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Some Uses of the
SPONTANEOUS POLARIZATION METHOD

A Thesis
by
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for a Professional Degree in Geophysical
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INTRODUCTION

As a partial fulfillment of the requirements in obtaining a Professional Degree in Geophysical Engineering at the California Institute of Technology, Spontaneous Polarization method of electrical exploration was chosen as the subject of this thesis. It is also known as "self potential electrical prospecting" and "natural currents method."

The object of this thesis is to present a spontaneous polarization exploration work done by the writer, and to apply analytical interpretation methods to these field results.

The writer was confronted with the difficulty of finding the necessary information in a complete paper about this method. The available papers are all too short and repeat the usual information, giving the same examples. The decision was made to write a comprehensive paper first, including the writer's experience, and then to present the main object of the thesis.

The following paper comprises three major parts:

- 1 - A comprehensive treatment of the spontaneous polarization method.
- 2 - Report of the field work.
- 3 - Analytical interpretation of the field work results.

The main reason in choosing this subject is that this method is the most reliable, easiest and requires the least equipment in prospecting for sulphide orebodies on unexplored, rough terrains.

The intention of the writer in compiling the theoretical and analytical information has been mainly to prepare a reference paper about this method.

The writer wishes to express his appreciation to Dr. G. W. Potapenko, Associate Professor of Physics at California Institute of Technology, for his generous help.

EARTH CURRENTS

There is always some difference in electric potential between any two points on the earth's surface. This difference, when measured between two points which are not very far away, is usually very small, such as 10 to 100 millivolts per kilometer. However, sometimes local abrupt potential gradients are observed. They may reach 4 millivolts per foot. These high potential differences are accompanied by quite large flows of current which can be measured by simple electrical equipment, making possible the detection of the body that causes the local electrical anomaly.

Almost all kinds of differences of equilibrium, such as chemical, magnetic, mechanical, thermal, give rise to natural currents. As the earth's crust is highly heterogeneous in every respect, these natural currents are expected everywhere. The earth potential between two grounded points consists of several components which can be divided into two main components:

1. Relatively steady components.

- a. Electrode potentials (minimized by using non-polarizable electrodes).
- b. Potentials due to local anomalies, such as oxidizing ore-bodies, local electro-capillary phenomena, etc.
- c. Regional potentials, called "telluric currents," amounting to 10 to 100 millivolts per kilometer, according to place and time of year.

2. Fluctuating potentials, caused mainly by magnetic activity.

From these considerations it seems that the detection of the body which is responsible for the local potentials, as in (1,b), is a very complicated problem. However, in reality this is a rather simple problem. The electrode potentials can be minimized and controlled. The localized phenomena, which are superposed over the widespread telluric currents, are much more intense, so that a good sized local anomaly has an insignificant "telluric" component.

The fluctuating component is closely related to magnetic activity and sunspots. It has yearly, monthly, daily and hourly components. The effect of these fluctuating earth currents is minimized by employing relatively short separations between the electrodes. In average areas the error caused by these currents is of minor importance if the spread is less than 500 feet.

Thus, the problem of geophysical exploration by measuring the earth currents is reduced to the study of the case 1,b. In the following paper electro-chemical and electro-capillary phenomena will be examined as the causes of the spontaneous polarization activities.

HISTORICAL BACKGROUND

Recognition of electrical activities related to orebodies goes back to 1830, when R. W. Fox stated that such currents were due to connections existing between different orebodies, or between different parts of the same orebody. He claimed that these phenomena in orebodies "bear a striking resemblance to galvanic combinations."

Later on, in 1870, W. Skey extended this subject by making laboratory experiments on minerals.

In 1880 Carl Barus, investigating the electrical activity over the Comstock lode, concluded that "the electromotive forces due purely to chemical difference and polarization of the terminals are of the same order as the data expressing the electric activity of the lode." The potentials thus found were very small, and electrical prospecting, measuring these natural currents, appeared to be impractical. Later, discovery of orebodies showing large potentials, up to one volt, proved the method to be one of the outstanding geophysical exploration methods.

GEOLOGICAL CONSIDERATIONS

It is advantageous to treat the subject first from a geological point of view, in order to get acquainted with what is observed in ore-bodies due to these electro-chemical phenomena accompanied by natural potentials.

It is well known that ore deposits are often changed in their upper parts by descending aerated meteoric waters, while the part below the water table is enriched by secondary processes. In the upper zone sulphates, carbonates, silicates, oxides, chlorides, arsenates, and native metals, and, in the lower zone, sulphides, tellurides, arsenides and antimonides are found.

This process of change, usually called weathering, is accompanied by electrical activities and gives rise to a large number of chemical reactions. Above the water table oxidation takes place; and the soluble products of oxidation are fixed as insoluble sulphides, by reducing reactions, in the secondary enrichment zone below the water table.

A quartz and pyrite mixture leaves a brown, spongy quartz and limonite mass called gossan. Copper minerals also yield gossan. As most of the veins contain these elements, a gossan at and near the surface is very common in sulphide veins.

In general, oxidation extends to the water table, but due to several other factors, besides variations in the water table level, the

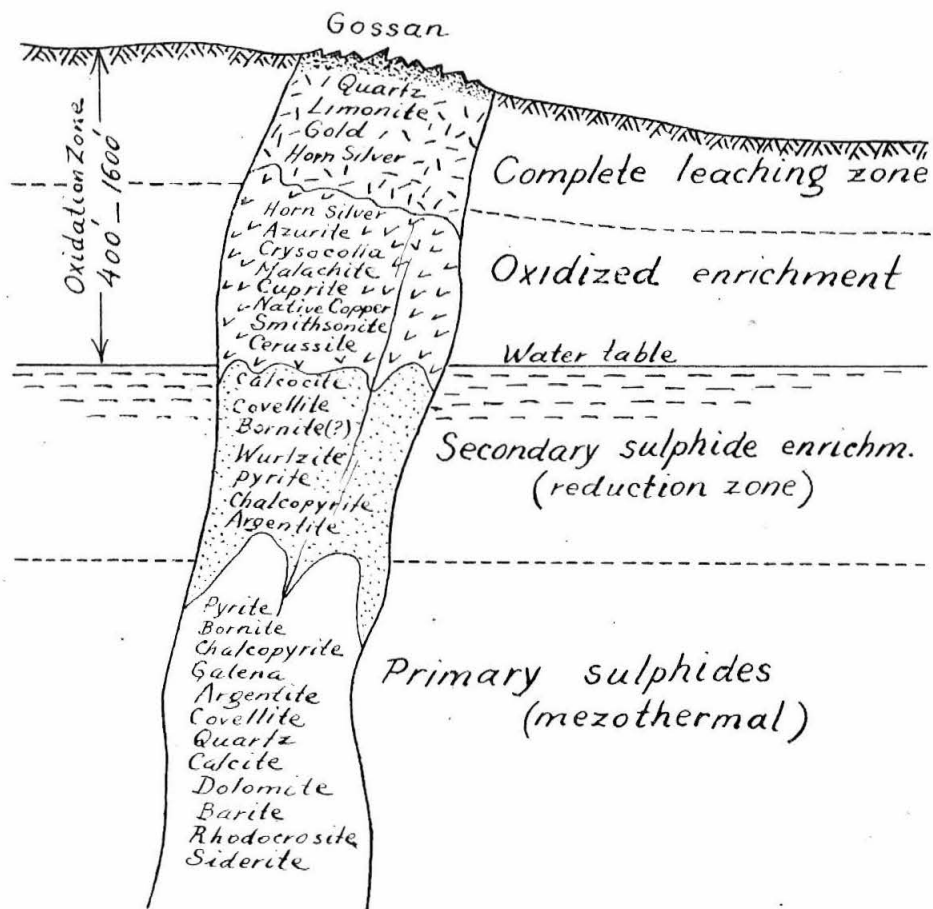


Fig. 1 - Cross-section of a Vein Undergoing Oxidation, and the Minerals Encountered in different zones.

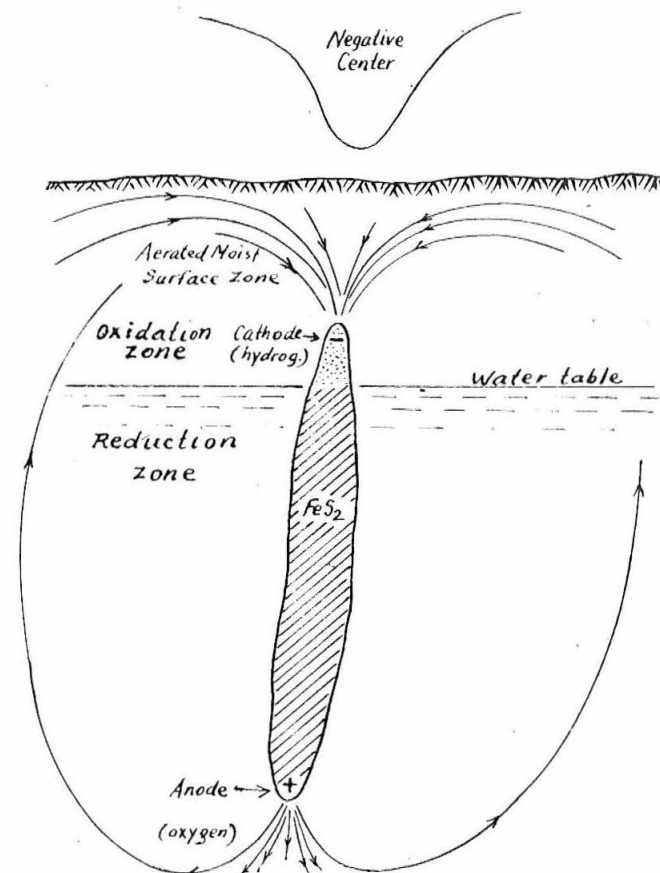


Fig. 1a - Electrical Activity on a Pyrite Vein

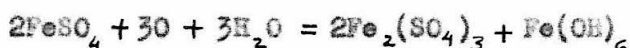
depth of oxidation shows a great variation. It is 2000 feet deep in Tintic, Utah, and 100 feet in Ductown, Tennessee.

According to Tolman, the upper zone can be divided into three sub-zones from the surface down to the water table. These are: (1) zone of complete oxidation; (2) zone of complete leaching; and (3) zone of oxide enrichment which lies immediately above the sulphides. The first subzone includes the gossan which shows limonite, hematite, residual silica, residual gold, and silver chloride.

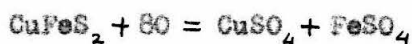
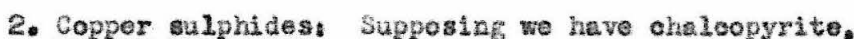
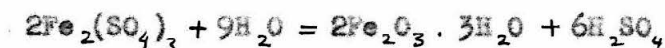
Some of the possible reactions taking place in the oxidized zone are as follows:



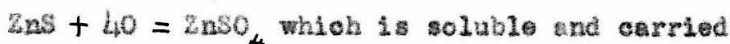
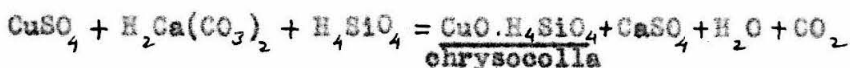
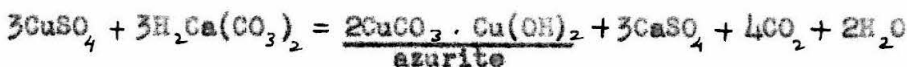
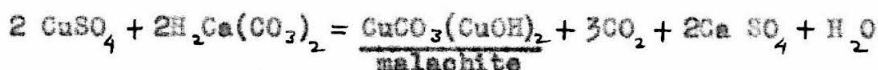
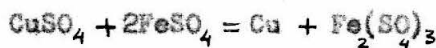
Ferrous sulphate oxidizes further, giving ferric sulphate:



Ferric sulphate yields limonite as follows:



Copper sulphate may be retained in the oxidizing zone as follows:



down. If pyrite is present, the ferric sulphate complicates the reactions, and smithsonite (ZnCO_3) is formed.

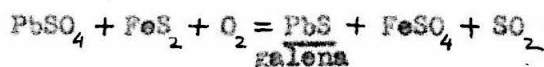
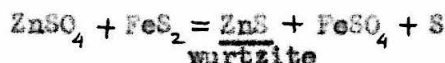
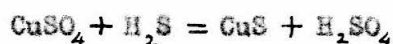
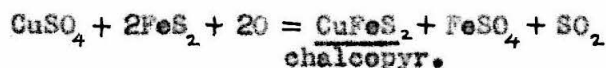
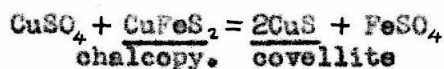
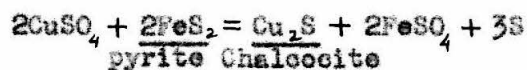
Ore minerals, after having been converted into soluble sulphates in the oxidation zone, are carried down below the water table. They come in contact with reducing agents which reduce them again to insoluble sulphides, giving rise to a secondary enrichment. The extent, or even presence of this secondary enrichment zone depends on several factors; such as climate, altitude, relief, permeability, geologic history of the locality, and chemical and mineral composition.

The exact equations of the chemical reactions in the secondary sulphide enrichment zone are not very well known. Precipitation below the water table may be due to: (1) reduction of sulphates to metallic sulphides by carbonaceous matter; (2) reduction by hydrogen sulphide; and (3) reaction of salts with sulphides.

Reactions of secondary sulphide deposition involve several intermediate steps. For example, the change from pyrite to chalcocite takes place possibly as follows:

Pyrite → Chalcopyrite → Bornite → Covellite → Chalcocite

Some of the enrichment zone reactions are as follows:



CAUSES OF SPONTANEOUS POLARIZATION

I. Electro-chemical Considerations

Before going into the detail let us summarize the electrical phenomena connected with orebodies. The following, Schlumberger's explanation, is very precise and suitable for this purpose:

"Let us consider a pyrite lens buried in the ground, more or less vertically. We have here the equivalent of a homogeneous metal, dipping in an asymmetric electrolyte, for the water imbibed in the ground is not everywhere of the same composition. Towards the surface and above the hydrostatic level, the water is aerated and rich in oxygen, whereas at depth, it is not, because it is not renewed. The pyrite lens and the surrounding rocks therefore constitute an immense natural battery of the type called a 'gas battery.' The current moves therein, flowing down towards the ore and coming up again through the ground so as to converge towards the top of the orebody. Owing to the electrolysis there is a formation of hydrogen at the cathode---that is to say, at the top of the mass where the water is aerated---and a formation of oxygen at the anode---that is, at the bottom where the water is not aerated. The gases thus liberated are not diffused and nascent hydrogen combines with the dissolved oxygen so as to form water. As to the oxygen deposited at the anode, it dissolves in the water, lacking in oxygen at this spot.

It is seen that all this process finally results in the unification of oxygen in the moisture in the ground and that as a whole it constitutes a method of diffusion of this element in the earth's crust.

If the buried metallic body is oxidizable, as in the case of pyrite, the battery effect is considerably reinforced by the chemical energy liberated by the oxidization reaction taking place at the apex. It can also be said, in other words, that the oxidizable ore plays the part of depolarizer of the natural battery. The oxidization of the gossan favors the absorption of the nascent hydrogen by furnishing it with a reducing element, exactly as does the bioxide of manganese, which envelops the carbon of an ordinary electric battery. At depth the oxygen formed goes to the pyrite and oxidizes it, instead of being slowly diffused in the moisture of the surrounding formations. In this way, the asymmetry of the electrolyte subsists permanently, whereas it would have a tendency to disappear, or would be maintained only by slow diffusion, were the ore not oxidized. In short, the action of the electric current may be said to cause a downward movement of the oxidization due to atmospheric agents along the walls of the mass."

If a metal is dipped in a solution, a potential difference is created between the two media. However this cannot be measured, except by connecting another electrode to the first one, dipped in a solution having electrolyte contact with the previous one. If the metals of the two electrodes and the solutions are alike, no potential is observed. Any kind of asymmetry, either in solutions or in metals, causes a potential difference. Electrical activity in orebodies is the same phenomenon. It is caused usually by the asymmetry of the solutions surrounding the vein, and, to some extent, by the differences in the mineralogical and chemical composition of the orebody.

The potential of a metal against a solution depends only on the concentration of the ions of the particular metal in the solution.

$$E = \frac{R}{F} \cdot \frac{t}{n} \ln \frac{P'}{C}$$

where R = gas constant, 8.309 joule per degree C.

n = valency

F = Faraday's constant of electrolysis, 96,494.

t = absolute temperature.

P' = Constant for the metal. Electrolytic solution pressure.

C = ion concentration

When two electrodes of the same metal are dipped in different solutions having electrolytic contact by means of a syphon bridge, the potential difference between the two electrodes is

$$\Delta E = 1.98 \cdot 10^{-4} \frac{t}{n} \log \frac{C_2}{C_1} \text{ volts.}$$

For $t = 273 + 18^\circ$, $n = 1$, and $\frac{C_2}{C_1} = 100$, the voltage difference is 0.115 volts.

The voltage observed on orebodies may be as high as 1.5 volts which is the maximum. This is almost the same as the voltage furnished by a wet cell. Thus, it is reasonable to assume that the potentials observed over orebodies are not the result of the solution differences alone, but the differences in the orebody as well. To see the effect of the change in mineralogical composition, we have to write the equation in its original form, conserving the constants P'_1 and P'_2 , electrolytic solution pressure, of both metals.

$$\Delta E = 1.98 \cdot 10^{-4} \frac{t}{n} \log \frac{P'_1 C_2}{C_1 P'_2} \text{ volts.}$$

The voltages calculated by this equation are in better agreement with the observed values. Suppose there is no difference between the solutions above and below the water table, the ratio $\frac{C_2}{C_1}$ being equal to one. In this case the formula gives enough potentials to account for

the actual observations. Pyrite may have a solution pressure of 10^7 atm. and gossan around 10^{-30} atm., so that the ratio is quite large.

Laboratory measurements show that the variation shown by a mineral in passing from an acid to an alkaline solution is in fact generally greater than the differences shown by diverse minerals in the same solution. In general, acid and oxidizing solutions give the highest potentials, alkaline and reducing solutions the lowest. The most general statement that can be made about the potentials shown by minerals in different solutions is that the potentials are chiefly determined by the solutions. This is due to the fact that wide variations in the concentrations of the effective constituents are possible in different solutions, making the ratio C_2/C_1 large. In case of "unattackable" electrodes the ratio P_1'/P_2' is almost equal to one, so that the potentials shown by these electrodes is related to the oxidizing or reducing nature of the solutions in contact with them. However, it may be stated that all minerals are attacked by all solutions, but in widely varying degrees. Even pyrite, one of the least attackable sulphides, is affected slowly by acidified ferric sulphate. As pyrite is capable of slowly reducing ferric sulphate, any electric action which could arise from the oxidizing power of ferric sulphate would occur chiefly on account of the fact that pyrite enters into direct action very slowly. With the more attackable minerals the possibility of electric action would be less than with pyrite.

In order to present a general view of all possible combinations in cells of different types, it will be advantageous to assemble in one table the available data showing single potentials. The assumption is that the normal calomel electrode has a potential of 0.56 volt. The concentrations are normal unless otherwise stated.

It may be assumed that the potentials noted with the insoluble sul-

Single Potentials of Solutions and Solid Electrodes

Solutions (unattackable electrodes)	Potential
Gold solution (aurous)	+1.8 volts
Sulphuric acid saturated with oxygen	+1.47
Acid ferric sulphate	+1.11
Oxygen in water	+1.09
Sulphuric acid	+1.07
Silver solution	+1.06
Mercury solution (mercurous)	+1.03
Thousandth-normal sulphuric acid	+0.95
Acidified ferrous sulphate	+0.79
Pure water	+0.68
Neutral ferrous sulphate	+0.63
Sodium sulphite	+0.58
Cuprous chloride	+0.56
Potassium arsenite	+0.51
Thousandth-normal sodium hydroxide	+0.41
Sodium hydroxide	+0.29
Sodium hydrosulphide	-0.14
Hydrogen in water	-0.14
Sodium sulphide	-0.27
Hydrogen in sodium hydroxide	-0.51

Solid Conductor (in water or solution stated)	Potential
Lead dioxide in 0.5 normal sulphuric acid	+1.84
Manganese dioxide in sulphuric acid	+1.72
Pyrolusite in water	+1.11
Manganite in water	+0.96
Magnetite in water	+0.68
Marcasite in water	+0.62
Pyrite in water	+0.60
Pyrrhotite in water	+0.56
Galena in water	+0.46
Copper	+0.44
Lead	-0.04
Nickel	-0.12
Cadmium	-0.31
Iron	-0.32
Zinc	-0.66
Manganese	-0.97

phides are due to certain concentrations of sulphide ions. It is obvious that the ions formed by the solution of most minerals in water fall into the class of easily oxidizable or reducible substances. The mineral potentials are therefore "oxidation and reduction potentials." All oxidation and reduction potentials may be regarded as due to certain concentrations of oxygen and hydrogen. Water itself may be assumed to have certain concentrations of free oxygen and hydrogen in equilibrium with it. The calculation of the potential shown by hydrogen or oxygen electrodes in pure water gives 0.676 volt. Thus all neutral solutions showing a higher potential than 0.676 with an unattackable electrode should contain an excess of free oxygen, whereas those giving a lower potential should contain an excess of free hydrogen. Waters at the surface of the earth contain oxygen at a pressure of about one-fifth of an atmosphere. The oxygen in the ground water will be considerably reduced, and at a certain depth there will be practically no excess of oxygen. The most abrupt change occurs at the water table. Below that certain depth hydrogen will be in excess at concentrations increasing with the depth. The value 0.676 is a sort of natural zero of potential.

The following laboratory experiment is very useful to examine what takes place in orebodies. Suppose we have a container, with a porous partition, filled with water, to make two compartments, each having a metallic electrode, connected with a wire and a galvanometer. The water in one compartment is aerated with an air hose, whereas the other is not. A current flows from the aerated electrode A to non-aerated electrode B in the outside circuit. Since the tendency is to make the electrolytes uniform, a displacement of OH^- ions from the aerated electrolyte towards the non-aerated electrolyte and of H^+ ions in the opposite direction is expected. This immigration causes a current from A to B in the wire. The following

general equation may be written:

Electrode A - electron \rightleftharpoons oxidizing medium + electron.

Electrode B + electron \rightleftharpoons reducing medium - electron.

The electron exchange from B towards A makes the electrolyte concentration uniform. Electrode A becomes the positive terminal of the battery. The surface potential measurements are made in the electrolyte and the current flows towards the positive terminal in this region.

The system just described does not work continuously; it polarizes. If the polarization products are eliminated it works continuously. If pyrite is present, for instance, the oxidation of ferrous iron, sulphur, etc., below the water table and the reduction of iron hydroxides at the surface depolarizes the battery. These reactions take place since the upper zone is an oxidation zone, so that reduction by H^+ ions can take place, and the opposite holds for the lower part. In other words, the electrical phenomena tend to prevent the action of aerated water near the surface. In the case of anthracite the oxygen is eliminated by combining with the carbon. The voltage may be as high as one volt. If the vein consists of galena, the oxidation products, anglesite and cerussite, create resistive coatings, and at the same time the nascent hydrogen and oxygen are eliminated with difficulty. Thus, galena usually gives very little potential differences.

Gossan Formation: It has been explained above that the apex of the orebody shows a negative center, that is, this part is the point of entrance of the current into the orebody. At the same time this zone is a zone of oxidation. This is a contradiction, because the generated oxygen ions go to the lower part, below the water table, which is expected to be oxidized.

