OPTIMAL PROCUREMENT AND CONTRACTING WITH RESEARCH AND DEVELOPMENT

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DEDICATION

To my parents

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There is an ancient Chinese poem that goes

"White sun ends with the mountains.
Yellow River flows on into the sea.
To widen the ken of a thousand miles,
Up, up another flight of stairs."

白日依山尽
黄河入海流
欲穷千里目
更上一层楼

Baxter 338, Caltech

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ABSTRACT

Government procurement of a new good or service is a process that usually includes basic research, development, and production. Empirical evidences indicate that investments in research and development (R&D) before production are significant in many defense procurements. Thus, optimal procurement policy should not be only to select the most efficient producer, but also to induce the contractors to design the best product and to develop the best technology. It is difficult to apply the current economic theory of optimal procurement and contracting, which has emphasized production, but ignored R&D, to many cases of procurement.

In this thesis, I provide basic models of both R&D and production in the procurement process where a number of firms invest in private R&D and compete for a government contract. R&D is modeled as a stochastic cost-reduction process. The government is considered both as a profit-maximizer and a procurement cost-minimizer. In comparison to the literature, the following results derived from my models are significant. First, R&D matters in procurement contracting. When offering the optimal contract the government will be better off if it correctly takes into account costly private R&D investment. Second, competition matters. The optimal contract and the total equilibrium R&D expenditures vary with the number of firms. The government usually does not prefer infinite competition among firms. Instead, it prefers free entry of firms. Third, under a R&D technology with the constant marginal returns-to-scale, it is socially optimal to have only one firm to conduct all of the R&D and production. Fourth, in an independent private values environment with risk-neutral firms, an informed government should select one of four standard auction procedures with an appropriate announced reserve price, acting as if it does not have any private information.

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CHAPTER 1

PROCUREMENT CONTRACTING: AN OVERVIEW

1.1 PROCUREMENT: AN ECONOMIC APPROACH

Procurement of goods and services is a process of economic exchange. In this process, there is usually only one buyer, often the government (or government agency), but one or more potential suppliers. The buyer is interested in selecting an efficient supplier to provide goods or services with some specified technological requirements at the lowest cost. Defense procurements are examples in which the Department of Defense purchases military weapons systems from private companies. The key features in these procurements are the degree of technological requirements and the related risk and uncertainty in research, development, and production processes. Peck and Scherer (1962) and Scherer (1964) offer many examples of defense procurements and describe their key features in detail.

This type of exchange is different from exchange in traditionally perfect competitive and complete markets. First of all, the transaction occurs among a small number of parties (one buyer and a few suppliers), in which one party's action will have a positive effect on the prices of the goods being transacted. On the one hand, the government is generally the sole buyer, so that it has the bargaining power of a monopsonist. On the other hand, the supply curve is not as well defined as that in an impersonal market system. There are many situations where the goods (e.g., new weapons) being procured are unique so that only one supplier is necessary. The selected supplier then also has some bargaining power as a monopolist. Since it is very costly to develop a new weapon, for instance, the government usually pays a large amount of money to the supplier up front. In the later stage of the development, the supplier may simply say that he cannot complete the goods without further funds. The government can either negotiate a settlement with the supplier or lose what it has already spent. Therefore, prices in government procurement transactions are not determined as in a competitive market.

Second, due to the uncertain nature of the goods to be procured, investment in research and development (R&D) before production is necessary. In defense procurements, especially, both the government and suppliers finance a large amount of money up front. Some of the investments are specific to government procurements and hence are relationship-specific types as argued by Williamson (1985). The government and suppliers may be linked for a long period of time and it may be difficult for the government to switch to competing suppliers. A better procurement policy should then govern such a long term relationship, encourage private investments in R&D, and improve the government's welfare.

Thus, exchange in government procurement cannot be performed through a traditionally conceived impersonal Arrow-Debreu market, but rather by bargaining and contracting. The economics of the exchange is then governed by the type of contract. Examples are fixed-price contracts or cost-plus contracts that are commonly used in government procurements. It is possible to design contracts to achieve certain desired outcomes subject to incentive and informational constraints. For instance, a Pareto optimal contract can be obtained by maximizing one party's expected utility subject to incentive and informational constraints and subject to the other party (or parties) receiving a minimum expected utility level. A better contract should also give the contractor enough incentive to invest in research and development and to limit production costs.

Based on the bargaining and contracting approaches, recent advances in the economic theory of optimal procurement have provided a basic framework for understanding many interesting features of procurement, of which I will give a brief review in the next section.[1] Three issues have been extensively discussed: The selection of the most efficient contractor, inducing the chosen contractor to produce in an efficient way, and efficiently allocating risk between the government and the contractor.

These theories of optimal procurement and contracting have emphasized the production of the goods being procured. However, the design of the products and the development of the technologies that produce them have been ignored in the literature. Procurement of a new good, *as a process*, usually includes not just the production stage, but also the basic research and development stage. As we will see in Section 1.3, investments in research and development (R&D) before production are very significant in defense procurements. Optimal procurement policy should not only select the most efficient producer, but also induce the contractors to design the best product and develop the best technology. It is difficult to simply apply the current theory without considering R&D to many actual cases of defense procurements. A more general theory that links the different stages of the whole procurement process is needed. My thesis develops a unified economic theory of optimal procurement that covers the research, development, and production processes.

1.2 PROCUREMENT CONTRACTS IN PRINCIPAL-AGENT MODELS

Contracts, such as insurance contracts, labor contracts, production contracts, and even incomplete contracts, have been extensively studied by economists. Hart and Holmstrom (1985) offer a nice survey on the economic theory of contracts. As they argue, the rapid development of this theory "is partly a reaction to our rather thorough understanding of the standard theory of perfect competition under complete markets, but more importantly to the resulting realization that this paradigm is insufficient to accommodate a number of important economic phenomena."

Many situations of economic exchange can be characterized by a principal-agent relationship. The agent usually chooses an action that affects the welfare of both the agent and the principal. The principal sets a rule or a contract that, for instance, specifies the fee to be paid to the agent. There are three important phenomena that appear in determining the terms of the contract between the principal and the agent: 1) *adverse selection*; 2) *moral hazard*, and 3) *risk-sharing*. The first two effects result from asymmetry of information (hidden information and hidden action) between the principal and agent.[2] The latter is due to the nature of uncertainty and the parties' risk aversions. These three effects have been the subject of a large number of studies and commonly recognized by economists.[3]

Government procurement can also be simply described as a principal-agent relationship, in which the government is the principal and the suppliers are the agents. More than one agent may be involved. The government cannot directly observe any supplier's level of productivity or expected production costs. Thus, it does not know which supplier is more efficient. This hidden information causes an adverse selection problem. On the other hand, after a contractor is chosen, the government is unable to observe how much effort the contractor is making to reduce production costs during the production process. This hidden action (effort) may result in a moral hazard phenomenon. Furthermore, if the contractor is risk averse, there is an optimal risk sharing problem as shown by Cummins (1977) in national defense contracting. It is in the government's interest to offer a contract in which the government bears some of the risk of unpredictable costs.

In procurement contracting, however, there exists another phenomenon that may not be so significant in other contracting issues. That is, there is a significant competition effect in procurement contracting.

4) *Bidding-competition effect*:

In the provisions of many goods to be procured, usually, there exist economies of scale, so that efficient production requires only one supplier. When several suppliers are possible candidates to develop and build the goods, it has been argued, first by Demsetz (1968) in the natural monopoly context, that competitive bidding should be used.[4] When offering a procurement contract the government usually calls for bids from a number of potential suppliers and selects the lowest bidder. The rules of contracting are closely related to the initial competition for the contract. A better contracting rule may encourage initial participation in the bidding, and more competition may improve the government's welfare. Holt (1980) discussed the competition for procurement contracts to provide a unit of the good under different auction procedures. He focused on the effect of changes in procurement procedures and the number of bidders on expected procurement costs. In the case of risk averse suppliers, the expected procurement cost is lower in a sealed-bid auction than in an English auction. In both bidding procedures, more competition helps the government reduce the expected procurement cost.

When the government demands many units of the good, Riordan and Sappington (1987) and Dasgupta and Spulber (1989) discovered that the optimal quantity schedule for the government, not the price, is independent of the number of competing firms. This is a nice feature of variable-quantity procurements. The more firms that participate in the bidding process, the lower the expected procurement costs. Once again, more competition reduces the total cost of procurements. Therefore, competition is good for the government. These results may not hold when the firms have to search for information about production costs.

Competitive bidding for a contract is generally viewed as a way to elicit accurate information about suppliers' costs when each supplier is privately informed about its production cost. For instance, if a Vickrey auction procedure is used, it is a dominant strategy for each supplier to submit its true production cost. Competition certainly helps the government to select the most efficient supplier and hence to lower the expected procurement cost. If it is possible for the supplier to exert some effort to limit production costs in the production stage, however, moral hazard occurs and competition may not give the winning supplier enough incentive to reduce production costs. It has been shown by McAfee and McMillan (1986) that there is a tradeoff between stimulating more competition in the initial bidding stage and giving the winning contractor more incentives to reduce costs in the production stage. They also showed that, in determining the optimal linear form of contracts, this bidding-competition effect works in the same direction as the risk-sharing effect and, together, these two trade off against the moral hazard effect. The bidding-competition effect is positive for all finite numbers of competing suppliers and vanishes as the number of suppliers goes to infinity.

When the production cost is ex post observable to the government, it can be used for contracting. McAfee and McMillan (1987a) and Laffont and Tirole (1987) showed that if the suppliers are risk neutral then the optimal contract should take the linear form. That is, the optimal incentive contract is linear both in the ex ante predicted cost and in the ex post observed cost. This is a combination of fixed-price contract and cost-plus contract, which overcomes both the hidden information and the hidden action problem.

1.3 EMPIRICAL OBSERVATIONS AND FAILURE OF THE THEORY

Previous contractual approaches take government procurement as a standard principal-agent relationship and catch some interesting features of procurements. But they greatly simplify the problem of *procurement as a process*. At the beginning of many procurements, neither the government nor the potential contractors have much information about the goods to be procured or the technology that produces them. The government either must find this information by itself or ask the contractors to find it. For example, United States defense acquisitions are generally characterized by the following three-stage process: concept design (or research), development, and production. These three stages are closely related. Usually, many firms invest for concept design, a few firms are chosen to be the developers, and then one firm is selected to be the supplier of the goods. During these stages, each interested firm invests lots of money in research and development (R&D).

According to the Independent R&D budget data published by the Department of Defense, a significant fraction of company-sponsored R&D is procurement-related and largely defense-related. An example from Lichtenberg (1988) showed that in 1983 major defense contractors reported having spent \$3.9 billion dollars in IR&D costs, which is 9.2 percent of company-funded R&D (\$42.6 billion dollars) in that year. Contractors were reimbursed for \$1.6 billion dollars of this expenditure by DOD and NASA. Thus, the private contractors actually spent \$2.3 billion on the military projects. In other words, 60 percent of total R&D expenditures were from contractors' own funds.[5]

Most of the time, the government is buying not only the goods, but also information about the goods and the technology. An important feature of defense procurements is that they often generate information for the government about improving the technical performance and lowering the manufacturing cost. As Lichtenberg found, a $1 increase in competitive procurement is estimated to induce 54 cents of additional private R&D investment. In other words, competitive procurement stimulates considerable private R&D investment. Thus, private R&D investments are significant in defense procurements. Meanwhile, since R&D is extremely uncertain and very costly, only a small number of firms can afford it. Some firms even drop out during the R&D process. Overall, the following three issues are significant and important in government procurements: R&D, especially precontract R&D, a small number of bidders, and the government's information about its demand.

(i) Precontract R&D:

In order to compete with rival bidders, potential contractors actually conduct some R&D before contracting, to collect information about demand and technology, to reduce production costs, or to develop better products. An example is the government procurement of a new generation of jet fighters, one of the few big weapons contracts of the 1990's.[6] There are two teams, one led by the Northrop Corporation with the partner McDonnell Douglas, the other by the Lockheed Corporation including partners Boeing and General Dynamics. The two teams compete for a contract to build new jet fighters. The Air Force is paying $691 million dollars to the two teams that will each design and build two prototypes. In fact, each team has spent almost as much of its own money as the Air Force's in the program, more than $600 million dollars. This is considered one of the riskiest competitions ever. The example indicates that private R&D investments before contracting are significant and that they should certainly affect the government design of the optimal production contract.[7]

Given this precontract R&D behavior, should the procurement contract be different from that without R&D? For instance, suppose the government offers a cost-plus contract, should it cover R&D costs? If the R&D costs are covered by the government contract, the contractor may not have an incentive to lower R&D costs. On the other hand, if the R&D costs are not covered, the contractor may not have enough incentive to invest in R&D because R&D is costly. Perhaps the solution would be for the government to subsidize part of the cost of private R&D investment through procurements. The question is: what should be the optimal procurement contract in the presence of precontract R&D, and does competitive procurement encourage private R&D and lower the costs to the government?

(ii) A small number of bidders:

It has been argued by Besen and Terasawa (1987) that in many important cases of defense procurement the assumption that there are numerous bidders competing for a procurement contract may be a poor one. There are some situations in which only one or a small number of suppliers are plausible candidates to the prime contractor. In other words, only a small number of firms find the procurement contract interesting or profitable given the current technology and other available information. Because of the high degree of uncertainty and the high costs of precontract R&D, some firms cannot afford to invest in R&D and to make bids. In the example of government procurement of new jet fighters above, only two joint venture teams of defense contractors are interested in the development of the projects and competing for the contract.[8]

The number of bidders in competitive procurements may depend on the R&D

technology and the contracting rules that the government offers. After observing the contracting rules, each firm makes the following decisions: whether to invest in R&D, how much to spend, whether to submit a bid, and what to bid. These decisions are simultaneously determined and depend on the R&D process, the costs of R&D, and the type of competitive procedure. Several questions are of interest: How is the number of informed firms determined under free entry and what policies encourage participation in R&D and bidding? Is free entry of firms an optimal policy for the buyer and for society?

(iii) Informed buyer:

Another important issue that has been ignored in the procurement literature is that the government may be well informed about its demand. Very often the government knows how many units of the goods it is going to purchase and how much it is willing to pay. This private information can be used strategically by the government in its design of procurement contracts. Also, if certain information is valuable to the government, it may invest in R&D and collect the information by itself. It would be important to understand how the government should use its demand information and whether it should collect other valuable information.

Wilson (1977) first discussed the information acquisition problem in a discriminatory sealed-bid auction model with common values. In his model, each bidder obtains, exogenously and costlessly, private information in the form of a realization of a random variable that is correlated with the value of the object. He showed that the random sale price converges almost surely to the object's true value as the number of bidders becomes arbitrarily large. That is, bidding can serve as a basis for competitive price formation. Milgrom (1979) generalized this result by offering some necessary and sufficient conditions. Matthews (1984) considered the amount of information obtained as a choice variable and proved that an equilibrium sale price converges to the true value if and only if small amounts of information are costless. But all these information acquisitions were not productive.

Rob (1986) modeled R&D activity in the procurement context as a seraching behavior. He actually emphasized the second-sourcing problem. In other words, he examined whether awarding the whole project to a single supplier is a good policy for the government. Suppose that the chosen contractor agrees to disclose the technological information that results from its R&D effort. This is the key assumption in his model. Then he showed that it is optimal for the government to award only a fraction of the project to a single supplier while the remainder is competitively purchased. Therefore, second-sourcing is optimal for the government. But some experimental results by Guler and Plott (1988) indicate that second-sourcing actually raises the costs to the government. The first winner will have less incentive to produce a cost-effective product. For one thing, the company will be reluctant to put a great deal of effort into developing new technology for potentially commercial use. In addition, the company does not want to release any technology information.

Guler (1990), in a recent study, compared two forms of sealed-bid auctions (first-price and second-price) when bidders make investment decisions prior to the auctioning of a fixed price production contract. He distinguished observability and unobservability of investment decisions and showed that the two auctions are equivalent under symmetric equilibrium, but not the same under asymmetric equilibria. He also found many interesting features of investment and bidding equilibria. Some experimental results are also reported.

1.4 RESOLUTION: A SUMMARY OF MY RESULTS

The rest of my thesis will be devoted to the analysis of the three significant issues that I described in Section 1.3. The main results are the following.

In Chapter 2, I provide a basic model of both R&D and production in procurement processes where an exogenously fixed number of firms invest in private R&D and compete for a government procurement contract. The R&D stage is modeled as a stochastic cost-reduction process. Since the R&D outcome is only observed by the firm, this model can actually be viewed as an endogenous hidden information (adverse selection) model. I characterize the incentive procurement contract that maximizes the government's expected welfare. Explicit consideration of the R&D process changes the standard results of contracts in several ways.

First, I consider the case of one firm that conducts R&D and produces. If R&D is costly and if the traditional Baron-Myerson (1982) contract is used, then the government will buy too little from the contractor and pay too little. Potential gains from the R&D and production process are not fully realized under the traditional contract. Raising the price paid encourages private R&D investment and also raises the government's expected welfare.

Second, the contractor earns positive expected profits under the optimal incentive contract. In other words, the government awards a production contract to the most innovative firm, which will allow the firm to earn positive economic profits at the production stage. It is such positive profits that encourage the firm to invest in costly R&D activity.

Third, in the case of many firms, the number of firms matters due to the endogeneity of R&D investments and the endogeneity of R&D rewards (production contract). Unlike the invariance results found by Riordan-Sappington (1987) and Sah-Stigliz (1987), the optimal procurement contract, the total equilibrium expenditure on R&D, and the pace of innovation depend on the number of competing firms in the industry.

Finally, the government prefers more than one firm to invest in private R&D and to bid for the production contract. Too much competition, however, may discourage private R&D investment and lead to a reduction in government welfare. The implication of my analysis in this chapter is that when offering a procurement contract the government gains if it correctly takes into account the firms' pre-contract costly R&D behavior.

In Chapter 3 I intend to explain why the number of bidders in many competitive procurements is small and how it is determined. I also study whether free-entry is an optimal policy both for the buyer and for society. The key features of the model in this chapter include pre-contract R&D, an endogenous number of firms, and a first-price sealed-bid procurement auction. The process is analyzed from the point of view of both the buyer and society. I also compare two types of R&D technologies: a fixed-scale process, either R&D is done or it is not, and a variable scale process with constant marginal returns to R&D expenditure.

I first characterize the unique symmetric perfect free-entry equilibrium that includes a number of active firms in R&D, an R&D investment strategy tuple, and a bidding strategy tuple. As the buyer's reservation price increases, both the equilibrium number of active firms and the total R&D investment increase. That is, raising the reservation price encourages participation both in R&D activity and in the bidding process, and also stimulates more R&D investments.

Meanwhile, the equilibrium number of active firms always decreases with the fixed cost of R&D. With a positive fixed cost of R&D, only a finite number of firms decide to invest in R&D. As this fixed cost approaches zero, there will be an infinite number of firms interested in R&D activity. If the fixed cost of R&D is very high then only a few firms find R&D and production profitable. This is why the number of bidders is very small in many cases of defense procurements.

I also find that, under the variable constant marginal returns of R&D technology, it is socially optimal for only one firm to do all of the R&D and production. However, since the buyer considers only his own costs of procurement, the buyer will prefer to allow free entry, and the number of firms will usually be larger than is socially optimal. For the fixed scale R&D technology, the buyer's choices are socially optimal if the buyer's opportunity cost is high (for example, when there are no substitutes available). On the other hand, if the opportunity cost is low, the buyer will choose a reservation price lower than the socially optimal value and a number of firms no larger than the socially optimal number. That distortion will be lower when the R&D cost is higher. These observations imply that the type of R&D technology plays an important role in determining optimal R&D and procurement policies both for the government and for society.

Since the buyer prefers a free-entry policy and since under free-entry the buyer ex ante expects to pay all of the R&D costs through the optimal production contract, it may be a good idea for him to do R&D himself. Meanwhile, the buyer may also have superior information about his demand. The question is how the buyer should use his private information when offering procurement contracts. This is a mechanism design problem by an informed principal that was first studied by Myerson (1983) in a general context.

For comparison with past literature, in Chapter 4 I study the optimal selling scheme for an informed seller instead of procurement with an informed buyer.[9] The qualitative results will hold in the procurement setting. I show that, in an independent private values model with many symmetric risk-neutral buyers, the seller should choose one of the four standard auction schemes with an appropriate preannounced reservation price just as if he is not privately informed. In other words, it is in the seller's interest to reveal his private information through an announced reservation price.

This result will also hold in the variable quantity bargaining model discussed by Spulber (1988). However, Vincent (1989) finds an example of common values model in which the auctioneer has an incentive to keep his private information secret. The seller never announces any reservation price in the auction. It still remains an open question under what conditions in a general model an informed principal should signal his private information in the mechanism design.

NOTES

1 Besen and Terasawa (1987) provide a nice survey on this literature of procurement and contracting and also discuss the difficulties of directly applying the lessions of these theories to the acquisitions of major weapons systems.

2 The terms *adverse selection* and *moral hazard* have been borrowed from the practice and theory of insurance. As Arrow (1985) argued, these terms are "really applicable only to special cases." He suggested to use more informative names: "hidden information" and "hidden action." In the hidden information models, the agent has made some observation, which the principal has not. The agent usually uses that information strategically in making decisions. In the hidden action models, the agent exerts some effort that cannot be observed and verified by the principal. Effort is a disutility to the agent, but has a value to the principal. These two types of models differ both in their economic implications and in their solution techniques.

3 For example, adverse selection effect has been studied in a market context by Akerlof (1970), Rothschild and Stiglitz (1976), and Maskin and Riley (1984a), in a regulatory policy design context by Baron and Myerson (1982), Baron and Besanko (1984), and Laffont and Tirole (1986), in the design of optimal auctions by Myerson (1981), Matthews (1983), and Maskin and Riley (1984b), and also in a general mechanism design literature. Moral hazard and risk-sharing effect have been studied by Ross (1973), Holmstrom (1979), and Shavell (1979), Grossman and Hart (1983), and surveyed by Hart and Holmstrom (1985).

4 Demsetz (1968) suggested that competitive bidding for the franchise to supply a good or service could solve the traditional natural monopoly problem. Since the

winning firm may charge a monopoly price, which will cause inefficient production, regulation on pricing policy is required. This argument is the basic idea of the regulatory mechanism design followed by Loeb and Magat (1979), and Baron and Myerson (1982). For a survey on the recent development, see Caillaud et al. (1988).

5 It should be possible to identify empirically how much money the private contractors spent on R&D investment before contracting and how much after contracting. Investments (effort) after the contract is signed have been studied in the literature of contract theory. Examples are Laffont and Tirole (1986, 1987) and McAfee and McMillan (1986, 1987a). Precontract R&D investments or information acquisition, however, have not been modelled and hence are not well understood by economists.

6 An article in the New York Times on December 27, 1989, reported this government procurement story.

7 Similar observations are also available in other contexts. Hendricks, Porter, and Boudreau (1987) studied federal auctions for leases on the Outer Continental Shelf from 1954 to 1969. They considered the first-price, sealed-bid, common values model of auctions. They found that potential firms do make decisions on how much information to collect before participating in competitive bidding.

8 Also, as Hendricks et al. (1987) indicated in the study of federal auctions, only a fraction of the set of potential bidders typically choose to submit bids in the auctions and the number of bidders is positively correlated with the value of a tract.

9 If the contractual commitment is not an issue and it is possible to specify a complete contract, then a procurement contract auction is the same as the standard

auction problem. As Spulber (1989) argued, however, the effectiveness of the auction mechanism is limited by the extent of the contractual commitment. Competitive bidding for contracts involve both a price and a promise to perform the services. He showed, in a simple model, that without proper incentives for performance, hidden information causes the auction to unravel.

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CHAPTER 2

INCENTIVE PROCUREMENT CONTRACTS WITH COSTLY R&D *

2.1 INTRODUCTION

It is commonly believed that there is a serious cost overrun problem with defense procurements in the United States when a sole supplier is chosen. It is also commonly believed that the introduction of competition among suppliers could generate substantial price reductions. For most defense procurements, a costly research and development (R&D) effort is required of the competitors before production of the product. After a competition, winning contractors may include their R&D costs in the price of the product, but losing contractors must absorb their R&D costs. Thus, requiring competition to reduce costs may actually discourage R&D and lead instead to higher costs. How do we deal with this problem? This chapter is concerned with the design of the optimal incentive procurement contract when R&D is an important prior condition to production.

Recent advances in the economic theory of optimal procurement mainly deal with only two issues. The first concerns the selection of the contractors to produce the item being procured at the lowest cost. The item could be manufacturing facilities, new weapons systems or electricity generation plants. The second concerns attempts to induce the chosen contractor to produce the item at the lowest cost. Asymmetry of information between the buyer (the government) and the potential firms is often emphasized. It is usually assumed that potential firms have some private information about production costs at the time they are chosen and that many of the decisions made at the production stage by the chosen contractor are too costly for the buyer to observe or audit. These decisions affect the final cost of production and hence influence both the buyer's and the contractor's welfare. Potential contractors are allowed to bid for an incentive contract or bid from a menu of incentive contracts. Since the nature of the item is known by both the buyer and the contractors in advance, incentive procurement contracts are usually designed either to minimize the buyer's total cost for a fixed level of output or to maximize the buyer's surplus when the purchase quantity is variable. In both cases the contracts also give the firms incentives to reveal their private information and/or make desirable decisions at the production stage. The earlier literature on this topic can be found in Demsetz (1968) and Loeb and Magat (1979). It was extended later on by Baron and Myerson (1982), Laffont and Tirole (1986), McAfee and McMillan (1986, 1987), and others. In particular, Riordan and Sappington (1987) and Dasgupta and Spulber (1989) have argued that because of competition, more efficient outcomes are possible than if there is only a single supplier. Research and development issues have not been analyzed in this literature on procurement. Besen and Terasawa (1987) have recently provided a selective survey of the literature.[1]

At the beginning of many procurements, neither the buyer nor the potential contractors have much information about either the technology or the item itself. Therefore, the buyer must find this information by himself or ask the contractors. For example, United States defense acquisitions are usually characterized by a three-stage process: concept design, development, and production. Most of the time, what the government is buying is not only the item itself, but also information about the item and the technology. Procurements generate information for the government about improving the technical performance and lowering the manufacturing cost. There is usually a tradeoff between encouraging production efficiency and encouraging R&D. These features of procurements make it very difficult to apply the current theory, which only considers the production stage, to many actual cases of defense procurements. We need to link the different stages of the procurement process and to investigate R&D behavior in procurements.

There exists a large literature on R&D races for a patent with a fixed rent in private markets (Loury (1979), Lee and Wilde (1980), Dasgupta and Stiglitz (1980),[2] and recently Sah and Stiglitz (1987)). Using stochastic racing models and dynamic game theoretic models, they emphasize the effects of market structure on private marginal returns to firms from innovations, and investigate the relationship between marginal private returns and social returns from innovations. In particular, Sah and Stiglitz (1987) have provided a set of conditions under which the total expenditure and the pace of innovation in an industry are invariant to the number of firms. These approaches cannot be applied to procurement cases directly because, in procurements, the government is able to manage and control the supplier's R&D behavior indirectly through the choice of the prize for innovation.[3] Traditional R&D models treat the prize as exogenous, but the government can offer a production contract as the prize for which the firms compete. The marginal private returns and social returns from R&D depend on the quantity to be procured, which is specified in the production contract. The results of R&D races in private markets may be changed in the procurement case.

We will link the R&D stage and the procurement stage and concentrate on the use of R&D to reduce production costs. The outcome of such R&D is information about the production technology. This information is stochastically related to the R&D effort. We first consider a principal-agent model of the procurement process in Section

2.2. The production stage is characterized by a linear-cost technology with an unknown marginal cost. The R&D outcome is the potential marginal cost that the firm can affect by exerting some effort. The procurement process is modeled as follows: first, the government announces a menu of production contracts and commits itself to offer the same contracts at the latter stage. Second, after observing the general contracts, the firm invests in R&D. Third, the firm observes the marginal cost outcome and selects a contract it prefers from the contract menu. Finally, the firm produces and gets paid according to the contract it has accepted. We have assumed that the government makes a full commitment before R&D. If the government can not make a credible commitment, both the firm and the government might have incentives to renegotiate after the R&D is done and change to different contracts. We do not consider the renegotiation issue in detail in this chapter.[4]

As a benchmark, the first-best solution is briefly discussed in Section 2.2.2. In Section 2.2.3, we discuss the nonobservability of the R&D investments and the R&D outcome by the government and characterize the incentive compatible procurement contracts. We show that under any incentive compatible contract, the larger the quantity procured, the more effort the firm exerts in its R&D activity. Thus, increasing the procured amount is one way to encourage R&D activities. Section 2.2.4 characterizes the optimal incentive procurement contract that maximizes the government's expected welfare. Moral hazard exists in this situation due to nonobservability of R&D investment by the government. R&D has an effect on the optimal production level opposite to the adverse selection effect. With costly R&D, the government offers a higher and steeper payment schedule, compared to the traditional Baron-Myerson (1982) contract where R&D is costless. Also, the optimal contract generates positive expected profits for the firm. It is such positive expected profits that encourage the firm to do R&D.

Section 2.3 extends the analysis to the case where there are many identical firms competing for procurement contracts. The potential R&D outcomes by different firms are assumed to be independent. We first discuss Nash equilibrium behavior with respect to R&D expenditures given an arbitrary incentive production contract. When the R&D technology exhibits constant marginal returns to expenditures, we find an invariance result similar to Sah and Stiglitz (1987). But when the R&D technology exhibits diminishing marginal returns to expenditures, we find that Sah and Stiglitz's invariance result does not hold and that more competition could reduce the potential marginal production cost to its lower bound. The specific nature of R&D technology plays an important role in determining the relationship between R&D and the structure of the industry.

We then characterize the optimal incentive production contract and the optimal level of R&D investments for the competition case in Section 2.3.3. When the R&D technology exhibits constant marginal returns to expenditures, we show that the optimum procurement quantity schedule is dependent on the number of competing firms. This contrasts with Dasgupta and Spulber (1989) and Riordan and Sappington (1987) who find that the optimal quantity schedule does not depend on the number of the firms. No R&D behavior was considered in their models. We also find that total expenditure on R&D and the pace of innovation depend on the number of firms because of the buyer's control of the prize for innovation. Thus, Sah and Stiglitz's invariance result does not hold when the prize is endogenous. In other words, the number of firms really matters to the optimal procurement. We also find that the government prefers more than one firm to participate in private R&D activity and to bid for the production contract.

In summary, we have provided a model of both R&D and production in procurement processes where one or more firms invest in private R&D and compete for a government procurement contract. The optimal incentive procurement contract has been characterized to maximize the government's expected welfare. We have found the following interesting results: 1) If the traditional Baron-Myerson (1982) contract is used where there is costly R&D, the government buys too little from the contractor and pays too little. Raising the price paid encourages private R&D investment and raises the government's expected welfare. 2) The contractor earns positive expected profits under the optimal incentive contract. It is such positive profits that encourage the firm to invest in private R&D. 3) Unlike the invariance results found by Riordan and Sappington (1987) and Sah and Stiglitz (1989), the optimal incentive production contract, the total equilibrium expenditure on R&D, and the pace of innovation in our model depend on the number of competing firms in the industry. 4) The government prefers more than one firm to invest in private R&D and to bid for the production contract. But too much competition may discourage private R&D investment and lead to a reduction in government welfare.

