Appendix B

Particles projected area as a way to infer the effective volume fraction

One of the limitations of measuring the volume fraction with the height reached by the particles is the lack of information about the concentration gradient. For this reason measurements of the particles projected area are made. In this way, differences in concentration along the vertical axes are taken into account. The problem with this method is that it gives no information regarding the volume fraction. One way to infer the volume fraction from the projected area is to use the projected area fraction of a known volume fraction to calibrate the measurements. Considering that the particles are homogeneously distributed for the experiments with a density ratio equal to 1, their projected area can be used to calibrate the measurements for settling particles. The methodology for measuring the projected area is described next.

The images from the visualization of the flow are first filtered by a FFT bandpass filter to correct for differences in lighting, then by an unsharp mask to filter the blur parts of the image. It is considered that only the sharp parts of the picture correspond to the particles next to the test section wall. The weight of the mask is chosen manually, and therefore this is a source of uncertainty. Once the image is processed, sixteen different automatic thresholds are applied. Each method segments the image into black and white and measures the area fraction. Then the method that best segments the image is considered. Note that this is another source of uncertainty since the method is chosen based on a subjective observation. Figure B.1 shows an example of the process of preparing the image and measuring the projected area fraction.

To infer the volume fraction from the projected area fraction, it is necessary to have a known volume fraction as a reference; however the assumption of the particles being homogeneously distributed for density ratios equal to one is not strictly true. The projected area fractions for $\phi = 10$, 20, and 30% and $\rho_p/\rho = 1$ show some dependance on Stokes numbers, where the projected area fraction seems to increase with St. For this reason an image that corresponds to a Stokes number around 40 and density ratio equal to one is selected and used to calibrate the projected area

