Chapter 9

APPLICATIONS: ENERGY HARVESTING

Nonlinear systems have been shown to outperform linear systems for energy harvesting under certain conditions[135] (see section 2.4). These methods can outperform linear systems due to the ability of a nonlinear system to transfer energy between frequencies. However, nonlinear energy harvesting systems utilize their multi-stability as an advantage over linear systems, and this means that dissipative mechanisms become important. Therefore the influence of the electrical circuit on the mechanical response is essential to more efficient energy harvesting systems. In current vibrational energy harvesting systems based on nonlinearity, the effect of electromechanical damping on the system is either ignored or so small that its effect is negligible. The following two examples present nonlinear mechanical systems that are driven to instability. This instability can only be stabilized by adding a dissipation. In this way, we use the electrical circuit to beneficially stabilize the nonlinear dynamics rather than adversely affect the state and performance of the system.

9.1 Energy harvesting inspired by thermal machines

We propose an energy harvesting system inspired by classic thermal machines. Natural energy sources present themselves as "random" or heat energy sources that must be converted to usable work energy. For the example of a steam engine, hot gas expands against a piston, transferring energy to a piston, the gas is cooled and contracts. This contraction is at a lower temperature than the expansion, and therefore when the system returns to its original state, making one full cycle, energy is transferred to the piston. This cyclical process is often expressed in a pressure volume (PV) diagram, in which the area enclosed is the work done through a single cycle (Figure 9.1). We could also look at work done in a force-displacement cycle. We can replace the steam engine or other thermal machine, which extracts energy from the temperature difference of a hot and cold gas, with a nonlinear hysteretic spring.

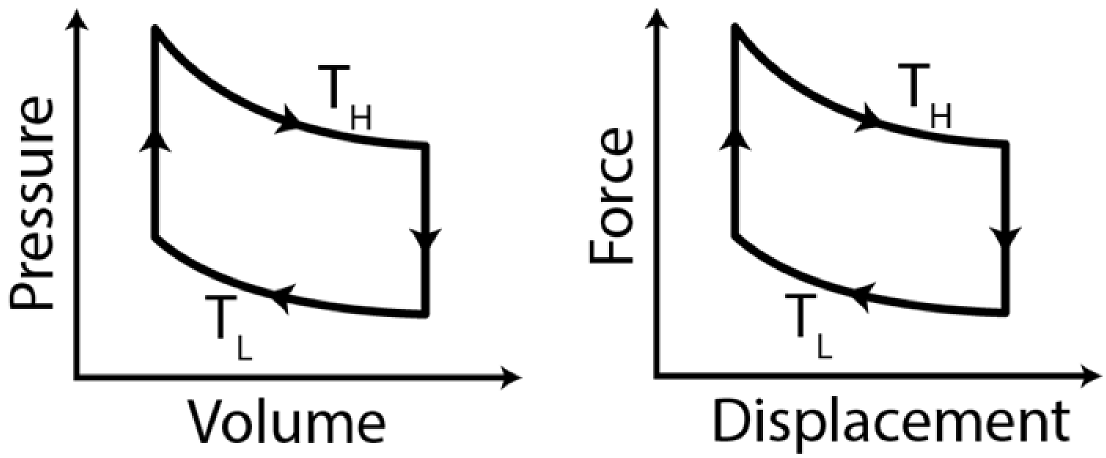

Figure 9.1: (Left) PV diagram for a traditional thermal machine such as a steam engine. (Right) We propose an energy harvesting system inspired by a thermal machine, in which the constituent harvesting is now a nonlinear hysteretic spring.

This hysteresis response is very similar to the hysteresis response that we observed in the tunable stiffness experiment, except that the cycle goes in the other direction. In the extraordinary tunable stiffness experiment, we demonstrate a damping response where the hysteresis originates from the two state solution of the nonlinear system past the bifurcation. If we can engineer a system with a hysteretic loop that goes in the opposite direction to the granular crystals, then the system acts as an energy harvester or "nonlinear thermal machine" (Figure 9.1).

