CALIFORNIA INSTITUTE OF TECHNOLOGY

PASADENA CALIFORNIA

T H E S I S

F O R M U L A S, T A B L E S and G R A P H S

F O R

T R A P E Z O I D A L R E I N F O R C E D C O N C R E T E B E A M S

DEVELOPED BY

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1. DEVELOPMENT OF FORMULAS

A. Assumptions

1. The union between the steel and the concrete is sufficient to cause the two materials to act as one material.

2. No initial stresses are considered in either the concrete or the steel due to temperature or shrinkage.

3. The applied forces are parallel to each other and perpendicular to the neutral surface of the beam before bending.

4. Sectional planes before bending remain planes after bending within the elastic limit of the steel.

B. The following notations will be employed in referring to Fig. 1.

Let f_c = Maximum intensity of compressive stress in the concrete under a given load.

f_s = Maximum intensity of tensile stress in the metal under the same load (the area of the reinforcement is assumed to be so small with reference to the total area of cross-section of the beam that the stress in the metal is practically uniform).

C = Total compression in concrete at a section of the beam.

T = Total tension of the steel at a section of the beam.

E_c - Represents the modulus of elasticity of concrete in compression.

E_s - Represents the modulus of elasticity of steel in tension.

n = The ratio of E_s/E_c .

d = The distance from compression surface to axis of reinforcement.

k = The proportionate depth of neutral axis from below the compression surface.

a_s = The area cross-section of steel.

p = The "steel-ratio" = the ratio of the area of steel to the area of concrete.

b = The width of beam at compression surface.

b_1 = The width of beam at axis of tension steel.

M_c = The resisting moment as determined by the concrete.

M_s = The resisting moment as determined by the steel.

M = The bending moment or resisting moment in general.

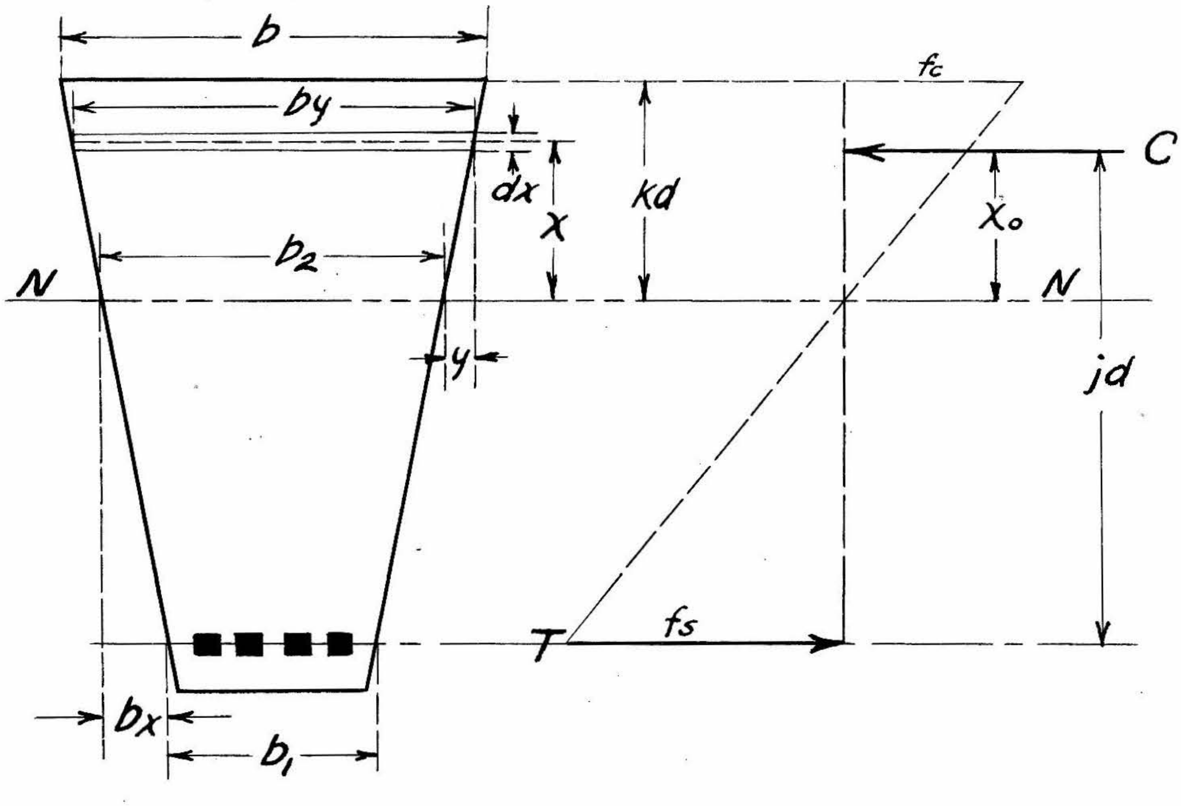


Fig. I

To find X_o and C

$$b_y = b_2 + 2y = b_2 + \frac{(b-b_2)x}{kd}$$

Let f_c' = force on elementary area $dy\,dx$.

$$f_c' = \frac{f_c x}{kd}$$

Let M_1 = Moment of all forces above N-N

$$M_1 = \int f_c' b_y x\,dx$$

$$= \int \frac{f_c x}{kd}\left[b_2 + \frac{(b-b_2)x}{kd}\right] x\,dx$$

$$= \int_{kd}^{0} \frac{f_c x^2 b_2\,dx}{kd} + \frac{f_c x^3}{(kd)^2}(b-b_2)\,dx$$

$$= \frac{f_c b_2 (kd)^2}{3} + \frac{f_c (b-b_2)(kd)^2}{4}$$

$$= f_c (kd)^2 \left(\frac{b_2+3b}{12}\right)$$

$$C = \int f_c' b_y\,dx$$

$$= \int \left[\frac{f_c x b_2}{kd} + \frac{f_c x^2}{(kd)^2}(b-b_2)\right] dx$$

$$= \int_{0}^{kd} \frac{f_c x b_2\,dx}{kd} + \frac{f_c x^2}{kd^2}(b-b_2)\,dx.$$

$$= \left.\frac{f_c x^2 b_2}{2kd}\right]_0^{kd} + \left.\frac{f_c x^3}{3(kd)^2}\right]_0^{kd} (b-b_2)$$