If we repeat the above experiment putting the electrodes A and B in separate containers connected with a syphon bridge having variable

cross section, and if the resistance of the syphon is small enough we observe that the non-aerated electrode B oxidizes, since the transport of oxygen ions takes place toward B. If we increase the resistance of the syphon, after a while the electrode A begins to oxidize, the current passing in the same direction. Similarly, orebodies are sometimes high resistance batteries with a small output and therefore incapable of preventing the action of the air in the gossan zone, that is, the arrival of hydrogen ions on the apex is insufficient to prevent oxidation. This is the reason for gossan formation. Anyhow, insufficient hydrogen ions or an excess of dissolved air in the upper part leads to gossan formation. In the case of good conducting and homogenous veins the spontaneous polarization oxidation happens only near the water table, a little above or below, oxidizing the metallic sulphides. The part above the water table oxidizes, because very little hydrogen ions come to this part and the solutions are enough aerated to have an excess of oxygen. The part below the water table receives little oxygen ions and the solutions also contain a small amount of dissolved air, the result being an oxidizing medium. As we go lower, the amount of oxygen ions decreases rapidly and hydrogen prevails, the medium being reducing.

Oxidation due to the minor heterogeneities of an orebody is another problem. The oxidation of galena in pyrite-galena ores is a well known fact. This can be illustrated by an experiment. Suppose we have a container filled with water and two electrodes, one galena and one pyrite, partially dipped into it and connected by a wire. The galena electrode is oxidized and coated with $PbSO_4$, the current flowing from pyrite to galena in the outside circuit. Pyrite remains unchanged. The formation of small spontaneous polarization batteries, consisting of small grains or crystals, protects some parts of the orebody and accelerates oxidation

of other parts. Accessory batteries, similar to the ones formed by different mineral grains may also be formed if a homogenous orebody is immersed in a medium which is heterogenous locally. These three forms of electrical phenomenon are superimposed, the major part being played by the one resulting from the asymmetry between each side of the water table. The parasite circuits, superimposed on the general direction of the current which is from top to bottom in the orebody, are generated especially in the oxidizing zone, and attack some components of the ore.

The consumption of ore to maintain a strong spontaneous polarization current is amazingly small. Let us assume that the potential difference in an orebody is one volt and the resistance of the orebody and the surrounding electrolyte is one ohm. The current will be one ampere, that is 86,400 coulombs per day. The reactions are very complicated, but if we consider that the oxidation product of pyrite is limonite and acid sulphuric, the oxidation of iron would require 300,000 coulombs per gramme-atom which would consume 15 kgs. of pyrite a year to maintain such a strong spontaneous polarization which is a maximum. It is obvious that the phenomenon is very stable in the course of at least a hundred years. Changes from time to time may occur as a result of rains, etc. which are temporary or cyclic.

It is possible to reproduce spontaneous polarization phenomena in laboratories on artificial orebodies. A piece of conductive ore is buried lengthwise in a mixture of sand and grey clay, about one fourth of its length being above the surface. Then the remaining part is covered by non-clayey washed sand. The whole thing is boiled to remove the air, and is left to cool. It is watered from time to time in order to maintain the moisture in the formation. The part above the clay surface represents the aerated part, since air penetrates into it. Potential measurements on the surface

of the sand show the same facts observed on natural orebodies.

The conditions necessary for the formation of spontaneous polarization can be summarized as follows:

1. Continuity of the electric conductivity; that is, the mineral particles should be electrically connected to each other. A deposit constituted by small kernels surrounded by an insulating matrix does not give any potential on the surface. In this case several parasite batteries are formed but the resultant is zero. Conductivity of the mineral constituent being known, the electric continuity of a deposit can be tested on the deposit itself. This is done by means of a sensitive galvanoscope and a dry battery. The circuit battery-galvanoscope is closed by the ore and the deviation of the galvanoscope indicates the electrical continuity. All the metallic sulphides, except zinc sulphide, are good conductors. However, an orebody as a whole may represent any degree of conductivity due to filled fissures and fractures. Experiments on small samples may lead to erroneous conclusions in both ways, because a network of veinlets may render an orebody conductive. The presence of copper increases the possibility of such veinlet systems, making the orebody conductive. Galena usually occurs in separate masses which have no electrical connection between them.

2. Chemical asymmetry around the conductive orebody is the original cause of the creation of the potential. This condition is usually provided by protrusion of the orebody above the water table into a more oxidizing medium than at depth.

The water table usually follows the general trend of the topography, that is, it is shallower under valleys than under hills. It changes from season to season, as a result of water supply. Erosion, causing disappearance in relief, uplift, and drying up of the climate lowers it. The lower-

ing of the water table is called "negative movement." In a geologically sinking region, and where the climate becomes more rainy, the opposite occurs. Rising of the water table is called "positive movement." Spontaneous phenomena are stronger in negative movement zones than in positive movement zones, for fresh metallic sulphides are continually being brought up. When the movement is positive, on the contrary, the water invades the gossan where no further oxidation can take place, and the spontaneous phenomenon ceases.

In the Alps, erosion is so fast that gossans have no time to form, and fresh sulphides are very near the surface. In this case potentials created are very large and marked.

In flat marshy districts where veins are flooded, and the reducing overburden formed of vegetable matter in decomposition absorbs the oxygen of the water coming down, practically no spontaneous phenomenon is observed.

3. Suitable mineralogical composition of the orebody is another condition. The mineralogical nature plays a triple part: (a) ion concentration, C , (b) electrolytic solution pressure, P , and (c) depolarizing effect which is the most important. The strongest reactions are always found on pyrite, chalcopyrite, pyrrhotite and iron pipes although these elements have low potentials against water. The fact, as explained before, is due to their depolarizing effect. Galena shows a very slight degree of spontaneous polarization.

II. Electrofiltration Potentials

When an electrolyte filters through a porous cell wall, an electromotive force is generated. This difference of potential is expressed as

follows:

$$E = K \frac{D \cdot \Delta P}{4 \pi \mu \sigma}$$

where

σ = specific conductivity of the electrolyte.

μ = coefficient of viscosity of the electrolyte.

D = dielectric constant of the electrolyte.

ΔP = drop in pressure through the wall.

K = constant depending upon the electrolyte and the chemical nature of the wall.

Electrofiltration activity like this causes large natural potentials frequently. They may be as high as one-and-a-half volts. It may be due to (1) electro-capillary by the descent of water, that is, infiltration of water by gravity, and (2) electro-capillary by ascension of water (capillary ascension and evaporation). Usually, descent of water gives negative centers and ascension of water gives positive centers.

These phenomena are explained by E. Földini in the following way:

"A porous body may be considered as an agglomeration of small capillary tubes. It is admitted that the wall of these tubes has the power of fixing the ions of the liquid to its surface by absorption and that this absorption is greater for the ions of a certain sign than for those of the other. The result is the formation of a sort of electric coating attached to the wall. Because of the attraction due to the absorbed ions, a coating of contrary sign is formed, attached to the liquid. This latter coating should moreover not be considered as infinitely thin, but rather as an ionic atmosphere, the density of which decreases upon leaving the wall. It is becoming more and more admitted that the negative ions are connected with the solid and the positive ions with the liquid. When the latter is in movement, it carries with it the electric coating attached to it, whence the appearance of a difference of potential between the two sides of the

wall."

"Finally, it is to be noted that the potential differences measured are smaller than those given by the formula. The concentration of ions being greater in the double coating, the conductivity of the liquid is increased. One would therefore have to introduce into the formula a conductivity σ' , greater than σ . Furthermore, the filtration potential develops an active electro-osmotic reaction which tends to send the liquid in the opposite direction to its initial movement. The filtration speed is therefore decreased and has a smaller value than that which would result from the fall of pressure ΔP ."

1. Electrofiltration by descension: Suppose we have a vertical bed of a permeable matter, such as sandstone, interbedded between two masses of impermeable shale. Meteoric water filtrates downward in the sand faster than in the shale, thus we have a relative motion of water in the sand with respect to the shale walls. This motion carries with it the positive charges, which results in a downward flow of current, creating a negative center on the surface. The sandstone bed acts like a generator similar to an oxidizing orebody. The distinction between the electrofiltration phenomenon and the spontaneous polarization is that, in a shaft sunk at the negative center, larger negative potentials are observed as we go downward in case of an orebody, and smaller negative potentials in case of a sand bed. The potential along the sand becomes zero at a point midway between the surface and the water table.

2. Electro-capillary by ascension (evaporation): Evaporation on the surface causes slight positive centers. For instance, a newly ploughed field is positive by 10 or 20 millivolts with respect to the neighboring surface. This results from stimulated evaporation over turned-up earth and electro-capillary ascension of water toward this evaporation zone.

This fact can be observed in laboratory by dipping partially a column of dried clay into saturated solution of copper sulphate. After the liquid has moistened the column by capillary ascension, the difference of potential between the liquid surface and the various points of the column is observed. The positive potential on the column increases as the distance from the liquid increases.

III. Diffusion Potentials

Frequently superimposed with electrofiltration potentials are the diffusion potentials. In some cases they even overshadow the electrofiltration potentials. They are produced by the contact of two different solutions, such as fresh water (meteoric) and saline connate water solution in sandy or other porous layers. Theoretically, the electromotive force, ΔE , generated between two solutions of the ion concentrations C and C is given by

$$\Delta E = 1.98 \cdot 10^{-4} \frac{t}{n} \frac{l_A - l_C}{l_A + l_C} \log \frac{C_1}{C_2} \text{ volts}$$

where l_A, l_C = Mobilities of the anions and cations.

n = valency

Suppose we have two NaCl solutions having an ion concentration ratio of $C_1/C_2 = 10$. The potential difference is 11.6 millivolts at standard temperature.

FORM AND INTENSITY OF THE ANOMALIES

In the following the current density and potential curves for sphere, rod, and sheet will be considered, supposing that these simple geometric forms approximate the shapes of orebodies. The necessity for a mathematical analysis of the simple geometric forms comes from the fact that the circumstances encountered in the nature are so complicated that a mathematical analysis of the actual cases is practically impossible. The only solution is investigation on models, similar to the actual ones. Thus, the analytical investigation of the simplest cases becomes necessary. The interpolation of the analytical results helps in determination of quantitative facts about the orebody.

I. Polarized Sphere

This problem has been calculated by A. Petrowsky. Application of such a system is small, that is, it does not approximate the natural orebodies to a great extent.

The problem is as follows:

A sphere of radius r_0 , infinitely conductive, is buried in the ground. The depth to the center is h . By any causes the sphere is polarized, that is, it develops an electromotive force of its own which has the greatest value along a certain diameter (axis of polarization), inclined at an angle α to the vertical, and diminishes by cosine law depending on the angle θ , which is formed by the given direction and the axis of polarization.

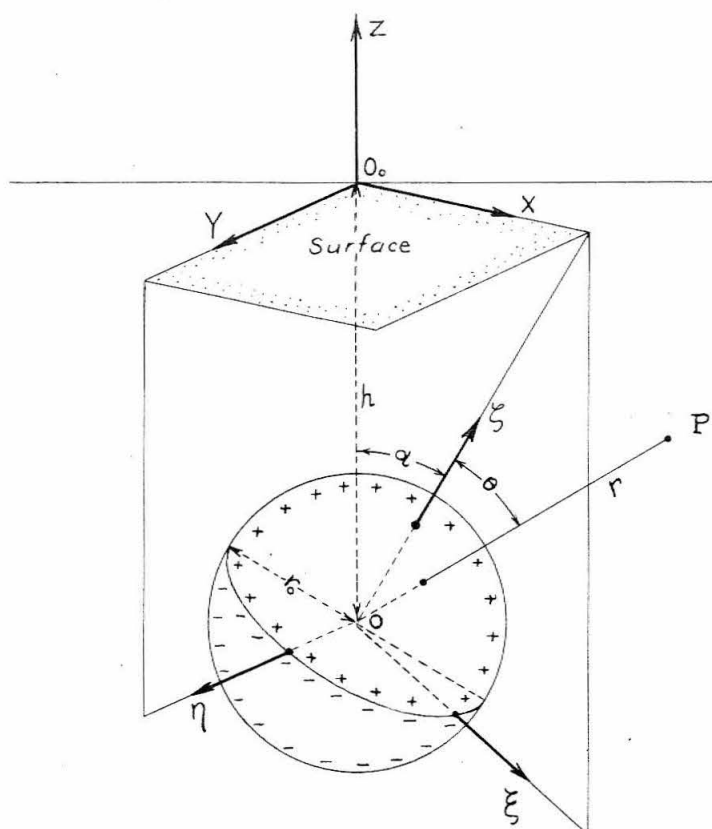


Fig. 1

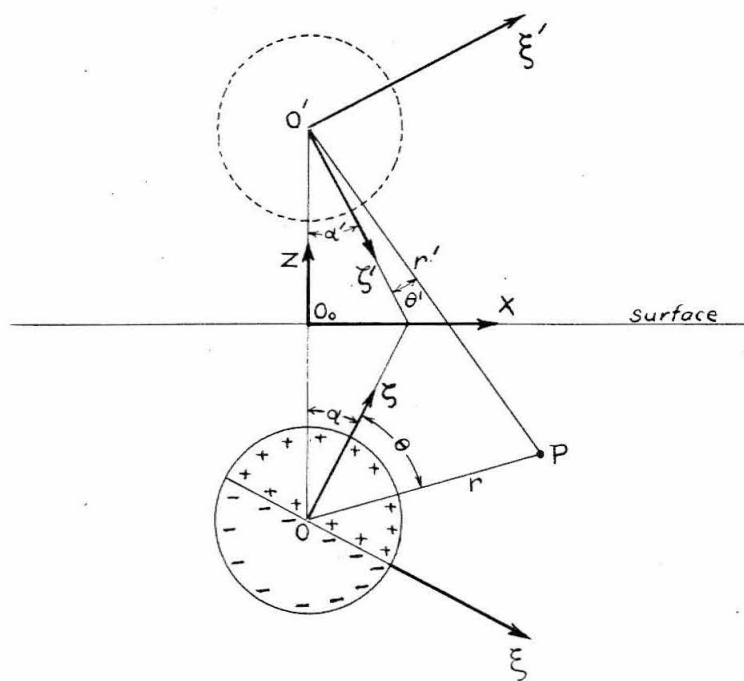


Fig. 2

$$e = E \cos \theta \quad (1)$$

At the same time a current is formed which spreads over the whole ground. The problem is the determination of (1) the projection of the center, O_0 , on the surface, (2) the depth, h , and (3) the radius r_0 .

It is assumed that each of the media surrounding the sphere, that is, air and ground, is homogeneous and isotropic, and h is considerably greater than r_0 .

Electric Field of a Hidden Sphere: Let us suppose that the plane dividing the media is absent and that the whole space is filled with a homogeneous medium.

Let us assume a coordinate system as shown (fig. 1b). The system is symmetrical with respect to the axis $O\zeta$ which coincides with the axis of polarization. Axis $O\eta$ is horizontal, and ξ is in the vertical plane of $O O_0 \zeta$. In this case the electric potential U is the function of the radius r and the polar angle θ , and the value of U must satisfy the Laplace equation.

$$\frac{\partial^2 U}{\partial r^2} + \frac{2}{r} \frac{\partial U}{\partial r} + \frac{\partial^2 U}{r^2 \partial \theta^2} + \frac{\cot \theta}{r^2} \frac{\partial U}{\partial \theta} = 0 \quad (2)$$

Moreover, this function must be finite, single-valued, and continuous, and must satisfy the following conditions:

- a) For $r = r_0$ it must take the form $U = \frac{E}{2} \cos \theta$.
- b) For $r = \infty$ it must be zero, $U = 0$.

The solution of these differential equations will be in the form

$$U = U'(r) \cos \theta + U''(r) \quad (3).$$

Differentiating this equation and introducing in the equation (2) we obtain the differential equation which is divided into two common differential equations

$$\frac{d^2 U}{dr^2} + \frac{2}{r} \frac{dU}{dr} - \frac{2}{r^2} U = 0 \quad (4)$$

$$\frac{d^2 U''}{dr^2} + \frac{2}{r} \frac{dU''}{dr} = 0 \quad (5)$$

Integrating the differential equations (4) and (5), and taking the sum of integrals, we obtain

$$U = \left[Ar + \frac{B}{r^2} \right] \cos \theta + \frac{C}{r} + D \quad (6)$$

Two equations can be written, one for $r > r_0$, U_{ex} , and one for $r < r_0$, U_{in} .

As we will not have to deal with U_{in} , let us consider U_{ex} alone.

$$U_{ex} = \left(A_{ex} r + \frac{B_{ex}}{r^2} \right) \cos \theta + \frac{C_{ex}}{r} + D$$

The coefficients are found by means of the conditions (a) and (b). The final equation is

$$U_{ex} = \frac{E r_0^2}{2r^2} \cos \theta \quad (7)$$

Now let us suppose that the space is divided into two homogeneous media by means of the surface. In such a case the ascending current lines become horizontal when they reach the surface. Thus we have another condition, (c): on the parting surface, the electric force must have a tangent component only.

Let us assume another system of coordinates as shown (fig.1b). In this case the new boundary condition (c) can be written

$$Z=0, \quad \frac{\partial W}{\partial Z} = 0 \quad (8)$$

where W electric potential after the introduction of the condition (c), outside of the sphere.

The potential W can be found approximately in the following way.

Let us consider the electric image of the sphere with respect to the surface, as shown in the figure 2. The image sphere O' is polarized in ad-

dition along the line $O'\zeta'$. The electric potential produced by the imaginary sphere O' at any point P will be

$$U'_{ex} = \frac{Er_o^2}{2r'^2} \cos \theta' \quad (9) \quad \text{then}$$

$$W = U_{ex} + U'_{ex} = \frac{Er_o^2}{2} \left[\frac{\cos \theta}{r^2} + \frac{\cos \theta'}{r'^2} \right] \quad (10)$$

and, in the cartesian coordinate system,

$$W = \frac{Er_o^2}{2} \left[\frac{x \sin \alpha + (h+z) \cos \alpha}{[x^2 + y^2 + (h+z)^2]^{3/2}} + \frac{x \sin \alpha + (h-z) \cos \alpha}{[x^2 + y^2 + (h-z)^2]^{3/2}} \right] \quad (11)$$

As we are interested in the measurements made on the surface, that is for $z=0$,

$$W = Er_o^2 \frac{x \sin \alpha + h \cos \alpha}{[x^2 + y^2 + h^2]^{3/2}} \quad (12)$$

The potential profile along the OX axis will be

$$W_x = Er_o^2 \frac{x \sin \alpha + h \cos \alpha}{[x^2 + h^2]^{3/2}} \quad (13)$$

and the potential profile along the O_y axis

$$W_y = Er_o^2 \frac{h \cos \alpha}{[y^2 + h^2]^{3/2}} \quad (14)$$

Now let us consider the current density.

$$\text{Current density} = i = \frac{1}{A} = \frac{1}{S} \left[-\bar{i} \frac{\partial U}{\partial x} - \bar{j} \frac{\partial U}{\partial y} - \bar{k} \frac{\partial U}{\partial z} \right] \quad (15)$$

$$\text{or simply} \quad i = -\frac{1}{S} \bar{F}$$

Since the vertical gradient is zero in our case,

$$\bar{F} = \bar{i} \frac{\partial W}{\partial x} + \bar{j} \frac{\partial W}{\partial y} \quad (16)$$

where \bar{F} = potential gradient, field strength, or electric force.

Taking the partial derivatives of \bar{F} , we find its projections along the axes x and y. Introducing the denotations

$$u = \frac{x}{h},$$

$$v = \frac{y}{h},$$

we have

$$F_x = \frac{E r_0^2}{h^3} \cdot \frac{3 u \cos \alpha - (1 + v^2 - 2 u^2) \sin \alpha}{[1 + u^2 + v^2]^{5/2}} \quad (17)$$

$$F_y = \frac{E r_0^2}{h^3} \frac{(\cos \alpha + u \sin \alpha) 3 v}{[1 + u^2 + v^2]^{5/2}} \quad (18)$$

$$W = \frac{E r_0^2}{h^2} \frac{\cos \alpha + u \sin \alpha}{[1 + u^2 + v^2]^{3/2}} \quad (19)$$

In fig. 3 are shown the values of F_x , that is, the current density profile, along the X axis, in the direction of the plane of polarization. On the axis of the abscissae are plotted the values of u , on the Y axis those of F_x , the coefficient being assumed to be equal to 1000, $\left(\frac{E r_0^2}{h^3} = 1000\right)$. The curves correspond to different values of α . A very significant fact about these curves is that, in case of a vertically polarized sphere, i.e. when $\alpha = 0^\circ$, one of the two zero points is right above the center, the other zero point being at infinity. The maximum and minimum points in this case are numerically equal, their distance from the O_0 point being equal to the half of the depth.

$$\left(\frac{x}{h}\right)_{\substack{\text{max.} \\ \text{min.}}} = \pm 0.5 \quad (20)$$

Figure 4 shows the profile along a line parallel to the plane of polarization, at a distance h from the O_0 point.

Figure 5 is the profile along the Y axis, perpendicular to the plane of polarization, passing through the O point. These curves have only one zero point, one minimum and one maximum. The axis of the abscissae is the asymptote to all of them. The zero point is always over the center, the curves being symmetrical with respect to this point. The maxima are always at a distance equal to the half of the depth

$$v_m = \left(\frac{y}{h}\right)_{\substack{\text{max.} \\ \text{min.}}} = \pm 0.5 \quad (21)$$

When $\alpha = 90^\circ$, the current density is always zero along the profile.

