I. CURRENT COMMUTATORS IN QUANTUM ELECTRODYNAMICS

II. ELECTROMAGNETIC MASS DIFFERENCES OF THE PSEUDOSCALAR MESON AND BARYON OCTETS

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ABSTRACT

This thesis is in two parts. In the first section, the operator structure of the singular terms in the equal-time commutator of space and time components of the electromagnetic current is investigated in perturbation theory by establishing a connection with Feynman diagrams. It is made very plausible that the singular term is a c number. Some remarks are made about the same problem in the electrodynamics of a spinless particle.

In the second part, an SU(3) symmetric multi-channel calculation of the electromagnetic mass differences in the pseudoscalar meson and baryon octets is carried out with an attempt to include some of the physics of the crossed (pair annihilation) channel along the lines of the recent work by Ball and Zachariasen. The importance of the tensor meson Regge trajectories is emphasized. The agreement with experiment is poor for the isospin one mass differences, but excellent for those with isospin two.

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I. CURRENT COMMUTATORS IN QUANTUM ELECTRODYNAMICS

I.1. INTRODUCTION

As first pointed out by Schwinger,[1] singular terms must be expected in the vacuum expectation values of equal-time commutators of space and time components of the electromagnetic current. He gave an explicit proof of this for the case of the noninteracting Dirac field by defining the current operator as the limit of the product of field operators evaluated at noncoincident points. Johnson[2] demonstrated that this was the case for interacting fields on the basis of Lorentz invariance and current conservation. Since current commutators have been applied widely with considerable success during the last several years,[3] it is of more than academic interest to examine the structure of the singular (or Schwinger) terms.

A c number Schwinger term will contribute solely to disconnected graphs of the type shown in Fig. 1.1 and has no physical consequences since it is eliminated by subtracting from the commutator its vacuum expectation value.

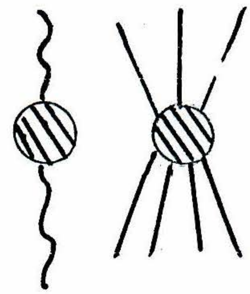


Fig. 1.1. Contribution of a c number Schwinger term.

In order to investigate whether the Schwinger term is an operator or a c number, we calculate some off-diagonal matrix elements of current commutators in perturbation theory in quantum electrodynamics. In the Appendix we discuss the electrodynamics of a spinless particle which is harder to interpret and of less interest than the spin-$\frac{1}{2}$ theory because the current is not analogous to a quark current. The derivation of the basic relations is not meant to be mathematically rigorous; the order of taking limits and performing integrals is freely interchanged. Our object is to determine what results are obtained by employing the usual techniques of reduction formulas and Feynman diagrams.

I.2. FORMALISM

We can calculate the equal-time commutator by writing the current, defined by $\Box A_\mu = j_\mu$,[4] in terms of the renormalized Heisenberg fields and employing the equal-time commutation rules

$$\begin{aligned} \left\{\psi_\alpha(\vec{x},t),\ \psi_\beta^\dagger(\vec{x}',t)\right\} &= \delta^3(\vec{x}-\vec{x}')\delta_{\alpha\beta}, \\ \left\{\psi_\alpha(\vec{x},t),\ \psi_\beta(\vec{x}',t)\right\} &= 0, \end{aligned} \tag{2.1}$$

in a straightforward manner. The current is $j_\mu(x) = Z_1/Z_3\, e\bar{\psi}\gamma_\mu\psi$ and this gives immediately

$$[j_\nu(\vec{x},0),\ j_0(0)] = 0. \tag{2.2}$$

Therefore, we define the matrix element of the Schwinger term as $\langle\alpha^{(-)}|[j_\nu(\vec{x},0),j_0(0)]|\beta^{(+)}\rangle$ computed from the Feynman amplitude.

Without ever writing the current as a singular product of fields, we can establish a connection between the matrix element $\langle\alpha^{(-)}|[j_\nu(\vec{x},0),j_0(0)]|\beta^{(+)}\rangle$ and the Feynman amplitude for the process $\beta \to \alpha + \gamma + \gamma$ in the following two different ways.[5]

The S-matrix element is[6]

$$\langle\alpha;k_1,\epsilon_1;k_2,\epsilon_2^{(-)}|\beta^{(+)}\rangle = 1 - i(2\pi)^4\delta^4(p_\alpha+k_1+k_2-p_\beta)$$

$$\times \frac{\epsilon_1^\mu}{(2\omega_1)^{\frac{1}{2}}}\ \langle\alpha;k_2,\epsilon_2^{(-)}|j_\mu(0)|\beta^{(+)}\rangle. \tag{2.3}$$

We define the Feynman amplitude $\mathcal{M}$ by

$$\mathcal{M} = -i\epsilon_1^\mu(2\omega_2\Pi 2E_\alpha 2E_\beta)^{\frac{1}{2}}\ \langle\alpha;k_2,\epsilon_2^{(-)}|j_\mu(0)|\beta^{(+)}\rangle \tag{2.4}$$

$$= -\epsilon_1^{\mu}\epsilon_2^{\nu}(\Pi 2E_\alpha 2E_\beta)^{\frac{1}{2}}\int d^4x\, e^{ik_2\cdot x}\Box\langle\alpha^{(-)}|T[A_\nu(x)j_\mu(0)]|\beta^{(+)}\rangle \tag{2.5}$$

$$\equiv \epsilon_1^{\mu}\epsilon_2^{\nu}\mathcal{M}_{\mu\nu} \tag{2.6}$$

and we break up $\mathcal{M}_{\mu\nu}$ according to $\mathcal{M}_{\mu\nu} \equiv \mathcal{M}_{\mu\nu}^{(T)} + \mathcal{M}_{\mu\nu}^{(P)}$, where

$$\mathcal{M}_{\mu\nu}^{(T)} = -(\Pi 2E_\alpha 2E_\beta)^{\frac{1}{2}}\int d^4x\, e^{ik_2\cdot x}\langle\alpha^{(-)}|T[j_\nu(x)j_\mu(0)]|\beta^{(+)}\rangle, \tag{2.7}$$

$$\mathcal{M}_{\mu\nu}^{(P)} = -(\Pi 2E_\alpha 2E_\beta)^{\frac{1}{2}}\int d^3\vec{x}\, e^{-i\vec{k}_2\cdot\vec{x}}$$
$$\times\langle\alpha^{(-)}|[\dot{A}_\nu(\vec{x},0) + i\omega_2 A_\nu(\vec{x},0), j_\mu(0)]|\beta^{(+)}\rangle. \tag{2.8}$$

Now let $\vec{k}_2$ remain fixed as $\omega_2 \to \infty$ and use the identity

$$j_\nu(x) = e^{iHx_0}j_\nu(\vec{x},0)e^{-iHx_0} \tag{2.9}$$

$$= j_\nu(\vec{x},0) + ix_0[H, j_\nu(\vec{x},0)] + \frac{1}{2!}(ix_0)^2\Big[H,[H,j_\nu(\vec{x},0)]\Big] + \ldots \tag{2.10}$$

to perform the integration over x_0 in (2.7). A series of decreasing integral powers of ω_2 results with the leading term $\frac{1}{\omega_2}X_{\mu\nu}$, where

$$X_{\mu\nu} = -i(\Pi 2E_\alpha 2E_\beta)^{\frac{1}{2}}\int d^3\vec{x}\, e^{-i\vec{k}_2\cdot\vec{x}}\langle\alpha^{(-)}|[j_\nu(\vec{x},0), j_\mu(0)]|\beta^{(+)}\rangle. \tag{2.11}$$

Inverting the Fourier transform, we find

$$\langle\alpha^{(-)}|[j_\nu(\vec{x},0), j_\mu(0)]|\beta^{(+)}\rangle$$
$$= \frac{i}{(\Pi 2E_\alpha 2E_\beta)^{\frac{1}{2}}}\int\frac{d^3\vec{k}_2}{(2\pi)^3}\, e^{i\vec{k}_2\cdot\vec{x}}X_{\mu\nu}(\vec{k}_2, p_\alpha, p_\beta). \tag{2.12}$$

This is our first fundamental result. The other relation between the commutator and the amplitude is based on current conservation.

Applying translational invariance,

$$j_\nu(x) = e^{iP\cdot x} j_\nu(0) e^{-iP\cdot x}, \tag{2.13}$$

and four-momentum conservation,

$$p_\beta = p_\alpha + k_1 + k_2, \tag{2.14}$$

to (2.7) yields

$$\mathcal{M}^{(T)}_{\mu\nu} = -(\Pi 2E_\alpha 2E_\beta)^{\frac{1}{2}} \int d^4x\, e^{-ik_1\cdot x}$$

$$\times \langle \alpha^{(-)} | T[j_\nu(0) j_\mu(-x)] | \beta^{(+)} \rangle . \tag{2.15}$$

Therefore,

$$R_\nu \equiv \frac{k_1^\mu \mathcal{M}^{(T)}_{\mu\nu}}{(\Pi 2E_\alpha 2E_\beta)^{\frac{1}{2}}} = - \int d^4x (i\partial^\mu e^{-ik_1\cdot x}) \langle \alpha^{(-)} | T[j_\nu(0) j_\mu(-x)] | \beta^{(+)} \rangle . \tag{2.16}$$

We now integrate by parts and use current conservation, $\partial^\mu j_\mu(x) = 0$, to obtain

$$R_\nu = i \int d^3\vec{x}\, e^{i\vec{k}_1\cdot\vec{x}} \langle \alpha^{(-)} | [j_\nu(0), j_o(-\vec{x},0)] | \beta^{(+)} \rangle \tag{2.17}$$

$$= i \int d^3\vec{x}\, e^{-i\vec{k}_2\cdot\vec{x}} \langle \alpha^{(-)} | [j_\nu(\vec{x},0), j_o(0)] | \beta^{(+)} \rangle . \tag{2.18}$$

Inverting the Fourier transform and employing current conservation, this time in the form $k_1^\mu \mathcal{M}_{\mu\nu} = 0$, which implies $k_1^\mu \mathcal{M}^{(T)}_{\mu\nu} = -k_1^\mu \mathcal{M}^{(P)}_{\mu\nu}$, we get finally

$$\langle \alpha^{(-)} | [j_\nu(\vec{x},0), j_o(0)] | \beta^{(+)} \rangle$$

$$= \frac{i}{(\Pi 2E_\alpha 2E_\beta)^{\frac{1}{2}}} \int \frac{d^3\vec{k}_2}{(2\pi)^3} e^{i\vec{k}_2\cdot\vec{x}} k_1^{\mu} \mathcal{M}^{(\Gamma)}_{\mu\nu}(k_2, p_\alpha, p_\beta). \tag{2.19}$$

Equation (2.19) is the second desired formula.

In utilizing Eq. (2.12) or Eq. (2.19), $\mathcal{M}_{\mu\nu}$ is written down from the Feynman rules and its asymptotic form is computed in the limit $\omega_2 \to \infty$ with $\vec{k}_2$ fixed. Equation (2.8) shows that $\mathcal{M}^{(P)}_{\mu\nu}$ is at most a linear polynomial in ω_2 and is easily isolated as the part of $\mathcal{M}_{\mu\nu}$ which does not tend to zero in this limit.

Equation (2.19) may puzzle the reader for two reasons. First, it is not obvious that it agrees with Eq. (2.12) and, second, the right-hand side appears to depend on ω_2, which the left-hand side clearly must not. According to (2.8), $\mathcal{M}^{(P)}_{\mu\nu}$ has the form

$$\mathcal{M}^{(P)}_{\mu\nu} = A_{\mu\nu}(\vec{k}_2, p_\alpha, p_\beta) + \omega_2 B_{\mu\nu}(\vec{k}_2, p_\alpha, p_\beta). \tag{2.20}$$

So, we have as $\omega_2 \to \infty$, $\vec{k}_2$ fixed,

$$\mathcal{M}_{\mu\nu} \to \omega_2 B_{\mu\nu} + A_{\mu\nu} + \omega_2^{-1} X_{\mu\nu} + O(\omega_2^{-2}). \tag{2.21}$$

Now, using $k_1 = p_\beta - p_\alpha - k_2 \equiv q - k_2$, we can expand $k_1^{\mu} \mathcal{M}_{\mu\nu}$ in powers of ω_2^{-1} to obtain

$$0 = k_1^{\mu} \mathcal{M}_{\mu\nu} = -\omega_2^{\,2} B_{o\nu} + \omega_2\left(q_o B_{o\nu} - A_{o\nu} + k_1^{\,i} B_{i\nu}\right)$$
$$+ \left(q_o A_{o\nu} - X_{o\nu} + k_1^{\,i} \lambda_{i\nu}\right) + O(\omega_2^{-1}). \tag{2.22}$$

This yields, of course,

$$B_{o\nu} = 0,$$
$$q_o B_{o\nu} - A_{o\nu} + k_1^{\,i} B_{i\nu} = 0, \tag{2.23}$$
$$X_{o\nu} = q_o A_{o\nu} + k_1^{\,i} A_{i\nu}.$$

But, from (2.20),

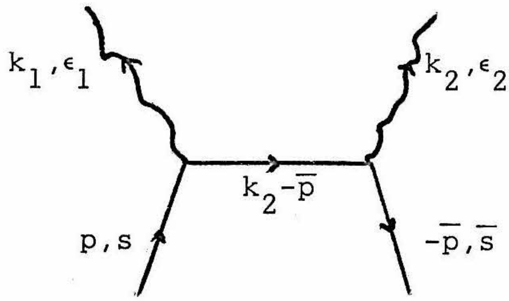
$$k_1^{\mu} \mathcal{M}_{\mu\nu}^{(P)} = -\omega_2^{\,2} B_{o\nu} + \omega_2\left(q_o B_{o\nu} - A_{o\nu} + k_1^{\,i} B_{i\nu}\right) + \left(q_o A_{o\nu} + k_1^{\,i} A_{i\nu}\right). \tag{2.24}$$

Substituting (2.23) into (2.24) demonstrates immediately that $k_1^{\mu} \mathcal{M}_{\mu\nu}^{(P)} = X_{o\nu}$ and this resolves both of our apparent difficulties at the same time.

In order to apply our formalism, we must study reactions with two photons in the final state. Processes which can be obtained from these by crossing naturally would give no new information. The simplest cases to consider to lowest order are pair annihilation and photon-photon scattering. The latter is fourth order, but converges without renormalization and the entire amplitude can be calculated by symmetrizing the contribution of a single Feynman diagram. Only a very complicated operator Schwinger term might give a null result in both of these situations. For example, a term with the operator structure $e^2\bar{\psi}\psi$ will show up in both computations and one like e^4A^2 will be exhibited in the second.

I.3. RESULTS

For electron-positron annihilation, the amplitude is calculated to lowest order from the following diagrams.



$+ \quad (k_1, \epsilon_1 \longleftrightarrow k_2, \epsilon_2)$

Fig. 3.1. Pair annihilation.

These give

$$\mathcal{M}_{\mu\nu} = \bar{v}(\bar{p},\bar{s})\left[(-ie\gamma_\nu)\frac{i}{\not{k}_2-\not{\bar{p}}-m}(-ie\gamma_\mu) + (-ie\gamma_\mu)\frac{i}{\not{p}-\not{k}_2-m}(-ie\gamma_\nu)\right]u(p,s). \tag{3.1}$$

Equation (3.1) clearly indicates that as $\omega_2 \to \infty$, $\vec{k}_2$ fixed, $\mathcal{M}_{\mu\nu} \to O(\omega_2^{-1})$, which means $\mathcal{M}^{(P)}_{\mu\nu} = 0$ and thus (2.19) gives no Schwinger term. It is also easy to check that $X_{o\nu} = 0$ so that (2.12) gives the same result, as it must.

For Delbrück scattering, Eq. (2.19) is much more convenient than Eq. (2.12) because it requires only the finding of the asymptotic behavior of the amplitude. Removing all the photon polarizations, we can express the Feynman amplitude in the form

$$\mathcal{M}_{\mu\nu\lambda\sigma} = T_{\mu\nu\lambda\sigma}(k_1, k_2, k_3, k_4) + T_{\mu\nu\sigma\lambda}(k_1, k_2, k_4, k_3) + T_{\mu\lambda\nu\sigma}(k_1, k_3, k_2, k_4), \tag{3.2}$$

where

$$T_{\mu\nu\lambda\sigma}(k_1, k_2, k_3, k_4) = -2\int \frac{d^4p}{(2\pi)^4}\, \mathrm{tr}\Big[(-ie\gamma_\nu)\frac{i}{\not{p}-m}(-ie\gamma_\lambda)\frac{i}{\not{p}-\not{k}_3-m} \times (-ie\gamma_\sigma)\frac{i}{\not{p}-\not{k}_3-\not{k}_4-m}(-ie\gamma_\mu)\frac{i}{\not{p}+\not{k}_2-m}\Big] \tag{3.3}$$

is <u>twice</u> the amplitude corresponding to the Feynman diagram below.

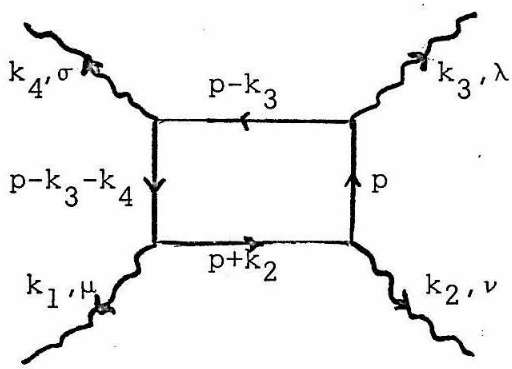


Fig. 3.2. Delbrück scattering.

