

# Abstract

This thesis presents experimental measurements of the rheological behavior of liquid-solid mixtures at moderate Reynolds (defined by the shear rate and particle diameter) and Stokes numbers, ranging from  $3 \leq Re \leq 1.6 \times 10^3$  and  $0.4 \leq St \leq 195$ . The experiments use a specifically designed Couette cylindrical rheometer that allows for probing the transition from transporting a pure liquid to transporting a dense suspension of particles. Measurements of the shear stress are presented for a wide range of particle concentration (10 to 60% in volume) and for particle to fluid density ratio ( $\rho_p/\rho$ ) between 1 and 1.05. The effective relative viscosity exhibits a strong dependence on the solid fraction for all density ratios tested. For  $\rho_p/\rho = 1$  the effective viscosity increases with Stokes number ( $St$ ) for volume fractions ( $\phi$ ) lower than 40% and becomes constant for higher  $\phi$ . When the particles are denser than the liquid, the effective viscosity shows a stronger dependence on  $St$ . An analysis of the particle resuspension for the case with  $\rho_p/\rho = 1.05$  is presented and used to predict the local volume fraction where the shear stress measurements take place. When the local volume fraction is considered, the effective viscosity for settling and no settling particles is consistent, indicating that the effective viscosity is independent of differences in density between the solid and liquid phase. Shear stress measurements of pure fluids (no particles) were performed using the same rheometer, and a deviation from laminar behavior is observed for gap Reynolds numbers above 4000, indicating the presence of hydrodynamic instabilities associated with the rotation of the outer cylinder. The increase on the effective viscosity with Stokes numbers observed for mixtures with  $\phi \leq 30\%$  appears to be affected by such hydrodynamic instabilities. The effective viscosity for the current experiments is considerably higher than the one reported in non-inertial suspensions.

*Experiment - where theory comes to die*

-Sidney R. Nagel

# Contents

<b>Acknowledgments</b>	<b>iv</b>
<b>Abstract</b>	<b>vi</b>
<b>1 Introduction</b>	<b>1</b>
1.1 Rheology of non-inertial suspensions . . . . .	1
Effective viscosity models . . . . .	2
Non-Newtonian behavior for concentrated non-inertial suspensions . . . . .	4
1.2 Rheology of inertial suspensions . . . . .	5
Previous experiments <sup>1</sup> . . . . .	7
Particle interactions . . . . .	16
Particle settling . . . . .	16
1.3 Secondary flows . . . . .	18
Effect of rough boundaries . . . . .	19
1.4 Thesis outline . . . . .	19
<b>2 Experimental setup</b>	<b>21</b>
2.1 Rheometer . . . . .	21
2.2 Torque measurements . . . . .	24
Optical probe calibration . . . . .	24
Springs calibration . . . . .	26
Angular speed measurements . . . . .	27
2.3 Error analysis . . . . .	31
2.4 Pure fluid torque measurements . . . . .	32
2.5 Particles . . . . .	36
Polystyrene: density ratio = 1 and 1.05 . . . . .	36
Polyester: density ratio=1.2 and 1.4 . . . . .	38
2.6 Visualizations . . . . .	40

---

<sup>1</sup>Some of the material that is summarized in this section is taken from a paper by Koos, Linares-Guerrero, Hunt, and Brennen (2012)

