Chapter 8

Conclusions

This thesis presents rheological measurements of liquid-solid flows at Stokes and Reynolds numbers where inertial effects are important. The results for the case with $\rho_p/\rho = 1$ are summarized in the following section followed by a summary of the hydrodynamic instability effects. The effect of particle settling is summarized in section 8.2 . A discussion about the differences between the current work and the experimental results found for inertial and non-inertial suspensions is presented in Section 7.7. Comparisons with numerical results for inertial suspensions are discussed as well and emphasis has been focused on the possible reasons for the discrepancies observed. Finally, topics of possible future interest are discussed in Section 8.4.

8.1 Particles with matched density

Experiments with $\rho_p/\rho = 1$ were performed for a volume fraction range of $10\% \leq \phi \leq 50\%$ using polystyrene particles immersed in an aqueous glycerin solution. The range of Stokes numbers tested for this case is $2.5 \leq St \leq 116.3$ (equivalently, the Reynolds number range is $22.6 \leq Re \leq 1.04 \times 10^3$). For $\phi \geq 40\%$, the measured torque exhibits a linear dependance on the shear rate. When normalizing the measured torque by the fluid torque predicted by laminar theory, the dependance on Stokes number is negligible. This indicates that for $\phi \geq 40\%$ the flow exhibits a Newtonian behavior where the effective relative viscosity is given by the normalized torques and it increases with volume fraction but remains constant with the Stokes number and equivalently with the Reynolds number:

$$\frac{M}{M_{laminar}} = f(\phi) \quad \text{for} \quad \phi \geq 40\% \quad \text{and} \quad \rho_p/\rho = 1.$$

For $\phi \leq 30\%$ the measured torques exhibit a non-linear dependance on the shear rate. The normalized torques for these lower volume fractions increase not only with ϕ but also with Stokes and equivalently Reynolds number:

$$\frac{M}{M_{laminar}} = g(\phi, St) \quad \text{for} \quad \phi \leq 30\% \quad \text{and} \quad \rho_p/\rho = 1.$$

To study if this dependance is a result of an increase in the particle interactions or the presence of hydrodynamics instabilities, an analysis of fluid and particle inertia was done. The torque measurements for pure fluid indicate the presence of hydrodynamic instabilities for modified gap Reynolds higher than 3×10^3. If the liquid-solid mixture is considered to be a Newtonian fluid and its effective viscosity is used to define the modified gap Reynolds number, it is possible to determine if the range of Reynolds number tested lie within the range of Reynolds number where hydrodynamic instabilities are present.

In this hypothetical scenario the viscosity of the liquid-solid mixture is considered to be independent of the shear rate and it only depends on the particle volume fraction. The effective relative viscosity of the mixture is then considered to be equal to the normalized torques corresponding to the lowest Stokes number:

$$\frac{\mu'_{min}}{\mu}(\phi) = \frac{M}{M_{laminar}}(St_{min})$$

where St_{min} is the lowest Stokes number tested for the corresponding loading fraction and μ'_{min}/μ is the hypothesized effective relative viscosity. This is based under the assumption that at the lowest Stokes number, the measurements are not affected by hydrodynamics instabilities. The measured torques are then normalized by a laminar torque that considers the effective viscosity of the mixture instead of the fluid viscosity. Similarly, an effective Reynolds number is defined where the effective viscosity of the mixture is considered instead of the fluid viscosity.

The measured torques normalized with the effective laminar torque for $\phi \geq 40\%$ are close to one and the corresponding effective Reynolds number range for these experiments are within the modified gap Reynolds number region where the pure fluid exhibits a laminar behavior. For $\phi \leq 30\%$, the normalized torques deviates from unity and their corresponding effective Reynolds number is within the region where hydrodynamic instabilities were observed.

Based on this analysis the inertial effects are present for effective Reynolds numbers ($Re^*_{b,\ eff}$) higher than 400:

$$\frac{M}{M_{laminar}} = g(\phi, St) \quad \text{for} \quad Re^*_{b,\ eff} > 400 \Rightarrow \text{hydrodynamic instabilities effects}$$

and for effective Reynolds numbers lower than 400, the effects of hydrodynamics instabilities are negligible:

$$\frac{M}{M_{laminar}} = f(\phi) \quad \text{for} \quad Re^*_{b,\ eff} < 400$$

8.2 Effect of particle settling

Experiments with particles denser than the suspending liquid were performed where $\rho_p/\rho = 1.05$. The range of Stokes number tested for this case is $2.6 \leq St \leq 195$ and the range of Reynolds number is $22 \leq Re \leq 1.7 \times 10^3$. In this set of experiments, the particles used were the same polystyrene particles, but water was used as the suspending liquid. For this case the particle distribution is no longer homogeneous and depending on the shear condition the particles settle ($\dot{\gamma} = 0$) or re-suspend. Therefore, the volume fraction at the test section is no longer equal to the loading fraction ($\bar{\phi}$)

$$\bar{\phi} = \frac{\text{Volume of particles}}{\text{Volume of annulus}} \neq \phi \quad \text{for} \quad \rho_p/\rho = 1.05.$$

To determine the effective volume fraction for these experiments, visualizations of the flow were performed. The particle resuspension was characterized by measuring the height reached by the particles for different loading fractions and at different Stokes numbers. Based on these measurements the volume fraction could be inferred. When the inferred volume fraction is plotted as a function of Stokes number, it collapses into one curve for all the loading fractions tested. This indicates that the resuspension process is independent of the loading fraction.