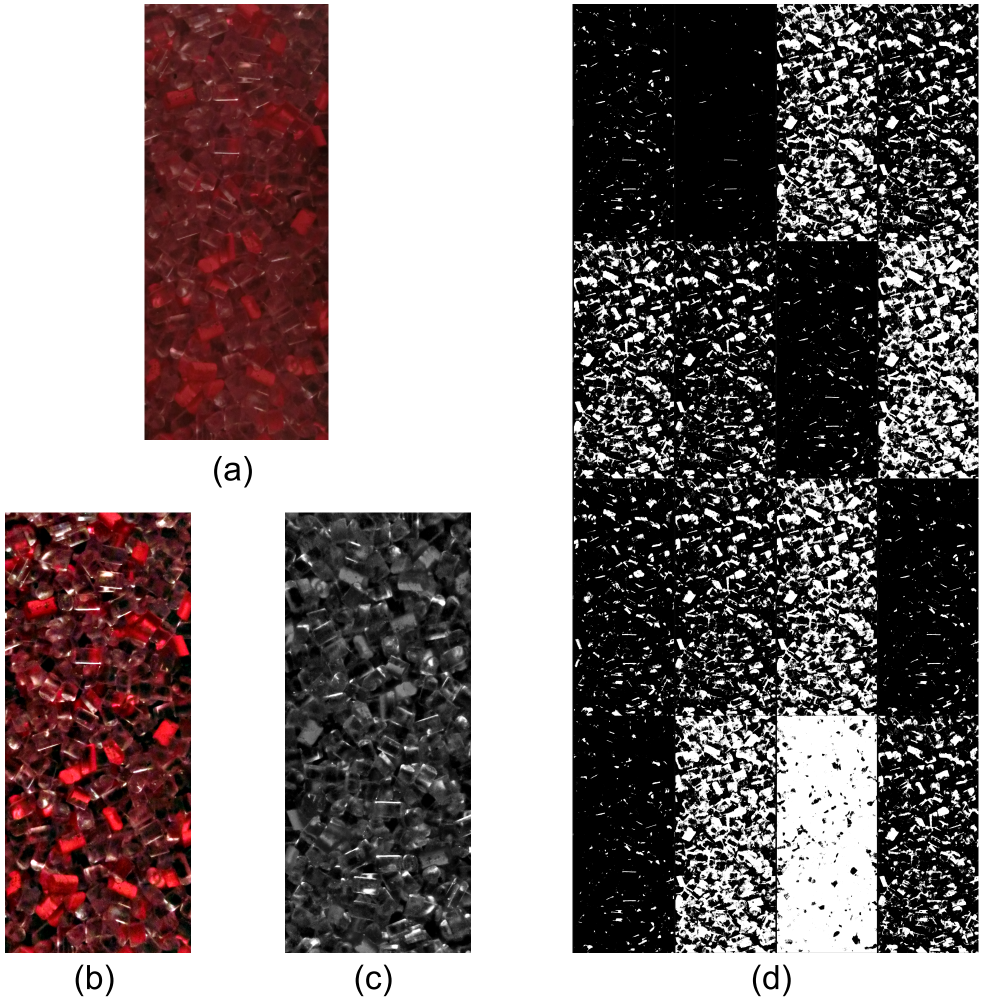

Figure B.1: Image processing example. (a) the image is filtered to correct for uneven lighting with a FFT bandpass filter, (b) the image is filer to correct for blurriness, (c) the image is convert to grayscale, and finally the image is segmented using 16 automatic thresholds methods and the method that best fit the data is chosen.

measurements. The volume fraction is not only inferred for the experiments with settling particles but also for the case with matched density. Figure B.2 shows the effective relative viscosity as a function of inferred volume fraction using the projected area for $\rho_p/\rho = 1$. The threshold method used to segment the images varied between loading fractions because a change in the light source position occurred every time the experiment was loaded with higher volume fractions. For most of the images corresponding to the same loading fraction the same threshold method was used to reduce bias in the results. For $\bar{\phi} = 10\%$, the inferred volume fraction shows the highest variability for $\rho_p/\rho = 1$. Notice that the inferred volume fraction can be higher than the loading fraction for high Stokes numbers; this can occur if there are differences in the particles density where some of the particles would sink while others would float, increasing in this way the particle concentration at the test section. Tests with polystyrene particles immersed in a matched density liquid were performed to study the neutrally buoyant condition. It was found that under no shear, approximately half of the particles would sink while the rest would float, and under slight shear of the flow the particles would distribute in an approximately homogeneous way. For higher loading fractions the change in volume fraction is negligible.

