TWO TOPICS IN ELEMENTARY PARTICLE PHYSICS

I. CROSSING AS A GROUP AND ELIMINATION OF EXOTIC CHANNELS

II. REAL PARTS OF MESON-NUCLEON FORWARD SCATTERING AMPLITUDES

Thesis by

Amos Yahil

In Partial Fulfillment of the Requirements

For the Degree of

Doctor of Philosophy

California Institute of Technology
Pasadena, California

1970

(Submitted 9 April, 1970)

To six friends who helped

me stand my ground.

To Yehudit and Yehuda

To Ruth and Joseph

and

To Carol and Ed.

ACKNOWLEDGMENTS

I am indebted to the members of the theoretical group at Caltech, who in their research and teaching guide us to reach our own aims. It is a special pleasure to thank my advisor Professor S.C. Frautschi for many hours of patient advice to an unruly student. I benefited from discussions with Professor G. Zweig on the first part of my thesis. The second half was suggested by and done in collaboration with Professor D. Horn.

I am grateful for the financial assistance generously offered to me throughout my stay at Caltech, both in the form of teaching assistantships and by the Richard P. Feynman Graduate Fellowship.

ABSTRACT

I. Crossing transformations constitute a group of permutations under which the scattering amplitude is invariant. Using Mandelstem's analyticity, we decompose the amplitude into irreducible representations of this group. The usual quantum numbers, such as isospin or SU(3), are "crossing-invariant". Thus no higher symmetry is generated by crossing itself. However, elimination of certain quantum numbers in intermediate states is not crossing-invariant, and higher symmetries have to be introduced to make it possible. The current literature on exchange degeneracy is a manifestation of this statement. To exemplify application of our analysis, we show how, starting with SU(3) invariance, one can use crossing and the absence of exotic channels to derive the quark-model picture of the tensor nonet. No detailed dynamical input is used.

II. A dispersion relation calculation of the real parts of forward $\pi^{\pm}p$ and $K^{\pm}p$ scattering amplitudes is carried out under the assumption of constant total cross sections in the Serpukhov energy range. Comparison with existing experimental results as well as predictions for future high energy experiments are presented and discussed. Electromagnetic effects are found to be too small to account for the expected difference between the $\pi^{-}p$ and $\pi^{+}p$ total cross sections at higher energies.

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I. INTRODUCTION

Regge pole theory[1)] has enabled us to understand the scattering process $A + B \rightarrow C + D$ at high energies and low momentum transfers in terms of exchanges in the crossed channel $A + \bar{C} \rightarrow \bar{B} + D$. (See Fig. 1 for definition of kinematic variables.) To leading order in s, the contribution of an individual Regge exchange to the scattering amplitude is given by

$$A(\nu,t) = - \beta(t) \frac{(-\nu)^{\alpha(t)} + \tau\nu^{\alpha(t)}}{\sin \pi\alpha(t)\ \Gamma(1 + \alpha(t))} \cdot \qquad (1)$$

A Regge pole is a pole in the j-plane at $j = \alpha(t)$ with residue $\beta(t)$. It corresponds to the exchange of a whole "trajectory" of particles. These have identical quantum numbers except angular momentum. The existence of exchange forces causes even- and odd-j partial waves to have different analytic continuations in the j-plane. A Regge trajectory is thus characterized by an additional quantum number $\tau = \pm 1$ called signature, which specifies in which of the two amplitudes the pole appears. (1) clearly exhibits this. Even signature ($\tau = 1$) implies poles in $A(\nu,t)$ at t values for which $\alpha(t) = 0,2,4,\ldots$. Similarly, poles are inferred at $\alpha(t) = 1, 3,5,\ldots$ if the signature is odd ($\tau = -1$). The absence of exchange forces manifests itself by the existence of two exchange degenerate trajectories with equal residues, but opposite signatures and parities.

$$\alpha_1(t) = \alpha_2(t) \qquad \beta_1(t) = \beta_2(t) \qquad \tau_1 = -\tau_2 \qquad P_1 = -P_2 \qquad (2)$$

Their sum cancels the second (exchange) term in (1).

Signature has additional meaning when considering the crossed reaction $A + \bar{D} \to C + \bar{B}$, whose amplitude is $A(-\nu,t)$. We define the symmetric and antisymmetric amplitudes

$$A^{(\pm)}(\nu,t) = \frac{1}{2}\,(A(\nu,t) \pm A(-\nu,t)). \tag{3}$$

Even signature contributes only to $A^{(+)}$, and odd one only to $A^{(-)}$.

A connection is made between the high energy Regge behavior of the amplitude and low energy scattering by means of analyticity in the energy variable ν (see Fig. 2). Each amplitude splits into a sum of an amplitude with a right-hand cut, one with a left-hand one, and possibly a third, which is entire in ν (a subtraction term).

$$A^{(\tau)}(\nu,t) = A_s^{(\tau)}(\nu,t) + A_u^{(\tau)}(\nu,t) + \text{subtraction term.} \tag{4}$$

As previously indicated, the asymptotic expansion (1) is already split as it stands. The first term has the right-hand (direct) cut, while the second has the left-hand (exchange) one.

$$A_s^{(\tau)}(\nu,t) = -\frac{1}{2}\,\beta(t)\,\frac{(-\nu)^{\alpha(t)}}{\Gamma(1+\alpha(t))\,\sin\pi\alpha(t)}$$

$$A_u^{(\tau)}(\nu,t) = -\frac{1}{2}\,\tau\,\beta(t)\,\frac{\nu^{\alpha(t)}}{\Gamma(1+\alpha(t))\,\sin\pi\alpha(t)} \tag{5}$$

Dolen, Horn and Schmid[2)] first realized that the asymptotic expansion (5) had to be equally good along the entire circle $|\nu| = N$. This enables explicit integration of the amplitude along that circle. Consider the <u>finite</u> closed contour P in Fig. 2. The amplitude $A_s^{(\tau)}(\nu,t)$ is regular inside it, and therefore Cauchy's theorem gives

$$\oint_P \nu^n A_s^{(\tau)}(\nu,t)d\nu = 0 \qquad n = 0,1,2,... \tag{6}$$

Evaluating the integral along the circle $|\nu| = N$ explicitly one has

$$\int_0^N \nu^n \, \mathrm{Im}\, A_s^{(\tau)}(\nu,t) = \frac{1}{2} \sum_i \frac{\beta_i(t)\, N^{\alpha_i(t) + n + 1}}{(\alpha_i(t) + n + 1)\, \Gamma(\alpha_i(t) + 1)} \quad n = 0,1,2,... \tag{7}$$

A similar analysis of $A^{(\tau)}{}_u(\nu,t)$ gives*

$$\int_0^N \nu^n \, \mathrm{Im}\, A^{(\tau)}(\nu,t) = \sum_i \frac{\beta_i(t)\, N^{\alpha_i(t) + n + 1}}{(\alpha_i(t) + n + 1)\, \Gamma(\alpha_i(t) + 1)} \quad n = 0,1,2,... \tag{8}$$

The sum extends over all Regge trajectories with signature τ. The values of n can be extended to negative integers, provided one handles the additional singularity at $\nu = 0$.

$$\int_0^N \frac{1}{\nu^{m+1}} \, \mathrm{Im}\, A^{(\tau)}(\nu,t) = \sum_i \frac{\beta_i(t)\, N^{\alpha(t) - m}}{(\alpha(t) - m)\, \Gamma(\alpha(t) + 1)} + \frac{\pi}{m!} \left(\frac{\partial}{\partial \nu}\right)^m A^{(\tau)}(\nu,t)\Big|_{\nu = 0} \quad m = 0,1,2,... \tag{9}$$

Equations (8) and (9) form constraints which analyticity and Regge asymptotics impose on the amplitude. They are known as finite energy sum rules (FESR). The asymptotic Regge amplitude gains meaning also at low energies. It is a local average of the amplitude. This is known as duality between direct channel scattering and crossed

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* The subtraction term does not contribute to (8) because it is analytic in ν.

channel exchanges. If the high energy behavior is not described by simple Regge poles, the right-hand side of (8) and (9) will have to be modified. The integration along the circle $|\nu| = N$ will be complicated. However, there is still duality between low and high energy scattering.

The next step was made by Freund and Harari[3]. They conjectured that the amplitude may be split into a sum of a "resonating" amplitude and a "diffractive" one.

$$A(\nu,t) = A_D(\nu,t) + A_R(\nu,t) \cdot \qquad (10)$$

Each satisfies FESR separately. The diffractive comprises only background scattering at low energies and diffraction (Pomeron exchange) at high energies. The high energy cross section arising from this amplitude is constant at high energies. The resonating amplitude consists of pure resonances without background at low energies, and is dual to the exchange of all usual Regge trajectories at high energies.* Those lead to cross sections which fall like a power of ν.

A direct analysis of this assumption has been done in πN and KN scattering[4]. However, the most convincing evidence comes from the interesting experimental correlation, which is explained by, and motivated this hypothesis. Experiment has so far failed to reveal resonances whose SU(3) quantum numbers are not contained in the usual quark model[5]. Mesons seem to fall into $\underset{\sim}{1}$ and $\underset{\sim}{8}$, while baryons

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* To distinguish diffractive and non-diffractive scattering we reserve the name Regge trajectories to those which are associated with the known t channel resonances.

can be classified in $\underset{\sim}{1}$, $\underset{\sim}{8}$, and $\underset{\approx}{10}$. Other representations are known as exotic. The exotic KN and NN channels show not only a lack of resonances at low energies, but they have constant cross sections at high energies.* On the other hand, πN, $\bar{K}N$ and $\bar{N}N$ have both low energy cross sections which fall approximately as

$$\sigma(\nu) = a + b/\sqrt{\nu}. \tag{11}$$

In an exotic channel the resonating amplitude has no right-hand cut at low $\nu > 0$. By FESR it has none at high energies either. Thus the total cross section, which is proportional to the imaginary part of the amplitude, is purely diffractive, i.e., flat. It was pointed out above how this comes about in the Regge language. The right-hand cut of the resonating amplitude vanishes by cancellation between pairs of exchange degenerate trajectories.

Veneziano[7)] has meantime written down an explicit example of a function which has Regge asymptotics at high energies, and shows resonance behavior at low energies.

The division into resonating and diffractive amplitudes is clearly only a first approximation. For one thing, unitarity, being a nonlinear relation, is bound to mix the two. Secondly, this approximation ignores Regge cuts[8)]. The latter have a logarithmic dependence on energy, and complicated FESR. In fact the two objections are related. Absorption corrections[9)] correspond to Regge

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* The NN channel is also considered exotic. The cross section is not really absolutely flat, but slowly varying in comparison with that of $\bar{N}N$.

cuts, and are mixed amplitudes consisting of double exchanges of an ordinary trajectory and a Pomeron. Thus one expects this hypothesis to hold only at low t, where Regge poles are a good approximation to the data, and provided that cuts are not strong. Throughout this thesis we will work in that limit.

In Part I we offer a different mathematical treatment of the Freund-Harari conjecture in meson-meson scattering leading to known results. We investigate the consequences of crossing symmetry and Mandelstam's analyticity in both kinematic variables[10]. Our analysis is independent of the Regge pole hypothesis, or any other dynamical scheme, except the Freund-Harari assumption.

Crossing symmetry means invariance of the amplitude, up to a sign, under the formal permutation of the quantum numbers of identical particles. If their energies have the same sign, the physical channel is unaltered, and we have the Pauli principle. If they have opposite signs we are transformed into another physical channel. Crossing symmetry then gains meaning only when we specify how to continue the amplitude through the unphysical region, i.e., when we invoke analyticity. If there are k identical particles, the crossing transformations will form the group of permutations on k objects, S_k.

Now, in addition we may have conservation of internal quantum numbers such as I-spin or SU(3). Particles belonging to a given multiplet will then be considered identical, and crossing will apply to them. Of course, one has to permute the internal quantum numbers too. If the internal symmetry is approximate, we can assume crossing to be good to the same extent. By this generalization the internal quantum numbers become "crossing invariant". To be sure, selection rules are obtained from the generalized Pauli principle (for instance an $I = 1$ resonance in a $\pi\pi$ channel must have odd j.) But no higher symmetries are generated by crossing. (The existence of an $I = 1$ resonance tells us nothing about possible $I = 0$ resonances.) The elimination of the right-hand cut for an exotic s channel is a statement on the analytic structure of the amplitude. It means that there is only a third double spectral function F(t,u). However, this constraint is not "crossing invariant". To make it so, one has to introduce higher symmetries. Exchange degeneracy in Regge theory is a manifestation of this statement[11)]. In another example we prove that resonances with $J^{PC} = 0^{++}, 2^{++}, 4^{++}$ ... form degenerate nonets whose couplings to the pseudoscalars are exactly those of the quark model[12)]. This result comes about in the limit of exact SU(3), without the need for detailed dynamical assumptions[13)].