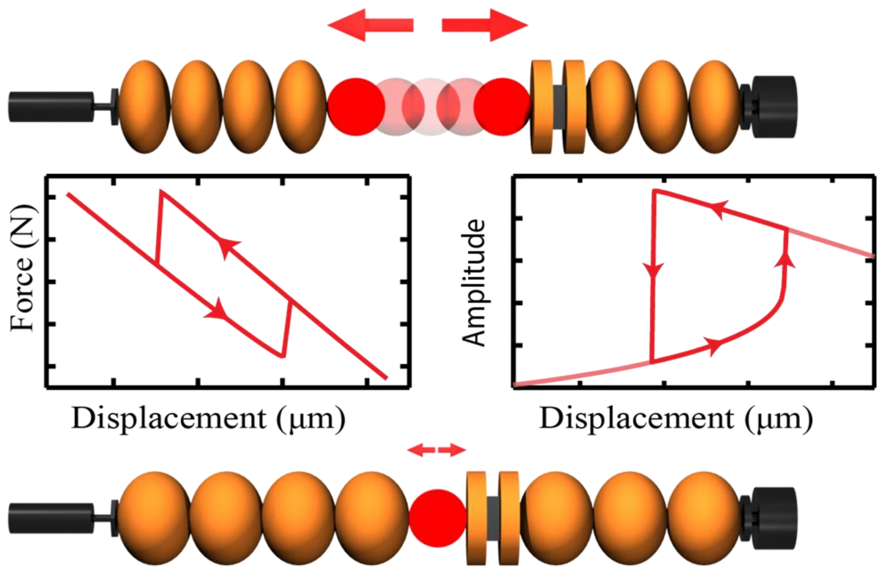

Figure 9.2: (Left) Force-displacement for the hysteretic damping mechanism observed in a granular crystal. (Right) The nonlinear amplitude response of the defect mode that results in the force-displacement relation to the left.

We propose a nonlinear energy harvesting system based on a nonlinear spring with bifurcation dynamics similar to the granular chain. In the granular example with hysteretic damping, the force interaction of the nonlinear defect mode can be approximated by a Taylor expansion, $F \sim kx + \alpha x^2 + \beta x^3$. The term with alpha describes the thermal expansion of the defect mode causing an extra force at the boundary. This results in a dependence of the force-displacement response on the amplitude-displacement response. This means a bi-stability in the amplitude response also causes a hysteresis. However, this particular system follows the low amplitude solution in compression and the high amplitude solution in extension, exactly the opposite of what we need for an energy harvester.

By changing the directionality of the bistable amplitude response, we can also change the hysteresis to go from damping to harvesting. In nonlinear dynamical systems, such as the

Duffing oscillator, it is well known that changing the sign of the cubic term, β, also changes the direction of the softening or hardening response of the resonance. In the granular crystal, the cubic term is negative, resulting in a softening potential. However, if the cubic term were positive, the potential would be hardening, and the amplitude loop could be reversed. We therefore search for a model system that has a hardening nonlinearity to demonstrate this idea of energy harvesting inspired by thermal machines.

Strings are systems with known hardening nonlinear responses[136]. Therefore we study a guitar string clamped between a cantilever and a fixed wall (Fig. 9.3). In our simplified model there is an energy source that drives the vibration of the string, which transfers energy to the cantilever. We start by considering a harmonic driving source, and although this is a gross approximation to sources in real systems, it provides intuition into the phenomenology.

When the string is driven with a harmonic excitation, it reaches a steady state amplitude. This vibration amplitude causes an increase of the average length of the string. This results in an increased tension on the string, causing two effects. The first is the hardening, or stiffening, nonlinearity that causes the third order nonlinear term to be positive, $\beta > 0$. The second is the additional tension on the cantilever which causes it to displace. This is schematically represented in the right panel of Figure 9.3. The displacement of the cantilever also has a back action on the string by causing a detuning, or softening. These two effects happen at different speeds and together result in the energy harvesting response of the nonlinear string-cantilever system.

