

Bibliography

- [1] G. C. Cho, J. Dodds, and J. C. Santamarina. Particle shape effects on packing density, stiffness, and strength: Natural and crushed sands. *Journal of Geotechnical and Geoenvironmental Engineering*, 132(5):591–602, 2006.
- [2] J. E. Andrade and C. F. Avila. Granular element method (GEM): linking inter-particle forces with macroscopic loading. *Granular Matter*, 14:1–13, 2012.
- [3] R. Hurley, E. Marteau, G. Ravichandran, and J. E. Andrade. Extracting inter-particle forces in opaque granular materials: Beyond photoelasticity. *Journal of the Mechanics and Physics of Solids*, 63(0):154 – 166, 2014.
- [4] J. E. Andrade, I. Vlahinić, K.-W. Lim, and A. Jerves. Multiscale ‘tomography-to-simulation’ framework for granular matter: the road ahead. *Géotechnique Letters*, 2, 2012.
- [5] J. Bonet and R. D. Wood. *Nonlinear Continuum Mechanics for for Finite Element Analysis*. Cambridge University Press, Cambridge, UK, 1997.
- [6] C. S. Desai and H. J. Siriwardane. *Constitutive Laws for Engineering Materials*. Prentice-Hall, Inc., 1984.
- [7] K. C. Ellison and J. E. Andrade. Liquefaction mapping in finite elements simulations. *Journal of Geotechnical and Geoenvironmental Engineering*. In press. doi:10.1061/(ASCE)GT.1943-5606.0000122, 2009.
- [8] R. I. Borja and J. E. Andrade. Critical state plasticity, Part VI: Meso-scale finite element simulation of strain localization in discrete granular materials. *Computer Methods in Applied Mechanics and Engineering*, 195:5115–5140, 2006.

- [9] E. Grueschow and J. W. Rudnicki. Elliptic yield cap constitutive modeling for high porosity sandstone. *International Journal of Solids and Structures*, 42(16-17):4574–4587, 2005.
- [10] M. T. Manzari and Y. F. Dafalias. A critical state two-surface plasticity model for sands. *Géotechnique*, 43:255–272, 1997.
- [11] Y. F. Dafalias and E. P. Popov. A model of nonlinearly hardening materials for complex loadings. *Acta Mechanica*, 21:173–192, 1975.
- [12] F. L. DiMaggio and I. S. Sandler. Material model for granular soils. *Journal of the Engineering Mechanics Division-ASCE*, 97:935–950, 1971.
- [13] M. Oda, T. Takemura, and M. Takahashi. Microstructure in shear band observed by microfocus X-ray computed tomography. *Géotechnique*, 54:539–542, 2004.
- [14] K. A. Alshibli, S. Sture, N. C. Costes, M. L. Frank, F. R. Lankton, S. N. Batiste, and R. A. Swanson. Assessment of localized deformations in sand using X-ray computed tomography. *Geotechnical Testing Journal, ASCE*, 23:274–299, 2000.
- [15] R. A. Ketcham and W. D. Carlson. Acquisition, optimization and interpretation of X-ray computed tomographic imagery: applications to the geosciences. *Computers & Geosciences*, 27:381–400, 2001.
- [16] L. Wang, J. Y. Park, and Y. Fu. Representation of real particles for DEM simulation using X-ray tomography. *Construction and Building Materials*, 21:338–346, 2005.
- [17] S. A. Hall, M. Bornert, J. Desrues, Y. Pannier, N. Lenoir, G. Viggiani, and P. Bésuelle. Discrete and continuum analysis of localized deformation in sand using X-ray micro CT and volumetric digital image correlation. *Géotechnique*, 60:315–322, 2010.
- [18] R. V. Martins, L. Margulies, S. Schmidt, H. F. Poulsen, and T. Leffers. Simultaneous measurement of the strain tensor of 10 individual grains embedded in an Al tensile sample. *Materials Science and Engineering A*, 387-389:84–88, 2004.
- [19] S. A. Hall, J. Wright, T. Pirling, E. Andò, D. J. Hughes, and G. Viggiani. Can intergranular force transmission be identified in sand? *Granular Matter*, Doi:10.1007/s10035-011-0251-x, 2011.