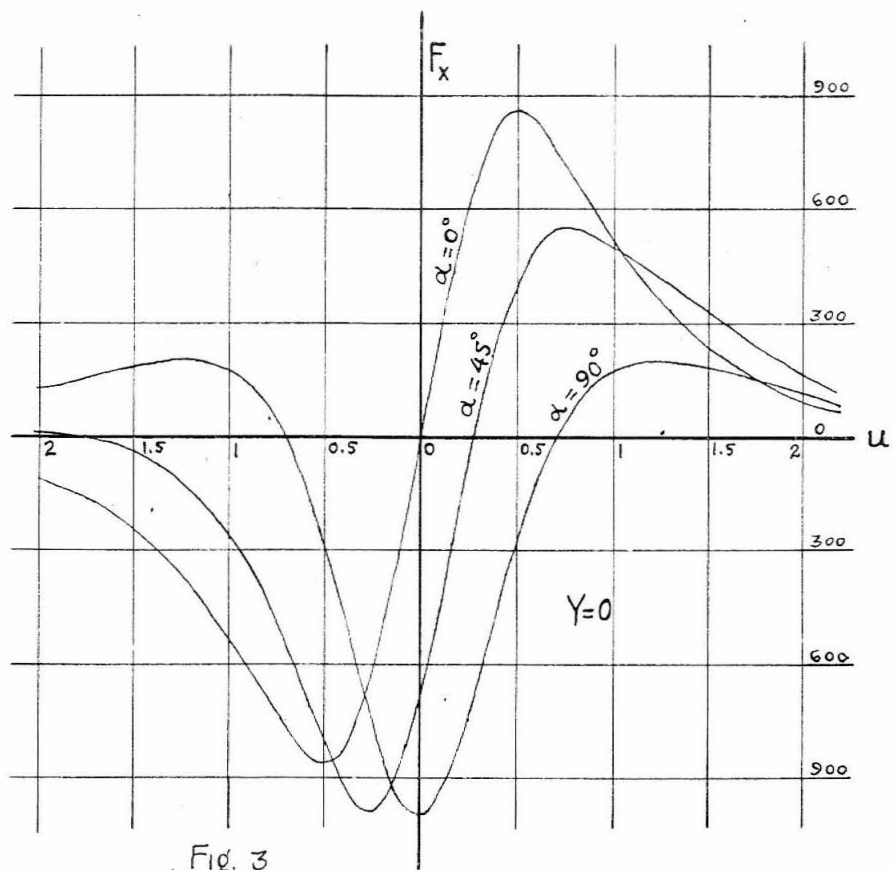


Fig. 3
Current density profile along the plane of polarization

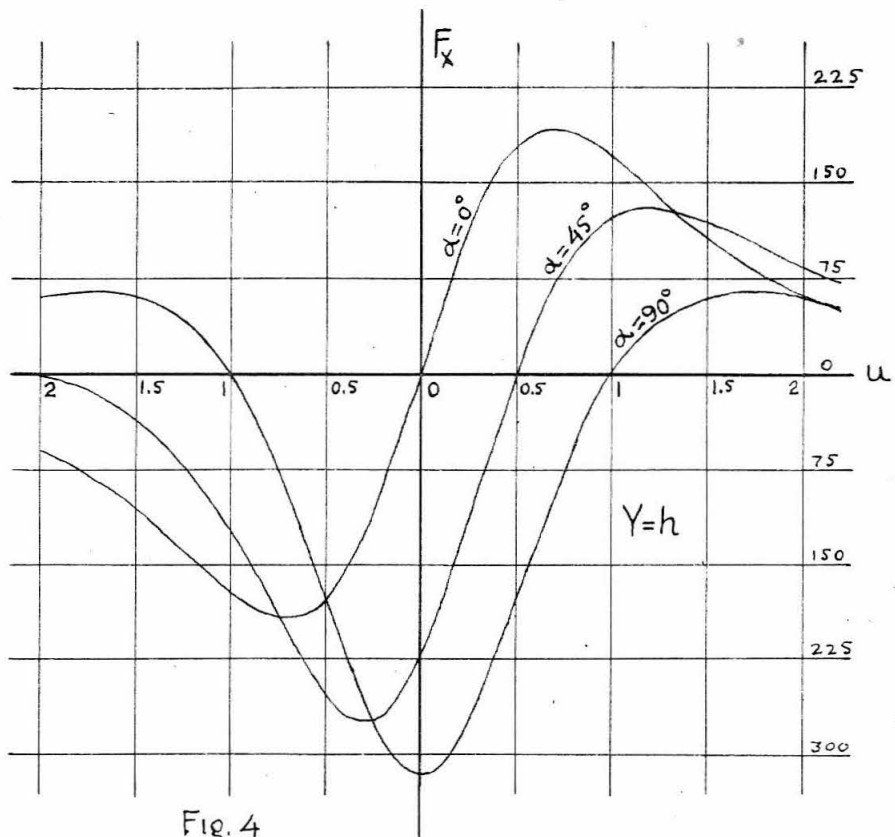


Fig. 4
Current density profile along a line parallel to the plane of polar.

Figure 6 gives the current density values along a profile perpendicular to the plane of polarization, parallel to the Y axis at a distance h. These curves are similar to the previous ones, except that the maxima are removed further.

Now let us find the expression for any profile, inclined to the X axis, at the same time to the plane of polarization, with an angle ϕ , not passing through the point O_0 . Introducing a new system of coordinates, X' and Y' , having their origin at O_0 , and making the angle ϕ with the X and Y system.

$$u' = \frac{x'}{h} \quad (22)$$

$$v' = \frac{y'}{h} \quad (23)$$

$$u = u' \cos \phi - v' \sin \phi$$

$$v = u' \sin \phi + v' \cos \phi$$

Substituting these values in equation (19) we have

$$W = \frac{E r_0^2}{h^2} \frac{\cos \alpha + u' \cos \phi \sin \alpha - v' \sin \phi \sin \alpha}{[1 + u'^2 + v'^2]^{3/2}} \quad (24)$$

$$F_{x'} = - \frac{\partial W}{\partial x'} = - \frac{\partial W}{h \partial u'}$$

$$F_{x'} = \frac{E r_0^2}{h^3} \frac{3 u' \cos \alpha - (1 + v'^2 - 2 u'^2) \cos \phi \sin \alpha - 3 u' v' \sin \phi \sin \alpha}{[1 + u'^2 + v'^2]^{5/2}} \quad (25)$$

The expression for $F_{y'}$ is of no use since a change in ϕ will be sufficient for any direction.

Figure 7 is the current density profile along the line inclined to the plane of polarization at an angle $\phi = 45^\circ$, passing through the point O_0 .

Figure 8 is the most general case, the profile having the same angle $\phi = 45^\circ$ with the plane of polarization, but not passing through the point O_0 , Y being equal to h.

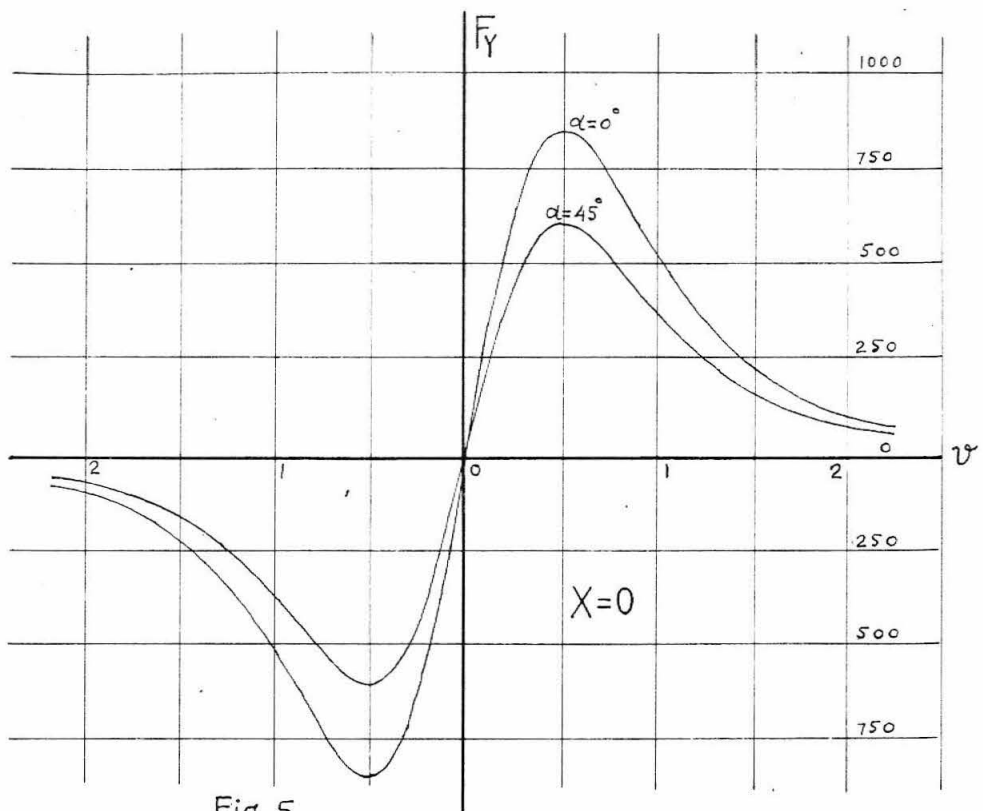


Fig. 5
Current density profile along the Y axis

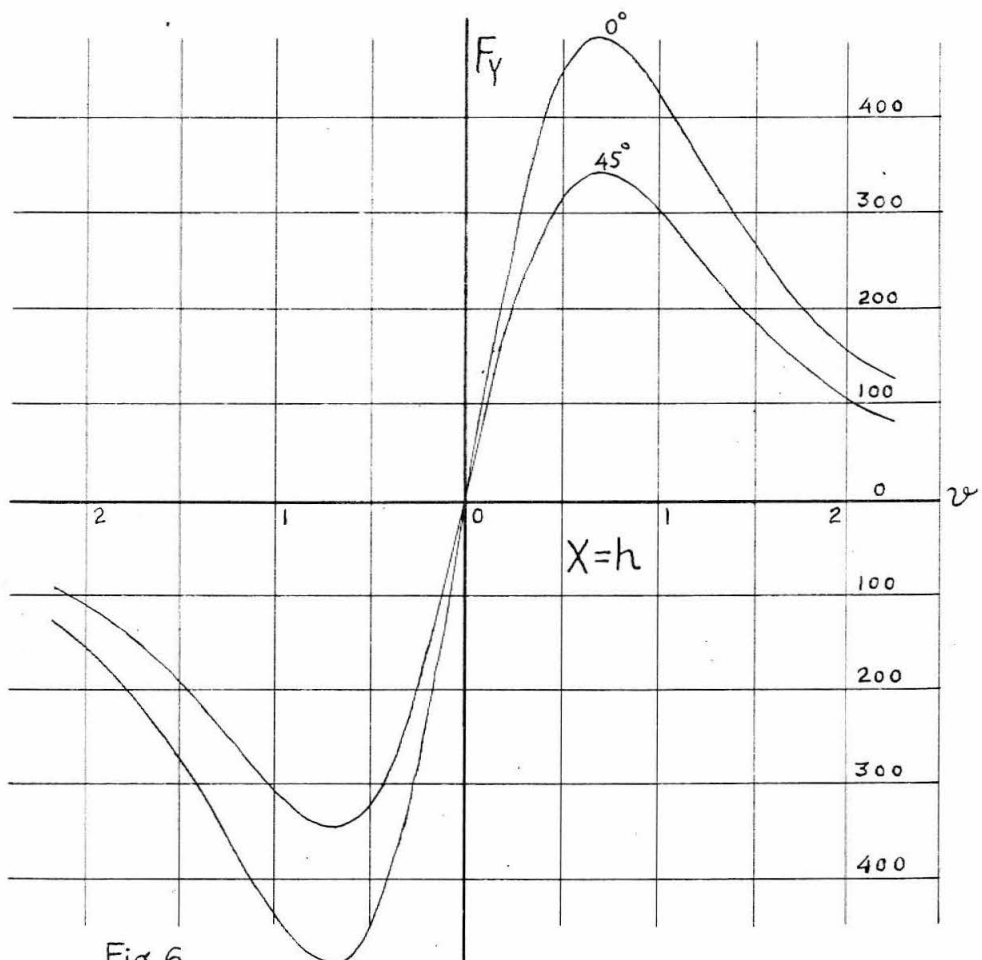


Fig. 6
Current density profile along a line parallel to the Y axis

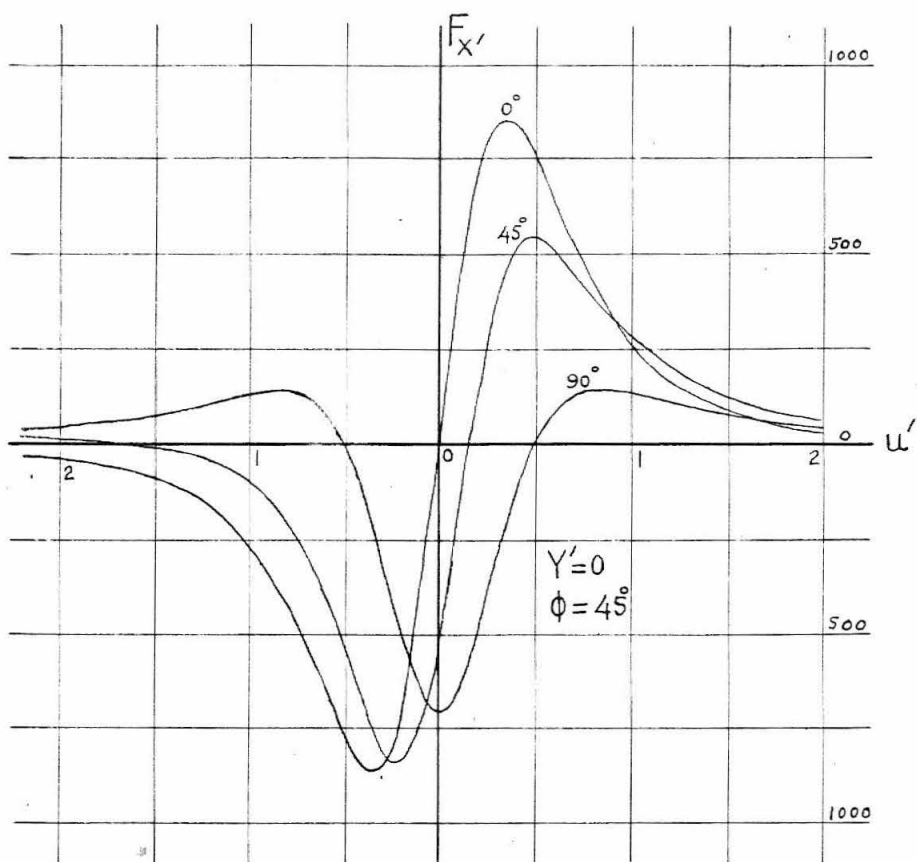


Fig. 7
Current density profile along an inclined line passing through O_0 .

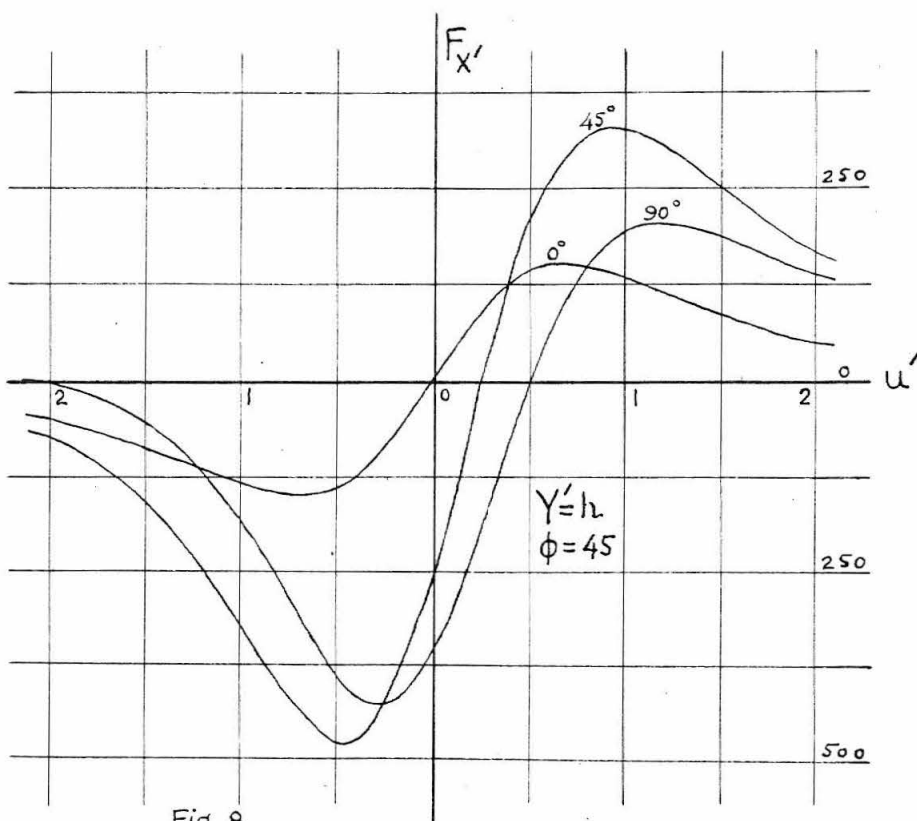


Fig. 8
Current density profile along an inclined line not passing through O_0 .

Location of the O_o - point, h and α :

Let us suppose an arbitrary direction, and make current density profiles parallel to this direction. We obtain curves having, in general, two zero points and three maxima. The geometrical loci of these points on the surface determine the position of the sphere.

Case 1: Profiles parallel to the plane of polarization (to the X axis). From the equation (17) it follows that the locus of the zero points is

$$u^2 - \frac{1}{2} v^2 + \frac{3}{2} \cot \alpha \cdot u - \frac{1}{8} = 0 \quad (26)$$

This is the equation of an hyperbola, its center, being in the plane of polarisation, has the following coordinates:

$$\begin{aligned} x_c &= \frac{3}{4} \cot \alpha \cdot h \\ y_c &= 0 \end{aligned} \quad (27)$$

The real semi-axis is in the plane of polarization, and has the length a , which is

$$a = \sqrt{\frac{h^2}{2} + \frac{9}{16} \cot^2 \alpha \cdot h^2} \quad (28)$$

The asymptotes are defined by the equations

$$\begin{aligned} y &= \sqrt{2} x \\ y &= -\sqrt{2} x \end{aligned} \quad (29)$$

and are inclined to the x axis at $54^\circ 45'$ angles.

Figure 9 shows the zero-point hyperbolas for various values of α .

From the equation (17) we obtain the loci of the maximum points, given with the following equation:

$$1 + v^2 = \frac{2 u^2 (2 + \tan \alpha \cdot u)}{1 + 3 \tan \alpha \cdot u} \quad (30)$$

This is a curve of third order, having three branches, each being symmetrical with respect to the x axis. The middle branch has an asymptote parallel to the y axis.

Figure 10 gives the maximum-point curves for different values of α .

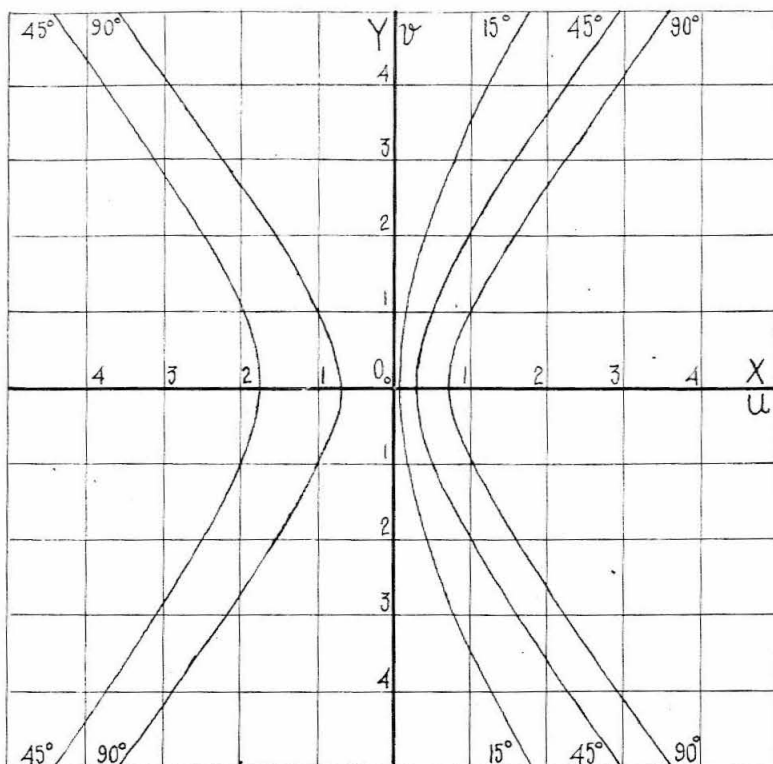


Fig. 9. — Zero-point curves along profiles parallel to the plane of polarization.

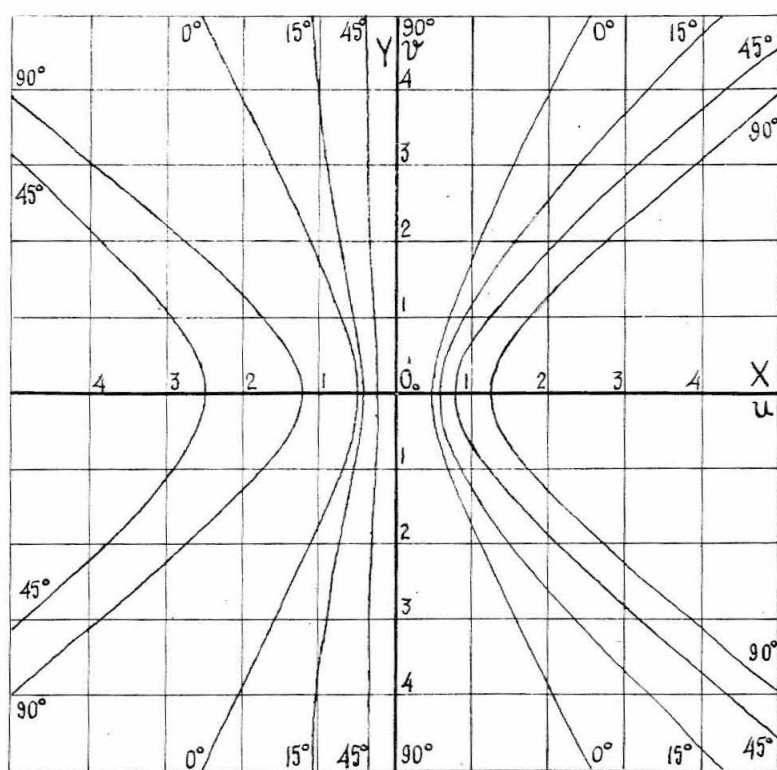


Fig. 10. — Maximum-point curves along profiles parallel to the plane of polarization.

Case 2: Profiles perpendicular to the plane of polarization. From the equation (18) it follows that the zero points are on the straight lines

$$v = 0, \quad (31)$$

$$\cos \alpha + u \sin \alpha = 0 \quad (32)$$

The first one is the x axis, the second one perpendicular to it at a distance $u = -\cot \alpha$. Current density is zero along the second line.

From the equation (18) again we obtain the geometrical locus of the maximum points,

$$4v^2 - u^2 = 1 \quad (33)$$

which is a hyperbola, the center of which being at the O_0 point. The real semi-axis is perpendicular to the plane of polarization, and is equal to the half depth.

$$a = \frac{h}{2} \quad (34)$$

Case 3: Profiles at an angle ϕ to the plane of polarization. From the equation (25), it follows that the locus of the zero points is defined by the equation

$$u'^2 - \frac{1}{2}v'^2 - \frac{3}{2}\tan \phi \cdot u'v' + \frac{3\cot \alpha}{2\cos \phi} u' - \frac{1}{2} = 0 \quad (35)$$

which is a hyperbola. The real semi-axis is inclined to the line of survey at an angle θ , which is defined by:

$$\theta = -\frac{\phi}{2}, \quad (36)$$

and its value is

$$a = h \sqrt{\frac{2\cos \phi \left(1 + \frac{9\cot^2 \alpha}{8 + \sin^2 \phi}\right)}{3 + \cos \phi}} \quad (37)$$

The maxima curves are of the third order, and do not represent characteristics; thus, they have no great value.

Now let us see what we can do in determining the O_0 , h , and α with

the results obtained in the three cases.

Case 1: Asymptote of the middle branch of the maxima curve is parallel to the imaginary axis of the zero hyperbola: the profile is parallel to the plane of polarization.

Drawing the asymptotes we find the center of the hyperbola which is in the plane of polarization and has coordinates

$$x_c = \frac{3}{4} \cot \alpha \cdot h \quad (27)$$

The point K where the asymptote to the middle branch of the maxima curve intersects the X axis has the following coordinate:

$$x_k = \frac{1}{3} \cot \alpha \cdot h \quad (38)$$

The distance C K d can be measured, and is equal to

$$d = \frac{5}{12} \cot \alpha \cdot h \quad (39)$$

From (27) and (39) we get

$$x_c = \frac{9}{5} d \quad (40)$$

Which determines the point O_0 , since the point C is known.

From (28)

$$h = \sqrt{2 a^2 - \frac{16d}{25} d^2} \quad (41)$$

By measuring a, the real semi-axis, and the distance d we find the depth.

The inclination angle α is found from (39)

$$\cot \alpha = \frac{12 d}{5 h} \quad (42)$$

The expression for r_0 depends on E which is not very well defined.