Using Mandelstam's analyticity[10)], we decompose the amplitude into irreducible representations of the symmetric group S_3. Our generalized approach makes the elimination of exotic channels very transparent. We apply this method to the SU(2) invariant $\pi\pi$ scattering amplitude, and show how exchange degeneracy comes about. Next we treat pseudoscalar-pseudoscalar scattering in the limit of exact SU(3). In addition to generalizing exchange degeneracy, we derive the previously mentioned results for the even-j resonances. We also briefly discuss the possible use of larger symmetric groups in analyzing the general n-point function. Although our formalism can be easily generalized, it is not a useful approach to multi-body dynamics because most resonances decay via two particle intermediate states. The physics is already contained in the four-point function.

The recent Serpukhov[14)] experiments at energies of 30-70 BeV show that all measured meson-nucleon total cross sections stay constant above 30 BeV. This contradicts previous expectations from Regge pole theory. The expansion (1) is clearly in trouble above 30 BeV. Three main lines of thought have emerged to deal with this "catastrophe".

1) The experiment is wrong. This possibility is not very likely, since the statistics are good and considerable care was taken to minimize systematic errors.

2) Regge cuts are strong, and the logarithmic dependence on ν is sufficiently rapid to create a swift flattening of the total cross section. There are several versions of this approach[15)].

3) A new physics sets in around 30 BeV, and the Regge approximation is no longer valid. This may or may not be combined with the second possibility (or even with the first!).

The attractiveness of such suggestions depends on

a) The theoretical ideas underlying each explanation.

b) The number of parameters needed to fit the data.

c) The ability of the theory to predict correctly future experiments.

Two features of Regge theory have to be modified to accommodate the new data. The first is the Pomeranchuk theorem[16). This states, under very general assumptions, that if in an elastic process $A + B \to A + B$

$$\mathrm{Re}\, A(\nu,0)/[\mathrm{Im}\, A(\nu,0) \log \nu] \xrightarrow[\nu \to \infty]{} 0 \qquad (12)$$

then

$$\sigma^{(-)}(\nu) = \frac{1}{2}\left(\sigma_T(AB) - \sigma_T(\bar{A}B)\right) \xrightarrow[\nu \to \infty]{} 0 \qquad (13)$$

Below 30 BeV the data[6)] looked as though the total cross sections approached each other, with $\sigma^{(-)}$ falling like a power of ν. The Serpukhov data seem to upset this*. Theories invoking Regge cuts have $\sigma^{(-)}$ fall logarithmically with ν. In addition they have $\sigma_T(\nu)$ approach its asymptotic value extremely slowly. Barger and Phillips[15)] in their fit state that the πN total cross section will reach asymptotia, to within the experimental error $\pm$ 0.2 mb, only at $\nu = 10^{98}$ BeV. By that time we lose our interest in religiously preserving the Pomeranchuk theorem. One realizes that there is nothing holy about assumption (12), and hence the conclusion (13) need not hold. In fact, it is quite compatible to have Re $A(\nu,t)$ grow logarithmically, while Im $A(\nu,t)$ approaches a constant. Some authors[17)] have investigated the consequences of this rise in the real part of the amplitude and its final dominance. It will be pointed out that this is again a very academic question. The logarithmic growth is so slow that we are pushed to tremendous energies before that stage

* There is not yet a positively charged meson beam at Serpukhov. In order for $\sigma_T(M^-p) - \sigma_T(M^+p)$ to continue falling like a power, $\sigma_T(M^+p)$ will have to start rising. $\sigma_T(\pi^-n)$, which equals $\sigma_T(\pi^+p)$ by I-spin invariance, does not seem to do this, although there is some uncertainty about the Glauber shadow corrections.

is attained. It is very naive to assume that we understand physics throughout this range, and can extrapolate our results all the way to infinite energies.

One of the nice features of Regge theory and the expansion (1) is that it gives both the magnitude and the phase of the amplitude at high energies in a manner that is compatible with dispersion theory. The reason is, of course, simple. Expression (1) is an analytic function of ν with the correct cut structure. In fact, it was precisely this analyticity which was used to derive the FESR.

What effect does the new asymptotic amplitude have, through analyticity, on the amplitude below the Serpukhov range? The former can be described by means of an additional cut starting at some $\nu = \Lambda$. For positive n there is no change in the FESR for $N < \Lambda$. To see this note that the amplitude is linear in the various cuts contributing to it. Recalling our previous derivation, we see that the new cut starting at $\nu = \Lambda$ cannot contribute to the FESR, since the amplitude it generates is analytic for $|\nu| \leq N < \Lambda$ *.

However, the real part of the amplitude is modified by the new cut. If we are to preserve analyticity, the real part of the amplitude can no longer be described by the Regge approximation (1). Furthermore, breaking the Pomeranchuk theorem necessitates an additional subtraction term in the odd amplitude.

We conclude that Regge theory can still be maintained as a

* For $n < 0$ one has contributions from the singularity at $\nu = 0$. Similarly the new cut affects the continuous moment sum rules[18], which we have not considered.

parameterization of the imaginary part of the amplitude below 30 BeV, but not of the real part. In addition, the positive moment FESR continue to hold.

Horn[19] has come up with a specific suggestion. Following the Freund-Harari hypothesis, he conjectured that possibly we have reached an ionization point beyond which there are no more resonances. The situation above that point is analogous to that in an exotic channel. The imaginary part of the amplitude is purely diffractive. The limitation of this picture is that it has not offered an alternative dynamics. It is merely an intuition, which may serve as a basis for a search of the underlying theory.

We do not attempt such an investigation. In Part II we address ourselves to a simpler question. We take the new Serpukhov data at face value, and look for their effect on the real parts of forward meson-nucleon scattering amplitudes via dispersion relations. It should be emphasized that, in order to evaluate the real part, it is only necessary to speculate on the behavior of the total cross sections up to energies which are, say, an order of magnitude greater. A different extrapolation beyond there does not necessarily affect the dispersion calculation, unless the change is very drastic.

We discuss the phase of the forward $\pi^{\pm}p$ scattering amplitudes, as well as the forward differential cross section of πN charge exchange (CEX). We find an estimate for the upper limit of electromagnetic effects in these amplitudes, and conclude that it is too small to account for the expected difference between $\sigma_T(\pi^- p)$ and $\sigma_T(\pi^+ p)$ at the higher energies. We discuss the fits to available data, and make predictions for future high energy experiments. We treat the $K^{\pm}p$ scattering in a similar way. Although the experimental data on the real parts of the $K^{\pm}p$ amplitudes are not very accurate, they favor the existence of an additional non-Regge term. This could be the subtraction constant needed if the difference between $\sigma_T(K^-p)$ and $\sigma_T(K^+p)$ persists at higher energies.

I. CROSSING AS A GROUP AND ELIMINATION OF EXOTIC CHANNELS

1. General Formalism

Consider meson-meson scattering in which at least three of the particles are identical. The three channels -- s, t, and u -- all transform into each other under crossing. These transformations comprise the symmetric group S_3. The invariant amplitudes $A_i(s,t,u)$ are a basis for a representation of S_3, because crossing symmetry implies a linear relation of the type

$$A_i(p(s,t,u)) = \sum_j D(p^{-1})_{ji} \, A_j(s,t,u) \qquad (1)$$

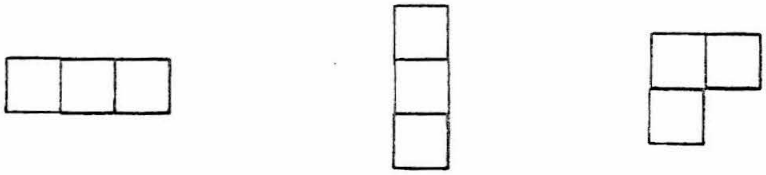
for each permutation $p \in S_3$. Note that the symmetry group is not enlarged if all four particles are identical. A priori the symmetry group is S_4. However, certain crossing transformations do not cross any channels, e.g., the simultaneous interchange of particles A and B and C and D. Thus this subgroup of transformations, D_2, is always represented by the identity transformation on the invariant amplitudes. We are interested only in investigating inequivalent transformations, and therefore consider only the quotient group S_4/D_2, which is S_3. To put it more physically: you cannot do better than permute all three channels. Note, however, that this equivalence subgroup is a special feature of the four-point function. It does not exist for the general n-point function.

As pointed out before, in order to explore the consequence of crossing symmetry, one has to invoke analyticity. We write each invariant amplitude in a Mandelstam form[10)]

$$A(s,t,u) = F_1(s,t) + F_2(t,u) + F_3(u,s)$$
$$+ a_1(s,t-u) + a_2(t,u-s) + a_3(u,s-t). \qquad (2)$$

The a-type functions have a cut from threshold to infinity in the first variable, and are entire in the second. The F-type functions have the cuts in both variables, but we have subtracted from them all parts that can be stacked on to the a-type functions. There is a question whether the decomposition (2) is unique. We have not been able to find the weakest assumptions needed to imply this, but guess that they may be rather general, because of the different cut structure of each function in the product of the complex s and t planes. The decomposition is certainly unique if each function can be written in the form of a dispersion integral. Without further ado, we accept the assumption of uniqueness. We confine our attention to F-type functions only. Our work carries through whether a-type functions are added or not. Note,however, that the latter can give only fixed poles in the j-plane.

It is worth looking for the most general basis of F-type functions for the three irreducible representations of S_3. We will identify these representations by means of their Young tableaux.

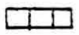
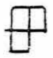


We refer the reader to Ref. 20 for a review of the properties of finite groups and their representations, with particular emphasis on S_3. We use the cycle notation to refer to elements of S_n. A cycle $(a_1 a_2 \dots a_m)$ in S_n is a chain of permutations as follows: put a_1 in the position of a_2, a_2 in that of a_3, and so on, and finally a_m back in that of a_1. Each permutation is a combination of cycles. We do not write one-element cycles. Thus (st) is a shorthand notation for (st)(u).

In explicitly reducing a representation it is useful to consider the generators of the group. These are a subset of the group generating all other elements by closure. Therefore, a representation is completely determined by the mapping of the generators on matrices. S_3 is generated by two elements (not any two!), say, (st) and (su). For the sake of a unified notation we envelop all three irreducible representations of S_3 in a single reducible four-dimensional one, thus defining "standard" representations. We map the generators as follows

$$D((st)) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \qquad D((su)) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & \bar{w} \\ 0 & 0 & w & 0 \end{pmatrix} \qquad (3)$$

where $w = e^{2\pi i/3}$ and $\bar{w} = w^2 = w^{-1}$. It is seen that the spaces spanned by the basis vectors $\{\epsilon_1\}$, $\{\epsilon_2\}$ and $\{\epsilon_3, \epsilon_4\}$ transform under S_3 as

  and  , respectively.

We write each basis vector ϵ_i in the Mandelstam form (2), using only F-type functions. When we apply the generators (3) we find relations among F-type functions of different variables. For instance, let

$$\epsilon_1 = F_1(s,t) + F_2(t,u) + F_3(u,s) \qquad (4)$$

From (3) we know

$$\epsilon_1 = (st)\epsilon_1 = F_1(t,s) + F_2(s,u) + F_3(u,t)$$
$$= (su)\epsilon_1 = F_1(u,t) + F_2(t,s) + F_3(s,u). \qquad (5)$$

The uniqueness of the Mandelstam decomposition allows only one solution

$$F_1 = F_2 = F_3 \quad \text{and} \quad F_1(x,y) = F_1(y,x) \qquad (6)$$

Other basis vectors are evaluated in the same way. The result is

$$\begin{aligned}
\epsilon_1 &= f_1(s,t) + f_1(t,u) + f_1(u,s) \\
\epsilon_2 &= g_1(s,t) + g_1(t,u) + g_1(u,s) \\
\epsilon_3 &= f_2(s,t) + w\, f_2(t,u) + \bar{w}\, f_2(u,s) \\
&\quad + g_2(s,t) + w\, g_2(t,u) + \bar{w}\, g_2(u,s) \qquad (7)\\
\epsilon_4 &= f_2(s,t) + \bar{w}\, f_2(t,u) + w\, f_2(u,s) \\
&\quad - g_2(s,t) - \bar{w}\, g_2(t,u) - w\, g_2(u,s)
\end{aligned}$$

where f_i and g_i are symmetric and antisymmetric, respectively, under the interchange of their arguments, but otherwise arbitrary F-type functions. A similar analysis can be done also for the a-type functions.

