INVESTIGATION OF STRESS DISTRIBUTION ON THIN METAL SHEET WITH HOLES

Thesis by

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SUMMARY

This paper presents the investigation of the ultimate strength of a plate under shearing stresses when the plate contains lightening holes. The optimum arrangement of holes and the most desirable size of hole has also been investigated. Stress-strain curves have been made which show the points at which the plates reach the buckling point, the elastic limit, and the point of permanent deformation.

APPRECIATION

The author wishes to express his appreciation of the assistance furnished by members of the Staff of the Guggenheim Graduate School of Aeronautics at the California Institute of Technology. In particular he wishes to thank Doctors Karman, Klein, and Sechler for their helpful suggestions and criticisms.

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INTRODUCTION

All-metal construction is rapidly replacing the older types of structures in the airplane industry and the necessity of further knowledge on use of thin metal structures is quite evident. The use of lightening holes in metal ribs and metal beams is quite common practice today but data on the effects of such holes on the ultimate load of such structures are still lacking.

The theoretical two-dimensional solution of the stress distribution in metal panels with circular and elliptical holes under shearing stress has been presented in several papers. The authors have used Airy's method to calculate the maximum stress at the circumference of the holes, and the influence of the holes on the rigidity of the structure. The chief result obtained is that the maximum stress at the circumference of the hole is four times the uniform stress applied to the sheet, and this rule can be applied to a strip of finite width if the diameter of the hole is small in comparison to the width.

Mr. M. Hirota (Reference 1) has treated this problem theoretically and has found expressions for the radial, tangential, and shearing stresses around the hole when the plate was subjected to a shearing stress. These equations are as follows:

$$\sigma r = (2P + 8P\ a^2 r^{-2} - 6P\ a^4 r^{-4}) \operatorname{Sin}\theta \operatorname{Cos}\theta$$

$$\sigma t = (2P + 6P\ a^4 r^{-4}) \operatorname{Sin}\theta \operatorname{Cos}\theta$$

$$\tau = (-P\ -2P\ a^2 r^{-2} + 3p\ a^4 r^{-4})\ (\operatorname{Cos}^2\theta - \operatorname{Sin}^2\theta)$$

where P is the shear stress applied at the edges of the sheet in lbs. /sq. in., "a" is the radius of the hole, and "r" is the radius of the element considered.

From the above we get the equation of the ring stress at the edge of the hole ($r = a$) as,

$$\sigma_t \, (r = a) \; = \; 8P \sin\theta \cos\theta$$

This value has a maximum at $\theta = 45^\circ$, at which point

$$\sigma_t = 4P$$

We would expect the equations of Hirota to hold up to the point of elastic buckling and they would be useful to determine the stresses up to this point. However, after buckling, the equations will very likely break down as the buckling causes a complete redistribution of stress, the exact distribution pattern of which is not known. Consequently it was thought desirable by the author to attempt to find some experimental data that might be useful in further research on the allowable shear loads beyond the buckling limit.

APPARATUS

The apparatus used is shown by a drawing in Fig. 1-1 and photographs of the testing method used are shown in several places in this paper. The testing apparatus consists primarily of a U-shaped outer piece which is bolted and soldered to the test specimen, and fastened to the upper support of a small Riehle tension machine.

The center of the specimen is bolted and soldered rigidly to a central clamp which is fastened to the lower head of the tension machine. Thus, two specimens, each 10" x 2" may be loaded in shear along the 10" edges and the results obtained will be an average of the two specimens tested.

Strain gauges (Ames dial gauges reading to 0.01 mm) are attached as shown and are used to measure the shear deformations of the center clamp with respect to the outer U-shaped member. Guides are placed at the top and bottom to prevent the central clamp from moving out of the original plane of the specimen.

The actual technique of attaching the sheets to the clamps took considerable time to develop, but it was finally solved by lightly tinning both sides of the clamp, inserting the specimen, and then bolting it up solid. The thin sheet between the center and outside clamps was protected by metal blocks and the whole clamping unit heated until the solder had melted and joined the clamps and sheet. Then, after removing the protecting blocks from the specimens they were ready for testing.

The specimens used were cold rolled "shim stock" steel, 0.0075" thick and 6" wide. Specimens were cut from this material and tested in a small tension machine in order to obtain the elastic properties of the material. The average values for the whole series are

$E = 29.5 \times 10^6$

$e = 43{,}200$ #/□" elastic limit

$u = 65{,}250$ #/□" ultimate strength.

Holes were made in the shear specimens by drilling between steel plates. This method eliminated distortion around the edges of the holes or turning under of the material around the hole.

Test Program and Results

The following table will indicate the tests made:

Series	No. holes	Diameter	Spacing between Centers
1	no holes	Plate sheet only	
2	1	Varied from.2" to 1"	
3	2	½"	5/8", ¾", 1", 1¼", and 1½"
4	3	½"	5/8", 1" and 1½"
5	4	½"	5/8", 1" and 1½"
6	2	1"	1½" and 2"
7	3	1"	1½" and 2"

All specimens were tested to destruction ~~determine~~ and deflection readings were taken throughout the run.

From these readings stress-strain curves were plotted and are shown in fig. 3-13 inc. Fig. 13-16 inc. summarize the important features of these stress strain curves.

A number of tests were first made on shear panels without holes in order to check the theoretical ultimate loads as given by Wagner (Reference 2.)

The equation given by Wagner is - -

$$f = \frac{2P}{h\,t}\;\frac{1}{\sin 2\alpha}$$

f = Unit tension stress

P = Applied shearing load

h = Thickness of the specimen

α = Angle of waves which occur along "tension field"

but in most cases $\alpha = 45°$ then

$$f = \frac{2\,p}{h\,t}$$

Solving for P

$$P = \frac{f\,h\,t}{2}$$

and with

$$f = 65{,}250^{\#/\square''} \quad , \ h = 10'' \qquad t = .00725''$$

$$P = \frac{65250 \times 10 \times .00725}{2}$$

$$= 2365 \text{ lbs.}$$

As compared to an average test value of P = 2100 lbs., a variation of 10%.

Figures 2, 3, and 4 show the effect of panels with holes of different diameters. The theoretical curve in fig. 2 is based upon the assumption that the percentage reduction in allowable load for an increase in hole diameter would be equal to the percentage increase in stress found by the equation of Hirota. The experimental value for the sheet, with no holes, is taken as the zero point of both curves. Figures 3 and 4 show that the stiffness of the panel decreases with an increase in the hole diameter which means that beams with lightening holes in the webs will have a larger deflection than beams with continuous sheet webs.

Figures 5-12 which show the effect of multiple holes on the stress strain curves of the panels under shear. The curve for the plane panel is included in these figures for comparison.

Figure 13 and 14 show the summary of the test results in which the shear load at various critical stresses is plotted against the number of holes.

The most important results of this investigation, however, is shown in figures 15 and 16 in which the strength / weight ratio is plotted as a function of the other variables. From these curves it is easily seen that one lightening hole in a panel is definitely bad from a strength / weight ratio standpoint, and that two holes at nearly any spacing gives a strength / weight ratio nearly as high as that obtained by the solid sheet. Three and four holes reduce the ratio to a value considerably below that of the solid sheet.

The above effect has not yet been satisfactorily explained and is a problem that might well be investigated further. The investigation would attempt to find why two such holes have a tendency to give less stress concentration than one hole of the same diameter. It is a stress distribution problem and an insight into the stress distribution might be gained by photoelastic methods. This could not apply directly to regions beyond the buckling stress but might give us indication of the probable stress distribution around multiple holes.

CONCLUSION

The failure of the sheet with one or more lightening holes takes ce at the point where the "tension field" has developed, or approximately 45° to the applied shear load. In a few cases failure started at the edge o e sheet near the clamp but this type of failure did not give much lower valu than those specimens in which failure started at the hole.

The sheet with 2 holes seems to give the best all around characteristics, however the drop-off in strength for more than two holes is not serious.

One hole, however, gives the lowest strength weight ratio of any, and therefore should not be used wherever it can be avoided. It is suggested that for panels under shear, lightening holes should be made up in groups of two or more.

The spacing of the holes has very little effect on the stiffness, yield point, or ultimate strength of the panels under shear.

From the results obtained it would seem as though lightening holes do not perform the duties that they were originally intended to perform, i.e., give a sheet metal panel that would be lighter for a given strength. In fact, the introduction of one lightening hole drops the strength weight ratio off so much (from 10-40%) that it is distinctly detrimental. It must be remembered, however, that modern practice never uses lightening holes with plain or unsupported, edges.

The edges are normally flanged and this flanging way have an effect on the total load which would partly counteract the stress concentrations due to the presence of the hole. There have been only a few experimental and theoretical investigations made on this point so further work is indicated. One recent paper (Reference 3) has investigated flanged holes and has found quite similar experimental results. In these tests, only a few experiments gave strength / weight ratios for sheets with holes that were higher than the strength / weight ratio of the plain sheet. Only one type of flange was investigated, however, and more experiments will have to be made before anything definite can be said about the optimum flange shape and its effect.

REFERENCE

(1) Uber den Spannungszustand in einem durchlochten Streifen.
By Morimichi Hirota. Doctor thesis at University Aachen, 1932.

(2) N.A.C.A. Techinal Memorandums 604, 605, and 606.
"Flat sheet metal Girder with very thin metal web.
By Herbert Wagner - Part I, II, III - From Z.F.M. 1929.