Determination of O_0 , h and α may be made graphically as shown on figure (11).

$$C a : C b = 5 : 9 \quad (43)$$

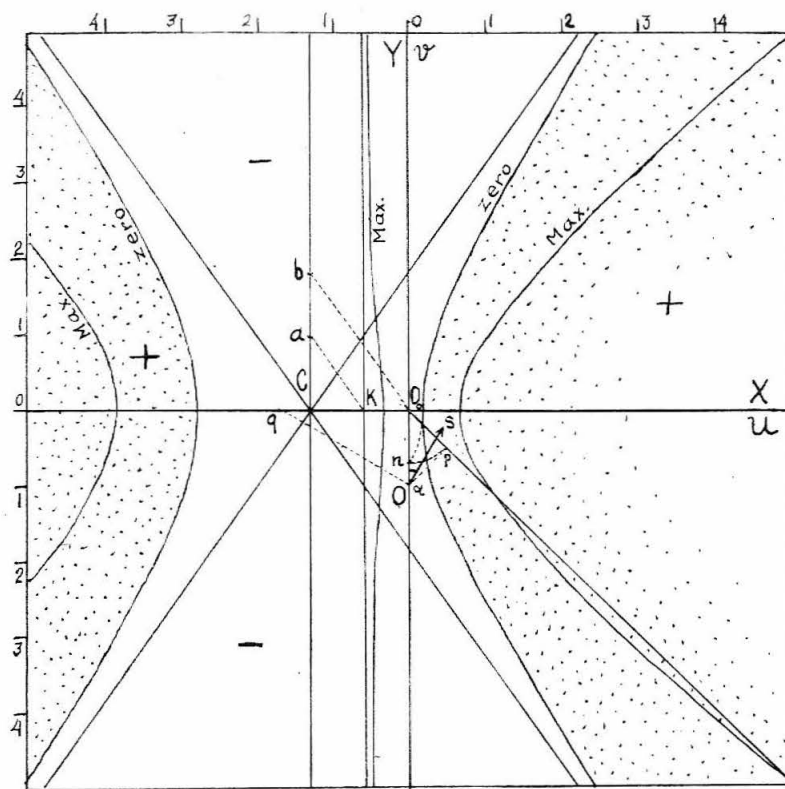


Fig. 11. — Graphical determination of O_0 , h , and d in Case 1.

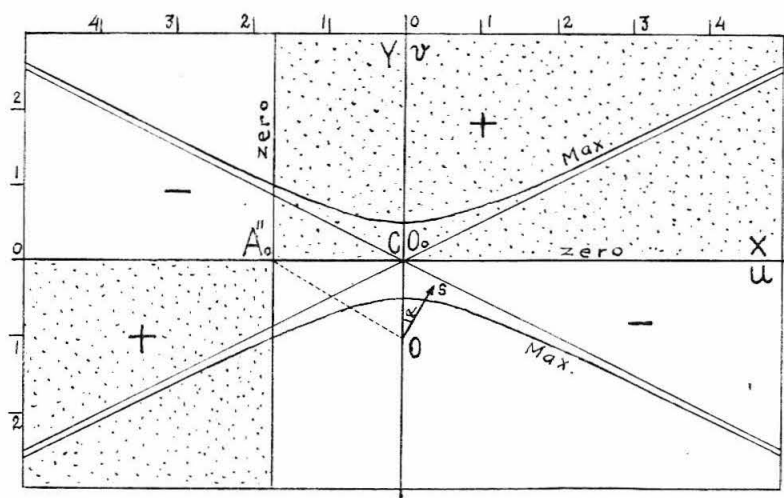


Fig. 12. — Graphical determination of O_0 , h , and d in Case 2.

a K is parallel to b O₀, thus the point O₀ is found. Draw a line O₀ p bisecting the angle C O₀ W. From C draw a circle with a radius equal to the real semi-axis. From O₀ draw the circle n p: from p erect perpendicular p O. The depth h is equal to O O₀.

Case 2: The zero-point curves are represented by two straight lines perpendicular to each other, and the maxima curves are as shown on figure 12: the profile is taken along the lines perpendicular to the plane of polarization. By tracing the asymptotes we find C which coincides with O₀, and the real semi-axis, which gives h.

$$h = 2 a \quad (34)$$

Measuring the distance A'' C = d, we have from (31) and (32)

$$\cot \alpha = \frac{d}{h} = - \frac{d}{2a} \quad (44)$$

Case 3: If there is no symmetry, the line of survey is inclined to the plane of polarization with an angle $\phi = 2 \theta$, where θ is the angle between the hyperbola representing the zero-point curve and the line of survey. Thus we find the direction of the plane of polarization. By repeating the survey in the direction of the plane of polarization the problem is reduced to the first case, or to the second. In practice, first a potential survey of the area is done as it will be explained later. From the equipotential lines the direction of the plane of polarization is usually apparent, if there is any. Then current density profiles along any desired direction are found from the potential data, without repeating the survey.

When dealing with natural currents, the poles are reversed; a negative source on the upper end and a positive source on the lower end.

The current density curve may be plotted from the smoothed potential curve, by taking the slopes at numerous points. It is also obtainable directly in the field from voltage readings with constant electrode separation,

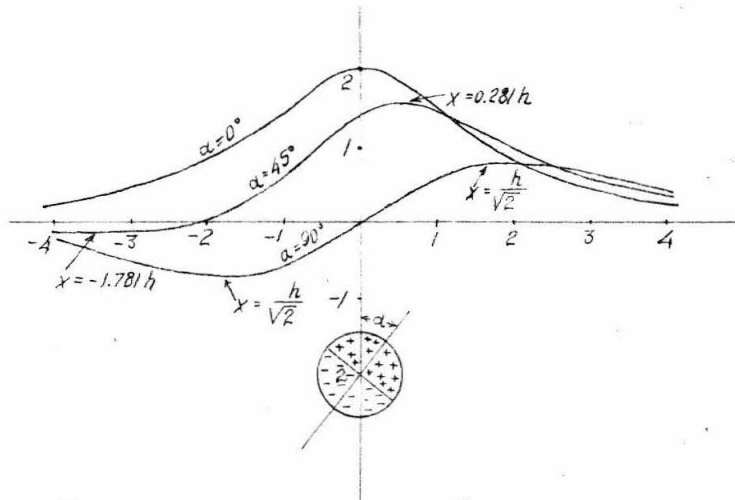


Fig. 12a - Potential curves for polarized sphere
(profiles along the plane of polarization ; depth = 2)

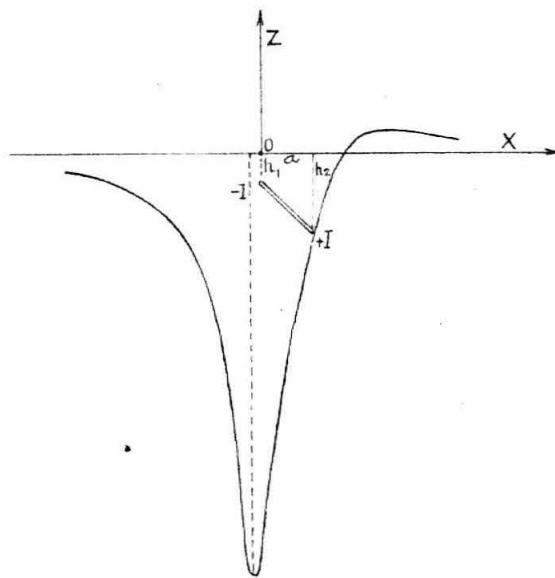


Fig. 12b - Potential profile due to a
polarized rod (along the polar. plane)

dividing the voltage reading by the electrode separation.

II. Polarized Rod

Sometimes an orebody may be assumed to be similar to a polarized rod with a negative current source on its upper end and a positive source on its lower end. Let us suppose that the orebody is in the XZ plane, so that the vertical distance of its upper end from the surface is h_1 , that of its lower end is h_2 , its projection on the X axis is a , and the distances of its ends from a surface point are r_1 and r_2 (see fig. 12 b). The coordinates of this point are XY and the origin is the projection of the upper end. It will be assumed that the potential at any point may be regarded as the resultant potential of a current $-I$ at the upper point and $+I$ at the lower point in a semi-infinite medium of resistivity ξ ohm cm. Taking into account the images of the upper and lower points with respect to the ground surface, the potential for the upper negative source is

$$V_1 = -\frac{\xi I}{2\pi r_1} \quad (45)$$

and that for the lower source is

$$V_2 = \frac{\xi I}{2\pi r_2} \quad (46)$$

so that the total potential is

$$V = -\frac{\xi I}{2\pi} \left(\frac{1}{r_1} - \frac{1}{r_2} \right) \quad (47)$$

Substituting

$$r_1 = \sqrt{x^2 + y^2 + h_1^2} \quad \text{and} \quad r_2 = \sqrt{(x - a)^2 + y^2 + h_2^2}$$

the potential becomes

$$V = -\frac{\xi I}{2\pi} \left[\frac{1}{\sqrt{x^2 + y^2 + h_1^2}} - \frac{1}{\sqrt{(x - a)^2 + y^2 + h_2^2}} \right] \quad (48)$$

It is seen that the shape of the potential surface depends only on the

depth, inclination and size of the orebody. The main features of a potential profile in the XZ plane are that the potential gradient in the direction of the dip is greater than in the opposite direction, and the negative center is not immediately over the upper end of the body, but is displaced away from the source of current. This displacement depends on the position of the lower positive source. When it is far removed, or when the rod is vertical the negative center is right below the negative source.

III. Polarized Sheet

A polarized sheet bears probably the closest similarity to natural orebodies among the simple geometrical shapes. This case has been calculated by Edge and Laby.

The assumptions are made that (1) the sheet has a negligible thickness; (2) the upper and lower edges are parallel to the surface; (3) the upper edge is the current sink and the lower edge is the current source, the distribution of current being uniform along these edges which are lines; (4) the country rock is homogenous; (5) the distortion of the potential surfaces, due to the orebody itself, may be neglected.

Let us suppose that the system shown on figure 13 exists, the upper and lower edges, pg and ts , being parallel to the surface and to the X axis. The projection of the upper edge coincides with the X axis, the O -point being in the middle. P is a point on the surface, for which the potential will be calculated. PQ is a perpendicular from P to the upper edge. The angles θ_1 and θ_2 refer to the upper edge, and θ'_1 and θ'_2 to the lower edge. The length of the sheet is $2l$. The total current is I amps.

The potential at the point $P(xy)$ on the surface due to an element

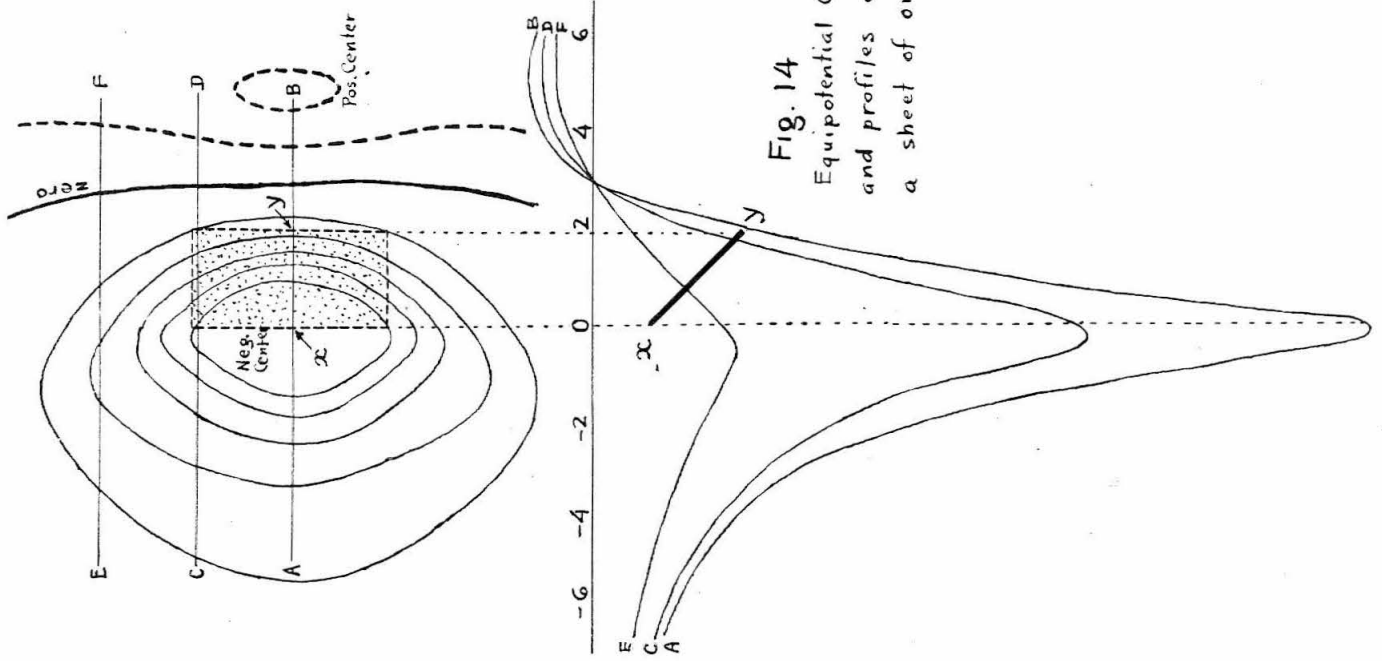


Fig. 14
Equipotential Contours
and profiles due to
a sheet of ore

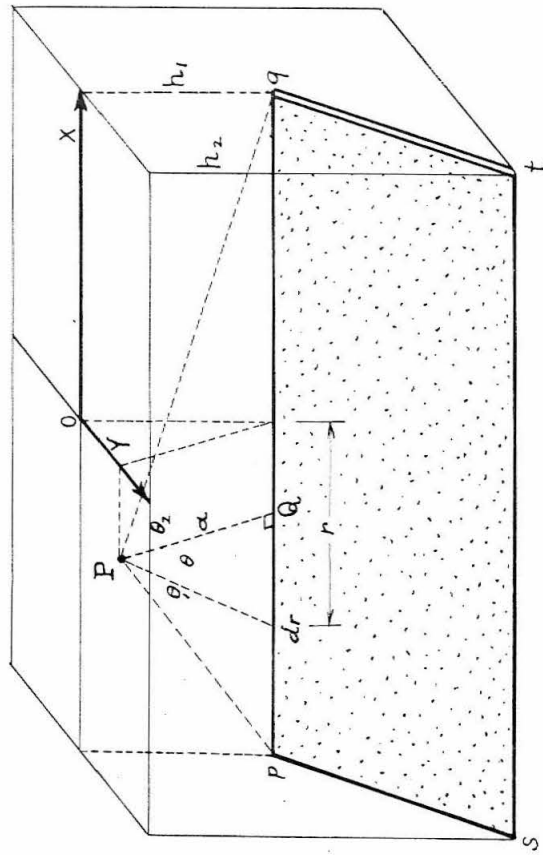


Fig. 13
Diagram of the polarized sheet.

dr of the upper edge and its image is given by

$$dV_1 = -\frac{I \rho}{4 \pi l} \cdot \frac{dr}{\sqrt{y^2 + h_1^2 + (r-x)^2}} \quad (49)$$

where h_1 is the depth of the upper edge, ρ is the resistivity of the ground, and r is the coordinate of the segment dr . The total potential due to the upper edge is

$$V_1 = -\frac{I \rho}{4 \pi l} \int_{-1}^{+1} \frac{dr}{\sqrt{a^2 + (r-x)^2}} \quad (50)$$

where $a = PQ = \sqrt{h_1^2 + y^2}$ (51)

Putting $(r-x) = a \tan \theta$ the final expression becomes

$$V_1 = -\frac{I \rho}{4 \pi l} \left\{ \log \tan \left(\frac{\pi}{4} + \frac{\theta_2}{2} \right) - \log \tan \left(\frac{\pi}{4} + \frac{\theta_1}{2} \right) \right\} \quad (52)$$

and for the lower edge

$$V_2 = \frac{I \rho}{4 \pi l} \left\{ \log \tan \left(\frac{\pi}{4} + \frac{\theta_2'}{2} \right) - \log \tan \left(\frac{\pi}{4} + \frac{\theta_1'}{2} \right) \right\} \quad (53)$$

The total potential at P is

$$V = V_1 + V_2 = -\frac{I \rho}{4 \pi l} \left\{ \log \tan \left(\frac{\pi}{4} + \frac{\theta_2}{2} \right) - \log \tan \left(\frac{\pi}{4} + \frac{\theta_1}{2} \right) - \log \tan \left(\frac{\pi}{4} + \frac{\theta_2'}{2} \right) + \log \tan \left(\frac{\pi}{4} + \frac{\theta_1'}{2} \right) \right\} \quad (54)$$

An example is given (fig. 14) for a particular case where

$h : h : l = 1 : 3 : 2$, the dip being 45 degrees.

APPLICATION OF THE SPONTANEOUS POLARIZATION PHENOMENA
TO EXPLORATION WORK

I. Natural Bodies Showing Spontaneous Polarization.

The degree to which natural orebodies respond has been found by experience, and the order is as follows:

1. Pyrite and pyrrhotite veins or bodies give the strongest reactions. When the conditions are favorable, the potential difference may be over one volt.

2. Anthracites, very poor in volatile matters, are good conductors and give strong reactions over one volt. A positive center, instead of a negative one, is found.

3. Graphite, very conductive and oxidizable, may show potentials over one volt (positive center).

4. Copper sulphides (chalcopyrite, chalcosite, covellite) constitute the best adopted ores to this method. Maximum reaction is one volt.

5. Magnetite is also conductive and oxidizable.

6. Cobalt ores give somewhat weak reactions.

7. Galena gives very weak reaction if the ore is continuous, otherwise the phenomenon is absent.

Besides these natural bodies, there are several more features causing similar effects. Some of them are:

1. Any kind of electro-capillary phenomenon resulting from (a) heterogeneous porosity, (b) irregular vegetation distribution, (c) ground wa-

ters, (d) differential evaporation, etc.

2. Faults and fractured zones, which allow the meteoric waters to penetrate easily, give rise to both oxidation and electro-capillary currents. Oxidation of organic matter along the gouge zone is sometimes greater than the electro-capillary effect.

3. Metals in placer deposits may show spontaneous polarization if the mineral continuity is provided, such as Cienaga, Columbia, river gravels containing pyrrhotite-impregnated boulders.

4. Formation boundaries cause small potential differences. Very accurate measurement of these potentials caused by differences in the conductivity of solutions filling the pores may reveal the boundary covered by overburden. In case of metamorphosed sediments it is sometimes possible to make a detailed study of the geologic structures, such as graphitic shale beds.

5. Corroding pipes also generate currents. Although this problem has a great commercial value in engineering, it has little importance in geological exploration (see Heiland 679-681).

II. Methods of Application

The field procedure varies with the kind of terrain and the amount of anomaly. Before going into the detail, let us summarize the methods used in practice.

1. Potential measurements.

a. Potential profiles

b. Equipotential lines

c. Potential gradients

2. Current measurements.

Current density profiles (potential gradient profiles).

3. Location of equipotential lines by null method, using either potentiometer or galvanometer.

1. The usual practice is to survey two base lines at right angles to each other, and then survey two series of regularly spaced traverse lines, each series being parallel to one of the base lines. Their intersections form a grid-work of stations. The intersection of the base lines is usually taken as the base station to which the readings are referred. In this case, each station, which is represented by the intersection of two profiles, has two potential values, and the average is taken in plotting either the potential profiles or equipotential contours.

Another way of surveying an unknown area is to make radial traverses in such a manner that any hidden ore-body of important dimensions may be crossed or closely approached.

If the area is known to some extent, such as the indications provided by gossans, it is sufficient to make only one series of straight line traverses in a direction at right angles to the supposed strike of the mineralized zones, intersecting the most promising surface indications.

After locating an important anomaly by means of these reconnaissance profiles a more detailed survey is made over the critical area with small electrode spreads, such as 25 feet.

After recording the potential value and its sign between the stations, the results are summed and plotted with distances as abscissae and the potential values as ordinates, in a similar way to a topographical section. It is generally preferable to express the final results by a map of equipotential curves.

This kind of profiling method is suitable for dry and rough terrain

where direct location of equipotential lines is not easy. If the reconnaissance shows a well defined gradient of 0.5 to one millivolt per foot, time may be saved by mapping equipotential lines as will be explained later.

Sometimes it is advantageous to plot the readings in the form of potential gradients. This is especially suitable for small and widespread anomalies, and in locating faults. The following way of plotting the potential gradients has been used successfully by Dr. Potapenko in locating faults.

Let us suppose we have nine stations, 25 feet apart, along a straight profile which is supposed to be perpendicular to the strike of the fault to be located. The instrument is set up at the station 1 and the overlapping potential measurements 1-2, 1-3, 1-4, 1-5 are taken. These values are plotted in millivolts and a straight line is drawn to indicate the average slope of the broken line 1-2-3-4-5. Then the tangent of this angle is plotted as the gradient value at the station 3. Thus, four readings are represented by a single point. The apparatus is brought to the station 3, and the readings 3-4, 3-5, 3-6, 3-7 are taken, overlapping the last two readings of the previous set. The gradient of these readings is related to the station 5. A sudden change in gradient indicates the fault. This way the effect of the regional anomaly is minimized and small anomalies are pronounced.

In order to interpret the results by means of negative and positive centers or values, we have to know the zero value in the area. The equipotential contour map defines the value of the potentials measured with regard to the base station, calculated by progressive addition of potentials to this point which is supposed to be zero during the survey. It is advisable to assume the average potential over a large area, where there is no

local anomaly, as the zero potential for the surveyed area. In a mountainous district, especially when it is dry, it is very difficult to estimate the zero value. It is very easy in clayey soil in plains.

2. The method of current measurements is carried on in a similar way to potential profiling, except that a microammeter is used to measure the current, instead of a potentiometer. The magnitude and the direction of the natural currents are recorded. The current readings are plotted with the plus and minus values as ordinates, and the distances from the starting point as abscissae. It was found by experience that the total resistance in the circuit is about 1000 ohms. Hence a reading of one microampere corresponds to a potential difference of one millivolt. This means that the apparatus is quite sensitive. When passing over grounds free from anomalies, the readings will be small and variable in sign. On traversing a vein the values will increase to a maximum over the vein and then abruptly reverse to an approximately equal value as the center of the vein is passed. The readings will gradually decrease and assume the asymptotic value. The shape of the profile curves over a spherical orebody is given in figures 3 to 8. Practically the deposit will be located almost directly below the point on the ground where the reversal takes place.

3. If the area is accessible, fairly level, and reasonably moist to give good contact, the equipotential lines can be traced directly. Lines of equal potentials are traced by locating points between which no current flows. This is done by means of a potentiometer, a galvanometer, or a microammeter. A high-resistance milliammeter, zero center, with a range of 300 to 400 milliamperes and reading to an accuracy of $\frac{1}{2}$ milliampere, is given by Heiland as the most suitable, since it is faster. This does not seem to be reasonable, as it will be explained later under the heading "Galvanometers." The

galvanometer of the potentiometer can be used, setting the potential rheostat to zero. The use of a potentiometer for this purpose is more difficult, since the direction and magnitude is not indicated directly as in microammeter, but it gives more accurate results, due to the fact that the potential readings are independent of the electrode resistance which is not constant.

This kind of surveying must be preceded by a reconnaissance profiling so that the operation is confined to the anomalous region. In this procedure, the microammeter and one electrode are placed at a point on the reconnaissance traverse at which the gradient is steep and regular. The assistant then takes the second electrode, connected to the microammeter by a 100-foot cable, and applies it to the ground at a succession of trial points situated approximately on the circumference of a circle, the radius of which is determined by the 100-foot cable. The operator, by watching the direction and amount of deflection in the microammeter for each of these trial points, will then be able to direct the assistant to an equipotential point at which no deflection is observed. A stake is driven into the ground at this point, which is then occupied by the operator. If the equipotential line closes on the starting point within a few feet, the result may be considered satisfactory. As soon as an equipotential line has been traced the approximate location of a negative center will be known. Its precise position is established by surveying other equipotential lines at arbitrary intervals.

III. Equipment

The complete standard equipment necessary for spontaneous polarization measurements consists of a pair of non-polarizable electrodes, potentiometer

and flexible cable. However, sometimes milliammeters and galvanometers are used as explained previously.