$$= f_c kd \left(\frac{b_2+2b}{6}\right)$$

$$X_o = \frac{M}{C}$$

$$= \frac{\dfrac{f_c (kd)^2 (b_2+3b)}{12}}{\dfrac{f_c (kd)(b_2+2b)}{6}}$$

$$= \frac{kd(b_2+3b)}{2(b_2+2b)}$$

But $b_2 = b_1 + (b-b_1)(1-k)$
$= b + k(b_1-b)$
$= b - k(b-b_1)$

$$\therefore X_0 = \frac{kd}{2}\left[\frac{b+k(b_1-b)+3b}{b+k(b_1-b)+2b}\right]$$

$$= \frac{kd}{2}\left[\frac{4b+k(b_1-b)}{3b+k(b_1-b)}\right]$$

When $b_1 = b$ (Rectangular beam)

$$X_0 = \frac{kd}{2}\left[\frac{4b+k\times 0}{3b+0}\right] = \frac{kd}{2}\times\frac{4}{3} = \frac{2}{3}kd$$

$$p = \frac{a_s}{A} = \qquad A = \frac{b_1+b}{2}d$$

$$a_s = \frac{p(b+b_1)d}{2}$$

$\Sigma H = 0$ or $C = T$

$$C = \frac{f_c kd}{6}(b_2+2b) = T = a_s f_s$$

$$\frac{f_c kd}{6}(b_2+2b) = \frac{f_s pd(b+b_1)}{2}$$

But $b_2 = b + k(b_1-b)$

$$\therefore \frac{f_c kd}{6}(b+k(b_1-b)) = \frac{f_s pd(b+b_1)}{2}$$

or $$\frac{f_c kd}{6}\left[3b+k(b_1-b)\right] = \frac{f_s pd(b+b_1)}{2}$$

But $f_s = f_c n\left(\frac{1-k}{k}\right)$

$$\therefore k = \frac{f_c n}{f_s + nf_c}$$

$$\therefore \frac{nf_c^2 d}{6(f_s+nf_c)}\left[3b - \frac{nf_c(b_1-b)}{f_s+nf_c}\right] = \frac{f_s pd(b+b_1)}{2}$$

$$\text{or}\quad \frac{(nf_c^2 d)(3bf_s+3bnf_c)-nb_1f_c+nbf_c}{6(f_s+nf_c)^2} = \frac{f_s p d(b+b_1)}{2}$$

$$\therefore p = \frac{2(3nbf_sf_c^2+4n^2bf_c^3-n^2b_1f_c^3}{3(f_s+nf_c)^2 f_s(b+b_1)}$$

$$= \frac{nf_c^2(3bf_s+4nbf_c-nb_1f_c)}{3f_s(f_s+nf_c)^2(b+b_1)}$$

When $b_1 = b$ (Rectangular beam)

$$p = \frac{nf_c^2(3bf_s+4nbf_c-nbf_c)}{3f_s(f_s+nf_c)^2(b+b)}$$

$$= \frac{nf_c^2 3(bf_s+nbf_c)}{6bf_s(f_s+nf_c)^2}$$

$$= \frac{nf_c^2}{2f_s(f_s+nf_c)} = \frac{nf_c^2}{2(f_s^2+nf_sf_c)} \quad \text{OK.}$$

$$jd = d-(kd-x_0) = d-kd+x_0$$

$$= d-kd+\frac{kd}{2}\left[\frac{4b+k(b_1-b)}{3b+k(b_1-b)}\right]$$

$$= d-kd\left\{1-\frac{1}{2}\left[\frac{4b+k(b_1-b)}{3b+k(b_1-b)}\right]\right\}$$

$$\therefore j = 1-k\left\{1-\frac{1}{2}\left[\frac{4b+k(b_1-b)}{3b+k(b_1-b)}\right]\right\}$$

When $b_1 = b$ (Rectangular beam)

$$j = 1-k\left(-\frac{1}{2}\times\frac{4}{3}+1\right) = 1-\frac{1}{3}k \quad \text{O.K.}$$

Solving for k from $C = T$

$$\frac{f_c kd}{6}\left[3b + k(b_1 - b)\right] = \frac{nf_c(1-k)}{2k} pd(b + b_1)$$

or $$\frac{3kb}{6} + \frac{k^2}{6}(b_1 - b) = \frac{np(1-k)(b+b_1)}{2k}$$

or $$kb + \frac{k^2}{3}(b_1 - b) = \frac{pn(1-k)(b+b_1)}{k}$$

or $$\frac{k^3}{3}(b_1 - b) + k^2 b - pn(1-k)(b+b_1) = 0$$

When $b_1 = b$

$$\frac{k^3}{3}(b_1 - b) = 0$$

$$\therefore \quad k^2 b - pn(1-k)2b = 0$$

or $$k^2 b + 2kpnb - 2pnb = 0$$

$$k = \frac{-2pnb \pm \sqrt{(2pn)^2 + 4b(2pn)b}}{2b}$$

$$= -pn \pm \sqrt{(pn)^2 + 2pn} \qquad \text{O.K.}$$

$$M_c = Cjd = \frac{f_c kd}{6}\left[3b + k(b_1 - b)\right] jd$$

$$= \frac{f_c kjd^2}{6}\left[3b + k(b_1 - b)\right]$$

$$M_s = Tjd = \frac{pf_s jd^2}{2}(b_1 + b)$$

RESULTS and FORMULAS

$$p = \frac{2a_s}{(b+b_1)d} = \frac{nf_c^2(3bf_s+4nbf_c-nb_1f_c)}{3f_s(f_s+nf_c)^2(b+b_1)}$$

$$a_s = \frac{p(b+b_1)d}{2}$$

$$K \qquad \frac{K^3}{3}(b_1-b)+K^2(b)-pn(1-K)(b+b_1)=0$$

$$j = 1-K\left\{1-\frac{1}{2}\left[\frac{4b+K(b_1-b)}{3b+K(b_1-b)}\right]\right\}$$