The invariant amplitudes A_i will in general form a reducible representation of S_3. We reduce them explicitly, i.e., express them as linear combinations of the ϵ_i. Using (7) we then have the most general F-function form for the A_i. They will obey crossing symmetry by construction. In the next two sections we use this technique to study I-spin invariant $\pi\pi$ scattering and SU(3) invariant pseudoscalar-pseudoscalar scattering.

2. $\pi\pi$ Scattering

As a first application we look at $\pi\pi \to \pi\pi$. The most general SU(2) invariant amplitude is written[21)]

$$M_{\gamma\delta,\alpha\beta}(s,t,u) = A_1(s,t,u)\delta_{\alpha\beta}\delta_{\gamma\delta} + A_2(s,t,u)\delta_{\alpha\gamma}\delta_{\delta\beta} + A_3(s,t,u)\delta_{\alpha\delta}\delta_{\beta\gamma} \tag{8}$$

where $\alpha,\ldots,\delta$ are the SU(2) indices of the π's in the usual way. Crossing symmetry is seen to ensue (see Fig. 1):

1) under the interchange of lines B and C

$$\begin{aligned} A_1(t,s,u) &= A_2(s,t,u) \\ A_2(t,s,u) &= A_1(s,t,u) \\ A_3(t,s,u) &= A_3(s,t,u) \end{aligned} \tag{9}$$

2) under the interchange of lines B and D

$$\begin{aligned} A_1(u,t,s) &= A_3(s,t,u) \\ A_2(u,t,s) &= A_2(s,t,u) \\ A_3(u,t,s) &= A_1(s,t,u) \end{aligned} \tag{10}$$

The two transformations correspond, of course, to our generators (st) and (su). By the definition (1) we therefore have

$$D((st)) = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad D((su)) = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \tag{11}$$

Note that (1) defines $D(p^{-1})$. In our case both generators are equal to their inverses. From the group structure we know

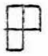
$$D((sut)) = D((st)(su)) = D((st))D((su)) \cdot \qquad (12)$$

Therefore

$$D((sut)) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \qquad (13)$$

The characters of the classes are thus

$$\chi(C_1) = 3 \quad \chi(C_2) = 0 \quad \chi(C_3) = 1 \cdot \qquad (14)$$

Using the orthogonality law[22], this representation is seen to reduce to  +  . There is no general prescription for explicit reduction. We know that A_i are linear combinations of ϵ_1, ϵ_3 and ϵ_4, and look for the right coefficients to have the A_i transform like (9) and (10). The result is

$$\begin{aligned} A_1 &= \epsilon_1 + \bar{w}\,\epsilon_3 + w\,\epsilon_4 \\ A_2 &= \epsilon_1 + w\,\epsilon_3 + \bar{w}\,\epsilon_4 \\ A_3 &= \epsilon_1 + \epsilon_3 + \epsilon_4 \cdot \end{aligned} \qquad (15)$$

Using (7) one can therefore write the most general F-function form for the A_i. After slightly renormalizing the arbitrary functions, one gets

$$\begin{aligned} A_1(s,t,u) &= A_2(u,s,t) = A_3(t,u,s) \\ &= f_1(s,t) + f_1(t,u) + f_1(u,s) + f_2(s,t) + f_2(u,s) - 2f_2(t,u) \\ &\quad + g_2(s,t) - g_2(u,s) \end{aligned} \qquad (16)$$

This representation in itself is not very interesting, since we have described three functions by three other ones. However, elimination of the right-hand cut in the I = 2 channel is very restrictive. The projection operators for the SU(2) representations in a given channel, say the s, are:

$$\Lambda^{I\,=\,2}_{\gamma\delta,\alpha\beta} = \frac{1}{2}\,(\delta_{\alpha\gamma}\delta_{\delta\beta} + \delta_{\alpha\delta}\delta_{\beta\gamma}) - \frac{1}{3}\,\delta_{\alpha\beta}\delta_{\gamma\delta}$$

$$\Lambda^{I\,=\,1}_{\gamma\delta,\alpha\beta} = \frac{1}{2}\,(\delta_{\alpha\gamma}\delta_{\delta\beta} - \delta_{\alpha\delta}\delta_{\beta\gamma}) \tag{17}$$

$$\Lambda^{I\,=\,0}_{\gamma\delta,\alpha\beta} = \frac{1}{3}\,\delta_{\alpha\beta}\delta_{\gamma\delta}.$$

The amplitude thus decomposes as follows

$$M_{\gamma\delta,\alpha\beta} = (A_2 + A_3)\,\Lambda^{I\,=\,2}_{\gamma\delta,\alpha\beta} + (A_2 - A_3)\,\Lambda^{I\,=\,1}_{\gamma\delta,\alpha\beta}$$
$$+ (3A_1 + A_2 + A_3)\,\Lambda^{I\,=\,0}_{\gamma\delta,\alpha\beta}\;. \tag{18}$$

The amplitude to scatter in a given I spin is therefore

$$A^{s}_{I\,=\,2} = A_2 + A_3$$
$$A^{s}_{I\,=\,1} = A_2 - A_3 \tag{19}$$
$$A^{s}_{I\,=\,0} = 3A_1 + A_2 + A_3.$$

According to our assumption $A^{s}_{I\,=\,2}$ is real for physical s, i.e.,

$$A^{s}_{I\,=\,2} = F(t,u)\,. \tag{20}$$

Substituting (16) into (19) we find that (20) implies

$$2f_1 - f_2 - g_2 = 0\,. \tag{21}$$

But the symmetric and antisymmetric amplitudes vanish separately, so that

$$2f_1 = f_2 \quad \text{and} \quad g_2 = 0 \cdot \tag{22}$$

The amplitude is thus described by a single symmetric function. We get the known result[23)]

$$\begin{aligned} A^s_{I\,=\,2} &= f(t,u) \\ A^s_{I\,=\,1} &= f(s,t) - f(u,s) \\ A^s_{I\,=\,0} &= \frac{3}{2}\,[f(s,t) + f(u,s)] - \frac{1}{2}\,f(t,u). \end{aligned} \tag{23}$$

As it stands (23) does not have much physical content. This comes about only when a dynamical statement is made about f(s,t). Exchange degeneracy is derived when Regge asymptotics are introduced. For large x and finite y

$$f(x,y) = -\,\beta(y)\,\frac{1}{\Gamma(\alpha(y)+1)\,\sin\pi\alpha(y)}\,(-\nu_y)^{\alpha(y)} \tag{24}$$

Thus at high t (or u) and finite s

$$\begin{aligned} A^s_{I\,=\,1} &= \frac{-\,\beta(s)}{\Gamma(\alpha(s)+1)\,\sin\pi\alpha(s)}\left[\left(\frac{u-t}{4\mu}\right)^{\alpha(s)} - \left(\frac{t-u}{4\mu}\right)^{\alpha(s)}\right] \\ A^s_{I\,=\,0} &= \frac{-\,3\beta(s)/2}{\Gamma(\alpha(s)+1)\,\sin\pi\alpha(s)}\left[\left(\frac{u-t}{4\mu}\right)^{\alpha(s)} + \left(\frac{t-u}{4\mu}\right)^{\alpha(s)}\right] \end{aligned} \tag{25}$$

f(x,y) is presumably very small when both x and y are very large.

Secondly, note that the I = 1 amplitude has no contribution from a third double spectral function due to the vanishing of $g_2(t,u)$. Mandelstam's well known result about cuts in the j-plane[8)] thus precludes one for this amplitude. Fixed poles at nonsense-wrong-signature points[24] arise only when the external particles have spin. The same conclusion about the third double spectral function holds in $\rho\rho$ scattering, and we expect no fixed pole there.

3. Pseudoscalar-Pseudoscalar Scattering

A similar analysis can be carried out for SU(3) invariant amplitudes. We seek to write the SU(3) couplings in a manner that exhibits transparent crossing features. There are eight independent couplings in $\underset{\sim}{8} \times \underset{\sim}{8} \rightarrow \underset{\sim}{8} \times \underset{\sim}{8}$. We choose to consider nine couplings.

1) Three f·f type $\quad f_{\alpha\beta}\cdot f_{\gamma\delta}$, $f_{\alpha\gamma}\cdot f_{\delta\beta}$ and $f_{\alpha\delta}\cdot f_{\beta\gamma}$

2) Three d·d type $\quad d_{\alpha\beta}\cdot d_{\gamma\delta}$, $d_{\alpha\gamma}\cdot d_{\delta\beta}$ and $d_{\alpha\delta}\cdot d_{\beta\gamma}$

s) Three d·f type $\quad d_{\alpha\beta}\cdot f_{\gamma\delta}$. $d_{\alpha\gamma}\cdot f_{\delta\beta}$ and $d_{\alpha\delta}\cdot f_{\beta\gamma}$

$\alpha,\ldots,\delta$ are the octet indices of the external particles. The couplings are scalar products of the usual f and d tensors of the SU(3) algebra, e.g.

$$f_{\alpha\beta}\cdot f_{\gamma\delta} = \sum_{m=1}^{8} f_{\alpha\beta m} f_{\gamma\delta m} = \sum_{m=1}^{8} \langle[\lambda_\alpha,\lambda_\beta]\lambda_m\rangle\langle[\lambda_\gamma,\lambda_\delta]\lambda_m\rangle^* \tag{26}$$

We use Gell-Mann's notation[25], and $\langle\ \rangle$ stands for 1/4 Tr. Note that we can also write

$$f_{\alpha\beta}\cdot f_{\gamma\delta} = -\frac{1}{2}\langle\lambda_\alpha[\lambda_\beta,[\lambda_\gamma,\lambda_\delta]]\rangle\ . \tag{27}$$

The Jacobi identity

$$[\lambda_\beta,[\lambda_\gamma,\lambda_\delta]] + [\lambda_\gamma,[\lambda_\delta,\lambda_\beta]] + [\lambda_\delta,[\lambda_\beta,\lambda_\gamma]] = 0 \tag{28}$$

thus implies

$$f_{\alpha\beta}\cdot f_{\gamma\delta} + f_{\alpha\gamma}\cdot f_{\delta\beta} + f_{\alpha\delta}\cdot f_{\beta\gamma} = 0. \tag{29}$$

This is the desired linear relation among the nine couplings, which are otherwise independent. Our couplings clearly treat all three channels on an equal footing. Instead of considering all intermediate representations in one channel, we look at octets only, but in all three channels. The projection operators for SU(3) representations in a given channel were worked out by Cutkosky[26]. We list his results in the Appendix, and connect between them and Rosner's quark diagrams[27].

Under certain circumstances some couplings are known not to contribute. In pseudoscalar-pseudoscalar scattering d·f couplings are forbidden. (In PP $\to$ PV only d·f couplings are allowed.) The most general SU(3) invariant amplitude is written

$$\begin{aligned} M_{\gamma\delta,\alpha\beta}(s,t,u) &= A_1(s,t,u)d_{\alpha\beta}\cdot d_{\gamma\delta} + A_2(s,t,u)d_{\alpha\gamma}\cdot d_{\delta\beta} \\ &+ A_3(s,t,u)d_{\alpha\delta}\cdot d_{\beta\gamma} + B_1(s,t,u)f_{\alpha\beta}\cdot f_{\gamma\delta} \\ &+ B_2(s,t,u)f_{\alpha\gamma}\cdot f_{\delta\beta} + B_3(s,t,u)f_{\alpha\delta}\cdot f_{\beta\gamma} . \end{aligned} \tag{30}$$

The linear relation (29) between the f·f couplings implies an arbitrariness in the B_i. We remove this by the constraint

$$B_1 + B_2 + B_3 = 0 \tag{31}$$

The A_i have the same crossing properties as in the SU(2) case. The B_i transform as follows:

1) under the interchange of lines B and C

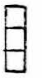
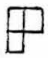
$$B_1(t,s,u) = - B_2(s,t,u)$$

$$B_2(t,s,u) = - B_1(s,t,u)$$

$$B_3(t,s,u) = - B_3(s,t,u) \tag{32}$$

2) under the interchange of lines B and D:

$$\begin{aligned} B_1(u,t,s) &= - B_3(s,t,u) \\ B_2(u,t,s) &= - B_2(s,t,u) \\ B_3(u,t,s) &= - B_1(s,t,u) \end{aligned} \tag{33}$$

