Essays on Correlated Equilibrium and Voter Turnout

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To my family, teachers, and friends.

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Abstract

This thesis consists of three essays in the areas of political economy and game theory, unified by their focus on the effects of pre-play communication on equilibrium outcomes.

Communication is fundamental to elections. Chapter 2 extends canonical voter turnout models, where citizens, divided into two competing parties, choose between costly voting and abstaining, to include any form of communication, and characterizes the resulting set of Aumann's correlated equilibria. In contrast to previous research, high-turnout equilibria exist in large electorates and uncertain environments. This difference arises because communication can coordinate behavior in such a way that citizens find it incentive compatible to follow their correlated signals to vote more. The equilibria have expected turnout of at least twice the size of the minority for a wide range of positive voting costs.

In Chapter 3 I introduce a new equilibrium concept, called subcorrelated equilibrium, which fills the gap between Nash and correlated equilibrium, extending the latter to multiple mediators. Subcommunication equilibrium similarly extends communication equilibrium for incomplete information games. I explore the properties of these solutions and establish an equivalence between a subset of subcommunication equilibria and Myerson's quasi-principals' equilibria. I characterize an upper bound on expected turnout supported by subcorrelated equilibrium in the turnout game.

Chapter 4, co-authored with Thomas Palfrey, reports a new study of the effect of com-

munication on voter turnout using a laboratory experiment. Before voting occurs, subjects may engage in various kinds of pre-play communication through computers. We study three communication treatments: *No Communication*, a control; *Public Communication*, where voters exchange public messages with all other voters, and *Party Communication*, where messages are exchanged only within one's own party. Our results point to a strong interaction effect between the form of communication and the voting cost. With a low voting cost, party communication increases turnout, while public communication decreases turnout. The data are consistent with correlated equilibrium play. With a high voting cost, public communication increases turnout. With communication, we find essentially no support for the standard Nash equilibrium turnout predictions.

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Chapter 1

Introduction

In this thesis I address several research questions from the broad areas of political economy and game theory. How can communication between competing groups of economic agents help coordinate their actions? More specifically, how does communication among voters affect turnout? To gain a better understanding of the main principles of group coordination under communication, I develop a generalization of Aumann's correlated equilibrium, and apply it to the canonical model of voter turnout. I explore this topic in three chapters.

In Chapter 2 I seek a rational solution to the renowned turnout paradox. Canonical voter turnout models, where citizens supporting two competing parties rationally and independently choose between costly voting and abstaining, based on how likely they are to be pivotal to the election outcome, provide unsatisfactory explanations – predicting turnout rates close to zero. In an apparent contradiction to this prediction, large election turnout rates are markedly higher. I re-examine these results in the

presence of unrestricted pre-play communication, and characterize the resulting set of correlated equilibria. In contrast to previous research, high-turnout equilibria exist in large electorates and uncertain environments. This difference arises because communication can be used to coordinate behavior in such a way that voters find it incentive compatible to always follow their signals past the communication stage, and vote more as a result. The equilibria have expected turnout of at least twice the size of the minority for a wide range of positive voting costs, and show intuitive comparative statics on turnout: it varies with the relative sizes of different groups, and decreases with the cost of voting. This research provides a general micro-foundation for group-based theories of voter mobilization, or voting driven by communication on a network.

In Chapter 3 I introduce a new equilibrium concept, called subcorrelated equilibrium, which extends Aumann's correlated equilibrium to the case of multiple mediators and fills the gap between Nash and correlated equilibrium. Subcommunication equilibrium similarly extends communication equilibrium for incomplete information games. I explore the general properties of these solution concepts. In particular, I establish an equivalence between a subset of subcommunication equilibria and Myerson's quasi-principals' equilibria. I apply subcorrelated equilibrium to the analysis of voter turnout games, where voters preferring one of the two competing parties can correlate their strategies within, but not across, their parties, and characterize an upper bound on expected turnout that can be supported by a subcorrelated equilibrium.

Chapter 4, co-authored with Thomas Palfrey, reports a new study of the effect of communication on voter turnout using a laboratory experiment. Before voting

occurs, subjects may engage in various kinds of pre-play communication through computers. Theoretically, pre-play communication in a voter turnout game admits equilibria with higher total turnout, as compared with no communication, by inducing correlation between voters' turnout decisions. Experimentally, we study three communication treatments: *No Communication*, a control; *Public Communication*, where voters exchange public messages with all other voters, and *Party Communication*, where messages are exchanged only within one's own party. We also vary the common voting cost and the relative party sizes, generating additional comparative static predictions. Our results point to a strong interaction effect between the form of communication and the voting cost. With a low voting cost, party communication increases turnout, while public communication decreases turnout. The data are consistent with the correlated equilibrium play. With a high voting cost, public communication increases turnout. With communication, we find essentially no support for the standard Nash equilibrium turnout predictions.

Subcorrelated and subcommunication equilibria developed in this thesis have many applications beyond the turnout games. The particular interesting ones left for future research include models of strategic network formation and endogenous party government.

Chapter 2

Correlated Equilibria in Voter Turnout Games

What drives voter turnout is a fundamental question in political economy. Canonical models, which rely on voters rationally and independently deciding whether to turn out based on how likely they are to be pivotal to the election outcomes, provide unsatisfactory explanations (Downs (1957), Riker and Ordeshook (1968), Palfrey and Rosenthal (1985), Myerson (2000)). In particular, these models fail to rationalize the high turnout rates observed in very large elections. Intuitively, as the electorate grows large, the probability that any individual voter is pivotal goes to zero, so with voting incurring a cost, very few people should turn out. This flaw has led many scholars to seek alternative, behavioral explanations.[1]

This study re-examines these results in the presence of communication, broadly defined – between candidates, media, and voters – and shows that this can support

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[1]See, e.g., Feddersen and Sandroni (2006), Bendor et al. (2011), Ali and Lin (2013).

high turnout in large elections while maintaining the assumption that voters' incentives are purely instrumental. The key difference is that communication allows for strategies such that equilibrium behavior is still optimal for each individual voter, but such that voters' turnout decisions are now correlated, rather than independent as in the standard game-theoretic analysis. That is, communication allows us to examine correlated equilibria (Aumann, 1974, 1987). These equilibria are behaviorally more plausible than Nash since they model voters' knowledge of the other voters' equilibrium strategies as a result of communication and learning, and so can apply to electorates with less than fully informed voters, like the U.S. (Bartels, 1996).

As suggested above, the forms of communication allowed in the model are very general. The only necessary condition is that the communication results in some amount of correlation in voters' decisions. As such, the model provides a very rich space in which communication can be from a few senders to many receivers – as it would be with the media or parties communicating with voters – or between a very large number of senders and receivers. In this sense, the model can provide a micro-foundation for group-based voter mobilization: as mobilization efforts induce correlation in decisions, they provide a mechanism for turnout that does not rely on group-based utilities or coercion (Uhlaner (1989), Schram and van Winden (1991), Cox (1999)). Moreover, as correlation could be induced by any signal – even signals like weather, which would not be thought of as having political content – the model incorporates mechanisms that would not play any role in standard rational choice explanations.[2]

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[2]See Gomez, Hansford, and Krause (2007) who demonstrate not only that the bad weather on

The intuition underlying the highest-turnout correlated equilibrium is straightforward. To see this, suppose there are two parties, A and B, who compete in an election decided by majority rule. Citizens (potential voters) are not indifferent between the parties, so there are n_A citizens that support party A and $n_B < n_A$ citizens that support party B. Each citizen decides to vote based only on the tradeoff between her potential effect on the election outcome and the cost of voting. A voter will only affect the outcome when pivotal, that is, when her vote would change the election from her least favored party winning to a tie, or from a tie to her most favored party winning. As in standard models, in any equilibrium, the probability that a voter is pivotal, multiplied by the benefit she gets from changing the outcome of the election, must be greater than or equal to the cost of voting. Thus turnout is highest when the election results in a tie, either directly or in expectation.

Without communication, citizens will make turnout decisions independently. The largest tie would require all of minority citizens (n_B citizens), and the exact same number of majority citizens (n_B out of n_A citizens) to participate. In such a case, every recruited citizen would be pivotal with the same probability, and so, as long as it is high enough, would have incentives to turn out as required by this strategy. But the remaining $n_A - n_B$ majority citizens would deviate by also turning out, so this is not an equilibrium. In fact, except for few very special cases, there are no equilibria where all citizens use pure strategies.

With communication, however, turnout decisions can be correlated. The party sup-

the election day decreases turnout, but also that it affects Democrats and Republicans differently.

ported by the minority of the citizens signals all of its supporters to vote. The party with the majority support uses a more complicated communication protocol. In some fraction of elections, p, the majority party creates a pivotal situation by sending a signal to vote to n_B of its supporters and no signal to the rest of majority citizens. In the remaining fraction of elections, the majority party sends to all of its supporters a signal to vote with probability $\frac{n_B}{n_A}$, and no signal with probability $1 - \frac{n_B}{n_A}$. Therefore, each minority citizen will be pivotal with probability p. As long as p is high enough, all minority citizens will find it in their interest to turn out and vote. On the other hand, the majority citizens, based on the signal from their party, will not know for sure whether or not they are in the pivotal situation. For the value of p corresponding to the correlated equilibrium, majority citizens will also find it in their interest to follow the signal of their party, and to avoid the cost of voting by abstaining if they receive no signal. It is easy to see that in this correlated equilibrium the expected turnout will be quite high: twice the size of the minority. If the minority is large enough, voter turnout could thus be close to 100%.

The upper bound on turnout of twice the size of the minority, highlighted in the example above, is sometimes closely approached by the actual elections. To take a recent high profile elections, the 2014 referendum on Scottish independence gathered 1,617,989 votes in favor of independence.[3] Internet, telephone, and face-to-face opinion polls, averaged over the last two months before the referendum day indicated that about 42.07% of Scots supported independence, which translates into about

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[3]Source: www.bbc.com/news/events/scotland-decides/results

1,802,023 citizens.[4] Assuming that polls more or less perfectly revealed the majority and minority supports, this means that nearly 90% of minority citizens turned out, which is close to the full minority turnout in the example. Moreover, the total turnout was 3,619,915 citizens, which is almost exactly twice the size of the minority (up to a third decimal point).

The remainder of the Chapter is organized as follows. Subsection 2.0.1 provides a literature overview. Section 3.1 describes the basic model, which assumes complete information and homogenous voting costs. Subsections 2.1.1.1 and 2.1.1.2 present and discuss the main results for this case. Subsection 2.1.1.3 presents efficiency analysis for the basic model. Section 2.2 extends the basic model to the case of heterogeneous voting costs and shows that the main results continue to hold. Section 2.3 explores the effects of private information about voting costs. Section 2.4 discusses how our results extend the related findings in the existing literature. Section 3.3 concludes.

2.0.1 Related literature

This study directly relates to two strands of the voluminous literature on formal models of turnout. One is the pivotal voter model, in particular, Palfrey and Rosenthal (1983, 1985). The other is group-based models that build upon the pivotal voter analysis, e.g. Morton (1991). Our model combines these approaches, and so contributes to the literatures on the turnout paradox and voter mobilization.

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[4]Source: whatscotlandthinks.org/questions/should-scotland-be-an-independent-country-1

The turnout paradox, that is, the unsupportable rational choice prediction of turnout rate close to zero in large elections, was first formulated by Downs (1957) in the context of a decision theoretic voting model, which was extended later by Tullock (1967) and Riker and Ordeshook (1968). It would be impossible to mention here all the relevant papers that have been published on the topic since those early studies, so we have to restrict ourselves to the most closely related works. We refer the reader to Feddersen (2004) and Geys (2006) for very well-written recent literature surveys. See also Palfrey (2013) for a recent survey of laboratory experiments in political economy, including experiments testing different theories of turnout (*Ibid.*, Section 4).

The pivotal voter model of Palfrey and Rosenthal (1983) argues that voters' decisions to turn out are strategic, so the probability of being pivotal must be determined endogenously in equilibrium. Under complete information and common voting cost, Palfrey and Rosenthal (1983) found several classes of high-turnout Nash equilibria. Under incomplete information about voting costs, though, Palfrey and Rosenthal (1985) showed that non-zero turnout rate in large elections is not sustainable in the (quasi-symmetric) Bayesian Nash equilibrium: only voters with non-positive voting costs will vote in the limit as the majority and minority groups get large. Myerson (1998, 2000) introduced a very general approach to the analysis of large games with population uncertainty. However his "independent actions" assumption, which results in the number of players being a Poisson random variable, does not allow correlation between players' strategies. Barelli and Duggan (2013) prove existence of a pure strategy Bayesian Nash equilibrium in games with correlated types and

interdependent payoffs. Their Example 2.4, an application of their main purification theorem, is a more general version of the costly voting game under incomplete information than the one we consider in Section 2.3. Unlike them, we study the strategic form incomplete information correlated equilibria of this game that differ from Bayesian Nash equilibria with correlated types, and focus on characterizing the bounds on expected turnout rather than equilibrium existence.

Although the pivotal voter model prediction about expected turnout fails under incomplete information, the comparative static predictions are largely supported in laboratory experiments: see, e.g. Levine and Palfrey (2007). More recent work falling within this approach focused on welfare effects associated with turnout, comparison of mandatory and voluntary voting rules, and the effect of polls (e.g., Börgers (2004), Goeree and Grosser (2007), Diermeier and Van Mieghem (2008), Krasa and Polborn (2009), Taylor and Yildirim (2010)). Campbell (1999) finds that decisive minorities (i.e., those with lower voting costs or with greater expected benefits) are more likely to win in a quasi-symmetric equilibrium, even if their expected share in the electorate is small. His main point of departure from Palfrey and Rosenthal (1985) is introducing correlation between voter types (i.e., party preference) and voting cost. In this respect, he extends Ledyard (1984) who assumed that types and costs are distributed independently.

Kalandrakis (2007, 2009) looks at general turnout games with complete information and heterogeneous costs, and shows that almost all Nash equilibria of these games are regular and robust to small amounts of incomplete information. These findings

can be compared to our results in Sections 2.2 and 2.3. Another closely related paper is Myatt (2012), who investigates how adding aggregate uncertainty about candidates' popularity could be used to solve the turnout paradox. His main result can be viewed as adding a modicum of correlation in an asymptotic approximation of the high-turnout quasi-symmetric Nash equilibrium characterized in Palfrey and Rosenthal (1985) to rule out zero equilibrium turnout rate as the electorate grows large. Similarly to those equilibria, it requires the common voting cost to be high enough, and predicts a tie in the equilibrium. Myatt (2012, Proposition 2) shows that the same logic can be applied to mixed-pure Nash equilibria, but characterizes the expected turnout only for a special case of the candidates' popularity density. Our results allow for correlation directly in the solution concept.

There are other prominent approaches to modeling voter behavior that aim at solving the turnout paradox (e.g., the ethical voter model of Feddersen and Sandroni (2006); see also the recent extensions by Evren (2012) and Ali and Lin (2013); or adaptive learning models, e.g., Bendor et al. (2011); or models based on uncertainty about candidates, e.g., Sanders (2001); or the quality of voters' private signals, e.g., McMurray (2013)). While these and similar models highlight a number of important aspects of voting in mass elections, they do not explicitly consider correlations in voters' actions. Our approach in this Chapter is different: we deliberately maintain the stark rational choice setting to show that even in this case the high-turnout equilibria can be supported once correlation among voters is accounted for.[5]

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[5]The effects of communication on turnout may be also indirect. For example, Ortoleva and Snowberg (2015) find, inter alia, that voter overconfidence, even conditional on ideology, increases turnout. Communication among voters might be a possible way that overconfidence builds up in

Unlike the pivotal voter model, where the individual voter is a central unit of analysis, group-based models operate at the level of groups of voters. An early example is Becker (1983), who models competition among pressure groups for political influence non-strategically as independent utility maximization by each group subject to a joint budget constraint. Uhlaner (1989) emphasizes the role of groups in voting decisions, but does not characterize the equilibrium of the model. Morton (1991) shows that with fixed candidates' positions, positive turnout can be obtained in equilibrium with two groups, but in the general equilibrium framework, where candidates' positions can shift, the paradox prevails. Schram (1991) and Schram and van Winden (1991) develop a model with two groups and opinion leaders in each group, who produce social pressure on others to turn out. The individual voters are modeled as consumers of social pressure. It is shown that it is optimal for the producers of social pressure to do it, but to explain why consumers of social pressure would find it optimal to follow the leaders a civic duty argument is used. Shachar and Nalebuff (1999) develop a model of a pivotal leader, and structurally estimate it using voting data for U.S. presidential elections. See Rosenstone and Hansen (1993), Cox (1999), and references therein for an overview of empirical findings related to party mobilization models.

Overall, group-based models get around the turnout paradox by assuming the existence of a small number of group leaders who control voter mobilization decisions by allocating resources or by means of social pressure. The exogenous mapping from mobilization efforts to voter turnout is assumed. The micro-foundation for the control mechanism as well as the origins of group leaders are not usually modeled. In

the first place.

our case, both of these mechanisms arise naturally as coordination mechanisms in the form of pre-play communication among voters. Communication in turn induces correlation among the voters' strategies that can lead to surprisingly high turnout.

There is growing field and laboratory experimental evidence that communication among voters, and between political activists and voters, taken in a wide variety of forms (e.g., public opinion polls, get-out-the-vote campaigns, and so forth) *critically influences turnout rates.* A book-length treatment of field experiments studying effects of get-out-the-vote campaigns on turnout is Gerber and Green (2008), and one of influential earlier papers is Gerber and Green (2000). Gerber et al. (2011) show that effects of TV advertising may be strong but short-lived. See also Lassen (2005) on a related topic of voter information affecting turnout.[6] Recently, DellaVigna et al (2014) emphasize the social pressure aspect of turnout, also studied in Gerber, Green, and Larimer (2008), while Barber and Imai (2014) show that even the neighborhood composition itself may matter for turnout. A recent work by Sinclair (2012) emphasizes the role of networks in political behavior, arguing that networks not only provide information, but also directly influence citizens' actions. See also Rolfe (2012). This approach is complementary to our work: while we do not explicitly model social connections among voters, one can easily imagine how such network links could serve as channels of pre-play communication.

Laboratory experiments include, e.g., Grosser and Schram (2006), who study the effects of communication in the form of neighborhood information exchange between

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[6]McMurray (2012) notes that models that avoid the turnout paradox by introducing consumption benefits, at the same time nullify the empirical relation between voter information and turnout.

an early voter (sender) and a late voter (receiver) from the same neighborhood. Grosser and Schram (2010), and Agranov et al. (2013) study the effects of polls on turnout and welfare. In particular, Agranov et al. (2013) show that while polls do not have negative welfare effects, they overestimate voter turnout. The authors also find evidence for voting with the winner, where a voter is more likely to turn out if she thinks her preferred candidate is more likely to win.

2.1   The Model

The set of voters is denoted N, with $|N| = n \geq 3$. There are two candidates, A and B. The decision making rule is simple majority with ties broken randomly. Each player $i \in N$ has type[7] $t_i \in \{A, B\}$ representing her political preference: if $t_i = A$ then i prefers candidate A to candidate B, if $t_i = B$ then the preference is reversed. Denote by N_A, with $|N_A| = n_A$, the group of voters who prefer candidate A, and N_B, with $|N_B| = n_B$, the group preferring candidate B. Throughout the Chapter we assume that $n_A > n_B$, and will refer to N_A and N_B as majority and minority, respectively. Thus in the usual parlance, candidate A is the *favorite*, while candidate B is the *underdog*.

Each voter has two pure actions: to vote for the preferred candidate (action 1) or abstain (action 0).[8] Thus i's action space is $S_i = \{0, 1\}$. The set of voting profiles is

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[7]We do not explicitly include i's private voting cost in her type for convenience reasons and always refer to i's voting costs separately.

[8]Voting for a less preferred candidate is always dominated, and can be dispensed with.

$S = S_1 \times \cdots \times S_n$, i.e. $S = \{(s_i)_{i \in N} | s_i \in \{0, 1\}\}$. Voting is costly, and utility of voting net of voting cost is normalized to 1 if the preferred candidate wins, $1/2$, if there is a tie, and 0 otherwise. Instead of explicitly modeling candidates as players of this game, we use a representation with a centralized mediator giving out recommendations to voters, who either maximizes or minimizes total expected turnout. As will be clear from Proposition 1, our main result, this does not matter for the empirically relevant case of the large minority with $n_B > \frac{1}{2} n_A$. In Chapter 4 we analyze the general case where this representation matters.

2.1.1 Complete information and homogenous voting costs

In this section we assume that N_A and N_B are commonly known. Furthermore, assume that the participation cost is the same for all voters and fixed at $c \in (0, 1/2)$.[9] In a more general case with heterogeneous costs, considered in Section 2.2, we discuss how one could allow *some* voters, e.g., those who view voting as a social duty, to have negative voting costs. In the case of a negative *common* cost, however, letting $c < 0$ results in a trivial equilibrium with everybody voting, so for the rest of this section we only consider non-negative values of c.

Definition 1. *A* correlated equilibrium *is a probability distribution*[10] $\mu \in \Delta(S)$ *such*

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[9]If $c \geq \frac{1}{2}$ ($c \leq 0$), the problem is trivial, with abstaining (voting) being everyone's dominant strategy.

[10]Aumann (1987) calls this object a *correlated equilibrium distribution*; this distinction is immaterial.

that for all $i \in N$, for all $s_i \in \{0,1\}$, and all $s_i' \in \{0,1\}$

$$\sum_{s_{-i} \in S_{-i}} \mu(s_i, s_{-i}) \left(\mathcal{U}_i(s_i, s_{-i}) - \mathcal{U}_i(s_i', s_{-i})\right) \geq 0 \tag{2.1}$$

where $\mathcal{U}_i(s_i, s_{-i})$ is the utility of voter i at a strategy profile (s_i, s_{-i}).

To get some intuition for this definition, assume for a moment that all joint strategy profiles have a strictly positive probability, and divide both sides of (2.1) by $\text{Prob}(s_i) = \sum_{s_{-i} \in S_{-i}} \mu(s_i, s_{-i})$. Since $\text{Prob}(s_{-i}|s_i) = \mu(s_i, s_{-i})/\text{Prob}(s_i)$, correlated equilibrium can be interpreted as a probability distribution over joint strategy profiles where at every profile player i's choice is a weak best response under the posterior distribution conditional on that choice. Conditioning is used here to obtain the others' posteriors about player i's choice, which must be correct in equilibrium. Notice also that Nash equilibrium is a special case of correlated equilibrium, where μ is the product of n independent probability distributions, each one over the corresponding player's action space. Thus Nash equilibrium rules out any correlation between players' actions.

Call (2.1) voter i's incentive compatibility (IC) constraints. Since each player has only two (pure) strategies, we only need to consider those inequalities in (2.1) where $s_i' \neq s_i$; thus for each of n players we will only need two inequalities making it $2n$ inequalities in total (plus the feasibility constraints on μ). Denote $\mathcal{D}(N_A, N_B, c)$ the set of solutions to such a system. Formally,

$$\mathcal{D}(N_A, N_B, c) = \{\mu \in \Delta(S)| \text{ for all } i \in N, (2.1) \text{ holds}\} \tag{2.2}$$

$\mathcal{D}(N_A, N_B, c)$ is a convex compact set, and since any Nash equilibrium is a correlated equilibrium, $\mathcal{D}(N_A, N_B, c)$ is also non-empty. It will be convenient to explicitly rewrite (2.2) as the set of distributions $\mu \in \Delta(S)$ such that $\forall i \in N$ the following two inequalities hold

$$\sum_{s_{-i} \in S^{-i}} \mu(0, s_{-i}) \left(\mathcal{U}_i(0, s_{-i}) - \mathcal{U}_i(1, s_{-i})\right) \geq 0 \tag{2.3}$$

$$\sum_{s_{-i} \in S^{-i}} \mu(1, s_{-i}) \left(\mathcal{U}_i(1, s_{-i}) - \mathcal{U}_i(0, s_{-i})\right) \geq 0 \tag{2.4}$$

Substituting the expression for the voter's utility with normalized benefit minus voting cost, conditions (2.3)-(2.4) reduce to

$$c \sum_{s_{-i} \in V_D^i} \mu(0, s_{-i}) + \left(c - \frac{1}{2}\right) \sum_{s_{-i} \in V_P^i} \mu(0, s_{-i}) \geq 0 \tag{2.5}$$

$$-c \sum_{s_{-i} \in V_D^i} \mu(1, s_{-i}) + \left(\frac{1}{2} - c\right) \sum_{s_{-i} \in V_P^i} \mu(1, s_{-i}) \geq 0 \tag{2.6}$$

where for any $i \in N_j$, $j \in \{A, B\}$

$$V_P^i = \left\{ (s_k)_{k \in N \setminus \{i\}} \Big| \sum_{k \in N_j \setminus \{i\}} s_k = \sum_{k \in N_{-j}} s_k \text{ or } \sum_{k \in N_j \setminus \{i\}} s_k = \sum_{k \in N_{-j}} s_k - 1 \right\} \tag{2.7}$$

$$V_D^i = \left\{ (s_k)_{k \in N \setminus \{i\}} \Big| \sum_{k \in N_j \setminus \{i\}} s_k > \sum_{k \in N_{-j}} s_k \text{ or } \sum_{k \in N_j \setminus \{i\}} s_k < \sum_{k \in N_{-j}} s_k - 1 \right\} \tag{2.8}$$

are the sets of profiles where player i is pivotal, and not pivotal, respectively. In the latter case, we call player i a *dummy*, hence the subscript.

Conditions (2.5)-(2.6) have a simple interpretation. They say that in any corre-
lated equilibrium, unlike in the Nash equilibrium, for each player there are two
best response conditions: one, (2.6), is conditional on voting, and the other, (2.5),
conditional on abstaining. These conditions are equivalent to the following two re-
strictions:

$$c \geq \frac{1}{2}\text{Prob}(i \text{ is pivotal} \mid i \text{ abstains})$$
$$c \leq \frac{1}{2}\text{Prob}(i \text{ is pivotal} \mid i \text{ votes})$$

Thus, a correlated equilibrium in this game is given by a probability distribution over
joint voting profiles where at every profile each player finds it incentive compatible
to follow her prescribed choice conditional on this profile realization.

Out of many possible correlated equilibria, we focus on the boundaries of the set:
we study the equilibria that maximize (max-turnout) and minimize (min-turnout)
expected turnout. Formally, a max-turnout equilibrium solves the following linear
programming problem:

$$\text{maximize } f(\mu) = \sum_{s \in S} \left(\mu(s) \sum_{i \in N} s_i \right) \tag{2.9}$$
$$\text{s.t.} \qquad \mu \in \mathcal{D}(N_A, N_B, c)$$

for $0 < c < 1/2$. Correspondingly, a min-turnout equilibrium solves

$$\text{minimize } f(\mu) \text{ s.t. } \mu \in \mathcal{D}(N_A, N_B, c) \tag{2.10}$$

A potential difficulty in deriving the analytical solution to these problems lies in the $2n$ incentive compatibility constraints (2.5)-(2.6) that must be simultaneously satisfied. Fortunately, it is possible to overcome this problem. The simplification comes from the observation that for all correlated equilibria that maximize or minimize turnout, there exists a "group-symmetric" probability distribution that delivers the same expected turnout.

Let $\mu(z_i, a, b)$ denote the probability of any joint profile where player i plays strategy z_i, and, among the other $n - 1$ players, a players turn out in group N_A and b players turn out in group N_B. Define a set of group-symmetric probability distributions as follows.

$$\mathcal{M} = \{\mu \in \mathcal{D}(N_A, N_B, c)|$$
$$\forall i \in N_A, \forall a \in \{1, \ldots, n_A - 1\}, \forall b \in \{0, \ldots, n_B\} : \mu(0_i, a, b) = \mu(1_i, a - 1, b)$$
$$\forall k \in N_B, \forall b \in \{1, \ldots, n_B - 1\}, \forall a \in \{0, \ldots, n_A\} : \mu(0_k, a, b) = \mu(1_k, a, b - 1)\}$$

In words, the distributions in $\mathcal{M}$ place the same probability on all such profiles that have the same number of players turning out from either side, and differ only by the identity of those who turn out and those who abstain. Thus the identity of the voter does not matter as long as the total number of this voter's group votes is the same, given the fixed number of votes on the other side.

Lemma 1. *For any distribution $\mu^* \in \mathcal{D}(N_A, N_B, c)$ that solves problem (2.9) or (2.10), there exists an equivalent group-symmetric probability distribution σ^* that also delivers a solution to the same problem. Formally, $f(\sigma^*) = f(\mu^*)$ and $\sigma^* \in \mathcal{M}$.*

Proof. See 2.A.1. ∎

Lemma 1 allows a substantial simplification of the problem without any loss of generality, reducing $2n$ inequalities down to just four: two for a member of group N_A and two more for a member of group N_B; and reducing the number of variables (unknown profile probabilities) from the original 2^n profiles down to $(n_A + 1)(n_B + 1)$, which is the maximal number of profiles with different probabilities under group-symmetric distributions.

Before describing the general characterization of solutions to (2.9) and (2.10), we walk through the simplest possible example with 3 voters, which serves to illustrate both Lemma 1 and the main results of the Chapter.

Example 1. Suppose $N = \{1, 2, 3\}$. Let $N_A = \{1, 2\}$ and $N_B = \{3\}$. There are eight possible voting profiles: from $(0, 0, 0)$ with no one voting to $(1, 1, 1)$ with full turnout. Denote (s_i, s_j, s_k) a strategy profile where $i, j \in N_A$ and $k \in N_B$. Then for each $i \in N_A$,

$$\mathcal{U}_i(s_i, s_{-i}) = \begin{cases} 1 - s_i c & \text{if } (s_i, s_j, s_k) \in \{(0,1,0), (1,0,0), (1,1,0), (1,1,1)\} \\ \frac{1}{2} - s_i c & \text{if } (s_i, s_j, s_k) \in \{(0,0,0), (0,1,1), (1,0,1)\} \\ 0 & \text{if } (s_i, s_j, s_k) = (0,0,1) \end{cases}$$

Similarly, for $k \in N_B$,

$$
\mathcal{U}_k(s_k, s_{-k}) = \begin{cases} 1 - c & \text{if } (s_i, s_j, s_k) = (0, 0, 1) \\ \frac{1}{2} - s_k c & \text{if } (s_i, s_j, s_k) \in \{(0,0,0), (0,1,1), (1,0,1)\} \\ -s_k c & \text{if } (s_i, s_j, s_k) \in \{(0,1,0), (1,0,0), (1,1,0), (1,1,1)\} \end{cases}
$$

Denote $\mu_{s_i s_j s_k} = \mu(s_i, s_j, s_k)$ to simplify notation. Now conditions (2.5)-(2.6) reduce to the following system of linear inequalities, where we also add the standard probability requirements:

$$
c\mu_{010} + \left(c - \frac{1}{2}\right)(\mu_{000} + \mu_{001} + \mu_{011}) \geq 0 \tag{2.11}
$$

$$
-c\mu_{110} + \left(\frac{1}{2} - c\right)(\mu_{100} + \mu_{101} + \mu_{111}) \geq 0 \tag{2.12}
$$

$$
c\mu_{100} + \left(c - \frac{1}{2}\right)(\mu_{000} + \mu_{001} + \mu_{101}) \geq 0 \tag{2.13}
$$

$$
-c\mu_{110} + \left(\frac{1}{2} - c\right)(\mu_{010} + \mu_{011} + \mu_{111}) \geq 0 \tag{2.14}
$$

$$
c\mu_{110} + \left(c - \frac{1}{2}\right)(\mu_{000} + \mu_{010} + \mu_{100}) \geq 0 \tag{2.15}
$$

$$
-c\mu_{111} + \left(\frac{1}{2} - c\right)(\mu_{001} + \mu_{011} + \mu_{101}) \geq 0 \tag{2.16}
$$

$$
\forall s \in \{0, 1\}^3 \quad \mu_s \geq 0 \tag{2.17}
$$

$$
\sum_{s \in \{0,1\}^3} \mu_s = 1 \tag{2.18}
$$

The solutions have the following properties.[11] In any correlated equilibrium the

---

[11]Recall that we restricted c to lie in (0,0.5). We can now provide the rationale behind this assumption. If $c > 0.5$, the unique correlated equilibrium has $\mu_{000} = 1$, i.e., no one votes. This follows because once $c > \frac{1}{2}$, inequalities (2.12), (2.14), and (2.16) can only hold if $\mu_{100} = \mu_{101} = \mu_{110} = \mu_{111} = 0$, $\mu_{010} = \mu_{011} = 0$, and $\mu_{001} = 0$, which implies $\mu_{000} = 1$. If $c = 0.5$, any probability distribution with $\mu_{110} = 0$ and $\mu_{111} = 0$ is a correlated equilibrium: inequalities (2.12), (2.14), and

constraints can be rewritten as

$$\frac{\frac{1}{2} - c}{c}(\mu_{000} + \mu_{010} + \mu_{100}) \leq \mu_{110} \leq \frac{\frac{1}{2} - c}{c}(\mu_{111} + \min\{\mu_{100} + \mu_{101}, \mu_{010} + \mu_{011}\}) \quad (2.19)$$

$$\mu_{010} \geq \frac{\frac{1}{2} - c}{c}(\mu_{000} + \mu_{001} + \mu_{011}) \quad (2.20)$$

$$\mu_{100} \geq \frac{\frac{1}{2} - c}{c}(\mu_{000} + \mu_{001} + \mu_{101}) \quad (2.21)$$

$$\mu_{111} \leq \frac{\frac{1}{2} - c}{c}(\mu_{001} + \mu_{011} + \mu_{101}) \quad (2.22)$$

$$\sum_{s \in \{0,1\}^3} \mu_s = 1 \quad (2.23)$$

$$\mu_{000}, \mu_{001}, \mu_{010}, \mu_{011}, \mu_{100}, \mu_{101}, \mu_{111} \in [0, 1), \mu_{110} \in (0, 1) \quad (2.24)$$

This system has many solutions, and $\mu_{000} < 1$ implies that all have positive expected turnout. Notice that in (2.20)-(2.22) the probabilities of profiles with more votes are bounded from above by the probabilities of profiles with less votes, while in (2.19) it is the other way round. These relations are important for the extreme correlated equilibria, because they determine the constraints that bind at an optimum.

We next identify the max-turnout equilibria that solve the following linear program:

$$\text{maximize} \sum_{s \in \{0,1\}^3} (s_i + s_j + s_k)\mu_{s_i s_j s_k} \quad \text{s.t.} \quad \mu \in \mathcal{D}(2, 1, c) \quad (2.25)$$

A solution to (2.25) always exists since $\mathcal{D}(2, 1, c) \neq \emptyset$. We will denote such a solution μ^*. Since the objective function does not depend on $\mu_{000} \geq 0$, (2.23) implies that

---

(2.16) can only hold if $\mu_{110} = \mu_{111} = 0$, while all remaining inequalities are trivially satisfied. If $c = 0$, then any probability distribution with $\mu_{000} = \mu_{001} = \mu_{011} = \mu_{101} = \mu_{010} = \mu_{100} = 0$ is a correlated equilibrium; thus it is any mixture between μ_{111} and μ_{110}.

$\mu^*_{000} = 0$. Using this fact and (2.23), we can rewrite the objective in (2.25) as

$$\sum_{s \in \{0,1\}^3} (s_i + s_j + s_k)\mu_{s_i s_j s_k} = 1 + (\mu_{011} + \mu_{101} + \mu_{110}) + 2\mu_{111} \qquad (2.26)$$

We next show that at μ^* the value of the objective function is 2 for any $0 < c < 0.5$. Lemma 1 implies that without loss of generality we can let $\mu_{010} = \mu_{100}$ and $\mu_{011} = \mu_{101}$. Hence (2.20) and (2.21) reduce to the same constraint, and (2.19)-(2.23) imply[12]

$$\mu_{010} \leq \mu_{111} + \mu_{011}$$
$$\mu_{010} \geq \frac{\frac{1}{2} - c}{c}(\mu_{001} + \mu_{011})$$
$$\mu_{111} \leq \frac{\frac{1}{2} - c}{c}(\mu_{001} + 2\mu_{011})$$
$$\sum_{s \in \{0,1\}^3} \mu_s = 1$$

where the first inequality follows from (2.19) with $\mu^*_{000} = 0$. This implies

$$\mu_{111} \leq 2\mu_{010} - \frac{\frac{1}{2} - c}{c}\mu_{001}$$

Then in (2.26) the right hand side is at most $1 + (\mu_{011} + \mu_{101} + \mu_{110} + \mu_{010} + \mu_{100} + \mu_{111}) - \frac{\frac{1}{2} - c}{c}\mu_{001} = 2 - \frac{\mu_{001}}{2c}$. Now we can see that to achieve the upper bound of two, it is necessary to put $\mu^*_{001} = 0$. Thus we let $\mu^*_{000} = \mu^*_{001} = 0$, and put $\mu^*_{111} = 2\mu^*_{010}$.

---

[12]For the sake of brevity, we omit the non-negativity constraints on μ.

Then constraints (2.19)-(2.23) reduce to

$$\frac{\frac{1}{2}-c}{c}2\mu^*_{010} \leq \mu^*_{110} \leq \frac{\frac{1}{2}-c}{c}(3\mu^*_{010} + \mu^*_{011}) \tag{2.27}$$

$$\mu^*_{010} \geq \frac{\frac{1}{2}-c}{c}\mu^*_{011} \tag{2.28}$$

$$2\mu^*_{010} \leq \frac{\frac{1}{2}-c}{c}2\mu^*_{011} \tag{2.29}$$

$$\sum_{s\in\{0,1\}^3} \mu^*_s = 1 \tag{2.30}$$

From the last two inequalities it follows that $\mu^*_{011} = \mu^*_{101} = \frac{c}{\frac{1}{2}-c}\mu^*_{010}$. Re-arranging,

$$\frac{\frac{1}{2}-c}{c}2\mu^*_{010} \leq \mu^*_{110} \leq \frac{\frac{1}{2}-c}{c}\left(3 + \frac{c}{\frac{1}{2}-c}\right)\mu^*_{010} \tag{2.31}$$

$$\mu^*_{011} = \frac{c}{\frac{1}{2}-c}\mu^*_{010} \tag{2.32}$$

$$2\mu^*_{011} + \mu^*_{110} + 4\mu^*_{010} = 1 \tag{2.33}$$

Replacing $\mu^*_{110} = 1 - \frac{1-c}{1/2-c}2\mu^*_{010}$ from (2.33) and re-arranging, we obtain the following system, which, if holds, delivers the value of two to the objective function:

$$\mu^*_{000} = \mu^*_{001} = 0$$

$$\mu^*_{111} = 2\mu^*_{010}$$

$$\mu^*_{011} = \frac{c}{\frac{1}{2}-c}\mu^*_{010}$$

$$\mu^*_{110} = 1 - 2\frac{1-c}{\frac{1}{2}-c}\mu^*_{010}$$

$$\frac{\frac{1}{2}-c}{c}2\mu^*_{010} \leq 1 - 2\frac{1-c}{\frac{1}{2}-c}\mu^*_{010} \leq \frac{\frac{1}{2}-c}{c}\left(3 + \frac{c}{\frac{1}{2}-c}\right)\mu^*_{010}$$

This system has at least one solution for all $c \in (0, 0.5)$. In particular, we can put

$$\mu^*_{000} = \mu^*_{001} = 0$$

$$\mu^*_{010} = \mu^*_{100} = c(1 - 2c)$$

$$\mu^*_{111} = 2c(1 - 2c)$$

$$\mu^*_{011} = \mu^*_{101} = 2c^2$$

$$\mu^*_{110} = 4c^2 - 4c + 1$$

One can verify that for this distribution, all original constraints hold, and the value of the objective function is two. Hence for any cost $0 < c < 0.5$, we can find a correlated equilibrium with expected turnout being exactly two out of three voters, i.e. twice the size of the minority. We will see shortly that this is a general property of the max-turnout correlated equilibria.

2.1.1.1 Max-turnout equilibria

Let us now turn to the general case. Recall that we want to solve the following problem for $0 < c < 1/2$:

$$\text{maximize } f(\mu) = \sum_{s \in \{0,1\}^n} \left(\mu(s) \sum_{i \in N} s_i \right) \tag{2.34}$$

$$\text{s.t.} \qquad \mu \in \mathcal{D}(N_A, N_B, c)$$

Let $f^* \equiv f(\mu^*)$ be the value of the objective at the optimum in (2.34). Our first

main result is the analytic solution to the max-turnout problem for all costs in the specified range.

Proposition 1. *Suppose $0 < c < 0.5$, $n_A, n_B \geq 1$, and $n_A > n_B$. Then the follow-ing[13] holds:*

(i) if $n_B \geq \lceil \frac{1}{2} n_A \rceil$, then $f^* = 2n_B$;

(ii) if $n_B < \lceil \frac{1}{2} n_A \rceil$, then

$$f^* = 2n_B + \frac{(n_A - 2n_B)(1 - 2c)}{1 + 2c\left(\frac{n_A(n_A-1)}{n_A+n_B(n_A-1)} - 1\right)} = 2n_B + \phi(c),$$

where $\phi(c) \in (0, n_A - 2n_B)$ and is decreasing in c. Alternatively, f^* can be expressed as

$$f^* = n_A \times \frac{2cn_B(n_A - 1) + n_B(n_A - 1) + n_A(1 - 2c)}{2c(n_A - n_B)(n_A - 1) + n_B(n_A - 1) + n_A(1 - 2c)} = n_A \times \xi(c)$$

where $\xi(c)$ is decreasing in c, and

a) $\xi(c) \in (0, 1)$ for all $0 < c < \frac{1}{2}$;

b) $\xi(c) \to \frac{2n_B}{n_A}$ as $c \to \frac{1}{2}$, so $f^* \to 2n_B$;

c) $\xi(c) \to 1$ as $c \to 0$, so $f^* \to n_A$.

Remark 1. The proof of Proposition 1 is in Appendix 2.A.2. Lemma 1 is fundamental in proving this result, allowing to establish the optimum and characterize the max-turnout equilibrium support under a group-symmetric distribution (see Corol-

---

[13]In terms of notation, $\lceil x \rceil$ stands for the smallest integer not less than x.

lary 1 below). The intuition for the result is as follows. To maximize turnout, the largest probability mass must be placed on the voting profile where everyone votes. However, since $n_A > n_B$, the *voting* players from N_B are not pivotal at this profile, so for those players constraint (2.6) binds at the optimum. This implies that constraint (2.5) for *abstaining* players in N_A binds at the optimum, because from (2.6) for players in N_B binding, the probability of the largest profile can be expressed via the probabilities of profiles where the *voting* players from N_B are pivotal, and those are precisely the profiles where *abstaining* players from N_A are pivotal. The key difference between cases (i) and (ii) only concerns the behavior of constraint (2.5) for players in N_B and constraint (2.6) for players in N_A. Using these binding constraints and the total probability constraint allows us to get a constructive characterization of the optimum.

Proposition 1 shows that all max-turnout correlated equilibria exhibit a substantial turnout of at least $2n_B$ for all common costs in the range where neither voting nor abstention is a dominant strategy, and for groups of different sizes. Max-turnout equilibria have a very natural interpretation: when the group sizes are so different that the minority have a priori low chances of winning even when the majority group votes at random (i.e., $n_B < \lceil \frac{1}{2} n_A \rceil$), the cost of voting matters and the maximal expected turnout is *decreasing* in cost. When the group size difference is not that large, the maximal expected turnout equals twice the size of the minority and does not depend on cost, as if voting was costless.

In addition to the maximal expected turnout, we also characterize the support of

the optimal group-symmetric distributions. Using Lemma 1, we can, without loss of generality, describe the profiles in the support as (a, b) where a (b) is the total number of voters from N_A (N_B, respectively) who turn out at this profile.

Corollary 1. *A correlated equilibrium with maximal expected turnout can be implemented via a group-symmetric distribution with the following support $\tilde{S} \subset S$:*

(i) if $n_B \geq \lceil \frac{n_A+1}{2} \rceil$, then

$$\tilde{S} = \left\{ (a, n_B) \in \mathbb{Z}^2 | a \in \{0, \ldots, n_B - 2\} \cup \{n_B, \ldots, n_A\} \right\};$$

(ii) if $n_B < \lceil \frac{1}{2} n_A \rceil$, then

$$\tilde{S} = \left\{ (a+1, a) \in \mathbb{Z}^2 | a \in \{0, \ldots, n_B\} \right\} \cup \{(n_B, n_B)\} \cup \{(n_A, 0)\}$$

Proof. See 2.A.2. ∎

In words, when $n_B > \lceil \frac{1}{2} n_A \rceil$, the equilibrium support consists of everyone in the minority voting except at the profile $(n_B - 1, n_B)$, and the majority mixing between all profiles. When $n_B < \lceil \frac{1}{2} n_A \rceil$, the support consists only of the profiles where the minority has exactly one vote less than the majority, the largest tied profile, and a single extreme profile with the full turnout by the majority and full abstention by the minority, $(n_A, 0)$.

Group-symmetric distributions allow to characterize the correlated equilibria with maximal expected turnout without loss of generality, but this characterization is

not unique: it is possible that an asymmetric probability distribution also delivers a solution to the max-turnout problem. However, the group-symmetric distribution has an attractive implementation property: all voters in a group are treated equally. Namely, one way to think about a group-symmetric correlated equilibrium is to imagine a mediator selecting a profile with a given total number of votes on each side according to the group-symmetric equilibrium distribution, μ^*, and then randomly recruiting the required number of voters on each side according to the selected profile, giving a recommendation to vote to those selected, and a recommendation to abstain to the rest. Thus the group-symmetric max-turnout equilibria involve interim randomization on the part of the mediator.

Remark 2. Based on the profiles that have positive probability in equilibrium, it is instructive to compare the correlated equilibria identified in case (i) with the mixed-pure Nash equilibria of Palfrey and Rosenthal (1983): indeed, according to Corollary 1, just like in those equilibria, voters in N_B should vote for sure, and voters in N_A should mix. The similarity ends here, however. First, the max-turnout mixed-pure Nash equilibria have expected turnout increasing in the cost. Second, in the mixed-pure equilibria of Palfrey and Rosenthal, all voters of the mixing group vote with the same probability $q \in (0, 1)$. Hence the probability of a profile (a, n_B) is $\binom{n_A}{a} q^a (1 - q)^{n_A - a}$. In the correlated equilibria from case (i), the probability of the same profile is $\binom{n_A}{a} \mu_{a, n_B}$, where μ delivers a maximum to the objective in (2.34). For the two probability distributions to coincide, it requires $\mu_{a, n_B} = q^a (1 - q)^{n_A - a}$ for all $a \in [0, n_A]$. But since $\binom{n_A}{n_B} \mu_{n_B, n_B} = 2c$ (see Corollary 2 below) and $\mu_{n_B - 1, n_B} = 0$, there is no $q \in (0, 1)$ that would satisfy this condition.

Remark 3. If one restricts the equilibrium support in case (i) to the following three profiles: full turnout, largest tie, and any single profile of the form (a, n_B) for $a \in \{0, \ldots, n_B - 2\}$, the group-symmetric max-turnout equilibrium is unique. This follows from equations (2.70) and (2.72) in 2.A.2. Our example in the introduction is a special case of this restricted equilibrium support with $a = 0$.

In view of Corollary 1, we can compute the probability that the election results in a tie, denoted π_{n_B, n_B}, since (n_B, n_B) is the only tied profile in the support of the equilibrium distribution. It is also interesting to see how the probability of the tie changes with the size of the electorate. There are several ways to model the limiting case when the electorate grows large. We present here the results for the simplest case, which is keeping the ratio $\frac{n_B}{n_A} = \alpha$ fixed at some $\alpha \in (0, 1]$ as $n_B, n_A \to \infty$.

Corollary 2. *(i) if $n_B \geq \lceil \frac{n_A + 1}{2} \rceil$, then*

$$\pi_{n_B, n_B} = 2c$$

(ii) if $n_B < \lceil \frac{1}{2} n_A \rceil$, then

$$\pi_{n_B, n_B} = \frac{2c}{1 + \left(\frac{1}{2c} - 1\right)\left(\frac{1}{n_A - 1} + \frac{n_B}{n_A}\right)}$$

(iii) for any fixed c, as $n_A, n_B \to \infty$ with $\frac{n_B}{n_A} = \alpha \in (0, 1)$, for $\alpha \in (0, 0.5)$ we have $\pi_{n_B, n_B} \to \frac{2c}{1 + \alpha\left(\frac{1}{2c} - 1\right)}$, and for $\alpha \in (0.5, 1)$, $\pi_{n_B, n_B} \to 2c$.

Proof. See equations (2.71) and (2.93) in 2.A.2. ∎

Corollary 2 shows that the probability of the tied outcome only depends on the cost and the relative size of the competing groups, and is increasing in the cost. There is one caveat: the tie probability is derived under the assumption of a group-symmetric probability distribution. For an asymmetric probability distribution that also delivers a solution to the max-turnout problem, Corollary 2 holds as long as the equilibrium support stays the same.

Another important property concerns the probability that the majority wins. Given Corollaries 1 and 2, it is not surprising that there are again two cases for the max-turnout equilibria:

Corollary 3. *The probability the majority wins in a correlated equilibrium with maximal expected turnout, π_m, is restricted as follows.*

(i) if $n_B \geq \lceil \frac{n_A+1}{2} \rceil$, then

$$1 - c \geq \pi_m > \frac{1}{2}$$

For the special case in Remark 3,

$$\pi_m = \frac{n_B}{n_A} + c \left(1 - \frac{2n_B}{n_A} \right)$$

(ii) if $n_B < \lceil \frac{1}{2} n_A \rceil$, then

$$\pi_m = 1 - \frac{c}{1 + \left(\frac{1}{2c} - 1 \right) \left(\frac{1}{n_A - 1} + \frac{n_B}{n_A} \right)}$$

Proof. See 2.A.3. ■

Corollary 3 shows that the probability that majority wins is decreasing in the cost for a small minority (case (ii)). As $c \to 0.5$, $\pi_m \to 0.5$ from above. Furthermore, for all costs in $(0, 0.5)$ the majority wins with probability at least 0.5. In case (i), when $n_B \geq \lceil \frac{n_A+1}{2} \rceil$, the upper bound on this probability is decreasing in the cost, but the situation is a bit more complicated, since π_m is non-monotone in the cost for a fixed pair of groups sizes n_A and n_B. The reason is the non-monotone behavior of the binomial coefficients as well as the sensitivity of the linear program to the changes in the constraint coefficients. The total probability mass fluctuates along the profiles of the form (a, n_B) for $a \in \{0, \ldots, n_B - 2\} \cup \{n_B, \ldots, n_A\}$ depending on the cost, and so does the probability of the majority winning.

Our next proposition shows that as the size of the electorate grows large, the max-turnout correlated equilibria remain divided into the same two categories: the cost-independent case with the maximal expected turnout being twice the size of the minority, and the cost-dependent case, where the maximal expected turnout includes an additional term.

Proposition 2. *Fix $c \in (0, 0.5)$ and let $n_A, n_B \to \infty$ with $\frac{n_B}{n_A} = \alpha \in (0, 1]$.*

(i) If $\alpha \geq 0.5$, then

$$\lim_{n_A, n_B \to \infty} \frac{f^*}{n} = \frac{2\alpha}{1 + \alpha}$$

(ii) If $\alpha < 0.5$, then

$$\lim_{n_A, n_B \to \infty} \frac{f^*}{n} = \frac{2\alpha}{1 + \alpha} + \frac{(1 - 2\alpha)(1 - 2c)}{(1 + \alpha)(1 - 2c\left(1 - \frac{1}{\alpha}\right))}$$

Proof. See 2.A.4. ■

2.1.1.2 Min-turnout equilibria

Concluding the section on the basic model, let us briefly address the lower bound
on the expected turnout. This case is different in that now we are looking for a
solution that minimizes the linear objective function subject to the same constraints
(2.5)-(2.6).

Denote the minimal expected turnout in this problem by

$$f_* \equiv f(\mu_*) = \min_{\mu \in \mathcal{D}(N_A, N_B, c)} \sum_{s \in \{0,1\}^n} \left(\mu(s) \sum_{i \in N} s_i \right) \tag{2.35}$$

Proposition 3. *Suppose* $0 < c < 0.5$, *and* $n_A, n_B \geq 1$. *Then* $f_* = 2 - \psi(c)$, *where*
$\psi(c) \in (0, 2)$.