1. Non-polarizable electrodes: The success of the operations depends largely upon the behavior of the non-polarizing electrodes. It is necessary that the two electrodes be identical, metal and solution be chemically pure. Such an electrode consists of a porous cup which contains a rod of metal and a saturated (sometimes diluted) solution of a salt of the metal. The metal will go into solution or will be precipitated, depending upon the direction of the current. The electro-chemical process is reversible and the electrodes do not change their properties appreciably with the passing of small quantities of current. Some polarization occurs due merely to concentration changes in the solution near the electrodes. Some diffusion potentials also occur due to the diffusion between the concentrated solution and the ground solutions. To eliminate this, some investigators used electrodes having two chambers, an inner one with the saturated solution and an outer chamber with a more dilute solution.

The theory of potentials caused by electro-chemical and electro-capillary phenomena has been discussed previously. It is seen that direct contact with the ground by means of solid conductors is impracticable in the spontaneous polarization method, since even platinum electrodes produce contact polarization effects of the order of one or two hundred millivolts which is usually much higher than the potentials to be measured. Moreover, these polarization potentials are of a very irregular character.

For general purposes, the electrodes may be considered to be satisfactory if their internal polarization effects do not exceed one millivolt.

Several types of non-polarizing electrodes are summarized below:

a. Carl Barus type: Zinc electrode in a saturated solution of $ZnSO_4$ contained by a bulb of semi-pervious animal gut.

b. Schlumberger type: A flat bottomed porous magnesia pot containing a saturated solution of copper sulphate into which a perforated tube of copper is inserted. The long copper tube is also filled with the solution, together with crystals of copper sulphate. A complete description and design of this type is given by Edge and Laby (page 239).

c. I.G.E.S. pattern (Schlumberger): This pattern was devised to increase stability and portability, minimizing evaporation of the solution. The pot has one lower and one upper chamber, the copper pipe being only in the upper chamber. Except the bottom lid which closes the lower chamber, the pot is completely glazed. Two percent of pure gelatin is added to the upper chamber. The lower chamber is filled with a hot solution of gelatin which is prepared by soaking the dry sheets in a cold concentrated solution of copper sulphate. The soft sheets are then melted in a beaker over a water-bath. The hot material is then poured into the lower chamber and the porous lid is cemented. This kind of electrodes requires no attention, except that their terminals should be connected in a bath when not in use.

d. Ambron type: multiple chamber, one inside the other. The innermost chamber contains the electrode metal and a saturated solution. The outer chambers contain weak solutions, the last one having the weakest solutions. Usually two chambers are used. This type eliminates the diffusion potentials but requires too much attention, and is not practical.

e. Leonard type: no porous pots; the ground is watered by a weak solution and the metal, which has a large surface area, is inserted into the wet part of the ground.

f. C.I.T. type: A flat bottom porous pot contains the $ZnSO_4$ solution (not saturated; the writer used a concentration of two pounds of $ZnSO_4$ per gallon of pure water). A rubber neck, to which the zinc electrode and the wire lead is attached, is screwed to the neck of the pot, and provides

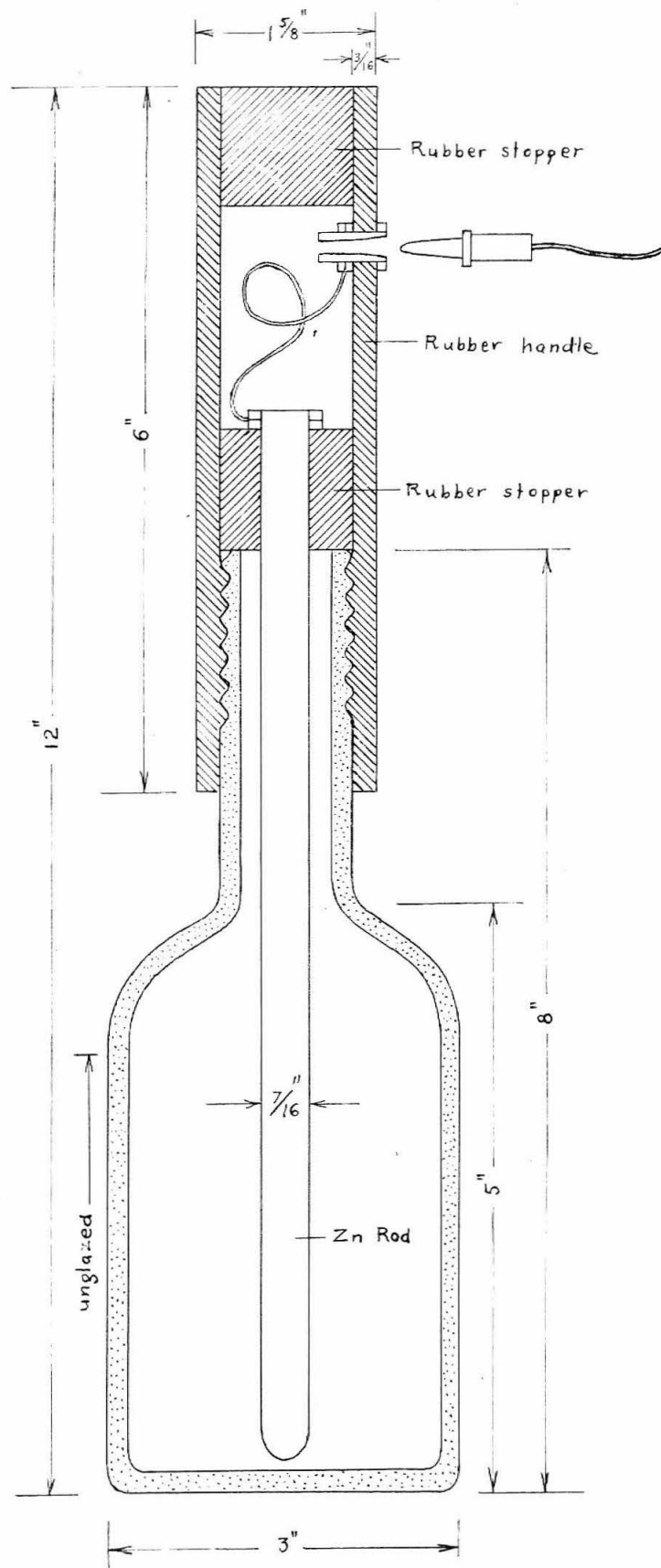


Fig.15 - C.I.T. Non-polarizable Electrode

a handle (see figure 15). The use of Zn - ZnSO_4 electrodes in California is justified, because in California the ground, in general, is such that the polarization of electrodes depends upon zinc ions more than copper ions; therefore it is easier to counteract this effect with Zn and ZnSO_4 electrodes. Another advantage is that ZnSO_4 does not sterilize the ground, whereas CuSO_4 does in a very pronounced way. The use of a diluted solution minimizes the diffusion potentials, but it creates large spurious potentials when considerable current is drawn, as the concentration change near the electrodes is more pronounced. Therefore, if the equipment is good and if care is taken not to draw current, a low concentration solution is advised. Another fact about the dilute solution electrodes is that after a day's work the concentration difference between the two electrodes, caused by diffusion and evaporation, is large enough to create spurious potentials the next day, and the electrodes must be equalized overnight. Whereas, in case of saturated solution, this difference does not exist, since the presence of excess crystals compensate this continuously.

In order that the electrodes should be well equilibrated they must be completely filled with liquid and the contacts liquid to tube or rod must not vary when they are inclined. The presence of air in the copper tube gives rise to oxidation potentials.

The two electrodes should at about the same temperature (for the Schlumberger electrodes a difference of 10 degrees Centigrade gives rise to a disturbance of 5 millivolts). It is therefore necessary to avoid leaving one electrode in the sun, the other being in the shade. It is also necessary that the two porous pots have the same degree of moisture. When a wet point is touched with one electrode and a dry point with the other, the dry electrode becomes negative. This is due to unequal distribution of the liquid in the porous pots.

In dry country it is necessary to prepare special contact surfaces before the electrodes are applied to the ground. A shallow hole, a few inches in diameter and two inches in depth, provides a good contact after it is wetted by a cup full of water or weak solution.

The behavior of the electrodes should be checked at frequent intervals. Sometimes internal polarization may be set up unexpectedly owing to some unforeseen cause, and if this remains undetected the potential measurements will have no value. It is recommended that the potentials of the first five stations occupied at the commencement of a day's work should be checked by a second observation with reversed electrodes. It will be found that after the first few stations have been completed the electrodes will be behaving normally. It is obvious that the electrodes should be numbered in order to have a record of them, and to place them in the same direction, since there is always some electrode polarization.

When not in use, the pair of electrodes should be left standing in a dish of concentrated solution with their terminals connected, so that the voltaic equilibrium will be established or kept for the next use. Sometimes it is necessary to leave the pair this way for a few weeks.

2. Potentiometer: The field potentiometers incorporate a galvanometer in a compact and robust form which should be set up quickly without any special care for levelling. The use of a standard cell is not necessary, since the error introduced by standardizing an ordinary dry cell is beyond the limits of field accuracy.

The following is the description of a potentiometer which was used by the writer and proved to be very practical. It was designed by Dr. Potapenko, adapting the design given in Edge and Laby (page 237). Figure 16 shows the circuit. The galvanometer G is a Weston portable instrument, model 440, having a sixty-division scale with central zero, each division be-

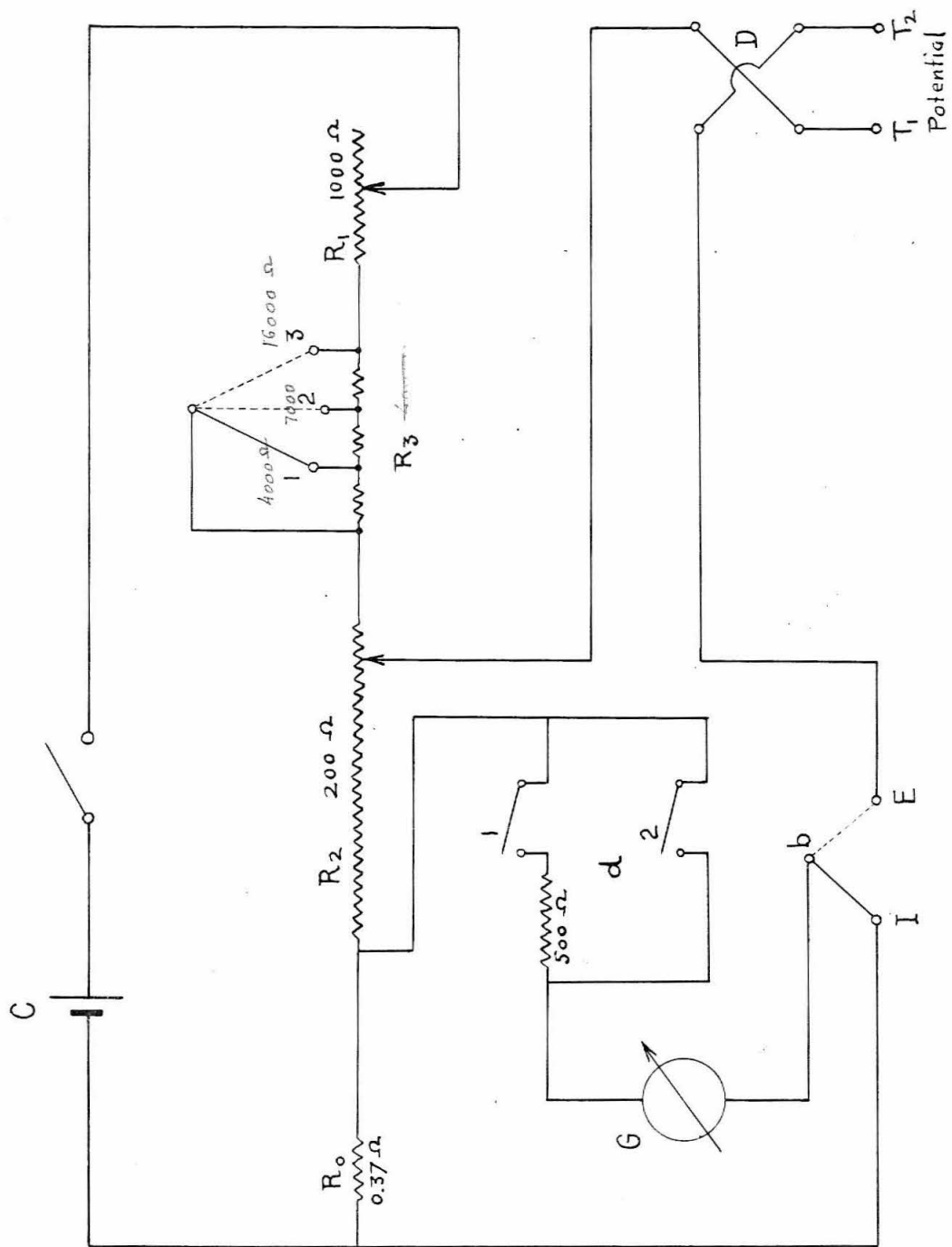


Fig.16 – Potentiometer Circuit

ing equivalent to 0.25 microampere. It has a resistance of 150 ohms. It serves the double purpose of balancing the ground potentials and for standardizing the current supplied to the potentiometer circuit by the small dry cell C. The circuit d is a safety device. First the contact 1, which has a large resistance, is pressed. If the deflection is not too large the contact 2 is pressed, drawing the total current to provide more sensitivity. The variable resistance R_2 has 18 divisions. When R_3 is on 1, each division corresponds to 1 millivolt; when on 2, each division is 2 millivolts, likewise for 3. Thus, the instrument has a range of 54 millivolts. The resistance R_0 is such that the galvanometer shows 10 divisions (2.5 microamperes) when one milliampere is passing through the potentiometer (R_2 , the name potentiometer is usually given to the rheostat alone) when b is on I and R_3 on 1. Likewise the galvanometer should show 20 divisions when R on 2, and 30 divisions when on 3. As the cell has a variable and unknown value, a variable resistance R_1 is added. R_2 is calibrated so that it shows the potential directly.

Operation: First b is switched on I and the circuit d is closed by pressing 2 or both. It must be remembered that if only 2 is pressed in standardizing, the same should be done when taking potential readings, in order to have the same constant resistance in the potentiometer circuit. The indicator is brought on 10, 20 or 30 divisions depending upon the position of R_3 , by means of R_1 . Then b is switched to E to take potential readings. The galvanometer is brought to zero by means of R_2 . The R_2 reading, multiplied by the number of the position of R_3 (1, 2 or 3) gives the potential in millivolts. The polarity of the instrument with respect to the direction of the profile can be found experimentally by creating known potentials with a dry cell in a dish of water and taking potential measurements on the water surface.

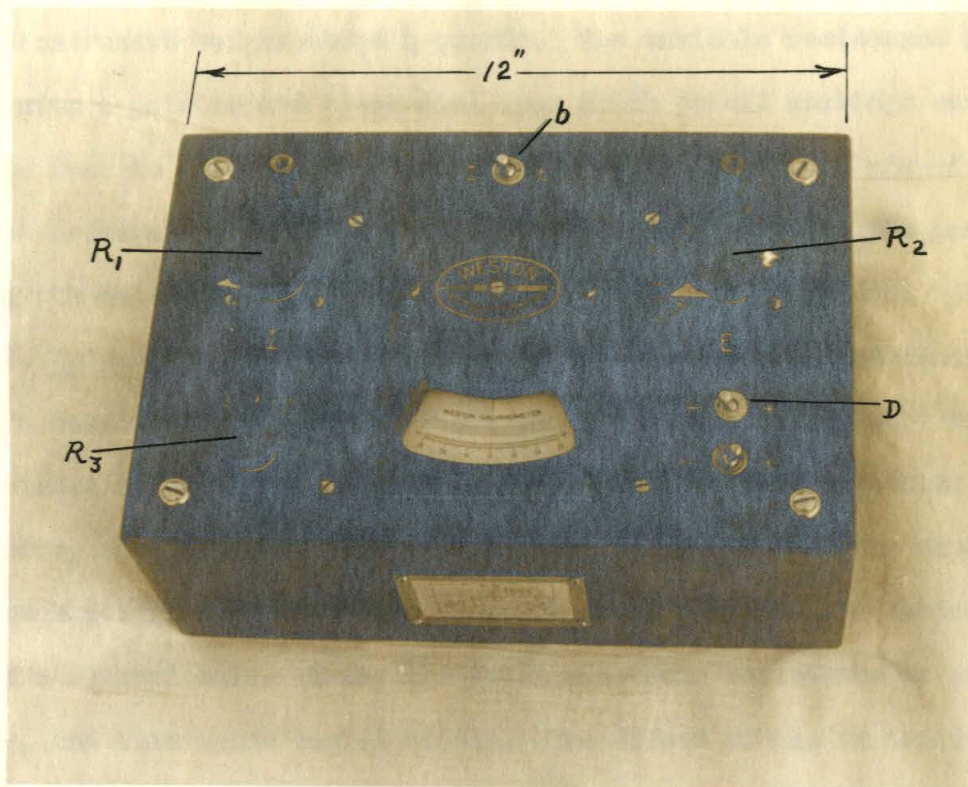


Fig. 16 a – C.I.T. Potentiometer, Outside View

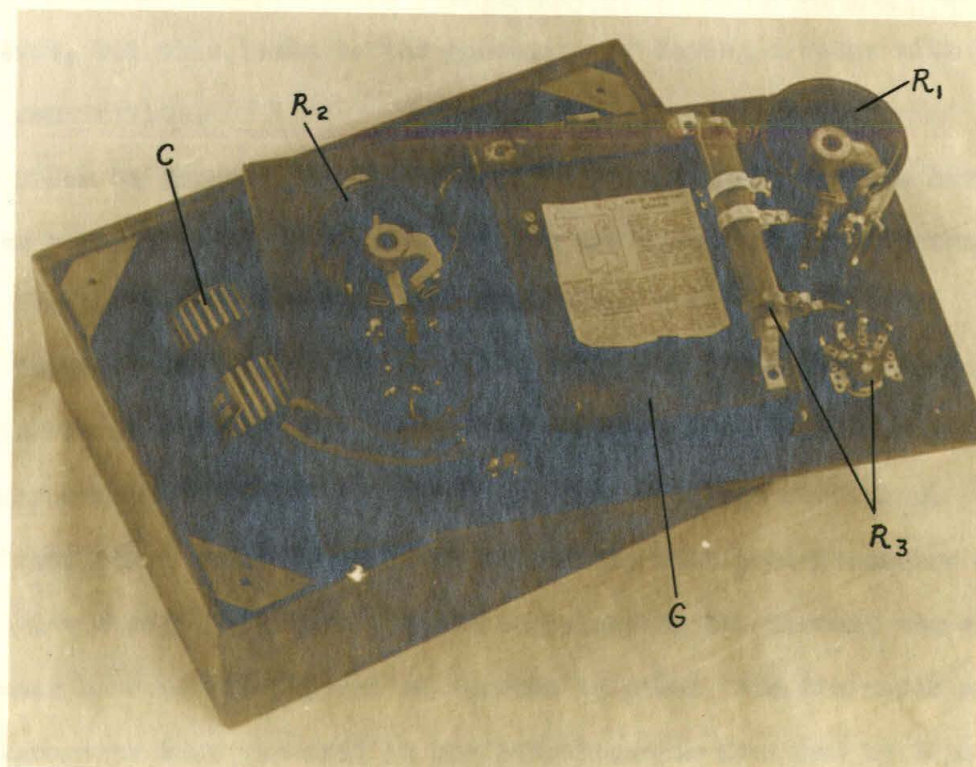


Fig. 16 b – C.I.T. Potentiometer, Inside View

The instrument weighs about 4 pounds. The variable resistance R_v is provided with a pointer and blank dial upon which pencil markings can be done. The base is provided with a tripod attachment, but the use of a tripod is not necessary. The component parts are all mounted on the panel, excepting the cell C which is held at the bottom by a brass clip.

3. Galvanometers: The galvanometer should be of rugged construction and have a sensitivity of 0.25 to 2 microamperes per readable unit of scale. The resistance of the galvanometer (in case of current measurements) should be 1000 ohms, or higher, in order to minimize the effects of the electrode resistance which is about 500 ohms for each electrode when the latter is placed in a watered hole. Thus, the total electrode resistance is around 1000 ohms, and this value varies widely. The effect of the variation is minimized by the addition of the constant galvanometer resistance. Besides, as long as any current is drawn into the circuit, the true picture is distorted. In order to draw less current we have to have high resistance in the circuit, but this leads to the necessity of having a meter with a high voltage sensitivity.

In order to increase the sensitivity of the galvanometer we have to use a low resistance, but then the results are distorted. Therefore, it is a compromise depending upon the conditions.

It is preferable to use a rugged pivotless instrument of the pointer type with the coil suspended between taut bronze ribbons. The suspension type instruments must be properly leveled when readings are taken.

Galvanometers which are used in connection with potentiometers do not have to have a high resistance, since in balancing the circuit the electrode resistances have no effect, and no current is drawn into the meter anyway. The galvanometer that was used in the potentiometer designed by Edge and Laby (I.G.E.S.) is a good example. It is a Weston portable instrument hav-

ing a 60 division scale with central zero (30-0-30), each division being 1 mm. in width and approximately equivalent to 0.25 microampere. It has a resistance of 154 ohms. It does not require special attention for leveling. Its transportation is safe as long as the movements of the galvanometer coil is damped by means of a connection.

In locating equipotential lines, a high-resistance milliammeter, with an accuracy of 0.5 milliampere, is advised by some writers. Let us suppose the total resistance in the circuit is 1000 ohms. A reading of 0.5 milliampere corresponds to a potential difference of 500 millivolts, which is too much. Therefore such an instrument is not sensitive enough for natural current measurements. A polar, D. C. millivoltmeter having a resistance of 5 ohms and a sensitivity of 10 microvolts per division is suitable for this purpose. The zero point should be such that the reversal of current will be noticed. With such an instrument one division (10^{-5} volts) corresponds to a current drawal of 10^{-8} amps when the resistance is 1000 ohms.

A microammeter having a resistance of 30 ohms was used for current measurements by Ives and Keys. The use of such a low resistance meter for this purpose is not reliable.

With a microammeter, potential differences can also be found in the following way: Let us suppose the circuit has an unknown resistance of R_0 (microammeter and the electrode resistances). First, a reading I is taken with R_0 only in the circuit, so that

$$I = \frac{V}{R_0}$$

Then a variable resistor R is placed in series with the electrodes and adjusted until one-half of the former reading is obtained, so that

$$\frac{1}{2} I = \frac{V}{R_0 + R}$$

Hence, $R_0 = R$ and $V = RI$

IV. Interpretation

Although the field work of spontaneous polarization method is relatively simple, its interpretation is usually complicated. The results may be erroneous, spurious or deceiving. The sources of error are very abundant. Final interpretation should give careful consideration to the following facts which are the main sources of error. The phenomena causing the errors, and their corrections, were discussed previously. The following is rather a comprehensive list of the sources of error:

1. Polarization of electrodes: A correction for polarization of electrodes may be determined and deducted.
2. Regional gradient: By surveying a comparatively large area around the anomaly and smoothing the anomalous part the regional component is estimated and subtracted.
3. Fluctuating earth currents: This effect is minimized by using short spreads.
4. Electro-capillary currents: In some cases, electro-capillary currents can exceed in strength those generated by the oxidation of orebodies. However, it is generally possible to make a distinction between them; either by simple reasoning or by examinations of the geological and geomorphological conditions. Even a determination of potential gradient with depth can be made by digging a few shallow holes, permitting to decide in doubtful cases. In temperate climates, the electro-capillary reaction rarely reach those of oxidizing orebodies, but in dry countries, where evaporation is pronounced, the effect of electro-capillary currents may be very disturbing when investigating weak negative centers. In this case the error is minimized by choosing the wettest season. The electro-capillary currents are usually caused by the following features:
 - a. Irregular vegetation distribution causing differential infil-

tration and evaporation of meteoric waters.

b. Porosity differences between rocks, causing the same effects as in (a).

c. Topographical features giving rise to localization of rain water infiltration, and to underground water flows.

