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THESIS

# Analysis of a Concrete Arch Highway Bridge Reinforced with Structural Steel.

By

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# Description of Bridge at Springfield, Mass.

An article in the Engineering News-Record for March 30, 1923, describes a new concrete arch bridge across the Connecticut River between Springfield and West Springfield, Mass.

There are seven arch spans with span lengths varying between 110 and 176 ft. The rise of the arches varies from 19.1 to 29.7 ft. The width is 80 ft.

The arches are made of five ribs, three grouped near the middle, and one at each side. The reinforced concrete roadway is carried on columns resting on pedestals on the arch ribs. The ribs are parabolic in shape, are 4 ft. 6 in. wide, and of constant depth in each span, but vary in depth with the span lengths, from 4 to 5 ft.

The reinforcement consists of heavy riveted structural steel lattice girders, with a small number of hoops and longitudinal rods. The girders were erected as three-hinged arches and later spliced at the crown to act as two-hinged arches. From the steel work were then hung the forms in which the concrete of the arch ribs was placed.

The adjustment of the steel ribs was secured by heavy screws through the cast steel shoes, by which the distance between the face of the granite skewback and the base of the shoe could be varied. After adjustment, this space was filled with an alloy of antimony and lead which does not shrink on cooling. The concrete was poured in 20 ft. sections from each end alternately in order to avoid distortion, each rib being poured in one day's continuous pouring. The structural steel frame of the arch ribs supports the concrete until it becomes set, after which the steel acts as reinforcement in supporting additional loads.. This method of construction makes it possible to develop an initial stress in the steel in addition to its stress as reinforcement, a result which cannot be accomplished with rod reinforcement.

The working stresses established for the design for live, dead, impact and temperature stresses were 16000 lb. per square inch in steel and 600 lb. per square inch in concrete. The ratio between the moduli of elasticity was taken as 12. In one of the ribs of the longest span the maximum stresses were as follows:

| | | |
|---|---|---|
| Steel.—Construction stresses due to weight of ribs and forms | | 9820 lb. per sq. in. |
| Stresses due to remaining dead-load, live-loads, and temperature | | 6570 lb. per sq. in. |
| | Total | 16390 " " " " |
| Concrete.—Maximum stress | | 593 " " " " |

The stress, due to the loading, in rod reinforcement at this particular section could not have exceeded 593 lb. multiplied by the ratio between the moduli of elasticity, or 12X 593 = 7116 lb. per sq. in. Hence the economy of material due to the use of the above type of structural steel rib is apparent.

Inasmuch as the steel ribs would cost more per pound than rod reinforcement, it was necessary in order to determine the relative economy of the two types of reinforcement to make comparative studies of the relative cost of ribs reinforced with rods and ribs reinforced with riveted members such as those used, due allowance being given not only to the additional cost per pound of steel ribs, but to the saving in false work and the reduction in the cost of the piers due to the lesser weight of the ribs. These studies showed a marked saving for the steel rib type; they also showed it to be economical to make each rib in any one span constant in cross-section for its entire length. Hence the maximum fiber stress at the different sections varies somewhat.

An additional and important argument in favor of this type of rib was the anticipated rapidity and ease with which such ribs could be erected, and the consequent lessening of danger to the permanent structure from floods and ice, an important factor in the Connecticut River. These anticipations were fully realized during construction.

No direct allownace was made for stresses caused by shrinkage of the rib concrete during setting. Such shrinkage would tend to bring it into tension and to cause some compression in the steel ribs.

The application of partial dead-load to the rib during construction caused stresses in the diagonals which disappeared when the rib concrete was completely poured, Consequently, except for the condition existing during construction, the steel rib flanges act as longitudinal reinforcing bars in the ordinary reinforced-concrete member, and the diagonals serve as shear reinforcement.

Most of the work was done with 1:6 concrete with a ¾-in.

maximum aggregate. Tests showed a compressive strength of 2260 lb. per sq. in. after 28 days.

This thesis will be concerned with the analysis of stresses in the ribs of a selected arch bridge of the type describ ed above. The span and rise have been made to fit the Linda Vista crossing of the Arroyo Seco in the City of Pasadena. The trial cross-sectional area and amount of reinforcement have been selected by comparison with the Connecticut River bridge described on the preceding pages.

In the design and construction of the bridge considered in this thesis, the following procedure will be followed:

Construction:

(1) The abutments will be constructed and fitted with adjustable cast steel shoes.

(2) The steel ribs will be swung into place by cranes, and fastened with pins at both ends and the crown. Wind bracing will be provided by steel lattice girders between ribs which will be covered with concrete. Steel hoop and longitudinal rod reinforcement will be fastened in place around the steel ribs.

(3) Forms for the rib concrete will be built around the steel rib and the concrete will be poured in 20'

sections from each end simultaneously. The three hinges will be left uncovered.

(4) Columns and roadway will then be constructed.

(5) The hinges will be filled with concrete and reinforcing rods.

Design:

(1) The shape of the arch axis will be determined by computing the coordinates of points on a parabola.

(2) The forces due to the load of the steel and concrete of the arch rib will be computed. A curve of moments and normal thrusts will be constructed. A steel rib will be designed to carry these loads with a max. stress of about 7000# per sq. in.

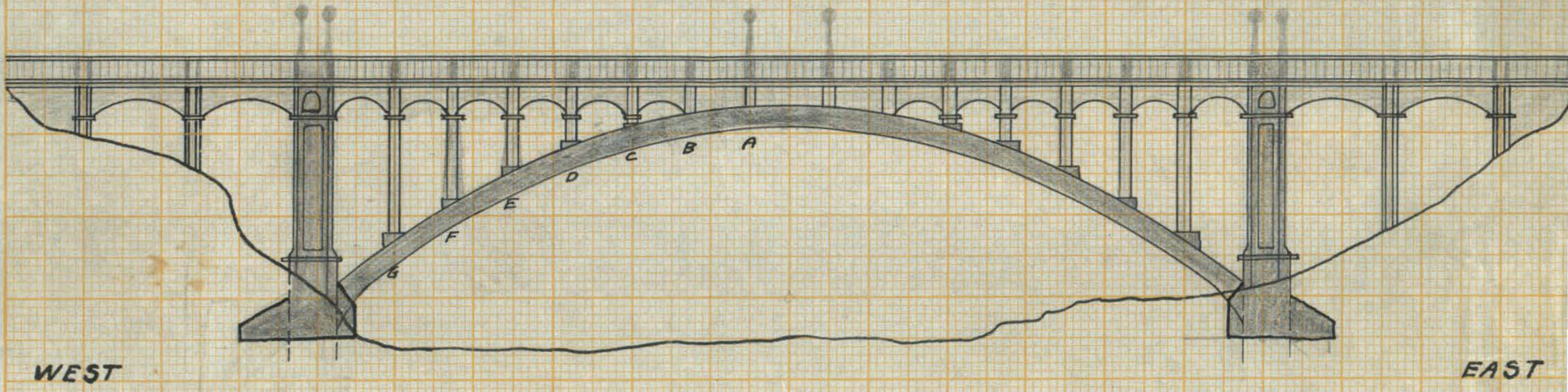
(3) The dead weight of the columns and roadway will be estimated.

(4) The forces due to the load of the columns and roadway will be computed. A curve of moments and normal thrusts will be constructed.

(5) As the hinges will be closed before any live load is applied, an analysis of the fixed arch will be made to find values for the moment, shear and thrust at the crown, for live load only.

(6) The maximum unit stress in the concrete at certain sections with live lead in various positions will be computed and plotted.

(7) Thrust diagrams for the same loadings will be constructed and the maximum unit stresses computed and plotted, as a comparison with the values first obtained.




PROFILE OF NEW LINDA VISTA BRIDGE SITE

ARROYO SECO PASADENA CALIF.

SCALE: 1" = 40'

## SHAPE OF ARCH AXIS

To find coordinates of points on the axis:

Span = 230'                    Rise = 44'

$X^2 = A\,Y$

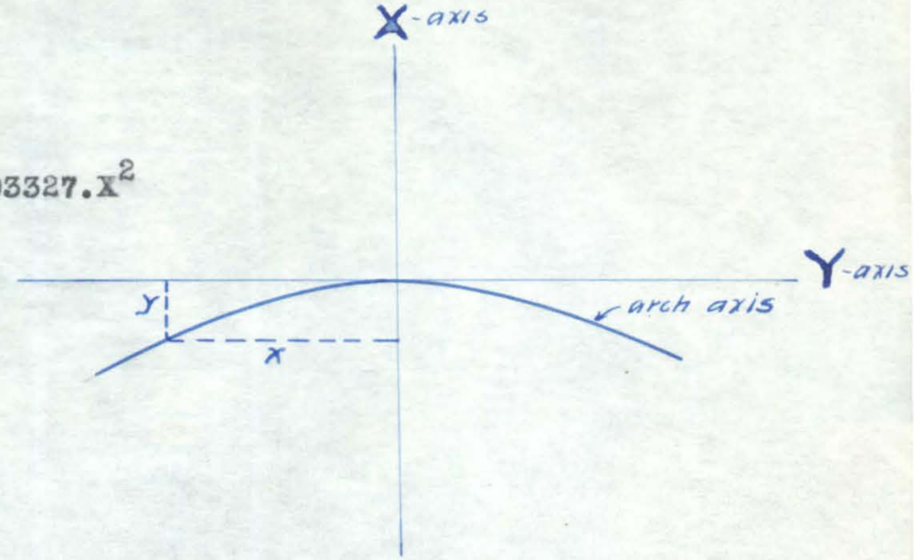
$X = 0,\ Y = 0$

$X = 115,\ Y = 44$

$X^2 = 13225$

$A = \frac{13225}{44} = 300.6$

$Y = \frac{1}{300.6} \cdot X^2 = .003327.X^2$

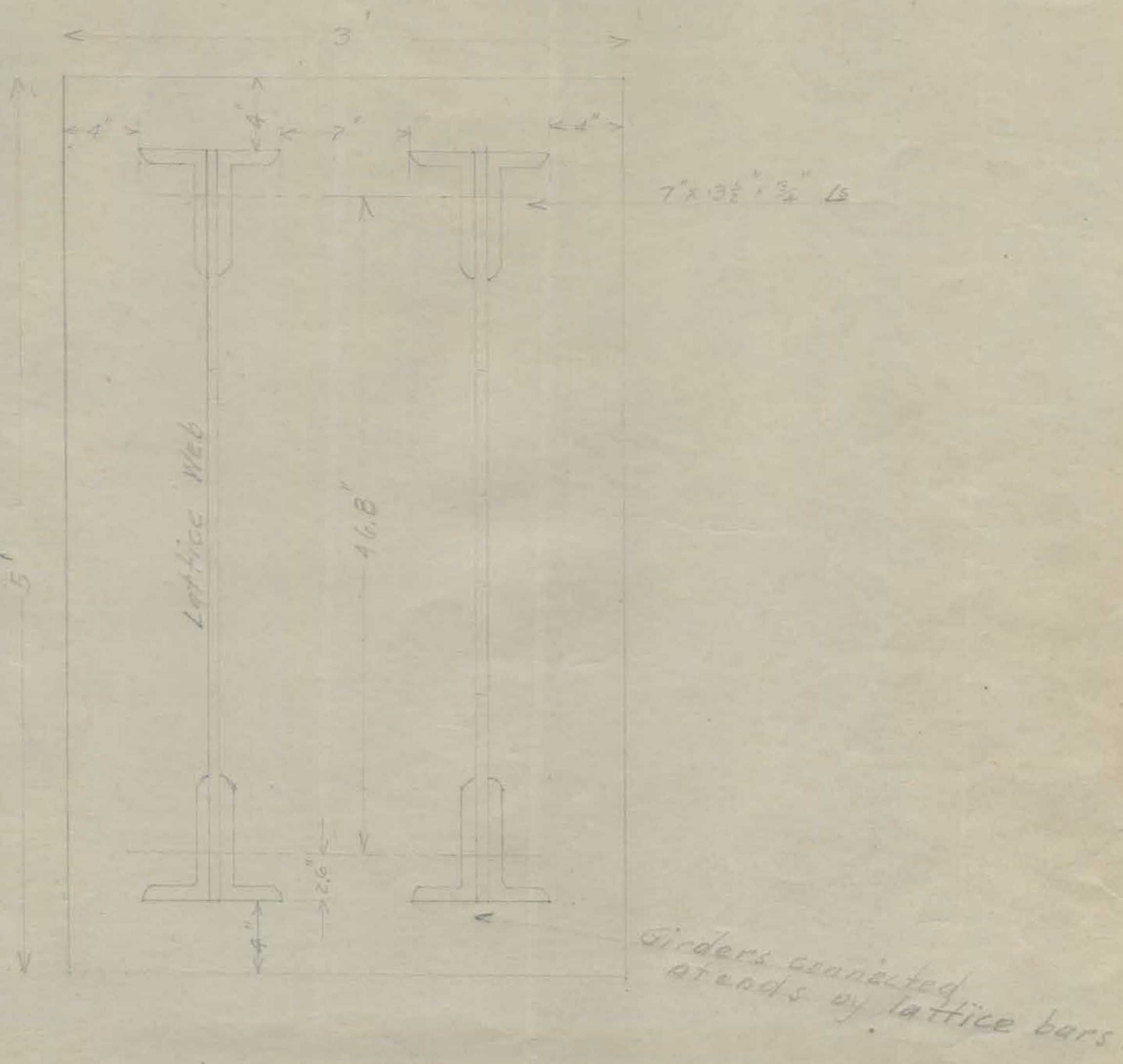



| X | $X^2$ | Y |
|---|---|---|
| 0 | 0 | 0 |
| 10 | 100 | 0.33 |
| 20 | 400 | 1.33 |
| 30 | 900 | 3.00 |
| 40 | 1600 | 5.33 |
| 50 | 2500 | 8.33 |
| 60 | 3600 | 12.00 |
| 70 | 4900 | 16.30 |
| 80 | 6400 | 21.30 |
| 90 | 8100 | 26.95 |
| 100 | 10000 | 33.27 |
| 110 | 12100 | 40.30 |
| 115 | 13225 | 44.00 |

## SHAPE OF CROSS-SECTION

A trial section 3'x 5' will be used.

TRIAL SECTION FOR ARCH RIB.



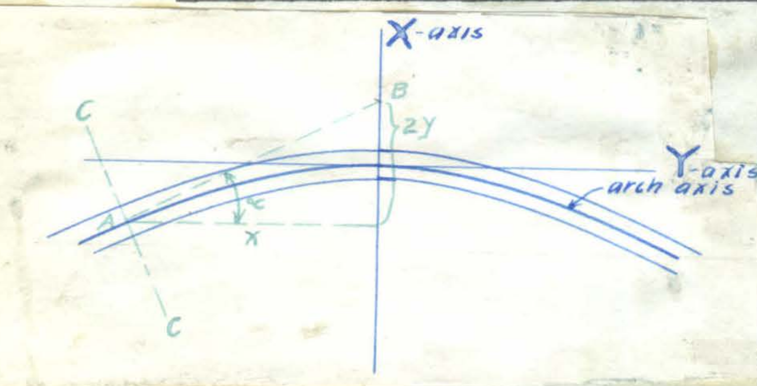
## COMPUTATION OF FORCES ON RIB STEEL DUE TO DEAD WEIGHT OF THE RIB

$$s = \frac{N}{A} \pm \frac{Nvc}{I} = \frac{N}{A} \pm \frac{Mc}{I}$$

see sheet.

s = max. unit stress in steel.
N = thrust normal to any cross-section.
M = bending moment on any section.
V = eccentricity of N.
A = area of any cross-section.
C = dist. from neutral axis to outer fiber.
I = moment of inertia of resisting section.

~~See drawing #~~
Length of arch rib axis from crown to abutment equals 125', by scale. (1"=5' scale)
Divide the axis into 5' lengths. Draw a line through each dividing point normal to the axis at that point. This divides the rib into sections which are about 5' x 5', and whose centers of gravity are approx. on the axis halfway between the dividing points.
If the rib is made 3' thick, the weight of each section will be 5' x 5' x 3' x 150# = 11,250#




C-C is a section normal to the axis.
A-B is a tangent to the axis at any section C-C. α is the angle between the tangent and the horizontal.