$$M_c = \frac{f_c kjd^2}{6}\left[3b+K(b_1-b)\right]$$

$$M_s = \frac{pf_sjd^2}{2}(b+b_1)$$

Solving for values of p $\qquad n=15$

$b_1=0.2b$

$$p=\frac{nf_c^2(3bf_s+4\times15bf_c-15\times0.2bf_c}{3f_s(f_s+nf_c)^2(b+0.2b)}$$

$$=\frac{15f_c^2(3bf_s+57f_c)}{3f_s(f_s+15f_c)^2\times1.2b}$$

$$=\frac{15f_c^2(f_s+19f_c)}{f_s(f_s+15f_c)^2\times1.2}$$

$b_1=0.4b$

$$p=\frac{15f_c^2(f_s+17f_c)}{f_s(f_s+15f_c)^2\times1.6}$$

$b_1=0.6b$

$$p=\frac{15f_c^2(f_s+16f_c)}{f_s(f_s+15f_c)^2\times1.8}$$

$b_1=0.8b$

$$p=\frac{15f_c^2(f_s+16f_c)}{f_s(f_s+15f_c)^2\times1.8}$$

For Values See Following Table.

TABLE I

	f_c	f_s	f_c^2	f_c^2/f_s	f_s+15f_c		D/G	$19f_c$	$18f_c$	$17f_c$	$16f_c$	$\frac{15(f_s+19f_c)}{1.2}$	$\frac{15}{1.4}(f_s+18f_c)$	$\frac{15}{1.6}(f_s+17f_c)$	$\frac{15}{1.8}(f_s+16f_c)$	Values of p			
																$b_r=.2b$	$b_r=.4b$	$b_r=.6b$	$b_r=.8b$
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
1		12×10^3		13.33	18×10^3	324×10^6	4.11×10^{-8}					245×10^3	206×10^3	176.4×10^3	153×10^3	1.006	.847	.725	.629
2		14 "	16×10^4	11.42	20	400	2.86					270	227	195	170	.771	.649	.556	.486
3	400	16 "		10.00	22	480	2.06	7600	7200	6800	6400	295	248	214	186.5	.609	.512	.442	.385
4		18 "		8.90	24	576	1.55					320	269	232.5	203.5	.494	.416	.359	.314
5		20 "		8.00	26	676	1.28					345	290	251.5	220	.441	.371	.322	.281
6		12 "		20.80	19.5	380	5.48					269	225	192	166.6	1.475	1.235	1.052	.910
7		14 "	25×10^4	1783	21.5	462	3.85					294	246.5	216	183.3	1.132	.950	.832	.706
8	500	16 "		15.60	23.5	552	2.83	9500	9000	8500	8000	319	268	230	200	.902	.758	.650	.565
9		18 "		13.88	25.5	650	2.14					344	289.5	248.5	216.6	.733	.617	.529	.461
10		20 "		12.50	27.5	756	1.65					369	311	267.5	233.3	.611	.515	.442	.386
11		12 "		30.00	21	441	6.80					292.5	244.5	208	180	1.990	1.660	1.414	1.224
12		14 "	36×10^4	25.70	23	529	4.86					317.5	266	227	196.5	1.545	1.294	1.104	.955
13	600	16 "		22.50	25	625	3.60	11400	10800	10200	9600	292.5	287.5	246	214	1.234	1.035	.886	.771
14		18 "		20.00	27	729	2.74					367.5	309	265	230	1.007	.897	.726	.630
15		20 "		18.00	29	841	2.14					392.5	330.5	283	246.5	.890	.708	.606	.528
16		12 "		40.8	22.5	506	8.06					316	263.5	224	193	2.550	2.125	1.806	1.555
17		14 "	49×10^4	35.0	24.5	600	5.83	13300	12600	11900	11200	341	285	243	210	1.990	1.660	1.417	1.225
18	700	16 "		30.6	26.5	702	4.36					366	306.5	262	227	1.596	1.335	1.142	.990
19		18 "		27.2	28.5	812	3.34					391	328	281	2435	1.300	1.090	.933	.808
20		20 "		24.5	30.5	930	2.63					416	349.5	300	260	1.094	.920	.789	.684
21		12 "		53.3	24.0	576	9.26					340	283	240	216.6	3.150	2.620	2.222	2.006
22		14 "	64×10^4	45.7	26	676	6.76	15200	14400	13600	12800	365	304.5	259	223.3	2.470	2.060	1.750	1.510
23	800	16 "		40.0	28	784	5.10					390	326	278	240	1.990	1.663	1.418	1.224
24		18 "		35.5	30	900	3.95					415	347.5	297	256.6	1.640	1.374	1.174	1.013
25		20 "		32.0	32	1024	3.12					440	369	316	273.3	1.372	1.153	.986	.854