(3) Uber das Verhalten von Leichtmetallblechstreifen
mit Kreisronden, randgebordelten Lochern bei
Schubbeanspruchung. By Von Karl Schussler.
Bericht aus dem Aerodynamischen Institut der
Technischen Hochschule Aachen, 1934.

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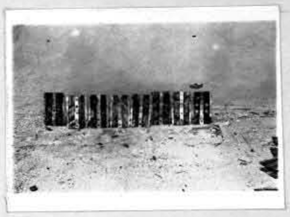
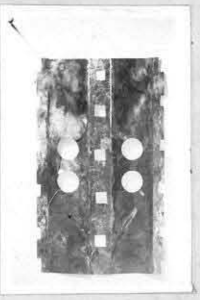
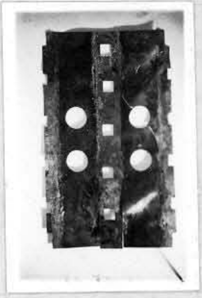
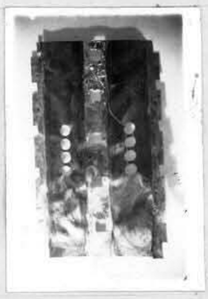
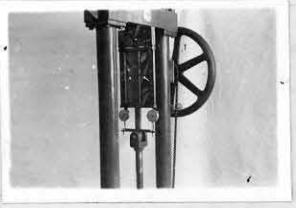
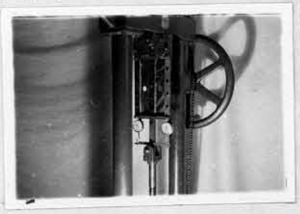
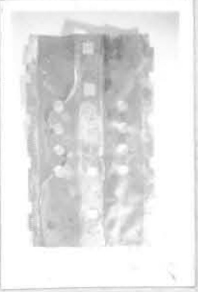
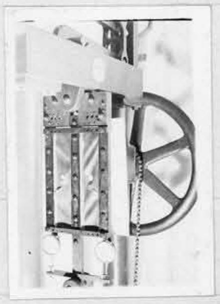






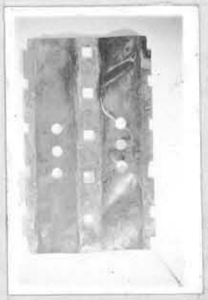
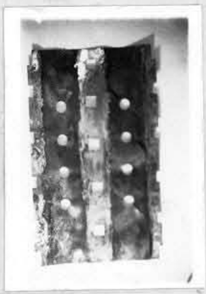
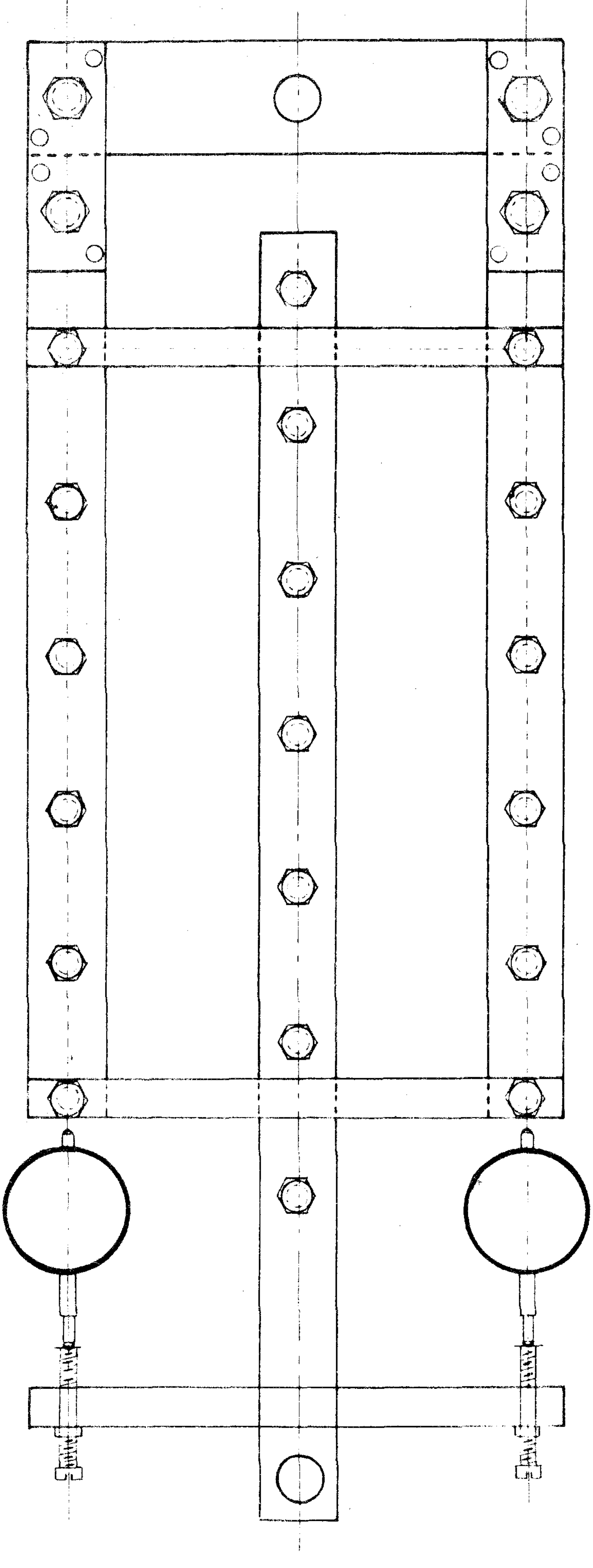



Fig. 1-1

TEST ARRANGEMENT



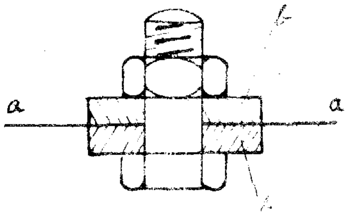


Fig. 1-2
CROSS SECTION OF CLAMPS
AND TEST STRIPS

MATERIAL	COLD ROLLED "SHIM STOCK" STEEL.
THICKNESS	0.00725 IN.
ELASTIC LIMIT IN TENSION	43,200 #/□"
ULTIMATE STRENGTH IN TENSION	65,250 #/□"
MODULUS OF ELASTICITY IN TENSION	29.5×10^6

