FLAVOR SU(3) PREDICTIONS FOR CHARMED BARYON AND B-MESON DECAYS

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ABSTRACT

The predictions of the SU(3) flavor symmetry of the strong interactions for the weak decay of charmed baryons and B-mesons are detailed. It is hoped that comparison between these predictions and experiment will shed some light on the underlying dynamics involved in these weak decays. Although only a few decay modes of the charmed baryons and B-mesons have been studied experimentally it is hoped that the next generation of B-factories and even Z-decays at LEP will provide enough events to test these predictions.

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INTRODUCTION

In the near future it is likely that branching ratios for many of the exclusive decays of charmed baryon and B-meson will be measured. The decay of B-mesons provides a unique laboratory for studying weak interactions in so far as they can decay not only to mesons but more interestingly to charmed and uncharmed baryons. The decays of D-mesons have been studied extensively and continue to be an active area of research. For kinematic reasons the D-mesons can decay only to mesonic final states and not to states containing baryons. Unlike the B-meson decays, the decay of D-mesons through radiative loop-induced weak operators is highly suppressed. Consequently, the study of B-meson decays provides a new way to isolate the effects of these one-loop processes.

Analysis of the available experimental data on D-meson decays indicates that the predictions of the $SU(3)_f$ flavor symmetry are not well satisfied in nature. The deviations from the predictions are not well understood and are attributed to final state interactions (FSI), but could conceivably be due to an intrinsic breakdown of $SU(3)_f$. The data indicates that there are significant final state interactions but is not precise enough to exclude the possibility of intrinsic violations of $SU(3)_f$. Therefore it would be useful to look for deviations in the decay of charmed baryons from the predictions of $SU(3)_f$ where it is hoped that the FSI will be smaller than in the mesonic sector.

The use of $SU(3)$ flavour symmetry in strange decays in the early sixties proved to be an immensely powerful tool in our understanding of weak interactions and weak currents. The octet dominance ($\Delta I = \frac{1}{2}$) rule in such decays was not understood until the mid seventies when it was realized that short distance strong interactions gave rise to an enhancement of the octet over the 27 component of the Hamiltonian by a factor of ~ 5. This is still smaller than the observed enhancement of ~ 20. Below scale of 1 GeV the strong interactions are not well understood and it is possible that the enhancement from the strong interactions could in fact be larger than this factor of ~ 5. Also, one-loop penguin diagrams give rise to a Hamiltonian that transforms as a 8 with $\Delta I = \frac{1}{2}$ and it is conceivable that these diagrams dominate the weak decays below 1 GeV.

In this thesis the predictions of $SU(3)_f$ are presented for the decays of charmed baryons and B-mesons. These relations will be useful in determining the underlying mechanisms responsible for the weak decays. Analagous to the strange decays, short distance strong interactions give rise to an enhancement of the sextet component of the Hamiltonian over the $\overline{15}$ in charm decays. The $SU(3)_f$ relations between the decay modes of charmed baryons may enable this enhancement to be observed. There are similar observations to be made in the B-meson decays. Penguin diagrams lead to $\Delta I = 0$ selection rule for some of the B-meson decay modes. By comparison with experimental data, the relative importance of such diagrams compared to the tree level Hamiltonian can be ascertained.

The results in Chapters 4 and 5 are published (with Mark B. Wise) in *Nuc. Phys.* **B326**, 15 (1989) and *Phys. Rev.* **D39**, 3346 (1989) respectively. The results of Chapter 3 have been submitted (with Roxanne P. Springer) to Physical Review D.

CHAPTER 1. WEAK INTERACTIONS

The subject of weak interactions covers a vast area of research from the elusive Higgs boson to determination of the Helium abundance of the universe. In this chapter I will not attempt to address even a small fraction of this work, but I hope to introduce the areas relevent to the decay of hadrons containing heavy quarks. The first section deals with the standard model of electroweak interactions which became popular in the early to mid seventies after it was shown to be renormalizable. This theory is not fundamental in the sense that it contains many free parameters that must be determined experimentally but there is a hope that the ultimate theory of everything will predict these numbers. On the other hand there is a remarkable agreement between the experimental observations and theoretical predictions which, with the possible exception of the Higgs sector and neutrino masses, leads one to believe that this gives a complete description of electroweak interactions below ~ 100 GeV. The second section introduces the concept of a flavor symmetry when some of the quarks have masses much smaller or heavier than the scale of strong interactions. In section 1.3 the methodology for constructing of the predictions of a flavor symmetry is presented by explicit calculation of the rates for semileptonic hyperon decay in terms of two reduced matrix elements. These predictions are then compared with the large body of experimental information on the subject. The agreement between theory and experiment is surprisingly good, indicating that flavor symmetry may be a useful tool in understanding weak decays of other hadrons. In section 1.4 the method by which explicit $SU(3)_f$ breaking is implemented is discussed. The breaking is a result of the inequality of masses of the three light quarks. It is hoped that the relevent expansion parameter for the breaking is $m_s/1$ GeV. The semileptonic decay of the hyperons is presented as an explicit example.

1.1 The Standard Model of Electroweak Interactions

The standard model [1.1] for strong, weak and electromagnetic interactions is based on the gauge group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$. The minimal particle content consists of the gauge bosons and the matter fields shown in Table 1.1.