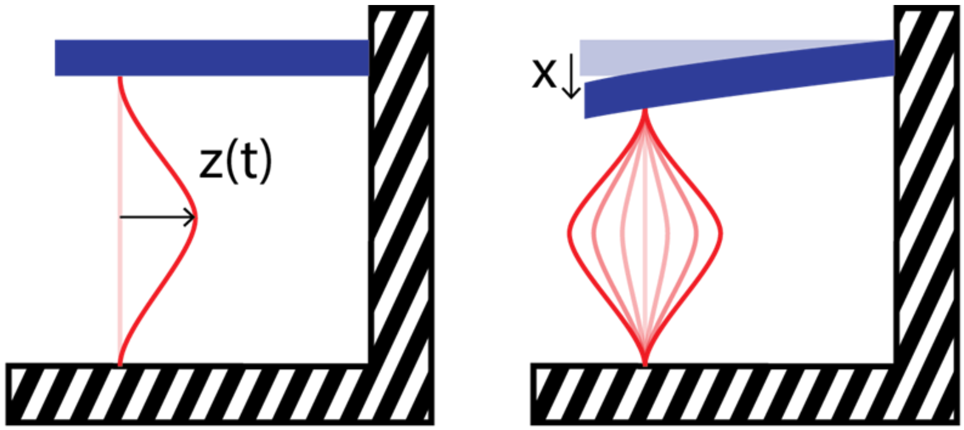

Figure 9.3: The string-cantilever system. (Left) The vibration coordinate of the string is defined as z(t). (Right) The coordinate of the cantilever is defined as x(t). As the amplitude of the string vibration increases there is an additional tension pulling on the cantilever.

In order to support the qualitative argument above, we begin with a partial differential equation describing the motion of a beam[137] of which a string is a limiting case and attempt to reduce this to a more manageable second order ordinary differential equation. We follow a derivation similar to Postma et al.[137]

$$EI\frac{\partial^4 w}{\partial x^4} - \frac{EA}{L}\left[X + \frac{1}{2}\int_0^L \left(\frac{\partial w}{\partial x}\right)^2 dx\right]\frac{\partial^2 w}{\partial x^2} + \rho A\frac{\partial^2 w}{\partial t^2} - iB = 0 \tag{9.1}$$

The equation includes a term for bending, tension, inertia, and the excitation applied to the beam, where $E, I, A, L, X, \rho, i,$ and B are the Young's modulus, moment of inertia, cross sectional area, length, initial tension, density, current through the beam, and perpendicular magnetic field. Using a Galerkin method, we apply a test function,

$$\phi(x) = \left(\sqrt{\frac{2}{3}}\right)\left[1 - cos\left(\frac{2\pi x}{L}\right)\right], \tag{9.2}$$

and reduce the PDE to a second order ode,

$$m_z\ddot{z} + b_z\dot{z} + (k_z + 2\alpha x)z + \beta_z z^3 = F(t), \tag{9.3}$$

where each of the parameters listed depend on the material constants chosen in (Eq. 9.1), and where $m_z = \rho AL, k_z = (2\pi)^4\, EI/(3L^3)\,, \alpha = 2\pi^2 EA/(3L^2)\,, \beta_z = (2\pi)^4\, EA/(18L^3),$ and b_z is a viscous dissipation added to the system to reflect the dissipation in the real system. The equation is the same as a for a driven-damped Duffing oscillator[8]. Note that in this equation $\beta_z > 0,$ which means that the dynamics are stiffening instead of softening, as in the case of the granular chain. As the string starts to vibrate to larger amplitudes the increase in time-averaged length is also accompanied by an increase in the average tension on the string. This also causes a detuning to higher frequencies when the string is driven to higher amplitudes, hence the stiffening nature of the string. When the equation is driven to higher amplitudes around the natural resonance of the system, the amplitude response for the Duffing oscillator is hysteretic. The detuning of the system, i.e., the difference of the drive frequency from the natural frequency of the system controls the amplitude of the response. Equation (9.3) also includes a coupling between the string and cantilever element. As the cantilever relaxes, this affects the linear resonant frequency. We can model the cantilever as a linear oscillator driven by the amplitude of the string,