- [20] M. M. Frocht. *Photoelasticity*, volume 1. John Wiley & Sons Ltd., New York, 1941.
- [21] M. M. Frocht. *Photoelasticity*, volume 2. John Wiley & Sons Ltd., New York, 1941.
- [22] J. E. Andrade, C. F. Avila, N. Lenoir, S. A. Hall, and G. Viggiani. Multiscale modeling and characterization of granular matter: from grain scale kinematics to continuum mechanics. *Journal of the Mechanics and Physics of Solids*, 59:237–250, 2011.
- [23] J. Christoffersen, M. M. Mehrabadi, and S. Nemat-Nasser. A micromechanical description of granular material behavior. *Journal of Applied Mechanics*, 48:339–344, 1981.
- [24] P. A. Cundall and O. D. L. Strack. A discrete numerical model for granular assemblies. *Géotechnique*, 29:47–65, 1979.
- [25] B. Sukumaran and A. K. Ashmawy. Quantitative characterisation of the geometry of discrete particles. *Géotechnique*, 51:619–627, 2001.
- [26] X. Garcia, L. T. Akanji, M. J. Blunt, S. K. Matthai, and J. P. Latham. Numerical study of the effects of particle shape and polydispersity on permeability. *Physical Review E*, 80:021304, 2009.
- [27] K. Iwashita and M. Oda. Rolling resistance at contacts in simulation of shear band development by dem. *Journal of engineering mechanics*, 124(3):285–292, 1998.
- [28] K. Iwashita and M. Oda. Micro-deformation mechanism of shear banding process based on modified distinct element method. *Powder Technology*, 109(1):192–205, 2000.
- [29] A. Tordesillas and D. C. Incorporating rolling resistance and contact anisotropy in micromechanical models of granular media. *Powder Technology*, 124(1):106–111, 2002.
- [30] A. Tordesillas, J. Peters, and M. Muthuswamy. Role of particle rotations and rolling resistance in a semi-infinite particulate solid indented by a rigid flat punch. *ANZIAM Journal*, 46:C260–C275, 2005.
- [31] M. J. Jiang, H.-S. Yu, and D. Harris. A novel discrete model for granular material incorporating rolling resistance. *Computers and Geotechnics*, 32(5):340–357, 2005.

- [32] X. Li, X. Chu, and Y. T. Feng. A discrete particle model and numerical modeling of the failure modes of granular materials. *Engineering Computations*, 22(8):894–920, 2005.
- [33] W. Zhang, J. Wang, and M. Jiang. Dem-aided discovery of the relationship between energy dissipation and shear band formation considering the effects of particle rolling resistance. *Journal of Geotechnical and Geoenvironmental Engineering*, 139(9):1512–1527, 2013.
- [34] J. Ai, J.-F. Chen, J. M. Rotter, and J. Y. Ooi. Assessment of rolling resistance models in discrete element simulations. *Powder Technology*, 206(3):269–282, 2011.
- [35] A.K. Ashmawy, B. Sukumaran, and A. V. Hoang. Evaluating the influence of particle shape on liquefaction behavior using discrete element method. In *Proceedings of the thirteenth international offshore and polar engineering conference (ISOPE 2003) Honolulu, Hawaii*, May 2003.
- [36] X. Garcia, J.-P. Latham, J. Xiang, and J.P. Harrison. A clustered overlapping sphere algorithm to represent real particles in discrete element modelling. *Géotechnique*, 59:779–784, 2009.
- [37] P. A. Cundall. Formulation of a three-dimensional distinct element model - Part I: A scheme to detect and represent contacts in a system composed of many polyhedral blocks. *International Journal of Rock Mechanics and Mining Sciences*, 25(3):107–116, 1988.
- [38] D. Zhao, E. G. Nezami, Y. M. A. Hashash, and J. Ghaboussi. Three-dimensional discrete element simulation for granular materials. *Engineering Computations*, 23:749–770, 2006.
- [39] F. Alonso-Marroquin and H. J. Herrmann. Calculation of the incremental stress-strain relation of a polygonal packing. *Physical Review E*, 66:021301, 2002.
- [40] F. Dubois and M. Jean. The non smooth contact dynamic method: recent LMG90 software developments and application. In Peter Wriggers and Udo Nackenhorst, editors, *Analysis and Simulation of Contact Problems*, volume 27 of *Lecture Notes in*

Applied and Computational Mechanics, pages 375–378. Springer Berlin Heidelberg, 2006.