Table 1.1: Matter fields in the Standard Model

Field	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	Spin
$Q_L^i = \begin{pmatrix} u_L^i \\ d_L^i \end{pmatrix}$	3	2	1/3	1/2
u_R^i	3	1	4/3	1/2
d_R^i	3	1	-2/3	1/2
$L_L^i = \begin{pmatrix} \nu_L^i \\ e_L^i \end{pmatrix}$	1	2	-1	1/2
e_R^i	1	1	-2	1/2
$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$	1	2	1	0

The superscript i on the matter fields is a generation index and takes the values $i \in \{1,2,3\}$. The subscripts L and R denote left handed and right handed respectively. Anomaly cancellation in the fermionic triangle diagrams uniquely constrains the hypercharge assignment Y of each matter field. The complex scalar field ϕ is necessary to break the $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ symmetry to $SU(3)_c \otimes U(1)_Q$ which describes nature at scales below ~ 100 GeV. Electromagnetism is realized in the unbroken $U(1)_Q$ symmetry and the $SU(3)_c$ symmetry is that of the strong interactions.

The Lagrange density for the system is composed of five terms

$$\mathcal{L} = \mathcal{L}_{\rm F} + \mathcal{L}_{\rm H} + \mathcal{L}_{\rm GB} + \mathcal{L}_{\rm Y} + \mathcal{L}_{\rm GF} \quad . \tag{1.1}$$

These terms correspond to the contributions from the fermions, Higgs, gauge bosons, Yukawa interactions and the gauge fixing components respectively.

Firstly, we will examine the Higgs sector which is responsible for the symmetry breaking [1.2]. The Lagrange density is

$$\mathcal{L}_{\rm H} = D^\mu \phi^\dagger D_\mu \phi - V(\phi) \quad , \tag{1.2}$$

where D_μ is the covariant derivative defined as

$$D_\mu = \partial_\mu - i\frac{1}{2} g \tau^a W_\mu^a - i\frac{1}{2} g' Y B_\mu \quad . \tag{1.3}$$

The $W_\mu^a(x)$ and $B_\mu(x)$ are the gauge fields associated with the $SU(2)_L$ and $U(1)_Y$

symmetries respectively and τ^a are the Pauli spin matrices. The potential $V(\phi)$ can be written as

$$V(\phi) = \lambda\left(\phi^\dagger\phi - v^2\right)^2 \quad , \tag{1.4}$$

which has a global minimum at $\phi^\dagger\phi = v^2$. The global $SU(2)_L \otimes U(1)_Y$ invariance of the lagrangian allows us to rotate an arbitrary vacuum expectation value (vev) of the field $\langle\phi\rangle$ into the form $\langle\phi\rangle = \binom{0}{v}$. Upon the field redefinition $\phi^+ \to \phi^+$ and $\phi^0 \to v + \phi^0$ Eq. (1.4) becomes

$$V(\phi) = \lambda\left(\phi^{+*}\phi^+ + \phi^{0*}\phi^0 + 2v\mathrm{Re}\phi^0\right)^2 \quad . \tag{1.5}$$

Notice that there are no terms quadratic in ϕ^+ or $\mathrm{Im}\phi^0$ indicating that these are massless modes. These are the Goldstone bosons associated with the breaking of the global $SU(2)_L \otimes U(1)_Y$ to $U(1)_Q$. However, a local $SU(2)_L \otimes U(1)_Y$ gauge transformation can be found that removes these fields at the expense of giving mass to three of the four gauge bosons associated with the $SU(2)_L \otimes U(1)_Y$ symmetry. The field $\mathrm{Re}\phi^0$ cannot be gauged away and is found to have mass of $\sqrt{2\lambda}v$, this is the Higgs particle.

The kinetic term for the scalar field in Eq. (1.2) gives rise to the mass term for the gauge bosons. Evaluating this at the vev it can be shown that

$$D^\mu\langle\phi\rangle^\dagger D_\mu\langle\phi\rangle = \frac{1}{4}v^2\left[g^2|W_\mu^1 + iW_\mu^2|^2 + |gW_\mu^3 - g'B_\mu|^2\right] \quad . \tag{1.6}$$

If we define

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \pm W_\mu^2) \quad \text{and} \quad Z_\mu^0 = \frac{1}{\sqrt{g^2+g'^2}}(gW_\mu^3 - g'B_\mu) \quad , \tag{1.7a, b}$$

then we can rewrite the mass terms as

$$M_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2}M_Z^2 Z_\mu Z^\mu = \frac{1}{2}g^2v^2W_\mu^+W^{-\mu} + \frac{1}{4}(g^2+g'^2)v^2 Z_\mu Z^\mu \quad , \tag{1.8}$$

Notice that the combination of fields orthogonal to that of the Z^0

$$A_\mu = \frac{1}{\sqrt{g^2 + g'^2}}(g' W^3_\mu + g B_\mu) \quad , \tag{1.9}$$

remains massless, this is identified with the photon field of electromagnetism. The generator associated with this field is

$$Q = \frac{1}{2}\left(\tau^3 + Y\right) \quad . \tag{1.10}$$

This is the only combination of generators that the vacuum doesn't transform under. The Weinberg angle θ_W is introduced as a free parameter that relates the $U(1)_Y$ and $SU(2)_L$ coupling constants viz $g' = g \tan\theta_W$

