gamma + \frac{1}{e} - 1\right) \quad (4.6)$$

The effective efficiency for takeoff is then,

$$e_1 = \frac{1}{\left(\gamma + \frac{1}{e} - 1\right)} \quad (4.7)$$

Using data from reference 6 for γ, e_1 has been plotted against the ratio h/b for various values of e, in Fig.1, where

h = height of wing from ground

b = wing span

Knowing e and h/b for a given airplane, e_1 is found from Fig.1, and the effective aspect ratio for takeoff is calculated,

$$AR_1 = ARe_1 \quad (4.8)$$

Before the optimum C_{L_o} for takeoff can be found, the coefficient of rolling friction, μ, must be estimated for the particular field in question. If no specific data is available, values may be taken from reference 6.

Having AR_1 and μ, C_{L_o} and the corresponding C_{Di_o} may be found from Fig.2, or calculated as follows,

$$C_{L_o} = \frac{\pi}{2} AR_1 \mu \qquad C_{D_{i_o}} = \frac{\mu}{2} C_{L_o} \quad (4.9)$$

In case C_{L_o} is near or above C_{Lm}, the airplane should maintain a reduced C_L during the takeoff run, since the drag increases rapidly near the stall. The C_L used for takeoff should lie on that portion of the C_D vs C_L^2 curve which is approximated closely by the straight line assumption.

The factor ϵ is then found by taking the ratio of the C_L held during the takeoff run to the optimum C_{L_o} found from equation (4.9) or Fig.2. From Fig.3 find the term $(2\epsilon - \epsilon^2)$ by which to multiply $C_{D_{i_o}}$ to substitute in λ_a.

(C) Thrust variation

Since static thrust data is at present incomplete and often leads to erroneous results in takeoff calculations, the initial thrust for use in this analysis will be determined by extrapolation in the following manner:

In general,

$$T = \frac{550\ P\eta}{V} \tag{4.10}$$

where
T = propeller thrust, pounds
P = engine horsepower
η = propulsive efficiency
V = velocity, feet per second

During the takeoff run the power is taken as constant since the propeller speed is practically constant for fixed pitch propellers, (see reference 1) as well as for constant speed propellers.

The ratio of thrust at any speed to the thrust at takeoff speed is then,

$$\frac{T}{T_T} = \frac{550P\eta}{V}\ \frac{V_T}{550P\eta_T} = \frac{\eta/\eta_T}{V/V_T} \tag{4.11}$$

The ratio T/T_T is then plotted against V^2/V_T^2, and the best fitting straight line extrapolated to zero speed to get the initial thrust.

Fixed pitch propeller

When a fixed pitch propeller is used, the propeller r.p.m. must be determined. This may be estimated from curves given for particular types of propellers in reference 1, or may be calculated for any propeller by the following method.

For a given propeller blade angle, the C_s vs J curve is practically a straight line in the takeoff region, so for any given propeller the ratio $\frac{C_s}{J}$ is found on this straight portion, and denoted by $\left(\frac{C_s}{J}\right)_T$.

Letting the subscript $(\ )_T$ represent the takeoff condition, and the subscript $(\ )_o$ represent the design condition of the propeller, then,

$$\left(\frac{C_s}{J}\right)_T = \frac{0.638\,\sigma_T^{1/5} N_T^{3/5} D}{88\, P_T^{1/5}} \tag{4.12}$$

Assuming the power to vary linearly with the propeller speed, the power for takeoff is,

$$P_T = \frac{P_o}{N_o} N_T R_p \tag{4.13}$$

where R_p is the ratio of the power at takeoff altitude to the power at design altitude at constant r.p.m. For non-supercharged engines, this ratio may be calculated with the use of charts given in reference 1. For supercharged engines, the powers used in calculating R_p are the actual available powers at the two altitudes for a given r.p.m.

Substituting The value for P_T from equation (4.13) into equation (4.12),

$$\left(\frac{C_s}{J}\right)_T = \frac{0.638\, D N_T^{2/5} \sigma_T^{1/5}}{88\left(\frac{P_o}{N_o}\right)^{1/5} R_p^{1/5}} = \left(\frac{0.638\,\sigma_o^{1/5} N_o^{3/5} D}{88\, P_o^{1/5}}\right)\left(\frac{N_T}{N_o}\right)^{2/5}\left(\frac{\sigma_T}{\sigma_o R_p}\right)^{1/5} \tag{4.14}$$

From equation (4.12) it is seen that,

$$\left(\frac{0.638\,\sigma_o^{1/5} N_o^{3/5} D}{88\, P_o^{1/5}}\right) = \left(\frac{C_s}{J}\right)_o \tag{4.15}$$

which corresponds to the design condition of the propeller at the design altitude. Equation (4.20) then becomes,

$$\left(\frac{C_s}{J}\right)_T = \left(\frac{C_s}{J}\right)_o \left(\frac{N_T}{N_o}\right)^{2/5} \left(\frac{\sigma_T}{\sigma_o}\right)^{1/5} \left(\frac{1}{R_P}\right)^{1/5} \tag{4.16}$$

The ratio of the takeoff propeller speed to the design speed is,

$$\frac{N_T}{N_o} = \frac{\left(\frac{C_s}{J}\right)_T}{\left(\frac{C_s}{J}\right)_o} \left(\frac{\sigma_o}{\sigma_T}\right)^{1/2} R_P^{1/2} \tag{4.17}$$

The power for takeoff is,

$$P_T = P_o \left(\frac{N_T}{N_o}\right) R_P \tag{4.18}$$

C_{s_T} and J_T may now be calculated for the takeoff speed, and the corresponding propulsive efficiency, η_T, determined.

Since both P_T and N_T are constant during the takeoff run, C_s is proportional to the velocity in the takeoff region, so η can be found on the correct blade angle curve for $C_s = 0.9\, C_{s_T}$, $0.8\, C_{s_T}$, etc., to as small a value of C_s as possible, corresponding to $V/V_T = 0.9$, 0.8, etc., from which the variation in the thrust ratio is calculated using equation (4.11).

T/T_T is then plotted against V^2/V_T^2, and a straight line faired through the points and extrapolated to zero speed to get the initial thrust ratio T_o/T_T. The thrust at takeoff speed is,

$$\frac{T_T}{W} = \frac{375\, P_T\, \eta_T}{W\, V_T} \tag{4.19}$$

where V_T is in miles per hour.

The initial thrust-weight ratio is then,

$$\frac{T_o}{W} = \frac{T_T}{W}\left(\frac{T_o}{T_T}\right) \tag{4.20}$$

The tail load correction must be first applied to T_0/W before using the takeoff formulas and charts.

The above method of thrust determination applies to any set of propeller characteristics. For use of the N.A.C.A. data, reference 8 may be used to determine the thrust variation for fixed pitch propellers.

Constant speed propellers

When a constant speed propeller is to be used, the power available for takeoff will be,

$$P_T = P_0 R_p \tag{4.21}$$

Since the propeller speed for takeoff is N_0, C_{s_T} and J_T can be immediately calculated, from which the thrust variation is found as described for the fixed pitch propeller, except that instead of moving down a constant blade angle curve on the C_s vs J curves, a straight line from the condition at takeoff speed to the origin is followed, since both C_s and J are proportional to the velocity in this case. If the C_s vs J curves were exactly straight in the takeoff region, the blade angle would not change during takeoff with a constant speed propeller. If a constant blade angle is assumed during takeoff reference 8 may be used also for the case of the constant speed propeller thrust variation.