The factor 2 occurs in (3.3) because the six distinct graphs obtained by fixing one of the photons are equal in pairs related by ordering the photons clockwise or counterclockwise around the Fermion box. More physically, this is the same as letting the electron run around the loop in the opposite sense, or charge conjugating the diagram. (The +1 here becomes a −1 for closed Fermion loops with an odd number of vertices and hence these diagrams vanish as demanded by Furry's theorem.)

Gauge invariance requires that $\mathcal{M}$ depend only on the field strengths,[7] which implies

$$\mathcal{M}_{\mu\nu\lambda\sigma}(0,0,0,0) = 0. \tag{3.4}$$

In more detail, suppose that at low energies $\mathcal{M}$ can be expanded in a power series in photon momenta:

$$\mathcal{M}_{\mu\nu\lambda\sigma}(k_1,k_2,k_3,k_4) = a_{\mu\nu\lambda\sigma} + (\text{terms linear, quadratic, etc. in the k's}). \tag{3.5}$$

Then

$$k_1^\mu \mathcal{M}_{\mu\nu\lambda\sigma}(k_1,k_2,k_3,k_4) = 0, \tag{3.6}$$

for all k_1 such that $k_1^2 = 0$, yields at once

$$k_1^\mu a_{\mu\nu\lambda\sigma} = 0, \tag{3.7}$$

for all such k_1. By selecting suitable null vectors k_1 it is easy to show that (3.7) implies

$$a_{\mu\nu\lambda\sigma} = 0, \tag{3.8}$$

which proves (3.4).

Let us use (3.2) and (3.3) to evaluate $\mathcal{M}_{\mu\nu\lambda\sigma}(0,0,0,0)$.[8] Notice that

$$S\gamma_\mu S = -\frac{\partial S}{\partial p^\mu}, \qquad \text{where} \qquad S(p) = \frac{1}{\not{p}-m}. \tag{3.9}$$

Equations (3.2), (3.3) and (3.9) give

$$\mathcal{M}_{\mu\nu\lambda\sigma}(0,k_2,k_3,k_4) = -e^4\int\frac{d^4p}{(2\pi)^4}\left(-\frac{\partial}{\partial p^\mu}\right)\mathrm{tr}\Big[S(p)\gamma_\lambda S(p+k_3)\gamma_\sigma S(p-k_3)\gamma_\nu$$

$$+ S(p)\gamma_\lambda S(p+k_3)\gamma_\nu S(p-k_4)\gamma_\sigma\Big]. \tag{3.10}$$

At this point it is essential to observe that the bracket behaves like p^{-3} as $p\to\infty$. Since the integrand is a total derivative which can be converted to a surface integral at infinity, we need to consider only large p. This permits the great simplification of setting $k_2 = k_3 = k_4 = 0$. Hence

$$\mathcal{M}_{\mu\nu\lambda\sigma}(0,k_1,k_2,k_3) = \mathcal{M}_{\mu\nu\lambda\sigma}(0,0,0,0) \tag{3.11}$$

$$= e^4\int\frac{d^4p}{(2\pi)^4}\frac{\partial}{\partial p^\mu}\mathrm{tr}\Big[S(p)\gamma_\lambda S(p)\gamma_\sigma S(p)\gamma_\nu + S(p)\gamma_\lambda S(p)\gamma_\nu S(p)\gamma_\sigma\Big] \tag{3.12}$$

$$= e^4\int\frac{d^4p}{(2\pi)^4}\frac{\partial}{\partial p^\mu}\frac{\partial}{\partial p^\lambda}\frac{\partial}{\partial p^\sigma}\mathrm{tr}[S(p)\gamma_\nu] \tag{3.13}$$

$$= 4e^4\int\frac{d^4p}{(2\pi)^4}\frac{\partial}{\partial p^\mu}\frac{\partial}{\partial p^\lambda}\frac{\partial}{\partial p^\sigma}\left(\frac{p_\nu}{p^2-m^2}\right) \tag{3.14}$$

$$= A\left(g_{\mu\nu}g_{\lambda\sigma} + g_{\mu\lambda}g_{\nu\sigma} + g_{\mu\sigma}g_{\nu\lambda}\right). \tag{3.15}$$

Equation (3.13) comes from repeated application of (3.9) and (3.15) follows from the symmetry and Lorentz covariance of $\mathcal{M}_{\mu\nu\lambda\sigma}(0,0,0,0)$. Equation (3.15) gives

$$A = \frac{1}{24} \mathcal{M}_{\mu\nu}{}^{\mu\nu}(0,0,0,0) \tag{3.16}$$

$$= \frac{e^4}{6} \int \frac{d^4p}{(2\pi)^4} \frac{\partial}{\partial p^\mu} \frac{\partial}{\partial p_\mu} \frac{\partial}{\partial p_\nu} \left(\frac{p_\nu}{p^2-m^2} \right) \tag{3.17}$$

$$= \frac{e^4}{6} \int \frac{d^4p}{(2\pi)^4} \left[\frac{-48m^4}{(p^2-m^2)^4} \right] \tag{3.18}$$

$$= \frac{-ie^4}{12\pi^2} \ . \tag{3.19}$$

Equation (3.19) contradicts (3.4) and thereby demonstrates that we must use the gauge invariant amplitude

$$\overline{\mathcal{M}}_{\mu\nu\lambda\sigma}(k_1,k_2,k_3,k_4) \equiv \mathcal{M}_{\mu\nu\lambda\sigma}(k_1,k_2,k_3,k_4) - \mathcal{M}_{\mu\nu\lambda\sigma}(0,0,0,0). \tag{3.20}$$

One has to enforce the gauge invariance of the vacuum polarization tensor in the same manner. In this case gauge invariance fails because the amplitudes T diverge logarithmically, even though their sum $\mathcal{M}$ is convergent.

Now, in computing $\mathcal{M}^{(P)}_{\mu\nu\lambda\sigma}(k_1,k_2,k_3,k_4)$, we may put $k_3 = k_4 = 0$. By Eq. (3.11)

$$\mathcal{M}_{\mu\nu\lambda\sigma}(k_1,k_2,0,0) = \mathcal{M}_{\mu\nu\lambda\sigma}(0,0,0,0). \tag{3.21}$$

Therefore,

$$\mathcal{M}^{(P)}_{\mu\nu\lambda\sigma}(k_1,k_2,k_3,k_4) = \mathcal{M}_{\mu\nu\lambda\sigma}(0,0,0,0), \tag{3.22}$$

so that, due to (3.20),

$$\overline{\mathcal{M}}^{(P)}_{\mu\nu\lambda\sigma}(k_1, k_2, k_3, k_4) = 0 \tag{3.23}$$

and again we get no Schwinger term.

I.4. CONCLUDING REMARKS

If we are prepared to ignore the fact that the manner in which gauge invariance is imposed through Eq. (3.20) is purely formal and its effect on the current is unclear, we can postulate with confidence that the Schwinger term is a c number. By considering the limit of a nonlocal field theory, Boulware reached the same conclusion in a paper published very soon after the completion of this work.[9] Later still, Brandt[10] gave a new definition of the equal-time commutator by writing each current in the commutator as the limit of a nonlocal product of renormalized fields and taking the limit after employing the equal-time commutation relations of the fields. He finds a Schwinger term with the operator structure A^2 and on the basis of his work challenges our heuristic assumption that the time-ordered product vanishes at infinity. In that case the simple methods proposed here and by Bjorken[5] are invalid.

We could also use Eq. (2.12) for photon-photon scattering, but since it necessitates finding the next to dominant terms of the amplitude, the calculation would be much more tedious and its independence of regularization only apparent because we must, in principle, regularize the complete amplitude before finding its limiting behavior. Pair annihilation to fourth order, besides being very messy, has an infrared divergence which cancels out in the cross section, but is not expected to do so in the commutator. As discussed in Chapter 2, it is very unlikely, in any case, that a Schwinger term which gives a null result in Delbrück scattering would appear here.

Applications of equal-time commutators have been founded on postulating the validity in the real world of equal-time commutators computed in some model such as the quark model. In this manner, sum rules, low energy theorems and Ward identities have been derived.[3] After hypothesizing the commutator, the high energy behavior of various processes can be deduced by reversing the arguments given in Chapter 2. Radiative corrections to weak interactions[5,11] and the leptonic decays of the η meson[12] have been studied using these methods. Young[13] has calculated the π^0 lifetime and the $\omega\rho\pi$ coupling constant and Bjorken[5] has obtained a lower bound for electron-nucleon inelastic scattering at high momentum transfer. Most interesting for us, in view of the fact that the second part of this thesis deals with electromagnetic mass differences, is that equal-time commutators determine the high energy behavior of the Cottingham formula.[5,11,14,15] In order to compute the matrix element for forward Compton scattering which appears in the formula, the commutators are evaluated between nucleon states of the same momentum. Since disconnected graphs of the type shown in Fig. 1.1 are specifically excluded from the Cottingham formula, c number Schwinger terms clearly do not contribute here. A nonzero matrix element of a q number Schwinger term in the commutator [j, j] would give rise to a quadratic divergence in the mass difference. Such Schwinger terms in more complicated commutators would cause a logarithmic divergence.

APPENDIX

THE SPINLESS THEORY

In the electrodynamics of a spinless particle, with the current

$$j_\mu(x) = \frac{Z_1}{Z_3}\left[ie(\varphi\partial_\mu\varphi^* - \varphi^*\partial_\mu\varphi) - 2e^2 A_\mu\varphi^*\varphi\right], \qquad (A.1)$$

the canonical commutation relations

$$\begin{aligned}
\left[\varphi(\vec{x},t),\ \varphi(\vec{x}',t)\right] &= 0, \\
\left[\varphi(\vec{x},t), \varphi^*(\vec{x}',t)\right] &= 0, \\
\left[\varphi(\vec{x},t),\ \dot{\varphi}(\vec{x}',t)\right] &= 0, \qquad (A.2)\\
\left[\varphi(\vec{x},t), \varphi^*(\vec{x}',t)\right] &= i\delta^3(\vec{x}-\vec{x}'),
\end{aligned}$$

yield directly

$$\left[j_i(\vec{x},0), j_o(0)\right] = -2ie^2\left(\frac{Z_1}{Z_3}\right)^2\left[\delta^3(\vec{x})(\varphi^*\partial_i\varphi + \varphi\partial_i\varphi^*) - \varphi^*(0)\varphi(0)\partial_i\delta^3(\vec{x})\right]. \qquad (A.3)$$

We now define the matrix element of the Schwinger term as $\langle\alpha^{(-)}|[j_i(\vec{x},0), j_o(0)]|\beta^{(+)}\rangle$, computed from Eq. (A.3), subtracted from the same quantity calculated from Eq. (2.12) or Eq. (2.19).

The amplitude for pair annihilation written down from the Feynman diagrams shown below is

$$\mathcal{M}_{\mu\nu} = \left[-ie(k_2-2\bar{q})_\nu\right] \frac{i}{(k_2-\bar{q})^2-\mu^2} \left[-ie(k_2-\bar{q}+q)_\mu\right]$$

$$+ \left[-ie(q-k_2-\bar{q})_\mu\right] \frac{i}{(q-k_2)^2-\mu^2} \left[-ie(2q-k_2)_\nu\right] + 2ie^2 g_{\mu\nu}. \qquad \text{(A.4)}$$

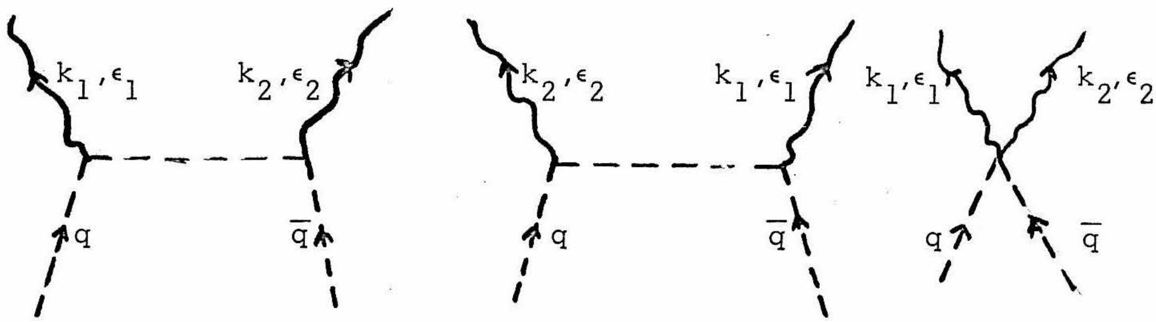


Fig. A.1. Pair annihilation.

From (A.4) it is easy to see that

$$X_{oi} = 2ie^2 k_{1i},$$

$$\mathcal{M}^{(P)}_{oi} = 0, \qquad \text{(A.5)}$$

$$\mathcal{M}^{(P)}_{ji} = 2ie^2 g_{ji},$$

so that (A.3), (2.12) and (2.19) all result in

$$\langle 0|[j_i(\vec{x},0), j_o(0)]|q,\bar{q}\rangle = \frac{2e^2}{(4\omega_q\omega_{\bar{q}})^{\frac{1}{2}}} [i\partial - (q+\bar{q})]_i \, \delta^3(\vec{x}). \qquad \text{(A.6)}$$

Photon-photon scattering is complicated by the presence of seagull diagrams. Some of these are independent of k_2 and, therefore, would have to be evaluated exactly if we used Eq. (2.19). In Fig. A.2 we display the Feynman graphs for this process.

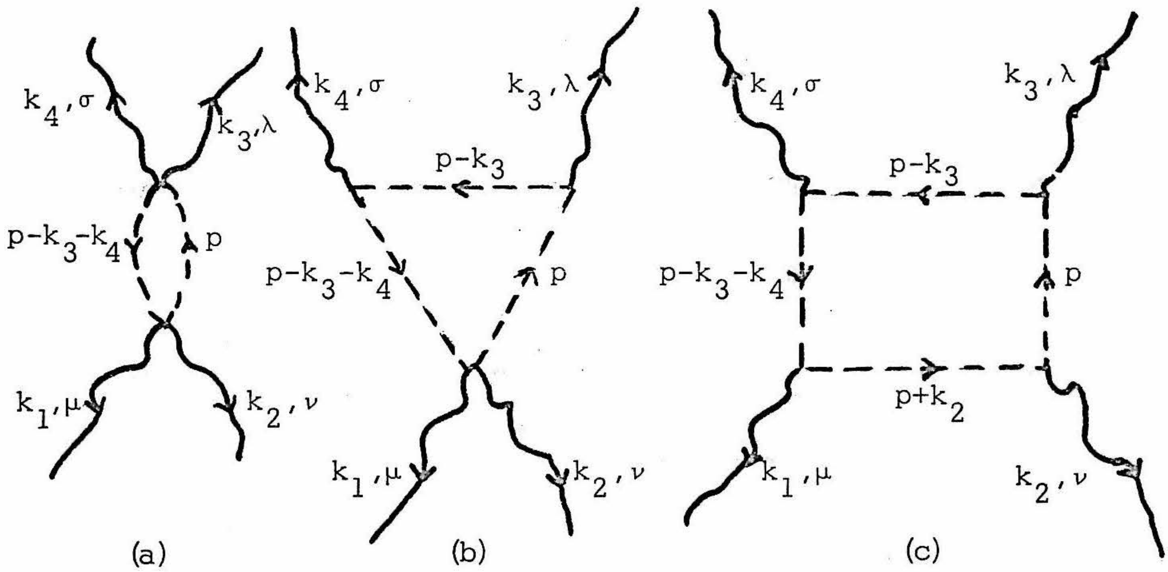


Fig. A.2. Photon-photon scattering.