The kinetic term of the scalar field gives rise to interaction terms between the Goldstone bosons and the longitudinal component of the gauge bosons. These interactions can be removed continuously by addition of the t'Hooft gauge fixing term [1.3]

$$\mathcal{L}_{\rm GF} = \frac{1}{2\zeta}\sum_a \left(\partial^\mu W^a_\mu - i\frac{1}{2}g\zeta[\langle\phi\rangle^\dagger \tau^a \phi - \phi^\dagger \tau^a \langle\phi\rangle]\right)^2$$

$$+ \frac{1}{2\zeta}\left(\partial^\mu B_\mu - ig'\zeta[\langle\phi\rangle^\dagger Y \phi - \phi^\dagger Y \langle\phi\rangle]\right)^2 \quad . \tag{1.11}$$

In the unitary gauge where $\zeta = \infty$ the Goldstone bosons have an infinite mass and therefore decouple from the theory. However, in Feynman gauge where $\zeta = 1$ they have the same mass as the corresponding gauge field.

The contribution to the Lagrange density from the gauge fields $\mathcal{L}_{\rm GF}$ is given by

$$\mathcal{L}_{\rm GF} = -\frac{1}{4}W^{a\mu\nu}W^a_{\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} - \frac{1}{4}G^{a\mu\nu}G^a_{\mu\nu} \quad . \tag{1.12}$$

The field tensors $W^a_{\mu\nu}$, $B_{\mu\nu}$ and $G^a_{\mu\nu}$ are given by

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\epsilon^{abc}W^b_\mu W^c_\nu \quad , \tag{1.13a}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \quad , \tag{1.13b}$$

and

$$G^a_{\mu\nu} = \partial G^a_\mu - \partial_\nu G^a_\mu + g_s f^{abc} G^b_\mu G^c_\nu \quad , \tag{1.13c}$$

where ϵ^{abc} is the totally antisymmetric tensor, f^{abc} are the antisymmetric structure constants of $SU(3)$ and G^a_μ are the gluon fields associated with the strong interactions.

The kinetic terms of the fermions in Eq. (1.1) gives rise to a Lagrange density of

$$\mathcal{L}_{\rm F} = i\overline{Q}^i_L \gamma^\mu D_\mu Q^i_L + i\overline{u}^i_R \gamma^\mu D_\mu u^i_R + i\overline{d}^i_R \gamma^\mu D_\mu d^i_R + i\overline{L}^i_L \gamma^\mu D_\mu L^i_L + i\overline{e}^i_R \gamma^\mu D_\mu e^i_R \quad , \tag{1.14}$$

where the covariant derivative defined for the quark fields in Eq. (1.3) also has a strong interaction contribution and becomes

$$D_\mu = \partial_\mu - i\frac{1}{2} g \tau^a W^a_\mu - i\frac{1}{2} g' Y B_\mu - i g_s (T^a)^\alpha_\beta G^a_\mu \quad , \tag{1.15}$$

where $(T^a)^\alpha_\beta$ are the generators of $SU(3)$. The color indices will be suppressed for the rest of this discussion. In terms of the gauge field mass eigenstates, the covariant derivative can be written

$$D_\mu = \partial_\mu - i g \tau^+ W^+_\mu - i g \tau^- W^-_\mu - i \frac{1}{\cos\theta_W} (\frac{1}{2}\tau^3 - \sin^2\theta_W Q) Z_\mu - i g \sin\theta_W Q A_\mu \quad , \tag{1.16}$$

where $\tau^\pm = \frac{1}{\sqrt{2}}(\tau^1 \pm i\tau^2)$.

The terms discussed above do not give the fermions of the theory mass terms. To do this we need to introduce Yukawa couplings between the fermions and the complex scalar field. The most general set of renormalsable Yukawa couplings that can be written down is

$$\mathcal{L}_{\rm Y} = g^{ij}_u \overline{u}^i_R \phi^T \epsilon Q^j_L + g^{ij}_d \overline{d}^i_R \phi^\dagger Q^i_L + g^{ij}_e \overline{e}^i_R \phi^\dagger L^j_L \quad , \tag{1.17}$$

where the unknown couplings $g^{ij}_{u,d,e}$ give rise to mixing between the generations and

$$\epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad . \tag{1.18}$$

Mass terms for the fermions are generated when the scalar field develops a vev.

If $M_{u,d,e}$ are the mass matrices for the "up-type" quarks, "down-type" quarks and charged leptons respectively then these are related to the Yukawa coupling constants by

$$M^{ij}_{u,d,e} = g^{ij}_{u,d,e} v \quad . \tag{1.19}$$

Assuming that there are only three generations of particles [1.4] and using the generation basis $(u,c,t),(d,s,b)$ and (e,μ,τ) we can redefine the fermionic fields and diagonalize the mass matrices $M_{u,d,e}$. Let

$$u^i_L \rightarrow V^{ij}_{uL} u^j_L \quad , \quad u^i_R \rightarrow V^{ij}_{uR} u^j_R \quad , \quad d^i_L \rightarrow V^{ij}_{dl} d^i_L \qquad d^i_R \rightarrow V^{ij}_{dR} d^j_R \quad , \tag{1.20a, b, c, d}$$

and similar transformations for the leptons. These unitary matrices diagonalize the mass matrices to $(M_u \rightarrow V^\dagger_{uR} M_u V_{uL})$

$$M_u = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \quad , \quad M_d = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}$$

and

$$M_e = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} \quad . \tag{1.21a, b, c}$$