A simple analysis, analogous to that done for the A_i, shows that the B_i span a vector space which reduces to  +  . But  is just $B_1 + B_2 + B_3$, which we have constrained to be zero. Explicit reduction gives

$$\begin{aligned} B_1 &= w\,\epsilon_3 - \bar{w}\,\epsilon_4 \\ B_2 &= \bar{w}\,\epsilon_3 - w\,\epsilon_4 \\ B_3 &= \epsilon_3 - \epsilon_4 . \end{aligned} \tag{34}$$

One can now write the most general F-function form of the invariant amplitudes, and explore the consequences of eliminating exotic $\underline{10}$, $\underline{\overline{10}}$ and $\underline{27}$ channels. This is done precisely as in the SU(2) case. The result is that all invariant amplitudes are again describable by a single symmetric function

$$\begin{aligned} A_1(s,t,u) &= A_2(s,t,u) = A_3(s,t,u) \\ &= f(s,t) + f(t,u) + f(u,s) \\ B_1(s,t,u) &= \frac{2}{3}\,[f(s,t) - f(u,s)] \\ B_2(s,t,u) &= \frac{2}{3}\,[f(t,u) - f(s,t)] \\ B_3(s,t,u) &= \frac{2}{3}\,[f(u,s) - f(t,u)] \end{aligned} \tag{35}$$

Leaving out contributions of the third double spectral function, we get the following couplings in the non-exotic channels

$$
\begin{aligned}
M_{\gamma\delta,\alpha\beta}(s,t,u)_{\underset{\sim}{8}_s} &= [f(s,t) + f(u,s)]\; d_{\alpha\beta}\cdot d_{\gamma\delta} \\
M_{\gamma\delta,\alpha\beta}(s,t,u)_{\underset{\sim}{8}_a} &= [f(s,t) - f(u,s)]\; f_{\alpha\beta}\cdot f_{\gamma\delta} \qquad (36) \\
M_{\gamma\delta,\alpha\beta}(s,t,u)_{\underset{\sim}{1}} &= [f(s,t) + f(u,s)]\; d_{\alpha\beta 0} d_{\gamma\delta 0}
\end{aligned}
$$

where

$$d_{\alpha\beta 0} = \langle \{\lambda_\alpha, \lambda_\beta\}\lambda_0 \rangle = \sqrt{\frac{2}{3}}\;\delta_{\alpha\beta} \qquad (37)$$

and λ_0 is defined following Gell-Mann[25)]

$$\lambda_0 = \sqrt{\frac{2}{3}}\; 1\,. \qquad (38)$$

Wherever $f(s,t) + f(u,s)$ has a pole in s, we will have a degenerate octet and singlet forming a nonet. Thus all even-j resonances appear in nonets. The couplings of the octet and the singlet can be combined to give

$$M_{\gamma\delta,\alpha\beta}(s,t,u)_{\underset{\sim}{9}} = [f(s,t) + f(u,s)] \sum_{m=0}^{8} \langle \{\lambda_\alpha, \lambda_\beta\}\lambda_m \rangle \langle \{\lambda_\gamma, \lambda_\delta\}\lambda_m \rangle^* \qquad (39)$$

They are seen to be those of the quark model. In particular we have Zweig's connectedness rule[5)] which implies that the state made of $\bar{\lambda}\lambda$ quarks does not decay into pions. No dynamical statement concerning the form of $f(s,t)$ was made here to derive this result.

One can again use (24) to get exchange degeneracy in Regge theory. Our amplitude then becomes identical to that of Rosner[27)]. Note that the odd signature amplitude again cannot have a third double spectral function, since f(t,u) is symmetric in t and u. Thus the previous result in the case of SU(2) generalizes to the entire octet.

4. The N-Point Function

In discussing the four-point function, we saw that the amplitude was in general a mixture of representations of S_3. Elimination of exotics did not change this feature. One expects the same to hold for the n-point function. Although we used the full S_3 symmetry as a tool of investigation, our results really referred only to a given channel. The singlet and the symmetric octet turn out to be degenerate because they have the same symmetry pattern, viz. they are both symmetric under the interchange of the two external octets into which they decay. Similarly there is exchange degeneracy between the representations having one symmetry and those having the other one.

In principle our method can be generalized to an arbitrary n-point function, and use made of higher symmetric groups. However, we do not expect new physics to come out of such an approach. To illustrate this point, consider the A_2 decay into three pions. It is dominated by the $\rho\pi$ mode[28)]. The states formed from three I = 1 pions are given by the following decomposition

$$1 \times 1 \times 1 = 3 + 2(2) + 1(3) + 0 \qquad (40)$$

We are looking for a mode with total I = 1 and with two of the three pions also in an I = 1 state. There are two such modes. However, they are physically equivalent. The Pauli principle (or crossing symmetry if we are dealing with an A_2 exchange) allows only a definite combination of the two modes.

The physics is contained in the observation that the decay seems to proceed via a two particle intermediate state. When one of the latter decays, it no longer remembers the resonance from which it originated. There may be some final state interaction, but it should be viewed as a small perturbation of the two-particle decay scheme. Thus all n-point functions effectively reduce to a series of four-point functions. In particular no new information can be found about the consequences of eliminating exotic channels.

It may be remarked in passing that, if one attempts to describe baryons as bound states of physical quarks[29)], larger symmetry groups are called in. This is because two quarks do not form strongly bound subsystems. The interaction is a three body force. It is not clear that one should believe in physical quarks. At any rate, we are very far from the sophistication of assigning analytic properties to such an interaction.

II. REAL PARTS OF FORWARD MESON-NUCLEON SCATTERING AMPLITUDES

5. General Formalism

We use dispersion relations to analyze a forward scattering amplitude, whose discontinuity is determined by total cross sections of two channels related by crossing (e.g., π^+p and π^-p or K^+p and K^-p). We refer the reader to Ref. 30 for the conventional formulation of dispersion relations and previous calculations. One usually separates the symmetric amplitude $A^{(+)} = \frac{1}{2}[A(\pi^-p) + A(\pi^+p)]$ from the antisymmetric one $A^{(-)} = \frac{1}{2}[A(\pi^-p) - A(\pi^+p)]$, and writes the dispersion relations

$$A^{(+)}(\nu) = A^{(+)}(\mu) + \frac{f^2k^2}{M\left[1 - \left(\frac{\mu}{2M}\right)^2\right]\left[\nu^2 - \left(\frac{\mu^2}{2M}\right)^2\right]} + \frac{k^2}{2\pi^2}\int_\mu^\infty d\nu' \frac{\nu'\,\sigma^{(+)}(\nu')}{k'(\nu'^2 - \nu^2 - i\epsilon)} , \tag{1}$$

$$A^{(-)}(\nu) = \frac{2f^2\,\nu}{\nu^2 - \left(\frac{\mu^2}{2M}\right)^2} + \frac{\nu}{2\pi^2}\int_\mu^\infty d\nu' \frac{k'\,\sigma^{(-)}(\nu')}{\nu'^2 - \nu^2 - i\epsilon} . \tag{2}$$

M is the nucleon mass and μ the meson mass. $\sigma^{(\pm)} = \frac{1}{2}[\sigma_T(\pi^-p) \pm \sigma_T(\pi^+p)]$ ν and k are the meson's laboratory energy and momentum, respectively. f^2 specifies the strength of the Born term, and is equal to 0.082. $A^{(+)}(\nu = \mu)$ is the only subtraction constant. It is known to be zero within experimental errors, in agreement with Adler's PCAC self-consistency condition[31]. In writing (2), one obviously makes the assumption that $\sigma^{(-)}$ goes asymptotically to zero. This is the point which we now want to change. Following the approach of Ref. 19, we assume that both $\sigma_T(\pi^- p)$ and $\sigma_T(\pi^+ p)$ remain constant from about 30 BeV on. This then implies that they have different values, and $\sigma^{(-)}$ is a non-zero constant. We want to see what the predictions of these assumptions for the real part are. Having to introduce a subtraction into (2), we therefore replace it by

$$A^{(-)}(\nu) = \frac{2f^2\,\nu}{\nu^2 - \left(\frac{\mu^2}{2M}\right)^2} + \frac{\nu}{2\pi^2}\int_{\mu}^{\kappa} d\nu' \, \frac{k'\,\sigma^{(-)}(\nu')}{\nu'^2 - \nu^2 - i\epsilon}$$
$$+ \frac{\nu k^2}{2\pi^2}\int_{\kappa}^{\infty} d\nu' \, \frac{\sigma^{(-)}(\nu')}{k'(\nu'^2 - \nu^2 - i\epsilon)} + c\,\frac{\nu}{M^2} \quad . \tag{3}$$

Note that, instead of performing a subtraction on the entire integral, we divide it into two parts. One is written in an unsubtracted form, and the other in a subtracted one. This is done for practical purposes. It avoids stressing the low-energy input and thus increasing the errors in the calculation. The number c depends on the choice of κ. Equation (3) also demonstrates the fact that the real part at low energy is not necessarily affected by the new assumptions on the high-energy behavior. We are actually able to reproduce at low energies (say, below 4 BeV) the same results previously obtained by the use of (2) with any reasonably decreasing fit to $\sigma^{(-)}$.

To illustrate the changes brought about by the assumptions on the behavior of the total cross section, let us discuss a mathematical example that is very similar to the actual situation in $K^{\pm}p$. Let us denote the two reactions in question by A and B (analogous to $K^{+}p$ and $K^{-}p$, respectively). Assume first that (case I):

$$\text{Im } A_{I} = a\,\nu, \quad \text{Im } B_{I} = a\,\nu + b\sqrt{\nu}, \qquad 0 < \nu < \infty. \tag{4}$$

It is then easy to find that

$$\text{Re } A_{I} = -\,b\sqrt{\nu}, \quad \text{Re } B_{I} = 0\ . \tag{5}$$

This is the expected result for $K^{\pm}p$ if one uses a Regge representation with a regular Pomeron and two pairs of exchange degenerate trajectories with intercepts at 1/2. If we now make the analogous assumption to that of Ref. 19, we have (case II):

$$\text{Im } A_{II} = a\,\nu, \quad \text{Im } B_{II} = \begin{cases} a\,\nu + b\sqrt{\nu} & 0 < \nu < \Lambda \\ (a + \frac{b}{\sqrt{\Lambda}})\,\nu & \Lambda < \nu < \infty \end{cases}\ . \tag{6}$$

It is then readily established that

$$\text{Re } A_{II} = -\,\frac{2b\sqrt{\nu}}{\pi}\arctan\sqrt{\frac{\Lambda}{\nu}} + \frac{b\nu}{\pi\sqrt{\Lambda}}\log|\nu + \Lambda| - \frac{c}{M^2}\,\nu\ ,$$

$$\text{Re } B_{II} = \frac{b\sqrt{\nu}}{\pi}\log\left|\frac{\sqrt{\nu} - \sqrt{\Lambda}}{\sqrt{\nu} + \sqrt{\Lambda}}\right| - \frac{b\nu}{\pi\sqrt{\Lambda}}\log|\nu - \Lambda| + \frac{c}{M^2}\,\nu\ . \tag{7}$$

It is now interesting to note that although Eqs. (5) and (7) are very different from one another, it is still possible to find a value of c that will show a similar behavior for low ν. Thus it is possible that even though $\text{Im } B_{I} \neq \text{Im } B_{II}$ for $\nu > \Lambda$, one still finds that the real parts of the various amplitudes can roughly agree for $\nu < \Lambda$.

To illustrate this point numerically, we choose a = b = 3.6, $\Lambda = 22$. (These values are close to those indicated by experiment if

ν is measured in BeV and the amplitudes in BeV^{-1}.) We find such an agreement between I and II for c = 1.1. We present in Fig. 3 the results for $\alpha(A) = \text{Re } A/\text{Im } A$ and $\alpha(B) = \text{Re } B/\text{Im } B$, since this is the customary way in which the data are given in πN experiments*.

Note that after the value $\nu \simeq 100$ the logarithmic part in Re A_{II} and Re B_{II} is taking over. Nevertheless it does not reach a sizable amount even at high ν values. To quote a number -- at $\nu = 10^6$ we find $\alpha(B_{II}) = -0.49$ and $\alpha(A_{II}) = 0.59$. We will find a similar behavior in the next section when discussing the πN problem.

6. Real Parts of $\pi^{\pm}p$ Amplitudes

In Ref. 19, the $\pi^{\pm}p$ total cross sections were fitted to a form

$$\sigma_{\pm} = a_{\pm} + b_{\pm}/\sqrt{\nu} \quad . \tag{8}$$

An ionization point was then assumed to appear at $\nu = 30$ BeV, resulting in the flattening off of the cross sections at that point. This meant that $2\sigma^{(-)} = \sigma(\pi^- p) - \sigma(\pi^+ p) > 1$ mb even at high energies. In Ref. 19, $\sigma^{(-)}$ was assumed to remain a constant for $\nu \geq 30$ BeV.