The entire amplitude can be decomposed as follows.

$$\mathcal{M}_{\mu\nu\lambda\sigma} = \mathcal{M}^{(R)}_{\mu\nu\lambda\sigma} + \mathcal{M}^{(S)}_{\mu\nu\lambda\sigma} + \mathcal{M}^{(T)}_{\mu\nu\lambda\sigma}, \tag{A.7}$$

$$\mathcal{M}^{(R)}_{\mu\nu\lambda\sigma} = R_{\mu\nu\lambda\sigma} + R_{\mu\lambda\nu\sigma} + R_{\mu\sigma\lambda\nu}, \tag{A.8}$$

$$\mathcal{M}^{(S)}_{\mu\nu\lambda\sigma} = S_{\mu\nu\lambda\sigma} + S_{\mu\lambda\nu\sigma} + S_{\mu\sigma\lambda\nu} + S_{\nu\lambda\mu\sigma} + S_{\nu\sigma\lambda\mu} + S_{\lambda\sigma\mu\nu}, \tag{A.9}$$

$$\mathcal{M}^{(T)}_{\mu\nu\lambda\sigma} = T_{\mu\nu\lambda\sigma} + T_{\mu\nu\sigma\lambda} + T_{\mu\lambda\nu\sigma}, \tag{A.10}$$

where the momenta are understood to be permuted just as the indices are and

$$R_{\mu\nu\lambda\sigma} = \int \frac{d^4p}{(2\pi)^4} \frac{i^2(2ie^2 g_{\mu\nu})(2ie^2 g_{\lambda\sigma})}{(p^2-\mu^2)[(p-k_3-k_4)^2 - \mu^2]}, \tag{A.11}$$

$$S_{\mu\nu\lambda\sigma} = 2\int \frac{d^4p}{(2\pi)^4} \frac{i^3(2ie^2 g_{\mu\nu})(-ie)^2(2p-k_3)_\lambda(2p-2k_3-k_4)_\sigma}{(p^2-\mu^2)[(p-k_3)^2-\mu^2][p-k_3-k_4)^2-\mu^2]}, \tag{A.12}$$

$$T_{\mu\nu\lambda\sigma} = 2\int \frac{d^4p}{(2\pi)^4} \frac{i^4(-ie)^4(2p+k_2-k_3-k_4)_\mu(2p+k_2)_\nu}{(p^2-\mu^2)[(p-k_3)^2 - \mu^2][(p-k_3-k_4)^2 - \mu^2]}$$

$$\times \frac{(2p-k_3)_\lambda(2p-2k_3-k_4)_\sigma}{[(p+k_2)^2 - \mu^2]}. \tag{A.13}$$

$R_{\mu\nu\lambda\sigma}$, $S_{\mu\nu\lambda\sigma}$ and $T_{\mu\nu\lambda\sigma}$ correspond to Fig. A.2(a), (b) and (c), respectively, and the source of the factor 2 in S and T is the same as in Delbrück scattering for spin-$\frac{1}{2}$. The calculation via Eq. (2.12) is considerably more tedious than the spin-$\frac{1}{2}$ case and occasionally it is necessary, in order to avoid spurious singularities, to break up the region of integration over Feynman parameters and then to approximate the integrand differently in the two regions. As an example of this, we would write

$$\int_0^1 dz \int_z^1 dx \int_z^x dy = \int_\epsilon^1 dz \int_z^1 dx \int_z^x dy + \int_0^\epsilon dz \int_0^1 dx \int_0^x dy, \tag{A.14}$$

where $0 < \epsilon << 1$. A typical denominator obtained after introducing Feynman parameters is

$$a^2 = \mu^2 - z(1-z)k_2^2 - y(1-y)k_3^2 - x(1-x)k_4^2 - 2z(1-y)k_2 \cdot k_3$$

$$- 2z(1-y)k_2 \cdot k_4 - 2y(1-x)k_3 \cdot k_4 . \qquad (A.15)$$

In the first volume in (A.14) $a^2 \approx -z(1-z)\omega_2^2$, whereas in the second region $a^2 \approx \mu^2 - 2y(1-x)k_3 \cdot k_4 - z\omega_2^2$. If we take $k_3 \cdot k_4 = 0$ to save labor, we obtain

$$X_{oi} = \frac{ie^4}{18\pi^2}\left[\epsilon_{3i}\epsilon_{4o}(9\omega_3 - \omega_4) + \epsilon_{4i}\epsilon_{3o}(9\omega_4 - \omega_3)\right] \qquad (A.16)$$

and hence

$$\langle k_3, \epsilon_3; k_4, \epsilon_4 | [j_i(\vec{x}, 0), j_o(0)] | 0 \rangle = -\frac{1}{(4\omega_3\omega_4)^{\frac{1}{2}}} \frac{e^4}{18\pi^2}\left[\epsilon_{3i}\epsilon_{4o}(9\omega_3 - \omega_4)\right.$$

$$\left. + \epsilon_{4i}\epsilon_{3o}(9\omega_4 - \omega_3)\right] \delta^3(\vec{x}) . \qquad (A.17)$$

We have not succeeded in comparing this with Eq. (A.3) because sandwiching the latter between the same states diverges. Consider, for example,

$$\frac{e^2}{(4\omega_3\omega_4)^{\frac{1}{2}}} \epsilon_3^{\mu} \epsilon_4^{\mu} A_{\mu\nu} \equiv \langle k_3, \epsilon_3; k_4, \epsilon_4 | \varphi(0) \varphi^*(0) | 0 \rangle \qquad (A.18)$$

$$= \sum_n \langle k_3, \epsilon_3; k_4, \epsilon_4 | \varphi | n \rangle \langle n | \varphi^* | 0 \rangle . \qquad (A.19)$$

To lowest order $|n\rangle = |q\rangle$ and $\langle q | \varphi^* | 0 \rangle = \frac{1}{(2\omega_q)^{\frac{1}{2}}}$,

$$\langle k_3, k_4 | \varphi | q \rangle = \frac{\langle k_3, k_4 | J | q \rangle}{\mu^2 - (k_3 + k_4 - q)^2} , \qquad (A.20)$$

where $(\Box + \mu^2)\varphi(x) = J(x)$. Thus $A_{\mu\nu}$ is expressed as an integral over the pair annihilation amplitude

$$\langle k_3, k_4 | J | q \rangle = \frac{-ie^2}{(8\omega_q \omega_3 \omega_4)^{\frac{1}{2}}} \left[\frac{-i\epsilon_4 \cdot (2q-2k_3-k_4)\epsilon_3 \cdot (2q-k_3)}{(q-k_3)^2 - \mu^2} \right.$$

$$\left. + \frac{-i\epsilon_3 \cdot (2q-2k_4-k_3)\epsilon_4 \cdot (2q-k_4)}{(q-k_4)^2 - \mu^2} + 2ig_{\mu\nu} \right], \qquad \text{(A.21)}$$

$$A_{\mu\nu} = \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{1}{2\omega_q} \frac{1}{2[q\cdot(k_3+k_4) - k_3\cdot k_4]} \left[\frac{(2q-2k_3-k_4)_\nu (2q-k_3)_\mu}{2q\cdot k_3} \right.$$

$$\left. + \frac{(2q-2k_4-k_3)_\mu (2q-k_4)_\nu}{2q\cdot k_4} + 2g_{\mu\nu} \right], \qquad \text{(A.22)}$$

which diverges.

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3. See, e.g., S. L. Adler and R. F. Dashen, Current Algebras (W. A. Benjamin, Inc., New York, 1968) and reprints and references therein.

4. Throughout this thesis we use the time-like metric

$$a_\mu b^\mu = g_{\mu\nu} a^\mu b^\nu = a^o b^o - a^i b^i$$

and the following representation of the γ-matrices.

$$\gamma^o = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \quad \gamma_5 = i\gamma^o\gamma^1\gamma^2\gamma^3,$$

$$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu].$$

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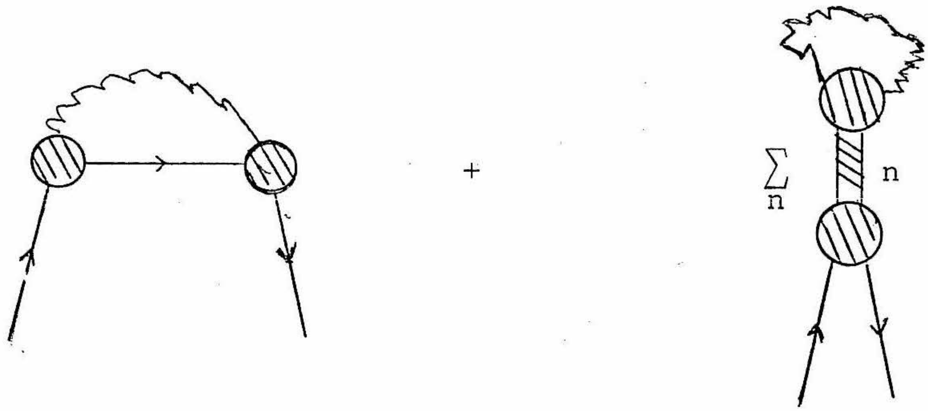
II. ELECTROMAGNETIC MASS DIFFERENCES OF THE PSEUDOSCALAR MESON AND BARYON OCTETS

II.1. INTRODUCTION

Feynman and Speisman[1] were the first to attempt to calculate the neutron-proton mass difference on the assumption that it is purely electromagnetic in origin. These authors obtained finite results by using cutoffs, which can be interpreted as electromagnetic form factors, to take into account some of the effects of the strong interactions. Although naively one would expect the Coulomb interaction to dominate and thereby make the proton heavier than the neutron, they could obtain the correct sign for the mass splitting by taking the cutoffs sufficiently high. Cini, Ferrari and Gatto[2] rephrased their argument in terms of dispersion theory and related the mass difference, given by the nucleon proper self-energy part evaluated on the mass shell, to an integral over the forward Compton scattering amplitude. They demonstrated that the experimental form factors fell off too rapidly at high momentum transfer to yield the required sign reversal. Therefore, it became imperative to search for other relevant consequences of the strong interactions.

The first advance in this direction was made by Harari,[4] who, because of the work of Cini, et al. and that of Cottingham,[3] studied the high energy behavior of the forward spin-nonflip Compton scattering amplitude. He observed that, according to the Regge pole theory, at high energies this amplitude is proportional to $\nu^{\alpha(o)}$, where ν is the customary direct channel energy variable and $\alpha(0)$ is the $t=0$ intercept of the leading Regge trajectory with the appropriate quantum numbers. For $I = 1$, these quantum numbers are $I = 1$, $C = 1$, $G = -1$ and $P = (-1)^J$. This is the trajectory of the A_2 meson which has[5] $\alpha_{A_2}(0) \approx 0.4 > 0$. Hence the dispersion relation for this amplitude requires a subtraction and a Feynman-Speisman type computation, being based on its saturation with low mass intermediate states, is not expected to work. On the other hand, for isospin two, no mesons have been observed with $(\text{mass})^2 \lesssim 2\,(\text{Bev})^2$ and this lead de Alfaro, Fubini, Rossetti and Furlan[6] to conclude that $\alpha_{I=2}(0) < 0$. Harari, then, could assert that low-lying intermediate states should dominate the $I = 2$ mass differences and the usual simple-minded estimates ought to be reasonably accurate. This, in fact, is the case.

The next physical effect to be stressed was the feedback on the electromagnetic self mass of the electromagnetic mass shifts internal to various Feynman diagrams.[7] This was then related to the existence of a nucleon-antinucleon bound state with zero residue—a ghost—with $J^P = 0^+$, $I^G = 1^-$ where the trajectory of the A_2 meson crosses the t-axis at negative mass squared.[8] This idea is illustrated schematically in Fig. 1.1. The details of the extinction of the bound



(a) (b)

Fig. 1.1. Born approximation, (a), and rescattering, (b), contributions to the mass difference.

state and the connection of the specific ghost-killing mechanism with the sign reversal will be discussed in the fourth chapter. We may interpret the ghost as the "tadpole" which Coleman and Glashow[9] postulated in order to propose a successful phenomenological explanation of the correct octet mass shifts. The SU(3) properties of the tadpole model follow because the A_2 is a member of the octet which also comprises the f_o and the K(1420). So, the octet of extinct bound states is a consequence of strong interaction dynamics rather than a new elementary particle. These concepts are amplified in a single-channel calculation of the neutron-proton mass difference by Ball and Zachariasen.[10]

We shall perform a multichannel computation of the 0^- and $\frac{1}{2}^+$ electromagnetic mass differences relying heavily on the apparatus developed in Reference 10. The next section recapitulates the required results of Ball and Zachariasen and describes the rest of the essential physics of the calculation. The Born approximation and the D-function, by means of which we include some of the strong rescattering effects, are treated in Chapters 3 and 4, respectively. The last two parts combine these in order to deduce the mass differences in our model and discuss the implications of the results.

II.2. METHOD

Ball and Zachariasen (hereinafter denoted by BZ) commence by writing the Cottingham[3] formula for the electromagnetic mass shift of a nucleon (which is exact to first order in the fine structure constant α) crossed into the pair annihilation channel. Including the factor $\frac{1}{2}$, which comes from the fact that the right hand side of Eq. (2.1) includes both Fig. 2.1(b) and (c), while either one is sufficient to yield Fig. 2.1(a), omitted by BZ, this reads

$$\delta M = \tfrac{1}{2} \int \frac{d^4q}{(2\pi)^4} \left(\frac{-i}{q^2}\right) T_{\mu}{}^{\mu}(q,-q;p,-p). \tag{2.1}$$

(BZ make a cancelling error of a factor 2 in the Born term, so all their numerical results are unaffected.) $T_{\mu\nu}(q,\overline{q};p,\overline{p})$ is the covariant T-matrix for the process $N\overline{N} \to \gamma\gamma$, for virtual photons of mass

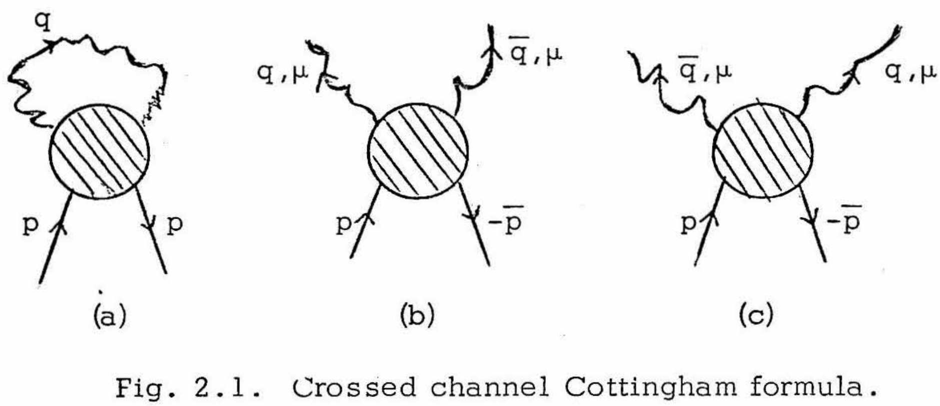


Fig. 2.1. Crossed channel Cottingham formula.

$q^2 (=\bar{q}^2)$, where p and $\bar{p}$ are the nucleon and antinucleon four-momenta, q and $\bar{q}$ are the photon four-momenta and $T_{\mu\nu}$ is normalized so that the S-matrix is given by

$$S = 1 - i(2\pi)^4 \delta^4 (q + \bar{q} - p - \bar{p}) \epsilon_\mu \epsilon_\nu \frac{\bar{v}_{\bar{p}} T^{\mu\nu} u_p}{\sqrt{16 E \bar{E} \omega \bar{\omega}}} . \tag{2.2}$$

Naturally, the Fermions are on the mass shell.

BZ sandwich (2.1) between the spinors $\bar{v}(\bar{p}, -\sigma)$ and $u(p, \sigma)$, which are written down explicitly in Appendix A, and sum over Fermion helicity σ. With

$$t = (q + \bar{q})^2 = (p + \bar{p})^2 \tag{2.3}$$

and

$$\nu = \frac{q \cdot p}{M} \tag{2.4}$$

there results

$$\delta M^2 = \frac{i}{2} \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2} T(0, \nu; q^2), \tag{2.5}$$

where

$$T(t, \nu; q^2) = - \frac{p}{2M} \sum_{\lambda, \mu} T_{\lambda\lambda, \mu\mu}(t, \nu; q^2) \tag{2.6}$$

and

$$\sum_{\lambda=-1}^{1} \sum_{\mu=-\frac{1}{2}}^{\frac{1}{2}} T_{\lambda\lambda, \mu\mu}(t, \nu; q^2) = - \sum_{\sigma} \bar{v}(\bar{p}, -\sigma) T_\mu{}^\mu (q, \bar{q}; p, \bar{p}) u(p, \sigma) . \tag{2.7}$$

As was first done by Cottingham,[3] one can rotate the contour of integration over q_o in (2.5) and perform the integration over the direction of $\vec{q}$. Thus, one finds

$$\delta M^2 = -\frac{1}{16\pi^3}\int_{-\infty}^{0}\frac{dq^2}{q^2}\int_{-\sqrt{-q^2}}^{\sqrt{-q^2}} d\nu(-q^2-\nu^2)^{\frac{1}{2}}\,T(0,i\nu;q^2). \tag{2.8}$$

Each of the helicity amplitudes $T_{\lambda\lambda,\mu\mu}$ has an especially simple t-channel partial wave expansion,[11] namely,

$$T_{\lambda\lambda,\mu\mu}(t,\nu;q^2) = \sum_J (2J+1)P_J(z)T^J_{\lambda\lambda,\mu\mu}(t;q^2), \tag{2.9}$$

where z is the cosine of the center of mass scattering angle for $N\overline{N} \rightarrow \gamma\gamma$, so that

$$\nu = \frac{t}{4M} - \frac{[(t-4M^2)(t-4q^2)]^{\frac{1}{2}}}{4M}\,z\,. \tag{2.10}$$

Equation (2.6) implies at once that $T(t,\nu;q^2)$ has the same partial wave expansion,

$$T(t,\nu;q^2) = \sum_J (2J+1)P_J(z)T^J(t;q^2). \tag{2.11}$$

BZ now define in the physical region for $N\overline{N} \rightarrow \gamma\gamma$, the quantity

$$X(t;q^2) = \int_{-1}^{1} dz(1-z^2)^{\frac{1}{2}}\,T(t,\nu;q^2) \tag{2.12}$$

and prove by analytic continuation to $t = 0$ that

$$X(0;q^2) = -\frac{1}{q^2}\int_{-\sqrt{-q^2}}^{\sqrt{-q^2}} d\nu(-q^2-\nu^2)^{\frac{1}{2}}\,T(0,i\nu;q^2). \tag{2.13}$$

Substituting (2.13) into (2.8) gives

$$\delta M^2 = \frac{1}{16\pi^3}\int_{-\infty}^{0} dq^2 X(0;q^2), \tag{2.14}$$

so that substituting (2.11) into (2.12) and this into (2.14) presents us with the grand result, the t-channel partial wave expansion of the mass shift,

$$\delta M^2 = \frac{1}{16\pi^3} \sum_J C_J \int_{-\infty}^{0} dq^2 T^J(0;q^2), \tag{2.15}$$

where

$$C_J = (2J+1) \int_{-1}^{1} dz \sqrt{1-z^2}\; P_J(z). \tag{2.16}$$

Obviously, $C_J = 0$ for odd J. For J even we find

$$C_J = \frac{2J+1}{1-J} \frac{2\pi}{(\frac{1}{2}J)!\,(\frac{1}{2}J+1)!} (\tfrac{1}{2} - \tfrac{1}{2}J)^2 (\tfrac{3}{2} - \tfrac{1}{2}J)^2 \cdots (\tfrac{1}{2})^2. \tag{2.17}$$

The first four C_J's are the following:

$$\begin{aligned} C_0 &= \frac{\pi}{2} = 1.57, \\ C_2 &= -\frac{5\pi}{16} = -0.98, \\ C_4 &= -\frac{9\pi}{128} = -0.22, \\ C_6 &= -\frac{65\pi}{2048} = -0.10. \end{aligned} \tag{2.18}$$

Hence, even without reference to the low energy approximations used later, we see there is a measure of reason for keeping only low partial waves.