$$m_x \ddot{x} + b_x \, \dot{x} + k_x x = -\alpha z^2, \quad (9.4)$$

where k_x, b_x, and m_x are the bending stiffness, damping, and effective mass of the cantilever beam, respectively. In addition, the cantilevers mass and stiffness must be designed so that the motion is slow compared to the string, allowing the string to remain on the steady state solution. The cantilever's motion has the effect of modulating the strings frequency. This allows us to move the bifurcated string system around the hysteretic loop through the motion of the cantilever (Fig. 9.4).

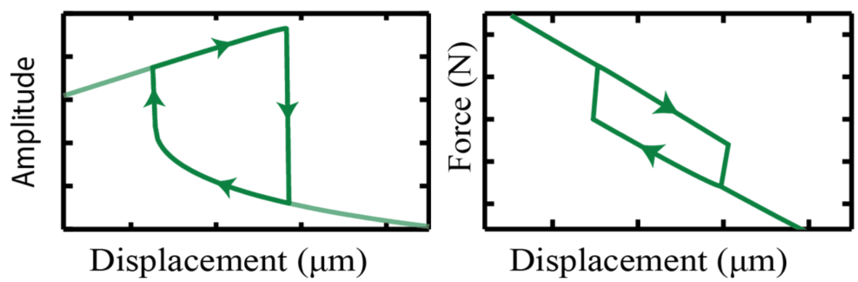

Figure 9.4: The displacement of the cantilever modulates the resonance frequency of the string. (Left) This results in a amplitude response for the string that is modulated by the cantilevers displacement, and (Right) a force-displacement relation for the cantilever that has a similar hysteresis to a thermal machine.

As the cantilever increases its displacement, $x(t)$, the amplitude of the string, $z(t)$, follows the high amplitude solution until the system falls over the bifurcation. As the displacement of the cantilever decreases, the amplitude force response follows the low amplitude solution. Equation (9.4) shows that the cantilever is driven by the strings amplitude. This results in a hysteretic force displacement curve for the cantilever, in which the string transfers energy to the cantilever through each cycle. To verify this we simulate the coupled system with no dissipation for the cantilever, b_x=0 (Fig. 9.5).