The results from the flow visualization were used to analyze the torque measurements for the experiments with $\rho_p/\rho = 1.05$. The range of loading fraction tested is $10\% \leq \bar{\phi} \leq 60\%$. For $\bar{\phi} \geq 30\%$, the normalized torques decrease with Stokes number and for $\phi \geq 40\%$, the normalized torques reach a plateau for Stokes numbers above a critical value. The value for this critical Stokes number increases with the loading fraction. The visualization of the flow for these experiments show that this dependance is a settling effect. At low Stokes numbers the particles settle, which leads to an increase in the effective volume fraction. As the Stokes numbers increase, the particles start fluidizing and thus decreasing the effective volume fraction. A decrease in the effective volume fraction leads to a decrease in the normalized torques. The normalized torques for $\bar{\phi} = 30\%$ stop decreasing at the Stokes number where the complete fluidization of the particles occurs and start increasing with Stokes numbers above the fluidization threshold. A different dependance is observed for $\bar{\phi} \leq 20\%$, where the normalized torques increase with the Stokes number instead of decreasing. At these low loading fractions the particles settle at the bottom of the rheometer and do not reach the test section. As the Stokes number increases, the particles fluidize and start covering the test cylinder, which leads to an increase in the effective volume fraction and thus an increase in the normalized torques. Comparisons between the normalized torques for pure fluid and $\bar{\phi} = 10\%$ show that for gap Reynolds numbers lower than 5×10^4, the normalized torques for pure fluid and $\bar{\phi} = 10\%$ are the same. This indicates that there is no presence of particles in the test section for these gap Reynolds numbers. The visualization of the flow confirms this hypothesis. Measurements of the height reached by the particles allow to determine the Stokes numbers at which the particles are partially covering the test section. Corrections to the measured torque for the case where the particles are partially covering the test section were done. Such corrections do not seem to coincide with the measured torques for the case where the test section is fully covered. This suggests that the contribution from the fluid considered in the correction might be over-estimated.

The effective volume fraction of the torque experiments is predicted using the particle resuspension analysis. When the effective volume fraction is considered, the normalized torques for both cases with different density ratio coincide indicating that the normalized torques are independent of the density ratio.

When only the cases where the particles are completely fluidized are considered, a similar behavior to the case with $\rho_p/\rho = 1$ is observed. The normalized torques for $\phi \leq 30\%$ exhibit a dependance on the Stokes number. The effective Reynolds number for these experiments is higher than 400, indicating that such dependance is due to the presence of hydrodynamic instabilities.

Flow over a porous medium

As mentioned earlier in this section, when the loading fraction is lower than 30%, the particles settle and do not reach the test section. To further study these low loading fractions, experiments with settling particles over a porous medium were performed. The lower section of the rheometer was filled with glass beads up to a height of approximately 11 cm ($\approx$ 2 cm below the test cylinder) and the polystyrene particles were placed on top. In this way, the polystyrene particles are brought up and their presence in the test section is guaranteed even for a loading fraction of 10%. The measured torque for these experiments increases with the shear rate and exhibits a drop for shear rates above a critical value. This drop is not observed for the case without porous medium. The torque was measured with increasing and decreasing shear rates to study the dependance on the shear rate history. The torques measured with increasing shear rates coincide with the torques measured with decreasing shear rates. This indicates that these flows exhibit no hysteresis.

In the same way as for the case without porous medium, visualizations of the flow over a porous medium were performed and used to analyze the torque measurements for this set of experiments. The drop in the measured torque observed for the case with porous medium occurs at the same Stokes number where the particles are fully fluidized. The normalized torques decrease with Stokes number and reach a plateau when the particles are fully fluidized. This behavior was observed for all the loading fractions tested with the exception of $\bar{\phi} = 10\%$. For this particular loading fraction and when the particles are fully fluidized, the normalized torques increase instead of becoming constant.

The effect of settling for these experiments is more marked than for the case without porous medium. The reason for this might be the presence of a vertical gradient in the particle concentration, where the particles located at the bottom are more packed than at the top. By having a porous medium, the particles bottom layer is brought in closer proximity to the test section making the change in effective volume fraction more pronounced.

When the effective volume fraction is considered, the normalized torques for the case with a porous medium are slightly higher than for the case without porous medium. The reason for these differences might be due to the presence of a vertical gradient in the volume fraction. It is also possible that the presence of the porous medium affects the normalized torque but the reason for this effect is not clear.