Any breaking of the Pomeranchuk theorem results in a logarithmic rise of the real part of the amplitude, notably of $A^{(-)}(\nu)$[17,19]. Hence, $\alpha_{\pm}(\nu) = \text{Re } A_{\pm}(\nu)/\text{Im } A_{\pm}(\nu)$ does not tend to 0 as $\nu \to \infty$. Once the logarithmic behavior begins to dominate, α rises in absolute value, with α_+ and α_- taking opposite signs. The strength of the

* It is unfortunate that the letter α is used for this ratio. It is not to be confused with a Regge trajectory.

logarithmic term is proportional to the value of $\sigma^{(-)}$.

The dispersion integrals were evaluated on a computer. In order to do the principal part integration, it is necessary to have a smooth fit to the data points, since the integral is sensitive to discontinuities near $\nu' = \nu$*. For $\nu \leq 4$ BeV we used the fit of Ref. 30. The data between 4 and 30 BeV[33,34] can be fitted in a variety of ways. We first fitted each cross section separately to a form

$$\sigma_{\pm} = a_{\pm} + b_{\pm}\ \nu^{n-1}. \tag{9}$$

In such fits, $a_{-} - a_{+}$ was invariably greater than 1 mb, and the choice of n was a matter of taste. We then tried a fit satisfying the Pomeranchuk theorem,

$$\sigma_{\pm} = a + b\ \nu^{n-1} \pm c\ \nu^{m-1}. \tag{10}$$

This was done in order to be able to compare the premise of a cutoff with the assumption that the Serpukhov data might be wrong, and that the Pomeranchuk theorem might be right after all.

---

* The principal part integral is performed as follows[32]:

$$P\int_A^B dx\ \frac{f(x)}{x-y} = \int_A^B dx\ \frac{f(x)-f(y)}{x-y} + f(y)\ P\int_A^B \frac{dx}{x-y}$$

$$= \int_A^B dx\ \frac{f(x)-f(y)}{x-y} + f(y)\ \log\left|\frac{B-y}{A-y}\right| .$$

The first integral one has a regular integrand. If $A \leq y \leq B$ one substitutes $f'(y)$ at $x = y$. This is easy if $f(x)$ is given in functional form. If only discrete values are known, it is advisable to have them equally spaced in x, so that one can use Simpson's formula. To that approximation

$$f'(x_n) = \frac{f(x_{n+1}) - f(x_{n-1})}{x_{n+1} - x_{n-1}} .$$

The data of Citron et al.[33)] do not seem to fit smoothly to those of Foley et al.[34)]. We had to settle for a slightly low value of n. We chose

$$n = 0.25, \qquad m = 0.6$$

$$a = 22.5 \qquad b = 18.9 \qquad c = -2.45$$

where ν is measured in BeV and σ in mb. Applying to fit (10) a cutoff at 30 BeV, we got for ν above cutoff

$$2\sigma^{(-)} = \sigma(\pi^-p) - \sigma(\pi^+p) = 1.3 \text{ mb} .$$

This number is consistent with the result of Ref. 14. In doing the same with fit (9), we got $2\sigma^{(-)}$ above cutoff to depend on the fit. $\sigma(\pi^-p)$ is, of course, determined by the Serpukhov data, but there is a slight freedom of play in $\sigma(\pi^+p)$. First one has to choose the cutoff point. We assumed it to be the same as in π^-p (30 BeV). Since this is 8 BeV higher than the last data point, the extrapolation depends on the fit. It was pointed out above that the strength of the logarithmic term in the real part of the amplitude is proportional to $\sigma^{(-)}$. If we constrained fit (9) to satisfy $2\sigma^{(-)} = 1.3$ mb, our numerical dispersion calculations with it gave the same results for the real parts as fit (10). We adopted the latter for the purpose of testing the sensitivity of the calculation to the possible breaking of the Pomeranchuk theorem. We called case I that which assumes (10) to be good for all ν. In case II we applied the cutoff, so that for $\nu \geq 30$ BeV both cross sections were constant. The two cases are illustrated in Fig. 4. Note that if further structure appears in σ_T at much higher energies, it may have negligible effects on our calculation.

The calculated ratios $\alpha_{\pm}(\nu) = \mathrm{Re}\, A_{\pm}(\nu)/\mathrm{Im}\, A_{\pm}(\nu)$ for the $\pi^{\pm}p$ amplitudes are plotted in Fig. 5, together with the data[35]. In case I there is no free parameter in the dispersion relations (1) and (2). In case II there is the arbitrariness of c in (3), which can be chosen to best fit the data. (We used $\kappa = 4$ BeV.)

If one assumes exact charge independence, one can evaluate the forward CEX differential cross section. The predictions are plotted together with the data[36] in Fig. 6 and Fig. 7. We note that in case I the prediction seems to be too high by about 30 percent at, say, 20 BeV. If we attribute the discrepancy to I-spin violation of the electromagnetic amplitude, we find it to be 20 percent of the total $A^{(-)}$ amplitude. With $2\sigma^{(-)}(\nu = 20) \sim 1.5$ mb, we would thus have $2\sigma_{EM}^{(-)} \lesssim 0.3$ mb. Since we do not expect the electromagnetic effects to vary strongly with energy, we may conclude that the ansatz of the Pomeranchuk theorem is good only up to $2\sigma^{(-)}(\infty) \lesssim 0.3$ mb.

In case II we can adjust c so as to get a very good fit to the CEX data (c = 0.35). Alternatively, we can fix c to fit the $\alpha_{\pm}$ data. Choosing here c = 0.35, we find a good fit to α_{+} but a poor one to α_{-}. This is an improvement over case I. A change to c = 0.25 results in an equivalent over-all fit to $\alpha_{\pm}$, with a poorer fit to α_{+} and a better one to α_{-}. Note that such a change contributes oppositely to α_{+} and α_{-}. Checking the CEX prediction with c = 0.25, we find it too low by about 40 percent. This corresponds to $2\sigma_{EM}^{(-)} \lesssim 0.5$ mb.

Note that the small deviations that we found are a feature of our calculated real parts. Point by point, the experimental $\alpha_{\pm}(\nu)$, within their errors, are consistent with the CEX data without any I-spin violation. This was already pointed out by Foley et al.[35]. Although we can fit the data with no I-spin breaking, we cannot rule out $2\sigma_{EM}^{(-)} \lesssim 0.5$ mb. However, this is still too small to account for the expected constant difference between $\sigma_T(\pi^- p)$ and $\sigma_T(\pi^+ p)$. We have to conclude, then, that this difference is a genuine strong interaction effect.

The main difference between the two dispersion calculations I and II sets in around 100 BeV. At that point, the logarithmic part of Re $A^{(-)}$ in case II begins to dominate. Instead of going to zero, $\alpha_+(\nu)$ becomes positive and increases, while $\alpha_-(\nu)$ turns over and becomes more negative. The CEX forward cross section begins to rise again. On an absolute scale, both effects are small. We should be able to see the CEX forward cross section flattening, but for the real part to dominate the amplitude we will need fantastically high energies. By that time, a new physics may very well set in. It was pointed out in Ref. 19, as well as in Ref. 17, that if Re A/Im A grows logarithmically, then one has to have the forward elastic peak shrink like $\log^2 s$ to avoid a conflict with unitarity. Strictly speaking, such a conflict would arise only at such large values of ν that the whole problem looks rather academic. Nevertheless, the same conclusion about the shrinkage arises of course from the assumption that σ_{el} does not rise with energy, which might very well be the case.

Finally, a word about errors and low-energy behavior. The cross sections are accurate to about 1 percent. This leads to errors of approximately ± 0.003 in $\alpha_{\pm}(\nu)$. A change in $\sigma^{(-)}$ above cutoff causes a bigger correction. Varying the high-energy cross sections above 30 BeV does not change the low-energy ($\nu \leq 4$ BeV) dispersion calculations. There, our results agree with those of Ref. 30.

7. Real Parts of $K^{\pm}p$ Amplitudes

We calculated the real parts of $K^{\pm}p$ forward scattering amplitudes in the same way as for $\pi^{\pm}p$. The data between threshold and $\nu = 3.3$ BeV[37)] were slightly smoothed. Above that point, the following fit was made:

$$\sigma(K^{+}p) = a \quad , \qquad (11)$$

$$\sigma(K^{-}p) = a + b/\sqrt{\nu} \quad .$$

$$a = 17.2 \qquad b = 17.4$$

The dispersion relations were evaluated for cases I and II as in πp, with the cutoff in case II taken at 20 BeV. The errors involved here are much bigger than in πp. The uncertainties in the subthreshold singularities do not allow a good determination of the real parts at low energies. In particular, the $Y^{*}(1405)$ is an S-wave, and thus is not quenched kinematically. We estimate its effect to be six times as big as the Born term in πN.* This would be approximately 5 - 10 percent of the real part at $\nu = 5$ BeV. An additional unknown is the

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* We approximated the resonance by a pole. The coupling $g^{2}/4\pi = 0.32$ is estimated from the Dalitz-Tuan model[38)].

subtraction term of the symmetric amplitude, $A^{(+)}(\nu = \mu)$. However, their combined effect remains constant, while the imaginary part grows like ν, so that their contribution to $\alpha(K^{\pm}p)$ should fall like $1/\nu$. In case II there is the further difficulty of evaluating the subtraction constant c in the antisymmetric amplitude $A^{(-)}$. The CEX reactions are not related by a simple I-spin rotation. Nor has a direct experimental determination of $\alpha_{\pm}(\nu)$ by Coulomb interference been done. The only existing test is the forward elastic differential cross section. This is a measurement of $1 + \alpha^2$.

$$\left.\frac{d\sigma}{dt}\right|_{t = 0} = \frac{\sigma_T^2}{16\pi}(1 + \alpha^2) \tag{12}$$

If α is small, its determination becomes difficult. Fortunately there exists relatively accurate K^+p data[39], which suggests $|\alpha(K^+p)| \sim 0.55 \pm 0.15$ for $\nu \sim 7 - 15$ BeV. The error in α was evaluated by allowing the $d\sigma/dt$ data to vary within their error bars, and evaluating the variation in α through (12). If we allow a further variation of one standard deviation on $d\sigma/dt$, we can set a lower limit on α of ~ 0.25. The K^-p data[40] is consistent with $|\alpha(K^-p)| = 0$, but an upper limit of ~ 0.3 has to be allowed within error bars. An additional standard deviation increases this limit to ~ 0.5. The calculated values of $\alpha(K^{\pm}p)$, together with the experimental limits are plotted in Figure 8.

Case I seems to disagree with the data. In case II we can explain the discrepancy by means of the subtraction term. To fit $\alpha(K^+p)$, we can choose either one of two values, depending on the sign of α, which cannot be determined by this method. We find for $\kappa = 3.3$,

$$c = \begin{cases} 2 & \alpha(K^+p) < 0, \quad \alpha(K^-p) > 0 \\ -1.6 & \alpha(K^+p) > 0, \quad \alpha(K^-p) < 0 . \end{cases}$$

$c = -1.6$ is ruled out because it gives $\alpha(K^-p) \sim -0.65$. Hence we conclude that $\alpha(K^+p) < 0$ and $\alpha(K^-p) > 0$. The data points for $\alpha(K^+p)$ were plotted under this assumption in Fig. 8. The errors are clearly very large and allow us safely to ignore the subthreshold singularities.

The general features of πp dispersion relations appear also in Kp. The logarithmic behavior is magnified because $2\sigma^{(-)} \sim 4$ mb. However, at present energies the bulk of the real part seems to come from the subtraction term, and not from the logarithmic one. In fact, these appear to have opposite signs. Thus we expect $|\alpha|$ actually to fall until very high energies, when α changes signs and $|\alpha|$ begins to grow again. As in πp, the real part does not dominate until extremely high energies.

The difference between the pion and the kaon amplitudes lies in the energy range below the cutoff point. The usual Regge picture -- which assumes the Pomeranchuk theorem to hold -- is compatable with experiment for the pions, but appears not to be so for the kaons. In the latter case, the existence of an additional real term seems to be implied by the data.