TABLE I

	Values of y				Values of k				$b_1=0.2b$			$b_1=0.4b$			$b_1=0.6b$			$b_1=0.8b$	
	0.2	0.4	0.6	0.8	0.2	0.4	0.6	0.8	1-0.4k	3-0.8k	j	1-0.3k	3-0.6k	j	1-0.2k	3-0.4k	j	1-0.1k	3-0.2k
1	.679	.884	.943	2.547	.358	.352	.302	.338	.857	2.714	.887	.844	2.789	.894	.940	2.879	.902	.966	2.932
2	.520	.682	.723	1.968	.321	.316	.270	.304	.872	2.743	.898	.905	2.810	.898	.946	2.892	.912	.970	2.939
3	.411	.538	.575	1.559	.290	.286		.276	.884	2.768	.907	.914	2.828	.908				.972	2.945
4	.333	.437	.468	1.272															
5	.298	.390	.419	1.138															
6	.996	1.297	1.368	3.686	.417	.408	.352	.392	.834	2.666	.871	.878	2.765	.870	.930	2.652	.876	.961	2.922
7	.764	.998	1.082	2.859	.376	.368	.320	.354	.850	2.699	.882	.890	2.779	.882	.936	2.872	.887	.965	2.929
8	.609	.796	.845	2.288	.342	.336	.288	.324	.863	2.726	.892	.899	2.798	.892	.942	2.885	.906	.968	2.935
9	.495	.648	.688	1.867	.314	.309		.298	.874	2.749	.900	.907	2.815	.900				.970	2.940
10	.412	.541	.575	1.563	.290	.286		.277	.884	2.768	.907	.914	2.828	.907				.972	2.944
11	1.343	1.743	1.838	4.957	.467	.457	.396	.438	.813	2.626	.856	.863	2.726	.856	.921	2.884	.873	.956	2.912
12	1.043	1.359	1.435	3.868	.425	.416	.359	.399	.830	2.660	.867	.875	2.750	.868	.928	2.856	.884	.960	2.920
13	.833	1.087	1.152	3.126	.389	.382	.328	.366	.844	2.689	.878	.885	2.771	.878	.934	2.869	.893	.963	2.927
14	.680	.889	.944	2.552	.358	.354	.302	.338	.857	2.714	.888	.894	2.789	.888	.940	2.879	.900	.966	2.932
15	.567	.743	.788	2.138	.332	.325	.280	.316	.867	2.734	.895	.904	2.805	.895	.944	2.888	.909	.968	2.937
16	1.721	2.231	2.348	6.298	.510	.498	.434	.476	.796	2.592	.843	.851	2.751	.846	.913	2.826	.860	.952	2.905
17	1.343	1.743	1.842	4.961	.467	.456	.396	.438	.813	2.626	.856	.867	2.726	.856	.921	2.842	.872	.956	2.912
18	1.077	1.402	1.485	4.010	.430	.421	.364	.404	.828	2.656	.866	.874	2.747	.866	.927	2.854	.882	.960	2.919
19	.876	1.144	1.213	3.272	.397	.390	.336	.374	.841	2.682	.875	.883	2.766	.876	.933	2.866	.891	.963	2.925
20	.738	.966	1.026	2.770	.370	.364	.313	.350	.852	2.704	.883	.891	2.782	.884	.937	2.875	.898	.965	2.930
21	2.126	2.751	2.889	8.124		.535	.468	.520				.839	2.679	.833	.906	2.813	.849	.948	2.896
22	1.667	2.163	2.275	6.116	.504	.494	.428	.472	.798	2.597	.845	.852	2.704	.844	.914	2.829	.862	.953	2.906
23	1.343	1.746	1.843	4.957	.467	.457	.396	.438	.813	2.626	.846	.863	2.726	.856	.921	2.842	.872	.956	2.912
24	1.107	1.443	1.526	4.103	.434	.426	.368	.408	.826	2.653	.865	.872	2.744	.864	.926	2.853	.880	.959	2.918
25	.926	1.211	1.282	3.459	.406	.398	.343	.382	.838	2.675	.873	.881	2.761	.873	.931	2.863	.888	.962	2.924

TABLE I

	$b_1=0.8b$ $b_1=0.2b$ $b_1=0.4b$					$b_1=0.6b$		$b_1=0.8b$	
	j	$0.6f_s$	$\frac{Mf_s}{bd^2}$	$0.7f_s$	$\frac{Mf_s}{bd^2}$	$0.8f_s$	$\frac{Mf_s}{bd^2}$	$0.9f_s$	$\frac{Mf_s}{bd^2}$
1	.888	7200	64.25	8400	63.60	9600	62.78	10800	60.32
2	.900	84	58.16	98	57.22	112	56.79	126	55.11
3	.909	96	53.03	112	52.07	128		144	50.39
4		108		126		144		162	
5		120		140		160		180	
6	.872	72	91.97	84	90.10	96	88.47	108	84.90
7	.883	84	83.87	98	82.00	112	82.65	126	78.43
8	.893	96	77.24	112	75.73	128	75.37	144	72.65
9	.902	108	71.25	126	69.97	144		162	67.36
10	.909	120	67.70	140	65.39	160		180	63.16
11	.856	72	122.65	84	119.33	96	118.50	108	113.16
12	.869	84	112.52	98	110.06	112	109.36	126	104.57
13	.880	96	104.04	112	101.76	128	101.27	162	97.70
14	.889	108	96.58	126	94.75	144	94.09	180	90.73
15	.896	120	90.22	140	88.71	160	88.14	108	85.16
16	.844	72	154.77	84	151.01	96	149.11	126	141.74
17	.856	84	143.09	98	139.25	112	138.39	144	132.12
18	.867	96	132.69	112	129.48	128	128.93	162	123.60
19	.877	108	122.85	126	120.23	144	119.71	162	114.80
20	.885	120	116.20	140	113.82	160	113.33	180	109.12
21	.830	72		84	183.32	96	181.09	108	179.82
22	.845	84	175.32	98	170.39	112	168.95	126	160.77
23	.856	96	163.53	112	159.44	128	158.27	144	150.88
24	.867	108	153.21	126	149.58	144	148.77	162	142.27
25	.874	120	143.73	140	140.92	160	140.09	180	134.35

Solving for Values of k $\quad n=15$

$$\frac{K^3}{3}(b_1-b) + K^2 b - pn(1-K)(b+b_1) = 0$$

$b_1 = 0.2\,b$

$$K^3 - 3.75K^2 - 67.3pK + 67.3p = 0$$

$b_1 = 0.4\,b$

$$K^3 - 5K^2 - 105pK + 105p = 0$$

$b_1 = 0.6\,b$

$$K^3 - 7.5K^2 - 130pK + 130p = 0$$

$b_1 = 0.8\,b$

$$K^3 - 15.0K^2 - 405pK + 405p = 0$$

These equations may be written

$$K^3 - xK^2 - yK + y = 0$$

or $\quad y = \dfrac{K^2(x-K)}{1-K}$

Where

$x = -\dfrac{3b}{(b_1-b)}$ $\quad$ For various values of b_1 $\quad x = C_1$

$y = -15p \times \dfrac{3(b+b_1)}{(b_1-b)}$ $\quad$ For various values of b_1 $\quad y = C_1 \times p$

Values of k, x and y in table II were found as follows.