Upon performing this transformation on the fermion kinetic term we see that the neutral currents remain diagonal by the unitarity of the transformations. However, the charged currents do not remain diagonal. In the mass eigenstate basis the charged current interaction becomes

$$\mathcal{L}_{cc} = \frac{g}{2\sqrt{2}} (\overline{u}, \overline{c}, \overline{t}) \gamma^\mu (1-\gamma_5) K \begin{pmatrix} d \\ s \\ b \end{pmatrix} W^+_\mu + \text{h.c.} \quad , \tag{1.22}$$

where K is the Kobayashi-Maskawa matrix [1.5]. Since this matrix is not determined theoretically we can parametrize it in terms of four parameters $\theta_1, \theta_2, \theta_3$ and δ as follows

$$K = V_{uL}^{\dagger} V_{dL} = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix} \quad , \tag{1.23}$$

where $s_k = \sin\theta_k$ and $c_k = \cos\theta_k$. The angles θ_k are chosen to lie in the first quadrant where their sines and cosines are positive. Experimental information on nuclear β-decay, semileptonic hyperon decays and semileptonic kaon decays [1.6] implies that

$$s_1 \approx 0.22 \quad . \tag{1.24}$$

The magnitude of the angles $\theta_{2,3}$ are extracted from experimental information on the lifetime of the B-meson and semileptonic B decays. This gives

$$(s_2^2 + s_3^2 + 2 s_2 s_3 c_\delta)^{\frac{1}{2}} \approx 0.05 \quad , \text{ and } \quad s_3 \leq 0.5 \quad . \tag{1.25}$$

1.2 Flavor Symmetry

Let us now consider the strong interaction between quarks, suppressing electroweak indices the Lagrange density is

$$\mathcal{L}_{\text{QCD}} = \sum_i \overline{q}_i^\alpha \gamma^\mu (\partial_\mu - i g_s (T^a)_\alpha^\beta G_\mu^a) q_\beta^i + m_i \overline{q}_i q^i \quad . \tag{1.26}$$

For a system of n quarks of equal mass there would be an exact $SU(n)_f$ flavor symmetry. Further, if m quarks had masses much less than the QCD scale (the scale at which QCD becomes strong and the scale that determines the dynamics of the system) then the system would have an approximate $SU(m)_f$ flavor symmetry. This is the situation realized in nature [1.7]. The mass of the three lightest quarks are $m_u \sim 5$ MeV/c^2, $m_d \sim 10$ MeV/c^2 and $m_s \sim 125$ MeV/c^2 while the QCD scale is ~ 1 GeV/c^2. Since the difference between the mass of the up and down quarks is small (~ 5 MeV/c^2), as well as being much less than the QCD scale an $SU(2)_f$ symmetry is expected and indeed one is observed experimentally. This is the well known isospin symmetry and it is observed to be very good. The mass of the strange quark, although much bigger than that of both the up and down quarks is still much smaller than the QCD scale, consequently one expects an approximate $SU(3)_f$ symmetry to exist. Since the dynamics of the system are determined by the QCD scale an $SU(4)_f$ flavor symmetry is not present since the charm quark mass of 1.5 GeV/c^2 is much greater than the QCD scale. In systems containing a heavy quark and light quarks there is a further $SU(3)_f$ symmetry involving the charm, bottom and top quarks [1.8]. The masses of these quarks are all much greater than the QCD scale and as far as the QCD dynamics are concerned the heavy quark acts as a static color source [1.9]. Consequently the dynamics is essentially independent of the mass of the heavy quark.

The weak charged current, whose interaction Lagrange density is given in Eq. (1.22) in general has non-trivial transformations under the $SU(3)_f$ group. This becomes apparent by examining the strangeness changing process $s \rightarrow uW^-$ which transforms as a $3 \otimes \overline{3}$ under $SU(3)_f$. This group structure decomposes to $8 \oplus 1$, and since the quark flavor is changed at this vertex this particular current transforms as an 8 under $SU(3)_f$. We can use the $SU(3)_f$ symmetry of the strong interactions to relate decays rates between reactions if the particles involved lie within the same $SU(3)_f$ multiplets.

1.3 $SU(3)_f$ Predictions for Semileptonic Hyperon Decay

The method used in constructing $SU(3)_f$ predictions between decay rates is demonstrated by examining semileptonic hyperon decays first discussed by Cabibbo [1.10]. The convention for $SU(3)_f$ representation will be that the triplets are denoted by column vector with upper indices and the antitriplets are row vectors with lower indices,

$$q^i = \begin{pmatrix} u \\ d \\ s \end{pmatrix} \quad \text{and} \quad \overline{q}_i = (\overline{u}, \overline{d}, \overline{s}) \quad . \qquad (1.27a, b)$$