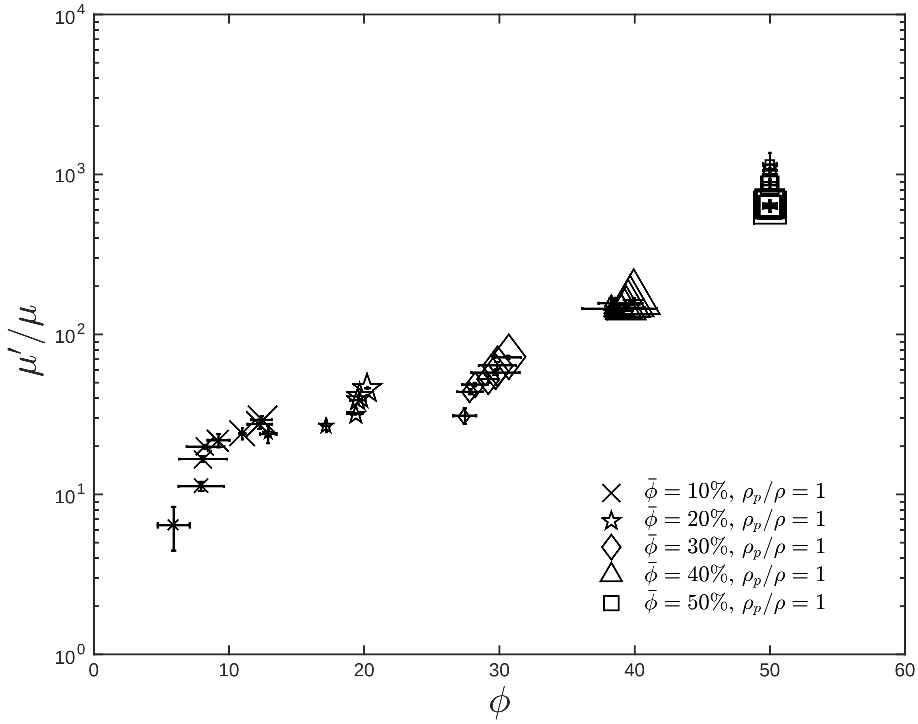

Figure B.2: μ'/μ as a function of the inferred volume fraction from projected area fraction for $\rho_p/\rho = 1$. The size of the symbols correspond to Stokes numbers magnitude.

Figure B.3 shows the inferred volume fraction from projected area fraction for $\rho_p/\rho = 1.05$. The change in volume fraction is larger than for the case with matched density. Similar to what was observed in the volume fraction predicted from the height reached by the particles, the volume fraction increases with Stokes numbers for loading fractions lower than 30% and decreases with St for loading fractions higher or equal than 30%.

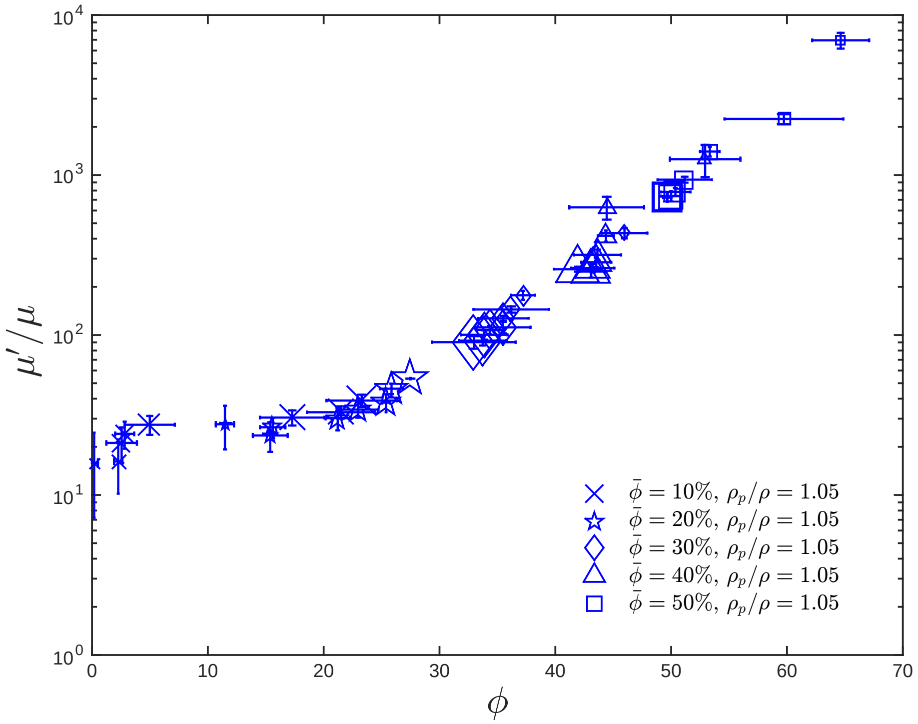

Figure B.3: Effective relative viscosity as a function of the inferred volume fraction from projected area fraction for $\rho_p/\rho = 1.05$. The size of the symbols correspond to Stokes numbers magnitude.

Figure B.4 shows the comparison between the effective relative viscosity for the two density ratios studied as a function of the inferred volume fraction. The effective relative viscosity coincides better for the volume fraction obtain from the projected area fraction than from the particles' height measurements. This might be due to an improve accuracy in the predicted volume fraction by considering a particles concentration gradient.