APPENDIX

In this appendix we list Cutkosky's results[26)] for SU(3) projection operators, in $\underset{\sim}{8} \times \underset{\sim}{8} \rightarrow \underset{\sim}{8} \times \underset{\sim}{8}$ and relate his couplings to those of Rosner[27)]. We use the normalization

$$[\lambda_\alpha, \lambda_\beta] = 2if_{\alpha\beta\gamma}\lambda_\gamma \tag{1}$$

$$\{\lambda_\alpha, \lambda_\beta\} = \frac{4}{3}\,\delta_{\alpha\beta} + 2d_{\alpha\beta\gamma}\lambda_\gamma \ . \tag{2}$$

Repeated indices are to be summed from 1 to 8. In this normalization

$$f_{\alpha nm}f_{\beta nm} = 3\delta_{\alpha\beta} \tag{3}$$

$$d_{\alpha nm}d_{\beta nm} = \frac{5}{3}\,\delta_{\alpha\beta} \tag{4}$$

$$d_{\alpha nm}f_{\beta nm} = 0 \ . \tag{5}$$

We use the shorthand notation $f_{\alpha\beta}\cdot f_{\gamma\delta}$ for $f_{\alpha\beta m}f_{\gamma\delta m}$. These scalar products form our couplings. They correspond to intermediate octets in all three channels. Projection operators for other representations in a given channel are linear combinations of these couplings. Our couplings satisfy the following useful identities

$$f_{\alpha\beta}\cdot f_{\gamma\delta} + f_{\alpha\gamma}\cdot f_{\delta\beta} + f_{\alpha\delta}\cdot f_{\beta\gamma} = 0 \tag{6}$$

$$f_{\alpha\beta}\cdot d_{\gamma\delta} + f_{\alpha\gamma}\cdot d_{\delta\beta} + f_{\alpha\delta}\cdot d_{\beta\gamma} = 0 \tag{7}$$

$$d_{\alpha\beta}\cdot d_{\gamma\delta} + d_{\alpha\gamma}\cdot d_{\delta\beta} + d_{\alpha\delta}\cdot d_{\beta\gamma} = \frac{1}{3}\,(\delta_{\alpha\beta}\delta_{\gamma\delta} + \delta_{\alpha\gamma}\delta_{\delta\beta} + \delta_{\alpha\delta}\delta_{\beta\gamma}) \ . \tag{8}$$

$$f_{\alpha\beta}\cdot d_{\gamma\delta} = d_{\alpha\delta}\cdot f_{\beta\gamma} - d_{\alpha\gamma}\cdot f_{\delta\beta} \tag{9}$$

$$\delta_{\alpha\beta}\delta_{\gamma\delta} = \frac{1}{2}\,(f_{\alpha\delta}\cdot f_{\beta\gamma} - f_{\alpha\gamma}\cdot f_{\delta\beta}) + \frac{3}{2}\,(d_{\alpha\gamma}\cdot d_{\delta\beta} + d_{\alpha\delta}\cdot d_{\beta\gamma}) \ . \tag{10}$$

The projection operators for SU(3) representations in the s channel for an elastic process are

$$\Lambda^{\underset{\sim}{8}s}_{\gamma\delta,\alpha\beta} = \frac{3}{5} d_{\alpha\beta}\cdot d_{\gamma\delta} \tag{11}$$

$$\Lambda^{\underset{\sim}{8}a}_{\gamma\delta,\alpha\beta} = \frac{1}{3} f_{\alpha\beta}\cdot f_{\gamma\delta} \tag{12}$$

$$\Lambda^{\underset{\sim}{1}}_{\gamma\delta,\alpha\beta} = \frac{1}{8} \delta_{\alpha\beta}\delta_{\gamma\delta} \tag{13}$$

$$Q_{\gamma\delta,\alpha\beta} = \frac{i}{\sqrt{5}} (f_{\alpha\beta}\cdot d_{\gamma\delta} - d_{\alpha\beta}\cdot f_{\gamma\delta}) \tag{14}$$

$$\Lambda^{\underset{\sim}{27}}_{\gamma\delta,\alpha\beta} = \frac{1}{2} (\delta_{\alpha\gamma}\delta_{\delta\beta} + \delta_{\alpha\delta}\delta_{\beta\gamma}) - \Lambda^{\underset{\sim}{8}s}_{\gamma\delta,\alpha\beta} - \Lambda^{\underset{\sim}{1}}_{\gamma\delta,\alpha\beta} \tag{15}$$

$$\Lambda^{\underset{\sim}{10}}_{\gamma\delta,\alpha\beta} = \frac{1}{4} (\delta_{\alpha\gamma}\delta_{\delta\beta} - \delta_{\alpha\delta}\delta_{\beta\gamma}) - \frac{1}{2} \Lambda^{\underset{\sim}{8}a}_{\gamma\delta,\alpha\beta} - \frac{\sqrt{5}}{3} Q_{\delta\beta,\alpha\gamma} \tag{16}$$

$$\Lambda^{\overline{\underset{\sim}{10}}}_{\gamma\delta,\alpha\beta} = \frac{1}{4} (\delta_{\alpha\gamma}\delta_{\delta\beta} - \delta_{\alpha\delta}\delta_{\beta\gamma}) - \frac{1}{2} \Lambda^{\underset{\sim}{8}a}_{\gamma\delta,\alpha\beta} + \frac{\sqrt{5}}{3} Q_{\delta\beta,\alpha\gamma} \tag{17}$$

$Q_{\gamma\delta,\alpha\beta}$ is the projection operator for an intermediate octet with f coupling in the initial state and d in the final or vice-versa. For an elastic process the two are related by time reversal invariance, and only the combination $Q_{\gamma\delta,\alpha\beta}$ is allowed.

An alternate coupling scheme of Rosner[27)] uses the idea that each meson is built of a quark and an antiquark. His couplings are of the type $\langle\lambda_\alpha\lambda_\beta\lambda_\gamma\lambda_\delta\rangle$ where $\langle\ \rangle$ stands for 1/4 Tr. This means physically that the quark of particle α annihilates the antiquark of particle δ, the quark of δ, the antiquark of γ, and so on. A different coupling scheme has to be used in meson-baryon or in baryon-baryon scattering, since baryons are composed of three quarks. The connection between the Rosner scheme and ours is straightforward with the use of the following identities.

$$f_{\alpha\beta} \cdot f_{\gamma\delta} = -\frac{1}{2} \langle [\lambda_\alpha, \lambda_\beta][\lambda_\gamma, \lambda_\delta] \rangle \tag{18}$$

$$i d_{\alpha\beta} \cdot f_{\gamma\delta} = \frac{1}{2} \langle \{\lambda_\alpha, \lambda_\beta\}[\lambda_\gamma, \lambda_\delta] \rangle \tag{19}$$

$$i f_{\alpha\beta} \cdot d_{\gamma\delta} = \frac{1}{2} \langle [\lambda_\alpha, \lambda_\beta]\{\lambda_\gamma, \lambda_\delta\} \rangle \tag{20}$$

$$d_{\alpha\beta} \cdot d_{\gamma\delta} = \frac{1}{2} \langle \{\lambda_\alpha, \lambda_\beta\}\{\lambda_\gamma, \lambda_\delta\} \rangle - \frac{8}{3} \langle \lambda_\alpha \lambda_\beta \rangle \langle \lambda_\gamma \lambda_\delta \rangle \tag{21}$$

In particular

$$\langle \lambda_\alpha \lambda_\beta \lambda_\delta \lambda_\gamma \rangle + \langle \lambda_\alpha \lambda_\gamma \lambda_\delta \lambda_\beta \rangle \tag{22}$$

$$= d_{\alpha\beta} \cdot d_{\gamma\delta} + \frac{2}{3} \delta_{\alpha\beta} \delta_{\gamma\delta} + f_{\alpha\beta} \cdot f_{\gamma\delta} \tag{23}$$

$$\langle \lambda_\alpha \lambda_\beta \lambda_\delta \lambda_\gamma \rangle - \langle \lambda_\alpha \lambda_\gamma \lambda_\delta \lambda_\beta \rangle \tag{24}$$

$$= i(f_{\alpha\beta} \cdot d_{\gamma\delta} - d_{\alpha\beta} \cdot f_{\gamma\delta}) \quad . \tag{25}$$

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FIGURE CAPTIONS

(Figures 3 through 8)

Figure 3: α, the ratio of real and imaginary parts of the various amplitudes discussed in the mathematical example of Section 5 . The subtraction constant c is chosen so that for $\nu < \Lambda$, $\alpha_I \approx \alpha_{II}$.

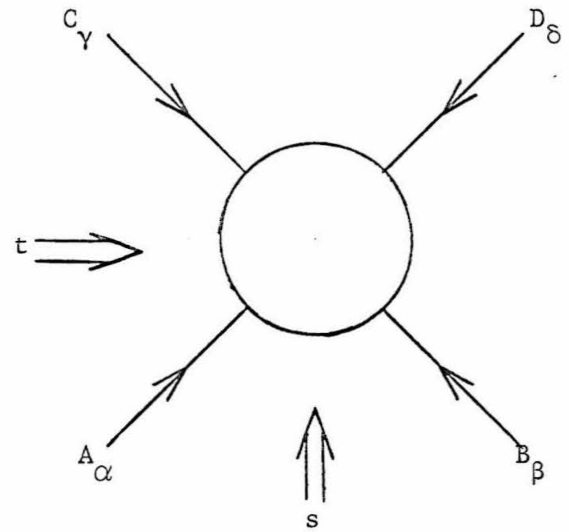
Figure 4: $\pi^{\pm}p$ total cross sections and fit (10). Errors plotted are the sum of the statistical and the systematic. The statistical errors of Allaby et al., are also indicated. The errors of Citron et al. are mainly systematic, and only representative data points of this group have been included.

Figure 5: Predicted $\alpha(\pi^{\pm}p) = \text{Re } A(\pi^{\pm}p)/\text{Im } A\ (\pi^{\pm}p)$ and experimental data of Foley et al.[35]. I and II refer to the choice of high-energy cross sections. (See Fig. 4.) c is the subtraction constant.

Figure 6: Forward differential πN charge exchange cross sections predicted assuming exact I spin conservation, and data of Mannelli et al.[36].

Figure 7: Blow-up of Figure 6. The discrepancy between the fit and the data is an indication of the amount of I-spin violating electromagnetic effect. On the basis of this deviation, we conclude $2\sigma_{EM}^{(-)} \lesssim 0.5$ mb.

Figure 8: $\alpha(K^{\pm}p) = \text{Re } A(K^{\pm}p)/\text{Im } A(K^{\pm}p)$ and experimental limits deduced from the forward elastic differential cross sections[39,40]. The sign of $\alpha(K^{+}p)$ was determined from the dispersion relations. (See text.)



$$s = (p_A + p_B)^2$$

$$t = (p_A + p_C)^2$$

$$u = (p_A + p_D)^2$$

$$\nu = \frac{s - u}{4m_B}$$

Figure 1: Diagram of a four-point function, and definition of kinematic variables. All momenta treated on the same footing. Incoming ones have positive energies, and the outgoing negative ones. $\alpha, \ldots, \delta$ are internal quantum numbers.

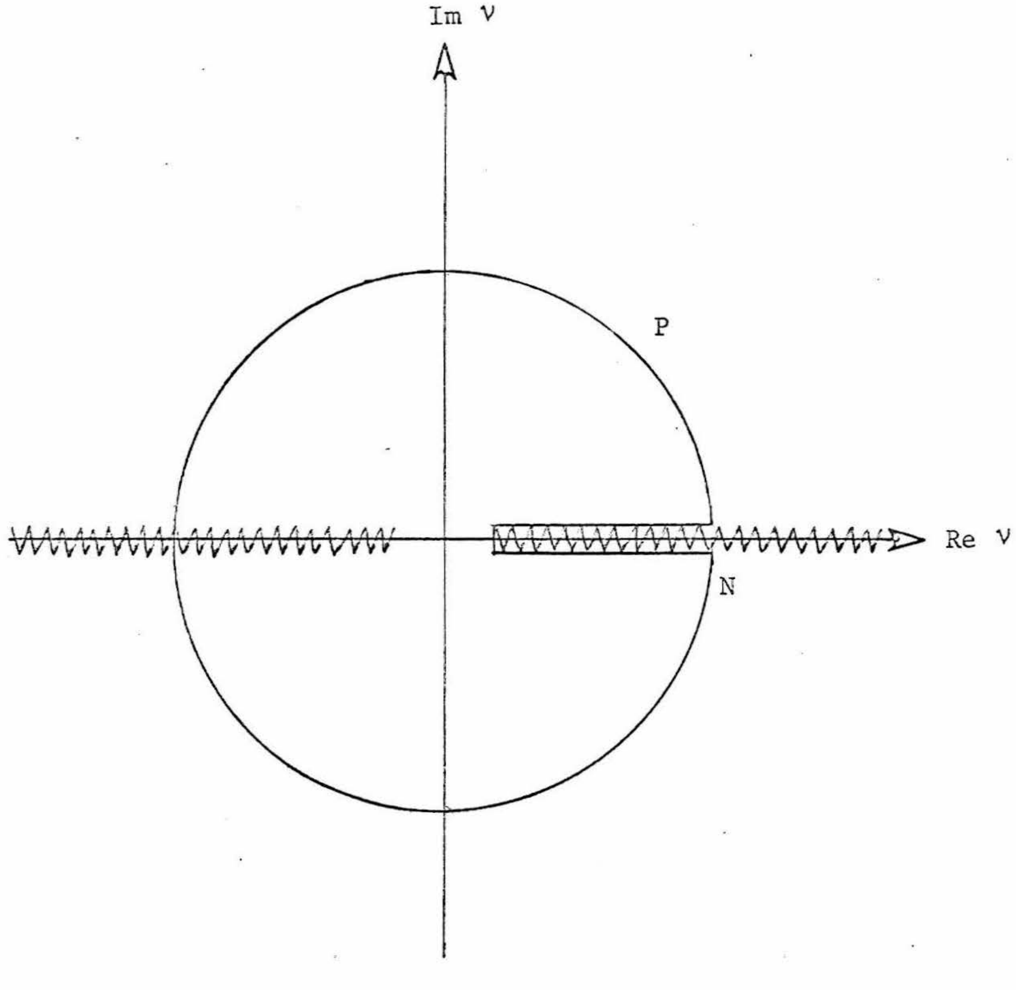


Figure 2: The complex ν plane, and the contour P (see text).