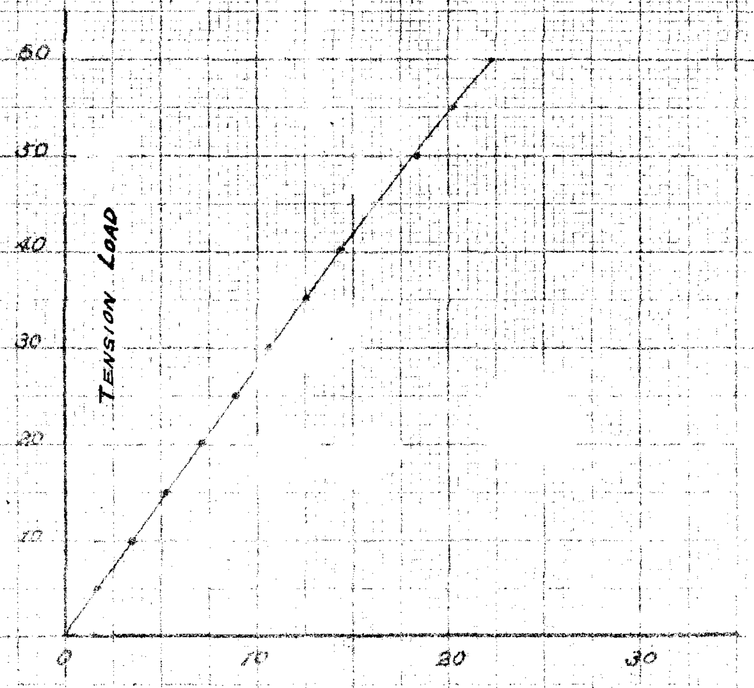


FIG. I

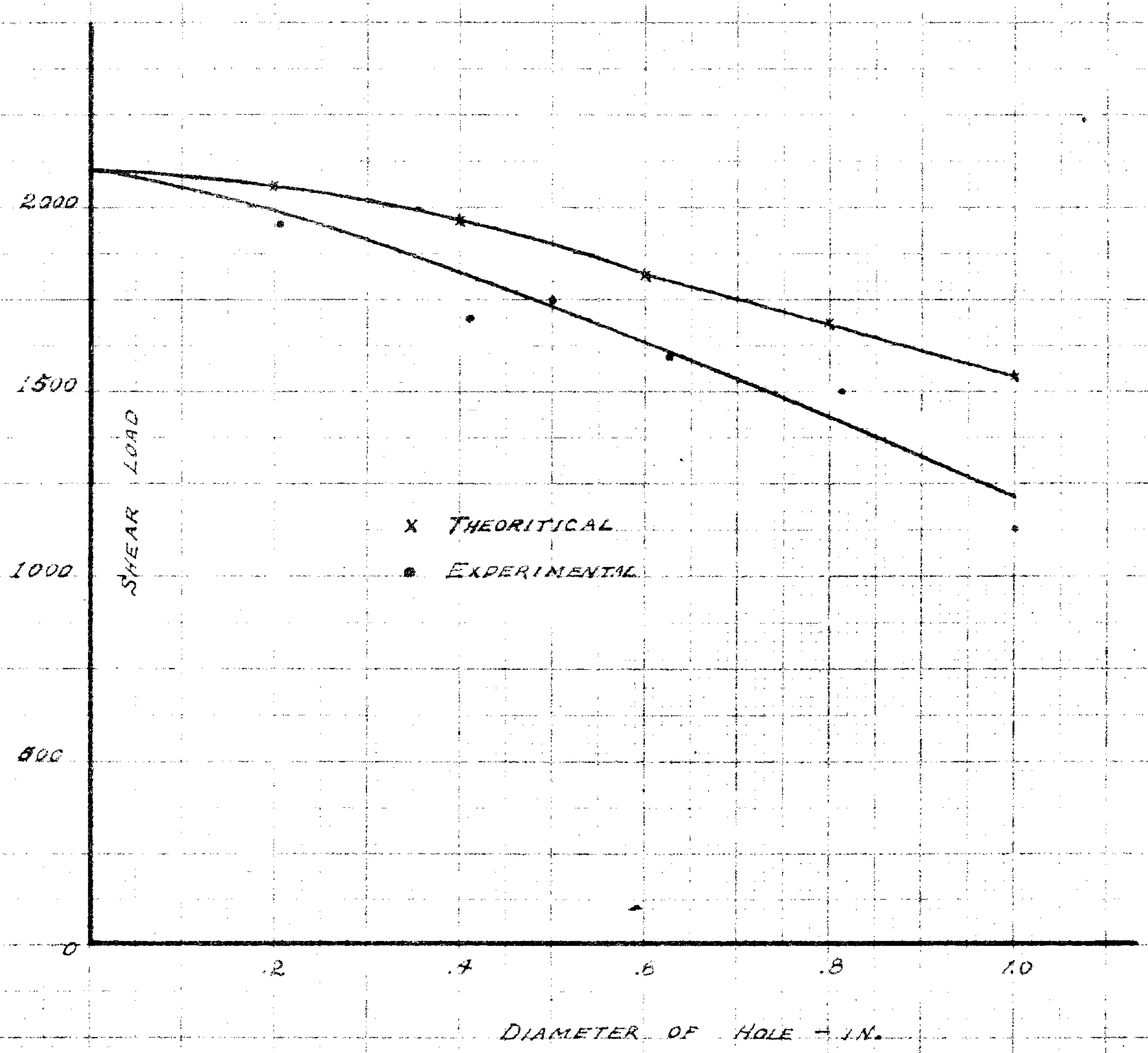

2000
1500
1000
500
0
SHEAR LOAD
X THEORITICAL
● EXPERIMENTAL
.2
.4
.6
.8
1.0
DIAMETER OF HOLE - IN.

FIG. 2

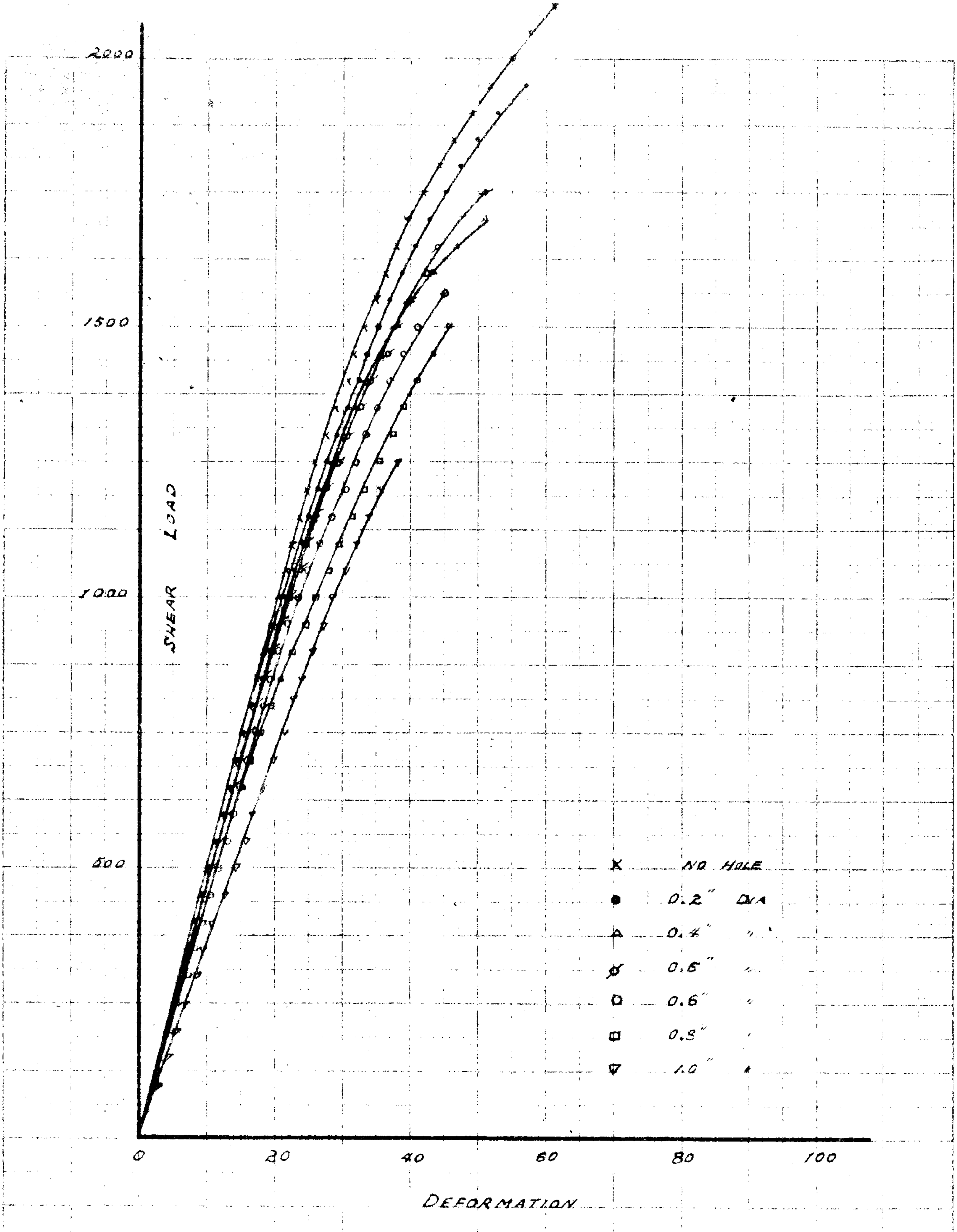

2000
1500
1000
500
SHEAR LOAD
0
20
40
60
80
100
DEFORMATION
X NO HOLE
● 0.2" DIA
△ 0.4" "
ø 0.5" "
O 0.6" "
□ 0.8" "
▽ 1.0" "