It is through Eq. (2.15) that the attempt is made to include the relevant phenomena of the $N\overline{N}$ system at zero total energy.

Equation (2.15) holds unaltered for the pseudoscalar mass shifts with the definition

$$T(t, \nu; q^2) = \sum_{\lambda=-1}^{1} T_{\lambda\lambda}(t, \nu; q^2) \tag{2.19}$$

$$= -T_{\mu}^{\ \mu}(q, \overline{q}; k, \overline{k}), \tag{2.20}$$

where k and $\overline{k}$ are the meson momenta. To obtain mass differences, we have merely to subtract the mass shifts for appropriately chosen particle-antiparticle pairs.

The whole analysis explicated above generalizes straightforwardly to the multichannel case. Letting each Latin index denote a particle, the basic formula (2.15) becomes

$$\delta M_{ij}^{\ 2} = \frac{1}{16\pi^3} \sum_J C_J \int_{-\infty}^{0} dq^2 T_{ij}^{\ J}(0; q^2). \tag{2.21}$$

Equation (2.21) can be employed to compute the transition masses $m_{\pi^0\eta}$ and $m_{\Sigma^0\Lambda}$ (corresponding to terms $-m^2_{\pi^0\eta}\varphi_{\pi^0}\varphi_{\eta}$ and $-m_{\Sigma^0\Lambda}\overline{\psi}_{\Sigma^0}\psi_{\Lambda}$ in the effective Lagrangian), but since these particles are not degenerate, in a lowest order perturbation calculation in α, it suffices to evaluate the diagonal elements of the mass-shift matrix. The modification of the masses arising from diagonalizing that matrix is second order in α. Using the experimental values,[12] the contributions to the Σ^0 and π^0 shifts are 0.06 Mev and less than 10^{-4} Mev, respectively.

The first approximation made in applying Eq. (2.21) has been mentioned already: we shall keep only the first few partial waves, specifically, $J = 0, 2, 4, 6$. The negligible contributions of the $J = 4$ and $J = 6$ partial waves will demonstrate that this simplification is justified. Beyond that, we shall replace T^J by the Born approximation B^J (Fig. 1.1(a)) for all partial waves other than $J=0$. Other than the obvious desire to save labor, we have three reasons for this. The first is the low energy approximation we are going to make; second, there is no known physical phenomenon, corresponding to the vanishing of the A_2 trajectory for the $J=0$ channel, in the higher partial waves which would give significant deviations from this substitution; and lastly, the C_J's are decreasing in size as J increases. So (2.21) becomes

$$\delta M_{ij}^2 = \overline{\delta M_{ij}^2} + (\delta M_{ij}^2)_{J=0} - \overline{(\delta M_{ij}^2)}_{J=0}, \tag{2.22}$$

with

$$\delta M_{ij}^2 = \sum_J (\delta M_{ij}^2)_J, \quad \overline{\delta M_{ij}^2} = \sum_J \overline{(\delta M_{ij}^2)}_J \tag{2.23}$$

and

$$(\delta M_{ij}^2)_J = \frac{C_J}{16\pi^3} \int_{-\infty}^{0} dq^2 T_{ij}^J(0;q^2), \tag{2.24}$$

$$\overline{(\delta M_{ij}^2)}_J = \frac{C_J}{16\pi^3} \int_{-\infty}^{0} dq^2 B_{ij}^J(0;q^2). \tag{2.25}$$

The problem has been reduced to calculating $T^{o}_{ij}(0;q^2)$. If we restrict ourselves to two-particle intermediate states, the unitarity condition for this amplitude is particularly simple. This comes about because in the center of mass system the component of orbital angular momentum in the direction of motion of the two particles is zero, so that $J=0$ implies the two particles must have the same helicity. Therefore, the $J^P=0^+$ amplitudes, obtained by summing over equal helicities, couple to no others. The unitarity condition, then, in matrix form referred to the states $|ij\rangle$, reads (suppressing the superscript 0)

$$\text{Im}\,T(t;q^2) = T^*(t;q^2)\rho(t)h(t), \tag{2.26}$$

where $\rho(t)$ is the diagonal phase space matrix and $h(t)$ is the strong interaction amplitude for our two-body channels with $J^P = 0^+$.

The unitarity condition for the amplitude $h(t)$ is

$$\text{Im}\,h(t) = h^*(t)\rho(t)h(t), \tag{2.27}$$

with the same $\rho(t)$, since the latter depends solely on the intermediate states which are identical in (2.26) and (2.27).

We can show easily that if we separate the left and right cuts of $h(t)$ in the standard manner by including them in the matrices $N(t)$ and $D(t)$, respectively,[13]

$$h = ND^{-1}, \tag{2.28}$$

then the amplitude $F(t) = T(t)D(t)$ has no right cut.[14] Equation (2.26) can be rewritten in the form

$$T = T^{*}(1 + 2i\rho h) \tag{2.29}$$

and (2.27) and (2.28) together give

$$\mathrm{Im}D = -\rho\, N. \tag{2.30}$$

Then on the right cut

$$\mathrm{Im}F = \frac{1}{2i}(TD - T^{*}D^{*}) \tag{2.31}$$

$$= T^{*}[\frac{1}{2i}(D-D^{*}) + \rho\, hD] \tag{2.32}$$

$$= T^{*}(\mathrm{Im}D + \rho\, N) \tag{2.33}$$

$$= 0, \tag{2.34}$$

where we used successively (2.29), (2.28) and (2.30). This theorem permits us to write the dispersion relation

$$T_{ij}(t;q^2) = \sum_{\substack{kl\\mn}} \frac{1}{\pi}\int_{-\infty}^{t_{kl}} dt' \frac{[\,\mathrm{Im}T_{kl}(t';q^2)]D_{kl,mn}(t')[D^{-1}(t)]_{mn,ij}}{t' - t}, \tag{2.35}$$

provided we assume that $T(t;q^2) \to 0$ as $t \to \infty$. Without the D-function in this dispersion relation we would be calculating the mass differences entirely from exchanges and finding the usual incorrect signs. The D-function takes into account the rescattering effects indicated in Fig. 1.1(b). Since we believe most of the contribution to (2.35) comes from low energies, we shall approximate ImT by ImB under the integral sign. We shall discuss this in more detail in Chapter 5.

In any case, with this simplification, the calculation of the masses is direct.

Using SU(3) symmetry will facilitate our computations tremendously. We shall take particular care in the pseudoscalar octet to try to assure that the errors due to this approximation should not exceed about 25%. The first step, then, is to expand the amplitudes T_{ij} in irreducible representations of SU(3). We know that

$$\underset{\sim}{8} \times \underset{\sim}{8} = \underset{\sim}{27} + \underset{\sim}{10} + \overline{\underset{\sim}{10}} + \underset{\sim}{8} + \underset{\sim}{8}' + \underset{\sim}{1}, \tag{2.36}$$

so that

$$T_{ij} = \sum_{d_\gamma, d_{\gamma'}, \nu} \langle \gamma\gamma \,|\, d_\gamma \nu \rangle \langle d_\gamma \nu \,|\, T \,|\, d_{\gamma'} \nu \rangle \langle d_{\gamma'} \nu \,|\, ij \rangle, \tag{2.37}$$

where d_γ and $d_{\gamma'}$ denote the representations in (2.36) and $|\nu\rangle = |YII_z\rangle$. Since the photon is the U-spin singlet member of an octet, the state

$$|\gamma\gamma\rangle = |U=0, U=0\rangle \tag{2.38}$$

$$= \tfrac{1}{2}(\sqrt{3}\,|010\rangle - |000\rangle)\tfrac{1}{2}(\sqrt{3}\,|010\rangle - |000\rangle) \tag{2.39}$$

is symmetric in SU(3). Hence d_γ must be a symmetric representation of SU(3) and only the following amplitudes survive in the expansion (2.37):

$$\langle 1|T|1\rangle = T_1,$$

$$\langle 8|T|8\rangle = T_8,$$

$$\langle 8|T|8'\rangle = T_{8'}, \tag{2.40}$$

$$\langle 27|T|27\rangle = T_{27}.$$

By SU(3) symmetry these amplitudes are independent of ν, the direction in unitary spin space. Clearly, the singlet term contributes equally to all mass shifts and hence can be neglected in calculating the mass differences. With the help of Eqs. (2.39) and (2.40) and McNamee and Chilton's[15] SU(3) Clebsch-Gordan coefficients we can write out (2.37) explicitly for the baryon octet. We employ the same phase conventions as they do for physical states and display it in Appendix B. With those definitions, the unitary singlet is completely symmetric.

$$T_{p\bar{p}} = -\frac{3}{20}T_8 + \frac{1}{4}\sqrt{\frac{1}{5}}\,T_{8'} + \frac{3}{20}T_{27},$$

$$T_{n\bar{n}} = \frac{3}{20}T_8 - \frac{1}{4}\sqrt{\frac{1}{5}}\,T_{8'} - \frac{3}{20}T_{27},$$

$$T_{\Sigma^+\bar{\Sigma}^+} = \frac{1}{2}\sqrt{\frac{1}{5}}\,T_{8'} - \frac{1}{4}T_{27}, \tag{2.41}$$

$$T_{\Sigma^0\bar{\Sigma}^0} = \frac{1}{2}T_{27},$$

$$T_{\Sigma^-\bar{\Sigma}^-} = -\frac{1}{2}\sqrt{\frac{1}{5}}\,T_{8'} - \frac{1}{4}T_{27},$$

$$T_{\Xi^0\overline{\Xi}^0} = \frac{3}{20} T_8 + \frac{1}{4}\sqrt{\frac{1}{5}}\, T_{8'} - \frac{3}{20} T_{27},$$

$$T_{\Xi^-\overline{\Xi}^-} = -\frac{3}{20} T_8 - \frac{1}{4}\sqrt{\frac{1}{5}}\, T_{8'} + \frac{3}{20} T_{27}.$$

From (2.41) we get the isospin amplitudes exhibited below.

I = 1:

$$T_{p\overline{p}} - T_{n\overline{n}} = -\frac{3}{10} T_8 + \frac{1}{2}\sqrt{\frac{1}{5}}\, T_{8'} + \frac{3}{10} T_{27},$$

$$T_{\Sigma^+\overline{\Sigma}^+} - T_{\Sigma^-\overline{\Sigma}^-} = \sqrt{\frac{1}{5}}\, T_{8'},$$

$$T_{\Xi^0\overline{\Xi}^0} - T_{\Xi^-\overline{\Xi}^-} = \frac{3}{10} T_8 + \frac{1}{2}\sqrt{\frac{1}{5}}\, T_{8'} - \frac{3}{10} T_{27}. \qquad (2.42)$$

I = 2:

$$T_{\Sigma^+\overline{\Sigma}^+} + T_{\Sigma^-\overline{\Sigma}^-} - 2T_{\Sigma^0\overline{\Sigma}^0} = -\frac{3}{2} T_{27}.$$

There is a simple partial check on Eq. (2.41): the Σ^-, Ξ^0, Ξ^- amplitudes can be deduced from the Σ^+, n, p amplitudes by means of the symmetry of the Clebsch-Gordan coefficients. The isospin one amplitudes in (2.42) must, and do, satisfy the Coleman-Glashow[16] relation

$$(T_{\Sigma^-\overline{\Sigma}^-} - T_{\Sigma^+\overline{\Sigma}^+}) + (T_{p\overline{p}} - T_{n\overline{n}}) = T_{\Xi^-\overline{\Xi}^-} - T_{\Xi^0\overline{\Xi}^0}. \qquad (2.43)$$

Equation (2.42) is identical for the pseudoscalar octet, except for the fact that Bose statistics requires $T_{8'} = 0$ since with J = 0 the particles must be in a symmetric unitary spin state.

In order to keep this calculation within reasonable proportions, we discard all intermediate states other than two 0^- octets and two $\frac{1}{2}^+$ octets. We shall discuss the likely validity of this drastic measure in view of our results in Chapter 6. Let us, then, display the matrices in the coupled many-channel formalism. The superscripts B and P stand for the baryon-antibaryon and meson-meson channels, respectively.

$$T^8 = \begin{pmatrix} T_8^B & T_{8'}^B & T_8^P \end{pmatrix},$$

$$T^{27} = \begin{pmatrix} T_{27}^B & T_{27}^P \end{pmatrix},$$

$$h^8 = \begin{pmatrix} h_{88}^{BB} & h_{88'}^{BB} & h_{88}^{BP} \\ h_{8'8}^{BB} & h_{8'8'}^{BB} & h_{8'8}^{BP} \\ h_{88}^{PB} & h_{88'}^{PB} & h_{88}^{PP} \end{pmatrix}, \tag{2.44}$$

$$h^{27} = \begin{pmatrix} h_{27}^{BB} & h_{27}^{BP} \\ h_{27}^{PB} & h_{27}^{PP} \end{pmatrix}.$$

The matrices of Eq. (2.44) will be the ones appearing in the unitarity conditions (2.26) and (2.27) and hence in the fundamental equation, (2.35), which will now read

$$T_\alpha(t;q^2) = \sum_{\beta\gamma} \frac{1}{\pi} \int_{-\infty}^{t_\beta} dt' \frac{[\mathrm{Im} B_\beta(t';q^2)] D_{\beta\gamma}(t')}{t' - t} [D^{-1}(t)]_{\gamma\alpha} \tag{2.45}$$

for both the octet and 27 amplitudes.

In order to be specific, we shall write out explicitly the normalization of our amplitudes.

$$\begin{aligned} T^{\gamma P} &= \sum_{\lambda=-1}^{1} T^{\gamma P}_{\lambda\lambda}, \\ T^{\gamma B} &= -\frac{p}{\sqrt{2}M} \frac{1}{\sqrt{2}} \sum_{\lambda,\mu} T^{\gamma B}_{\lambda\lambda,\mu\mu}, \\ h^{PP} &= h^{PP}, \\ h^{PB} &= -\frac{p}{\sqrt{2}M} \frac{1}{\sqrt{2}} \sum_{\mu} h^{PB}_{\mu\mu}, \\ h^{BB} &= \left(-\frac{p}{\sqrt{2}M} \frac{1}{\sqrt{2}}\right)^2 \sum_{\mu\mu'} h^{BB}_{\mu\mu,\mu'\mu'}, \end{aligned} \tag{2.46}$$

where the sums are all over helicities, as before, p is the baryon momentum in the center of mass and M is the baryon mass. The same definitions (2.46) hold with or without the partial wave analysis (2.9). The amplitudes defined in (2.46) will turn out to have no kinematic singularities. The baryon and meson phase space factors are

$$\rho_B = \frac{M^2}{2\pi} \frac{1}{\sqrt{t(t - 4M^2)}} \qquad \text{and} \qquad \rho_P = \frac{1}{16\pi}\sqrt{\frac{t - 4\mu^2}{t}} \; . \tag{2.47}$$

The Cottingham formula includes driving terms, the effect of the electromagnetic shift of internal masses in Feynman diagrams and the effect of the electromagnetic modification of strong coupling constants. In Dashen's calculation,[17] the coupling constant shifts do not contribute to the mass difference. We give examples of diagrams which include internal mass and coupling constant shifts below.

Fig. 2.2. Examples of diagrams which include mass, (a), and coupling constant, (b), shifts.

We can now understand schematically why with our t-channel unitarity approximation we are taking into account some internal mass shifts, but no coupling constant shifts. The diagrams of Fig. 2.3 are unitarity diagrams with the intermediate particles on the mass shell.

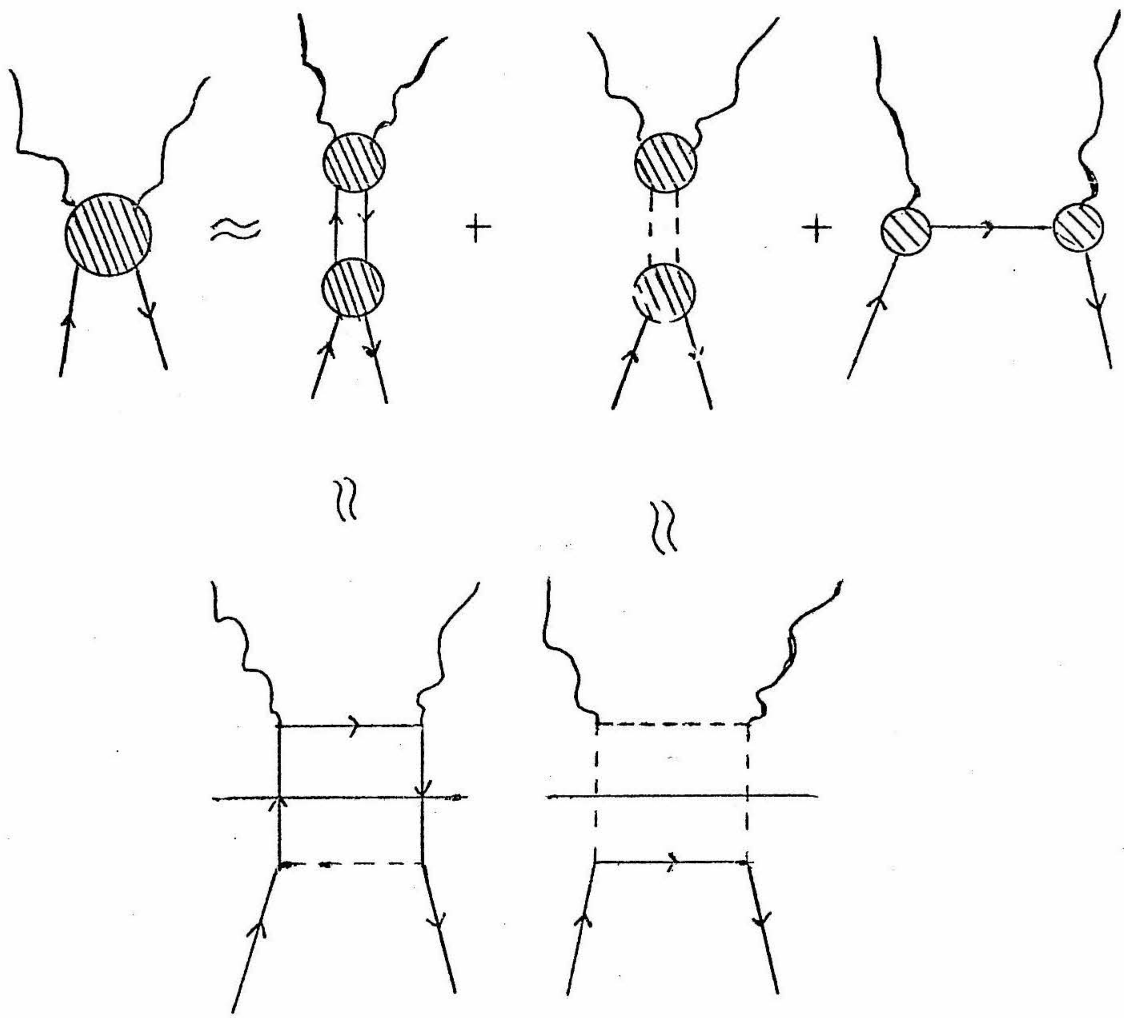


Fig. 2.3. Born term and internal mass shift type terms in mass difference computation.