Before proceeding to our formal analysis, we would like to discuss several recent papers that have tried to link R&D and production in procurements. Rob (1986) has included the R&D process in his model of procurement contracts by viewing R&D activity as searching behavior. The optimal stopping rule allows him to derive the average actual production cost, which depends on the unit searching cost and the cutoff level. The government prespecifies a quantity and price for the project to minimize its outlay on the project. The chosen contractor agrees to disclose the technological information that results from its R&D effort, which is the key assumption in his model. As a result, it is optimal for the government to award only a fraction of the project to a single supplier while the remainder is competitively purchased. This is called "educational" or "learning" buy. We will see in Section 2.3.1 that Rob's model of R&D behavior as a search process can be viewed as a special case of our model. The R&D technology in his paper exhibits constant marginal returns to expenditures.

Besen and Terasawa (1988) have also developed a simple and different model of research and development and production that captures some features of defense acquisitions. In their model, the level of technical performance and the amount of hardware to be procured are fixed. In addition, the target cost in the production contract will be the maximum level acceptable to the government. The contracts in both stages are linear functions of target cost and actual cost, and are exogenously given. They find that production will not be carried out efficiently, that cost overruns will be commonplace, and that contractors can be expected to incur losses during research and development. Our paper differs from theirs in that we will design the optimal incentive procurement contract instead of assuming a linear contract exogenously.

Dasgupta (1987) also considers a two-period procurement model with one buyer and n identical firms. Given a second-period sealed-bid auction, each firm chooses an investment level and the buyer chooses the reserve price simultaneously. At the Nash equilibrium, there is underinvestment relative to the social optimum (cooperative solution) because of "opportunistic" behavior. But for most defense procurements, the government and suppliers need not move simultaneously. In our model, we consider a Stackelberg game in which the government is the leader. In this case the government has indirect control of the supplier's R&D decisions through the optimal choice of production contracts.

2.2 A PRINCIPAL-AGENT MODEL

2.2.1 The Model

In this section, we consider only one firm and model (defense) procurement as a two-stage process. In the first stage, research and development (R&D) is conducted by a firm, but no goods are procured or acquired. The outcome of this stage is better knowledge about the cost of producing the item. In the second stage, the item is produced by the firm. The output of the item can be observed by both the firm and government. We assume that there is no uncertainty in the production stage.[5] We will refer to the government as the principal and the firm as the agent in this section.

The agent produces a good at a constant marginal cost that is unknown to both the agent and the principal in the first stage. But the agent can take an action x in the R&D process and find out what the marginal cost is, where x can be either money spent or a level of effort such as assigning the best engineers in the firm to this R&D project. Let $x \in [0, \bar{x}]$, where $\bar{x} > 0$ is the budget. The R&D outcome (marginal cost) is uncertain, so we represent it by a random variable Y. We assume that Y is generated according to the following production function:

$$Y = f(x, \Omega),$$

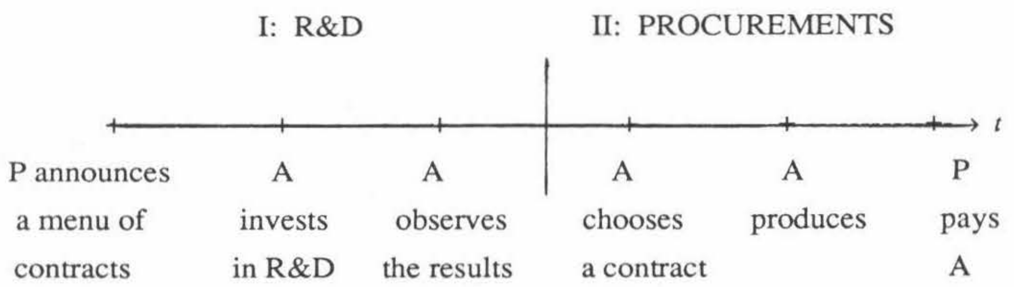
where Ω is a random variable with known support, $Pr(\Omega \leq \omega) = G(\omega)$, $f_1 \leq 0$, $f_{12} \neq 0$, $f_{11} \geq 0$. The realized R&D output is $y = f(x, \omega)$. The more effort the agent spends, the lower the marginal cost it may find.

Individual actions directed towards innovation cannot be observed by the principal and hence cannot be contracted upon. The agent may not have an incentive to invest much in the R&D stage. Thus, there may exist a moral hazard problem due to the incongruity of the incentives of the agent and the principal.

Both the principal and the agent are assumed to be risk-neutral. Thus the problem of risk-sharing is not an issue. The utility for the principal from procuring an amount Q and paying P will be $W = B(Q) - P$, where P and Q are nonnegative real variables and $B(Q)$ is the benefit function, $B'(Q) > 0$, $B''(Q) < 0$. The utility U for the agent is a function of the income R (or the profit $R = P - yQ$) and the level of effort x, which is assumed to be additively separable. That is, $U = R - cx$, where cx is the R&D cost,[6] $c \geq 0$ is a known constant. Everything except ω and x is common knowledge.

The timing of the game is the following. First, the principal announces a menu of general contracts and commits itself to offer the same contract at the latter stage. Second, after observing the general contracts, the agent invests in R&D. Third, the agent observes the R&D outcomes and selects a contract it prefers from the contract menu (announces its type). Finally, the agent produces and gets paid according to the contract it has accepted (see Figure 1).

Since it is too costly for the principal to observe or audit the R&D output, the agent may not want to submit true information about the R&D output. This asymmetry of information results in another incentive problem. By the revelation principle (see the Appendix), we can concentrate on the mechanisms for which the principal asks the agent to report its R&D result and which lead the agent to report its private information truthfully. Based on this marginal cost information, the principal will offer the agent a procurement contract that specifies an amount $Q(y)$ to be procured and a total payment $P(y)$ to the agent.



P — Principal A — Agent

FIGURE 1: The Timing of the Two-stage Game

To proceed formally, we introduce the following notation. Since Y is a random variable, let

$$H(y \mid x) \equiv Pr(Y \leq y) = \int_{f(x,\omega) \leq y} dG(\omega)$$

be the cumulative distribution of Y and $h(y \mid x) \equiv H_y(y \mid x)$ be the associated density with the support $[\underline{y}, \bar{y}]$, $0 \leq \underline{y} \leq \bar{y}$. We assume the support does not move with x. We also assume that for every $x \in (0, \bar{x})$, $H_x(y \mid x) > 0$ for all $y \in (\underline{y}, \bar{y})$, so that a change in x has a nontrivial effect on the distribution of y. Specifically, it will shift the distribution of y to the left in the sense of first-order stochastic dominance (see Figure 2). The more effort the agent spends, the higher the probability that the agent will find a marginal cost less than y.

It is also assumed that $H_{xx} < 0$ for all $x \in (0, \bar{x})$ and $y \in (\underline{y}, \bar{y})$, and that $H(\bar{y} \mid x) = 1$ for any x, so $\bar{y}$ represents the current common knowledge of the marginal cost. That is, both the agent and principal know that the good can be produced at the marginal cost $\bar{y}$. We also assume that $H(y \mid 0) = 0$ for all $y < \bar{y}$. If the agent does not spend any effort on R&D, the current marginal cost $\bar{y}$ won't be reduced. The principal can always procure $\bar{Q} \geq 0$ at the marginal cost $\bar{y}$ such that $B'(\bar{Q}) = \bar{y}$. The principal gets the total surplus $\bar{S} = B(\bar{Q}) - \bar{y}\,\bar{Q}$ while the agent earns zero. It is easy to see that $\bar{S} \geq 0$ since $B(Q)$ is concave function. We can assume $\bar{Q} > 0$ for simplicity.

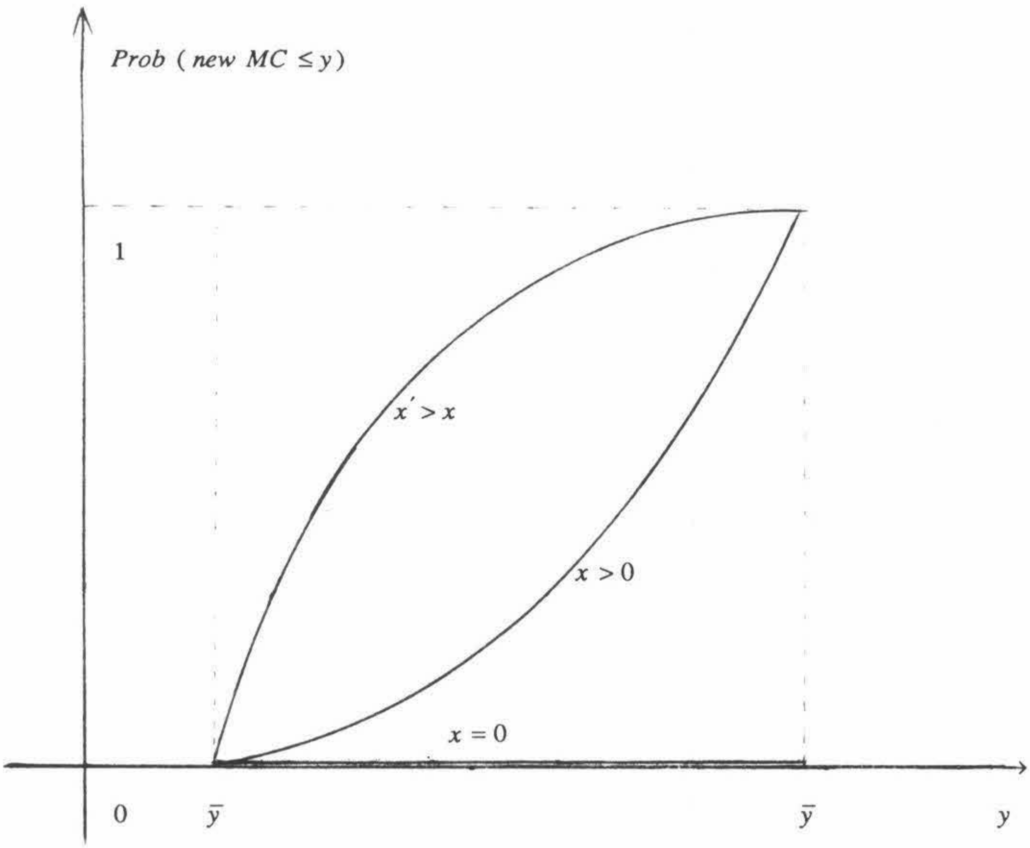


FIGURE 2: R&D Technology

An example of a distribution that satisfies the assumptions is $H(y \mid x) = 1 - (1-y)^x$, $y \in [0, 1]$, $x \in [0, \bar{x}]$. Let x be the integer number of times the agent draws its cost from a uniform distribution on $[0, 1]$, then $H(y \mid x)$ represents the distribution of the lowest-order statistic. That is, the agent chooses the lowest cost from the x drawings. In this case, the R&D process is like searching behavior and thus the optimal investment level x is the optimal stopping level of the drawing. This is similar to the optimal stopping approach of Rob (1986). Another example of a distribution that satisfies our assumptions is $H(y \mid x) = 1 - e^{-\alpha(x)y}$, $y \in [0, \infty)$, and $x \geq 0$, where $\alpha(x)$ is a positive, increasing, and concave function. Here, the R&D outcome is subject to an exponential distribution and the R&D technology exhibits decreasing marginal returns to expenditures. Since $E(Y \mid x) = 1/\alpha(x)$, the more effort the agent exerts, the lower the expected marginal cost the agent observes. This is the distribution of innovation that appears in the literature on stochastic R&D races (see Loury 1979, Reinganum 1988, and others). The difference is that in their models y represents the uncertain date at which the R&D project will be successfully completed.

2.2.2 The First-Best Solution

Before analyzing incentive procurement contracts, we look at the first-best solution. Suppose that the effort x is observable to both the principal and the agent, and the R&D outcome y can also be observed by both parties at the end of the R&D stage. A Pareto-optimum $[Q^*(y), x^*]$ can be computed by maximizing the principal's expected welfare given that the agent's expected profit is no less than a certain level $\bar{\pi}$. We assume $\bar{\pi} = 0$ without loss of generality. The first order conditions give the following equations:

$$B'(Q^*(y)) = y \tag{1}$$

$$\int_{\underline{y}}^{\bar{y}} Q^*(y) H_x(y \mid x^*) dy - c = 0. \tag{2}$$

Since the R&D output is observable, the principal can procure efficiently. That is, the procurement amount Q^* is chosen such that the marginal benefit equals the marginal cost. For this optimal quantity $Q^*(y)$, the marginal social benefit of investment x, which is $\int_{\underline{y}}^{\bar{y}} Q^*(y) H_x(y \mid x^*) dy$, equals the marginal cost of the investment x. Therefore, we have both production efficiency and investment efficiency when investment and R&D outcomes are observable to both parties. The optimal payment transfer $P^*(y)$ to the agent is determined such that the agent earns exactly zero profit. The principal gets the total surplus $\bar{S} + S^*$, where $S^* = \int_{\underline{y}}^{\bar{y}} Q^*(y) H(y \mid x^*) dy - cx^*$ is the surplus from the R&D under the efficient arrangement of production. We assume that S^* is positive, that is to say, the R&D is meaningful.

2.2.3 Unobservable R&D Investment and R&D Outcomes

It is a common phenomenon that R&D outcomes cannot be observed or verified by the principal directly. It is also costly for the principal to audit the agent to get this information. Thus, each agent has private information about the R&D outcome and reports this information strategically (e.g., the agent may not tell the truth). This results in an adverse selection problem. Also, the principal can not observe the agent's investment level. This results in a moral hazard problem.

If the principal only cares about efficiency, then he can delegate the production decision and R&D decision to the agent and allow the agent to keep the entire social surplus. Loeb and Magat (1979) have discussed this situation in detail without R&D. The result is also true in the case of costly R&D. The agent will choose the quantity and investment level for R&D to maximize the sum of its expected profits and the principal's expected welfare. Since both the agent and the principal are risk neutral, this full delegation results in the first-best solution. The principal does not get any benefit. This solution is not employed in practice for obvious political reasons.

If the principal's objective is to maximize its own expected welfare, the full delegation of decisions is not optimal. The principal should behave monoposonistically. An incentive procurement contract is needed to maximize the principal's expected welfare such that the agent wants to reveal its private information and to invest in R&D. By the revelation principle,[7] we need only to consider incentive compatible direct revelation mechanisms. Given a revelation contract $[Q(y), P(y)]$, the agent chooses effort x and gets a realized R&D output $y = f(x, \omega)$. Then the agent reports y', which depends on y: we can denote $y' = \phi(y)$. The expected utility for the agent if it chooses $\phi(y)$ and x is

$$EU(\phi, y; x) = \int_{\underline{y}}^{\bar{y}} \left[P(\phi(y)) - yQ(\phi(y)) \right] h(y \mid x) dy - cx,$$

where $R(y, y') = P(y') - yQ(y')$ is the profit for the agent from reporting y' given that y is the true R&D output. The agent will choose its R&D strategy x and its reporting strategy $\phi(y)$ to maximize its expected utility.

We can consider the agent's optimal choice of the reporting strategy $\phi(y)$ first. Given any x, the agent will choose ϕ to maximize the above expected utility subject to $\phi(y) \in [\underline{y}, \bar{y}]$ for any $y \in [\underline{y}, \bar{y}]$. We are interested in incentive compatible contracts $[Q(y), P(y)]$ that give the agent an incentive to report the true R&D outcome. That is, we want $\phi(y) = y$ for all y to be the agent's optimal strategy under this contract $[Q(y), P(y)]$. Solving this simple optimal control problem, we obtain the following

Lemma 1: A contract $[Q(y), P(y)]$ is incentive compatible if and only if $Q'(y) \le 0$ and $R'(y) = -Q(y)$ for all $y \in [\underline{y}, \bar{y}]$, where $R(y) = P(y) - yQ(y)$.

The proof is standard and is given in the Appendix. Lemma 1 offers a necessary and sufficient condition for a production contract $[Q(y), P(y)]$ to be incentive compatible. We can see that incentive compatibility requires the monotonicity of $Q(y)$ and $P(y)$. The lower the marginal cost the agent finds, the bigger the procurement amount it will produce, and the higher the payment it gets. We also have a decreasing, convex compensation rule $R(y)$. Reducing the marginal cost results in a greater profit share at an increasing rate. This gives the agent an incentive both to reduce the marginal cost and to report the true R&D outcome. From Lemma 1, we can also calculate the compensation rule $R(y) = R(\bar{y}) + \int_y^{\bar{y}} Q(\tilde{y})d\tilde{y}$ for any y. If $Q(y) \ge \tilde{Q}(y)$ for all y and $R(\bar{y}) \ge \tilde{R}(\bar{y})$, then $R(y) \ge \tilde{R}(y)$ for any $y < \bar{y}$. Thus, under an incentive contract, a higher procurement amount generally gives the agent a higher profit share.

We now consider the agent's optimal R&D strategy. Let $EU(x)$ be the agent's expected utility under the truthful reporting of the R&D output given the effort level x. That is,

$$EU(x) = \int_{\underline{y}}^{\bar{y}} R(y)h(y \mid x)dy - cx.$$

Integrating the right-hand side by parts and using Lemma 1, we get

$$EU(x) = R(\bar{y}) + \int_{\underline{y}}^{\bar{y}} Q(y)H(y \mid x)dy - cx.$$

At the beginning of the R&D stage, the agent chooses an investment level $\tilde{x} \in [0, \bar{x}]$ to maximize ex ante profit $EU(\tilde{x})$. That is, the optimal investment strategy x satisfies

$$x \in \text{argmax} \;\; R(\bar{y}) + \int_{\underline{y}}^{\bar{y}} Q(y)H(y \mid \tilde{x})dy - c\tilde{x}. \tag{3}$$

$\tilde{x} \in [0, \bar{x}]$

Assume that Inada's 'derivative conditions' are satisfied; that is, $\lim_{x \to 0} H_x(y \mid x) = +\infty$ and $\lim_{x \to \bar{x}} H_x(y \mid x) = 0$ for all $y \in (\underline{y}, \bar{y})$. Then for any $c > 0$ and $Q(y)$ such that $Q(y) \geq 0$ for all y and $Q(y) > 0$ for a nonzero measure subset of $[\underline{y}, \bar{y}]$, the solutions for (3) are interior solutions. The Inada conditions mean that there is a great potential to increase the probability of finding the marginal production cost less than y in the initial investment, and the increasing rate of that probability diminishes when the investment level reaches a certain upper bound $\bar{x}$. Thus, a certain level of investment between zero and the upper bound $\bar{x}$ is optimal for the agent.

For these interior solutions of (3), the first order condition gives

$$\int_{\underline{y}}^{\bar{y}} Q(y) H_x(y \mid x) dy - c = 0, \tag{4}$$

$x \in (0, \bar{x})$. Since we assume that $H_{xx} < 0$ for $y \in (\underline{y}, \bar{y})$, $EU(x)$ is strictly concave in x if $Q(y) \geq 0$ and $Q(y) \neq 0$ on a nonzero measure subset of $[\underline{y}, \bar{y}]$. The second order condition for (3) is satisfied and the interior solution of (4) is unique. Therefore, we can use the first order approach and substitute (4) for (3). In the extreme case where $c = 0$, the solution for (3) is the boundary $\bar{x}$. Remember $\bar{x}$ is known by the principal.

Before considering the optimal procurement contract, we discuss the effect of the quantity $Q(y)$ on the R&D strategy. Given $Q(y) \geq 0$ and $Q(y) \neq 0$, let $x^*(Q)$ be the solution for (4) and $EU = EU(x^*(Q))$ be the agent's expected utility under $[Q(y), x^*(Q)]$. Then we can show the following.

Lemma 2: Given any two incentive compatible contracts $[Q(y), P(y)]$ and $[\tilde{Q}(y), \tilde{P}(y)]$, suppose $Q(y) \geq \tilde{Q}(y) \geq 0$ for all $y \in [\underline{y}, \bar{y}]$. Then i) $x^*(Q) \geq x^*(\tilde{Q})$, and ii) if $R(\bar{y}) = \tilde{R}(\bar{y})$

then $EU \geq E\tilde{U}$.

The proof is by contradiction and is given in the Appendix. Lemma 2 implies that a bigger procurement project, in the sense of higher Q at every y, makes the agent invest more in the R&D process and earn a higher expected profit. The agent always prefers a larger sized project. Therefore, a simple way to encourage R&D is to increase the size of the project or to procure more from the same agent. Another obvious implication of Lemma 2 is that, relative to the first-best solution, underprocurement results in underinvestment in R&D.

2.2.4 The Optimal Incentive Procurement Contract

Now, we are ready to look at the principal's optimization problem and characterize optimal incentive procurement contracts when the principal's objective is to maximize its own expected welfare. The principal will choose $Q(y)$, $P(y)$, and x to maximize its expected welfare subject to the incentive compatibility constraint, the ex ante and interim individual rationality constraints, and the agent's R&D decision constraint. We will initially ignore the global incentive compatibility constraint $Q'(y) \leq 0$. If the final solution does not satisfy this global incentive constraint, we need to use Guesnerie and Laffont's (1984) technique to get the optimal incentive contract. We do not repeat their arguments here. Under the local incentive compatibility constraint, the payment $P(y)$ can be solved in terms of $Q(y)$ by using $P(y) = yQ(y) + R(y)$ and $R(y) = R(\bar{y}) + \int_y^{\bar{y}} Q(\tilde{y})d\tilde{y}$. Thus, the optimal incentive contract is determined by solving the following optimization problem:

$$(P) \quad \text{Max} \quad \int_{\underline{y}}^{\bar{y}} \left[B(Q(y)) - Q(y)\left[y + \frac{H(y \mid x)}{h(y \mid x)} \right] \right] h(y \mid x)dy - R(\bar{y})$$

$Q(y), R(\bar{y}), x$

$$s.t. \quad (IIR) \quad R(\bar{y}) + \int_{y}^{\bar{y}} Q(\tilde{y}) d\tilde{y} \geq 0$$

$$(R\&D) \quad \int_{\underline{y}}^{\bar{y}} Q(y) H_x(y \mid x) dy - c = 0 \tag{4}$$

$$(EIR) \quad \int_{\underline{y}}^{\bar{y}} Q(y) H(y \mid x) dy + R(\bar{y}) - cx \geq 0 \tag{5}$$

The principal will choose $R(\bar{y})$ as low as possible under the interim individual rationality constraint (IIR). In other words, $R(\bar{y}) = 0$. The agent earns zero profit if its marginal production cost is the publicly known $\bar{y}$. The principal offers a contract $[\bar{P}, \bar{Q}]$ and gets the social surplus $\bar{S} = B(\bar{Q}) - \bar{y}\,\bar{Q}$, where $\bar{P} = \bar{y}\,\bar{Q}$ and $\bar{Q}$ is determined by $B'(\bar{Q}) = \bar{y}$.

Since the objective function EW is concave and differentiable in $Q(y)$ and x, and since EU is linear in $Q(y)$, concave and differentiable in x, the sufficient conditions for (P) are satisfied (see Theorem 8.C.5 in Takayama 1985). Let $\hat{Q}(y)$, $R(\bar{y})$, and $\hat{x}$ be the solution to the optimization problem (P) and $E\hat{U} = \int_{\underline{y}}^{\bar{y}} \hat{Q}(y) H(y \mid \hat{x}) dy + R(\bar{y}) - c\hat{x}$ be the agent's expected profit under the optimum contract. Then, because of the (IIR) constraint, $\hat{R}(\bar{y}) = 0$. We can show the following:

Proposition 1: Suppose $c > 0$, then $0 < \hat{x} < \bar{x}$ and $E\hat{U} > 0$.

Proof: First, we want to show $\hat{x} > 0$. Suppose $\hat{x} = 0$. Then $E\hat{W} = \bar{S}$. We know $Q^*(y) > 0$ for all y and $Q^*(y)$ is decreasing in y. There exists at least one $\tilde{Q}(y) > 0$ such that $\tilde{Q}(y)$ is decreasing in y and a little lower than $Q^*(y)$ for $y < \bar{y}$ and the same as $Q^*(y)$ at $y = \bar{y}$. Then $B'(\tilde{Q}(y)) > B'(Q^*(y)) = y$ because $B(Q$ is strictly concave. Let $\tilde{x}$ satisfy $\int_{\underline{y}}^{\bar{y}} \tilde{Q}(y) H_x(y \mid \tilde{x}) dy = c0$ and $\int_{\underline{y}}^{\bar{y}} \tilde{Q}(y) H(y \mid \tilde{x}) dy - c\tilde{x} \geq 0$. Then $\tilde{x} > 0$ since $\lim_{x \to 0} H_x(y \mid x) = +\infty$. Thus,

$$E\tilde{W} = \bar{S} - \int_{\underline{y}}^{\bar{y}} \left[B'(\tilde{Q}(y)) - y \right] \tilde{Q}'(y) H(y \mid \bar{x}) dy > \bar{S} = E\hat{W}.$$

That is, the principal could choose $[\tilde{Q}(y), \bar{x}]$, which gives him higher welfare than $\bar{S}$. Thus, $\hat{x} = 0$ cannot be the optimal solution to (P). Therefore, $\hat{x} > 0$.

Second, we claim $\hat{Q}(y)$ is positive at least on a non-zero measure subset of $[\underline{y}, \bar{y}]$. Otherwise, we will have $\int_{\underline{y}}^{\bar{y}} \hat{Q}(y) H_x(y \mid \hat{x}) dy = 0$. Then from the $(R\&D)$ constraint, $c\hat{x} = 0$ and hence $c = 0$. This contradicts the assumption $c > 0$. Thus the claim holds. Furthermore, using Inada's 'derivative conditions' and the assumption $c > 0$, we know $x < \bar{x}$.

Finally, we can show $E\hat{U} > 0$. From the (EIR) constraint, $E\hat{U} \geq 0$. We only need to show $E\hat{U} \neq 0$. Suppose $E\hat{U} = 0$. Let $\delta(x) = \int_{\underline{y}}^{\bar{y}} \hat{Q}(y) H(y \mid x) dy - cx$, then $\delta(x)$ is continuous over $[0, \bar{x}]$, $\delta(\hat{x}) = 0$, and $\delta'(\hat{x}) = 0$. Since $\hat{Q}(y) \geq 0$ for all y and $\hat{Q}(y) > 0$ on a nonzero measure subset, and since $H_{xx} < 0$ for all $x \in (0, \bar{x})$ and $y \in (\underline{y}, \bar{y})$, $\delta(x)$ is strictly concave in x. Thus, $\hat{x}$ is a maximum point of $\delta(x)$. For $x \in (0, \hat{x}]$, $\delta(x) \leq \delta(\hat{x}) = 0$, and $\delta(0) = 0$. In summary, we obtain $\delta(\hat{x}) = 0$, $\delta(0) = 0$, and $\delta(x) \leq 0$ for any $x \in (0, \hat{x})$. These together contradict the continuity and strict concavity of $\delta(x)$. Therefore, $E\hat{U} = \delta(\hat{x}) > 0$.

Q.E.D.

The principal offers a production contract that allows the agent to invest a positive amount in R&D and to earn positive expected profits . This is a nice way to reward the agent for innovation since the R&D outcome cannot be observed or verified directly by the principal. It is easy to show that the result in Proposition 1 will not be true when either the R&D investment or the R&D outcome are observable to the principal. The principal will be able to extract the full surplus from the agent in these cases. Therefore, it is the interaction of the non-observability of the R&D investment and non-observability of the R&D outcome by the principal that allows the risk-neutral agent to earn positive profits.

The interim individual rationality (IIR) constraint also played a key role in Proposition 1. If the (IIR) constraint is relaxed, $R(\bar{y})$ can be negative. Since the choice of $R(\bar{y})$ does not affect the solution of $Q(y)$ and x, the principal could choose $R(\bar{y}) = \int_{\underline{y}}^{\bar{y}} \hat{Q}(y)H(y \mid \hat{x})dy - c\hat{x}$. The agent earns a zero expected profit. The principal extracts the whole surplus. The agent with a higher cost observation will end up with a negative ex post profit if it accepts the contract, and hence will drop out before production unless the principal can force the agent to produce.

Let λ and μ be the multipliers associated with the constraints (EIR) and $(R\&D)$, respectively. At the optimum $[\hat{Q}(y), \hat{R}(\bar{y}), \hat{x}]$, $\lambda E\hat{U} = 0$. From Proposition 1, $E\hat{U} > 0$. Thus, $\lambda = 0$. Then the optimal quantity $\hat{Q}(y)$, optimal investment level $\hat{x}$, and $\hat{\mu}$ are simultaneously determined by the following equations:

$$\int_{\underline{y}}^{\bar{y}} \hat{Q}(y)H_x(y \mid \hat{x})dy - c = 0, \tag{4}$$

$$B'(\hat{Q}(y)) - y = \frac{H(y \mid \hat{x})}{h(y \mid \hat{x})} - \hat{\mu}\frac{H_x(y \mid \hat{x})}{h(y \mid \hat{x})}, \tag{6}$$

$$\int_{\underline{y}}^{\bar{y}} \left[B'(\hat{Q}(y)) - y\right] \hat{Q}'(y)H_x(y \mid \hat{x})dy = \hat{\mu}\int_{\underline{y}}^{\bar{y}} \hat{Q}(y)H_{xx}(y \mid \hat{x})dy. \tag{7}$$

Consider the extreme case when $c = 0$. In this case, R&D is costless. The more the agent invests, the higher profit it earns because the marginal benefit of investment is positive. Thus, the agent will invest the upper bound level $\bar{x}$ in R&D, which is known by the principal. The $(R\&D)$ constraint in (P) is then not binding. The agent observes y from the distribution $H(y \mid \bar{x})$ without any cost. Then, the optimal procurement amount $\hat{Q}_0(y)$ is determined by

$$B'(\hat{Q}_0(y)) = y + \frac{H(y \mid \bar{x})}{h(y \mid \bar{x})}. \tag{8}$$

This is exactly the Baron and Myerson (1982) solution. Because of information asymmetry, there is an information cost $H(y\,|\,\bar{x})\,/\,h(y\,|\,\bar{x})$ paid by the principal under the optimal incentive contract in order to induce the agent to reveal its private information y. The principal chooses the quantity $\hat{Q}_0(y)$ such that the marginal benefit of the quantity equals the marginal production cost plus the marginal information cost. There is an adverse selection effect (also see Baron and Myerson 1982 in the regulation context). This effect results in underprocurement as compared to the case where y is observable to the principal.

Suppose $c > 0$; that is, R&D is costly. The agent invests in R&D to balance the benefits and costs. The $(R\&D)$ constraint is binding in this case. Formally, we have

Lemma 3: Suppose $H_x(y\,|\,x) > 0$, $1 + \dfrac{\partial(H/h)}{\partial y} \geq 0$, and $\dfrac{\partial(H_x/h)}{\partial y} \geq 0$ for all $x \in (0, \bar{x})$ and $y \in (\underline{y}, \bar{y})$. Then $\hat{\mu} > 0$.