FIG. 3

FIG. 4

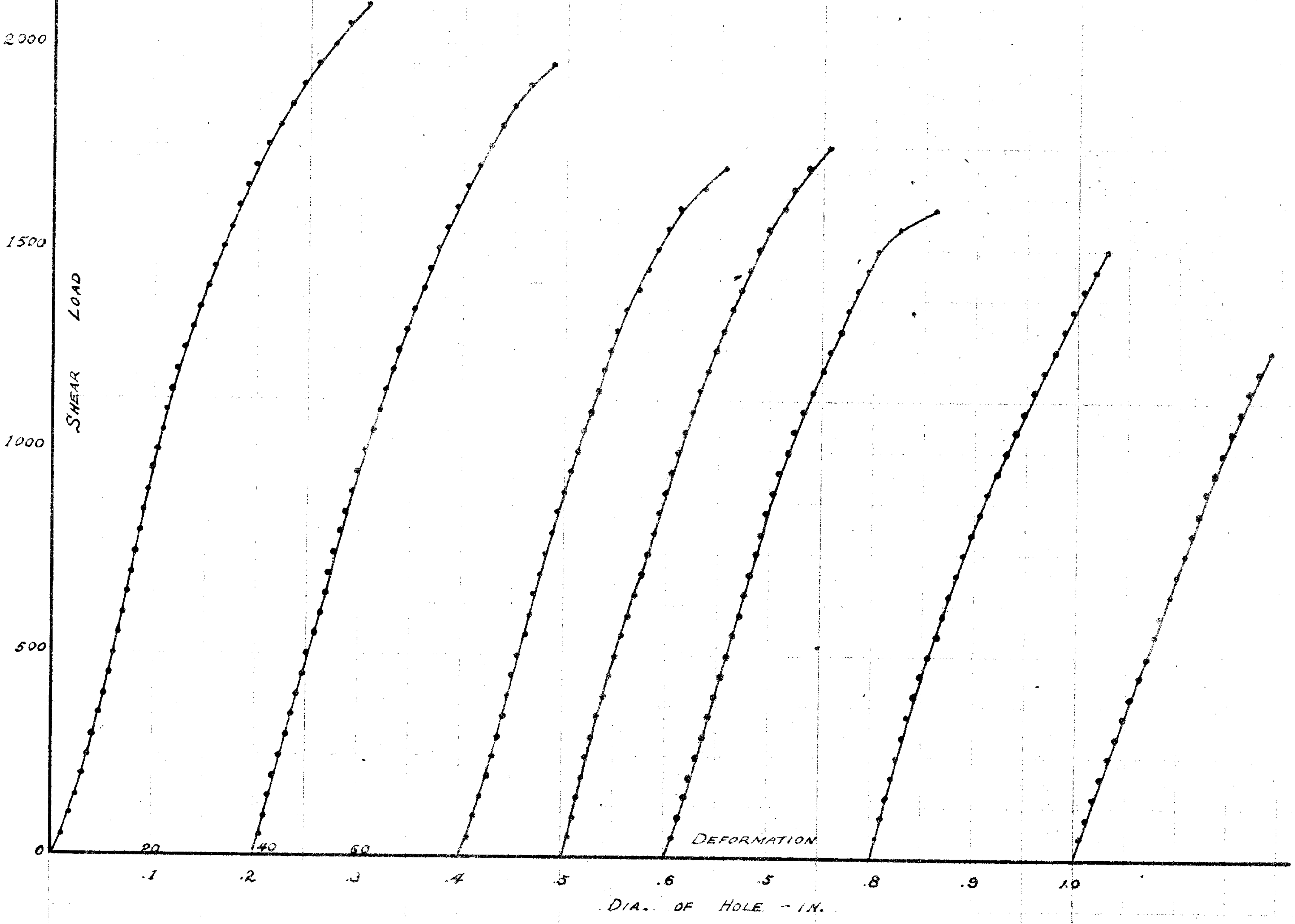
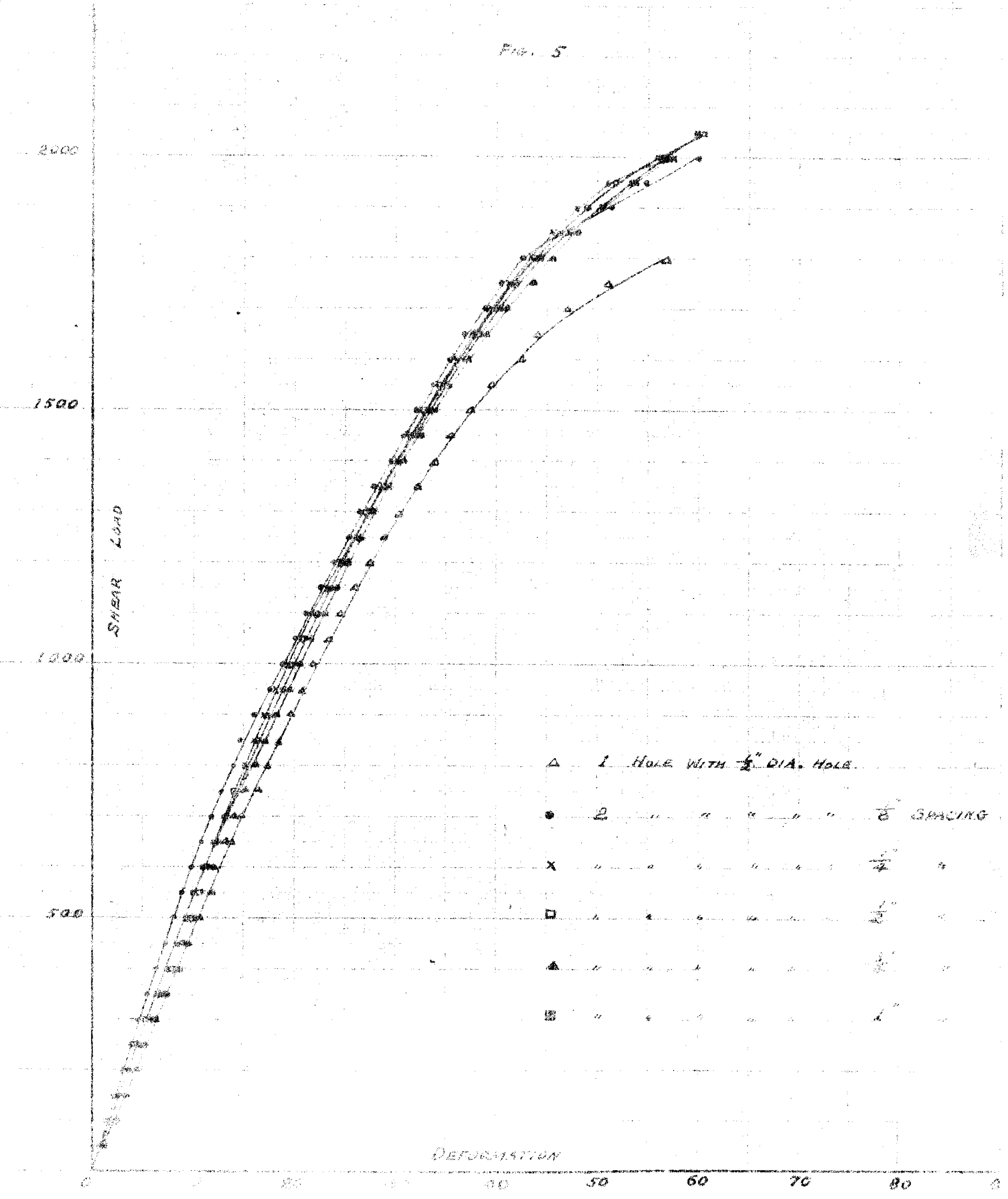

SHEAR LOAD
2000
1500
1000
500
0
20
40
60
DEFORMATION
.1
.2
.3
.4
.5
.6
.5
.8
.9
1.0
DIA. OF HOLE - IN.

Fig. 5



2000
1500
1000
500
Shear Load
Deformation
0
10
20
30
40
50
60
70
80
90
△ 1 Hole with 1/4" Dia. Hole.
● 2 " " " " " 1/8" Spacing
X " " " " " " 1/4" "
□ " " " " " " 3/4" "
▲ " " " " " 1/2" "
▨ " " " " " 1" "

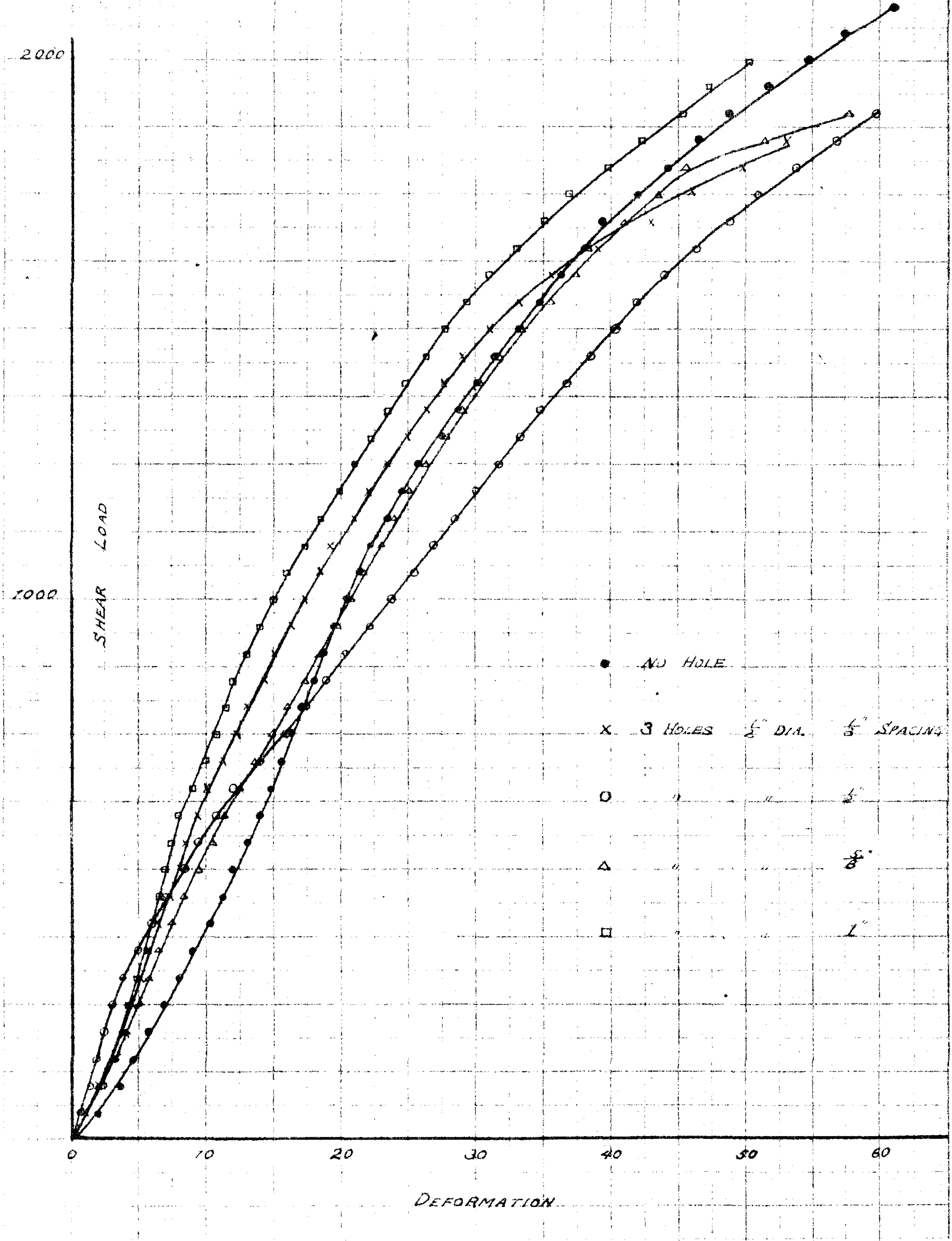

2000
1000
SHEAR LOAD
0
10
20
30
40
50
60
DEFORMATION
● NO HOLE
X 3 HOLES 1/2" DIA. 1/3" SPACING
O " " 1/2"
△ " " 5/8"
□ " " 1"