From an examination of a number of thrust calculations, it was found that the variation in takeoff distance was negligible for a given airplane if different thrust variations were used as long as the areas under thrust vs square of velocity curves were the same. This means that in fairing a straight line through calculated points on the thrust curve for any particular case, an attempt should be made to draw a line having the same area under it as does the actual thrust curve, attaching equal importance to a unit area near zero speed and a unit area near takeoff speed.

V-USE OF FLAPS FOR TAKEOFF

In considering the effect of flaps on takeoff, it is convenient to put $V_T^2 = 2\, l_w/\rho C_{L_m}$, where $l_w = W/S$ = wing loading, whereupon the distance for takeoff is,

$$S = \frac{l_w}{\rho g C_{L_m}\left(\frac{T_0}{W} - \mu\right)} \Phi(\lambda_a) \qquad (5.0)$$

In the following analysis the subscript $(\ )_F$ is used to indicate the condition with flaps, while no subscript is used for the condition without flaps, or with flaps at 0°.

The term $a = (T_0 - T_T)/W$ which appears in λ_a is the decrease in the thrust-weight ratio from zero speed to the takeoff speed corresponding to that for which T_T was calculated. If T_T is calculated for the takeoff speed without flaps, the term "a" for the flapped condition becomes,

$$a_F = \left(\frac{T_0 - T_T}{W}\right)\frac{V_{T_F}^2}{V_T^2} = \left(\frac{T_0 - T_T}{W}\right)\frac{C_{L_m}}{C_{L_{m_F}}} \qquad (5.1)$$

since the thrust curve is cut off at the point $V = V_{T_F}$.

The acceleration parameter for takeoff with flaps is then,

$$\lambda a_F = \frac{\left(\frac{C_{D_{PF}} - C_{D_{i_o}}}{C_{Lm_F}} + \frac{T_o - T_T}{W}\frac{C_{Lm}}{C_{Lm_F}}\right)}{\left(\frac{T_o}{W} - \mu\right)} \qquad (5.2)$$

If the airplane can maintain the attitude for the optimum lift coefficient during takeoff with flaps, $C_{D_{i_o}}$ is the same as for without flaps, assuming the same efficiency factor with flaps. In some cases with flaps at high angles, the optimum C_{L_o} cannot be obtained without the thrust axis being at a negative angle to the horizontal. In such cases a higher C_L will have to be maintained during the takeoff run, corresponding, for instance, to the thrust axis horizontal or at some definite positive angle, since it is undesirable to have the tail too high during takeoff because of nosing over possibilities. The factor ϵ will have to be found and the correction applied to $C_{D_{i_o}}$ as previously explained if flaps necessitate a C_L other than the optimum for takeoff.

The takeoff distance with flaps is,

$$S_F = \frac{\ell w}{\rho g\left(\frac{T_o}{W} - \mu\right)} \frac{1}{C_{Lm_F}} \Phi(\lambda a_F) \qquad (5.3)$$

The ratio of the takeoff distance with flaps to that without flaps is then,

$$\frac{S_F}{S_o} = \frac{C_{Lm}}{\Phi(\lambda a)}\left[\frac{\phi(\lambda a_F)}{C_{Lm_F}}\right] \qquad (5.4)$$

Since the term $\left[\frac{C_{Lm}}{\Phi(\lambda a)}\right]$ is constant for all flap angles, the flap angle resulting in the shortest takeoff distance will be that corresponding to the minimum value of $\left[\frac{\Phi(\lambda a_F)}{C_{Lm_F}}\right]$.

λa_F is calculated for several flap angles from equation

(5.2) and the corresponding values of $\Phi(\lambda_{a_F})$ determined from Fig.6. $\Phi(\lambda_{a_F})$ is then plotted against $C_{L_{m_F}}$. A line from the origin drawn tangent to this curve gives the minimum value of $\frac{\Phi(\lambda_{a_F})}{C_{L_{m_F}}}$, and the corresponding best flap angle is determined from the value of $C_{L_{m_F}}$ at the point of tangency.

VI EXAMPLE OF TAKEOFF PERFORMANCE CALCULATION

Airplane characteristics used in this example were taken from reference 10, in which complete full-scale wind-tunnel data are presented for the Fairchild F-22 parasol monoplane.

Propeller characteristics were taken from large scale fascimiles of the charts given in reference 11.

Airplane data

$W = 1467$ lb.	$P_o = 95$ @ 2100 r.p.m.	$C_{L_m} = 1.32$
$S = 162$ sq. ft.	$D = 7$ ft. (2-blade)	$h/b = 0.3$
$b = 30$ ft.	$b_t = 10$ ft.	$AR = 5.55$

From a plot of C_D vs C_L^2, we readily determine from equation (4.3),

$$e = 0.87$$

$$C_{D_p} = 0.06$$

From Fig.1, using $h/b = 0.3$, find $e_1 = 1.015$, from which,

$$AR_1 = 1.015 \times 5.55 = 5.63$$

Assuming $\mu = 0.05$, from Fig.2 find,

$$C_{L_o} = 0.45$$

$$C_{D_{i_o}} = 0.011$$

Thrust variation

Corresponding to a maximum velocity of 93.5 m.p.h., find,

$C_{s_0} = 1.125$

$J_0 = V/ND = 0.56$ $\qquad\qquad C_{s_0}/J_0 = 2.01$

From propeller characteristics, the blade angle is 17°. The ratio $\frac{C_s}{J}$ is measured on the straight portion of the 17° C_s vs J curve,

$$\left(\frac{C_s}{J}\right)_T = 1.935$$

Since the propeller was designed at sea level, and the takeoff is being calculated also for sea level,

$\sigma_0 = \sigma_T = R_p = 1.0$, and from equation (4.17),

$N_T/N_0 = (1.935/2.01)^{5/2} = 0.909$, from which the takeoff propeller r.p.m. is,

$$N_T = 0.909 \times 2100 = 1910 \text{ r.p.m.}$$

The power available for takeoff is,

$$P_T = 0.909 \times 95 = 86.3 \text{ hp.}$$

The takeoff speed corresponding to $C_{L_m} = 1.32$ is 51.5 m.p.h., from which we find,

$$C_{s_T} = 0.654$$

$$J_T = 0.339$$

$$\eta_T = 0.550$$

The thrust-weight ratio at takeoff speed is found from equation (4.19),

$$\frac{T_T}{W} = \frac{375 \times 86.3 \times 0.550}{1467 \times 51.5} = 0.236$$

The thrust variation is then found as previously described, and tabulated in the table on page 28. Propulsive efficiencies are found from the propeller characteristics on the 17° blade angle curve corresponding to the tabulated values of C_s. The ratio T/T_T is found from equation (4.11).

