Nonlinear Effects in Granular Crystals with Broken Periodicity

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To my family and friends

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ABSTRACT

When studying physical systems, it is common to make approximations: the contact interaction is linear, the crystal is periodic, the variations occurs slowly, the mass of a particle is constant with velocity, or the position of a particle is exactly known are just a few examples. These approximations help us simplify complex systems to make them more comprehensible while still demonstrating interesting physics. But what happens when these assumptions break down? This question becomes particularly interesting in the materials science community in designing new materials structures with exotic properties In this thesis, we study the mechanical response and dynamics in granular crystals, in which the approximation of linearity and infinite size break down. The system is inherently finite, and contact interaction can be tuned to access different nonlinear regimes. When the assumptions of linearity and perfect periodicity are no longer valid, a host of interesting physical phenomena presents itself. The advantage of using a granular crystal is in its experimental feasibility and its similarity to many other materials systems. This allows us to both leverage past experience in the condensed matter physics and materials science communities while also presenting results with implications beyond the narrower granular physics community. In addition, we bring tools from the nonlinear systems community to study the dynamics in finite lattices, where there are inherently more degrees of freedom. This approach leads to the major contributions of this thesis in broken periodic systems. We demonstrate the first defect mode whose spatial profile can be tuned from highly localized to completely delocalized by simply tuning an external parameter. Using the sensitive dynamics near bifurcation points, we present a completely new approach to modifying the incremental stiffness of a lattice to arbitrary values. We show how

using nonlinear defect modes, the incremental stiffness can be tuned to anywhere in the force-displacement relation. Other contributions include demonstrating nonlinear breakdown of mechanical filters as a result of finite size, and the presents of frequency attenuation bands in essentially nonlinear materials. We finish by presenting two new energy harvesting systems based on our experience with instabilities in weakly nonlinear systems.

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