Proof: The proof is by contradiction. If $\hat{\mu} \leq 0$, then from (6), we obtain $B'(Q(y)) - y > 0$, and

$$B''(\hat{Q}(y))\hat{Q}'(y) = 1 + \frac{\partial(H/h)}{\partial y} - \hat{\mu}\frac{\partial(H_x/h)}{\partial y}$$

which implies that $\hat{Q}'(y) < 0$ by the assumptions. Given $\hat{Q}(y)$, taking the derivative of EW with respect to x and integrating by parts, we get

$$\frac{\partial EW}{\partial x}\,|_{\hat{x}} = -\int_{\underline{y}}^{\bar{y}} \left[B'(\hat{Q}(y)) - y\right]\hat{Q}'(y) H_x(y\,|\,\hat{x})dy.$$

It is easy to see this term is positive because $H_x > 0$ and $\hat{Q}'(y) < 0$. Since $H_{xx} < 0$, then equation (7) implies that $\hat{\mu}$ should be positive. This contradicts the previous hypothesis. Thus $\hat{\mu} > 0$ and $\dfrac{\partial EW}{\partial x}\,|_{\hat{x}} > 0$ by (7).

Q.E.D.

The principal wants more R&D but can not control x. The preferences for investment are not consistent between the agent and the principal. Moral hazard exists because the principal can not observe the agent's investment decision. Since $H_x(y \mid \hat{x}) > 0$ for any $y \in (\underline{y}, \bar{y})$, then $-\hat{\mu}\dfrac{H_x(y \mid \hat{x})}{h(y \mid \hat{x})} < 0$. This negative term (moral hazard) has an opposite effect on $\hat{Q}(y)$ to the adverse selection effect represented by the information cost term $\dfrac{H(y \mid \hat{x})}{h(y \mid \hat{x})}$. From (6), we can see that the combination of the two opposite effects determines the extent to which production is carried out inefficiently (see Figure 3).

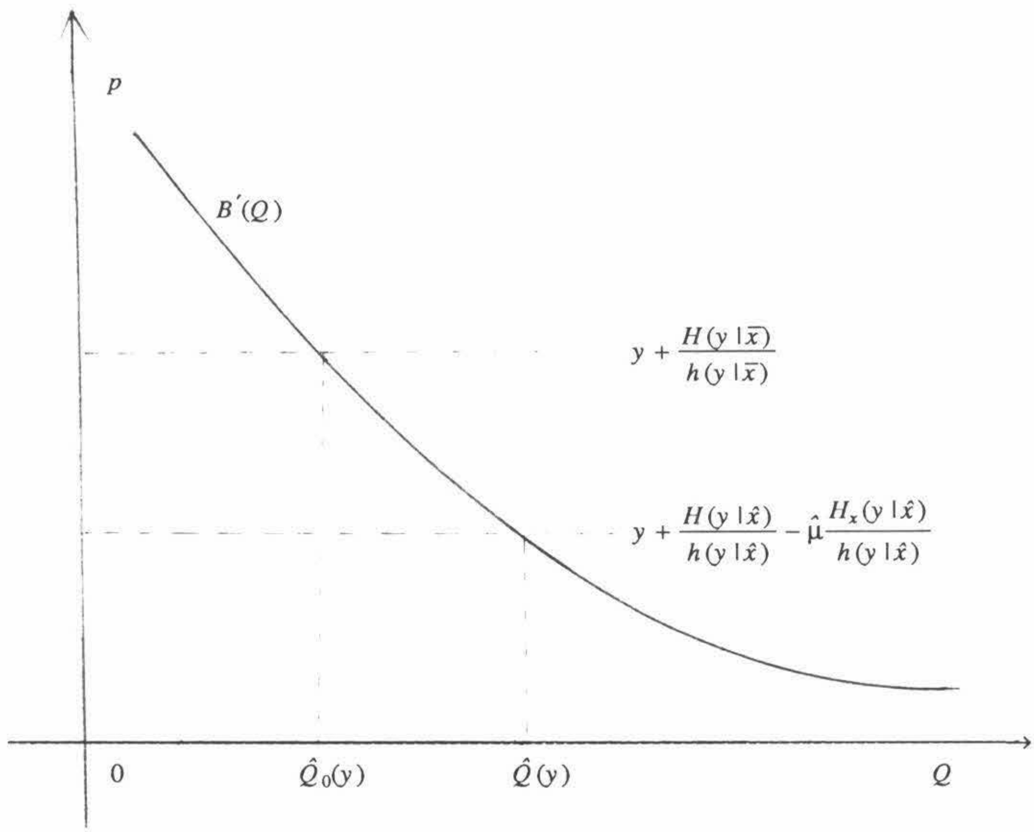


FIGURE 3: Optimal Quantity

Since $\hat{\mu}$ is the multiplier for the $(R\&D)$ constraint, it can be easily shown that $\frac{\partial E\hat{W}(c)}{\partial c} = -\hat{\mu}$ for $c > 0$, where $E\hat{W}(c)$ is the principal's expected welfare at the optimal solution $[\hat{Q}(y), R(\bar{y}), \hat{x}]$. Under the conditions in Lemma 3, $\hat{\mu} > 0$ and hence $\frac{\partial E\hat{W}(c)}{\partial c} < 0$. Using the optimal contract, the more costly the R&D is, the worse-off the principal is. Thus, the principal actually pays indirectly for part of the R&D cost.

Now, we compare these two cases: $c = 0$ and $c > 0$. In the following, we let $\hat{Q}(y)$ and $\hat{x}$ be the solution to (P) when $c > 0$. We have

Proposition 2: Suppose $\frac{\partial(H/h)}{\partial x} \geq 0$ for any $x \in (0, \bar{x})$ and $y \in [\underline{y}, \bar{y}]$, and the assumptions in Lemma 3 hold. Then i) $\hat{Q}(y) > \hat{Q}_0(y)$ for any $y \in (\underline{y}, \bar{y})$, $\hat{Q}(\underline{y}) = \hat{Q}_0(\underline{y})$, and $\hat{Q}(\bar{y}) \geq \hat{Q}_0(\bar{y})$; ii) $\hat{P}(y) > \hat{P}_0(y)$ for any $y \in [\underline{y}, \bar{y})$ and $\hat{P}(\bar{y}) \geq \hat{P}_0(\bar{y})$.

Proof: From Proposition 1, $0 < \hat{x} < \bar{x}$. Since $\frac{\partial(H/h)}{\partial x} \geq 0$ by the assumption, then

$$\frac{H(y \mid \bar{x})}{h(y \mid \bar{x})} \geq \frac{H(y \mid \hat{x})}{h(y \mid \hat{x})} > \frac{H(y \mid \hat{x})}{h(y \mid \hat{x})} - \hat{\mu}\frac{H_x(y \mid \hat{x})}{h(y \mid \hat{x})}$$

for all $y \in (\underline{y}, \bar{y})$. Comparing (6) with (8), we know $B'(\hat{Q}(y)) < B'(\hat{Q}_0(y))$ and hence $\hat{Q}(y) > \hat{Q}_0(y)$ for all $y \in (\underline{y}, \bar{y})$ since $B(Q)$ is strictly concave. When $y = \underline{y}$, $B'(\hat{Q}(\underline{y})) = B'(\hat{Q}_0(\underline{y}))$ from (6) and (8) and thus $\hat{Q}(\underline{y}) = \hat{Q}_0(\underline{y})$. When $y = \bar{y}$, $H(\bar{y} \mid \bar{x}) / h(\bar{y} \mid \bar{x}) \geq H(\bar{y} \mid \hat{x}) / h(\bar{y} \mid \hat{x})$ from the assumption. Comparing (6) with (8), we obtain $\hat{Q}(\bar{y}) \geq \hat{Q}_0(\bar{y})$.

Since $\hat{P}(y) = y\hat{Q}(y) + \int_{\underline{y}}^{\bar{y}} \hat{Q}(t)dt$ and $\hat{P}_0(y) = y\hat{Q}_0(y) + \int_{\underline{y}}^{\bar{y}} \hat{Q}_0(t)dt$, then it is easy to see $\hat{P}(y) > \hat{P}_0(y)$ for any $y \in [\underline{y}, \bar{y})$ and $\hat{P}(\bar{y}) \geq \hat{P}_0(\bar{y})$.

Q.E.D.

The assumption $\frac{\partial(H/h)}{\partial x} \geq 0$ in Proposition 2 means that the hazard rate H/h faced by the principal due to the nonobservability of R&D investment increases when the agent increases the level of investment. In this case, if R&D is costly, the principal offers a higher quantity schedule and a higher payment schedule as well. The principal prefers a relatively bigger production project if R&D is costly.

When $\hat{Q}'(y) < 0$ for every $y \in (\underline{y}, \bar{y})$, there exists an inverse function $y = \hat{Q}^{-1}(Q)$. Then the payment $\hat{P}(y)$ can be written as $P = \hat{P}(\hat{Q}^{-1}(Q)) \equiv P(Q)$ given any procurement amount Q. The agent will reveal its cost information y by choosing the quantity Q. A separating incentive procurement contract $[\hat{Q}(y), \hat{P}(y)]$ can be implemented by a simple nonlinear payment schedule $P = P(Q)$ under which the agent chooses the quantity to produce. When $\hat{Q}'(y) = 0$ over a subinterval of $[\underline{y}, \bar{y}]$, the optimal contract specifies pooling. The principal can not distinguish the different types and offers the same production contract.

Compare two seperating contracts $[\hat{Q}(y), \hat{P}(y)]$ and $[\hat{Q}_0(y), \hat{P}_0(y)]$. Let $P(Q) \equiv \hat{P}(\hat{Q}^{-1}(Q))$, $P_0(Q) \equiv \hat{P}_0(\hat{Q}_0^{-1}(Q))$, $Q_h \equiv \hat{Q}(\underline{y})$, and $Q_l \equiv \hat{Q}(\bar{y})$, then

Proposition 3: Suppose the assumptions in Proposition 2 hold, then $P'(Q) > P_0'(Q)$ for any $Q \in (Q_l, Q_h)$ and $P(Q) > P_0(Q)$ for any $Q \in [Q_l, Q_h]$.

Proof: First, we show $P'(Q) > P_0'(Q) > 0$ for any $Q \in (Q_l, Q_h)$. By the definition of $P(Q)$ and the self-selection property (or incentive compatibility), we obtain

$$P'(Q) = \frac{\hat{P}'(\hat{Q}^{-1}(Q))}{\hat{Q}'(\hat{Q}^{-1}(Q))} = \hat{Q}^{-1}(Q).$$

Similarly, $P_0'(Q) = \hat{Q}_0^{-1}(Q) > 0$. From Proposition 2, $\hat{Q}^{-1}(Q) > \hat{Q}_0^{-1}(Q)$ for any $Q \in (Q_l, Q_h)$. Thus, $P'(Q) > P_0'(Q) > 0$ for any $Q \in (Q_l, Q_h)$.

Second, we prove $P(Q) > P_0(Q)$ for any $Q \in [Q_l, Q_h]$. Since both $P(Q)$ and $P_0(Q)$ are continuous and increasing in Q, and since $P(Q)$ is steeper than $P_0(Q)$, we only need to show $P(Q_h) > P_0(Q_h)$ and $P(Q_l) > P_0(Q_l)$. In fact, since $\hat{Q}^{-1}(Q_h) = \underline{y}$, we have $P(Q_h) = \hat{P}(\hat{Q}^{-1}(Q_h)) = \hat{P}(\underline{y})$ and $P_0(Q_h) = \hat{P}_0(\hat{Q}_0^{-1}(\overline{Q})) = \hat{P}_0(\underline{y})$. By Proposition 2, $\hat{P}(\underline{y}) > \hat{P}_0(\underline{y})$ and thus $P(Q_h) > P_0(Q_h)$. Since $Q_l = \hat{Q}(\bar{y})$, then $\tilde{y} \equiv \hat{Q}_0^{-1}(Q_l) < \bar{y}$ by Proposition 2. Then $P(Q_l) = \hat{P}(\bar{y}) = \bar{y}\hat{Q}(\bar{y}) = \bar{y}Q_l$ and

$$P_0(Q_l) = \hat{P}_0(\tilde{y}) = \tilde{y}\hat{Q}_0(\tilde{y}) + \int_{\tilde{y}}^{\bar{y}} \hat{Q}_0(y)dy$$

$$< \tilde{y}\hat{Q}_0(\tilde{y}) + (\bar{y} - \tilde{y})\hat{Q}_0(\tilde{y}) = \bar{y}\hat{Q}_0(\tilde{y}) = \bar{y}Q_l.$$

Thus, $P(Q_l) > P_0(Q_l)$.

Q.E.D.

Proposition 3 implies that with more costly R&D the principal should offer a higher and steeper payment schedule (see Figure 4). For any given quantity Q, the principal pays more in the case of costly R&D than in the case of costless R&D. In other words, if the Baron-Myerson-type contract was used when R&D was costly, the principal would buy too little from the agent and pay too little to the agent. Raising the price paid raises the principal's welfare. Therefore, whether R&D is costly or not certainly affects the principal's decision and the principal's welfare as well. When designing an incentive procurement contract, the principal cannot ignore the agent's private R&D investment behavior.

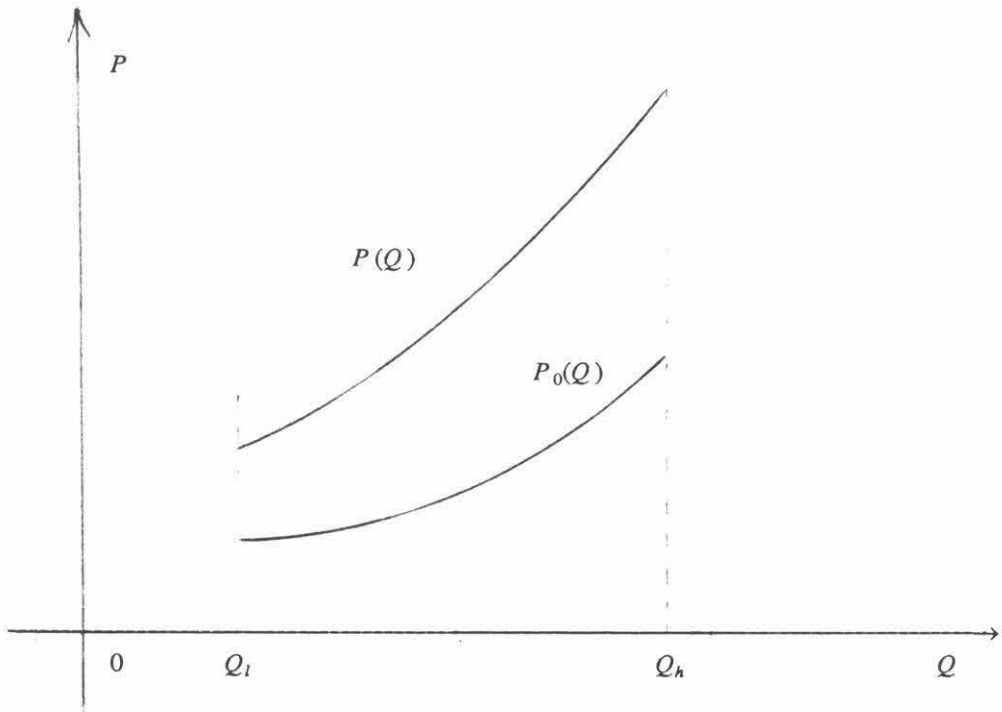


FIGURE 4: Nonlinear Payment Schedule

As we discussed before, because of an adverse selection effect, the asymmetry of information results in an underprocurement relative to the first-best solution. R&D (or moral hazard) has an opposite effect on the determination of procurement quantity to the adverse selection effect. When R&D is costly, the optimal quantity schedule $\hat{Q}(y)$ may still be different from the first-best quantity schedule $Q^*(y)$. To end this section, we will consider a special class of R&D technologies and illustrate how $\hat{Q}(y)$ may be different from the first-best level $Q^*(y)$:

Proposition 4: Suppose $H(y \mid x) = 1 - [1 - F(y)]^x$, where $x \geq 0$, $F(y)$ is an arbitrary cumulative distribution with the support $[\underline{y}, \bar{y}]$ and the density function $f(y)$, and $f(y)/[1 - F(y)]$ is nonincreasing over $(\underline{y}, \bar{y})$. Then

i) $\hat{\mu} > 0$;

ii) if $\hat{x} \geq \hat{\mu}$, then $\hat{Q}(y) < Q^*(y)$ for all $y \in (\underline{y}, \bar{y})$;

iii) if $\hat{x} < \hat{\mu}$, then there exists $y_0 \in (\underline{y}, \bar{y})$ such that $\hat{Q}(y_0) = Q^*(y_0)$, $\hat{Q}(y) > Q^*(y)$ for $y \in (\underline{y}, y_0)$, and $\hat{Q}(y) < Q^*(y)$ for $y \in (y_0, \bar{y})$.

See the Appendix for the proof. The conditions in ii) and iii) are not primitive conditions. They depend on the structure of the benefit function $B(Q)$ and distribution function $F(y)$. Because of the interaction of the adverse selection effect and the moral hazard effect, there is a possibility that the optimal quantity $\hat{Q}(y)$ is higher than the first-best quantity $Q^*(y)$ for lower marginal costs. But for higher marginal costs, the optimal quantity $\hat{Q}(y)$ is lower than the first-best quantity. That is, there is underprocurement relative to the first-best when the marginal cost is relatively high.

2.3 COMPETITION FOR A PROCUREMENT CONTRACT

Some defense procurements are organized so that many suppliers compete for the right to be the sole contractor on the production of the good to be procured. In order to get the procurement contract, potential firms will invest in R&D activities and become informed about the potential product and technologies before the competitive bidding starts. How does the procurement contract affect R&D behavior? Does the optimal quantity to be procured depend on the number of bidders? Does the total expenditure on R&D and the pace of innovation depend on the structure of the industry? Does competition improve production efficiency and R&D efficiency? In this section, we extend the analysis in Section 2 and discuss the effects of competition on R&D expenditures and procurement contracts. We will first describe a basic model that is an extension of the principal-agent model in Section 2.2, and study the Nash equilibrium behavior of R&D expenditures for an arbitrary incentive procurement contract. Then we characterize the optimal incentive contract and discuss properties of this contract.

2.3.1 The Model

Suppose that there is one buyer and n firms, where n is exogenous.[8] The benefit function for the buyer $B = B(Q)$ is the same as before. The production cost function for firm i is $C_i(Q) = y_i Q$, where y_i is constant marginal cost and is unknown to the buyer and all firms before R&D. But each firm can observe its own marginal cost by investing in R&D. Suppose that firm i invests (capital) x_i and observes y_i which is drawn from a cumulative distribution $H(y_i \mid x_i)$ with the density function $h(y_i \mid x_i)$ and the support $[\underline{y}, \bar{y}]$. The R&D outputs among different firms are independently and identically distributed.[9] The R&D cost for firm i is cx_i, where $c > 0$ is a known constant. Each firm's R&D

output is only observed by itself, but not observed by the buyer and other firms. Therefore, each firm has private information about its R&D output and will use this information strategically if it is optimal to do so.

At the beginning of the R&D stage, the buyer announces a payment schedule $Q = Q(b)$ and promises that the firm with the lowest bid $\tilde{b}$ by an exogenously given date will get the contract $[Q(\tilde{b}), Q(\tilde{b})\tilde{b}]$.[10] We want to determine the optimal payment schedule $Q = \hat{Q}(b)$ for the buyer. Under this contract, firm i's strategy will be $b_i = b_i(y_i), i = 1, \ldots, n$. Since firms are symmetric and the contract is also symmetric, it is reasonable to consider symmetric equilibrium $b_i = b(y_i)$ only. If as is usually the case, the bidding function $b(y_i)$ is increasing function, then the firm with the lowest bid $\tilde{b}$ is the firm with the lowest marginal cost $\tilde{y}$. In these cases, we only have to consider incentive compatible direct revelation contracts $[Q(y), P(y)]$, where $P(y) = Q(y)b(y)$ and y is the lowest marginal cost. The timing of the game is then as follows: First, the buyer announces production contracts $[Q(y), P(y)]$ and promises that the firm with the lowest (reported) marginal cost $\tilde{y}$ by an exogenously given date will be awarded a contract $[Q(\tilde{y}), P(\tilde{y})]$. Second, each firm invests and observes its marginal cost y_i at the given date. Then the firm with the lowest marginal cost $\tilde{y}$ gets the contract. Finally, the winning firm produces $Q(\tilde{y})$ and gets paid by $P(\tilde{y})$.

We want truthful reporting to be a noncooperative Nash equilibrium. Given that other firms report true information, firm i reports y_i', which depends on its true information y_i. We denote its strategy as $y_i' = \phi_i(y_i)$. Then the probability of firm i winning is

$$K(y_i' \mid x_{-i}) \equiv Pr(y_i' \leq Y_j, j \neq i, j = 1, \cdots, n)$$

$$= \Pi_{j \neq i} \left[1 - H(y_i' \mid x_j)\right],$$

where Y_j is a random variable and represents firm j's R&D outcome (marginal cost). The ex ante profits for firm i from using strategy $\phi_i(y_i)$ and x_i given that the other firms use x_{-i} and y_{-i} is

$$EU_i(\phi_i(y) \mid x_1, \ldots, x_n) = \int_{\underline{y}}^{\bar{y}} \left[P(\phi_i(y)) - yQ(\phi_i(y)) \right] K(\phi_i(y) \mid x_{-i}) h(y \mid x_i) dy - cx_i .$$

Similar to Lemma 1 in Section 2.2.3, the necessary and sufficient conditions for firm i to tell the truth ($\phi_i(y) = y$ for all y) are that $Q'(y)/Q(y) \le \sum_{j \ne i} h_j / (1 - H_j)$ and that the payment $P(y)$ is

$$P(y) = yQ(y) + \int_{y}^{\bar{y}} Q(\tilde{y}) K(\tilde{y} \mid x_{-i}) d\tilde{y} \; / \; K(y \mid x_{-i}). \tag{9}$$

For the truth-telling Nash equilibrium, we can simply write firm i's ex ante profits $EU_i(x_1, \ldots, x_n)$ as

$$EU_i(x_1, \ldots, x_n) = \int_{\underline{y}}^{\bar{y}} Q(y) K(y \mid x_{-i}) H(y \mid x_i) dy - cx_i . \tag{10}$$

We will look at noncooperative Nash equilibrium behavior in R&D expenditures for both an arbitrary incentive contract and the optimal incentive contracts in the next sections. Since most of the results will depend on the structure of the R&D technology, we first classify R&D technology:

Condition (A): For any $y \in (\underline{y}, \bar{y})$, $-\log[1 - H(y \mid x)]$ is linear in $x (x \ge 0)$.

Condition (B): For any $y \in (\underline{y}, \bar{y})$, $-\log[1 - H(y \mid x)]$ is strictly concave in $x (x \ge 0)$.

Lemma 4: Condition (A) is equivalent to $H(y \mid x) = 1 - [1 - F(y)]^x$, where $x \ge 0$ and $F(y)$ is an arbitrary cumulative distribution function over $[\underline{y}, \bar{y}]$.

The proof is quite straightforward and is omitted here. Suppose that a firm has a prior distribution $F(y)$ about its marginal production cost y. The firm can do an experiment with a fixed cost c and observe marginal cost y^1 which is drawn from the distribution $F(y)$. If this experiment can be repeated k times independently, then the firm observes a sequence of the marginal cost realizations $(y^1, \ldots, y^k)$. The firm will choose the minimum marginal cost y^m. From statistical theory, y^m is a realization of a random variable (the minimum-order statistic) with the distribution $H(y \mid k) = 1 - [1 - F(y)]^k$. Thus, we have an independent search process. Rob (1986) used a similar optimal search model of R&D behavior in procurements.

In general, we can allow $k = x$ to be a continuous variable and to represent the expenditure on R&D. This R&D process of marginal cost reduction is just an independent search process. The first x dollars have the same effect on the minimum marginal cost as the last x dollars. To some extent, this process is subject to constant marginal returns in the number of experiments or expenditures. Similarly, condition (B) represents R&D processes that exhibit diminishing marginal returns to expenditures. Diminishing marginal returns to scale may be a good description of most R&D processes in reality. A relatively simple form that satisfies (B) is

Condition (B_1): $H(y \mid x) = 1 - [1 - F(y)]^{\alpha(x)}$, where $x \geq 0$, $F(y)$ is an arbitrary cumulative distribution function over $[\underline{y}, \bar{y}]$, $\alpha'(x) > 0$, $\alpha''(x) < 0$, $\alpha(0) = 0$, and $\alpha'(0) = +\infty$.

It is easy to check that (B_1) satisfies condition (B), so a (B_1) technology exhibits diminishing marginal returns to expenditures on R&D. In this case, with the belief that y is drawn from a distribution $F(y)$, the firm could not make an independent experiment and then simply take the lowest marginal cost observation. The later experiments are not as productive as the earlier experiments. Let $F(y) = 1 - e^{-y}$ over $[0, \infty)$, then

$H(y \mid x) = 1 - e^{-\alpha(x)y}$ which is also an exponential distribution. This distribution is adopted in the literature on stochastic R&D races (see Reinganum 1988 for a survey). Diminishing marginal returns to scale is also assumed in this literature.

2.3.2 Arbitrary Incentive Contracts

Consider an arbitrary direct revelation (possibly non-optimal) contract $[Q(y), P(y)]$, which is assumed to be independent of n in this section. Under this contract, if truth-telling is a Nash equilibrium, then each firm's expected profits can be written as (10). At the beginning of the R&D stage, each firm chooses an investment level to maximize its expected profits. If Inada's 'derivative conditions' are satisfied, the Nash equilibrium $(x_1, \ldots, x_n)$ satisfies the first order condition:

$$\int_{\underline{y}}^{\bar{y}} Q(y)K(y \mid x_{-i})H_{x_i}(y \mid x_i)dy - c = 0 \tag{11}$$

for $i = 1, 2, \ldots, n$, where H_{x_i} is the derivative of $H(y_i \mid x_i)$ with respect to x_i. Since EU_i is concave in x_i, the second order conditions are satisfied.

Under condition (A), the equilibrium condition (11) becomes

$$-\int_{\underline{y}}^{\bar{y}} Q(y)[1 - F(y)]^x \log[1 - F(y)]dy = c \tag{12}$$

which is the same for all firms, where $x = \sum_{i=1}^{n} x_i$ is the total expenditure on R&D. Given $Q(y)$, the equilibrium condition (12) determines x. Both symmetric and asymmetric Nash equilibria on R&D expenditures exist and the total expenditure $x = \sum_{i=1}^{n} x_i$ determined by (12) is independent of n, the number of firms. The total expenditure is the same under different equilibria. The expected minimum marginal cost is

$$E(y^m \mid x) = -\int_{\underline{y}}^{\bar{y}} y d \prod_{i=1}^{n} [1 - H(y \mid x_i)]$$

$$= \underline{y} + \int_{\underline{y}}^{\bar{y}} [1 - F(y)]^x dy.$$

It is also independent of the number of firms. Therefore, we get the same invariance results as Sah and Stiglitz (1987) do.

In our model, firms compete for a profitable production contract instead of a prize with a fixed rent as in Sah and Stiglitz and other models in the R&D races literature. An incentive production contract $[Q(y), P(y)]$ can generate a profit $R(y) = P(y) - yQ(y)$, which decreases with the R&D outcome y (the marginal cost). The winning firm is not just awarded a prize, it will get a better prize if its R&D outcome is of higher quality. This can be viewed as being similar to a variable patent system in private markets. The reason we have the invariance result here is quite intuitive. Remember that each firm has the same R&D technology and does R&D independently. The R&D technology exhibits constant marginal returns to expenditures and it is just like an independent search process. Therefore, when one firm does k experiments, it has the same effect on the observed minimum marginal production cost as if k firms each did one experiment. When the prize is predetermined and independent of the number of firms, the total number of experiments or total R&D expenditure for all firms is independent of the number of firms. Under this specified environment, one firm will do what n firms will do. Therefore, it is not surprising to get the invariance result under the particular R&D technology. Later, we will show that if the R&D technology exhibits diminishing marginal returns to expenditures or if the buyer chooses the incentive contract optimally, the above invariance result does not hold. We would conjecture that if the R&D technologies among firms are dependent, the invariance result does not hold either.

If the R&D process exhibits diminishing marginal returns to expenditures on R&D, Nash equilibrium behavior on R&D expenditure will be different from above. We first show that each firm will invest the same amount in equilibrium.

Proposition 5: Under condition (B), i) only a symmetric Nash equilibrium on R&D expenditures exists; ii) the individual expenditure $x(n)$ decreases with n; and iii) $\lim_{n \to \infty} x(n) = 0$.

Proof: We first show that any Nash equilibrium $(\hat{x}_1, \ldots, \hat{x}_n)$ at the R&D stage is symmetric. If not, there exists $i \neq j$ such that $\hat{x}_i \neq \hat{x}_j$, $\hat{x}_i > 0$ and $\hat{x}_j > 0$. Since $\hat{x}_i$ and $\hat{x}_j$ satisfy (11), we get

$$\int_{\underline{y}}^{\bar{y}} Q(y) \prod_{k=1}^{n} [1 - H(y \mid \hat{x}_k)] \left[\frac{H_{x_i}(y \mid \hat{x}_i)}{1 - H(y \mid \hat{x}_i)} - \frac{H_{x_j}(y \mid \hat{x}_j)}{1 - H(y \mid \hat{x}_j)} \right] dy = 0.$$

Condition (B) implies that $H_x(y \mid x) / [1 - H(y \mid x)]$ is decreasing in x for all $x > 0$ and $y \in (\underline{y}, \bar{y})$. Then the above equation cannot be true. This contradiction implies that any Nash equilibrium is symmetric.

Let $x(n)$ be the individual R&D expenditure, then the equilibrium condition (11) becomes

$$\int_{\underline{y}}^{\bar{y}} Q(y)[1 - H(y \mid x(n))]^{n-1} H_x(y \mid x(n)) dy = c. \tag{11'}$$

Considering n as a real variable and taking the derivatives of both sides of (11′) with respect to n, we obtain

$$\frac{\partial x(n)}{\partial n} \int_{\underline{y}}^{\bar{y}} Q(y)[1 - H(y \mid x(n))]^{n-2} \Big[[1 - H(y \mid x(n))] H_{xx}(y \mid x(n)) + (1-n) H_x^2(y \mid x(n)) \Big] dy$$

$$+ \int_{\underline{y}}^{\bar{y}} Q(y)[1 - H(y \mid x(n))]^{n-1} H_x(y \mid x(n)) \log[1 - H(y \mid x(n))] dy = 0.$$

Condition (B) implies that $(1-H)H_{xx}+H_x^2<0$ for all $x>0$ and $y \in (\underline{y}, \bar{y})$. Thus, $\frac{\partial x(n)}{\partial n} \leq 0$; that is, $x(n)$ decreases with n.