Proof. See 2.A.5. ■

As Proposition 3 shows, the lower turnout bound is not very interesting. For all
cases, the minimal expected turnout is between 0 and 2, depending on the cost,
and the exact formula for $\psi(c)$ is complicated, since, unlike the maximum case, the
equilibrium distribution support also depends on the cost, as shown in the Appendix.
On the other hand, the result is intuitive: the minimum turnout case is total cost-
minimizing, so to remove the individual incentives to turn out it is sufficient to
have the equilibrium distribution place all the probability mass onto the uncontested

profiles where either side wins for sure. Such profiles need no more than two agents voting.[14]

2.1.1.3 Correlated equilibria and efficiency

In this section we rely on the results we have obtained in the basic model to draw some general implications about the effects of correlated strategies on welfare.

Firstly, we note that since the set of expected correlated equilibrium payoffs is convex, there is always an equilibrium with the total expected turnout between the minimum and the maximum.

Proposition 4. *For any $0 < c < 0.5$ and $t \in [f_*(c), f^*(c)]$, there exists a correlated equilibrium with the total expected turnout equal to t.*

Proof. See 2.A.6. ∎

Next, we ask which correlated equilibria are socially optimal. That is, we are looking for equilibria that maximize expected social welfare, understood as a sum of all individuals' expected utilities. Given a correlated equilibrium μ, after some simple algebra, the expected welfare can be formally written as follows.

$$W(\mu) = (n_A - n_B)\Pr(majority\ wins) + n_B - cT(\mu) \tag{2.36}$$

---

[14]There is an exception to this rule when the voting cost is approaching zero, but even if profiles with total turnout larger than 2 have positive probabilities in equilibrium, their effect on the objective is completely compensated by the profiles with turnout between 0 and 2. See 2.A.5 for details.

where $T(\mu)$ is the total expected turnout under μ. The expression in (2.36) nicely demonstrates the relation between total expected turnout and welfare: increasing total turnout reduces welfare if the probability that majority wins is kept constant, but it may increase welfare if the increased turnout leads to a higher probability that majority wins.

Given our results on max turnout equilibria in Section 2.1.1.1, we can now establish some welfare properties of such equilibria.

Proposition 5. *Suppose $0 < c < 0.5$ and $n_A > n_B$. Denote W^* the expected welfare at a max-turnout equilibrium.*

(i) if $n_B \geq \lceil \frac{n_A+1}{2} \rceil$, then $W^* = (n_A - n_B)\Pr(majority\ wins) + n_B(1 - 2c)$; and $\frac{n_A+n_B}{2} - 2cn_B < W^* \leq n_A - c(n_A + n_B)$;

(ii) if $n_B < \lceil \frac{1}{2}n_A \rceil$, then

$$W^* = n_A(1 - c)\left(1 + \frac{2cn_B(n_A - 1)}{2c(n_A - n_B)(n_A - 1) + n_B(n_A - 1) + n_A(1 - 2c)}\right);$$

(iii) In both cases, W^* is decreasing in the voting cost.

Correlated equilibria that maximize total welfare have lower expected turnout than the max-turnout equilibria. A welfare-maximizing correlated equilibrium would require the probability that majority wins as large as possible (ideally, equal to 1) and turnout as low as possible (ideally, 0). In this case the maximum welfare equals n_A. However, there is a tradeoff between the probability majority wins and the expected turnout: majority cannot win for sure in any correlated equilibrium.

Lemma 2. *For any $0 < c < \frac{1}{2}$, there does not exist a correlated equilibrium with majority winning for sure.*

Proof. See 2.A.7. ∎

Remark 4. It is interesting to note that if voting costs are different in different groups, it *is* possible to have a correlated equilibrium with majority winning for sure. In particular, if there are two group costs, c_A and c_B, then for $c_A < c_B$ both IC constraints for voters in N_A and non-voters in N_B can be satisfied. The welfare-maximizing equilibria in such case have the probability majority wins equal to one, and all probability mass on the profiles with one and two voters from N_A and zero voters from N_B.

When looking for a welfare-maximizing correlated equilibrium, Lemma 2 implies that the probability majority wins enters (2.36) non-trivially and must be traded off with the total expected turnout. Similarly to Lemma 1, there is no loss of generality involved from considering only group-symmetric probability distributions. We can now establish the equilibrium support for welfare-maximizing equilibria, and characterize the optimum. Formally, the problem is now

$$\text{maximize } W(\mu) \text{ s.t. } \mu \in \mathcal{D}(N_A, N_B, c) \tag{2.37}$$

Proposition 6. *Assume $n_A > 2$.*

i) There is a unique cutoff cost c_* such that for any $0 < c < c_*$ the maximal expected

welfare implementable in a correlated equilibrium is

$$W(\mu^*, c) = n_A - c + \frac{\left[c - \frac{n_A + n_B + 2n_B(\frac{1}{2}-c)}{2} - \frac{\left(\frac{1}{2}-c\right)^2(1+n_B)}{c}\right]}{\frac{(c+\frac{1}{2}(n_B+1))(\frac{1}{2}-c)}{c^2} + \frac{n_B}{2c} + 1}$$

and the corresponding equilibrium support profiles are $(a+1, a), a \in [0, n_B]$, (n_B, n_B), *and* $(2, 0)$.

ii) for $c > c_*$ *such that Condition A (see below) holds, the maximal expected welfare implementable in a correlated equilibrium is*

$$\tilde{W}(\mu^*, c) = n_A - c + \frac{\left[c(1 + n_B) + n_B[n_B - n_A - 1] - \frac{\left(\frac{1}{2}-c\right)^2(1+n_B)}{c}\right]}{\frac{n_B - (c+\frac{1}{2})}{\frac{1}{2}-c} + \frac{(c+\frac{1}{2}(n_B+1))(\frac{1}{2}-c)}{c^2}}$$

and the corresponding equilibrium support profiles are $(a + 1, a), a \in [0, n_B]$, $(0, 1)$, *and* $(2, 0)$.

iii) for $c > c_*$ *such that Condition A does not hold, the maximal expected welfare implementable in a correlated equilibrium is*

$$\bar{W}(\mu^*, c) = n_A - c + \frac{\left(\frac{1}{2} - c\right)[n_B(n_B - n_A) - c(n_B - 1)]}{n_B - \left(c + \frac{1}{2}\right)}$$

and the corresponding equilibrium support profiles are $(0, 1), (1, 0),$ *and* $(2, 0)$.

Proof. See 2.A.8 ∎

Remark 5. The unique cutoff cost c_* is determined by equation (2.113) in the proof.

Condition A in the statement of Proposition 6 is the following cubic inequality in the voting cost:

$$c^3 \left(n_A + \frac{n_B - 5}{2} \right) + \frac{c^2}{2} \left((n_A - n_B)(n_B - 1) + 3 - n_B \right)$$
$$- \frac{c}{4} \left(\frac{n_B + 1}{2} + (n_A - n_B)(2n_B + 1) \right) + \frac{(n_A - n_B)(n_B + 1)}{8} > 0$$

This inequality is equivalent to having $\tilde{W}(\mu^*, c) > \bar{W}(\mu^*, c)$.

Proposition 6 characterizes welfare-optimal equilibria and shows that those are generally different from either min- or max-turnout equilibria, although the expected turnout in the welfare-maximizing case is close to the minimal expected turnout.

2.2 Complete Information and Heterogeneous Voting Costs

We have assumed so far that the cost of voting is common for all players. This assumption may seem too strong, so in this section we are going to relax it and see if the main results continue to hold.

Assume that each voter $i \in N$ has a voting cost $c_i \in (0, 0.5)$ and the costs are commonly known. In this cost range, no voter has a dominant strategy to always

vote or always abstain. The correlated equilibrium conditions (2.5)-(2.6) now take the following form: $\forall i \in N$,

$$c_i \sum_{s_{-i} \in V_D^i} \mu(0, s_{-i}) + \left(c_i - \frac{1}{2}\right) \sum_{s_{-i} \in V_P^i} \mu(0, s_{-i}) \geq 0 \qquad (2.38)$$

$$-c_i \sum_{s_{-i} \in V_D^i} \mu(1, s_{-i}) + \left(\frac{1}{2} - c_i\right) \sum_{s_{-i} \in V_P^i} \mu(1, s_{-i}) \geq 0 \qquad (2.39)$$

where, as before, V_P^i (V_D^i) is the set of voting profiles where player i is a pivotal(dummy, respectively). Denote $\mathcal{D}(N_A, N_B, (c_i)_{i \in N})$ the set of probability distributions over $\Delta(S)$ that satisfy (2.38)-(2.39).

With heterogeneous costs, the group-symmetric distribution construction (see Lemma 1), may entail some loss of generality. Since voting costs are different, the expected turnout can be increased, compared to the group-symmetric case, if the probability distribution over profiles is adjusted so that each profile probability takes into account not only the total number of those players voting at this profile, but also their voting costs. E.g., profiles where players with higher costs are voting might be optimally assigned smaller probability than profiles with the same total turnout, but where players with lower costs are voting.[15]

Without loss of generality, let us order all players in group N_A $(N_B$, respectively) by their voting costs from low to high. Denote $\underline{c}_A$, $\underline{c}_B$ the lowest costs in the respective groups. Similarly, denote $\bar{c}_A$, $\bar{c}_B$ the highest costs. A joint cost profile $\boldsymbol{c}_{[\underline{c}_A, \bar{c}_A, \underline{c}_B, \bar{c}_B]}$

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[15]Nevertheless, there is an important special case with two common group costs, c_A and c_B, where one can prove an analog of Lemma 1. We do not analyze it here.

is any cost assignment $(c_i)_{i \in N}$ to the players in N such that $\forall i \in N_j$, $j \in \{A, B\}$, $\underline{c}_j \leq c_i \leq \bar{c}_j$. Denote the maximal expected turnout in the turnout problem with heterogeneous costs by

$$h^* \equiv f(\mu^*) = \max_{\mu \in \mathcal{D}(N_A, N_B, (c_i)_{i \in N})} \sum_{s \in \{0,1\}^n} \left(\mu(s) \sum_{i \in N} s_i \right) \tag{2.40}$$

In the present version of the manuscript, we restrict our analysis to the case of symmetric distributions and demonstrate that our results under homogenous costs can be replicated as a special case. The main goal of this exercise is to show that the maximal expected turnout remains at high levels under heterogeneous costs, even if the set of admissible probability distributions is restricted to be symmetric.

2.2.1 Symmetric distributions

In this subsection, we require the probability distributions to be group-symmetric. Analogously to Lemma 1, define

$\mathcal{M}_H := \{\mu \in \mathcal{D}(N_A, N_B, (c_i)_{i \in N}) |$

$\qquad \forall i \in N_A, \forall b \in \{0, \ldots, n_B\}, \forall a \in \{1, \ldots, n_A - 1\} : \mu(0_i, a, b) = \mu(1_i, a - 1, b)$

$\qquad \forall k \in N_B, \forall b \in \{1, \ldots, n_B - 1\}, \forall a \in \{0, \ldots, n_A\} : \mu(0_k, a, b) = \mu(1_k, a, b - 1)\}$

In words, $\mathcal{M}_H$ is the set of group-symmetric probability distributions over joint profiles which are also correlated equilibria for complete information and heterogeneous

costs. Denote the maximal expected turnout in the turnout problem with heterogeneous costs and group-symmetric distributions by

$$\tilde{h}^* := \max_{\mu \in \mathcal{M}_H} \sum_{s \in \{0,1\}^n} \left(\mu(s) \sum_{i \in N} s_i \right) \tag{2.41}$$

Clearly, $h^* \geq \tilde{h}^*$. We will now show that an analog of Proposition 1 holds under the condition $\underline{c}_A = \bar{c}_B$.

Proposition 7. *Suppose $0 < c_i < 0.5$ for all $i \in N$. Require $\mu \in \mathcal{M}_H$. Then the following expressions for $\tilde{h}^*$ provide the optimal value to the objective in the max turnout problem with heterogeneous costs and group-symmetric distributions if and only if $\underline{c}_A = \bar{c}_B = c$ and*

(i) $n_B > \lceil \frac{1}{2} n_A \rceil$, with $\tilde{h}^* = 2 n_B$;

(ii) $n_B < \lceil \frac{1}{2} n_A \rceil$, with

$$\tilde{h}^* = n_A \times \frac{2\bar{c}_A n_B (n_A - 1) + n_B (n_A - 1) + n_A (1 - 2c)}{2\bar{c}_A [n_A - n_B](n_A - 1) + n_B (n_A - 1) + n_A (1 - 2c)}$$
$$= n_A \times \xi(c, \bar{c}_A)$$

where $\xi(c, \bar{c}_A)$ is decreasing in both c and $\bar{c}_A$, and

a) $\xi(c, \bar{c}_A) \in (0,1)$ for all $0 < c \leq \bar{c}_A < \frac{1}{2}$;

b) $\xi(c, \cdot) \to \frac{2n_B}{n_A}$ as $c \to \frac{1}{2}$, so $\tilde{h}^* \to 2n_B$;

c) $\xi(\cdot, \bar{c}_A) \to 1$ as $\bar{c}_A \to 0$, so $\tilde{h}^* \to n_A$.

Furthermore, $2n_B < \tilde{h}^* < n_A$.

Proof. See 2.A.9.   ∎

Proposition 7 is our second main result. It shows that the maximal expected turnout under correlated equilibria and group-symmetric distributions behaves similarly to the case of a single voting cost, and essentially depends on two things: the relative sizes of the groups and the bounds of the support of the cost distribution. The intuition for the result is similar to Proposition 1. Maximizing turnout implies that constraint (2.39) for players in N_B binds at the optimum. This in turn implies that constraint (2.38) for players in N_A binds at the optimum. Now the binding constraint (2.39) for players in N_B crucially depends on $\bar{c}_B$, because once it holds for the voters with the highest costs in group N_B, it automatically holds for voters in N_B with lower costs. On the other hand, the binding constraint (2.38) for players in N_A crucially depends on $\underline{c}_A$, because once it holds for the voters with the lowest costs in group N_A, it automatically holds for voters in N_A with higher costs. The effects of the two constraints cancel each other out if and only if $\underline{c}_A = \bar{c}_B$. Once this condition holds, the key difference between cases (i) and (ii) under symmetric distributions only concerns the behavior of constraint (2.38) for players in N_B and constraint (2.39) for players in N_A, just like in Proposition 1.

In the proof of Proposition 7 we show that when $\underline{c}_A = \bar{c}_B$, the equilibrium distribution support is the same as in Proposition 1, so Corollary 1 holds without change. For the sake of completeness let us also provide here the expressions for the probability

of the largest tie, π_{n_B, n_B}. The only change from Corollary 2 concerns the case of small minority.

Corollary 4. *Suppose $n_A > n_B \geq 1$, $0 < c_i < 0.5$ for all $i \in N$, and $\underline{c}_A = \bar{c}_B = c$. Assuming symmetric distributions,*

(i) if $n_B > \lceil \frac{1}{2} n_A \rceil$ then

$$\pi_{n_B, n_B} = 2c$$

(ii) if $n_B < \lceil \frac{1}{2} n_A \rceil$, then

$$\pi_{n_B, n_B} = \frac{2}{\frac{1}{c}\left[1 + \frac{1}{2\bar{c}_A(n_A-1)} + \frac{n_B}{n_A}\left(\frac{1}{2\bar{c}_A} - 1\right)\right] - \frac{1}{\bar{c}_A(n_A-1)}}$$

Proof. See equations (2.130) and (2.151) in the proof of Proposition 7 in 2.A.9. ∎

Notice that if $c = \bar{c}_A$, the expression for case (ii) coincides with its analog in Corollary 2.

What happens when $\underline{c}_A \neq \bar{c}_B$? In Appendix 2.A.9 we show that if $\bar{c}_B < \underline{c}_A$, then the maximal expected turnout exceeds the value of $\tilde{h}^*$ for both cases of Proposition 7 and for any admissible combination of the other cost thresholds. At first sight this might look counterintuitive: $\bar{c}_B < \underline{c}_A$ implies that the majority group find it costlier to vote than the minority group, so they should vote less. However, the higher voting cost of the majority group also implies that it will be easier to satisfy their IC constraints for abstention, as well as the minority group IC constraints for voting. Thus in the group-symmetric max-turnout correlated equilibrium, the competitive profiles with

higher total turnout will be assigned higher probabilities, producing higher expected turnout. As $\underline{c}_B \to \frac{1}{2}$, $\bar{c}_B \to \underline{c}_A$, so the maximal expected turnout converges to $\tilde{h}^*$ from above. Similarly, when $\bar{c}_B > \underline{c}_A$, the maximal expected turnout is lower than the value of $\tilde{h}^*$ for both cases of Proposition 7. Nevertheless, as $\min\{\underline{c}_A, \underline{c}_B\} \to \frac{1}{2}$, $\underline{c}_A \to \bar{c}_B$, so the maximal expected turnout converges to $\tilde{h}^*$ from below. Therefore, the result of Proposition 7 is, in a sense, a limiting case when the lowest cost threshold increases towards $\frac{1}{2}$ and symmetric distributions are assumed.

One can also imagine the case where some voters have costs greater than $\frac{1}{2}$ or less than 0. These cases are not very interesting from the analysis point of view: if voter i has a dominant strategy to abstain due to $c_i > \frac{1}{2}$ (violating constraint (2.39) for any probability distribution that places a positive probability on profiles with i voting), her presence in the list of players does not affect at all the outcome of the election, so we can redefine $N \equiv N \setminus \{i\}$. A more elaborate way to handle this problem requires the use of an asymmetric probability distribution, which would distinguish i from the other players in her group and assign probability zero to all profiles with i voting. We do not fully analyze this case, but we conjecture that allowing for high-cost voters will not substantially change our results.

If voter i has a dominant strategy to vote due to $c_i < 0$, then simply removing this voter results in a loss of generality. The case of negative costs requires some special handling, but it is tractable in our framework. First of all, without additional assumptions about the distribution of such costs across groups, one can nevertheless argue that, under the veil of ignorance, voters with negative costs are just as likely

to belong to either of the groups, so we would expect their votes to cancel each other out. Notwithstanding this argument, we would like to consider the case of negative costs for some voters for the following reasons. First, it suggests a turnout model that incorporates some additional factors, like citizen duty, which may be important for some voters. Second, we need to consider the negative costs to be able to directly compare our results with Palfrey and Rosenthal (1985), who in their Assumption 2 explicitly include them. It is important to understand whether we get a high-turnout equilibria due to our solution concept being the correlated equilibrium, or due to a different assumption about the cost support.

Let $\mathcal{L} \subset N$ be the set of voters with (strictly) negative costs. We restrict the set of admissible joint distributions to those that place probability zero on voters in $\mathcal{L}$ receiving a recommendation to abstain and probability one on voters in $\mathcal{L}$ receiving a recommendation to vote. With this modification, we can replace the actual group sizes, n_A and n_B with their modified versions, $\tilde{n}_A$ and $\tilde{n}_B$, which take into account the voters from $\mathcal{L}$ so that $\tilde{n}_A = n_A - \mathcal{L}_A$ and $\tilde{n}_B = n_B - \mathcal{L}_B$. This is as if the actual group sizes are shifted by a constant. It is clear that our results hold for the modified game.

2.3   Incomplete Information

Incomplete information in the voter turnout game was introduced by Ledyard (1981), and further explored in Palfrey and Rosenthal (1985). Under incomplete information,

Palfrey and Rosenthal (1985, Theorem 2) established that in the quasi-symmetric Bayesian Nash equilibrium only voters with non-positive voting costs will vote in the limit as n_A, n_B get large. There are several ways to introduce the incomplete information into the basic model, but not all of them are suitable for the analysis of high-turnout correlated equilibria. In this section we consider the simplest version.

In general, player i's *type* is a pair (t_i, c_i) of her political type (candidate preference) and the corresponding cost of voting. The political type directly affects the utilities of all voters through the resulting split into majority and minority, but the voting cost type only affects the utility of a specific player. In this section we assume, for simplicity, that voters' political types are common knowledge.[16] We use t to denote the fixed commonly known joint political type where each voter i has political type t_i. The costs of voting are stochastic: each voter $i \in N$, draws her private cost of voting, c_i, from a commonly known *discrete*[17] distribution F_{t_i} with support $\{\underline{c}_{t_i}, \ldots, \bar{c}_{t_i}\}$, where $0 < \underline{c}_{t_i} \leq \frac{1}{2}$ and $0 < \bar{c}_{t_i} < 1$. The assumption about the support range helps rule out uninteresting equilibria, e.g. those with everyone voting for sure, or those with everyone abstaining for sure. We assume c_i is distributed independently of all other voters' costs c_{-i} (and types t_{-i}). Distributions F_A and F_B determine the set of admissible joint cost profiles, characterized by the tuple of respective cost bounds

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[16]This is a strong assumption. There are ways of relaxing it (Myerson, 1998, 2000), but they are inconsistent with the variant of the incomplete information correlated equilibrium we consider in this Chapter. We conjecture that this assumption can be relaxed in a communication equilibrium (Myerson, 1986; Forges, 1986), and leave it for future research.

[17]Typically it is assumed in the literature that the cost distributions are absolutely continuous. We do not make this assumption to avoid dealing with measurability issues in the definition of a strategic form correlated equilibrium below. See Cotter (1991) for a detailed discussion of these issues.

$(\underline{c}_A, \bar{c}_A, \underline{c}_B, \bar{c}_B)$ as

$$\mathcal{C}_{(\underline{c}_A, \bar{c}_A, \underline{c}_B, \bar{c}_B)} \equiv \{(c_i)_{i \in N} | \underline{c}_{t_i} \leq c_i \leq \bar{c}_{t_i}\} \tag{2.42}$$

We write $\mathcal{C}^{-i}_{(\underline{c}_A, \bar{c}_A, \underline{c}_B, \bar{c}_B)}$ to refer to the set of admissible cost profiles for players other than i. Denote $\pi(c)$ the probability of a joint cost profile $c = ((c_i)_{i \in N}) \in \mathcal{C}_{(\underline{c}_A, \bar{c}_A, \underline{c}_B, \bar{c}_B)}$. The independently distributed costs then imply that

$$\pi(c) \equiv \left(\prod_{\{i \in N : t_i = A\}} F_A(c_i) \right) \left(\prod_{\{i \in N : t_i = B\}} F_B(c_i) \right)$$

Since the political types are fixed by assumption, we omit the respective component in the definition of players' strategies and for each $i \in N$ define a pure strategy $s_i : \{\underline{c}_{t_i}, \ldots, \bar{c}_{t_i}\} \to \{0, 1_A, 1_B\}$ as a function that maps voter i's cost into an action (abstain, vote for candidate A, or vote for candidate B, respectively). We assume that voters never vote for the candidate of the opposite political type, so we abuse notation and merge 1_A and 1_B into 1 meaning the act of voting for the "correct" candidate. The set of all pure strategies for player $i \in N$ is a *finite* set $\mathcal{S}_i = \{0, 1\}^{\{\underline{c}_{t_i}, \ldots, \bar{c}_{t_i}\}}$, i.e. the set of all functions from cost types into actions. Let $\mathcal{S} \equiv \times_{i \in N} \mathcal{S}_i$ be the set of all joint strategies.

The utility of player i from a joint strategy $s(c) \equiv (s_j(c_j)_{j \in N})$ when player i's voting cost is c_i (and the joint political type is t) takes the following form:

$$
u_i(s(c)|c_i) = \begin{cases} 1 - s_i(c_i)c_i & \text{if} \displaystyle\sum_{\{j \in N | t_j = t_i\}} s_j(c_j) > \sum_{\{j \in N | t_j \neq t_i\}} s_j(c_j) \\[3ex] \frac{1}{2} - s_i(c_i)c_i & \text{if} \displaystyle\sum_{\{j \in N | t_j = t_i\}} s_j(c_j) = \sum_{\{j \in N | t_j \neq t_i\}} s_j(c_j) \\[3ex] -s_i(c_i)c_i & \text{if} \displaystyle\sum_{\{j \in N | t_j = t_i\}} s_j(c_j) < \sum_{\{j \in N | t_j \neq t_i\}} s_j(c_j) \end{cases}
$$

Let us now discuss the solution concept. There are quite a few alternative definitions of the correlated equilibrium in games with incomplete information (see in particular Forges (1993, 2006, 2009), Section 8.4 of Bergemann and Morris (2013) and Milchtaich (2013)), which are often far from being equivalent. The sets of expected payoffs corresponding to specific definitions are (sometimes) partially ordered by inclusion. We use the strategic form incomplete information correlated equilibrium, as defined in Forges (1993, 2006). This is the strongest definition in the sense that it results in the smallest set of expected payoffs compared, for example, to the communication equilibrium (Myerson (1986), Forges (1986)). Hence if we can obtain a substantial turnout in the strategic form correlated equilibrium, then we can also obtain it in any of the more general definitions of the correlated equilibrium under incomplete information.

A *strategic form incomplete information correlated equilibrium*(SFIICE) is a probability distribution $q \in \Delta(S)$ that selects a pure strategy profile $s = (s_i)_{i \in N}$ with probability $q(s)$, such that when recommended s_i and knowing her type, no player has an incentive to deviate, given that other players follow their recommendations.

Formally, $q \in \Delta(S)$ is a SFIICE if for all $i \in N$, all $c_i \in \{\underline{c}_{t_i}, \ldots, \bar{c}_{t_i}\}$, all $a_i \in \{0, 1\}$, and any $s_i \in \mathcal{S}_i$ such that $s_i(c_i) = a_i$, we have

$$\sum_{\left\{c_{-i} \in \mathcal{C}^{-i}_{(\underline{c}_A, \bar{c}_A, \underline{c}_B, \bar{c}_B)}\right\}} \pi(c) \sum_{a_{-i}} \left(\sum_{\{s_{-i}(c_{-i}) = a_{-i}\}} q(s_i, s_{-i})\right) \left[u_i(a_i, a_{-i}) - u_i(a_i', a_{-i})\right] \geq 0$$

for all $a_i' \in \{0, 1\}$.

It will be convenient to explicitly rewrite these conditions as the set of distributions $q \in \Delta(S)$ such that for all $i \in N$, all $c_i \in \{\underline{c}_{t_i}, \ldots, \bar{c}_{t_i}\}$, and all $s_i \in \mathcal{S}_i$ such that $s_i(c_i) = 0$ we have

$$\sum_{c_{-i}} \pi(c) \left[c_i \sum_{a_{-i} \in V_D^i} \left(\sum_{\{s_{-i}(c_{-i}) = a_{-i}\}} q(s_i, s_{-i}|s_i(c_i) = 0)\right) \right.$$
$$\left. + \left(c_i - \frac{1}{2}\right) \sum_{a_{-i} \in V_P^i} \left(\sum_{\{s_{-i}(c_{-i}) = a_{-i}\}} q(s_i, s_{-i}|s_i(c_i) = 0)\right) \right] \geq 0 \qquad (2.43)$$

and for all $s_i \in S_i$ such that $s_i(c_i) = 1$ we have

$$\sum_{c_{-i}} \pi(c) \left[-c_i \sum_{a_{-i} \in V_D^i} \left(\sum_{\{s_{-i}(c_{-i}) = a_{-i}\}} q(s_i, s_{-i}|s_i(c_i) = 1)\right) \right.$$
$$\left. + \left(\frac{1}{2} - c_i\right) \sum_{a_{-i} \in V_P^i} \left(\sum_{\{s_{-i}(c_{-i}) = a_{-i}\}} q(s_i, s_{-i}|s_i(c_i) = 1)\right) \right] \geq 0 \qquad (2.44)$$

where, as before, V_P^i and V_D^i are the set of joint action profiles such that player i is pivotal and dummy, respectively, and the summation over the others' costs is understood to be over cost profiles in $\mathcal{C}^{-i}_{(\underline{c}_A, \bar{c}_A, \underline{c}_B, \bar{c}_B)}$. The induced probability distribution over action profiles at every cost profile $c \in \mathcal{C}_{(\underline{c}_A, \bar{c}_A, \underline{c}_B, \bar{c}_B)}$ is given by

$$\nu(a|c) \equiv \sum_{\{s \in S | \forall i \in N : s_i(c_i) = a_i\}} q(s) \tag{2.45}$$

The max-turnout problem under incomplete information now takes the following form:

$$g^* \equiv \max_{q \in \mathcal{D}(N_A, N_B, F_A, F_B)} \sum_{\left\{c \in \mathcal{C}_{(\underline{c}_A, \bar{c}_A, \underline{c}_B, \bar{c}_B)}\right\}} \pi(c) \left(\sum_{a \in \{0,1\}^n} \nu(a|c) \left(\sum_{i \in N} a_i \right) \right) \tag{2.46}$$

Full characterization of the solution to this problem is not our goal in this section. Rather, we just want to show a possibility result, that correlated equilibria with substantial turnout can survive in the incomplete information case. The next proposition delivers the desired result.

Proposition 8. *Suppose $n_A, n_B \geq 1$ and $n_A > n_B$. Let F_A, F_B be any discrete distributions over players' voting costs, $\{\underline{c}_A, \ldots, \bar{c}_A\}$, and $\{\underline{c}_B, \ldots, \bar{c}_B\}$, respectively, such that $\bar{c}_B \leq \underline{c}_A \in (0, 0.5)$, $0 < \bar{c}_A < 0.5$, and $0 < \underline{c}_B < 0.5$. Then $g^* \geq \tilde{h}^*$, where $\tilde{h}^*$ is defined in (2.41).*

Proof. See 2.A.10. ∎

This result holds for large electorates as well.[18]

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[18]In particular, we mean the case where the ratio between group sizes remains fixed as their sizes increase to ∞, with fixed cost supports.

2.4 Discussion

Since Nash equilibria are also correlated equilibria, it is important to understand what exactly the analysis of correlated equilibria adds to the existing results in the literature.

Under complete information and common voting cost, this Chapter extends Palfrey and Rosenthal (1983), who characterized two classes of the Nash equilibria that exhibit substantial turnout and survive when the electorate becomes large.[19] Palfrey and Rosenthal call those *mixed-pure strategy equilibria* and *symmetric totally-mixed strategy equilibria*, respectively. The former equilibria require all voters in one group mixing between voting and abstention with some common probability, whereas voters in the other group are further divided into two subgroups such that all voters in one subgroup vote for sure, and all voters in the other subgroup abstain for sure. The latter equilibria require voters in each group mixing with the same group-specific probability. Both of these equilibrium classes have a counterintuitive property: the expected turnout is *increasing* in cost. Furthermore, symmetric totally-mixed equilibria only exist when the cost is large enough and both groups have the same size. This unfortunate dependence on both groups having exactly the same size translates directly into the incomplete information case, and, in a sense, is the primary reason why no high-turnout equilibria survive even slightest uncertainty in Palfrey and Rosenthal (1985) when the electorate size gets large. The corresponding result in

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[19]This latter criterion is important: Palfrey and Rosenthal (1983) identify many other equilibria that have nice properties, but do not survive in large electorates.

this Chapter (see Proposition 1) has neither of these shortcomings.

Under heterogeneous voting costs, we can compare our Proposition 7 with Taylor and Yildirim (2010, Proposition 2). They find that under incomplete information, in large electorates the limit expected turnout and the probability of winning are completely determined by the lowest voting costs in each group. In contrast, the max-turnout correlated equilibrium puts a joint restriction both on the lowest voting cost in one group and the highest voting cost in the other. This is the effect of two opposing incentive compatibility constraints. In a quasi-symmetric Bayesian Nash equilibrium in cutpoint strategies, which is typically considered in the literature, the two constraints for each group merge into one at the critical cost. Another related result is Kalandrakis (2007), who proves that under complete information and heterogeneous costs almost all Nash equilibria are regular, and there exists at least one monotone Nash equilibrium, where players with higher costs participate with weakly lower probabilities. In our group-symmetric max-turnout correlated equilibria a similar logic allows to restrict attention to the lowest and highest costs in each group.

Under incomplete information, we extend Palfrey and Rosenthal (1985). Their high-turnout equilibria do not survive uncertainty when the electorate size gets large. In contrast, our high-turnout correlated equilibria persist under certain conditions on cost supports (see Proposition 8). This result can be also compared with Kalandrakis (2009), who basically shows that high-turnout Nash equilibria of the complete information game with a common positive cost can persist under incomplete information.

Assuming that densities of the private voting cost, private benefit, or both, converge to a point mass that corresponds to a complete information turnout game with a positive common voting cost, Kalandrakis (2009, Theorem 4) permits introducing incomplete information with respect to individual voting cost, the size of each candidate's support, or both. The crucial difference from Palfrey and Rosenthal (1985)'s negative result on high-turnout equilibria under incomplete information is that Kalandrakis holds the size of the electorate fixed, and varies the uncertainty level, while Palfrey and Rosenthal hold the uncertainty level fixed and vary the total size of the electorate. A natural restriction on Kalandrakis' results comes from the fact that the Nash equilibria of the complete information game can be approximated by Bayesian equilibria only for sufficiently small perturbations. Thus while Kalandrakis (2009) established regularity of the class of asymmetric Nash equilibria which was typically dismissed in the literature due to lack of tractability, he does not resolve the turnout paradox. Our results, in a sense, provide a link between those two papers. We show in Proposition 8 that group-symmetric max-turnout correlated equilibria can be preserved under incomplete information about voting costs, while correlation allows to maintain high-turnout for large electorates, as long as cost supports are fixed.

One potential criticism of our model concerns the idea of maximizing the expected total turnout without separate considerations for turnout in each group of supporters. It is not clear a priori whether our model can be consistent with the models of the group-based ethical voter approach, if we assume that both groups independently maximize the turnout within their own members. However, our results in Proposition 1 show that when the minority is large, the same level of maximal ex-

pected total turnout can be achieved when groups maximize their members' turnout independently. We relegate more general analysis to Chapter 3, which explicitly addresses coordination among groups in a new equilibrium concept.

Another limitation of the model comes from assuming the common knowledge of party sizes and party supports. Relaxing this assumption is an important extension we leave for future research. We conjecture that the highest-turnout correlated equilibria remain robust to this change in the model assumptions, however it seems that it requires a more restrictive solution concept than the one we used: the communication equilibrium (Forges, 1986; Myerson, 1986).

2.5   Concluding Remarks

This is the first study to introduce and characterize the set of correlated equilibria in the voter turnout games. The solution concept of the correlated equilibrium, developed by Aumann (1974, 1987), allows us to explicitly take into account the possibilities of pre-play communication between voters. Communication expands the set of equilibrium outcomes in turnout games thereby providing a micro-foundation for group-based mobilization, as well as a solution to the turnout paradox that does not require ad hoc assumptions about voters' utility.

We analyzed the correlated equilibrium turnout in three main settings, varying the information structure (complete and incomplete) and the assumptions on agents' voting costs (homogenous and heterogeneous).

Under complete information and homogenous voting cost, we fully characterized the turnout bounds in terms of the correlated equilibria that maximize and minimize the expected turnout. These bounds provide a theoretical constraint on the levels of turnout that can be achieved if there are no restrictions on pre-play communication, and also characterize the range of expected turnout implementable in a correlated equilibrium. The set of correlated equilibria includes all equilibria arising under any of the more restricted communication protocols, e.g., voter communication in networks.

We found that there are two classes of the max-turnout correlated equilibria, determined by the relative sizes of the two competing groups. If the minority is at least half the size of the majority, the resulting expected turnout is twice the size of the minority and does not depend on the cost. If the minority is less than half the size of the majority, the resulting expected turnout is a decreasing function of the voting cost that starts at the size of the majority for low costs and goes down to twice the size of the minority for high costs. We also characterized the equilibrium distribution support and several key election statistics (probabilities of a tie and of the majority winning). In contrast to the high-turnout Nash equilibria, the high-turnout correlated equilibria possess intuitive properties. For example, the majority group is more likely to win for all costs, and the tie probability is increasing in the cost. We also characterize the correlated equilibria that maximize social welfare. Those are generally different from the minimal turnout equilibria, but exhibit a similar range of expected turnout.

We then showed that the high-turnout equilibria under complete information and homogenous voting cost have analogs under heterogeneous costs, which may also remain feasible correlated equilibria under incomplete information about voting costs (assuming certain additional conditions about the cost support).

Our results emphasize the important role of communication in turnout games. A natural question remains: why is the correlated equilibrium a reasonable solution concept? How, exactly, the correlated equilibria we describe in this Chapter can be implemented? The answer to the first question is given by Aumann (1987) and Hart and Mas-Colell (2000). Correlated equilibrium can be interpreted as an "expression of Bayesian rationality": if it is common knowledge that every player maximizes expected utility given her (subjective) beliefs about the state of the world, the resulting strategy choices form a correlated equilibrium. Furthermore, correlated equilibrium can be obtained as a result of a simple dynamic procedure driven by players' regret over past period observations.

The answer to the second question typically invokes describing a direct mechanism where an impartial mediator, such as a leader, gives recommendations to players. However it is important to realize that a correlated equilibrium can be also implemented without the mediator, as a Nash (or even sequential) equilibrium of the expanded game with simple communication.[20] Laboratory experiments are a useful tool for understanding the effects of unmediated communication on turnout in a controlled setting. In Chapter 4 we show that these effects are nuanced: with

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[20]See Forges (1990), Gerardi (2004), and Gerardi and Myerson (2007).

a low voting cost, party-restricted communication increases turnout, while public communication decreases turnout; while with a high voting cost, public communication increases turnout. From a theoretical perspective, establishing a realistic communication scheme that is "minimally necessary" for implementing high-turnout correlated equilibria remains a promising extension that we leave for future research.

2.A   Appendix: Proofs

2.A.1   Proof of Lemma 1

Proof. Fix any $i \in N_A$ and consider two voting profiles: $x_1 := (0_i, a, b)$ and $x_2 := (1_i, a-1, b)$ such that the total number of votes in group N_A is a, the total number of votes in group N_B is b, and in the first profile voter i abstains, while in the second profile i turns out to vote and somebody else from N_A abstains. We will construct the equivalent symmetric distribution iteratively. At step 1, we let $\sigma_1^*(s) = \mu^*(s)$ for all profiles $s \neq x_1, x_2$. The objective in either (2.9) or (2.10) does not depend on voters' identities, only on the total number of votes in each profile. Since the total number of votes at either x_1 or x_2 is the same and equals $a+b$, it does not matter for the objective how σ_1^* distributes the probability mass among x_1 and x_2 compared to μ^* as long as $\mu^*(x_1) + \mu^*(x_2) = \sigma_1^*(x_1) + \sigma_1^*(x_2)$. Hence we can let $\sigma_1^*(x_1) = \sigma_1^*(x_2) = \frac{1}{2}(\mu^*(x_1) + \mu^*(x_2))$. Clearly, this argument holds for any $a \in \{1, \ldots, n_A - 1\}$, any $b \in \{0, \ldots, n_B\}$, and any $i \in N_A$, and a similar argument holds for any $k \in N_B$ and profiles $(0_k, a, b)$ and $(1_k, a, b-1)$, respectively. We can now iteratively construct σ^*, where at each step $t \geq 2$ we define x_1^t and x_2^t by one of the remaining combinations of (a, b, i), and let $\sigma_t^*(s) = \sigma_{t-1}^*(s)$ for all profiles $s \neq x_1^t, x_2^t$. Once we have considered all combinations, we obtain σ^*, for which by construction $f(\sigma^*) = f(\mu^*)$ and $\sigma^* \in \mathcal{M}$. It remains to show that all IC constraints are satisfied at σ^*. To see this, let's roll back to σ_1^* and show that the IC constraints are satisfied at each iteration. Notice that the sets V_D^i and V_P^i in (2.8) and (2.7) do not depend on other voters' identities, but only on the total number of votes on each side of the profile, hence $x_1 \in V_P^i$ if and only if $x_2 \in V_P^\ell$ for any $\ell \in N_A, \ell \neq i$ such that ℓ votes at x_1 and abstains at x_2 (since $a \in \{1, \ldots, n_A - 1\}$, there must exist at least one such player). Similarly, $x_2 \in V_P^i$ if and only if $x_1 \in V_P^\ell$ for any such ℓ. These relations hold for all (x_1, x_2) with $a \in \{1, \ldots, n_A - 1\}$, and any $b \in \{0, \ldots, n_B\}$. By assumption, IC constraints (2.5)-(2.6) hold for all $i \in N$ under μ^*. Since the voting cost is the same for everyone in N_A, and μ^* is optimal, the corresponding IC constraints must be of the same kind

(slack or binding) for both i and ℓ under μ^*, and, moreover, they must put the same restriction on the total probability that i is pivotal at x_1 as they put on the total probability that ℓ is pivotal at x_2, i.e.

$$\sum_{\substack{s_{-i}\in V_P^i \\ |s_{-i}|=a+b}} \mu^*(0_i, 1_\ell, s_{-i\cup\ell}) = \sum_{\substack{s_{-\ell}\in V_P^\ell \\ |s_{-\ell}|=a+b}} \mu^*(0_\ell, 1_i, s_{-i\cup\ell})$$

and

$$\sum_{\substack{s_{-i}\in V_P^i \\ |s_{-i}|=a-1+b}} \mu^*(1_i, 0_\ell, s_{-i\cup\ell}) = \sum_{\substack{s_{-\ell}\in V_P^\ell \\ |s_{-\ell}|=a-1+b}} \mu^*(1_\ell, 0_i, s_{-i\cup\ell})$$

But then redistributing this probability mass symmetrically under σ^* does not violate the IC for $i \in N_A$, $a \in \{1, \ldots, n_A - 1\}$, and any $b \in \{0, \ldots, n_B\}$. Similarly, we can prove that this redistribution does not violate the IC for $k \in N_B$ and $b \in \{1, \ldots, n_B - 1\}, a \in \{0, \ldots, n_A\}$. ∎

2.A.2   Proof of Proposition 1

Proof. Using the fact that all profile probabilities sum up to one and $\mu_{0,0} = 0$ at the optimum, rewrite the objective in (2.34) as

$$1 + \sum_{\{s|\sum s_i=2\}} \mu(s) + 2 \sum_{\{s|\sum s_i=3\}} \mu(s) + \ldots$$
$$+ (n-2) \sum_{\{s|\sum s_i=n-1\}} \mu(s) + (n-1)\mu(1, \ldots, 1) \qquad (2.47)$$

Since $\sum_s \mu(s) = 1$, the above expression is maximized if the largest possible probability is placed on the outcomes with more turnout.[21] In particular, the maximal possible value of n is achieved when $\mu(1, \ldots, 1) = 1$.

---

[21]Indeed, each consecutive term in the expanded sum has a greater marginal effect on the value of the objective than the previous term.

Since $n_A > n_B$, the full turnout profile, $(1, \ldots, 1)$ is in V_A. By Lemma 1, it is sufficient to consider symmetric distributions. To simplify the notation, we denote the probability of any profile with a, b total votes for A, B, respectively, by $\mu_{a,b} \equiv \mu(\#A = a, \#B = b)$, without further reference to an individual player. We are going to use these $(n_A + 1)(n_B + 1)$ probabilities as our decision variables. When we distinguish between individual voters among those in the profile (a, b), however, there are going to be $\binom{n_A}{a}\binom{n_B}{b}$ different profiles (each having the same probability $\mu_{a,b}$ in the symmetric distribution). Hence the total probability constraint is now written as

$$\sum_{a=0}^{n_A} \sum_{b=0}^{n_B} \binom{n_A}{a}\binom{n_B}{b}\mu_{a,b} = 1 \tag{2.48}$$

One may wonder how the symmetric distribution can be implemented. In the mediator setup, we can think of it in the following way: a mediator picks a voting profile (a, b) with probability $\binom{n_A}{a}\binom{n_B}{b}\mu_{a,b}$, and then randomly recruits the respective number of voters on each side. These voters receive a recommendation to vote. The remaining voters receive a recommendation to abstain.

Using the symmetry, we can rewrite constraints (2.5)-(2.6) for players in N_A (N_B, respectively) as the following system of four inequalities with respect to $(n_A+1)(n_B+1)$ variables of the form $\mu_{a,b}$:

$$\sum_{a=1}^{n_A-1} \sum_{b=0}^{\min\{a-1,n_B\}} \binom{n_A-1}{a}\binom{n_B}{b}\mu_{a,b} + \sum_{a=0}^{n_B-2}\sum_{b=a+2}^{n_B} \binom{n_A-1}{a}\binom{n_B}{b}\mu_{a,b} \geq$$
$$\frac{\frac{1}{2}-c}{c}\left(\sum_{a=0}^{n_B}\binom{n_A-1}{a}\binom{n_B}{a}\mu_{a,a} + \sum_{a=0}^{n_B-1}\binom{n_A-1}{a}\binom{n_B}{a+1}\mu_{a,a+1}\right) \tag{2.49}$$

$$\sum_{a=2}^{n_A} \sum_{b=0}^{\min\{a-2,n_B\}} \binom{n_A-1}{a-1}\binom{n_B}{b}\mu_{a,b} + \sum_{a=1}^{n_B-1}\sum_{b=a+1}^{n_B} \binom{n_A-1}{a-1}\binom{n_B}{b}\mu_{a,b} \leq$$
$$\frac{\frac{1}{2}-c}{c}\left(\sum_{a=0}^{n_B}\binom{n_A-1}{a}\binom{n_B}{a}\mu_{a+1,a} + \sum_{a=1}^{n_B}\binom{n_A-1}{a-1}\binom{n_B}{a}\mu_{a,a}\right) \tag{2.50}$$

and

$$\sum_{a=2}^{n_A}\sum_{b=0}^{\min\{a-2,n_B-1\}}\binom{n_A}{a}\binom{n_B-1}{b}\mu_{a,b} + \sum_{a=0}^{n_B-2}\sum_{b=a+1}^{n_B-1}\binom{n_A}{a}\binom{n_B-1}{b}\mu_{a,b} \geq$$

$$\frac{\frac{1}{2}-c}{c}\left(\sum_{a=0}^{n_B-1}\binom{n_A}{a}\binom{n_B-1}{a}\mu_{a,a} + \sum_{a=0}^{n_B-1}\binom{n_A}{a+1}\binom{n_B-1}{a}\mu_{a+1,a}\right) \qquad (2.51)$$

$$\sum_{a=2}^{n_A}\sum_{b=1}^{\min\{a-1,n_B\}}\binom{n_A}{a}\binom{n_B-1}{b-1}\mu_{a,b} + \sum_{a=0}^{n_B-2}\sum_{b=a+2}^{n_B}\binom{n_A}{a}\binom{n_B-1}{b-1}\mu_{a,b} \leq$$

$$\frac{\frac{1}{2}-c}{c}\left(\sum_{a=0}^{n_B-1}\binom{n_A}{a}\binom{n_B-1}{a}\mu_{a,a+1} + \sum_{a=1}^{n_B}\binom{n_A}{a}\binom{n_B-1}{a-1}\mu_{a,a}\right) \qquad (2.52)$$

We will refer to the first and the third inequality above as the odd incentive compatibility (IC) constraints, and to the second and the fourth inequality as the even IC constraints, distinguished by the group.

Since we assumed $n_A > n_B$, at the largest turnout profile $\mu(1,\dots,1) \equiv \mu_{n_A,n_B}$ voters from N_B (as well as voters from N_A, if $n_A > n_B + 1$) are dummies. This implies that the even IC constraint for N_B is always binding at the optimum. As for the even IC constraint for N_A, we can show that for $n_A > n_B \geq \lceil\frac{1}{2}n_A\rceil$ it is always slack. To see this, notice that the even IC for N_A requires

$$\mu_{n_A,n_B} \leq \frac{\frac{1}{2}-c}{c}\left(\sum_{a=0}^{n_B-1}\binom{n_A-1}{a}\binom{n_B}{a}\mu_{a+1,a} + \sum_{a=1}^{n_B}\binom{n_A-1}{a-1}\binom{n_B}{a}\mu_{a,a}\right)$$

$$-\left[\sum_{b=1}^{\lceil n_B-1\rceil}\binom{n_B}{b}\mu_{n_A,b} + \sum_{a=3}^{n_B}\sum_{b=1}^{a-2}\binom{n_A-1}{a-1}\binom{n_B}{b}\mu_{a,b}\right.$$

$$\left.+ \sum_{a=n_B+2}^{n_A-1}\sum_{b=1}^{n_B}\binom{n_A-1}{a-1}\binom{n_B}{b}\mu_{a,b} + \sum_{a=1}^{n_B-2}\sum_{b=a+2}^{n_B}\binom{n_A-1}{a-1}\binom{n_B}{b}\mu_{a,b}\right]$$

$$-\sum_{a=1}^{n_B-1}\binom{n_A-1}{a-1}\binom{n_B}{a+1}\mu_{a,a+1} - \sum_{b=1}^{n_B}\binom{n_A-1}{n_B}\binom{n_B}{b}\mu_{n_B+1,b} - \sum_{a=2}^{n_B}\binom{n_A-1}{a-1}\mu_{a,0}$$

$$+\frac{1}{2c}\binom{n_A-1}{n_B}\mu_{n_B+1,n_B} - \sum_{a=n_B+1}^{n_A}\binom{n_A-1}{a-1}\mu_{a,0} \qquad (2.53)$$

The binding even IC for N_B requires

$$
\begin{aligned}
\mu_{n_A,n_B} = \frac{\frac{1}{2}-c}{c} & \left(\sum_{a=0}^{n_B-1} \binom{n_A}{a}\binom{n_B-1}{a}\mu_{a,a+1} + \sum_{a=1}^{n_B}\binom{n_A}{a}\binom{n_B-1}{a-1}\mu_{a,a} \right) \\
& - \left[\sum_{b=1}^{n_B-1}\binom{n_B-1}{b-1}\mu_{n_A,b} + \sum_{a=3}^{n_B}\sum_{b=1}^{a-2}\binom{n_A}{a}\binom{n_B-1}{b-1}\mu_{a,b} \right. \\
& \left. + \sum_{a=n_B+2}^{n_A-1}\sum_{b=1}^{n_B}\binom{n_A}{a}\binom{n_B-1}{b-1}\mu_{a,b} + \sum_{a=1}^{n_B-2}\sum_{b=a+2}^{n_B}\binom{n_A}{a}\binom{n_B-1}{b-1}\mu_{a,b} \right] \\
& - \sum_{a=1}^{n_B-1}\binom{n_A}{a+1}\binom{n_B-1}{a-1}\mu_{a+1,a} - \sum_{b=1}^{n_B}\binom{n_A}{n_B+1}\binom{n_B-1}{b-1}\mu_{n_B+1,b} \\
& \hspace{6cm} - \sum_{b=2}^{n_B}\binom{n_B-1}{b-1}\mu_{0,b} \qquad (2.54)
\end{aligned}
$$

Comparing the right hand sides of these two expressions, we see that in every single term of (2.53), except for the two terms on the last line, the total profile turnout matches exactly the total profile turnout in the corresponding term of (2.54). There are three possibilities. If the RHS of (2.54) is strictly less than the RHS of (2.53), the even IC for N_A is slack, so we are done. The RHS of (2.54) cannot be strictly greater, since then the even IC for N_A does not hold at all, and so we are not at the optimum of the constrained maximization program. The critical case is when the two RHS are the same; but this holds at the optimum if and only if the sum of the last two terms of (2.53) is zero (otherwise, since the profiles in the last two terms of (2.53) are not matched in (2.54), and so are not restricted by (2.54), the RHS of (2.53) in the optimum can be increased without increasing the RHS of (2.54), which is optimal when $n_B \geq \lceil \frac{1}{2}n_A \rceil$).

As long as $n_B \geq \lceil \frac{1}{2}n_A \rceil$, we have $2n_B \geq n_A$ and $2n_B + 1 > n_A$, so

$$
\frac{1}{2c}\binom{n_A-1}{n_B}\mu_{n_B+1,n_B} - \sum_{a=n_B+1}^{n_A}\binom{n_A-1}{a-1}\mu_{a,0} > 0
$$

That is, the sum of the two terms on the last line of (2.53) is strictly positive at the

optimum. This follows, since the total turnout of the first term, $2n_B + 1$, exceeds the total turnout of the largest term in the above sum, which is n_A, achieved at $\mu_{n_A,0}$. Therefore, the RHS of (2.54) is strictly less at the optimum than the RHS of (2.53) and so the even IC for N_A is slack, given that the even IC for N_B is binding. Now, if $n_B < \lceil \frac{1}{2} n_A \rceil$, then $2n_B + 1 \leq n_A$, so it is easy to see that the even IC constraints for both groups are binding at the optimum.