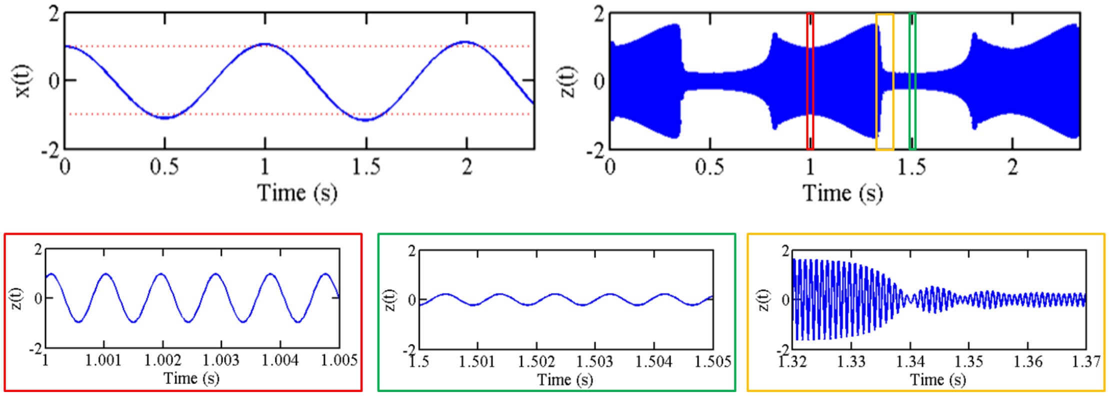

Figure 9.5: Cantilever (Top Left) time series and string (Top Right) time series. The string is driven by a harmonic signal, while the cantilever causes a slow variation of the resonance frequency. (Bottom Left) String time series for the high amplitude solution. (Bottom Middle) String time series for the low amplitude solution. (Bottom Right) String time series showing the transient dynamics..

In Figure 9.5 we simulate the coupled spring cantilever system with the cantilever given an initial potential energy in the form of a nonzero starting displacement. The string is driven harmonically with its frequency modulated by the slow movement of the cantilever. Figure 9.5 shows how the string's displacement cyclically moves from a high amplitude to a low amplitude solution, following the position of the cantilever. Each time through this cycle, the string transfers energy to the cantilever system. This can be seen in the growing amplitude of the cantilever.

What makes this nonlinear energy harvesting system different from many other solutions is the effect of electrical coupling. Without the electrical coupling the amplitude of the cantilever will grow, and the system is not stable. It is only the additional presence of damping that stabilizes the system. In this example the electrical coupling is a necessary component instead of having a detrimental effect.

9.1.1 Conclusion and outlook

While these results are preliminary and present a paradigm for developing energy harvesting materials, there are a few things that we have learned. The first is that we have demonstrated energy transfer between two different frequency modes of completely arbitrary frequency. The energy is transferred from high to low frequencies, where the frequencies are determined by the natural frequencies of the two structures. Second is that the energy transfer is synchronous. This means that if we attached a second string to the cantilever it can be pushed through its own hysteresis in phase with the other string. This is because the modulation of each string is determined by the cantilever itself.

9.2 Parametrically driven energy harvesting

The second approach that we present for an energy harvesting system is based on the parametric type resonances observed in the finite granular chains. We look to use these instabilities as an instance in which the mechanical dissipation does not balance the energy injected into the system through excitations. By introducing a coupled electrical circuit, we propose a mechanism to stabilize the mechanical system and more efficiently harvest energy.

9.2.1 Project goal

We propose a novel concept for energy harvesting based on nonlinear dynamical systems that will more efficiently harvest energy from both small and large mechanical excitations. The design approach we propose relies on creating mechanical systems that are both nonlinear and parametric, a combination that leads to a host of advantages over traditional energy harvesting approaches.

There are three primary objectives of this project: (i) to develop a theoretical model for energy harvesting in a parametric nonlinear system that will be compared with results from traditional linear systems; (ii) implement a prototype device, a sphere in contact with a vibrating surface, to harvest mechanical energy through electromechanical coupling; and (iii) to use Finite Element Methods design nonlinear geometries that also contain parametric resonances, and can implement the theory across a range of power and size scales.

9.2.2 Introduction

Mechanical resonances are basic physical phenomena, present in all engineering systems and structures. Resonance occurs when a structure experiences a periodic force at one of its natural frequencies, which depends on the design geometry and constituent materials. When a structure is at resonance there is a buildup of energy, and it generally undergoes large oscillations and deformations. In most design efforts, resonances are commonly avoided due to their potential to cause catastrophic failure of infrastructures, for example the Tacoma Narrows Bridge collapse in 1940. The Taipei 101 building, the tallest in the world until 2010, uses a tunable mass damper to protect the structure against resonances that can arise from wind or earthquakes. In this proposed work, we look at resonances as a means to achieve large oscillations for renewable energy conversion. Initially, we planned to explore a proof-of-concept device that will harvest energy from ambient sources (see Table 1). Then, since the approach would only rely on fundamental physical principles, the design would be appropriately modified to produce energy on a larger scale.