$M_{cc} = V_L \times (115 - X) - H_L(44 - Y) - \Sigma(PL)$
$N_{cc} = \Sigma P \sin\alpha + H_c \cos\alpha$

see sheet 9

$\Sigma P$ = sum of loads between C-C and the crown
$H_c$ = horizontal thrust at crown.

Each half-rib includes 25 sections.
25 x 11,250 = 281,250# = V = vert. reaction at end.

See sheet # 8 for moment arms.

$H_L = H_c$
By equating moments about the crown:
$+ H \times 44 + 1495.7 \times 11.25 - 115 \times 281.25 = 0$

$$H_L = + \frac{32343.7 - 16826.6}{44} = + \frac{15517}{44} = + 352.6 \text{ thousand lb.}$$

By equating moments about the left end:

$$H_C = \frac{1384.8 \times 11.25}{44} = \frac{15579}{44} = 354.0 \text{ thousand lb.}$$

Average = 353.3 thousand lb.

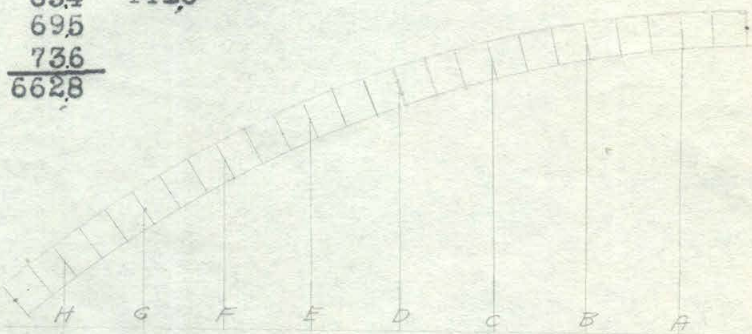
$\Sigma L$

Sum of moment arms of dead weight loads of the arch rib about:
(Moment arms were scaled from 1" = 5' drawing of arch rib.)

(Points half-way between centers of ~~columns~~ 5' sections)

(25-5' sections)

| Crown | Left end | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|---|
| 2.5 | 2.0 | 2.5 | 2.5 | 2.4 | 2.4 | 2.3 | 2.2 | 2.1 | 2.0 |
| 7.6 | 6.1 | 7.5 | 7.4 | 7.2 | 7.1 | 6.8 | 6.6 | 6.3 | 6.1 |
| 12.6 | 10.2 | 12.4 | 12.3 | 12.0 | 11.7 | 11.3 | 10.9 | 10.5 | 8.1 |
| 17.6 | 14.4 | 17.3 | 17.1 | 16.7 | 16.3 | 15.7 | 15.2 | 14.6 | |
| 22.5 | 18.6 | 22.2 | 21.9 | 21.4 | 20.8 | 20.1 | 19.4 | 18.7 | |
| 27.4 | 22.9 | 27.1 | 26.7 | 26.0 | 25.3 | 24.4 | 23.6 | 52.2 | |
| 32.3 | 27.2 | 31.9 | 31.4 | 30.6 | 29.7 | 28.7 | 27.7 | | |
| 37.2 | 31.6 | 36.7 | 36.1 | 35.1 | 34.11 | 32.9 | 31.8 | | |
| 42.0 | 36.0 | 41.5 | 40.7 | 39.6 | 38.4 | 37.1 | 137.4 | | |
| 46.8 | 40.5 | 46.2 | 45.3 | 44.0 | 42.7 | 41.2 | | | |
| 51.6 | 45.5 | 50.9 | 49.8 | 48.4 | 46.9 | 45.3 | | | |
| 56.3 | 49.6 | 55.5 | 54.3 | 52.7 | 51.1 | 265.8 | | | |
| 61.0 | 54.2 | 60.1 | 58.7 | 57.0 | 55.2 | | | | |
| 65.6 | 58.9 | 64.6 | 63.1 | 61.2 | 59.3 | | | | |
| 70.2 | 63.6 | 69.1 | 67.4 | 65.4 | 441.0 | | | | |
| 74.7 | 68.4 | 73.5 | 71.7 | 69.5 | | | | | |
| 79.2 | 73.2 | 77.9 | 75.9 | 73.6 | | | | | |
| 83.6 | 78.0 | 82.2 | 80.1 | 662.8 | | | | | |
| 88.0 | 82.9 | 86.5 | 84.2 | | | | | | |
| 92.3 | 87.8 | 90.7 | 88.3 | | | | | | |
| 96.6 | 92.7 | 94.9 | 934.9 | | | | | | |
| 100.8 | 97.6 | 99.0 | | | | | | | |
| 105.0 | 102.6 | 103.1 | | | | | | | |
| 109.1 | 107.6 | 1253.3 | | | | | | | |
| 113.2 | 112.7 | | | | | | | | |
| 1495.7 | 1384.8 | | | | | | | | |
| Between crown and left end | Between left end and crown | Between A and left end | Between B and left end | Between C and left wnd | BTWE Btween D and left end | Between E and left end | Between F and left end | Between G and left end | Between H and left end |



# THRUSTS AND BENDING MOMENTS.

$$M_{cc} = V_L \times (115 - X) - H_L(44 - Y) - \Sigma(PL)$$

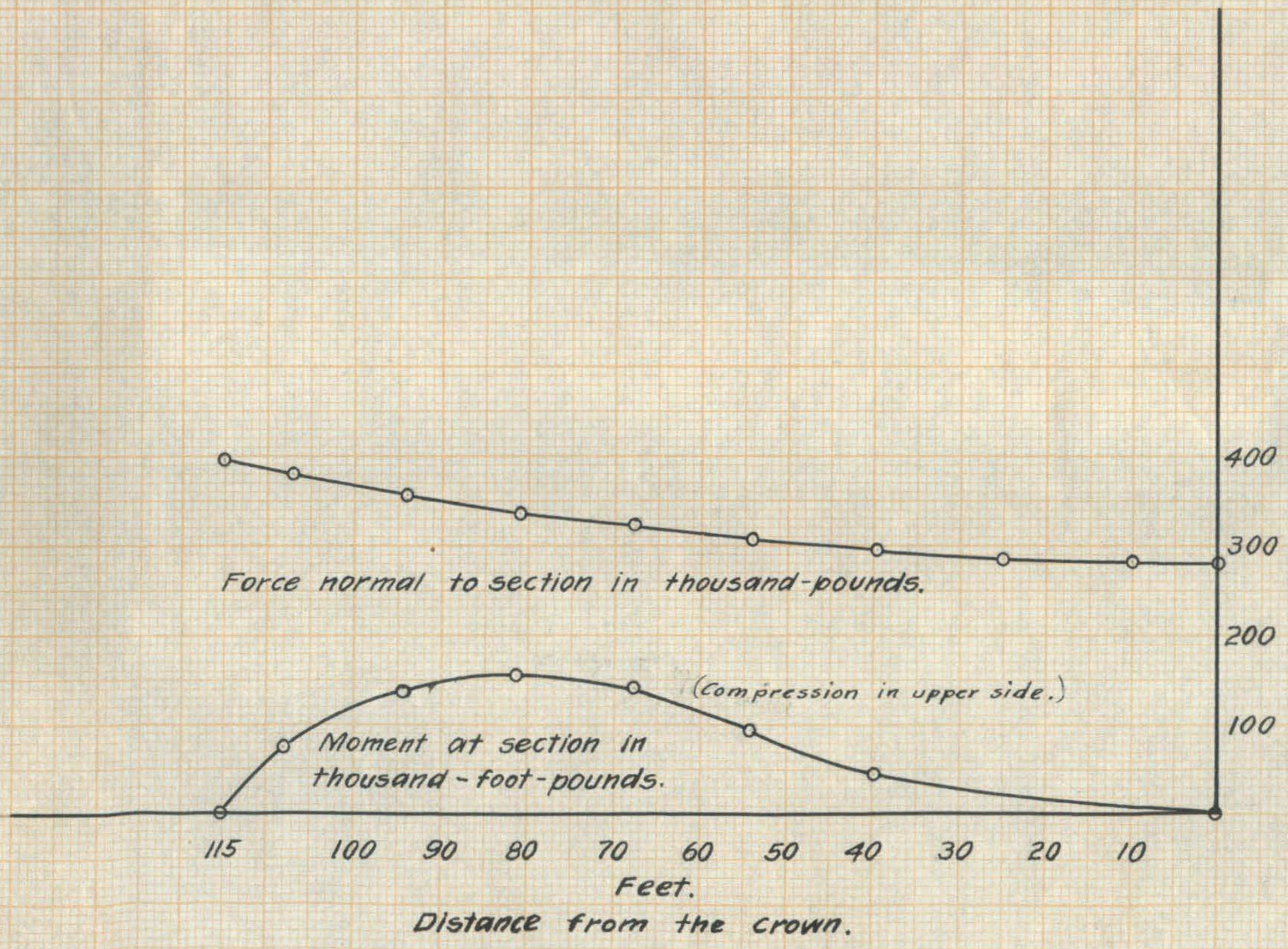
| X | (115-X) | (44-Y) | X | (115-x) | (44-Y) |
|---|---|---|---|---|---|
| 0 | 115.0 | 44.00 | 67.8 | 47.2 | 28.71 |
| 10.0 | 105.0 | 43.67 | 81.3 | 33.7 | 22.01 |
| 24.9 | 90.1 | 41.94 | 94.4 | 20.6 | 14.35 |
| 39.6 | 75.4 | 38.78 | 106.9 | 8.1 | 5.98 |
| 53.9 | 61.1 | 34.33 | 115.0 | 0.0 | 0.00 |

| X | $V_L \times (115-x)$ | $H_L(44-y)$ | P | $\Sigma(L)$ ← see sheet 8. | | | | $M_{cc}$ |
|---|---|---|---|---|---|---|---|---|
| 0 | 281.2 x 115 | - 353.3 x 44 | - 112.5 x | 1495.7 | = 32343.7 | - 16826.7 | - 15517 | = 0 |
| 10.0 | 281.2 x 105 | - 353.3 x 43.67 | - 112.5 x | 1253.3 | = 29526 | -15429 | - 14100 | = - 3 |
| 244 | 281.2 x 90.1 | - 353.3 x 41.94 | - 112.5 x | 934.9 | = 25336 | -14817 | - 10518 | = + 1 |
| 39.6 | 281.2 x 75.4 | - 353.3 x 38.78 | - 112.5 x | 662.8 | = 21202 | -13701 | - 7456 | = + 45 |
| 53.9 | 281.2 x 61.1 | - 353.3 x 34.33 | - 112.5x | 441.0 | = 17181 | -12129 | - 4961 | = + 91 |
| 67.8 | 281.2 x 47.2 | - 353.3 x 28.71 | - 112.5 x | 265.8 | = 13273 | -10143 | - 2990 | = + 140 |
| 81.3 | 281.2 x 33.7 | - 353.3 x 22.01 | - 112.5 x | 137.4 | = 9476 | - 7776 | - 1546 | = + 154 |
| 94.4 | 281.2 x 20.6 | - 353.3 x 19.35 | - 112.5 x | 52.2 | = 5793 | - 5070 | - 587 | = + 136 |
| 106.9 | 281.2 x 8.1 | - 353.3 x 5.98 | - 112.5 x | 8.1 | = 2278 | - 2113 | - 91 | = + 74 |
| 115.0 | 281.2 x 0.0 | - 353.3 x 0.0 | - 112.5 x | 0.0 | = 0 | - 0 | - - | = 0 |

Plotted on 9-A

| SECTION | $X^2$ | X | Y | 2Y | $\tan \alpha$ | $\sin \alpha$ | $\cos \alpha$ | $\Sigma P$ | $\Sigma P \sin \alpha$ | $H_c \cos \alpha$ | $N_{cc}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| A | 100.0 | 10.0 | 0.333 | 0.666 | .0666 | .0665 | .9978 | 22.50 | 1.5 | 280.6 | 282.1 |
| B | 620.0 | 24.9 | 2.063 | 4.126 | .1658 | .1636 | .9865 | 56.25 | 9.2 | 277.5 | 286.7 |
| C | 1568.2 | 39.6 | 5.217 | 10.434 | .2640 | .2552 | .9668 | 90.00 | 22.9 | 272.0 | 294.9 |
| D | 2905.2 | 53.9 | 9.666 | 19.332 | .3585 | .3375 | .9415 | 123.80 | 41.8 | 264.7 | 306.5 |
| E | 4596.8 | 67.8 | 15.29 | 30.58 | .4510 | .4110 | .9116 | 157.40 | 64.7 | 256.5 | 321.2 |
| F | 6609.7 | 81.3 | 21.99 | 43.98 | .5410 | .4759 | .8796 | 191.20 | 91.0 | 247.5 | 338.5 |
| G | 8911.4 | 94.4 | 29.65 | 59.30 | .6280 | .5318 | .8469 | 225.00 | 119.6 | 238.4 | 358.0 |
| H | 11428 | 106.9 | 38.02 | 76.04 | .7110 | .5796 | .8150 | 258.70 | 150.0 | 229.0 | 379.0 |
| | 13225 | 115.0 | 44.00 | 88.00 | .7650 | .6078 | .7942 | 281.25 | 171.0 | 223.0 | 394.0 |

FORCES ON ARCH DUE TO WEIGHT OF RIB.



Initial Stress in Steel

stress calculated at three sections $x = 81.3$
$x = 106.9$
$x = 115.0$

$$f_s = \frac{N}{A_s} + \frac{M \cdot c}{I_s}$$

$x=81.3$ $N = 338000^{\#}$ (see sheet 9)
$A_s = 8 \times 7.31 = 58.48$ sq.in. ($= .40659$ sq.ft. see sheet 6-A)
$M = 154000$ ft.lbs. (see sheet 9)
$= 1850000$ in.lbs.
$c = 26''$
$I_s = 8(I_o + Ae^2) = 8[36 + 7.31 \times 23.4^2)]$
$= 8[36 + (7.31 \times 547.56)] = 8(36 + 4000)$
$= 32288$ (inches)

$$f_s = \frac{338000}{58.48} + \frac{1850000 \times 26}{32288}$$

$= 5700 + 1490 = \underline{7190}^{\#}$ per sq.in.

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$x=106.9$ $N = 379000$
$M = 74000$ ft.lb. $= 888000$ in.lb.

$$f_s = \frac{379000}{58.48} + \frac{888000 \times 26}{32288}$$

$= 6480 + 715 = \underline{7195}^{\#}$ per sq.in.