As for the odd IC constraints, we can show that the situation is the opposite: the odd IC for N_A is always binding at the optimum, while the odd IC for N_B only binds when $n_B > \lceil \frac{1}{2} n_A \rceil$ (for even n_A) or $n_B \geq \lceil \frac{1}{2} n_A \rceil$ (for odd n_A). To see this, notice that in the binding constraint (2.54) all profiles such that a non-voter from N_A is a dummy have the negative sign, so we want to reduce them as much as possible in the optimum. The only subset of profiles where a non-voter from N_A is a dummy which is not directly restricted by (2.54) has the form $\sum_{a=1}^{n_A-1} \binom{n_A-1}{a-1} \mu_{a,0}$. But these profiles are restricted by (2.53). If the latter is binding, the restriction is trivial. Suppose not, then if we reduced all directly restricted by (2.54) probabilities to their lower limit of zero and the odd IC for N_A still was not binding, then constraint (2.53) (slack by assumption) would imply that $\mu_{n_A,n_B} < 0$. Therefore, the odd IC for N_A must bind at the optimum.

Let us now turn to the odd IC constraint for N_B. The odd IC for N_A is binding as

we just demonstrated, so we can rewrite (2.49) and (2.51), respectively, as

$$
\sum_{a=n_B+1}^{n_A-1} \binom{n_A-1}{a} \mu_{a,n_B} - \frac{\frac{1}{2}-c}{c} \binom{n_A-1}{n_B} \mu_{n_B,n_B}
$$

$$
+ \binom{n_A-1}{n_B}\binom{n_B}{n_B-1} \mu_{n_B,n_B-1} + \sum_{a=0}^{n_B-2} \binom{n_A-1}{a} \mu_{a,n_B}
$$

$$
+ \sum_{a=0}^{n_B-2} \binom{n_A-1}{a+1}\binom{n_B}{a} \mu_{a+1,a} + \sum_{a=2}^{n_B+1}\sum_{b=0}^{a-2} \binom{n_A-1}{a}\binom{n_B}{b} \mu_{a,b}
$$

$$
+ \sum_{a=n_B+2}^{n_A-1}\sum_{b=0}^{n_B-1} \binom{n_A-1}{a}\binom{n_B}{b} \mu_{a,b} + \sum_{a=0}^{n_B-3}\sum_{b=a+2}^{n_B-1} \binom{n_A-1}{a}\binom{n_B}{b} \mu_{a,b} =
$$

$$
\frac{\frac{1}{2}-c}{c} \left(\sum_{a=0}^{n_B-1} \binom{n_A-1}{a}\binom{n_B}{a} \mu_{a,a} + \sum_{a=0}^{n_B-1} \binom{n_A-1}{a}\binom{n_B}{a+1} \mu_{a,a+1} \right) \quad (2.55)
$$

and

$$
\sum_{b=0}^{n_B-1} \binom{n_B-1}{b} \mu_{n_A,b}
$$

$$
+ \sum_{a=0}^{n_B-2} \binom{n_A}{a}\binom{n_B-1}{a+1} \mu_{a,a+1} + \sum_{a=2}^{n_B+1}\sum_{b=0}^{a-2} \binom{n_A}{a}\binom{n_B-1}{b} \mu_{a,b}
$$

$$
+ \sum_{a=n_B+2}^{n_A-1}\sum_{b=0}^{n_B-1} \binom{n_A}{a}\binom{n_B-1}{b} \mu_{a,b} + \sum_{a=0}^{n_B-3}\sum_{b=a+2}^{n_B-1} \binom{n_A}{a}\binom{n_B-1}{b} \mu_{a,b} \ge
$$

$$
\frac{\frac{1}{2}-c}{c} \left(\sum_{a=0}^{n_B-1} \binom{n_A}{a}\binom{n_B-1}{a} \mu_{a,a} + \sum_{a=0}^{n_B-1} \binom{n_A}{a+1}\binom{n_B-1}{a} \mu_{a+1,a} \right) \quad (2.56)
$$

Comparing these two expressions, we see that, except for the terms on the first two lines of (2.55) and those on the first line of (2.56), in every remaining profile of (2.55) the total turnout matches exactly the total turnout in the corresponding term of (2.56).

Suppose $n_B < \lceil \frac{1}{2} n_A \rceil$, then $2n_B < n_A$. We want to show that at the optimum

$$\sum_{b=0}^{n_B-1} \binom{n_B-1}{b} \mu_{n_A,b} + \frac{1}{2c} \binom{n_A-1}{n_B} \mu_{n_B,n_B} >$$

$$\sum_{a=n_B+1}^{n_A-1} \binom{n_A-1}{a} \mu_{a,n_B} + \binom{n_A-1}{n_B} \mu_{n_B,n_B}$$

$$+ \binom{n_A-1}{n_B} \binom{n_B}{n_B-1} \mu_{n_B,n_B-1} + \sum_{a=0}^{n_B-2} \binom{n_A-1}{a} \mu_{a,n_B} \qquad (2.57)$$

The case of $n_A - 1 < n_B + 1$ is not possible, since then $n_A = n_B + 1$, but $2n_B < n_A$ implies $n_B < 1$. So $n_A - 1 \geq n_B + 1$, then $n_A \geq n_B + 2$. Notice that on the RHS of (2.57) (at profiles with probability μ_{a,n_B} in the first sum) the total turnout in each profile equals $n_B + n_A - k$, where $k \geq 1$, matching the corresponding turnout in each profile on the LHS of (2.57) (at profiles with probability $\mu_{n_A,b}$) as long as $n_A - k \geq n_B + 1$ (since $a \geq n_B + 1$ in the first sum). Once $n_A - k = n_B + 1$, there are no more profiles left in the first sum of the RHS of (2.57), but there remain profiles with probability $\mu_{n_A,b}$ in the corresponding sum on the LHS of (2.57) as long as $0 \leq n_B - k \leq n_B - 1$, since we have $0 \leq b \leq n_B - 1$. Writing the largest possible $k^* = n_A - n_B - 1$, we see that since $n_A \geq n_B + 2$ by assumption, we indeed have $k^* \geq 1$. Therefore, the LHS of (2.57) contains the profiles with larger turnout that are unmatched by the profiles on the RHS of (2.57): at the very least, the corresponding probabilities are $\mu_{n_A,0}$ and $\mu_{n_A,1}$. So at the optimum (2.57) holds; hence, the odd IC for N_B is slack.

Now if n_A is even, we can extend this result to the case where $n_B = \lceil \frac{1}{2} n_A \rceil$, since then $2n_B = n_A$, so even though $\mu_{n_A,1}$ becomes matched by the first probability in the sum on the RHS, μ_{n_B+1,n_B}, we still have $\mu_{n_A,0}$ unmatched on the LHS. However, if n_A is odd, then $n_B = \lceil \frac{1}{2} n_A \rceil$ implies $2n_B = n_A + 1$, so $\mu_{n_A,0}$ becomes matched by μ_{n_B,n_B-1}, and hence the odd IC for N_B is binding at the optimum.

Finally, if $n_B > \lceil \frac{1}{2} n_A \rceil$, then $2n_B \geq n_A + 1$, so all profiles on the LHS of (2.57) are matched by the corresponding profiles on the RHS, so the odd IC for N_B is binding.

Table 2.1 summarizes our findings on binding and slack constraints in the maximization problem. To finish the proof, we need to consider three cases, corresponding to

Table 2.1: IC Constraints at the Optimum (Max-Turnout Equilibria)

	$n_B < \lceil \frac{1}{2} n_A \rceil$	$n_B = \lceil \frac{1}{2} n_A \rceil$	$n_B > \lceil \frac{1}{2} n_A \rceil$
Odd IC for N_A (2.49)		always binds	
Even IC for N_A (2.50)	binds	slack	slack
Odd IC for N_B (2.51)	slack	slack for even n_A; binds for odd n_A	binds
Even IC for N_B (2.52)		always binds	

Note: $n_A > n_B$, $0 < c < \frac{1}{2}$

the columns of Table 2.1.

First, suppose $n_B > \lceil \frac{1}{2} n_A \rceil$. Then the odd IC constraint for N_A binding implies

$$
\sum_{a=1}^{n_B+1} \sum_{b=0}^{a-1} \binom{n_A - 1}{a} \binom{n_B}{b} \mu_{a,b} + \sum_{a=n_B+2}^{n_A-1} \sum_{b=0}^{n_B} \binom{n_A - 1}{a} \binom{n_B}{b} \mu_{a,b}
$$

$$
+ \sum_{a=0}^{n_B-2} \sum_{b=a+2}^{n_B} \binom{n_A - 1}{a} \binom{n_B}{b} \mu_{a,b} - \frac{\frac{1}{2} - c}{c} \left(\sum_{a=0}^{n_B} \binom{n_A - 1}{a} \binom{n_B}{a} \mu_{a,a} \right.
$$

$$
\left. + \sum_{a=0}^{n_B-1} \binom{n_A - 1}{a} \binom{n_B}{a + 1} \mu_{a,a+1} \right) = 0, \qquad (2.58)
$$

the odd IC constraint for N_B binding implies

$$
\sum_{a=2}^{n_B+1} \sum_{b=0}^{a-2} \binom{n_A}{a} \binom{n_B - 1}{b} \mu_{a,b} + \sum_{a=n_B+2}^{n_A} \sum_{b=0}^{n_B-1} \binom{n_A}{a} \binom{n_B - 1}{b} \mu_{a,b}
$$

$$
+ \sum_{a=0}^{n_B-2} \sum_{b=a+1}^{n_B-1} \binom{n_A}{a} \binom{n_B - 1}{b} \mu_{a,b} - \frac{\frac{1}{2} - c}{c} \left(\sum_{a=0}^{n_B-1} \binom{n_A}{a} \binom{n_B - 1}{a} \mu_{a,a} \right.
$$

$$
\left. + \sum_{a=0}^{n_B-1} \binom{n_A}{a + 1} \binom{n_B - 1}{a} \mu_{a+1,a} \right) = 0, \qquad (2.59)
$$

and the even IC constraint for N_B binding implies

$$
\mu_{n_A,n_B} = \frac{\frac{1}{2}-c}{c}\left(\sum_{a=0}^{n_B-1}\binom{n_A}{a}\binom{n_B-1}{a}\mu_{a,a+1} + \sum_{a=1}^{n_B}\binom{n_A}{a}\binom{n_B-1}{a-1}\mu_{a,a}\right)
$$
$$
-\left[\sum_{b=1}^{n_B-1}\binom{n_B-1}{b-1}\mu_{n_A,b} + \sum_{a=2}^{n_B}\sum_{b=1}^{a-1}\binom{n_A}{a}\binom{n_B-1}{b-1}\mu_{a,b}\right.
$$
$$
\left. + \sum_{a=n_B+1}^{n_A-1}\sum_{b=1}^{n_B}\binom{n_A}{a}\binom{n_B-1}{b-1}\mu_{a,b} + \sum_{a=0}^{n_B-2}\sum_{b=a+2}^{n_B}\binom{n_A}{a}\binom{n_B-1}{b-1}\mu_{a,b}\right] \tag{2.60}
$$

At the optimum, μ_{n_A,n_B} must be as large as possible. This implies that the terms in the first parentheses must be as large as possible, and in particular, the last term in the second sum, $\binom{n_A}{n_B}\mu_{n_B,n_B}$, since it has the largest turnout among the terms with the positive sign. Now, from the odd IC for N_A,

$$
\binom{n_A-1}{n_B-1}\cdot\mu_{n_B-1,n_B} = \frac{c}{\frac{1}{2}-c}\left[\sum_{a=1}^{n_B+1}\sum_{b=0}^{a-1}\binom{n_A-1}{a}\binom{n_B}{b}\mu_{a,b}\right.
$$
$$
\left. + \sum_{a=n_B+2}^{n_A-1}\sum_{b=0}^{n_B}\binom{n_A-1}{a}\binom{n_B}{b}\mu_{a,b} + \sum_{a=0}^{n_B-2}\sum_{b=a+2}^{n_B}\binom{n_A-1}{a}\binom{n_B}{b}\mu_{a,b}\right]
$$
$$
-\left(\sum_{a=0}^{n_B}\binom{n_A-1}{a}\binom{n_B}{a}\mu_{a,a} + \sum_{a=0}^{n_B-2}\binom{n_A-1}{a}\binom{n_B}{a+1}\mu_{a,a+1}\right) \tag{2.61}
$$

Substituting, re-arranging and simplifying the terms (notice that $\mu_{0,0}=0$ by optimality),

$$\mu_{n_A,n_B} = \frac{\frac{1}{2} - c}{c} \left(- \sum_{a=0}^{n_B-2} \binom{n_A}{a} \binom{n_B-1}{a+1} \frac{n_A+1}{n_A-n_B+1} \mu_{a,a+1} \right.$$

$$\left. + \sum_{a=1}^{n_B} \binom{n_A}{a} \binom{n_B}{a} \mu_{a,a} \left(\frac{a(n_A+1) - n_A n_B}{n_B(n_A-n_B+1)} \right) \right)$$

$$+ \sum_{a=0}^{n_B-2} \sum_{b=a+2}^{n_B} \binom{n_A}{a} \binom{n_B}{b} \left(\frac{(n_B-b)(n_A+1) + (b-a-1)n_B}{n_B(n_A-n_B+1)} \right) \mu_{a,b}$$

$$+ \sum_{a=n_B+2}^{n_A-1} \sum_{b=1}^{n_B} \binom{n_A}{a} \binom{n_B}{b} \left(\frac{n_B(n_A+b-a) - b(n_A+1)}{n_B(n_A-n_B+1)} \right) \mu_{a,b}$$

$$+ \sum_{a=2}^{n_B} \sum_{b=1}^{a-1} \binom{n_A}{a} \binom{n_B}{b} \left(\frac{n_B(n_A+b-a) - b(n_A+1)}{n_B(n_A-n_B+1)} \right) \mu_{a,b}$$

$$+ \sum_{b=1}^{n_B} \binom{n_A}{n_B+1} \binom{n_B}{b} \left(\frac{(n_A-n_B)(n_B-b) - (n_B+b)}{n_B(n_A-n_B+1)} \right) \mu_{n_B+1,b}$$

$$+ \frac{n_A}{n_A-n_B+1} \sum_{a=1}^{n_A-1} \binom{n_A-1}{a} \mu_{a,0} - \sum_{b=1}^{n_B-1} \binom{n_B-1}{b-1} \mu_{n_A,b} \qquad (2.62)$$

The binding odd IC for N_B allows us to express μ_{n_A,n_B-1} as

$$\mu_{n_A,n_B-1} = \frac{\frac{1}{2} - c}{c} \left(\sum_{a=1}^{n_B-1} \binom{n_A}{a} \binom{n_B-1}{a} \mu_{a,a} + \sum_{a=0}^{n_B-1} \binom{n_A}{a+1} \binom{n_B-1}{a} \mu_{a+1,a} \right)$$

$$- \left(\sum_{a=2}^{n_B+1} \sum_{b=0}^{a-2} \binom{n_A}{a} \binom{n_B-1}{b} \mu_{a,b} + \sum_{b=0}^{n_B-2} \binom{n_B-1}{b} \mu_{n_A,b} \right)$$

$$+ \sum_{a=n_B+2}^{n_A-1} \sum_{b=0}^{n_B-1} \binom{n_A}{a} \binom{n_B-1}{b} \mu_{a,b} + \sum_{a=0}^{n_B-2} \sum_{b=a+1}^{n_B-1} \binom{n_A}{a} \binom{n_B-1}{b} \mu_{a,b} \right)$$

Plugging μ_{n_A,n_B-1} into the expression for μ_{n_A,n_B} above, we obtain

$$\mu_{n_A,n_B} = \frac{\frac{1}{2}-c}{c}\frac{1}{n_A-n_B+1}\binom{n_A}{n_B}\mu_{n_B,n_B} + \sum_{a=n_B+1}^{n_A-1}\binom{n_A}{a}\left(\frac{n_B-a-1}{n_A-n_B+1}\right)\mu_{a,n_B}$$

$$+\frac{\frac{1}{2}-c}{c}\sum_{a=1}^{n_B-1}\binom{n_A}{a}\binom{n_B}{a}\left(\frac{(a-n_B)(2n_A-n_B+1)+a}{n_B(n_A-n_B+1)}\right)\mu_{a,a}$$

$$+\frac{(n_A+1)(2-\frac{1}{2c})-n_B}{n_A-n_B+1}\sum_{a=0}^{n_B-2}\binom{n_A}{a}\binom{n_B-1}{a+1}\mu_{a,a+1}$$

$$+\sum_{a=1}^{n_B-1}\binom{n_A}{a+1}\binom{n_B}{a}\left(\frac{-\frac{\frac{1}{2}-c}{c}(n_B-a)(n_A-n_B+1)+n_A(n_B-a)-(n_B+a)}{n_B(n_A-n_B+1)}\right)\mu_{a+1,a}$$

$$+\left(\frac{c(n_A-1)-\left(\frac{1}{2}-c\right)(n_A-n_B+1)}{c(n_A-n_B+1)}\right)n_A\mu_{1,0} + \sum_{a=0}^{n_B-2}\binom{n_A}{a}\left(\frac{n_B-a-1}{n_A-n_B+1}\right)\mu_{a,n_B}$$

$$+\sum_{a=0}^{n_B-3}\sum_{b=a+2}^{n_B-1}\binom{n_A}{a}\binom{n_B}{b}\left(\frac{(n_B-b)(2n_A-n_B+1)+(b-a-1)n_B}{n_B(n_A-n_B+1)}\right)\mu_{a,b}$$

$$+\sum_{a=n_B+2}^{n_A-1}\sum_{b=1}^{n_B-1}\binom{n_A}{a}\binom{n_B}{b}\left(\frac{(n_B-b)(2n_A-n_B+1)+(b-a-1)n_B}{n_B(n_A-n_B+1)}\right)\mu_{a,b}$$

$$+\sum_{a=3}^{n_B}\sum_{b=1}^{a-2}\binom{n_A}{a}\binom{n_B}{b}\left(\frac{(n_B-b)(2n_A-n_B+1)+(b-a-1)n_B}{n_B(n_A-n_B+1)}\right)\mu_{a,b}$$

$$+\sum_{a=2}^{n_A-1}\binom{n_A}{a}\frac{2n_A-n_B+1-a}{n_A-n_B+1}\mu_{a,0} + \sum_{b=0}^{n_B-2}\binom{n_B}{b}\left(\frac{n_B-2b}{n_B}\right)\mu_{n_A,b}$$

$$+\sum_{b=1}^{n_B-1}\binom{n_A}{n_B+1}\binom{n_B}{b}\left(\frac{(n_B-b)(2n_A-2n_B+1)-(n_B+b)}{n_B(n_A-n_B+1)}\right)\mu_{n_B+1,b} \quad (2.63)$$

It is important to determine the signs of all the terms in the above expression. It is easy to see that the first term (the largest tied profile) is positive, the next one negative. The terms with $\mu_{a,a}, a \in \{1, \ldots, n_B-1\}$, are negative, as well as the terms with $\mu_{a,a+1}, a \in \{0, \ldots, n_B-2\}$.[22] The terms with $\mu_{a+1,a}, a \in \{1, \ldots, n_B-1\}$, are

---

[22]To see this, note that for $0 < c < 0.25$ we have $2 - 1/2c$ negative, which is sufficient. When

negative too.[23] The term with $\mu_{1,0}$ can be positive depending on the cost (but has the lowest possible total turnout). The remaining terms on the same line are positive. All terms on the next three lines (terms with $\mu_{a,b}$ for $a \in \{0, \ldots, n_B - 3\}, b \in \{a + 2, \ldots, n_B - 1\}$; $a \in \{n_B + 2, \ldots, n_A - 1\}, b \in \{1, \ldots, n_B - 1\}$; and $a \in \{3, \ldots, n_B\}, b \in \{1, \ldots, a - 2\}$) are positive.[24] The terms with $\mu_{a,0}, a \in \{2, \ldots, n_A - 1\}$ are all positive. The last term on the same line (with $\mu_{n_A,b}$) is positive for $b \in [0, \lfloor \frac{n_B}{2} \rfloor]$ and negative for $b \in [\lfloor \frac{n_B}{2} \rfloor + 1, n_B - 2]$. The terms on the last line (terms with $\mu_{n_B+1,b}, b \in \{1, \ldots, n_B - 1\}$) are all positive, since even for $b = n_B - 1$, the numerator is positive.

We can now start optimizing by setting $\mu_{a,b} = 0$ for all negative terms with total turnout smaller than $n_B + n_B$. That is, in (2.63) we set

$$\mu_{a,a} = 0, a \in \{0, \ldots, n_B - 1\} \tag{2.64}$$

$$\mu_{a,a+1} = 0, a \in \{0, \ldots, n_B - 2\} \tag{2.65}$$

$$\mu_{a+1,a} = 0, a \in \{1, \ldots, n_B - 1\} \tag{2.66}$$

$0.25 < c < 0.5$, the difference $2 - 1/2c$ is positive, but since $n_B < n_A + 1$,

$$(n_A + 1)\left(2 - \frac{1}{2c}\right) - n_B < n_B\left(2 - \frac{1}{2c}\right) - n_B = n_B\left(1 - \frac{1}{2c}\right) < 0.$$

[23]This follows, since the numerator of the expression in the parentheses multiplied by $\mu_{a+1,a}$ can be rewritten as

$$\left(1 - \frac{1}{2c}\right)(n_B - a)(n_A - n_B + 1) - (n_A + 1)(n_B - a - 1) < 0.$$

[24]The case when $b \geq a + 2$ is obvious. The next one ($a \in \{n_B + 2, \ldots, n_A - 1\}, b \in \{1, \ldots, n_B - 1\}$) follows from observing that already at $a = n_A - 1, b = 1$, the numerator is

$$(n_B - 1)(2n_A - n_B + 1) + (1 - n_A)n_B = n_B(n_A - n_B + 3) - n_A - (n_A + 1)$$
$$> n_B(n_A - n_B + 3) - 2n_B - n_A > n_B(n_A - n_B + 3) - 4n_B + 1$$
$$= n_B(n_A - n_B - 1) + 1 > 0.$$

The terms for $a \in \{3, \ldots, n_B\}, b \in \{1, \ldots, a - 2\}$ are all positive, since even if we take the largest $a = n_B$, the numerator is positive: $(n_B - b)(2(n_A - n_B) + 1) - n_B > 0 \Leftrightarrow b < n_B - \frac{n_B}{2(n_A - n_B) + 1}$, which always holds.

Given the slack even IC for N_A at the optimum when $n_B > \lceil \frac{1}{2} n_A \rceil$ (see Table 2.1), we must have

$$
\begin{aligned}
\mu_{n_A,n_B} <{} & \frac{\frac{1}{2}-c}{c}\binom{n_A-1}{n_B-1}\mu_{n_B,n_B} + \left(\frac{1}{2c}-1\right)\binom{n_A-1}{n_B}\mu_{n_B+1,n_B} \\
& + \frac{\frac{1}{2}-c}{c}\left(\sum_{a=0}^{n_B-1}\binom{n_A-1}{a}\binom{n_B}{a}\mu_{a+1,a} + \sum_{a=1}^{n_B-1}\binom{n_A-1}{a-1}\binom{n_B}{a}\mu_{a,a}\right) \\
& - \left[\sum_{b=1}^{n_B-1}\binom{n_B}{b}\mu_{n_A,b} + \sum_{a=3}^{n_B}\sum_{b=1}^{a-2}\binom{n_A-1}{a-1}\binom{n_B}{b}\mu_{a,b}\right. \\
& + \sum_{a=n_B+2}^{n_A-1}\sum_{b=1}^{n_B}\binom{n_A-1}{a-1}\binom{n_B}{b}\mu_{a,b} + \sum_{a=1}^{n_B-2}\sum_{b=a+2}^{n_B}\binom{n_A-1}{a-1}\binom{n_B}{b}\mu_{a,b} \\
& + \sum_{a=1}^{n_B-1}\binom{n_A-1}{a-1}\binom{n_B}{a+1}\mu_{a,a+1} + \sum_{b=1}^{n_B-1}\binom{n_A-1}{n_B}\binom{n_B}{b}\mu_{n_B+1,b} \\
& \left. + \sum_{a=2}^{n_B}\binom{n_A-1}{a-1}\mu_{a,0} + \sum_{a=n_B+1}^{n_A}\binom{n_A-1}{a-1}\mu_{a,0}\right]
\end{aligned}
\tag{2.67}
$$

Using (2.64)-(2.66), we obtain

$$
\begin{aligned}
\mu_{n_A,n_B} <{} & \frac{\frac{1}{2}-c}{c}\binom{n_A-1}{n_B-1}\mu_{n_B,n_B} + \frac{\frac{1}{2}-c}{c}\binom{n_A-1}{n_B}\mu_{n_B+1,n_B} \\
& + \frac{\frac{1}{2}-c}{c}\mu_{1,0} - \left[\sum_{b=1}^{n_B-1}\binom{n_B}{b}\mu_{n_A,b} + \sum_{a=3}^{n_B}\sum_{b=1}^{a-2}\binom{n_A-1}{a-1}\binom{n_B}{b}\mu_{a,b}\right. \\
& + \sum_{a=n_B+2}^{n_A-1}\sum_{b=1}^{n_B}\binom{n_A-1}{a-1}\binom{n_B}{b}\mu_{a,b} + \sum_{a=1}^{n_B-2}\sum_{b=a+2}^{n_B}\binom{n_A-1}{a-1}\binom{n_B}{b}\mu_{a,b} \\
& + \binom{n_A-1}{n_B-2}\mu_{n_B-1,n_B} + \sum_{b=1}^{n_B-1}\binom{n_A-1}{n_B}\binom{n_B}{b}\mu_{n_B+1,b} \\
& \left. + \sum_{a=2}^{n_B}\binom{n_A-1}{a-1}\mu_{a,0} + \sum_{a=n_B+1}^{n_A}\binom{n_A-1}{a-1}\mu_{a,0}\right]
\end{aligned}
\tag{2.68}
$$

Replacing the LHS of this expression with (2.63) and re-arranging, we obtain

$$\frac{n_A(n_A - 1)}{n_A - n_B + 1}\mu_{1,0} + \sum_{a=0}^{n_B-2} \binom{n_A - 1}{a} \frac{n_B - 1}{n_A - n_B + 1}\mu_{a,n_B}$$

$$+ \sum_{a=0}^{n_B-3} \sum_{b=a+2}^{n_B-1} \binom{n_A}{a}\binom{n_B}{b} \left(\frac{(n_B - b)(2n_A - n_B + 1) + (b - a - 1)n_B}{n_B(n_A - n_B + 1)} + \frac{a}{n_A} \right) \mu_{a,b}$$

$$+ \sum_{a=n_B+2}^{n_A-1} \sum_{b=1}^{n_B-1} \binom{n_A}{a}\binom{n_B}{b} \left(\frac{(n_B - b)(2n_A - n_B + 1) + (b - a - 1)n_B}{n_B(n_A - n_B + 1)} + \frac{a}{n_A} \right) \mu_{a,b}$$

$$+ \sum_{a=3}^{n_B} \sum_{b=1}^{a-2} \binom{n_A}{a}\binom{n_B}{b} \left(\frac{(n_B - b)(2n_A - n_B + 1) + (b - a - 1)n_B}{n_B(n_A - n_B + 1)} + \frac{a}{n_A} \right) \mu_{a,b}$$

$$+ \sum_{a=2}^{n_A-1} \binom{n_A}{a} \left(\frac{2n_A - n_B + 1 - a}{n_A - n_B + 1} + \frac{a}{n_A} \right) \mu_{a,0} + \sum_{b=0}^{n_B-2} \binom{n_B}{b} \frac{2(n_B - b)}{n_B}\mu_{n_A,b}$$

$$+ \sum_{b=1}^{n_B-1} \binom{n_A}{n_B + 1}\binom{n_B}{b} \left(\frac{(n_B - b)(2n_A - 2n_B + 1) - (n_B + b)}{n_B(n_A - n_B + 1)} + \frac{n_B + 1}{n_A} \right) \mu_{n_B+1,b} <$$

$$\frac{\frac{1}{2} - c}{c} \binom{n_A - 1}{n_B} \frac{n_B - 1}{n_A - n_B + 1}\mu_{n_B,n_B} + \frac{\frac{1}{2} - c}{c} \binom{n_A - 1}{n_B} \mu_{n_B+1,n_B}$$

$$+ \frac{\frac{1}{2} - c}{c}(n_A + 1)\mu_{1,0} + \sum_{a=n_B+1}^{n_A-1} \binom{n_A}{a} \left(\frac{a + 1 - n_B}{n_A - n_B + 1} \right) \mu_{a,n_B}$$

$$- \left[\binom{n_B}{n_B - 1}\mu_{n_A,n_B-1} + \binom{n_A - 1}{n_B - 2}\mu_{n_B-1,n_B} \right] \quad (2.69)$$

Notice that all terms to the left of the inequality sign are positive and enter (2.63) with positive signs. The terms to the right of the inequality sign are all positive except the last parenthesis. Since those terms in the parentheses are not restricted by (2.63) (due to our constraint substitution), we optimally set them equal to zero. In addition we set $\mu_{1,0} = 0$ since this allows to increase the remaining terms on the RHS that have larger turnout.

Taking into account the signs of the terms in (2.63), and given (2.69), we see that the RHS of (2.63) is optimized whenever we increase the RHS of (2.69). Therefore, in the optimum the sum of the terms on the LHS of (2.69) is as small as possible. It

cannot be zero, though, due to the binding odd IC constraint for N_A (2.58). Indeed, this constraint determines the maximal allowed increase to μ_{n_B,n_B} via the sum of $\sum_{a=n_B+1}^{n_A-1} \mu_{a,n_B}$ and $\sum_{a=0}^{n_B-2} \mu_{a,n_B}$ (taken with appropriate coefficients).

Hence in the optimum, the support of the equilibrium distribution only includes the profiles of the form (a, n_B) for $a \in \{0, \ldots, n_B - 2\} \cup \{n_B, \ldots, n_A\}$. Therefore, the optimal probability of the largest profile is

$$
\mu_{n_A,n_B} = \frac{\frac{1}{2} - c}{c} \frac{1}{n_A - n_B + 1} \binom{n_A}{n_B} \mu_{n_B,n_B} + \sum_{a=n_B+1}^{n_A-1} \binom{n_A}{a} \left(\frac{n_B - a - 1}{n_A - n_B + 1} \right) \mu_{a,n_B}
$$
$$
+ \sum_{a=0}^{n_B-2} \binom{n_A}{a} \left(\frac{n_B - a - 1}{n_A - n_B + 1} \right) \mu_{a,n_B} \tag{2.70}
$$

The only remaining constraint is that the total sum of probabilities is one, which, given (2.70), can be written as

$$
\sum_{a=0}^{n_B-2} \binom{n_A}{a} \mu_{a,n_B} + \sum_{a=n_B}^{n_A-1} \binom{n_A}{a} \mu_{a,n_B} + \frac{\frac{1}{2} - c}{c} \frac{1}{n_A - n_B + 1} \binom{n_A}{n_B} \mu_{n_B,n_B}
$$
$$
+ \sum_{a=n_B+1}^{n_A-1} \binom{n_A}{a} \left(\frac{n_B - a - 1}{n_A - n_B + 1} \right) \mu_{a,n_B} + \sum_{a=0}^{n_B-2} \binom{n_A}{a} \left(\frac{n_B - a - 1}{n_A - n_B + 1} \right) \mu_{a,n_B} = 1
$$

Rewriting,

$$
\frac{n_A}{n_A - n_B + 1} \left(\sum_{a=0}^{n_B-2} \binom{n_A - 1}{a} \mu_{a,n_B} + \sum_{a=n_B+1}^{n_A-1} \binom{n_A - 1}{a} \mu_{a,n_B} \right)
$$
$$
+ \frac{\frac{1}{2c} + n_A - n_B}{n_A - n_B + 1} \binom{n_A}{n_B} \mu_{n_B,n_B} = 1
$$

Using the binding odd IC constraint for N_A, (2.58), we obtain

$$\sum_{a=n_B+1}^{n_A-1} \binom{n_A-1}{a} \mu_{a,n_B} + \sum_{a=0}^{n_B-2} \binom{n_A-1}{a} \mu_{a,n_B} - \frac{\frac{1}{2}-c}{c} \binom{n_A-1}{n_B} \mu_{n_B,n_B} = 0$$

Substituting into the previous expression, we obtain

$$\mu_{n_B,n_B} = \frac{2c}{\binom{n_A}{n_B}} \tag{2.71}$$

Hence

$$\sum_{a=n_B+1}^{n_A-1} \binom{n_A-1}{a} \mu_{a,n_B} + \sum_{a=0}^{n_B-2} \binom{n_A-1}{a} \mu_{a,n_B} = \frac{(1-2c)(n_A-n_B)}{n_A} \tag{2.72}$$

Plugging these expressions into the objective function and simplifying, we rewrite (2.47) as

$$f^* = 1 + \sum_{a=0}^{n_B-2} (n_B + a - 1)\binom{n_A}{a}\mu_{a,n_B} + \sum_{a=n_B+1}^{n_A-1} (n_B + a - 1)\binom{n_A}{a}\mu_{a,n_B}$$

$$+ (n_B + n_B - 1)\binom{n_A}{n_B}\mu_{n_B,n_B} + (n_A + n_B - 1)\mu_{n_A,n_B}$$

$$= 1 + \sum_{a=0}^{n_B-2} (n_B + a - 1)\binom{n_A}{a}\mu_{a,n_B} + \sum_{a=n_B+1}^{n_A-1} (n_B + a - 1)\binom{n_A}{a}\mu_{a,n_B}$$

$$+ (n_B + n_B - 1)2c + \frac{n_A + n_B - 1}{n_A - n_B + 1}\left[1 - 2c + \sum_{a=n_B+1}^{n_A-1}\binom{n_A}{a}(n_B - a - 1)\mu_{a,n_B} \right.$$

$$\left. + \sum_{a=0}^{n_B-2}\binom{n_A}{a}(n_B - a - 1)\mu_{a,n_B} \right]$$

$$= 1 + (2n_B - 1)2c + \frac{(1-2c)(n_A + n_B - 1)}{n_A - n_B + 1} + \frac{2(n_B-1)n_A}{n_A - n_B + 1}\left[\sum_{a=0}^{n_B-2}\binom{n_A-1}{a}\mu_{a,n_B} \right.$$

$$\left. + \sum_{a=n_B+1}^{n_A-1}\binom{n_A-1}{a}\mu_{a,n_B} \right]$$

$$= 4cn_B + (1-2c)\left[1 + \frac{n_A + n_B - 1}{n_A - n_B + 1} + \frac{2(n_B-1)n_A(n_A - n_B)}{(n_A - n_B + 1)n_A} \right]$$

$$= 4cn_B + 2(1-2c)n_B\frac{n_A - n_B + 1}{n_A - n_B + 1} = 2n_B$$

So, the maximal expected turnout is twice the size of the minority.

This completes the proof of case (i), with the exception of the knife-edge case of $n_B = \lceil \frac{1}{2}n_A \rceil$. We address this case after finishing the proof of case (ii).

Now suppose $n_B < \lceil \frac{1}{2}n_A \rceil$. Then $2n_B < n_A$. Due to the odd IC for N_A and even IC for N_B binding, we can express the probability of the largest profile as

$$\mu_{n_A,n_B} = \frac{\frac{1}{2}-c}{c}\left(-\sum_{a=0}^{n_B-2}\binom{n_A}{a}\binom{n_B-1}{a+1}\frac{n_A+1}{n_A-n_B+1}\mu_{a,a+1}\right.$$

$$+\sum_{a=1}^{n_B}\binom{n_A}{a}\binom{n_B}{a}\mu_{a,a}\left(\frac{a(n_A+1)-n_An_B}{n_B(n_A-n_B+1)}\right)\right)$$

$$+\sum_{a=0}^{n_B-2}\sum_{b=a+2}^{n_B}\binom{n_A}{a}\binom{n_B}{b}\left(\frac{(n_B-b)(n_A+1)+(b-a-1)n_B}{n_B(n_A-n_B+1)}\right)\mu_{a,b}$$

$$+\sum_{a=n_B+2}^{n_A-1}\sum_{b=1}^{n_B}\binom{n_A}{a}\binom{n_B}{b}\left(\frac{n_B(n_A+b-a)-b(n_A+1)}{n_B(n_A-n_B+1)}\right)\mu_{a,b}$$

$$+\sum_{a=2}^{n_B}\sum_{b=1}^{a-1}\binom{n_A}{a}\binom{n_B}{b}\left(\frac{n_B(n_A+b-a)-b(n_A+1)}{n_B(n_A-n_B+1)}\right)\mu_{a,b}$$

$$+\sum_{b=1}^{n_B}\binom{n_A}{n_B+1}\binom{n_B}{b}\left(\frac{(n_A-n_B)(n_B-b)-(n_B+b)}{n_B(n_A-n_B+1)}\right)\mu_{n_B+1,b}$$

$$+\frac{n_A}{n_A-n_B+1}\sum_{a=1}^{n_A-1}\binom{n_A-1}{a}\mu_{a,0}-\sum_{b=1}^{n_B-1}\binom{n_B-1}{b-1}\mu_{n_A,b} \tag{2.73}$$

On the other hand, the even IC for N_A binding implies

$$\mu_{n_A,n_B} = \frac{\frac{1}{2}-c}{c}\left(\sum_{a=0}^{n_B}\binom{n_A-1}{a}\binom{n_B}{a}\mu_{a+1,a}+\sum_{a=1}^{n_B}\binom{n_A-1}{a-1}\binom{n_B}{a}\mu_{a,a}\right)$$

$$-\left[\sum_{b=1}^{n_B-1}\binom{n_B}{b}\mu_{n_A,b}+\sum_{a=3}^{n_B}\sum_{b=1}^{a-2}\binom{n_A-1}{a-1}\binom{n_B}{b}\mu_{a,b}\right.$$

$$\left.+\sum_{a=n_B+2}^{n_A-1}\sum_{b=1}^{n_B}\binom{n_A-1}{a-1}\binom{n_B}{b}\mu_{a,b}+\sum_{a=1}^{n_B-2}\sum_{b=a+2}^{n_B}\binom{n_A-1}{a-1}\binom{n_B}{b}\mu_{a,b}\right]$$

$$-\sum_{a=1}^{n_B-1}\binom{n_A-1}{a-1}\binom{n_B}{a+1}\mu_{a,a+1}-\sum_{b=1}^{n_B-1}\binom{n_A-1}{n_B}\binom{n_B}{b}\mu_{n_B+1,b}$$

$$-\sum_{a=2}^{n_A}\binom{n_A-1}{a-1}\mu_{a,0} \tag{2.74}$$

Comparing these two expressions and taking into account that the odd IC for N_B is

slack, we see that at the optimum,

$$\mu_{a,a+1} = 0, a \in \{0, \ldots, n_B - 1\} \tag{2.75}$$

$$\mu_{a,a} = 0, a \in \{0, \ldots, n_B - 1\} \tag{2.76}$$

$$\mu_{n_B+1,b} = 0, b \in \{1, \ldots, n_B - 1\} \tag{2.77}$$

$$\mu_{a,b} = 0, a \in \{n_B + 2, \ldots, n_A - 1\}, b \in \{0, \ldots, n_B\} \tag{2.78}$$

$$\mu_{a,b} = 0, a \in \{3, \ldots, n_B + 1\}, b \in \{1, \ldots, a - 2\} \tag{2.79}$$

$$\mu_{a,b} = 0, a \in \{0, \ldots, n_B - 2\}, b \in \{a + 2, \ldots, n_B\} \tag{2.80}$$

Given (2.75)-(2.80), we can rewrite (2.74) as

$$\mu_{n_A,n_B} = \frac{\frac{1}{2} - c}{c} \sum_{a=0}^{n_B} \binom{n_A - 1}{a} \binom{n_B}{a} \mu_{a+1,a} + \frac{\frac{1}{2} - c}{c} \binom{n_A - 1}{n_B - 1} \mu_{n_B,n_B}$$
$$- \sum_{b=1}^{n_B-1} \binom{n_B}{b} \mu_{n_A,b} - \sum_{a=2}^{n_B+1} \binom{n_A - 1}{a - 1} \mu_{a,0} - \mu_{n_A,0} \tag{2.81}$$

We also rewrite (2.73) as

$$\mu_{n_A,n_B} =$$
$$\sum_{a=0}^{n_B} \binom{n_A}{a + 1} \binom{n_B}{a} \frac{n_B(n_A - 1) - a(n_A + 1)}{n_B(n_A - n_B + 1)} \mu_{a+1,a} - \sum_{b=1}^{n_B-1} \binom{n_B - 1}{b - 1} \mu_{n_A,b}$$
$$+ \frac{(\frac{1}{2} - c) n_A}{c n_B(n_A - n_B + 1)} \binom{n_A - 1}{n_B - 1} \mu_{n_B,n_B} + \frac{n_A}{n_A - n_B + 1} \sum_{a=2}^{n_B+1} \binom{n_A - 1}{a} \mu_{a,0} \tag{2.82}$$

Now (2.81) and (2.82) imply that in the optimum

$$\mu_{n_A,b} = 0, b \in \{1, \ldots, n_B - 1\} \tag{2.83}$$

In addition, the slack odd IC for N_B, given (2.75)-(2.80), takes the form

$$\mu_{n_A,0} + \sum_{a=2}^{n_B+1} \binom{n_A}{a} \mu_{a,0} > \frac{\frac{1}{2} - c}{c} \sum_{a=0}^{n_B-1} \binom{n_A}{a + 1} \binom{n_B - 1}{a} \mu_{a+1,a} \tag{2.84}$$

Together with $2n_B \leq n_A - 1$, this implies that at the optimum $\mu_{a,0} = 0, a \in [2, n_B+1]$, and hence the support of the distribution includes only the profiles of the form $(a+1, a), a \in [0, n_B]$, (n_B, n_B) and $(n_A, 0)$. In particular, $\mu_{n_A, n_B} = 0$, since from (2.84) and (2.81), $\mu_{n_A,0}$ offsets μ_{n_B+1,n_B} and μ_{n_B,n_B} (from the maximization point of view, the profiles with higher turnout must receive larger probability weights).

Hence we can rewrite (2.81) as

$$\mu_{n_A,n_B} = 0 =$$

$$\frac{\frac{1}{2} - c}{c} \left(\sum_{a=0}^{n_B} \binom{n_A - 1}{a} \binom{n_B}{a} \mu_{a+1,a} + \binom{n_A - 1}{n_B - 1} \mu_{n_B,n_B} \right) - \mu_{n_A,0} \qquad (2.85)$$

The probability constraint now can be written as

$$\sum_{a=0}^{n_B} \binom{n_A}{a+1} \binom{n_B}{a} \mu_{a+1,a} + \binom{n_A}{n_B} \mu_{n_B,n_B}$$

$$+ \frac{\frac{1}{2} - c}{c} \left(\sum_{a=0}^{n_B} \binom{n_A - 1}{a} \binom{n_B}{a} \mu_{a+1,a} + \binom{n_A - 1}{n_B - 1} \mu_{n_B,n_B} \right) = 1$$

Simplifying,

$$\sum_{a=0}^{n_B} \binom{n_A}{a+1} \binom{n_B}{a} \left(\frac{n_A + (a+1)(\frac{1}{2c} - 1)}{n_A} \right) \mu_{a+1,a}$$

$$+ \binom{n_A - 1}{n_B - 1} \left(\frac{n_A}{n_B} + \frac{1}{2c} - 1 \right) \mu_{n_B,n_B} = 1 \qquad (2.86)$$

From (2.82),

$$0 = \binom{n_A}{n_B} \frac{\frac{1}{2c} - 1}{n_A - n_B + 1} \mu_{n_B,n_B}$$

$$+ \sum_{a=0}^{n_B} \binom{n_A}{a+1} \binom{n_B}{a} \left(\frac{n_B(n_A - 1) - a(n_A + 1)}{n_B(n_A - n_B + 1)} \right) \mu_{a+1,a} \qquad (2.87)$$

Thus

$$\mu_{n_B,n_B} = -\sum_{a=0}^{n_B} \binom{n_A}{a+1}\binom{n_B}{a}\left(\frac{n_B(n_A-1)-a(n_A+1)}{n_B(\frac{1}{2c}-1)\binom{n_A}{n_B}}\right)\mu_{a+1,a} \qquad (2.88)$$

Now we can rewrite (2.86) as

$$\sum_{a=0}^{n_B} \binom{n_A}{a+1}\binom{n_B}{a}\mu_{a+1,a}\left[\frac{n_A+(a+1)(\frac{1}{2c}-1)}{n_A}\right.$$
$$\left.-\left(\frac{n_A}{n_B}+\frac{1}{2c}-1\right)\left(\frac{n_B(n_A-1)-a(n_A+1)}{n_A(\frac{1}{2c}-1)}\right)\right] = 1 \qquad (2.89)$$

Simplifying,

$$\sum_{a=0}^{n_B} \binom{n_A}{a+1}\binom{n_B}{a}\mu_{a+1,a}\left[1+\frac{(a+1)(\frac{1}{2c}-1)-n_B(n_A-1)+a(n_A+1)}{n_A}\right.$$
$$\left.-\frac{n_B(n_A-1)-a(n_A+1)}{n_B\left(\frac{1}{2c}-1\right)}\right] = 1 \qquad (2.90)$$

In addition, the binding odd IC for N_A implies

$$\sum_{a=0}^{n_B} \binom{n_A}{a+1}\binom{n_B}{a}\mu_{a+1,a}\left(n_B-a-\frac{n_B}{n_A}\right) = 0 \qquad (2.91)$$

The binding even IC for N_B is implies

$$\sum_{a=1}^{n_B} \binom{n_A}{a+1}\binom{n_B-1}{a-1}\mu_{a+1,a} = \frac{\frac{1}{2}-c}{c}\binom{n_A}{n_B}\mu_{n_B,n_B} \qquad (2.92)$$

Using these expressions together with (2.86) and (2.88), we can (after some tedious

algebra) express

$$\mu_{n_B,n_B} = \frac{2c}{\binom{n_A}{n_B}\left(1 + \left(\frac{1}{2c} - 1\right)\left(\frac{1}{n_A-1} + \frac{n_B}{n_A}\right)\right)} \tag{2.93}$$

Now, from (2.85),

$$\begin{aligned}
\mu_{n_A,0} &= \frac{\frac{1}{2}-c}{c}\left(\sum_{a=0}^{n_B}\binom{n_A-1}{a}\binom{n_B}{a}\mu_{a+1,a} + \binom{n_A-1}{n_B-1}\mu_{n_B,n_B}\right) \\
&= \frac{\frac{1}{2}-c}{c}\left[\binom{n_A}{n_B}\mu_{n_B,n_B}\left(\frac{1}{2c} - 1\right)\left(\frac{1}{n_A-1} + \frac{n_B}{n_A}\right) + \binom{n_A}{n_B}\frac{n_B}{n_A}\mu_{n_B,n_B}\right] \\
&= \frac{\frac{1}{2}-c}{c}\binom{n_A}{n_B}\mu_{n_B,n_B}\left[\frac{1}{2c}\left(\frac{1}{n_A-1} + \frac{n_B}{n_A}\right) - \frac{1}{n_A-1}\right] \\
&= \frac{2c}{\frac{n_A+\frac{1}{2c}-2}{n_A-1} + \frac{n_B\left(\frac{1}{2c}-1\right)}{n_A}}\left(\frac{1}{2c} - 1\right)\left[\frac{1}{n_A-1}\left(\frac{1}{2c} - 1\right) + \frac{1}{2c}\frac{n_B}{n_A}\right]
\end{aligned}$$

$$\tag{2.94}$$

Plugging these expressions into the objective function and simplifying, we rewrite (2.47) as

$$f^* = 1 + \sum_{a=0}^{n_B} 2a \binom{n_A}{a+1}\binom{n_B}{a}\mu_{a+1,a} + (2n_B - 1)\binom{n_A}{n_B}\mu_{n_B,n_B} + (n_A - 1)\mu_{n_A,0}$$

$$= 1 + \sum_{a=0}^{n_B} 2a\binom{n_A}{a+1}\binom{n_B}{a}\mu_{a+1,a}$$

$$-(2n_B - 1)\binom{n_A}{n_B}\sum_{a=0}^{n_B}\binom{n_A}{a+1}\binom{n_B}{a}\left(\frac{n_B(n_A - 1) - a(n_A + 1)}{n_B(\frac{1}{2c} - 1)\binom{n_A}{n_B}}\right)\mu_{a+1,a}$$

$$+(n_A - 1)\frac{\frac{1}{2} - c}{c}\left(\sum_{a=0}^{n_B}\binom{n_A - 1}{a}\binom{n_B}{a}\mu_{a+1,a}\right.$$

$$\left.-\binom{n_A - 1}{n_B - 1}\sum_{a=0}^{n_B}\binom{n_A}{a+1}\binom{n_B}{a}\left(\frac{n_B(n_A - 1) - a(n_A + 1)}{n_B(\frac{1}{2c} - 1)\binom{n_A}{n_B}}\right)\mu_{a+1,a}\right)$$

$$= 1 + \sum_{a=0}^{n_B}\binom{n_A}{a+1}\binom{n_B}{a}\mu_{a+1,a}\left[2a - (2n_B - 1)\frac{n_B(n_A - 1) - a(n_A + 1)}{n_B(\frac{1}{2c} - 1)}\right.$$

$$\left.+(n_A - 2n_B + 2n_B - 1)\frac{(a+1)(\frac{1}{2c} - 1)}{n_A} - (n_A - 2n_B + 2n_B - 1)\frac{n_B(n_A - 1) - a(n_A + 1)}{n_A}\right]$$

$$= 1 + \sum_{a=0}^{n_B}\binom{n_A}{a+1}\binom{n_B}{a}\mu_{a+1,a}\left[2a - (2n_B - 1)\frac{n_B(n_A - 1) - a(n_A + 1)}{n_B(\frac{1}{2c} - 1)}\right.$$

$$+(2n_B - 1)\frac{(a+1)(\frac{1}{2c} - 1)}{n_A} - (2n_B - 1)\frac{n_B(n_A - 1) - a(n_A + 1)}{n_A}$$

$$\left.+(n_A - 2n_B)\frac{(a+1)(\frac{1}{2c} - 1)}{n_A} - (n_A - 2n_B)\frac{n_B(n_A - 1) - a(n_A + 1)}{n_A}\right]$$

$$= 1 + 2n_B - 1 + \sum_{a=0}^{n_B}\binom{n_A}{a+1}\binom{n_B}{a}\mu_{a+1,a}\left[2a - 2n_B + 1\right.$$

$$\left.+(n_A - 2n_B)\frac{(a+1)(\frac{1}{2c} - 1) - n_B(n_A - 1) + a(n_A + 1)}{n_A}\right]$$

$$= 2n_B + \frac{n_A - 2n_B}{2cn_A}\left(1 + n_B - \frac{n_B}{n_A}\right)\sum_{a=0}^{n_B}\binom{n_A}{a+1}\binom{n_B}{a}\mu_{a+1,a}$$

$$= 2n_B + \frac{n_A - 2n_B}{2cn_A}\left(1 + n_B - \frac{n_B}{n_A}\right)\left(1 - \mu_{n_A,0} - \binom{n_A}{n_B}\mu_{n_B,n_B}\right)$$

$$= 2n_B + \frac{n_A - 2n_B}{2cn_A}\left(1 + n_B - \frac{n_B}{n_A}\right)$$

$$\times\left(\frac{\frac{n_A + \frac{1}{2c} - 2}{n_A - 1} + \frac{n_B(\frac{1}{2c} - 1)}{n_A} - 2c\left(\frac{1}{2c} - 1\right)\left[\frac{1}{n_A - 1}\left(\frac{1}{2c} - 1\right) + \frac{1}{2c}\frac{n_B}{n_A}\right] - 2c}{\frac{n_A + \frac{1}{2c} - 2}{n_A - 1} + \frac{n_B(\frac{1}{2c} - 1)}{n_A}}\right)$$

$$= 2n_B + \frac{(n_A - 2n_B)(1 - 2c)}{1 + 2c\left(\frac{n_A(n_A - 1)}{n_A + n_B(n_A - 1)} - 1\right)}$$

$$= 2n_B + \phi(c)$$

$$= n_A \times \frac{2cn_B(n_A - 1) + n_B(n_A - 1) + n_A(1 - 2c)}{2c(n_A - n_B)(n_A - 1) + n_B(n_A - 1) + n_A(1 - 2c)}$$

$$= n_A \times \xi(c)$$

This completes the proof of case (ii).