FIG. 6

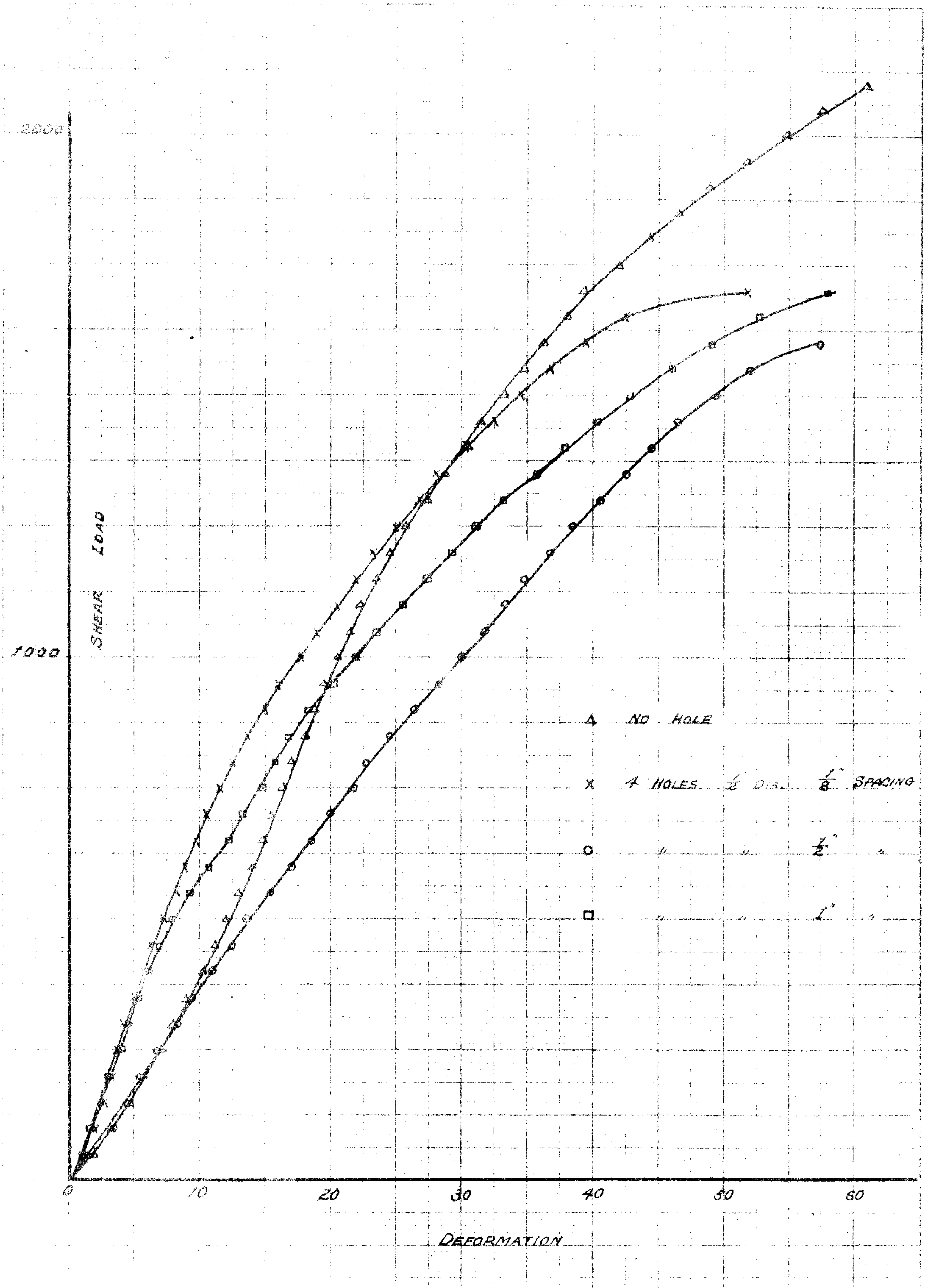

2000
1000
SHEAR LOAD
0
10
20
30
40
50
60
DEFORMATION
△ NO HOLE
× 4 HOLES ½ DIA. ⅛" SPACING
○ " " " ½" "
□ " " " 1" "

FIG. 7

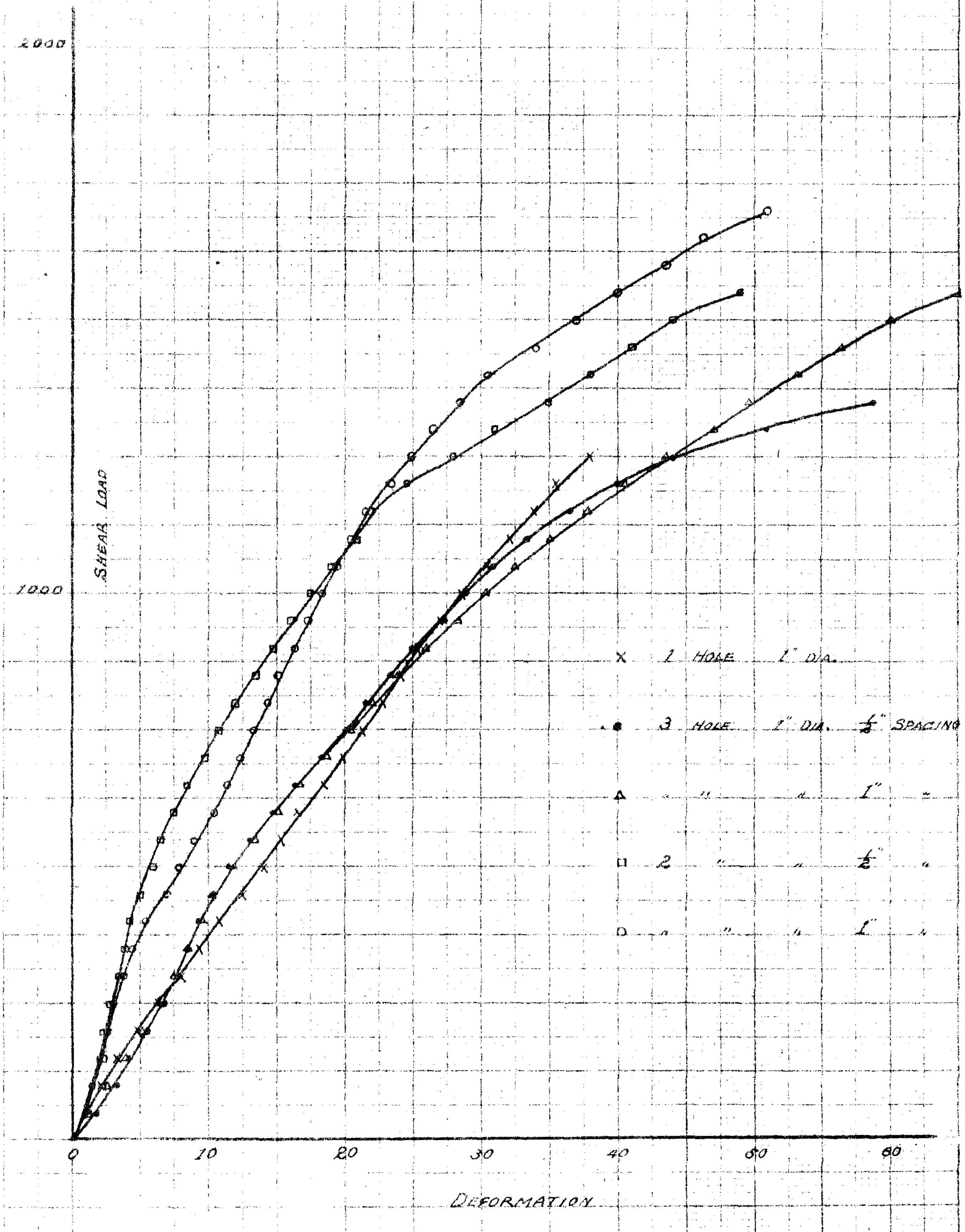

2000
1000
SHEAR LOAD
0
10
20
30
40
50
60
DEFORMATION
X 1 HOLE 1" DIA.
● 3 HOLE 1" DIA. ½" SPACING
△ " " " 1" "
□ 2 " " ½" "
○ " " " 1" "