5. Increased conductivity of the surface after a rain, distorting the potential distribution.

6. Topography:

a. It controls the intensity of the electro-chemical phenomenon.

b. Due to the fact that in the earth's electrical field, localities of higher elevation are at a different potential, some uphill currents are present. However, the errors introduced by these uphill currents are very small and negligible.

c. Descriptive errors introduced by topography: In many cases the negative center is shifted downhill. This effect is apparent when a cross section of the orebody is studied, drawing the current lines, considering the slope.

7. Geological and structural conditions:

a. Fault zones filled with wet gouge or other conducting materials distort the surface potentials. Water seepage along the fractured fault zone generates electro-filtration currents. Oxidation is pronounced along the aerated fault zone.

b. Formation differences: (1) Resistive surface material (as in many desert areas) shields the galvanic currents; so that when they come up to the surface their magnitude may be too small to be measured. (2) In marshy plains, where the organic surface material is being oxidized, the aerated water is reduced before reaching the orebody, so that no galvanic current is produced. (3) Rock boundaries play a major part. Contacts

of different rocks containing different ground waters give rise to potential differences. Another effect is the attraction of current into the more conductive formation. This results in shifting of the negative center towards the more conductive formation, and in distortion of the regional gradient. Still another effect is the porosity difference which was mentioned before.

8. Changes in temperature causing fluctuations in the electrical output of the orebody, if the latter is very shallow.

9. Freezing and thawing weather.

10. Leakage currents from power circuits: In mining areas, where direct currents are used extensively, leakage and ground-return of power circuits generate intense spurious currents. However, even if A. C. is used, some D. C. component is always present due to unequal loading.

11. Nearby power lines causing induction currents.

12. Artificial conductors (rails, pipes) distorting the potential distribution.

13. Oxidizing underground iron installations generating spontaneous polarization currents.

14. Underground openings causing increased aeration, water infiltration towards the openings, and distortion of potential distribution due to absence of conducting material.

The spontaneous polarization method is rather a qualitative one. The quantitative data, that is, dip, depth, location and size determinations, are no more than estimations. These determinations can be achieved in two ways: (1) Assuming a simple geometrical shape, and calculating the quantities; and (2) interpolating from curves calculated for geometric forms. In case of a spherical orebody, calculated curves and the calculating methods

were discussed in detail. Polarized bar and sheet were also discussed to some extent. If the equipotential lines are somewhat circular and not elongated, the assumption of a spherical shape is justified. If the elongation is in the direction of the positive and negative centers, a polarized bar can be assumed. Usually the anomalies are elongated in a direction perpendicular to the direction of negative and positive centers. In this case a polarized sheet must be assumed.

The curves to interpolate, and the methods of calculation of the dip, depth and size for polarized bar and sheet were not given in detail. As it is seen from the mathematical interpretation of spherical orebodies and from a few field measurements, the anomalies are usually so irregular that the loci of zero points and maximum points are far from being hyperbolas, and the determination of asymptotes is a very rare case. Hence, the quantitative determinations would be rare and highly unreliable. However, if the conditions are favorable, it is convenient and helpful to make mathematical interpretations. A qualitative analysis of spontaneous polarization results is generally sufficient to locate the position of the body.

Interpretation should be accompanied by a proper control of geology. The best way is to draw areal maps to the same scales as the potential contour maps. The reliability of the results is increased greatly if the ore occurrences in the district are investigated. The sphalerite found with pyrite in the Big Bend area is a good example for this. The geophysical work must be followed by drilling, trenching or other direct exploration work to determine the commercial value of the ore. There is no way of knowing the commercial value of the ore from the spont. pol. currents.

V. Conclusions

The practice has shown that the spontaneous polarization method is the quickest, handiest and most reliable as far as sulphide orebodies are concerned, provided that it is used rationally, with due attention to spurious effects. Its field of application is being extended rapidly. Before starting an extensive spontaneous polarization work, a few test profiles over most promising areas and a preliminary geological examination is very essential. These examinations may indicate possibility or impossibility of measurable natural currents. Usually five essential points are considered:

1. The nature of the ore.
2. The conductivity and continuity of the orebody.
3. The depth of water table.
4. The depth of oxidation.
5. The intensity of the electro-capillary currents.

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REPORT OF THE SPONTANEOUS POLARIZATION WORK

over the Property of

SURCEASE MINING COMPANY

HOEFLING BROTHERS, Inc.

Butte County, California

INTRODUCTION

The areas surveyed lie between the latitudes N 39° 40' - N 39° 45' and longitudes W 121° 30' - W 121° 25', Township 21 N, Range 5 E. It is accessible by the State Highway 24 and the county road branching eastward. It is about 20 miles north of Oroville and 35 miles east of Chico, in Butte County. The elevation is between 2,200 and 2,400 feet, with a considerable relief and rather smooth topography. It can be said that the area is on the "piedmont" of the Sierra Nevada Mountains. The geology is given in detail by the report of International Geophysics. The work was done in 1934 under the directions of J. J. Jakosky. It consisted of geological mapping, electrical (resistivity) investigation and magnetic survey around the Surcease Vein belonging to Hoefling Brothers, Inc., Surcease Mining Company. In short, the prominent country rock is a green amphibolite. It appears like diorite intrusives or andesite flows. In many places it is schistose, grading into an amphibolite schist which is highly altered all over the area. Minor acidic intrusions, quartz veins and considerable granodiorite (or quartz-porphyry) intrusions are found. An abundance of oxidized capping (gossan) is very striking. They have a consistent trend of NW -SE.

The area is covered with pine trees and manzanita brush. The soil is very soft, and reddish clay around the Barton Hill covers the outcrops. The drainage is toward the Feather River which makes a horse-shoe around the area. The rainfall is considerable, probably around 50 - 60 inches.

The work started on October 14, 1944, and ended on November 2, 1944.

The survey was carried on by the writer, assisted by Mr. Fernando Nieri.

The writer extends his appreciation for the advice given by Dr. J. P. Buwalda, Chairman of the Division of the Geological Sciences, and by Dr. G. W. Potapenko, Associate Professor of Physics, of the California Institute of Technology.

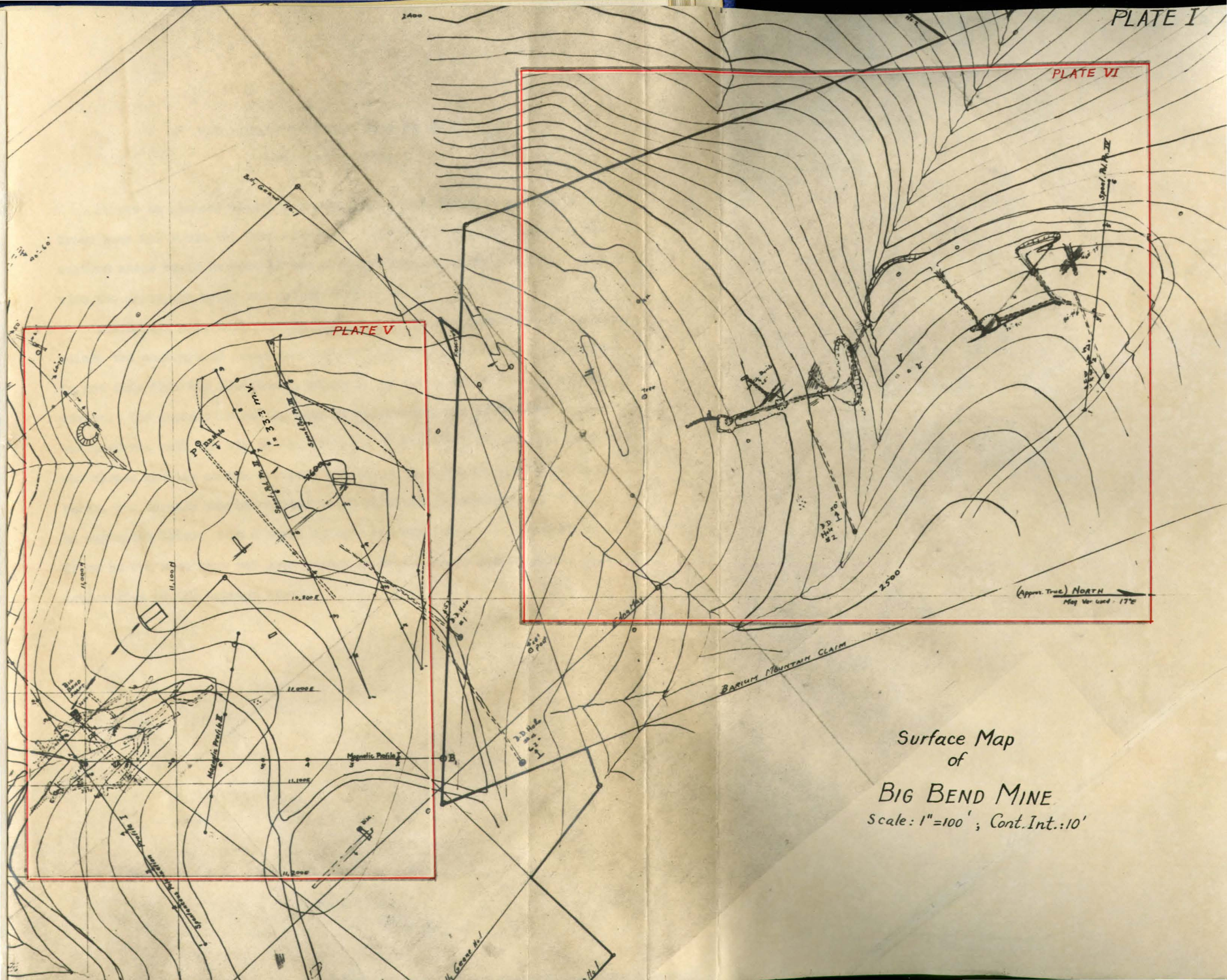
Grateful acknowledgement is given to Messrs. A. E. Jones, H. H. Bein, and W. E. Messner for their generous co-operation and information concerning the geology of the district.

PLATE VI

PLATE V

(Approx. True) NORTH
Mag. Var. used: 17°E

Surface Map
of
BIG BEND MINE
Scale: 1"=100'; Cont. Int.: 10'



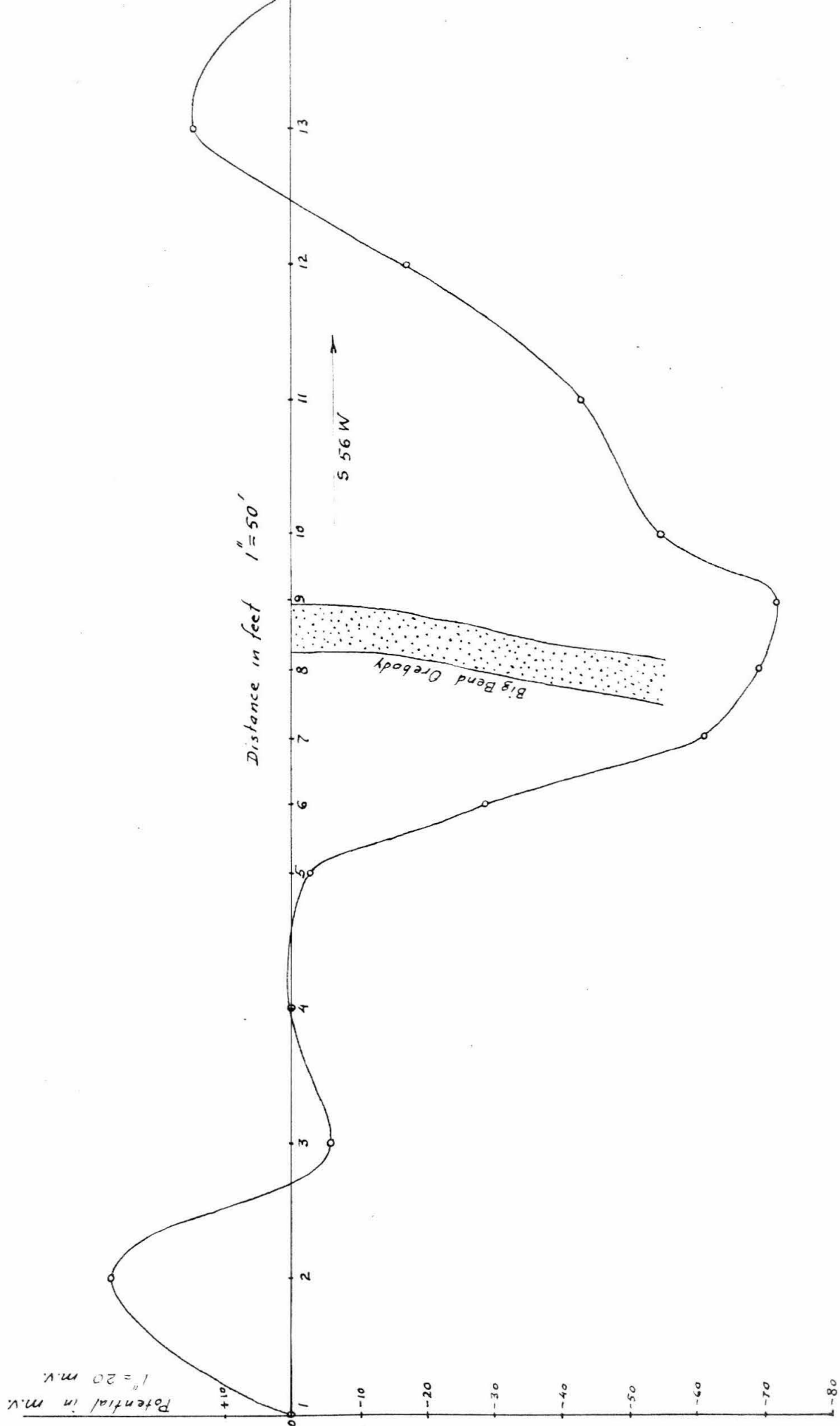
PRELIMINARY TEST WORK OVER THE KNOWN

BIG BEND OREBODY

It was so planned that the spontaneous polarization method would be tried over the known Big Bend orebody. Further exploration work over the unknown areas would be done if the orebody responds to this method. Otherwise the surveyors would not do any further work.

The spont. pol. test profile No. 1 had to be run over the edge to avoid mainly the underground pipes causing oxidation current. It was impossible to get away from the 100'-level, since it had to be perpendicular to the strike. The anomaly is small, some 80 millivolts, but it is very regular and conforms with the dip of the orebody. Subtracting the effects of drifts and stopes causing an air contact, and the oxidizing underground pipes and rails, the anomaly may be reduced to 50 millivolts. However, as it will be explained later, such a small anomaly was expected. The curve which is given on the next page was good enough to decide an extensive spont. pol. exploration of the unknown areas.

Fig. 16c - Spontaneous Polarization Profile I
across the Big Bend Orebody



EXPLORATION WORK

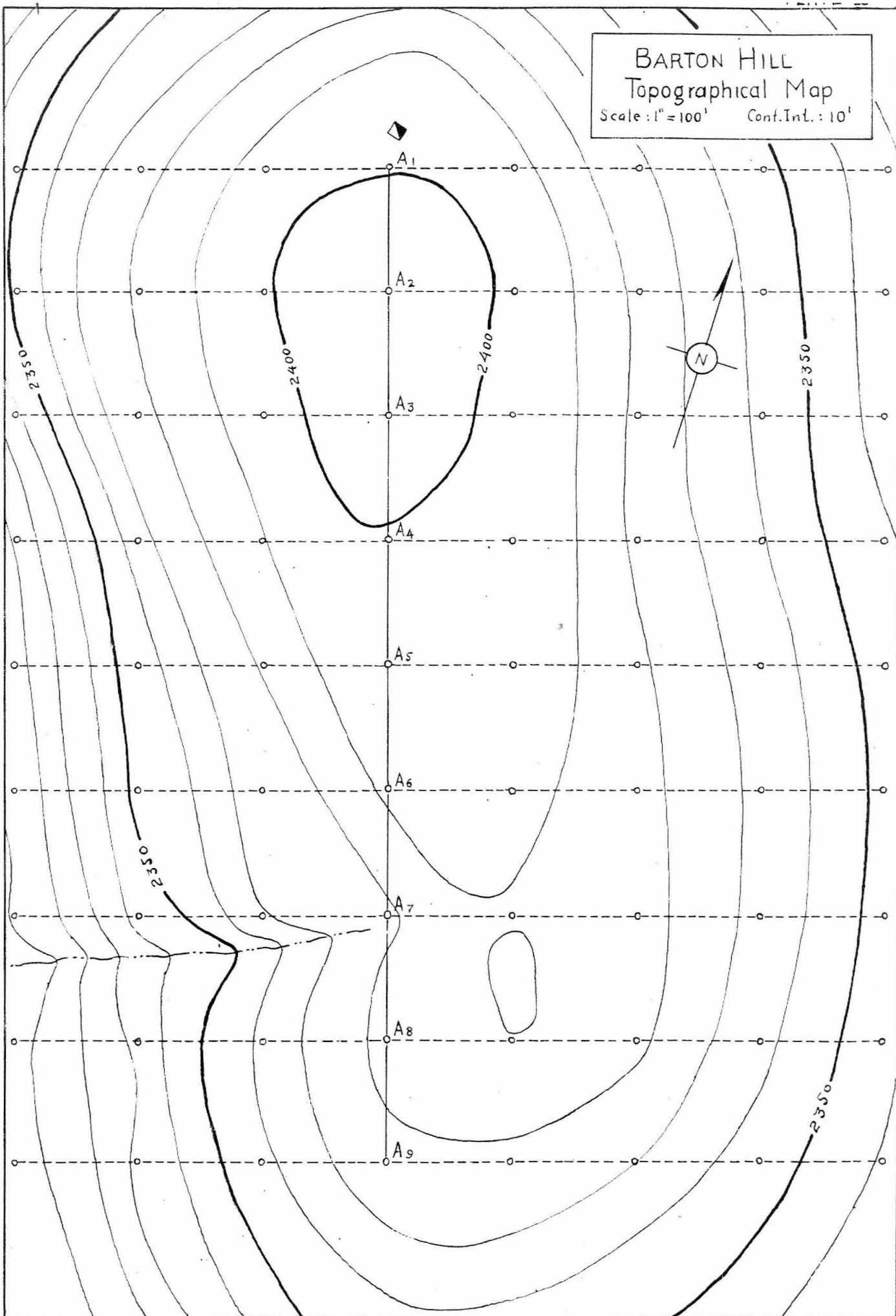
I. PROCEDURE

The complete electrical equipment used for this method consisted of a simple portable potentiometer, which was built by Dr. Potapenko, Associate Professor of Physics at California Institute of Technology, adapting the design given by Broughton Edge in I. C. E. S. The potentiometer was connected by long, insulated wire leads to a pair of non-polarizing electrodes. The non-polarizing electrodes consisted of porous pots filled with a solution of zinc sulphate into which zinc rods are inserted. The minimum concentration of the solution is three-quarters of a pound zinc sulphate per gallon of distilled water. It was found that two pounds per gallon gave better results. Since the ground was dry, it was necessary to scrape a small hole and water it before the porous pot was replaced in it.

Because the area over Barton Hill was unknown as to the mineral deposits, a series of straight line reconnaissance traverses running N 73° E were made. This direction was supposed to be perpendicular to the strike of the mineralized zones, because the whole district presents faults and outcrops running roughly north south. Besides, the schistosity of the highly altered amphibolite schist near the prospecting shaft on the Barton Hill also showed a north south direction. Unfortunately, the anomalies happened to be running parallel to the profiles run N 73° E at hundred-foot intervals. Later on, as the workings in the prospecting shaft advanced, it was seen that the schistosity direction assumed an irregular direction at the bottom of the twenty-five feet deep shaft.

BARTON HILL Topographical Map

Scale: 1" = 100' Cont. Int.: 10'



The most promising surface indications, such as the altered schist around the shaft, were intersected. The best way would be to lay out another system of profiles again at hundred foot intervals, perpendicular to the previous ones, so that any hidden orebody of important dimensions might be crossed or closely approached. This was not advised, due to the economic conditions. Because the area is covered with thick brush, the profiles had to be opened by a bulldozer. The base line had to be in the middle of the surveyed area, due to the steep grade of both sides where the bulldozer could not work along the contours. The traverse lines were surveyed by Mr. H. Bein, and pegs were laid out at hundred-foot intervals. The topographical map of the area was also made by Mr. H. Bein while the survey was being done.

The reconnaissance survey along these traverses was usually carried on at fifty-foot intervals; however, twenty-five even twelve-and-a-half-foots intervals have been used frequently at critical points and when the anomaly was significantly great.

Two important anomalies were found, and more detailed work has been done in these zones, using twenty-five-foot intervals regularly, and twelve-and-a-half-foot intervals frequently. These areas will be discussed later in connection with "Results and Interpretation." A fifty-foot cable was used all of the time, the apparatus moving fifty feet each time, overlapping the twenty-five-foot readings, which were made very accurately.

After measuring the potentials between the stations, the results were summed and plotted with distances as abscissae and the potential values in millivolts as ordinates. The potential values throughout the traverse system were so adjusted that they all have reference to the base station, indicated by two concentric circles on the map, on the north-west corner of the area, to which an arbitrary zero potential was given.

After plotting the profiles and smoothing them, the equipotential lines were drawn.

II. DIFFICULTIES ENCOUNTERED IN THE WORK

1 - The main part of the ore found in the Big Bend orebody is sphalerite, which does not respond to this method at all. If it was not the pyritic component of the mineral deposit, this method would not show anything. Since the amount of pyrite, chalcocite, and chalcopyrite is small with respect to the rest of the ore mass, the anomalies are expected to be small.

2 - The topography of Barton Hill is very pronounced, the grade being as much as twenty-five percent on both sides, the elevation difference between the valley and the top being about two-hundred feet. This feature implies a deep water table over the hill. Thus, the oxidation zone is considerably deep, and its effect on the surface may be very weak.

3 - Barton Hill is covered with thick manzanita brush, making the survey inflexible.

4 - The potential changes over Barton Hill were very irregular. The causes may be due to:

- a) The ground is soft and impregnated by plant roots.
- b) The distribution of vegetation is not homogeneous, causing heterogeneous moisture distribution giving rise to spurious negative centers.
- c) The area is highly fractured, causing the potential values to jump.

5 - The profile traverses were opened by a bulldozer stripping the whole vegetation. The moisture changed as the survey proceeded, due to drying of the opened area.

6 - During the survey it rained twice, and work had to be done right after the rain, and even while it was raining. Percolation of rain water causes spurious potentials. Another effect of rain is the creation of a

conductive superficial layer distorting the general distribution of potential.

7 - Topography is considerably pronounced. It gives rise to irregular distribution of surface potentials. It may also cause shifting of negative centers, resulting in shift of predicted location of orebodies.

8 - Practically no geological work has been done over Barton Hill, and a proper geological control is necessary in interpreting results. Areal maps should have been drawn to the same scales as the geophysical potential contours. Difference in geological formations, contact zones, and fault zones filled with gouges cause severe distortion of the surface potentials.

III. RESULTS AND INTERPRETATION

Barton Hill

The indications are very weak and erratic. Four negative centers were encountered; two of them are worth exploration by boring.