- [41] A. A. Peña, P. G. Lind, and H. J. Herrmann. Modeling slow deformation of polygonal particles using DEM. *Particuology*, 6:506 – 514, 2008.
- [42] C. Ericson. *Real-Time Collision Detection (The Morgan Kaufmann Series in Interactive 3-D Technology) (The Morgan Kaufmann Series in Interactive 3D Technology)*. Morgan Kaufmann Publishers Inc., San Francisco, CA, USA, 2004.
- [43] L. Pournin and T. M. Liebling. A generalization of distinct element method to three dimensional particles with complex shapes. In *Proceedings of Powders & Grains (Balkema, Leiden, 2005)*, 2005.
- [44] S. A. Galindo-Torres and D. M. Pedroso. Molecular dynamics simulations of complex-shaped particles using voronoi-based spheropolyhedra. *Phys. Rev. E*, 81:061303, 2010.
- [45] G. T. Houlsby. Potential particles: a method for modelling non-circular particles in DEM. *Computers & Geotechnics*, 36:953–959, 2009.
- [46] J. Harkness. Potential particles for the modelling of interlocking media in three dimensions. *International Journal for Numerical Methods in Engineering*, 80(12):1573–1594, 2009.
- [47] I. Vlahinić, E. Andò, G. Viggiani, and J. E. Andrade. Towards a more accurate characterization of granular media: extracting quantitative descriptors from tomographic images. *Granular Matter*, pages 1–13, 2013. doi:10.1007/s10035-013-0460-6.
- [48] T. J. R. Hughes, J. A. Cottrell, and Y. Bazilevs. Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement. *Computer Methods in Applied Mechanics and Engineering*, 194:4135–4195, 2005.
- [49] J. E. Andrade, K.-W. Lim, C. F. Avila, and I. Vlahinić. Granular element method for computational particle mechanics. *Computer Methods in Applied Mechanics and Engineering*, 241-244:262–274, 2012.

- [50] K.-W. Lim and J. E. Andrade. Granular element method for three-dimensional discrete element calculations. *International Journal for Numerical and Analytical Methods in Geomechanics*, 38(2):167–188, 2014.
- [51] K.-W. Lim, K. Krabbenhoft, and J. E. Andrade. A contact dynamics approach to the granular element method. *Computer Methods in Applied Mechanics and Engineering*, 268:557–573, 2014.
- [52] K.-W. Lim, K. Krabbenhoft, and J. E. Andrade. On the contact treatment of non-convex particles in the granular element method. *Computational Particle Mechanics*, 1(3):257–275, 2014.
- [53] R. Hart, P. A. Cundall, and J. Lemos. Formulation of a three-dimensional distinct element model - Part II: Mechanical calculations for motion and interaction of a system composed of many polyhedral blocks. *International Journal of Rock Mechanics and Mining Sciences*, 25(3):117–125, 1988.
- [54] O. R. Walton and R. L. Braun. Simulation of rotary-drum and repose tests for frictional spheres and rigid sphere clusters. *DOE/NSF Workshop on Flow of Particulates*, pages 1–17, 1993.
- [55] D. J. Evans and S. Murad. Singularity free algorithm for molecular dynamics simulation of rigid polyatomics. *Molecular Physics*, 34(2):327–331, 1977.
- [56] H. Goldstein, C. P. Poole, and J. L. Safko. *Classical Mechanics (3rd Edition)*. Addison Wesley, 3rd edition, 2001.
- [57] X. Tu and J. E. Andrade. Criteria for static equilibrium in particulate mechanics computations. *International Journal for Numerical Methods in Engineering*, 75:1581–1606, 2008.
- [58] H. Hinrichsen and D. E. Wolf. *The Physics of Granular Media*. Wiley-VCH, Weinheim, 2004.
- [59] P. Lötstedt. Coulomb friction in two-dimensional rigid body systems. *ZAMM - Journal of Applied Mathematics and Mechanics / Zeitschrift für Angewandte Mathematik und Mechanik*, 61(12):605–615, 1981.