Thrust Calculations for Fairchild F-22

(1)	(2)	(3)	(4)	(5)	(6)
V/V_T	C_s	η	η/η_T	T/T_T	$(V/V_T)^2$
1.0	.654	.550	1.000	1.000	1.00
.9	.588	.500	.909	1.011	.81
.8	.523	.450	.818	1.023	.64
.7	.458	.398	.723	1.033	.49
.6	.393	.344	.626	1.043	.36
.5	.327	.289	.525	1.050	.25
.4	.262	.232	.422	1.055	.16
.3	.196	.175	.318	1.060	.09

This thrust variation has been plotted in figure 10. From the intersection of the straight line faired through the calculated points with the zero speed axis, read: $T_o/T_T = 1.066$

The initial thrust-weight ratio is,

$$T_o/W = 1.066 \times .236 = .252$$

From Fig.4 is found the tail drag correction, using a ratio of propeller diameter to tail span of $D/b_t = 0.7$,

$$\Delta T_o/W = -.016$$

The corrected initial thrust-weight ratio is,

$$T_o/W = .252 - .016 = .236$$

For this particular case,

$$a = (T_o - T_T)/W = .236 - .236 = 0$$

Assuming $\mu = .05$, λ_a is now calculated:

$$\lambda_a = \frac{\left(\frac{(.06 - .011)}{1.32} + 0\right)}{(.236 - .05)} = .200$$

From Fig.6,

$$\Phi(\lambda_a) = 1.116$$

$$\psi(\lambda_a) = 1.076$$

Equation (1.39) gives the takeoff distance in zero wind,

$$S = 532 \text{ ft.}$$

Equation (1.40) gives the takeoff time in zero wind,

$t = 13.6$ seconds

Assuming a 10 m.p.h. headwind, $V_W/V_T = 10/51.5 = .194$

From Fig.7, using $\lambda_a = .200$, find

$S_W/S_o = .66$, from which the takeoff distance with a 10 m.p.h. wind becomes,

$S_W = .66 \times 532 = 351$ ft.

From Fig.8, find $t_W/t_o = .82$, and

$t_W = .82 \times 13.6 = 11.2$ seconds

Determination of Best Flap Angle for takeoff:

λ_{a_F} must first be determined for each flap angle for which data is available, from which $\Phi(\lambda_{a_F})$ is found from Fig.6. In calculating λ_{a_F}, when the thrust axis is at a negative angle for $C_{L_o} = .45$, use C_L corresponding to the thrust axis horizontal. For example, at a flap angle $\delta_F = 20^\circ$ (Fig.5, ref.10) $C_L = .45$ is found at an angle θ, of the thrust axis, $= -3.5^\circ$, so use $C_L = .685$ at $\theta = 0^\circ$. At $C_L = .685$ read $C_D = .115$. Instead of plotting C_D vs C_L^2 for $\delta_F = 20^\circ$ to obtain C_{D_p}, find C_{D_i} from Fig.2 corresponding to $C_L = .685$ and for the effective aspect ratio away from the ground; $AR_e = AR \times e = 5.55 \times .87 = 4.83$.

$C_{D_i} = .03$

$C_{D_{pF}} = C_D - C_{D_i} = .115 - .03 = .085$

$C_{D_{i_o}} = .011$ (same as without flaps)

$\epsilon = .685/.45 = 1.524$

From Fig.3, read $(2\epsilon - \epsilon^2) = .66$

$C_{L_{mF}} = 1.59$ at $\delta_F = 20^\circ$ (Fig.7, ref.10)

$(T_o/W - \mu) = .236 - .05 = .186$ (same as without flaps)

$$\lambda_{a_F} = \frac{(.085 - .011 \times .66)}{(1.59 \times .186)} = .254$$

$\Phi(\lambda_{a_F}) = 1.153$

Results of similar calculations for the remaining flap angles are tabulated in the table below:

δ_F	θ	C_L	λ_{a_F}	$\Phi(\lambda_{a_F})$	$C_{L_{m_F}}$
0	1	.450	.200	1.116	1.32
20	0	.685	.254	1.153	1.59
40	0	.900	.463	1.342	1.79
59	0	1.065	.555	1.455	1.88

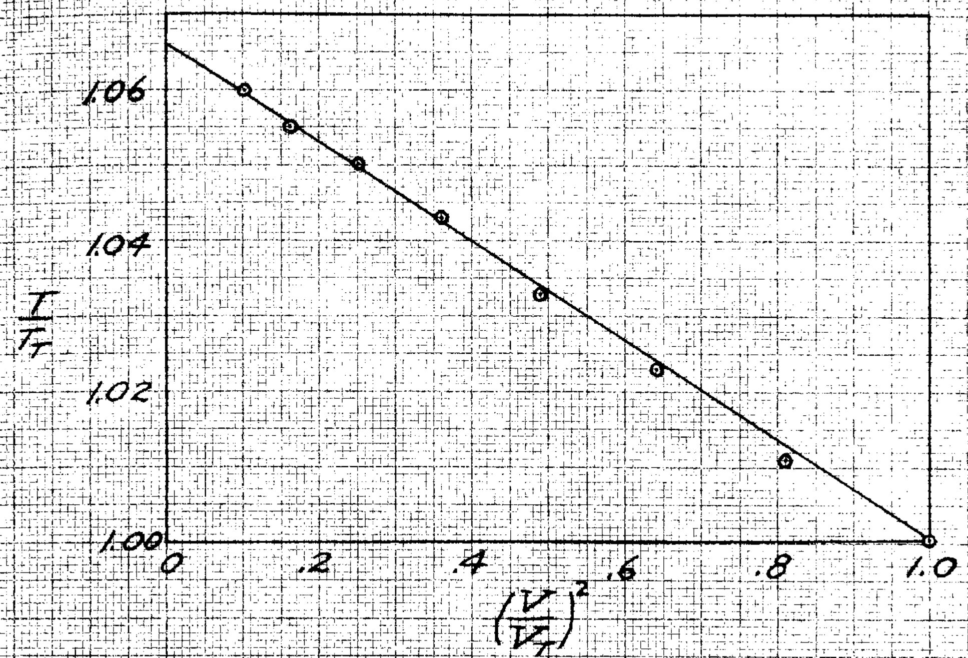
$\Phi(\lambda_{a_F})$ is plotted against $C_{L_{m_F}}$ in Fig.11. A line drawn from the origin tangent to the curve gives the minimum value of $\Phi(\lambda_{a_F})/C_{L_{m_F}}$, and consequently the shortest takeoff distance as previously explained. The best $C_{L_{m_F}}$ for takeoff is that corresponding to the point of tangency in Fig.11, which is seen to be $C_{L_{m_F}} = 1.64$. From Fig.7, ref.11, the optimum flap angle to hold for takeoff is $\delta_F = 24^\circ$ for $C_{L_{m_F}} = 1.64$.

The ratio of the takeoff distance with the flap in the optimum position is found by reading off $\Phi(\lambda_{a_F}) = 1.19$ at the point of tangency in Fig.11, and using equation (5.4),