Finally, there remains the knife-edge case of $n_B = \lceil \frac{1}{2} n_A \rceil$. When n_A is odd, the odd IC for N_B is binding, so the proof of case (i) given above works just the same, giving the maximal expected turnout of $2n_B$. When n_A is even, the odd IC for N_B is slack, so the proof of case (ii) directly applies. The value of the objective function at the optimum has $\phi(c) = 0$. However, in contrast to case (ii), the support of the symmetric distribution is different and in fact, may include all profiles except the tied ones with turnout less than n_B, and the profiles of the form $(a, a+1), a \in [0, n_B - 1]$. There is no simple analytic expression available for the equilibrium support, so we verified our conclusions for this case using computer simulations. ∎

2.A.3 Proof of Corollary 3

Proof. Case (ii). In this case, the only profile in the support where the majority can lose is (n_B, n_B), so

$$\pi_m = 1 - \frac{1}{2} \binom{n_A}{n_B} \mu_{n_B, n_B} = 1 - \frac{c}{1 + \left(\frac{1}{2c} - 1\right)\left(\frac{1}{n_A - 1} + \frac{n_B}{n_A}\right)}$$

Case (i). Since in the equilibrium distribution support all the profiles where the majority wins are of the form (a, n_B) for $a \in [n_B + 1, n_A]$ plus the largest tied profile, it is easy to see that

$$\pi_m = \frac{1}{2} \binom{n_A}{n_B} \mu_{n_B, n_B} + \sum_{a=n_B+1}^{n_A - 1} \binom{n_A}{a} \mu_{a, n_B} + \mu_{n_A, n_B} \tag{2.95}$$

The first term above equals c from (2.71), but we do not have enough constraints to identify the remaining terms in the sum individually. The rest of the proof characterizes the bounds on these terms producing the result in the statement.

As Table 2.1 shows, at the optimum there are three binding constraints plus the total

sum of probabilities constraint. It turns out that the first binding constraint, (2.58), becomes equation (2.72), which we repeat here for convenience:

$$\sum_{a=0}^{n_B-2} \binom{n_A-1}{a} \mu_{a,n_B} + \sum_{a=n_B+1}^{n_A-1} \binom{n_A-1}{a} \mu_{a,n_B} = (1-2c)\left(1-\frac{n_B}{n_A}\right) \qquad (2.96)$$

The second binding constraint, (2.59), becomes an identity as it does not contain any profiles from the equilibrium support. The third binding constraint, (2.60), reduces to the total probability constraint. Namely,

$$\sum_{a=0}^{n_B-2} \binom{n_A}{a} \mu_{a,n_B} + \binom{n_A}{n_B} \mu_{n_B,n_B} + \sum_{a=n_B+1}^{n_A} \binom{n_A}{a} \mu_{a,n_B} = 1 \qquad (2.97)$$

From (2.71), $\frac{1}{2}\binom{n_A}{n_B}\mu_{n_B,n_B} = c$. Hence from the total probability constraint, $1 - \pi_m - c \geq 0$ and the first inequality in the statement follows.

The two binding constraints we are left with, (2.96) and (2.97), are not enough to determine π_m even knowing μ_{n_B,n_B}.[25] Nevertheless, from (2.96) and (2.97) we can express

$$\mu_{n_A,n_B} = \frac{n_B}{n_A}(1-2c) - \sum_{a=0}^{n_B-2} \frac{a}{n_A}\binom{n_A}{a}\mu_{a,n_B} - \sum_{a=n_B+1}^{n_A-1} \frac{a}{n_A}\binom{n_A}{a}\mu_{a,n_B}$$

Note that at the optimum $\mu_{n_A,n_B} > 0$. We want to show that $\pi_m > 0.5$ for all $c \in (0, 0.5)$. Suppose by way of contradiction that $\pi_m \leq 0.5$ for some cost c in this range. Then from (2.95)

$$\mu_{n_A,n_B} + \sum_{a=n_B+1}^{n_A-1} \binom{n_A}{a}\mu_{a,n_B} \leq \frac{1}{2}(1-2c)$$

---

[25]For the special case identified in Remark 3, there are just two profiles in the equilibrium support other than (n_B, n_B): $(0, n_B)$ and (n_A, n_B), hence we can derive their probabilities using (2.96) and (2.97). We obtain $\mu_{0,n_B} = (1-2c)\left(1-\frac{n_B}{n_A}\right)$, and therefore $\pi_m = \frac{n_B}{n_A} + c\left(1-\frac{2n_B}{n_A}\right)$.

Correspondingly, from (2.97)

$$\sum_{a=0}^{n_B-2} \binom{n_A}{a} \mu_{a,n_B} \geq \frac{1}{2}(1-2c)$$

This implies that the total probability mass is greater on the lower turnout profiles than on the higher turnout ones. Denote T the expected turnout at the optimal probability distribution. From Proposition 1, $T = 2n_B$. Then

$$2n_B = \sum_{a=0}^{n_B-2}(a+n_B)\binom{n_A}{a}\mu_{a,n_B} + 2c \cdot 2n_B + \sum_{a=n_B+1}^{n_A}(a+n_B)\binom{n_A}{a}\mu_{a,n_B}$$

$$= (1-2c)\left(\frac{1}{2}+\varepsilon\right)\bar{\mu}_L + 2c \cdot 2n_B + (1-2c)\left(\frac{1}{2}-\varepsilon\right)\bar{\mu}_H,$$

where $\bar{\mu}_L$ is the mean expected turnout at the lower turnout profiles, $\bar{\mu}_L \in (0, 2n_B - 2)$; $\bar{\mu}_H$ is the mean expected turnout at the higher turnout profiles, $\bar{\mu}_H \in (2n_B + 1, n_A + n_B)$, and $\varepsilon \in [0, 0.5)$ is such that $\pi_m = \frac{1}{2} - \varepsilon$. But then

$$2n_B = (1-2c)\left[\left(\frac{1}{2}+\varepsilon\right)(\bar{\mu}_L+\bar{\mu}_H) - 2\varepsilon\bar{\mu}_H\right] + 2c \cdot 2n_B < 2n_B,$$

since due to $2n_B > n_A$ and turnout maximization, $\left(\frac{1}{2}+\varepsilon\right)(\bar{\mu}_L+\bar{\mu}_H) - 2\varepsilon\bar{\mu}_H < 2n_B$. Contradiction, so $\pi_m > \frac{1}{2}$. ∎

2.A.4   Proof of Proposition 2

Proof. The results follow by taking the limits of the expressions for the maximal expected turnout obtained in Proposition 1, divided by $n \equiv n_A + n_B$. To make sure the proof of Proposition 1 works in the first place, notice that the incentive compatibility constraints (2.49)-(2.52) are well-behaved for all n and bounded. ∎

2.A.5 Proof of Proposition 3

Proof. The minimum case is different, because the smallest (and so potentially optimal) profile $(0,0) \in V_T$ for all $n_A, n_B \geq 1$. Nevertheless, the symmetric distribution construction derived in the proof of Proposition 1 can be applied here just as well. Notice first that $\mu_{0,0} \geq 0$ at the optimum. Using the latter and the fact that all profile probabilities sum up to one, rewrite the objective in (2.35) as

$$1 - \mu_{0,0} + \sum_{\{s| \sum s_i = 2\}} \mu(s) + 2 \sum_{\{s| \sum s_i = 3\}} \mu(s) + \ldots$$
$$+ (n-2) \sum_{\{s| \sum s_i = n-1\}} \mu(s) + (n-1)\mu_{n_A, n_B} \tag{2.98}$$

To minimize this expression, we want to increase $\mu_{0,0}$ as much as possible and set all remaining probabilities to their lowest possible level. Notice that profiles $(0,1)$ and $(1,0)$ are not directly present in (2.98), and profiles with total turnout of exactly two have the same (absolute) marginal effect on the objective as $\mu_{0,0}$.

The odd ICs for N_A and N_B, (2.49) and (2.51), respectively, restrict $\mu_{0,0}$ from above[26], and the exact bound depends on the ratio $\frac{\frac{1}{2}-c}{c}$. This ratio approaches zero when the cost increases towards 0.5, so for large enough cost, minimization requires placing the largest probability mass onto $(0,0)$ at the expense of other voting profiles (in particular, with total turnout of three or more), and so the minimal expected turnout approaches zero. The opposite happens when the cost is close to 0, because then $\frac{\frac{1}{2}-c}{c} \to \infty$, and hence (2.49) and (2.51) both require their right hand sides being close to zero. Since $n_A \geq n_B$, this is achieved by setting $\mu_{a,a} = 0$ for $a \in \{0, \ldots, n_B - 1\}$. The probability of the largest tied profile, μ_{n_B, n_B}, can be positive at the optimum for c close to zero, because $\frac{\frac{1}{2}-c}{c}\binom{n_A - 1}{n_B - 1}\mu_{n_B, n_B}$ restricts $\mu_{2,0}$ from above in the even IC for N_A, (2.50), and $\frac{\frac{1}{2}-c}{c}\binom{n_A}{n_B}\mu_{n_B, n_B}$ restricts $\mu_{0,2}$ from above in the even IC for N_B, (2.52), whereas for $n_A = n_B$, μ_{n_B, n_B} is not at all restricted by the odd IC constraints

---

[26]If $n_A = n_B$, the right hand sides of the ICs for N_A in (2.49) and (2.51) must be a bit adjusted to have the indices in the first summation go up to $n_B - 1$ instead of n_B.

(see ft.26), and for $n_A > n_B$, μ_{n_B,n_B} is only restricted by the odd IC for N_A. But in all cases, the value of the objective does not exceed 2: it is always optimal to shift the largest possible probability mass onto the profiles with total turnout between 0 to 2, so even when c is close to 0, the (possible) presence of non-zero terms with turnout exceeding 2 is compensated by their small equilibrium probabilities. The analytic expression for the missing cost-dependent part of the expected turnout, $\psi(c)$, can be now straightforwardly, though rather tediously (due to the equilibrium distribution support depending on the cost) derived using the same approach we applied in the proof of Proposition 1. For c close to 0, all IC constraints are binding, while for c close to 0.5, all but the even IC for N_B bind. ∎

2.A.6 Proof of Proposition 4

Proof. Any $t \in [f_*(c), f^*(c)]$ can be written as a linear combination of $f_*(c)$ and $f^*(c)$: $t = \lambda f_*(c) + (1 - \lambda)f^*(c)$ for some $\lambda \in [0, 1]$. Since $f_*(c), f^*(c)$ are expected turnouts in a min-turnout and max-turnout correlated equilibria (CE), and the set of CE payoffs is convex, t is also an expected turnout in some correlated equilibrium given by probability distribution $\lambda\mu^* + (1 - \lambda)\mu_*$. ∎

2.A.7 Proof of Lemma 2

Proof. By way of contradiction, suppose there is a correlated equilibrium with majority winning for sure. Then a profile $(0, 0)$ is not in equilibrium support. The only way the IC constraint for voters in N_B can be satisfied with a positive voting cost is to restrict the total probability mass to only those profiles where no one from N_B ever votes.[27] This leaves admissible only profiles with voters from N_A voting. Denote $\nu(1_i, a - 1, b)$ the equilibrium probability of a joint profile where $i \in N_A$ votes and

---

[27]Strictly speaking, in this case the conditional probability that a voting player from N_B is pivotal is not well-defined, so the corresponding IC constraint is vacuously satisfied.

there are $a - 1$ other players from N_A voting and b players from N_B voting.[28] The IC constraint for voter i from N_A (see (2.6)) takes the following form:

$$\sum_{a=2}^{n_A} \nu(1_i, a - 1, 0) \leq \frac{\frac{1}{2} - c}{c} \nu(1_i, 0, 0)$$

At the same time, the IC constraint for any non-voter from N_B (see (2.5)) can be written as

$$\sum_{a=2}^{n_A} \sum_{i \in N_A} \nu(1_i, a - 1, 0) \geq \frac{\frac{1}{2} - c}{c} \sum_{i \in N_A} \nu(1_i, 0, 0)$$

Clearly, both constraints cannot be satisfied simultaneously. Hence there is no correlated equilibrium with majority winning for sure. ∎

2.A.8 Proof of Proposition 6

Proof. Welfare maximizing correlated equilibria have $\Pr(majority\ wins)$ as large as possible and expected turnout as small as possible, so profiles of the form $(a + 1, a), a \in [0, n_B]$ must be in the support. The IC for non-voters in N_B is now binding at the optimum and implies

$$\binom{n_A}{2}\mu_{2,0} + \binom{n_B - 1}{1}\mu_{0,1} =$$
$$\frac{\frac{1}{2} - c}{c}\left[\sum_{a=0}^{n_B - 1}\binom{n_A}{a}\binom{n_B - 1}{a}\mu_{a,a} + \sum_{a=0}^{n_B - 1}\binom{n_A}{a+1}\binom{n_B - 1}{a}\mu_{a+1,a}\right], \qquad (2.99)$$

hence the equilibrium profiles must also include either $(2, 0)$, or $(0, 1)$, or both. Note though that at $(0, 1)$ majority loses, so including this profile decreases the probability that majority wins as well as expected welfare. It is also important that in (2.99) the probability of tied profiles restricts the probability mass distributed among the welfare-optimal profiles $(a + 1, a), a \in [0, n_B - 1]$, so there can be at most one tied

---

[28]This is a shorthand notation, which should be understood as a sum of probabilities of all joint profiles where i is voting and all remaining players behave as described.

profile in the equilibrium: (n_B, n_B).

The odd IC constraint for N_A implies

$$\sum_{a=0}^{n_B} \binom{n_A - 1}{a + 1}\binom{n_B}{a}\mu_{a+1,a} + \binom{n_A - 1}{2}\mu_{2,0} \geq \frac{\frac{1}{2} - c}{c}\left[\binom{n_A - 1}{n_B}\mu_{n_B, n_B} + \binom{n_B}{1}\mu_{0,1}\right]$$

This constraint is always slack at the optimum since it does not restrict the total probability of the welfare-maximizing profiles on the left hand side.

The even IC constraint for N_A implies

$$\binom{n_A - 1}{1}\mu_{2,0} \leq \frac{\frac{1}{2} - c}{c}\left(\sum_{a=0}^{n_B}\binom{n_A - 1}{a}\binom{n_B}{a}\mu_{a+1,a} + \binom{n_A - 1}{n_B - 1}\mu_{n_B, n_B}\right) \qquad (2.100)$$

and is binding at the optimum due to (2.99). Finally, the even IC constraint for N_B implies

$$\sum_{a=1}^{n_B}\binom{n_A}{a + 1}\binom{n_B - 1}{a - 1}\mu_{a+1,a} \leq \frac{\frac{1}{2} - c}{c}\left(\mu_{0,1} + \binom{n_A}{n_B}\mu_{n_B, n_B}\right) \qquad (2.101)$$

This constraint can be satisfied if either $(0, 1)$ or (n_B, n_B) are in the support[29], in which case the constraint is binding as long as some of the profiles $(a+1, a), a \in [1, n_B]$ are in the support. If neither $(0, 1)$ nor (n_B, n_B) is in the support, the constraint requires all profiles $(a + 1, a), a \in [1, n_B]$ not to be in the support as well.

Rewriting (2.99), we obtain

$$\binom{n_A}{2}\mu_{2,0} + \binom{n_B - 1}{1}\mu_{0,1} = \frac{\frac{1}{2} - c}{c}\sum_{a=0}^{n_B - 1}\binom{n_A}{a + 1}\binom{n_B - 1}{a}\mu_{a+1,a} \qquad (2.102)$$

---

[29]It is not optimal to include both profiles in the support, because then there is simultaneously a decrease in the probability majority wins and an increase in the expected turnout.

From (2.100),

$$(n_A - 1)\mu_{2,0} = \frac{\frac{1}{2} - c}{c}\left(\sum_{a=0}^{n_B}\binom{n_A - 1}{a}\binom{n_B}{a}\mu_{a+1,a} + \binom{n_A - 1}{n_B - 1}\mu_{n_B,n_B}\right) \qquad (2.103)$$

so we can rewrite (2.102) as

$$\frac{\frac{1}{2} - c}{c}\left[\sum_{a=0}^{n_B-1}\left[\binom{n_A}{a+1}\binom{n_B - 1}{a} - \frac{\binom{n_A}{2}}{n_A - 1}\binom{n_A - 1}{a}\binom{n_B}{a}\right]\mu_{a+1,a}\right] =$$
$$\frac{\frac{1}{2} - c}{c}\frac{\binom{n_A}{2}}{n_A - 1}\left(\binom{n_A - 1}{n_B}\mu_{n_B+1,n_B} + \binom{n_A - 1}{n_B - 1}\mu_{n_B,n_B}\right) + (n_B - 1)\mu_{0,1}$$

or

$$\frac{\frac{1}{2} - c}{2c}n_A\mu_{1,0} + \frac{\frac{1}{2} - c}{2c}\sum_{a=1}^{n_B}\binom{n_A}{a+1}\binom{n_B}{a}\left(1 - a\left(1 + \frac{2}{n_B}\right)\right)\mu_{a+1,a} =$$
$$\frac{\frac{1}{2} - c}{c}\binom{n_A}{n_B}\frac{n_B}{2}\mu_{n_B,n_B} + (n_B - 1)\mu_{0,1} \qquad (2.104)$$

Notice that all terms in the sum on the first line are negative if profiles $(a+1, a), a \in [1, n_B]$ are in the support.

The total probability constraint takes the following form:

$$\sum_{a=0}^{n_B}\binom{n_A}{a+1}\binom{n_B}{a}\mu_{a+1,a} + \binom{n_A}{2}\mu_{2,0} + \binom{n_B}{1}\mu_{0,1} + \binom{n_A}{n_B}\mu_{n_B,n_B} = 1$$

Plugging in the expression for $\mu_{2,0}$ from (2.103), we obtain

$$\sum_{a=0}^{n_B}\binom{n_A}{a+1}\binom{n_B}{a}\mu_{a+1,a}\left[1 + \frac{(\frac{1}{2} - c)(a+1)}{2c}\right]$$
$$+ \binom{n_A}{n_B}\mu_{n_B,n_B}\left[\frac{n_B(\frac{1}{2} - c)}{2c} + 1\right] + n_B\mu_{0,1} = 1 \qquad (2.105)$$

Using the total probability constraint, the total welfare minus the fixed n_B term can

be written as follows

$$W - n_B = (n_A - n_B) \left[1 - \frac{1}{2} \binom{n_A}{n_B} \mu_{n_B,n_B} - n_B \mu_{0,1} \right]$$

$$-c \left[\sum_{a=0}^{n_B} (2a+1) \binom{n_A}{a+1} \binom{n_B}{a} \mu_{a+1,a} + 2 \binom{n_A}{2} \mu_{2,0} + 2 n_B \binom{n_A}{n_B} \mu_{n_B,n_B} + n_B \mu_{0,1} \right]$$

Suppose all profiles $(a+1, a), a \in [1, n_B]$ are in the support, then IC constraint (2.101) is binding:

$$\sum_{a=0}^{n_B} a \binom{n_A}{a+1} \binom{n_B}{a} \mu_{a+1,a} = \frac{\frac{1}{2} - c}{c} \left(n_B \mu_{0,1} + n_B \binom{n_A}{n_B} \mu_{n_B,n_B} \right) \qquad (2.106)$$

Thus we can rewrite the above expression for welfare as

$$W - n_B = (n_A - n_B) \left[1 - \frac{1}{2} \binom{n_A}{n_B} \mu_{n_B,n_B} - n_B \mu_{0,1} \right]$$

$$-c \left[\frac{2(\frac{1}{2} - c)}{c} \left(n_B \mu_{0,1} + n_B \binom{n_A}{n_B} \mu_{n_B,n_B} \right) + 1 + \binom{n_A}{2} \mu_{2,0} + (2 n_B - 1) \binom{n_A}{n_B} \mu_{n_B,n_B} \right]$$

Simplifying, this equals

$$= n_A - n_B - c + \binom{n_A}{n_B} \mu_{n_B,n_B} \left[c - \frac{n_A + n_B}{2} \right] + n_B \mu_{0,1} \left[2c + n_B - n_A - 1 \right] - c \binom{n_A}{2} \mu_{2,0}$$

Plugging in the expression for $\mu_{2,0}$, we obtain

$$n_A - n_B - c + \binom{n_A}{n_B} \mu_{n_B,n_B} \left[c - \frac{n_A + n_B}{2} - \frac{n_B(\frac{1}{2} - c)}{2} \right] + n_B \mu_{0,1} \left[2c + n_B - n_A - 1 \right]$$

$$- \frac{\frac{1}{2} - c}{2} \left(\sum_{a=0}^{n_B} \binom{n_A}{a+1} \binom{n_B}{a} (a+1) \mu_{a+1,a} \right)$$

After tedious algebraic manipulations with the binding IC constraints, we can express

the sum of all $(a+1, a)$ profile probabilities as follows.

$$\frac{\frac{1}{2} - c}{2c} \sum_{a=0}^{n_B} \binom{n_A}{a+1}\binom{n_B}{a} \mu_{a+1,a} = \mu_{0,1}\left[n_B - 1 + \left(\frac{\frac{1}{2} - c}{c}\right)^2 (1 + \frac{n_B}{2})\right]$$

$$+ \binom{n_A}{n_B}\mu_{n_B,n_B}\left[\left(\frac{\frac{1}{2} - c}{c}\right)^2 (1 + \frac{n_B}{2}) + \frac{n_B(\frac{1}{2} - c)}{2c}\right]$$

Using this expression together with (2.106) to substitute the respective terms in the formula for welfare, we obtain

$$W = n_A - c + \binom{n_A}{n_B}\mu_{n_B,n_B}\left[c - \frac{n_A + n_B}{2} - \frac{n_B(\frac{1}{2} - c)}{2}\right] + n_B\mu_{0,1}\left[2c + n_B - n_A - 1\right]$$

$$-c\left[\mu_{0,1}\left[n_B - 1 + \left(\frac{\frac{1}{2} - c}{c}\right)^2 (1 + \frac{n_B}{2})\right] + \binom{n_A}{n_B}\mu_{n_B,n_B}\left[\left(\frac{\frac{1}{2} - c}{c}\right)^2 (1 + \frac{n_B}{2}) + \frac{n_B(\frac{1}{2} - c)}{2c}\right]\right]$$

$$- \frac{\frac{1}{2} - c}{2}\frac{\frac{1}{2} - c}{c}\left(n_B\mu_{0,1} + n_B\binom{n_A}{n_B}\mu_{n_B,n_B}\right)$$

Simplifying, we can finally write

$$W^* = n_A - c + \binom{n_A}{n_B}\mu_{n_B,n_B}\left[c - \frac{n_A + n_B + 2n_B(\frac{1}{2} - c)}{2} - \frac{(\frac{1}{2} - c)^2 (1 + n_B)}{c}\right] +$$

$$\mu_{0,1}\left[c(1 + n_B) + n_B\left[n_B - n_A - 1\right] - \frac{(\frac{1}{2} - c)^2 (1 + n_B)}{c}\right] \qquad (2.107)$$

From the binding IC constraints we obtain

$$\frac{\frac{1}{2} - c}{2c} \sum_{a=0}^{n_B} \binom{n_A}{a+1}\binom{n_B}{a} \mu_{a+1,a} = \mu_{0,1}\left[n_B - 1 + \left(\frac{\frac{1}{2} - c}{c}\right)^2 (1 + \frac{n_B}{2})\right]$$

$$+ \binom{n_A}{n_B}\mu_{n_B,n_B}\left[\left(\frac{\frac{1}{2} - c}{c}\right)^2 (1 + \frac{n_B}{2}) + \frac{(\frac{1}{2} - c)\, n_B}{2c}\right]$$

We can now write down the total probability constraint as follows

$$
\frac{c+\frac{1}{2}}{\frac{1}{2}-c}\left[\mu_{0,1}\left[n_B-1+\left(\frac{\frac{1}{2}-c}{c}\right)^2(1+\frac{n_B}{2})\right]+\binom{n_A}{n_B}\mu_{n_B,n_B}\left[\left(\frac{\frac{1}{2}-c}{c}\right)^2(1+\frac{n_B}{2})+\frac{(\frac{1}{2}-c)\,n_B}{2c}\right]\right]
$$

$$
+\frac{(\frac{1}{2}-c)^2}{2c^2}\left(n_B\mu_{0,1}+n_B\binom{n_A}{n_B}\mu_{n_B,n_B}\right)+\binom{n_A}{n_B}\mu_{n_B,n_B}\left[\frac{n_B(\frac{1}{2}-c)}{2c}+1\right]+n_B\mu_{0,1}=1
$$

Simplifying,

$$
\mu_{0,1}\left[\frac{n_B-(c+\frac{1}{2})}{\frac{1}{2}-c}+\frac{(c+\frac{1}{2}(n_B+1))(\frac{1}{2}-c)}{c^2}\right]
$$

$$
+\binom{n_A}{n_B}\mu_{n_B,n_B}\left[\frac{(c+\frac{1}{2}(n_B+1))(\frac{1}{2}-c)}{c^2}+\frac{n_B}{2c}+1\right]=1
$$

We can now estimate the effects of including either $(0,1)$ or (n_B,n_B) in the equilibrium support on welfare. First, let $\mu_{0,1}=0$, then

$$
\binom{n_A}{n_B}\mu_{n_B,n_B}=\frac{1}{\frac{(c+\frac{1}{2}(n_B+1))(\frac{1}{2}-c)}{c^2}+\frac{n_B}{2c}+1}
\tag{2.108}
$$

Plugging in to (2.107), we obtain

$$
W^*_{n_B,n_B}=n_A-c+\frac{\left[c-\frac{n_A+n_B+2n_B(\frac{1}{2}-c)}{2}-\frac{(\frac{1}{2}-c)^2(1+n_B)}{c}\right]}{\frac{(c+\frac{1}{2}(n_B+1))(\frac{1}{2}-c)}{c^2}+\frac{n_B}{2c}+1}
\tag{2.109}
$$

Second, let $\mu_{n_B,n_B}=0$, then

$$
\mu_{0,1}=\frac{1}{\left[\frac{n_B-(c+\frac{1}{2})}{\frac{1}{2}-c}+\frac{(c+\frac{1}{2}(n_B+1))(\frac{1}{2}-c)}{c^2}\right]}
\tag{2.110}
$$

Plugging in to (2.107), we obtain

$$W_{0,1}^* = n_A - c + \frac{\left[c(1+n_B) + n_B\left[n_B - n_A - 1\right] - \frac{\left(\frac{1}{2}-c\right)^2(1+n_B)}{c} \right]}{\frac{n_B - (c+\frac{1}{2})}{\frac{1}{2}-c} + \frac{(c+\frac{1}{2}(n_B+1))(\frac{1}{2}-c)}{c^2}} \quad (2.111)$$

Comparing $W_{0,1}$ and W_{n_B,n_B}, we see that (n_B, n_B) is in the support iff

$$\frac{\left[c - \frac{n_A+n_B+2n_B(\frac{1}{2}-c)}{2} - \frac{\left(\frac{1}{2}-c\right)^2(1+n_B)}{c} \right]}{\frac{(c+\frac{1}{2}(n_B+1))(\frac{1}{2}-c)}{c^2} + \frac{n_B}{2c} + 1} > \frac{\left[c(1+n_B) + n_B\left[n_B - n_A - 1\right] - \frac{\left(\frac{1}{2}-c\right)^2(1+n_B)}{c} \right]}{\frac{n_B - (c+\frac{1}{2})}{\frac{1}{2}-c} + \frac{(c+\frac{1}{2}(n_B+1))(\frac{1}{2}-c)}{c^2}}$$

which is equivalent to

$$2c\left[2c - cn_A - \frac{n_B+1}{2} \right] > \frac{(\frac{1}{2}-c)c(n_B+1)\left[cn_B\left[n_B - n_A\right] + c - \frac{n_B+1}{4} \right]}{c^2(\frac{3}{2}n_B - 1) - c\frac{2n_B+1}{4} + \frac{n_B+1}{8}}$$

It is straightforward to check that the denominator on the RHS is always positive for $n_B \geq 1$ and $c \in (0, 0.5)$, so we can rewrite

$$\left(2c^2(2 - n_A) - c(n_B + 1) \right)\left(c^2(\frac{3}{2}n_B - 1) - c\frac{2n_B+1}{4} + \frac{n_B+1}{8} \right) >$$
$$(\frac{c}{2} - c^2)(n_B + 1)\left[cn_B\left[n_B - n_A\right] + c - \frac{n_B+1}{4} \right]$$

This expression reduces to the following quadratic inequality:

$$c^2(2 - n_A)(3n_B - 2) - c\left[(n_B + 1)(n_B\left(\frac{3}{2} + n_A - n_B \right) - n_A) + \frac{n_A - 2}{2} \right]$$
$$+ \frac{(n_B + 1)(n_B - n_A)(\frac{1}{2} - n_B)}{2} > 0 \quad (2.112)$$

The discriminant of (2.112) is

$$D \equiv (n_B + 1)^2 \left(n_B \left(\frac{3}{2} + n_A - n_B \right) - n_A \right)^2 + \frac{(n_A - 2)^2}{4}$$
$$+ (n_A - 2)(n_B + 1) \left[\frac{n_B}{2} + (n_A - n_B)(1 + 6n_B(n_B - 1)) \right],$$

which is always positive for $n_A > 2 \geq n_B \geq 1$, so the cutoff cost is uniquely[30] determined by

$$c_* = \min \left\{ 0.5, \frac{(n_B + 1)(n_B(\frac{3}{2} + n_A - n_B) - n_A) + \frac{n_A - 2}{2} - \sqrt{D}}{2(2 - n_A)(3n_B - 2)} \right\} \quad (2.113)$$

Hence for $0 < c < c_*$ profile (n_B, n_B) is in the equilibrium support, and the optimal welfare is given by (2.109). For $c_* < c < 0.5$, profile (n_B, n_B) is not in the equilibrium support, but profile $(0, 1)$ is, and the optimal welfare is given by (2.111). These expressions were derived under the assumption that profiles $(a + 1, a), a \in [1, n_B]$ are in equilibrium support. To conclude the proof, we need to consider the case where the cost is so high that these profiles are not in the support, and constraint (2.101) is slack. In this case, of course, (n_B, n_B) is not in the equilibrium support.

Suppose that profiles $(a + 1, a), a \in \{1, \ldots, n_B\}$ are not in the equilibrium support. Then from (2.104),

$$\frac{\frac{1}{2} - c}{c} \mu_{1,0} = \frac{2(n_B - 1)}{n_A} \mu_{0,1}$$

Using the total probability constraint, we can now write

$$\binom{n_A}{1} \mu_{1,0} + \binom{n_A}{2} \mu_{2,0} + \binom{n_B}{1} \mu_{0,1} = 1$$

or

$$\mu_{0,1} \left[\frac{2(n_B - 1)}{\frac{1}{2} - c} + 2n_B - 1 \right] = 1$$

_____

[30]The other critical cost value is always negative, so outside the range of $(0, 0.5)$.

So we obtain

$$\mu_{0,1} = \frac{\frac{1}{2} - c}{n_B - \left(\frac{1}{2} + c\right)}$$

$$\mu_{1,0} = \frac{2c(n_B - 1)}{n_A \left[n_B - \left(\frac{1}{2} + c\right)\right]}$$

$$\mu_{2,0} = \frac{\left(\frac{1}{2} - c\right)(n_B - 1)}{\binom{n_A}{2}\left[n_B - \left(\frac{1}{2} + c\right)\right]}$$

and the optimal welfare is

$$W^* = \frac{n_B(n_B - c)(\frac{1}{2} - c) + 2c(n_B - 1)(n_A - c) + (n_A - 2c)\left(\frac{1}{2} - c\right)(n_B - 1)}{n_B - \left(\frac{1}{2} + c\right)} \quad (2.114)$$

We can now compare W^* with $W^*_{0,1}$ to obtain the condition on the cost for which $(a+1, a), a \in [1, n_B]$ are not in the support: this is so iff

$$n_A - c + \frac{\left(\frac{1}{2} - c\right)c^2 \left[c(1 + n_B) + n_B\left[n_B - n_A - 1\right] - \frac{\left(\frac{1}{2} - c\right)^2(1 + n_B)}{c}\right]}{c^2(n_B - (c + \frac{1}{2})) + (c + \frac{1}{2}(n_B + 1))(\frac{1}{2} - c)^2} <$$

$$\frac{n_B(n_B - c)(\frac{1}{2} - c) + 2c(n_B - 1)(n_A - c) + (n_A - 2c)\left(\frac{1}{2} - c\right)(n_B - 1)}{n_B - \left(\frac{1}{2} + c\right)}$$

which is equivalent to the following cubic inequality:

$$c^3 \left(n_A + \frac{n_B - 5}{2}\right) + \frac{c^2}{2}\left((n_A - n_B)(n_B - 1) + 3 - n_B\right)$$

$$- \frac{c}{4}\left(\frac{n_B + 1}{2} + (n_A - n_B)(2n_B + 1)\right) + \frac{(n_A - n_B)(n_B + 1)}{8} < 0$$

∎

2.A.9 Proof of Proposition 7

Proof. The proof closely follows the proof of Proposition 1. Assume that the optimal correlated equilibrium distribution is symmetric, as defined there. Although the voting costs are heterogeneous, it is easy to see that once constraints (2.38) hold for the players with the lowest cost in each group, and constraints (2.39) hold for the players with the highest cost, the incentive compatibility constraints for all players will hold automatically.

Using symmetry, we obtain the following system of four inequalities with respect to $(n_A + 1)(n_B + 1)$ variables of the form $\mu_{a,b}$:

$$\sum_{a=1}^{n_A-1} \sum_{b=0}^{\min\{a-1,n_B\}} \binom{n_A-1}{a}\binom{n_B}{b}\mu_{a,b} + \sum_{a=0}^{n_B-2} \sum_{b=a+2}^{n_B} \binom{n_A-1}{a}\binom{n_B}{b}\mu_{a,b} \geq$$
$$\frac{\frac{1}{2}-\underline{c}_A}{\underline{c}_A}\left(\sum_{a=0}^{n_B}\binom{n_A-1}{a}\binom{n_B}{a}\mu_{a,a} + \sum_{a=0}^{n_B-1}\binom{n_A-1}{a}\binom{n_B}{a+1}\mu_{a,a+1}\right) \quad (2.115)$$

$$\sum_{a=2}^{n_A} \sum_{b=0}^{\min\{a-2,n_B\}} \binom{n_A-1}{a-1}\binom{n_B}{b}\mu_{a,b} + \sum_{a=1}^{n_B-1} \sum_{b=a+1}^{n_B} \binom{n_A-1}{a-1}\binom{n_B}{b}\mu_{a,b} \leq$$
$$\frac{\frac{1}{2}-\bar{c}_A}{\bar{c}_A}\left(\sum_{a=0}^{n_B}\binom{n_A-1}{a}\binom{n_B}{a}\mu_{a+1,a} + \sum_{a=1}^{n_B}\binom{n_A-1}{a-1}\binom{n_B}{a}\mu_{a,a}\right) \quad (2.116)$$

and

$$\sum_{a=2}^{n_A} \sum_{b=0}^{\min\{a-2,n_B-1\}} \binom{n_A}{a}\binom{n_B-1}{b}\mu_{a,b} + \sum_{a=0}^{n_B-2}\sum_{b=a+1}^{n_B-1}\binom{n_A}{a}\binom{n_B-1}{b}\mu_{a,b} \geq$$

$$\frac{\frac{1}{2}-\underline{c}_B}{\underline{c}_B}\left(\sum_{a=0}^{n_B-1}\binom{n_A}{a}\binom{n_B-1}{a}\mu_{a,a} + \sum_{a=0}^{n_B-1}\binom{n_A}{a+1}\binom{n_B-1}{a}\mu_{a+1,a}\right) \quad (2.117)$$

$$\sum_{a=2}^{n_A}\sum_{b=1}^{\min\{a-1,n_B\}}\binom{n_A}{a}\binom{n_B-1}{b-1}\mu_{a,b} + \sum_{a=0}^{n_B-2}\sum_{b=a+2}^{n_B}\binom{n_A}{a}\binom{n_B-1}{b-1}\mu_{a,b} \leq$$

$$\frac{\frac{1}{2}-\bar{c}_B}{\bar{c}_B}\left(\sum_{a=0}^{n_B-1}\binom{n_A}{a}\binom{n_B-1}{a}\mu_{a,a+1} + \sum_{a=1}^{n_B}\binom{n_A}{a}\binom{n_B-1}{a-1}\mu_{a,a}\right) \quad (2.118)$$

We will refer to the first and the third inequality above as the odd incentive compatibility (IC) constraints, and to the second and the fourth inequality as the even IC constraints, distinguished by the group.

Since $n_A > n_B$, at the largest turnout profile μ_{n_A,n_B} all voters from N_B (as well as voters from N_A, if $n_A > n_B + 1$) are dummies. Hence the even IC constraint for N_B is always binding at the optimum.

The even IC constraint for N_A requires

$$\mu_{n_A,n_B} \leq \frac{\frac{1}{2}-\bar{c}_A}{\bar{c}_A}\left(\sum_{a=0}^{n_B-1}\binom{n_A-1}{a}\binom{n_B}{a}\mu_{a+1,a} + \sum_{a=1}^{n_B}\binom{n_A-1}{a-1}\binom{n_B}{a}\mu_{a,a}\right)$$

$$-\left[\sum_{b=1}^{n_B-1}\binom{n_B}{b}\mu_{n_A,b} + \sum_{a=3}^{n_B}\sum_{b=1}^{a-2}\binom{n_A-1}{a-1}\binom{n_B}{b}\mu_{a,b}\right.$$

$$+\sum_{a=n_B+2}^{n_A-1}\sum_{b=1}^{n_B}\binom{n_A-1}{a-1}\binom{n_B}{b}\mu_{a,b} + \sum_{a=1}^{n_B-2}\sum_{b=a+2}^{n_B}\binom{n_A-1}{a-1}\binom{n_B}{b}\mu_{a,b}\right]$$

$$-\sum_{a=1}^{n_B-1}\binom{n_A-1}{a-1}\binom{n_B}{a+1}\mu_{a,a+1} - \sum_{b=1}^{n_B}\binom{n_A-1}{n_B}\binom{n_B}{b}\mu_{n_B+1,b} - \sum_{a=2}^{n_B}\binom{n_A-1}{a-1}\mu_{a,0}$$

$$+\frac{1}{2\bar{c}_A}\binom{n_A-1}{n_B}\mu_{n_B+1,n_B} - \sum_{a=n_B+1}^{n_A}\binom{n_A-1}{a-1}\mu_{a,0} \quad (2.119)$$

The binding even IC for N_B requires

$$\mu_{n_A,n_B} = \frac{\frac{1}{2} - \bar{c}_B}{\bar{c}_B} \left(\sum_{a=0}^{n_B-1} \binom{n_A}{a}\binom{n_B-1}{a}\mu_{a,a+1} + \sum_{a=1}^{n_B} \binom{n_A}{a}\binom{n_B-1}{a-1}\mu_{a,a} \right)$$

$$- \left[\sum_{b=1}^{n_B-1} \binom{n_B-1}{b-1}\mu_{n_A,b} + \sum_{a=3}^{n_B}\sum_{b=1}^{a-2} \binom{n_A}{a}\binom{n_B-1}{b-1}\mu_{a,b} \right.$$

$$+ \sum_{a=n_B+2}^{n_A-1}\sum_{b=1}^{n_B} \binom{n_A}{a}\binom{n_B-1}{b-1}\mu_{a,b} + \sum_{a=1}^{n_B-2}\sum_{b=a+2}^{n_B} \binom{n_A}{a}\binom{n_B-1}{b-1}\mu_{a,b} \left. \right]$$

$$- \sum_{a=1}^{n_B-1} \binom{n_A}{a+1}\binom{n_B-1}{a-1}\mu_{a+1,a} - \sum_{b=1}^{n_B} \binom{n_A}{n_B+1}\binom{n_B-1}{b-1}\mu_{n_B+1,b}$$

$$- \sum_{b=2}^{n_B} \binom{n_B-1}{b-1}\mu_{0,b} \qquad (2.120)$$

These expressions immediately imply that the *odd* IC for N_A is always binding at the optimum. To see this, notice that in the binding constraint (2.120) all profiles relevant for the odd IC for N_A, i.e. those where a non-voter from N_A is a dummy, have the negative sign and so must be reduced as much as possible at the optimum. The only subset of profiles where a non-voter from N_A is a dummy that is not directly restricted by (2.120) has the form $\sum_{a=1}^{n_A-1} \binom{n_A-1}{a-1}\mu_{a,0}$. But these profiles are restricted by (2.119). If the latter is binding, the restriction is trivial. Suppose not, then if we reduced all directly restricted by (2.120) probabilities to their lower limit of zero and the odd IC for N_A was still not binding, then constraint (2.119) (slack by assumption) would imply that $\mu_{n_A,n_B} < 0$. Therefore, the odd IC for N_A must

bind at the optimum, so we can write it as

$$\sum_{a=n_B+1}^{n_A-1} \binom{n_A-1}{a} \mu_{a,n_B} - \frac{\frac{1}{2} - \underline{c}_A}{\underline{c}_A} \binom{n_A-1}{n_B} \mu_{n_B,n_B}$$

$$+ \binom{n_A-1}{n_B} \binom{n_B}{n_B-1} \mu_{n_B,n_B-1} + \sum_{a=0}^{n_B-2} \binom{n_A-1}{a} \mu_{a,n_B}$$

$$+ \sum_{a=0}^{n_B-2} \binom{n_A-1}{a+1} \binom{n_B}{a} \mu_{a+1,a} + \sum_{a=2}^{n_B+1} \sum_{b=0}^{a-2} \binom{n_A-1}{a} \binom{n_B}{b} \mu_{a,b}$$

$$+ \sum_{a=n_B+2}^{n_A-1} \sum_{b=0}^{n_B-1} \binom{n_A-1}{a} \binom{n_B}{b} \mu_{a,b} + \sum_{a=0}^{n_B-3} \sum_{b=a+2}^{n_B-1} \binom{n_A-1}{a} \binom{n_B}{b} \mu_{a,b} =$$

$$\frac{\frac{1}{2} - \underline{c}_A}{\underline{c}_A} \left(\sum_{a=0}^{n_B-1} \binom{n_A-1}{a} \binom{n_B}{a} \mu_{a,a} + \sum_{a=0}^{n_B-1} \binom{n_A-1}{a} \binom{n_B}{a+1} \mu_{a,a+1} \right) \quad (2.121)$$

Thus for all cost thresholds the even IC for N_B and the odd IC for N_A are binding. Proceeding exactly as in the proof of Proposition 1, we can express μ_{n_A,n_B} from (2.120) (the binding even IC for N_B) and then substitute the term $\binom{n_A-1}{n_B-1}\mu_{n_B-1,n_B}$ using (2.121) (the binding odd IC for N_A). The resulting expressions are the modified versions of (2.60), (2.61), and (2.62), respectively:

$$\mu_{n_A,n_B} = \frac{\frac{1}{2} - \bar{c}_B}{\bar{c}_B} \left(\sum_{a=0}^{n_B-1} \binom{n_A}{a} \binom{n_B-1}{a} \mu_{a,a+1} + \sum_{a=1}^{n_B} \binom{n_A}{a} \binom{n_B-1}{a-1} \mu_{a,a} \right)$$

$$- \left[\sum_{b=1}^{n_B-1} \binom{n_B-1}{b-1} \mu_{n_A,b} + \sum_{a=2}^{n_B} \sum_{b=1}^{a-1} \binom{n_A}{a} \binom{n_B-1}{b-1} \mu_{a,b} \right.$$

$$\left. + \sum_{a=n_B+1}^{n_A-1} \sum_{b=1}^{n_B} \binom{n_A}{a} \binom{n_B-1}{b-1} \mu_{a,b} + \sum_{a=0}^{n_B-2} \sum_{b=a+2}^{n_B} \binom{n_A}{a} \binom{n_B-1}{b-1} \mu_{a,b} \right], \quad (2.122)$$

$$\binom{n_A - 1}{n_B - 1} \cdot \mu_{n_B-1,n_B} = \frac{\underline{c}_A}{\frac{1}{2} - \underline{c}_A} \left[\sum_{a=1}^{n_B+1} \sum_{b=0}^{a-1} \binom{n_A - 1}{a} \binom{n_B}{b} \mu_{a,b} \right.$$

$$+ \sum_{a=n_B+2}^{n_A-1} \sum_{b=0}^{n_B} \binom{n_A - 1}{a} \binom{n_B}{b} \mu_{a,b} + \sum_{a=0}^{n_B-2} \sum_{b=a+2}^{n_B} \binom{n_A - 1}{a} \binom{n_B}{b} \mu_{a,b} \right]$$

$$- \left(\sum_{a=0}^{n_B} \binom{n_A - 1}{a} \binom{n_B}{a} \mu_{a,a} + \sum_{a=0}^{n_B-2} \binom{n_A - 1}{a} \binom{n_B}{a+1} \mu_{a,a+1} \right), \quad (2.123)$$

and

$$\mu_{n_A,n_B} = \frac{\frac{1}{2} - \bar{c}_B}{\bar{c}_B} \left(- \sum_{a=0}^{n_B-2} \binom{n_A}{a} \binom{n_B - 1}{a+1} \frac{n_A + 1}{n_A - n_B + 1} \mu_{a,a+1} \right.$$

$$\left. + \sum_{a=1}^{n_B} \binom{n_A}{a} \binom{n_B}{a} \frac{a(n_A + 1) - n_B n_A}{n_B(n_A - n_B + 1)} \mu_{a,a} \right)$$

$$- \left[\sum_{b=1}^{n_B-1} \binom{n_B - 1}{b - 1} \mu_{n_A,b} + \sum_{a=2}^{n_B} \sum_{b=1}^{a-1} \binom{n_A}{a} \binom{n_B - 1}{b - 1} \mu_{a,b} \right.$$

$$\left. + \sum_{a=n_B+1}^{n_A-1} \sum_{b=1}^{n_B} \binom{n_A}{a} \binom{n_B - 1}{b - 1} \mu_{a,b} + \sum_{a=0}^{n_B-2} \sum_{b=a+2}^{n_B} \binom{n_A}{a} \binom{n_B - 1}{b - 1} \mu_{a,b} \right]$$

$$+ \frac{\frac{1}{2} - \bar{c}_B}{\bar{c}_B} \frac{\underline{c}_A}{\left(\frac{1}{2} - \underline{c}_A\right)} \frac{n_A}{n_A - n_B + 1} \left[\sum_{a=1}^{n_B+1} \sum_{b=0}^{a-1} \binom{n_A - 1}{a} \binom{n_B}{b} \mu_{a,b} \right.$$

$$\left. + \sum_{a=n_B+2}^{n_A-1} \sum_{b=0}^{n_B} \binom{n_A - 1}{a} \binom{n_B}{b} \mu_{a,b} + \sum_{a=0}^{n_B-2} \sum_{b=a+2}^{n_B} \binom{n_A - 1}{a} \binom{n_B}{b} \mu_{a,b} \right] \quad (2.124)$$

The key difference between (2.124) and (2.62) is that in (2.124), all terms in the last square brackets have an additional multiplier $\frac{\frac{1}{2} - \bar{c}_B}{\bar{c}_B} \frac{\underline{c}_A}{\left(\frac{1}{2} - \underline{c}_A\right)}$. The equivalence between (2.124) and (2.62) holds if and only if this multiplier equals 1, that is, if and only if $\underline{c}_A = \bar{c}_B$. If these cost thresholds are different, the equilibrium probability of the largest profile, as well as other profiles in the equilibrium support, will be different than in either of the cases considered in Proposition 1, and therefore, the corresponding maximal expected turnout will be different from f^*, the maximal expected turnout in Proposition 1. How much different depends on the relation between $\underline{c}_A$

and $\bar{c}_B$. Suppose that $\underline{c}_A < \bar{c}_B$. Then a simple contradiction argument implies

$$\frac{\frac{1}{2} - \bar{c}_B}{\bar{c}_B} \frac{\underline{c}_A}{\left(\frac{1}{2} - \underline{c}_A\right)} < 1 \tag{2.125}$$

In this case, maximization implies placing a smaller probability mass on the largest turnout profile than in Proposition 1.[31] Therefore, the expected turnout is lower than in Proposition 1 for all costs satisfying this condition. It is easy to show that if $\underline{c}_A > \bar{c}_B$, the opposite inequality holds in (2.125) and the equilibrium distribution places a larger mass on μ_{n_A,n_B}, so the expected turnout is higher than in Proposition 1.

We will now show that if $\underline{c}_A = \bar{c}_B$, the equilibrium distribution support does not change, and the maximal expected turnout corresponds to f^* from Proposition 1.[32]

The remaining IC constraints, the even IC for N_A and the odd IC for N_B, generally exhibit more complicated bind/slack properties. Unlike the homogenous cost case, their behavior at the optimum depends on the relations between the cost thresholds. We claim, however, that if $\underline{c}_A = \bar{c}_B = c$, the IC constraints exhibit the same behavior as in Proposition 1: for $n_B > \lceil \frac{1}{2} n_A \rceil$ the even IC for N_A is slack for all admissible values of c and the remaining cost thresholds, $\bar{c}_A$ and $\underline{c}_B$, and the odd IC for N_B is binding. For $n_B < \lceil \frac{1}{2} n_A \rceil$ the even IC for N_A is binding, and the odd IC for N_B is slack.

Let $n_B > \lceil \frac{1}{2} n_A \rceil$. Comparing (2.120) and (2.119), we see that the RHS of (2.120) cannot be strictly greater than the RHS of (2.119), since if this was the case, the even IC for N_A would not hold at all at the optimum of the constrained maximization program. If the RHS of (2.120) is strictly less than the RHS of (2.119), then the even IC for N_A is slack, as we claim. The critical case is when the two RHS are the

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[31]Note that this is true even if $\mu_{n_A,n_B} = 0$ at the optimum, because in this case decreasing the RHS of (2.124) means smaller probability of the next largest profile in the equilibrium support.

[32]This is exactly so for case (i), where the expected turnout does not depend on the cost. For case(ii), which is cost dependent, we will show that the expected turnout exhibits the same cost-dependent dynamics as in Proposition 1.

same. Since by assumption $n_B > \lceil \frac{1}{2} n_A \rceil$, we have $2n_B \geq n_A + 1$, so $2n_B + 1 > n_A$. Then the total turnout of the first term on the last line of (2.119), $2n_B + 1$, exceeds the total turnout of the largest term in the remaining summation on that line, which is n_A, achieved at $\mu_{n_A,0}$. Hence at the optimum

$$\frac{1}{2\bar{c}_A} \binom{n_A - 1}{n_B} \mu_{n_B+1,n_B} - \sum_{a=n_B+1}^{n_A} \binom{n_A - 1}{a - 1} \mu_{a,0} > 0$$

Since $\underline{c}_A = \bar{c}_B = c$, a simple contradiction argument implies that

$$\frac{\frac{1}{2} - \bar{c}_A}{\bar{c}_A} \leq \frac{\frac{1}{2} - \bar{c}_B}{\bar{c}_B} \tag{2.126}$$

At the same time, $\frac{\frac{1}{2} - \bar{c}_A}{\bar{c}_A} < \frac{1}{2\bar{c}_A}$, and the largest turnout in the parentheses on the first line of (2.119), $2n_B$, is less than $2n_B + 1$, the turnout of the term with coefficient $\frac{1}{2\bar{c}_A}$ on the last line of (2.119). Optimization implies that the effect of this latter term must exceed the effect of the former. Therefore at the optimum the RHS of (2.120) is strictly less than the RHS of (2.119), and so the even IC for N_A is slack.