Furthermore, $\{x(n)\}$ is a monotonic decreasing sequence with the lower bound 0. Then there exists a limit $x_0 \geq 0$ of the sequence when n approaches infinity. We will show that $x_0 = 0$. If not, then $H(y \mid x_0) > 0$ for $y > \underline{y}$ by the assumption and hence $[1-H(y \mid x(n))]^{n-1} \to 0$ for $y > \underline{y}$ when $n \to \infty$. Let $n \to \infty$ in equation (11'), we get $c = 0$, which contradicts to $c > 0$ by the assumption. Therefore, $x(n) \to x_0 = 0$.

Q.E.D.

When more firms enter the R&D race game, each existing firm will invest less in R&D. In the limit, the individual expenditure approaches zero.

It is not clear how the total expenditure and the pace of innovation depend on the number of firms under diminishing marginal returns to expenditure. In a special case of (B), we find the following dependence result, which differs from Sah and Stiglitz (1987):

Proposition 6: Under (B_1), i) $E(y^m \mid x(n))$ decreases with n; ii) $\lim_{n \to \infty} E(y^m \mid x(n)) = \underline{y}$, where $E(y^m \mid x(n)) = \underline{y} + \int_{\underline{y}}^{\bar{y}} [1-F(y)]^{n\alpha(x(n))} dy$ is the expected minimum marginal cost.

Proof: Under condition (B_1), the Nash equilibrium on R&D expenditures is symmetric by Proposition 5. Let $x(n)$ be the individual equilibrium expenditure, then the equilibrium condition (11) can be written as

$$\int_{\underline{y}}^{\bar{y}} Q(y)[1-F(y)]^{n\alpha(x(n))} \log[1-F(y)] dy = -c/\alpha'(x(n)). \qquad (11'')$$

By Proposition 5, $x(n)$ decreases with n and $x(n) \to 0$ when $n \to \infty$. Since $\alpha(x)$ is strictly concave function, (11″) implies that $n\alpha(x(n))$ increases with n. It is easy to calculate the expected minimum marginal cost as the following:

$$E(y^m \mid x(n)) = -\int_{\underline{y}}^{\bar{y}} y d[1 - F(y)]^{n\alpha(x(n))}$$

$$= \underline{y} + \int_{\underline{y}}^{\bar{y}} [1 - F(y)]^{n\alpha(x(n))} dy$$

which decreases with n. Since $\alpha'(0) = +\infty$ from the assumption, then $\alpha'(x(n)) \to +\infty$. From (11″), it must be $n\alpha(x(n)) \to +\infty$. Thus, $E(y^m \mid x(n)) \to \underline{y}$.

Q.E.D.

Because of diminishing marginal returns to expenditures the cost reduction by each firm is limited. But different firms can do R&D independently. When more and more firms invest in R&D, minimum marginal costs are expected to be reduced. In the limit, the expected minimum marginal cost could reach the lower bound $\underline{y}$. But at the same time, the total expenditures on R&D may increase with the number of firms. As a particular example, let $\alpha(x) = x^{\alpha}$, $0 < \alpha < 1$, and let $Q(y) = -f(y)/\log[1 - F(y)]$. Assume that $f'(y) < 0$ for $y \in (\underline{y}, \bar{y})$, then it is easy to check that $Q'(y) \le 0$. The global incentive condition is satisfied. In this case, the equilibrium condition (11) becomes

$$nx(n) + x(n)^{1-\alpha} = \alpha/c.$$

When $\alpha = 1$, the total expenditure is $nx(n) = 1/c$, which is independent of n as we showed before. But for $0 < \alpha < 1$, $nx(n)$ varies with n. From Proposition 5, we know that $x(n)$ decreases with n and $x(n) \to 0$ when $n \to +\infty$. From the above equation, we obtain that $nx(n)$ increases with n and $nx(n) \to \alpha/c$ when $n \to +\infty$. Thus, the total expenditure $nx(n)$ increases with the number of firms and has a finite limit in this example. Therefore, when the R&D technology exhibits diminishing marginal returns to expenditures, more competition with a finite amount of total expenditures on R&D could reduce the marginal production cost to the lower bound.

2.3.3 The Optimal Incentive Contract

Under the incentive condition (9), we can easily calculate the buyer's expected utility $EW(Q(y), x_1, \ldots, x_n)$. The buyer will choose quantity $Q(y)$ and R&D expenditures $(x_1, \ldots, x_n)$ to maximize its expected utility EW subject to the firms' individual rationality constraints, self-selection constraints, and the Nash equilibrium conditions on R&D expenditures. An optimal incentive contract will be determined by the following optimization problem:

$$(P_n) \quad \underset{Q(y), x_1, \ldots, x_n}{\text{Max}} \int_{\underline{y}}^{\bar{y}} \left[\left[B(Q(y)) - yQ(y) \right] \sum_{i=1}^{n} K(y \mid x_{-i}) h(y \mid x_i) - Q(y) \sum_{i=1}^{n} K(y \mid x_{-i}) H(y \mid x_{-i}) \right] dy$$

$$s.t. \quad \int_{\underline{y}}^{\bar{y}} Q(y) K(y \mid x_{-i}) H_{x_i}(y \mid x_i) dy - c = 0, \quad i = 1, \ldots, n \tag{11}$$

$$\int_{\underline{y}}^{\bar{y}} Q(y) K(y \mid x_{-i}) H(y \mid x_i) dy - cx_i \geq 0, \quad i = 1, \ldots, n \tag{13}$$

where the global incentive constraint is ignored. Guesnerie and Laffont's (1984) argument can be used if the global incentive constraint does not hold for the solution to (P_n). We can see that when $n = 1$ the optimization problem (P_n) is just (P), which we discussed in Section 2.2.4. EU_i is strictly concave in x_i, $EU_i = 0$ when $x_i = 0$, and $EU_i > 0$ for some $x_i > 0$ and for any nontrivial distribution $H(y \mid x)$. Thus, for an interior solution $x_i > 0$ of (P_n), the individual rationality constraint (13) must be nonbinding. In order to encourage each firm to do R&D, positive ex ante profits are required.

Let μ_i and λ_i be the multipliers for (11) and (13), respectively, then $\lambda_i = 0$ for all i because of the nonbinding constraints (13). The necessary conditions for the optimal procurement quantity $Q(y)$ and R&D expenditures $(x_1, \ldots, x_n)$ are

$$B'(Q(y)) = y + \frac{\sum_{i=1}^{n} K(y \mid x_{-i})H(y \mid x_i)}{\sum_{i=1}^{n} K(y \mid x_{-i})h(y \mid x_i)} - \frac{\sum_{i=1}^{n} \mu_i K(y \mid x_{-i})H_{x_i}(y \mid x_i)}{\sum_{i=1}^{n} K(y \mid x_{-i})h(y \mid x_i)} \tag{14}$$

$$\int_{\underline{y}}^{\bar{y}} \left[B'(Q(y)) - y\right] Q'(y)K(y \mid x_{-i})H_{x_i}(y \mid x_i)dy = \mu_i \int_{\underline{y}}^{\bar{y}} Q(y)K(y \mid x_{-i})H_{x_i x_i}(y \mid x_i)dy \tag{15}$$

Thus, equation systems (9), (11), (14), and (15) simultaneously determine the optimal contract $[Q(y), P(y)]$, the optimal R&D expenditures $(x_1, \ldots, x_n)$, and the multipliers $(\mu_1, \ldots, \mu_n)$.

Lemma 5: If $1 + \frac{\partial}{\partial y}\left[\frac{\sum_i K(y \mid x_{-i})H(y \mid x_i)}{\sum_i K(y \mid x_{-i})h(y \mid x_i)}\right] > 0$ and $\frac{\partial}{\partial y}\left[\frac{K(y \mid x_{-i})H_{x_i}(y \mid x_i)}{\sum_i K(y \mid x_{-i})h(y \mid x_i)}\right] \geq 0$ for all $y, x_1, \ldots, x_n$ and all i, then there exists at least one j such that $\mu_j > 0$.

Proof: The proof is by contradiction and similar to the proof of Lemma 3. If not, then $\mu_j \leq 0$ for all j. From (14), we obtain $B'(Q(y)) - y > 0$ and

$$B''(Q(y)) = 1 + \frac{\partial}{\partial y}\left[\frac{\sum_i K(y \mid x_{-i})H(y \mid x_i)}{\sum_i K(y \mid x_{-i})h(y \mid x_i)}\right] - \sum_i \left[\mu_i \frac{\partial}{\partial y}\left[\frac{K(y \mid x_{-i})H_{x_i}(y \mid x_i)}{\sum_i K(y \mid x_{-i})h(y \mid x_i)}\right]\right]$$

By the assumption, the above equation implies $Q'(y) < 0$. Thus,

$$\frac{\partial EW}{\partial x_i} = -\int_{\underline{y}}^{\bar{y}} \left[B'(Q(y)) - y\right] Q'(y)K(y \mid x_{-i})H_{x_i}(y \mid x_i)dy > 0$$

Since $H_{x_i x_i} < 0$, then (15) implies $\mu_i > 0$. This is a contradiction. Therefore, there exists at least one j such that $\mu_j > 0$.

Q.E.D.

It may not be easy to interpret the assumptions in Lemma 5. But if we restrict our attention to technologies (A) or (B_1) we can provide a standard condition. Under technology (A) or (B_1), from (15) it can be shown that $\mu_1 = \cdots = \mu_n \equiv \mu$. Then the assumptions in Lemma 5 become much simpler:

Lemma 6: Under R&D technology (A) or (B_1), if $f(y)/[1-F(y)]$ is nonincreasing in y, then $\mu > 0$.

The proof is similar to the proof of Lemma 5 and is omitted here. Since $h(y \mid x)/[1-H(y \mid x)] = \alpha(x) f(y)/[1-F(y)]$ under technology (A) or (B_1), the condition in Lemma 6 means that for any investment level x the hazard rate $h(y \mid x)/[1-H(y \mid x)]$ is nonincreasing in y, which is the standard regularity condition. Under condition (A), (14) and (15) become

$$B'(Q(y)) = y + \frac{1-F(y)}{f(y)x}\left[\sum_{i=1}^{n}[1-F(y)]^{-x_i} - n + n\mu\log[1-F(y)]\right] \tag{14'}$$

$$\int_{\underline{y}}^{\bar{y}}\left[B'(Q(y)) - y\right] Q'(y)[1-F(y)]^{x}\log[1-F(y)]dy$$

$$= \mu\int_{\underline{y}}^{\bar{y}} Q(y)[1-F(y)]^{x}\log^2[1-F(y)]dy \tag{15'}$$

Thus, the optimal procurement quantity, equilibrium R&D expenditures, and μ are simultaneously determined by (12), (14′), and (15′). We have the following results:

Proposition 7: Under condition (A), if $f(y)/[1-F(y)]$ is nonincreasing in y, then the optimal procurement quantity depends on the number of potential firms, as do the total expenditure and the pace of innovation.

The proof is in the Appendix. Therefore, in the presence of costly R&D investments, the optimal procurement quantity is dependent on the number of competing firms . This contrasts with Riordan and Sappington (1987), and Dasgupta and Spulber (1989) who find that the optimal quantity schedule does not depend on the number of firms in awarding monopoly franchises and in normal procurements, respectively. But no R&D behavior was considered in their models. In the presence of R&D behavior, the design of incentive contracts is based upon the buyer's belief about the firms' private information (the R&D outcome). Thus, investment behavior actually has some influence on procurement contracts. The number of potential firms plays an important role in determining the optimal procurement quantity and the investment level. But how the optimal quantity and the R&D expenditure depend on the number of firms in this case is not clear.

Let $EW(n)$ be the principal's expected welfare under the optimal incentive contract, then we know

Proposition 8: Under conditions in Lemma 6, $\frac{dEW(n)}{dn}|_{n=1} > 0$.

In other words, the principal prefers that more than one firm participate in R&D and bid for the procurement contract. It is not clear whether the government is always better off when more firms participate in R&D and compete for the contract. What we know is the following:

Proposition 9: Suppose that the conditions in Proposition 7 hold, and that the optimal solution $Q_n(y)$ and $\sum_{i=1}^{n} x_i$ have limits $Q_0(y) \geq 0$ and $x_0 \geq 0$ when $n \to \infty$, respectively, and $Q_n(y)$ decreases with y. Then $B'(Q_0(y)) = y$ and $\lim_{n \to \infty} E(y^m | x(n)) = y_0 > \underline{y}$.

We have ex post production efficiency in the limit, that is, marginal benefit equals marginal cost given the technology, but the expected minimum marginal cost might increase with the number of firms because $Q_n(y)$ and hence $nx(n)$ might decrease with n. The expected minimum marginal cost may not reach the lower bound $\underline{y}$ in the limit. Thus, in the case of costly R&D, more competition improves production efficiency, but may discourage R&D. Production may be socially more efficient but at a relatively higher R&D cost.[11]

2.4 CONCLUDING REMARKS

In this chapter, we have created a basic model of the R&D and production process. We have examined what should happen if the government manages and controls the R&D stage indirectly by awarding an appropriate production contract. The optimal production contract is characterized to maximize the government's expected welfare given that each firm has incentives to invest in R&D and to report its true private information. There exists inefficiency in procurements because of unobservable R&D outcomes and moral hazard in R&D activities. Costly R&D has an opposite effect on the production decision to the adverse selection effect (Baron and Myerson 1982). If the Baron-Myerson type contract is used in this case, the government buys too little and pays too little as compared with its optimum. Therefore, the government prefers to take into account the pre-contract R&D behavior when offering a procurement contract.

The analysis of the competition case shows that the number of competing firms matters. The optimal incentive production contract offered by the government depends on the number of firms. This is in sharp contrast to Riordan and Sappington (1987) and Dasgupta and Spulber (1989) where no R&D behavior has been considered. The total expenditures on R&D and the pace of innovation in the industry also depend on the number of firms. This is different from the invariance result of Sah and Stiglitz (1987) where firms do R&D and compete for a fixed rent in a Bertrand market. The government prefers more than one firm to invest in private R&D and to compete for the production contract. In general, competition among a large number of suppliers in procurements may encourage production efficiency, but discourage R&D. It depends on the structure of the R&D technology.

APPENDIX TO CHAPTER 2

Application of the Revelation Principle: We first consider the case of one agent. In general, the principal announces a mechanism $[Q(m), P(m), x]$, where $m \in M$ is the message that the agent sends to the principal, $Q: M \rightarrow R_+$ and $P: M \rightarrow R_+$ are the quantity to be produced and the payment paid to the agent, and $x \in$ is the principal's proposal to the agent. Given this mechanism, the agent takes an action $\tilde{x}$ and observes his type y. Then the agent sends a message $m = m(y, \tilde{x}) \in M$ to the principal. Finally, the principal asks the agent to produce $Q(m(y, \tilde{x}))$ and pays him $P(m(y, \tilde{x}))$. Since P, Q, and x are continuous variables and are chosen from convex sets, we only need to consider pure strategies of $Q(.)$, $P(.)$, and x. The principal chooses a mechanism $[Q(m), P(m), x]$ to maximize

$$EW = \int_{\underline{y}}^{\bar{y}} \left[B\left[Q(m(y, \tilde{x}))\right] - P(m(y, x))\right] dH(y \mid x)$$

subject to

$$[\tilde{m} = m(y, x), \tilde{x} = x] \in argmax\ EU = \int_{\underline{y}}^{\bar{y}} \left[P(\tilde{m}(y, \tilde{x})) - yQ(\tilde{m}(y, \tilde{x}))\right] dH(y \mid \tilde{x}) - c\tilde{x}. \quad (*)$$

A mechanism is direct if and only if $M = [\underline{y}, \bar{y}]$. An optimal incentive compatible direct revelation mechanism $[Q(y), P(y), x]$ is the one that maximizes

$$EW = \int_{\underline{y}}^{\bar{y}} \left[B(Q(y)) - P(y)\right] dH(y \mid x)$$

subject to

$$[\phi(y) = y, \tilde{x} = x] \in argmax\ \ EU = \int_{\underline{y}}^{\bar{y}} \left[P(\tilde{\phi}(y)) - yQ(\tilde{\phi}(y))\right] dH(y \mid \tilde{x}) - c\tilde{x}. \quad (**)$$

The Revelation Principle in this context says that, given any (agent's) optimal strategy $[m(y, x), x]$ in any mechanism $[Q(m), P(m), x]$ with m *memeber* M, there exists an incentive compatible direct revelation mechanism $[Q^*(y), P^*(y), x^*]$ in which the principal obtains the same expected welfare as in the given optimal strategy of the given mechanism. The same is true for the agent.

To see why this is true, we first notice that, due to the sequential nature ofthe agent's decision making, the agent's optimal reporting strategy $m = m(y, x)$ is actually independent of his action x. That is, after the agent sunk x, he only cares about the benefit from the mechanism, which depends on the production cost y he observes and the message m he sends. Let $m = m(y)$. Given the mechanism $[Q(m), P(m), x]$ and the agent's optimal strategy $[m(y), x]$, we define $Q^*(y) \equiv Q(m(y))$, $P^*(y) = P(m(y))$, and $x^* = x$. Then, under the new mechanism, the principal has the payoff

$$EW^* = \int_{\underline{y}}^{\bar{y}} \left[B(Q^*(y)) - P^*(y) \right] dH(y \mid x^*)$$

$$= \int_{\underline{y}}^{\bar{y}} \left[B\left[Q(m(y)) \right] - P(m(y)) \right] dH(y \mid x)$$

$$= EW$$

That is, the principal has the same expected welfare under these two mechanisms. Now, suppose $[Q^*(y), P^*(y), x^*]$ does not satisfy the incentive compatibility condition (**). That means $[\tilde{\phi}(y) = y, \tilde{x} = x^*]$ does not maximize

$$EU = \int_{\underline{y}}^{\bar{y}} \left[P^*(\tilde{\phi}(y)) - yQ^*(\tilde{\phi}(y)) \right] dH(y \mid \tilde{x}) - c\tilde{x}$$

$$= \int_{\underline{y}}^{\bar{y}} \left[P\left[m(\tilde{\phi}(y)) \right] - yQ\left[m(\tilde{\phi}(y)) \right] \right] dH(y \mid \tilde{x}) - c\tilde{x}$$

$$= \int_{\underline{y}}^{\bar{y}} \left[P(m_1(y)) - yQ(m_1(y)) \right] dH(y \mid \tilde{x}) - c\tilde{x}$$

where $m_1(y) = m(\tilde{\phi}(y))$. That is, $[m_1(y) = m(y), \tilde{x} = x]$ does not maximize

$$\int_{\underline{y}}^{\bar{y}} \left[P(m_1(y)) - yQ(m_1(y)) \right] dH(y \mid \tilde{x}) - c\tilde{x}$$

That violates condition (*). Thus, $[Q^*(y), P^*(y), x^*]$ is incentive compatible. Similarly, it is easy to see that the agent has the same expected profit under these two mechanisms.

In the case of n agents, the principal announces a mechanism $[Q(m), P(m), x]$, where $m = (m_1, \ldots, m_n) \in M$ are the messages that the agents send to the principal, $Q: M \rightarrow R_+^n$ and $P: M \rightarrow R_+^n$ are the quantity to be produced and the payment paid to each agent, and $x \in R_+^n$ is the principal's proposal to the agents. Given this mechanism, agent i takes an action x_i and observes his type y_i, which is drawn from a distribution $H_i(y_i \mid x_i)$. The principal chooses a mechanism $[Q(m), P(m), x]$ with $m \in M$ to maximize

$$EW = E_y \sum_{i=1}^{n} \left[B\left[Q_i(m(y, \tilde{x})) \right] - P_i(m(y, x)) \right]$$

subject to

$$[\tilde{m}_i = m_i(y, x), \tilde{x}_i = x_i] \in argmax\ EU = E_y \left[P_i(\tilde{m}_i, m_{-i}) - y_i Q_i(\tilde{m}_i, m_{-i}) \right] - c\tilde{x}_i$$

for all $i = 1, \ldots, n$. Similarly, the optimal strategies $m = m(y, x)$ for the agents are independent of their actions x. *The Revelation Principle* holds as in the case of one agent.

Q.E.D.

Proof of Lemma 1: First, if $\phi(y) = y$ is the agent firm's optimal strategy, the necessary condition is

$$\frac{\partial R(y, \phi) h(y \mid x)}{\partial \phi} \mid_{\phi = y} = 0$$

for all x and y, i.e., $\frac{\partial R(y,\phi)}{\partial \phi} \mid_{\phi=y} = 0$ for all y. The second order condition can be written as $\frac{\partial^2 R(y,\phi)}{\partial \phi^2} \mid_{\phi=y} \leq 0$ for all y. Combining these two conditions, we get $Q'(y) \leq 0$ for all $y \in [\underline{y}, \bar{y}]$. Let $R(y) \equiv R(y,y)$, then $P(y) = yQ(y) + R(y)$ and $R'(y) = \frac{\partial R(y,\phi)}{\partial y} \mid_{\phi=y} = -Q(y)$.

On the other hand, given that $Q'(y) \leq 0$ and $P(y) = yQ(y) + R(y)$ for all $y \in [\underline{y}, \bar{y}]$, we can show that $R(y) \geq R(y,\phi)$ for any y, $\phi \in [\underline{y}, \bar{y}]$. In fact, since $R(y) = R(\bar{y}) + \int_y^{\bar{y}} Q(\tilde{y})d\tilde{y}$, for $\phi > y$ we have

$$R(y) - R(\phi) = \int_y^{\phi} Q(\tilde{y})d\tilde{y}$$

$$\geq Q(\phi)(\phi - y) = R(y,\phi) - R(\phi)$$

where the inequality is true because $Q(y)$ is nonincreasing in y. We get $R(y) \geq R(y,\phi)$. The same is true for any $\phi < y$. Thus, for any $y \in [\underline{y}, \bar{y}]$, once $\phi(y) \in [\underline{y}, \bar{y}]$, we have $R(y) \geq R(y, \phi(y))$, which implies

$$\int_{\underline{y}}^{\bar{y}} R(y)h(y \mid x)dy - cx \geq \int_{\underline{y}}^{\bar{y}} R(y,\phi(y))h(y \mid x)dy - cx$$

since $h(y \mid x) \geq 0$ for all x, y. Therefore, truth-telling is the optimal strategy for the agent.

Q.E.D.

Proof of Lemma 2: Suppose the conclusion in Lemma 2 does not hold, i.e., $x^*(Q) < x^*(\tilde{Q})$. From the first-order condition, we get

$$\int_{\underline{y}}^{\bar{y}} Q(y)H_x(y \mid x^*(Q))dy = \int_{\underline{y}}^{\bar{y}} \tilde{Q}(y)H_x(y \mid x^*(\tilde{Q}))dy$$

$$< \int_{\underline{y}}^{\bar{y}} Q(y)H_x(y \mid x^*(\tilde{Q}))dy$$

because $H_x > 0$. This implies that

$$\int_{\underline{y}}^{\bar{y}} Q(y)\Big[H_x(y \mid x^*(Q)) - H_x(y \mid x^*(\tilde{Q}))\Big]\, dy < 0.$$

But $H_{xx} < 0$ and $x^*(Q) < x^*(\tilde{Q})$ imply $H_x(y \mid x^*(Q)) \geq H_x(y \mid x^*(\tilde{Q}))$ for all $y \in (\underline{y}, \bar{y})$. The above inequality cannot be true. This is a contradiction. Thus, $x^*(Q) \geq x^*(\tilde{Q})$.

Let EU and $E\tilde{U}$ be the expected utilities for the firm under these two contracts, respectively. When $R(\bar{y}) = \tilde{R}(\bar{y})$ we get

$$\begin{aligned} E\tilde{U} - EU &= \int_{\underline{y}}^{\bar{y}} \Big[\tilde{Q}(y)H(y \mid x^*(\tilde{Q})) - Q(y)H(y \mid x^*)\Big]\, dy - cx^*(\tilde{Q})) + cx^*(Q)) \\ &\leq \int_{\underline{y}}^{\bar{y}} Q(y)\Big[H(y \mid x^*(\tilde{Q})) - H(y \mid x^*(Q))\Big]\, dy - c\Big[x^*(\tilde{Q})) - x^*(Q))\Big] \end{aligned}$$

Using Taylor's expansion, there exists $\xi \in [0, \bar{x}]$ such that

$$H(y \mid x^*(\tilde{Q})) - H(y \mid x^*(Q)) = H_x(y \mid x^*(Q))\delta + H_{xx}(y \mid \xi)\delta^2 / 2,$$

where $\delta = x^*(\tilde{Q}) - x^*(Q)$. Substituting these two equations into the above inequality, we get

$$E\tilde{U} - EU \leq \Big[\int_{\underline{y}}^{\bar{y}} Q(y)H_x(y \mid x^*(Q)) - c\Big]\,\delta + \frac{1}{2}\delta^2 \int_{\underline{y}}^{\bar{y}} Q(y)H_{xx}(y \mid \xi)dy.$$

The first term on the right-hand side equals zero by the first order condition, and the second term is no larger than zero. Thus, $EU \geq E\tilde{U}$.

Q.E.D.

Proof of Proposition 4: Since $H(y \mid x) = 1 - [1 - F(y)]^x$ for all $x \geq 0$ and $y \in [\underline{y}, \bar{y}]$, then

$$\frac{H(y \mid x)}{h(y \mid x)} = \frac{[1 - F(y)]^{1-x} - [1 - F(y)]}{x\, f(y)},$$

$$\frac{H_x(y \mid x)}{h(y \mid x)} = -\,\frac{[1 - F(y)]\log[1 - F(y)]}{x\, f(y)},$$

$$\frac{\partial\Big[H(y \mid x)/h(y \mid x)\Big]}{\partial y} = \frac{1}{x}\,[1 - (1-x)[1 - F(y)]^{-x}] - \frac{f'(y)}{xf^2(y)}\Big[[1 - F(y)]^{1-x} - [1 - F(y)]\Big]$$

$$\geq [1 - F(y)]^{-x} \geq 0$$

$$\frac{\partial\left[H_x(y \mid x)/h(y \mid x)\right]}{\partial y} = \frac{1}{x} + \frac{1}{x}\log[1 - F(y)]\left[1 + \frac{f'(y)[1 - F(y)]}{f^2(y)}\right] \geq 0.$$

The above two inequalities hold because the assumption $\frac{f(y)}{1 - F(y)}$ is nonincreasing in y. The conditions in Lemma 3 are satisfied and thus $\hat{\mu} > 0$ by Lemma 3. Now let

$$\phi(y) = H(y \mid \hat{x}) - \hat{\mu}H_x(y \mid \hat{x})$$

$$= 1 - [1 - F(y)]^{\hat{x}} + \hat{\mu}[1 - F(y)]^{\hat{x}}\log[1 - F(y)].$$

Then $\phi(\underline{y}) = 0$, $\phi(\bar{y}) = 1$, and

$$\phi'(y) = [1 - F(y)]^{\hat{x}-1} f(y)\left[\hat{x} - \hat{\mu} - \hat{\mu}\hat{x}\log[1 - F(y)]\right]$$

If $\hat{x} \geq \hat{\mu}$, then $\phi'(y) > 0$ for all $y \in (\underline{y}, \bar{y})$. Thus, $\phi(y) > \phi(\underline{y}) = 0$. That is, $H(y \mid \hat{x}) - \hat{\mu}H_x(y \mid \hat{x}) > 0$ for all $y \in (\underline{y}, \bar{y})$. Since $B''(Q) < 0$, from (1) and (6) we obtain $\hat{Q}(y) < Q^*(y)$ for all $y \in (\underline{y}, \bar{y})$.

If $\hat{x} < \hat{\mu}$, then there exists a $\tilde{y} \in (\underline{y}, \bar{y})$ such that

$$F(\tilde{y}) = 1 - e^{-\frac{\hat{\mu} - \hat{x}}{\hat{\mu}\hat{x}}}$$

Thus, $\phi'(y) < 0$ for $y < \tilde{y}$ and $\phi'(y) > 0$ for $y > \tilde{y}$. Since $\phi(\underline{y}) = 0$, then $\phi(y) < 0$ for some $y > \underline{y}$. But we know $\phi(\bar{y}) = 1$. Thus there exists a $y_0 \in (\underline{y}, \bar{y})$ such that $\phi(y_0) = 0$, $\phi(y) < 0$ for $y < y_0$, and $\phi(y) > 0$ for $y > y_0$. Therefore, from (1) and (6) we have $\hat{Q}(y_0) = Q^*(y_0)$, $\hat{Q}(y) > Q^*(y)$ for $y \in (\underline{y}, y_0)$, and $\hat{Q}(y) < Q^*(y)$ for all $y \in (y_0, \bar{y}]$.

Q.E.D.

Proof of Proposition 7: We show this result for symmetric Nash equilibrium of R&D

expenditure. Under condition (A), the optimal quantity $Q(y)$, equilibrium R&D expenditures, and μ are simultaneously determined by (12), (14′), and (15′). If the optimal quantity schedule $Q(y)$ is independent of n for any y, then from (12) we know that the total expenditure x is also independent of n. This together with (15′) implies that μ is independent of n. Consider the symmetric equilibrium of R&D expenditure. Let $x(n)$ be the equilibrium individual expenditure. Taking the derivative of both sides of (14′) with respect to n, we get

$$0=\left[ze^{z}-e^{z}+1+\frac{\mu z}{x(n)}\right]\frac{\partial x(n)}{\partial n}\frac{1}{x(n)^{2}} \tag{14''}$$

for all $y \in (\underline{y}, \hat{y})$, where $\hat{y}(>\underline{y})$ represents the marginal cost level of the marginal firm and is independent of n, and $z=z(y)=-x(n)\log[1-F(y)]>0$ for $y \in (\underline{y}, \bar{y})$. Let $\phi(z)=ze^{z}-e^{z}+1+\frac{\mu z}{x(n)}$, then

$$\frac{d\phi[z(y)]}{dy}=(ze^{z}+\frac{\mu}{x_n})\frac{x_n f(y)}{1-F(y)}$$

cannot be zero for all $y \in (\underline{y}, \hat{y})$ since $\mu>0$ by Lemma 6. Thus $\phi(z(y))$ cannot be zero for all $y \in (\underline{y}, \hat{y})$. Since $x=nx(n)$ is independent of n, $\frac{\partial x_n}{\partial n}$ is not zero. Thus, (14″) cannot be true. The contradiction implies that $Q(y)$ does depend on n.

From (12) again, the total expenditure in R&D x will depend on n. Since the expected minimum marginal cost equals $E(y^{m}\mid x)=\underline{y}+\int_{\underline{y}}^{\bar{y}}[1-F(y)]^{x}dy$, it also depends on n.

Q.E.D.