All other representations can be constructed from the triplet and antitriplet. The lowest lying baryons are elements of an octet and are represented by

$$h^i_j = \begin{pmatrix} \Sigma^0/\sqrt{2}+\Lambda^0/\sqrt{6} & \Sigma^+ & p \\ \Sigma^- & -\Sigma^0/\sqrt{2}+\Lambda^0/\sqrt{6} & n \\ \Xi^- & \Xi^0 & -2\Lambda^0/\sqrt{6} \end{pmatrix} \quad , \tag{1.28}$$

where the conjugate operator $\overline{h}^i_j$ can be found by "barring" the elements and transposing. Below the weak scale, the operator responsible for the semileptonic decay of light quarks is given by

$$\mathcal{O} = \left(\frac{g}{2\sqrt{2}}\right)^2 \frac{1}{M_W^2} \left[\cos\theta_1 \overline{d}^\alpha \gamma^\mu (1-\gamma_5) u_\alpha \overline{\nu} \gamma_\mu (1-\gamma_5) e\right.$$

$$\left. - \sin\theta_1 \cos\theta_3 \overline{s}^\alpha \gamma^\mu (1-\gamma_5) u_\alpha \overline{\nu} \gamma_\mu (1-\gamma_5) e\right] + \text{H.C.} \quad . \tag{1.29}$$

There are no QCD corrections to this operator from momentum scales between the weak scale and the QCD scale as the operator has the form of a conserved current as far as the color indices are concerned. This operator transforms as an octet under $SU(3)_f$ and is represented by a 3×3 matrix as

$$H^i_j = \begin{pmatrix} 0 & 1 & -s_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad , \tag{1.30}$$

where we have used the small angle approximation for the mixing angles, $c_1 = c_2 = c_3 = 1$.

We are interested in evaluating matrix elements of the form

$$\langle h' | H_{\text{eff}} | h \rangle \quad , \tag{1.31}$$

and by using the Wigner-Eckart theorem can decompose this into a Clebsch-Gordan coefficient and an irreducible matrix element. The effective Hamiltonian H_{eff} is constructed from h^i_j, $\overline{h}^i_j$ and H^i_j by forming all the possible tensor contractions of the three operators that transform as singlets under $SU(3)_f$. There are only two distinct contractions and so the effective Hamiltonian becomes

$$H_{\text{eff}} = \alpha h^a_b \overline{h}^b_c H^c_a + \beta \overline{h}^a_b h^b_c H^c_a \quad , \tag{1.32}$$

where α and β are the reduced matrix elements. After doing the contractions the operator involving the Cabibbo-allowed processes can be written as

$$H_{\text{eff}} = \alpha \left[\Sigma^- \left(\overline{\Lambda}^0/\sqrt{6} + \overline{\Sigma}^0/\sqrt{2}\right) + \left(\Lambda^0/\sqrt{6} - \Sigma^0/\sqrt{2}\right)\overline{\Sigma^+} + \overline{p}n\right]$$

$$+ \beta \left[\overline{\Sigma^+}\left(\Lambda^0/\sqrt{6} + \Sigma^0/\sqrt{2}\right) + \Sigma^- \left(\overline{\Lambda}^0/\sqrt{6} - \overline{\Sigma}^0/\sqrt{2}\right) + \Xi^-\overline{\Xi^0}\right] \quad . \tag{1.33}$$

A similar expression can be found for the Cabibbo-suppressed decays. We interpret, for example, Λ as an operator that annihilates a Λ particle and $\overline{\Lambda}$ as an operator that creates a Λ particle. We can tabulate the rate for each process in terms of the two unknown reduced matrix elements

Table 1.2: Rates for the Semileptonic Decay of Hyperons

Process	Rate
$n \to pe^-\overline{\nu_e}$	$\lvert\alpha\rvert^2$
$\Sigma^- \to \Lambda^0 e^-\overline{\nu}_e$	$\frac{1}{6}\lvert\alpha+\beta\rvert^2$
$\Sigma^- \to \Sigma^0 e^-\overline{\nu}_e$	$\frac{1}{2}\lvert\alpha-\beta\rvert^2$
$\Sigma^0 \to \Sigma^+ e^-\overline{\nu}_e$	$\frac{1}{2}\lvert-\alpha+\beta\rvert^2$
$\Xi^- \to \Xi^0 e^-\overline{\nu}_e$	$\lvert\beta\rvert^2$
$\Xi^- \to \Lambda^0 e^-\overline{\nu}_e$	$s_1^2\frac{1}{6}\lvert-\alpha+2\beta\rvert^2$
$\Xi^- \to \Sigma^0 e^-\overline{\nu}_e$	$s_1^2\frac{1}{2}\lvert-\alpha\rvert^2$
$\Xi^0 \to \Sigma^+ e^-\overline{\nu}_e$	$s_1^2\lvert-\alpha\rvert^2$
$\Lambda^0 \to pe^-\overline{\nu}_e$	$s_1^2\frac{1}{6}\lvert 2\alpha-\beta\rvert^2$
$\Sigma^0 \to pe^-\overline{\nu}_e$	$s_1^2\frac{1}{2}\lvert-\beta\rvert^2$
$\Sigma^- \to ne^-\overline{\nu}_e$	$s_1^2\lvert-\beta\rvert^2$

In such decays there are at most six different Lorentz structures that contribute to the decay amplitude, which can be written as