<b>3</b>	<b>Previous experiments on smooth and rough walls</b>	<b>42</b>
3.1	Calibration . . . . .	43
3.2	Previous smooth walls measurements . . . . .	43
3.3	Previous rough walls experiments . . . . .	48
	Depletion layer thickness . . . . .	51
	Slip velocity measurements . . . . .	53
3.4	Summary . . . . .	54
<b>4</b>	<b>Rheological measurements with rough walls</b>	<b>57</b>
4.1	Motivation . . . . .	57
4.2	Polystyrene particles with matched fluid density . . . . .	57
4.3	Polystyrene particles with $\rho_p/\rho = 1.05$ . . . . .	63
4.4	Summary . . . . .	68
<b>5</b>	<b>Rheological measurements with rough walls over a porous medium</b>	<b>72</b>
5.1	Motivation . . . . .	72
5.2	Modification of the experiment and porous medium configuration . . . . .	72
5.3	Torque measurements of polystyrene particles over a porous medium with $\rho_p/\rho = 1.05$	73
	Hysteresis . . . . .	78
5.4	Normalized torque for polystyrene particles over a porous medium with $\rho_p/\rho = 1.05$	80
5.5	Summary . . . . .	84
<b>6</b>	<b>Particle resuspension</b>	<b>86</b>
6.1	Motivation . . . . .	86
6.2	Particle settling . . . . .	86
	Particle settling over a porous medium . . . . .	91
6.3	Random loose packing . . . . .	93
	Random loose packing of polystyrene particles over a porous medium . . . . .	96
6.4	Particle resuspension for $\rho_p/\rho = 1.05$ . . . . .	98
	Particle resuspension over a porous medium ( $\rho_p/\rho = 1.05$ ). . . . .	101
6.5	Summary . . . . .	110
<b>7</b>	<b>Discussion</b>	<b>111</b>
7.1	Inertial and particle concentration effects on mixtures with $\rho_p/\rho = 1$ . . . . .	111
7.2	Direct comparisons between $\rho_p/\rho = 1$ and $\rho_p/\rho = 1.05$ . . . . .	116
7.3	Prediction of the effective volume fraction . . . . .	124
7.4	Inertial and particle concentration effects on mixtures with $\rho_p/\rho = 1$ . . . . .	126
7.5	Flow over a porous medium $\rho_p/\rho = 1.05$ . . . . .	130

Effect of resuspension on flow over a porous medium . . . . .	131
Direct comparisons between flow with and without a porous medium base . . . . .	132
Effective volume fraction prediction for flow over a porous medium with $\rho_p/\rho = 1.05$	141
Inertial and particle concentration effects on flow over a porous medium with $\rho_p/\rho = 1.05$	144
7.6 Corrected torque for partial filling . . . . .	145
7.7 Comparison between current and previous experimental and numerical results . . . .	149
7.8 Summary . . . . .	153
<b>8 Conclusions</b>	<b>155</b>
8.1 Particles with matched density . . . . .	155
8.2 Effect of particle settling . . . . .	157
Flow over a porous medium . . . . .	158
8.3 Comparison with previous results . . . . .	159
8.4 General comments and future work . . . . .	159
<b>A Inferred torque for zero shear rate</b>	<b>162</b>
<b>B Particles projected area as a way to infer the effective volume fraction</b>	<b>165</b>
<b>C Rheological measurements with polyester particles</b>	<b>171</b>
C.1 Motivation . . . . .	171
C.2 Polyester particles with $\rho_p/\rho = 1.4$ . . . . .	171
C.3 Polyester particles with $\rho_p/\rho = 1.2$ . . . . .	174
C.4 Direct comparison between same particles but different density ratios . . . . .	177
C.5 Flow visualization for polyester particles . . . . .	179
Visualizations for $\rho_p/\rho = 1.4$ . . . . .	183
Visualizations for $\rho_p/\rho = 1.2$ . . . . .	186
C.6 Discussion . . . . .	194

# List of Tables

1.1	Previous experiments properties . . . . .	13
2.1	Rheometer properties and dimensions . . . . .	23
2.2	Particles properties . . . . .	37
3.1	Properties of the particles used in the smooth wall experiments performed by Koos (2009) . . . . .	44
7.1	Critical Stokes numbers for fluidization . . . . .	118