$$\frac{S_F}{S_o} = \frac{1.32}{1.116} \times \frac{1.19}{1.64} = 0.858$$

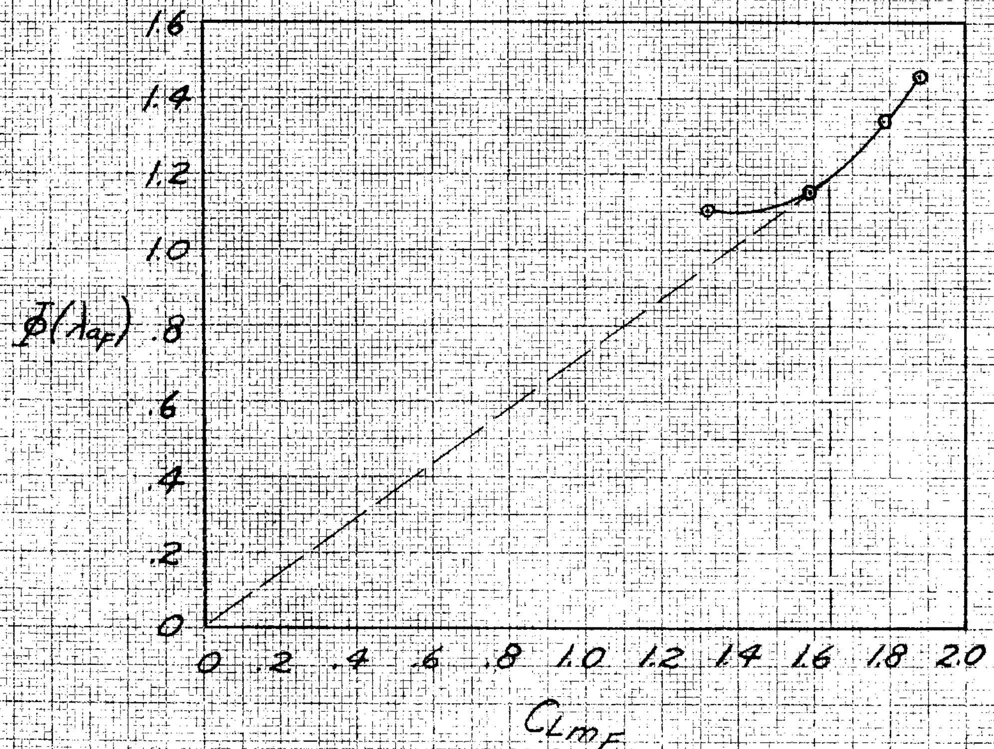
The takeoff distance at the best flap angle is,

$$S_F = 0.858 \times 532 = 457 \text{ ft.}$$



(FIG. 10)

THRUST VARIATION FOR FAIRCHILD F-22



(FIG. 11)

DETERMINATION OF BEST $C_{L_{m_F}}$
FOR TAKEOFF OF FAIRCHILD F-22

VII - CONCLUSIONS

In general, precise estimation of takeoff performance by the use of any purely theoretical analysis cannot be realized due to a number of factors entering into the problem which are unable to be taken into account. Some of these factors are: piloting technique; variations in the field surface; and fluctuations in wind velocity. This analysis shows that considerable variations in C_L from the optimum value result in relatively small increases in the takeoff distance, so if the airplane is maintained at approximately the attitude for the optimum C_L, the personal element should be practically eliminated. From comparison of flight test results with the corresponding calculations using the charts and formulas presented in this report, an empirical correction factor might be determined for general application to account for the remaining indeterminate factors mentioned above.

Figures 7 and 8, giving the effect of wind on takeoff, should prove useful for the reduction of flight test data. For most modern airplanes, the approximate expressions corresponding to $\lambda_a = 0$ are of sufficient accuracy for this purpose, and require no use of either airplane or engine-propeller characteristics. It is interesting to note that these approximate variations of takeoff time and distance with wind were found empirically, in reference 2, for seaplanes. In the present analysis these expressions were shown to hold only for cases where the net accelerating force was nearly constant during the takeoff run. Since the net accelerating force varies considerably for a seaplane during takeoff, the expressions would not be expected to give consistently accurate results for the effect of wind on seaplane takeoff. Examination of the experimental points plotted in reference 2 substantiates this conclusion.

Airplanes attempting to pass the Department of Commerce takeoff requirements are penalized 25 ft. on the takeoff distance per mile an hour headwind present during the test. The well known fact that this penalty is inconsistent and nearly always unusually severe may be verified by comparison of several cases with Fig.7. The approximate expression developed here would seem to be a simple and more logical basis for such penalties.

From the dependence of the takeoff distance on the term $C_{D_{i_o}}$ in λ_a, the conclusion is drawn that the effect of the ground on takeoff is slight, even in the case of low wing monoplanes. The takeoff distance of the Fairchild F-22 was increased less than 0.4% when calculated neglecting the effect of the ground.

The introduction of the takeoff acceleration parameter, λ_a, greatly facilitates the determination of the dependence of takeoff performance on the various airplane and engine-propeller characteristics.