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \overline{u}(h') \left[f_1(q^2)\gamma^\mu + i f_2(q^2)\sigma^{\mu\nu}q_\nu + f_3(q^2)q^\mu \right.$$

$$\left. + g_1(q^2)\gamma^\mu\gamma_5 + i g_2(q^2)\sigma^{\mu\nu}q_\nu\gamma_5 + g_3(q^2)q^\mu\gamma_5 \right] u(h)$$

$$\overline{u}(e)\gamma_\mu(1-\gamma_5)v(\nu_e) \quad , \tag{1.34}$$

where θ is the Cabibbo angle, G_F is Fermi's coupling constant and q^μ is the four momentum transfer in the decay. The transversality of the vector current from CVC ($\partial_\mu V^\mu \rightarrow q_\mu V^\mu$) means that $q^2 f_3(q^2) = 0$ from which we conclude that $f_3(q^2) = 0$. The operator $\sigma^{\mu\nu}\gamma_5$ is odd under the G-parity operation (as is γ_5). Therefore, since the strong interactions cannot induce G-parity odd operators, the form factor $g_2(q^2)$ is set to zero (for a discussion see ref[1.11]). When this hadronic current is contracted with a leptonic current (e or μ) the contraction involving $g_3(q^2)q^\mu$ picks up a factor of $m_{e,\mu}$. This term is small compared to the scale of the hyperon system, and we neglect it. The weak magnetism term $f_2(q^2)\sigma^{\mu\nu}q_\nu$ is suppressed by the mass of the hyperon system and is consequently negligible. This leaves two possible lorentz structures contributing to the amplitude. They are the vector and axial vector operators. Therefore the transition amplitude can be written as

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \overline{u}(h') \left[f_1(q^2)\gamma^\mu + g_1(q^2)\gamma^\mu\gamma_5 \right] u(h)\overline{u}(e)\gamma_\mu(1-\gamma_5)u(\nu_e) \quad . \tag{1.35}$$

In the limit as $q^2 \rightarrow 0$ we let the form factors become $C_V = f_1(0)$ and $C_A = g_1(0)$. Both of these form factors can be decomposed into reduced matrix elements and Clebsch-Gordan coefficients.

The vector charges are generators of the $SU(3)_f$ group and so transitions between octet elements k and j are equal to the structure constants of $SU(3)$ f_{ijk}. Further, since $\Sigma^{\pm,0}$ and Λ^0 are in different multiplets of the $SU(2)$ isospin subgroup of $SU(3)$

the vector current must vanish for the transition $\Sigma \rightarrow \Lambda$. This is verified experimentally [1.6] where it is found that the ratio of vector to axialvector contributions to the decay amplitude is $C_V/C_A = 0.01 \pm 0.1$. Consequently, we see from Table 1.2 that this condition determines that $\alpha_V = -\beta_V$ but does not constrain the axialvector parameters. Neutron β-decay is used to determine the the reduced matrix element for the vector transition. The matrix element for the process is

$$\langle p|\tau^+|n\rangle = \sqrt{I(I+1) - I_z(I_z+1)} = 1 \quad . \tag{1.36}$$

Therefore, we find that $\alpha_V = 1$ and the vector component of all the other transitions are related. Both the vector and axial vector form factors have been determined precisely for decay $\Lambda^0 \rightarrow pe^-\overline{\nu}_e$ where it is found that the vector coupling C_V is $|C_V| = 1.229 \pm 0.035$ and the axial vector coupling C_A is $|C_A| = 0.903 \pm 0.046$, The $SU(3)_f$ prediction for the vector coupling is $C_V = \frac{1}{\sqrt{6}}(-2\alpha_V + \beta_V) = -1.22$ which is in remarkable agreement with the experimentally determined value. Theoretically, this result is not that suprising as it was shown by Ademollo and Gatto [1.12] that the vector couplings are protected, up to second order in the breaking parameter, from violations in $SU(3)_f$.

The measured value of C_V/C_A for the $n \rightarrow pe^-\overline{\nu}_e$ and $\Sigma^- \rightarrow ne^-\overline{\nu}_e$ decays determines that $\alpha_A = 1.25$ and $\beta_A = 0.36 \pm 0.05$. These values then predict that C_A for the $\Lambda \rightarrow pe^-\overline{\nu}_e$ is -0.87 ± 0.02 which is to be compared with an experimentally determined value of $C_A = -0.903$ which is again in very good agreement. In these decay processes predictions based on $SU(3)_f$ are correct at about the 5% level.

1.4 $SU(3)_f$ Breaking

We know from the pattern of masses of the light hadrons that $SU(3)_f$ is only an approximate symmetry. The symmetry is broken by the nondegeneracy of the light quark masses, $m_s \gg m_u, m_d$. One may hope that the relevent expansion parameter is $m_s/1\text{GeV}$, and use perturbation theory to estimate the effect of the breaking on the $SU(3)_f$ relations. This will be demonstrated explicitly for the case of semileptonic hyperon decays. The mass term in the QCD Lagrange density given in Eq. (1.26) induces a perturbing term that transforms as an octet under $SU(3)_f$ and can be written as

$$P_b^a = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & m_s \end{pmatrix} \quad , \tag{1.37}$$

where m_s is the strange quark mass in units of GeV/c^2. The Ademollo-Gatto theorem [1.12] protects the vector relations from breaking terms, but not the axial vector relations. To find the effect of the breaking we form all the possible contractions of h_b^a, $\overline{h}_b^a$, H_d^c and P_b^a. Therefore, the effective $SU(3)_f$ breaking Hamiltonian is

$$\begin{aligned} H_{\text{eff}}^{\text{break}} &= a.H_b^a \overline{h}_c^b H_d^c P_a^d + b.\overline{h}_b^a h_c^b H_d^c P_a^d + c.\overline{h}_b^a H_c^b h_d^c P_a^d \\ &+ d.h_b^a H_c^b \overline{h}_d^c P_a^d + e.h_b^a \overline{h}_c^b P_d^c H_a^d + f.h_b^a \overline{h}_a^b P_d^c H_c^d \\ &+ g.h_b^a P_a^b \overline{h}_d^c H_c^d + h.h_b^a H_a^b P_d^c \overline{h}_c^d \quad . \end{aligned} \tag{1.38}$$