Mechanical energy conversion and production typically falls into one of two categories depending on the scale. (i) For power sources that are more than a few watts, e.g., wind or hydrodynamic power, conventional turbine technology is used. (ii) When power levels are less than a watt, the process is typically considered energy harvesting or scavenging. For this, one of the most prolific designs is a cantilever-based piezoelectric device. The efficiency of such cantilever-designs is limited by the piezoelectric materials, which can harvest only a small fraction of energy dispersed over the whole structure[138], and only work over a small frequency bandwidth. In this proposal, we are interested in designing new and improved approaches for harvesting energy from small ambient mechanical excitations, using nonlinear dynamical systems. The fundamental concepts proposed here as a proof-of-principle could be extended to designing energy production solutions also for large power sources.

Nonlinear dynamical systems, for example simple pendula, bending beams, and many optical materials, have an amplitude dependent response. The dynamic response of these systems is different for small and large mechanical excitations. The design approach we propose relies on creating mechanical systems that are both nonlinear and parametric, a combination that leads to a host of advantages over traditional linear dynamical systems. For example, nonlinear parametric systems can be driven at resonance to amplify their mechanical deformation, and subsequently their oscillation can be accurately stabilized through geometric nonlinearities. The simplest example of a nonlinear parametric system is a spherical particle confined by rigid walls and driven by a mechanical vibration. This system is highly tunable, from weakly to strongly nonlinear, and is extremely easy to implement. We will use this simple toy-model to provide a theoretical and experimental proof of concept for new energy harvesting approaches, evaluate their efficiency, and propose scalable designs for practical engineering solutions. We will design and test an energy harvesting system based on a single spherical particle confined between rigid boundaries. We will then demonstrate how its dynamic response can provide a completely new approach to localizing and harvesting mechanical energy. In the last few months of the planned work, we will propose designs adaptable to larger mechanical sources and different ambient conditions.

Vibration source	**Peak frequency (Hz)**	**Acceleration amplitude (m/s^2)**
Washing machine	109	0.5
External windows	100	0.7
Car engine compartment	200	12
Vehicles	5-2000	0.5-110
Refrigerator	240	0.1

Table 9.1: Example of common mechanical energy sources for small energy harvesting/scavenging. Source: "Piezoelectric Energy Harvesting Devices for Low Frequency Vibration Applications" Dongna Shen, PhD Thesis, Auburn University (2009)

9.2.3 Theoretical investigation

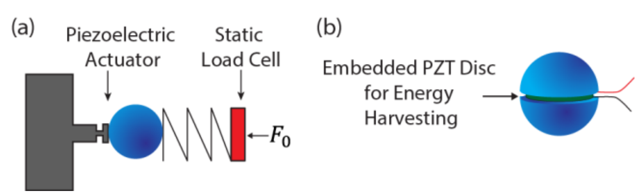

Figure 9.6: Experimental setup for a Spherical Bead Compressed Against a Piezoelectric Actuator. (a) A prototype schematic for the proof of concept energy harvesting device. This is mathematically represented in eq. 1. (b) An embedded piezoelectric sensor acts as a high impedence voltage source.

The fundamental response of both nonlinear and parametrically driven systems is well understood and has been extensively studied[52]. However, these phenomena have not yet been combined and investigated to design structures for enhanced energy harvesting and production. A sphere compressed with a force, F_0, against a vibrating surface (Fig. 9.6), is characterized by both a nonlinear mechanical response (deriving from the geometry of the contact between the sphere and the vibrating surface) and a parametric excitation (the ambient vibration). These physical phenomena and the response of this simple system can be described by the following model[35]:

$$m\ddot{u} = F_0\left[1 - u + \left(\frac{A}{F_0}\right)^{\frac{2}{3}} B\ \cos(\omega t)\right]_+^{3/2} - F_0 - \gamma\dot{u}\,, \tag{9.5}$$

where m is the mass of the sphere, u is the displacement of the bead, γ is the bead's dissipation coefficient, and A is the Hertzian spring constant. B and ω are the vibration source's amplitude and frequency, respectively. The bracket , $[x]_+ = \max(0, x)$, indicates a tensionless behavior in which there is no force attracting the bead to the vibrating surface after they separate.