All of the equations were of the form ($k^3 - xk^2 - yk + y = 0$), there being but four different values of x (all greater than 1) for the whole set of problems, and the values of k being known to range between .270 and .550. The equation was rewritten in the form $k^2(k-x) = y(k-1)$, or since both x and 1 were greater than k, in the form $k^2(x-k) = y(1-k)$, whence dividing thru by 1-k, $y = k^2(x-k)/(1-k)$. The values of y given by the succesive values of k = .270, .280, .290, and so on to k = .550 were next computed and tabulated. The computation was easily preformed on a 16 place Monroe by setting x-k up on the left of the keyboard, 1-k on the right, and building 1-k up to k^2 in the middle of the lower dial, whence y appeared at the left of the lower dial. For example, for k = .43 (x-k) = 3.32, 1-k = .57, and k^2 = .1849(found in a table of squares), keyboard 3320000057 and by forward turns of the crank build 57 up to 1849(023) in the middle of the lower dial, when y = 1.0769(749) appears at the left of the lower dial. Copy y = 1.077, clear dials, and change keyboard setup to 3310000056 for k = .44 (k-k =3.31, 1-k = .56, and k^2 = .1936). Build 56 up to 1935(976) in the middle of the lower dial and when y = 1.1443(001) appears at the left.

Values of y for intermediate 3 place values of k were then computed by proportional interpolation, that is, for example, having found y = 1.077 for k = .430 and y = 1.144 for k = .440, dial 10770 and keyboard 67 (1144 - 1077 obtained mentally), one forward turn of the crank gives y = 1.084 for k = .431, another turn gives y = 1.090 for k = .432, and so on to y = 1.144 for k = .440, thus automatically checking the correctness of the mental subtraction, and dialling the value of y for k = .440 in readiness for the interpolation of the next 9 values of y.

TABLE II

K	K^2	X				X − K				1−K	y			
		0.2	0.4	0.6	0.8	0.2	0.4	0.6	0.8		0.2	0.4	0.6	0.8
.270	72.9×10^{3}	$X_{0.2} = \frac{3b}{0.2b-b} = \frac{30}{8} = 3.75$	$X_{0.4} = \frac{3b}{0.4b-b} = \frac{30}{6} = 5.00$	$X_{0.6} = \frac{3b}{0.6b-b} = \frac{30}{4} = 7.50$	$X_{0.8} = \frac{3b}{0.8b-b} = \frac{30}{2} = 15.00$	3.48	4.73	7.23	14.73	.73	.348	.472	.722	1.471
2											.354	.480	.735	1.497
4											.360	.489	.748	1.524
6											.366	.497	.760	1.550
8											.372	.506	.773	1.577
.280	78.4					3.47	4.72	7.22	14.72	.72	.378	.514	.786	1.603
2											.384	.523	.800	1.631
4											.391	.532	.813	1.659
6											.397	.540	.827	1.686
8											.404	.549	.840	1.714
.290	84.1					3.46	4.71	7.21	14.71	.71	.410	.558	.854	1.742
2											.417	.567	.868	1.772
4											.424	.576	.883	1.801
6											.430	.586	.897	1.831
8											.437	.595	.912	1.860
.300	90.0					3.45	4.70	7.20	14.70	.70	.444	.604	.926	1.890
2											.451	.614	.946	1.921
4											.458	.624	.956	1.952
6											.465	.6.33	.971	1.984
8											.472	.643	.986	2.015
.310	96.1					3.44	4.69	7.19	14.69	.69	.479	.653	1.001	2.046
2											.486	.663	1.017	2.079
4											.494	.674	1.033	2.112
6											.501	.684	1.049	2.145
8											.509	.695	1.065	2.178

TABLE II

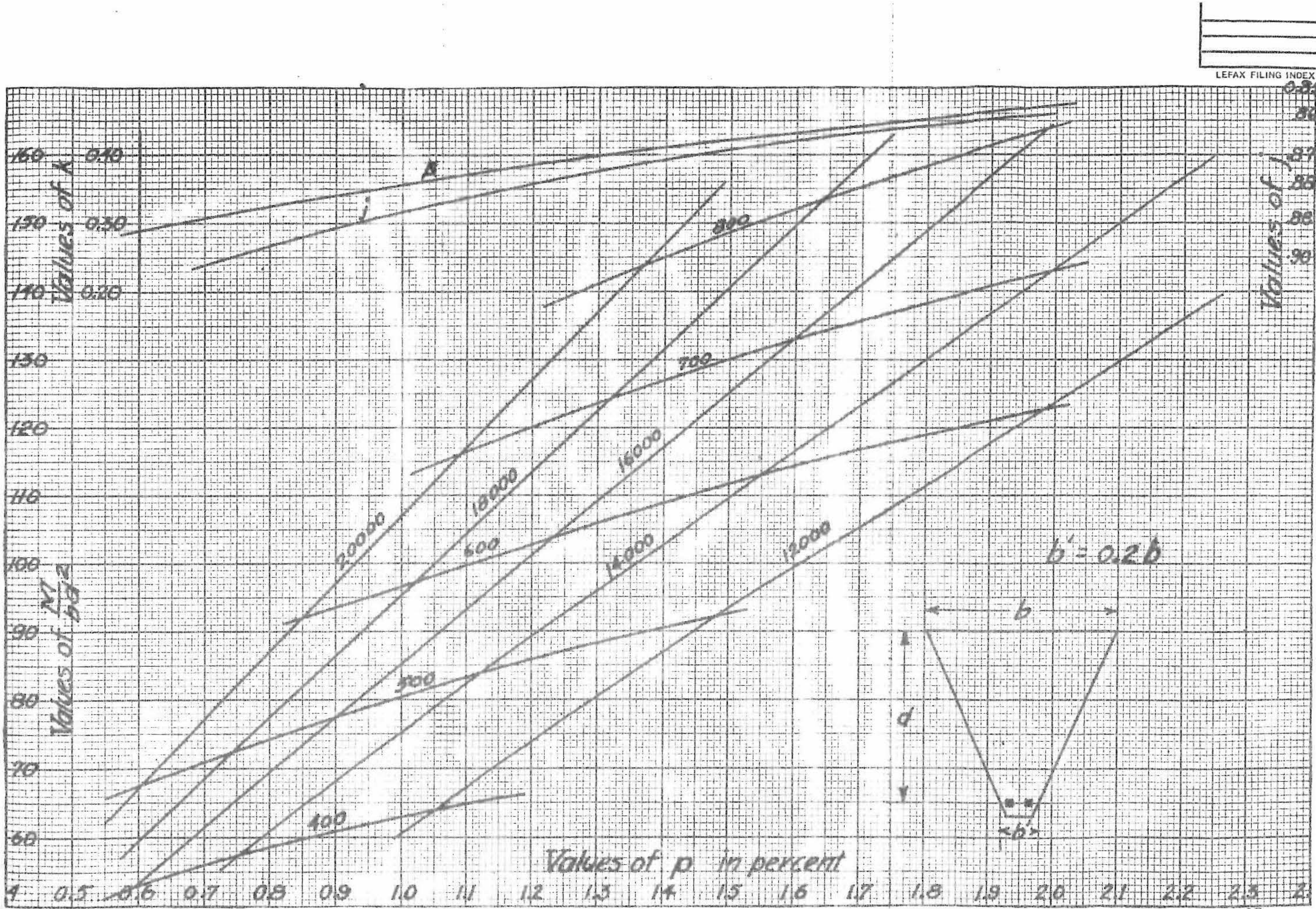
K	K^2	X				X − K				1 − k	Y			
	$\times 10^3$	0.2	0.4	0.6	0.8	0.2	0.4	0.6	0.8		0.2	0.4	0.6	0.8
.320	102.4					3.43	4.68	7.18	14.68	.68	.516	.705	1.081	2.211
2											.524	.716	1.098	2.246
4											.532	.727	1.115	2.280
6											.540	.737	1.131	2.315
8											.548	.748	1.148	2.349
.330	108.9					3.42	4.67	7.17	14.67	.67	.556	.759	1.165	2.384
2											.564	.772	1.182	2.421
4											.572	.785	1.201	2.458
6											.581	.797	1.218	2.494
8											.589	.810	1.236	2.531
.340	115.6					3.41	4.66	7.16	14.65	.66	.597	.823	1.254	2.568
2											.606	.834	1.273	2.607
4		3.75	5.00	7.50	15.00						.615	.844	1.292	2.645
6											.623	.855	1.310	2.684
8											.632	.865	1.329	2.722
.350	122.5					3.40	4.65	7.15	14.64	.65	.641	.876	1.348	2.761
2											.650	.889	1.368	2.802
4											.659	.902	1.387	2.843
6											.668	.914	1.407	2.883
8											677	.927	1.426	2.924
.360	129.6					3.39	4.64	7.14	14.63	.64	.686	.940	1.446	2.965
2											.696	.953	1.467	3.008
4											.706	.966	1.487	3.051
6											.715	.980	1.508	3.093
8											.725	.993	1.528	3.136