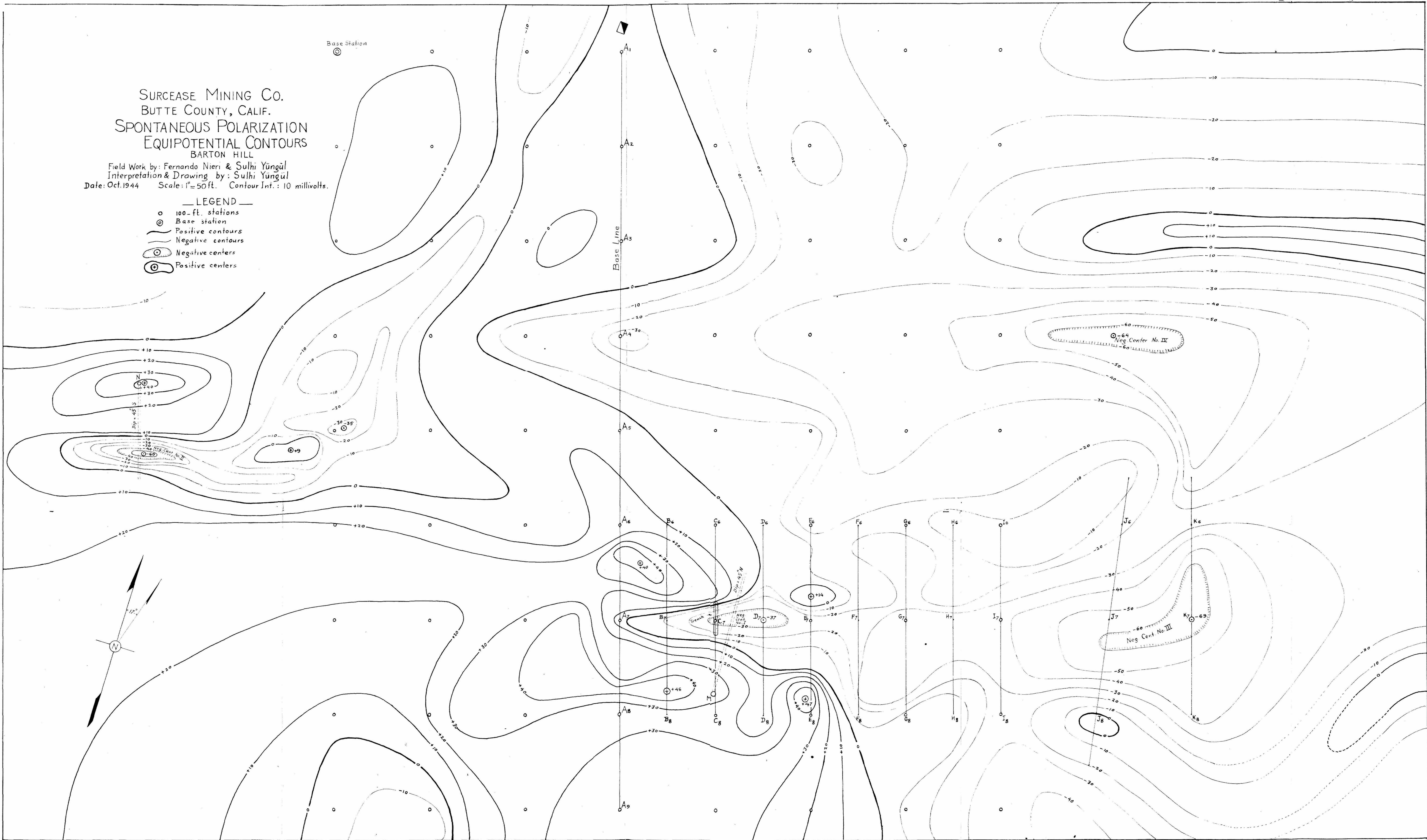
Negative Center No. 1: The total amount of the anomaly; that is, the difference between the negative center and the positive centers surrounding it, is about 70 millivolts, which is less than usual indication. The regional gradient is up hill following the profile No. 7, and this negative center is a minor anomaly over a regional anomaly. The positive centers to the south are stronger than those to the north, indicating that the structure causing the negative center dips southwest.

After locating this anomaly by the reconnaissance survey, a detailed work was done along the eleven profiles crossing the anomaly at right angles. These profiles are $A_6 - A_8$, $B_6 - B_8$, $C_6 - C_8$, and so on up to $K_6 - K_8$. They are shown on the following plate. The profiles F, G, and H had to be surveyed right after a heavy rain, and little consideration was given to this part of the work. A 175 feet long drill hole, drilled at the point M, dipping 45° toward the direction $N 5^\circ W$, will come underneath the negative center at a depth of 70 feet or will strike the orebody at a shallower depth, if the latter is dipping as predicted. If it is dipping northward, there is a possibility of missing it. It is not very likely that the drill hole will strike the orebody at the sulphide zone. Since the dip indications are very slight, a long drill hole extending down to the sulphide zone is not advised. The soil over the negative center is soft and free from outcrops, making a smooth sag extending down the hill. A very significant feature is that on the west side of the hill is a gully running down along the same direction as the sag over the anomaly, and the general trend of the anomaly itself. The

SURCEASE MINING CO.
BUTTE COUNTY, CALIF.
SPONTANEOUS POLARIZATION
EQUIPOTENTIAL CONTOURS
BARTON HILL

Field Work by: Fernando Nieri & Sulhi Yüngül
Interpretation & Drawing by: Sulhi Yüngül
Date: Oct. 1944 Scale: 1"=50 ft. Contour Int.: 10 millivolts.

- LEGEND —
- 100-ft. stations
 - ⊙ Base station
 - Positive contours
 - - - Negative contours
 - ⊖ Negative centers
 - ⊕ Positive centers



possibilities are:

- 1 - The anomaly may indicate an orebody.
- 2 - It may be due to a fractured fault zone filled with fine clay gouge.
- 3 - It may also be due to a very soft formation squeezed between harder formations, causing percolation currents as a result of moisture.

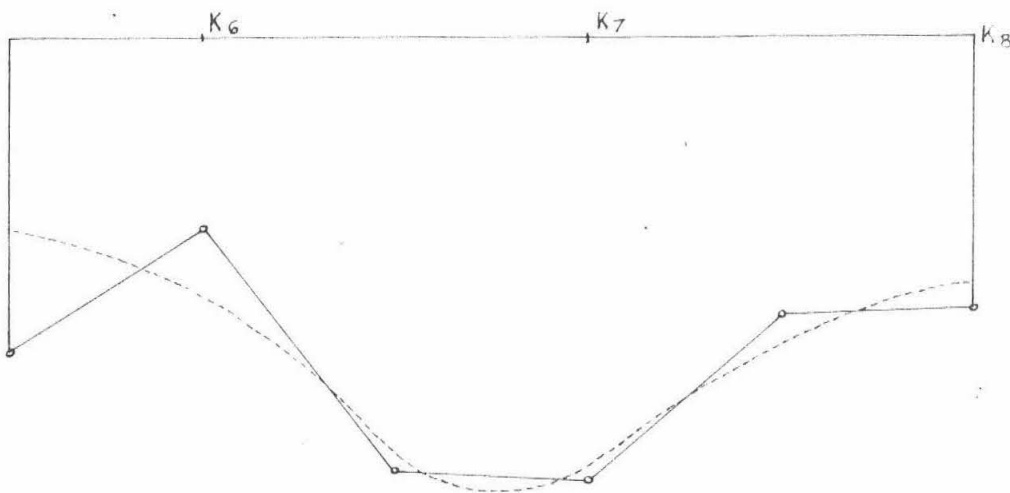
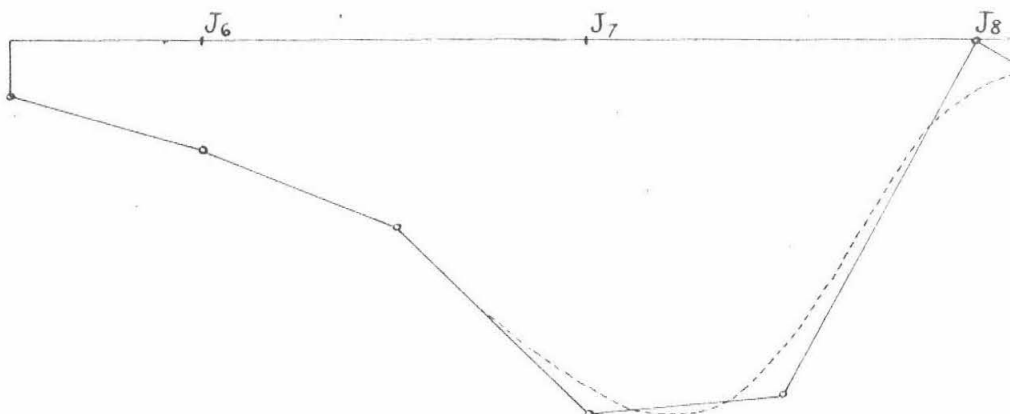
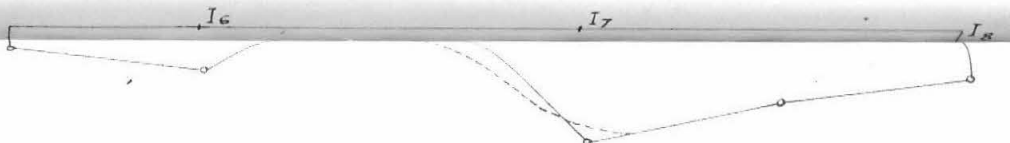
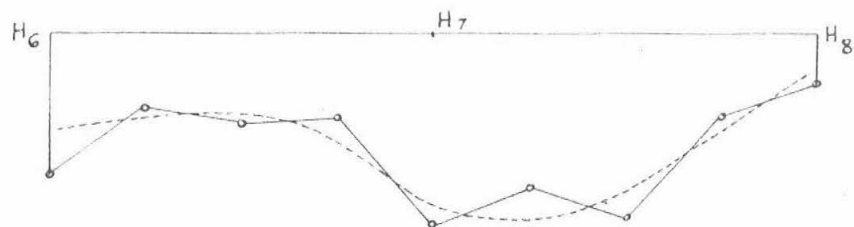
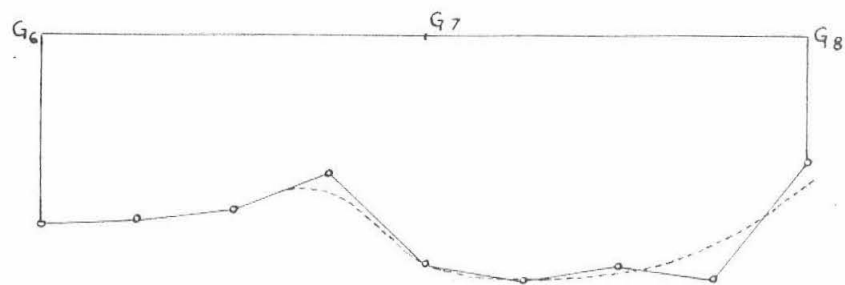
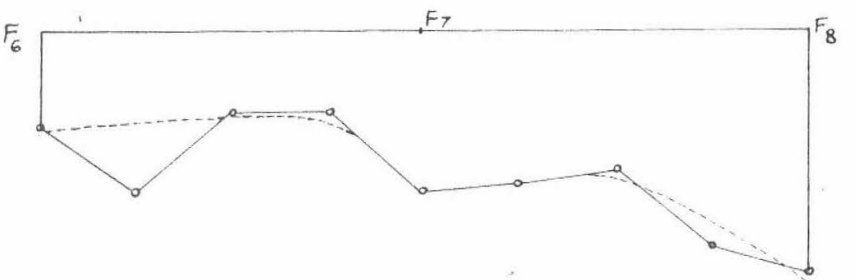
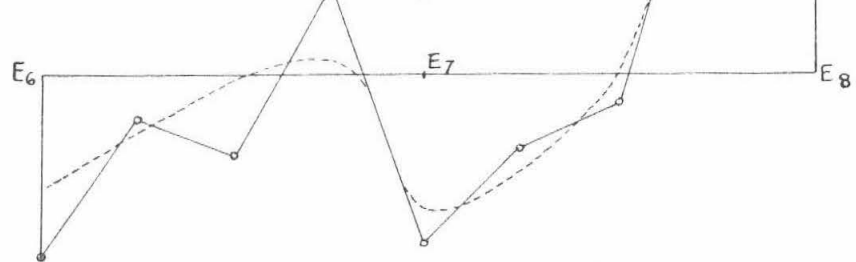
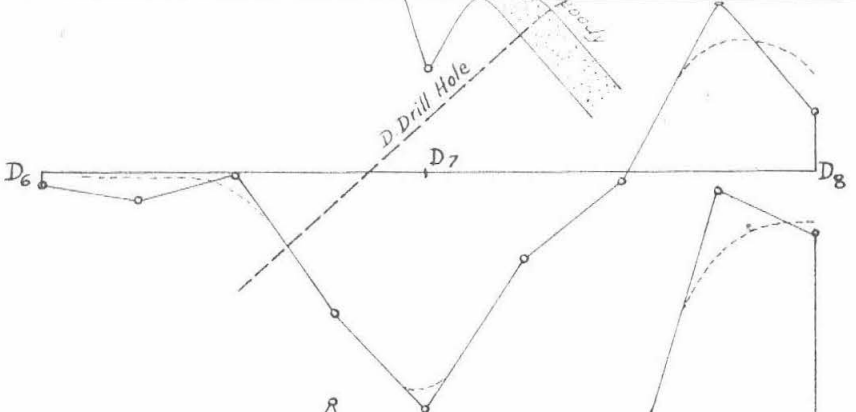
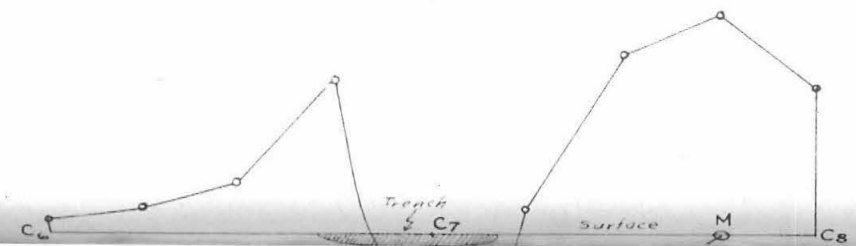
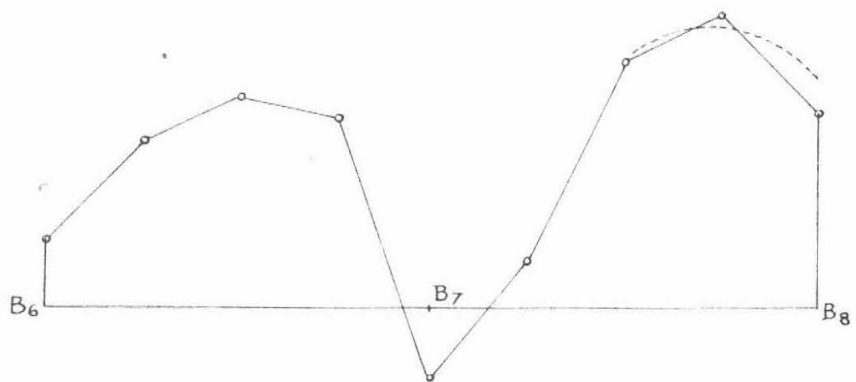
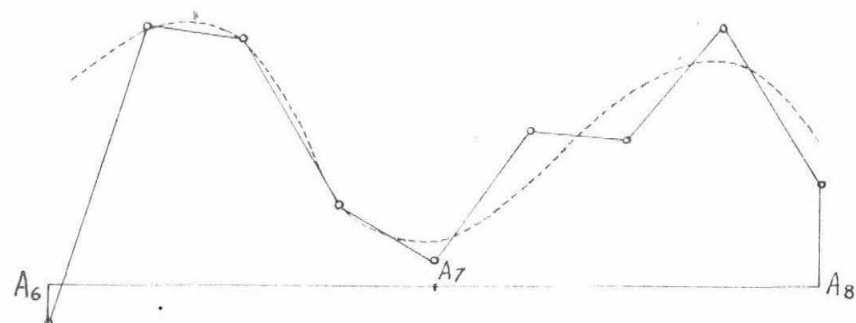
The indication is good enough to take the chance of doing the following exploration work which is advised:

- 1 - A trench 30 feet long and from 5 to 10 feet deep, extending north south over the profile $C_6 - C_8$, the center of it being at C_7 .
- 2 - A drill hole, as explained before, if the gossan found in the trench conforms with the predicted location and dip of the orebody. If not, the direction and location of the hole may be changed.
- 3 - The exact location and dip are determined by the trench and the drill hole. Another core drilling intersecting the vein at a greater depth may reveal the sulphide zone.

*Spontaneous Polarization Profiles
between Pr. 6 & 8 (Neg. Cent. No. I)*

Hor. Scale: 1" = 50 ft.

Ver. Scale: 1" = 30 m.v.



Negative Center No. II: This negative center was encountered along the westward extension of profile No. 5. It gave the impression that the anomaly ran east west. Three 350 feet long traverses parallel to the reconnaissance profiles were opened at 50-foot intervals to investigate this anomaly. The fourth and fifth profiles were also extended down. The expected results were not obtained during the survey. After plotting the results, it was found that the anomaly had an east west trend. The results of profiles running parallel to the anomaly are not very reliable. The writer recommends further survey over this region, with profiles running N 17° W. If the survey of the area is not possible, the following considerations and advice are given.

The anomaly is about 80 millivolts. Its dimensions seem to be small. The positions of the negative and positive centers show a body dipping toward south with a low dip angle, striking N 73° E.

Advice:

- 1 - Trenching if possible, over the negative center in north south direction.
- 2 - A 150 feet long drill hole at the point N, with a dip of 45°, toward the direction of S 17° E.

Negative Centers Nos. III and IV: The low potential values over these zones are due to the regional gradient. More geophysical work, even trenching is advisable, but no drilling is advised.

Edna May Claim

During the last day of the survey , two profiles, namely the spont. pol. profiles No. II and III on the Big Bend Mine Surface Map, were run in a hurry. The indications are very significant, but definite exploration work advice cannot be given since the amount of work done is not sufficient. The indication:

1 - The body causing the anomaly is between the Big Bend Mine and the profile No. III.

2 - It dips toward west, contrary to the fact that the gossan on the surface dips to the east.

A detailed work between the profile No. III and the Big Bend Mine is advised, running profiles S 50° W, or parallel to Nos. II and III. If this further work cannot be done, a core drilling is advised at the point P as shown on the Big Bend Mine Surface Map.

ADDITIONAL SPONTANEOUS POLARIZATION WORK

AT SURCEASE MINE

The following field work was carried on by Mr. Hugh H. Bein, Geologist and Metallurgist of the Surcease Mine, under the directions of the writer who advised this work in the preceding report. The results were also interpreted by the writer.

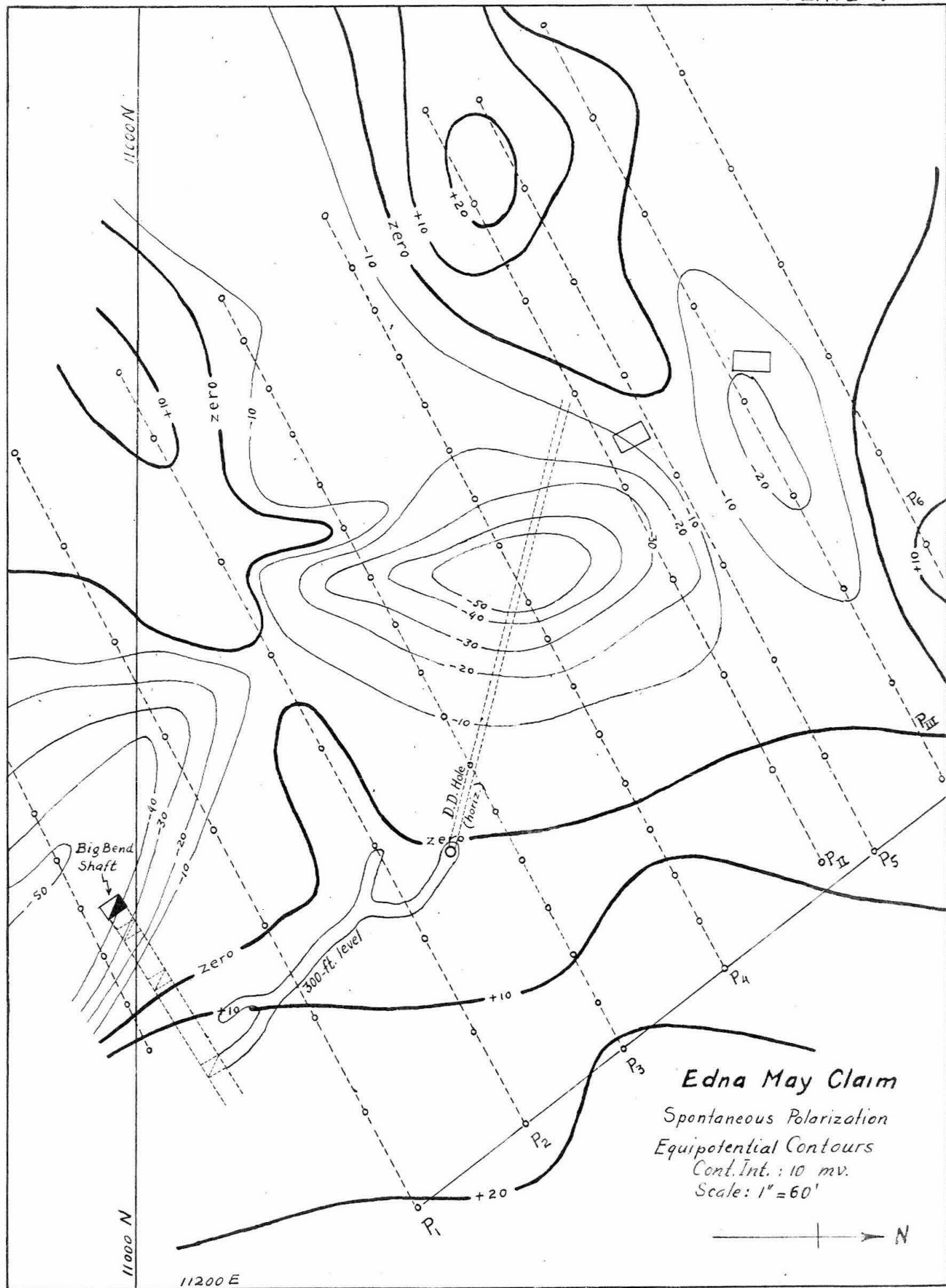
Drill holes drilled on these and the previously described anomalies revealed sulphide and pyrite bodies. Unfortunately, due to obvious reasons, it has been impossible to obtain and publish a detailed result of the drilling campaign.