The most important part of this model is the exponent of 3/2. It results in two important physical phenomena for our proposed approach. First, the vibrating surface does not just provide acceleration to the sphere, but also acts as a parametric drive periodically varying the spring constant of the system. Second, the force between the spherical bead and flat vibrating surface is nonlinear, meaning that the effective spring constant of the system also depends on the amplitude of the vibration.

A common example of a parametric drive is a child on a swing, where the child periodically adjusts his position to increase the amplitude of his swinging. In certain frequency bands, called parametric tongues, the dynamics leads to oscillations that grow exponentially in amplitude, even in the presence of dissipation. This phenomena is very exciting, especially in the context of energy harvesting. The structure continues to gain energy until the system collapses or is stabilized. This type of oscillator is oftentimes referred to as a self-exciting oscillation and was identified as the source that led to the ultimate collapse of the Tacoma Narrows Bridge[139]. Figure 9.7a shows a parametric tongue observed in a driven single spherical particle. There is sudden discontinuous change in the dynamics above a threshold driving amplitude, a bifurcation, in which the oscillation amplitude of the bead is clearly larger. In fact, after this sudden change, as much as 21 times more energy is being transferred from the vibrating surface to the bead.

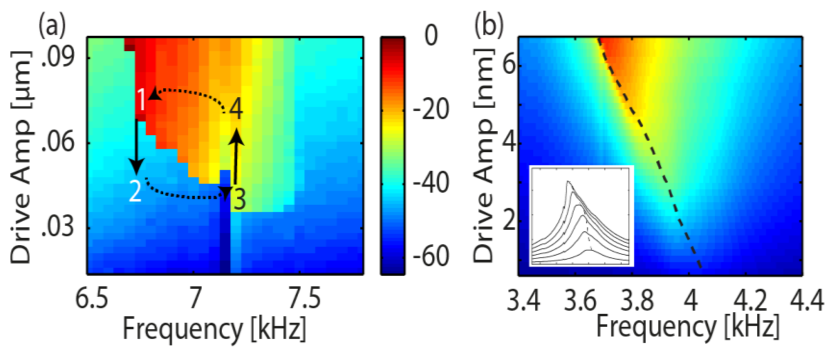

Figure 9.7: Nonlinear Detuning and Parametric Energy Harvesting. a, The parametric tongue separating regions of low and high amplitude oscillations for a single sphere particle. We would like to test the indicated cycle as a mechanism to collect mechanical energy. Initially the system would start at (1). By removing energy the system loses amplitude (2), causing the system to retune (3). This causes the amplitude to increase (4) until the nonlinearity again detunes the system (upper dotted arrow). b, The detuning of the natural frequency of the nonlinear spherical particle serves to stabilize and the dynamics. At higher amplitudes, the natural frequency decreases. (colorbar units are in dBs)

In addition, Figure 9.7a shows that the dynamics are stable above the cutoff drive amplitude. This means there is not a collapse or destructive deformation due to an exponentially increasing amplitude. Instead nonlinearity acts to prevent catastrophic events. Figure 9.7b shows the dependence of the natural frequency of the system on amplitude. As the amplitude increases, the natural frequency decreases. This pushes the system to the edge of the parametric tongue frequency band, and prevents continuous growth in amplitude.

Using this proven fundamental physics, we will design systems that self-correct, balance, and protect themselves as a new approach for efficient energy harvesting.

9.3 Author contributions

The concept for the string-cantilever energy conversion system was developed by Joseph Lydon and Marc Serra-Garcia together. The parametrically driven system was developed by Joseph Lydon, and Chiara Daraio contributed to the writing of the manuscript.