Proof of Proposition 8: Under technology (B_1), the Nash equilibrium of R&D expenditures is asymmetric by Proposition 5. Under technology (A), we consider symmetric equilibrium only. At the optimum solution $[\hat{Q}(y), \hat{R}(\bar{y}), \hat{x}]$ to (P_n), the buyer's expected welfare can be written as

$$EW(n) = n\int_{\underline{y}}^{\bar{y}}\left[B(\hat{Q}(y)) - y\hat{Q}(y)\right][1 - H(y\mid\hat{x})]^{n-1}dH(y\mid\hat{x}) \tag{8.1}$$

$$- n\int_{\underline{y}}^{\bar{y}}\hat{Q}(y)[1 - H(y\mid\hat{x})]^{n-1}H(y\mid\hat{x})dy.$$

Viewing n as a continuous variable at this moment, we obtain

$$\frac{dEW(n)}{dn} = \left[\frac{\partial EW}{\partial\hat{Q}(.)}\frac{\partial\hat{Q}(.)}{\partial n}\right] + \frac{\partial EW}{\partial\hat{x}}\frac{\partial\hat{x}}{\partial n} + \frac{\partial EW}{\partial n}, \tag{8.2}$$

where

$$\left[\frac{\partial EW}{\partial\hat{Q}(.)}\frac{\partial\hat{Q}(.)}{\partial n}\right] = n\int_{\underline{y}}^{\bar{y}}[1 - H(y\mid\hat{x})]^{n-1}\frac{\partial\hat{Q}(y)}{\partial n}\left[B'(\hat{Q}(y)) - y - \frac{H(y\mid\hat{x})}{h(y\mid\hat{x})}\right]dH(y\mid\hat{x})$$

$$= -n\hat{\mu}\int_{\underline{y}}^{\bar{y}}[1 - H(y\mid\hat{x})]^{n-1}H_x(y\mid\hat{x})\frac{\partial\hat{Q}(y)}{\partial n}dy. \tag{8.3}$$

The last equality holds due to (14). Integrating (8.1) by parts, we can rewrite $EW(n)$ as

$$EW(n) = B(\overline{Q}) - \underline{y}\overline{Q} + \int_{\underline{y}}^{\bar{y}}\left[B'(\hat{Q}(y)) - y\right]\hat{Q}'(y)[1 - H(y\mid\hat{x})]^{n}dy$$

$$-\int_{\underline{y}}^{\bar{y}}\hat{Q}(y)[1 - H(y\mid\hat{x})]^{n-1}\left[1 + (n-1)H(y\mid\hat{x})\right]dy. \tag{8.4}$$

Then

$$\frac{\partial EW}{\partial\hat{x}} = -n\int_{\underline{y}}^{\bar{y}}\left[B'(\hat{Q}(y)) - y\right]\hat{Q}'(y)[1 - H(y\mid\hat{x})]^{n-1}H_x(y\mid\hat{x})dy$$

$$-(n-1)\int_{\underline{y}}^{\bar{y}}\hat{Q}(y)[1 - H(y\mid\hat{x})]^{n-1}H_x(y\mid\hat{x})dy$$

$$+(n-1)\int_{\underline{y}}^{\bar{y}}\hat{Q}(y)[1 - H(y\mid\hat{x})]^{n-2}H_x(y\mid\hat{x})\left[1 + (n-1)H(y\mid\hat{x})\right]dy$$

$$= -n\hat{\mu}\int_{\underline{y}}^{\bar{y}}\hat{Q}(y)[1 - H(y\mid\hat{x})]^{n-1}H_{xx}(y\mid\hat{x})dy$$

$$+ n(n-1)\int_{\underline{y}}^{\bar{y}} \hat{Q}(y)[1-H(y\mid\hat{x})]^{n-2}H_x(y\mid\hat{x})H(y\mid\hat{x})dy. \tag{8.5}$$

Taking the derivatives of both sides of (11) with respect to n, we get

$$\int_{\underline{y}}^{\bar{y}} \frac{\partial\hat{Q}(y)}{\partial n}[1-H(y\mid\hat{x})]^{n-1}H_x(y\mid\hat{x})dy + \int_{\underline{y}}^{\bar{y}} \hat{Q}(y)[1-H(y\mid\hat{x})]^{n-1}H_{xx}(y\mid\hat{x})dy\frac{\partial\hat{x}}{\partial n}$$

$$-(n-1)\int_{\underline{y}}^{\bar{y}} \hat{Q}(y)[1-H(y\mid\hat{x})]^{n-2}H_x^2(y\mid\hat{x})dy\frac{\partial\hat{x}}{\partial n}$$

$$+\int_{\underline{y}}^{\bar{y}} \hat{Q}(y)[1-H(y\mid\hat{x})]^{n-1}H_x(y\mid\hat{x})\log[1-H(y\mid\hat{x})]dy = 0. \tag{8.6}$$

Combining (8.3) and (8.5) with (8.6), we obtain

$$\left[\frac{\partial EW}{\partial\hat{Q}(.)}\frac{\partial\hat{Q}(.)}{\partial n}\right] + \frac{\partial EW}{\partial\hat{x}}\frac{\partial\hat{x}}{\partial n} = -n(n-1)\hat{\mu}\int_{\underline{y}}^{\bar{y}} \hat{Q}(y)[1-H(y\mid\hat{x})]^{n-2}H_x^2(y\mid\hat{x})dy\frac{\partial\hat{x}}{\partial n}$$

$$+ n\hat{\mu}\int_{\underline{y}}^{\bar{y}} \hat{Q}(y)[1-H(y\mid\hat{x})]^{n-1}H_x(y\mid\hat{x})\log[1-H(y\mid\hat{x})]dy$$

$$+ n(n-1)\int_{\underline{y}}^{\bar{y}} \hat{Q}(y)[1-H(y\mid\hat{x})]^{n-2}h(y\mid\hat{x})H_x(y\mid\hat{x})dy\frac{\partial\hat{x}}{\partial n} \tag{8.7}$$

On the other hand, we have

$$\frac{\partial EW}{\partial n} = \int_{\underline{y}}^{\bar{y}} \left[B'(\hat{Q}(y)) - y\right]\hat{Q}'(y)[1-H(y\mid\hat{x})]^{n}\log[1-H(y\mid\hat{x})]dy$$

$$-\int_{\underline{y}}^{\bar{y}} \hat{Q}(y)[1-H(y\mid\hat{x})]^{n-1}\left[1+(n-1)H(y\mid\hat{x})\right]\log[1-H(y\mid\hat{x})]dy$$

$$-\int_{\underline{y}}^{\bar{y}} \hat{Q}(y)[1-H(y\mid\hat{x})]^{n-1}H(y\mid\hat{x})dy \tag{8.8}$$

Let $H(y\mid x) = 1-u(y)^{\alpha(x)}$, where $u(y) = 1-F(y)$, $\alpha(x)\geq 0$, $\alpha'(x) > 0$, $\alpha''(x)\leq 0$, and $\alpha(0) = 0$. When $\alpha(x) = x$, $H(y\mid\hat{x})$ represents technology (A). When $\alpha''(x) < 0$, $H(y\mid x)$ represents technology (B_1). Substituting this distribution into (8.7) and (8.8), and combining with (8.2), we obtain the following

$$\frac{dEW(n)}{dn} = (n-1)\frac{\partial(n\alpha(\hat{x}))}{\partial n}\int_{\underline{y}}^{\bar{y}} \hat{Q}(y)u(y)^{(n-1)\alpha(\hat{x})}g_1(y)\log u(y)dy$$

$$+\hat{\mu}\frac{\alpha(\hat{x})\alpha''(\hat{x})}{\alpha'(\hat{x})}\int_{\underline{y}}^{\bar{y}}\hat{Q}(y)u(y)^{n\alpha(\hat{x})}\log u(y)dy$$

$$+\int_{\underline{y}}^{\bar{y}}\hat{Q}(y)u(y)^{(n-1)\alpha(\hat{x})}g_2(y)dy \tag{8.9}$$

where

$$g_1(y)=u(y)^{\alpha(\hat{x})}-1-\hat{\mu}\alpha'(\hat{x})u(y)^{\alpha(\hat{x})}\log u(y)$$

$$g_2(y)=u(y)^{\alpha(\hat{x})}-1-\alpha(\hat{x})\log u(y)$$

Let $n=1$ in (8.9), we have

$$\frac{dEW(n)}{dn}\mid_{n=1}=\hat{\mu}\frac{\alpha(\hat{x})\alpha''(\hat{x})}{\alpha'(\hat{x})}\int_{\underline{y}}^{\bar{y}}\hat{Q}(y)u(y)^{\alpha(\hat{x})}\log u(y)dy+\int_{\underline{y}}^{\bar{y}}\hat{Q}(y)g_2(y)dy \tag{8.10}$$

Since $\hat{\mu}>0$ by Lemma 6, $\alpha''(\hat{x})\le 0$ by assumptions, and $\log u(y)<0$ and $g_2(y)>0$ for all $y\in(\underline{y},\bar{y})$, (8.10) implies $\frac{dEW(n)}{dn}\mid_{n=1}>0.$

Q.E.D.

Proof of Proposition 9: We consider symmetric equilibrium only. First, since $\sum_{i=1}^{n} x_i = nx(n)$ has a finite limit $x_0\ge 0$ by the assumption, then when $n\to\infty$ we obtain

$$E(y^m\mid x(n))=\underline{y}+\int_{\underline{y}}^{\bar{y}}[1-F(y)]^{nx(n)}dy\ \rightarrow\ \underline{y}+\int_{\underline{y}}^{\bar{y}}[1-F(y)]^{x_0}dy=y_0>\underline{y}$$

Second, by Lemma 6, $\mu_n>0$. Suppose that μ_n has a limit μ_0. Then $\mu_0\ge 0$. We will show that $\mu_0=0$ and $\frac{\mu_n}{x_n}\rightarrow 1$. Combining (14′) with (15′), we obtain

$$\int_{\underline{y}}^{\bar{y}}Q_n'(y)[1-F(y)]^{x+1}\log[1-F(y)]\left[\sum_{i=1}^{n}[1-F(y)]^{-x_i}-n+n\mu_n\log[1-F(y)]\right]\frac{1}{nx(n)f(y)}dy$$

$$=\mu_n\int_{\underline{y}}^{\bar{y}}Q_n(y)[1-F(y)]^{x}\log^2[1-F(y)]dy$$

Let $n \to \infty$, then

$$\left[\lim \frac{\mu_n}{x(n)} - 1\right] \int_{\underline{y}}^{\bar{y}} Q_0(y)[1-F(y)]^{x_0}\log^2[1-F(y)]dy$$

$$= \mu_0 \int_{\underline{y}}^{\bar{y}} Q_0(y)[1-F(y)]^{x_0}\log^2[1-F(y)]dy$$

Since $Q_n(y)$ decreases with y by the assumption, then $Q_n'(y) \leq 0$ for all y and n, and hence $Q_0'(y) \leq 0$. If $\mu_0 > 0$, then the above equation implies that $\lim\limits_{n\to\infty} \frac{\mu_n}{x(n)} - 1 \leq 0$. That is, $\mu_n \leq x(n)$ when n is large enough. Thus, when $n \to \infty$, $\mu_n \to 0$ since $x_n \to 0$. Therefore, $\mu_0 = 0$ and hence $\lim \frac{\mu_n}{x(n)} = 1$. Thus, (14′) implies that $B'(Q_0(y)) = y$.

Q.E.D.

NOTES

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1 The economic theory of procurement has much in common with the economic theory of regulation and auctioning. Because of asymmetric information, designing incentive procurement contracts is similar to regulatory mechanism design. Caillaud, Guesnerie, Rey, and Tirole (1988) review the recent literature on government regulation under asymmetric information.

2 For a more complete survey of the recent literature on research, development, and diffusion, see Reinganum (1988).

3 Research and development in defense procurements is characterized by a particularly high degree of uncertainty. First, the level of innovation is uncertain. Second, the government cannot easily verify the outcome of the innovation activity. Third, R&D decisions or efforts directed towards innovation by the firms are difficult for the government to observe directly. Because of these problems the government has difficulties in rewarding and encouraging innovation activity efficiently. But, one way to reward successful innovation is to create a prize that is related to the production of the item being procured.

4 Laffont and Tirole (1988), and Fudenberg and Tirole (1988) have recently discussed the renegotiation issue in an agency model with moral hazard and in a repeated adverse selection model, respectively. Our model of R&D and

production includes both adverse selection and moral hazard problems. Because of this, the design of renegotiation-proof contracts is different from that in Fudenberg and Tirole (1988) and Laffont and Tirole (1988). The optimal production contract here should not only be renegotiation-proof, but also give the firm incentives to reveal its private information and to invest enough in R&D. Therefore, it is important to investigate commitment and renegotiation in procurement and contracting in a two-stage model with both adverse selection and moral hazard.

5 It is often possible for the firm who wins the procurement contract to continue to exert some effort in order to reduce production cost further. If the production cost is ex post observable to the government, incentive contracts could be designed to give the chosen contractor an incentive to reduce production cost in both the R&D and production stage. In this case, incentive contracts can be based on both the R&D outcome and the realized production cost. Similar to Laffont and Tirole (1986), and McAfee and McMillan (1987), it can be shown that a menu of linear contracts in both the expected cost and the ex post observed cost is optimal in the case of R&D and production uncertainty. But R&D changes the coefficients of the linear contracts.

6 We can always rescale the variable x such that the R&D cost is linear in x.

7 In the Appendix, we offer a proof that the Revelation Principal applies to our model.

8 If the number of potential firms is endogenous, it can be shown that there exists a free entry equilibrium under which the equilibrium number of firms, the level of investment in R&D, and the break-even level of production cost are simultaneously determined.

9 The innovation processes of different firms may not be the same, but usually have some common elements of technological uncertainty, one of which might be the general difficulty of cost reduction. Because of these common factors the potential R&D outcomes among different firms may be correlated. It may then be possible for the government to extract the full surplus from the contracting firms by designing appropriate incentive contracts. But this full extraction of surplus may discourage R&D. We do not discuss this issue here in detail.

10 In general, production contracts could be allowed to depend on all bids $(b_1, \ldots, b_n)$ or messages. However, we consider here a variable quantity auction in which the lowest bid firm wins the contract and the quantity depends on the lowest bid only. Without R&D, some regularity conditions are sufficient for this special contract to be optimal within general mechanisms (see Riordan and Sappington 1987). It remains an open question whether, with R&D, offering a contract $[Q_i(b_1, \ldots, b_n), P_i(b_1, \ldots, b_n)]$ to the ith firm improves the government's expected welfare.

11 We should be careful about this implication because some non-primitive conditions have been used in Proposition 9. Certainly, more research is needed to make this point clear.

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CHAPTER 3

ENTRY AND R&D COSTS IN PROCUREMENT CONTRACTING *

3.1 INTRODUCTION

In competitive procurements and contracting, potential firms usually participate in R&D activities. Some firms may not find it profitable to enter into competitive bidding just because precontract R&D is very costly. As Besen and Terasawa (1987) argued, in many government procurement cases, only a small number of potential firms choose to submit bids. Also, according to Hendricks, Porter, and Boudreau (1987), potential firms decide how much information to collect before participating in competitive bidding. Thus, precontract R&D behavior and a small number of bidders are significant and important in procurement contracting. The firms's decisions to acquire information and to submit bids usually depend on the R&D process, the costs of R&D, the costs of preparing bids, and the type of competitive bidding procedure in place.

The existing literature on auctions and procurements,[1] except for French and McCormick (1984), and McAfee and McMillan (1987a), typically assumes that the number of bidders is exogenous and constant,[2] and that each bidder has certain private information. Under these basic assumptions, different auction procedures are compared and the optimal auction mechanisms are also designed in this literature. When the number of bidders goes up, as Holt (1980) showed in the government procurement of a unit of goods, the expected procurement cost to the government decreases. When the government with incomplete knowledge about the firms' production costs demands many units of the goods, it should procure the goods at the level at which the marginal benefit equals the marginal virtual cost.[3] The government discriminates as a monopsonist. Asymmetry of information causes a welfare loss for the government. The more the firms compete for the procurement contract, the less the welfare loss. As the number of firms goes to infinity, the welfare loss disappears; hence the most efficient outcome is reached. Therefore, competition is good for the government.

R&D and entry behavior in auction and procurement processes, however, have not been examined carefully in this literature. Although French and McCormick (1984), and McAfee and McMillan (1987a) have considered fixed entry costs and entry equilibria, prebidding R&D decisions have not been formally modelled.[4] On the one hand, if fewer firms participate in the competitive bidding, the contract will be more profitable to the winning firm and each firm will tend to invest more in R&D. If the expected profit of the winning firm is positive, then more firms will enter the auction. On the other hand, the buyer may want to control the firms' R&D decisions through the choice of the contract auction rules. Thus, the following questions are of interest. What is the equilibrium number of bidders under free entry and how does each potential firm make precontract R&D decisions? Is free entry of firms an optimal policy for the buyer? Moreover, is it socially optimal? These are the questions that I intend to answer in this chapter.

I provide a model of competitive procurement with precontract R&D. The number of firms is viewed as an endogenous variable in the model. I distinguish active firms (or informed firms) from actual bidders. A firm is active if it invests in R&D and becomes informed about demand and production cost. An actual bidder is a firm that submits a bid in the auction for the production contract. Similar to Chapter 2, the R&D activity by each firm is formally modelled as a stochastic process with certain R&D

costs. I also allow for each firm a bid-preparation cost similar to that in Samuelson (1985). These R&D costs and bid-preparation costs affect the number of informed firms and the number of actual bidders. Under free entry, the equilibrium number of informed firms, the expected number of actual bidders, and the level of investment in R&D, are *simultaneously* determined and depend on the R&D costs, bid-preparation costs, and the type of auction.

The next section describes the model and the equilibrium concept I am going to use. Then I show the existence and uniqueness of symmetric perfect free-entry equilibrium under the first-price sealed-bid auction with a given reservation price. Some interesting comparative analysis is provided at the symmetric perfect free-entry equilibrium.

First, as the buyer's reservation price increases, both the equilibrium number of active firms and the total level of R&D investments increase. Thus, raising the reservation price can encourage participation in R&D and stimulates R&D investments.

Second, the total equilibrium expenditure on R&D among all firms decreases with the marginal cost of R&D. As the marginal cost of R&D goes to zero, the total expenditure by all firms goes to infinity no matter how large the fixed cost of R&D. On the other hand, as the marginal cost of R&D is relatively high, it will be very costly for any firm to conduct any R&D activity. Thus, the marginal cost of R&D is the key determinant of the total R&D expenditure.

Third, without a fixed cost of R&D, free entry causes an infinite number of firms to enter the R&D process. But with a positive fixed cost of R&D, only a finite number of firms will invest in R&D. The higher the fixed cost of R&D, the fewer the equilibrium number of active firms. In other words, the fixed cost of R&D plays an important role in determining the equilibrium number of active firms and the expected number of actual bidders.

From the point of view of the buyer's optimal strategy, I show the following: First, consider the fixed-scale R&D technology. If the buyer's opportunity cost of procuring the good somewhere else is relatively high, no reservation price is necessary for the buyer. If the buyer's opportunity cost is relatively low, however, he should choose a reservation price that is lower than the opportunity cost. This optimal reservation price is higher than the reservation price the buyer would have chosen if there were no fixed R&D cost. Although there exists a distortion of the efficient outcome, the presence of positive fixed costs of R&D makes that distortion smaller. In both cases, the optimal number of informed firms for the buyer enter the procurement process under free entry. That is, free entry is optimal for the buyer conditional on the appropriate choice of a reservation price. Second, when R&D is subject to constant marginal returns to scale on expenditure, if the buyer is able to control each firm's R&D investment costlessly, he does not want to leave the R&D decisions and entry decisions to the firms. In general, he wants each firm to invest more in R&D than it wants to. Thus, when R&D decisions are not observable by the buyer, a moral hazard problem arises. Taking each firm's R&D decision as a constraint, the buyer still prefers free entry of firms through an appropriate choice of a reservation price that may be either higher than or lower than his true opportunity cost.

Social optimality is characterized in Section 3.5 by the minimization of the total expected social costs of procurement. Under either the variable, constant marginal return R&D technology, or the fixed-scale R&D technology, social optimality requires the buyer to set his reservation price equal to the minimum of his opportunity cost and the highest possible production cost observation among all firms. In contrast to the buyer's preferences, when each firm's R&D decision is subject to constant marginal returns to scale on expenditure, society prefers only one firm to conduct all of the R&D and production. The comparison between the buyer's optimum and the social optimum shows that the type of R&D technology plays an important role in determining the optimal R&D and procurement policies for the buyer and for society.

3.2 THE MODEL

There is a single buyer (e.g., the government) who seeks to procure one unit of a certain novel good or service. The buyer wants to minimize the expected total costs of this procurement.

There are many potential firms; each of which has the potential to produce a unit of the good at an unknown cost y. Firms are assumed to be symmetric in this chapter. Each firm can invest in R&D for information about cost reduction and will observe a potential cost y. I consider two types of R&D technology: a fixed-scale process, in which either R&D is done upon a fixed cost or it is not, and a variable scale process with constant marginal returns to R&D expenditure.

In the first case, each firm will make a take-it-or-leave-it decision whether to reduce its production cost. If the firm decides to invest in R&D then it observes a production cost y at a fixed cost C of investment and has a belief that other firms' observations of production cost are drawn independently from a cumulative distribution $F(y)$ with the support $[\underline{y}, \bar{y}]$. This is similar to the case studied by French and McCormick (1984) and McAfee and McMillan (1987). They consider the fixed cost C as an entry fee in the competitive bidding.

The fixed-scale R&D technology is simply a one-shot experiment process. I am more interested in a relatively sophisticated R&D technology in this chapter. Suppose that a firm repeats the fixed-scale experiment many times. In each experiment, the firm observes a production cost y, which is drawn from a cumulative distribution $F(y)$ with the support $[\underline{y}, \bar{y}]$. Except for a fixed cost C_2 at the beginning of the experiments, each experiment costs the firm C_1. If the firm repeats this experiment k times independently, it will observe k numbers of production cost $(y_1, \ldots, y_k)$ at the cost $C_1 k + C_2$. The minimum cost level y_m of $y_1, \ldots, y_{k-1}$, and y_k is subject to the distribution of the lowest-order statistic:

$$H(y \mid k) = 1 - [1 - F(y)]^k.$$

Certainly, the minimum cost y_m is the outcome the firm picks from this process of repeated experiments.

In general, let x be a variable that represents the firm's investment level in R&D. Based upon x, each firm observes a production cost y, which is assumed to be independently drawn from an identical cumulative distribution

$$H(y \mid x) = 1 - [1 - F(y)]^x \tag{1a}$$

with the fixed support $[\underline{y}, \bar{y}]$, $\bar{y} > \underline{y} \geq 0$, where $F(y)$ is a continuously differentiable cumulative distribution function with support $[\underline{y}, \bar{y}]$ and density function $f(y)$, and $x \in [0, +\infty)$ is the level of investment in R&D.[5] I assume $f(y) > 0$ for all $y \in [\underline{y}, \bar{y}]$ and $y + F(y)/f(y)$ is increasing in y. Let $G(y) \equiv 1 - F(y)$ for all y. The R&D cost is assumed to be linear and the same for all firms:

$$C(x) = C_1 x + C_2, \tag{1b}$$

where $C_1 \geq 0$ is the marginal cost of R&D investment and $C_2 \geq 0$ is the fixed cost of R&D.

I model the competitive procurement as a three-stage process. In the first stage, the buyer announces and commits to the general rules of procurements. I consider the first-price sealed-bid auction with an announced reservation price.[6] In the second stage, R&D is conducted during which each firm invests in R&D and acquires information about the production cost. In the final stage, a competitive bidding procedure is conducted in which the buyer procures the good via the sealed-bid auction announced at the beginning.

More specifically, this process can be described as follows. First, the buyer announces the rules of the sealed-bid auction including a reservation price r, which is no higher than the highest possible cost level $\bar{y}$. The lowest bid will be accepted if it is below r. Second, the firm will calculate its expected profit from bidding and decide to invest in R&D if this profit is no less than its R&D cost. Third, based on the observed production cost information y, the firm will bid unless the expected profit is less than the bid preparation cost K that each has to pay to participate in the bidding process. The winner is then chosen as the contractor for production.

The buyer is able to procure the good elsewhere at the cost y_0 if the lowest bid is higher than the reservation price r. A special case is when y_0 is very high, which means that there are no substitutes available for the buyer. Let $y_m = \min(\bar{y}, y_0)$ represent the minimum of the highest possible production cost $\bar{y}$, which the firms observe, and the buyer's opportunity cost y_0.

Suppose that each firm learns its production cost without any cost of R&D. Given a fixed number of firms and their types, each firm submits a bid upon paying a cost $K > 0$. This is the case in Samuelson (1985). Therefore, the present study can also be viewed as a generalization of the models by French and McCormick (1984), McAfee and McMillan (1987), and Samuelson (1987).

In the next section, I will analyze a free-entry equilibrium for a particular reservation price of the buyer in a sealed-bid auction. A perfect free-entry equilibrium of the final two-stage game consists of a market structure n, an investment strategy $(x_1, \ldots, x_n)$, and a bidding strategy $(B_1(y_1), \ldots, B_n(y_n))$ such that the following apply: (i) the bidding strategy $(B_1(y_1), \ldots, B_n(y_n))$ is a Bayes-Nash equilibrium, (ii) the investment strategy $(x_1, \ldots, x_n)$ is a noncooperative Nash equilibrium, (iii) each of n firms in the market must anticipate nonnegative profits, and (iv) $n+1$ firms earn negative profits. Entries of all firms are simultaneous decisions, not sequential. Under the fixed-scale R&D technology, equilibria in both (i) and (ii) are symmetric. Under the variable constant marginal return R&D technology, however, asymmetric equilibria may exist.[7] I will only consider symmetric equilibria in both (i) and (ii) and call $(n, x, B(y))$ a symmetric perfect free-entry equilibrium. I will show that, for a given reservation price, there exists a unique symmetric perfect free-entry equilibrium. In Section 3.4, we will see that the buyer prefers the free entry of firms and I will calculate which symmetric perfect free-entry equilibrium he should select by choosing an appropriate reservation price. Considerations from the point of view of social optimality are discussed in Section 3.5.

3.3 SYMMETRIC PERFECT FREE-ENTRY EQUILIBRIUM

Given the rules of the sealed-bid auction with a reservation price r, suppose that a firm i believes that n firms including itself might invest in R&D and compete in the contract auction. I will show how the equilibrium number of firms is determined later on. For variable scale R&D technology (1a) and (1b), firm i invests x_i in R&D at a cost of $C(x_i)$ and privately learns the new production cost y_i of supplying the good, which is independently drawn from the distribution $H(y_i \mid x_i)$. Consider symmetric noncooperative Nash equilibria $x_i = x_j = x$ for all i and j. Firm costs are generated independently from a common distribution function $H(y \mid x)$ with the support $[\underline{y}, \overline{y}]$.

Suppose that in the auction, firm i uses a strategy $B_i = B_i(y_i)$,[8] which is strictly increasing in y_i, $i = 1, \ldots, n$. Firm i with cost observation y_i will generate the following profit from bidding by submitting B_i:

$$\pi_i(B_i, y_i) = (B_i - y_i) Prob\,(\text{winning})$$

$$= (B_i - y_i)\Big[1 - H(B_j^{-1}(B_i) \mid x)\Big]^{n-1}.$$

Consider symmetric bidding strategies $B_i(y) = B_j(y) = B(y)$ for any i and j as the Bayes-Nash equilibrium. By the Envelope Theorem, at the Bayes-Nash equilibrium

$$\frac{d\pi(B(y), y)}{dy} = -\,[1 - H(y \mid x)]^{n-1}. \tag{2}$$

The submission of a bid requires the expenditure of K in preparation costs. Free-exit implies $\pi(B(y), y) - K \geq 0$. Thus, the firm will not bid if costs are above some break-even level $\hat{y}$. The marginal firm $\hat{y}$ is indifferent between entering a bid or not. If the marginal firm makes a bid, the optimal bid is the reservation price $B(\hat{y}) = r$. The probability that the marginal firm $\hat{y}$ wins is $[1 - H(\hat{y} \mid x)]^{n-1} = [1 - F(\hat{y})]^{(n-1)x} \equiv G(\hat{y})^{(n-1)x}$

and the marginal firm's expected profit will be $\pi(B(\hat{y}), \hat{y}) = (r - \hat{y})G(\hat{y})^{(n-1)x}$. Thus, the marginal firm $\hat{y}$ is determined by the following free-exit condition (FE):

$$(r - \hat{y})G(\hat{y})^{(n-1)x} = K. \tag{FE}$$

Then, from (2) and (FE), we have

$$\pi(B(y), y) = K + \int_y^{\hat{y}} G(t)^{(n-1)x}\,dt$$

for all $y \le \hat{y}$. The firm with cost $y > \hat{y}$ will not bid because its expected profit from bidding will be less than the bid preparation cost K. From (FE), $\hat{y}$ is strictly lower than the reservation price r because of the positive bid preparation cost. If $K = 0$ then $\hat{y} = r$. That is, if no such cost existed, the marginal firm would be the firm with a cost observation equal to the reservation price.

On the other hand, we know that, at the symmetric bidding equilibrium $B = B(y)$,

$$\pi(B(y), y) = (B(y) - y)G(y)^{(n-1)x}.$$

Comparing the above two expressions, we can easily write the equilibrium bidding function $B(y)$ as the following:

$$B(y) = y + \frac{K}{G(y)^{(n-1)x}} + \frac{\int_y^{\hat{y}} G(t)^{(n-1)x}\,dt}{G(y)^{(n-1)x}} \tag{3}$$

for all $y \in [\underline{y}, \hat{y})$ and $B(\hat{y}) = r$. Because of the bid-preparation cost K and the firms' private information about y, each firm intends to bid a higher level than the true production cost y. The equilibrium bid function consists of the true production cost y, the information cost, and the bid preparation cost. From (3), the equilibrium bidding function $B(y)$ is completely determined by n, x, and $\hat{y}$.

Suppose firm i invests x_i in R&D. Since $(n-1)x$ in the expression of $\pi(B(y), y)$ is the total expenditure on R&D by the other $n-1$ firm and independent of x_i, firm i's total expected profits from bidding will be

$$\int_{\underline{y}}^{\hat{y}} \left[\pi(B(y), y) - K \right] dH(y \mid x_i) = \int_{\underline{y}}^{\hat{y}} G(t)^{(n-1)x} H(t \mid x_i) dt$$

given that each other firm chooses x. At the symmetric Nash equilibrium, firm i will choose $x_i = x$ such that the marginal expected profit equals the marginal cost of investment in R&D. Formally, we have

$$\int_{\underline{y}}^{\hat{y}} G(t)^{nx} \ln G(t) dt + C_1 = 0. \tag{R\&D}$$

The second order condition is satisfied because $-\int_{\underline{y}}^{\hat{y}} G(t)^{(n-1)x + x_i} \ln^2 G(t) dt < 0$ for all $x_i \geq 0$. Let

$$E\pi_n(x, \hat{y}) = \int_{\underline{y}}^{\hat{y}} G(t)^{(n-1)x} H(t \mid x) dt - C_1 x - C_2 \tag{4}$$

be the firm's ex ante expected profit given the symmetric equilibrium strategies of both investment and bidding. Each potential firm enters the R&D process if its expected profit is nonnegative. That is, equilibrium entry gives

$$E\pi_n(x, \hat{y}) \geq 0. \tag{EEa}$$

And any additional entrant $n+1$ earns negative profits:

$$E\pi_{n+1}(x', \hat{y}') < 0, \tag{EEb}$$

where x' and $\hat{y}'$ are the individual R&D expenditure and the break-even cost level that are determined by (FE) and (R&D) when $n+1$ firms simultaneously enter the R&D process.