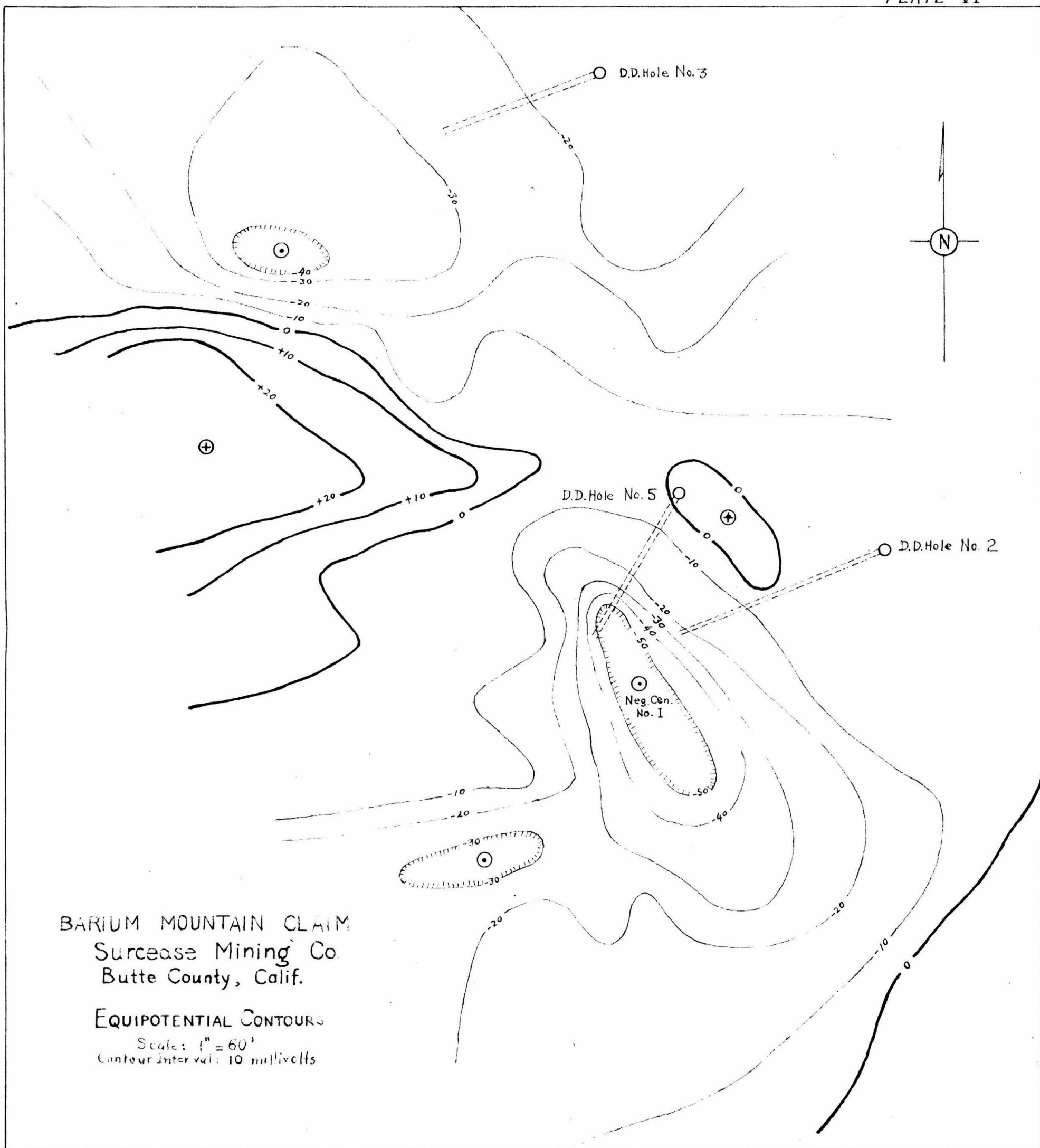
1 - Edna May Claim

The orebody is small and vertical. The system of profiles and the equipotential contours are shown on PLATE V. As this anomaly will be interpreted analytically later, any further discussion is of no value. A horizontal drill hole, drilled from the 300-foot level of the Big Bend mine, was advised.

2 - Barium Mountain Claim

The drill holes Nos. 2 and 3 were drilled before the geophysical work and showed no ore. After the geophysical work the drill hole No. 5 was drilled. This anomaly also will be discussed analytically in the next part. The PLATE VI shows the equipotential lines and the drill holes.





BARIUM MOUNTAIN CLAIM
Surcease Mining Co.
Butte County, Calif.

EQUIPOTENTIAL CONTOURS

Scale: 1" = 60'
Contour interval: 10 millivolts

MATHEMATICAL INTERPRETATION
OF SOME EXAMPLES

As explained before, mathematical interpretation of field results can be accomplished in very few cases, although, in general, some estimations can always be made. The following examples are so chosen as to give a variety of assumptions and estimating procedures. With the exception of the Hope anomaly (Vancouver, British Columbia), the examples are from the Surcease mine (Butte County, California) surveyed by the writer.

1 - Barton Hill Anomaly
Negative Center No. II

This is a very good example of what can be done mathematically if the conditions are favorable. No interpolation and estimation is necessary. Assumption of a sphere is justified. The maxima curve and the zero-point-curve over the negative center are highly regular. One branch of the positive center zero-points curve does not conform with the rest. The procedure is shown on figure 17 (See PLATE III). Seven profiles, parallel to the plane of polarization, have been taken. The zero points are the maximum potential points along these profiles, and the maximum points are those at which the potential profile is the steepest, all found from the equipotential contours.

Results: $h = 29$ feet, $\alpha = 31^{\circ}$. This shallow depth and the mag-

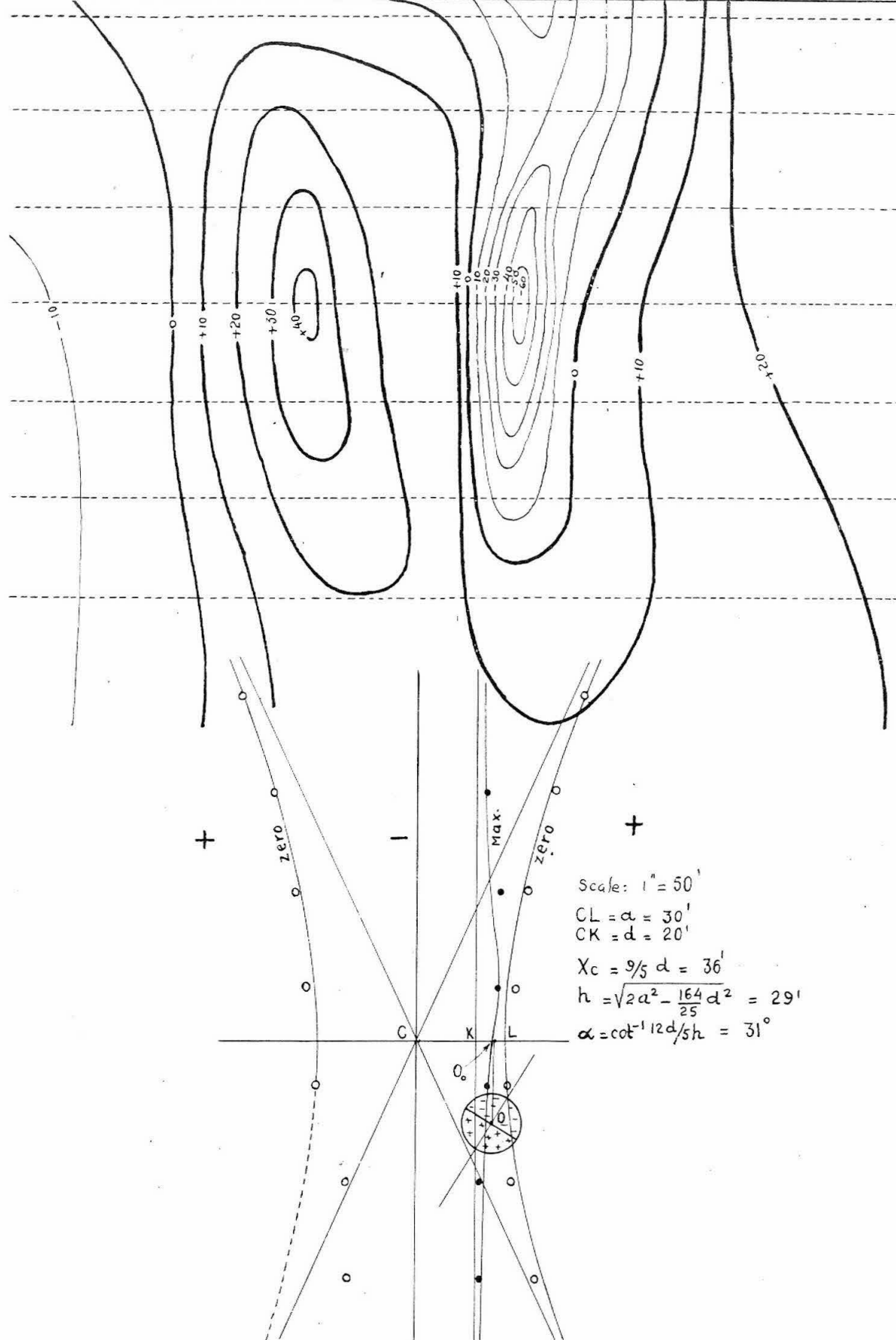


Fig. 17— Mathematical interpretation of the
Negative Center No. II (Barton Hill, Surcease Mine)

nitude of the anomaly indicates that the body causing the anomaly is very small.

2 - Big Bend Anomaly

In this case we have a system of very irregular equipotential contours, yet the shape of the anomaly is such that the assumption of a sphere is closer to the reality than the assumption of a sheet or rod. In any case the profile along the plane of polarization is the strongest and most regular, showing definite maxima and inflection points. Now let us see what we can do with a single profile in the plane of polarization. By interpolating the curves given in figure 3 the angle α and the ratio AB/h are found (fig. 18). Measuring AB from the profile, h is obtained. The O_0 -point can also be interpolated from figure 3, taking both the zero points and the middle maxima points into account, so that the final value is an average of both values. Another line of investigation is to approximate the asymptote to the middle maxima curve to measure d , then $X_0 = 9/5 d$ and $\alpha = \cot^{-1} 12 d/5 h$. Thus, the interpolations and the calculations are repeated until they are in close agreement.

Results: $\alpha = 60^\circ$; $\frac{AB}{h} = 1.70$; $AB = 180$ feet ; $h = 106$ feet; $X_0/h = 0.6$, $X_B = 64$ feet.

These results are in perfect agreement with the facts about the Big Bend orebody. Of course h is not the depth to the center of gravity of the sulphide mass, but to the water table in the vicinity of which, above and below, the oxidation takes place. The water table in the Big Bend orebody is about 100 feet deep. The location of the O_0 -point and the dip angle α are in good agreement with the facts (see PLATES I and V).

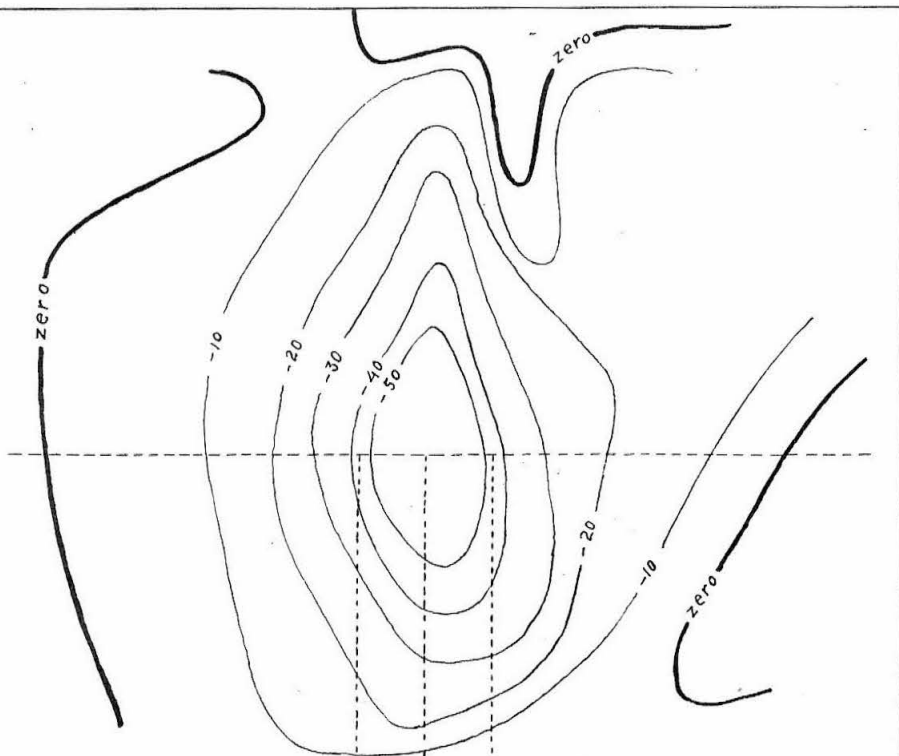


Fig. 19.
Edna May
Anomaly

Scale: 1"=60'

$$\begin{aligned}\alpha &= 0^\circ \\ \overline{EF}/h &= 0.98 \\ \overline{EF} &= 42' \\ h &= 43'\end{aligned}$$

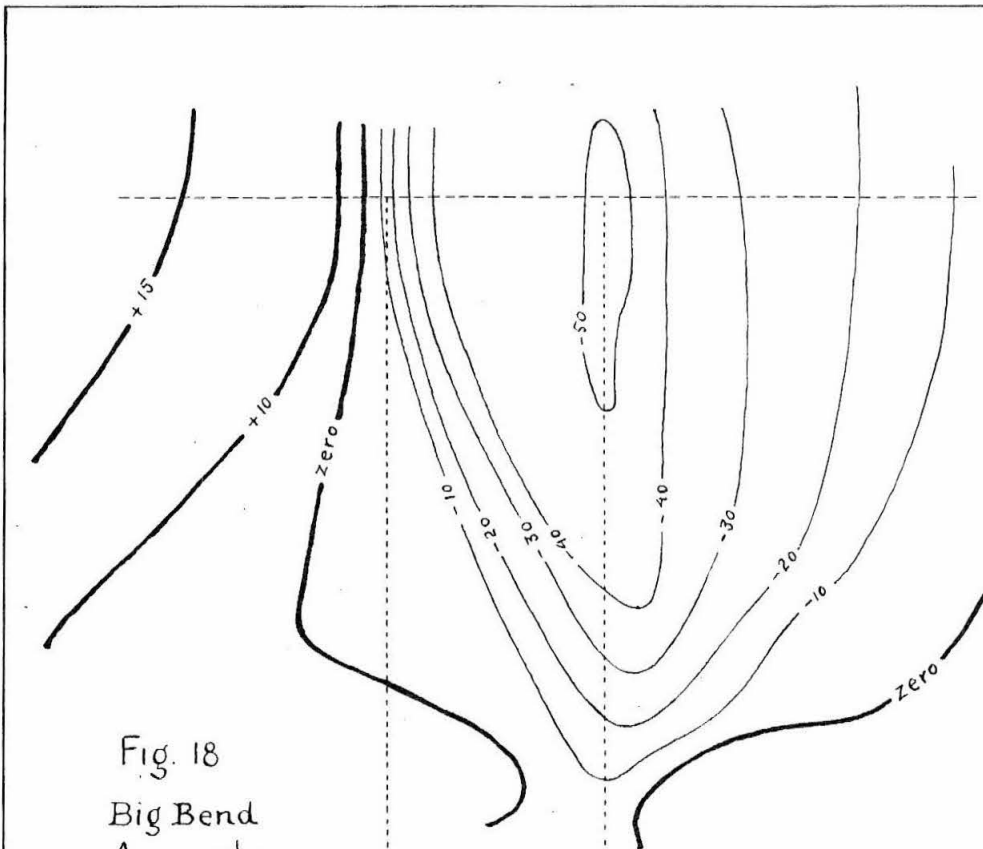


Fig. 18
Big Bend
Anomaly

Scale: 1"=60'

$$\begin{aligned}\alpha &= 60^\circ \\ \overline{AB}/h &= 1.70 \\ \overline{AB} &= 180' \\ h &= 106' \\ X_B/h &= 0.6 \\ X_B &= 64'\end{aligned}$$

3 - Edna May Anomaly.

The conditions are the same as in the Big Bend anomaly, that is, the body is assumed to be spherical and the equipotential lines are such that the interpretation depends on a single profile passing through the plane of polarization.

The current density profile (fig. 19) shows that one of the zero points is thrown to infinite. This means that the axis of polarization is vertical, $\alpha = 0^\circ$. In this case the distance a , real semi-axis, is infinite, the zero-point hyperbola having been reduced to a straight line. The distance EF between the two maximum points is measured. Referring to figure 10, the ratio EF/h is measured for $\alpha = 0^\circ$.

$$EF/h = 0.98 \quad EF = 42 \text{ feet} \quad h = 43 \text{ feet.}$$

The 0° -point is already known, it is the point where the current density is zero, the negative center itself (See PLATES I and V).

4 - Barium Mountain Anomaly

Negative Center No. I

From a study of the topographical map (see PLATE I) it is seen that the gap between the two zero-potential curves is due to the steep gulch, creating electrocapillary potentials. Therefore, we are justified in joining these curves (dotted heavy lines). This correction gives us quite a different plane of polarization (fig. 21).

As the equipotential lines are elongated along the plane of polarization, a polarized rod can be assumed. However, we will assume a polarized sphere, which is not far off, as the calculations for a polarized rod

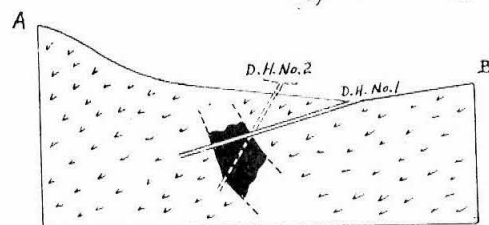
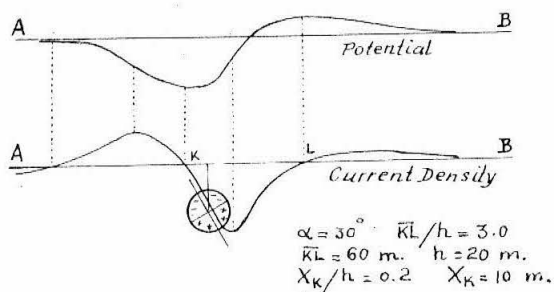
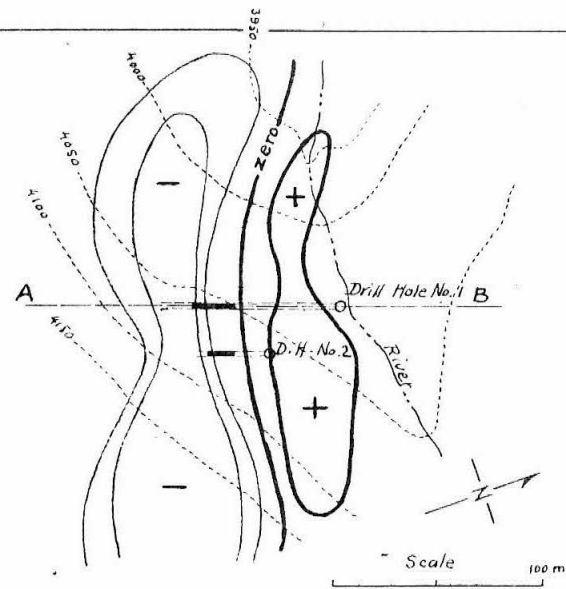


Fig. 20
Hope Anomaly
Vancouver, Br. Columbia

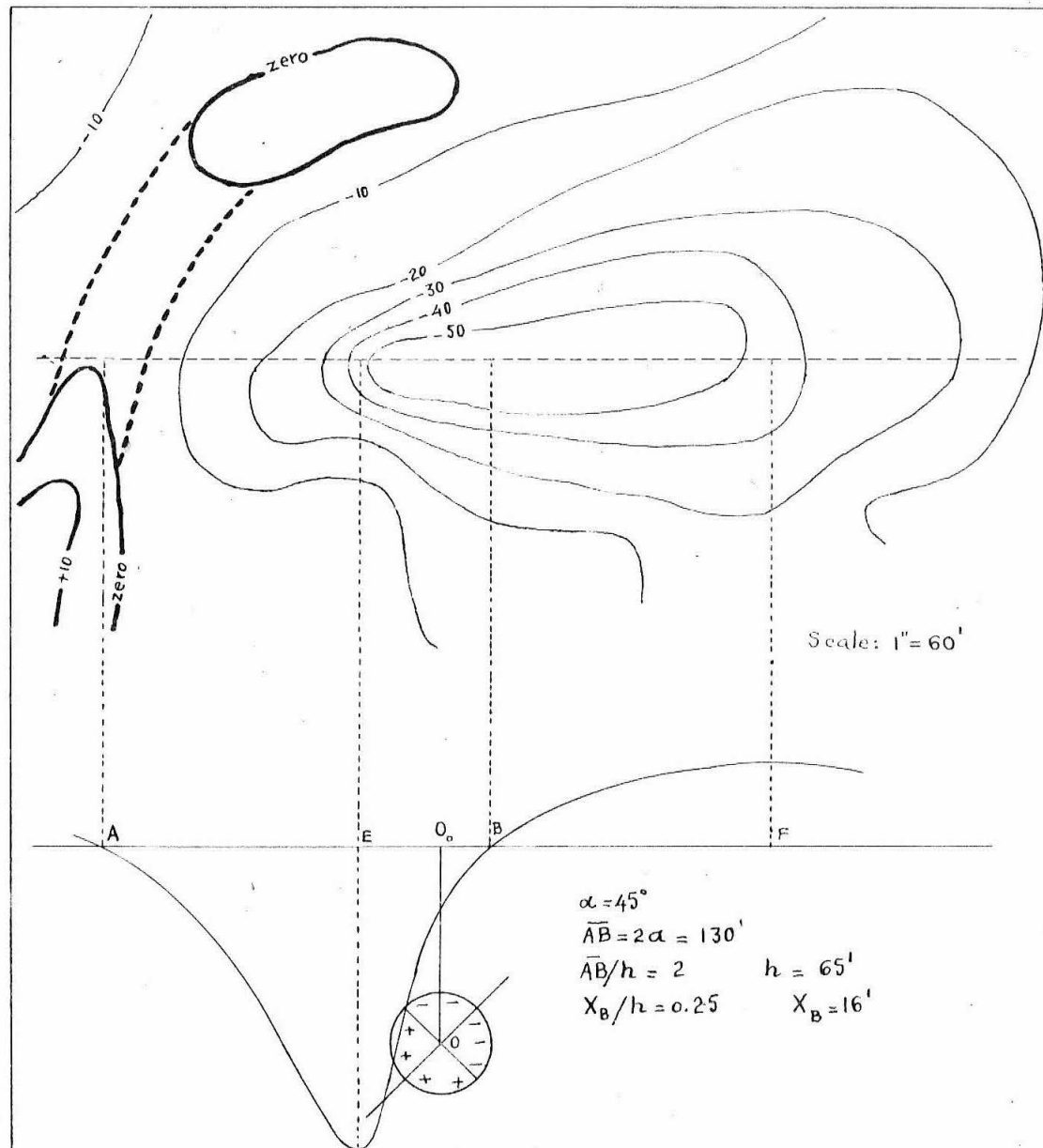


Fig. 21
Barium Mountain Anomaly (No. I)
Surcease Mine

are not available.

Interpretation is similar to that of the Big Bend anomaly.

Results: $\alpha = 45^\circ$, $AB = 2a = 130$ feet , $AB/h = 2$, $h = 65$ feet
 $X_B/h = 0.25$, $X_B = 16$ feet.

5 - Hope Anomaly

Vancouver, British Columbia

This is a good example for a polarized sheet. As we do not have the calculations to interpret such an anomaly mathematically, again we will make use of the polarized sphere calculations. This anomaly can be approximated to a line of polarized spheres. The error resulting from this approximation is not too great, compared with the errors caused by other factors.

This example has been taken from a survey carried out near New Hope, in British Columbia, for British Columbia Nickel Mines. The survey led to the discovery of a large nickel-bearing pyrrhotite orebody having no surface indications. Nickelferous pyrrhotite with a proved tonnage exceeding 500,000 tons occurs in the vicinity of a pyroxenite dike in granodiorites (or peridotites). The potential anomalies exceeded 300 millivolts. The dip was estimated and the drill holes 1 and 2 cut through a body of ore 25 meters wide (see fig. 20).

Assuming a sphere, taking the profile AB, the interpretation is done in a similar way to the previous ones.

Results: $\alpha = 30^\circ$, $KL/h = 3.0$, $KL = 2a = 60$ m. , $h = 20$ m.
 $X_K/h = 0.2$ $X_K = 10$ m.