II.3. BORN APPROXIMATION

First, we use Eq. (2.42), in the notation of Eq. (2.44), to express, for the baryons, the SU(3) amplitudes in terms of the individual particle amplitudes. We find,

$$T_1^8 = \frac{10}{3}\left(T_{n\bar{n}} - T_{p\bar{p}}\right) + T_{\Sigma^+\bar{\Sigma}^+} - \frac{7}{3}\,T_{\Sigma^-\bar{\Sigma}^-} + \frac{4}{3}\,T_{\Sigma^0\bar{\Sigma}^0}\,,$$

$$T_2^8 = \sqrt{5}\left(T_{\Sigma^+\bar{\Sigma}^+} - T_{\Sigma^-\bar{\Sigma}^-}\right), \tag{3.1}$$

$$T_1^{27} = -\frac{2}{3}\left(T_{\Sigma^+\bar{\Sigma}^+} + T_{\Sigma^-\bar{\Sigma}^-} - 2T_{\Sigma^0\bar{\Sigma}^0}\right).$$

For each pair of particles, by writing the vertex in terms of the standard form factors $F_1(q^2)$ and $F_2(q^2)$, we can evaluate the contribution of Fig. 3.1.

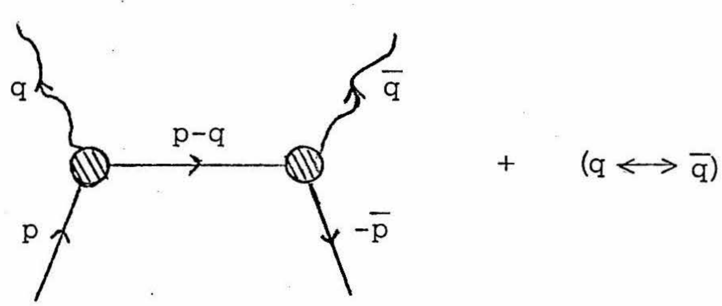


Fig. 3.1. Contribution to the baryon Born term from octet exchange.

$$B_\mu^{\ \mu} = i(-i)\left(F_1\gamma_\mu - iF_2\sigma_{\mu\nu}\bar{q}^\nu\right)\frac{i}{\not{p} - \not{q} - M}$$

$$\times (-i)\left(F_1\gamma^\mu - iF_2\sigma^{\mu\lambda}q_\lambda\right) + (q \longleftrightarrow \bar{q}). \tag{3.2}$$

This amplitude is simplified by means of the well-known rules for manipulating γ-matrices and then the helicity sum in (2.7) is performed using the trick explained in Appendix A. The result, (2.6) is then analyzed into partial waves through (2.11). One obtains

$$B^J(t;q^2) = -\frac{4M^2}{pq}\,Q_J\left(\frac{q^2 - \frac{1}{2}t}{2pq} - i\epsilon\right)\mathfrak{F}(t;q^2), \tag{3.3}$$

where

$$p = (\tfrac{1}{4}t - M^2)^{\frac{1}{2}}, \qquad q = (\tfrac{1}{4}t - q^2)^{\frac{1}{2}} \tag{3.4}$$

and

$$\mathfrak{F}(t;q^2) = \left(1 + \frac{q^2}{2M^2} - \frac{t}{2M^2}\right)[F_1(q^2)]^2$$

$$+ \left(\frac{3q^2}{M} - \frac{t}{M} - \frac{q^2 t}{4M^3}\right)F_1(q^2)F_2(q^2)$$

$$+ \left(2q^2 + \frac{q^4}{4M^2} + \frac{t^2}{16M^2} - t - \frac{q^2 t}{4M^2}\right)[F_2(q^2)]^2. \tag{3.5}$$

This agrees with the result of BZ (except for the factor two mentioned before). We should state that in deriving Eq. (3.3) the non-pole terms in z were dropped. BZ calculate that these would contribute less than 0.2 Mev to the neutron-proton mass difference. This does not necessarily imply, however, that non-pole terms arising from other form factors, permitted by the intermediate baryon's being off shell, are also negligible. In the dispersion language these terms are associated with subtractions in ν. Ignoring them in view of the lack of experimental data will receive some justification from our results.

With the SU(3) relations among the form factors[16]

$$F_{\Sigma^+} = F_p, \quad F_\Lambda = \tfrac{1}{2} F_n, \quad F_{\Xi^0} = F_n,$$

$$F_{\Xi^-} = F_{\Sigma^-} = -(F_p + F_n), \quad F_{\Sigma^0} = -\tfrac{1}{2} F_n, \tag{3.6}$$

and

$$F_{\Lambda\Sigma^0} = -\frac{\sqrt{3}}{2} F_n,$$

we obtain the SU(3) decomposition of the Born term from (3.1) and (3.3).

$$[B^J(t;q^2)]_1^8 = -\frac{28M^2}{3pq} Q_J(r)\left(\mathcal{F}_{n\bar{n}} - 2\mathcal{F}_{p\bar{p}} - \mathcal{F}_{p\bar{n}}\right),$$

$$[B^J(t;q^2)]_2^8 = \frac{4\sqrt{5}\, M^2}{pq} Q_J(r)\left(\mathcal{F}_{n\bar{n}} + \mathcal{F}_{p\bar{n}}\right), \tag{3.7}$$

$$[B^J(t;q^2)]_1^{27} = \frac{2}{7} [B^J(t;q^2)]_1^8,$$

where

$$r = \frac{q^2 - \frac{1}{2}t}{2pq} - i\epsilon$$

and M is the mean octet mass,[18]

$$M = \tfrac{1}{2}\left[\tfrac{1}{2}(m_N + m_\Xi) + \tfrac{1}{4}(3m_\Lambda + m_\Sigma)\right] = 1.14 \text{ Bev}. \tag{3.8}$$

Since the decuplet also lies low in mass and is essential to the understanding of low energy baryon-meson physics, we calculated the contribution of decuplet exchange. In Fig. 3.1, the exchanged particle now is to be interpreted as a member of the decuplet and the blob as the form factor for the photoexcitation of the baryon octet. It is an excellent approximation to assume that this vertex is dominated by a magnetic dipole transition.[19,20] Then we can write it in explicitly gauge invariant form as

$$-e\,\frac{C_3(q^2)}{m_\pi}\;\gamma_5\left(q_\nu\gamma_\mu - \not{q}g_{\mu\nu}\right), \tag{3.9}$$

where μ and ν are the photon and $3/2^+$ polarizations, respectively. The spin 3/2 propagator is given by Gasiorowicz,[21] with the overall sign wrong. Now we have

$$\begin{aligned} B_\mu^{\ \mu} = {} & i(-e)\,\frac{C_3(q^2)}{m_\pi}\;\gamma_5\left(\overline{q}_\nu\gamma_\mu - \overline{\not{q}}g_{\mu\nu}\right)\frac{i(\not{k}+M^*)}{k^2-M^{*2}} \\ & \times\left[-g^{\nu\lambda}+\tfrac{1}{3}\gamma^\nu\gamma^\lambda+\frac{2}{3M^{*2}}\,k^\nu k^\lambda-\frac{1}{3M^*}\left(k^\nu\gamma^\lambda-k^\lambda\gamma^\nu\right)\right] \\ & \times(-e)\,\frac{C_3(q^2)}{m_\pi}\,\gamma_5\left(q_\lambda\gamma^\mu - \not{q}g_\lambda^\mu\right) + (q\longleftrightarrow\overline{q}), \end{aligned} \tag{3.10}$$

where $k = p - q$ and M^* is the mean decuplet mass,

$$M^* = \frac{1}{4}\left(m_\Delta + m_{\Sigma^*} + m_{\Xi^*} + m_{\Omega^-}\right) = 1.46 \text{ Bev}. \tag{3.11}$$

With the same manipulations as those described after Eq. (3.2), we obtain

$$B^J(t;q^2) = -\frac{4\pi\alpha\,|C_3(q^2)|^2}{3m_\pi^2 pq}\, Q_J(x)\,\tilde{\mathscr{F}}(t;q^2), \tag{3.12}$$

where

$$x = \frac{q^2 - \frac{1}{2}t + M^2 - M^{*2}}{2pq},$$

$$\tilde{\mathscr{F}}(t;q^2) = \sum_{n=-2}^{+1} M^{*n} A_n(t;q^2),$$

$$A_{-2} = \left(q^2 - M^{*2} - M^2\right)\left(M^{*2} - M^2 + q^2\right)^2,$$

$$A_{-1} = \frac{t}{M}\left(q^2 - M^{*2} - M^2\right)\left(M^{*2} - M^2 + q^2\right),$$

$$A_o = -2\left[2M^{*2}(\tfrac{1}{2}t - q^2) + (M^{*2} - M^2)(q^2 - M^2 + M^{*2})\right],$$

$$A_{+1} = \frac{1}{M}\left[4(t - 3M^2)(\tfrac{1}{2}t - q^2) + t(M^{*2} - M^2)\right].$$

Using the SU(3) results[12]

$$C_{\Delta^+ p} = C_3, \qquad C_{\Delta^o n} = C_3, \qquad C_{\Xi^{o*}\Xi^o} = -C_3,$$

$$C_{\Sigma^{+*}\Sigma^+} = -C_3, \qquad C_{\Sigma^{o*}\Lambda^o} = \frac{\sqrt{3}}{2} C_3, \quad C_{\Sigma^{o*}\Sigma^o} = -\frac{1}{2} C_3,$$

$$C_{\Sigma^{-*}\Sigma^-} = 0, \qquad C_{\Xi^{-*}\Xi^-} = 0, \tag{3.13}$$

we arrive at the decomposition, analogous to (3.7), for the decuplet contribution

$$[B^J(t;q^2)]_1^8 = \frac{4}{3} B^J(t;q^2),$$

$$[B^J(t;q^2)]_2^8 = \sqrt{5}\ B^J(t;q^2), \tag{3.14}$$

$$[B^J(t;q^2)]_1^{27} = -\frac{1}{3} B^J(t;q^2),$$

where $B^J(t;q^2)$ is given by (3.12).

Naturally, (3.7) and (3.14) must be added in order to calculate the Born approximation to the baryon mass differences. Equation (3.14) does not contribute to the $I=1$ neutron-proton mass difference because the only non-strange member of the decuplet, the Δ, has $I=3/2$ and hence cannot couple to a nucleon and an $I=0$ photon. For an $I=1$ mass difference, one of the photons must have $I=1$ and the other $I=0$.

Lastly, we have to evaluate the Born term for mesons.[38] Since we are excluding non-pole terms, the seagull diagram will not contribute and Fig. 3.1 yields

$$B_\mu^{\ \mu} = i[-iF_\pi(q^2)](2k-q)_\mu \frac{i}{(k-q)^2-\mu^2}[-iF_\pi(q^2)](k-\bar{k}-q)^\mu + (q \leftrightarrow \bar{q}), \tag{3.15}$$

where $k^\mu, \bar{k}^\mu$ are the meson four-momenta. Hence

$$B^J(t;q^2) = [F_\pi(q^2)]^2 \left(\frac{\frac{1}{2}t + q^2 - 4\mu^2}{kq}\right) Q_J(x), \tag{3.16}$$

where

$$k = (\tfrac{1}{4}t - \mu^2)^{\frac{1}{2}} \quad \text{and} \quad x = \frac{q^2 - \frac{1}{2}t}{2kq}.$$

Numerical investigation shows that (3.16), at low q^2 and t, changes by less than 10% as μ varies from m_π to m_K. So we are justified in using a mean meson mass $\mu = 0.49$ Bev, deduced from

$$\mu^2 = \tfrac{1}{2}\left[m_K^2 + \tfrac{1}{4}(3m_\eta^2 + m_\pi^2)\right]. \tag{3.17}$$

In the meson case, the Coleman-Glashow relations (3.6) reduce to

$$F_{\pi^+} = F_{K^+} = -F_{K^-} = -F_{\pi^-}$$

and

$$F_{K^0} = F_{\overline{K}^0} = F_\eta = F_{\pi^0} = F_{\eta\pi^0} = 0. \tag{3.18}$$

Therefore, finally we obtain

$$\begin{aligned} [B^J(t;q^2)]_3^8 &= -\frac{14}{3} B^J(t;q^2), \\ [B^J(t;q^2)]_2^{27} &= -\frac{4}{3} B^J(t;q^2), \end{aligned} \tag{3.19}$$

where $B^J(t;q^2)$ is given by Eq. (3.16).

In order to arrive at the numerical values of the Born approximation to the mass differences, we employ the following experimental form factors:[10,19,20,22]

$$\begin{aligned} F_1^P(q^2) &= eF(q^2), & F_1^n(q^2) &= 0, \\ F_2^P(q^2) &= 1.79\left(\frac{e}{2M}\right)F(q^2), & F_2^n(q^2) &= -1.91\left(\frac{e}{2M}\right)F(q^2), \\ F_\pi(q^2) &= eF(q^2), & C_3(q^2) &= 0.37\, F(q^2), \end{aligned} \tag{3.20}$$

where

$$F(q^2) = \frac{1}{(1 - q^2/q_o^2)^2} , \quad q_o^2 = 0.72 \ (\text{Bev})^2 .$$

When calculating $\delta\mu_i$ from $\delta\mu_i^2$ for mesons, it is essential, due to the large percentage symmetry breaking, to remember that, as stated after Eq. (3.16), $\delta\mu_i^2$ is nearly independent of μ^2. So we use $\delta\mu_i = 1/2\mu_i \ \delta\mu_i^2$, where μ_i is the exact mass. For the baryons (this is only relevant if we try to get accuracy better than 20%) it turns out that $\delta m_i^2/M$ is nearly independent of M, so we take

$$\delta m_i = \frac{m_i}{M} \frac{\delta m_i^2}{2m_i} = \frac{\delta m_i^2}{2M} .$$

The results of the numerical integrations are given in Table 3.1, below. For the baryons, the octet and decuplet contributions are exhibited separately in the first and second lines for each splitting and their sum is displayed in the third line. All numbers are in Mev and the experimental column is taken from Reference 18. The table makes apparent the poor agreement for I=1, the good agreement for I=2 and the excellent convergence of the partial wave analysis.

TABLE 3.1

BORN APPROXIMATION TO THE MASS DIFFERENCES

I	J	0	2	4	6	Total	Experiment
1	$\overline{m}_p - \overline{m}_n$	+0.496 0 +0.496	+0.139 0 +0.139	-0.021 0 -0.021	+0.007 0 +0.007	+0.621 0 +0.621	-1.294 ± 0.010
	$\overline{m}_{\Sigma^+} - \overline{m}_{\Sigma^-}$	-0.302 -0.95 -1.25	-0.057 -0.05 -0.62	+0.006 0 +0.006	-0.002 0 -0.002	-0.355 -1.00 -1.36	-7.92 ± 0.30
	$\overline{m}_{\Xi^o} - \overline{m}_{\Xi^-}$	-0.795 -0.95 -1.75	-0.196 -0.05 -0.25	+0.027 0 +0.027	-0.008 0 -0.008	-0.972 -1.00 -1.97	-6.55 ± 0.88
	$\overline{m}_{K^+} - \overline{m}_{K^o}$	+0.691	+0.137	-0.016	+0.005	+0.817	-3.94 ± 0.27
2	$\overline{m}_{\Sigma^+} + \overline{m}_{\Sigma^-} - 2\overline{m}_{\Sigma^o}$	+1.29 -0.48 +0.81	+0.335 -0.02 +0.315	-0.047 0 -0.047	+0.015 0 +0.015	+1.59 -0.50 +1.09	+1.80 ± 0.54
	$\overline{m}_{\pi^+} - \overline{m}_{\pi^o}$	+2.49	+0.495	-0.058	+0.018	+2.95	+4.603 ± 0.026

II.4. D-FUNCTION

The last piece of information required before applying the basic result (2.45) is the $J^P = 0^+$ D-function for our baryon-antibaryon and meson-meson channels. We know that the A_2 trajectory passes through zero at $t = t_o \approx -0.6\ (\mathrm{Bev})^2$.[23,24] In order to understand what this implies for the D-function, suppose, for the moment, that we have only one sense and one nonsense channel. Then, near the pole, we can write the partial wave amplitude as

$$T^J = \frac{1}{J-\alpha}\begin{pmatrix} \beta_{SS} & \sqrt{\alpha}\,\beta_{SN} \\ \sqrt{\alpha}\,\beta_{SN} & \beta_{NN} \end{pmatrix} = ND^{-1}, \tag{4.1}$$

where we extracted explicitly the $\sqrt{\alpha}$, suggested by models, from the sense-nonsense residue. Factorization[25] gives

$$\alpha\beta_{SN}^2 = \beta_{SS}\beta_{NN}. \tag{4.2}$$

Equation (4.2) allows four possibilities as $\alpha \to 0$.