# List of Figures

1.1	Flow pattern for suspension rheology . . . . .	6
1.2	Predicted effective relative viscosity for inertial suspensions from numerical studies . .	14
1.3	Diagram of previous and current experimental work done in inertial suspensions in terms of $Re$ . . . . .	15
1.4	Diagram of previous and current experimental work done in inertial suspensions in terms of $St$ . . . . .	17
2.1	Schematic of the concentric-cylinder rheometer . . . . .	22
2.2	Torque measurement system schematic . . . . .	25
2.3	Diagram of an MTI KD-300 fonic sensor output as a function of target displacement.	26
2.4	Fonic sensor calibration curve . . . . .	27
2.5	Springs calibration curves . . . . .	28
2.6	Highest sensitivity spring calibration curve . . . . .	28
2.7	Sketch of the rotational speed measurement system . . . . .	29
2.8	Rotational speed as a function of time . . . . .	30
2.9	Rotational speed measurements from 3 different devices . . . . .	30
2.10	Pure fluid torque measurements . . . . .	33
2.11	Ratio of torques for pure fluid as a unction of $Re_b$ . . . . .	34
2.12	Ratio of torques for pure fluid as a unction of $Re_b^*$ . . . . .	35
2.13	Pure fluid torque measurements normalized with laminar Couette flow as a function of $Re_b^*$ . . . . .	37
2.14	Polystyrene particles . . . . .	38
2.15	Rheometer rough walls . . . . .	39
2.16	Polyester particles . . . . .	39
2.17	Visualization cylinder setup . . . . .	40
2.18	Sample of polystyrene particles used for visualization purposes . . . . .	41
3.1	Previous torque measurements results with smooth walls . . . . .	45
3.2	Previous normalized torques results with smooth walls . . . . .	46

3.3	Previous results for nylon and SAN particles apparent relative viscosity . . . . .	46
3.4	Previous results of the apparent relative viscosity for different non-settling particles in aqueous glycerine . . . . .	47
3.5	Ratio of measured torques from previous rough walls experiments. . . . .	49
3.6	Previous results of effective relative viscosity as a function of $\phi$ . . . . .	50
3.7	Depletion layer thicknesses calculated from previous results of Koos (2009). . . . .	52
3.8	Depletion layer thicknesses ( $\delta$ ) calculated from Koos (2009) particle velocity measurements . . . . .	54
3.9	Depletion layer thicknesses calculated from previous measurements of the apparent and effective viscosity . . . . .	55
4.1	Measured torque as a function of shear rate for $\phi = 10, 20,$ and $30\%$ . $\rho_p/\rho = 1$ . . . . .	58
4.2	Measured torque as a function of shear rate for $\phi = 40,$ and $50\%$ . $\rho_p/\rho = 1$ . . . . .	59
4.3	Normalized torques as a function of Stokes number for $\phi = 10, 20$ and $30\%$ . $\rho_p/\rho = 1$ . . . . .	60
4.4	Normalized torques as a function of Stokes number for $\phi = 40$ and $50\%$ . $\rho_p/\rho = 1$ . . . . .	61
4.5	Normalized torques as a function of Stokes number for all $\phi$ tested. $\rho_p/\rho = 1$ . . . . .	62
4.6	Effective relative viscosity as a function of $\phi$ . $\rho_p/\rho = 1$ . . . . .	62
4.7	Effective relative viscosity as a function of $\phi$ for current and previous experiments. $\rho_p/\rho = 1$ . . . . .	63
4.8	Measured torques as a function of shear rate for $\bar{\phi} = 10$ and $20\%$ . $\rho_p/\rho = 1.05$ . . . . .	64
4.9	Measured torques as a function of shear rate for $\bar{\phi} = 30\%$ with $\rho_p/\rho = 1.05$ . . . . .	65
4.10	Measured torques as a function of shear rate for $\bar{\phi} = 40\%$ with $\rho_p/\rho = 1.05$ . . . . .	65
4.11	Measured torques as a function of shear rate for $\bar{\phi} = 50\%$ with $\rho_p/\rho = 1.05$ . . . . .	66
4.12	Measured torques as a function of shear rate for $\bar{\phi} = 60\%$ with $\rho_p/\rho = 1.05$ . . . . .	66
4.13	Measured torques as a function of shear rate for all $\bar{\phi}$ tested with $\rho_p/\rho = 1.05$ . . . . .	67
4.14	Normalized torques as a function of Stokes number for $\bar{\phi} = 10$ and $20\%$ . $\rho_p/\rho = 1.05$ . . . . .	68
4.15	Normalized torques as a function of Stokes number for $10 \leq \bar{\phi} \leq 60\%$ . $\rho_p/\rho = 1.05$ . . . . .	69
4.16	Normalized torques as a function of Stokes number for all $\bar{\phi}$ tested with $\rho_p/\rho = 1.05$ . . . . .	70
4.17	Effective relative viscosity as a function of $\bar{\phi}$ . $\rho_p/\rho = 1.05$ . . . . .	70
5.1	Scheme of the apparatus configuration for the experiments with flow over a porous medium . . . . .	74
5.2	Measured torques as a function of the shear rate for flow over a porous medium for $\bar{\phi} = 10\%$ . . . . .	75
5.3	Measured torques as a function of the shear rate for flow over a porous medium for $\bar{\phi} = 20\%$ . . . . .	76