$x=115.0$ $N = 394000$ $M = 0$

$$f_s = \frac{394000}{58.48} = \underline{6730}^{\#} \text{ per sq.in.}$$

## Estimate of Deadweight of Superstructure

| | Dead weight per Column (Columns 15'4" c-c) |
|---|---|
| The railing will be assumed to weigh 300# per lineal foot. | 15.33 x 300 = 4,600 lb. |
| The roadway will be assumed to have a total width of 32 feet, including sidewalks, with a thickness of 11 inches. The weight per square foot will be about 140 #. | 16x15.33x140 = 34,400 |
| Floor beams will be assumed to be placed 5 feet apart. Beams and girders will be assumed to weigh 300# per lineal foot. | 52 x 300 = 15,600 |
| | Total 54,600 |
| An additional allowance of 10% will be made | 5400 |
| | 60,000 |

The columns will be assumed to be 18 inches square. They will therefore weigh about 340 lb. per foot of height.

| COLUMN | HEIGHT | WEIGHT | TOTALWEIGHT ON COLUMN |
|---|---|---|---|
| A | 5 | 1700 | 61700 |
| B | 7 | 2400 | 62400 |
| C | 9 | 3100 | 63100 |
| D | 14 | 4800 | 64800 |
| E | 20 | 6800 | 66800 |
| F | 29 | 9900 | 69900 |
| G | 38 | 12900 | 72900 |
| | | | 461600 |

COMPUTATION OF $\Sigma$(PL) OF SUPERSTRUCTURE LOADS ABOUT CROWN.

| Column | Load (Thousand-lbs.) | Mom. Arm (Feet) | (PL) |
|---|---|---|---|
| A | 61.7 | 7.67 | 470 |
| B | 62.4 | 23.00 | 1440 |
| C | 63.1 | 38.33 | 2440 |
| D | 64.8 | 53.67 | 3480 |
| E | 66.8 | 69.00 | 4610 |
| F | 69.9 | 84.33 | 5890 |
| G | 72.9 | 99.67 | 7270 |
| | | $\Sigma$(PL) | =25600 (Thousand-foot-lbs.) |

$\Sigma$(PL) to left of section A-B

| | | | |
|---|---|---|---|
| B | 62.4 | 7.67 | 479 |
| C | 63.1 | 23.00 | 1450 |
| D | 64.8 | 38.33 | 2484 |
| E | 66.88 | 53.67 | 3585 |
| F | 69.9 | 69.00 | 4823 |
| G | 72.9 | 84.33 | 6148 |
| | | $\Sigma$(PL) = | 18970 |

$\Sigma$(PL) to left of sec. B-C

| | | | |
|---|---|---|---|
| C | 63.1 | 7.67 | 480 |
| D | 64.8 | 23.00 | 1490 |
| E | 66.8 | 38.33 | 2560 |
| F | 69.9 | 53.67 | 3750 |
| G | 72.9 | 69.00 | 5030 |
| | | $\Sigma$(PL) = | 13310 |

$\Sigma$(PL) to left of sec. C-D

| | | | |
|---|---|---|---|
| D | 64.8 | 7.67 | 500 |
| E | 66.8 | 23.00 | 1540 |
| F | 69.9 | 38.33 | 2680 |
| G | 72.9 | 53.67 | 3910 |
| | | $\Sigma$(PL) = | 8630 |

$\Sigma$(PL) to left of sec. D-E

| | | | |
|---|---|---|---|
| E | 66.8 | 7.67 | 510 |
| F | 69.9 | 23.00 | 1610 |
| G | 72.9 | 38.33 | 2800 |
| | | $\Sigma$(PL) = | 4920 |

$\Sigma$(PL) to left of sec. E-F

| | | | |
|---|---|---|---|
| F | 69.9 | 7.67 | 540 |
| G | 72.9 | 23.00 | 1680 |
| | | $\Sigma$(PL) = | 2220 |

$\Sigma$(PL) to left of sec. F-G

| | | | |
|---|---|---|---|
| G | 72.9 | 7.67 | 560 = $\Sigma$(PL) |

# MOMENT OF INERTIA
(of Concrete and steel section)

To calc I - in biquadratic inches

$$I = I_c + (n-i)\times Is$$

$$I_c = \frac{bd^3}{12} = \frac{36 \times 60^3}{12} = 3 \times 216000 = 648000$$

$$(n-1) = 14$$

$I_s = 32288$ (see sheet ~~12a~~ 10)

$$I_s \times (n-1) = 452000$$

$$I = 452000 + 648000 = 1100000$$

In biquadratic feet.

$$I_c = \frac{bd^3}{12} = \frac{3 \times 5^3}{12} = \frac{125}{4} = 31.25$$

$$(n-1)\times Is = \frac{452000}{124} = \frac{452000}{20736} = 21.8$$

$$I = I_c + (n-1)\times I_s = 21.8 + 31.25 = 53.05$$

$$\frac{1}{I} = .01884$$

Computation of Stresses Due to Dead Weight

of Superstructure

$V_L$ = 461,600# (See sheet 11 )
Comp. of Reactions:
Equation of moments about the crown:

$+H_L \times 44 + \Sigma(PL) - V_L \times 115 = 0$ See sheet 12 for $\Sigma(PL)$

$461.6 \times 115 - 25600 = H_L \times 44$

$53100 - 25600 = H_L \times 44$

$H_L = \dfrac{27500}{44} = 625$ thousand lb.

$H_c = H_L$

Comp. of B.M. at sections halfway between columns:
Equation of moments to left of section
$M = V_L \times (115-X) - H_L\,(44-y) - \Sigma(PL)$

| X | $X^2$ | y | 115-X | 44-y | X | $X^2$ | Y | 115-X | 44-y |
|---|---|---|---|---|---|---|---|---|---|
| 15.33 | 235.009 | .7819 | 99.67 | 43.2181 | 61.33 | 376.137 | 12.514 | 53.67 | 31.490 |
| 30.67 | 940.649 | 3.1295 | 84.33 | 40.8705 | 76.67 | 587.829 | 19.557 | 38.33 | 24.443 |
| 46.00 | 2116. | 7.0399 | 69.00 | 36.9601 | 92.00 | 846.400 | 28.160 | 23.00 | 15.840 |

$V_L \times (115-X) - H_L \times (44-y) - \Sigma(PL)$

| Sec | |
|---|---|
| A-B | 461.6 x 99.67 - 625 x 43.2181 - 18980. = |
| B-C | 461.6 x 84.33 - 625 x 40.8705 - 13310. = |
| C-D | 461.6 x 69.00 - 625 x 36.9601 - 8630. = |
| D-E | 461.6 x 53.67 - 625 x 31.490 - 4920. = |
| E-F | 461.6 x 38.33 - 625 x 24.443 - 2220. = |
| F-G | 461.6 x 23.00 - 625 x 15.840 - 560. = |

~~-625 x~~

(Thousand Ft-lb. Plotted on sheet 16)

| | | | | | M | Fcm (lbs.) |
|---|---|---|---|---|---|---|
| A-B | 46028. | - 27011. | - 18970. | = ~~0~~ | + 47 = M | 8 |
| B-C | 38927. | - 25544. | - 13310. | = ~~0~~ | + 43 | 22 |
| C-D | 31850. | - 23100. | - 8630. | = ~~100~~ | + 120 | 39 |
| D-E | 24774. | - 19681. | - 4920. | = ~~200~~ | + 163 | 53 |
| E-F | 17693. | - 15277. | - 2220. | = ~~200~~ | + 196 | 64 |
| F-G | 10617. | - 9900. | - 560. | = ~~140~~ | + 157 | 51 |
| | | | | | 65 | 21 |

$f_{cm} = \dfrac{Mc}{I}$

$f_{cm} = \dfrac{196000 \times 2.5 \times .01884}{144} = 64^{\#}$

Dead Load

$\tan\alpha = \frac{2Y}{X}$

| | | | | | | Thousand-lb seepage | | | Plotted on sheet 16 | |
|---|---|---|---|---|---|---|---|---|---|---|
| Sec | X | 2Y | $\tan\alpha$ | $\sin\alpha$ | $\cos\alpha$ | $\Sigma P$ | $\Sigma P\sin\alpha$ | $H_c\cos\alpha$ | N | $f_{cn}$ |
| A-B | 15.33 | 1.56 | .1018 | .101 | .995 | 617 | 6.24 | 622. | 628. | 211 |
| B-C | 30.67 | 6.26 | .204 | .199 | .980 | 1241 | 24.7 | 613. | 638. | 214 |
| C-D | 46.00 | 14.00 | .306 | .292 | .956 | 1872 | 54.6 | 598. | 653. | 219 |
| D-E | 61.33 | 25.00 | .408 | .378 | .926 | 2520 | 95.2 | 579. | 674. | 226 |
| E-F | 76.67 | 39.00 | .510 | .454 | .891 | 3188 | 145. | 557. | 702. | 236 |
| F-G | 92.00 | 56.40 | .613 | .522 | .853 | 3887 | 203. | 533. | 736. | 247 |
| | 107.33 | | | | | | | | 765. | 257 |

$N = \Sigma P\sin\alpha + H_c\cos\alpha$

Area of steel = 58.48 sq. in.

$(n-1) = 14$

$14 \times 58.48 = 818$ sq. in

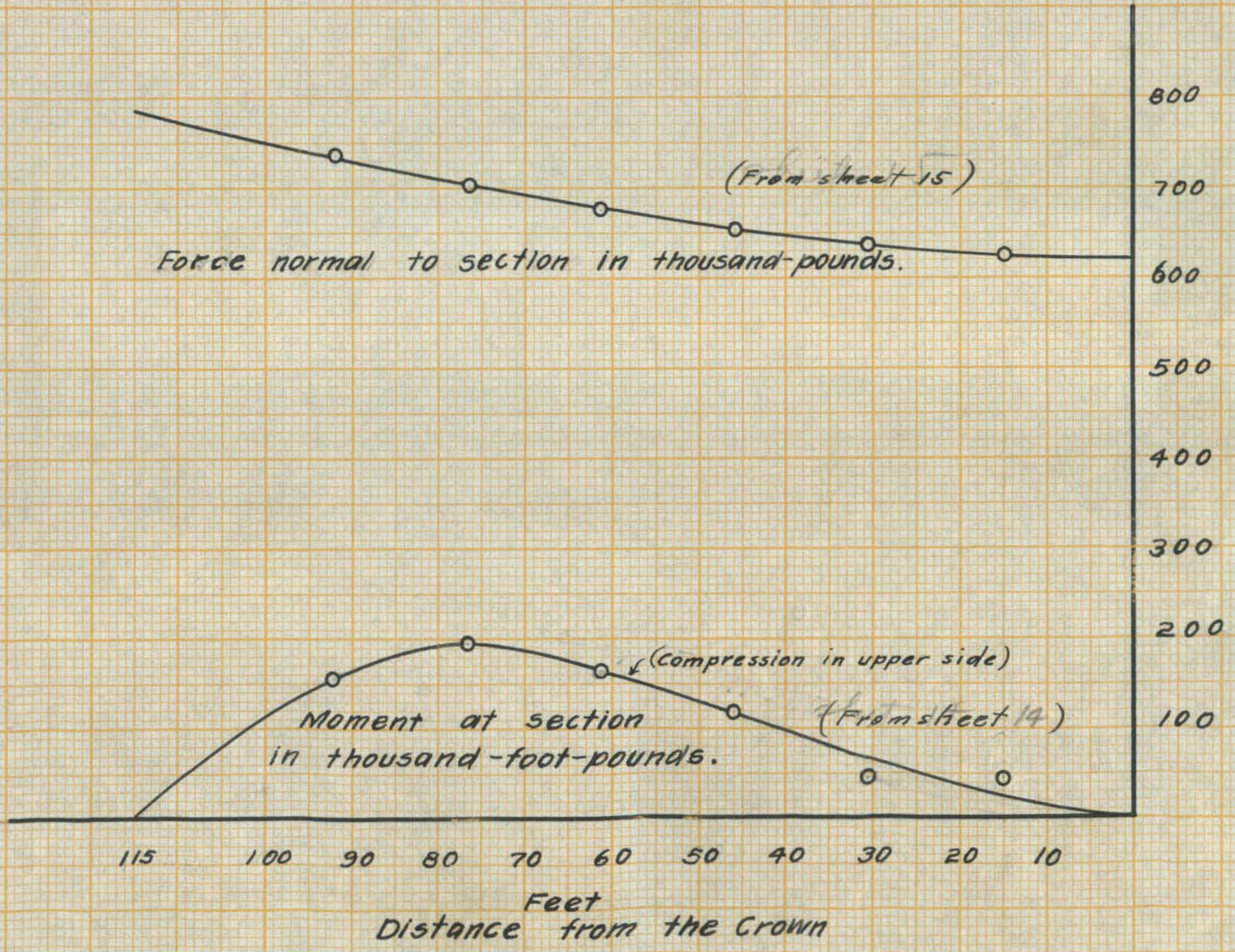
Area of concrete = $60 \times 36 = 2160$ sq. in.

$2160 + 818 = 2978$ sq. in.

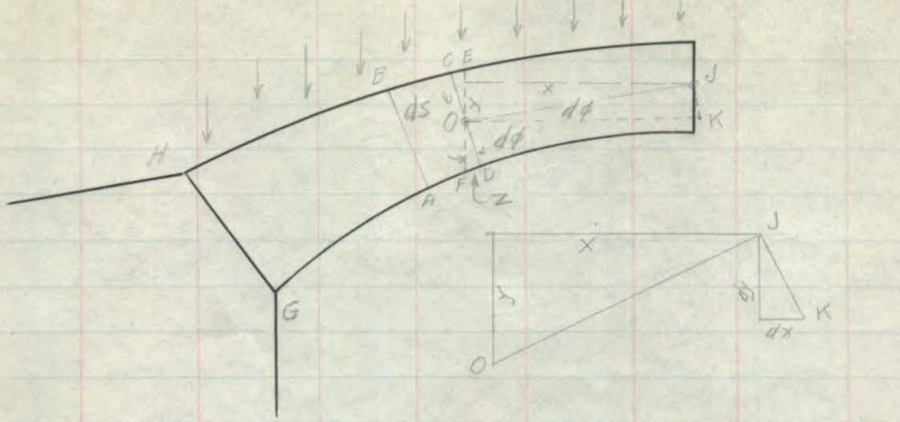
$f_{cn} = \frac{N}{2978}$

$\frac{628000}{2978} = 211^{\#}$

FORCES ON ARCH DUE TO WEIGHT OF COLUMNS AND ROADWAY.



Derivation of formulas - Spalding's "Masonry Structures" - p. 295.




The arch is considered as fixed at GH.

unit fiber elongation = $\frac{z}{ds} = \frac{e\,d\phi}{ds}$ $\quad z = e\,d\phi \quad d\phi = \frac{z}{e} \quad d\phi \cdot JO = JK$

unit stress = $f_m = \frac{Me}{I}$

Modulus of elasticity = $E \quad \frac{E}{f_m} = \frac{ds}{z} \qquad f_m = \frac{E\,z}{ds} = \frac{eE\,d\phi}{ds} = \frac{Me}{I}$

Equating $\frac{Ee\,d\phi}{ds} = \frac{Me}{I} \qquad d\phi = \frac{M\,ds\,e}{IEe}$

$\frac{JK}{JO} = \frac{dy}{x} = d\phi \qquad \frac{JK}{JO} = \frac{dx}{y} = d\phi$

$dy = x\,d\phi = \frac{Mx\,ds}{EI} \qquad dx = y\,d\phi = \frac{My\,ds}{EI}$

$d\phi = \frac{M\,ds}{EI} \qquad dy = \frac{Mx\,ds}{EI} \qquad dx = \frac{My\,ds}{EI}$

$\int_c^L \phi_L = -\int_c^R \phi_R \qquad \int_c^L dy_L = \int_c^R dy_R \qquad \int_c^L dx_L = -\int_c^R dx_R$

$\frac{1}{E}\int_c^L \frac{M\,ds}{I} = -\frac{1}{E}\int_c^R \frac{M\,ds}{I} \qquad \frac{1}{E}\int_c^L \frac{Mx\,ds}{I} = \frac{1}{E}\int_c^R \frac{Mx\,ds}{I}$

$$\frac{1}{E}\int_c^L \frac{My\,ds}{I} = -\int_c^R \frac{My\,ds}{I}$$

$M_L$ = B.M. on any section in left half.
$M_R$ = " " " " right ".
$m_L$ = " " " " left ", caused by loads between that section & crown
$m_R$ = " " " " right ", " " " " " " " ".
$M_c$ = " at crown
$V_c$ = shear at crown
$H_c$ = thrust at crown.
B.M. at any section = B.M. at any other section + moments of intermediate loads.

$M_L = M_c + V_c x + H_c y - m_L$ $\qquad$ $M_R = M_c - V_c x + H_c y - m_R$

By substitution and combination with the equations on the preceding page, the equations on sheet 18 may be obtained.

MECHANICAL ANALYSIS OF ELASTIC ARCH

In the analysis of arches by the application of the equations obtained from the theory of elasticity, the use of methods employing arbitrary divisions of the arch axis appears to be supplanting the older method of dividing the arch axis in sections so as to make the value of $ds/I$ a constant, because of the increase in accuracy of the former. And when the stresses in an arch are to be investigated for more than two or three positions of live load, the labor involved is lessened if influence lines for the crown thrust, moment, and shear are first obtained.

In the following method, the arbitrary divisions are reduced to differential dimensions and the results consequently approach in accuracy those of direct integration. It gives influence line values, is comparatively simple, and largely mechanical. Instead of replacing the integrals in the fundamental equations for arches by summation signs and taking finite values of $ds$, the values of the functions involved are plotted to scale and integration is performed by quadrature; i.e., by finding the area under the curves of these functions by planimeter or similar means.