(i) Choosing sense mechanism:

$$\beta_{SS} \to 1, \qquad \beta_{NN} \to \alpha .$$

In this case the sense-sense amplitude T^0 has a pole and this must be ruled out for a ghost $(t < 0)$.

(ii) Gell-Mann mechanism:[26]

$$\beta_{SS} \to \alpha , \qquad \beta_{NN} \to 1.$$

Thus the trajectory chooses nonsense and the sense-sense amplitude has no singularity.

(iii) No compensation mechanism:[27]

$$\beta_{NN} \to \alpha, \qquad \beta_{SS} \to \alpha^2.$$

The trajectory couples to the nonsense-nonsense amplitude with vanishing residue.

(iv) Chew mechanism:[28]

$$\beta_{NN} \to \alpha^2, \quad \beta_{SS} \to \alpha.$$

This is just the reverse of the no compensation mechanism. Here the dynamics operates to make the residue of the sense-sense amplitude vanish as $\alpha \to 0$.

BZ observe that, in the single channel case, if the sense D-function had a zero at t_0, then over most of the range of integration in (2.45) $D(t)/D(0)$ would be negative and the correct sign would be obtained for the neutron-proton mass difference. We shall see that the zero occurs only if the ghost killing mechanism is that of Chew. For this purpose, we employ the N/D equations in the form[29]

$$N(t) = B(t) + \frac{1}{\pi}\int \frac{dt'}{t'-t}\,[B(t') - B(t)]\rho\,(t')N(t'),$$

$$D(t) = 1 - \frac{1}{\pi}\int \frac{dt'}{t'-t}\,\rho(t')N(t'). \tag{4.3}$$

The essential structure of Eq. (4.3) is the following.

$$(1 + X)N = B, \tag{4.4}$$

$$D = 1 + YN, \tag{4.5}$$

where X and Y are integral operators, X being "proportional" to B and Y being diagonal.

If B has diagonal elements vanishing like $\sqrt{\alpha}$, $(1 + X)$, therefore $(1+X)^{-1}$ and, by Eq. (4.4), N will have the same property. Equation (4.5) then implies that D will, as well. Putting such an N and D into Eq. (4.1) yields a set of four equations. If we demand $D_{11} \to 0$ as $\alpha \to 0$, these equations select choosing sense—unacceptable for a ghost—or the Chew mechanism.

In the multichannel case the source of the sign change in (2.45) is not so perspicuous, but we must have $\det D(t_o) = 0$ if the Chew mechanism does indeed apply.

Chu and Roy,[23] by considering finite energy sum rules for a sense-nonsense amplitude in photoproduction, are able to rule out the Gell-Mann mechanism for the coupling of the A_2 trajectory to $N\overline{N}$. Such an amplitude, of course, does not distinguish between the no compensation and Chew mechanisms. Fits to πN scattering[27,30] seem to indicate that the P' trajectory chooses no compensation and, on the basis of SU(3) symmetry, this would compel us to assume the same dynamical ghost killing mechanism for the A_2. This conclusion receives further support from the recent application of finite energy sum rules to KN scattering by Graham and Huq.[31] These authors deal with the A_2 trajectory directly and find that the no compensation mechanism is present. On the other hand, the reaction $\pi^- p \rightarrow \eta n$ appears to require the A_2 to couple via the Chew mechanism.[24]

Thus, the experimental evidence seems to favor no compensation, but not conclusively. If the application of the Chew mechanism by BZ turns out to contradict experiment, the original, single channel, physical understanding of the sign reversal will disappear and we shall have considerably less confidence that the tensor meson trajectories suffice to explain the mass differences.

In order to find a simple model for the D-function which satisfies unitarity, analyticity and the symmetry of the T-matrix, we shall employ the Balázs method[32] with a single pole. The residue matrix will be determined by matching to single particle exchanges. Assuming that the amplitude is dominated by the "tadpole," due to the vanishing of the tensor meson trajectory, not only at $t=t_0$, but even at $t=0$, we shall fix the pole position and the three matching points, one each for $B\overline{B} \rightarrow B\overline{B}$, $B\overline{B} \rightarrow PP$ and $PP \rightarrow PP$, by requiring, at $t=0$, that

$$h^{BB}_{88'}/h^{BB}_{88} \approx h^{BB}_{8'8'}/h^{BB}_{88'} \approx h^{BP}_{8'8}/h^{BP}_{88} \approx -2, \tag{4.6}$$

the F/D ratio of the A_2 coupling to baryons determined from high energy scattering data.[33] This value is well-known to be consistent with that necessary to fit the observed electromagnetic mass differences with the tadpole model.[34]

Demanding even a crude simulation of (4.6) in our model leaves no freedom. The determinant of the octet D-function automatically develops a zero around $t \approx -0.2$ (Bev)^2, so that we are not forced to employ the Chew mechanism as a fitting criterion. The experimental data determine only the product of the A_2 couplings to baryons and mesons.[24] Since the t-dependence of the Balázs method is completely different from the Regge form, the extrapolation of the couplings from t_o to 0 is dubious. For these reasons, as well as the lack of leeway left after attempting to match the D/F ratios, we did not try to force our residues to agree with the magnitude of the A_2 couplings.

We proceed to evaluate rough estimates of the amplitudes h, denoted by B, defined in Eqs. (2.44) and (2.46). The SU(3) phases, projection operators and crossing matrices are defined in Appendix B.

$PP \to PP$:

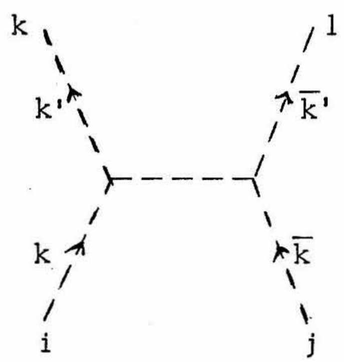
The S-wave interaction may be replaced by the exchange of a scalar meson σ, assumed to be a unitary singlet.[35] The direct channel diagram, then, will not alter the mass differences. The effective Lagrangian, in the eight-component formalism, is

$$\mathcal{L} = -\tfrac{1}{2} g_{\sigma\pi\pi} \sigma \sum_i P_i^2 . \tag{4.7}$$

The results of Furlan and Rossetti, based on the saturation of current algebra with a few single particle states, agree with those of Brown and Singer, who fit three pion η and K decays. These authors quote[35]

$$m_\sigma \approx 400 \text{ Mev} \quad \text{and} \quad \frac{g^2_{\sigma\pi\pi}}{4\pi} \approx 0.108\,(\text{Bev})^2. \tag{4.8}$$

In Fig. 4.1 we show the Feynman diagrams for σ exchange, writing explicitly the octet indices of the mesons.



$$+ \quad (k',k) \longleftrightarrow (\bar{k}',l)$$

Fig. 4.1. σ exchange.

Separating out the singlet projection operator, using the SU(3) octet-octet crossing matrix, and analyzing into partial waves, we get the σ contribution

$$B^8_{33} = B^{27}_{22} = -\frac{g^2_{\sigma\pi\pi}}{\vec{k}^2} Q_0\left(1 + \frac{m_\sigma^2}{2\vec{k}^2}\right). \tag{4.9}$$

The other obvious force is provided by the exchange of the octet of vector mesons coupling to the unitary spin current. Bose statistics forbid the singlet vector meson from coupling to a pseudoscalar pair.

The Lagrangian is

$$\mathcal{L} = i f_{\rho\pi\pi} V_i^{\mu} P^T F_i \partial_{\mu} P, \tag{4.10}$$

where[18] $m_\rho = 765$ Mev and $\Gamma_{\rho\pi\pi} = 130$ Mev give

$$\frac{f^2_{\rho\pi\pi}}{4\pi} = 2.5. \tag{4.11}$$

With V exchange in Fig. 4.1, this yields, as above,

$$\begin{aligned} B^8_{33} &= \tfrac{1}{2} B, \\ B^{27}_{22} &= -\tfrac{1}{3} B, \end{aligned} \tag{4.12}$$

where

$$B = -3f^2_{\rho\pi\pi}\left(\frac{2t + m_V^2 - 4\mu^2}{\vec{k}^2}\right) Q_0\left(1 + \frac{m_V^2}{2\vec{k}^2}\right)$$

and m_V is the mean octet vector meson mass

$$m_V = m_{K^*} = 0.890 \text{ Bev}. \tag{4.13}$$

Owing to $\varphi - \omega$ mixing, here we do not average the two sides of the mass formula as we did for the pseudoscalar and spin $\frac{1}{2}$ octets. In the calculations leading to (4.12) we dropped all non-pole terms in order to be consistent with the prescription used in the evaluation of the Born terms. This is also in accord with the no subtraction idea in dispersion theory and the fact that compositeness and Regge poles exclude Kronecker deltas in the angular momentum from appearing in the amplitude.

Numerical comparison of (4.9) and (4.12) demonstrates that, at $t=0$, the vector contribution is at least four times larger than that of the σ and, for $t\neq 0$, much larger than that. In view of the roughness of our computations, this justifies neglecting σ exchange.

$B\overline{B} \rightarrow PP$:

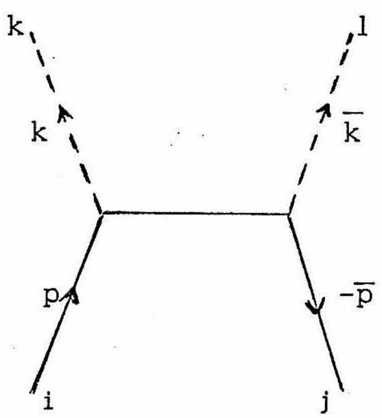
Here the dominant low energy processes are baryon octet and decuplet exchange. For octet exchange, the Lagrangian is

$$\mathcal{L} = -2ig\overline{B}\gamma_5[\alpha D_i + (1-\alpha)F_i]BP_i, \tag{4.14}$$

with[12]

$$\frac{g^2}{4\pi} = 15 \quad \text{and} \quad \alpha \approx 0.7.$$

The Feynman diagrams are drawn in Fig. 4.2.



$+ \ (k,k) \longleftrightarrow (\overline{k},l)$

Fig. 4.2 Baryon exchange.

After the usual prescription is followed, we obtain

$$B^{27}_{12} = B^{27} = -[\tfrac{1}{3}\alpha^2 - (1-\alpha)^2]B,$$

$$B^{8}_{13} = B^{8}_{31} = [\tfrac{1}{2}\alpha^2 - \tfrac{3}{2}(1-\alpha)^2]B, \qquad (4.15)$$

$$B^{8}_{23} = B^{8}_{32} = \sqrt{5}\,\alpha(1-\alpha)B,$$

where

$$B = 8g^2 x\, Q_0(x),$$

$$x = \frac{\mu^2 - \frac{1}{2}t}{2pk}.$$

The amplitude in (4.15) is symmetric by time reversal invariance. Computing $PP \rightarrow B\overline{B}$ checks this part of the calculation independently.

Decuplet exchange is easily formulated in terms of quark indices. The coupling is unique and the coupling constant can be deduced from the width of the 33 resonance.[20]

$$\mathcal{L} = \lambda \epsilon^{ade} \overline{\Delta}^{\mu}_{abc} B^{b}_{d} \partial_{\mu} P^{c}_{e} + \text{h.c.}, \qquad (4.16)$$

where Δ^{μ}_{abc} represents the $3/2^+$ decuplet field and

$$\lambda = 15.4\ (\text{Bev})^{-1}. \qquad (4.17)$$

We find, for the decuplet term,

$$B^{27}_{12} = B^{27}_{21} = \tfrac{1}{3}B,$$

$$B^{8}_{13} = B^{8}_{31} = 2B, \qquad (4.18)$$

$$B^{8}_{23} = B^{8}_{32} = \sqrt{5}\,B,$$

where

$$B = \frac{\lambda^2}{3Mpk} Q_o(x)\Big\{\Big[M(\mu^2 - M^{*2} - M^2) + 2M^*(\tfrac{1}{4}t - M^2)\Big]$$

$$\times \Big[(\tfrac{1}{2}t - \mu^2) + \frac{1}{4M^{*2}}(M^{*2} - M^2 + \mu^2)^2\Big] + \frac{1}{8}t\Big(M^* t + \frac{1}{M^*}$$

$$\times \Big[(M^{*2} - \mu^2)(M^{*2} + M^2 - \mu^2) - M^2(3M^{*2} - M^2 + \mu^2)\Big]\Big)\Big\},$$

$$x = \frac{\mu^2 - \frac{1}{2}t + M^2 - M^{*2}}{2pk} .$$

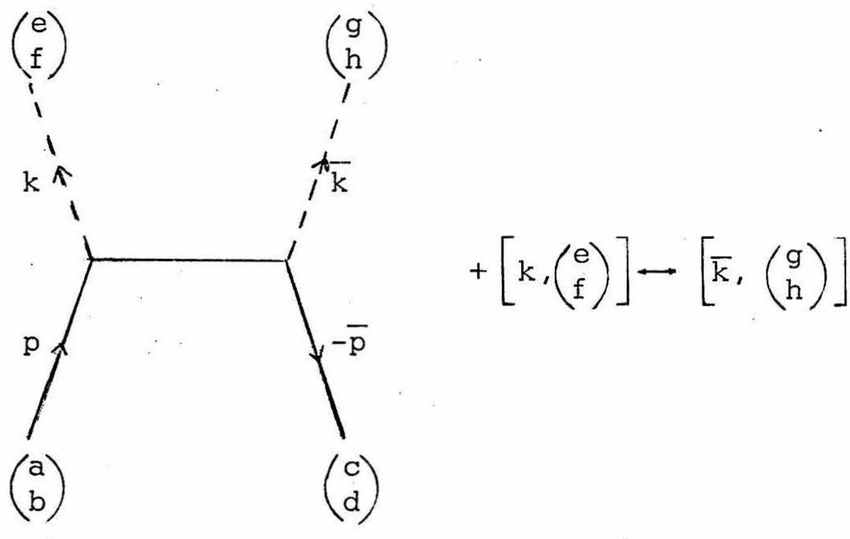


Fig. 4.3. Decuplet exchange.

$B\overline{B} \to B\overline{B}$:

The calculation for this part of the amplitude is the same as before except for the fact that performing the spin sums requires a generalization of the previous technique. This is explained in

Appendix A. Ball, Scotti, and Wong[36] fit nucleon-nucleon scattering for $s \geqq 4M^2$ and $t < 0$—precisely the region of interest to us—by means of single meson exchanges. We take their values of the coupling constants and of the σ mass. For σ, P and V exchange, the Feynman diagrams look like Fig. 4.4.

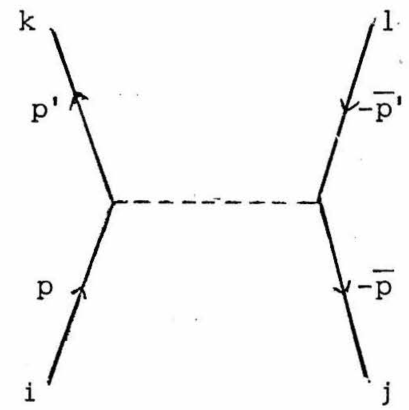


Fig. 4.4. Baryon-antibaryon interaction.