5.4	Measured torques as a function of the shear rate for flow over a porous medium for $\bar{\phi} = 30\%$ . . . . .	77
5.5	Measured torques as a function of the shear rate for flow over a porous medium for $\bar{\phi} = 40$ and $50\%$ . . . . .	77
5.6	Measured torques for $\uparrow \dot{\gamma}$ and $\downarrow \dot{\gamma}$ for flow over a porous medium with $\bar{\phi} = 10\%$ . . . . .	78
5.7	Measured torques for $\uparrow \dot{\gamma}$ and $\downarrow \dot{\gamma}$ for flow over a porous medium with $\bar{\phi} = 20\%$ . . . . .	79
5.8	Measured torques for $\uparrow \dot{\gamma}$ and $\downarrow \dot{\gamma}$ for flow over a porous medium with $\bar{\phi} = 30\%$ . . . . .	79
5.9	Measured torques for $\uparrow \dot{\gamma}$ and $\downarrow \dot{\gamma}$ for flow over a porous medium with $\bar{\phi} = 40$ and $50\%$ . . . . .	80
5.10	Normalized torques as a function of Stokes number for flows over a porous medium with $\phi = 10\%$ . . . . .	81
5.11	Normalized torques as a function of Stokes number for flows over a porous medium with $\phi = 20\%$ . . . . .	81
5.12	Normalized torques as a function of Stokes number for flows over a porous medium with $\phi = 30\%$ . . . . .	82
5.13	Normalized torques as a function of Stokes number for flows over a porous medium with $\phi = 40\%$ . . . . .	83
5.14	Normalized torques as a function of Stokes number for flows over a porous medium with $\phi = 50\%$ . . . . .	83
5.15	Normalized torques as a function of Stokes number for flows over a porous medium for all $\bar{\phi}$ tested . . . . .	84
6.1	Time dependent settling process . . . . .	87
6.2	Settled particles marked surface contour . . . . .	88
6.3	Settled particles height ( $h_s$ ) normalized with the total annulus height ( $h_t$ ) as a function of time for $\phi = 25\%$ . . . . .	89
6.4	Settling rate for different loading fractions . . . . .	90
6.5	Settled particles height as a function of $\bar{\phi}$ . . . . .	91
6.6	Normalized height as a function of $\bar{\phi}$ for flow over a porous medium . . . . .	92
6.7	Normalized settled particles height with modified total height for flow with and without porous medium . . . . .	93
6.8	Normalized settled particles height with total height for flow with and without porous medium . . . . .	94
6.9	Measured random loose packing . . . . .	95
6.10	Measured random loose packing for flow over porous medium . . . . .	96
6.11	Comparison between measured random loose packing for flow with and without a porous medium . . . . .	97