Examining the Cabibbo-allowed decays, we see that $H_b^a P_c^b = 0$ reducing the number of contributing elements to four. Expanding Eq. (1.38) gives

$$H_{\text{eff}}^{\text{break}} = \left(\frac{m_s}{1\text{GeV}/\text{c}^2}\right)\left[c.\overline{p}n + d.\overline{\Xi}^0\Xi^- - g.\sqrt{\frac{2}{3}}\overline{\Sigma}^+\Lambda^0 - h.\sqrt{\frac{2}{3}}\overline{\Lambda}^0\Sigma^-\right] \quad , \tag{1.39}$$

which implies that the isospin relation $\Gamma(\Sigma^- \to \Sigma^0 e^- \overline{\nu}_e) = \Gamma(\Sigma^0 \to \Sigma^+ e^- \overline{\nu}_e)$ is not altered to first order in the breaking parameter. This is to be expected since in the quark model picture the strange quark is merely a spectator in this decay. Therefore, assuming that the reduced matrix elements are not significantly larger than α and β, the axial vector relations are expected to be good at the $\sim 10\%$ level.

In general it is found that $SU(3)_f$ symmetry works typically at the 30% level in low energy physics. In the subsequent chapters we will investigate the predictions of $SU(3)_f$ for the decays of hadrons containing a bottom or charmed quark.

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CHAPTER 2. D-MESON DECAYS

The D mesons, D^+, D^0 and D_s^+, are the simplest bound system with a heavy quark constituent. It is found that the D meson masses are [1.8]

$$M_{D^+} = 1869.3\pm0.6\text{MeV} \quad , \quad M_{D^0} = 1864.5\pm0.6\text{MeV} \quad \text{and} \quad M_{D_s^+} = 1969.3\pm1.1\text{MeV} \quad . \tag{2.1a, b, c}$$

The mesons are not heavy enough to decay to final states containing baryons. They decay nonleptonically to mesonic final states or semileptonically to states with one or more mesons and either a $e\nu_e$ or $\mu\nu_\mu$ pair. The lifetime of these mesons has been observed to be [1.8]

$$\tau_{D^+} = 1.069 \pm 0.033\text{ps} \quad , \quad \tau_{D^0} = 0.428 \pm 0.011\text{ps} \quad \text{and} \quad \tau_{D_s^+} = .436 \pm 0.036\text{ps} \quad . \tag{2.2a, b, c}$$

2.1 Charm Changing Weak Operators

The interaction Lagrange density in Eq. (1.22) determines the transformation properties of the effective Hamiltonian for the weak decay of charmed hadrons. The $\Delta c = -1$ nonleptonic decays arise from the weak Hamiltonian with flavor quantum numbers $(c\overline{s})(d\overline{u})$ for Cabibbo allowed decays, $s_1[(c\overline{d})(d\overline{u}) - (c\overline{s})(s\overline{u})]$ for Cabibbo suppressed decays and $s_1^2(c\overline{d})(s\overline{u})$ for doubly Cabibbo suppressed

These operators are different components of the same Hamiltonian which can be decomposed into irreducible representations of $SU(3)_f$. An example of this is the decomposition of the component responsible for the Cabibbo-allowed decays (denoted with a superscript (a)),

$$(c\overline{s})(d\overline{u}) = \mathcal{O}_6^{(a)} + \mathcal{O}_{\overline{15}}^{(a)} \quad , \tag{2.3}$$

where

$$\mathcal{O}_6^{(a)} = \frac{1}{2}[(c\overline{s})(d\overline{u}) - (c\overline{u})(d\overline{s})] \tag{2.4a}$$

transforms as a 6 under flavor $SU(3)_f$ and

$$\mathcal{O}_{\overline{15}}^{(a)} = \frac{1}{2}[(c\overline{s})(d\overline{u}) + (c\overline{u})(d\overline{s})] \tag{2.4b}$$

transforms as a $\overline{15}$ under flavor $SU(3)_f$. The Cabibbo-suppressed operator has a similar decomposition into $\mathcal{O}_6^{(s)}$ and $\mathcal{O}_{\overline{15}}^{(s)}$, as does the doubly-Cabibbo-suppressed operator into $\mathcal{O}_6^{(ds)}$ and $\mathcal{O}_{\overline{15}}^{(ds)}$. Perturbative QCD corrections arising from momentum scales between the W-boson mass and the charmed quark mass give rise to an enhancement of the coefficient of $\mathcal{O}_6$ over the coefficient of $\mathcal{O}_{\overline{15}}$ by

$$\left[\frac{\alpha_s(m_b)}{\alpha_s(m_W)}\right]^{\frac{18}{23}}\left[\frac{\alpha_s(m_c)}{\alpha_s(m_b)}\right]^{\frac{18}{25}} \sim 2.5 \quad , \tag{2.5}$$

in the effective weak Hamiltonian [2.1]. Consequently it is possible, analagous to octet dominance in the weak decay of strange particles, that the sextet component of the Hamiltonian may dominate charmed meson decay.