Since the equilibrium bidding strategy $B(y)$ is completely determined by $(n, x, \hat{y})$, we only have to consider $(n, x, \hat{y})$ for a symmetric perfect free-entry equilibrium. Therefore, for any given reservation price r, equations (3), (FE), (EEa), (EEb), and (R&D) *simultaneously* determine the symmetric perfect free-entry equilibrium $(n_e, x_e, \hat{y}_e)$ with the bidding function in (3), where n_e is the equilibrium number of informed firms, x_e is the each firm's equilibrium investment level in R&D, and $\hat{y}_e$ is the break-even cost level that the informed firm will bid if its cost is no higher than $\hat{y}_e$. The total expenditure in R&D at the equilibrium is $n_e x_e$, denoted by $\bar{x}_e$. At the equilibrium, each informed firm invests x_e in R&D and the firms with cost observations higher than $\hat{y}$ submit bids. The number of firms that actually submit bids is random and subject to a binomial distribution. Thus, the average (or expected) number of actual bidders is $n_a = n_e\, H(\hat{y}_e \mid x_e)$, which depends on the equilibrium number of informed firms, the investment level in R&D, and the break-even cost level. When there is a bid preparation cost, $\hat{y}_e < \bar{y}$ and hence the average number of actual bidders is less than the number of informed firms.

I need to show the existence and uniqueness of symmetric perfect free-entry equilibrium for a given reservation price. I first consider the special case where there is no bid preparation cost. Then, from (FE), $\hat{y}$ is the same as the reservation price r, and (R&D) is a one-variable equation that determines the total investment level $\bar{x}_e$. Substituting $\bar{x}_e$ into (EEa) and (EEb), it should be easy to solve for the individual investment level x_e and the number of firms n_e. I allow the number of firms n to be a continuous variable at this moment and adjust the solution later on. Each firm enters the R&D process until its expected profit is zero and hence the equilibrium-entry conditions (EEa) and (EEb) can be represented by the equality

$$\int_{\underline{y}}^{\hat{y}} G(t)^{(n-1)x} H(t \mid x)\,dt - C_1 x - C_2 = 0. \tag{EE}$$

Then the following are true:

Proposition 1: In the case of $K = 0$, there exists a unique solution $(n_e, x_e, \hat{y}_e)$, with $n_e \in (0, +\infty)$, $x_e \in (0, +\infty)$, and $\hat{y}_e = r$, to the system of equations (FE), (EE), and (R&D) if and only if $C_2 > 0$ and $0 < C_1 < -\int_{\underline{y}}^{r} \ln G(t) dt$. Furthermore, at the equilibrium

(a) $\dfrac{\partial \bar{x}_e}{\partial r} > 0$, $\dfrac{\partial x_e}{\partial r} < 0$, and $\dfrac{\partial n_e}{\partial r} > 0$;

(b) $\dfrac{\partial \bar{x}_e}{\partial C_1} < 0$ and $\dfrac{\partial x_e}{\partial C_1} < 0$; and

(c) $\dfrac{\partial \bar{x}_e}{\partial C_2} = 0$, $\dfrac{\partial n_e}{\partial C_2} < 0$, $\dfrac{\partial x_e}{\partial C_2} > 0$, and $\dfrac{\partial n_a}{\partial C_2} < 0$.

Proof: Since $K = 0$, by definition $\hat{y}_e = r$. Then (R&D) and (EE) form a recursive system. Let

$$\phi(\bar{x}) \equiv \int_{\underline{y}}^{r} G(t)^{\bar{x}} \ln G(t) dt + C_1,$$

then $\phi(0) = \int_{\underline{y}}^{r} \ln G(t) dt + C_1$, $\phi(+\infty) = C_1$, and $\phi'(\bar{x}) > 0$ for all $\bar{x} > 0$. By continuity, $\phi_1(\bar{x})$ has a unique positive root and hence condition (R&D) uniquely determines a solution $0 < \bar{x}_e = n_e x_e < +\infty$ if and only if $0 < C_1 < -\int_{\underline{y}}^{r} \ln G(t) dt$. Similarly, let

$$\psi(x) \equiv \int_{\underline{y}}^{r} G(t)^{\bar{x}_e - x} dt - \int_{\underline{y}}^{r} G(t)^{\bar{x}_e} dt - C_1 x - C_2,$$

then $\psi(0) = -C_2$, $\psi(+\infty) = +\infty$, and $\psi'(x) > 0$ for all $x > 0$ because of equation (R&D). Then by continuity, $\psi(x)$ has a unique positive root x_e if and only if $\psi(0) = -C_2 < 0$, i.e., $C_2 > 0$. Let $n_e = \bar{x}_e / x_e$. Therefore, (FE), (EE), and (R&D) determine a unique solution $(n_e, x_e, \hat{y}_e)$ with $0 < n_e < +\infty$, $0 < x_e < +\infty$, and $\hat{y}_e = r$.

Now, taking the derivatives of both sides of equation (R&D) with respect to $\hat{y}_e = r$, I have

$$\delta_1 \frac{\partial \bar{x}_e}{\partial r} + G(r)^{\bar{x}_e} \ln G(r) = 0, \tag{5}$$

where $\delta_1 = \int_{\underline{y}}^{r} G(t)^{\bar{x}_e} \ln^2 G(t) dt > 0$. Since $0 < G(t) < 1$ for all $t \in (\underline{y}, \bar{y})$, equation (5) implies $\frac{\partial \bar{x}_e}{\partial r} > 0$ for any $r \in (\underline{y}, \bar{y})$.

Taking the derivatives of both sides of equation (EE) with respect to r and using equation (R&D), I get

$$\delta_2 \frac{\partial (n_e - 1)x_e}{\partial r} + G(r)^{(n_e - 1)x_e}[1 - G(r)^{x_e}] = 0, \tag{6}$$

where $\delta_2 = \int_{\underline{y}}^{r} G(t)^{(n_e - 1)x_e}[1 - G(t)^{x_e}] \ln G(t) dt < 0$. Equation (6) implies $\frac{\partial (n_e - 1)x_e}{\partial r} > 0$. From (5) and (6), I can solve

$$\frac{\partial x_e}{\partial r} = \frac{1}{\delta_1 \delta_2} \int_{\underline{y}}^{r} [G(t)G(r)]^{(n_e - 1)x_e} \phi_1(t) \ln G(t) dt, \tag{7}$$

where $\phi_1(t) = [1 - G(r)^{x_e}]G(t)^{x_e} \ln G(t) - [1 - G(t)^{x_e}]G(r)^{x_e} \ln G(r)$ for $t \in [\underline{y}, r]$. It can be shown $\phi_1(t) < 0$ for all $t \in (\underline{y}, r)$. Since $\delta_1 > 0$ and $\delta_2 < 0$, then (7) implies $\frac{\partial x_e}{\partial r} < 0$ which also implies $\frac{\partial n_e}{\partial r} > 0$. Otherwise $\frac{\partial \bar{x}_e}{\partial r} < 0$, which contradicts with (5).

Similarly, I can show (b) and (c) hold by taking the derivatives of both sides of equation (EE) and (R&D) with respect to C_1 and C_2, respectively.

Q.E.D.

For a fixed number of bidders, if the buyer sets a higher reservation price, each bidder's expected profit from bidding would be higher. Then there was a greater number of bidders whose fixed costs of R&D could be covered by this expected profit. But as the number of bidders went up, each bidder's expected profit from bidding would go down. Each firm would then invest less in R&D. Proposition 1 shows that, at the free-entry equilibrium, as the buyer's reservation price increases, both the equilibrium number of active firms and the total level of R&D investments increase. In other words, raising the reservation price encourages participations in R&D activity and stimulates R&D investments over all.

As the marginal cost of expenditure on R&D increases, both the individual and the total expenditure on R&D decrease. The condition $C_1 < -\int_{\underline{y}}^{r} \ln G(t)dt$ in Proposition 1 is required so that the total expenditure on R&D is positive. That is, in order to have some R&D activity in the industry, the marginal cost of R&D cannot be too high. On the other hand, when the marginal cost of R&D approaches zero, the total expenditure on R&D approaches infinity no matter how large the fixed cost of R&D is.

The fixed cost of R&D is sunk and does not affect the total expenditure, but does affect the equilibrium number of informed firms and the average number of actual bidders as well. As this fixed cost decreases, the equilibrium number of firms increases. In the limit, as the fixed cost of R&D approaches zero, the equilibrium number of active firms n_e approaches infinity and each firm invests almost zero in R&D. To avoid this limit case, the buyer could introduce a positive entry fee that each firm would pay prior to undertaking an expenditure in R&D. The higher the entry fee is, the less the number of firms is in the equilibrium. Therefore, the reservation price (possibly with a entry fee) and the fixed cost of R&D are important determinants of the equilibrium number of firms.

If n_e is an integer, each informed firm gets exactly zero expected profit at the equilibrium. If n_e is not an integer, the equilibrium needs to be adjusted. Let $[n_e]$ represent the largest integer that is less than or equal to n_e. If $[n_e]$ firms become active, the total expenditure on R&D does not change. Then each firm invests $x_e' = \overline{x}_e / [n_e]$ on R&D and $x_e' > x_e$. Since $E\pi_{n_e}(x_e, r) = 0$ from (4) and (EE), we have

$$E\pi_{[n_e]}(x_e', r) - E\pi_{n_e}(x_e, r) = \int_{\underline{y}}^{r} G(t)^{\bar{x}_e - x_e'}dt - \int_{\underline{y}}^{r} G(t)^{\bar{x}_e - x_e}dt - C_1(x_e' - x_e)$$

$$= \frac{1}{2}(x_e' - x_e)^2 \int_{\underline{y}}^{r} G(t)^{\bar{x}_e - \xi}\ln^2 G(t)dt > 0,$$

where $x_e' \geq \xi \geq x_e$ and the second equality holds because of Taylor's expansion and equation (R&D). Thus each firm earns a positive expected profit. The above expression can also be used to estimate how much expected economic profits each firm is able to earn.

On the other hand, if more than $[n_e]$ firms become active, each firm would invest x on R&D, which is strictly less than x_e. A similar argument implies that each firm would earn a negative profit. Thus, $([n_e], x_e', r)$ with (3) is the correct symmetric perfect free-entry equilibrium in this case.

In more general cases where $K > 0$, I am also able to show the existence and uniqueness of a symmetric perfect free-entry equilibrium. For any given number of firms that invest in R&D, let us first look at the firms' R&D behavior and exit decisions. Let $\alpha = n$ be a continuous variable parameter, $\alpha \geq 1$. Given any α, consider the solution $(x_\alpha, \hat{y}_\alpha)$ to the equations system (FE) and (R&D) and let $\pi(\alpha) \equiv E\pi_\alpha(x_\alpha, \hat{y}_\alpha)$ be each firm's expected profit for a given reservation price r when there are α firms becoming active. Also let $\bar{x}_\alpha = \alpha x_\alpha$. Then I have

Proposition 2: In the case of $K > 0$, suppose $\underline{y} < r - K \leq \bar{y}$, and $0 < C_1 < -\int_{\underline{y}}^{r-K} \ln G(t)dt$. Then for any $\alpha \in [1, +\infty)$ there exists a unique solution $(x_\alpha, \hat{y}_\alpha)$, with $x_\alpha > 0$ and $\hat{y}_\alpha \in (\underline{y}, r - K)$, to the equations system (FE) and (R&D). Furthermore,

(a) $x_\alpha, \bar{x}_\alpha, \hat{y}_\alpha$, and $\pi(\alpha)$ are all continuous and strictly decreasing in $\alpha \in (1, +\infty)$;

(b) $\hat{y}_\alpha \to \hat{y}_\infty \in (\underline{y}, r - K)$, $x_\alpha \to 0$, $\bar{x}_\alpha \to \bar{x}_\infty > 0$, and $\pi(\alpha) \to -C_2$ when $\alpha \to +\infty$;

(c) $\hat{y}_\alpha \to r - K$, $x_\alpha \to x_1$, $\bar{x}_\alpha \to x_1$, and $\pi(\alpha) \to \pi(1)$ when $\alpha \to 1$.

Proof: First of all, we show the existence and uniqueness of the solution to (FE) and (R&D). If $\alpha = 1$, then, from (FE), $\hat{y}_1 = r - K$. Equation (R&D) determines a unique solution $x_1 > 0$ since $0 < C_1 < -\int_{\underline{y}}^{r-K} \ln G(t)dt$ by the assumption.

If $\alpha > 1$, then $r - \hat{y} > 0$ from (FE) since $K > 0$. Then (FE) gives

$$\phi(\hat{y}) \equiv (\alpha - 1)x = \frac{\ln K - \ln(r - \hat{y})}{\ln G(\hat{y})} \tag{8}$$

which also implies $\phi(\underline{y}) = +\infty$, $\phi(r - K) = 0$, and $\phi'(\hat{y}) < 0$ for any $\hat{y} \in (\underline{y}, r - K)$. Substitute (8) into equation (R&D) and let

$$\psi(\hat{y}) \equiv \int_{\underline{y}}^{\hat{y}} G(t)^{\alpha\phi(\hat{y})/(\alpha-1)} \ln G(t)dt + C_1.$$

Then $\psi(\underline{y}) = C_1 > 0$, $\psi(r - K) = \int_{\underline{y}}^{r-K} \ln G(t)dt + C_1 < 0$ by the assumption, and

$$\psi'(\hat{y}) = G(\hat{y})^{\alpha\phi(\hat{y})/(\alpha-1)} \ln G(\hat{y}) + \frac{\alpha}{\alpha - 1}\phi'(\hat{y}) \int_{\underline{y}}^{\hat{y}} G(t)^{\alpha\phi(\hat{y})} \ln^2 G(t)dt < 0$$

for any $\hat{y} \in (\underline{y}, r - K)$. By the continuity of $\psi(\hat{y})$, there exists a unique root $\hat{y}_\alpha$ of ψ. Substituting $\hat{y} = \hat{y}_\alpha$ into (8), we can calculate $x_\alpha = \phi(\hat{y}_\alpha)/(\alpha - 1) > 0$. Thus, for $\alpha > 1$, there exists a unique solution $(x_\alpha, \hat{y}_\alpha)$ to the equations system (FE) and (R&D) with $x_\alpha > 0$ and $y_\alpha \in (\underline{y}, r - K)$.

Second, we prove that (a) holds. It is easy to see x_α, $\bar{x}_\alpha$, $\hat{y}_\alpha$, and $\pi(\alpha)$ are all continuously differentiable in $\alpha \in (1, +\infty)$. Taking the derivatives of both sides of (FE) and (R&D) with respect to α, respectively, we obtain

$$\rho(\hat{y}_\alpha)\frac{\partial \hat{y}_\alpha}{\partial \alpha} + \left[(\alpha - 1)\frac{\partial x_\alpha}{\partial \alpha} + x_\alpha\right] K \ln G(\hat{y}_\alpha) = 0 \tag{9}$$

and

$$G(\hat{y}_\alpha)^{(\alpha-1)x_\alpha} \ln G(\hat{y}_\alpha)\frac{\partial \hat{y}_\alpha}{\partial \alpha} + \left[\alpha\frac{\partial x_\alpha}{\partial \alpha} + x_\alpha\right] \int_{\underline{y}}^{\hat{y}_\alpha} G(t)^{\alpha x_\alpha} \ln^2 G(t)dt = 0, \tag{10}$$

where

$$\rho(\hat{y}) = \frac{\partial}{\partial \hat{y}}\left[(r-\hat{y})G(\hat{y})^{(\alpha-1)x}\right] = -G(\hat{y})^{(\alpha-1)x} - x(\alpha-1)(r-\hat{y})G(\hat{y})^{(\alpha-1)x-1}f(\hat{y}) < 0$$

for all $\hat{y} \in (\underline{y}, r-K)$. From (9) and (10), we can calculate $\frac{\partial \hat{y}_\alpha}{\partial \alpha}$ as the following:

$$\left[(\alpha-1)KG(\hat{y}_\alpha)^{(\alpha-1)x_\alpha}\ln^2 G(\hat{y}_\alpha) - \alpha\rho(\hat{y}_\alpha)\int_{\underline{y}}^{\hat{y}_\alpha} G(t)^{\alpha x_\alpha}\ln^2 G(t)dt\right]\frac{\partial \hat{y}_\alpha}{\partial \alpha}$$

$$= x_\alpha K \ln G(\hat{y}_\alpha)\int_{\underline{y}}^{\hat{y}_\alpha} G(t)^{\alpha x_\alpha}\ln^2 G(t)dt$$

which clearly implies $\frac{\partial \hat{y}_\alpha}{\partial \alpha} < 0$ for all $\alpha > 1$. Then from (10) we know $\frac{\partial x_\alpha}{\partial \alpha} < 0$ and $\frac{\partial \bar{x}_\alpha}{\partial \alpha} < 0$ for all $\alpha > 1$. At the same time, from (9), we get $\frac{\partial(\alpha-1)x_\alpha}{\partial \alpha} > 0$ for $\alpha > 1$.

Using equations (4) and (R&D), we can calculate

$$\frac{d\pi(\alpha)}{d\alpha} = \frac{\partial \hat{y}_\alpha}{\partial \alpha}G(\hat{y}_\alpha)^{\bar{x}_\alpha - x_\alpha}H(\hat{y}_\alpha \mid \bar{x}_\alpha) + \frac{\partial(\alpha-1)x_\alpha}{\partial \alpha}\int_{\underline{y}}^{\hat{y}_\alpha} G(t)^{\bar{x}_\alpha - x_\alpha}H(t \mid \bar{x}_\alpha)\ln G(t)dt.$$

The results we obtained above imply $\frac{d\pi(\alpha)}{d\alpha} < 0$ for all $\alpha > 1$.

Third, let α approach infinity. The equation $\psi(\hat{y}_\alpha) = 0$ becomes

$$\int_{\underline{y}}^{\hat{y}_\infty} G(t)^{\phi(\hat{y}_\infty)}\ln G(t)dt + C_1 = 0, \tag{11}$$

which determines a unique solution $\hat{y}_\infty \in (\underline{y}, r-K)$. Then (8) implies $(\alpha-1)x_\alpha \to \phi(\hat{y}_\infty)$, $x_\alpha \to 0$, and $\bar{x}_\alpha \to \phi(\hat{y}_\infty) \equiv \bar{x}_\infty > 0$ when $\alpha \to +\infty$. Using these results, we can easily see $\pi(\alpha) \to -C_2$ when $\alpha \to +\infty$.

Finally, we prove (c). Since $\hat{y}_\alpha$ is continuous and strictly decreasing in α for all $\alpha > 1$ and has an upper bound $r-K$, then when α approaches 1, $\hat{y}_\alpha$ has a limit, denoted by $\hat{y}_0$ with $\hat{y}_0 \le r-K$. Suppose $\hat{y}_0 < r-K$. In the following, we can show that there exists

$\tilde{\alpha} > 1$ such that $\hat{y}_{\tilde{\alpha}} = \hat{y}_0$. Then $\hat{y}_\alpha > \hat{y}_0$ for all $\alpha \in (1, \tilde{\alpha})$ and hence $\hat{y}_0$ cannot be the limit of $\hat{y}_\alpha$. This is a contradiction. Thus, $\hat{y}_0 = r - K$.

In fact, if $\hat{y}_0 < r - K$ then $\phi(\hat{y}_0) > 0$, where $\phi(\hat{y})$ is defined by (8). Let

$$v(\alpha) \equiv \int_{\underline{y}}^{\hat{y}_0} G(t)^{\alpha\phi(\hat{y}_0)/(\alpha-1)} \ln G(t)dt + C_1,$$

Then $v'(\alpha) < 0$ for all $\alpha > 1$ and $v(\alpha) \to C_1 > 0$ when $\alpha \to 1$. Since $\hat{y}_0 > \hat{y}_\infty$ and $\phi(\hat{y}_0) < \phi(\hat{y}_\infty)$, using (11), we get

$$v(+\infty) = \int_{\underline{y}}^{\hat{y}_0} G(t)^{\phi(\hat{y}_0)} \ln G(t)dt + C_1$$

$$= \int_{\hat{y}_\infty}^{\hat{y}_0} G(t)^{\phi(\hat{y}_0)} \ln G(t)dt + \int_{\underline{y}}^{\hat{y}_\infty} \left[G(t)^{\phi(\hat{y}_0)} - G(t)^{\phi(\hat{y}_\infty)} \right] \ln G(t)dt < 0.$$

Thus, there exists $\tilde{\alpha} > 1$ such that $\psi(\tilde{\alpha}) = 0$. Let $x_{\tilde{\alpha}} = \tilde{\alpha}\phi(\hat{y}_0) / (\tilde{\alpha} - 1)$, then $(x_{\tilde{\alpha}}, \hat{y}_{\tilde{\alpha}})$ is the unique solution to (FE) and (R&D), where $\hat{y}_{\tilde{\alpha}} = \hat{y}_0$.

Since $\hat{y}_0 = r - K$ and $\phi(\hat{y}_0) = 0$, equation (8) implies $(\alpha - 1)x_\alpha \to 0$ when $\alpha \to 1$. From equation $\psi(\hat{y}_\alpha) = 0$, we know $\hat{x}_\alpha \to x_1$ when $\alpha \to 1$. Thus, it is easy to see $\pi(\alpha) \to \pi(1)$ when $\alpha \to 1$.

Q.E.D.

For any given number of active firms, the R&D expenditure and the break-even cost level are uniquely determined. When more firms become active, each active firm's expected profit decreases. More competition makes the procurement contract less profitable to each firm. Each firm intends to invest less in R&D. The total expenditure on R&D among all firms is also lower. If there was no bid-preparation cost, the total R&D expenditure would not change with the number of active firms. More firms increase the total bid-preparation costs and discourage R&D over all.

As the number of active firms goes to infinity, each firm invests almost zero on R&D although the total R&D expenditure approaches a positive amount. Each firm's expected profit approaches $-C_2$. Thus if there is no fixed cost of R&D then free entry causes an infinite number of firms to enter the R&D process. If there is a positive fixed R&D cost then only a finite number of firms will decide to enter the procurement process. I will make this point more precise in the next proposition. Therefore, the fixed cost of R&D C_2 is the key determinant of the free-entry equilibrium number of firms although the latter is also affected directly or indirectly by the marginal cost of R&D C_1, the bidding preparation cost K, the reservation price r, and the distribution of production cost $H(y \mid x)$.

If $\pi(1) \leq 0$ for any reservation price r, no firm can make any profit by conducting R&D and production. This is not an interesting case. I assume $\pi(1)$ is positive for a given reservation price r. That is, when there is only one firm participating in R&D activity and bidding for the procurement contract, that firm is able to earn positive profits. Under free entry, at least one firm will then enter the R&D and bidding process. Each firm will earn a profit $\pi(\alpha)$, which is strictly decreasing in the number of firms entered α. Firms enter until this profit equals zero. If the fixed cost of R&D is positive, the equilibrium number of active firms should be finite. Formally, I have

Proposition 3: Suppose K, C_1, C_2, $\pi(1)$ are all positive and $\underline{y} < r - K \leq \bar{y}$. Then there exists a unique symmetric perfect free-entry equilibrium $(n_e, x_e, \hat{y}_e)$ with (3), where the integer $n_e \geq 1$, $x_e > 0$, and $\underline{y} < \hat{y}_e < r - K$.

Proof:[9] I first want to show that there exists a unique solution to (FE), (R&D), and (EE). This is equivalent to showing that there exists a unique $\alpha \geq 1$ such that $\pi(\alpha) = 0$.

Let

$$u(x) \equiv \int_{\underline{y}}^{r-K} H(t \mid x)dt - C_1 x - C_2$$

for $x \geq 0$, then $\pi(1) = \max u(x)$. The assumptions $\pi(1) > 0$ and $C_2 > 0$ imply that there exists $x_1 > 0$ such that $\pi(1) = u(x_1)$ and $u'(x_1) = 0$. Since $u''(x) < 0$ for all $x \geq 0$, then $u'(0) > u'(x_1) = 0$. That is, $C_1 < -\int_{\underline{y}}^{r-K} \ln G(t)dt$. Thus, the assumptions in Proposition 2 are satisfied. According to Proposition 2, $\pi(\alpha)$ is continuous and strictly decreasing over $\alpha \in [1, +\infty)$ with $\pi(+\infty) = -C_2 < 0$. Then the assumption $\pi(1) > 0$ implies that there is a unique $\alpha^* > 1$ such that $\pi(\alpha^*) = 0$.

Let $n_e = [\alpha^*]$ be the largest integer that is less than or equal to α^*, and let $x_e = x_{[\alpha^*]}$ and $\hat{y}_e = \hat{y}_{[\alpha^*]}$. Since $[\alpha^*] \leq \alpha^* < [\alpha^*] + 1$, $\pi(\alpha^*) = 0$, and $\pi(\alpha)$ is strictly decreasing in α, then

$$E\pi_{n_e}(x_e, \hat{y}_e) = \pi([\alpha^*]) \geq 0$$

and

$$E\pi_{n_e + 1} = \pi([\alpha^*] + 1) < 0.$$

Thus, $(n_e, x_e, \hat{y}_e)$ satisfy (FE), (R&D), (EEa), and (EEb) with the integer $n_e \geq 1$, $x_e > 0$, and $\hat{y}_e \in (\underline{y}, r - K)$. That is, with (3), $(n_e, x_e, \hat{y}_e)$ is a unique symmetric perfect free-entry equilibrium.

Q.E.D.

3.4 OPTIMALITY FROM THE BUYER'S POINT OF VIEW

Now, go back to the first stage of the three-stage game and look at the buyer's optimality problem. I want to know whether there exists a reservation price under which the free-entry equilibrium characterized in the last section is optimal for the buyer.

For any given number of active firms and variable scale R&D technology, the distribution of production cost y of the winning firm at the symmetric equilibrium is $1-[1-H(y\,|\,x)]^n = 1-G(y)^{nx}$. The buyer's ex ante expected costs in the competitive procurement are

$$\int_{\underline{y}}^{\hat{y}} B(y)d(1-G(y)^{nx}) = \int_{\underline{y}}^{\hat{y}} yd(1-G(y)^{nx}) + nKH(\hat{y}\,|\,x) + nC(x) + nE\pi_n(x,\hat{y}) \tag{12}$$

where $E\pi_n(x,\hat{y})$ is a firm's expected profits under the symmetric equilibria, defined by (4). The buyer's expected costs in the competitive procurement with R&D include the expected minimum production cost, the total R&D costs among all firms, the total expected bid-preparation costs, and the total expected profits among all firms.

Under free entry, each firm enters the R&D and bidding processes until its expected profit $E\pi_n(x,\hat{y})$ equals zero. Therefore, the winner's expected profits $\int_{\underline{y}}^{\hat{y}} (B(y)-y)d(1-G(y)^{nx})$ from the competitive bidding are equal to the total costs on both R&D and bid preparation among all of the firms. In other words, if free entry is allowed, the rents for the firms from contracting are dissipated by precontract R&D and bid-preparation activities. The question, as I will answer in this and the next sections, is whether these R&D activities are good for the buyer and society.

At the symmetric free-entry equilibrium under a given reservation price, what the buyer has to pay is not just the expected minimum production cost, but also the total

R&D cost $n_e(C_1x_e + C_2)$ of all informed firms, and the total bid preparation cost $n_a K = n_e KH(\hat{y}_e \mid x_e)$ of all actual bidders as well. One might have thought that the buyer has only to pay the R&D costs of the winner. But since firms are assumed to be symmetric and to adopt the same investment and bidding strategy, each has an equal probability to be the winner. Therefore, the buyer actually ex ante expects to pay all of the costs of R&D among active firms.

Remember that the buyer can procure the good elsewhere at the cost y_0 if the lowest bid is higher than the reservation price r. Because of the bid preparation cost, $B(\hat{y}) = r$ and the firm with cost y bids if and only if $y \le \hat{y}$. The buyer actually procures the good at cost y_0 elsewhere with probability $[1 - H(\hat{y} \mid x)]^n = G(\hat{y})^{nx}$. Thus, the buyer's total expected costs will be, remember $\hat{y} = \hat{y}(r, n, x)$ from (FE),

$$EBC(r, n, x) = \int_{\underline{y}}^{\hat{y}} B(y)d(1 - G(y)^{nx}) + y_0 G(\hat{y})^{nx}$$

$$= \underline{y} + (y_0 - \hat{y})G(\hat{y})^{nx} + nK - nKG(\hat{y})^x$$

$$+ n\int_{\underline{y}}^{\hat{y}} G(t)^{(n-1)x} dt - (n-1)\int_{\underline{y}}^{\hat{y}} G(t)^{nx} dt. \tag{13}$$

The buyer wants to minimize his total ex ante expected costs of procurements $EBC(r, n, x)$ by selecting r, n, and possibly x. Since the buyer has to pay all the costs in (12), as a tradeoff, he may want to set $\hat{y}$ less than his opportunity cost y_0.

Consider a fixed-scale R&D technology. That is, each firm either invests in R&D at a cost $C > 0$ or does not invest. If the firm invests in R&D, it observes its production y and believes that other investing firms' production cost observations are independently drawn from the same cumulative distribution $F(y)$ with the support $[\underline{y}, \bar{y}]$. I also assume that there is no bid preparation cost before the competitive bidding; then $\hat{y} = r$ from (FE). Thus, the buyer's expected cost can be simply written as

$$EBC(r,n) = \underline{y} + (y_0 - r)G(r)^n + \int_{\underline{y}}^{r} G(t)^n dt + n\int_{\underline{y}}^{r} F(t)G(t)^{n-1}dt. \tag{14}$$

Suppose $C = 0$. That is, each potential firm can observe its own production cost y without any expense and believes that other firms' production costs are drawn independently from the same distribution $F(y)$. The buyer then chooses (r, n) to maximize his expected profit (14). It can be easily shown that the following are true: First, the buyer should choose the optimal reservation price $r = r_0$ such that

$$r_0 + \frac{F(r_0)}{f(r_0)} = y_0 \tag{15}$$

if $y_0 < \bar{y} + 1/f(\bar{y})$ and $r_0 = \bar{y}$ otherwise.[10] The optimal reservation price r_0 for the buyer in (15) is independent of the number of firms and is strictly less than his opportunity cost y_0. It is possible that the buyer procures somewhere else at cost y_0 even though the winner in the competitive bidding offers a lower cost than y_0. Thus, because of asymmetry of information between the buyer and firms, the buyer finds it in his interest to distort the outcome away from the efficient allocation. Second, since r_0 is independent of n, $EBC(r_0, n) \geq \underline{y}$, and $EBC(r_0, n)$ goes to $\underline{y}$ when n approaches infinity, the buyer prefers an infinite number of firms to bid for the contract. Since each firm has a positive expected profit $\int_{\underline{y}}^{r_0} F(t)G(t)^{n-1}dt$, free entry will cause an infinite number of firms to be in the competitive bidding process and drive the production cost to the lowest bound $\underline{y}$. Therefore, the buyer prefers free entry in this case.