6.12	Particles resuspension as a function of St for $\bar{\phi} = 25\%$ . . . . .	99
6.13	Normalized resuspension height as a function of St for $\bar{\phi} = 25\%$ . . . . .	100
6.14	Normalized resuspension height as a function of St . . . . .	101
6.15	Particle packing as a function of St . . . . .	102
6.16	Particle packing normalized with $\phi_{RLP}$ . . . . .	102
6.17	Resuspension height normalized by the modified annulus height ( $h_{tm}$ ) as a function of St	103
6.18	Resuspension height normalized by the total modified annulus height ( $h_{tm}$ and $h_t$ ) . .	104
6.19	Resuspension height normalized by the total annulus height ( $h_t$ ) as a function of St .	105
6.20	Particles packing for different $\bar{\phi}$ as a function of St for the case of flow over a porous medium . . . . .	106
6.21	Normalized particles packing by measured random loose packing for different $\bar{\phi}$ . . . .	107
6.22	Particles packing for different $p\bar{h}i$ as a function of St . . . . .	108
6.23	Particles packing normalized with measured random loose packing for different $p\bar{h}i$ as a function of St . . . . .	109
7.1	Normalized torques as a function of $Re_b$ for pure fluid and mixtures with $\phi = 10, 20,$ and $30\%$ and $\rho_p/\rho = 1$ . . . . .	112
7.2	Effective relative viscosity as function of $\phi$ for current and previous work of Koos et al. (2012) . . . . .	113
7.3	Minimum effective relative viscosity as function of $\phi$ for current experiments compared with $\mu'/\mu$ from previous work of Koos et al. (2012) . . . . .	113
7.4	Measure toques normalized by effective laminar torque as a function of effective Reynolds number . . . . .	114
7.5	Measure toques normalized by $M_{eff, laminar}$ as a function of $Re_{b, eff}$ compared with normalized pure fluid torque measurements . . . . .	115
7.6	Normalized torques for 10% loading fractions as a function of St for $\rho_p/\rho = 1$ and 1.05	116
7.7	Comparison between normalized torques for pure fluid and $\bar{\phi} = 10\%$ with $\rho_p/\rho = 1.05$	117
7.8	Image sequence of the flow for $\bar{\phi} = 10\%$ . . . . .	118
7.9	Comparison between normalized torques for different density ratio with $\bar{\phi} = 20\%$ . . .	119
7.10	Comparison between normalized torques for different density ratios with $\bar{\phi} = 30\%$ . .	120
7.11	Normalized torques with $\bar{\phi} = 40, 50$ and $60\%$ for $\rho_p/\rho = 1$ and $\rho_p/\rho = 1.05$ . . . . .	121
7.12	Comparison between normalized torques for $\rho_p/\rho = 1$ and $\rho_p/\rho = 1.05$ . . . . .	121
7.13	Relative effective viscosity as a function of $\phi$ and $\bar{\phi}$ for $\rho_p/\rho = 1$ and $\rho_p/\rho = 1.05$ without settling effects . . . . .	122
7.14	Relative effective viscosity as a function of $\phi$ and $\bar{\phi}$ for $\rho_p/\rho = 1$ and $\rho_p/\rho = 1.05$ for all St tested . . . . .	123