- [60] P. Lötstedt. Mechanical systems of rigid bodies subject to unilateral constraints. *SIAM Journal on Applied Mathematics*, 42(2):pp. 281–296, 1982.
- [61] J. J. Moreau. Some numerical methods in multibody dynamics: application to granular materials. *Eur. J. Mech. A Solids*, 13:93–114, 1994.
- [62] M. Jean. The non-smooth contact dynamics method. *Computer Methods in Applied mechanics and Engineering*, 177(3-4):235–257, 1999.
- [63] J. C. Simo and T. J. R. Hughes. *Computational Inelasticity*. Springer-Verlag, New-York, 1998.
- [64] P. Lötstedt. Numerical simulation of time-dependent contact and friction problems in rigid body mechanics. *SIAM Journal on Scientific and Statistical Computing*, 5(2):370–393, 1984.
- [65] D. Baraff. Coping with friction for non-penetrating rigid body simulation. *ACM SIGGRAPH Computer Graphics*, 25(4):31–40, 1991.
- [66] D. Baraff. Fast contact force computation for nonpenetrating rigid bodies. In *SIGGRAPH '94 Conference Proceedings*, pages 23–32. ACM, 1994.
- [67] D. Stewart and J. C. Trinkle. An implicit time-stepping scheme for rigid body dynamics with coulomb friction. *International Journal for Numerical Methods in Engineering*, 39:2673–2691, 1996.
- [68] M. Anitescu and F. A. Potra. Formulating dynamic multi-rigid-body contact problems with friction as solvable linear complementarity problems. *Nonlinear Dynamics*, 14:231–247, 1997.
- [69] C. E. Lemke. Bimatrix equilibrium points and mathematical programming. *Management Science*, 11(7):pp. 681–689, 1965.
- [70] S. Boyd and L. Vandenberghe. *Convex Optimization*. Cambridge University Press, New York, NY, USA, 2004.
- [71] S. C. Billups, S. P. Dirkse, and M. C. Ferris. A comparison of large scale mixed complementarity problem solvers. *Computational Optimization and Applications*, 7(1):3–25, 1997.

- [72] K. Krabbenhoft, A.V. Lyamin, J. Huang, and M. Vicente da Silva. Granular contact dynamics using mathematical programming methods. *Computers and Geotechnics*, 241-244:262–274, 2012.
- [73] C. Noguier-Lehon, B. Cambou, and E. Vincens. Influence of particle shape and angularity on the behaviour of granular materials: a numerical analysis. *International Journal for Numerical and Analytical Methods in Geomechanics*, 27(14):1207–1226, 2003.
- [74] S. McNamara and H. Herrmann. Measurement of indeterminacy in packings of perfectly rigid disks. *Phys. Rev. E*, 70:061303, Dec 2004.
- [75] M. Renouf, F. Dubois, and P. Alart. A parallel version of the non smooth contact dynamics algorithm applied to the simulation of granular media. *Journal of Computational and Applied Mathematics*, 168(12):375 – 382, 2004.
- [76] L. Staron and E. J. Hinch. Study of the collapse of granular columns using two-dimensional discrete-grain simulation. *Journal of Fluid Mechanics*, 545:1–27, 2005.
- [77] A. Taboada, K.-J. Chang, F. Radjai, and F. Bouchette. Rheology, force transmission, and shear instabilities in frictional granular media from biaxial numerical tests using the contact dynamics method. *Journal of Geophysical Research: Solid Earth*, 110(B9), 2005.
- [78] G. Saussine, C. Cholet, P.E. Gautier, F. Dubois, C. Bohatier, and J.J. Moreau. Modelling ballast behaviour under dynamic loading. part 1: A 2d polygonal discrete element method approach. *Computer Methods in Applied Mechanics and Engineering*, 195:2841 – 2859, 2006.
- [79] A. Ries, D. E. Wolf, and T. Unger. Shear zones in granular media: Three-dimensional contact dynamics simulation. *Phys. Rev. E*, 76:051301, Nov 2007.
- [80] L. Staron and E. J. Hinch. The spreading of a granular mass: role of grain properties and initial conditions. *Granular Matter*, 9:205–217, 2007.
- [81] R. Farhang and R. Vincent. Contact dynamics as a nonsmooth discrete element method. *Mechanics of Materials*, 41(6):715 – 728, 2009.