The equations for the crown thrust, shear, and moment may be obtained either by the method of least work or by deflections, and are as follows:

$$H_o = \frac{\int\frac{ds}{I}\int\frac{m\,y\,ds}{I} - \int\frac{m\,ds}{I}\int\frac{y\,ds}{I}}{2\left[\int\frac{ds}{I}\int\frac{y^2\,ds}{I} - \left(\int\frac{y\,ds}{I}\right)^2\right]} \qquad H^1 = \frac{\Delta\int\frac{ds}{I}}{2\left[\int\frac{ds}{I}\int\frac{y^2\,ds}{I} - \left(\int\frac{y\,ds}{I}\right)^2\right]}$$

$$V_o = \frac{\int\frac{(m_L - m_R)\,x\,ds}{I}}{2\int\frac{x^2\,ds}{I}}$$

$$M_o = \frac{\int\frac{m\,ds}{I} - 2H_o\int\frac{y\,ds}{I}}{2\int\frac{ds}{I}} \qquad M^1 = \frac{-H^1\int\frac{y\,ds}{I}}{\int\frac{ds}{I}}$$

For a symmetrical arch the limits of integrals are all for half the arch.

$ds$ = differential length of the arch axis

$I$ = moment of inertia of any section ($I$ of concrete area + $E_s/E_c$ times $I$ of steel area)

$x$, $y$ = co-ordinates of any point on the arch axis with the crown as origin, and all taken as positive

$l$ = length of span

$E_c$ = modulus of elasticity of concrete

$m$ = bending moment at any point due to external loads considering each half of the arch as a cantilever. Where $m$ occurs without subscripts, it includes the bending moments at symmetrical points on both halves of the arch. In all cases $m$ has been taken positive for downward loads.

$\Delta$ = $c\ t\ l\ E_c$ in the case of temperature change

= $p\ l$ in the case of rib shortening due to thrust

= $k\ l\ E_c$ in the case of shrinkage

19



c = coefficient of expansion

t = temperature rise or fall, taken as positive for a rise

p = average intensity of compression on equivalent concrete area (area of concrete + $E_s/E_c$ times area of steel)

k = coefficient for shrinkage

$H_o$ = thrust at the crown, positive for compression

$V_o$ = shear at the crown, positive when acting as in Fig. 0

$M_o$ = moment at the crown, positive when causing compression in the top

H' = thrust at crown due to temperature change or rib shortening

M' = moment at crown due to rib shortening or temperature change

$\propto$ = angle between tangent to arch axis at any point and the horizontal.

Since ds = dx/cos $\propto$, the term ds/I in the above equations may be replaced by dx/I' where I' = I cos $\propto$. Note that $\propto$ is also the angle which the radius of curvature at any point makes with the vertical.

Since influence values are sought, the value of m due to a unit load may be replaced by (x - na) for values of x greater than na. For values of x less than na, m = 0. If we consider the left half of the arch $(m_L - m_R) = (x - na)$ as $m_R = 0$.

Procedure

The arch is first divided into any convenient number (n) of equal horizontal divisions (a) in length. It is not necessary that (a) should be an aliquot part of the span, but it is somewhat simpler if (a) is a number of even feet in length. When (a) can be made 10 feet, the reduction of some of the data can be done by shifting the decimal point.

Values of 1/I', y/I', y /I' are computed for each of the n sections. These are all the preliminary computations needed. The steps in the analysis then are -

1a Plot the values of $1/I'$ as ordinates with corresponding values of (n) as abscissas and connect these points with a smooth curve, A, Fig. 1. The area of a strip pq is then $dx/I'$ and the area between curve A and the axis of ordinates is equal to $\int dx/I'$.

1b Plot n times the value of $1/I'$ on the same base at each corresponding value of (n) and connect the points thus found with a smooth curve, B. Since the ordinates of curve B are values of $n/I'$, the area of a strip p'q' is $ndx/I'$, and as $an = x$ this area is $xdx/aI'$. So the area between curve B and the axis is $1/a \int xdx/I'$.

1c Connect all points representing $2/I'$, $3/I'$, $4/I'$, etc., beginning at curve B and extending to the right to the springing line.

Assume a unit load placed at $n = 2$, the value of m at any point x is $1(x - na)$ which may be written $a(x/a - n)$, and the value of $mdx/I'$ may be expressed as $a(x/a - n)dx/I'$.

Since the length of the strip oq' is $2/I'$ and of p'q', $x/aI'$, the length of p'o is $(x/a - n)dx/I'$, and the area between curve B and curve $A_2$ is $1/a \int mdx/I'$ for a unit load at $n = 2$, the limits being from $n = 2$ to the springing line.

For a unit load at any other value of n, the value of $1/a \int mdx/I'$ may be found in the same way by evaluating the area between curve B and the curve $A_n$.

2a With a new base, Fig. 2, plot curve B using a smaller scale for the ordinates, which will give curve , giving values of $n/I'$.

2b Plot n times these values of $n/I'$ at each corresponding value of n and connect the points thus found with a smooth curve, D.

Since the ordinates of curve D are values of $n^2/I'$, the area of a strip rs is $n^2dx/I'$, and as $an = x$, this area $= x^2dx/a^2I'$. So the area between curve D and the axis is $1/a^2 \int x^2dx/I'$.

2c Connect all points representing $2n/I'$, $3n/I'$, $4n/I'$, etc., beginning at curve D and extending to the right to the springing line.

Assume a unit load placed at $n = 2$. As in 1c above, the value of m is $a(x/a - n)$. Hence $mx = ax(x/a - n) = a^2(x^2/a^2 - n^2)$, and $mxdx/I' = a^2(x^2/a^2 - n^2)dx/I'$. $x^2dx/a^2I'$ is the area of strip rs and $n\,dx/I'$ is the area of strip rt when $n = 2$. Hence the area of strip ts is $(1/a^2)(mxdx/I')$ and the area between curve D and curve $C_2$ is equal to $(1/a^2)$ $mxdx/I'$, the limits being from $n = 2$ to the springing line.

For a unit load at any other value of n, the value of $(1/a^2)\int mxdx/I'$ may be found in the same way by evaluating the area between curve D and curve $C_n$.

3a Plot the values of $y/I'$ as ordinates from a third base using the corresponding values of n as abscissas, and connect these points with a smooth curve E, in Fig. 3.

The area between this curve and the axis of ordinates is equal to $\int ydx/I'$.

3b Plot n times the value of $y/I'$ on the same base at each corresponding value of n and connect the points thus found with a smooth curve, F.

3c Connect all points represeenting $2y/I'$, $3y/I'$, $4y/I'$, etc., beginning at curve F and extending to the right to the springing line.

Assume a unit load at $n = 2$. The value of my is then $y(x - na)$ or $ay(x/a - n)$. And $mydx/I' = ay(x/a - n)dx/I'$. The area of strip uw is $(xy/aI')dx$ and of strip uz, $ndx/I'$. Hence the area of strip zw is $1/a(mydx/I')$ and the area between curves F and $E_2 = 1/a \int mydx/I'$ for a unit load at $n = 2$.

For a unit load at any other value of n, the value of $1/a \int mydx/I'$ may be found in the same way by evaluating the area between curve F and curve $E_n$.

4 Plot values of $y^2/I'$ as ordinates on a fourth base, Fig. 4. The area between this curve and its axis of ordinates is the $\int y^2dx/I'$.

5 The values obtained by evaluating the various areas in 1 to 4 above may now be entered in a table similar to that below and the remainder of the work obtaining the influence values for $H_c$, $V_c$, and $M_o$ is indicated at the top of the columns of this table, the figures in parentheses referring to the steps in the above outline.

6 Computing $\Delta$ involves no difficulty, and as all the other terms in the equation for H' have already been found, the crown thrust and moment due to temperature or rib shortening are readily obtained by substituting in the expressions for these quantities.

Tabular computations in solving for $H_o$, $V_o$, and $M_o$ for influence values

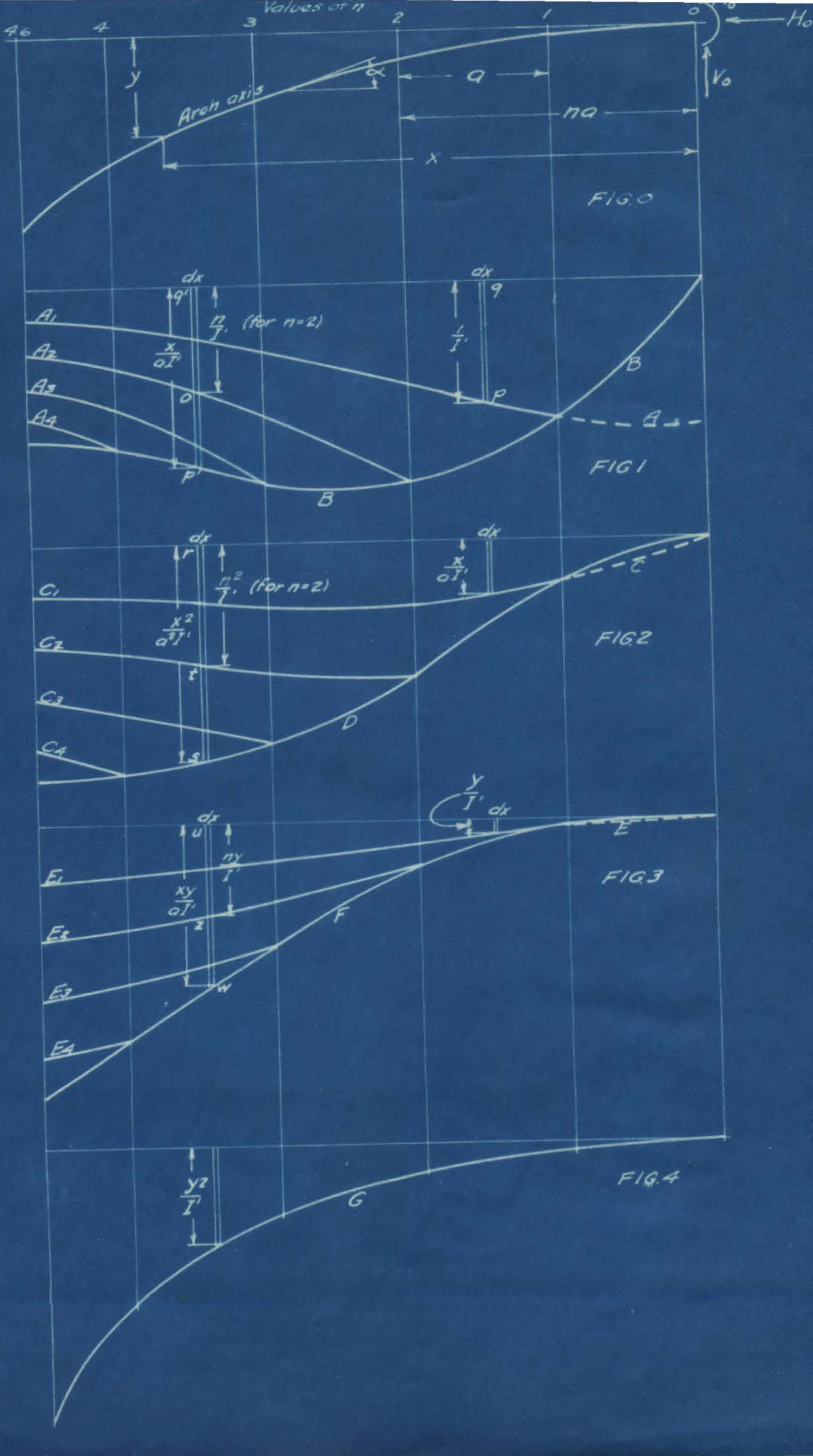
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | (2c) | (2b) | (3c) | (1c) | (1a) | (3a) | | $H_o$ | | | | $M_o$ |
| | $\frac{1}{a^2}\int \frac{m\,x\,dx}{I'}$ | ½ $\frac{Col.1}{2 \times 116.7}$ | $\frac{1}{a}\int \frac{m\,y\,dx}{I'}$ | $\frac{1}{a}\int \frac{m\,dx}{I'}$ | $13.3\,a \times Col.3$ | $55.2\,a \times Col.4$ | Col.5 - Col.6 | $\frac{Col.7}{10520}$ | $a \times Col.4$ | $55.2 \times 2H_o$ | Col.9 - Col.10 | $\frac{Col.11}{2 \times 13.3}$ |
| 0 | 116.7 | .500 | 252 | 31.4 | 33500 | 17300 | 16200 | 1.54 | 314 | 170 | 144 | -3.41 |
| 1 | 71.5 | .320 | 167 | 15.4 | 22200 | 8500 | 13700 | 1.30 | 154 | 143 | 11 | -0.41 |
| 2 | 36.1 | .161 | 960 | 6.1 | 12750 | 3360 | 9390 | .89 | 61 | 98 | -57 | -1.39 |
| 3 | 12.0 | .054 | 400 | 1.6 | 5320 | 880 | 4440 | .42 | 16 | 47 | -31 | -1.17 |
| 4 | 1.8 | .008 | 80 | 0.2 | 1060 | 110 | 950 | .09 | 2 | 10 | -8 | -0.30 |
| 4.6 | 0.0 | .000 | 0 | 0.0 | 0 | 0 | 0 | .00 | 0 | 0 | 0 | 0.00 |

a = 10

(2b) $\frac{1}{a^2}\int \frac{x^2\,dx}{I'} = 116.7$ (1a) $\int \frac{dx}{I'} = 13.3$

(3a) $\int \frac{y\,dx}{I'} = 55.2$ (4) $\int \frac{y^2\,dx}{I'} = 624$

$$2\left[\int \frac{y^2\,dx}{I'} \int \frac{dx}{I'} - \left(\int \frac{y\,dx}{I'}\right)^2\right] = 2\left[624 \times 13.3 - (55.2)^2\right] = 10520$$


Values of n
4.6
4
3
2
1
0
$H_o$
$V_o$
y
Arch axis
α
a
na
x
FIG.0
dx
q'
q
$\frac{n}{I'}$ (for n=2)
$\frac{1}{I'}$
$\frac{x}{aI'}$
$A_1$
$A_2$
$A_3$
$A_4$
o
p
p'
B
A
FIG 1
r
$\frac{n^2}{I'}$ (for n=2)
$\frac{x}{aI'}$
$\frac{x^2}{a^2I'}$
$C_1$
$C_2$
$C_3$
$C_4$
t
s
C
D
FIG.2
$\frac{y}{I'}$
u
$\frac{ny}{I'}$
$\frac{xy}{aI'}$
$E_1$
$E_2$
$E_3$
$E_4$
z
w
E
F
FIG.3
$\frac{y^2}{I'}$
G
FIG.4