7.15	Relative effective viscosity as a function of $\phi$ and $\bar{\phi}$ for $\rho_p/\rho = 1$ and $\rho_p/\rho = 1.05$ without settling effects compared with Koos et al. (2012). . . . .	123
7.16	Relative effective viscosity as a function St with $\bar{\phi} = 10$ and 20% for $\rho_p/\rho = 1$ and $\rho_p/\rho = 1.05$ without settling effects . . . . .	124
7.17	Image sequence of the flow for $\bar{\phi} = 20\%$ . . . . .	125
7.18	Ratio of $\mu'/\mu$ as a function of predicted effective volume fraction for $\rho_p/\rho = 1.05$ . . .	126
7.19	Ratio of $\mu'/\mu$ as a function of $\phi$ and predicted effective volume fraction for $\rho_p/\rho = 1$ and $\rho_p/\rho = 1.05$ . . . . .	127
7.20	Ratio of $\mu'/\mu$ as a function of $\phi$ and predicted effective volume fraction for $\rho_p/\rho = 1$ and $\rho_p/\rho = 1.05$ compared with Koos et al. (2012). . . . .	128
7.21	Measured torques normalized with effective laminar torque as a function of $Re_{b, eff}$ for $\rho_p/\rho = 1.05$ . . . . .	129
7.22	Measured torques normalized with effective laminar torque as a function of $Re_{b, eff}^*$ for $\rho_p/\rho = 1.05$ using minimum value of $\mu'/\mu$ . . . . .	130
7.23	Particle resuspension visualization of flow over porous medium for $\bar{\phi} = 10\%$ compared with ratio of $\mu'/\mu$ vs St . . . . .	133
7.24	Particle resuspension visualization of flow over porous medium for $\bar{\phi} = 20\%$ compared with ratio of $\mu'/\mu$ vs St . . . . .	134
7.25	Particle resuspension visualization of flow over porous medium for $\bar{\phi} = 30\%$ compared with ratio of $\mu'/\mu$ vs St . . . . .	135
7.26	Particle resuspension visualization of flow over porous medium for $\bar{\phi} = 40\%$ and 50% .	136
7.27	Effective relative viscosity for flow over porous media for $\bar{\phi} = 40\%$ and 50% . . . . .	137
7.28	Flow over porous medium normalized torques as a function of St for $\bar{\phi} = 10$ compared with no porous medium with $\bar{\phi} = 20\%$ . . . . .	138
7.29	Flow over porous medium normalized torques as a function of St for $\bar{\phi} = 20$ compared with no porous medium with $\bar{\phi} = 30\%$ . . . . .	139
7.30	Flow over porous medium normalized torques as a function of St for $\bar{\phi} = 30$ compared with no porous medium with $\bar{\phi} = 40\%$ . . . . .	140
7.31	Flow over porous medium normalized torques as a function of St for $\bar{\phi} = 40$ and 50% compared with no porous medium with $\bar{\phi} = 50\%$ . . . . .	140
7.32	Particles normalized heights for flow with and without porous medium . . . . .	141
7.33	Effective relative viscosity for flow over porous media as a function of the effective volume fraction . . . . .	142
7.34	Effective relative viscosity as a function of the effective volume fraction for flow with and without porous medium . . . . .	143