- [82] N. Estrada, E. Azéma, F. Radjai, and A. Taboada. Identification of rolling resistance as a shape parameter in sheared granular media. *Phys. Rev. E*, 84:011306, Jul 2011.
- [83] P.-Y. Lagree, L. Staron, and S. Popinet. The granular column collapse as a continuum: validity of a two-dimensional Navier-Stokes model with a $\mu(I)$ -rheology. *Journal of Fluid Mechanics*, 686:378–408, 2011.
- [84] L. Piegl and W. Tiller. *The NURBS book (2nd ed.)*. Springer-Verlag New York, Inc., New York, NY, USA, 1997.
- [85] C.-K. Shene. CS3621 Introduction to Computing with Geometry. <http://www.cs.mtu.edu/~shene/COURSES/cs3621/NOTES/>, July 2011.
- [86] J. Lowther J. Fisher and C.-K. Shene. If you know B-Splines well, you also know NURBS! *SIGCSE Bull.*, 36:343–347, March 2004.
- [87] M. G. Cox. The numerical evaluation of B-splines. Technical report, National Physics Laboratory DNAC 4, 1971.
- [88] C. De Boor. On calculation with B-splines. *Journal of Approximation Theory*, 6:50–62, 1972.
- [89] Robert McNeel & Associates. Rhinoceros (Rhino). <http://www.rhino3d.com/>.
- [90] B. Saint-Cyr, J.-Y. Delenne, C. Voivret, F. Radjai, and P. Sornay. Rheology of granular materials composed of nonconvex particles. *Phys. Rev. E*, 84:041302, 2011.
- [91] F. Ludewig and N. Vandewalle. Strong interlocking of nonconvex particles in random packings. *Phys. Rev. E*, 85:051307, 2012.
- [92] T.A. Laursen. *Computational Contact and Impact Mechanics: Fundamentals of Modeling Interfacial Phenomena in Nonlinear Finite Element Analysis*. Springer, Berlin, 2002.
- [93] I. Temizer, P. Wriggers, and T.J.R. Hughes. Contact treatment in isogeometric analysis with NURBS. *Computer Methods in Applied Mechanics and Engineering*, 200(9 - 12):1100 – 1112, 2011.