Figure B.5 shows the effective relative viscosity as a function of the inferred volume fraction. The change in volume fraction is similar to the one observed for the case without porous medium but same density ratio.

Finally, the effective relative viscosity results for the case with and without porous medium and difference density ratios are shown in Figure B.6. Unlike the results found using the particles height, the effective relative viscosity for the flow over porous medium coincides with the cases without porous medium and density ratios of 1.2 and 1.4.

These results suggest that better fits are obtained by using the projected area fraction.

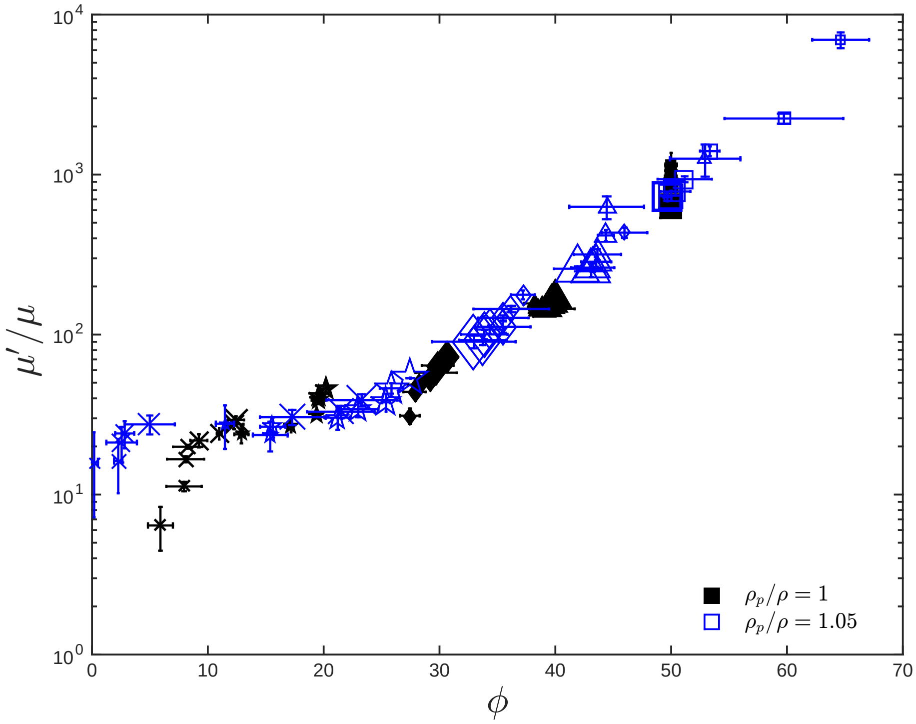

Figure B.4: Comparison between the effective relative viscosity as a function of the inferred volume fraction from projected area fraction for $\rho_p/\rho = 1$ and $\rho_p/\rho = 1.05$. The size of the symbols correspond to Stokes numbers magnitude.