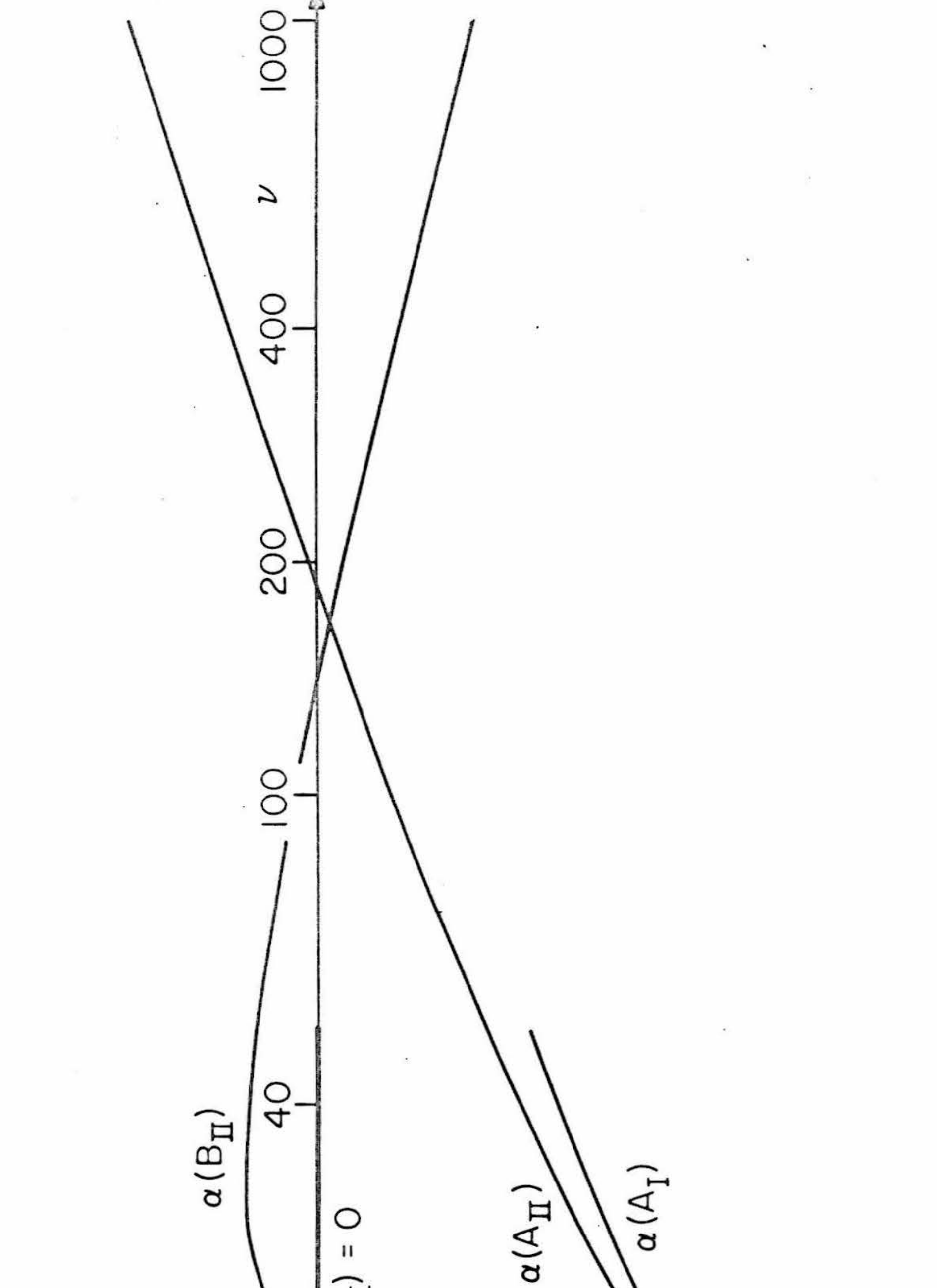


Figure 3

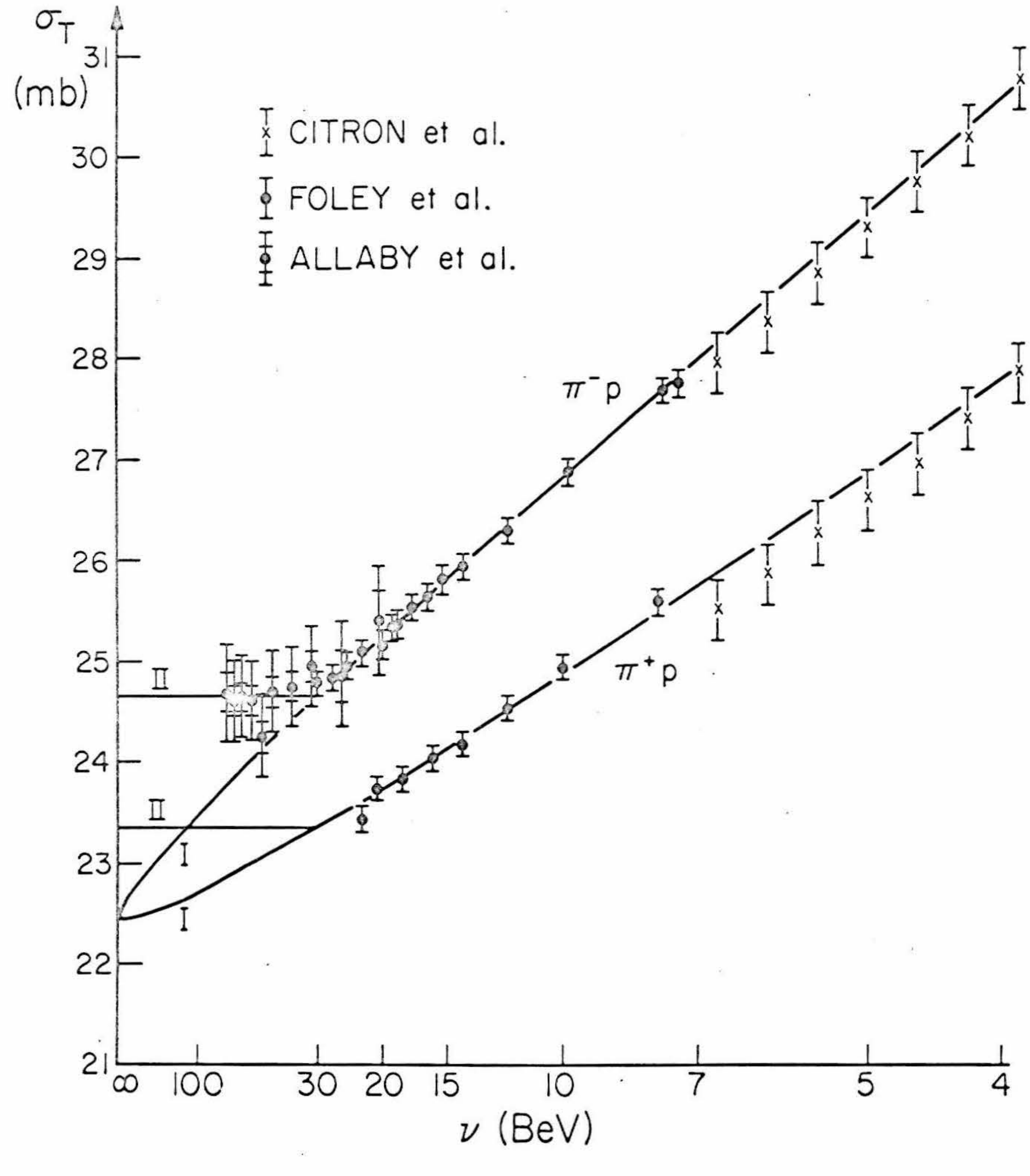

σ_T (mb)
31
30
29
28
27
26
25
24
23
22
21
CITRON et al.
FOLEY et al.
ALLABY et al.
$\pi^- p$
$\pi^+ p$
II
II
I
I
∞
100
30
20
15
10
7
5
4
ν (BeV)