- [94] D. R. Jones, C. D. Perttunen, and B. E. Stuckman. Lipschitzian optimization without the lipschitz constant. *Journal of Optimization Theory and Applications*, 79(1):157–181, 1993.
- [95] R. L. Graham. An efficient algorithm for determining the convex hull of a finite planar set. *Information processing letters*, 1(4):132–133, 1972.
- [96] J. M. Gablonsky. An implementation of the DIRECT algorithm, technical report crsc-tr98-29. Technical report, Center for Research in Scientific Computation, North Carolina State University, Raleigh, NC, 1998.
- [97] C. T. Kelley. *Iterative Methods for Optimization, number 18 in Frontiers in Applied Mathematics*. SIAM, Philadelphia, 1999.
- [98] J. Nocedal and S. J. Wright. *Numerical Optimization*. Springer, New York, USA, 2nd edition, 2006.
- [99] K. Krabbenhoft, A.V. Lyamin, J. Huang, and M. Vicente da Silva. Granular contact dynamics with particle elasticity. *Granular Matter*, 14:607–619, 2012.
- [100] J.F. Sturm. SeDuMi 1.02, a MATLAB toolbox for optimization over symmetric cones. *Optimization Methods and Software*, 11–12:625–653, 1999.
- [101] MOSEK ApS. MOSEK. <http://www.mosek.com/>.
- [102] J. J. Moreau. Bounded variation in time. In *Moreau, J.J.; Panagiotopoulos, P.D.; Strang, G (eds.) Topics in Nonsmooth Mechanics*, volume 1, pages 1–74. Basel-Boston-Stuttgart: Birkhauser Verlag, 1987.
- [103] J. J. Moreau. Unilateral contact and dry friction in finite freedom dynamics. In *Moreau, J.J.; Panagiotopoulos, P. (eds.) Nonsmooth Mechanics and Applications, CISM Courses and Lectures*, volume 302, pages 1–82. Springer, Berlin, 1988.
- [104] J. Huang, M. Vicente da Silva, and K. Krabbenhoft. Three-dimensional granular contact dynamics with rolling resistance. *Computers and Geotechnics*, In Press, 2012.
- [105] W. L. Wood. *Practical Time-Stepping Schemes*. Oxford University Press, Oxford, 1990.

- [106] P. Souloumiac, Y.M. Leroy, B. Maillot, and K. Krabbenhoft. Predicting stress distributions in fold-thrust and accretionary wedges by optimisation. *J. Geophys. Res.*, 114, B09404, 2009.
- [107] P. Souloumiac, K. Krabbenhoft, Y.M. Leroy, and B. Maillot. Failure in accretionary wedges with the maximum strength theorem. *Comput. Geosci.*, 19:793–811, 2010.
- [108] S. J. Wright. *Primal-dual interior-point methods*. Society for Industrial and Applied Mathematics, Philadelphia, PA, USA, 1997.
- [109] F. Radjai, L. Brendel, and S. Roux. Nonsmoothness, indeterminacy, and friction in two-dimensional arrays of rigid particles. *Phys. Rev. E*, 54:861–873, 1996.
- [110] J. H. Snoeijer, T. J. H. Vlugt, M. van Hecke, and W. van Saarloos. Force network ensemble: A new approach to static granular matter. *Phys. Rev. Lett.*, 92:054302, 2004.
- [111] S. McNamara, R. García-Rojo, and H. Herrmann. Indeterminacy and the onset of motion in a simple granular packing. *Phys. Rev. E*, 72:021304, 2005.
- [112] T. Unger, J. Kertész, and D. E. Wolf. Force indeterminacy in the jammed state of hard disks. *Phys. Rev. Lett.*, 94:178001, 2005.
- [113] R. D. Mindlin. Compliance of elastic bodies in contact. *Journal of Applied Mechanics*, 71:259–268, 1949.
- [114] R. Phillips. *Crystals, Defects and Microstructures: Modeling Across Scales*. Cambridge University Press, 2001.
- [115] M. Oda. Initial fabrics and their relations to mechanical properties of granular materials. *Soils and Foundations*, 12:17–36, 1972.
- [116] A. L. Rechenmacher and R. J. Finno. Digital image correlation to evaluate shear banding in dilative sands. *Geotechnical Testing Journal, ASCE*, 27:1–10, 2004.
- [117] K.A. Alshibli and A. Hasan. Spatial variation of void ratio and shear band thickness in sand using x-ray computed tomography. *Géotechnique*, 58(4):249–257, 2008.