Turning to the odd IC constraints, we can rewrite (2.117) as

$$\sum_{b=0}^{n_B-1} \binom{n_B - 1}{b} \mu_{n_A,b}$$
$$+ \sum_{a=0}^{n_B-2} \binom{n_A}{a} \binom{n_B - 1}{a + 1} \mu_{a,a+1} + \sum_{a=2}^{n_B+1} \sum_{b=0}^{a-2} \binom{n_A}{a} \binom{n_B - 1}{b} \mu_{a,b}$$
$$+ \sum_{a=n_B+2}^{n_A-1} \sum_{b=0}^{n_B-1} \binom{n_A}{a} \binom{n_B - 1}{b} \mu_{a,b} + \sum_{a=0}^{n_B-3} \sum_{b=a+2}^{n_B-1} \binom{n_A}{a} \binom{n_B - 1}{b} \mu_{a,b} \geq$$
$$\frac{\frac{1}{2} - \underline{c}_B}{\underline{c}_B} \left(\sum_{a=0}^{n_B-1} \binom{n_A}{a} \binom{n_B - 1}{a} \mu_{a,a} + \sum_{a=0}^{n_B-1} \binom{n_A}{a + 1} \binom{n_B - 1}{a} \mu_{a+1,a} \right) \tag{2.127}$$

Comparing (2.127) with (2.121), we again see that, except for the terms on the first two lines of (2.121) and those on the first line of (2.127), in every remaining profile

of (2.121) the total turnout matches exactly the total turnout in the corresponding term of (2.127). Since $\underline{c}_A = \bar{c}_B$, we have

$$\frac{\frac{1}{2} - \underline{c}_A}{\underline{c}_A} \leq \frac{\frac{1}{2} - \underline{c}_B}{\underline{c}_B} \tag{2.128}$$

Then at the optimum

$$\sum_{b=0}^{n_B-1} \binom{n_B-1}{b} \mu_{n_A,b} + \frac{1}{2c} \binom{n_A-1}{n_B} \mu_{n_B,n_B} \leq$$
$$\sum_{a=n_B+1}^{n_A-1} \binom{n_A-1}{a} \mu_{a,n_B} + \binom{n_A-1}{n_B} \mu_{n_B,n_B}$$
$$+ \binom{n_A-1}{n_B}\binom{n_B}{n_B-1} \mu_{n_B,n_B-1} + \sum_{a=0}^{n_B-2} \binom{n_A-1}{a} \mu_{a,n_B} \tag{2.129}$$

This follows, since $n_B > \lceil \frac{1}{2}n_A \rceil$, so $2n_B \geq n_A+1$ and all profiles on the LHS of (2.129) are matched by the corresponding profiles on the RHS. Together with (2.128), this implies that the odd IC for N_B is binding.

We can now proceed exactly as in the proof of Proposition 1. One can even show that the expression for the probability of the largest tie remains the same:

$$\mu_{n_B,n_B} = \frac{2c}{\binom{n_A}{n_B}}, \tag{2.130}$$

where $c = \underline{c}_A = \bar{c}_B$. Substituting the expressions for the probabilities of the largest profiles into the objective function, we obtain $h^* = 2n_B$. This completes the proof of case (i).

Now suppose $n_B < \lceil \frac{1}{2}n_A \rceil$, then $2n_B < n_A$. Analogously to case (ii) of Proposition 1, we have the even IC for N_A binding and the odd IC for N_B slack at the optimum, if $\underline{c}_A = \bar{c}_B = c$. Due to the odd IC for N_A and the even IC for N_B binding[33], we can express the probability of the largest profile as

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[33]These constraints bind for case (i) just as well.

$$\mu_{n_A,n_B} = \frac{\frac{1}{2}-c}{c}\left(-\sum_{a=0}^{n_B-2}\binom{n_A}{a}\binom{n_B-1}{a+1}\frac{n_A+1}{n_A-n_B+1}\mu_{a,a+1}\right.$$

$$\left.+\sum_{a=1}^{n_B}\binom{n_A}{a}\binom{n_B}{a}\mu_{a,a}\left(\frac{a(n_A+1)-n_An_B}{n_B(n_A-n_B+1)}\right)\right)$$

$$+\sum_{a=0}^{n_B-2}\sum_{b=a+2}^{n_B}\binom{n_A}{a}\binom{n_B}{b}\left(\frac{(n_B-b)(n_A+1)+(b-a-1)n_B}{n_B(n_A-n_B+1)}\right)\mu_{a,b}$$

$$+\sum_{a=n_B+2}^{n_A-1}\sum_{b=1}^{n_B}\binom{n_A}{a}\binom{n_B}{b}\left(\frac{n_B(n_A+b-a)-b(n_A+1)}{n_B(n_A-n_B+1)}\right)\mu_{a,b}$$

$$+\sum_{a=2}^{n_B}\sum_{b=1}^{a-1}\binom{n_A}{a}\binom{n_B}{b}\left(\frac{n_B(n_A+b-a)-b(n_A+1)}{n_B(n_A-n_B+1)}\right)\mu_{a,b}$$

$$+\sum_{b=1}^{n_B}\binom{n_A}{n_B+1}\binom{n_B}{b}\left(\frac{(n_A-n_B)(n_B-b)-(n_B+b)}{n_B(n_A-n_B+1)}\right)\mu_{n_B+1,b}$$

$$+\frac{n_A}{n_A-n_B+1}\sum_{a=1}^{n_A-1}\binom{n_A-1}{a}\mu_{a,0}-\sum_{b=1}^{n_B-1}\binom{n_B-1}{b-1}\mu_{n_A,b} \qquad (2.131)$$

On the other hand, the even IC for N_A binding implies

$$\mu_{n_A,n_B} = \frac{\frac{1}{2}-\bar{c}_A}{\bar{c}_A}\left(\sum_{a=0}^{n_B}\binom{n_A-1}{a}\binom{n_B}{a}\mu_{a+1,a}+\sum_{a=1}^{n_B}\binom{n_A-1}{a-1}\binom{n_B}{a}\mu_{a,a}\right)$$

$$-\left[\sum_{b=1}^{n_B-1}\binom{n_B}{b}\mu_{n_A,b}+\sum_{a=3}^{n_B}\sum_{b=1}^{a-2}\binom{n_A-1}{a-1}\binom{n_B}{b}\mu_{a,b}\right.$$

$$\left.+\sum_{a=n_B+2}^{n_A-1}\sum_{b=1}^{n_B}\binom{n_A-1}{a-1}\binom{n_B}{b}\mu_{a,b}+\sum_{a=1}^{n_B-2}\sum_{b=a+2}^{n_B}\binom{n_A-1}{a-1}\binom{n_B}{b}\mu_{a,b}\right]$$

$$-\sum_{a=1}^{n_B-1}\binom{n_A-1}{a-1}\binom{n_B}{a+1}\mu_{a,a+1}-\sum_{b=1}^{n_B-1}\binom{n_A-1}{n_B}\binom{n_B}{b}\mu_{n_B+1,b}$$

$$-\sum_{a=2}^{n_A}\binom{n_A-1}{a-1}\mu_{a,0} \qquad (2.132)$$

Comparing (2.132) with (2.131), taking into account that $\bar{c}_A \geq c$ and the odd IC for

N_B is slack, we see that at the optimum, just like in case (ii) of Proposition 1,

$$\mu_{a,a+1} = 0, a \in \{0, \dots, n_B - 1\} \tag{2.133}$$

$$\mu_{a,a} = 0, a \in \{0, \dots, n_B - 1\} \tag{2.134}$$

$$\mu_{n_B+1,b} = 0, b \in \{1, \dots, n_B - 1\} \tag{2.135}$$

$$\mu_{a,b} = 0, a \in \{n_B + 2, \dots, n_A - 1\}, b \in \{0, \dots, n_B\} \tag{2.136}$$

$$\mu_{a,b} = 0, a \in \{3, \dots, n_B + 1\}, b \in \{1, \dots, a - 2\} \tag{2.137}$$

$$\mu_{a,b} = 0, a \in \{0, \dots, n_B - 2\}, b \in \{a + 2, \dots, n_B\} \tag{2.138}$$

Given (2.133)-(2.138), we can rewrite (2.132) as

$$\mu_{n_A,n_B} = \frac{\frac{1}{2} - \bar{c}_A}{\bar{c}_A} \sum_{a=0}^{n_B} \binom{n_A - 1}{a} \binom{n_B}{a} \mu_{a+1,a} + \frac{\frac{1}{2} - \bar{c}_A}{\bar{c}_A} \binom{n_A - 1}{n_B - 1} \mu_{n_B,n_B}$$
$$- \sum_{b=1}^{n_B-1} \binom{n_B}{b} \mu_{n_A,b} - \sum_{a=2}^{n_B+1} \binom{n_A - 1}{a - 1} \mu_{a,0} - \mu_{n_A,0} \tag{2.139}$$

We also rewrite (2.131) as

$$\mu_{n_A,n_B} = \sum_{a=0}^{n_B} \binom{n_A}{a + 1} \binom{n_B}{a} \frac{n_B(n_A - 1) - a(n_A + 1)}{n_B(n_A - n_B + 1)} \mu_{a+1,a}$$
$$- \sum_{b=1}^{n_B-1} \binom{n_B - 1}{b - 1} \mu_{n_A,b} + \frac{\left(\frac{1}{2} - c\right) n_A}{c n_B(n_A - n_B + 1)} \binom{n_A - 1}{n_B - 1} \mu_{n_B,n_B}$$
$$+ \frac{n_A}{n_A - n_B + 1} \sum_{a=2}^{n_B+1} \binom{n_A - 1}{a} \mu_{a,0} \tag{2.140}$$

Now (2.139) and (2.140) imply that at the optimum

$$\mu_{n_A,b} = 0, b \in \{1, \dots, n_B - 1\} \tag{2.141}$$

In addition, the slack odd IC for N_B, given (2.133)-(2.138), takes the form

$$\mu_{n_A,0} + \sum_{a=2}^{n_B+1} \binom{n_A}{a}\mu_{a,0} > \frac{\frac{1}{2}-\underline{c}_B}{\underline{c}_B} \sum_{a=0}^{n_B-1} \binom{n_A}{a+1}\binom{n_B-1}{a}\mu_{a+1,a} \qquad (2.142)$$

Together with $2n_B \leq n_A - 1$, this implies that at the optimum $\mu_{a,0} = 0, a \in \{2,\ldots,n_B+1\}$, and hence the support of the distribution includes only the profiles of the form $(a+1,a), a \in \{0,\ldots,n_B\}$, (n_B, n_B) and $(n_A, 0)$. In particular, $\mu_{n_A,n_B} = 0$, since from (2.142) and (2.139), $\mu_{n_A,0}$ offsets μ_{n_B+1,n_B} and μ_{n_B,n_B} (from the maximization point of view, the profiles with higher turnout must receive larger probability weights).

Hence we can rewrite (2.139) as

$$\mu_{n_A,n_B} = 0 = \frac{\frac{1}{2}-\bar{c}_A}{\bar{c}_A}\left(\sum_{a=0}^{n_B} \binom{n_A-1}{a}\binom{n_B}{a}\mu_{a+1,a} + \binom{n_A-1}{n_B-1}\mu_{n_B,n_B}\right) - \mu_{n_A,0} \quad (2.143)$$

The probability constraint can now be written as

$$\sum_{a=0}^{n_B} \binom{n_A}{a+1}\binom{n_B}{a}\mu_{a+1,a} + \binom{n_A}{n_B}\mu_{n_B,n_B} + \frac{\frac{1}{2}-\bar{c}_A}{\bar{c}_A}\left(\sum_{a=0}^{n_B} \binom{n_A-1}{a}\binom{n_B}{a}\mu_{a+1,a} + \binom{n_A-1}{n_B-1}\mu_{n_B,n_B}\right) = 1$$

Simplifying,

$$\sum_{a=0}^{n_B} \binom{n_A}{a+1}\binom{n_B}{a}\left(\frac{n_A+(a+1)(\frac{1}{2\bar{c}_A}-1)}{n_A}\right)\mu_{a+1,a} + \binom{n_A-1}{n_B-1}\left(\frac{n_A}{n_B}+\frac{1}{2\bar{c}_A}-1\right)\mu_{n_B,n_B} \quad (2.144)$$

From (2.140),

$$0 = \binom{n_A}{n_B}\frac{\frac{1}{2c}-1}{n_A-n_B+1}\mu_{n_B,n_B} + \sum_{a=0}^{n_B} \binom{n_A}{a+1}\binom{n_B}{a}\left(\frac{n_B(n_A-1)-a(n_A+1)}{n_B(n_A-n_B+1)}\right)\mu_{a+1,a} \quad (2.145)$$

Thus

$$\mu_{n_B,n_B} = -\sum_{a=0}^{n_B} \binom{n_A}{a+1} \binom{n_B}{a} \left(\frac{n_B(n_A-1) - a(n_A+1)}{n_B(\frac{1}{2c}-1)\binom{n_A}{n_B}} \right) \mu_{a+1,a} \qquad (2.146)$$

Now we can rewrite (2.144) as

$$\sum_{a=0}^{n_B} \binom{n_A}{a+1} \binom{n_B}{a} \mu_{a+1,a} \left[\frac{n_A + (a+1)(\frac{1}{2\bar{c}_A}-1)}{n_A} \right.$$
$$\left. - \left(\frac{n_A}{n_B} + \frac{1}{2\bar{c}_A} - 1 \right) \left(\frac{n_B(n_A-1) - a(n_A+1)}{n_A(\frac{1}{2c}-1)} \right) \right] = 1 \qquad (2.147)$$

Simplifying,

$$\sum_{a=0}^{n_B} \binom{n_A}{a+1} \binom{n_B}{a} \mu_{a+1,a} \left[1 + \frac{(a+1)(\frac{1}{2\bar{c}_A}-1)}{n_A} \right.$$
$$\left. - \frac{[n_B(n_A-1) - a(n_A+1)][2\bar{c}_A(n_A-n_B) + n_B]}{n_A n_B \bar{c}_A \left(\frac{1}{c} - 2 \right)} \right] = 1 \qquad (2.148)$$

In addition, the binding odd IC for N_A implies

$$\sum_{a=0}^{n_B} \binom{n_A}{a+1} \binom{n_B}{a} \mu_{a+1,a} \left(n_B - a - \frac{n_B}{n_A} \right) = 0 \qquad (2.149)$$

The binding even IC for N_B is implies

$$\sum_{a=1}^{n_B} \binom{n_A}{a+1} \binom{n_B-1}{a-1} \mu_{a+1,a} = \frac{\frac{1}{2}-c}{c} \binom{n_A}{n_B} \mu_{n_B,n_B} \qquad (2.150)$$

Using these expressions together with (2.144) and (2.146), we can (after some alge-

bra) express

$$\mu_{n_B,n_B} = \frac{2}{\binom{n_A}{n_B}\left(\frac{1}{c}\left[1 + \frac{1}{2\bar{c}_A(n_A-1)} + \frac{n_B}{n_A}\left(\frac{1}{2\bar{c}_A} - 1\right)\right] - \frac{1}{\bar{c}_A(n_A-1)}\right)} \tag{2.151}$$

It is interesting to compare this expression with its analog (2.93) in the proof of Proposition 1. Notice that the two coincide if $c = \bar{c}_A$. From (2.143),

$$
\begin{aligned}
\mu_{n_A,0} &= \frac{\frac{1}{2} - \bar{c}_A}{\bar{c}_A}\left(\sum_{a=0}^{n_B}\binom{n_A-1}{a}\binom{n_B}{a}\mu_{a+1,a} + \binom{n_A-1}{n_B-1}\mu_{n_B,n_B}\right) \\
&= \frac{\frac{1}{2} - \bar{c}_A}{\bar{c}_A}\left[\binom{n_A}{n_B}\mu_{n_B,n_B}\frac{(n_B(n_A-1)+n_A)\left(\frac{1}{2c}-1\right)}{n_A(n_A-1)} + \binom{n_A}{n_B}\frac{n_B}{n_A}\mu_{n_B,n_B}\right] \\
&= \frac{\frac{1}{2} - \bar{c}_A}{\bar{c}_A}\frac{1}{n_A}\binom{n_A}{n_B}\mu_{n_B,n_B}\left[\frac{(n_B(n_A-1)+n_A)\left(\frac{1}{2c}-1\right)}{n_A-1} + n_B\right] \\
&= \frac{\left(\frac{1}{2\bar{c}_A}-1\right)\left(\frac{1}{2c}\left(\frac{n_B(n_A-1)}{n_A}+1\right)-1\right)}{\left(\frac{1}{2\bar{c}_A}-1\right)\left(\frac{1}{2c}\left(\frac{n_B(n_A-1)}{n_A}+1\right)-1\right)+n_A\frac{1}{2c}-1} \tag{2.152}
\end{aligned}
$$

Plugging these expressions into the objective function and simplifying, we rewrite the analog of (2.47) for the case of heterogeneous costs as

$$h^* = 1 + 2\sum_{a=0}^{n_B} a\binom{n_A}{a+1}\binom{n_B}{a}\mu_{a+1,a} + (2n_B - 1)\binom{n_A}{n_B}\mu_{n_B,n_B} + (n_A - 1)\mu_{n_A,0}$$

$$= 1 + 2n_B\left(\frac{1}{2c} - 1\right)\binom{n_A}{n_B}\mu_{n_B,n_B} + (2n_B - 1)\binom{n_A}{n_B}\mu_{n_B,n_B}$$

$$+ (n_A - 1)\frac{\left(\frac{1}{2\bar{c}_A} - 1\right)}{n_A}\binom{n_A}{n_B}\mu_{n_B,n_B}\frac{(n_B(n_A - 1) + n_A)\left(\frac{1}{2c} - 1\right) + n_B(n_A - 1)}{n_A - 1}$$

$$= 1 + \binom{n_A}{n_B}\mu_{n_B,n_B}\left[\frac{n_B}{c} - 1 + \left(\frac{1}{2\bar{c}_A} - 1\right)\left(\frac{1}{2c}\left(n_B(1 - \frac{1}{n_A}) + 1\right) - 1\right)\right]$$

$$= 1 + \frac{2\left[\frac{n_B}{c} - 1 + \left(\frac{1}{2\bar{c}_A} - 1\right)\left(\frac{1}{2c}\left(n_B(1 - \frac{1}{n_A}) + 1\right) - 1\right)\right]}{\frac{1}{c}\left[1 + \frac{1}{2\bar{c}_A(n_A - 1)} + \frac{n_B}{n_A}\left(\frac{1}{2\bar{c}_A} - 1\right)\right] - \frac{1}{\bar{c}_A(n_A - 1)}}$$

$$= n_A \times \frac{2\bar{c}_A n_B(n_A - 1) + n_B(n_A - 1) + n_A(1 - 2c)}{2\bar{c}_A[n_A - n_B](n_A - 1) + n_B(n_A - 1) + n_A(1 - 2c)}$$

$$= n_A \times \xi(c, \bar{c}_A)$$

Finally, a simple proof by contradiction shows that $2n_B < h^* < n_A$ for all costs $0 < c \leq \bar{c}_A < \frac{1}{2}$. This completes the proof of case (ii).

■

2.A.10   Proof of Proposition 8

Proof. Set q to be a probability distribution in $\Delta(S)$ that chooses the cost-independent strategies (i.e., constant functions from types to actions) with probability 1. This allows us to write $s_i(c_i) = s_i$ for all $c_i \in (0, 0.5)$. Require in addition that for all $i \in N$, all $a_{-i} \in V_D^i$, and all $a_{-i} \in V_P^i$

$$\sum_{\{s_{-i}(c_{-i})=a_{-i}\}} q(s_i, s_{-i}|s_i(c_i) = 0) = \mu(0, a_{-i})$$

and

$$\sum_{\{s_{-i}(c_{-i})=a_{-i}\}} q(s_i, s_{-i}|s_i(c_i) = 1) = \mu(1, a_{-i}),$$

where $\mu(a) \in \Delta(S)$ is the probability distribution over joint action profiles that delivers the solution to the max-turnout problem under complete information with heterogeneous costs defined by $\mathcal{C}_{(\underline{c}_A, \bar{c}_A, \underline{c}_B, \bar{c}_B)}$ (see Proposition 7). Notice that for cost-independent strategies, the summations on the LHS of the above expressions are taken over a single strategy. Then for every player i, q selects the constant-zero strategy $s_i(c_i) = 0 \ \forall c_i \in (0, 0.5)$ with probability $\sum_{a_{-i}} \mu(0, a_{-i})$, and the constant-one strategy $s_i(c_i) = 1 \ \forall c_i \in (0, 0.5)$ with the complementary probability. Suppose first that $\bar{c}_B = \underline{c}_A$. Then Proposition 7 holds at *any* cost profile $c \in \mathcal{C}_{(\underline{c}_A, \bar{c}_A, \underline{c}_B, \bar{c}_B)}$ with the *same* equilibrium distribution over actions, $\mu(a) \in \Delta(S)$, because this distribution is completely determined by the cost bounds $(\underline{c}_A, \bar{c}_A, \underline{c}_B, \bar{c}_B)$ and the sizes of the groups, n_A and n_B. Therefore, none of the incentive compatibility constraints (2.38)-(2.39) is violated at an arbitrary $c \in \mathcal{C}_{(\underline{c}_A, \bar{c}_A, \underline{c}_B, \bar{c}_B)}$. Hence this is true for all admissible c, and both constraints (2.43)-(2.44) hold as well. Therefore, we can guarantee the expected turnout at least as large as $\tilde{h}^*$. If $\bar{c}_B < \underline{c}_A$, then, as discussed in the proof of Proposition 7, the maximum expected turnout exceeds $\tilde{h}^*$, so again, for any fixed cost profile $c \in \mathcal{C}_{(\underline{c}_A, \bar{c}_A, \underline{c}_B, \bar{c}_B)}$ we can satisfy conditions (2.38)-(2.39) with the same equilibrium distribution $\mu(a) \in \Delta(S)$. ∎

Chapter 3

Subcorrelated and Subcommunication Equilibria

How can communication between competing groups of economic agents help coordinate their actions? Many important situations in economics and political science can be modeled as a nexus of such agents and group coordination mechanisms. Examples include voter turnout, where citizens, supporting one of the two political parties, receive mobilization messages and then decide between costly voting and staying at home, local public good games, competitions between several teams, and so on.

To explore this question in an abstract framework, we introduce the concept of *subcorrelated equilibrium* (SCE), which extends Aumann's correlated equilibrium to the case of multiple mediators, and *subcommunication equilibrium* (SCME), which similarly extends Myerson's communication equilibrium. There is a partition of players into groups, and the subcorrelated equilibrium allows players' choices to be correlated within the group, but remain independent across different groups. Players

receive (correlated) signals from their groups and make optimal choices given the group structure, which is complicated by the fact that their utility depends on the actions of players in all other groups.

More specifically, to account for possible coordination among players, game theorists often add a pre-play communication stage, assuming that all players coordinate their actions via a single correlation device. We relax this assumption and design the subcorrelated equilibrium to provide positive answers to questions like: what happens if the correlation device is only available to certain groups of players? What if different groups of players coordinate their actions using different correlation devices? The subcorrelated equilibrium carves out a subset of correlated equilibria consistent with these additional restrictions on correlation devices; it thus fills the gap between Nash and correlated equilibrium.

The subcorrelated equilibrium can be extended to the incomplete information setting by introducing agents' types and replacing correlation devices with communication devices, which can now receive reports from the agents in their groups (in particular, their types) in addition to giving recommendations. This leads to the notion of the subcommunication equilibrium.

Subcorrelated and subcommunication equilibria are most closely related to equilibria among several principals, developed in Myerson (1982, Section 4). Myerson interprets communication devices in groups as principals controlling disjoint groups of agents, who maximize their expected utility subject to their agents' incentive compatibility constraints. Myerson's principals' equilibria, defined as a solution to this

mutual optimization problem may fail to exist in games with sufficient competition among principals, because in this case a principal could benefit from deviating to a non-equilibrium direct mechanism that is incentive compatible for her own agents, but violates the incentive compatibility constraints of some other principal's agents. As a solution to this problem, Myerson introduced principals' quasi-equilibria that exist in all finite games and are asymptotically optimal for all principals. The sub-communication equilibrium does not require different communication devices to be competing principals, but for the subset of SCME where this interpretation holds, we prove their equivalence to the set of principals' quasi-equilibrium of Myerson (1982). These equilibria can be obtained as a solution to the optimization problem where all principals maximize their expected utility subject to their agents' incentive compatibility constraints as well as the incentive compatibility constraints of all other principals' agents at the tuple of direct mechanisms under consideration.

Subcorrelated and subcommunication equilibria are related to several literatures. The competing mechanisms literature (e.g., Epstein and Peters (1999), Martimort and Stole (2002), and Peters and Troncoso-Valverde (2013)) assumes multiple principals controlling the same pool of agents, and that the principals know their own mechanisms only, whereas the agents observe the mechanisms offered by all principals and thus have some market information in addition to their type. Including market information into an agent's type leads to difficulties in the incomplete information settings, because players in a direct mechanism must report their type to each principal, which can lead to an infinite regress. See Peters and Troncoso-Valverde (2013). Subcommunication equilibria have multiple principals controlling different groups of

agents and using commonly known direct mechanisms. Another distinction (e.g., compared to Epstein and Peters (1999)) is that in our case, the outside option utility may depend on the actions of the other principals' agents.

The epistemic game theory literature that looks at correlated rationalizable equilibria in players' beliefs is relevant for the subcorrelated equilibrium. A related paper here is Tsakas (2014), who introduces the so called "correlated-belief equilibrium", which relaxed the assumption of independent marginal conjectures, implied by a Nash equilibrium. The main difference between the correlated-belief equilibrium and the subcorrelated equilibrium is due to the fact that the former is a special case of a *subjective* correlated equilibrium (Tsakas, 2014, Prop. 5), whereas the latter restricts the *objective* correlated equilibrium. See also Liu (2015) for a more general epistemic approach to the relation between the correlation devices, partition models, and belief hierarchies. Another related idea can also be found in Stein, Ozdaglar and Parrilo (2014). They introduce an equilibrium concept called "exchangeable equilibrium", where players' strategies are conditionally independent and identically distributed (iid) random variables. This is a weakening of the Nash equilibrium, where strategies are unconditionally iid. Still, this concept assumes a single correlation device, so it is a very different relaxation of the correlated equilibrium, compared to the subcorrelated equilibrium.

Finally, there is also a somewhat distant relation to the literature on cooperative solutions. For example, Ichiishi (1981) introduced a "social coalitional equilibrium", which has a similar flavor to the subcorrelated equilibrium. However in that paper

as well as in other extensions that followed (see, e.g., Ray and Vohra (1997) and references therein) players are assumed to behave cooperatively in groups (called coalitions) and non-cooperatively across groups. In contrast, in the subcorrelated equilibrium all behavior is non-cooperative. Furthermore, the main focus of Ichiishi (1981) is on proving existence of the social coalitional equilibrium, which in our setting follows immediately from the correlated equilibrium existence in finite games.

The remainder of the Chapter is organized as follows. Section 3.1 describes the main modeling framework. We define the subcorrelated equilibrium and its properties in Section 3.1.1. Section 3.1.2 describes the general case of subcommunication equilibrium, which is an extension of the subcorrelated equilibrium to the incomplete information setting. Section 3.2 applies the general ideas and definitions of subcorrelated and principals' equilibria in a less abstract setting of a voter turnout game. Section 3.3 concludes.

3.1   The Model

In this section we introduce the formal framework for subcorrelated and subcommunication equilibria. These concepts generalize correlated and communication equilibria under complete and incomplete information, respectively. Accordingly, we split the definitions and results into two subsections that differ by the information assumptions in the game. We start by providing the necessary notation.

There are two types of players: principals and agents. We assume that each principal

has at least one agent, and that each agent interacts with no more than one principal. Let I be the set of agents, and N, with $n := |N|$, the set of principals. For each principal $k \in N$, let $N_k \subseteq I$ be the set of k's agents, with $n_k := |N_k|$. Let $\tilde{N}_k := \{k\} \cup N_k$ be a set that consists of principal k and his n_k agents. Let $\mathcal{N} := \bigcup_{k=1}^{n} \tilde{N}_k = N \cup I$ be the set of all players in the game. An alternative way to represent the players in this game is to partition I into n disjoint groups of agents, where $1 \leq n \leq |I|$ is fixed, and each group is controlled by a different principal. Let $A^k := A_0^k \times A_1^k \times \ldots \times A_{n_k}^k$ be the joint action domain for principal k, who controls A_0^k, and his agents, numbered $\{1, \ldots, n_k\}$, who each control action domain A_i^k. Let $A := \times_{j=1}^{n} A^j$ be the joint action domain of all principals and their agents. We assume A is finite.

We are interested in the set of equilibrium outcomes in a game extended with general forms of pre-play communication. While there are infinitely many ways communication between players can be introduced, we are going to invoke the revelation principle and focus on direct mechanisms only.[1]

3.1.1   Complete information

We start by analyzing the complete information case. Before presenting the formal definitions, we briefly discuss the main modeling choices for the principals' behavior.

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[1]The standard interpretation of a direct mechanism is in terms of a centralized mediator (a communication device) who receives type reports from players, draws a joint action profile from a probability distribution and then privately informs each player about their respective profile component.

3.1.1.1   Principals' behavior

There are several alternative ways to model the principals' behavior.

First, we can leave the principals out of strategic interactions entirely, and model all pre-play communication among agents via a single correlation device.[2] This device can be thought of as an impartial mediator, drawing a joint action profile from a commonly known probability distribution, and then privately informing each agent about her respective profile component. The equilibrium conditions imposed on the set of admissible distributions correspond to the incentive compatibility of each agent's actions when all other agents follow their recommended actions. This solution concept is the (objective) *correlated equilibrium* (Aumann, 1974, 1987).

Second, we can take into account the division of the agents into groups, and model each principal as an independent correlation device for their group. That is, each principal serves as a mediator for his agents, drawing a voting profile from a probability distribution over group profiles, and then privately informing each agent in his group about her respective profile component. The equilibrium conditions now require in addition to incentive compatibility that the distribution over joint action profiles is a product of n independent distributions over group profiles. This solution concept is introduced in this chapter and called *subcorrelated equilibrium*; it selects a subset of correlated equilibria.

Third, we can think of the principals as independent players who maximize their

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[2]We focus on complete information case here, but the same logic applies to the incomplete information case, with correlation devices replaced by communication devices.

expected utilities by choosing an incentive compatible direct mechanism for their agents given the other principal's chosen mechanism. The equilibrium conditions now require in addition to the product structure that the mechanisms are optimal for each principal and incentive compatible for their agents. This solution concept, called *principals' equilibrium*, was introduced by Myerson (1982); it selects a subset of subcorrelated equilibria. The difficulty about this concept is that each agent's incentive compatibility constraints depend on all other agents, while each principal only cares about his own agents' constraints. As a result, principals' equilibria rarely exist.

Finally, we can keep the same structure as in a principals' equilibrium, but require that the mechanisms are optimal for each principal subject to the incentive compatibility constraint for their agents *as well as the other principals' agents*. We call this a *jointly feasible principals' equilibrium*; it selects a subset of subcorrelated equilibria. Restricting both principals to choose among the *jointly* incentive-compatible probability distributions prevents profitable deviations by a principal that only become possible due to lifting the additional restrictions imposed by the other principal's agents incentive compatibility. If the other principals' agents' IC constraints did not restrict the first principal's agents, this would mean that the other principals could have increased her expected utility, but decided not to do it. We will show in Section 3.1.2 that these equilibria always exist and have a tight relationship with Myerson's quasi-principals' equilibria, which relax the principals' equilibrium to ensure existence and asymptotic optimality for all principals.

In theory, the assumptions about principals' behavior and utility are primitives of the model, but there are many applications where these choices come naturally. In Section 3.2 we investigate one such example of a voter turnout game. In the game there are two candidates, competing in an election, and two (disjoint) groups of citizens each supporting one of the candidates. Thus candidates correspond to principals and supporting citizens to their agents. The voter turnout on the election day decides the winner, but voting is costly for the citizens, so it is only rational for them to vote when the chance their vote makes the difference is sufficiently high. Since principals are political candidates, there are several particularly attractive models of their utility. For example, candidates can be assumed to maximize expected welfare of their supporters, or their expected turnout, or the probability of winning the elections. In these cases, candidates' objectives directly oppose one another, so it is perhaps not surprising that such principals' equilibria do not exist. We characterize the subcorrelated equilibria of the turnout game that maximize the total expected turnout, and some jointly feasible principals' equilibria.

We will now present the formal definitions of the solution concepts discussed above, starting from a subcorrelated equilibrium.

3.1.1.2 Definitions and properties

In the complete information case, players' utilities are $\mathcal{U}_0^k : A \to \mathbb{R}$ for principal k and $\mathcal{U}_i : A \to \mathbb{R}$ for principal k's agent i. For any set X, denote $\Delta(X)$ the set of all probability distributions on X. A *direct mechanism* $\pi^k \in \Delta(A^k)$ is a distribution

where $\pi^k(a^k)$ is the probability that principal k chooses action a_0^k and recommends actions a_i^k for all $i \in N_k$. This model also captures the situation where principals are just mediators for their agents (i.e., correlation devices for each group): in this case we can set $A_0^k = \{0\}$ to ensure π^k does not depend on principal k's action and set $\mathcal{U}_0^k(a) = 0$ for any $a \in A$.

Definition 2. *A **subcorrelated equilibrium** (with n groups) is a probability distribution $\pi \in \Delta(A)$ such that*

1) For all agents, incentive compatibility (IC) constraints hold under π: $\forall k \in N, \forall\, i \in N_k, \forall\, a_i, a_i' \in A_i^k$

$$\sum_{a_{-i} \in A_{-i}} \pi(a_i, a_{-i}) \left[\mathcal{U}_i(a_i, a_{-i}) - \mathcal{U}_i(a_i', a_{-i}) \right] \geq 0, \tag{3.1}$$

and 2) π is generated by n independent probability distributions, with

$$\pi(a) = \prod_{k=1}^{n} \pi_k(a | \tilde{N}_k), \tag{3.2}$$

for all $a \in A$, where $\pi_k \in \Delta(A^k)$ and $a | \tilde{N}_k := ((a_i)_{i \in \tilde{N}_k})$ is group k joint action component.

Constraints (3.1) define a correlated equilibrium (Aumann, 1974, 1987). Eq. (3.2) just adds an additional requirement that the resulting joint distribution is a product of independently mixed group distributions, one for each group. Thus, in a subcorrelated equilibrium, players receive signals from their groups' correlation devices, and then best respond to a (possibly) correlated joint distribution. Notice that Definition

2 does not impose IC constraints on the principals. This is not necessary: when principals are active players, their IC hold by default since they are optimizing. When principals are just mediators, their IC hold trivially since they are non-strategic.

We first consider the case where all principals are mediators, since this results in the largest set of subcorrelated equilibria, and address the case with competing principals in the more general, incomplete information case presented in Section 3.1.2.

Denote SCE^n the set of all such subcorrelated equilibria with n mediators, CE the set of Aumann's correlated equilibria, and NE the set of Nash equilibria.

Proposition 9. *SCE^n has the following properties.*

1. *$SCE^n \neq \emptyset$ for finite games and $SCE^n \subseteq CE$. For $n = 1$, $SCE^n = CE$. For $n = |I|$, $SCE^n = NE$.*

2. *SCE^n is non-increasing in n: ceteris paribus, merging groups results in a weakly larger set of subcorrelated equilibria.*

3. *SCE^n is convex for $n = 1$, and sectionally convex for $n > 1$ with respect to sections through the $n-1$ dimensional joint mechanisms of all but one group. In particular, for $n = 2$, SCE^n is biconvex.[3]*

Proof. Properties 1 and 2 follow directly from the definition, so we will only prove Property 3. If $n = 1$, $SCE^1 = CE$, hence it is convex. Suppose $n > 1$. Let $\pi_1, \pi_2 \in SCE^n$. We will show that if $n - 1$ out of n generating probabilities are the

---

[3]A set $X \subseteq X_1 \times X_2$ is biconvex if for each $\ell \in \{1, 2\}$ and each $x_\ell \in X_\ell$, all sections of X through x_ℓ are convex, i.e., $\{x_{-\ell} \in X_{-\ell} | (x_\ell, x_{-\ell}) \in X\}$ is convex for each x_ℓ. See Aumann and Hart (1986).

same in π_1 and π_2, then $\pi_M \equiv \lambda\pi_1 + (1-\lambda)\pi_2 \in SCE^n$ for all $\lambda \in [0,1]$. Since π_M is a linear combination of π_1 and π_2, all agents' IC constraints hold at π_M. Since $n-1$ generating probabilities are the same in π_1 and π_2, without loss of generality let μ_k^1 and μ_k^2 be the different generating probabilities for players in group N_k. Then for any $a \in A$ we can write

$$\pi_M(a) = (\lambda\mu_k^1(a|\tilde{N}_k) + (1-\lambda)\mu_k^2(a|\tilde{N}_k)) \prod_{\ell=1}^{n-1} \mu_\ell(a|\tilde{N}_\ell) = \nu_k(a|\tilde{N}_k) \prod_{\ell=1}^{n-1} \mu_\ell(a|\tilde{N}_\ell),$$

where $\nu_k \in \Delta(\times_{i \in \tilde{N}_k} A_i)$ is a valid probability distribution on the set of joint profiles restricted to group $\tilde{N}_k$. Therefore, by definition $\pi_M \in SCE^n$. $\blacksquare$

3.1.2 Incomplete information

In this section we continue to explore the properties of the subcorrelated and principals' equilibria in the more general case, with incomplete information and communication devices.

Compared to the complete information case, we now introduce players' types. Let agents' types for principal k be in $T^k := T_1^k \times \ldots \times T_{n_k}^k$, and define $T := \times_{j=1}^n T^j$. Assume there is a common prior $P \in \Delta(T)$ over types. Players' utilities are now $\mathcal{U}_0^k : A \times T \to \mathbb{R}$ for principal k and $\mathcal{U}_i : A \times T \to \mathbb{R}$ for principal k's agent i. A *direct mechanism* $\pi^k \in \Delta(A^k \times T^k)$ is a distribution where $\pi^k(a^k|t^k)$ is the probability that principal k chooses action a_0^k and recommends actions a_i^k for all $i \in N_k$ given the reported type profile $t^k \in T^k$. Let $\pi \in \Delta(A \times T)$ be a joint probability distribution

over actions and types, then principal k's expected utility given π is

$$U_k(\pi) = \sum_{a \in A} \sum_{t \in T} \pi(a,t) \mathcal{U}_0^k(a,t)$$

We assume that π is commonly known.[4]

Definition 3. *A* subcommunication equilibrium *(with n groups) is a probability distribution $\bar{\pi} \in \Delta(A \times T)$ such that*

1) $\forall k \in N$, $\forall i \in N_k$ IC constraints hold under $\bar{\pi}$: $\forall \tau_i^k, \hat{\tau}_i^k \in T_i^k$, $\forall \delta_i^k : A_i^k \to A_i^k$,

$$\sum_{\substack{t \in T \\ t_i^k = \tau_i^k}} P(t) \sum_{a \in A} \left[\bar{\pi}(a|t)\mathcal{U}_i(a,t) - \bar{\pi}(a|\hat{\tau}_i^k, t_{-i,k})\mathcal{U}_i((\delta_i^k(a_i^k), a_{-i,k}), t) \right] \geq 0 \qquad (3.3)$$

and 2) there exists a tuple of n direct mechanisms $(\bar{\pi}^1, \ldots, \bar{\pi}^n)$, with $\bar{\pi}^k \in \Delta(A^k \times T^k)$ for each $k \in N$, such that for each $(a,t) \in A \times T$

$$\bar{\pi}(a,t) = P(t) \left(\prod_{k=1}^{n} \bar{\pi}^k(a^k|t^k) \right) \qquad (3.4)$$

We say that $\bar{\pi}$ is a subcommunication equilibrium generated by $(\bar{\pi}^1, \ldots, \bar{\pi}^n)$.

The first condition in Definition 3 is incentive compatibility for all principal k's agents assuming all other agents are honest and obedient. Constraints (3.3) define a *Communication equilibrium* (Forges, 1986; Myerson, 1986, 1991). The second condition is a partitioning requirement.

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[4]In contrast, the competing mechanisms literature assumes that mechanisms chosen by different principals may be only known to their agent(s).

The subcommunication equilibrium with principals as mediators inherits the properties of the respective subcorrelated equilibrium, and the analog of Proposition 9 applies.

3.1.2.1 Equilibria with multiple principals

We now turn to a more restrictive model of principals' behavior when they are not simply mediators but also active players in the game. This further restricts the set of subcorrelated/subcommunication equilibria.

First, using the product structure imposed by (3.4), we define the set of incentive-compatible direct mechanisms for principal $k \in N$ when the other principals are using mechanisms $(\bar{\pi}^1, \ldots, \bar{\pi}^{k-1}, \bar{\pi}^{k+1}, \ldots, \bar{\pi}^n)$ as follows.

$$F^k(\bar{\pi}^1, \ldots, \bar{\pi}^n) := \Big\{ \gamma^k \in \Delta(A^k \times T^k) | \forall i \in N_k, \forall \tau_i^k, \hat{\tau}_i^k \in T_i^k, \ \forall \delta_i^k : A_i^k \to A_i^k$$

$$\sum_{\substack{t \in T \\ t_i^k = \tau_i^k}} P(t) \sum_{a \in A} \left(\prod_{j \neq k} \bar{\pi}^j(a^j|t^j) \right) \left[\gamma^k(a^k|t^k)\mathcal{U}_i(a,t) - \gamma^k(a^k|\hat{\tau}_i^k, t_{-i}^k)\mathcal{U}_i((\delta_i^k(a_i^k), a_{-i,k}), t) \right] \geq 0 \Big\} \quad (3.5)$$

In words, $F^k(\bar{\pi}^1, \ldots, \bar{\pi}^n)$ is the set of all direct mechanisms γ^k for principal k such that γ^k is incentive compatible for principal k's agents given $(\bar{\pi}^1, \ldots, \bar{\pi}^n)$ and assuming all other agents are honest and obedient.

Define also

$$\phi^k(\bar{\pi}) := \arg\max_{\gamma^k \in F^k(\bar{\pi}^1, \ldots, \bar{\pi}^n)} U_k(\bar{\pi}^1, \ldots, \gamma^k, \ldots, \bar{\pi}^n) \quad (3.6)$$

In words, ϕ^k is a direct mechanism delivering the maximum possible expected utility

to principal k subject to IC constraints for principal k's agents, but without imposing IC constraints for other principals' agents at $\bar{\pi}$.

Definition 4. *A tuple of n direct mechanisms $\bar{\pi} := (\bar{\pi}^1, \ldots, \bar{\pi}^n)$ is a* principals' equilibrium *(Myerson, 1982) if for each $k \in N$, $\bar{\pi}^k = \phi^k(\bar{\pi})$.*

A principals' equilibrium is a straightforward generalization of the principal-agent model to the case of multiple principals controlling disjoint sets of agents.

We can now give the definition of a nontrivial principals' game.

Definition 5. *A game $G := \left(\mathcal{N}, P, (A_i)_{i \in \mathcal{N}}, (T_i)_{i \in \mathcal{N}}, (\mathcal{U}_i)_{i \in \mathcal{N}}\right)$ is a* nontrivial princi- pals' game *if for each principal k there is at least one agent of some other principal $j \neq k$, whose IC constraints are violated at k's optimal IC mechanism. That is, for each $k \in N$, $\exists j$ such that $\bar{\pi}^j \notin F^j(\bar{\pi}^1, \ldots, \phi^k(\bar{\pi}), \ldots, \bar{\pi}^n)$ for any $\bar{\pi}$.*

In the class of nontrivial principals' games we cannot separate optimal decisions of different principals, which leads to non-existence of principals' equilibria.

Proposition 10. *In a nontrivial principals' game principals' equilibria do not exist.*

Proof. Obvious from the definition of a nontrivial principals' game. ∎

Given this negative result, Myerson proposed a relaxation of the principals' equi- librium called *principals' quasi-equilibrium* that ensures existence in finite games (including nontrivial principals' games).

Definition 6. *A tuple of n direct mechanisms $\bar{\pi} = (\bar{\pi}^1, \ldots, \bar{\pi}^n)$ is a* principals' quasi-equilibrium *(Myerson, 1982) if there exists a sequence of direct mechanisms*

$\{(\pi_\ell^1, \ldots, \pi_\ell^n)\}_{\ell=1}^\infty$ *such that for all* $k \in N$

$$\bar{\pi}^k = \lim_{\ell \to \infty} \pi_\ell^k \tag{3.7}$$

$$U_k(\bar{\pi}) \geq \lim_{\ell \to \infty} \max_{\gamma^k \in F^k(\pi_\ell^1, \ldots, \pi_\ell^n)} U_k(\pi_\ell^1, \ldots, \gamma^k, \ldots, \pi_\ell^n) \tag{3.8}$$

$$\bar{\pi}^k \in F^k(\bar{\pi}^1, \ldots, \bar{\pi}^n) \tag{3.9}$$

where F^k *is defined by* (3.5).

We develop another relaxation of the principals' equilibrium, called the *jointly feasible principals' equilibrium*, which is consistent with the idea of a subcommunication equilibrium.

Definition 7. *A tuple of* n *direct mechanisms* $\bar{\pi} = (\bar{\pi}^1, \ldots, \bar{\pi}^n)$ *is a* jointly feasible principals' equilibrium *if it generates a subcommunication equilibrium such that for all* $k \in N$

$$U_k(\bar{\pi}) = \max_{\substack{\gamma^k \in F^k(\bar{\pi}^1, \ldots, \bar{\pi}^n) \\ \bar{\pi}^j \in F^j(\bar{\pi}^1, \ldots, \gamma^k, \ldots, \bar{\pi}^n), \ j \neq k}} U_k(\bar{\pi}^1, \ldots, \gamma^k, \ldots, \bar{\pi}^n) \tag{3.10}$$

Thus, in a jointly feasible principals' equilibrium, players maximize their expected utility subject to their agents' IC constraints *as well as those of other principals' agents*.

We will now show that Myerson's principals' quasi-equilibria and the jointly feasible principals' equilibria are equivalent. More specifically, the two concepts are equivalent in the class of nontrivial principals' games, as the following proposition demonstrates.

Proposition 11. *For all nontrivial principals' games, the set of jointly feasible principals' equilibria and the set of principals' quasi-equilibria coincide.*

Proof. ($\Leftarrow$) Let $\bar{\pi}$ be a principals' quasi-equilibrium, then condition (3.9) in Definition 6 holding for all principals implies that $\bar{\pi}$ is a subcommunication equilibrium, while conditions (3.7)-(3.8) imply that

$$
\begin{aligned}
U_k(\bar{\pi}) &\geq \lim_{\ell \to \infty} \max_{\gamma^k \in F^k(\pi_\ell^1, \ldots, \pi_\ell^n)} U_k(\pi_\ell^1, \ldots, \gamma^k, \ldots, \pi_\ell^n) \\
&\geq \lim_{\ell \to \infty} \max_{\substack{\gamma^k \in F^k(\pi_\ell^1, \ldots, \pi_\ell^n) \\ \bar{\pi}^j \in F^j(\bar{\pi}^1, \ldots, \gamma^k, \ldots, \bar{\pi}^n), \ j \neq k}} U_k(\pi_\ell^1, \ldots, \gamma^k, \ldots, \pi_\ell^n) \\
&= \max_{\substack{\gamma^k \in F^k(\bar{\pi}^1, \ldots, \bar{\pi}^n) \\ \bar{\pi}^j \in F^j(\bar{\pi}^1, \ldots, \gamma^k, \ldots, \bar{\pi}^n), \ j \neq k}} U_k(\bar{\pi}^1, \ldots, \gamma^k, \ldots, \bar{\pi}^n) \quad (3.11)
\end{aligned}
$$

Since (3.9) holds for all k at $\bar{\pi}$, we cannot have

$$
U_k(\bar{\pi}) > \max_{\substack{\gamma^k \in F^k(\bar{\pi}^1, \ldots, \bar{\pi}^n) \\ \bar{\pi}^j \in F^j(\bar{\pi}^1, \ldots, \gamma^k, \ldots, \bar{\pi}^n), \ j \neq k}} U_k(\bar{\pi}^1, \ldots, \gamma^k, \ldots, \bar{\pi}^n)
$$

hence (3.11) holds with equality and $\bar{\pi}$ is a jointly feasible principals' equilibrium according to Definition 7.

($\Rightarrow$) Let $\bar{\pi} = (\bar{\pi}^1, \ldots, \bar{\pi}^n)$ be any jointly feasible principals' equilibrium. By definition, $\bar{\pi}^k \in F^k(\bar{\pi}^1, \ldots, \bar{\pi}^n)$ for all k, so (3.9) holds trivially. For each $k \in \{1, \ldots, n\}$, let $\phi^k(\bar{\pi})$, defined by (3.6), be a direct mechanism delivering the maximum possible expected utility to principal k subject to IC constraints for principal k's agents, but without respect to other principals' agents' IC constraints at $\bar{\pi}$. By optimal-

ity of ϕ^k, at least some of the IC constraints must be binding for at least some of each principal k's agents (otherwise she could do better) at $\phi^k(\bar{\pi})$. If there was a tuple $(\phi^1, \ldots, \phi^n)$ such that $\phi^k(\phi^{-k}) = \phi^k$ for all k, we would have a principals' equilibrium. Since principals' equilibria do not exist in the class of nontrivial principals' games (Proposition 10), for each k there is at least one agent of some other principal $j \neq k$, whose IC constraints are violated at k's optimal IC mechanism, i.e. $\bar{\pi}^j \notin F^j(\bar{\pi}^1, \ldots, \phi^k(\bar{\pi}), \ldots, \bar{\pi}^n)$. Consider a sequence of direct mechanisms $\{(\pi_\ell^1, \ldots, \pi_\ell^n)\}_{\ell=1}^\infty$ defined as follows: for all $k \in \{1, \ldots, n\}$

$$\pi_\ell^k := \left(1 - \frac{1}{\ell}\right)\bar{\pi}^k + \frac{1}{\ell}\phi^k(\bar{\pi}), \qquad (3.12)$$

Since $\lim_{\ell \to \infty}(\pi_\ell^1, \ldots, \pi_\ell^n) = \bar{\pi}$ and (3.9) holds in the limit,

$$\lim_{\ell \to \infty} \max_{\gamma^k \in F^k(\pi_\ell^1, \ldots, \pi_\ell^n)} U_k(\pi_\ell^1, \ldots, \gamma^k, \ldots, \pi_\ell^n) \leq \max_{\substack{\nu^k \in F^k(\bar{\pi}^1, \ldots, \bar{\pi}^n) \\ \bar{\pi}^j \in F^j(\bar{\pi}^1, \ldots \nu^k, \ldots, \bar{\pi}^n), j \neq k}} U_k(\pi_\ell^1, \ldots, \nu^k, \ldots, \pi_\ell^n)$$

$$= U_k(\bar{\pi})$$

Hence all conditions (3.7)-(3.9) hold for $\{(\pi_\ell^1, \ldots, \pi_\ell^n)\}_{\ell=1}^\infty$, so $\bar{\pi}$ is a principals' quasi-equilibrium. ■

3.2 Application: Voter Turnout Games

In this section we apply the subcorrelated equilibrium framework in the context of a voter turnout game.[5]

There are two candidates, A and B, competing in an election, and two (disjoint) groups of citizens supporting one of the candidates: N_A of size n_A, and N_B of size n_B. The candidates' political positions are fixed and commonly known.[6] Each citizen $i \in I = N_A \cup N_B$, with $|I| = n_A + n_B$, has two pure actions: to vote for the preferred candidate (action 1) or abstain (action 0). Thus i's action space is $A_i = \{0, 1\}$. The set of joint voting profiles is $A = A_1 \times \cdots \times A_{|I|}$. Voting is costly with a common cost c, where $0 < c < \frac{1}{2}$.[7] A citizen's utility of voting (denoted $\mathcal{U}_i$) net of the voting cost is normalized to 1 if the preferred candidate wins, $1/2$, if there is a tie, and 0 otherwise. Assume also that $n_A, n_B \geq 1$, and $n_A > n_B$ so that A is supported by a majority of citizens.

We will analyze the subcorrelated equilibria in this turnout game. The citizens' IC constraints (conditions (3.1)) can be written as follows.[8]

$$c \geq \frac{1}{2}\Pr(i \text{ is pivotal} \mid i \text{ abstains})$$
$$c \leq \frac{1}{2}\Pr(i \text{ is pivotal} \mid i \text{ votes})$$

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[5]The game-theoretic analysis of voter turnout without communication was done in Palfrey and Rosenthal (1983, 1985). See Chapter 2 for the analysis of correlated equilibria in these games.

[6]Thus candidates have no action other than the choice of the direct mechanism for their supporters. This assumption can be relaxed in the general equilibrium model like Ledyard (1981).

[7]The common cost assumption can be relaxed.

[8]See Section 2.1.1.