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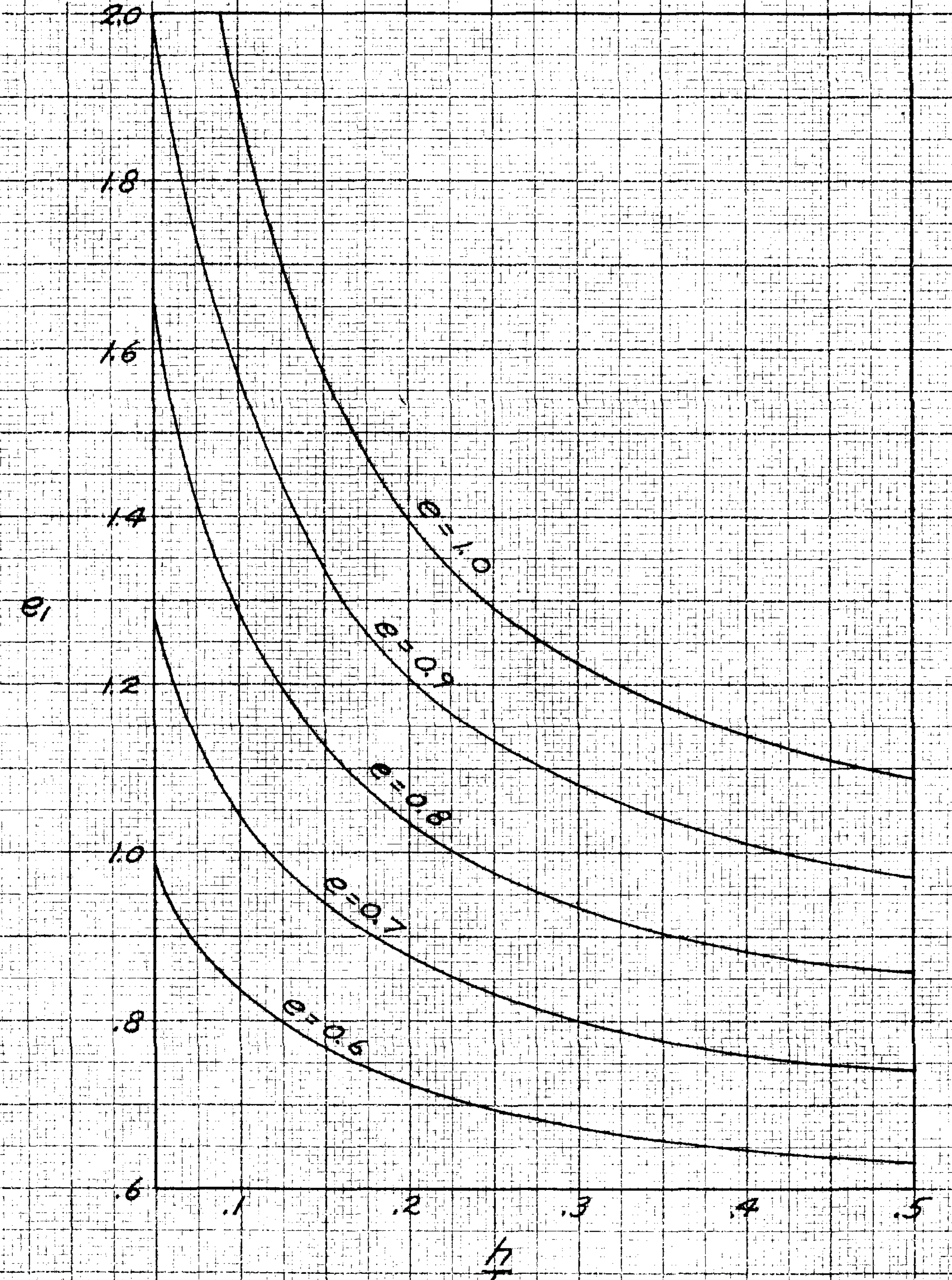
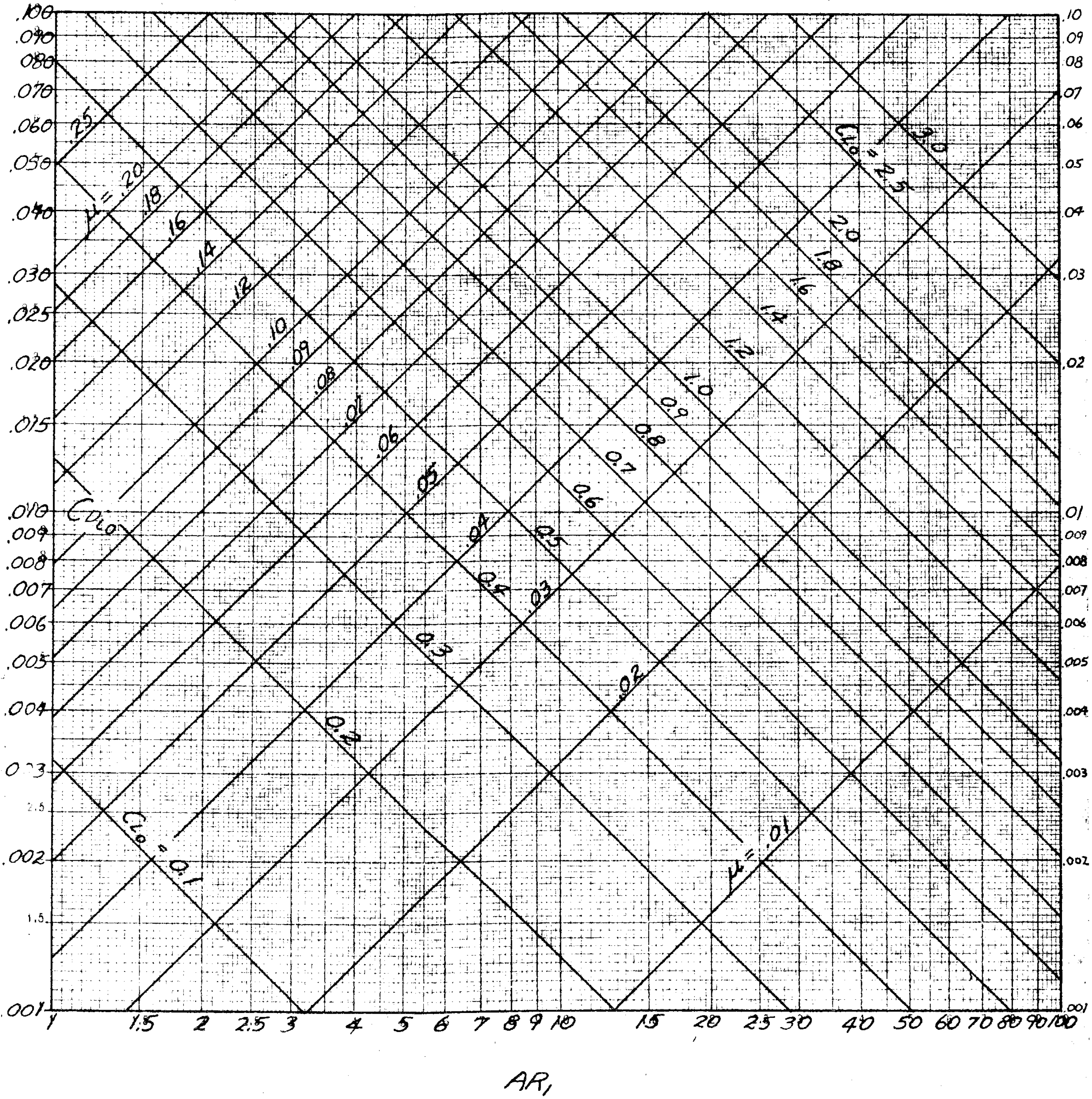
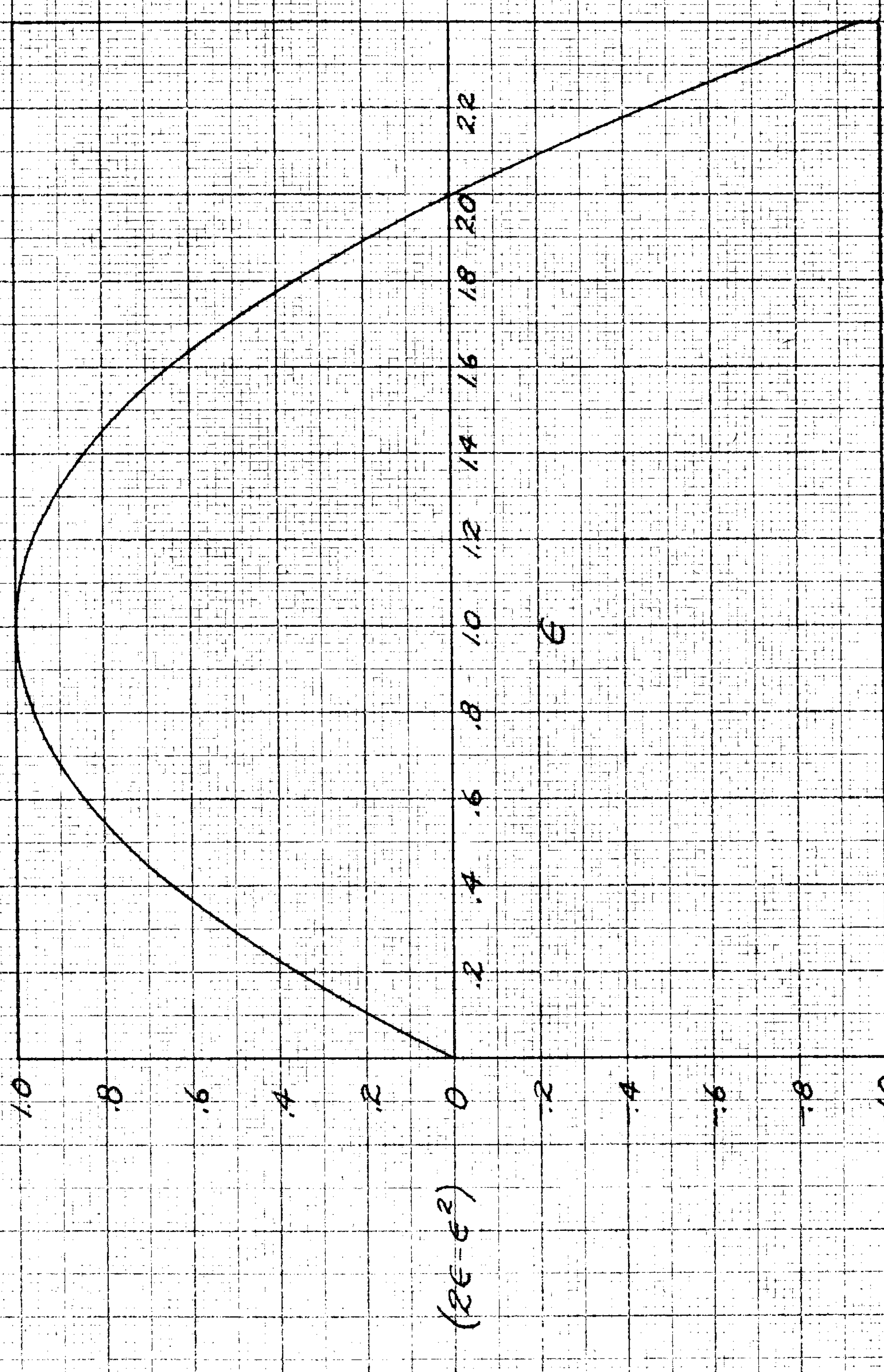