FIG. 8

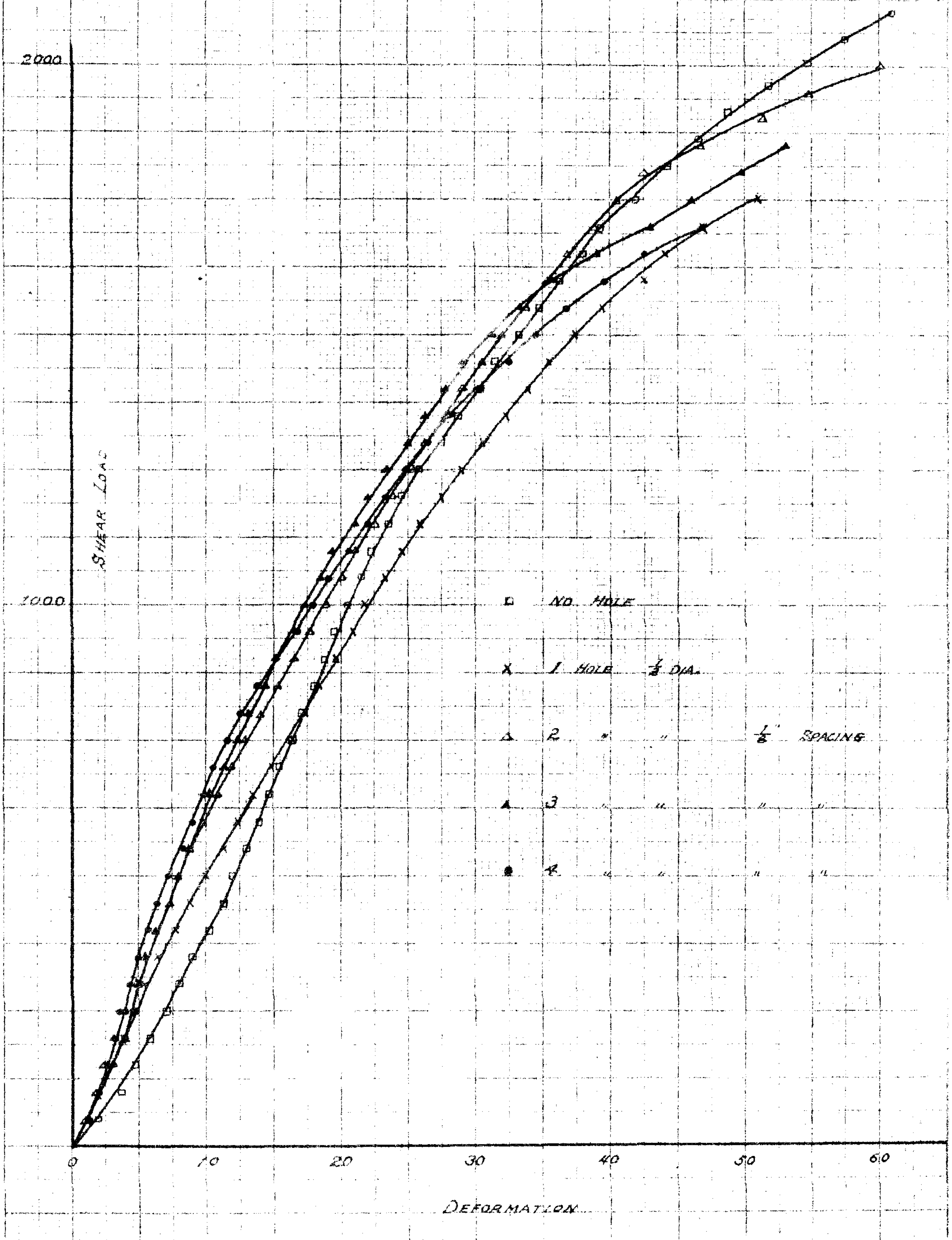

2000
1000
SHEAR LOAD
0
10
20
30
40
50
60
DEFORMATION
□ NO HOLE
X 1 HOLE 1/8 DIA.
△ 2 " " 1/8" SPACING
▲ 3 " " " "
● 4 " " " "

FIG. 9

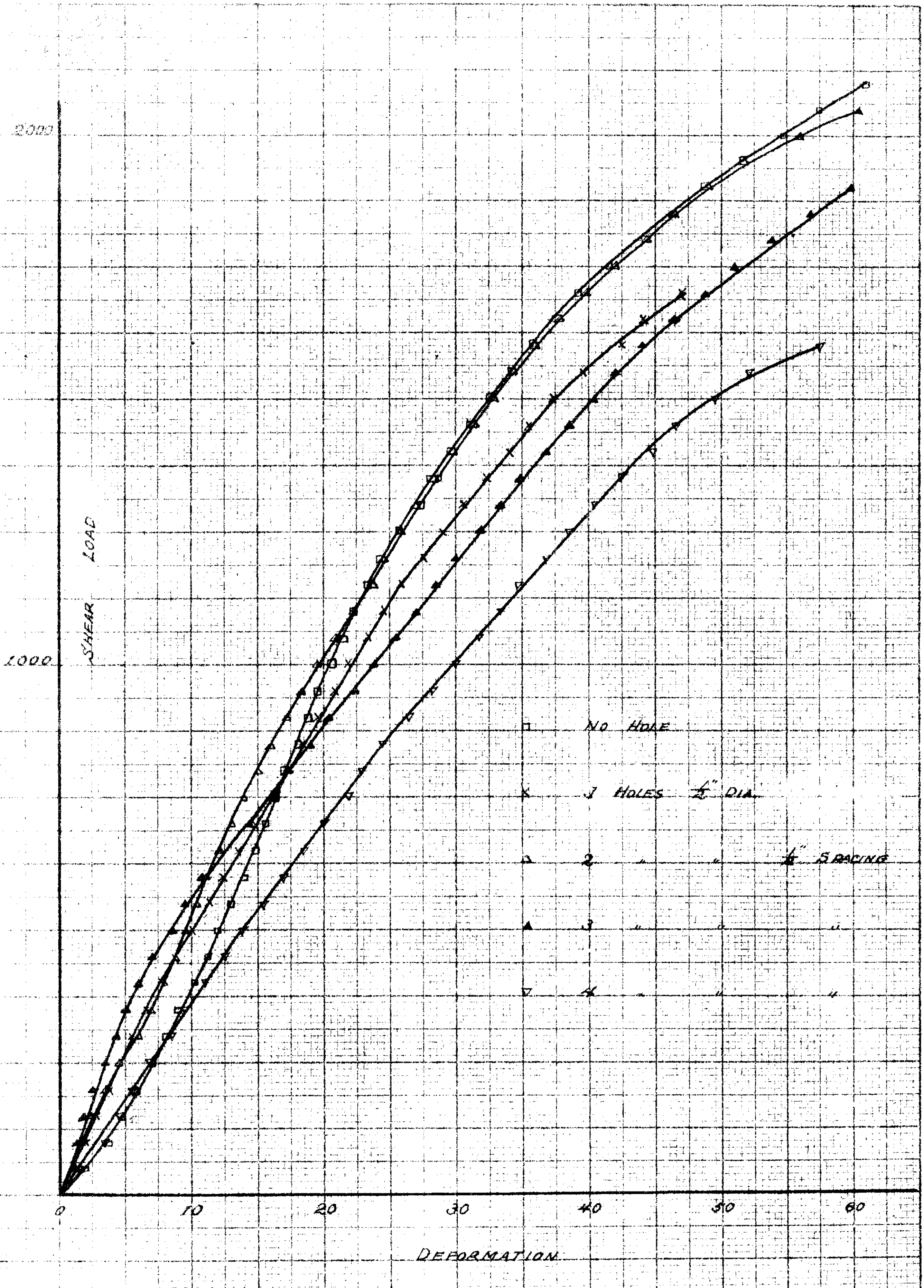

2000
1000
SHEAR LOAD
0
10
20
30
40
50
60
DEFORMATION
□ NO HOLE
× 1 HOLES 1/4" DIA.
△ 2 " " 1/4" SPACING
▲ 3 " " "
▽ 4 " " "

FIG. 10

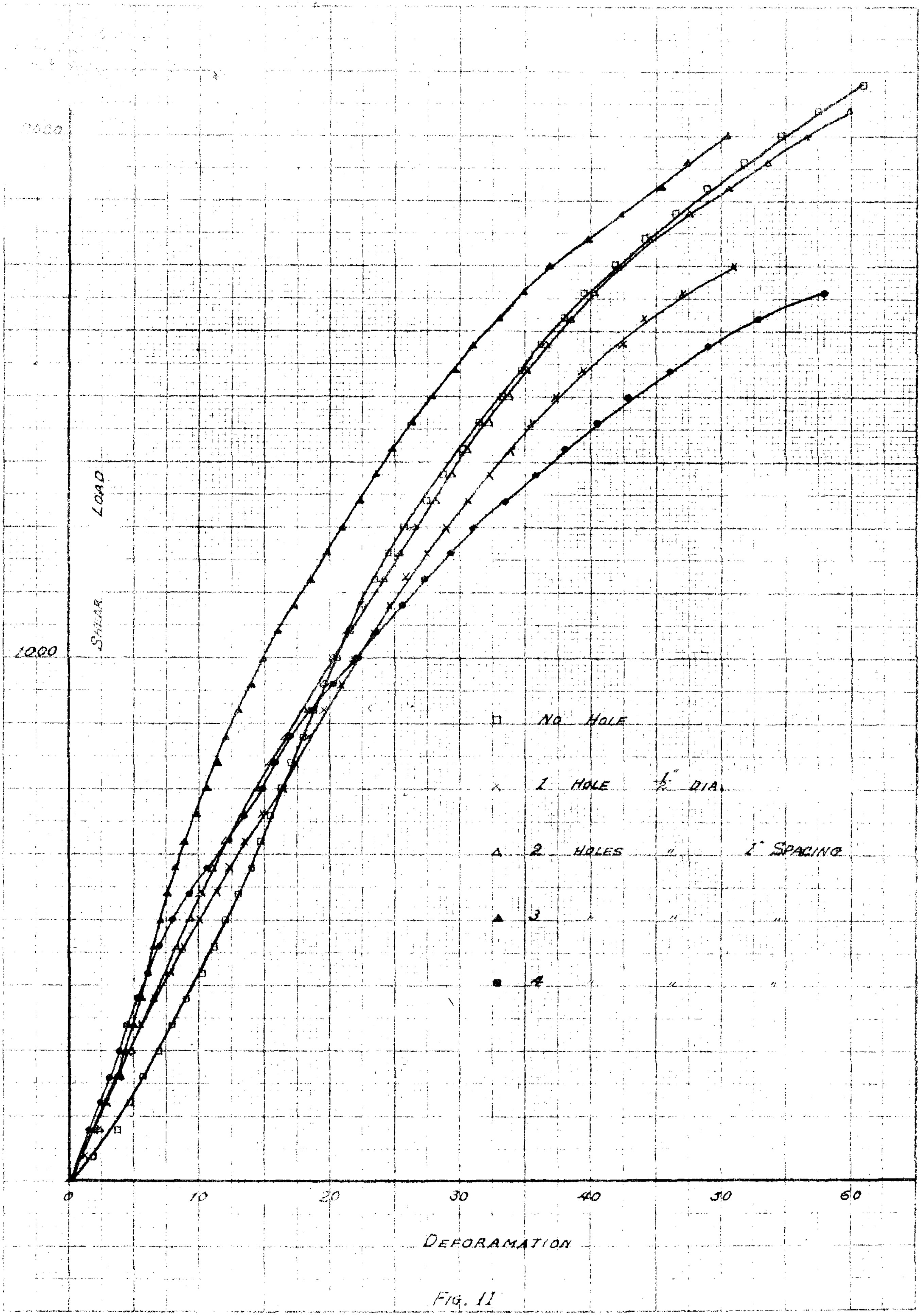

2000
1000
LOAD
SHEAR
0
10
20
30
40
50
60
DEFORAMATION
□ NO HOLE
× 1 HOLE ½" DIA.
△ 2 HOLES " 1" SPACING
▲ 3 " " "
● 4 " " "