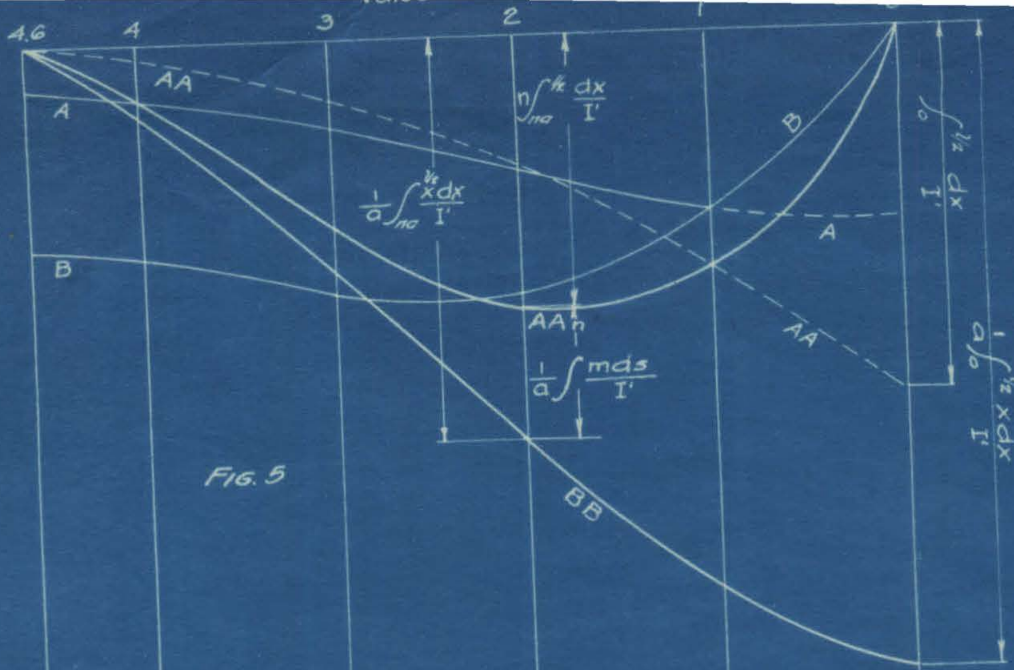



FIG. 5

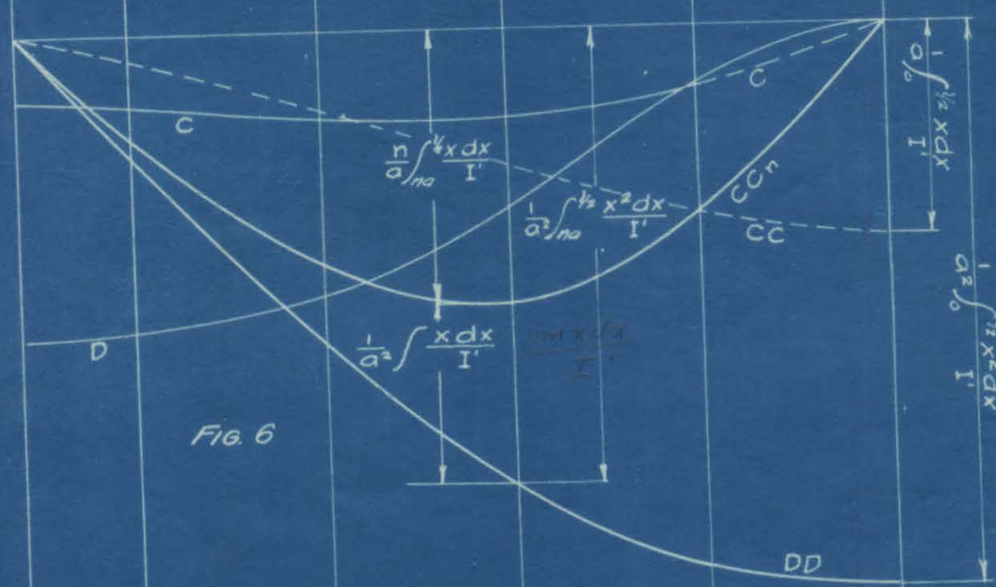



FIG. 6

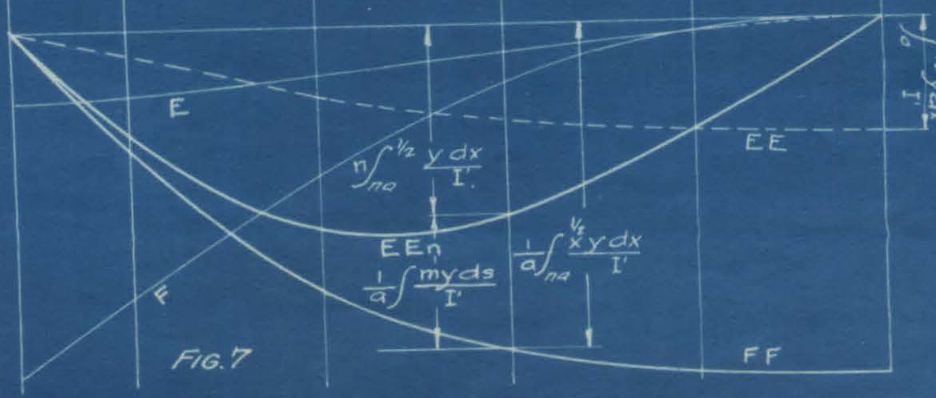



FIG. 7

# PART II

A modification of the method outlined in Part I which will simplify the work still further can be applied when an integraph is available. In the fundamental equations given at the beginning of Part I, the value of m for a unit load at a distance na from the origin is x - na and the integrals in which m occurs are integrated between the limits na and $\frac{l}{2}$ since for values of x less than na, m vanishes.

We may then write the various integrals involving m in the following forms:

$$\int \frac{m\,ds}{I} = \int \frac{x\,dx}{I'} - na\int \frac{dx}{I'} = a\left[\ \underset{(B)}{\int \frac{x\,dx}{a\,I'}} - n\underset{(A)}{\int \frac{dx}{I'}}\ \right] \qquad (1)$$

$$\int \frac{m\,x\,ds}{I} = \int \frac{x^2\,dx}{I'} - na\int \frac{x\,dx}{I'} = a^2\left[\ \underset{(D)}{\int \frac{x^2\,dx}{a^2\,I'}} - n\underset{(C)}{\int \frac{x\,dx}{a\,I'}}\ \right] \qquad (2)$$

$$\int \frac{m\,y\,ds}{I} = \int \frac{xy\,dx}{I'} - na\int \frac{y\,dx}{I'} = a\left[\ \underset{(F)}{\int \frac{xy\,dx}{a\,I'}} - n\underset{(E)}{\int \frac{y\,dx}{I'}}\ \right] \qquad (3)$$

The limits for all the above are $\frac{l}{2}$ and na.

Curves for all of the expressions under the integrals on the right of the above equations have been constructed in Part I and the letters in parentheses refer to these curves, which have been redrawn in Figures 5, 6, and 7.

With the integraph or by the approximate method given at the end of this paper, the integral curves of all the above are drawn beginning at the springing line, and these integral curves are designated with the double letters.

In a manner similar to that used in Part I, plot n times the values of the ordinates to curves AA, CC, and EE at the corresponding values of n, giving curves AAn, CCn, and EEn, the ordinates of which equal

$n\int\frac{dx}{I'}$ $\quad n\int\frac{x\,dx}{a\,I'}$ $\quad n\int\frac{y\,dx}{I'}$ respectively, these being the last terms in each of the three equations (1), (2), and (3).

Then the vertical intersepts between curves AAn and BB $= \frac{1}{a}\int\frac{m\,ds}{I}$

the vertical intersepts between curves CCn and DD $= \frac{1}{a}\int\frac{mx\,ds}{I}$

and the vertical intersepts between curves EEn and FF $= \frac{1}{a}\int\frac{my\,ds}{I}$

The ordinates at the crown, n = 0, to curves BB and DD equal

$\frac{1}{a}\int_0^{\frac{1}{2}}\frac{x\,dx}{I'}$ and $\frac{1}{a}\int_0^{\frac{1}{2}}\frac{x\,dx}{I'}$ respectively.

The ordinates at the crown, n = 0, to curves AA and EE equal

$\int_0^{\frac{1}{2}}\frac{dx}{I'}$ and $\int_0^{\frac{1}{2}}\frac{y\,dx}{I'}$ respectively.

Hence all the integrals required in the fundamental equations are now found except $\int_0^{\frac{1}{2}}\frac{y^2\,dx}{I'}$ for which it is simpler to planimeter the area under curve G of Fig. 4 than to construct an integral curve.

The remainder of the work of obtaining influence values for Ho, Vo, and Mo can best be done in tabular form as in Part I.

The following simple method of approximating an integral curve without the use of the integraph is given by Abdank-Abakanowicz.-

1 Let RS be any curve for which an integral curve is wanted.

2 Lay off a series of equally spaced vertical lines, shown in full.in Fig 8 and draw a series of broken lines midway between these.

3 Select an arbitrary distance AB equal in length to any arbitrary number of

the spaces between verticals.

4 Draw 1-1', 2-2', 3-3', etc.

5 Through any convenient point P on the vertical through R draw P1" parallel to 1-1', then 1"2" parallel to 2-2', then 2"3" parallel to 3-3', etc., until curve PQ has been completed. This is approximately the integral curve of RS and the ordinates measured from a line drawn parallel to the axis of RS through P ~~divides~~ multiplied by the value of AB give the area under the curve RS up to the point where the ordinate is measured.

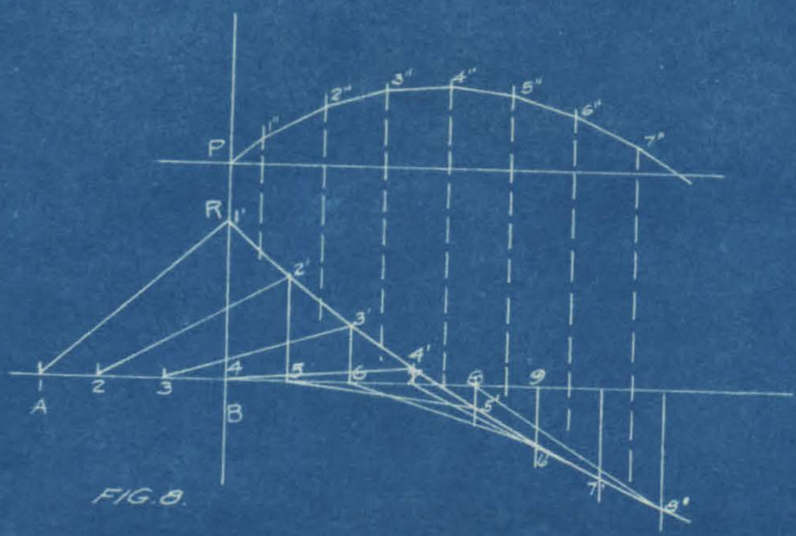



FIG. 8.

R. R. Marte

| A | N | X | Y | D | I | $\frac{1}{I}$ | 2Y | $\tan\alpha$ | $\cos\alpha$ | $\frac{1}{\cos\alpha}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 10' | 0 | 0 | 0 | 5' | see | .01884 | 0 | 0 | 1.0000 | 1.000 |
| | 1 | 10 | 0.333 | | sheet | | .666 | .0666 | .9978 | 1.002 |
| | 2 | 20 | 1.33 | | 13 | | 2.66 | .1330 | .9912 | 1.009 |
| | 3 | 30 | 3.00 | | I= | | 6.00 | .2000 | .9806 | 1.020 |
| | 4 | 40 | 5.33 | | 53.05 | | 10.66 | .2665 | .9664 | 1.034 |
| " | 5 | 50 | 8.33 | | | | 16.66 | .3332 | .9489 | 1.052 |
| | 6 | 60 | 12.00 | | | | 24.00 | .4000 | .9285 | 1.077 |
| | 7 | 70 | 16.30 | | | | 32.60 | .4657 | .9063 | 1.103 |
| | 8 | 80 | 21.30 | | | | 42.60 | .5325 | .8829 | 1.132 |
| | 9 | 90 | 26.95 | | | | 53.90 | .5990 | .8581 | 1.165 |
| | 10 | 100 | 33.27 | | | | 66.54 | .6654 | .8325 | 1.200 |
| | 11 | 110 | 40.30 | | | | 80.60 | .7330 | .8065 | 1.240 |
| | 115 | 115 | 44.00 | | | | 88.00 | .7650 | .7946 | 1.259 |

| X | $\frac{1}{I\cos\alpha} = I'$ | $\frac{Y}{I'}$ | $\frac{Y^2}{I'}$ | $\frac{n}{I'}$ | $\frac{n^2}{I'}$ | $\frac{nY}{I}$ |
|---|---|---|---|---|---|---|
| 0 | .01884 | .0000 | 0 | 0 | 0 | 0 |
| 10 | .01887 | .0063 | .0021 | .01887 | .01887 | .00628 |
| 20 | .01900 | .0253 | .0336 | .03800 | .07600 | .05050 |
| 30 | .01920 | .0576 | .1730 | .05760 | .17300 | .1730 |
| 40 | .01948 | .1038 | .5535 | .07800 | .31200 | .4156 |
| 50 | .01980 | .1650 | 1.373 | .09900 | .4950 | .8250 |
| 60 | .02028 | .2435 | 2.920 | .1218 | .7310 | 1.463 |
| 70 | .02080 | .3390 | 5.530 | .1456 | 1.0180 | 2.370 |
| 80 | .02130 | .4535 | 9.670 | .1704 | 1.363 | 3.630 |
| 90 | .02195 | .5920 | 15.96 | .1976 | 1.778 | 5.325 |
| 100 | .02260 | .7520 | 25.04 | .2260 | 2.260 | 7.520 |
| 110 | .02335 | .9420 | 38.00 | .2560 | 2.830 | 10.36 |
| 115 | .02370 | 1.0430 | 46.00 | .2726 | 3.140 | 12.00 |

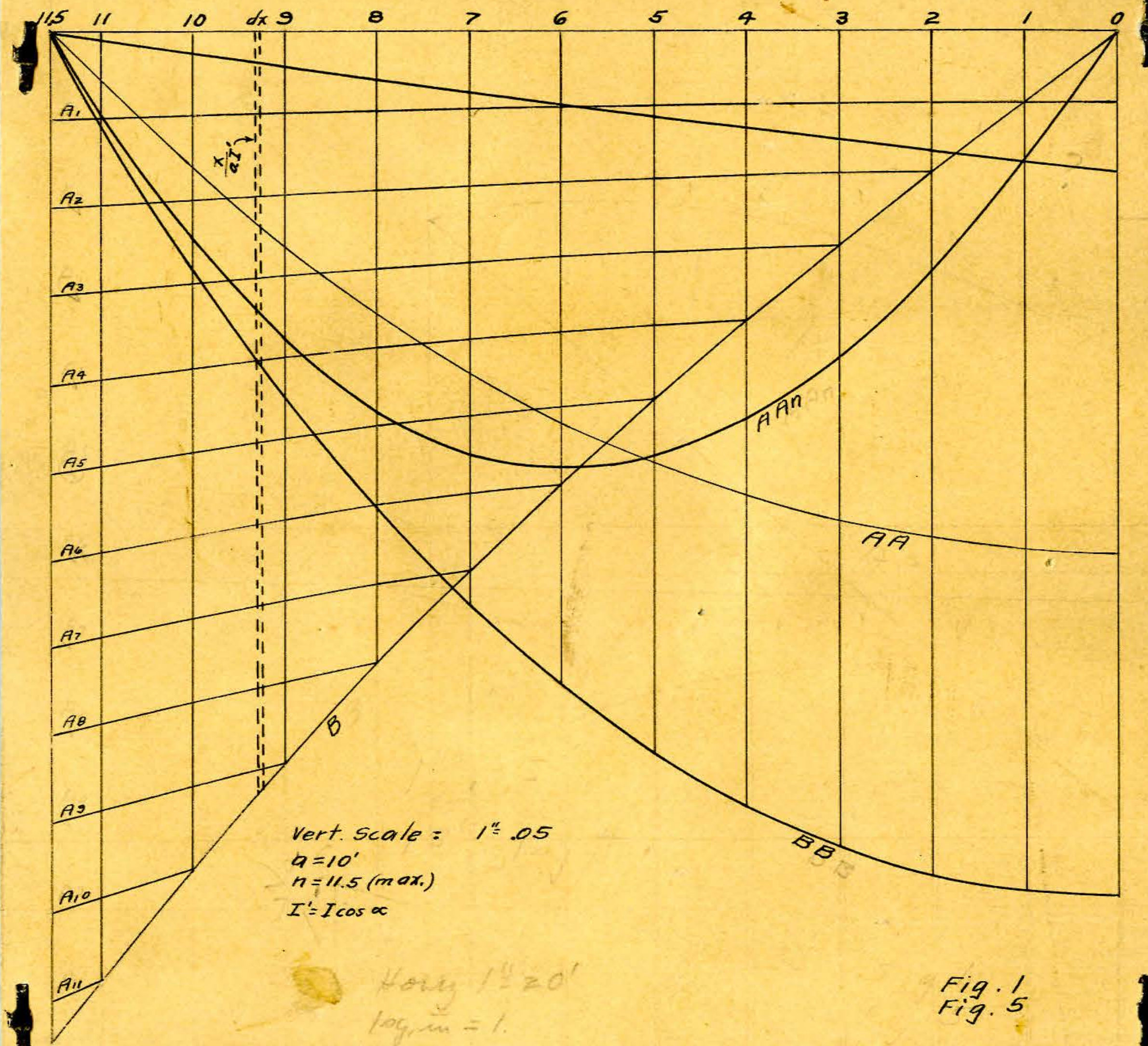

11.5
11
10
dx
9
8
7
6
5
4
3
2
1
0
A1
A2
A3
A4
A5
A6
A7
A8
A9
A10
A11
AAn
AA
B
BB
Vert. Scale: 1" .05
a=10'
n=11.5 (max.)
I'=I cos α
Horiz 1"=20'
log m = 1.