Now, suppose $C > 0$. In order to become informed about production costs, each firm has to pay a R&D cost $C > 0$ before the competitive bidding. This is the case considered by McAfee and McMillan (1987). But in their model, they assume that the opportunity cost of the procurement for the buyer is so high that no reservation price is needed. As we will see in the following, if the opportunity cost is relatively low, a reservation price is necessary for the buyer. The buyer chooses $r \in [\underline{y}, \bar{y}]$ and $n \geq 1$ to minimize his expected costs (14) subject to each firm's nonnegative profits constraint (EEa):

$$E\pi_n(r) \equiv \int_{\underline{y}}^{r} F(t)G(t)^{n-1}dt - C \geq 0. \tag{16}$$

Consider n as a real variable. It is easy to see $EBC(r, n)$ and $E\pi_n(r)$ are all continuous functions with respect to r and n. Since $E\pi_n(r)$ is increasing in r and decreasing in n, the constraint (16) with $r \in [\underline{y}, \bar{y}]$ and $n \geq 1$ forms a non-empty compact set in R_+^2 if $C < \int_{\underline{y}}^{y_m} F(t)dt$. Thus, there exists a solution to the buyer's optimization problem. Let r^* and n^* be the buyer's optimal reservation price and the optimal number of firms, respectively. Then we have

Proposition 4: Under a fixed-scale R&D technology, if $0 < C < \int_{\underline{y}}^{y_m} F(t)dt$, then i) $r^* = \bar{y}$ when $y_0 \geq \bar{y} + 1/f(\bar{y})$ and $r_0 < r^* < y_0$ when $y_0 \leq \bar{y}$; and ii) free entry causes the buyer's optimal number of firms n^* to enter the competitive procurement process.

Proof: The first order conditions give the following:

$$\phi_\lambda(r, n) \equiv -\frac{\partial EBC}{\partial r} + \lambda \frac{\partial E\pi_n}{\partial r} = 0,$$

$$\psi_\lambda(r, n) \equiv -\frac{\partial EBC}{\partial n} + \lambda \frac{\partial E\pi_n}{\partial n} = 0,$$

and $\lambda E\pi_n(r) = 0$ for interior solution $r = r^* \in (\underline{y}, \bar{y})$ and $n = n^* \in (1, +\infty)$, where $\lambda = \lambda^* \geq 0$ is the multiplier for the inequality constraint (13), and

$$\phi_\lambda(r, n) = nG(r)^{n-1}f(r)\left[y_0 - r - (1 - \frac{\lambda}{n})\frac{F(r)}{f(r)}\right]$$

$$\psi_\lambda(r, n) = -(y_0 - r)G(r)^n \ln G(r)$$

$$-\int_{\underline{y}}^{r} G(t)^{n-1}\Big[F(t) + \Big[1 + (n-1-\lambda)F(t)\Big]\ln G(t)\Big]\,dt.$$

First, consider the case $y_0 \ge \bar{y} + 1/f(\bar{y})$. For any $r < \bar{y}$ and $n \ge 1$, we have

$$\phi_\lambda(r, n) \ge nG(r)^{n-1}f(r)\left[\bar{y} + \frac{1}{f(\bar{y})} - r - \frac{F(r)}{f(r)} + \frac{\lambda F(r)}{nf(r)}\right] > 0$$

since $r + F(r)/f(r)$ is increasing in r. Thus, $r^* = \bar{y}$. We claim $n^* > 1$ in this case. In fact, if $n^* = 1$ then the first order condition gives $\psi_{\lambda^*}(\bar{y}, 1) \le 0$. Since $E\pi_1(\bar{y}) = \int_{\underline{y}}^{\bar{y}} F(t)dt - C > 0$ by the assumption and $\lambda^* E\pi_1(\bar{y}) = 0$, we have $\lambda^* = 0$. But $\psi_0(\bar{y}, 1) = -\int_{\underline{y}}^{\bar{y}} \Big[F(t) + \ln G(t)\Big]\,dt > 0$. This contradicts with $\psi_{\lambda^*}(\bar{y}, 1) \le 0$. Therefore $n^* > 1$. This with the first order condition implies $\psi_{\lambda^*}(\bar{y}, n^*) = 0$. Since $\psi_0(\bar{y}, n^*) > 0$, $\psi_n(\bar{y}, n^*) < 0$, and $\dfrac{d\psi_\lambda(\bar{y}, n^*)}{d\lambda} < 0$ for all $\lambda > 0$, equation $\psi_{\lambda^*}(\bar{y}, n^*) = 0$ implies $\lambda^* > 0$. Thus, $\lambda^* E\pi_{n^*}(\bar{y}) = 0$ implies $E\pi_{n^*}(\bar{y}) = 0$, which determines a unique $n^* > 1$. Then $\psi_{\lambda^*}(\bar{y}, n^*) = 0$ uniquely determines $\lambda^* \in (0, n^*)$. Therefore, $r^* = \bar{y}$ and n^* determined by $E\pi_{n^*}(\bar{y}) = 0$ are optimal for the buyer.

Second, consider the case $y_0 \le \bar{y}$. If $n^* = 1$, then the first order condition implies $\psi_{\lambda^*}(r^*, 1) \le 0$. Suppose $r^* = \bar{y}$ then $\phi_{\lambda^*}(\bar{y}, 1) = f(\bar{y})(y_0 - \bar{y}) - (1 - \lambda^*) \ge 0$. That is, $1 - \lambda^* \le F(\bar{y})(y_0 - \bar{y}) \le 0$, which implies $\lambda^* > 0$. This with the first order condition $\lambda^* E\pi_1(\bar{y}) = 0$ implies $E\pi_1(\bar{y}) = \int_{\underline{y}}^{\bar{y}} F(t)dt - C = 0$. This contradicts with the assumption. Thus it must be the case $r^* < \bar{y}$. Then $\phi_{\lambda^*}(r^*, 1) = 0$ holds, that is,

$$y_0 = r^* + (1 - \lambda^*)\frac{F(r^*)}{f(r^*)}. \tag{17}$$

Suppose $r^* \ge y_0$, then $E\pi_1(r^*) = \int_{\underline{y}}^{r^*} F(t)dt - C \ge \int_{\underline{y}}^{y_0} F(t) - C > 0$ by the assumption. Thus

$\lambda^* E\pi_1(\bar{y}) = 0$ implies $\lambda^* = 0$ and hence (17) implies $y_0 > r^*$. This contradicts to $r^* > y_0$. Therefore, $r^* < y_0 \le \bar{y}$. Then

$$0 \ge \psi_{\lambda^*}(r^*, 1) = -(y_0 - r^*)G(r^*)\ln G(r^*) - \int_{\underline{y}}^{r^*} \left[F(t) + \left[1 - \lambda^* F(t) \right] \ln G(t) \right] dt$$

implies $\lambda^* > 0$. We know $r^* > r_0$ from (17), where r_0 is determined by (15). In summary, we have shown that if $n^* = 1$ then $\lambda^* > 0$ and $r_0 < r^* < y_0$.

If $n^* > 1$, then the first order condition gives $\psi_{\lambda^*}(r^*, n^*) = 0$. For $n^* > 1$ and any $r > \underline{y}$, equation $\psi_\lambda(r, n^*) = 0$ determines a unique $\lambda = \lambda(r) \in (0, n^*)$, which is continuous at $r = \bar{y}$, and $\lambda(\bar{y}) \in (0, n^*)$. Because of the inequality $\bar{y} + (1 - \frac{\lambda(\bar{y})}{n^*})\frac{1}{f(\bar{y})} > \bar{y}$ and the continuity, we have

$$r + (1 - \frac{\lambda(r)}{n^*})\frac{F(r)}{f(r)} > \bar{y}$$

and

$$\phi_{\lambda(r)}(r, n^*) < n^* G(r)^{n^*-1} f(r)(y_0 - \bar{y}) \le 0$$

when r is close enough to $\bar{y}$ and $r < \bar{y}$. Thus, it must be the case $r^* < \bar{y}$. Then $\phi_{\lambda^*}(r^*, n^*) = 0$ holds, that is,

$$y_0 = r^* + (1 - \frac{\lambda^*}{n^*})\frac{F(r^*)}{f(r^*)}. \tag{18}$$

Combining (18) with $\psi_{\lambda^*}(r^*, n^*) = 0$, we obtain $\lambda^* = \lambda(r^*) \in (0, n^*)$. Then

$$E\pi_n(r^*) = \int_{\underline{y}}^{r^*} F(t)G(t)^{n^*-1} dt - C = 0 \tag{19}$$

and $r_0 < r^*$. In other words, equations (18), (19), and $\psi_{\lambda^*}(r^*, n^*) = 0$ simultaneously determine r^*, n^*, and λ^* and $0 < \lambda^* < n^*$, $r_0 < r^* < y_0$.

We have shown $\lambda^* > 0$ in both cases. That is, the firm's nonnegative profits constraint (16) is binding. Therefore, if free entry is allowed, the optimal number of firms n^* from the buyer's point of view enter the R&D process provided that the buyer chooses the optimal reservation price r^*.

Q.E.D.

If n^* is not an integer, then similar to the discussion in the last section, $[n^*]$ will be the optimal number of firms for the buyer. Each of $[n^*]$ firms earns a positive expected profit.

The condition $C < \int_{\underline{y}}^{y_m} F(t)dt$ in Proposition 4 is equivalent to $E\pi_1(y_m) > 0$, which means that, under the highest reservation price y_m, if only one firm conducts R&D and production, that firm earns a positive expected profit. In other words, conducting R&D and production is potentially profitable. Otherwise, there is no interest in analyzing the optimal policy for the buyer or society.

In a competitive procurement with a fixed cost of R&D, if the buyer's opportunity cost y_0 is relatively high, no reservation price is needed and the optimal number of firms enter the procurement process. That is the same as the result obtained by McAfee and McMillan (1987). In addition, they show that the sealed-bid auction without reservation price is an optimal mechanism.

If the opportunity cost y_0 is relatively low (lower than the highest possible production cost level $\bar{y}$), however, the optimal number of potential firms still enter the procurement process provided that the buyer chooses an optimal reservation price r^*, which is lower than the buyer's opportunity cost y_0. The optimal reservation price r^* is higher than the reservation price r_0 in the case where no such R&D cost exists. Thus, the distortion of the efficient outcome (see Section 3.5) due to asymmetry of information still exists, but the positive R&D cost reduces that distortion. The fact that each firm has to pay a positive cost to become informed reduces the asymmetry of information between the buyer and firms compared to the usual adverse selection models. I have also made a similar argument in Chapter 2.

Now, consider a variable scale R&D process subject to constant marginal return to scale on R&D expenditure, where expenditure x is an endogenous continuous variable. Suppose that the buyer is able to control the firm's R&D decision and treat x as observable. Thus the buyer can control r, n, and x. Suppose that there is no bid-preparation cost, then the buyer wants to choose (r, n, x) to solve his following optimization problem:

$$\underset{r,n,x}{Min}\ EBC(r,n,x) = \underline{y} + (y_0 - r)G(r)^{nx} + n\int_{\underline{y}}^{r} G(t)^{(n-1)x}dt - (n-1)\int_{\underline{y}}^{r} G(t)^{nx}dt \qquad (20)$$

$$s.t. \quad E\pi_n(r,x) = \int_{\underline{y}}^{r} G(t)^{(n-1)x} H(t \mid x)dt - C_1 x - C_2 \geq 0$$

for $r \in [\underline{y}, \overline{y}]$, $n \geq 1$, and $x \geq 0$. As before, I treat n as a real variable. Since $EBC(r, n, x)$ is continuous and the constraints form a compact set, there exists a solution to the above optimization problem (20). Would the buyer still be satisfied with the symmetric free-entry equilibrium with a reservation price as I characterized in the last section? In other words, would the buyer give each firm freedom to make decisions on R&D and entry even though he can control them?

Let $E\pi_1(r) = \max E\pi_1(r, x)$ over $x \in [0, +\infty)$ be the expected profit when there is only one firm to conduct R&D and to make a bid under a buyer's reservation price r. It is easy to see $E\pi_1(r)$ is increasing in r. I assume $E\pi_1(y_m) > 0$, that is, at the highest possible reservation price $r = y_m$, the sole firm that does both R&D and production should earn a positive expected profit. Then I have

Proposition 5: Suppose C_1, C_2, and $E\pi_1(y_m)$ are all positive, then there does not exist a reservation price under which the symmetric free-entry equilibrium solves the buyer's optimization problem (20).

Proof: Since $C_1 > 0$ and $C_2 > 0$ by the assumption, from the constraint of (20), $x = 0$ and $x = +\infty$ cannot be solutions to (20), nor $r = \underline{y}$ and $n = +\infty$. A necessary condition for the optimal interior solution (r, n, x) to (20) is that there exist $\lambda \geq 0$ such that

$$nxf(r)G(r)^{nx-1}\left[y_0 - r - (1 - \frac{\lambda}{n})\frac{H(r \mid x)}{h(r \mid x)}\right] \geq 0, \tag{21}$$

$$-x(y_0 - r)G(r)^{nx}\ln G(r) - \int_{\underline{y}}^{r} G(t)^{(n-1)x}\phi(t \mid x)dt$$

$$+ (\lambda - n)x\int_{\underline{y}}^{r} G(t)^{(n-1)x}H(t \mid x)\ln G(t)dt \leq 0, \tag{22}$$

$$-n(y_0 - r)G(r)^{nx}\ln G(r) + (\lambda - n)(n-1)\int_{\underline{y}}^{r} G(t)^{(n-1)x}H(t \mid x)\ln G(t)dt$$

$$+ \lambda\left[-\int_{\underline{y}}^{r} G(t)^{nx}\ln G(t)dt - C_1\right] = 0, \tag{23}$$

and $\lambda E\pi_n(r, n) = 0$, where $\phi(t \mid x) = 1 - G(t)^x + xG(t)^x \ln G(t) > 0$ for all $t > \underline{y}$. Suppose $n > 1$. If $r = \bar{y}$ then (22) with equality implies $0 < \lambda < n$, which with (23) implies

$$-\int_{\underline{y}}^{\bar{y}} G(t)^{nx}\ln G(t)dt - C_1 < 0. \tag{24a}$$

If $r < \bar{y}$, then substituting (21) into (22) with both equality we observe $\lambda < n$. Substituting (21) with equality into (23) and using $0 \leq \lambda < n$, we also obtain

$$-\int_{\underline{y}}^{r} G(t)^{nx}\ln G(t)dt - C_1 < 0. \tag{24b}$$

Therefore, any solution (r, n, x) with $n > 1$ to (20) will violate the equilibrium condition (R&D).

On the other hand, suppose $n = 1$ is a solution to (20) and satisfies equation (R&D), then (23) becomes

$$(y_0 - r)G(r)^x \ln G(r) = 0.$$

This together with (21) implies $r = y_m$. Then (22) becomes

$$-\int_{\underline{y}}^{y_m} \phi(t \mid x)dt + (\lambda - 1)x \int_{\underline{y}}^{y_m} H(t \mid x)\ln G(t)dt \le 0$$

which implies $\lambda > 0$. Thus, $E\pi_1(y_m) = 0$. This violates the assumption.

In summary, there does not exist a reservation price under which the symmetric free-entry equilibrium (n, x) determined by (R&D), (EEa), and (EEb) solves the buyer's optimization problem (20).

Q.E.D.

If the buyer can control each firm's R&D decision or the R&D investment is observable to him, he can force the firms to invest in R&D as described by (20). Proposition 5 says that there does not exist a reservation price under which the symmetric free-entry equilibrium reaches the buyer's optimum (20). In other words, the buyer's ideal optimum in (20) cannot be supported by any symmetric free-entry equilibrium. Therefore, the buyer would not want the firms to make their own R&D and entry decisions.

From (24), $\frac{\partial \pi}{\partial x} < 0$, each firm's marginal profit of R&D investment is negative at the buyer's optimum. That is, the buyer would require each active firm to invest more than it wants to. That implies that there will exist a moral hazard problem if the buyer is unable to control the R&D decision x or if x is unobservable to the buyer. Thus the buyer has to take each firm's R&D decision as a constraint. He should then solve the optimization problem (20) subject to an additional constraint (R&D). Solving this optimization problem, we know the following: If the opportunity cost y_0 is high, the buyer should choose $r = \bar{y}$. If y_0 is relatively low, he should choose r such that

$$y_0 = r + (1 - \frac{\lambda}{n})\frac{H(r \mid x)}{h(r \mid x)} - \frac{\mu}{n}\frac{H_x(r \mid x)}{h(r \mid x)}$$

where n, x, and the multipliers λ and μ are simultaneously determined by the other first-order conditions including (EE) and (R&D). It can also be shown that $\lambda > 0$ and $\mu > 0$. That is, both the nonnegative profits constraint (EEa) and the R&D decision constraint (R&D) are binding. The buyer controls free entry by offering a reservation price determined by the above equation. Because the effect of moral hazard interacts with the effect of asymmetric information, the optimal reservation price r for the buyer may be either higher than or lower than his opportunity cost y_0.

3.5 SOCIAL OPTIMALITY

The expected social costs include the expected production cost, the bid preparation cost, the R&D cost, and the opportunity cost:

$$ESC = \int_{\underline{y}}^{\hat{y}} y d(1 - G(y)^{nx}) + nKH(\hat{y} \mid x) + nC(x) + y_0 G(\hat{y})^{nx}. \tag{25}$$

Comparing (25) with (13), we know $ESC = EBC - nE\pi_n$. That is, the buyer cares, but society does not, about the firms' expected profits, which are the transfers from the buyer to the firms. What society cares about is the total combination of costs on R&D, production, and bid preparation. Under free entry, the expected social cost will be the same as the buyer's expected cost. The social optimization problem is to choose $(r, n, x, \hat{y})$ to minimize the expected social cost $ESC(r, n, x, \hat{y})$.

As discussed in Proposition 4, I first consider the fixed-scale R&D technology. I also assume that there is no bid-preparation cost in this case. Then the expected social costs of procurements can simply be written as

$$ESC(r, n) = \underline{y} + (y_0 - r)G(r)^n + \int_{\underline{y}}^{r} G(t)^n dt + nC. \tag{26}$$

The social planner wants to choose (r, n) to minimize the expected social cost function (26) subject to the firm's nonnegative profit constraint (16). I have

Proposition 6: Suppose $K = 0$, $C > 0$, and $E\pi_1(y_m) > 0$, then $r = y_m$ is socially optimal and each firm earns positive expected profits.

Proof: The proof is similar to the proof of Proposition 4. Because of the continuity of $ESC(r, n)$ and the compactness of the constraints, there exists a solution to the minimization problem (26) with (16). Let (r^*, n^*) be a solution. Let $L = -ESC(r, n) + \lambda E\pi_n(r)$, $\phi_\lambda(r, n) = \frac{\partial L}{\partial r}$, and $\psi_\lambda(r, n) = \frac{\partial L}{\partial n}$, where $\lambda \geq 0$ is the multiplier for the constraint (16). We can calculate

$$\phi_\lambda(r,n) = nG(r)^{n-1}f(r)\left[y_0 - r + \frac{\lambda}{n}\frac{F(r)}{f(r)}\right]$$

and

$$\psi_\lambda(r,n) = -(y_0 - r)G(r)^n \ln G(r) - C - \int_{\underline{y}}^{r} G(t)^{n-1}\left[G(t) - \lambda F(t)\right]\ln G(t)dt.$$

The first-order conditions give $\phi_\lambda(r,n)=0$, $\psi_\lambda(r,n)=0$, and $\lambda E\pi_n(r)=0$ for interior solution.

First, consider the case $y_0 \ge \bar{y}$, then

$$\phi_\lambda(r,n) \ge nG(r)^{n-1}f(r)\left[\bar{y} - r + \frac{\lambda}{n}\frac{F(r)}{f(r)}\right] > 0$$

for all $r < \bar{y}$. Thus $r^* = \bar{y}$. Suppose $n^* = 1$, then $\lambda = 0$ because $\lambda E\pi_1(\bar{y}) = 0$ and $E\pi_1(\bar{y}) > 0$ by the assumption. Suppose $n^* > 1$, then the first-order conditions give $\psi_\lambda(\bar{y}, n^*) = 0$. If $\lambda > 0$, then $\psi_\lambda(\bar{y}, n^*) = 0$ implies $\int_{\underline{y}}^{\bar{y}} G(t)^n \ln G(t)dt + C < 0$. But

$$\int_{\underline{y}}^{\bar{y}} G(t)^{n-1}\left[F(t) + G(t)\ln G(t)\right]dt > 0$$

and hence $E\pi_n(\bar{y}) = \int_{\underline{y}}^{\bar{y}} G(t)^{n-1}F(t)dt - C > 0$, which implies $\lambda = 0$. This contradicts to $\lambda > 0$. Thus, $\lambda = 0$. Then $\psi_0(\bar{y}, n^*) = 0$ can be written as $\int_{\underline{y}}^{\bar{y}} G(t)^{n^*}\ln G(t)dt + C = 0$. Thus $E\pi_{n^*}(\bar{y}) > 0$.

Second, consider the case $y_0 \ge \bar{y}$. Suppose $n^* = 1$. If $r^* = \bar{y}$ then $\phi_\lambda(\bar{y}, 1) = f(\bar{y})(y_0 - \bar{y}) + \lambda / n \ge 0$, which implies $\lambda > 0$ and hence $E\pi_1(\bar{y}) = 0$. But that contradicts to the assumption $E\pi_1(\bar{y}) > 0$. This contradiction means $r^* < \bar{y}$. Then $\phi_\lambda(r^*, 1) = 0$. That is, $y_0 = r^* - \lambda F(r^*) / f(r^*)$, which implies $r^* \ge y_0$. If $r^* > y_0$ then $E\pi_1(r^*) > E\pi_1(y_0) > 0$ by the assumption. Then $\lambda = 0$, which implies $r^* = y_0$. This is a contradiction. Thus,

$r^* = y_0$. Suppose $n^* > 1$, then the first-order condition gives $\psi_\lambda(r^*, n^*) = 0$. If $r^* = \bar{y}$, then $\psi_\lambda(\bar{y}, n^*) = 0$ becomes

$$C + \int_{\underline{y}}^{\bar{y}} G(t)^{n^*-1}\Big[G(t) - \lambda F(t)\Big] \ln G(t)dt = 0.$$

If $\lambda > 0$ then $C + \int_{\underline{y}}^{\bar{y}} G(t)^{n^*} \ln G(t)dt < 0$, which implies $E\pi_{n^*}(\bar{y}) = \int_{\underline{y}}^{\bar{y}} G(t)^{n^*-1}F(t)dt - C > 0$. Then $\lambda = 0$. This contradicts to $\lambda > 0$. Thus, $\lambda = 0$. Then $\phi_0(r, n^*) = n^*G(r)^{n^*-1}f(r)(y_0 - r) < 0$ when r is less than but very close to $\bar{y}$. Thus r^* cannot be $\bar{y}$. In other words, $r^* < \bar{y}$. Then $\phi_\lambda(r^*, n^*) = 0$. That is,

$$y_0 = r^* - \frac{\lambda}{n^*}\frac{F(r^*)}{f(r^*)}.$$

Now, if $\lambda > 0$ then $\psi_\lambda(r^*, n^*) = 0$ implies $C + \int_{\underline{y}}^{r^*} G(t)^{n^*} \ln G(t)dt < 0$, which also implies

$$E\pi_{n^*}(r^*) = \int_{\underline{y}}^{r^*} G(t)^{n^*-1}F(t)dt > 0.$$

Then $\lambda = 0$, which contradicts to $\lambda > 0$. Thus, $\lambda = 0$, which implies $r^* = y_0$ and $E\pi_{n^*}(y_0) > 0$.

In summary, we have shown $r^* = y_m$ and $E\pi_{n^*}(y_m) > 0$.

Q.E.D.

From Proposition 6, the social optimal reservation price is $r^* = y_m$ and society allows each firm to earn positive expected profits. The constraint (16) is not binding. The social optimal number of firms n^* is determined by minimizing $ESC(y_m, n)$ with respect to n. Since

$$\frac{\partial ESC(y_m, n)}{\partial n} = C + \int_{\underline{y}}^{y_m} G(t)^n \ln G(t)dt$$

and $ESC(y_m, n)$ is convex in n, then n^* is determined as the following: If $C + \int_{\underline{y}}^{y_m} G(t)\ln G(t)dt \geq 0$ then $n^* = 1$. If $C + \int_{\underline{y}}^{y_m} G(t)\ln G(t)dt < 0$ then $n^* > 1$ and satisfies

$$C + \int_{\underline{y}}^{y_m} G(t)^{n^*} \ln G(t) dt = 0.$$

Society may prefer more than one firm to conduct private R&D under the fixed-scale R&D technology.

The implications of Proposition 4 and 6 are the following: In the first case where the opportunity cost y_0 is high, the buyer should select a socially optimal reservation price $r = \bar{y}$, the highest possible cost observation by the firms. In other words, both the buyer and society agree that reservation price is not necessary. Let n_b^* be the optimal number of firms for the buyer, which is determined by Proposition 4, then $n_b^* > 1$ and $E\pi_{n_b^*}(\bar{y}) = 0$. Notice that $ESC(\bar{y}, n)$ is strictly convex in n and

$$ESC(\bar{y}, n) - ESC(\bar{y}, n-1) = -E\pi_n(\bar{y})$$

for all $n > 1$. Then $ESC(\bar{y}, n_b^*) = ESC(\bar{y}, n_b^* - 1)$. Thus, $n_b^* - 1 < n^* < n_b^*$. Consider the integer problem, the social optimal number of firms will be the same as the buyer's optimal number of firms. Therefore, free entry and the first-price sealed-bid auction without any reservation price achieve the social optimum. This is the result that was also observed by McAfee and McMillan (1987a).

In the second case where the opportunity cost y_0 is low (lower than the highest possible production cost observation $\bar{y}$), however, the buyer intends to offer a lower reservation price relative to the social optimum, i.e., $r_b^* < y_0$. That may cause less firms to enter the R&D process, relative to social optimum. In fact, since $n_b^* > 1$ and $E\pi_{n_b^*}(y_0) > E\pi_{n_b^*}(r_b^*) = 0$ from Proposition 4 and since

$$ESC(y_0, n) - ESC(y_0, n-1) = -E\pi_n(y_0)$$

for $n > 1$, we have $ESC(y_0, n_b^*) < ESC(y_0, n_b^* - 1)$. On the other hand,

$$\frac{\partial ESC(y_0, n)}{\partial n}\Big|_{n = n_b^* - 1} = \int_{\underline{y}}^{y_0} G(t)^{n_b^* - 1}\Big[F(t) + G(t)\ln G(t)\Big]\, dt - E\pi_{n_b^*}(y_0) < 0.$$

and $ESC(y_0, n)$ is convex in n. Thus, we have either $n_b^* - 1 < n^* < n_b^*$ or $n^* \geq n_b^*$. Consider the integer problem, the social optimal number of firms will be at least as large as the buyer's optimal number of firms. In summary, we have the following Corollary of Proposition 4 and 6:

Corollary: Under the fixed-scale R&D technology, if $E\pi_1(y_m) > 0$, then the buyer prefers free entry, society does not, and the following hold as well:

(i) If $y_0 \geq \bar{y} + 1/f(\bar{y})$ then the buyer's choices of a reservation price and a number of firms are socially optimal;

(ii) If $y_0 \leq \bar{y}$ then the buyer chooses a reservation price lower than the socially optimal value and a number of firms no larger than the socially optimal number.

In the case of variable scale R&D technology, which is subject to constant marginal returns to scale on expenditure, the social planner wants to minimize the expected social costs (25) subject to (16), (R&D), and (FE). Then we have

Proposition 7: Suppose C_1, C_2, and $E\pi_1(y_m - K)$ are all positive, then $n = 1$ with $r = y_m$ and $\hat{y} = y_m - K$ is socially optimal.

Proof: Let $\lambda \geq 0$ and μ be the multipliers for the constraint (16) and (R&D), respectively, and

$$L = -ESC(r, n, x, \hat{y}) + \lambda E\pi_n(r, x) + \mu\Big[-\int_{\underline{y}}^{\hat{y}} G(t)^{nx}\ln G(t)dt - C_1\Big]$$

We can calculate

$$\frac{\partial L}{\partial \hat{y}} = nh(\hat{y} \mid x)G(\hat{y})^{(n-1)x}\left[y_0 - \hat{y} + \frac{\lambda}{n}\frac{H(\hat{y} \mid x)}{h(\hat{y} \mid x)} + \frac{\mu}{n}\frac{H_x(\hat{y} \mid x)}{h(\hat{y} \mid x)}\right] - nKh(\hat{y} \mid x)$$

$$\frac{\partial L}{\partial x} = -n(y_0 - \hat{y})G(\hat{y})^{nx}\ln G(\hat{y}) - nKH_x(\hat{y} \mid x) - n\left[\int_{\underline{y}}^{\hat{y}} G(t)^{nx}\ln G(t)dt + C_1\right]$$

$$+\lambda(n-1)\int_{\underline{y}}^{\hat{y}} G(t)^{(n-1)x}H(t \mid x)\ln G(t)dt - n\mu\int_{\underline{y}}^{\hat{y}} G(t)^{nx}\ln^2 G(t)dt$$

$$\frac{\partial L}{\partial n} = -x(y_0 - \hat{y})G(\hat{y})^{nx}\ln G(\hat{y}) - KH(\hat{y} \mid x) - C_2$$

$$+\lambda x\int_{\underline{y}}^{\hat{y}} G(t)^{(n-1)x}H(t \mid x)\ln G(t)dt - \mu x\int_{\underline{y}}^{\hat{y}} G(t)^{nx}\ln^2 G(t)dt$$

Since $C_1 > 0$ and $C_2 > 0$, constraint (16) implies $x = 0$ and $x = +\infty$ cannot be a solution. Thus, the first-order condition gives $\frac{\partial L}{\partial x} = 0$. Then

$$n\frac{\partial L}{\partial n} = n\frac{\partial L}{\partial n} - x\frac{\partial L}{\partial x}$$

$$= -nC_2 - nK\left[1 - G(\hat{y})^x - xG(\hat{y})^x\ln G(\hat{y})\right] + \lambda x\int_{\underline{y}}^{\hat{y}} G(t)^{(n-1)x}H(t \mid x)\ln G(t)dt$$

$$< 0.$$

Therefore, $n^* = 1$. Then (FE) implies $\hat{y} = r - K$. Thus, the first-order conditions $\frac{\partial L}{\partial \hat{y}} = 0$ and $\frac{\partial L}{\partial x} = 0$ become

$$y_0 - r + \lambda\frac{H(r-K \mid x)}{h(r-K \mid x)} + \mu\frac{H_x(r-K \mid x)}{h(r-K \mid x)} = 0 \tag{27}$$

and

$$(y_0 - \hat{y})G(r-K)^x\ln G(r-K) + \mu\int_{\underline{y}}^{r-K} G(t)^x\ln^2 G(t)dt = 0. \tag{28}$$

Consider the case $y_0 \geq \bar{y}$. If $r^* < \bar{y}$ then $r^* < y_0$. Then (27) implies $\mu < 0$ and (28) implies $\mu > 0$. This contradiction means $r^* = \bar{y}$ and $\hat{y}^* = \bar{y} - K$.

Consider the case $y_0 < \bar{y}$. If $r^* < y_0$ then (27) implies $\mu < 0$ and (28) implies $\mu > 0$. This is a contradiction. If $y_0 < r^* \leq \bar{y}$ then (28) implies $\mu < 0$ and hence (27) implies $\lambda > 0$. Thus, $E\pi_1(r^* - K) = 0$. But $E\pi_1(r^* - K) > E\pi_1(y_0 - K) > 0$ by the assumption. This is also a contradiction. Therefore, $r^* = y_0$ and $\hat{y}^* = y_0 - K$.