8.3 Comparison with previous results

The effective viscosity for the current experiments is higher than the effective viscosity found in previous experimental work for inertial suspensions (Hanes and Inman, 1985; Prasad and Kytömaa, 1995). A possible reason for these differences is that the effective volume fraction in their measurements might be lower than the one reported due to settling. For both experimental studies, the torque was measured on the top of the annular shell and the particles were denser than the suspending liquid. Additionally, the work of Prasad and Kytömaa (1995) considers a lower Stokes regime which would increase the effect of settling in their measurements.

The effective viscosity for non-inertial suspensions is lower than the one found in the present work even for the cases where the normalized torques show no dependance on Stokes and Reynolds number. The reason for this is still not clear.

The numerical simulations for inertial suspensions show a dependance on Reynolds for volume fractions equal or higher than 10% (Kulkarni and Morris, 2008; Yeo and Maxey, 2013; Picano et al., 2013). This result agrees with the current experimental work; however, the effective viscosity found in these simulations is lower than the one found in the present experiments. Depending on the numerical work, a possible reason was proposed that include the neglecting of Reynolds stresses in the work of Yeo and Maxey (2013) and the presence of slip at the wall in the work of Picano et al. (2013).

8.4 General comments and future work

The main objective of the work presented in this thesis is to expand the Stokes and Reynolds number regime studied experimentally in liquid-solid flows where the inertia of both: the solid and liquid phase is important. A characterization of the particle settling and resuspension was presented through visualizations of the flow which allow to infer the effective volume fraction. Two density ratios: $\rho_p/\rho = 1$ and $\rho_p/\rho = 1.05$ were studied and when the effective volume fraction is considered, the difference in density ratio does not seem to affect the effective relative viscosity of the mixture;

however, this thesis considered only a 5% variation in density.

The characterization of the particle resuspension can have important implications in the modeling of systems involving settling and resuspension particles. Such systems are frequently found in many industrial processes (drilling muds, mixing of nuclear waste, handling of oil sands, mining processes, etc) and natural phenomena (debris-flows, lava flows, rock transport by rivers, etc). In the presence of settling, the materials are no longer homogeneous which can strongly affect their mechanical properties as shown by the results of the present work. *Viscous resuspension* has been previously examined by Leighton and Acrivos (1986). However, in most industrial processes involving sediments the inertia of the flow is significant and viscous resuspension is negligible (Wallner and Schaflinger, 1998). A model to predict the particle resuspension in inertial liquid-solid flows remains to be developed. This could be achieved by modifying the particle terminal velocity (v_{ter}) in the model develop by Leighton and Acrivos (1986) . For higher Reynolds numbers, the terminal velocity can be found from a balance of drag, gravitational and buoyancy forces. Additional modification to the diffusion coefficient might be also necessary to obtain more accurate results.

The presence of hydrodynamic instabilities seem to affect the current torque measurements for $\phi \leq 30\%$. The presence and effect of particle interactions that are not a result of secondary flows is still not resolved. An experimental setup with longer test apparatus might allow the study of liquid-solid flows that are not affected by hydrodynamics instabilities. Alternatively, experiments in a gravity free field would allow the study of moderate Stokes and Reynolds numbers regime for a wide range of volume fractions without having the effects of settling.

A more accurate measurement of the effective volume fraction can be achieved by analyzing the particle concentration gradient. Measurements of the particle velocity can confirm the existence of secondary flows in the test cylinder region and evidence the presence or absence of shear bands for the case with settling particles. Measurements of the particle collisions and its frequency can provide information of the effect of particle agitation in the bulk behavior of the mixture. Experiments with fluidized beds performed by Aguilar-Corona (2008) show that the frequency of collisions reaches a maximum for a volume fraction of approximately 30%. It is of interest to determine if similar results are found for the current experiments. Such findings can help to develop theoretical models or computer simulations.

Zenit et al. (1997) measured the collisional particle pressure for a vertical gravity-driven and fluidized bed, and similar to the findings of Aguilar-Corona (2008), the maximum particle pressure occurs at solid fractions of 30%. Measurements of the particle pressure in the current experiments can be made if pressure transducers are placed at the rheometer walls. Such measurements can shed a light into the relation between the normal and shear stress for liquid-solid flows with inertial effects.

In Appendix A, the results for experiments with polyester particles with $\rho_p/\rho = 1.2$ and $\rho_p/\rho =$

1.4 are presented. The visualization for these flows exhibit a particle concentration gradient in the radial direction. The particle resuspension analysis for these experiments is limited due to this radial migration of the particles. Experiments with particles of different size, shape and material where the centrifugal effects do not affect the radial migration of particles remain to be performed. It would be of interest to know if higher density ratios affect the effective viscosity of the mixture. Visualization of the flow for such experiments would allow to study the effects of particle shape and size in the particle resuspension process.