EFFECTIVE AIRPLANE EFFICIENCY
FACTOR FOR TAKEOFF
(FIG. 1)

OPTIMUM LIFT COEFFICIENT FOR TAKEOFF AND CORRESPONDING INDUCED DRAG COEFFICIENT

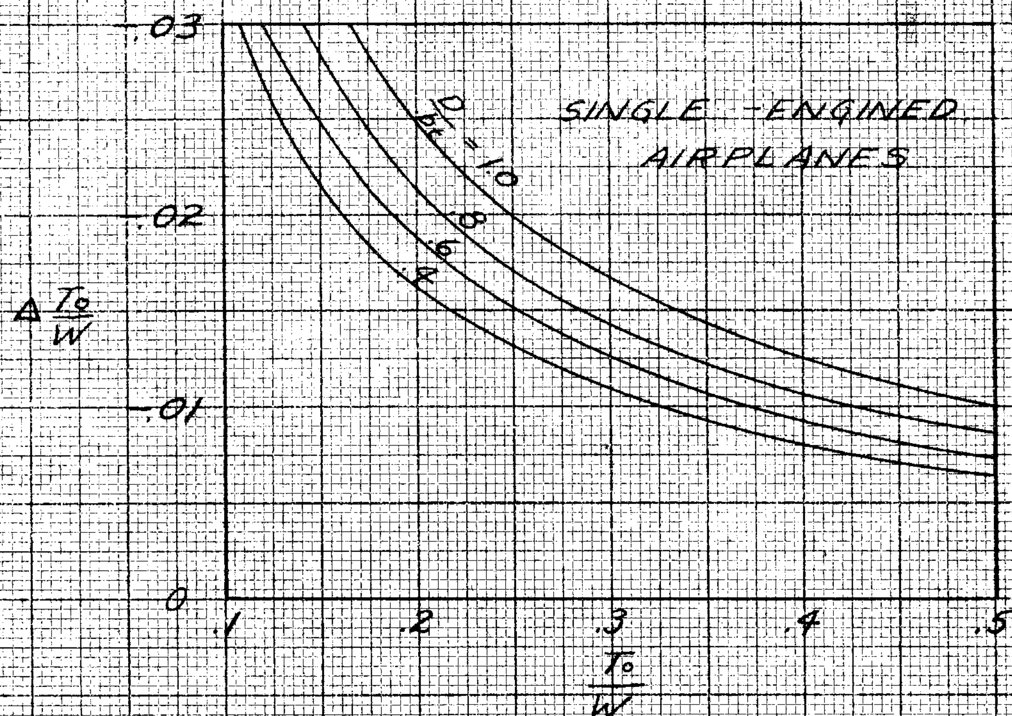


(FIG. 2)

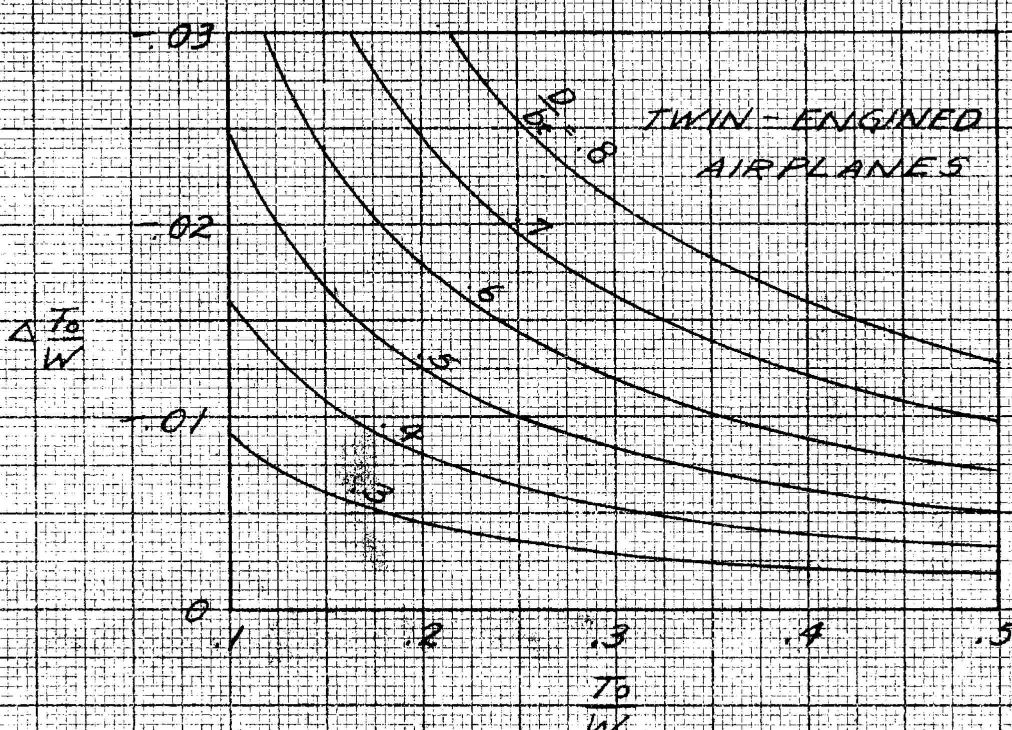


1.0
.8
.6
.4
.2
0
.2
.4
.6
.8
1.0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
ϵ
$(2\epsilon-\epsilon^2)$

(FIG 3)

