Chapter 1

INTRODUCTION

This research began with the idea that bifurcation phenomena in granular chains could be used to design materials to more efficiently harvest background vibrational energy. At times, this goal seemed out of reach. While the potential impact of granular mechanics on energy harvesting came in and out of focus, the research continued along a path that explores dynamical phenomena, some of which includes bifurcations, tunable band gaps, localized modes of lattices, tunable stiffness, finite size effects, and even vibrational bifurcations in guitar strings. In the end the research returned and finished with proposals for energy harvesting in nonlinear systems.

1.1 Motivation

Nonlinearity is ubiquitous and while oftentimes associated with damage or unwanted effects, it is also responsible for many of the remarkable phenomena found in materials. Thermal expansion occurs when atoms or molecules increase their average spacing due to an increased vibrational amplitude, a nonlinear affect arising from an asymmetric interaction potential[1,2]. The equipartition of energy requires that modes interact nonlinearly, exchanging energy[3]. If interactions were purely linear the equipartition would not occur and materials would not relax to a thermal equilibrium. Umklapp phonon scattering is inherently nonlinear and significantly influences the thermal conductivity at higher temperatures. The above examples demonstrate that nonlinearity has some clear benefits: energy can be transferred between modes and frequencies, the natural frequencies of the system may depend on strain or other external conditions[4-7], and the response of a nonlinear system can become extremely sensitive to varying parameters[8]. By leveraging these effects, we can look to use nonlinearity to design materials with targeted properties.

A fundamental theme of materials science is in understanding how materials derive properties from their structure. This could be how the different microstructures affect the Young's modulus and toughness, the effect of grain size on a solid's hardness[9], or how the crosslinking in a polymer affects it viscoelastic response. We are interested in another example of structure: materials with broken periodicity. Periodicity is the regular arrangement of atoms, particles, or other repeat unit on a lattice. In a truly periodic material this regularity repeats forever, which leads to remarkable phenomena such as frequency band gaps[10]. These band gaps prevent the propagation of waves in material and can be used to reflect or localize energy[11]. The discrete nature of periodicity leads to wave dispersion, in which different frequencies travel at different wave speeds.

Real materials are not completely periodic: there are grain boundaries to crystals, defect impurities, and an inherently finite size. This leads to a broken periodicity in which there is no longer complete translational symmetry. These imperfections in the periodicity of materials have an enormous effect on material properties and are a reason for the many interesting phenomena and effects in materials. A few examples of how defects effect material properties include electrical conductivity in semiconductors[12], thermal conductivity of graphene[13], and mechanical strength of carbon materials[14,15].

More recently, the concept of periodicity has been extended in the context of metamaterials[16], in which a periodic structure is engineered at a mesoscale into the system. So instead of looking for a material with a specific property, the goal is to tailor the structure of local interactions for particular properties. In mechanics, this resulted in materials with unexpected and extreme properties, including negative effective mass and modulus[16-21]. Since the scale of the engineered repeat units are larger than atomistic, the assumption of infinite size does not necessarily hold.

We expand ideas from metamaterials research by looking at nonlinearity as a different mechanism to tune material properties. In this thesis we look at granular crystals, a macroscopic array of nonlinearly interacting spherical particles where nonlinearity is introduced in the local interaction between granular particles. By arranging the particles in a one-dimensional array we can study a system with periodicity similar to that found in other materials. In a granular crystal, the nonlinearity originates from a change in the contact area as the neighboring particles are compressed[22]. This nonlinearity can be tuned across a complete range, encompassing linear, weakly nonlinear, and strongly nonlinear dynamics.

The finite size, mass defects, and resonant defects of granular crystals leads to broken periodicity. Nonlinearity in systems with broken periodicity is not well understood and offers an enhanced control over the vibration dynamics and propagation of waves in these structures. By using structural nonlinearity, we can design materials with extreme tunability. Although nonlinearity in other materials is oftentimes considered secondary, we study the physics that originates from the interaction of nonlinear localized modes with the extended modes of the acoustic band of the granular crystal. This research acts as a model for designing nonlinearity into materials for targeted, extreme, and unusual properties.

1.2 Goals and achievements

We describe new ways to use nonlinear modes for enhanced control over the propagation and localization of mechanical energy. In addition, we show how nonlinearity and defect modes can be used in practical applications such as fully tunable stiffness and energy harvesting. We use the one-dimensional granular crystal for theory and experiments as a model system with nonlinearity, discreteness, and broken periodicity.