Fig. 1
Fig. 5

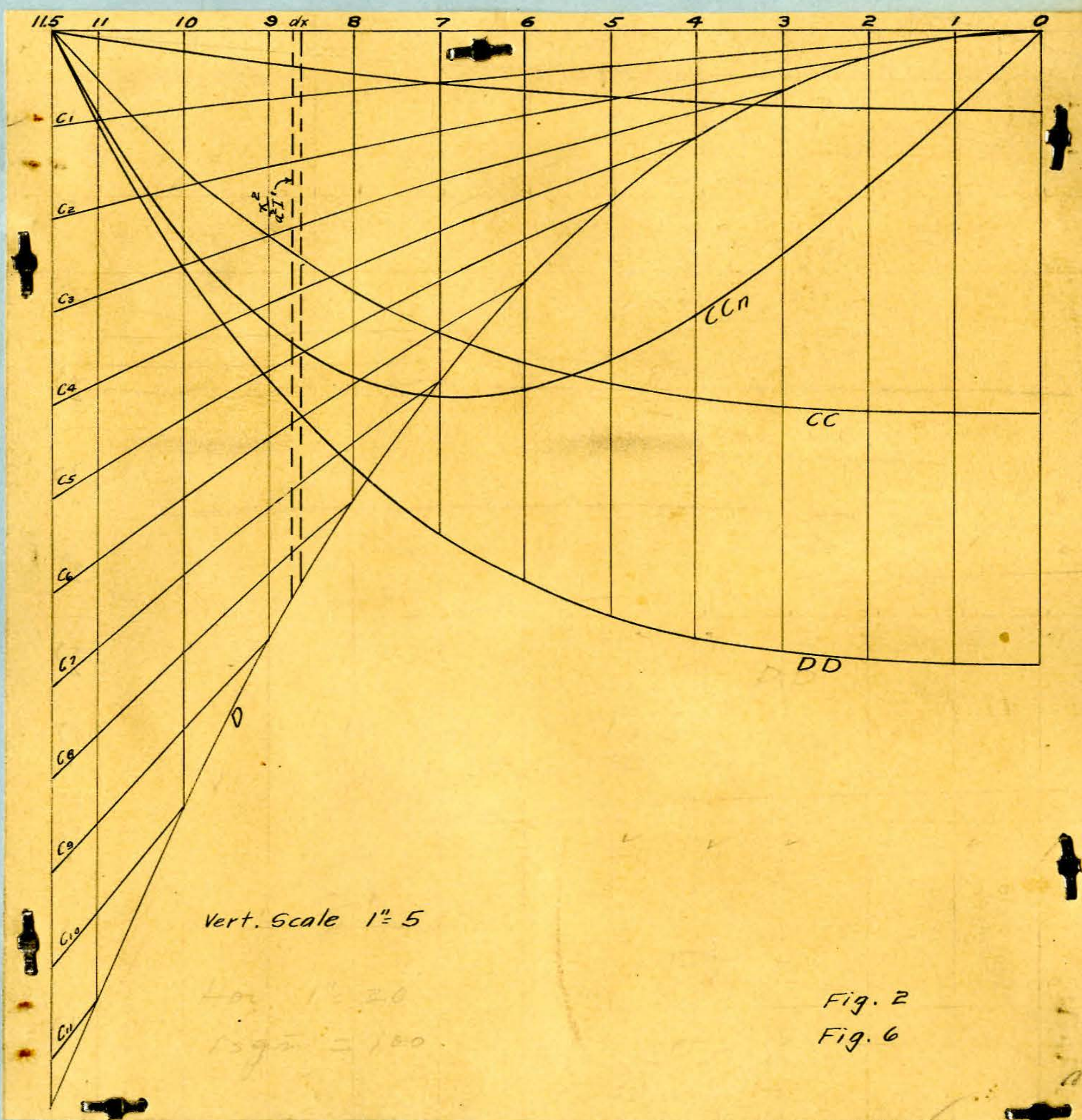
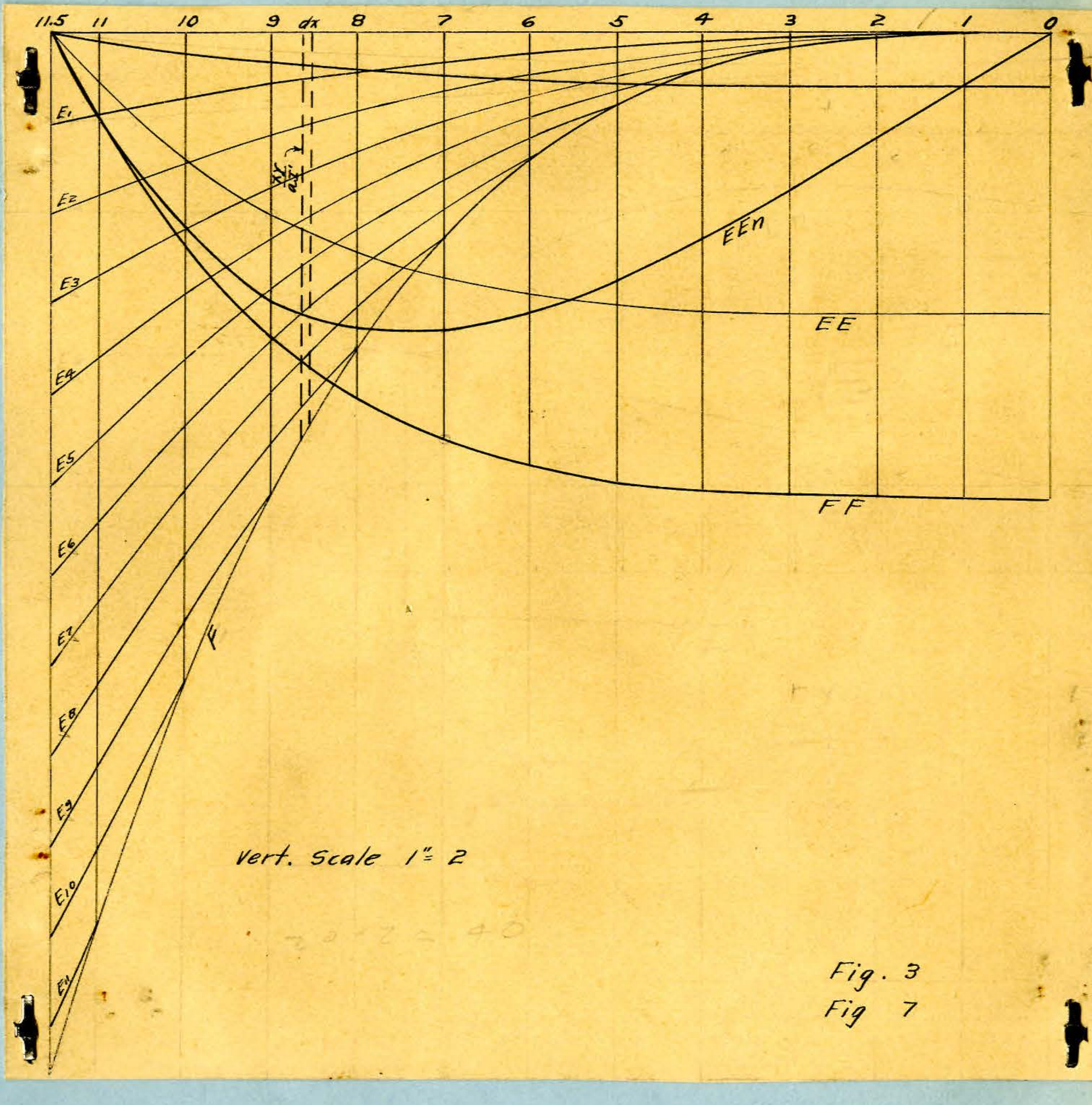

11.5 11 10 9 dx 8 7 6 5 4 3 2 1 0
C1
C2
C3
C4
C5
C6
C7
C8
C9
C10
C11
CCn
CC
DD
D
Vert. Scale 1" = 5

Fig. 2
Fig. 6



11.5
11
10
9
dx
8
7
6
5
4
3
2
1
0
E1
E2
E3
E4
E5
E6
E7
E8
E9
E10
E11
EEn
EE
FF
Vert. Scale 1" = 2


Fig. 3
Fig 7

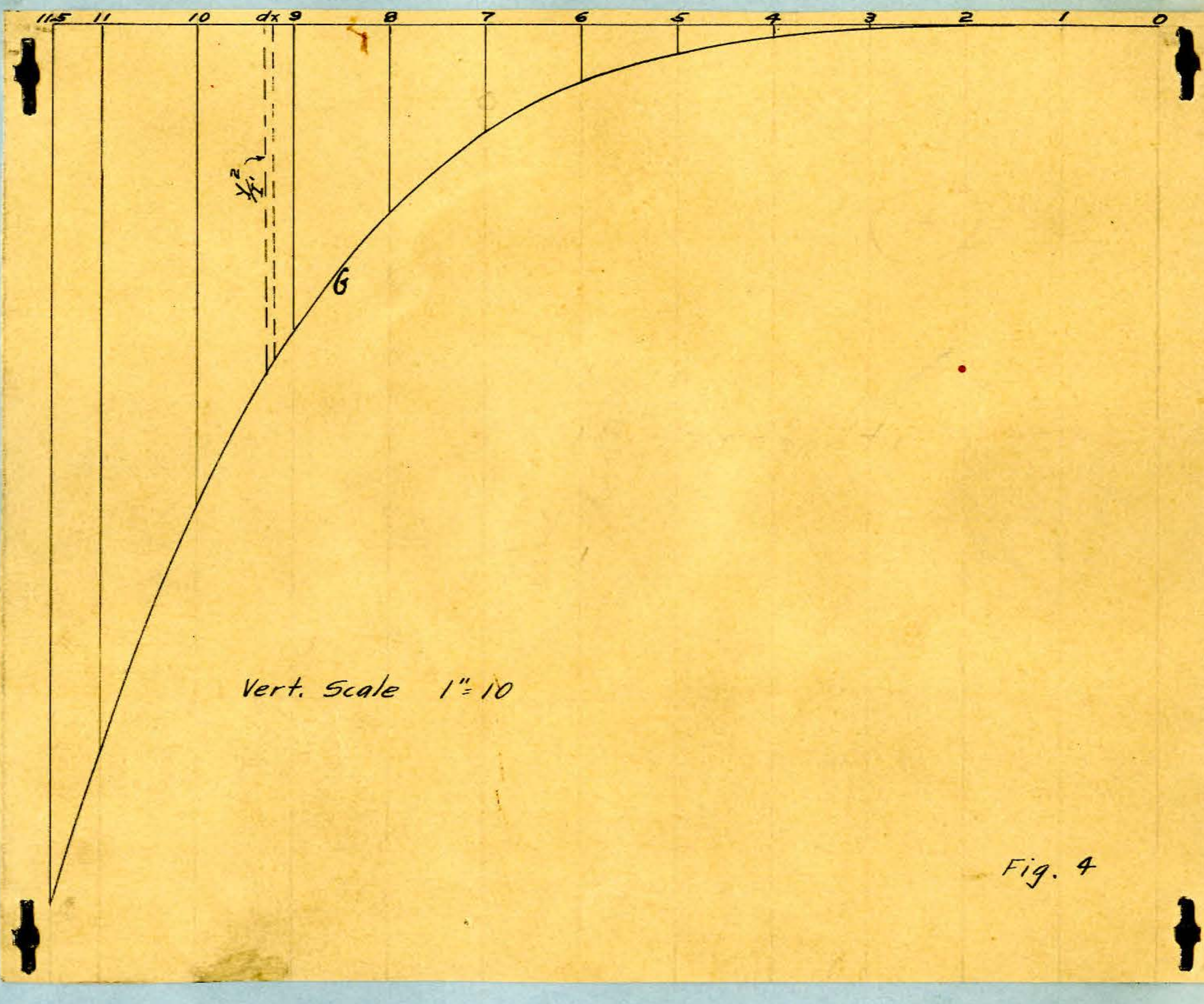


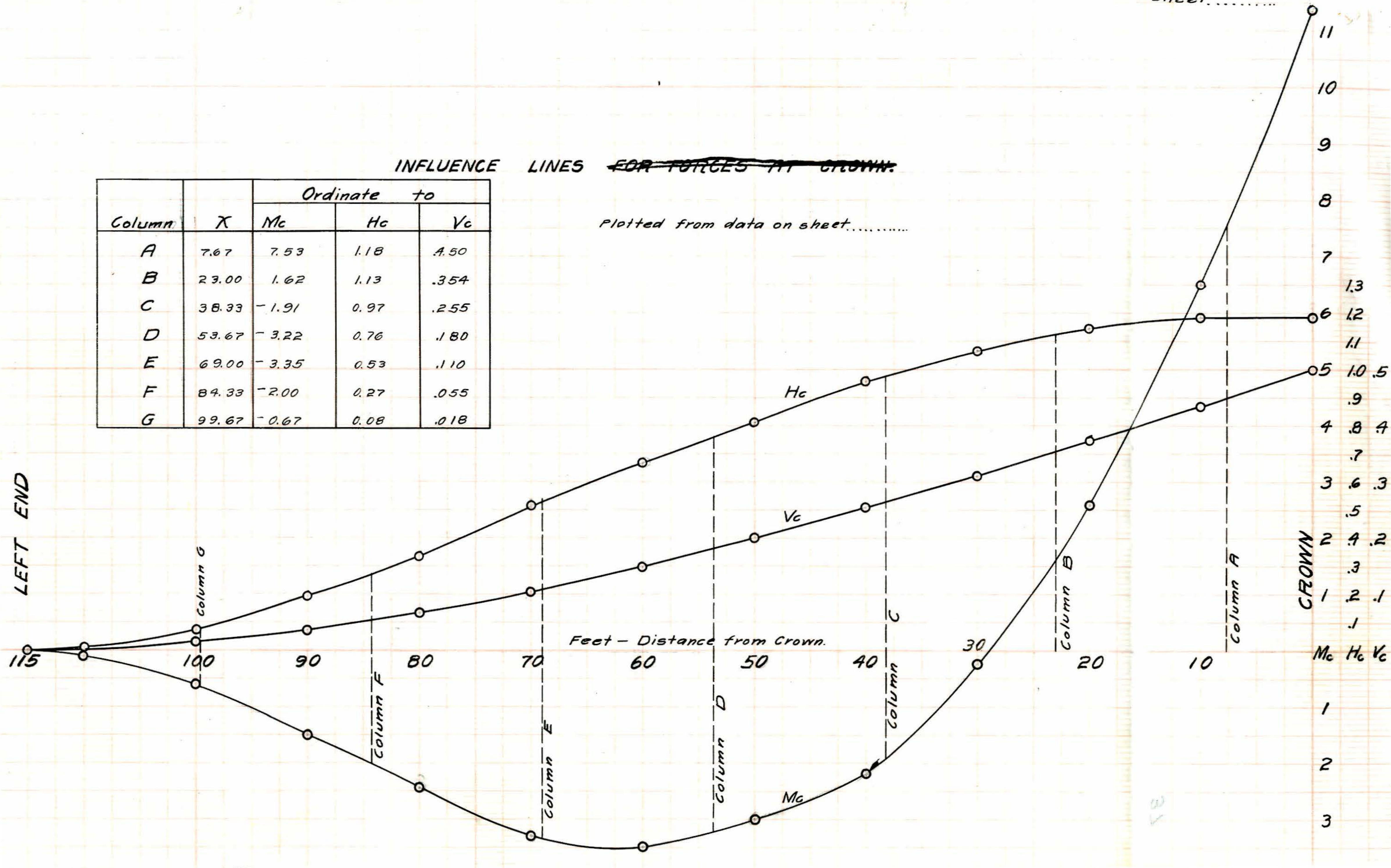

Fig. 4

Sheet.........