These inequalities relate the probability of being pivotal (i.e., making a tie or breaking a tie with by casting one's vote), conditional on the player's own action, and the voting cost. Thus, a *correlated equilibrium* in this game is given by a probability distribution over joint voting profiles where at every profile each citizen finds it incentive compatible to follow her prescribed choice conditional on this profile realization. A *subcorrelated equilibrium* requires in addition that the probability of any joint profile is a product of two independent probabilities of the respective group profiles. Formally,

$$\sum_{a_{-i} \in V_P^i} \gamma^j(0_i, a_{-i}|N_j)\gamma^{-j}(a_{-i}|N_{-j}) \le \frac{c}{\frac{1}{2}-c} \sum_{a_{-i} \in V_{NP}^i} \gamma^j(0_i, a_{-i}|N_j)\gamma^{-j}(a_{-i}|N_{-j}) \quad (3.13)$$

$$\sum_{a_{-i} \in V_{NP}^i} \gamma^j(1_i, a_{-i}|N_j)\gamma^{-j}(a_{-i}|N_{-j}) \le \frac{\frac{1}{2}-c}{c} \sum_{a_{-i} \in V_P^i} \gamma^j(1_i, a_{-i}|N_j)\gamma^{-j}(a_{-i}|N_{-j}) \quad (3.14)$$

for all $i \in N_j$, $j \in \{A, B\}$, where V_P^i (V_{NP}^i) are the sets of $|I| - 1$ dimensional "others' " voting profiles where player i is pivotal (non-pivotal, respectively), and $\gamma^j(z_i, a_{-i}|N_j)$ denotes the probability of n_j-dimensional group voting profile where player i plays action z_i, while the remaining group j members play action $a_{-i}|N_j$.

Denote $\mathcal{D}_{SC}(\cdot, \gamma^B, c)$ the set of probability distributions in $\Delta\left(\times_{i \in N_A} A_i\right)$ that satisfy IC constraints (3.13)-(3.14) for group N_A given the action distribution γ^B in group N_B and voting cost c, and similarly, $\mathcal{D}_{SC}(\gamma^A, \cdot, c)$ be the set of probability distributions in $\Delta\left(\times_{i \in N_B} A_i\right)$ that satisfy IC constraints (3.13)-(3.14) for group N_B, given the action distribution γ^A in group N_A. A subcorrelated equilibrium is generated by (γ^A, γ^B) if and only if $(\gamma^A, \gamma^B) \in \mathcal{D}_{SC}(\cdot, \gamma^B, c) \times \mathcal{D}_{SC}(\gamma^A, \cdot, c)$.

A *principals' equilibrium* in the turnout game, where each correlation device can be viewed as an independent expected utility-maximizing principal, is a pair $(\gamma^{A*}, \gamma^{B*})$ such that

$$\gamma^{A*} \in \arg \max_{\gamma \in \mathcal{D}_{SC}(\cdot, \gamma^{B*}, c)} U_A(\gamma, \gamma^{B*}) \tag{3.15}$$

$$\gamma^{B*} \in \arg \max_{\gamma \in \mathcal{D}_{SC}(\gamma^{A*}, \cdot, c)} U_B(\gamma^{A*}, \gamma) \tag{3.16}$$

It is easy to see that any principals' equilibrium $(\gamma^{A*}, \gamma^{B*}) \in \mathcal{D}_{SC}(\cdot, \gamma^{B*}, c) \times \mathcal{D}_{SC}(\gamma^{A*}, \cdot, c)$, hence it is also a subcorrelated equilibrium. As in the general case, for any fixed direct mechanism the other principal might be using, the expected utility of the current principal is linear in their own direct mechanism: $U_j(\gamma^j, \gamma^{-j})$ is linear in γ^j given γ^{-j}, $j \in \{A, B\}$.

There are several ways to define principals' utility in the turnout game, of which we will consider three. First, suppose two principals maximize the expected total welfare of their groups, i.e., the sum of expected utilities of the voters in their groups. We call such principals *benevolent*. Formally, expected utilities of principals A and B, respectively, at a joint distribution generated by (γ^A, γ^B) are

$$U_A(\gamma^A, \gamma^B) = \sum_{a=0}^{n_B-1} \sum_{b=a+1}^{n_B} \gamma^A(a)\gamma^B(b)[-ca] + \sum_{a=0}^{n_B} \gamma^A(a)\gamma^B(a) \left[\left(\frac{1}{2} - c \right) a + \frac{1}{2}(n_A - a) \right]$$
$$+ \sum_{a=1}^{n_A} \sum_{b=0}^{\min(a-1, n_B)} \gamma^A(a)\gamma^B(b) \left[(1 - c)\, a + (n_A - a) \right]$$

and

$$U_B(\gamma^A, \gamma^B) = \sum_{a=0}^{n_B-1} \sum_{b=a+1}^{n_B} \gamma^A(a)\gamma^B(b) \left[(1-c)\,b + (n_B - b)\right]$$

$$+ \sum_{a=0}^{n_B} \gamma^A(a)\gamma^B(a) \left[\left(\frac{1}{2} - c\right)a + \frac{1}{2}(n_B - a)\right] + \sum_{a=1}^{n_A} \sum_{b=0}^{\min(a-1,n_B)} \gamma^A(a)\gamma^B(b)\left[-cb\right].$$

For any fixed γ^B and $0 < c < \frac{1}{2}$, $U_A(\gamma, \gamma^B)$ is maximized at setting $\gamma^A(n_B + 1) = 1$.

Second, a relevant formulation of principals' utility concerns maximizing expected group turnout. Call such principals *turnout-motivated*. In this case, we have

$$U_A(\gamma^A, \gamma^B) = \sum_{a=0}^{n_A} a\gamma^A(a)$$

$$U_B(\gamma^A, \gamma^B) = \sum_{b=0}^{n_B} b\gamma^B(b)$$

Finally, one can also look at principals maximizing their probability of winning elections. Call such principals *office-motivated*. In this case, principal A's expected utility is

$$U_A(\gamma^A, \gamma^B) = \frac{1}{2} \sum_{a=0}^{n_B} \gamma^A(a)\gamma^B(a) + \sum_{a=1}^{n_A} \sum_{b=0}^{\min(a-1,n_B)} \gamma^A(a)\gamma^B(b)$$

and for principal B,

$$U_B(\gamma^A, \gamma^B) = \sum_{a=0}^{n_B-1} \sum_{b=a+1}^{n_B} \gamma^A(a)\gamma^B(b) + \frac{1}{2} \sum_{a=0}^{n_B} \gamma^A(a)\gamma^B(a)$$

For any fixed γ^B, $U_A(\gamma, \gamma^B)$ is maximized at setting $\sum_{a=n_B+1}^{n_A} \gamma^A(a) = 1$.

Our first result in this section shows that principals' equilibria of the kind considered above do not exist in the turnout game.

Proposition 12. *Neither benevolent, nor turnout-motivated, nor office-motivated principals' equilibria exist in the turnout game.*

Proof. The proof has three simple steps. First, any principals' equilibrium in a complete information game is a subcorrelated equilibrium. This trivially holds by definition. Hence in the equilibrium agents' IC constraints (3.13) -(3.14) must simultaneously hold for all voters in both groups. Second, as explained below, in any benevolent, turnout-motivated, or office-motivated principals' equilibrium, denoted $(\gamma^{A*}, \gamma^{B*})$, IC constraint (3.14) binds for group N_B, restricting the optimal mechanism of principal A, γ^{A*}. Third, it now follows that principal A always has a profitable deviation from $(\gamma^{A*}, \gamma^{B*})$ by disregarding the IC constraints for group N_B while his agents' IC constraints (3.13)-(3.14) continue to hold: A's expected utility can be increased due to lifting the restriction on γ^{A*}. Hence there cannot be any such principals' equilibrium, because the majority group principal would always find it more profitable (and consistent with his agents' IC constraints) to deviate.

The argument for the second step is as follows. Since $n_A > n_B$, for any fixed mechanism of principal B and $0 < c < \frac{1}{2}$, principal A can ensure N_A is more likely to win by increasing as much as possible (under the joint IC constraints for both groups, as required by the subcorrelated equilibrium) the probability mass on profiles with at least $n_B + 1$ votes from group N_A. Moreover, in any principals' equilibrium of the specified type, principal A actually wants to do it: if A was unconstrained, it would be

optimal for him to put all probability mass on profiles with exactly n_B+1 votes from N_A when benevolent, on the profile with n_A votes from N_A when turnout-motivated, and on any mixture between $n_B + 1$ and n_A votes from N_A when office-motivated, respectively. Therefore, A optimally puts as large probability on such profiles as allowed by the joint IC. But at any joint profile with at least $n_B + 1$ votes from N_A, any voter from N_B who also decides to turn out is non-pivotal. Therefore IC constraint (3.14) for voters in N_B has $\Pr(n_B + 1$ or more from N_A vote $)$ on the left hand side and must be binding in the optimum. ∎

We now briefly address the problem of maximizing the total expected turnout in a subcorrelated equilibrium. Namely, let

$$(\gamma^{A*}, \gamma^{B*}) \in \arg \max_{\substack{\gamma^A \in \mathcal{D}_{SC}(\cdot, \gamma^B, c) \\ \gamma^B \in \mathcal{D}_{SC}(\gamma^A, \cdot, c)}} \sum_{a \in \{0,1\}^{|I|}} \left(\gamma^A(a|N_A) \gamma^B(a|N_B) \sum_{i \in I} a_i \right), \qquad (3.17)$$

Let f^*_{SCE} be the maximal expected turnout in problem (3.17) and f^* the maximal expected turnout in the corresponding correlated equilibrium.[9]

It turns out that the max-turnout subcorrelated equilibria result in the same expected turnout as the max-turnout correlated equilibria for the special case of $n_B \geq \lceil \frac{1}{2} n_A \rceil$, but have strictly lower turnout when $n_B < \lceil \frac{1}{2} n_A \rceil$.

Proposition 13. *Suppose* $0 < c < \frac{1}{2}$. *Then:*

(i) if $n_B \geq \lceil \frac{1}{2} n_A \rceil$, *then* $f^*_{SC} = f^* = 2n_B$, *i.e., the max correlated equilibria and max subcorrelated equilibria result in the same expected turnout of twice the size of*

---

[9]As we discussed before, the correlated equilibrium differs from the subcorrelated one by not requiring the joint distribution to be a product mixture.

minority.

(ii) if $n_B < \lceil \frac{1}{2} n_A \rceil$, then

$$f_{SC}^* < f^* = 2n_B + \frac{(n_A - 2n_B)(1 - 2c)}{1 + 2c\left(\frac{n_A(n_A - 1)}{n_A + n_B(n_A - 1)} - 1\right)}$$

Proof. Follows from Proposition 1 in Chapter 2. ∎

Proposition 13 shows that for the case of large minority $(n_B \geq \lceil \frac{1}{2} n_A \rceil)$, there is no difference between the max correlated and max subcorrelated equilibria in terms of expected turnout.

Finally, let us turn to the difference between joint and individual group turnout optimization in jointly feasible principals' equilibria. The joint turnout optimization is assumed in problem (3.17); now we instead consider distributions $(\gamma^{A*}, \gamma^{B*})$ that solve

$$\gamma^{A*} \in \arg \max_{\substack{\gamma \in \mathcal{D}_{SC}(\cdot, \gamma^{B*}, c) \\ \gamma^{B*} \in \mathcal{D}_{SC}(\gamma, \cdot, c)}} \sum_{a \in \{0,1\}^{n_A}} \left(\gamma(a) \sum_{i \in N_A} a_i \right) \tag{3.18}$$

$$\gamma^{B*} \in \arg \max_{\substack{\gamma \in \mathcal{D}_{SC}(\gamma^{A*}, \cdot, c) \\ \gamma^{A*} \in \mathcal{D}_{SC}(\cdot, \gamma, c)}} \sum_{a \in \{0,1\}^{n_B}} \left(\gamma(a) \sum_{i \in N_B} a_i \right) \tag{3.19}$$

Thus, in (3.18)-(3.19), groups maximize their own expected turnout, not the total turnout. However, there is a one-directional relation between the solutions to the problem of maximizing total expected turnout in a subcorrelated equilibrium and the solution to problem (3.17).

Proposition 14. *Any solution to the problem of maximizing the total expected turnout in a subcorrelated equilibrium that solves* (3.17) *also solves* (3.18)-(3.19). *The opposite does not hold in general: there are solutions to* (3.18)-(3.19) *which do not solve* (3.17). *Nevertheless, there is always at least one solution to* (3.18)-(3.19) *that also solves* (3.17).

Proof. To see that solution to (3.17) also solves (3.18)-(3.19), note that for a fixed γ^{B*}, we can rewrite the right hand side of (3.17) as follows.

$$
\sum_{a \in \{0,1\}^{|I|}} \gamma^A(a|N_A)\gamma^{B*}(a|N_B) \left(\sum_{i \in N_A} a_i + \sum_{i \in N_B} a_i \right)
$$

$$
= \sum_{a \in \{0,1\}^{|I|}} \left[\gamma^{B*}(a|N_B) \left(\sum_{i \in N_B} a_i \right) \gamma^A(a|N_A) + \gamma^{B*}(a|N_B) \left(\gamma^A(a|N_A) \sum_{i \in N_A} a_i \right) \right]
$$

$$
= \sum_{a \in \{0,1\}^{n_B}} \gamma^{B*}(a) \left(\sum_{i \in N_B} a_i \right) + \left[\sum_{a \in \{0,1\}^{n_B}} \gamma^{B*}(a) \right] \left[\sum_{a \in \{0,1\}^{n_A}} \left(\gamma^A(a) \sum_{i \in N_A} a_i \right) \right]
$$

$$
= C(\gamma^{B*}) + D(\gamma^{B*}) \left[\sum_{a \in \{0,1\}^{n_A}} \left(\gamma^A(a) \sum_{i \in N_A} a_i \right) \right],
$$

where $C(\gamma^{B*}), D(\gamma^{B*}) \geq 0$ do not depend on γ^A. Similarly, for a fixed γ^{A*}, we can express the right hand side of (3.17) as

$$
\sum_{a \in \{0,1\}^{|I|}} \gamma^{A*}(a|N_A)\gamma^B(a|N_B) \left(\sum_{i \in N_A} a_i + \sum_{i \in N_B} a_i \right) = \tilde{C}(\gamma^{A*}) + \tilde{D}(\gamma^{A*}) \left[\sum_{a \in \{0,1\}^{n_B}} \left(\gamma^B(a) \sum_{i \in N_B} a_i \right) \right]
$$

Clearly, any solution to both of these equations also solves (3.18)-(3.19).

To show that the opposite does not hold, consider the following example. Let

$n_A = 3, n_B = 2$. Fix the common cost $c = 0.4$, and consider the following pair of distributions over the respective groups: $\gamma^A_{000} = \frac{1}{15}$, $\gamma^A_{001} = \gamma^A_{010} = \gamma^A_{100} = 0$, $\gamma^A_{011} = \gamma^A_{101} = \gamma^A_{110} = \frac{4}{15}$, and $\gamma^A_{111} = \frac{2}{15}$ for N_A; and $\gamma^B_{00} = \gamma^B_{01} = \gamma^B_{10} = 0$, $\gamma^B_{11} = 1$ for N_B. The expected turnout in either group is 2, so the total expected turnout is 4. It is easy to check that all IC constraints (3.13)-(3.14) hold for both groups. It is also easy to see why these distributions solve (3.17): from Proposition 1 in Chapter 2, the maximal expected turnout in the correlated equilibrium in this example is 4, so this is the largest total expected turnout in any subcorrelated equilibrium. Consequently, from the first part of this proof, (γ^A, γ^B) solve (3.18)-(3.19). Consider now a different pair of distributions (ν^A, ν^B) specified as follows: $\nu^A_{000} = 0$, $\nu^A_{001} = \nu^A_{010} = \nu^A_{100} = 0$, $\nu^A_{011} = \nu^A_{101} = \nu^A_{110} = \frac{4}{15}$, and $\nu^A_{111} = \frac{3}{15}$ for N_A; and $\nu^B_{00} = \frac{1}{5}$, $\nu^B_{01} = \nu^B_{10} = 0$, and $\nu^B_{11} = \frac{4}{5}$ for N_B. The only difference from (γ^A, γ^B) is that ν^A has moved $\frac{1}{15}$ of probability from the profile with zero group turnout to a profile with full group turnout, and ν^B has moved $\frac{1}{5}$ of probability in the opposite direction. Again, it is easy to check that all IC constraints (3.13)-(3.14) hold for these distributions as well. The expected turnout in group N_A is now 2.2, whereas the expected turnout in group N_B is 1.6. Hence the total expected turnout is $3.8 < 4$, and so (ν^A, ν^B) is not a solution to (3.17).

To see that (ν^A, ν^B) nevertheless solve (3.18)-(3.19), let's write out the joint IC constraints having fixed ν^A. Due to symmetry, it is sufficient to consider just one

player in each group, so we start from group N_B and player 1 in that group:

$$c\left(\frac{3\cdot 4}{15}\nu_{00}^B + \frac{3}{15}(\nu_{00}^B + \nu_{01}^B)\right) + \left(c - \frac{1}{2}\right)3\cdot\frac{4}{15}\nu_{01}^B \geq 0$$

$$-c\left(\frac{3\cdot 4}{15}\nu_{10}^B + \frac{3}{15}(\nu_{10}^B + \frac{3}{15}\nu_{11}^B)\right) + \left(\frac{1}{2} - c\right)3\cdot\frac{4}{15}\nu_{11}^B \geq 0$$

For $c = 0.4$, the first constraint does not restrict ν_{00}^B, while the second one requires $\nu_{10}^B = 0$, which by symmetry translates to $\nu_{01}^B = 0$. The IC constraints for group N_A now take the following form:

$$c\frac{4}{15}\nu_{00}^B + \left(c - \frac{1}{2}\right)\frac{4}{15}\nu_{11}^B \geq 0$$

$$-c\left(\frac{2\cdot 4}{15}\nu_{00}^B + \frac{3}{15}\nu_{00}^B\right) + \left(\frac{1}{2} - c\right)\left(2\cdot\frac{4}{15}\nu_{11}^B + \frac{3}{15}\nu_{11}^B\right) \geq 0$$

The first constraint at $c = 0.4$ reduces to $\nu_{00}^B \geq \frac{1}{4}\nu_{11}^B$. The second constraint reduces to $\nu_{00}^B \leq \frac{1}{4}\nu_{11}^B$. Hence $\nu_{00}^B = \frac{1}{4}\nu_{11}^B$, and the total probability constraint determines $\nu_{00}^B = \frac{1}{5}$, $\nu_{11}^B = \frac{4}{5}$. So for the given ν^A, ν^B solves (3.19). Similarly, one can show that for the given ν^B, ν^A solves (3.18). ∎

3.3 Concluding Remarks

This Chapter introduces the subcorrelated equilibrium as well as its generalization for the case of incomplete information called subcommunication equilibrium. These

equilibria refine the set of Aumann's correlated equilibria, and Myerson's and Forges' communication equilibria, respectively, and can serve as an intermediate solution concept between Nash and the correlated equilibrium, when there are several correlation devices available to groups of players. Unlike principals' equilibria of Myerson (1982), subcorrelated/subcommunication equilibrium does not necessarily require that different correlation devices represent competing principals, which ensures existence and makes it a potentially attractive solution concept for analysis of dynamic situations, e.g., where the game gradually evolves from some (equilibrium) status quo to another equilibrium, like in policy reforms. We establish an equivalence between a subset of subcorrelated equilibria and Myerson's quasi-principals' equilibria.

Chapter 4

Voting with Communication: An Experimental Study of Correlated Equilibrium

How does communication among voters affect turnout? This is a key question for modern political campaigns that actively employ media and social networks to reach and mobilize their supporters. Field experimental studies, which usually isolate a particular communication mechanism, have shown significant but mixed evidence (Gerber and Green, 2000; Gerber et al., 2011), which is perhaps not surprising, given the plethora of different ways people communicate. Effectiveness of political communication depends on complex interactions of different communication mechanisms, political actors, and institutional structures (Druckman, 2014).

In this Chapter, we attempt to explore the general principles behind these interactions in a laboratory experiment. There are two groups of voters of different sizes (think political parties) that compete against each other in a winner-take-all election.

Voters decide between turning out to vote for their preferred party and staying at home. Voting is costly, with a commonly known voting cost $c > 0$, same for everyone. Each voter's payoff depends on which party wins the elections, which is in turn defined by how many voters decide to turn out. A rational voter trades off expected benefits with expected cost, so she should only turn out when there is a sufficiently high chance of her vote changing the outcome.

Absent communication, each voter decides whether to participate independently of others. The game-theoretic analysis of this case (Palfrey and Rosenthal, 1983) shows that there is a multitude of Nash equilibria with a wide range of expected turnout.

With communication, the formal structure of the game changes dramatically, as individual turnout decisions can now be correlated. Allowing for correlation greatly expands the set of equilibria. In fact, the game with unrestricted communication actually admits an infinite number of equilibria, with expected total turnout ranging between zero and twice the size of the minority for all positive voting costs such that voting is rationalizable, i.e., abstention is not a dominant strategy (**?**).

We study the effects of unmediated pre-play communication on turnout. Before making their decisions, subjects engage in free-form communication in the form of chat, by broadcasting messages to subsets of players. We consider two cases: *public communication*, where players can exchange public messages visible to all participants; and *party communication*, where players can exchange messages that are only public within their own party (majority or minority). These communication patterns seem to have reasonable analogs in typical elections. For example, car bumper stickers

can be interpreted as public messages, while Facebook status updates (e.g., "I'm a Voter" button feature, studied in Bond et al (2012)), visible only to one's own group of friends or social connections, are examples of party-based public messages. A variation on correlated equilibrium, called *subcorrelated equilibrium* is used to characterize the equilibria with party communication (**?**). While our experiment (and the model) does not have any explicit centralized mobilization efforts per se, one can view the kind of decentralized communication studied here as corresponding to neighborhood information exchanges (Grosser and Schram, 2006) or conversations and interactions with family and friends, or via social media.

In addition to the communication treatment, we vary the other crucial parameters of the model: the voting cost ("low" cost with 10% and "high" cost with 30% of the maximum election benefit), and relative party sizes (landslide and close elections). The sensitivity of turnout to changes in these parameters under our restrictions on communication has not been investigated.

The results of the experiment lead to three main contributions.

First, we establish a causal link between the cost of voting, the communication mechanism, and the total turnout rate, and identify strong interaction effects between the structure of communication and the cost of voting. With a low voting cost, party communication increases total turnout, while public communication decreases total turnout. With a high voting cost, public communication increases total turnout. Thus, we identify a *cost/communication interaction effect*, whereby cost considerations appear an important factor in participation decisions under communication,

which ties in with some existing empirical results (Brady and McNulty, 2011; Hodler, Luechinger, and Stutzer, 2015).

Second, the experiment allows us to test, for the first time, the consistency of experimental data under communication with the correlated equilibrium. Correlated equilibria have been largely ignored in the experimental study of pre-play communication, yet are particularly well suited for the analysis of such games.[1] This is especially true with our design that includes both public and party communication mechanisms, which require somewhat different variations on the correlated equilibrium concept. We design several new tests to check for the consistency with correlated equilibrium, and find that voting cost plays an important role here as well: with low cost, we cannot reject the hypothesis that the data are generated by a correlated equilibrium, while with high cost this is no longer the case. This approach also allows us to test for consistency with (uncorrelated) Nash equilibrium, and these tests find no support for Nash equilibrium in our communication sessions.

Third, we find that turnout levels are strongly affected by the ex ante competitiveness of the election and the voting cost. Theoretically, turnout in each party is higher when parties are closer to the same size (*competition effect*), and when costs are lower (*cost effect*).[2] We observe the competition effect (with respect to the majority party only) in all our communication and cost variations, and the cost effect in all but the public communication treatment.

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[1]An exception is Moreno and Wooders (1998). See also Cason and Sharma (2007), and Duffy and Feltovich (2010) for studies of abstract games with recommended play.

[2]This theory has support from other experiments (Levine and Palfrey, 2007; Herrera, Morelli, and Palfrey, 2014).

The remainder of the Chapter is organized as follows. In Section 4.0.1 we provide a brief literature review. In Section 4.1 we describe the details of our experimental design. In Section 4.2 we present our findings at the electorate level, party level, and individual level. Section 4.3 concludes. Additional estimation details are in 4.A. Experimental instructions are in 4.B.

4.0.1   Related literature

This Chapter contributes to studies of the effects of communication on turnout using lab experiments. A laboratory setting provides a natural first step, allowing for tight control of key model parameters and actors' preferences that is nearly impossible in the field.

Several studies investigate the effects of restrictive communication mechanisms – like neighborhood information exchange and polls – on voter turnout. Grosser and Schram (2006) consider the effects of communication in the form of neighborhood information exchange. In their model, every two voters form a neighborhood with one being an early voter (sender) and one a late voter (receiver). They find that information exchange increases turnout, although these results seem to be sensitive to the analyzed sender-receiver protocol. Grosser and Schram (2010), and Agranov et al. (2013) study the effects of polls on turnout and welfare. In particular, Agranov et al. (2013) show that while polls do not have negative welfare effects, they overestimate voter turnout. The authors also find evidence for voting with the winner, where a voter is more likely to turn out if she thinks her preferred candidate is more

likely to win. In our experiment, subjects communicate by means of a free-form chat with public messages (either public within a party, or public within the whole electorate). In theory, with a slightly different experimental design, this mechanism can replicate the effects of both polls and neighborhood information exchanges. Schram and Sonnemans (1996a) study a social pressure turnout model of Schram and van Winden (1991) in the lab and find that communication increases turnout. In the model, there are two groups and opinion leaders in each group, who produce social pressure on others to turn out. One of the basic predictions of that model is that communication increases turnout. In their experiment communication was oral between the members of the same group for 5 minutes, after which five more rounds of the game without further communication were played, which is different from having pre-play communication each round, as in our study.

A closer related paper is Kittel, Luhan, and Morton (2014). They study 3-party elections with costly voting, varying voter preference types (swings, who have strict rankings over three parties, vs. partisans, who strictly prefer one party but are indifferent among the two less preferred ones), party labels (voters are assigned to labeled parties), and pre-play communication protocol (public across groups vs. public within groups, like in our experiment). They find that communication increases turnout and increases the proportion of strategic voting. The effects of the communication protocol on turnout depend on voter preferences (swing vs. partisan) and are nuanced. In particular, swing voters and partisans show different turnout rates: swings assigned to their second choice are more likely to turn out in the "all-chat" than in the "party-chat", while swings assigned to their first choice, as well as

partisans, show no difference. While we use similar communication treatments, our results are not directly comparable since 3-party elections introduce a completely different motive for voting.

There are also two strands of the literature that are less related to our study. One studies the effects of deliberation on jury voting, where abstention is not allowed (Guarnaschelli, McKelvey, and Palfrey, 2000; Goeree and Yariv, 2011).[3] Another studies the performance of the Palfrey and Rosenthal (1983) model in the lab, similar to us, but without communication among voters (Schram and Sonnemans, 1996b; Levine and Palfrey, 2007; Duffy and Tavits, 2008).

Summarizing the related literature analysis, our main methodological contribution in this Chapter consists of exploring the effects of unrestricted public communication protocols in the form of pre-play chat on voter turnout in the pivotal voter model, which also allows us to check consistency of the data with the correlated equilibrium.

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[3]Goeree and Yariv (2011) use a free communication protocol and find that communication significantly improves information aggregation and efficiency. Interestingly, although they allow subjects to send messages to any subset of other subjects, they observe that almost 100% of messages were public.

4.1 Experimental Design

4.1.1 Environment

In this section we provide some theoretical background for the pivotal voter model (Palfrey and Rosenthal, 1983) our experiment builds upon. There are n voters divided into two parties, A and B, with the number of supporters n_A and n_B, respectively, so that $n_A + n_B = n$. A is the majority party, thus $n_A > n_B$. Voters in each party decide between voting for their respective party (action 1) or abstaining (action 0). The election is decided by the simple majority with ties broken randomly. Voting is costly, with $c \in (0, 1/2)$ the common voting cost, same for everyone. The utility of voting net of voting cost is normalized to 1 if the preferred party wins, $1/2$, if there is a tie, and 0 otherwise. The game has complete information, the only uncertainty from a player's point of view comes from not knowing how exactly everybody else is going to choose. Each voter would ideally prefer her party to win the election without her actually voting, so the game combines a free-rider problem with a collective action problem in each party.

Rational voters trade off expected benefits from voting with expected costs, so the probability of their vote changing the election outcome becomes a major factor. In equilibrium, this so called pivotal probability is determined endogenously. The other important variables are probabilities of a tie and near tie (i.e., a tie$\pm$ one vote), and of the minority party winning (the upset rate). The equilibrium logic leads from the primitives of the model (party sizes and voting cost) to predictions

about these probabilities as well as the total turnout rates in each party and in the whole electorate. The turnout game has many Nash equilibria, and out of those, we focus on the ones with maximal expected total turnout. After all, the well-known turnout paradox arises precisely because people tend to vote too much relative to the equilibrium predictions.

In the experiment, we vary the sizes of the parties, n_A and n_B, the common voting cost c, and the communication protocol. The latter factor is especially important, because communication allows voters' actions to be correlated, which may, in theory, lead to higher turnout than predicted by the standard Nash equilibria. The main difference between Nash (NE) and correlated equilibrium (CE), developed in Aumann (1974, 1987) and formally defined below, is that in the latter, players' strategies can be correlated, while in a Nash all players decide whether to vote or abstain independently. Thus a Nash equilibrium is also a correlated equilibrium, but (in a formal sense) almost all correlated equilibria are not Nash equilibria.

Correlated equilibria in turnout games were analyzed in **?**, who characterized the bounds of the set of CE in these games. In particular, CE can have total expected turnout of up to twice the size of the minority party when the minority size is at least 50% that of the majority (the large minority case), and up to a higher value of the majority size in the remaining, small minority, case. These equilibria presume unrestricted communication between all players, so all strategies can be correlated in a CE.

Subcorrelated equilibrium (SCE), proposed in **?**, is a correlated equilibrium with ad-

ditional restrictions on the structure of admissible correlations: namely (in a turnout game), SCE allows players' choices to be correlated within their party, but requires independence of players' choices across parties. It turns out that in the large minority case, one can limit communication in this way (i.e., to remain unrestricted only within each party) and still get the twice the size of the minority theoretical upper bound on the expected total turnout. However, this restriction to within-party communication actually bites for the small minority case and predicts a lower turnout.

The above considerations led us to implement the following treatments. We chose to study electorates with $(n_A, n_B) \in \{(6,4), (7,3)\}$ because these cases correspond to "toss-up" and "landslide" elections, respectively, and result in different equilibrium predictions. We will sometimes refer to partitions $(6,4)$ and $(7,3)$ as large and small minority, respectively. We varied the voting cost between low ($c = 0.1$) and high ($c = 0.3$). With respect to communication, we added to the game a pre-play communication stage in the form of a computerized broadcast chat, where, before making decisions, players engage in pre-play communication by exchanging public messages to other players. We investigate two cases: *Public Communication* (PC), where players can exchange public messages visible to all participants; and *Group-restricted public communication* (GC), where players can exchange messages that are only public within their own group. We also run a control case of no communication (NC).

Table 4.1 summarizes the Nash equilibrium[4] turnout rates in this game, as well as the maximum expected turnout rates for correlated and subcorrelated equilibrium, for the four treatments in the party-size-voting-cost domain. Notation-wise, in Table 4.1, n_A (n_B) is the size of party A (B), c is the common voting cost, T_A (T_B) is expected equilibrium turnout rate in party A (B), T is total expected turnout rate, τ is probability of a tie, π is probability of a pivotal event (defined as tie$\pm$ one vote), ω_B is probability of the underdog winning (upset rate). The correlated equilibria with maximal expected total turnout are not unique, so τ, π, and ω_B can differ across different equilibria.

Table 4.1 shows several common patterns of theoretic predictions across treatments. We emphasize those about the total turnout in the following proposition.

Proposition 15. *In equilibria that maximize expected turnout, the total turnout rate is 1) increasing with correlation among voters (from Nash to Subcorrelated to Correlated equilibrium); 2) decreasing in the majority party size, except under high voting cost in a quasi-symmetric Nash; 3) non-increasing in the voting cost.*

4.1.2 Procedures

We ran a total of 6 sessions with a high common cost ($c = 0.3$) and another 6 sessions with a low common cost ($c = 0.1$), with the main focus on the effects

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[4]Given the anonymous random matching and the symmetric structure of the game for each party, we limit attention to Nash equilibrium where all members of the party mix with the same probability. This is standard in the analysis of data from turnout experiments (Levine and Palfrey, 2007). Other kinds of highly asymmetric equilibria are discussed in Palfrey and Rosenthal (1983). See also Schram and Sonnemans (1996b).

Table 4.1: Theoretic Predictions for Equilibria with Maximal Total Expected Turnout

n_A	n_B	c	T_A	T_B	T	τ	π	ω_B
			Nash Quasi-Symmetric					
6	4	.1	.625	.375	.525	.089	.298	.083
–	–	.3	.161	.253	.198	.322	.781	.518
7	3	.1	.521	.479	.508	.098	.319	.009
–	–	.3	.147	.380	.217	.310	.773	.545
			Subcorrelated Equilibria					
6	4	.1	.667	1.000	.800	.200	.636	.365
–	–	.3	.667	1.000	.800	.600	.790	.421
7	3	.1	.725	.482	.652	.096	.278	.048
–	–	.3	.544	.799	.620	.479	.691	.240
			Correlated Equilibria					
6	4	.1	.667	1.000	.800	.200	.636	.365
–	–	.3	.667	1.000	.800	.600	.790	.421
7	3	.1	.581	.089	.670	.059	.335	.030
–	–	.3	.414	.215	.628	.429	.764	.215

Notes: There are many other equilibria with smaller total turnout, which are not listed here.

of communication. For each cost, there were 2 sessions on each one of the three communication treatments: no communication (NC), group communication (GC), and public communication (PC). We used a within-subjects design for the relative size treatment in each session, and recruited 20 subjects per session to mitigate effects of shared histories in the presence of communication. The same communication mode (NC, GC, or PC) was used throughout the entire session. No subject participated in more than one session. For communication treatments we limited the duration of chat to 110 seconds.[5]

Each session consisted of 20 matches. In each match of the game, players in the majority and the minority group were asked to decide whether to abstain or to vote, knowing the common voting cost. To induce as neutral environment as possible, no voting context was mentioned, similarly to the design of Levine and Palfrey (2007). Majority and minority parties were called "type A" and "type B", respectively. Subjects chose between two abstract options, X and Y, which correspond to voting and abstention. The voting cost was implemented as a bonus payoff (i.e., choosing option Y would result in a higher individual payoff than choosing option X). An example of the user interface is shown in Figure 4.1 in 4.A.

Our randomization algorithm is as follows. First, all subjects are randomly split into majority and minority pools. E.g., for size $(6, 4)$ and 20 subjects in a session, majority pool has 12 people, and minority pool has 8. Next, for each of the two 10-subject electorates, two subgroups are formed by randomly picking the corresponding number

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[5]When analyzing chat logs, it was clear that this amount of time was more than enough for meaningful communication.

of subjects from each pool. This step is repeated for 10 matches (Part 1). Thus for 10 matches the party labels remain the same, but the subgroup composition changes each match. For the next 10 matches (Part 2), those from the former minority pool are assigned to the majority party for sure. Those from the former majority pool are randomly picked to fill the remaining parties' capacities in each group. We chose this scheme to mitigate the inherent inequality of payoffs across subjects, since in either size treatment the majority is more likely to win.[6]

We did not inform subjects about the number of matches in each part. Instructions for Part 2 were delivered only once Part 1 was over. Two rounds from each part were randomly selected and paid, so the payoff of each subject was the sum of payoffs in four rounds plus the show-up fee of $7. Overall, the 12-session, $2 \times 2 \times 3$ design used 240 subjects which generated a dataset with a total of 480 elections. Average payoff per subject was $28.81, sessions with communication lasted approximately 1.5 hours, sessions without communication took a bit less than one hour. Sample instructions are in 4.B.

Table 4.2 summarizes our design and experimental parameters.

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[6]There was one software issue in the session with (7,3)(6,4), low cost, and Public Communication. The randomization scheme used in Part 1 ((7,3) treatment) was actually corresponding to (6,4). That is, there were 7 people in group A, 3 people in group B, and their types were always correctly labeled, but the randomization algorithm was such that some of them did not keep their types constant the entire 10 matches: there were people from A switching to B and vice versa (as the algorithm was only keeping together the first 6 people in one group). 8 subjects out of 40 were affected in the sense that at least once during the 10 matches their types were switched, but since it was a public communication treatment, we do not see this is as an issue for the analysis.

Table 4.2: Design Summary

Communication	Cost $c = 0.1$		Cost $c = 0.3$	
	First 10 rounds	Next 10 rounds	First 10 rounds	Next 10 rounds
Group Chat	(6,4)	(7,3)	(6,4)	(7,3)
	(7,3)	(6,4)	(7,3)	(6,4)
Public Chat	(6,4)	(7,3)	(6,4)	(7,3)
	(7,3)	(6,4)	(7,3)	(6,4)
No Chat	(6,4)	(7,3)	(6,4)	(7,3)
	(7,3)	(6,4)	(7,3)	(6,4)

Notes: Table cells contain the sizes of (Majority, Minority) for each treatment. For each communication regime and voting cost, each session combined two size treatments and had 20 rounds total, with two independent electorates in each round. Sessions with communication lasted about 1.5 hours, sessions without communication took a bit less than 1 hour.

4.2 Results

In this section we describe our main experimental results. To save space and for expositional clarity, for many of our results we report additional supporting figures and estimation details (e.g., standard errors and p-values) in 4.A.[7]

Table 4.3 presents for each treatment the summary averages of turnout rates in each party and total turnout rate. With our $2 \times 2 \times 3$ design, the total number of elections in each of the 12 treatments was 40. To account for possible correlation across rounds and across 10-person groups (as group composition changed every round), we treated each group over 10 rounds as a panel, and computed panel-corrected standard errors

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[7]In addition to the turnout-related results reported in this section, we also looked at the effects on welfare, which in this model highlights the tradeoff between the probability the majority group wins, the voting cost, and the expected total turnout. Reducing the voting cost or the relative minority size increases total welfare in all treatments. For large minority, communication increases welfare under low voting cost, but decreases it under high voting cost. Detailed results are available from the authors upon request.

with a correction for first order autocorrelation within each panel.[8]

Table 4.3: Mean Turnout Rates by Treatment

No Communication									
N	n_A	n_B	c	$\hat{T}_A$		$\hat{T}_B$		$\hat{T}$	
40	6	4	.1	.600	(.030)	.574	(.040)	.587	(.032)
–	–	–	.3	.403	(.020)	.371	(.018)	.390	(.014)
–	7	3	.1	.443	(.021)	.377	(.044)	.423	(.015)
–	–	–	.3	.336	(.033)	.349	(.059)	.341	(.029)
Group Communication									
N	n_A	n_B	c	$\hat{T}_A$		$\hat{T}_B$		$\hat{T}$	
40	6	4	.1	.875	(.023)	.424	(.064)	.696	(.032)
–	–	–	.3	.483	(.055)	.380	(.086)	.442	(.046)
–	7	3	.1	.616	(.026)	.448	(.100)	.566	(.030)
–	–	–	.3	.378	(.091)	.251	(.042)	.342	(.054)
Public Communication									
N	n_A	n_B	c	$\hat{T}_A$		$\hat{T}_B$		$\hat{T}$	
40	6	4	.1	.541	(.037)	.361	(.081)	.471	(.043)
–	–	–	.3	.639	(.053)	.392	(.056)	.541	(.042)
–	7	3	.1	.387	(.048)	.221	(.038)	.337	(.034)
–	–	–	.3	.532	(.035)	.342	(.054)	.473	(.036)

Notes: N is the number of group decision observations. Panel-corrected AR(1)-corrected standard errors are in parentheses.

Table 4.3 reveals a number of differences across treatments, which we discuss and test below.[9] A quick comparison with the theoretic predictions for max-turnout equilibria in Table 4.1 shows that Nash equilibrium is not consistent with the behavior observed in our experiment: average total turnouts rates in the data are much higher (with one exception) than in the maximal turnout Nash equilibria predictions of Table 4.1, already in the No Communication treatment, and even more so in the Group Communication treatment. Similarly to the stylized facts from real world elections

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[8]We also ran the standard two-sample t-test for mean comparisons across treatments, which assumes independent observations in each sample, and found very similar results. The findings we report are thus more conservative.

[9]In addition to turnout rates, we summarized several important electoral characteristics: mean frequencies of ties, pivotal events and the frequency of upsets in Table 4.16 in 4.A.

and many past experimental findings, voters in the lab tend to over-vote compared to the Nash prediction. In contrast, a casual inspection of the turnout rates suggest that the data might be consistent with correlated and subcorrelated equilibrium predictions, although to establish this more rigorously requires a deeper analysis that we present below. Total turnout rates in Table 4.3 are somewhat lower than max-turnout correlated and subcorrelated equilibrium predictions in Table 4.1, but seem to satisfy the constraint of the turnout upper bound, and thus it might be possible to associate correlated and subcorrelated equilibria with expected turnout that would match the data. In order to check this, we develop and apply a formal direct test for consistency with correlated and subcorrelated equilibrium in Section 4.2.5.2.

The remainder of this section is organized as follows. In Sections 4.2.1-4.2.3, we statistically test the treatment effects of changes in communication protocol, voting cost, and relative party sizes. These behavioral effects are tested for using group-level decisions and do not rely on specific assumptions about equilibrium behavior. In Section 4.2.4 we look at the distribution of votes at the party and electorate levels. In Section 4.2.5 we look at the individual level data and check whether these patterns are consistent with equilibrium behavior.

4.2.1 Effects of communication

We start our analysis by looking at the main effect of interest: how communication affects turnout.

Result 1. *Group communication increases total turnout (with significant increase under the low voting cost). Public communication increases turnout under the high cost, but decreases turnout under the low cost.*

Support for Result 1. To test for the effect of communication on total (and party) turnout, we compared the mean turnout rates under two different communication modes while keeping party sizes and voting cost fixed.[10]

Table 4.4: Effects of Communication on Turnout

n_A	n_B	c	$\Delta\hat{T}_A$	$\Delta\hat{T}_B$	$\Delta\hat{T}$
Group Communication v. No Communication					
6	4	0.1	0.276***	−0.151**	0.109**
–	–	0.3	0.080	0.010	0.052
7	3	0.1	0.173***	0.071	0.143***
–	–	0.3	0.042	−0.099	0.001
Public Communication v. No Communication					
6	4	0.1	−0.058	−0.214**	−0.117**
–	–	0.3	0.236***	0.021	0.151***
7	3	0.1	−0.055	−0.155***	−0.086**
–	–	0.3	0.200***	−0.008	0.131***

Significance codes: *** < 0.01, ** < 0.05, * < 0.1

From Table 4.4, we see that GC increases total turnout, compared to NC for each size treatment and the low cost. The difference under the high cost is also positive, but not significant. By contrast, PC works in the opposite way: for each size treatment, PC increases total turnout, compared to NC, under the high cost, but decreases it under the low cost. ■

Result 2. *Communication affects parties differently: under the low cost, communication decreases minority party turnout (except Group communication and small minority). Under the high cost, Public communication increases majority party turnout,*

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[10]We report two-sided *p*-values in Table 4.17 in 4.A.

while Group communication shows no effect.

Support for Result 2. Looking at the turnout rate by parties in Table 4.4, we see that group communication increases majority turnout for each size treatment and the low cost, compared with no communication. As for public communication, PC increases majority party turnout for each majority size and the high cost, but decreases majority turnout under the low cost (the decrease is not significant though). Comparing GC and PC, we find a pattern similar to the one we observed for total turnout: for each size treatment, PC increases majority turnout, compared to GC, under the high cost, but decreases it under the low cost.

The effect of communication on minority turnout is less pronounced. GC decreases minority turnout, compared to NC for the large minority and the low cost ($p = 0.05$), and for the small minority and the high cost (not significantly). PC decreases minority turnout compared to NC for each size treatment and the low cost; this finding is in line with our results for total turnout. Finally, PC and GC are not significantly different with respect to minority turnout for each size and cost treatments, except for partition $(7, 3)$ and the low cost, where PC decreases minority turnout compared to GC. ∎

Next, we look at the effects of communication on several electoral characteristics: probability of ties, pivotal events, and upsets.

Result 3. *Communication uniformly decreases probabilities of ties and upsets[11], with significant effects under GC, low cost, and large minority. Communication also*

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[11]With one exception under GC, high cost, and large minority.

decreases the probabilities of pivotal events, with significant effects under PC and high cost, and under GC, low cost, and large minority.

Support for Result 3. We report the corresponding results in Table 4.5.

Table 4.5: Effects of Communication on Electoral Characteristics

Group Communication v. No Communication					
n_A	n_B	c	ΔTie	ΔPivotal	ΔUpset
6	4	0.1	-0.256**	-0.250**	-0.240*
–	–	0.3	-0.056	-0.028	0.057
7	3	0.1	-0.060	-0.053	-0.044
–	–	0.3	-0.075	-0.036	-0.063
Public Communication v. No Communication					
n_A	n_B	c	ΔTie	ΔPivotal	ΔUpset
6	4	0.1	-0.104	0.003	-0.122
–	–	0.3	-0.122	-0.400***	-0.030
7	3	0.1	-0.050	0.075	-0.031
–	–	0.3	-0.112	-0.300***	-0.113*

Significance codes: *** < 0.01, ** < 0.05, * < 0.1

From Table 4.5 we find that group communication uniformly decreases probabilities of ties and pivotal events. The effect is significant for the low cost and partition $(6, 4)$. Public communication also decreases the probabilities of ties, although the difference is not significant. The decrease in the probability of pivotal events is large and significant under the high voting cost. GC marginally decreases the upset probability, compared to NC, under the low cost and partition $(6, 4)$. PC does the same under the high cost and partition $(7, 3)$. ∎

4.2.2 Effects of changing the voting cost

Result 4. *Increasing the voting cost reduces the total turnout for each size and communication treatment, except public communication. Under public communication, increasing the cost increases the total turnout. These changes mostly occur via the effect on the majority party turnout. Increasing the cost increases the probability of pivotal events under group communication and no communication, and decreases it under public communication.*

Support for Result 4. We first look at the effect of increasing the voting cost on turnout and welfare (Table 4.6), and then report the effects on the electoral characteristics (Table 4.7).

Table 4.6: Effects of Cost on Turnout

			High Cost v. Low Cost		
n_A	n_B	Communication mode	$\Delta\hat{T}_A$	$\Delta\hat{T}_B$	$\Delta\hat{T}$
6	4	NC	-0.196^{***}	-0.204^{***}	-0.197^{***}
–	–	GC	-0.392^{***}	-0.043	-0.254^{***}
–	–	PC	0.098	0.031	0.070
7	3	NC	-0.107^{***}	-0.027	-0.081^{**}
–	–	GC	-0.238^{**}	-0.197^{*}	-0.225^{***}
–	–	PC	0.145^{**}	0.120^{*}	0.136^{***}

Significance codes: $^{***} < 0.01$, $^{**} < 0.05$, $^{*} < 0.1$.

From Table 4.6, we see that reducing the voting cost increases total turnout for each size and communication treatment except public communication. Under PC, counter-intuitively, reducing voting cost decreases total turnout. Breaking down the total turnout by parties, we see that these changes mostly occur via the effect on the majority party turnout, although for the large majority and communication

treatments, the change in the voting cost also affects (marginally significantly) the minority party turnout.

Table 4.7 presents the corresponding effects of the cost change on the electoral characteristics we considered before: the probability of ties, pivotal events, and upsets.

Table 4.7: Effects of Cost on Electoral Characteristics

		High Cost v. Low Cost			
n_A	n_B	Comm. mode	ΔTie	ΔPivotal	ΔUpset
6	4	NC	-0.067	0.127	-0.055
–	–	GC	0.133	0.349***	0.242
–	–	PC	-0.084	-0.276***	0.037
7	3	NC	0.126	0.200**	0.108
–	–	GC	0.134*	0.217***	0.088
–	–	PC	0.082	-0.174**	0.025

Significance codes: *** < 0.01, ** < 0.05, * < 0.1.

From Table 4.7 we see that reducing the voting cost does not significantly affect the probability of ties. Reducing the cost decreases the probability of pivotal events under group communication and no communication, and increases it under public communication. The probability of upsets is uniformly increasing in the cost (except no communication and large minority), but the changes are not significant. ∎

4.2.3   Effects of changing the relative party sizes

We now turn to the effects of changing the party sizes while keeping the electorate size fixed. The competition effect hypothesis (Levine and Palfrey, 2007) predicts that turnout in each party is decreasing in $n_A - n_B$. Thus increasing the minority party size from 3 to 4 should increase turnout in each party.

Result 5. *Increasing the minority party at the expense of the majority party increases the total turnout rate, significantly under the low cost. We observe the competition effect under the low cost and no communication, and partial competition effect (for majority party only) in the remaining treatments.*

Support for Result 5. The effects on turnout are reported in Table 4.8.

Table 4.8: Effects of Changing the Relative Party Size on Turnout

		Minority Size 4 v. Minority Size 3		
c	Comm. mode	$\Delta \hat{T}_A$	$\Delta \hat{T}_B$	$\Delta \hat{T}$
0.1	NC	0.156***	0.198***	0.165***
–	GC	0.259***	−0.024	0.130***
–	PC	0.154**	0.140	0.134**
0.3	NC	0.067*	0.021	0.049
–	GC	0.105	0.130	0.100
–	PC	0.106*	0.050	0.068

Significance codes: *** < 0.01, ** < 0.05, * < 0.1.

From Table 4.8, we note that increasing the minority party at the expense of the majority party increases the total turnout rate, significantly so under the low cost.

We observe the competition effect under the low cost and no communication, but not in the remaining treatments, primarily because the expected turnout in the minority party is not significantly different across party sizes. There are significant differences for the majority party, so we observe a partial competition effect.

Going from the small minority to the large minority does not significantly change the probability of ties, pivotal events, and upsets, as we show in Table 4.22 in 4.A. ■

4.2.4 Vote distributions

We now give a closer look at the distribution of votes for each party under different treatments. First, we plot the respective histograms, separated by the communication treatments in Figures 4.2 – 4.3 in 4.A. These figures illustrate the effects described in Sections 4.2.1-4.2.3.

Second, we look at the high turnout profiles, like those with n_A and $n_A - 1$ votes for the majority party (or n_B and $n_B - 1$ votes for the minority party). A high expected turnout implies an increased probability of the high turnout profiles, and we can check how it is affected by communication.

Result 6. *Communication increases the likelihood of high-turnout profiles in both majority and minority.*

Support for Result 6. The vote distributions by profile are reported in Tables 4.26 – 4.27 in 4.A. First, we look at the majority party. We find that for partition $(6, 4)$ and no communication, the majority group profiles with high turnout (i.e., 5 and 6 votes) are played 27.5% of the time under the low cost and 5% of the time under the high cost. With group communication, the corresponding profiles are played a staggering 87.5% of the time under the low cost and 23% of the time under the high cost. With public communication, these profiles are played 20% of the time under the low cost and 35% of the time under the high cost.

For partition $(7, 3)$ and no communication, the majority group profiles with high turnout (i.e., 6 and 7 votes) are never played under the low cost and are played just 2.5% of the time under the high cost. With group communication, the corresponding

profiles are played 15% of the time under the low cost and 10% of the time under the high cost. With public communication, these profiles are played 5% of the time under the low cost and 22.5% of the time under the high cost.

Thus, communication increases the likelihood of high-turnout profiles in the majority.

Second, we look at the minority party. For partition $(6, 4)$ and no communication, the minority group profile with high turnout (i.e., 4 votes) is played 7.5% of the time under the low cost and is never played under the high cost. With group communication, the corresponding profile is played 37.5% of the time under the low cost and 17.5% of the time under the high cost. With public communication, this profile is played 2.5% of the time under the low cost and 17.5% of the time under the high cost.

For partition $(7, 3)$ and no communication, the minority group profile with high turnout (i.e., 3 votes) is played 5% of the time under either cost treatment. With group communication, the corresponding profile is played 37.5% of the time under the low cost and 7.5% of the time under the high cost. With public communication, this profile is played 2.5% of the time under the low cost and 12.5% of the time under the high cost.