7.35	Effective relative viscosity as a function of the effective volume fraction for flow with and without porous medium and $\rho_p/\rho = 1.05$ and $\rho_p/\rho = 1$ . . . . .	143
7.36	Measured torques normalized by the effective laminar torque as a function of the effective Reynolds number for flow with and without porous medium. . . . .	144
7.37	Measured torques normalized by the effective laminar torque as a function of the $Re_{b, eff}^*$ for flow with and without porous medium. . . . .	145
7.38	Measured torque for pure fluid and its curve fit . . . . .	146
7.39	Corrected torque as a function of shear rate for the cases with partially covered test cylinder . . . . .	147
7.40	Normalized corrected torque as a function of $St$ . . . . .	148
7.41	Normalized corrected torque as a function of $St$ . . . . .	148
7.42	Comparison between present and previous experimental results . . . . .	150
7.43	Comparison between present and previous numerical results . . . . .	152
7.44	Comparison between $\mu'/\mu$ as a function of $Re$ for the present and previous numerical work . . . . .	153
A.1	inferred torque at the origin for $\rho_p/\rho = 1$ and $\rho_p/\rho = 1.05$ . . . . .	162
A.2	inferred torque at the origin for flow over porous medium and $\rho_p/\rho = 1.05$ . . . . .	163
A.3	inferred torque at the origin for flow with and without porous medium and $\rho_p/\rho = 1.05$ , and $\rho_p/\rho = 1$ as a function of $\bar{\phi}$ . . . . .	164
A.4	inferred torque at the origin for flow with and without porous medium and $\rho_p/\rho = 1.05$ as a function of $h_s/h_t$ . . . . .	164
B.1	Image processing example. . . . .	166
B.2	$\mu'/\mu$ as a function of the inferred volume fraction from projected area fraction for $\rho_p/\rho = 1$ . . . . .	167
B.3	$\mu'/\mu$ as a function of the inferred volume fraction from projected area fraction for $\rho_p/\rho = 1.05$ . . . . .	168
B.4	Inferred volume fraction from projected area fraction for $\rho_p/\rho = 1$ and $\rho_p/\rho = 1.05$ . . . . .	169
B.5	Inferred volume fraction from projected area fraction for flow over porous medium and $\rho_p/\rho = 1.05$ . . . . .	169
B.6	Inferred volume fraction from projected area fraction for flow with and without porous medium and $\rho_p/\rho = 1$ and $1.05$ . . . . .	170
C.1	Measured torques for polyester particles with $\rho_p/\rho = 1.4$ . . . . .	172
C.2	Measured torques for polyester particles with $\rho_p/\rho = 1.4$ and $\bar{\phi} = 30, 40, \text{ and } 50\%$ . . . . .	173

C.3	Normalized torques for polyester particles with $\rho_p/\rho = 1.4$ and all loading fractions tested. . . . .	173
C.4	Measured torques for polyester particles with $\rho_p/\rho = 1.2$ and $\bar{\phi} = 10$ and 20%. Suspending liquid is aqueous glycerine. . . . .	174
C.5	Measured torques for polyester particles with $\rho_p/\rho = 1.2$ and $\bar{\phi} = 10$ and 20%. Suspending liquid is salt water. . . . .	175
C.6	Measured torques for polyester particles with $\rho_p/\rho = 1.2$ and $\bar{\phi} = 30, 40,$ and 50%. Suspending liquid is aqueous glycerine. . . . .	176
C.7	Measured torques for polyester particles with $\rho_p/\rho = 1.2$ and $\bar{\phi} = 30, 40,$ and 50%. Suspending liquid is salt water. . . . .	176
C.8	Normalized torques for polyester particles with $\rho_p/\rho = 1.2$ and all loading fractions tested. The suspending liquid is aqueous glycerine. . . . .	177
C.9	Normalized torques for polyester particles with $\rho_p/\rho = 1.2$ and all loading fractions tested. The suspending liquid is salt water. . . . .	178
C.10	Comparison between the $M/M_{laminar}$ for $\rho_p/\rho = 1.4$ and $\rho_p/\rho = 1.2$ and $\bar{\phi} = 10$ and 20%. . . . .	179
C.11	Pure fluid normalized torques compared with low loading fractions normalized torques where polyester particles are not present in the test section. . . . .	180
C.12	Previous and present pure fluid normalized torques compared with low loading fractions normalized torques where polyester particles are not present in the test section. . . . .	181
C.13	Comparison among different density ratios for $\bar{\phi} = 30\%$ using polyester particles. . . . .	182
C.14	Comparison among different density ratios for $\bar{\phi} = 40\%$ using polyester particles. . . . .	182
C.15	Comparison among different density ratios for $\bar{\phi} = 50\%$ using polyester particles. . . . .	183
C.16	Flow visualization for $\bar{\phi} = 10\%$ and 20%, and $\rho_p/\rho = 1.4$ . . . . .	184
C.17	Flow visualization for $\bar{\phi} = 30\%$ , and $\rho_p/\rho = 1.4$ . . . . .	185
C.18	Flow visualization for $\bar{\phi} = 40\%$ , and $\rho_p/\rho = 1.4$ . . . . .	187
C.19	Flow visualization for $\bar{\phi} = 50\%$ , and $\rho_p/\rho = 1.4$ . . . . .	188
C.20	Flow visualization for $\bar{\phi} = 10\%$ , and $\rho_p/\rho = 1.2$ . . . . .	189
C.21	Flow visualization for $\bar{\phi} = 20\%$ , and $\rho_p/\rho = 1.2$ . . . . .	189
C.22	Flow visualization for $\bar{\phi} = 30\%$ , and $\rho_p/\rho = 1.2$ . The suspending liquid is salt water. . . . .	190
C.23	Flow visualization for $\bar{\phi} = 30\%$ , and $\rho_p/\rho = 1.2$ . The suspending liquid is aqueous glycerine. . . . .	191
C.24	Flow visualization for $\bar{\phi} = 40\%$ , and $\rho_p/\rho = 1.2$ . The suspending liquid is salt water. . . . .	192
C.25	Flow visualization for $\bar{\phi} = 40\%$ , and $\rho_p/\rho = 1.2$ . The suspending liquid is aqueous glycerine. . . . .	193
C.26	Flow visualization for $\bar{\phi} = 50\%$ , and $\rho_p/\rho = 1.2$ . The suspending liquid is salt water. . . . .	195