# INFLUENCE LINES ~~FOR FORCES AT CROWN.~~

Plotted from data on sheet..........

| Column | X | Ordinate to Mc | Hc | Vc |
|---|---|---|---|---|
| A | 7.67 | 7.53 | 1.18 | .450 |
| B | 23.00 | 1.62 | 1.13 | .354 |
| C | 38.33 | −1.91 | 0.97 | .255 |
| D | 53.67 | −3.22 | 0.76 | .180 |
| E | 69.00 | −3.35 | 0.53 | .110 |
| F | 84.33 | −2.00 | 0.27 | .055 |
| G | 99.67 | −0.67 | 0.08 | .018 |



| | Curve | N | Orig | R. | Diff | $R_2$ | Diff | Av. | Value |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | (1.2140 x 11.72 x 1 = 14.228) | sq. in |
| BB | $BB\text{-}AA_n$ | 0 | 3.738 | 4.952 | 1.214 | 6.166 | 1.214 | 1.2140 | 14.228 |
| | | 1 | 1.704 | 2.721 | 1.017 | 3.738 | 1.017 | 1.017 | 11.919 |
| | | 2 | 10.026 | 10.865 | .839 | 11.704 | .839 | .8390 | 9.833 |
| | | 3 | 8.675 | 9.352 | .677 | 10.026 | .674 | .6755 | 7.917 |
| | | 4 | 7.617 | 8.145 | .528 | 8.675 | .530 | .5290 | 6.200 |
| | | 5 | 4.247 | 4.648 | .401 | 5.055 | .402 | .4020 | 4.711 |
| | | 6 | 3.660 | 3.951 | .291 | 4.247 | .296 | .2910 | 3.411 |
| | | 7 | 3.264 | 3.463 | .199 | 3.660 | .197 | .1980 | 2.321 |
| | | 8 | 3.020 | 3.141 | .121 | 3.264 | .123 | .1220 | 1.430 |
| | | 9 | 2.894 | 2.957 | .063 | 3.020 | .063 | .0630 | .738 |
| | | 10 | 2.848 | 2.872 | .024 | 2.894 | .022 | .0230 | .270 |
| | | 11 | 2.838 | 2.841 | .003 | 2.843 | .002 | .0025 | .0293 |
| | | 115 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| A±A | | | 1.633 | 1.782 | .149 | 1.931 | .149 | .149 | 2.31 |
| | | | | | | | | (.9540 x 11.72 x 100 = 1118.09) | |
| D-D | $DD\text{-}CC_n$ | 0 | 3.498 | 4.452 | .954 | 5.406 | .954 | .9540 | 111.809 |
| | | 1 | 1.837 | 2.667 | .830 | 3.498 | .831 | .8305 | 97.335 |
| | | 2 | 5.866 | 6.581 | .715 | 7.291 | .710 | .7120 | 83.446 |
| | | 3 | 4.674 | 5.271 | .597 | 5.866 | .595 | .5960 | 69.851 |
| | | 4 | 3.704 | 4.189 | .485 | 4.674 | .485 | .4850 | 56.842 |
| | | 5 | 2.939 | 3.322 | .383 | 3.705 | .383 | .3830 | 44.888 |
| | | 6 | 2.373 | 2.655 | .282 | 2.939 | .284 | .2830 | 33.168 |
| | | 7n | 1.973 | 2.174 | .201 | 2.373 | .199 | .2000 | 23.440 |
| | | 8 | 1.720 | 1.846 | .126 | 1.973 | .127 | .1265 | 14.826 |
| | | 9 | 1.589 | 1.657 | .068 | 1.725 | .068 | .0680 | 7.970 |
| | | 10 | 1.537 | 1.563 | .026 | 1.589 | .026 | .0260 | 3.047 |
| | | 11 | 1.530 | 1.5335 | .00035 | 1.5307 | .00035 | .0035 | .4102 |
| | | 115 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C±C | | | 5.789 | 5.911 | .120 | 6.034 | .123 | .1225 | 14.357 |

| Curve FF-EE$_n$ | N | Orig. | R | Diff | $R_2$ | Diff | Av. | Valve | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | (.7000×11.72×40 = 328.16) | | |
| | 0 | 8060 | 8758 | .698 | 9460 | .702 | .7000 | 32816 | |
| | 1 | 6822 | 7442 | .620 | 8060 | .618 | .6190 | 29010 | |
| | 2 | 5743 | 6284 | .541 | 6822 | .538 | .5395 | 25292 | |
| | 3 | 43650 | 48225 | .4575 | 52840 | .4615 | .4590 | 21518 | |
| | 4 | 3604 | 3983 | .379 | 4365 | .382 | .3805 | 17838 | |
| | 5 | 2687 | 2990 | .303 | ~~3294~~ | .304 | .3035 | 14228 | |
| | 6 | 2220 | 2454 | .234 | 2687 | .233 | .2335 | 10946 | |
| | 7 | 1881 | 2050 | .169 | 2220 | .170 | .1695 | 7946 | |
| | 8 | 1667 | 1774 | .107 | 1881 | .107 | .1070 | 5016 | |
| | 9 | 15500 | 16085 | .0585 | 16670 | .0585 | .0585 | 2742 | |
| | 10 | 1507 | 1528 | .021 | 1550 | .022 | .0215 | 1008 | |
| | 11 | 1501 | 1504 | .0030 | 1507 | .0030 | .0030 | 1406 | |
| | 11.5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| EE | | 0232 | 0313 | .081 | 0393 | .080 | .0805 | 3774 | |
| | | | | | | | | (.329×15.5×200 = 1020) | |
| G | | 6703 | 7033 | .330 | 7361 | .328 | .329 | 510 | 1020. |

| (1) $n$ | (2) $\int \frac{m\,dx}{I'}$ | (3) $\int \frac{mx\,dx}{I'}$ | (4) $\int \frac{my\,dx}{I'}$ | (5) $\int_0^{l/2} \frac{dx}{I'}$ | (6) (4)×(5) | (7) $\int_0^{l/2} \frac{y\,dx}{I'}$ | (8) (2)×(7) | (9) (6)−(8) | (10) $K$ | (11) $\int_0^{l/2} \frac{x^2\,dx}{I'}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 142.28 | 1118.09 | 328.16 | 2.31 | 758.05 | 37.74 | 536.96 | 221.09 | 186.38 | 1118.09 |
| 1 | 119.19 | 973.35 | 290.19 | | 670.34 | | 449.82 | 220.52 | | |
| 2 | 98.33 | 834.46 | 252.92 | | 584.25 | | 371.10 | 213.15 | | |
| 3 | 79.17 | 698.51 | 215.18 | | 497.07 | | 298.79 | 198.28 | | |
| 4 | 62.00 | 568.42 | 178.38 | | 412.06 | | 233.99 | 178.07 | | |
| 5 | 47.11 | 448.88 | 142.28 | | 328.67 | | 177.79 | 150.88 | | |
| 6 | 34.11 | 331.68 | 109.46 | | 252.85 | | 128.73 | 124.12 | | |
| 7 | 23.21 | 234.40 | 79.46 | | 183.55 | | 87.59 | 95.96 | | |
| 8 | 14.30 | 148.26 | 50.16 | | 115.87 | | 53.97 | 61.90 | | |
| 9 | 7.38 | 79.70 | 27.42 | | 63.34 | | 27.85 | 35.49 | | |
| 10 | 2.70 | 30.47 | 10.08 | | 23.28 | | 10.19 | 13.09 | | |
| 11 | 0.29 | 4.102 | 1.406 | | 3.248 | | 1.106 | 2.14 | | |
| 115 | 0.00 | 0.00 | 0.00 | | 00 | | 0 | 0 | | |