Thus communication increases the likelihood of high-turnout profiles in the minority group as well. ∎

We also looked at the dynamics of party turnout with rounds of the game (see Figures 4.4 and 4.5 in 4.A). There are a few dominance relations for the majority

party turnout, but the pattern is less clear for the minority party, consistently with our results in Table 4.4.

4.2.5 Individual level data

Turning to the individual level data, we first classify subjects by their observed frequency to turn out conditional on their party.

Result 7. *Most subjects are responsive to their party type. There is more homogeneous behavior in treatments with communication than without communication. Contrary to the Underdog effect, subjects assigned to the majority party are, on average, more likely to turn out than subjects assigned to the minority party.*

Support for Result 7. Figure 4.6 in Appendix 4.A depicts individual turnout frequencies for each party. Since in Part 2 of each session, in each round subjects from the former majority party were assigned to the minority party with probability 1/2, some subjects were changing parties during the respective 10 rounds. Thus if party assignment does not influence their voting behavior, there will be coinciding droplines in a figure. We see that out of the 40 subjects who participated in each treatment, only few subjects were irresponsive to their party type. We also see more homogeneous behavior in treatments with communication than in the NC treatment. Finally, we see that subjects assigned to the majority party are, on average, more likely to turn out than subjects assigned to the minority party. This is contrary to the Underdog effect, which predicts a larger turnout rate in the minority party. ∎

4.2.5.1 Correlation in voting decisions

In this section, we investigate whether adding communication results in a "higher" correlation in voters' choices, and whether the data are consistent with correlated and subcorrelated equilibrium.

Assumptions and Hypotheses

For the analysis of correlation, we need frequency estimates of the all-electorate voting profiles from the data. The number of all possible profiles in a 10-person electorate is 2^{10}, making it impossible to estimate frequencies with our data; hence we make the following assumption to reduce the number of joint profiles. We combine together all profiles that have the same number of votes from each party and only differ by the identity of those voting and abstaining, and assume that such profiles are equally likely. For example, we assume that the following profiles have equal probability: a profile with all of the minority voting and with voters 1, 2, and 3 of the majority voting, and a profile with all of the minority voting and with voters 4, 5, and 6 of the majority voting. To state this assumption formally, let $\mu(z_i, a, b)$ denote the probability of any joint profile where player i plays strategy z_i, and, among the other $n - 1$ players, a players turn out in group N_A and b players turn out in group N_B (where "groups" correspond to majority and minority party, respectively, in the context of our experiment).

Assumption 1. *(Group-symmetric distributions). For all treatments, we only con-*

sider distributions over joint voting profiles which satisfy the following restrictions:

$$\forall i \in N_A, \forall a \in \{1, \ldots, n_A - 1\}, \forall b \in \{0, \ldots, n_B\} : \mu(0_i, a, b) = \mu(1_i, a - 1, b)$$

$$\forall k \in N_B, \forall b \in \{1, \ldots, n_B - 1\}, \forall a \in \{0, \ldots, n_A\} : \mu(0_k, a, b) = \mu(1_k, a, b - 1)$$

Using Assumption 1, we now have a total of $(n_A + 1)(n_B + 1)$ different profiles, i.e. 32 and 35 profiles in $(7, 3)$ and $(6, 4)$ treatments, respectively, which can be estimated from the data. Furthermore, this assumption is also plausible to hold in our data, since player ids within each party are randomly re-assigned every round. We can now simply write $\mu_{a,b}$ for the probability of a joint profile with a votes from party N_A and b votes from party N_B. We cannot verify Assumption 1 with our current data, so for the remainder of our analysis, we just assume that it holds throughout.

Given a group-symmetric distribution, we can hypothesize various additional incremental restrictions on its structure. We begin by formulating a group independence hypothesis, which says that admissible group-symmetric distributions over all-electorate voting profiles can be decomposed as a product of two independent distributions over party voting profiles.

Hypothesis 1. *(Group Independence). Distributions over joint voting profiles satisfy Assumption 1 and the following restriction:*

$$\forall a \in \{0, \ldots, n_A\}, \forall b \in \{0, \ldots, n_B\} : \mu_{a,b} = \gamma(a)\delta(b),$$

where $\gamma \in \Delta(\{0, \ldots, n_A\})$ and $\delta \in \Delta(\{0, \ldots, n_B\})$ are probability distributions over

group profiles.

The decomposition in Hypothesis 1 is necessary for the subcorrelated equilibrium, formally defined below. This is a relatively weak restriction. For instance, any Nash equilibrium would require a distribution over joint profiles to be decomposable as a product of n independent individual voting probabilities.

Hypothesis 1 can be strengthened at the individual level by requiring symmetry and independence, which is necessary for a quasi-symmetric Nash equilibrium.

Hypothesis 2. *(Symmetric Independent Voting). Distributions over joint voting profiles satisfy Hypothesis 1, and each voter votes independently with the same probability conditional on her party.*

Notice that Hypothesis 2 simultaneously requires independence and equal voting probabilities, so in principle it can be rejected due to a violation of either property. But since Assumption 1 combined with Hypothesis 1 holding implies the symmetry of all players in a party, we can focus on testing for independence only. Thus if we find that Hypothesis 2 holds in the No Communication treatment, but can be rejected in some communication treatment, this is evidence for communication introducing correlation.

If Hypothesis 2 holds, strategies can be described by two probabilities, p_A and p_B, for voters in the majority and minority parties, respectively. Furthermore, the prob-

ability of a majority group profile with a votes is written as

$$\gamma(a) = \binom{n_A}{a} p_A^a (1 - p_A)^{n_A - a}, \tag{4.1}$$

and probability of a minority group profile with b votes is written as

$$\delta(b) = \binom{n_B}{b} p_B^b (1 - p_B)^{n_B - b}. \tag{4.2}$$

The probability of a joint profile (a, b) with $a + b$ votes is simply $\mu_{a,b} = \gamma(a)\delta(b)$.

Hypothesis 2 can be strengthened even more by requiring that subjects actually play a quasi-symmetric Nash equilibrium.

Hypothesis 3. *(Quasi-symmetric Nash). Distributions over joint voting profiles satisfy Hypothesis 2 with probabilities p_A^* and p_B^* determined in a quasi-symmetric Nash equilibrium.*

The equilibrium conditions on p_A^* and p_B^* are given in Palfrey and Rosenthal (1983, pp.27-28). We focus on testing Hypothesis 3 for the max-turnout quasi-symmetric Nash equilibrium because the summary estimates in Table 4.3 indicate very high total turnout rates that cannot be generated by other Nash equilibria with lower turnout.

The technical details of the tests used for testing Hypotheses 1-3 are in 4.A.1. The main idea is to compare the estimated probabilities of the joint profiles $\hat{\mu}_{a,b}$ for each pair of vote counts (a, b) with the induced probabilities $\tilde{\mu}_{a,b}$ under the null of the respective Hypothesis. We compare the two distributions by means of a two-

sample Epps-Singleton test, which is a powerful non-parametric alternative to the Kolmogorov-Smirnov and Mann-Whitney tests suited for comparing discrete distributions, to see if the two are significantly different.

Result 8. *Without communication, we can view voters' turnout decisions as independent. Introducing communication results in correlated turnout decisions (except with public communication under the low voting cost). In communication treatments, there is essentially no support for Nash equilibrium play.*

Support for Result 8. First, we check Hypothesis 1. From Table 4.23 in 4.A.1, we see that Hypothesis 1 appears to hold in all treatments.

Next, we test Hypothesis 2. Table 4.9 shows that Hypothesis 2 is not rejected at 0.05 level in all no communication treatments by both Epps-Singleton and likelihood ratio tests. The likelihood ratio test rejects Hypothesis 2 for all communication treatments (except public communication under low cost). Epps-Singleton also rejects Hypothesis 2 under group communication and high cost, but does not reject it in the remaining communication treatments. Since the results of Epps-Singleton do not take into account the variance in the estimation of $\hat{p}_A$ and $\hat{p}_B$, we are more confident in the likelihood ratio test results in this case.

Thus subjects' turnout decisions under communication are correlated. This fact by itself does not imply that the subjects' play is inconsistent with Nash equilibrium, so we test Hypothesis 3 separately. Table 4.10 confirms our results above. We reject the max-turnout quasi-symmetric Nash equilibrium in all treatments except NC and low cost, and one GC treatment with low cost and large minority. ■

Table 4.9: Test for Symmetric Independent Voting: Estimated vs. Induced Distributions

Communication	n_A	n_B	c	ES test		LR test	
				W_2	p-val	LR	$\chi^2_{0.05}$
NC	6	4	0.1	1.173	.883	35.780	46.194 (32)
–	–	–	0.3	0.962	.916	35.631	–
GC	–	–	0.1	7.368	.118	137.566*	–
–	–	–	0.3	15.997**	.003	86.962*	–
PC	–	–	0.1	8.501	.075	45.554	–
–	–	–	0.3	6.922	.140	85.660*	–
NC	7	3	0.1	4.636	.327	16.694	42.557 (29)
–	–	–	0.3	3.979	.409	30.330	–
GC	–	–	0.1	1.335	.855	86.237*	–
–	–	–	0.3	9.488*	.049	51.049*	–
PC	–	–	0.1	2.153	.708	21.453	–
–	–	–	0.3	5.784	.216	51.053*	–

Notes: W_2 is two-sample Epps-Singleton test statistic for discrete data, computed using a modified version of external Stata routine **escftest**. LR is the likelihood ratio, corresponding $\chi^2_{0.05}$ critical value has $(n_A+1)(n_B+1)-1-2$ degrees of freedom (either 32 or 29). Significance codes: *** < 0.001, ** < 0.01, * < 0.05.

Table 4.10: Test for Max-Turnout Quasi-Symmetric Nash

Communication	n_A	n_B	c	ES test	
				W_2	p-val
NC	6	4	0.1	2.752	.600
–	–	–	0.3	48.595***	.000
GC	–	–	0.1	54.621***	.000
–	–	–	0.3	50.604***	.000
PC	–	–	0.1	10.563*	.032
–	–	–	0.3	179.778***	.000
NC	7	3	0.1	7.830	.098
–	–	–	0.3	26.193***	.000
GC	–	–	0.1	7.230	.124
–	–	–	0.3	26.705***	.000
PC	–	–	0.1	19.393***	.001
–	–	–	0.3	110.710***	.000

Notes: W_2 is two-sample Epps-Singleton test statistic for discrete data, computed using a modified version of external Stata routine **escftest**. Significance codes: *** < 0.001, ** < 0.01, * < 0.05.

4.2.5.2 Consistency with Correlated and Subcorrelated Equilibrium

Given the frequencies of all joint profiles, we can now check if the realized vote distribution forms a correlated equilibrium. Since we cannot reject Hypothesis 1 in our data, we can also test for a subcorrelated equilibrium. This kind of estimation becomes feasible under the following assumption.

Assumption 2. *(Fixed distribution) The realized voting outcomes are drawn every round from the* same *joint probability distribution (not necessarily an equilibrium one).*

Assumption 2 would be violated if, for example, outcomes from the first 5 rounds and from the last 5 rounds had come from two different distributions.

A *correlated equilibrium* (CE) in our game is a probability distribution over joint voting profiles where at every profile each player's choice is a best response under the posterior distribution conditional on that choice. Thus, unlike Nash, there are two best response conditions for each player: conditional on deciding to vote, and conditional on deciding to abstain. In the turnout game, CE is defined by the following $2n$ inequalities (in addition to the standard probability constraints) for each $i \in N$:

$$c \geq \frac{1}{2} \Pr(i \text{ is pivotal} \mid i \text{ abstains}) \tag{4.3}$$

$$c \leq \frac{1}{2} \Pr(i \text{ is pivotal} \mid i \text{ votes}) \tag{4.4}$$

Under the group-symmetric distributions, defined in Assumption 1, **?** shows that conditions (4.3)-(4.4) can be written as system of four inequalities, two for players in N_A, and two for players in N_B, with respect to $(n_A + 1)(n_B + 1)$ variables of the form $\binom{n_A}{a}\binom{n_B}{b}\mu_{a,b}$.

Let $\boldsymbol{\phi} = [\phi_A^0, \phi_A^1, \phi_B^0, \phi_B^1]$ be a 4×1 vector of the left hand sides of the incentive compatibility constraints (4.3)-(4.4). We show in 4.A.2 that one can write $\boldsymbol{\phi} = \boldsymbol{J\mu}$, where $\boldsymbol{J} \in \mathbb{R}_{4 \times (n_A+1)(n_B+1)}$ is a constant Jacobian matrix, and $\boldsymbol{\mu} \in \Delta(\{0, \ldots, n_A + 1\} \times \{0, \ldots, n_B + 1\})$ is a group-symmetric probability distribution over joint voting profiles. Then $\boldsymbol{\mu}$ is a group-symmetric correlated equilibrium if and only if $\boldsymbol{\phi} \leq \boldsymbol{0}$. Clearly, when $\boldsymbol{\mu}$ is estimated from the data, there might be some violations in individual components of $\boldsymbol{\phi}$, so we define $\boldsymbol{\nu} \equiv -\boldsymbol{\phi} = -\boldsymbol{J\mu}$, and test $H_0 : \boldsymbol{\nu_0} \geq \boldsymbol{0}$ $vs. \boldsymbol{\nu_0} \not\geq \boldsymbol{0}$ using a test from Wolak (1989). See 4.A.2 for details.

Result 9. *Estimated joint vote distributions are largely consistent with correlated equilibrium under the low cost, but not under the high cost. Inconsistency in the latter case is due to an insufficiently high frequency of being pivotal conditional on deciding to vote, which can be related to over-voting in the majority party and under-voting in the minority party relative to the equilibrium level.*

Support for Result 9. Table 4.11 presents the results of our test for constraint violations for all our treatments. We report the estimated profile frequencies and the estimates of $\boldsymbol{\phi}$ in corresponding Tables 4.24-4.25 in 4.A.

The results from Table 4.11 should be taken with caution, because they are based on a small number of observations. Based on these data, however, we find that

Table 4.11: Test for Consistency with Correlated Equilibrium

Communication	n_A	n_B	c	IU stat
NC	6	4	0.1	9.682**
–	–	–	0.3	24.747***
GC	–	–	0.1	2.760
–	–	–	0.3	9.548**
PC	–	–	0.1	5.213
–	–	–	0.3	64.553***
NC	7	3	0.1	0.065
–	–	–	0.3	11.031**
GC	–	–	0.1	0.195
–	–	–	0.3	16.564***
PC	–	–	0.1	5.861
–	–	–	0.3	58.619***

IU stat is the test statistic defined in (4.10). Significance codes: *** < 0.01, ** < 0.05, * < 0.1.

under the low cost, for all cases except one (no communication, low cost, and large minority) we cannot reject the the null of aggregate voting behavior being consistent with a correlated equilibrium. By contrast, under the high voting cost we soundly reject the correlated equilibrium hypothesis.[12]

The estimates of ϕ in Tables 4.24-4.25 in 4.A also show that inconsistencies under the high voting cost are due to violations of the incentive compatibility constraints (4.4), which restrict the pivotal probability conditional on deciding to vote. Thus, this probability in the data is not high enough under the high voting cost. The remaining constraints (4.3) are satisfied. ∎

Our findings so far suggest that the group communication case is somewhat special in that voting probabilities appear correlated within groups, but not across groups. This pattern would be consistent with the presence of a subcorrelated equilibrium in

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[12]Strictly speaking, this is a joint hypothesis of a correlated equilibrium and Assumptions 1–2.

our data. In this case, the moment inequality test procedure needs to be adjusted.[13]

A *subcorrelated equilibrium* (SCE) in our game is a correlated equilibrium for which in addition to constraints (4.3)-(4.4), Hypothesis 1 holds. Thus, in a subcorrelated equilibrium votes can be correlated within, but not across, the two parties. In order to check for consistency of our data with a Subcorrelated equilibrium, we computed an analog of Table 4.11 under Hypothesis 1. That is, instead of computing the joint distribution directly, we estimated $\hat{\gamma}$ and $\hat{\delta}$ from the data, and then computed an induced probability distribution $\tilde{\mu}_{a,b}$ as $\hat{\gamma}(a)\hat{\delta}(b)$ for each joint profile (a, b).

Result 10. *Assuming group independence (which is not rejected in the data), joint vote distributions are consistent with subcorrelated equilibrium under the low cost and group communication, but not under the high cost or other communication treatments (except one no communication case).*

Support for Result 10. Table 4.12 presents the results of our test for subcorrelated equilibrium. The estimated group frequencies and estimates of ϕ are in Tables 4.26 – 4.27 in 4.A.

From Table 4.12 we see that test results for subcorrelated equilibrium are broadly in line with results for correlated equilibrium: under the high voting cost we reject the null of consistency with subcorrelated equilibrium in all cases; but under the low voting cost we now also reject it under public communication. Thus, subcorrelated equilibrium, by imposing additional restrictions on the distribution over joint profiles, allows one to distinguish between group and public communication.

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[13]See 4.A.2 for details.

Table 4.12: Test for Consistency with Subcorrelated Equilibrium

Communication	n_A	n_B	c	IU stat
NC	6	4	0.1	11.656**
–	–	–	0.3	21.234***
GC	–	–	0.1	3.432
–	–	–	0.3	73.085***
PC	–	–	0.1	6.791*
–	–	–	0.3	68.663***
NC	7	3	0.1	1.268
–	–	–	0.3	21.406***
GC	–	–	0.1	0.662
–	–	–	0.3	31.523***
PC	–	–	0.1	8.802**
–	–	–	0.3	64.155***

IU stat is defined in (4.10). Significance codes:
*** < 0.01, ** < 0.05, * < 0.1.

Importantly, we see again (using estimates of ϕ in Tables 4.26 – 4.27) that the inconsistencies under the high voting cost are due to violations of the incentive compatibility constraints (4.4) for those players from both parties who turn out. This pattern is consistent with over-voting in the majority party and under-voting in the minority party, and is also evidenced from the anti-Underdog effect in our data. ∎

4.2.5.3 Analysis of Chat Data in Communication Treatments

In this section we take a deeper look at the actual communication between subjects. We recorded the messages subjects exchanged during each communication stage and employed an independent research assistant to classify and code the messages according to 10 general categories, listed in Table 4.13.

We begin our analysis by looking at the frequency distributions across communication

Table 4.13: Message Categories

Code	Description	Message examples
0	Disagreement	"no!", "unfair :(", "lol, no point"
1	Irrelevant	"hello", "lol", "omg"
2	Agreement to a proposed strategy	"ok", "cool", "yes", "alright", "sure"
3	General discussion about rules of the game	"if we all chose x we can win at the end", "if we are of type A should it always work to pick x if we all agree on it?"
4	Informative statement about history	"only one A chose Y last time"
5	Question to others: what are you going to do?	"do we want to play it safe or try the x gamble again?", "do we all want to try for x?"
6	Strategy suggestion about others/own decision/group decision	"choose x", "let's all pick x", "if you're 1-4 you pick x"
7	Own plan: will choose X	"I'll do X", "im gonna go x"
8	Own plan: will choose Y	"I'll do Y"
9	Ambiguous/contradictory	"gonna choose x or y.. I am not sure"

treatments. We observe a large fraction of meaningless messages in each treatment (code 1). There are consistently more irrelevant messages under public communication than under group communication. Other popular categories of messages include discussion about the rules of the game (code 3) and strategy suggestions (code 6). There is a small fraction of messages informative about history of play (code 4), messages expressing agreement (code 2), and a smaller fraction of messages related to questions to others and own plans (codes 5, 7, and 8, respectively). The remaining categories – disagreement and ambiguous messages – comprise on average less than 1.6% of total messages.

With this general distribution in mind, Table 4.14 lists the frequencies of messages by category pooled across all rounds. For clarity of presentation, we excluded the very large shares of irrelevant messages and messages discussing rules of the game (codes 1 and 3), as well as ambiguous messages (code 9), which are almost non-present in

the data.[14] We refer to remaining messages as relevant messages.

Table 4.14: Message Frequencies by Treatment

n_A	n_B	Cost	Comm. mode	N	Message Category						
					Strategy			Negotiation		History	Query
					(Strat.)	(Vote)	(Abst.)	(Disagr.)	(Agr.)		
6	4	0.1	GC	834	41.73	15.23	4.32	2.52	19.06	10.55	6.59
–	–	–	PC	654	64.68	1.53	4.89	1.68	7.80	13.91	5.50
–	–	0.3	GC	651	40.71	3.53	9.22	6.30	20.89	13.21	6.14
–	–	–	PC	944	67.06	5.51	2.65	1.17	9.11	11.97	2.54
7	3	0.1	GC	698	35.96	11.03	5.44	1.72	18.34	22.64	4.87
–	–	–	PC	620	56.94	1.13	4.35	2.10	5.48	23.87	6.13
–	–	0.3	GC	655	44.58	11.60	8.70	3.66	10.69	15.57	5.19
–	–	–	PC	536	46.64	8.40	4.29	3.17	11.19	19.59	6.72

Notes. Table cells contain for each message code percentages of N, the total number of relevant messages (i.e., excluding codes 1,3, and 9) in a given treatment.

We also break down message frequencies by round in Tables 4.28 – 4.29 in 4.A. Those tables show that the patterns in Table 4.14 are not a by-product of pooling across rounds.[15]

As the voting cost increases, we see a decrease in the total number of messages as well as relevant messages for all cases except one (PC and large minority). There is an increase in the share of strategy suggestions (except PC and small minority, and GC and large minority). The share of informative history messages decreases under public communication, and under group communication and small minority.

Comparing across communication treatments for a fixed voting cost, we see that public communication treatments have more strategy suggestions.[16] It is also inter-

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[14]These types of messages do not affect turnout rates, as we check below. For entire frequency distributions, see Tables 4.28 – 4.29 in 4.A.3.

[15]It is worth noting that we observe a somewhat increasing fraction of irrelevant messages over rounds.

[16]There is also a notable increase in the proportion of irrelevant messages compared to group communication and less discussion about the rules.

esting to note that group communication treatments have a larger share of agreement messages than public communication ones (with one exception).

To assess how messages in different categories affect total turnout, we estimated a simple linear relationship, regressing for each treatment the total number of messages per electorate in each code category on total turnout rate in that electorate. The estimates are reported in Table 4.15 (with standard errors and p-values reported in Table 4.30 in 4.A.) We also estimated an ordered probit model and found very similar results.[17]

Table 4.15: Effects of the Number of Relevant Messages in Each Category on Total Turnout Rate

n_A	n_B	Cost	Comm. mode	(Disagr.)	(Agr.)	(Hist.)	(Q?)	(Strat.)	(Vote)	(Abst.)
							Message Category			
6	4	0.1	GC	−0.044*	0.002	0.000	0.053***	0.014*	0.014	0.004
–	–	–	PC	0.061	0.055**	−0.024	0.014	−0.003	0.037	−0.064***
–	–	0.3	GC	−0.007	0.038***	0.009	−0.018	0.026***	0.063*	−0.011
–	–	–	PC	0.038	0.046***	0.012	−0.010	−0.001	0.046**	−0.098***
7	3	0.1	GC	0.025	0.003	0.010*	0.009	0.001	0.034**	−0.023
–	–	–	PC	0.021	0.008	−0.001	0.046	−0.005*	−0.028	−0.040
–	–	0.3	GC	−0.061	0.047*	0.004	−0.024	0.004	0.037***	−0.004
–	–	–	PC	−0.019*	0.021	0.005	0.028*	−0.004	0.044**	−0.073**

Notes. Table cells contain for each message code OLS estimates of the effects of the total number of messages per electorate in that category on total turnout rate in a given treatment. Significance codes: *** < 0.001, ** < 0.01, * < 0.05.

Table 4.15 reveals several common patterns among group and public communication treatments. Disagreement messages have no effect (except (GC, large minority, low cost) and (PC, small minority, high cost), where the effect is negative). Agreement

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[17]We report effects of irrelevant category messages (codes 1,3 and 9) in Table 4.30. We show there that including two most common message categories (irrelevant and discussion of rules) has no effect on turnout (except few cases). Including ambiguous messages (code 9) results in an implausibly large significant positive effect under PC and small minority, but given their small share in the total messages, this is likely a data artifact.

messages significantly increase turnout in half of the treatments, and increase turnout insignificantly in the remaining treatments. Strategy suggestions increase turnout under group communication and large minority, but decrease turnout under public communication, small minority, and low cost. In the remaining treatments the effect is positive for group communication, negative for public communication, but not significant. History-related messages show no effect (except low cost, GC, and small minority). Questions to others have a positive effect in two cases: under low cost, GC, and large minority, and under high cost, PC, and small minority. Direct statements about one's own plan are largely consistent with turnout outcomes: messages about voting mostly have a positive effect, while messages about abstention have a negative effect.

Given our analysis in the previous section, it seems plausible that inconsistency with correlated equilibrium under the high voting cost might be related to the violation of Assumption 2: the distribution over voting profiles might be noticeably changing over time (e.g., due to faster learning under the high cost). There is an increase in the share of strategy suggestions, with more suggestions creating more room for miscoordination, which is detrimental at a high cost. As a result, expected distribution over voting profiles shifts towards "safer", non-pivotal voting outcomes, violating incentive compatibility constraints for those turning out to vote in either party.

4.3 Concluding Remarks

This is the first study to investigate effects of unrestricted and party-restricted unmediated cheap talk communication on turnout in the laboratory. We studied how changes in communication structure, voting cost, and the relative party sizes affect turnout, welfare, and electoral characteristics.

There are a number of central findings. First, with communication, we find essentially no support for the standard Nash equilibrium turnout predictions.

Second, the results point to a strong interaction between the form of communication and the voting cost in terms of how these two factors influence overall turnout in elections. In particular, with a low voting cost, party communication increases turnout, while public communication decreases turnout. In contrast, with a high voting cost, public communication increases turnout.

A third finding is that these factors impact differentially the turnout rates of majority party versus minority party voters. We find that when the voting cost is high, the majority party voters turn out at relatively higher rates, while the minority party voters turn out at relatively lower rates.

Fourth, we develop and apply a test for the effect of communication on correlation and find that communication results in correlated decisions. Overall, as a result of this correlation, the effects of communication are large, and comparable in magnitude to the effects of changing the main exogenous parameters of the model that have

traditionally been viewed as key driving variables that influence turnout, i.e., voting cost and the competitiveness of the election.

These findings underscore the importance of developing rigorous theoretical models to explicitly take communication possibilities into account. Correlated and subcorrelated equilibrium provide the desirable framework by allowing players to coordinate under unrestricted and group-restricted communication protocols. Testing for these equilibria in our data is the first attempt at eliciting the general principles of coordination in competing groups under communication. The partial success of this framework suggests on the one hand that further study, perhaps with more complicated experiments with larger groups and more variation, is warranted. On the other hand, the data do not fully support the original hypothesis that turnout might approach the maximum theoretical bound implied by correlated equilibrium, so there is clearly room for further theoretical development in the modeling of communication in these games.

4.A Appendix: Additional Estimation Details

The figures and tables in this appendix present additional estimation details for results in Section 4.2.

Figure 4.1: User Interface (No Communication Treatment)

Table 4.16: Mean Frequencies by Treatment

No Communication									
N	n_A	n_B	c	Tie		Pivotal		Upset	
40	6	4	0.1	0.272	(.044)	0.477	(.086)	0.259	(.049)
–	–	–	0.3	0.205	(.068)	0.603	(.052)	0.204	(.058)
–	7	3	0.1	0.111	(.182)	0.300	(.085)	0.116	(.084)
–	–	–	0.3	0.237	(.086)	0.500	(.035)	0.223	(.046)
Group Communication									
N	n_A	n_B	c	Tie		Pivotal		Upset	
40	6	4	0.1	0.016	(.095)	0.227	(.069)	0.019	(.122)
–	–	–	0.3	0.149	(.062)	0.575	(.065)	0.261	(.084)
–	7	3	0.1	0.043	(.065)	0.247	(.056)	0.072	(.058)
–	–	–	0.3	0.177	(.045)	0.464	(.036)	0.161	(.062)
Public Communication									
N	n_A	n_B	c	Tie		Pivotal		Upset	
40	6	4	0.1	0.167	(.116)	0.479	(.082)	0.137	(.070)
–	–	–	0.3	0.083	(.066)	0.204	(.053)	0.174	(.064)
–	7	3	0.1	0.043	(.065)	0.375	(.070)	0.085	(.082)
–	–	–	0.3	0.125	(.104)	0.201	(.051)	0.110	(.042)

Notes: N is the number of group decision observations. Panel-corrected AR(1)-corrected standard errors are in parentheses

Table 4.17: Effects of Communication on Turnout

Group Communication v. No Communication									
N	n_A	n_B	c	$\Delta \hat{T}_A$		$\Delta \hat{T}_B$		$\Delta \hat{T}$	
40	6	4	0.1	0.276***	(0.000)	−0.151*	(0.050)	0.109*	(0.018)
–	–	–	0.3	0.080	(0.182)	0.010	(0.911)	0.052	(0.285)
–	7	3	0.1	0.173***	(0.000)	0.071	(0.519)	0.143***	(0.000)
–	–	–	0.3	0.042	(0.666)	−0.099	(0.180)	0.001	(0.993)
Public Communication v. No Communication									
N	n_A	n_B	c	$\Delta \hat{T}_A$		$\Delta \hat{T}_B$		$\Delta \hat{T}$	
40	6	4	0.1	−0.058	(0.225)	−0.214*	(0.022)	−0.117*	(0.032)
–	–	–	0.3	0.236***	(0.000)	0.021	(0.720)	0.151***	(0.001)
–	7	3	0.1	−0.055	(0.294)	−0.155**	(0.009)	−0.086*	(0.023)
–	–	–	0.3	0.200***	(0.000)	−0.008	(0.924)	0.131**	(0.006)
Public Communication v. Group Communication									
N	n_A	n_B	c	$\Delta \hat{T}_A$		$\Delta \hat{T}_B$		$\Delta \hat{T}$	
40	6	4	0.1	−0.334***	(0.000)	−0.063	(0.546)	−0.225***	(0.000)
–	–	–	0.3	0.156*	(0.045)	0.011	(0.913)	0.099	(0.114)
–	7	3	0.1	−0.228***	(0.000)	−0.226*	(0.039)	−0.229***	(0.000)
–	–	–	0.3	0.155	(0.117)	0.091	(0.186)	0.131*	(0.048)

Notes: Two-sided p-values in parentheses. Significance codes: *** < 0.001, ** < 0.01, * < 0.05

Table 4.18: Effects of Communication on Electoral Characteristics

				Group Communication v. No Communication					
N	n_A	n_B	c	ΔTie		ΔPivotal		ΔUpset	
40	6	4	0.1	−0.256**	(0.018)	−0.250*	(0.027)	−0.240	(0.073)
–	–	–	0.3	−0.056	(0.542)	−0.028	(0.737)	0.057	(0.581)
–	7	3	0.1	−0.068	(0.727)	−0.053	(0.607)	−0.044	(0.671)
–	–	–	0.3	−0.060	(0.539)	−0.057	(0.581)	−0.063	(0.419)
				Public Communication v. No Communication					
N	n_A	n_B	c	ΔTie		ΔPivotal		ΔUpset	
40	6	4	0.1	−0.104	(0.405)	0.003	(0.982)	−0.122	(0.158)
–	–	–	0.3	−0.122	(0.200)	−0.400***	(0.000)	−0.030	(0.728)
–	7	3	0.1	−0.068	(0.727)	0.075	(0.496)	−0.031	(0.796)
–	–	–	0.3	−0.112	(0.412)	−0.300***	(0.000)	−0.113	(0.071)
				Public Communication v. Group Communication					
N	n_A	n_B	c	ΔTie		ΔPivotal		ΔUpset	
40	6	4	0.1	0.151	(0.317)	0.253*	(0.022)	0.118	(0.404)
–	–	–	0.3	−0.066	(0.469)	−0.372***	(0.000)	−0.087	(0.415)
–	7	3	0.1	0.000	(1.000)	0.128	(0.158)	0.013	(0.898)
–	–	–	0.3	−0.052	(0.652)	−0.263***	(0.002)	−0.051	(0.503)

Notes: Two-sided p-values in parentheses. Significance codes: *** < 0.001,
** < 0.01, * < 0.05

Table 4.19: Effects of Cost on Turnout

				High Cost v. Low Cost					
N	n_A	n_B	Comm. mode	$\Delta\hat{T}_A$		$\Delta\hat{T}_B$		$\Delta\hat{T}$	
40	6	4	NC	−0.196***	(0.000)	−0.204***	(0.000)	−0.197***	(0.000)
–	–	–	GC	−0.392***	(0.000)	−0.043	(0.689)	−0.254***	(0.000)
–	–	–	PC	0.098	(0.136)	0.031	(0.756)	0.070	(0.246)
–	7	3	NC	−0.107**	(0.008)	−0.027	(0.712)	−0.081*	(0.017)
–	–	–	GC	−0.238**	(0.015)	−0.197	(0.075)	−0.224***	(0.001)
–	–	–	PC	0.145**	(0.017)	0.120	(0.070)	0.136**	(0.007)

Notes: Two-sided p-values in parentheses. Significance codes: *** < 0.001, ** < 0.01,
* < 0.05.

Table 4.20: Effects of Cost on Electoral Characteristics

				High Cost v. Low Cost					
N_{cl}	n_A	n_B	Comm. mode	ΔTie		ΔPivotal		ΔUpset	
40	6	4	NC	-0.067	(0.411)	0.127	(0.211)	-0.055	(0.469)
–	–	–	GC	0.133	(0.246)	0.349***	(0.000)	0.242	(0.107)
–	–	–	PC	-0.084	(0.530)	-0.276**	(0.006)	0.037	(0.697)
–	7	3	NC	0.126	(0.535)	0.200*	(0.034)	0.108	(0.266)
–	–	–	GC	0.134	(0.095)	0.217**	(0.002)	0.088	(0.303)
–	–	–	PC	0.082	(0.506)	-0.174*	(0.048)	0.025	(0.790)

Notes: Two-sided *p*-values in parentheses. Significance codes: *** < 0.001, ** < 0.01, * < 0.05.

Table 4.21: Effects of Changing the Relative Party Size on Turnout

			Majority Size 6 v. Majority Size 7					
N_{cl}	c	Comm. mode	$\Delta \hat{T}_A$		$\Delta \hat{T}_B$		$\Delta \hat{T}$	
40	0.1	NC	0.156***	(0.000)	0.198***	(0.001)	0.165***	(0.000)
–	–	GC	0.259***	(0.000)	-0.024	(0.840)	0.130**	(0.004)
–	–	PC	0.154*	(0.013)	0.140	(0.125)	0.134*	(0.016)
–	0.3	NC	0.067	(0.084)	0.021	(0.625)	0.049	(0.139)
–	–	GC	0.105	(0.326)	0.130	(0.181)	0.100	(0.162)
–	–	PC	0.106	(0.099)	0.050	(0.519)	0.068	(0.227)

Notes: Two-sided *p*-values in parentheses. Significance codes: *** < 0.001, ** < 0.01, * < 0.05.

Table 4.22: Effects of Changing the Relative Party Size on Electoral Characteristics

			Majority Size 6 v. Majority Size 7					
N_{cl}	c	Comm. mode	ΔTie		ΔPivotal		ΔUpset	
40	0.1	NC	0.161	(0.396)	0.177	(0.147)	0.143	(0.147)
–	–	GC	-0.027	(0.816)	-0.021	(0.819)	-0.054	(0.692)
–	–	PC	0.124	(0.354)	0.104	(0.336)	0.052	(0.636)
–	0.3	NC	-0.032	(0.773)	0.104	(0.104)	-0.020	(0.791)
–	–	GC	-0.028	(0.716)	0.111	(0.139)	0.100	(0.344)
–	–	PC	-0.042	(0.734)	0.003	(0.970)	0.064	(0.406)

Notes: Two-sided *p*-values in parentheses. Significance codes: *** < 0.001, ** < 0.01, * < 0.05.
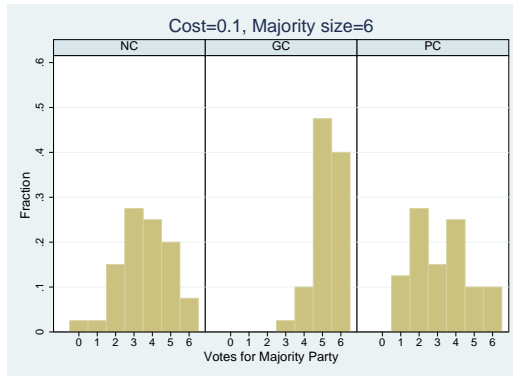
(a) Low Cost, partition $(6, 4)$
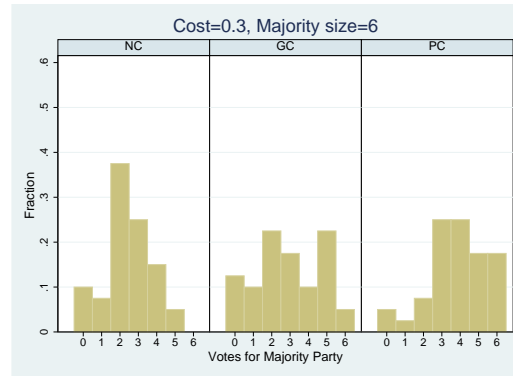
(b) High Cost, partition $(6, 4)$
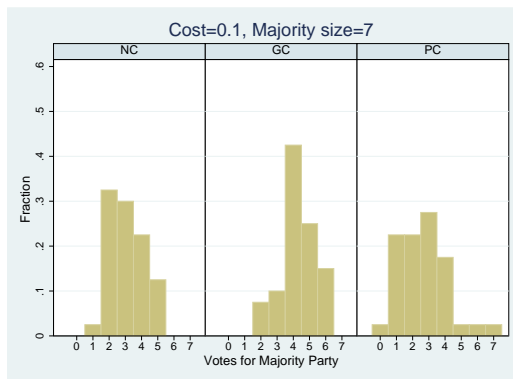
(c) Low Cost, partition $(7, 3)$
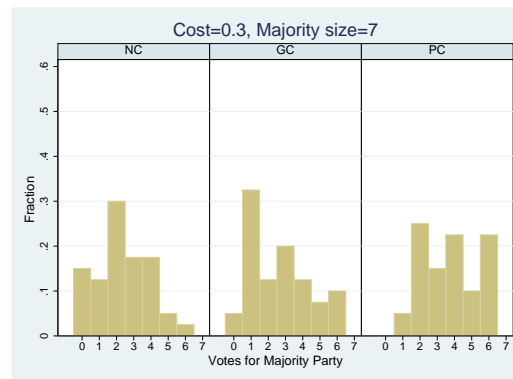
(d) High Cost, partition $(7, 3)$

Figure 4.2: Voting patterns for majority party
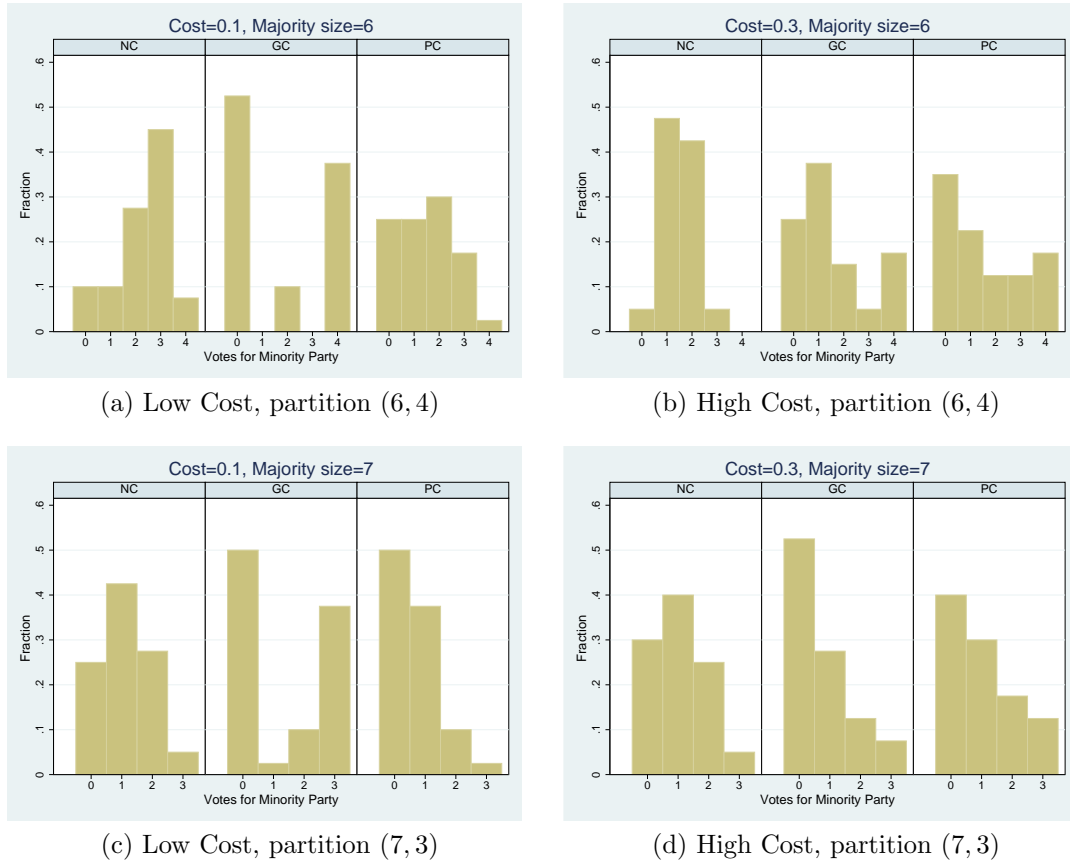
(a) Low Cost, partition $(6, 4)$

(b) High Cost, partition $(6, 4)$

(c) Low Cost, partition $(7, 3)$

(d) High Cost, partition $(7, 3)$

Figure 4.3: Voting patterns for minority party
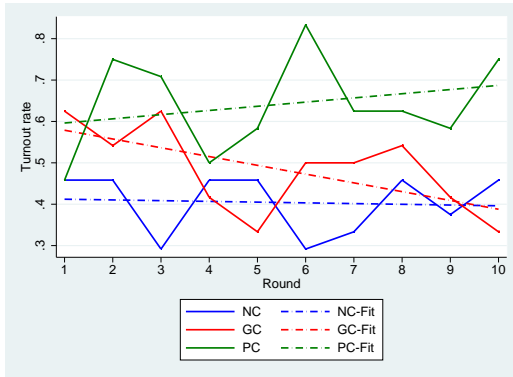
(a) Low Cost, partition $(6, 4)$
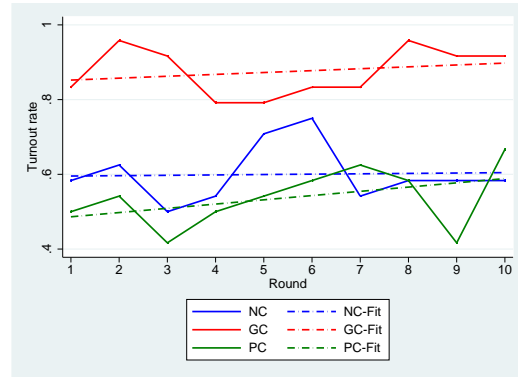
(b) High Cost, partition $(6, 4)$
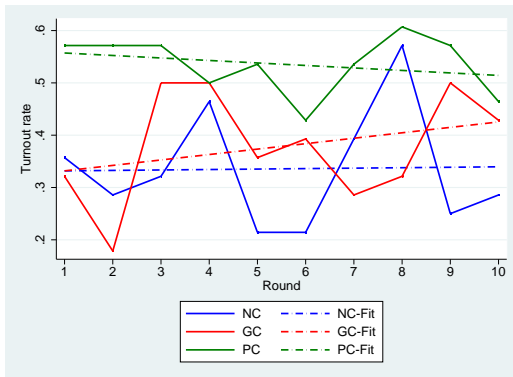
(c) Low Cost, partition $(7, 3)$
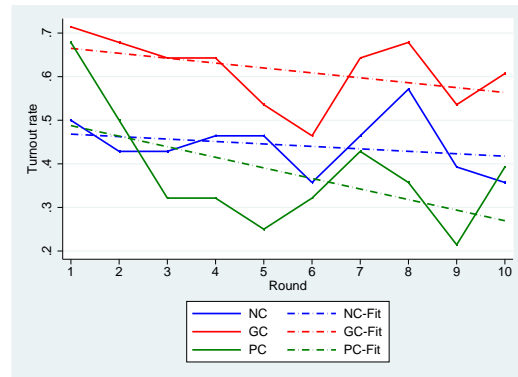
(d) High Cost, partition $(7, 3)$

Figure 4.4: Turnout dynamics for majority party
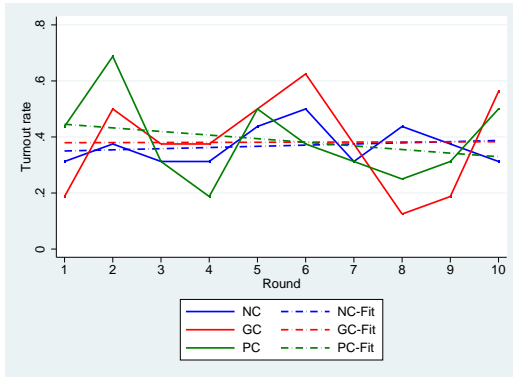
(a) Low Cost, partition $(6, 4)$
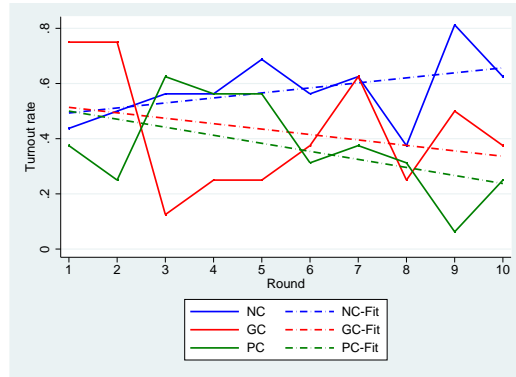
(b) High Cost, partition $(6, 4)$
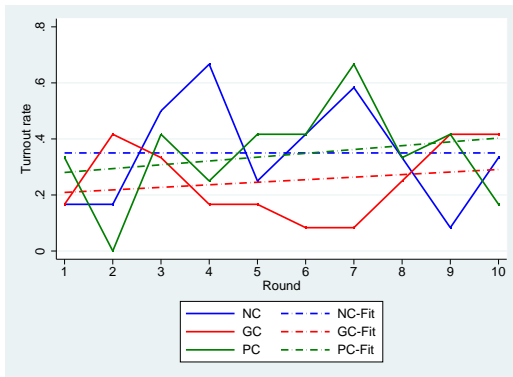
(c) Low Cost, partition $(7, 3)$
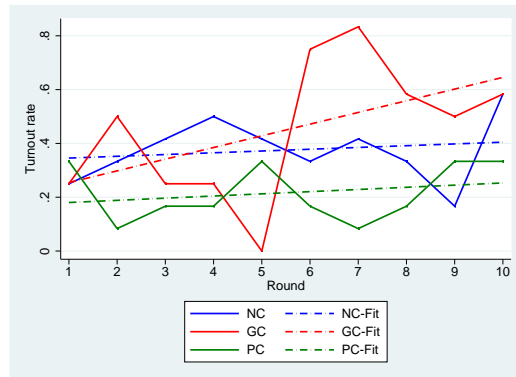
(d) High Cost, partition $(7, 3)$

Figure 4.5: Turnout dynamics for minority party
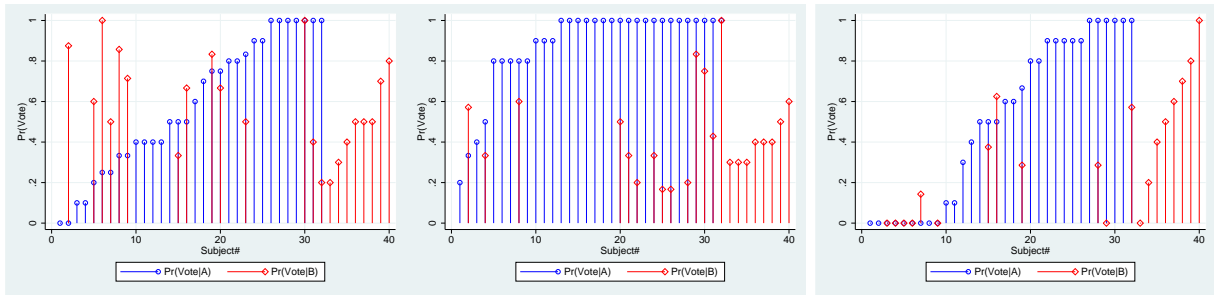
(a) NC, partition $(6,4)$, $c = 0.1$
(b) GC, partition $(6,4)$, $c = 0.1$
(c) PC, partition $(6,4)$, $c = 0.1$
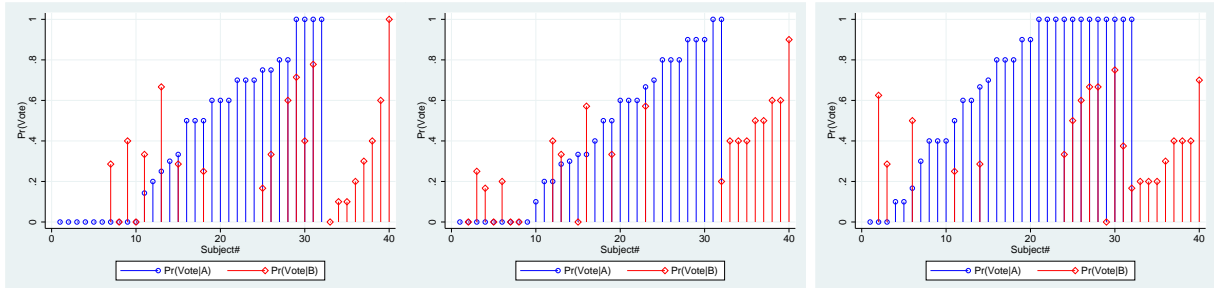
(d) NC, partition $(6,4)$, $c = 0.3$
(e) GC, partition $(6,4)$, $c = 0.3$
(f) PC, partition $(6,4)$, $c = 0.3$
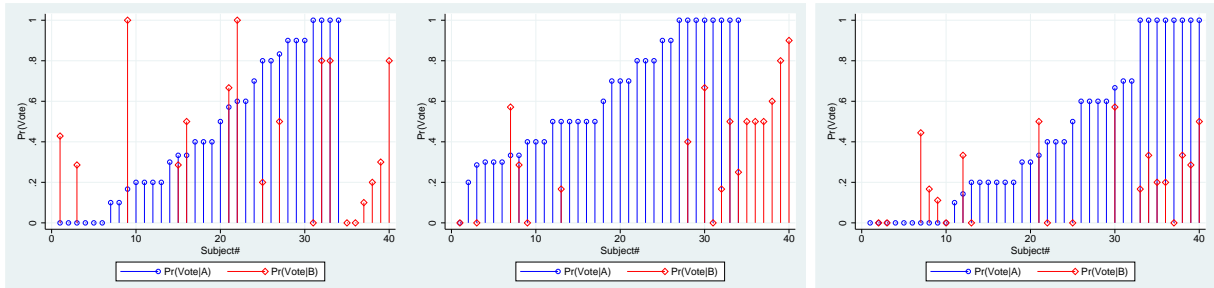
(g) NC, partition $(7,3)$, $c = 0.1$
(h) GC, partition $(7,3)$, $c = 0.1$
(i) PC, partition $(7,3)$, $c = 0.1$
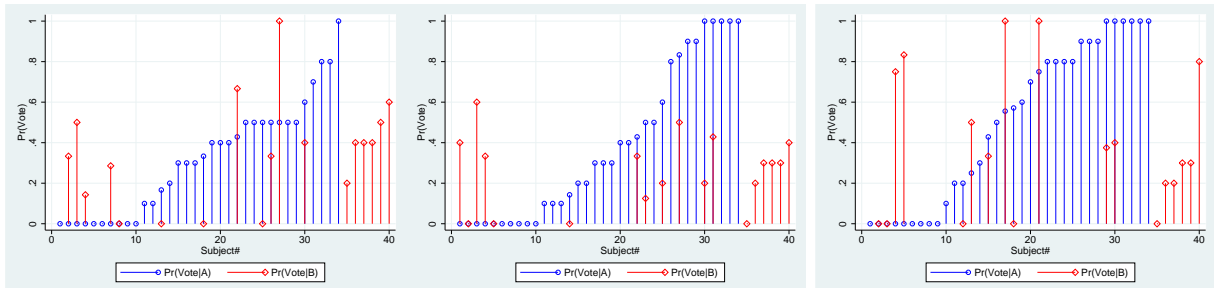
(j) NC, partition $(7,3)$, $c = 0.3$
(k) GC, partition $(7,3)$, $c = 0.3$
(l) PC, partition $(7,3)$, $c = 0.3$

Figure 4.6: Individual turnout frequencies

4.A.1   Tests for the presence of correlation

To test Hypothesis 1, we first estimate from the data the probabilities of the joint profiles $\mu_{a,b}$ for each pair of vote counts (a, b), obtaining the estimated distribution $\hat{\mu}$. Next, we estimate probability distributions over group profiles, $\hat{\gamma}$ and $\hat{\delta}$, separately for each party, and compute the induced probability distribution $\tilde{\mu}$ as a product of $\hat{\gamma}$ and $\hat{\delta}$, thereby satisfying Hypothesis 1. Finally, we compare $\tilde{\mu}$ with $\hat{\mu}$ by means of a two-sample Epps-Singleton test, which is a powerful non-parametric alternative to the Kolmogorov-Smirnov and Mann-Whitney tests suited for comparing discrete distributions, to see if the two distributions are significantly different.