Even though Ball, et al., deal only with the nucleon-nucleon system, we know that the σ meson they use must be an SU(3) singlet since if it were a member of an octet, they would be forced to include a $J^P = 0^+$, $I = 1$, $S = 0$ meson in their calculations. The Lagrangian is

$$\mathcal{L} = -g_{\sigma NN} \overline{B} B \sigma, \tag{4.19}$$

where

$$m_\sigma \approx 530 \text{ Mev} \quad \text{and} \quad \frac{g_{\sigma NN}^2}{4\pi} = 4.15.$$

The σ, then, contributes

$$B^{27}_{11} = B^{8}_{11} = B^{8}_{22} = -\frac{g^2_{\sigma NN}}{4M^2\vec{p}^2}\left[(M^2+\tfrac{1}{4}m_\sigma^2)t - M^2(4M^2-m_\sigma^2)\right]$$

$$\times Q_o\left(1+\frac{m_\sigma^2}{2\vec{p}^2}\right), \qquad (4.20)$$

$$B^{8}_{12} = B^{8}_{21} = 0.$$

Since $(m_\sigma/2M)^2 \approx 1/16$, this can be approximated to read

$$B^{27}_{11} = B^{8}_{11} = B^{8}_{22} \approx -g^2_{\sigma NN}Q_o\left(1+\frac{m_\sigma^2}{2\vec{p}^2}\right), \qquad (4.21)$$

$$B^{8}_{12} = B^{8}_{21} = 0.$$

For pseudoscalar exchange, the Lagrangian is just given by (4.14) and the result is

$$\begin{aligned} B^{27}_{11} &= [\tfrac{1}{3}\alpha^2 - (1-\alpha)^2]B, \\ B^{8}_{11} &= [-\tfrac{1}{2}\alpha^2 + \tfrac{3}{2}(1-\alpha)^2]B, \\ B^{8}_{22} &= [\tfrac{5}{6}\alpha^2 + \tfrac{3}{2}(1-\alpha)^2]B, \\ B^{8}_{12} &= B^{8}_{21} = \sqrt{5}\,\alpha(1-\alpha)B, \end{aligned} \qquad (4.22)$$

where

$$B = \frac{g^2\mu^2}{M^2}\,Q_o\left(1+\frac{\mu^2}{2\vec{p}^2}\right).$$

In Reference 36 on octet of vector mesons is used with an electric coupling which is pure F. The magnetic coupling, on the other hand, has a D/F ratio roughly identical to the one for the PBB coupling.[12]

This value yields a weak coupling to the isoscalar member of the octet, which agrees, using vector dominance, with the smallness of the isoscalar nucleon anomalous moment. Then the Lagrangian is

$$\mathcal{L} = -f_{\rho NN}\overline{B}\gamma_\mu F_i B V_i^\mu - \frac{g_{\rho NN}}{2M}\overline{B}\sigma_{\mu\nu}\left[\alpha D_i + (1-\alpha)F_i\right]B\left(\partial^\mu V_i^{\,\nu} - \partial^\nu V_i^\mu\right). \tag{4.23}$$

From the fit of Ball, et al.,

$$\frac{f_{\rho NN}^2}{4\pi} = 5.64\,, \tag{4.24}$$

and assuming that the ρ dominates the isovector electric and magnetic form factors of the nucleon leads to

$$\frac{g_{\rho NN}^2}{4\pi} = 19.3. \tag{4.25}$$

With (4.23) one finds the results

$$B_{11}^{27} = A\left\{X - 2Y - \left[\frac{1}{3}\alpha^2 - (1-\alpha)^2\right]Z\right\},$$

$$B_{11}^{8} = A\left\{-\frac{3}{2}X + 3Y - \left[-\frac{1}{2}\alpha^2 + \frac{3}{2}(1-\alpha)^2\right]Z\right\},$$

$$B_{22}^{8} = A\left\{-\frac{3}{2}X + 3Y - \left[\frac{5}{6}\alpha^2 + \frac{3}{2}(1-\alpha)^2\right]Z\right\},$$

$$B_{21}^{8} = B_{12}^{8} = \sqrt{5}\,\alpha\,A\left\{\frac{Y}{1-\alpha} - (1-\alpha)Z\right\},$$

where (4.26)

$$A = \frac{1}{2M\,\vec{p}^2}\,Q_0\left(1 + \frac{m_V^2}{2\vec{p}^2}\right),$$

$$X = f^2_{\rho NN}\left[4\vec{p}^4 + M^2(2\vec{p}^2 + m_V^2)\right],$$

$$Y = 3f_{\rho NN}g_{\rho NN}(1 - \alpha)m_V^2\vec{p}^2,$$

$$Z = \frac{g^2_{\rho NN}m_V^2}{2M^2}\left[\vec{p}^2(8\vec{p}^2 + m_V^2) + 2M^2m_V^2\right].$$

The total $B\overline{B} \to B\overline{B}$ amplitude, of course, is the sum of (4.21), (4.22) and (4.26).

Having the amplitudes B(t), we approximate them by a one-pole form. As before, the same equations hold for the 2×2 $\underset{\sim}{27}$ amplitudes and the 3×3 $\underset{\sim}{8}$ amplitudes.

$$B(t) = \frac{R}{t-t_1}. \tag{4.27}$$

In order to determine R^{27} and R^8, we use three different matching points t_{m_1}, t_{m_2}, and t_{m_3} for $PP \to PP$, $B\overline{B} \to PP$, and $B\overline{B} \to B\overline{B}$, respectively. In each case, the matching point must be to the right of the right end of the left cut for the reaction in question. From (4.12), (4.15), and (4.22), this implies

$$\begin{aligned} t_{m_1} &> 4\mu^2 - m_V^2 = 0.17\ (\text{Bev})^2, \\ t_{m_2} &> 4\mu^2 - \frac{\mu^4}{M^2} = 0.92\ (\text{Bev})^2, \\ t_{m_3} &> 4M^2 - \mu^2 = 5.0\ (\text{Bev})^2. \end{aligned} \tag{4.28}$$

Then the standard Balázs method[32] yields the D-function. N and D are defined by (2.28), so that, on the left cut,

$$\text{Im } N(t) = \text{Im } h(t)D(t) \tag{4.29}$$

$$= \text{Im } B(t)D(t) \tag{4.30}$$

$$= -\pi R D(t_1)\delta(t-t_1). \tag{4.31}$$

Therefore,

$$N(t) = \frac{RD(t_1)}{t - t_1} \tag{4.32}$$

and (no subtraction is required)

$$D(t) = 1 - \frac{1}{\pi}\int \frac{dt'}{t' - t}\,\rho(t')N(t'), \tag{4.33}$$

where $\rho(t)$ is the symmetric matrix given by

$$\rho_{ij} = \rho_i \delta_{ij} \theta(t - \overline{t}_i). \tag{4.34}$$

The phase space factors ρ_B and ρ_P are given by (2.47) and the thresholds are obviously $\overline{t}_B = 4M^2$ and $\overline{t}_P = 4\mu^2$. There is an apparent difficulty here insofar as the tip of the left cut for $B\overline{B} \to B\overline{B}$, $4M^2 - \mu^2$, is to the right of the two-meson threshold $4\mu^2$. For this process, however, only the $4M^2$ threshold matters in Eq. (4.33) and this gives no trouble. This phenomenon is discussed in more detail by Kayser.[37]

Equation (4.33) can be written much more simply by separating out the trivial t-dependence:

$$D(t) = 1 + I(t)C, \tag{4.35}$$

where

$$C = -\frac{1}{\pi} RD(t_1) \tag{4.36}$$

and

$$I(t) = \int \frac{\rho\,(t')dt'}{(t'-t)(t'-t_1)}\,. \tag{4.37}$$

Equation (4.35) evidently determines

$$D(t_1) = 1 - \frac{1}{\pi} I(t_1) R D(t_1). \tag{4.38}$$

Hence

$$D(t_1) = \left[1 + \frac{1}{\pi} I(t_1) R\right]^{-1}. \tag{4.39}$$

In order to avoid any possible confusion owing to the omission of indices, we display the matrices I(t).

$$I^8(t) = \begin{pmatrix} I_B(t) & 0 & 0 \\ 0 & I_B(t) & 0 \\ 0 & 0 & I_P(t) \end{pmatrix}$$

and (4.40)

$$I^{27}(t) = \begin{pmatrix} I_B(t) & 0 \\ 0 & I_P(t) \end{pmatrix},$$

where

$$I_B(t) = \int_{4M^2}^{\infty} \frac{\rho_B(t')dt'}{(t'-t)(t'-t_1)}$$

$$= \frac{M^2}{2\pi} \frac{1}{t_1 - t}\left[\frac{1}{tb} \log\left(\frac{b+1}{b-1}\right) - \frac{1}{t_1 b_1} \log\left(\frac{b_1+1}{b_1-1}\right)\right] \tag{4.41}$$

and

$$I_P(t) = \int_{4\mu^2}^{\infty} \frac{\rho_P(t')dt'}{(t'-t)(t'-t_1)}$$

$$= \frac{1}{16\pi}\,\frac{1}{t_1-t}\left[a\,\log\left(\frac{a+1}{a-1}\right) - a_1\,\log\left(\frac{a_1+1}{a_1-1}\right)\right]. \tag{4.42}$$

a, b, a_1, b_1 are defined below.

$$a = \sqrt{\frac{4\mu^2 - t}{-t}}\,, \qquad b = \sqrt{\frac{4M^2 - t}{-t}}\,,$$

$$a_1 = \sqrt{\frac{4\mu^2 - t_1}{-t_1}}\,, \qquad b_1 = \sqrt{\frac{4M^2 - t_1}{-t_1}}\,. \tag{4.43}$$

Parenthetically, we observe a useful partial check on our numerical calculations. By considering the matrix (use (4.36) and (4.39))

$$C^{-1} = -\pi[D(t_1)]^{-1}R^{-1} \tag{4.44}$$

$$= -\pi R^{-1} - I(t_1), \tag{4.45}$$

and noting that R, and hence R^{-1}, are symmetric, we see at once that C is symmetric.

The fitting procedure was described above. In Eq. (4.6) we employ

$$h(0) = \frac{RD(t_1)[D(0)]^{-1}}{-t_1}. \tag{4.46}$$

The matching points are varied as permitted by Eq. (4.28) and, in order to avoid a spurious singularity in (4.33), the pole position t_1

must be restricted to the left of the lower threshold, i.e.,

$$t_1 < 4\mu^2 = 0.96\ (\mathrm{Bev})^2. \tag{4.47}$$

All the fits had the same qualitative features and gave very similar answers for the mass splittings. We exhibit two of the fits. The difference between them gives a fair picture of the amount of variation present.

$\bar{t}$ is the location of the zero of the determinant of the octet D-function. The parameters of (4.6) are, all at $t=0$,

$$\begin{aligned} x &= h^8_{12}/h^8_{11}, \\ y &= h^8_{22}/h^8_{12}, \\ z &= h^8_{23}/h^8_{13}. \end{aligned} \tag{4.48}$$

Fit I:

$$t_1 = 0.9, \quad t_{m_1} = 2.2, \quad t_{m_2} = 1.3, \quad t_{m_3} = 7.0.$$

$$\bar{t} = -0.2, \quad x = -1.10, \quad y = -1.27, \quad z = -3.12.$$

$$C^{27} = \begin{pmatrix} -8.47 & 0.19 \\ 0.19 & -9.07 \end{pmatrix},$$

$$C^8 = -\begin{pmatrix} 207 & 99 & 23 \\ 99 & 168 & 20 \\ 23 & 20 & 16 \end{pmatrix}. \tag{4.49}$$

Fit II:

$$t_1 = 0.9 \qquad t_{m_1} = 1.4, \qquad t_{m_2} = 1.0, \qquad t_{m_3} = 6.9.$$

$$\bar{t} = -0.2, \qquad x = -1.18, \qquad y = -1.71, \qquad y = -4.17.$$

$$C^{27} = \begin{pmatrix} -8.15 & 0.28 \\ 0.28 & -7.35 \end{pmatrix}$$

$$C^{8} = -\begin{pmatrix} 114 & 18 & 6 \\ 18 & 98 & 5 \\ 6 & 5 & 15 \end{pmatrix}. \tag{4.50}$$

The ghost-killing should come from the dynamics, but, since our calculation is so crude, we attempted also to put it in by hand by subtracting the numerator matrix at t_o. The Balázs method can still be employed (with a subtraction of which the results are independent), but now the A_2 parameters could not be fitted at all.

II.5. RESULTS

In this chapter we insert the results of the last two sections into Eq. (2.45) to calculate the pair annihilation amplitudes and hence the mass differences. In the interest of clarity, let us ignore the decuplet exchange Born term for the moment. We define, referring to (3.7) and (3.19),

$$
\begin{aligned}
b_i^{\alpha}(t;q^2) &= \frac{1}{pq} Q_0(r), \qquad \text{for} \quad \alpha = 8,\ i = 1,2 \\
&\qquad\qquad\qquad\quad \text{and} \quad \alpha = 27,\ i = 1 \\
b_i^{\alpha}(t;q^2) &= \frac{1}{kq} Q_0(x) \qquad \text{for} \quad \alpha = 8,\ i = 3 \\
&\qquad\qquad\qquad\quad \text{and} \quad \alpha = 27,\ i = 2.
\end{aligned}
\tag{5.1}
$$

Then we can rewrite the Born term

$$B_i^{\alpha}(t;q^2) = b_i^{\alpha}(t;q^2)\,\mathcal{F}_i^{\alpha}(t;q^2). \tag{5.2}$$

Since we are dealing only with $J = 0$, we suppress the J superscript.

So the absorptive parts are

$$\operatorname{Im} B_i^{\alpha}(t;q^2) = \begin{cases} \frac{\pi}{2}\,(pq)^{-1}\mathcal{F}_i^{\alpha}(t;q^2) \\ \frac{\pi}{2}\,(kq)^{-1}\mathcal{F}_i^{\alpha}(t;q^2). \end{cases} \tag{5.3}$$

The $\mathcal{F}_i^{\alpha}(t;q^2)$ are polynomials in t and the functions $(pq)^{-1}$ and $(kq)^{-1}$ are sharply peaked at the right extremity of the left cut,

$$t_i^\alpha = \begin{cases} 4q^2 - \dfrac{q^4}{M^2} \\ 4q^2 - \dfrac{q^4}{\mu^2} , \end{cases} \tag{5.4}$$

as long as q^2 is small. α and i are always defined as in (5.1). If the unsubtracted dispersion relation were valid for the real amplitude, we could assume that most of the contribution, for low q^2, which dominates owing to the form factors, comes from low t. Then we could approximate (2.45) by evaluating the polynomial $\mathcal{F}_i^\alpha$ and the slowly-varying D(t') at the tip of the left cut, (5.4), and performing the integral over the remainder of the integrand. If $\mathrm{Im}B_i^\alpha$ is a good approximation to $\mathrm{Im}T_i^\alpha$ at low energies, this is still a valid rough estimate, in spite of the divergence of Eq. (2.45). This reasoning yields

$$T_i^\alpha(0;q^2) = \sum_{jk} \frac{1}{\pi} \int_{-\infty}^{t_j^\alpha} \frac{[\mathrm{Im}b_j^\alpha(t';q^2)]\mathcal{F}_j^\alpha(t';q^2)}{t'} D_{jk}^\alpha(t')\left\{[D^\alpha(0)]^{-1}\right\}_{ki} \tag{5.5}$$

$$\approx \sum_{jk} \mathcal{F}_j^\alpha(t_j^\alpha;q^2) D_{jk}^\alpha(t_j^\alpha)\left\{[D^\alpha(0)]^{-1}\right\}_{ki} \times \frac{1}{\pi}\int_{-\infty}^{t_j^\alpha} \frac{\mathrm{Im}\, b_j^\alpha(t';q^2)}{t'} \tag{5.6}$$

$$= \sum_{jk} b_j^\alpha(0;q^2)\mathcal{F}_j^\alpha(t_j^\alpha;q^2)D_{jk}^\alpha(t_j^\alpha)\left\{[D^\alpha(0)]^{-1}\right\}_{ki} . \tag{5.7}$$

To (5.7) must be added the contribution of decuplet exchange, (3.14), with the same D-function. It is handled in precisely the same manner and we note only that the left cut here extends to (using (3.12))

$$2\left(1+\frac{M^2}{M^{*2}}\right)q^2 - \left(1-\frac{M^2}{M^{*2}}\right)^2 M^{*2} - \frac{q^4}{M^{*2}} . \tag{5.8}$$

It is interesting, but quite disagreeable, to observe that, since $q^2 \leqq 0$, the lowest mass exchanges do not yield all the nearest singularities. So, $\mathrm{Im}\, B \neq \mathrm{Im}\, T$, even close to the tip of the left cut. To understand this, suppose a mass squared $\bar{s}$ is exchanged in the s-channel. (The u-channel gives the same result by crossing symmetry.) Then (5.8) gives the position of the tip of the left cut with $M^{*2} \to \bar{s}$. Therefore, it is located at

$$t_o(\bar{s}) = -\bar{s} - \frac{(M^2 - q^2)^2}{\bar{s}} + 2(M^2 + q^2). \tag{5.9}$$

This is a maximum when

$$\bar{s} = \bar{s}_o \equiv M^2 - q^2. \tag{5.10}$$

Then

$$t_o(\bar{s}) = 4q^2. \tag{5.11}$$

For very low q^2, this is just slightly to the right of $4q^2 - q^4/M^2$, the end of the $\frac{1}{2}^+$ exchange cut. This can happen if

$$\bar{s}_o = M^2 - q^2 \geqq (M+\mu)^2. \tag{5.12}$$

For the actual nucleon and pion masses, the catastrophe can occur if $-q^2 > 0.26\ (\mathrm{Bev})^2$. In the next section we shall discuss the possible significance of this phenomenon.

Before quoting the results of the final numerical integration for the $J=0$ term in the mass differences, we notice in (5.4) that, consistent with our continually used assumption that small t and small q^2 dominate,

$$|q^2| << 4M^2 = 5.2\ (\mathrm{Bev})^2. \tag{5.13}$$

Therefore,

$$4|q^2| \gg \frac{q^4}{M^2} \tag{5.14}$$

and we can replace the tip of the baryon octet exchange cut by $4q^2$. In the same way, we neglect q^4/M^{*2} in (5.8). These approximations are necessary for the convergence of the integral (2.15) for the masses.

$J = 0$ Mass Differences (Mev)

Fit I:

$I = 1$:

$$m_p - m_n = -3.90$$

$$m_{\Sigma^+} - m_{\Sigma^-} = -3.30$$

$$m_{\Xi^0} - m_{\Xi^-} = +0.55$$

$$m_{K^+} - m_{K^0} = +1.20$$

$I = 2$:

$$m_{\Sigma^+} + m_{\Sigma^-} - 2m_{\Sigma^0} = +1.98$$

$$m_{\pi^+} - m_{\pi^0} = +3.60.$$

Fit II:

I = 1:

$$m_p - m_n = -3.50$$

$$m_{\Sigma^+} - m_{\Sigma^-} = -3.61$$

$$m_{\Xi^0} - m_{\Xi^-} = -0.11$$

$$m_{K^+} - m_{K^0} = +1.35.$$

I = 2:

$$m_{\Sigma^+} + m_{\Sigma^-} - 2m_{\Sigma^0} = +1.98$$

$$m_{\pi^+} - m_{\pi^0} = +3.52.$$

We combine these with the results of Table 3.1 by means of Eq. (2.22). The experimental values are given in Table 3.1.