$2(5) = 4.62$

$\frac{1}{2(5)} = .2165$

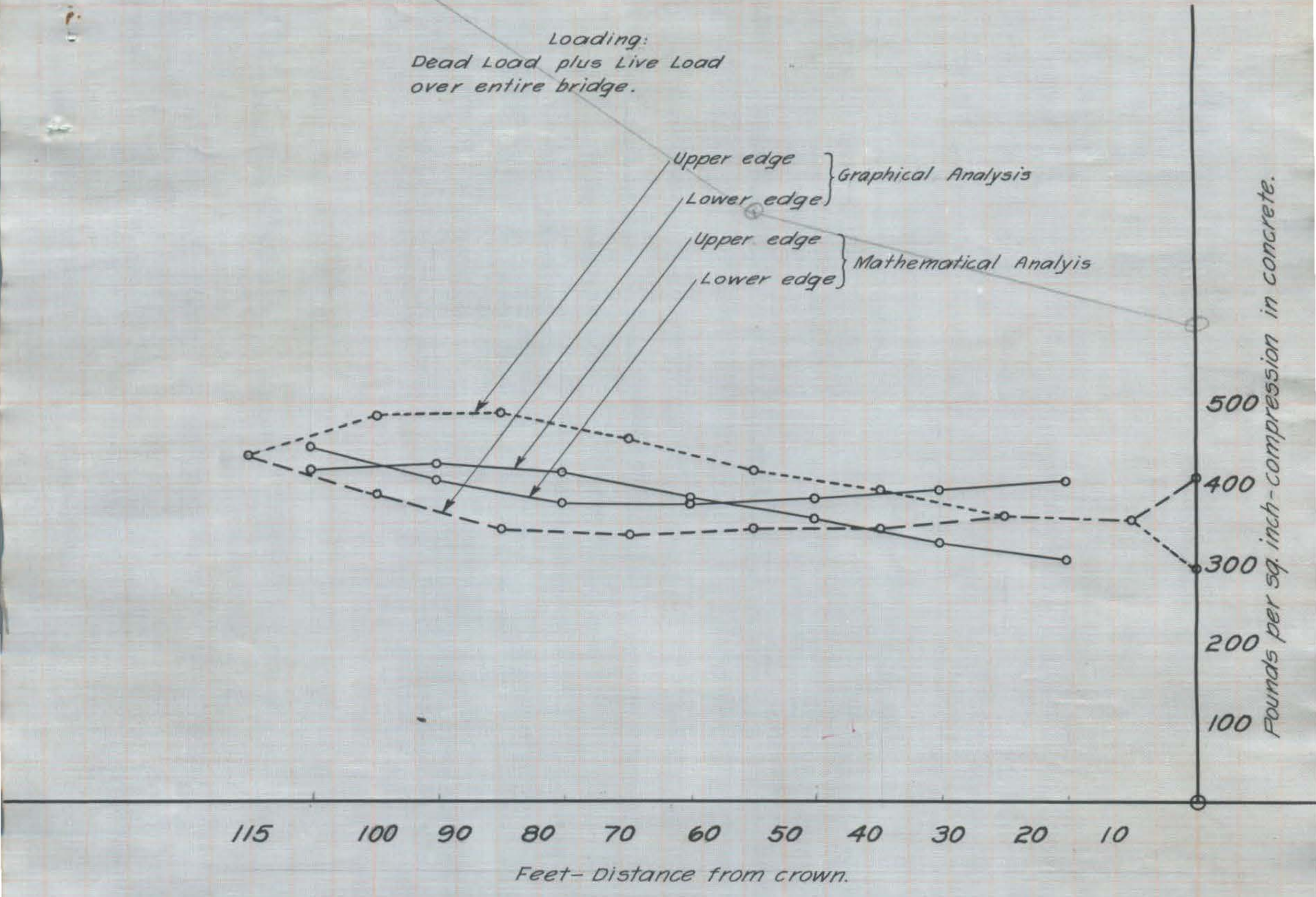
$2(7) = 75.48$

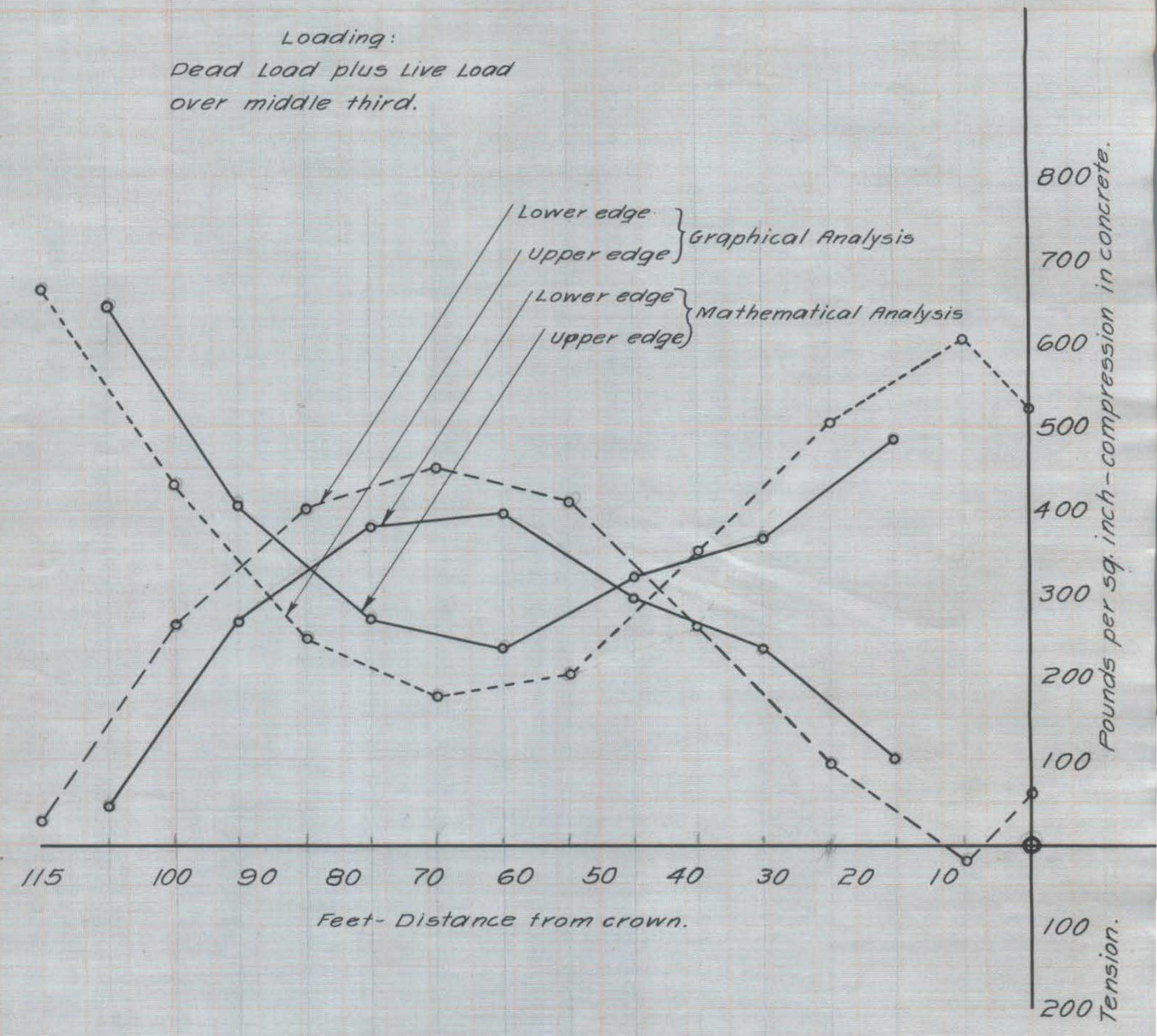
$\frac{1}{(10)} = .005365$

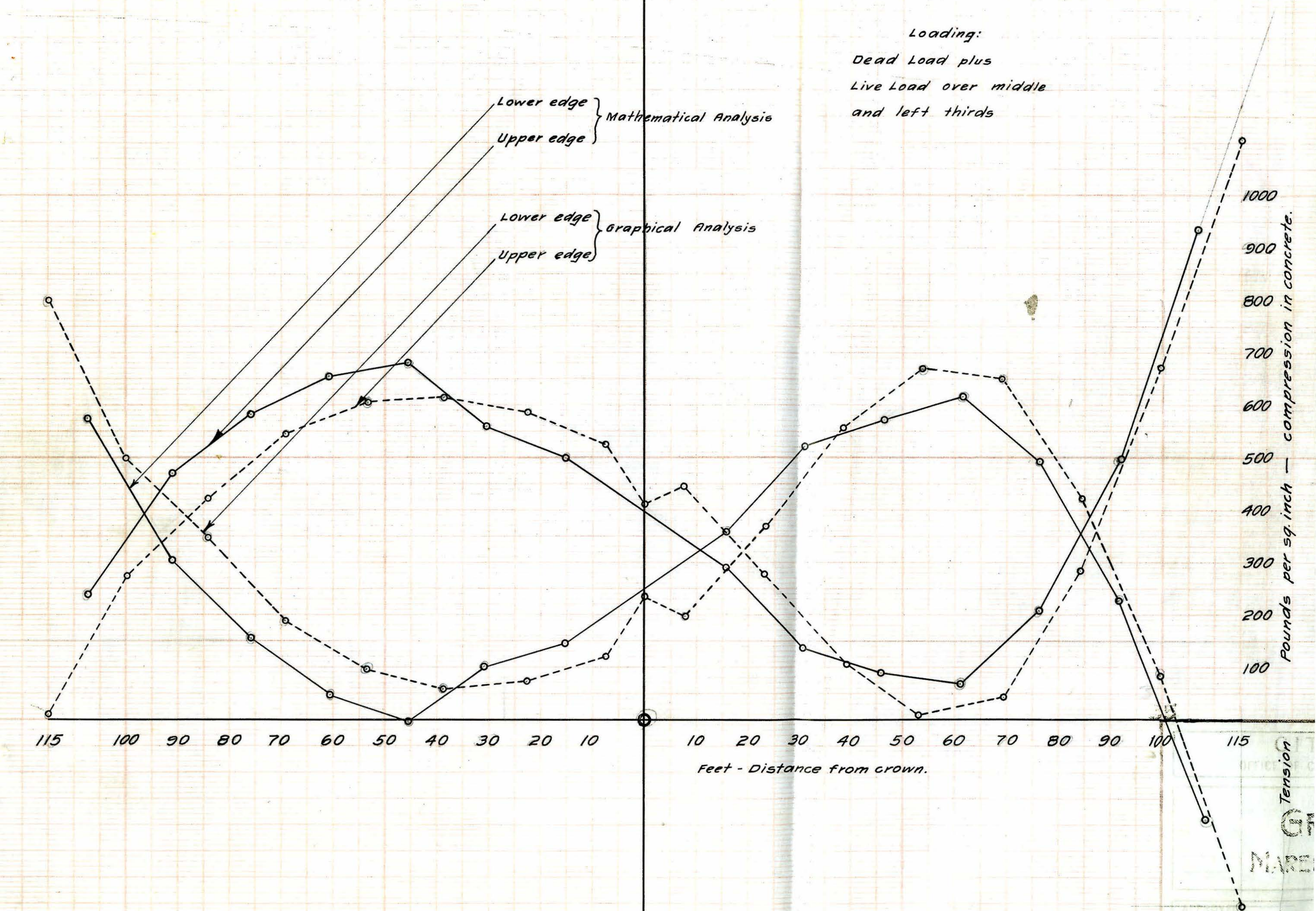
$2(11) = 22361.8$

K= 2x1020. x 2.31 -2 x (37.74)$^2$ =
= 4712.4 - 2848.6 = 1863.8

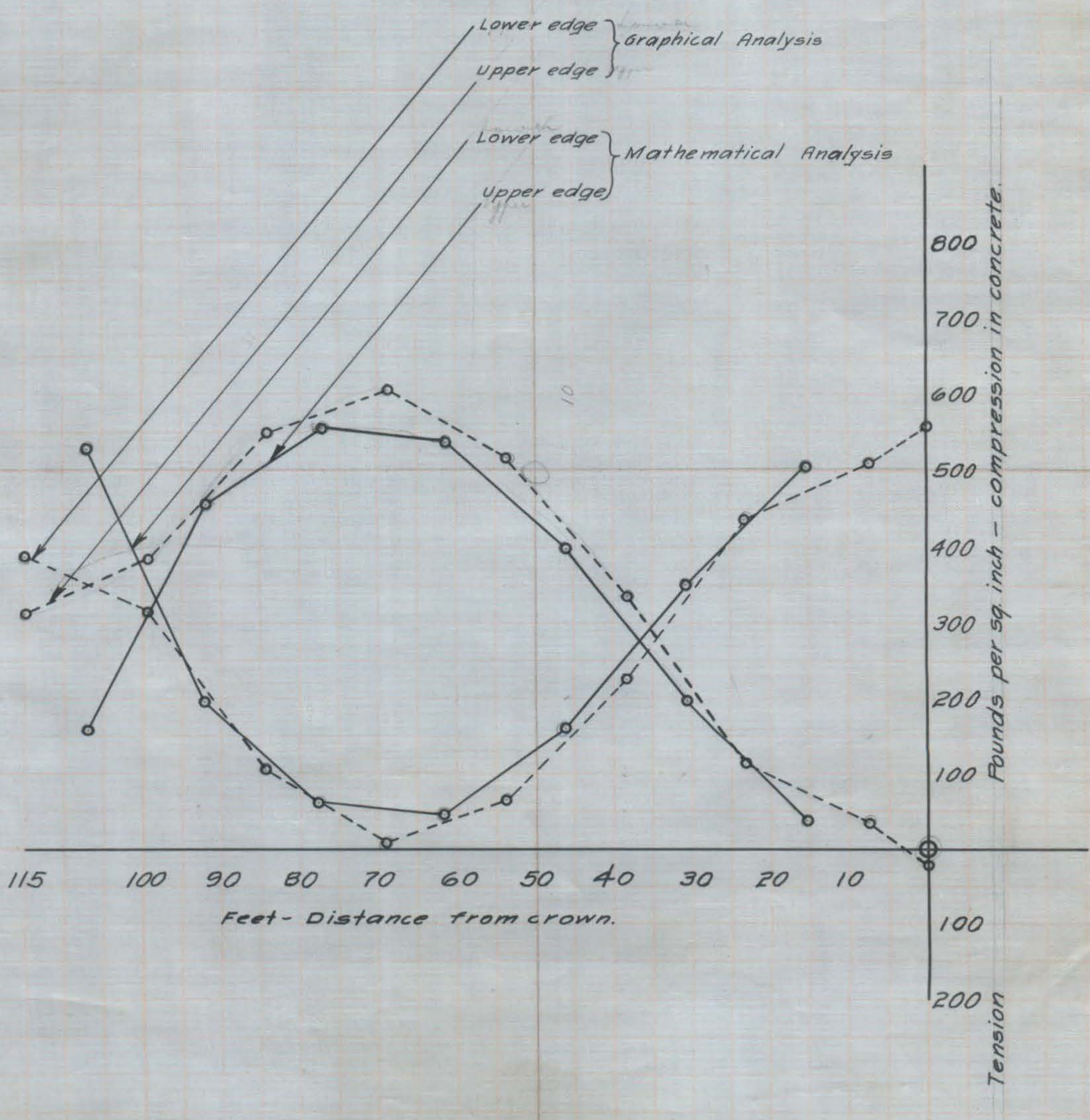
| (1) $n$ | (12) $H_c$ $\frac{(9)}{(10)}$ | (13) $V_c$ $\frac{(3)}{(2)} \times (11)$ | (14) 2·(12)·(7) | (15) (2)−(14) | (16) $M_c$ $\frac{15}{2(5)}$ |
|---|---|---|---|---|---|
| 0 | 1.186 | .5000 | 89.5 | 52.8 | 11.42 |
| 1 | 1.183 | .4353 | 89.3 | 29.9 | 6.5 |
| 2 | 1.144 | .3732 | 86.3 | 12.0 | 2.6 |
| 3 | 1.064 | .3124 | 80.3 | - 1.1 | -2.4 |
| 4 | .955 | .2542 | 72.1 | -10.1 | -2.2 |
| 5 | .809 | .2007 | 61.1 | -14.0 | -3.0 |
| 6 | .666 | .1483 | 50.3 | -16.2 | -3.51 |
| 7 | .515 | .1048 | 38.9 | -15.7 | -3.4 |
| 8 | .332 | .0663 | 25.1 | -10.8 | -2.3 |
| 9 | .190 | .0356 | 14.3 | - 6.9 | - 1.5 |
| 10 | .070 | .0136 | 5.3 | -2.6 | -0.6 |
| 11 | .011 | .0018 | 0.8 | - 0.5 | -0.1 |
| 115 | 0 | 00 | 0 | 0 | 0 |

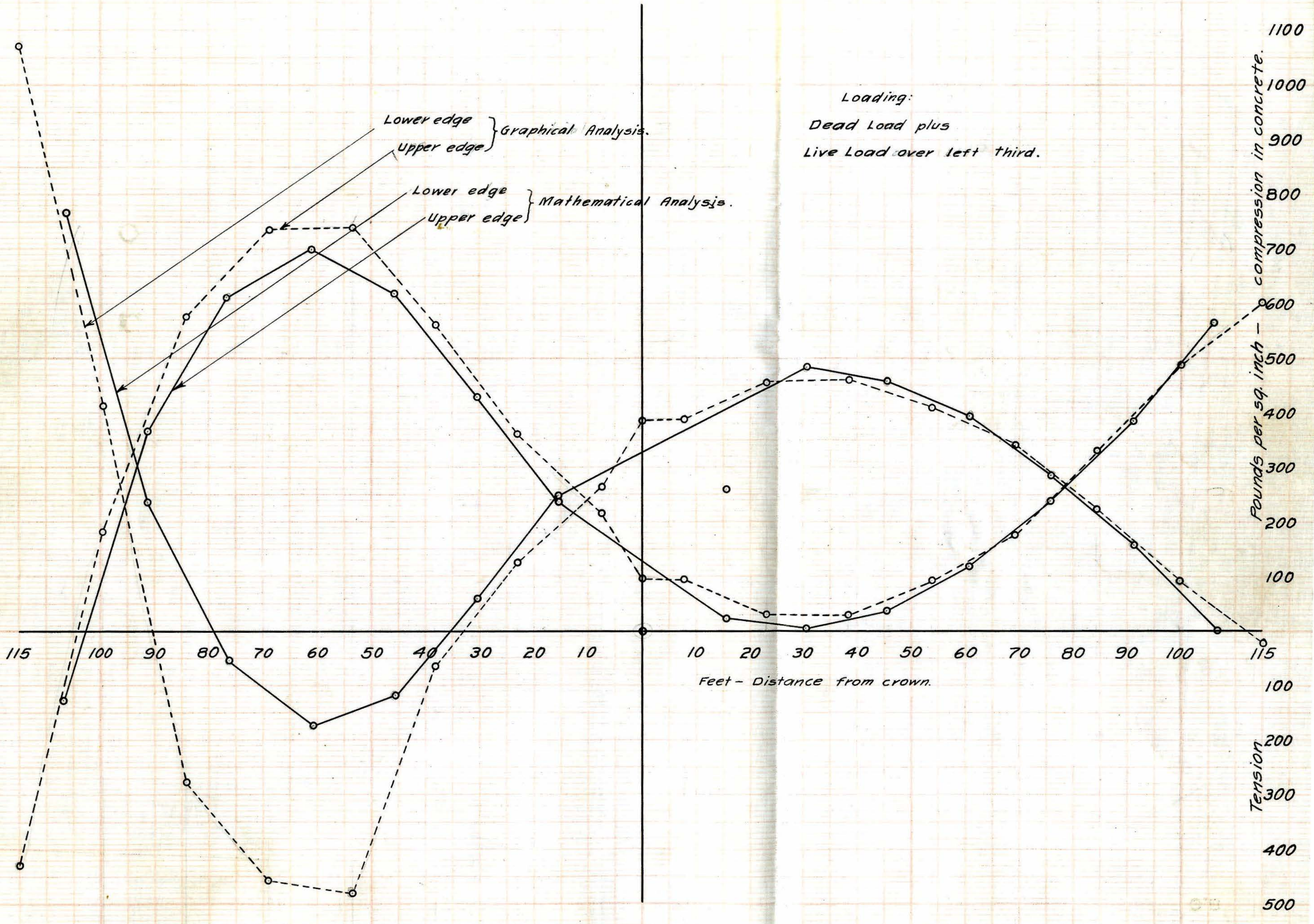

Loading:
Dead Load plus Live Load
over entire bridge.
Upper edge
Lower edge
Graphical Analysis
Upper edge
Lower edge
Mathematical Analyis
500
400
300
200
100
Pounds per sq. inch-compression in concrete.
115
100
90
80
70
60
50
40
30
20
10
Feet- Distance from crown.


Loading:
Dead Load plus Live Load
over middle third.
Lower edge
Upper edge
Graphical Analysis
Lower edge
Upper edge
Mathematical Analysis
800
700
600
500
400
300
200
100
100
200
Pounds per sq. inch-compression in concrete.
Tension.
115
100
90
80
70
60
50
40
30
20
10
Feet- Distance from crown.


Loading:
Dead Load plus
Live Load over middle
and left thirds
Lower edge
Upper edge
Mathematical Analysis
Lower edge
Upper edge
Graphical Analysis
Pounds per sq. inch — compression in concrete.
Tension
1000
900
800
700
600
500
400
300
200
100
115
100
90
80
70
60
50
40
30
20
10
10
20
30
40
50
60
70
80
90
100
115
Feet - Distance from crown.

Loading:
Dead Load plus
Live Load over both end thirds





Lower edge
Upper edge
Graphical Analysis.
Lower edge
Upper edge
Mathematical Analysis.
Loading:
Dead Load plus
Live Load over left third.
Pounds per sq. inch — compression in concrete.
Tension
1100
1000
900
800
700
600
500
400
300
200
100
100
200
300
400
500
115
100
90
80
70
60
50
40
30
20
10
10
20
30
40
50
60
70
80
90
100
115
Feet – Distance from crown.

Revised Sections.

| | Abutment | $x = 53.67$ | Crown |
|---|---|---|---|
| Max. Stress | 1170# | 740 | 600 |
| Loading | Middle + Left thirds | Left third. | Middle third. |

In the interests of greater economy of material, it would be best to maintain the constant depth of rib of 5' and vary the width to meet the maximum stress at the crown and at the abutment. The steel girder cross-section shown on sheet 6-a may be maintained, but the distance between the two girders will be greater at the abutment than at the crown.

$(n-1) I_s = 452{,}000$ see sheet 13
$I_c = 648{,}000$

Total $= 1{,}100{,}000$

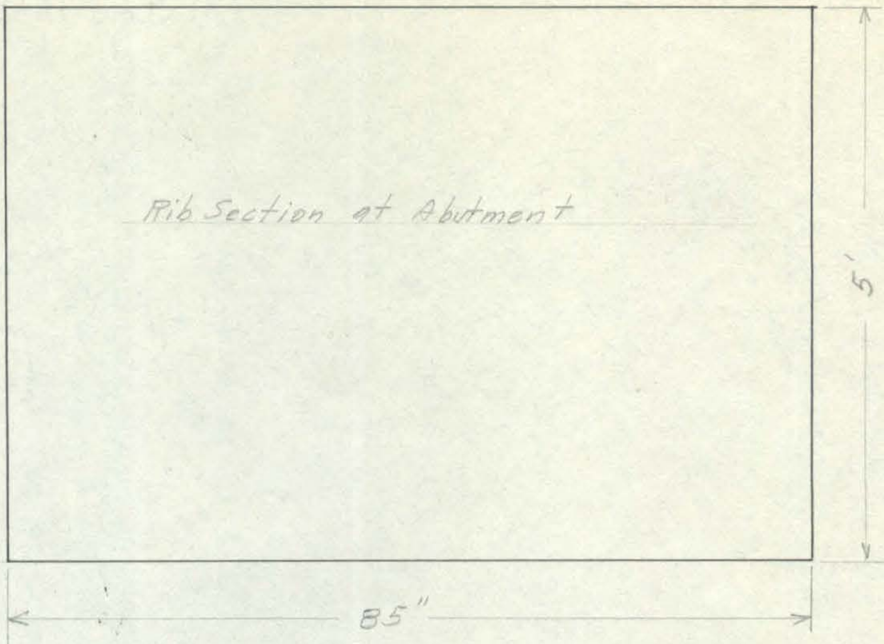
$\frac{600}{650} \times 1{,}100{,}000 = 1{,}015{,}000$
$452\,000$
$563{,}000 = I_c$ at crown

$\frac{1170}{650} \times 1{,}100{,}000 = 1{,}980{,}000$
$452\,000$
$1{,}528{,}000 = I_c$ at abutment.

$I_c = \frac{bd^3}{12}$ $\quad \frac{d^3}{12} = \frac{60^3}{12} = 18000$

$b \text{ (at crown)} = \frac{563000}{18000} = \underline{31''}$

$b \text{ (at abutment)} = \frac{1{,}528{,}000}{18000} = \underline{85''}$




Cross-Section of Steel Girder is as shown on sheet. 6-a.