We can test Hypothesis 2 as follows. First, estimate the probabilities of the joint profiles $\mu_{a,b}$ from the data for each pair of vote counts (a, b), obtaining the estimated distribution $\hat{\mu}$. Next, estimate p_A and p_B from the data as averages over individual subjects' voting probabilities in each group (depicted in Figure 4.6 for all subjects in all treatments), obtaining $\hat{p}_A$ and $\hat{p}_B$. Given these estimates, we compute the induced group probabilities $\tilde{\gamma}$ and $\tilde{\delta}$ by plugging $\hat{p}_A$ and $\hat{p}_B$ into (4.1) and (4.2), and compute the induced joint distribution under the null as $\tilde{\mu}_{a,b} = \tilde{\gamma}(a)\tilde{\delta}(b)$. Finally, we compare the resulting distribution with the actual distribution (i.e., $\tilde{\mu}$ with $\hat{\mu}$) by means of a two-sample Epps-Singleton test, obtaining the W_2 test statistic. The caveat of this approach is that one needs to account for variance in the estimates $\hat{p}_A$ and $\hat{p}_B$ when generating the induced distribution. Therefore we supplement the results of Epps-Singleton test with a maximum likelihood ratio test (cf. Moreno and Wooders (1998, pp.57-58)). Let $n_{a,b}$ be the number of times vote count (a, b) was observed, and $N = \sum_{a=0}^{n_A} \sum_{b=0}^{n_B} n_{a,b}$ be the total number of observations in a given treatment. The loglikelihood that a sample was generated by a multinomial distribution μ can be written as

$$\ell(\mu) = C + \sum_{a=0}^{n_A} \sum_{b=0}^{n_B} n_{a,b} \ln \mu_{a,b} \tag{4.5}$$

where C is a normalization constant. Under the null of Hypothesis 2, the loglikeli-

hood is maximized at

$$\hat{\mu}_{a,b}^0 = \binom{n_A}{a}\binom{n_B}{b}\left(\frac{\sum_{a,b}\frac{a}{n_A}n_{a,b}}{N}\right)^a\left(1-\frac{\sum_{a,b}\frac{a}{n_A}n_{a,b}}{N}\right)^{n_A-a}\left(\frac{\sum_{a,b}\frac{b}{n_B}n_{a,b}}{N}\right)^b\left(1-\frac{\sum_{a,b}\frac{b}{n_B}n_{a,b}}{N}\right)^{n_B-b}$$

Under the alternative hypothesis of μ being an arbitrary multinomial distribution, the loglikelihood is maximized at $\hat{\mu}_{a,b}^1 = \frac{n_{a,b}}{N}$. The null and the alternative have 2 and $(n_A + 1)(n_B + 1) - 1$ degrees of freedom, respectively. The likelihood ratio, $-2\left(\ell(\hat{\mu}^0) - \ell(\hat{\mu}^1)\right)$, is then asymptotically distributed as χ^2 with $(n_A + 1)(n_B + 1) - 1 - 2$ degrees of freedom, so we can compare the LR statistic with the χ^2 critical value at 0.05 level. These tests are also reported below.

To test Hypothesis 3, we plug the Nash equilibrium probabilities in (4.1) and (4.2) (e.g., we set p_A and p_B equal to T_A and T_B from Table 4.1, respectively) and again compare the predicted distribution with the actual one using Epps-Singleton test.

Test results

Table 4.23: Test for Group Independence

Communication	n_A	n_B	c	ES test	
				W_2	p-val
NC	6	4	0.1	4.552	.336
–	–	–	0.3	0.446	.978
GC	–	–	0.1	1.840	.765
–	–	–	0.3	0.443	.979
PC	–	–	0.1	0.413	.981
–	–	–	0.3	4.052	.399
NC	7	3	0.1	1.965	.742
–	–	–	0.3	1.666	.797
GC	–	–	0.1	1.302	.861
–	–	–	0.3	1.416	.842
PC	–	–	0.1	4.619	.329
–	–	–	0.3	3.060	.548

Notes: W_2 is two-sample Epps-Singleton test statistic for discrete data, computed using a modified version of external Stata routine **escftest**.

4.A.2 Tests for consistency with correlated and subcorrelated equilibrium

Correlated Equilibrium Under the group-symmetric distributions, defined in Assumption 1, we can rewrite conditions (4.3)-(4.4) as the following system of four inequalities, two for players in N_A, and two for players in N_B, with respect to $(n_A + 1)(n_B + 1)$ variables of the form $\binom{n_A}{a}\binom{n_B}{b}\mu_{a,b}$:

$$\phi_A^0 \equiv \frac{\frac{1}{2} - c}{c} \left(\sum_{a=0}^{n_B} \frac{\binom{n_A-1}{a}}{\binom{n_A}{a}} \binom{n_A}{a} \binom{n_B}{a} \mu_{a,a} + \sum_{a=0}^{n_B-1} \frac{\binom{n_A-1}{a}}{\binom{n_A}{a}} \binom{n_A}{a} \binom{n_B}{a+1} \mu_{a,a+1} \right) -$$

$$\left[\sum_{a=1}^{n_A-1} \sum_{b=0}^{\min\{a-1,n_B\}} \frac{\binom{n_A-1}{a}}{\binom{n_A}{a}} \binom{n_A}{a} \binom{n_B}{b} \mu_{a,b} + \sum_{a=0}^{n_B-2} \sum_{b=a+2}^{n_B} \frac{\binom{n_A-1}{a}}{\binom{n_A}{a}} \binom{n_A}{a} \binom{n_B}{b} \mu_{a,b} \right] \leq 0 \quad (4.6)$$

$$\phi_A^1 \equiv \sum_{a=2}^{n_A} \sum_{b=0}^{\min\{a-2,n_B\}} \frac{\binom{n_A-1}{a-1}}{\binom{n_A}{a}} \binom{n_A}{a} \binom{n_B}{b} \mu_{a,b} + \sum_{a=1}^{n_B-1} \sum_{b=a+1}^{n_B} \frac{\binom{n_A-1}{a-1}}{\binom{n_A}{a}} \binom{n_A}{a} \binom{n_B}{b} \mu_{a,b} -$$

$$\frac{\frac{1}{2} - c}{c} \left(\sum_{a=0}^{n_B} \frac{\binom{n_A-1}{a}}{\binom{n_A}{a+1}} \binom{n_A}{a+1} \binom{n_B}{a} \mu_{a+1,a} + \sum_{a=1}^{n_B} \frac{\binom{n_A-1}{a-1}}{\binom{n_A}{a}} \binom{n_A}{a} \binom{n_B}{a} \mu_{a,a} \right) \leq 0 \quad (4.7)$$

and

$$\phi_B^0 \equiv \frac{\frac{1}{2} - c}{c} \left(\sum_{a=0}^{n_B-1} \frac{\binom{n_B-1}{a}}{\binom{n_B}{a}} \binom{n_A}{a} \binom{n_B}{a} \mu_{a,a} + \sum_{a=0}^{n_B-1} \frac{\binom{n_B-1}{a}}{\binom{n_B}{a}} \binom{n_A}{a+1} \binom{n_B}{a} \mu_{a+1,a} \right) -$$

$$\left[\sum_{a=2}^{n_A} \sum_{b=0}^{\min\{a-2,n_B-1\}} \frac{\binom{n_B-1}{b}}{\binom{n_B}{b}} \binom{n_A}{a} \binom{n_B}{b} \mu_{a,b} + \sum_{a=0}^{n_B-2} \sum_{b=a+1}^{n_B-1} \frac{\binom{n_B-1}{b}}{\binom{n_B}{b}} \binom{n_A}{a} \binom{n_B}{b} \mu_{a,b} \right] \leq 0 \quad (4.8)$$

$$\phi_B^1 \equiv \sum_{a=2}^{n_A} \sum_{b=1}^{\min\{a-1,n_B\}} \frac{\binom{n_B-1}{b-1}}{\binom{n_B}{b}} \binom{n_A}{a} \binom{n_B}{b} \mu_{a,b} + \sum_{a=0}^{n_B-2} \sum_{b=a+2}^{n_B} \frac{\binom{n_B-1}{b-1}}{\binom{n_B}{b}} \binom{n_A}{a} \binom{n_B}{b} \mu_{a,b} -$$

$$\frac{\frac{1}{2} - c}{c} \left(\sum_{a=0}^{n_B-1} \frac{\binom{n_B-1}{a}}{\binom{n_B}{a+1}} \binom{n_A}{a} \binom{n_B}{a+1} \mu_{a,a+1} + \sum_{a=1}^{n_B} \frac{\binom{n_B-1}{a-1}}{\binom{n_B}{a}} \binom{n_A}{a} \binom{n_B}{a} \mu_{a,a} \right) \leq 0 \quad (4.9)$$

Let $\boldsymbol{\phi} = [\phi_A^0, \phi_A^1, \phi_B^0, \phi_B^1]$ be a 4×1 vector of the left hand sides of the incentive compatibility constraints (4.6)-(4.9). Notice that we can write $\boldsymbol{\phi} = \boldsymbol{J}\boldsymbol{\mu}$, where

$J \in \mathbb{R}_{4 \times (n_A+1)(n_B+1)}$ is a constant Jacobian matrix, and $\mu \in \Delta(\{0, \ldots, n_A + 1\} \times \{0, \ldots, n_B + 1\})$ is a group-symmetric probability distribution over joint voting profiles. Then μ is a group-symmetric CE if and only if $\phi \leq \mathbf{0}$.

We now apply the inequality-based testing procedure described in Wolak (1989). Define $\nu \equiv -\phi = -J\mu$, and let $\hat{\nu}$ be its estimate from the experimental data, obtained from K independent trials. Given μ, interpreted as a multinomial distribution with K trials and $(n_A + 1)(n_B + 1)$ outcomes, and a $(n_A + 1)(n_B + 1) \times (n_A + 1)(n_B + 1)$ variance-covariance matrix Σ, we can derive by the Delta method that $\hat{\nu} \overset{a}{\sim} N(\nu_0, \Omega)$ and $\Omega = J\Sigma J'$.

We want to test $H_0 : \nu_0 \geq \mathbf{0}$ $vs. \nu_0 \ngeq \mathbf{0}$. Following Wolak (1989), we define the test statistic as

$$IU \equiv (\hat{\nu} - \tilde{\nu})'\Omega^{-1}(\hat{\nu} - \tilde{\nu}), \tag{4.10}$$

where

$$\tilde{\nu} \equiv \arg \min_{\nu \geq \mathbf{0}} (\hat{\nu} - \nu)'\Omega^{-1}(\hat{\nu} - \nu).$$

In our computations, we estimate $\hat{\nu} = -J\hat{\mu}$, where $\hat{\mu}$ is a vector of estimated joint frequencies from K experimental trials, and replace Ω with its consistent estimate $\hat{\Omega} = J\hat{\Sigma}J'$. Suppose the IU test statistic computed from the data equals s, then Wolak (1989, Corollary 1) allows us to compute the p-value under the null hypothesis $\nu_0 \geq \mathbf{0}$ as follows.

$$\sup_{\nu_0 \geq \mathbf{0}} \Pr_{\nu, \Omega}[IU \geq s] = \sum_{k=0}^{4} \Pr(\chi_k^2 \geq s)w(4, 4 - k, \Omega),$$

where $\Pr(\chi_k^2 \geq s)$ denotes the probability that a χ_k^2 random variable exceeds s, and $w(4, 4 - k, \Omega)$ is the probability that exactly $4 - k$ out of 4 elements in $\tilde{\nu}$ are strictly positive. These weights can be computed by Monte Carlo simulations.[18]

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[18]For the case with up to 4 restrictions analytical expressions for the weights are available in Kudo (1963) and Shapiro (1985).

Subcorrelated Equilibrium In order to test the subcorrelated equilibrium hypothesis, we slightly modify the moment inequality procedure using the fact that now $\hat{\boldsymbol{\nu}} = -\boldsymbol{J}[\hat{\boldsymbol{\delta}} \otimes \hat{\boldsymbol{\gamma}}]$, where $\hat{\boldsymbol{\delta}} \in \Delta(\{0, \dots, n_B + 1\})$, $\hat{\boldsymbol{\gamma}} \in \Delta(\{0, \dots, n_A + 1\})$ have sample covariance matrices $\hat{\boldsymbol{\Sigma}}_1$ and $\hat{\boldsymbol{\Sigma}}_2$, respectively (each estimated as a covariance matrix of a corresponding multinomial distribution over group profiles), and $\otimes$ stands for Kronecker product. Using the Delta method, we obtain that $\hat{\boldsymbol{\nu}} \overset{a}{\sim} N(\boldsymbol{\nu_0}, \boldsymbol{\Omega})$ and

$$\boldsymbol{\Omega} = H \begin{bmatrix} \hat{\boldsymbol{\Sigma}}_1 & 0 \\ 0 & \hat{\boldsymbol{\Sigma}}_2 \end{bmatrix} H',$$

where

$$H_{4 \times (n_B + 1 + n_A + 1)} = \left[\boldsymbol{J}[\boldsymbol{I}_{[n_B+1]} \otimes \hat{\boldsymbol{\gamma}}] \quad \boldsymbol{J}[\hat{\boldsymbol{\delta}} \otimes \boldsymbol{I}_{[n_A+1]}] \right],$$

and $\boldsymbol{I}_{[x]}$ is an identity matrix of size $x \times x$. The rest of the testing procedure is the same: we compute the IU test statistic and test the null of $H_0 : \boldsymbol{\nu_0} \geq \boldsymbol{0}$ $vs. \boldsymbol{\nu_0} \ngeq \boldsymbol{0}$.

Table 4.24: Profile Frequencies and Test for Correlated Equilibrium, Partition $(6, 4)$

			No communication							
			$c = 0.1$					$c = 0.3$		
$a \backslash b$	0	1	2	3	4	$a \backslash b$	0	1	2	3	4
0	0	0	0	0.025	0	0	0.025	0	0.050	0.025	0
1	0	0.025	0	0	0	1	0	0.050	0.025	0	0
2	0	0	0.075	0.050	0.025	2	0.025	0.225	0.125	0	0
3	0.025	0	0.050	0.175	0.025	3	0	0.100	0.150	0	0
4	0.050	0.050	0.100	0.050	0	4	0	0.075	0.075	0	0
5	0.025	0	0.025	0.125	0.025	5	0	0.025	0	0.025	0
6	0	0.025	0.025	0.025	0	6	0	0	0	0	0
ϕ_A^0			0.621					-0.311		
ϕ_A^1			-0.379					0.071		
ϕ_B^0			0.263					-0.017		
ϕ_B^1			-0.613					0.223		
IU stat			$9.682^{**}, p = .032$					$24.747^{***}, p = .000$		

			Group communication							
			$c = 0.1$					$c = 0.3$		
$a \backslash b$	0	1	2	3	4	$a \backslash b$	0	1	2	3	4
0	0	0	0	0	0	0	0.050	0.075	0	0	0
1	0	0	0	0	0	1	0.050	0.025	0.025	0	0
2	0	0	0	0	0	2	0.025	0.150	0	0.025	0.025
3	0	0	0	0	0.025	3	0.050	0.050	0.050	0	0.025
4	0.050	0	0.025	0	0.025	4	0	0	0.025	0	0.075
5	0.225	0	0.075	0	0.175	5	0.075	0.075	0.050	0	0.025
6	0.250	0	0	0	0.150	6	0	0	0	0.025	0.025
ϕ_A^0			-0.021					-0.149		
ϕ_A^1			0.063					0.219		
ϕ_B^0			-0.575					-0.192		
ϕ_B^1			0.175					0.121		
IU stat			$2.760, p = .457$					$9.548^{**}, p = .034$		

			Public communication							
			$c = 0.1$					$c = 0.3$		
$a \backslash b$	0	1	2	3	4	$a \backslash b$	0	1	2	3	4
0	0	0	0	0	0	0	0	0.050	0	0	0
1	0.075	0.050	0	0	0	1	0.025	0	0	0	0
2	0.100	0.075	0.050	0.050	0	2	0	0	0	0.025	0.050
3	0.025	0.050	0.025	0.050	0	3	0.100	0.100	0	0.050	0
4	0.050	0.025	0.075	0.075	0.025	4	0.100	0.050	0.075	0	0.025
5	0	0.050	0.050	0	0	5	0.050	0.025	0.025	0.050	0.025
6	0	0	0.100	0	0	6	0.075	0	0.025	0	0.075
ϕ_A^0			0.246					-0.192		
ϕ_A^1			-0.296					0.531		
ϕ_B^0			0.556					-0.550		
ϕ_B^1			-0.319					0.231		
IU stat			$5.213, p = .198$					$64.553^{***}, p = 0.000$		

Notes: ϕ_j^i refers to incentive compatibility condition i for group N_j (see (4.6)-(4.9)).
IU stat is defined in (4.10). Significance codes: $^{***} < 0.01$, $^{**} < 0.05$, $^{*} < 0.1$.

Table 4.25: Profile Frequencies and Test for Correlated Equilibrium, Partition $(7,3)$

No communication									
	$c = 0.1$					$c = 0.3$			
$a\backslash b$	0	1	2	3	$a\backslash b$	0	1	2	3
0	0	0	0	0	0	0.050	0.075	0.025	0
1	0	0	0.025	0	1	0.050	0.075	0	0
2	0.125	0.100	0.075	0.025	2	0.100	0.075	0.125	0
3	0.075	0.150	0.050	0.025	3	0.025	0.125	0.025	0
4	0.025	0.125	0.075	0	4	0.050	0.025	0.075	0.025
5	0.025	0.050	0.050	0	5	0.025	0	0	0.025
6	0	0	0	0	6	0	0.025	0	0
7	0	0	0	0	7	0	0	0	0
ϕ_A^0	-0.021					-0.200			
ϕ_A^1	0.032					0.169			
ϕ_B^0	-0.083					-0.233			
ϕ_B^1	-0.208					0.128			
IU stat	0.065, $p = .992$					11.031**, $p = .018$			
Group communication									
	$c = 0.1$					$c = 0.3$			
$a\backslash b$	0	1	2	3	$a\backslash b$	0	1	2	3
0	0	0	0	0	0	0	0.025	0.025	0
1	0	0	0	0	1	0.150	0.150	0.025	0
2	0.025	0	0	0.050	2	0.050	0.050	0.025	0
3	0.050	0	0	0.050	3	0.150	0.025	0.025	0
4	0.175	0.025	0.075	0.150	4	0.075	0.025	0	0.025
5	0.150	0	0.025	0.075	5	0.050	0	0	0.025
6	0.100	0	0	0.050	6	0.050	0	0.025	0.025
7	0	0	0	0	7	0	0	0	0
ϕ_A^0	-0.064					-0.300			
ϕ_A^1	0.079					0.230			
ϕ_B^0	-0.550					-0.250			
ϕ_B^1	-0.050					0.097			
IU stat	0.195, $p = .966$					16.564***, $p = .002$			
Public communication									
	$c = 0.1$					$c = 0.3$			
$a\backslash b$	0	1	2	3	$a\backslash b$	0	1	2	3
0	0	0.025	0	0	0	0	0	0	0
1	0.125	0.050	0.025	0.025	1	0	0.025	0	0.025
2	0.125	0.100	0	0	2	0.125	0.025	0.075	0.025
3	0.125	0.100	0.050	0	3	0.075	0.050	0	0.025
4	0.075	0.075	0.025	0	4	0	0.125	0.075	0.025
5	0	0.025	0	0	5	0.050	0.025	0	0.025
6	0.025	0	0	0	6	0.150	0.050	0.025	0
7	0.025	0	0	0	7	0	0	0	0
ϕ_A^0	-0.175					-0.286			
ϕ_A^1	0.004					0.440			
ϕ_B^0	0.425					-0.561			
ϕ_B^1	0.008					0.161			
IU stat	5.861, $p = .195$					58.619***, $p = 0.000$			

Notes: ϕ_j^i refers to incentive compatibility condition i for group N_j (see (4.6)-(4.9)).
IU stat is defined in (4.10). Significance codes: *** < 0.01, ** < 0.05, * < 0.1.

Table 4.26: Group Profile Frequencies and Test for Subcorrelated Equilibrium, Partition $(6, 4)$

			No communication			
	$c = 0.1$				$c = 0.3$	
a	$\gamma(a)$	b	$\delta(b)$	a	$\gamma(a)$	b	$\delta(b)$
0	0.025	0	0.100	0	0.100	0	0.050
1	0.025	1	0.100	1	0.075	1	0.475
2	0.150	2	0.275	2	0.375	2	0.425
3	0.275	3	0.450	3	0.250	3	0.050
4	0.250	4	0.075	4	0.150	4	0
5	0.200			5	0.050		
6	0.075			6	0		
ϕ_A^0		0.419				-0.206	
ϕ_A^1		-0.496				0.098	
ϕ_B^0		0.253				-0.120	
ϕ_B^1		-0.466				0.136	
IU stat		$11.656^{**}, p = .013$				$21.234^{***}, p = .000$	

			Group communication			
	$c = 0.1$				$c = 0.3$	
a	$\gamma(a)$	b	$\delta(b)$	a	$\gamma(a)$	b	$\delta(b)$
0	0	0	0.525	0	0.125	0	0.250
1	0	1	0	1	0.100	1	0.375
2	0	2	0.100	2	0.225	2	0.150
3	0.025	3	0	3	0.175	3	0.050
4	0.100	4	0.375	4	0.100	4	0.175
5	0.475			5	0.225		
6	0.400			6	0.050		
ϕ_A^0		-0.039				-0.221	
ϕ_A^1		0.002				0.291	
ϕ_B^0		-0.569				-0.317	
ϕ_B^1		0.191				0.200	
IU stat		$3.432, p = .336$				$73.085^{***}, p = 0.000$	

			Public communication			
	$c = 0.1$				$c = 0.3$	
a	$\gamma(a)$	b	$\delta(b)$	a	$\gamma(a)$	b	$\delta(b)$
0	0	0	0.250	0	0.050	0	0.350
1	0.125	1	0.250	1	0.025	1	0.225
2	0.275	2	0.300	2	0.075	2	0.125
3	0.150	3	0.175	3	0.250	3	0.125
4	0.250	4	0.025	4	0.250	4	0.175
5	0.100			5	0.175		
6	0.100			6	0.175		
ϕ_A^0		0.345				-0.191	
ϕ_A^1		-0.122				0.445	
ϕ_B^0		0.306				-0.481	
ϕ_B^1		-0.299				0.173	
IU stat		$6.791^{*}, p = .088$				$68.663^{***}, p = 0.000$	

Notes: ϕ_j^i refers to incentive compatibility condition i for group N_j (see (4.6)-(4.9)). IU stat is defined in (4.10). Significance codes: $^{***} < 0.01$, $^{**} < 0.05$, $^{*} < 0.1$.

Table 4.27: Group Profile Frequencies and Test for Subcorrelated Equilibrium, Partition $(7, 3)$

	No communication						
	$c = 0.1$				$c = 0.3$		
a	$\gamma(a)$	b	$\delta(b)$	a	$\gamma(a)$	b	$\delta(b)$
0	0	0	0.250	0	0.150	0	0.300
1	0.025	1	0.425	1	0.125	1	0.400
2	0.325	2	0.275	2	0.300	2	0.250
3	0.300	3	0.050	3	0.175	3	0.050
4	0.225			4	0.175		
5	0.125			5	0.050		
6	0			6	0.025		
7	0			7	0		
ϕ_A^0	-0.062				-0.258		
ϕ_A^1	-0.135				0.176		
ϕ_B^0	0.189				-0.258		
ϕ_B^1	-0.120				0.131		
IU stat	1.268, $p = .756$				21.406***, $p = .000$		
	Group communication						
	$c = 0.1$				$c = 0.3$		
a	$\gamma(a)$	b	$\delta(b)$	a	$\gamma(a)$	b	$\delta(b)$
0	0	0	0.500	0	0.050	0	0.525
1	0	1	0.025	1	0.325	1	0.275
2	0.075	2	0.100	2	0.125	2	0.125
3	0.100	3	0.375	3	0.200	3	0.075
4	0.425			4	0.125		
5	0.250			5	0.075		
6	0.150			6	0.100		
7	0			7	0		
ϕ_A^0	-0.151				-0.325		
ϕ_A^1	0.044				0.255		
ϕ_B^0	-0.515				-0.262		
ϕ_B^1	0.097				0.090		
IU stat	0.662, $p = .858$				31.523***, $p = 0.000$		
	Public communication						
	$c = 0.1$				$c = 0.3$		
a	$\gamma(a)$	b	$\delta(b)$	a	$\gamma(a)$	b	$\delta(b)$
0	0.025	0	0.500	0	0	0	0.400
1	0.225	1	0.375	1	0.050	1	0.300
2	0.225	2	0.100	2	0.250	2	0.175
3	0.275	3	0.025	3	0.150	3	0.125
4	0.175			4	0.225		
5	0.025			5	0.100		
6	0.025			6	0.225		
7	0.025			7	0		
ϕ_A^0	0.066				-0.323		
ϕ_A^1	-0.001				0.412		
ϕ_B^0	0.488				-0.486		
ϕ_B^1	-0.152				0.192		
IU stat	8.802**, $p = .040$				64.155***, $p = 0.000$		

Notes: ϕ_j^i refers to incentive compatibility condition i for group N_j (see (4.6)-(4.9)). IU stat is defined in (4.10). Significance codes: *** < 0.01, ** < 0.05, * < 0.1.

4.A.3 Analysis of chat data

Table 4.28: Message Frequencies by Round under Group Communication

n_A	n_B	Cost	Round	N	Message Code									
					0	1	2	3	4	5	6	7	8	9
6	4	0.1	1	191	3.14	21.99	13.09	25.13	0.00	4.19	25.13	6.28	0.52	0.52
–	–	–	2	188	1.60	18.62	11.70	27.66	5.85	5.32	25.00	3.19	1.06	0.00
–	–	–	3	199	2.51	28.64	8.54	20.60	5.03	2.01	26.13	3.52	3.02	0.00
–	–	–	4	159	0.00	21.38	8.18	38.36	2.52	3.14	23.27	1.26	1.89	0.00
–	–	–	5	176	1.14	18.75	7.39	34.66	10.23	2.27	12.50	11.36	1.70	0.00
–	–	–	6	165	1.21	35.15	8.48	20.61	4.24	3.64	16.36	9.70	0.00	0.61
–	–	–	7	161	0.00	24.84	9.94	27.33	0.62	4.35	13.04	11.80	8.07	0.00
–	–	–	8	166	0.00	28.92	10.84	27.11	3.61	3.01	14.46	9.04	3.01	0.00
–	–	–	9	204	0.98	22.55	3.92	39.71	6.37	1.96	15.69	7.84	0.49	0.49
–	–	–	10	206	0.49	29.61	6.31	27.18	8.74	0.97	18.45	6.80	0.97	0.49
–	–	0.3	1	133	3.76	23.31	19.55	22.56	0.75	6.02	21.05	1.50	0.75	0.75
–	–	–	2	132	3.03	14.39	12.88	21.21	6.06	9.09	22.73	0.76	9.09	0.76
–	–	–	3	126	2.38	14.29	8.73	30.95	6.35	4.76	23.81	1.59	7.14	0.00
–	–	–	4	107	4.67	18.69	14.95	25.23	1.87	1.87	23.36	0.93	7.48	0.93
–	–	–	5	119	8.40	10.92	5.04	35.29	6.72	2.52	20.17	5.04	5.04	0.84
–	–	–	6	122	2.46	20.49	19.67	23.77	12.30	0.82	18.03	0.00	2.46	0.00
–	–	–	7	124	1.61	29.84	3.23	18.55	8.06	1.61	29.84	4.03	3.23	0.00
–	–	–	8	116	3.45	33.62	8.62	18.10	2.59	0.86	25.00	2.59	5.17	0.00
–	–	–	9	133	2.26	33.08	6.77	26.32	5.26	3.01	13.53	2.26	7.52	0.00
–	–	–	10	121	1.65	33.06	10.74	14.88	19.83	0.83	18.18	0.00	0.83	0.00
7	3	0.1	1	185	0.00	33.51	15.68	25.95	0.00	1.62	12.43	5.95	4.86	0.00
–	–	–	2	217	1.38	21.66	5.07	24.88	1.84	1.84	29.95	5.99	5.07	2.30
–	–	–	3	162	0.00	30.86	17.28	20.37	11.11	3.09	10.49	3.09	3.09	0.62
–	–	–	4	148	0.00	45.27	8.11	27.03	5.41	0.68	9.46	2.70	1.35	0.00
–	–	–	5	163	0.00	39.26	6.13	28.22	4.91	0.61	16.56	3.07	1.23	0.00
–	–	–	6	169	0.00	28.40	4.73	42.60	8.88	1.18	7.69	5.33	1.18	0.00
–	–	–	7	198	0.00	35.86	3.54	25.25	19.70	0.00	10.61	5.05	0.00	0.00
–	–	–	8	186	1.08	26.88	6.45	32.26	11.29	3.76	10.75	3.76	2.69	1.08
–	–	–	9	175	1.71	43.43	2.86	11.43	17.14	1.14	18.86	2.29	1.14	0.00
–	–	–	10	176	2.27	38.07	3.41	27.27	8.52	5.11	10.23	5.11	0.00	0.00
–	–	0.3	1	149	3.36	16.11	10.74	36.91	0.00	6.71	20.81	2.01	3.36	0.00
–	–	–	2	124	1.61	13.71	8.06	28.23	3.23	4.03	26.61	3.23	10.48	0.81
–	–	–	3	127	1.57	18.90	3.94	32.28	2.36	3.94	23.62	12.60	0.79	0.00
–	–	–	4	126	0.79	13.49	7.94	26.19	3.97	4.76	26.19	11.90	4.76	0.00
–	–	–	5	177	5.08	22.60	2.82	23.16	7.34	1.13	23.16	10.17	4.52	0.00
–	–	–	6	124	0.81	24.19	2.42	30.65	19.35	1.61	17.74	0.00	3.23	0.00
–	–	–	7	106	0.00	20.75	2.83	31.13	10.38	0.94	18.87	6.60	8.49	0.00
–	–	–	8	118	0.85	22.88	4.24	31.36	6.78	1.69	26.27	1.69	4.24	0.00
–	–	–	9	128	0.78	23.44	4.69	23.44	17.19	0.00	22.66	4.69	3.13	0.00
–	–	–	10	161	1.24	29.81	4.35	37.89	7.45	0.62	13.66	3.11	1.24	0.62

Notes. Table cells contain for each message code percentages of the total number of messages in a given round. For code category description see Table 4.13.

Table 4.29: Message Frequencies by Round under Public Communication

n_A	n_B	Cost	Round	N	Message Code									
					0	1	2	3	4	5	6	7	8	9
6	4	0.1	1	144	0.69	31.25	2.78	11.11	2.78	7.64	37.50	1.39	4.86	0.00
–	–	–	2	150	4.00	33.33	4.67	18.67	7.33	2.00	26.00	0.67	3.33	0.00
–	–	–	3	143	0.70	35.66	4.90	17.48	6.29	2.80	25.87	1.40	4.90	0.00
–	–	–	4	170	0.00	47.65	5.88	9.41	5.88	2.35	26.47	1.76	0.59	0.00
–	–	–	5	133	0.00	47.37	5.26	15.04	8.27	0.00	21.05	0.75	2.26	0.00
–	–	–	6	145	0.00	49.66	2.07	11.03	8.28	4.14	23.45	0.00	1.38	0.00
–	–	–	7	164	0.61	54.88	1.83	14.63	5.49	0.00	20.73	0.00	1.83	0.00
–	–	–	8	164	0.00	50.61	1.83	15.24	3.66	3.05	23.17	0.00	0.61	1.83
–	–	–	9	162	0.00	40.12	2.47	27.16	3.09	0.62	25.31	0.00	1.23	0.00
–	–	–	10	190	1.05	39.47	1.58	10.00	7.37	1.05	38.42	0.53	0.53	0.00
–	–	0.3	1	128	0.78	28.91	6.25	32.81	1.56	2.34	25.00	1.56	0.78	0.00
–	–	–	2	136	0.74	22.06	9.56	22.06	12.50	0.74	27.21	4.41	0.74	0.00
–	–	–	3	152	0.66	21.71	5.92	36.18	6.58	3.29	21.71	3.29	0.66	0.00
–	–	–	4	156	0.00	26.92	3.85	34.62	7.69	1.28	21.15	1.92	2.56	0.00
–	–	–	5	432	1.39	11.81	0.46	17.13	2.78	0.23	64.12	1.62	0.46	0.00
–	–	–	6	233	0.43	17.17	4.29	30.47	7.73	1.29	35.62	3.00	0.00	0.00
–	–	–	7	157	0.00	31.85	7.01	24.84	5.73	1.91	22.93	3.18	2.55	0.00
–	–	–	8	184	0.00	46.20	7.07	15.76	5.43	2.17	19.02	2.17	2.17	0.00
–	–	–	9	167	0.60	55.09	2.99	11.98	7.78	0.00	16.77	2.99	1.80	0.00
–	–	–	10	188	0.00	45.21	4.79	15.96	5.32	1.06	20.74	4.26	2.66	0.00
7	3	0.1	1	116	3.45	34.48	6.90	22.41	1.72	4.31	23.28	0.86	1.72	0.86
–	–	–	2	128	3.91	31.25	3.13	21.88	6.25	1.56	31.25	0.78	0.00	0.00
–	–	–	3	134	0.00	25.37	2.24	27.61	8.96	3.73	29.85	0.00	2.24	0.00
–	–	–	4	167	0.60	53.89	0.60	11.38	11.98	3.59	14.37	0.00	3.59	0.00
–	–	–	5	168	0.00	44.64	2.98	19.05	7.14	2.38	23.21	0.00	0.60	0.00
–	–	–	6	170	0.59	34.12	1.76	21.76	19.41	2.35	17.06	0.00	2.94	0.00
–	–	–	7	168	0.00	37.50	1.79	20.24	13.10	2.98	20.83	1.19	1.79	0.60
–	–	–	8	212	0.47	41.98	1.89	13.21	5.66	0.94	33.49	1.42	0.94	0.00
–	–	–	9	172	0.00	58.72	0.58	15.12	6.98	1.16	14.53	0.00	2.91	0.00
–	–	–	10	179	0.56	63.13	1.12	12.29	8.38	1.68	12.85	0.00	0.00	0.00
–	–	0.3	1	161	0.00	36.65	8.07	24.22	0.00	5.59	18.01	3.73	1.86	1.86
–	–	–	2	150	0.67	42.00	6.00	25.33	2.00	2.00	17.33	3.33	0.67	0.67
–	–	–	3	145	1.38	42.76	4.14	23.45	5.52	2.07	17.93	1.38	1.38	0.00
–	–	–	4	126	0.00	38.10	4.76	25.40	8.73	0.79	22.22	0.00	0.00	0.00
–	–	–	5	167	0.60	35.93	2.99	27.54	10.78	0.00	19.16	2.40	0.60	0.00
–	–	–	6	141	5.67	46.81	2.84	19.15	6.38	0.71	17.02	0.71	0.71	0.00
–	–	–	7	155	1.29	30.32	5.81	25.16	9.68	3.87	13.55	7.10	1.94	1.29
–	–	–	8	170	1.76	45.29	1.18	14.12	10.00	4.71	12.94	5.88	1.76	2.35
–	–	–	9	164	0.00	46.95	2.44	14.02	10.98	1.83	16.46	3.66	3.05	0.61
–	–	–	10	147	0.00	59.18	1.36	21.09	4.08	1.36	10.20	0.00	2.72	0.00

Notes. Table cells contain for each message code percentages of the total number of messages in a given round. For code category description see Table 4.13.

Table 4.30: Effects of the Number of Messages in Each Category on Total Turnout Rate

n_A	n_B	Cost	Chat mode	N	Message Code									
					0 (Disagr.)	1 (Junk)	2 (Agr.)	3 (Rules)	4 (Hist.)	5 (Q?)	6 (Strat.)	7 (Vote)	8 (Abst.)	9 (Amb.)
6	4	0.1	GC	40	−0.047*	0.004	0.006	0.003	−0.001	0.055***	0.012*	0.012	0.003	0.067
					(.018)	(.004)	(.011)	(.003)	(.011)	(.013)	(.006)	(.010)	(.008)	(.069)
					[.013]	[.305]	[.581]	[.375]	[.947]	[.000]	[.046]	[.207]	[.752]	[.335]
–	–	–	PC	–	0.079	0.001	0.059**	−0.014	−0.033*	0.002	−0.005	0.014	−0.067***	0.021
					(.053)	(.002)	(.020)	(.009)	(.014)	(.024)	(.004)	(.033)	(.017)	(.028)
					[.143]	[.596]	[.005]	[.130]	[.023]	[.936]	[.260]	[.674]	[.000]	[.455]
–	–	0.3	GC	–	0.008	−0.006	0.036***	−0.001	0.010	−0.005	0.025***	0.062*	−0.017	−0.165
					(.025)	(.007)	(.007)	(.009)	(.012)	(.025)	(.007)	(.027)	(.021)	(.109)
					[.744]	[.366]	[.000]	[.951]	[.431]	[.838]	[.001]	[.026]	[.401]	[.137]
–	–	–	PC	–	0.038	0.005*	0.046***	0.001	0.012	−0.010	−0.001	0.046**	−0.098***	0.000
					(.049)	(.002)	(.013)	(.004)	(.012)	(.025)	(.001)	(.015)	(.021)	(−)^a
					[.440]	[.025]	[.001]	[.797]	[.312]	[.701]	[.471]	[.005]	[.000]	[−]^a
7	3	0.1	GC	–	0.038	0.002	0.007	0.005	0.012*	0.014	−0.001	0.036**	−0.019	0.011
					(.033)	(.003)	(.008)	(.003)	(.005)	(.013)	(.003)	(.011)	(.017)	(.037)
					[.257]	[.415]	[.401]	[.102]	[.026]	[.270]	[.844]	[.002]	[.263]	[.769]
–	–	–	PC	–	0.033	0.003	0.017	0.006	0.000	0.044*	−0.005**	−0.051*	-0.045	0.251***
					(.036)	(.002)	(.027)	(.004)	(.005)	(.019)	(.002)	(.022)	(.023)	(.042)
					[.374]	[.195]	[.521]	[.198]	[.959]	[.023]	[.004]	[.026]	[.063]	[.000]
–	–	0.3	GC	–	−0.037	0.017**	0.058**	−0.004	0.007	−0.012	−0.000	0.034***	−0.010	−0.085
					(.029)	(.005)	(.017)	(.004)	(.007)	(.020)	(.007)	(.008)	(.008)	(.083)
					[.218]	[.002]	[.002]	[.350]	[.287]	[.540]	[.992]	[.000]	[.233]	[.313]
–	–	–	PC	–	−0.017*	0.002	0.021	0.007*	0.010	0.004	−0.003	0.009	−0.059**	0.117**
					(.008)	(.002)	(.012)	(.003)	(.008)	(.015)	(.005)	(.015)	(.021)	(.043)
					[.041]	[.370]	[.096]	[.044]	[.242]	[.808]	[.613]	[.546]	[.007]	[.009]

Notes. Table cells contain for each message code OLS estimates of the effects of the total number of messages per electorate in that code category on total turnout rate in a given treatment. For detailed message code description see Table 4.13. Standard errors (in parentheses) are computed using N electorate-round level observations. Corresponding p-values are in brackets. Significance codes: *** < 0.001, ** < 0.01, * < 0.05.

^a for this treatment, there were no messages in the respective code category, so no variance can be estimated.

4.B Appendix: Instructions

Thank you for agreeing to participate in this research experiment on group decision making. During the experiment we require your complete, undistracted attention. So we ask that you follow instructions carefully. Please do not open other applications on your computer, chat with other students, read books, or do homework. Also make sure to turn off your cell phone.

For your participation, you will be paid in cash, at the end of the experiment. Different participants may earn different amounts. What you earn depends partly on your decisions, partly on the decisions of others, and partly on chance. So it is important that you listen carefully and fully understand the instructions before we begin. There will be a short comprehension quiz after the upcoming practice session, which you all need to pass before we can begin the paid matches.

The entire experiment will take place through computer terminals, and all interaction among you will take place through the computers. It is important that you not talk or in any way try to communicate with other participants during the experiments, except according to the rules described in these instructions. We will start with a brief instruction period. During the instruction period, you will be given a complete description of the experiment and will be shown how to use the computers. If you have any questions during the instruction period, raise your hand and your question will be answered out loud so everyone can hear. If any difficulties arise after the experiment has begun, raise your hand, and an experimenter will come and assist you privately.

This experiment consists of two different parts. The instructions for part 2 will be delivered after part 1 is completed. At the end of the experiment you will be paid the sum of what you have earned in both parts, plus the show-up fee of $7. Everyone will be paid in private, and you are under no obligation to tell others how much you earned.

[Turn on the projector and start the multistage server]

Here are the instructions for part 1. Part 1 consists of several matches. Your earnings in part 1 will be determined as follows: the computer will randomly select two matches from part 1, and you will be paid what you earned in those two matches. All matches are equally likely to be chosen as the paid matches. Your earnings during the experiment are denominated in points. Your US dollar earnings are determined by multiplying your earnings from the paid matches in points by a conversion rate. In this experiment, the conversion rate is 0.07, meaning that 100 points is worth $7.

Please click on the Client Multistage icon. This window will appear (SHOW SCREEN 2 [client information].) Enter your computer name (e.g., SSEL01) in the box that appears and then click Submit. You will then see this screen. (SHOW SCREEN 3 [initializing].)

Please turn your attention to the screen at the front of the room. We will demonstrate how the matches are played. Please do not begin unless we tell you to do so. Please have your attention focused on the stage during this demonstration period.

Once everyone has logged in, you will be randomly assigned to one of two types: type A or type B. You will see this screen (SHOW SCREEN 4 [user interface, either with or without chat, depending on treatment].)

At the top of the screen is your player id number. This is your id within your type. Once the first practice match starts, you will be randomly assigned a type label (A or B) and an id within this type. You will have the same type label, but your player id may change from match to match. Below the screen informs you which type you are in and how many members there are of each type. As you can see, type A has 6 (reverseSequence: 7) members and type B has 4 (reverseSequence: 3) members.

[start here for COMMUNICATION treatments]

Next on the screen is a time counter showing how many seconds you are allowed to chat with the other [(GC): players of your type /(PC): players] before making your

choice. All matches will have two stages. At the first stage you can use the chat feature to communicate with other [(GC): players of your type /(PC): players] about the decision making problem. Each message you send or receive in this chat stage is visible to all players of [(GC): your type, but not to players of the other type. /(PC): both types]. At the second stage, everyone will be asked to independently choose between two options, as will be described shortly. Messages sent by you are displayed in red and have '[you]' at the identifying string. Messages sent by other players are displayed in black. All messages include the sender's player id and type label. During the communication stage we require you to be courteous and polite to other participants, and also preserve the anonymity of interaction. That is, you are not allowed to communicate any personal information that might identify you to other participants. Once the time counter reaches zero, the communication stage is over, and you will see this screen (SHOW SCREEN 5 [user interface when chat is over, GC or PC].)

[start here for CONTROL CASE (NO COMMUNICATION); continue here for other treatments]

Next on the screen is a table, describing how your earnings depend on your choice of either X or Y and on which type has the most members choosing X. The display in front of the room shows you what the screen looks like for a player of type A. You will choose either X or Y by highlighting the corresponding row label and clicking with your mouse. (SHOW SCREEN 5x and 5y and 6 [showing highlighting], use screens with chat for communication treatments.) After you and the other participants have all made your choices of X or Y in a match the screen will change to highlight the row corresponding to your own choice, and the column of the type which had the greatest number of players choosing X (SHOW screen for completed match). Your earnings from each match are computed in the following way. It is very important that you understand this, so please listen carefully. Suppose you choose X. If your type has more players choosing X than the other type, then you will earn 105 points, if both types have the same number of players choosing X, then you will earn 55 points, and if the other type has more players choosing X than your type, then you will earn 5

points. Alternatively, suppose you choose Y. If your type has more players choosing X than the other type, then you will earn 115 (high-cost: 135) points, if both types have the same number of players choosing X, then you will earn 65 (high-cost: 85) points, and if the other type has more players choosing X than your type, then you will earn 15 (high-cost: 35) points Here is an example: suppose that one player of type A chose X and two players of type B chose X. Then the B type has more players choosing X than the A type. Each player of type A who chose X earns 5 points; each player of type A who chose Y earns ten (high-cost: thirty) additional points making it 15 (high-cost: 35) points; the players of type B who chose X earn 105 points, and each player of type B who chose Y earns ten (high-cost: thirty) additional points making it 115 (high-cost: 135) points. The bottom of the screen contains a history panel. During the experiment, this panel will be updated to reflect the history of your past matches. For each match you can see the match number, your type in that match, your choice, your earnings from that match if it is chosen to be a paid match by the computer, and the number of each type choosing X. Your type will remain the same for all matches. However, the actual membership in your type will be randomly reshuffled after each match. Here is how the matching works (SHOW MATCHING SLIDE). There are 20 people in this room. First, we randomly divide you into two types, A and B. Next, we randomly pick 6 (reverseSequence: 7) people of type A, 4 (reverseSequence: 3) people of type B, randomly assign ids within each type and put them together in one group. Then we pick 6 (reverseSequence: 7) remaining people of type A, 4 (reverseSequence: 3) people of type B, randomly assign ids and put them together in the second group. Next match, we repeat the same process again. Thus, you will remain the same type, but your player id as well as the other players of your type in your group will change from match to match. If you have any questions at this time, please raise your hand and ask your question so that everyone in the room may hear it.

PRACTICE [BRING UP PAYMENT SCREEN FOR PRACTICE SESSION]

We will now give you a chance to get used to the computers with a short practice session. Please take your time, and do not press any keys or use your mouse until

instructed to do so. You will NOT be paid for this session; it is just to allow you to get familiar with the experiment and your computers. Please pull out your dividers. [start practice]

[NO COMMUNICATION] Everyone please choose X. Once everyone has made their selection, the results from this first practice match are displayed on your screen. The outcome of the match is now highlighted. The number of players who chose X is greater for type A, so the potential payoff from this match (if it is selected by the computer) is 105 points for players of type A. For players of type B, the potential payoff is 5 points, since for them it is the other type that has more players choosing X. Remember, you are not paid for this practice match. We will now proceed to the second practice match. Notice that you may have been assigned a new player id. Now please everyone chose Y. Once everyone has made their selection, the results from this second practice match are displayed on your screen. The number of players who chose X is the same (zero) for both types, so the potential payoff from this match (if it is selected by the computer) is 65 (high cost: 85) points for players of either type. We have now completed the practice session, and the quiz popped up.

[COMMUNICATION] Notice that the 110-second communication stage has started. To send a message [GC: to the other players of your type/ (PC): to other players], click on the text field, type in your message and either press Enter or click 'Send'. [(GC): Remember, each message you send or receive in this chat is visible to all players of your type, but not to the players of the other type. / (PC): Remember, each message you send or receive in this chat is visible to all players of both types.] During the communication stage we require you to be courteous and polite to other participants, and also preserve the anonymity of interaction. That is, you are not allowed to communicate any personal information that might identify you to other participants. After the communication stage is over, please wait for further instructions, and don't click anywhere.

[Wait for the subjects to chat]

Now that the communication stage is over, please everyone choose X. Once everyone has made their selection, the outcome of the match is highlighted. The number of players who chose X is greater for type A, so the potential payoff from this match (if selected by the computer) is 105 points for players of type A. For players of type B, the potential payoff is 5 points, since for them it is the other type that has more players choosing X. Remember, you are not paid for this practice match. We will now proceed to the second practice match. Notice that you may have been assigned a new player id. This time a 60-second communication stage has started, so that we can move on to the paid matches quicker. After the communication stage is over, please wait for further instructions.

[Wait for the subjects to chat]

Now please everyone chose Y. Once everyone has made their selection, the results from this second practice match are displayed on your screen. The number of players who chose X is the same (zero) for both types, so the potential payoff from this match (if it is selected by the computer) is 65 (high cost: 85) points for players of either type. We have now completed the practice session, and the quiz popped up.

[QUIZ] Please read each question carefully and check the correct answer. Once everyone has answered the questions correctly, you may all go on to the second stage of the quiz. After successfully completing the second round of questions, we will commence with the first paid session. If you have questions during the quiz, please raise your hand. [END QUIZ]

The next remaining matches in Part 1 will follow the same rules as the practice session. Let me summarize those rules before we start. Please listen carefully. In each match, 6 (reverseSequence: 7) players are assigned to type A, and 4 (reverseSequence: 3) players are assigned to type B. You may choose X or Y. As you can see on the table of this screen, if you choose X, your payoff will be 105 points if your type has more players choosing X than the other type, 5 points if your type has fewer players choosing X, and 55 points if both types have the same number of members choosing

X. If you choose Y, your match payoff will be 115 (high-cost: 135) points if your type has more players choosing X than the other type, 15 (high-cost: 35) points if your type has fewer players choosing X than the other type, and 65 (high-cost: 85) points, if both types have the same number of players choosing X. Computer will randomly select two matches from part 1, and in part 1 you will be paid what you earned in those two matches. All matches are equally likely to be chosen as paid matches. Are there any questions before we begin the paid matches? [Answer questions.]

[BRING UP PAYMENT SCREEN FOR part 1]

Please begin. (Play matches 1–10.) Part 1 is now over.

[SESSION 2] Here are the instructions for part 2 of the experiment.

[BRING UP PAYMENT SCREEN FOR SESSION 2]

The second part will be slightly different from the first part. Let me summarize those rules before we start. Please listen carefully. There will be a series of matches in this part. In each match, 7 (reverseSequence: 6) players will be assigned to type A, and 3 (reverseSequence: 4) players will be assigned to type B. In the first match your type label will be assigned as follows. [SHOW MATCHING SCREEN] If you were of type B during part 1, you will now be assigned to type A for all matches. If you were of type A during part 1, in each match there is an equal chance that you either remain assigned to type A, or will be assigned to type B. So if you were of type A during part 1, each match now you may have a different type label and player id. If you were in type B during part 1, your type label will remain the same for all matches in part 2. As before, the actual membership in your type will be randomly reshuffled after each match, so the other members of your type will change

from match to match, as well as your player id even if your keep the same type. In any case, there is always information at the top of the screen telling you your player id and which type you are. Your earnings in part 2 will be determined similarly as before. The computer will randomly select two matches from part 2, and you will be paid what you earned in those two matches. All matches are equally likely to be chosen as paid matches. Your total earnings from the experiment will be the sum of your earnings from both parts, plus the show-up fee. Are there any questions before we begin the second paid session? Please begin. (Play matches 1–10.) Part 2 is now over. The experiment is now completed. Thank you all very much for participating in this experiment. Please record your total payoff in U. S. dollars at the experiment record sheet. Please add your show-up fee of $7 and write down the total, rounded up to the nearest dollar. After you are done with this, please remain seated. You will be paid in the office at the back of the room one at a time. Please bring all your things with you when you go to the back office. You can leave the experiment through the back door of the office. Please refrain from discussing this experiment while you are waiting to receive payment so that privacy regarding individual choices and payoffs may be maintained.

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