A significant portion of this research is related to nonlinear dynamic responses, i.e., how weak and strong nonlinearity can be used to alter frequency band gaps and wave propagation[23-26]. The dynamics in this research project expand the complete range of nonlinear responses, starting from near linear and going to completely nonlinear and non-smooth.

We start by studying dynamics in the linear limit, where resonant defect mode profiles can be externally tuned from localized to completely delocalized. This is the first theory and experimental observation of defect modes that can be externally controlled, eliminating the need to replace a defect due to changing application demands. This is in direct contrast with modes introduced by mass defects, in which the localization typically depends on the size of the defect and cannot be altered later[27]. The simplicity of the physical model extends the impact of these results and potential applications to other material systems.

In the weakly nonlinear dynamical regime, we demonstrate how driving localized modes can be used to control macroscopic properties, providing the first example of fully tunable stiffness. By using the highly sensitive nature of nonlinearity combined with the thermal expansion we tune the incrmental lattice stiffness to arbitrary values at any point on the force-displacement relation.

We also demonstrate how mechanical filters based on periodicity break down due to relatively weak nonlinearities at finite sizes. While previous work had examined this effect in diatomic granular chains, where the instability was connected to nonlinear discrete breather solutions[28,29], we show this instability in homogenous granular crystals. As the size of the granular crystal decreases the presence of instabilities increases and energy is more readily transferred between frequencies. We explore how bifurcations and energy transfer depend on the damping, finite size, nonlinearity, and breakdown of evanescent waves. Because energy is transferred to new frequencies that are in the propagating band of the system, the bifurcations affect wave propagation and result in the breakdown of mechanical filtering, which relies on frequency band gaps[30-32]. We show how this dynamic response holds even when the dynamics become non-smooth and gaps open up between particles.

In the strongly nonlinear regime we explore frequency bands gaps in granular crystal that exhibit no linear speed of sound and therefore no linear coupling stiffness[33-35]. Since frequency band gaps in linear media are derived from the coupling stiffness, it is not necessarily clear how band gaps exhibit themselves in essentially nonlinear systems, where there is no possible linearization. While the dynamics are strongly nonlinear and quite different from linear media, we demonstrate that the waves propagate as separate pulses instead of harmonic waves, and that the concept of a band gap still holds.

Studying the weak nonlinearities in granular crystals led us to propose two new models for energy harvesting. In the both models a nonlinear oscillator is driven intentionally to instability. Instability causes non-conservative forces in the system to no longer balance, i.e., dissipation no longer balances the energy driven into the system. As the system departs from its steady state solution, there is more energy being injected into the system than dissipated. By holding the system in this instable region one could efficiently transfer energy between the source and harvester without altering the state of the system, presenting a new paradigm for nonlinear energy harvesting systems.

1.3 Structure of the thesis

This thesis is a collection of self-standing articles of nonlinearity in systems with broken periodicity[26,36,37]. The articles span from nearly linear dynamics, in which the nonlinearity enters as a tuning parameter, to the essentially nonlinear limit.

Chapter 2 presents the necessary background information and a review of previous literature in nonlinear granular chains. In chapter 3, we provide an overview of all the experimental and computational tools used in each experiment. Since the experimental setups for the self-standing results are similar but distinctly different, we first present a detailed description of experimental tools and their function and then describe the distinct experimental setups. In chapter 4, we follow the same structure as chapter 3, but for modeling tools. The following chapters discuss the results and potential applications of the work. In chapter 5, we start by studying dynamics in the linear limit, where resonant defect mode profiles can be externally controlled. This is the first theory and experimental observation of defect modes that can be externally controlled, eliminating the need to replace a defect due to changing application demands. In chapter 6, we study the tuning of a lattices mechanical response using driven defect modes, providing the first example of fully tunable stiffness. By using the highly sensitive nature of nonlinearity combined with the thermal expansion we show the first example of how lattice stiffness can be tuned to arbitrary and extreme values at any point on the force displacement relation. In chapter 7, we demonstrate how mechanical filters based on periodicity may break down due to relatively weak nonlinearities at finite sizes. In chapter 8, we explore frequency bands in strongly nonlinear wave propagation. In chapter 9, we finish by presenting applications of the nonlinear dynamics in two models for energy harvesting. Although the two approaches aim to solve different problems in energy harvesting, both rely on pushing the dynamics into regions of nonlinear instability for more efficient energy harvesting.