C.27 Flow visualization for  $\bar{\phi} = 50\%$ , and  $\rho_p/\rho = 1.2$ . The suspending liquid is aqueous glycerine. . . . . 196

# List of Symbols

$\bar{\phi}$	Loading volume fraction
$\delta$	Depletion layer thickness
$\dot{\gamma}$	Shear rate
$\mu_{app}$	Apparent suspension viscosity from smooth wall measurements
$\mu$	Suspending liquid viscosity
$\mu'$	Suspension effective viscosity
$\mu_{min}^*$	Lowest effective viscosity for each volume fraction $\rho_p/\rho = 1.05$
$\mu'_{min}$	Lowest effective viscosity for each volume fraction and $\rho_p/\rho = 1$
$\omega$	Rotational speed
$\phi$	Volume fraction
$\phi_m$	Maximum volume fraction
$\phi_{RCP}$	Random close packing
$\phi_{RLP}$	Random loose packing
$\psi$	Sphericity
$\rho$	Suspending liquid density
$\rho_p$	Particle density
$\tau$	Shear stress
$b$	Shear gap width
$d$	Particle diameter
$H$	Height of test cylinder

$h$	Height reached by particles
$h_s$	Height of settled particles
$h_T$	Total annulus height
$h_{tm}$	Height measured from porous medium surface to annulus top
$M$	Measured torque
$M_{eff, laminar}$	Effective laminar torque based on effective viscosity, $M_{eff, laminar} = 2\pi r_i^2 H \dot{\gamma} \mu'_{min}$
$M_{laminar}$	Torque from laminar theory, $M_{laminar} = 4\pi \mu H \omega r_i^2 r_o^2 / (r_o^2 - r_i^2)$
$Pe$	Péclet number, $Pe = 6\pi \mu d^3 \dot{\gamma} / kT$
$r_i$	Inner cylinder radius
$r_o$	Outer cylinder radius
$Re$	Reynolds number based on shear rate and particle diameter, $Re = \rho \dot{\gamma} d^2 / \mu$
$Re_{b, eff}^*$	Modified effective Reynolds, $Re_{b, eff}^* = \rho \omega b / \mu'_{min}$
$Re_b^*$	Modified gap Reynolds number based on rotational speed, $Re_b^* = \rho r_o \omega b / \mu$
$Re_b$	Gap Reynolds number based on shear rate, $Re_b = \rho \dot{\gamma} b^2 / \mu$
$Re_{b, eff}$	Effective Reynolds number, $Re_{b, eff} = \rho \omega r_o b / \mu'$
$Sc$	Schmidt number, $Sc = Pe / Re$
$St$	Stokes number, $St = \rho_p Re / 9\rho$
$V_p$	Volume of particles