FIG. 11

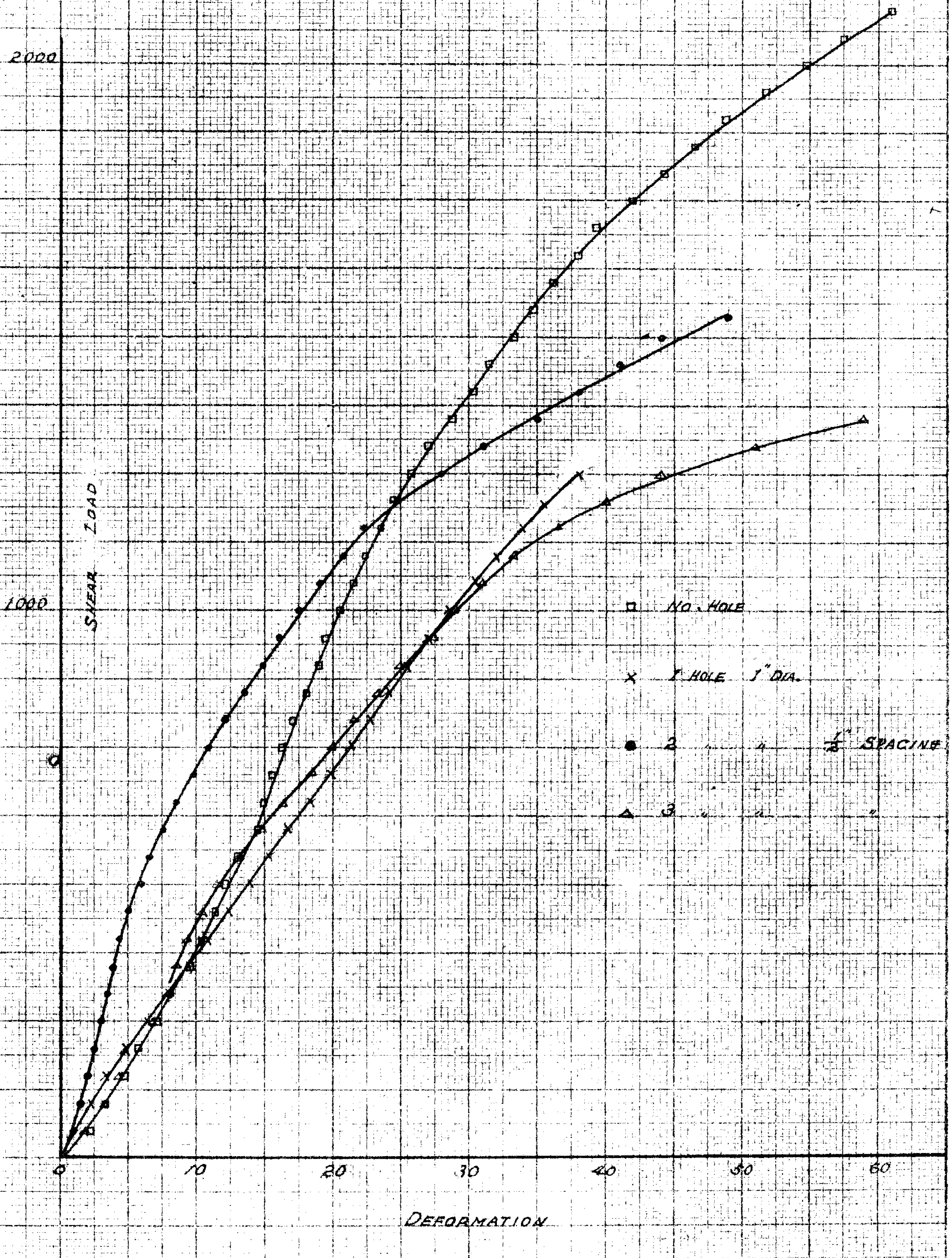

2000
1000
0
SHEAR LOAD
0
10
20
30
40
50
60
DEFORMATION
□ NO. HOLE
× 1 HOLE 1" DIA.
● 2 " " 1/2" SPACING
△ 3 " " "

FIG. 12

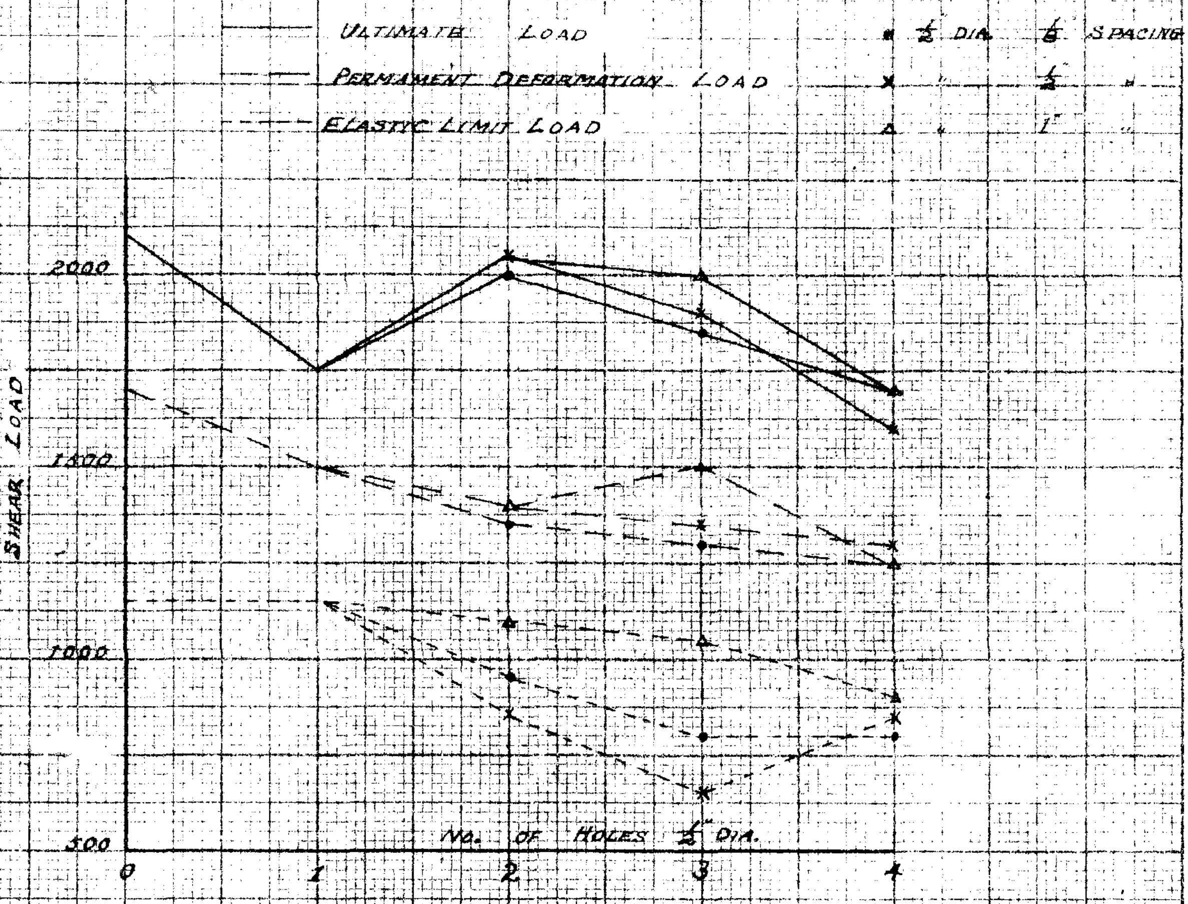

ULTIMATE LOAD
PERMANENT DEFORMATION LOAD
ELASTIC LIMIT LOAD
● 1/2 DIA. 1/8 SPACING
× " 1/2 "
△ " 1" "
SHEAR LOAD
2000
1500
1000
500
NO. OF HOLES 1/2" DIA.
0
1
2
3
4