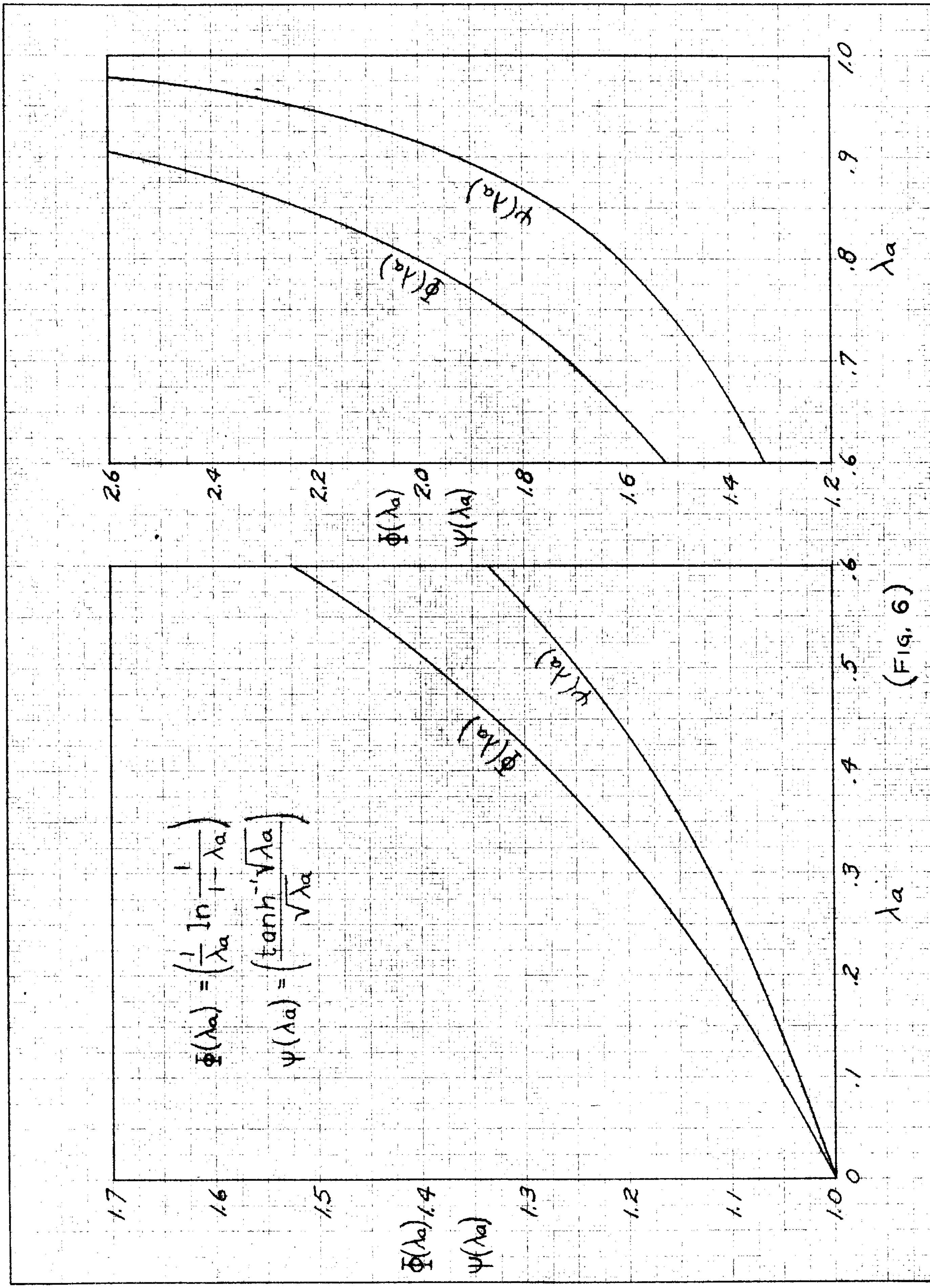


(FIG 4)



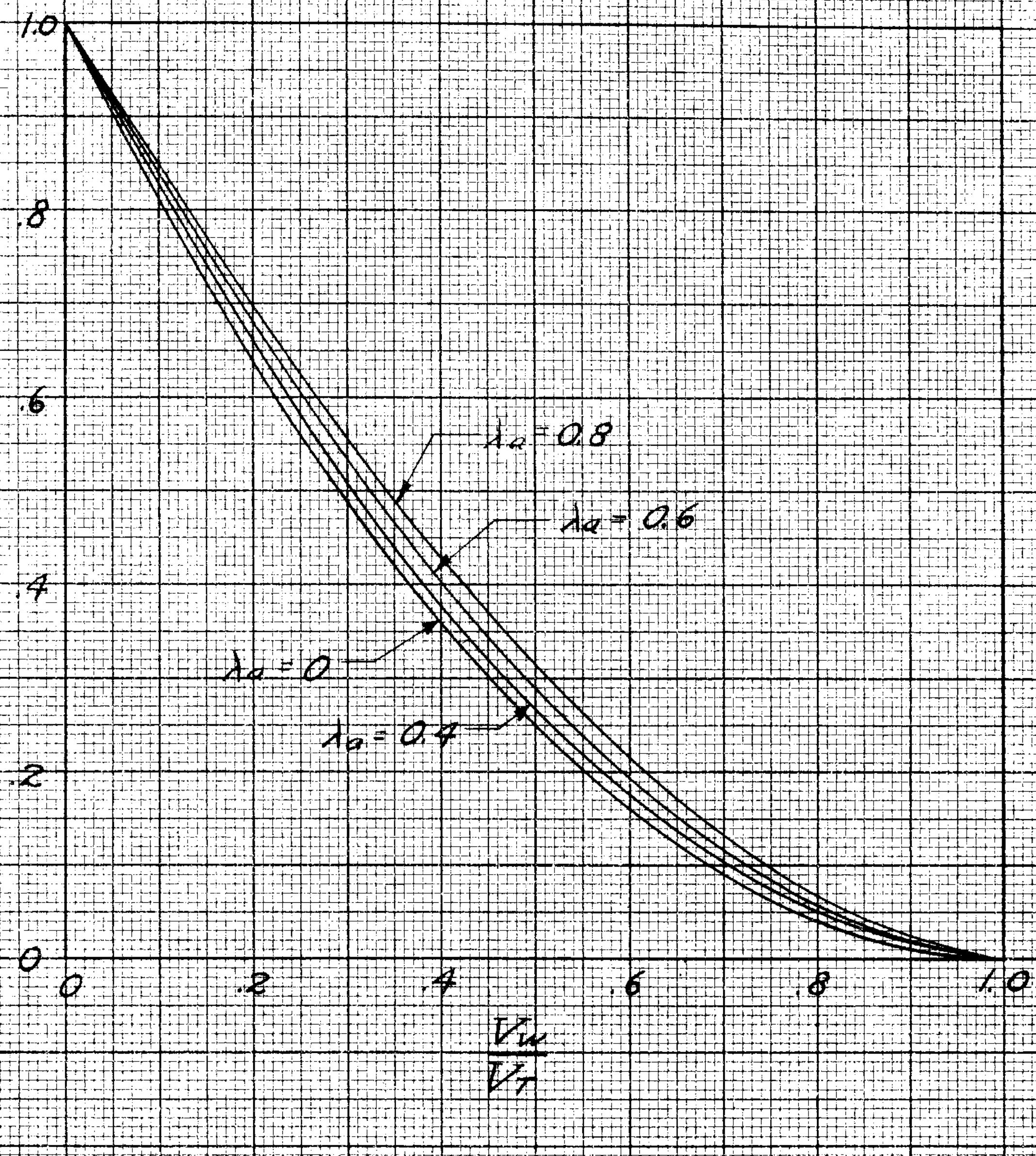
(FIG 5)