Q.E.D.

Even if there are R&D decisions and bid-preparation costs, setting the buyer's reservation price r equal to the minimum of the opportunity cost y_0 and the highest possible production cost $\bar{y}$ to be observed by the firms is socially optimal. The most interesting result is that society prefers only one firm to conduct R&D and production when R&D is subject to constant marginal returns to scale on expenditure. Remember that R&D is an independent drawing process and the R&D outcome of n firms will be the same as the R&D outcome of one firm that invests the same amount as all n firms. But, because of the fixed cost of R&D, more firms participating in R&D result in higher total R&D costs. Thus, for society, one firm conducting all of the R&D and production is more efficient. Contrary to the social optimum, the buyer usually prefers more than one firm to enter both R&D and bidding processes.

From the above comparison analysis, we have seen that the optimal policies from the point of view of the buyer and society are different under two types of R&D technology: the fixed-scale and the variable scale with constant marginal returns. When R&D is subject to diminishing or increasing marginal returns to scale on expenditure, we expect that some of these results will also change. Further research is needed on this topic. As far as we can tell, the type of R&D technology plays important roles in determining optimal R&D and procurement policies for the buyer and for society.

3.6 REMARKS

I have presented a model of private R&D and procurements with entry. From the above analysis, when R&D technology is subject to a constant marginal return to scale, society prefers to have only one firm conduct all of the R&D and production and the buyer usually prefers more firms to invest in private R&D. But the buyer has to pay the total R&D costs among all firms even if only one contractor is chosen for production. The buyer would like R&D to be conducted efficiently. This raises the question whether there exists alternative and more efficient ways to manage R&D activities. One way to accomplish this might be to have the buyer do the R&D himself and then release the R&D outcomes to potential firms. The buyer could also hire an agency (private or public) to conduct R&D and force the agency to transfer the R&D outcomes to potential producers. That would eliminate the duplication of effort that occurs when several firms conduct R&D at the same time. For instance, in some defense procurement cases, a government agency (e.g., NASA, DOD) conducts the basic research and may also develop the new products. Then, the government agency transfers the technology information to potential contractors for production and chooses the most efficient contractor to produce the product.

There are some disadvantages in releasing or transfering such R&D information. (i) Credibility: French and McCormick (1984) have argued that if the buyer does R&D himself, he has an incentive to provide optimistic information about the technology and demand because of the conflict of interest between the buyer and the firms. Unless this incentive can be controlled, the buyer's information may be ignored by the firms. (ii) Transferability: Some technological information (or physical capital, human capital, and so on) obtained by the buyer or the hired agency may not be easily transferable

(Williamson 1976, Laffont and Tirole 1988). (iii) Learning costs: it takes time or effort for the firms to understand the technological information or prototypes. There are learning costs that will be incurred before production can begin. It would be desirable to investigate and compare different arrangements of R&D management and to identify the advantages and disadvantages of each. Successful modelling will certainly help us better understand current practice in government R&D management.

NOTES

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1 For a survey on auctions and bidding, see McAfee and McMillan (1987c). For a survey on the economic theory of procurement and contracting, see Besen and Terasawa (1987).

2 McAfee and McMillan (1987b) allow the number of actual bidders to be stochastic, but the probability of any subset of potential bidders becoming the set of actual bidders is assumed to be exogenous and independent of their types. They have shown that the optimal auction is the same whether or not the risk-neutral bidders know who their competitors are.

3 See Riordan and Sappington (1987) and Dasgupta and Spulber (1989) for these procurement results.

4 Rob (1986) and I (in Chapter 2) have formally incorporated R&D activities into competitive procurement processes for a given number of firms and characterized the equilibrium investment level in R&D and the optimal procurement contract.

5 This R&D process of cost reduction is an independent one-shot drawing process. It is different from a sequential search process. The first x dollars have the same effect on the R&D outcome as the last x dollars. To some extent, this process is subject to constant marginal returns to expenditure in R&D. I consider this special technology to simplify analyzing the free entry equilibrium in this chapter.

The analysis should be extended to more general cases, such as the diminishing marginal return R&D technologies that I considered in the case of a fixed number of firms in Chapter 2.

6 I consider the first-price sealed-bid auction in this paper because it is often used in practical procurement processes. The second-price sealed-bid auction with the same reservation price will not change the firms' net expected profits from bidding and hence should give the same results. It would be interesting to look at the effects of oral auctions on the firms' R&D investment strategies (possibly asymmetric) by allowing some firms to have information advantages before R&D. I thank Preston McAfee for this interesting point.

7 Under the first-price sealed-bid auction, with some restrictive conditions that $[1 - F(y)]/f(y)$ is nonincreasing in y and the bidding firms have the same lowest bids, I can show that there does not exist any asymmetric (bidding and investment) equilibrium at both stage (i) and (ii). Under the second-price sealed-bid auction, the bidding firms bid their true observations of production costs, and asymmetric investment equilibria in stage (ii) always exists. Coordination is needed for equilibrium selection in this case. I ignore such problems in this chapter.

8 Since firm i is unable to observe the other firms' investment levels x_{-i}, its bidding strategy B_i can not depend on x_{-i} directly. Symmetric beliefs of each firm enable us to consider symmetric bidding strategies (also see Note 6). I thank Tom Palfrey for this helpful comment.

9 This proof is based on Proposition 2. An alternative proof of Proposition 3 is to construct a compact, convex set S and a continuous mapping from S to S, based

upon the equations (EE), (R&D), and (FE), and to use the Brouwer's fixed point theorem. Interested readers can get the manuscript of the second proof from the author.

10 See also Riley and Samuelson (1981) for a proof.

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CHAPTER 4

OPTIMAL SELLING SCHEME WITH AN INFORMED AUCTIONEER

4.1 INTRODUCTION

The optimal auction design literature examines optimal selling schemes for a monopolist with many potential buyers. If the environment is such that: i) the buyers are symmetric and risk-neutral; ii) the buyers' values of the object are private and independent, then it has been shown that four standard auctions (English, Dutch, first-price sealed-bid, second-price sealed-bid) with an appropriate announced reservation price are all optimal selling schemes for the seller.[1] To derive this result, it is also commonly assumed that the seller's value for the object is publicly known to all buyers. But the seller's value is often his private information. It is costly for the buyers to know the seller's value. The seller may want to use his private information strategically instead of revealing it to the buyers.[2] The question is how the seller should use his private information and what selling scheme is optimal for him in that situation. We might expect the seller not to reveal or only partially to reveal his private information and to offer a different selling scheme from the above four standard auctions but, as we will see in this chapter, it actually does not matter whether the seller has private information or not once the environment is such that: (i) the seller and buyers are risk-neutral and the buyers are symmetric; (ii) the buyers' values and the seller's value are private and independent. The seller should choose the same scheme he would if he does not have private information.

We assume, in this chapter, that the seller has private information about his value for the object and has the ability to select a selling scheme. Finding the optimal scheme for the seller is a mechanism design problem by an informed principal. Myerson (1983) provides a general framework and some solution concepts. But he does not offer any conditions under which the informed principal should choose the same mechanism as that chosen when the principal's information is public. Using the Myerson's approach, we first prove that there exists an incentive compatible and individual rational direct revelation mechanism that maximizes the seller's ex ante expected profit among all incentive compatible and individual rational mechanisms with respect to all buyers and the seller as well. This is the same mechanism that the seller would select if the seller's value were known to all buyers. Then we show that this mechanism is a strong solution and hence neutral optimum for the seller in the sense of Myerson (1983). Based on the Revelation Principle and the Inscrutability Principle, we can see that it is in the best interest of the seller to implement this strong solution. In the special case when there is only one buyer, this optimal mechanism is the same as the seller's offer scheme in Williams (1987). That is, when the seller has all of the bargaining ability, he makes an offer and the buyer either accepts the offer or rejects it.

Furthermore, the optimal direct revelation selling mechanism can be implemented by the Vickrey auction with a preannounced reservation price, which is an increasing function of the seller's value. According to the Revenue-Equivalence Theorem, for any given reservation value of the seller, any one English auction, Dutch auction, first-price sealed-bid auction, or Vickrey auction with the same reservation price yields the same expected revenue for the seller. Thus, any one of these four auctions with an optimal reservation price is the best selling scheme for the seller. The implication of the present analysis is that, in the standard independent-private value model with symmetric and risk-neutral buyers, the main results on the optimal auction design still hold whether or not the seller is informed about his value for the object to be sold.

4.2 THE MODEL AND PRELIMINARIES

Consider a monopolistic seller who seeks to sell one indivisible commodity he owns to n potential buyers. The seller knows his own valuation for the object, which cannot be observed by any buyer. But the buyers all believe that the seller's valuation, denoted by z, is drawn from a random distribution $G(z)$. This distribution has a positive density $g(z)$ on the bounded interval $[a, b]$. Similarly, let x_i represent buyer i's value, which is observed by buyer i only. The other buyers and the seller believe that x_i is independently drawn from the same distribution $F(x_i)$ with the density function $f(x_i)$ and support $[a, b]$. Let

$$I(x_i) = x_i - \frac{1 - F(x_i)}{f(x_i)} \quad \text{and} \quad J(z) = z + \frac{G(z)}{g(z)}.$$

We make the standard regularity assumptions: both $I(x_i)$ and $J(z)$ are continuously differentiable over $[a, b]$ and $I(x_i)$ is strictly increasing in x_i. Let $N = \{1, 2, \ldots, n\}$, $x = (x_1, \ldots, x_n)$, and $x_{-i} = (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n)$.

A direct revelation trading mechanism consists of n probability schedules and n payment schedules that determine the final distribution of the object and money given the declared valuations $(\hat{x}, \hat{z})$. Let $< p(\hat{x}, \hat{z}), r(\hat{x}, \hat{z}) >$ represent such a mechanism, where

$$p(\hat{x}, \hat{z}) = \Big[p_1(\hat{x}, \hat{z}), \ldots, p_n(\hat{x}, \hat{z}) \Big],$$

$$r(\hat{x}, \hat{z}) = \Big[r_1(\hat{x}, \hat{z}), \ldots, r_n(\hat{x}, \hat{z}) \Big],$$

$p_i(\hat{x}, \hat{z})$ is the probability that the seller sells the object to buyer i, and $r_i(\hat{x}, \hat{z})$ is the payment from buyer i to the seller. The probability that the seller sells the object is $\sum_{i=1}^{n} p_i(\hat{x}, \hat{z})$. Since the seller has only one object, the function $p(\hat{x}, \hat{z})$ must satisfy the following probability conditions:

$$\sum_{i=1}^{n} p_i(\hat{x}, \hat{z}) \leq 1, \quad \text{and} \quad p_i(\hat{x}, \hat{z}) \geq 0 \qquad \forall\, i \in N, \ \forall\, (\hat{x}, \hat{z}) \in [a, b]^{n+1}. \tag{1}$$

The seller and buyers each have a von Neumann-Morgenstern utility function that is additively separable and linear both in money and in the value of the object.[3] Thus the seller's expected utility, given that his true value is z and the vector of declared values $(\hat{x}, \hat{z})$, is

$$V(\hat{x}, \hat{z}, z) = \sum_{i=1}^{n} \left[r_i(\hat{x}, \hat{z}) - z p_i(\hat{x}, \hat{z}) \right].$$

Similarly, buyer i's expected utility is

$$U_i(\hat{x}, \hat{z}, x_i) = x_i p_i(\hat{x}, \hat{z}) - r_i(\hat{x}, \hat{z}).$$

Let $\bar{p}_i(x_i) = \underset{x_{-i}, z}{E}\, p_i(x, z)$, $\bar{r}_i(x_i) = \underset{x_{-i}, z}{E}\, r_i(x, z)$ be buyer i's expected probability of receiving an object and his expected payment to the seller, respectively, given that his value is x_i. Also, let $\underline{p}_i(z) = \underset{x}{E}\, p_i(x, z)$ and $\underline{r}_i(z) = \underset{x}{E}\, r_i(x, z)$. The expected utilities of the seller and buyer i conditional on their values are

$$V(z) = \sum_{i=1}^{n} \left[\underline{r}_i(z) - z \underline{p}_i(z) \right],$$

$$U_i(x_i) = x_i \bar{p}_i(x_i) - \bar{r}_i(x_i).$$

Incentive compatibility (IC) for the seller requires

$$V(z) \geq \sum_{i=1}^{n} \left[\underline{r}_i(\hat{z}) - z \underline{p}_i(\hat{z}) \right], \quad \forall\, z, \hat{z} \in [a, b]. \tag{2}$$

Incentive compatibility (IC) for the buyers requires

$$U_i(x_i) \geq x_i \bar{p}_i(\hat{x}_i) - \bar{r}_i(\hat{x}_i), \quad \forall\, x_i, \hat{x}_i \in [a, b], \ \forall\, i \in N. \tag{3}$$

Lemma : A trading mechanism $<p, r>$ is incentive compatible if and only if the following hold:

a) $\bar{p}_i(x_i)$ is weakly increasing in x_i and $\underline{p}_i(z)$ is weakly decreasing in z;

b) $V(b) + \sum_{i=1}^{n} U_i(a) = \underset{x,z}{E}\left[\sum_{i=1}^{n}\left[I(x_i) - J(z)\right] p_i(x, z)\right]$;

c) $\bar{r}_i(x_i) = x_i \bar{p}_i(x_i) - U_i(a) - \int_a^{x_i} \bar{p}_i(t_i) dt_i$,

$\sum_{i=1}^{n} \underline{r}_i(z) = V(b) + \sum_{i=1}^{n}\left[\int_z^b \underline{p}_i(t) dt + z\underline{p}_i(z)\right]$.

The proof is standard. See Myerson and Satterthwaite (1983), for example.

Individual rationality (IR) for the seller requires

$$V(z) \geq 0 \quad \forall z \in [a, b] \tag{4}$$

and individual rationality (IR) for the buyers requires

$$U_i(x_i) \geq 0 \quad \forall x_i \in [a, b], \forall i \in N. \tag{5}$$

From the Lemma, $V(z)$ is weakly decreasing in z and $U_i(x_i)$ is weakly increasing in x_i. Thus, with incentive compatibility, the individual rationality conditions (4) and (5) are equivalent to $V(b) \geq 0$ and $U_i(a) \geq 0$ for every buyer i, respectively.

Since the seller and all buyers have incomplete information, we can model the trading process as a Bayesian incentive problem in which the seller and n buyers are players. According to the Revelation Principle, for any Bayesian equilibrium of any mechanism that the players might play, there exists an equivalent incentive compatible and individual rational (IC-IR) direct revelation mechanism that satisfies (1) - (5). Therefore, there is no loss of generality in considering only IC-IR direct revelation mechanisms.

Since the seller, one of the players in the Bayesian game, has the ability to select mechanisms, he will select a mechanism that maximizes his conditional expected profit given his value z. Given that the seller knows his type z, the question is whether different types of the seller will choose different trading mechanisms. By the Inscrutability Principle (Myerson 1983), there is no loss of generality in requiring that all types of the seller choose the same trading mechanism. Then the seller's actual choice of mechanism will convey no information. Thus, we only have to consider direct revelation mechanisms that satisfy (1) - (5). Myerson (1983) provides a rationale for an informed principal to select a strong solution (undominated and safe mechanism) or a neutral optimum. But he does not show that the principal should choose the same mechanism as that chosen when his information is public. In the following, we will prove that in our setting the seller should choose the same auction scheme as he would when his value of the object is publically known.

4.3 THE SELLER'S OPTIMAL SELLING MECHANISM

For any $i \in N$ and $(x, z) \in [a, b]^{n+1}$, let

$$\bar{x}(z, x_{-i}) = \max\left[I^{-1}(z), \max_{j \neq i} x_j\right], \tag{6}$$

$$p_i^*(x, z) = \begin{cases} 1 & \text{if } x_i > \bar{x}(z, x_{-i}) \\ 0 & \textit{otherwise} \end{cases}, \tag{7}$$

and

$$r_i^*(x, z) = p_i^*(x, z)\bar{x}(z, x_{-i}), \tag{8}$$

where $I^{-1}(z)$ is the inverse function of $I(x_i)$, which exists because $I(x_i)$ is strictly increasing in x_i. $<p^*, r^*>$ is a direct revelation trading mechanism. From Myerson (1981), for any given z of the seller's value, $<p^*, r^*>$ maximizes the seller's conditional expected profit $V(z)$ subject to feasibility constraints (1), IC constraints (3) and IR constraints (5) for the buyers. Thus, for any given $z \in [a, b]$, $<p^*, r^*>$ is feasible and IC-IR for each buyer. If the seller's value were known to all buyers, the seller would select mechanism $<p^*, r^*>$.

Consider what happens if the seller selects a mechanism $<p, r>$ to maximize his ex ante expected profit subject to conditions (1) - (5), without caring about information released in that choice. Using the Lemma, we can calculate the seller's ex ante expected profit

$$EV = \int_a^b V(z)g(z)dz$$

$$= V(b) + \mathop{E}_{x,z}\left[\frac{G(z)}{g(z)}\sum_{i=1}^{n} p_i(x, z)\right].$$

Proposition 1: Mechanism $<p^*, r^*>$ maximizes the seller's ex ante expected profit among all IC-IR direct revelation trading mechanisms.

Proof: By the Lemma, the incentive conditions (2) and (3) are equivalent to a), b), and c) described in the Lemma. We can first ignore the feasibility constraints (1) and the incentive constraints 1) and 3) and will check these later. Then the seller's ex ante optimization problem described above is reduced to the following:

$$\underset{p,\, V(b)}{Max} \quad V(b) + \underset{x,z}{E}\left[\frac{G(z)}{g(z)} \sum_{i=1}^{n} p_i(x,z)\right]$$

$$s.t. \qquad \underset{x,z}{E}\left[\sum_{i=1}^{n}\left[I(x_i) - J(z)\right] p_i(x,z)\right] \geq V(b), \tag{9}$$

$$0 \leq p_i(x,z) \leq 1, \quad \forall\, x, z, i,$$

$$V(b) \geq 0.$$

The constraint (9) is binding, otherwise increasing $V(b)$ improves the seller's expected profit without violating all these constraints. Then

$$V(b) = \underset{x,z}{E}\left[\sum_{i=1}^{n}\left[I(x_i) - J(z)\right] p_i(x,z)\right].$$

In other words, $U_i(a) = 0$ for all $i \in N$. By ignoring constraint $V(b) \geq 0$, the seller's problem is further reduced to maximize

$$EV = \underset{x,z}{E}\left[\sum_{i=1}^{n}\left[I(x_i) - z\right] p_i(x,z)\right]$$

subject to $0 \leq p_i(x,z) \leq 1$ for all $(x,z) \in [a,b]^{n+1}$ and all $i \in N$. It is easy to see that the solution to this reduced problem is $p^*(x,z)$.

Now we can check all the constraints we ignored before. First, $p_i^*(x, z)$ is weakly increasing in x_i and weakly deceasing in z. Thus, $\bar{p}_i^*(x_i)$ is weakly increasing in x_i and $\underline{p}_i^*(z)$ is weakly decreasing in z. The incentive constraint a) in the Lemma is satisfied. Second, by the definition of $r_i^*(x, z)$, we know $<p^*, r^*>$ satisfies c) in the Lemma. Third, since

$$\int_a^{I(x_i)} \left[I(x_i) - J(z)\right] dG(z) = \int_a^{I(x_i)} \left[I(x_i) - z\right] dG(z) - \int_a^{I(x_i)} G(z)dz$$

$$= 0,$$

where the second equality holds from the integration of the first term by parts, then

$$V(b) = \mathop{E}_{x, z}\left[\sum_{i=1}^{n} \left[I(x_i) - J(z)\right] p_i^*(x, z)\right]$$

$$= \sum_{i=1}^{n} \mathop{E}_{x_{-i}} \int_{\max_{j \neq i} x_j}^{b} \left[\int_a^{I(x_i)} \left[I(x_i) - J(z)\right] dG(z)\right] dF(x_i)$$

$$= 0.$$

The seller's individual rationality condition is satisfied. Finally, it is easy to see that $\sum_{i=1}^{n} p_i^*(x, z) \leq 1$ for all x and z. In summary, mechanism $<p^*, r^*>$ maximizes the seller's ex ante expected profit subject to IC-IR constraints and feasibility constraints (1) - (5).

Q.E.D.

Since $<p^*, r^*>$ maximizes the seller's ex ante expected profit and $g(z)$ is positive over (a, b) by the assumption, by the seperating hyperplane theorem, $<p^*, r^*>$ is an undominated mechanism. It should be very reasonable for the seller to select this undominated mechanism if $<p^*, r^*>$ is also safe, in the sense that $<p^*, r^*>$ would be IC-IR even if the buyers knew the seller's value (see Myerson 1983). An undominated and safe mechanism is called a strong solution by Myerson. He also suggests another solution for the principal, i.e., neutral optimum, which is an incentive compatible mechanism that cannot be blocked by any theory of "blocking" that satisfies four basic axioms. We can show $< p^*, r^* >$ satisfies these criteria.

Proposition 2: Mechanism $< p^*, r^* >$ is a strong solution and neutral optimum for the seller.

Proof: First, from the seperating hyperplane theorem, $< p^*, r^* >$ is an undominated mechanism for the seller because it maximizes the seller's ex ante expected profit and the density function $g(z)$ is positive over (a, b). Second, in order to show $< p^*, r^* >$ is safe in the sense of Myerson (1983), we need to show that it is IC-IR for each buyer given the seller's value z and that it is IC-IR for the seller. From the last section, we know $< p^*, r^* >$ is IC-IR for the seller. On the other hand, from Myerson (1981), for any given $z \in [a, b]$, $< p^*, r^* >$ is IC-IR for each buyer. That is, $< p^*, r^* >$ is a safe mechanism. Therefore, $< p^*, r^* >$ is a strong solution for the seller. Third, by Theorem 5 in Myerson (1983), the strong solution $< p^*, r^* >$ is also a neutral optimum for the seller.

Q.E.D.

Thus, $< p^*, r^* >$ is the seller's reasonable choice of selling schemes. In a special case where there is only one buyer, $< p^*, r^* >$ is the seller's offer scheme in Williams (1987).

4.4 IMPLEMENTATION

A modified Vickrey auction is an auction in which all buyers and the seller submit bids and the object is transferred to the trader with the highest bid at a price equal to the second-highest bid. Under this auction, if the seller's bid is the highest then there is no trade. If there is only one bid from the buyers that is higher than the seller's bid then the object is transferred to that buyer with the highest bid at a price equal to the seller's bid. We consider Bayesian equilibria $(b_1(x_1), \ldots, b_n(x_n), s(z))$ under the modified auction rule.

Proposition 3: (i) Under the modified auction rule, the unique Bayesian equilibrium is $b_i(x_i) = x_i$ for all i and $s(z) = I^{-1}(z)$; (ii) the modified Vickrey auction implements $< p^*, r^* >$.

Proof: First, by the standard argument,[4] we know that truth-telling is a dominant strategy for each buyer under the modified Vickrey auction rule. That is, each buyer submits a bid b_i that is equal to his own value x_i of the object.

Second, given the dominant strategy of truth-telling by each buyer under the modified Vickrey auction rule, we want to show that the best response for a seller of type z is to submit a bid that equals $I^{-1}(z)$. The seller's expected profit, given his value z and his bid s, is

$$EV(s \mid z) = \underset{x}{E}\left[z \mid s > x_i, \forall\, i \in N \right] + \underset{x}{E}\left[\max\,(y_{n-1}, s) \mid s \leq y_n \right] \tag{10}$$

where y_n and y_{n-1} are the first and the second order statistics of $(x_1, \ldots, x_n)$, respectively, and have a joint density function

$$h(y_n, y_{n-1}) = n(n-1)F^{n-2}(y_{n-1}) f(y_{n-1}) f(y_n), \quad a \leq y_{n-1} \leq y_n \leq b.$$

The first term in (10) equals $zF^n(s)$ and the second term is

$$E_x\left[\max(y_{n-1}, s) \mid s \le y_n\right] = \int\limits_{a \le y_{n-1} \le y_n \le b,\, s \le y_n} \int \max(y_{n-1}, s) h(y_n, y_{n-1}) dy_n dy_{n-1}$$

$$= \int\limits_{s \le y_n \le b} \int\limits_{a \le y_{n-1} \le s} s h(y_n, y_{n-1}) dy_n dy_{n-1}$$

$$+ \int\limits_{s \le y_n \le b} \int\limits_{s \le y_{n-1} \le y_n} y_{n-1} h(y_n, y_{n-1}) dy_n dy_{n-1}$$

$$= b - sF^n(s) + \int_s^b \left[(n-1)F^n(y) - nF^{n-1}(y)\right] dy,$$

where the last equality follows from some algebra and integration by parts. Then,

$$EV(s \mid z) = zF^n(s) + b - sF^n(s) + \int_s^b \left[(n-1)F^n(y) - nF^{n-1}(y)\right] dy,$$

and

$$\frac{\partial EV(s \mid z)}{\partial s} = nF^{n-1}(s) f(s)\left[z - I(s)\right].$$

$EV(s \mid z)$ is maximized at $s = I^{-1}(z)$ since

$$\frac{\partial EV(s \mid z)}{\partial s} \begin{cases} > 0 & \text{if } s < I^{-1}(z) \\ = 0 & \text{if } s = I^{-1}(z). \\ < 0 & \text{if } s > I^{-1}(z) \end{cases}$$

Therefore, $s = I^{-1}(z)$ is the optimal strategy for the seller following the truth-telling strategy by each buyer.

Finally, given the equilibrium strategy $b_i(x_i) = x_i$ for all i and $s(z) = I^{-1}(z)$ under the modified Vickrey auction rule, buyer i receives the object if, and only if, x_i is higher than x_j for all $j \ne i$ and is higher than $I^{-1}(z)$ as well. That is, buyer i receives the object if, and only if, x_i is higher than the second highest bid $\bar{x}(z, x_{-i})$. This outcome is completely described by $p_i^*(x, z)$. If buyer i wins he will pay the seller an amount equal to the second highest bid $\bar{x}(z, x_{-i})$. Therefore, the modified Vickrey auction implements the seller's optimal direct revelation mechanism $< p^*, r^* >$.

Q.E.D.

We have shown that the modified Vickrey auction is the best selling scheme for the seller who is privately informed about his value of the object. Since the equilibrium strategy for each buyer under the modified Vickrey auction is a dominant strategy, the seller can simply make a bid first and then let each buyer submit a bid. In other words, the seller with value z can actually announce a reservation price r_0 that is equal to his equilibrium bid $I^{-1}(z)$ and then ask each buyer to make a sealed bid. Any bid below r_0 from the buyers is not acceptable to the seller. That is, the seller will keep the object if there is no acceptable bid. The buyer with the highest acceptable bid will get the object and pay the seller at the price of the second-highest acceptable bid (if there is only one such acceptable bid, the winning buyer just pays the seller's reservation price r_0). This is the Vickrey auction (or second-price sealed-bid auction).

Proposition 4: The Vickrey auction with an announced reservation price $r_0 = I^{-1}(z)$ is the optimal scheme for the seller of type z.

In this auction mechanism, the seller actually reveals his private information z through the announcement of the reservation price $r_0 = I^{-1}(z)$. Since $I(z) < z$ for any $z < b$, then $I^{-1}(z) > z$ for all $z < b$. The seller announces a reservation price higher than his true reserve value.

For any given z of the seller's value, by the Revenue-Equivalence Theorem (see McAfee and McMillan 1987), the English auction, the Dutch auction, and the first-price sealed-bid auction with the announced reservation price $r_0 = I^{-1}(z)$ yield the same expected revenue $V(z)$ for the seller as the Vickrey auction with the same reservation price $r_0 = I^{-1}(z)$. Therefore, even if the seller has a private value, any one of these four standard auctions is an optimal selling scheme provided it is supplemented by an announced reservation price r_0.

Consider the special case where there is only one buyer. If the seller has the ability to select a trading scheme, he will select a scheme in which both the buyer and seller submit sealed bids. If the buyer's bid is no less than the seller's bid the object is transferred to the buyer at the price of the seller's bid. This is the seller's offer auction under which the seller makes an offer and the buyer either takes the offer or rejects it. Williams (1987) shows that this seller's offer auction maximizes the expected return to the seller. When there are many buyers, the best scheme for the seller is not the seller's offer auction, but the Vickrey auction with an appropriate reservation price.

Finally, suppose that the seller does not have strong bargaining ability. The seller and buyers may select a mediator (or an auctioneer, arbitrator) to allocate the object. Suppose that the mediator recommends that the seller and all buyers use a first-price sealed-bid auction with no reservation price. It can be shown that the seller collects less ex ante expected profit EV_2 in this case than EV_1 when he has all of the bargaining ability. For example, in the case of symmetric uniform distribution, we can calculate EV_1 and EV_2 as the following:

$$EV_1 = 1 - \frac{1 + n2^{n+1}}{(n+1)(n+2)2^n},$$

$$EV_2 = 1 - \frac{2^n + n2^{n+1}}{(n+1)(n+2)2^n}.$$

It is clear $EV_1 > EV_2$ for all $n \geq 1$. Therefore, once the seller has the ability to select a trading mechanism, he does not want to transfer the object he owns to some mediator and then participate in the mechanism suggested by the mediator.

4.5 FINAL REMARKS

In the case of variable supply, the result will be similar. That is, the seller will choose the same mechanism he would if the seller's cost is completely known to all buyers. Spulber (1988) provided a similar bargaining model in the case of one buyer and one seller with private information on both sides.

In the above discussions, we have assumed that the values are independent and that both the seller and the buyers are risk-neutral. Because of these assumptions, the mechanism that maximizes the seller's conditional (interim) expected profit subject to IC-IR with respect to all buyers is the same as the mechanism that maximizes the seller's ex ante expected profit subject to IC-IR with respect to all buyers and the seller as well. The second property implies that the optimal mechanism is undominated while the first implies that the optimal mechanism is safe. Thus, we are able to obtain a strong solution that the seller will select even if he has private information.

In a principal-agent private-values model, Maskin and Tirole (1988) have shown that the principal will select the full information contract when both the principal and the agent have the quasi-utility functions. This is similar to the result we derived in this chapter. We believe that the assumptions of private values and quasi-utility functions are the key to obtain the invariance result. In a recent paper, Vincent (1989) finds an example of common-values model in which the auctioneer has an incentive to keep his private information secret. In other words, the auctioneer never announces any reservation price in the auction. This seems to be consistent with the empirical observation by Ashenfelter (1989) in auctions for wine and art. It remains an open question to what extent the invariance result I find in this chapter holds in a general environment or under what conditions an informed principal should signal his private information in the design of mechanisms.

NOTES

1 See McAfee and McMillan (1987) for an excellent survey on auction theory.

2 For example, Ashenfelter (1989) provides some evidence that the auctioneer keeps the reservation price secret in auctions for wine and art.

3 The assumption that utility function is linear in the value is not crucial. The results in this paper hold for any quasi-linear utility function.

4 See McAfee and McMillan (1987), for instance.

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