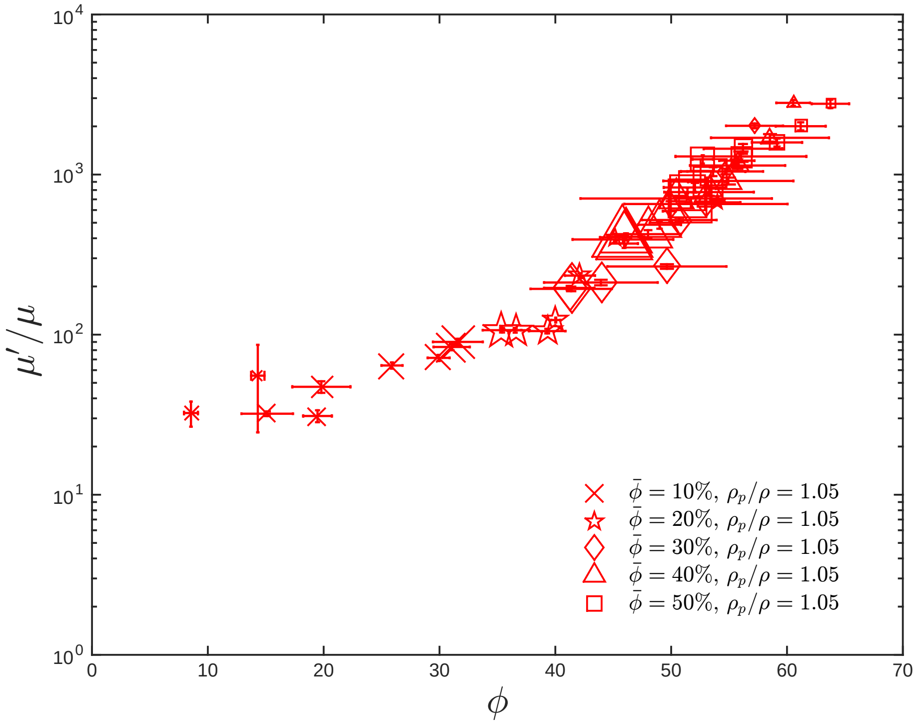

Figure B.5: Effective relative viscosity as a function of the inferred volume fraction from projected area fraction for flow over a porous medium and $\rho_p/\rho = 1.05$. The size of the symbols correspond to Stokes numbers magnitude.

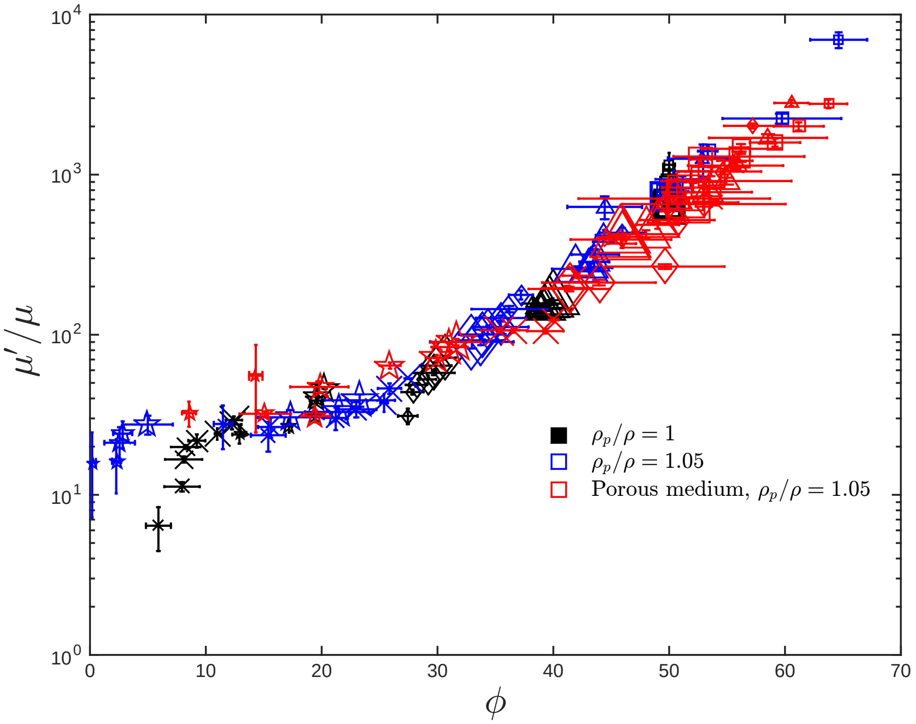

Figure B.6: Comparison between the effective relative viscosity as a function of the inferred volume fraction from projected area fraction for flow with and without a porous medium and $\rho_p/\rho = 1.05$ and $\rho_p/\rho = 1$. The size of the symbols correspond to Stokes numbers magnitude.