CORRECTION FOR TAIL DRAG



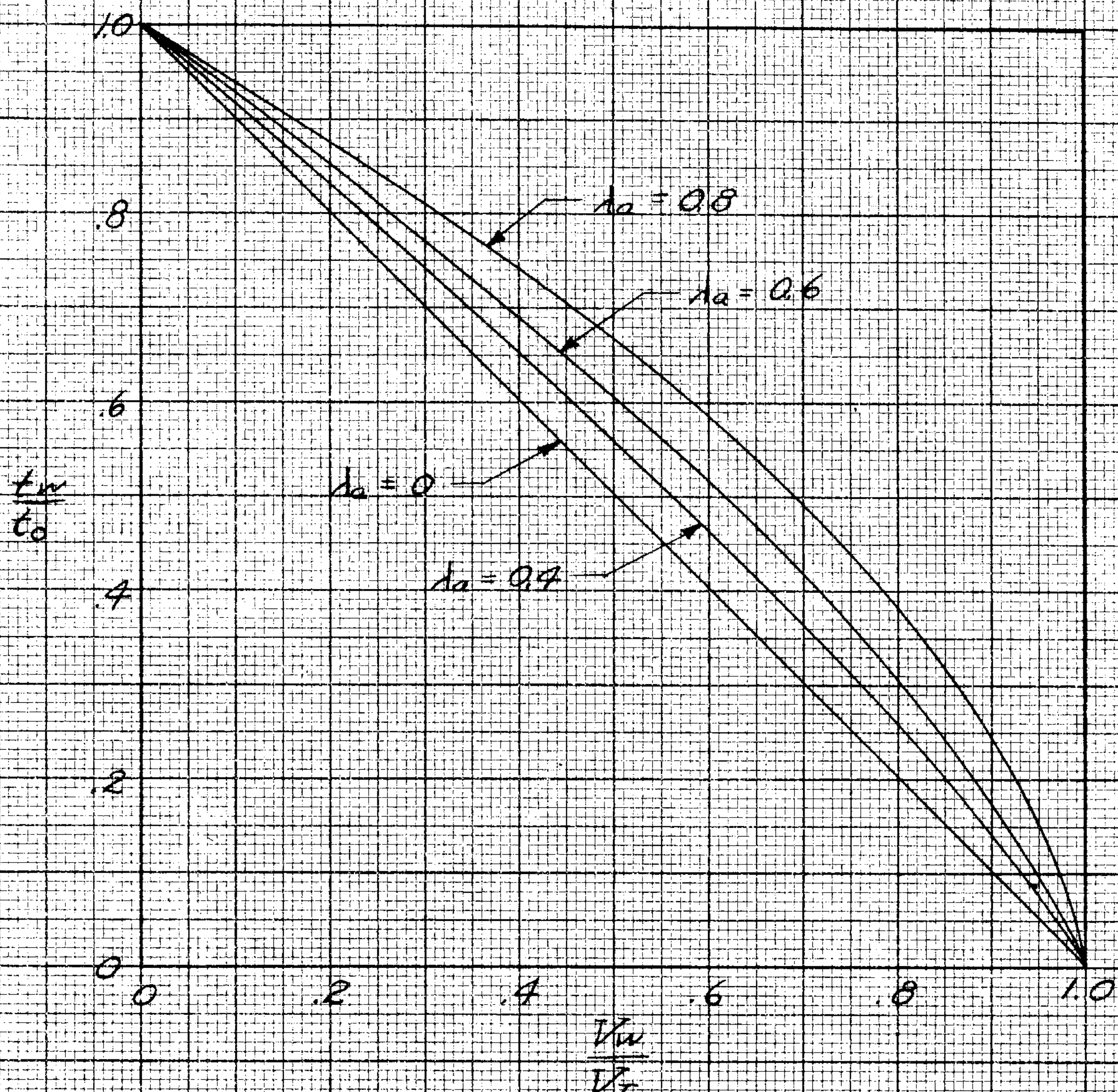
$\Phi(\lambda_a) = \left(\frac{1}{\lambda_a} \ln \frac{1}{1-\lambda_a}\right)$
$\Psi(\lambda_a) = \left(\frac{\tanh^{-1}\sqrt{\lambda_a}}{\sqrt{\lambda_a}}\right)$
$\Phi(\lambda_a)$
$\Psi(\lambda_a)$
λ_a
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
0
.1
.2
.3
.4
.5
.6
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6
.7
.8
.9
1.0

(FIG. 6)



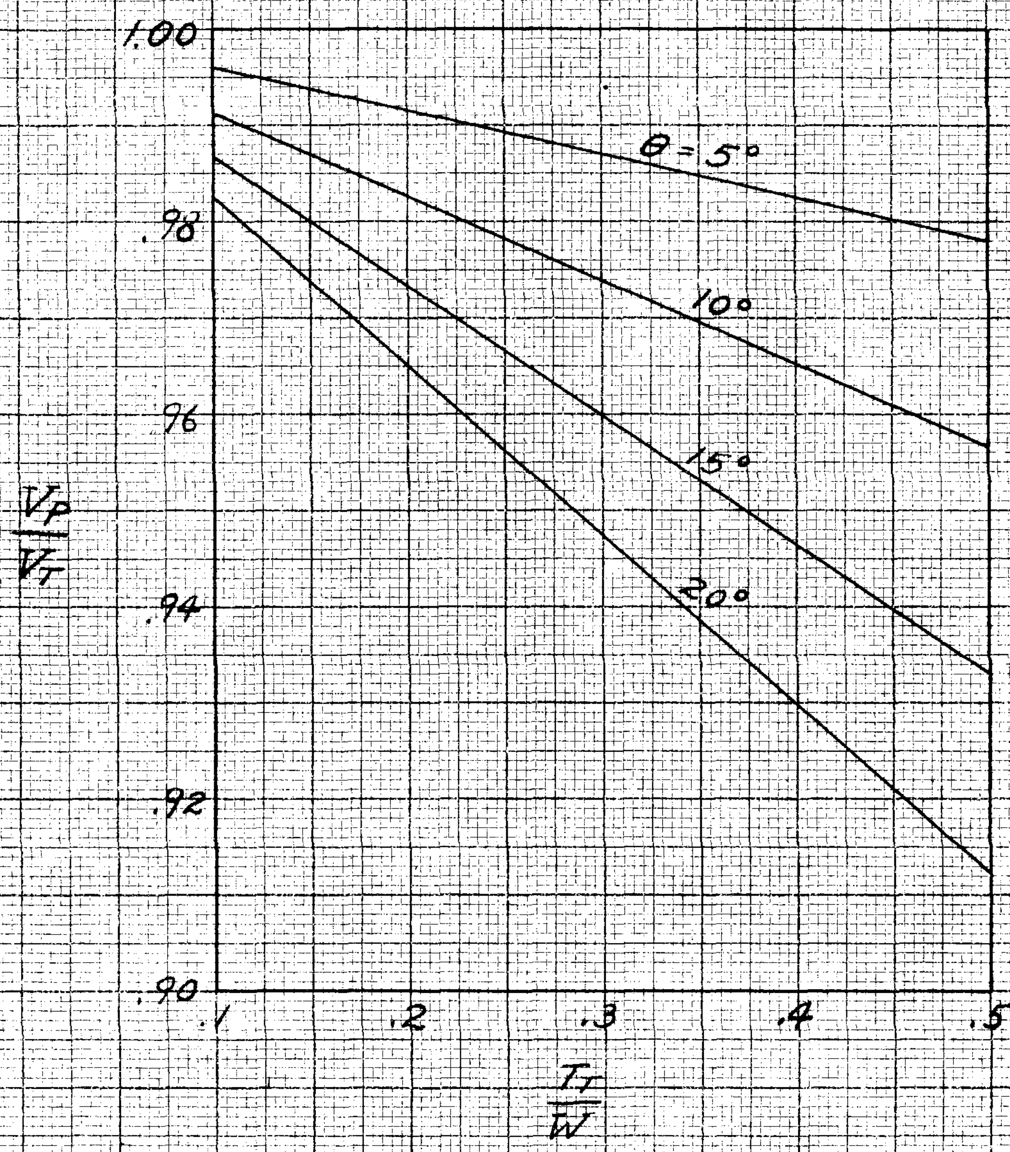
EFFECT OF WIND ON
TAKEOFF DISTANCE

(FIG 7)



EFFECT OF WIND ON
TAKEOFF TIME

(FIG. 8)



EFFECT OF POWER
ON TAKE OFF SPEED

(FIG. 9)