Corrected Total Mass Splittings (Mev)

Fit I:

I = 1:

$$m_p - m_n = -3.78$$

$$m_{\Sigma^+} - m_{\Sigma^-} = -3.46$$

$$m_{\Xi^0} - m_{\Xi^-} = +0.32$$

$$m_{K^+} - m_{K^0} = +1.33.$$

I = 2:

$$m_{\Sigma^+} + m_{\Sigma^-} - 2m_{\Sigma^0} = +2.26$$

$$m_{\pi^+} - m_{\pi^0} = +4.06.$$

<u>Fit II</u>:

I = 1:

$$m_p - m_n = -3.38$$

$$m_{\Sigma^+} - m_{\Sigma^-} = -3.72$$

$$m_{\Xi^0} - m_{\Xi^-} = -0.34$$

$$m_{K^+} - m_{K^0} = +1.48.$$

I = 2:

$$m_{\Sigma^+} + m_{\Sigma^-} - 2m_{\Sigma^0} = +2.26$$

$$m_{\pi^+} - m_{\pi^0} = +3.98$$

II.6. DISCUSSION

The results of the calculation exhibit the ancient dichotomy between the $I = 2$ and $I = 1$ mass differences. The $I = 2$ Σ mass splitting is within the experimental error and the pion mass difference is too small by a mere 15%—quite understandable in view of the large discrepancy between the actual pion mass and the mean pseudoscalar octet mass. The outcome of the isospin one computation, the raison d'être of the whole work, since the $I = 2$ mass differences are sufficiently well explained by the Born approximation, is a dismal failure. The correct sign for the nucleon mass difference is very likely an accident in view of the extreme disagreement for the Σ and Ξ, not to mention the kaon, for which not even the sign reversal is obtained.

We can segregate the sources of error into two classes. The first kind are "technical" in nature. These include dropping of the non-pole terms and the method of evaluating the dispersion integral (2.45). It seems highly unlikely that the way out of the predicament lies in this direction, however, because the isospin two mass splittings come out quite well and these manipulative maneuvers probably do not discriminate with respect to isospin.

As far as "physical" effects are concerned, there are two possibilities. First, the dynamics we have put in may be insufficient to explain the existence and properties of the tensor mesons. This could be due not only to the simplistic one-pole Balázs method, but also to the inclusion of too few exchanges and, possibly, not enough external channels. The fact that high mass exchanges in the pair annihilation amplitude give rise to cuts extending as far to the right as those due to low mass exchanges, supports this hypothesis. It does not imply, however, that better low energy approximations cannot work. One may need a Reggeized bootstrap calculation in order to deal with the tensor mesons. If poor description of these particles is the chief cause of the random agreement, a D-function based more directly on phenomenology should cure the difficulty. The information on the magnitude of the 2^+ trajectory couplings is incomplete, but one could try to employ the exchange degeneracy[33] with the better known 1^- trajectories.

It is also possible, indeed, not entirely implausible considering the experimental evidence for the no compensation mechanism for tensor meson trajectories, that the 2^+ mesons simply do not explain the mass differences. Then it is much harder to conceive of a simple, yet reasonable, way to parametrize the D-function. The dispersion relation (2.45) itself is based on elastic unitarity and, as suggested in the previous paragraph, this may be too crude an approximation.

APPENDIX A

SPIN SUMS

The two expressions required for the calculation of amplitudes in this thesis are

$$X = \sum_{\sigma} \overline{v}_{\overline{p},-\sigma} K u_{p,\sigma}, \tag{A.1}$$

$$Y = \sum_{\sigma,\sigma'} \left(\overline{v}_{\overline{p},-\sigma} K_2 v_{\overline{p}',-\sigma'} \right) \left(\overline{u}_{p',\sigma'} K_1 u_{p,\sigma} \right). \tag{A.2}$$

The Dirac spinors are

$$u_{p,\sigma} = (E+m)^{\frac{1}{2}} \begin{pmatrix} 1 \\ \dfrac{\vec{\sigma}\cdot\vec{p}}{E+m} \end{pmatrix} \chi_{\sigma} \tag{A.3}$$

$$= \frac{\not{p}+m}{(E+m)^{\frac{1}{2}}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \chi_{\sigma}, \tag{A.4}$$

$$v_{p,\sigma} = (E+m)^{\frac{1}{2}} \begin{pmatrix} \dfrac{\vec{\sigma}\cdot\vec{p}}{E+m} \\ 1 \end{pmatrix} (-i\sigma_y)\chi_{\sigma}^{*} \tag{A.5}$$

$$= \frac{\not{p}-m}{(E+m)^{\frac{1}{2}}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} (i\sigma_y)\chi_{\sigma}^{*}, \tag{A.6}$$

where χ_{σ} is the two-component Pauli spinor for spin σ along the unit vector $\hat{p}$. We take $\hat{p} = \hat{e}_z$ in order to have real χ_{σ}. Then we have, as well,

$$\sigma_x \chi_\sigma = \chi_{-\sigma} . \tag{A.7}$$

Therefore,

$$\overline{v}_{p,-\sigma} = \chi^{\dagger}_{-\sigma} (i\sigma_y) (0 \quad 1) \frac{\not{p}-m}{(E+m)^{\frac{1}{2}}} \tag{A.8}$$

$$= \chi^{\dagger}_{\sigma} (0 \quad -\sigma_z) \frac{\not{p}-m}{(E+m)^{\frac{1}{2}}} , \tag{A.9}$$

where we used (A.7) to obtain (A.9).

Since

$$(1 \quad 0)\gamma_z = (0 \quad \sigma_z), \tag{A.10}$$

therefore,

$$X = -r \sum_{\sigma} \chi^{\dagger}_{\sigma} (1 \quad 0)\gamma_z (\overline{\not{p}}-m) K (\not{p}+m) \begin{pmatrix} 1 \\ 0 \end{pmatrix} \chi_\sigma , \tag{A.11}$$

where

$$r = [(E+m)(\overline{E}+m)]^{-\frac{1}{2}} .$$

Noting that

$$P \begin{pmatrix} \chi_\sigma \\ 0 \end{pmatrix} = \begin{pmatrix} \chi_\sigma \\ 0 \end{pmatrix} \quad \text{and} \quad P \begin{pmatrix} 0 \\ \chi_\sigma \end{pmatrix} = 0, \tag{A.12}$$

where

$$P = \tfrac{1}{2}(1 + \gamma_o), \tag{A.13}$$

and defining the four-component rest spinors

$$w^{(1,2)} = \begin{pmatrix} \chi_{\uparrow,\downarrow} \\ 0 \end{pmatrix} \quad \text{and} \quad w^{(3,4)} = \begin{pmatrix} 0 \\ \chi_{\uparrow,\downarrow} \end{pmatrix}, \tag{A.14}$$

we find

$$X = -r \sum_{r=1}^{4} w^{(r)\dagger} \gamma_z (\bar{\not{p}}-m) K(\not{p}+m) P w^{(r)} \tag{A.15}$$

$$= -r \; \mathrm{tr}[\gamma_z (\bar{\not{p}}-m) K(\not{p}+m) \tfrac{1}{2}(1+\gamma_0)] \tag{A.16}$$

$$= -\tfrac{1}{2} r \; \mathrm{tr}(GK), \tag{A.17}$$

where

$$G = (\not{p}+m)(1+\gamma_0)\gamma_z(\not{p}-m). \tag{A.18}$$

In the frame $p^\mu = (E,0,0,p)$, $\bar{p}^\mu = (E,0,0,-p)$,

$$G = -2(E+m)(p + m\gamma_z + E\gamma_0\gamma_z), \tag{A.19}$$

and hence

$$X = \mathrm{tr}\Big[(p + m\gamma_z + E\gamma_0\gamma_z)K\Big]. \tag{A.20}$$

In order to calculate Y, we must generalize the spin flip matrix σ_x. With $\hat{n} = \hat{p}'$, the spinors are

$$\chi_\uparrow = \frac{1}{\sqrt{2(n_3+1)}} \begin{pmatrix} n_3 + 1 \\ n_1 + in_2 \end{pmatrix}$$

and (A.21)

$$\chi_\downarrow = \frac{1}{\sqrt{2(n_3+1)}} \begin{pmatrix} -n_1 + in_2 \\ n_3 + 1 \end{pmatrix},$$

so the spin flip matrix A satisfying

$$A\chi_{\sigma'} = \chi_{-\sigma'}, \tag{A.22}$$

is

$$A = \vec{\sigma} \cdot \hat{e}, \tag{A.23}$$

where

$$\hat{e} = \left(\frac{1+n_3-n_1^2}{1+n_3}, \quad \frac{-n_1 n_2}{1+n_3}, -n_1 \right). \tag{A.24}$$

Notice that $A = -ie^{i\pi\hat{e}\cdot\vec{\sigma}/2}$ and $\hat{e}\cdot\hat{n} = 0$, meaning that A is a rotation matrix about an axis perpendicular to $\hat{n}$ through 180°, as it must be. We shall find it convenient to work with real spinors, so that we take, without loss of generality, $n_2 = 0$, which implies

$$\hat{e} = (n_3, 0, -n_1). \tag{A.25}$$

Now consider

$$Z = \sum_{\sigma'} v_{\bar{p}', -\sigma'} \bar{u}_{p', \sigma'}. \tag{A.26}$$

$$(i\sigma_y)\chi_{-\sigma'} = (i\sigma_y)(A\chi_{\sigma'}) = \hat{p}'\cdot\sigma\chi_{\sigma'} \tag{A.27}$$

from (A.25). Using (A.4), (A.6), (A.27), we get

$$Z = r \sum_{\sigma'} (\bar{\not{p}}'-m) \begin{pmatrix} 0 \\ \hat{p}'\cdot\vec{\sigma} \end{pmatrix} \chi_{\sigma'}\chi_{\sigma'}^{\dagger} (1 \quad 0)(\not{p}'+m). \tag{A.28}$$

Completeness gives at once

$$Z = r(\bar{\not{p}}'-m) \begin{pmatrix} 0 & 0 \\ \hat{p}'\cdot\vec{\sigma} & 0 \end{pmatrix} (\not{p}'+m) \tag{A.29}$$

$$= r(\bar{\not{p}}'-m)\, \tfrac{1}{2}(\gamma^0 - 1)(\hat{p}'\cdot\vec{\gamma})(\not{p}'+m). \tag{A.30}$$

Substituting Z into (A.2) and using (A.20) yields

$$Y = [2(E+m)]^{-1}\, \mathrm{tr}\left\{\left[p + (m+E\gamma^{o})\gamma^{3}\right] K_2\, (\bar{p\!\!/}'-m)(\gamma^{o}-1)(\hat{p}'\cdot\vec{\gamma})(p\!\!/'+m)K_1\right\} \qquad (A.31)$$

$$= \mathrm{tr}\left\{\left[p + \gamma^{3}(m-E\gamma^{o})\right] K_2 \left[p + (m-E\gamma^{o})\hat{p}'\cdot\vec{\gamma}\right] K_1\right\} \qquad (A.32)$$

with

$$p'^{\mu} = (E,\vec{p}\,'), \quad \bar{p}'^{\mu} = (E,-\vec{p}\,').$$

APPENDIX B

SU(3) CONVENTIONS AND RESULTS

In the notation $|YII_z\rangle$ we define our states to be

$$\begin{array}{ll}
K^+, p, \overline{\Xi^-} & |1\ \tfrac{1}{2}\ \tfrac{1}{2}\rangle \\
K^o, n, \overline{\Xi^o} & |1\ \tfrac{1}{2}\ -\tfrac{1}{2}\rangle \\
\overline{K^o}, \Xi^o, \overline{n} & |-1\ \tfrac{1}{2}\ \tfrac{1}{2}\rangle \\
K^-, \Xi^-, \overline{p} & -|-1\ \tfrac{1}{2}\ -\tfrac{1}{2}\rangle \\
\pi^+, \Sigma^+, \overline{\Sigma^-} & -|0\ 1\ 1\rangle \\
\pi^o, \Sigma^o, \overline{\Sigma^o} & |0\ 1\ 0\rangle \\
\pi^-, \Sigma^-, \overline{\Sigma^+} & |0\ 1\ -1\rangle \\
\eta, \Lambda, \overline{\Lambda} & |0\ 0\ 0\rangle\ .
\end{array} \tag{B.1}$$

Let us consider $\underset{\sim}{8} \otimes \underset{\sim}{8} \to \underset{\sim}{8} \otimes \underset{\sim}{8}$ and single out the SU(3) index by means of a Greek letter.

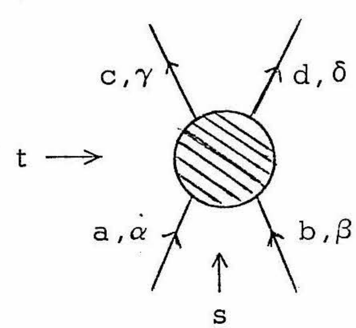


Fig. B.1. General scattering process.

The channels are defined below.

$$\begin{array}{lll} \text{s channel:} & a + b \longrightarrow c + d & \\ \text{t channel:} & a + \overline{c} \longrightarrow \overline{b} + d & \qquad (B.2) \\ \text{u channel:} & a + \overline{d} \longrightarrow c + \overline{b}. & \end{array}$$

The t and u channel crossing matrices are given by the relations

$$\langle \beta\delta | Q_\mu | \alpha\gamma \rangle = \sum_{\mu'} C^t_{\mu'\mu} \langle \gamma\delta | Q_{\mu'} | \alpha\beta \rangle , \tag{B.3}$$

$$\langle \gamma\beta | Q_\mu | \alpha\delta \rangle = \sum_{\mu'} C^u_{\mu'\mu} \langle \gamma\delta | Q_{\mu'} | \alpha\beta \rangle , \tag{B.4}$$

where μ is the SU(3) representation and Q_μ is the projection operator.

Now,

$$\langle \gamma\beta | Q_\mu | \alpha\delta \rangle = (-1)^U \langle \beta\gamma | Q_\mu | \alpha\delta \rangle \tag{B.5}$$

$$= (-1)^U \sum_{\mu'} C^t_{\mu'\mu} \langle \delta\gamma | Q_{\mu'} | \alpha\beta \rangle \tag{B.6}$$

$$= \sum_{\mu'} (-1)^{U+S} C^t_{\mu'\mu} \langle \gamma\delta | Q_{\mu'} | \alpha\beta \rangle . \tag{B.7}$$

Above, by $(-1)^U$ and $(-1)^S$ we mean the SU(3) symmetry of the final states in the u and s channels, respectively. Equation (B.6) comes from Eq. (B.3). Comparing (B.4) and (B.7) we see that

$$(-1)^{U+S} C^t_{\mu'\mu} = C^u_{\mu'\mu} . \tag{B.8}$$

These are exactly the conventions used by Carruthers[12] and his 8×8 crossing matrix obeys (B.8). One <u>cannot</u> employ here the 7×7 crossing matrix often quoted, e.g., by Gasiorowicz,[21] since it is obtained by treating $8' \to 8$ and $8 \to 8'$ as identical. As a matter of fact, the exchange contributions found from the 8×8 crossing matrix are negatives of each other, although, naturally, the direct channel amplitude is symmetric as required by time reversal invariance.

Keeping in mind the order of the states (B.2) and Fig. B.1, we write out the s channel octet projection operators.

$$
\begin{aligned}
P^s(8_{aa}) &= P_{\gamma\delta,\alpha\beta}(8_{aa}) = -\frac{1}{3} F_{\epsilon\gamma\delta} F_{\epsilon\alpha\beta}, \\
P^s(8_{ss}) &= P_{\gamma\delta,\alpha\beta}(8_{ss}) = \frac{3}{5} D_{\epsilon\gamma\delta} D_{\epsilon\alpha\beta}, \\
P^s(8_{as}) &= P_{\gamma\delta,\alpha\beta}(8_{as}) = -\frac{1}{\sqrt{5}} F_{\epsilon\gamma\delta} D_{\epsilon\alpha\beta}, \\
P^s(8_{sa}) &= P_{\gamma\delta,\alpha\beta}(8_{sa}) = \frac{1}{\sqrt{5}} D_{\epsilon\gamma\delta} F_{\epsilon\alpha\beta}.
\end{aligned}
\tag{B.9}
$$

$P(8_{as})$ and $P(8_{sa})$ satisfy

$$
\begin{aligned}
P(8_{as})P(8_{sa}) &= P(8_{aa}), \\
P(8_{sa})P(8_{as}) &= P(8_{ss}).
\end{aligned}
\tag{B.10}
$$

In order to find the projection operator for the $\underset{\sim}{10}$ representation in terms of quark indices, we calculate the SU(3) part of Fig. B.2.

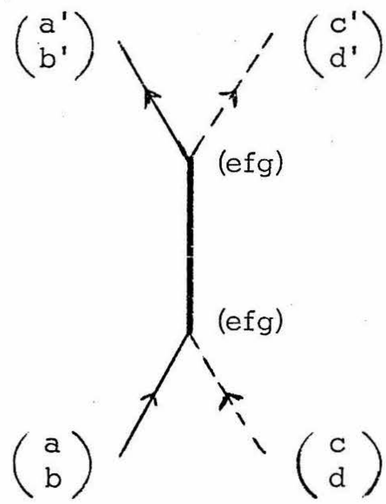


Fig. B.2. Direct channel $\underset{\sim}{10}$ contribution.

Normalizing, we get

$$P^{s}(10) = P_{\binom{a'}{b'}\binom{c'}{d'},\binom{a}{b}\binom{c}{d}}(10) = \tfrac{1}{2}\,\delta_a^{a'}\delta_c^{c'}\epsilon^{ebd}\epsilon_{eb'd'} . \tag{B.11}$$

Actually, it is necessary to symmetrize this with respect to a' and c'.

Thus we used

$$\frac{1}{2}\left[\frac{1}{2}\left(\delta_a^{a'}\delta_c^{c'} + \delta_a^{c'}\delta_c^{a'}\right)\right]\epsilon^{ebd}\epsilon_{eb'd'} . \tag{B.12}$$

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