FIG. 13

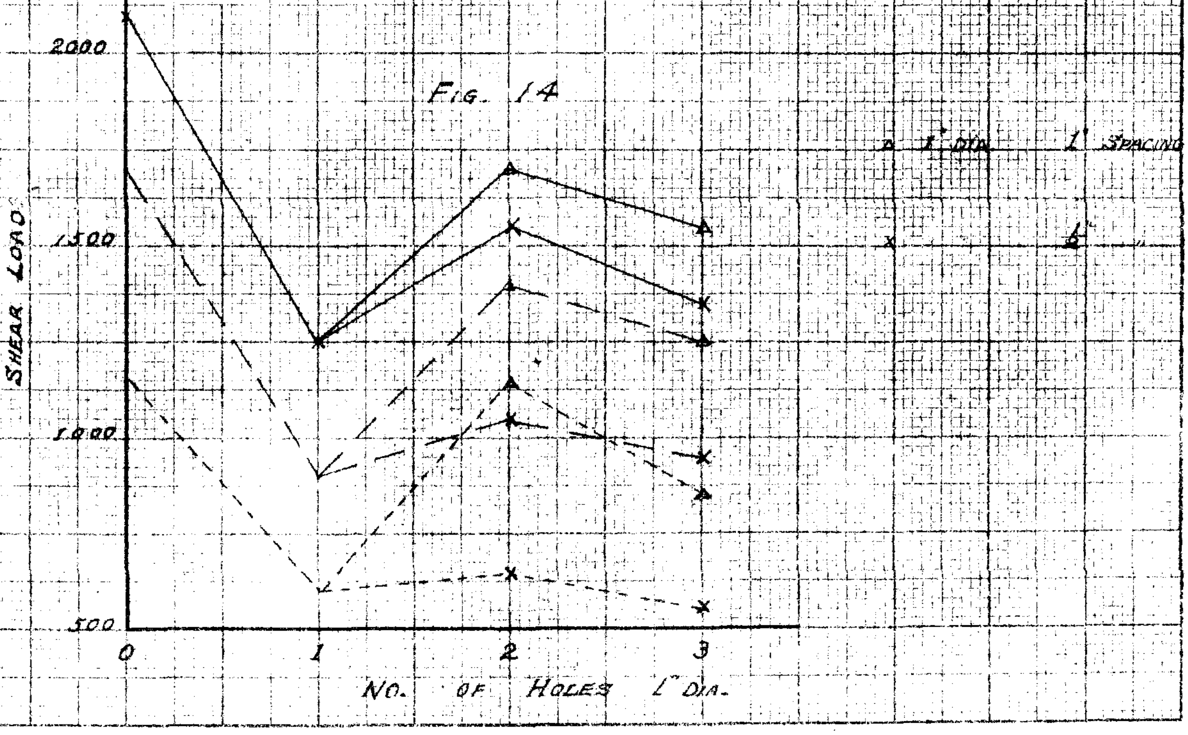
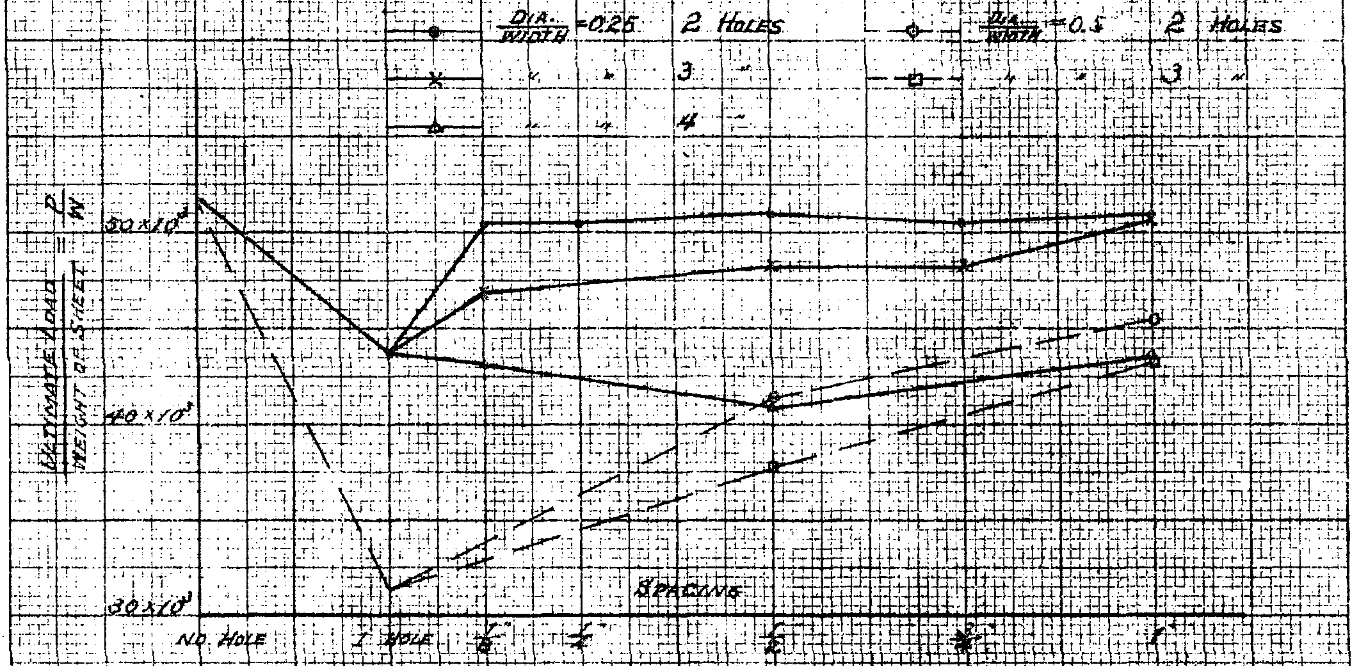

△ 1" DIA. 1" SPACING
× " 1/2" "
SHEAR LOAD
2000
1500
1000
500
0
1
2
3
NO. OF HOLES 1" DIA.

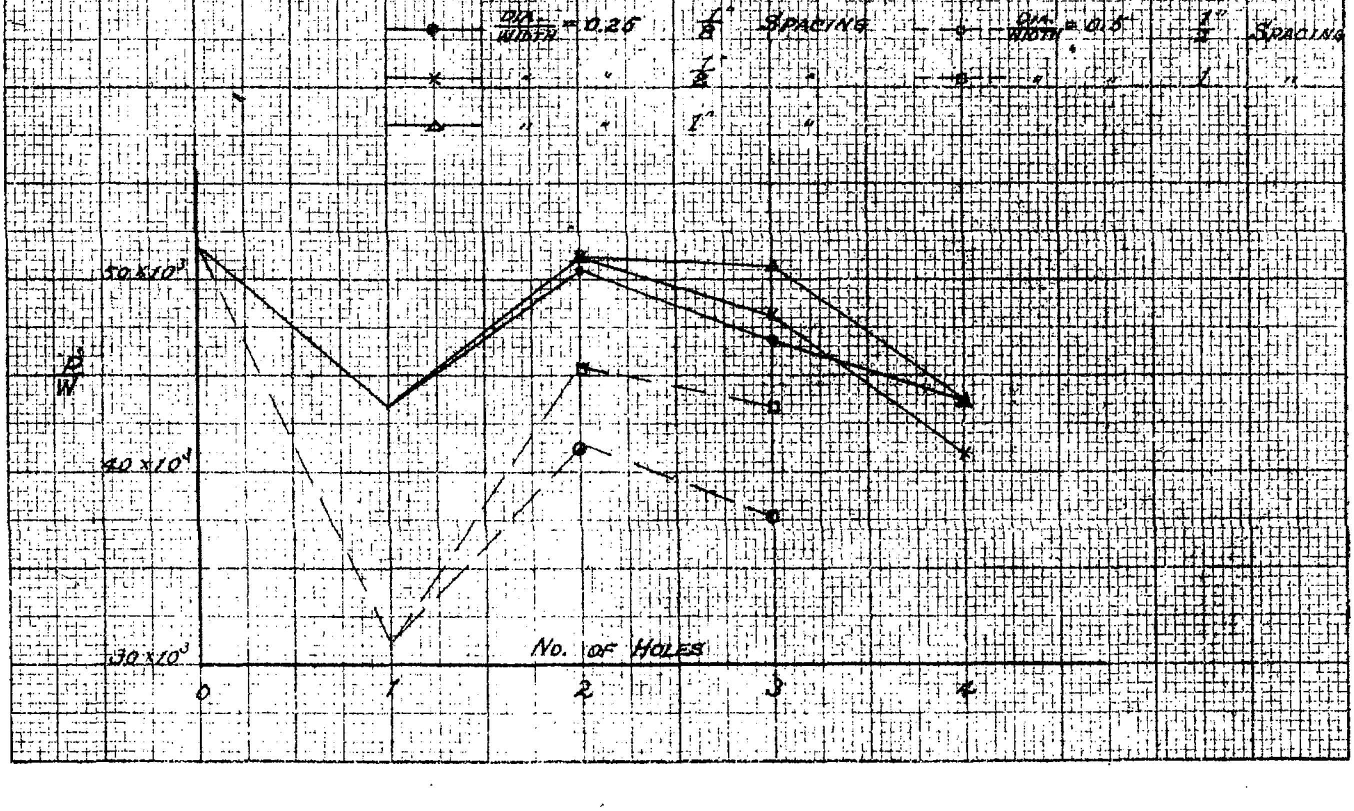
FIG. 14

Fig. 15



Dia./Width = 0.25 2 Holes
" " 3 "
" " 4 "
Dia./Width = 0.5 2 Holes
" " 3 "
Ultimate Load / Weight of Sheet = P/W
50×10³
40×10³
30×10³
Spacing
No. Hole
1 Hole
1/8"
1/4"
1/2"
3/4"
1"

Fig. 16



Dia./Width = 0.25 1/8" Spacing
" " 1/2" "
" " 1" "
Dia./Width = 0.5 1/2" Spacing
" " 1 "
P/W
50×10³
40×10³
30×10³
No. of Holes
0
1
2
3
4