TABLE II

K	K^2	X				X − K				1 − K	y			
	$\times 10^3$	0.2	0.4	0.6	0.8	0.2	0.4	0.6	0.8		0.2	0.4	0.6	0.8
.370	136.9					3.38	4.63	7.13	14.63	.63	.735	.993	1.528	3.179
2											.745	1.006	1.549	3.224
4											.754	1.019	1.571	3.269
6											.764	1.033	1.593	3.314
8											.773	1.046	1.614	3.360
.380	144.4					3.37	4.62	7.12	14.62	.62	.783	1.060	1.636	3.405
2											.794	1.088	1.681	3.453
4											.805	1.103	1.704	3.500
6											.816	1.118	1.727	3.548
8											.827	1.134	1.750	3.595
.390	152.1					3.36	4.61	7.11	14.61	.61	.838	1.149	1.773	3.643
2											.849	1.164	1.797	3.693
4		3.75	5.00	7.50	15.00						.860	1.180	1.821	3.743
6											.871	1.196	1.845	3.793
8											.882	1.211	1.869	3.843
.400	160.0					3.35	4.60	7.10	14.60	.60	.893	1.227	1.893	3.893
2											.905	1.243	1.918	3.946
4											.917	1.259	1.944	3.999
6											.928	1.276	1.969	4.051
8											.940	1.291	1.995	4.104
.410	168.1					3.34	4.59	7.09	14.59	.59	.952	1.308	2.020	4.157
2											.964	1.325	2.047	4.212
4											.976	1.342	2.073	4.268
6											.989	1.359	3.000	4.323
8											1.001	1.376	3.126	4.379

TABLE II

K	K^2	X				X − K				1−K	y			
	$\times 10^3$	0.2	0.4	0.6	0.8	0.2	0.4	0.6	0.8		0.2	0.4	0.6	0.8
.420	176.4					.58	3.33	4.58	7.08	14.58	1.013	1.393	2.153	4.434
2											1.026	1.411	2.181	4.492
4											1.039	1.429	2.209	4.551
6											1.051	1.446	2.237	4.609
8											1.064	1.464	2.265	4.668
.430	184.9					3.32	4.57	7.07	14.57	.57	1.077	1.482	2.293	4.726
2											1.090	1.501	2.322	4.788
4											1.104	1.520	2.351	4.849
6											1.117	1.538	2.381	4.911
8											1.130	1.557	2.411	4.972
.440	193.6					3.31	4.56	7.06	14.56	.56	1.144	1.576	2.440	5.034
2											1.158	1.596	2.471	5.099
4		3.75	5.00	7.50	15.00						1.172	1.616	2.502	5.163
6											1.187	1.635	2.534	5.228
8											1.201	1.655	2.565	5.292
.450	202.5					3.30	4.55	7.05	14.55	.55	1.215	1.675	2.596	5.357
2											1.230	1.696	2.629	5.425
4											1.245	1.716	2.661	5.493
6											1.259	1.737	2.694	5.562
8											1.274	1.758	2.726	5.630
.460	2.116					3.29	4.54	7.04	14.54	.54	1.289	1.779	2.759	5.698
2											1.305	1.801	2.793	5.770
4											1.320	1.823	2.827	5.841
6											1.336	1.844	2.861	5.913
8											1.351	1.866	2.896	5.984