Figure 4

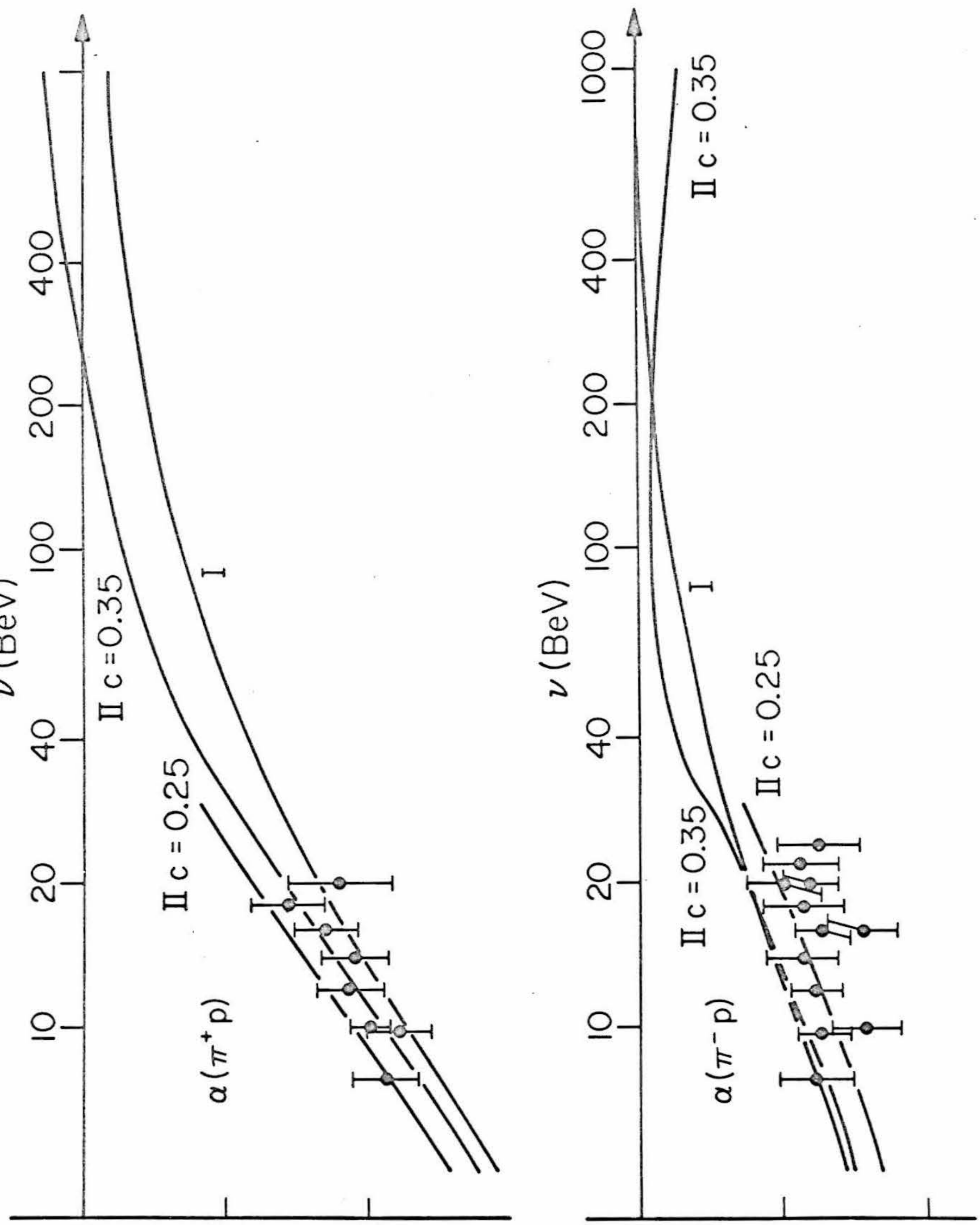
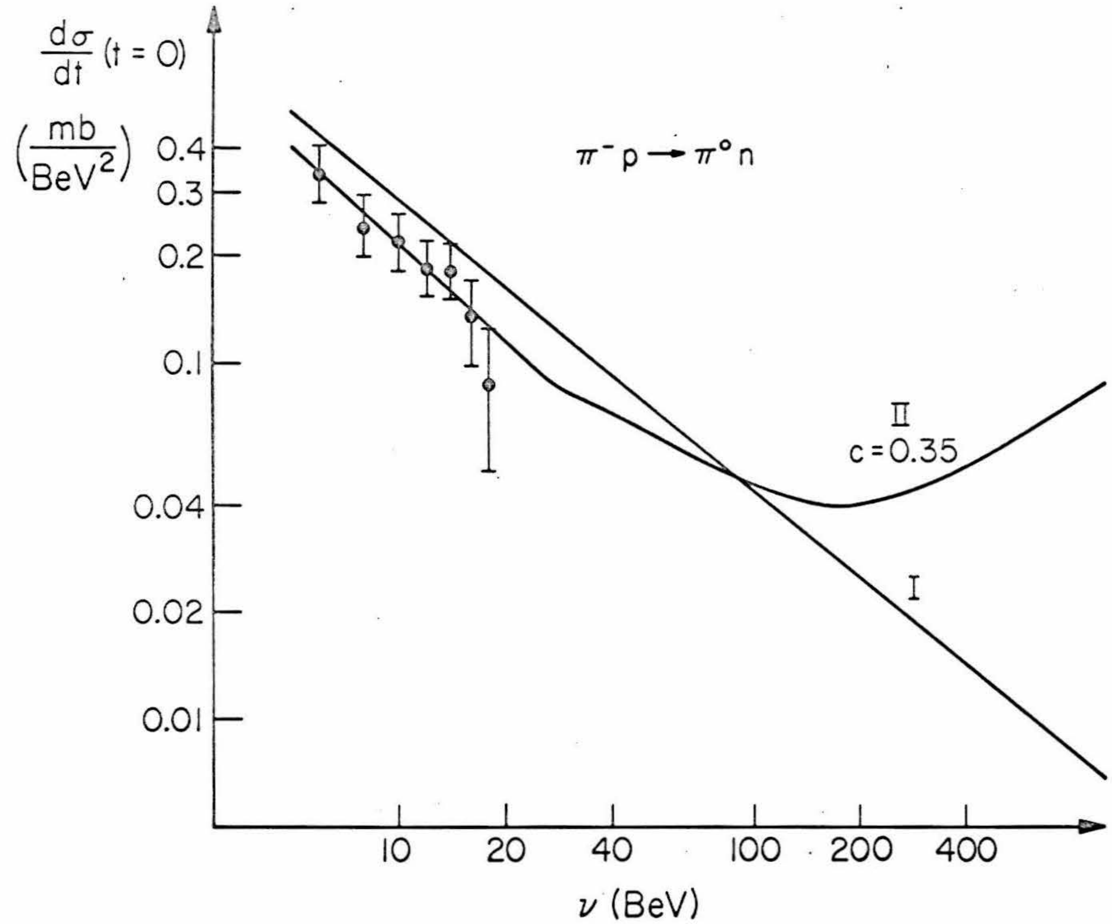
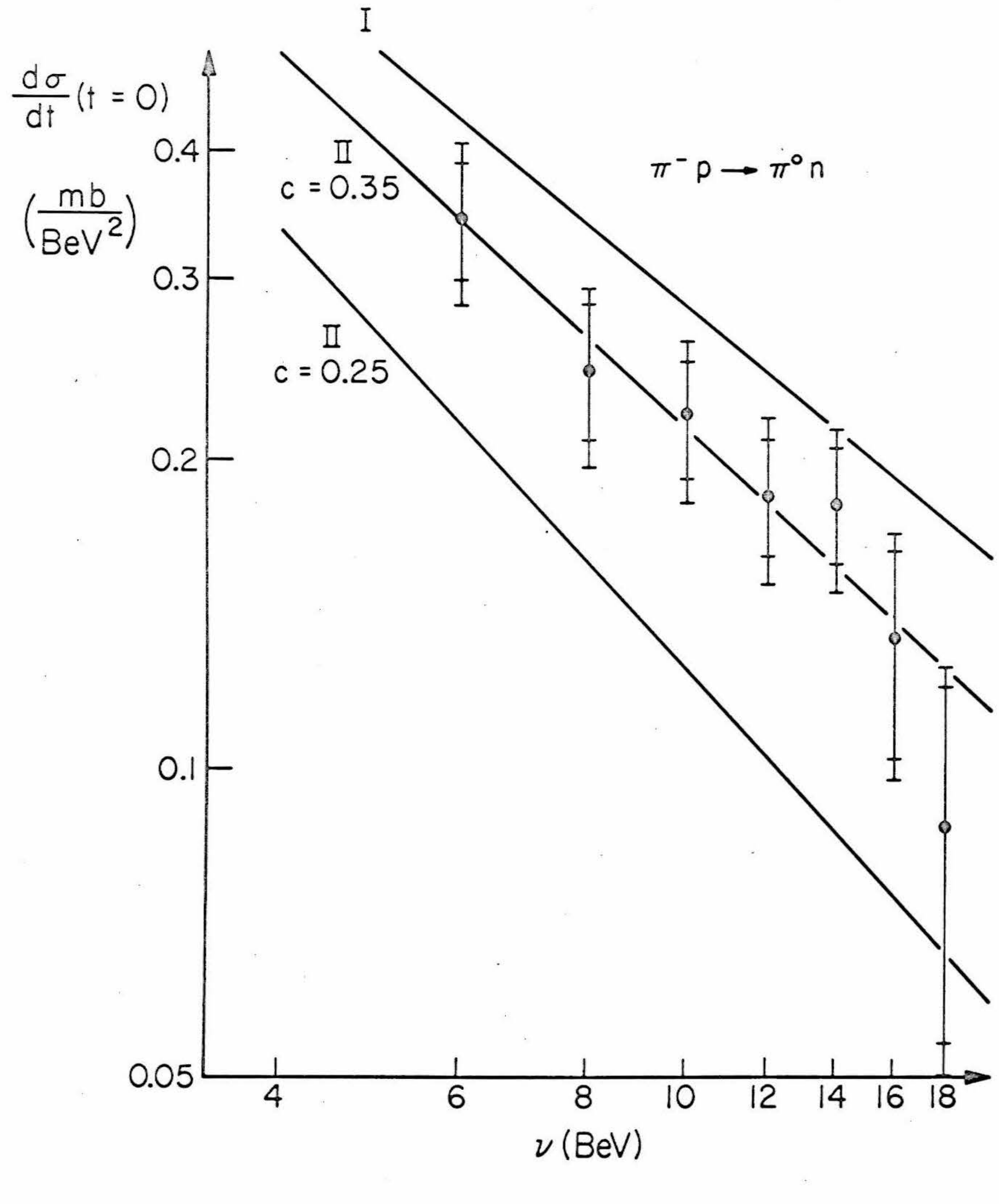


Figure 5



$\frac{d\sigma}{dt}$ (t = 0)
$\left(\frac{mb}{BeV^2}\right)$
0.4
0.3
0.2
0.1
0.04
0.02
0.01
$\pi^- p \rightarrow \pi^\circ n$
II
c = 0.35
I
10
20
40
100
200
400
ν (BeV)

Figure 6



dσ/dt (t = 0)
(mb/BeV²)
I
II
c = 0.35
II
c = 0.25
π⁻ p → π° n
0.4
0.3
0.2
0.1
0.05
4
6
8
10
12
14
16
18
ν (BeV)

Figure 7

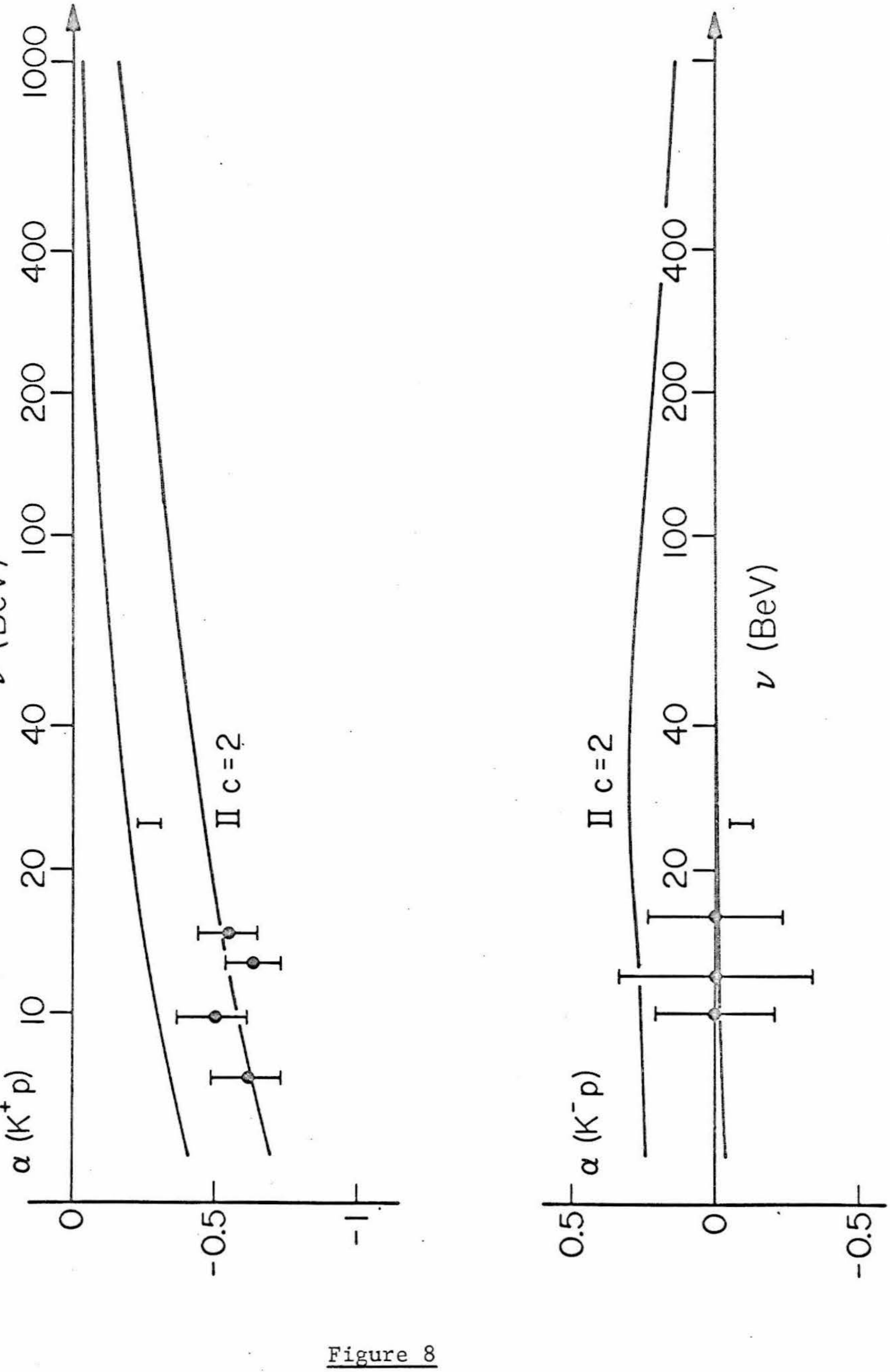

α (K⁺p)
ν (BeV)
I
II c=2
0
-0.5
-1
10
20
40
100
200
400
1000
α (K⁻p)
ν (BeV)
I
II c=2
0.5
0
-0.5
10
20
40
100
200
400

Figure 8