- [118] E. Andò, S. A. Hall, G. Viggiani, J. Desrues, and P. Besuelle. Grain-scale experimental investigation of localised deformation in sand: a discrete particle tracking approach. *Acta Geotechnica*, 7(1):1–13, 2012.
- [119] E. J. Garboczi. Three-dimensional mathematical analysis of particle shape using X-ray tomography and spherical harmonics: Application to aggregates used in concrete. *Cement and Concrete Research*, 32(10):1621–1638, 2002.
- [120] M. Saadatfar, M. L. Turner, C. H. Arns, H. Averdunk, T. J. Senden, A. P. Sheppard, R. M. Sok, W. V. Pinczewski, J. Kelly, and M. A. Knackstedt. Rock fabric and texture from digital core analysis. In *SPWLA 46th Annual Logging Symposium, New Orleans, Louisiana, June*, pages 26–29, 2005.
- [121] E. T. Bowman, K. Soga, and W. Drummond. Particle shape characterisation using fourier descriptor analysis. *Géotechnique*, 51(6):545–554, 2001.
- [122] G. Mollon and J. Zhao. 3D generation of realistic granular samples based on random fields theory and Fourier shape descriptors. *Computer Methods in Applied Mechanics and Engineering*, in press.
- [123] E. Weinan. *Principles of multiscale modeling*. Cambridge University Press, 2011.
- [124] C. Wellmann, C. Lillie, and P. Wriggers. Homogenization of granular material modeled by a three-dimensional discrete element method. *Computers and Geotechnics*, 35:394–405, 2007.
- [125] C. Dascalu and B. Cambou. Special issue on multiscale approaches to geomaterials. In C. Dascalau and B. Cambou, editors, *Acta Geotechnica*, volume 3. Springer, 2008.
- [126] H.A. Meier, P. Steinmann, and E. Kuhl. Towards multiscale computation of confined granular media - contact forces, stresses and tangent operators. *Technische Mechanik*, 28:32–43, 2008.
- [127] J. E. Andrade and X. Tu. Multiscale framework for behavior prediction in granular media. *Mechanics of Materials*, 41:652–669, 2009.

- [128] A. Koynov, I. Akseli, and A.M. Cuitino. Modeling and simulation of compact strength due to particle bonding using a hybrid discrete-continuum approach. *International Journal of Pharmaceutics*, 418:273–285, 2011.
- [129] R. A. Regueiro and B. Yan. Concurrent multiscale computational modeling for dense dry granular materials interfacing deformable solid bodies. *Springer Series in Geomechanics and Geoengineering*, pages 251–273, 2011.
- [130] T. K. Nguyen, G. Combe, D. Caillerie, and J. Desrues. FEM×DEM modelling of cohesive granular materials: Numerical homogenisation and multi-scale simulations. *Acta Geophysica*, 62(5):1109–1126, 2014.
- [131] N. Guo and J. Zhao. A coupled FEM/DEM approach for hierarchical multiscale modelling of granular media. *International Journal for Numerical Methods in Engineering*, 10.1002/nme.4702, 2014.
- [132] P. Soille. *Morphological Image Analysis, Principles and Applications*. Springer, Berlin, 2nd edition, 2004.
- [133] N. Lenoir, M. Bornert, J. Desrues, and G. Viggiani. 3D digital image correlation applied to X-ray microtomography images from triaxial compression tests on argillaceous rock. *Strain*, 43:193–205, 2007.
- [134] M. Babić. Discrete Particle Numerical Simulation of Granular Material Behavior. Technical Report 88-11, Department of Civil and Environmental Engineering, Clarkson University, 1988.
- [135] M. Anitescu. Optimization-based simulation of nonsmooth rigid multibody dynamics. *Mathematical Programming*, 105(1):113–143, 2006.
- [136] M. Anitescu and A. Tasora. An iterative approach for cone complementarity problems for nonsmooth dynamics. *Computational Optimization and Applications*, 47(2):207–235, 2010.
- [137] A. Tasora and M. Anitescu. A convex complementarity approach for simulating large granular flows. *Journal of Computational and Nonlinear Dynamics*, 5(3):031004, 2010.

- [138] A. Tasora and M. Anitescu. A matrix-free cone complementarity approach for solving large-scale, nonsmooth, rigid body dynamics. *Computer Methods in Applied Mechanics and Engineering*, 200(5):439–453, 2011.
- [139] A. Tasora and M. Anitescu. A complementarity-based rolling friction model for rigid contacts. *Meccanica*, 48(7):1643–1659, 2013.
- [140] C. Moler and C. V. Loan. Nineteen dubious ways to compute the exponential of a matrix, twenty-five years later. *SIAM Review*, 45(1):3–49, 2003.