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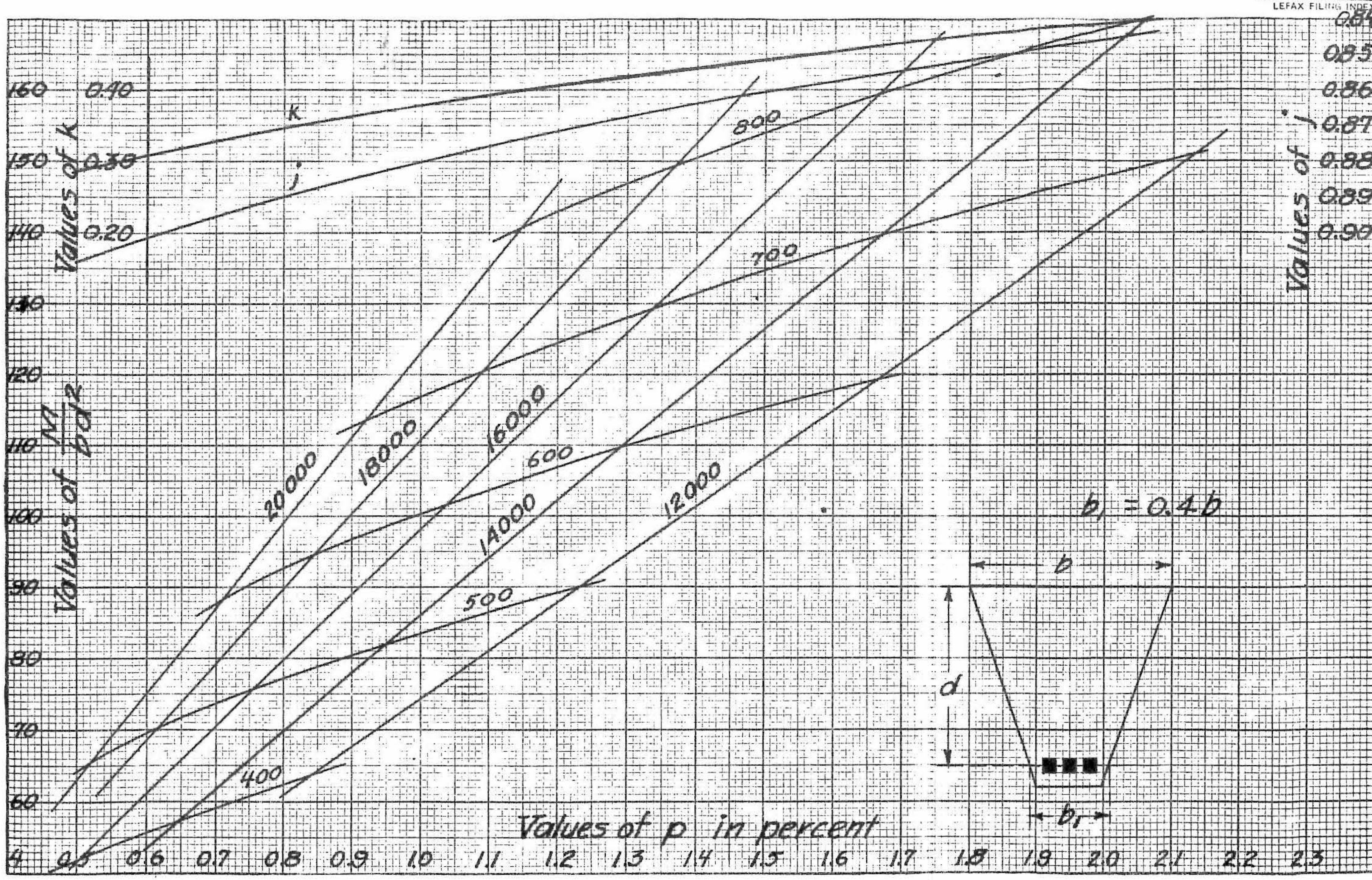
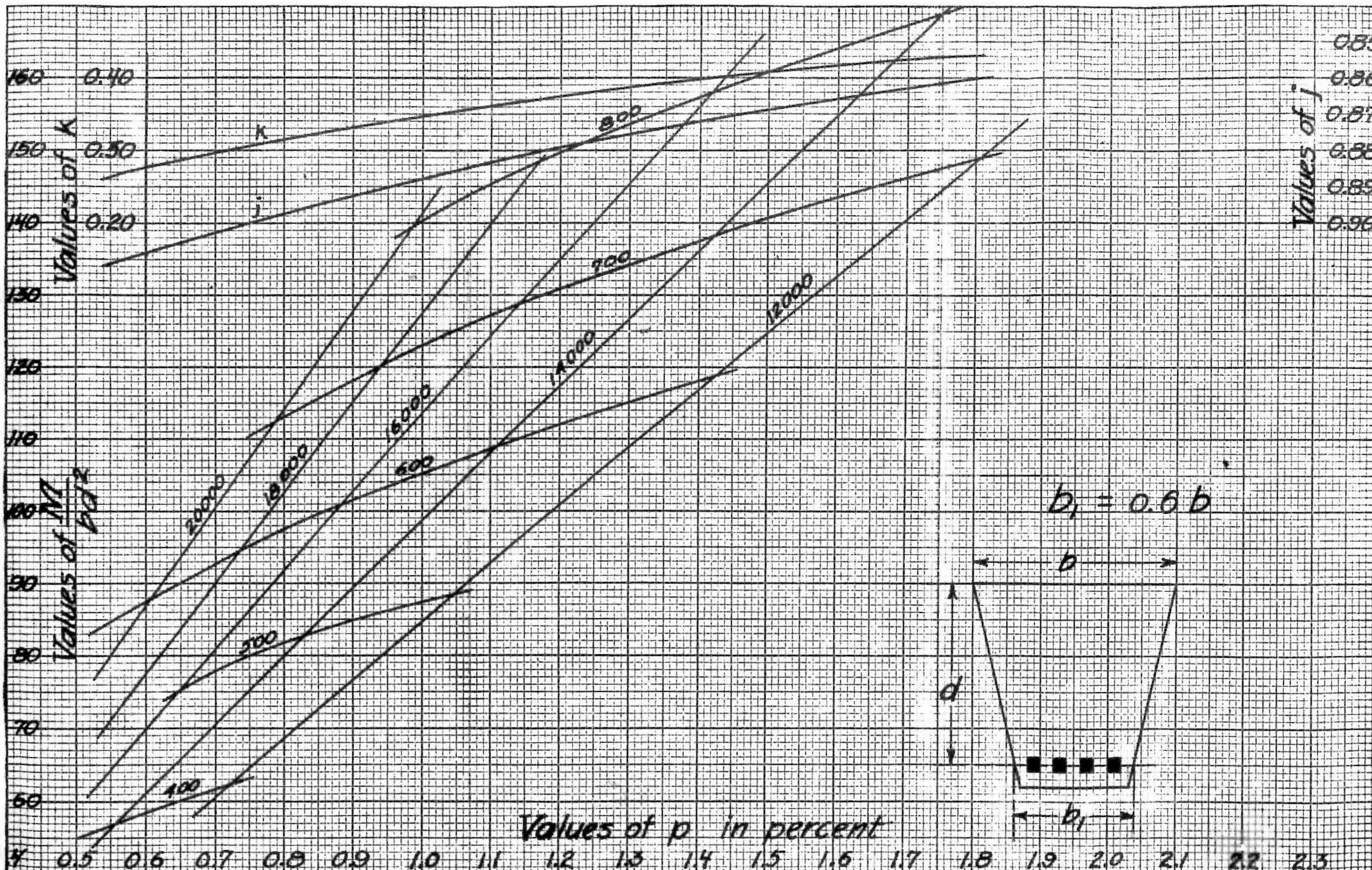
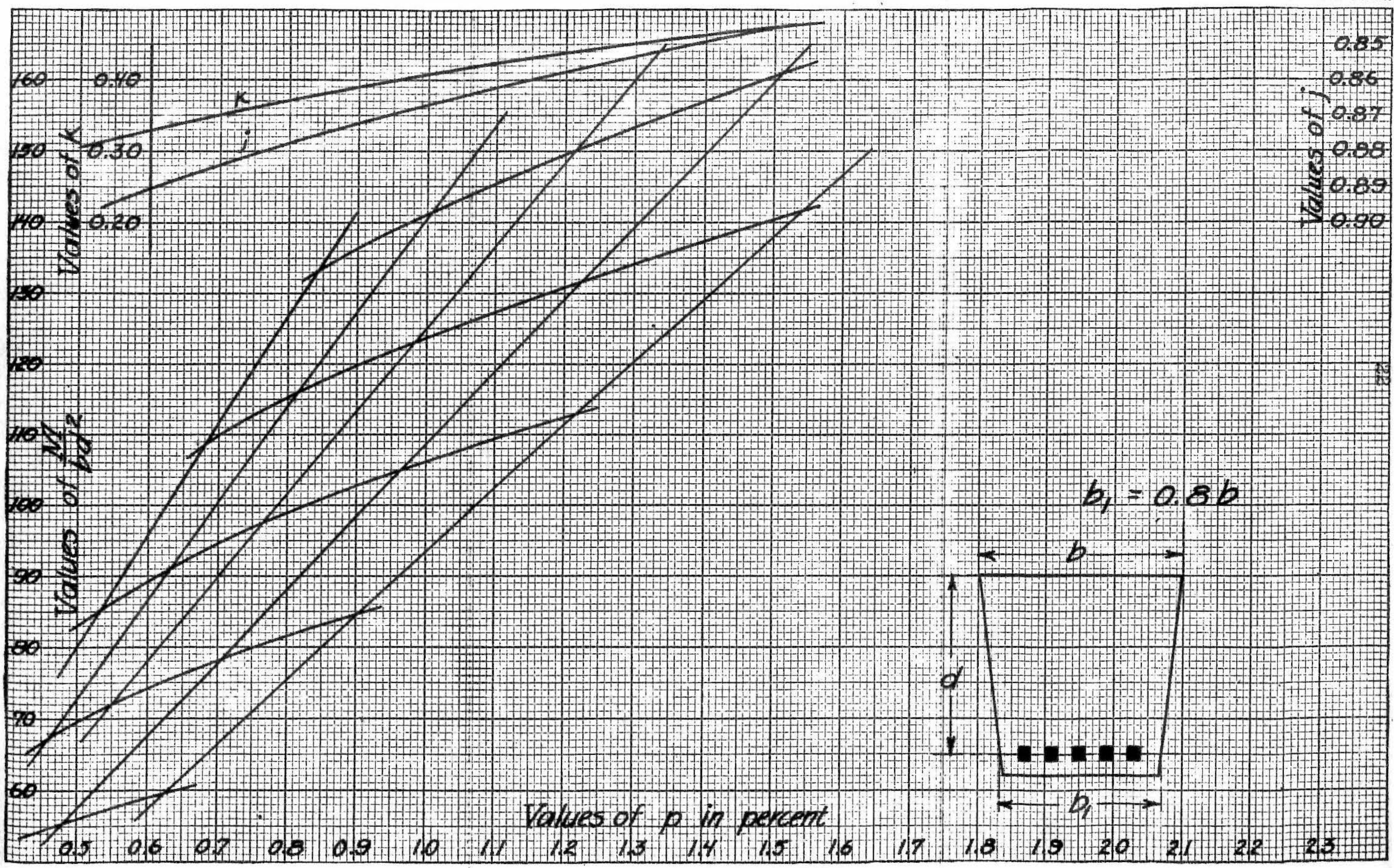


20 Lines = 1 Inch

20



LEFAX FILING INDEX
Values of k
Values of j
Values of $\frac{M}{bd^2}$
Values of p in percent
k
j
0.40
0.30
0.20
0.85
0.86
0.87
0.88
0.89
0.90
160
150
140
130
120
110
100
90
80
70
60
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0
2.1
2.2
2.3
800
700
600
300
400
20000
18000
16000
14000
12000
$b_1 = 0.6\ b$
b
d
b_1



Values of K
Values of j
K
j
0.40
0.30
0.20
0.85
0.86
0.87
0.88
0.89
0.90
160
150
140
130
120
110
100
90
80
70
60
Values of M/bd²
Values of p in percent
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0
2.1
2.2
2.3
b₁ = 0.8b
b
d
b₁