The operator $\mathcal{O}_{\overline{15}}$ can be represented in tensor notation as $H_c^{ab}(\overline{15})$, which is traceless and symmetric on its upper indices. It has non-zero elements

$$H_2^{13}(\overline{15}) = H_2^{31}(\overline{15}) = +1 \tag{2.6a}$$

for Cabibbo-allowed decays

$$H_2^{12}(\overline{15}) = H_2^{21}(\overline{15}) = -H_3^{13}(\overline{15}) = -H_3^{31}(\overline{15}) = s_1 \tag{2.6b}$$

for Cabibbo-suppressed decays and

$$H_3^{12}(\overline{15}) = H_3^{21}(\overline{15}) = -s_1^2 \tag{2.6c}$$

for doubly-Cabibbo-suppressed decays. Similarly, $\mathcal{O}_6$ can be represented in tensor notation as $H_{ab}(6)$ which is symmetric on its indices and has nonzero elements

$$H_{22}(6) = +2 \tag{2.7a}$$

for Cabibbo-allowed decays

$$H_{23}(6) = H_{32}(6) = -2s_1 \tag{2.7b}$$

for Cabibbo-suppressed decays and

$$H_{33}(6) = +2s_1^2 \tag{2.7c}$$

for doubly-Cabibbo-suppressed decays. These can be written with the same tensor structure as the $\overline{15}$ by contracting with the totally antisymmetric tensor ϵ_{abc}. These then can be written as $H_2^{13}(6) = -H_2^{31}(6) = -1$ for Cabibbo-allowed decays $H_2^{12}(6) = -H_2^{21}(6) = -H_3^{13}(6) = H_3^{31}(6) = -s_1$ for Cabibbo-suppressed decays and $H_3^{12}(6) = -H_3^{21}(6) = s_1^2$ for doubly-Cabibbo-suppressed decays.

The $\Delta c = -1$ semileptonic decays arise from the weak Hamiltonian with flavor quantum numbers $[(c\overline{s}) + s_1(c\overline{d})](l^-\overline{\nu}_l)$ where l denotes a lepton ($l = e, \mu$ but not τ). This operator transforms as a $\overline{3}$ under $SU(3)_f$ and has nonzero elements

$$H^3(\overline{3}) = 1 \tag{2.8a}$$

for Cabibbo-allowed decays and

$$H^2(\overline{3}) = s_1 \tag{2.8b}$$

for Cabibbo-suppressed decays.

2.2 Two-Body Nonleptonic D-Meson Decays

First, we will review the predictions for the decay mode $D \to MM$ where M represents a member of the pseudoscalar octet. These results have been derived previously [2.2]. The D mesons transform as an antitriplet under $SU(3)_f$ and can be represented as

$$D_a = (c\overline{u}, c\overline{d}, c\overline{s}) = (D^0, D^+, D_s^+) \quad . \tag{2.9}$$

The octet of the lightest pseudoscalar mesons is represented as

$$M_b^a = \begin{pmatrix} \pi^0/\sqrt{2} + \eta^0/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi^0/\sqrt{2} + \eta^0/\sqrt{6} & K^0 \\ K^- & \overline{K}^0 & -\sqrt{2/3}\eta^0 \end{pmatrix} \quad . \tag{2.10}$$

We have assumed that the $SU(3)_f$ octet isoscalar eigenstate is very nearly the mass eigenstate corresponding to the η^0 (i.e. we have neglected $\eta - \eta'$ mixing). There are three independent contractions that can be constructed from the two meson octets, the charmed meson antitriplet and the Hamiltonian. Therefore we can write the effective Hamiltonian as

$$H_{\text{eff}} = a.D_a H_c^{ab}(\overline{15}) M_b^d M_d^c + b.D_a M_b^a H_d^{bc}(\overline{15}) M_c^d + c.D_a H_c^{ab}(6) M_b^d M_d^c \quad , \tag{2.11}$$

where a, b and c are unknown reduced matrix elements [**Note**: We have used the identity $D_a M_b^a H_d^{bc}(6) M_c^d + D_a H_c^{ab}(6) M_b^d M_d^c = 0$ to reduce the number of reduced matrix elements from four to three]. The results of expanding this Hamiltonian can be found in Table 2.1. It is interesting to note that the decays $D_s^+ \to \pi^0\pi^+$ and $D^0 \to \overline{K}^0 K^0$ are forbidden by the $SU(3)_f$ symmetry.

There is only one relation amongst the Cabibbo-allowed decay rates and it is due to the full $SU(3)_f$ symmetry,

$$\Gamma(D^0 \to \overline{K}^0 \eta^0) = \frac{1}{3}\Gamma(D^0 \to \overline{K}^0 \pi^0) \quad . \tag{2.12}$$

However, if we assume that the sextet component of the Hamiltonian dominates the *decay rates then* the following relations arise

$$\Gamma(D^0 \to K^-\pi^+) = 6\Gamma(D^0 \to \overline{K}^0\eta^0) = 2\Gamma(D^0 \to \overline{K}^0\pi^0)$$

$$= \frac{3}{2}\Gamma(D_s^+ \to \pi^+\eta^0) = \Gamma(D_s^+ \to K^+\overline{K}^0) \quad . \tag{2.13}$$

Experimentally, the branching ratio of several of these decay modes have been measured. It is seen that [1.8, 2.3]

$$Br(D_s^+ \to \overline{K}^0 K^+) = (2.1 \pm 0.6) \times 10^{-2} \quad , \tag{2.14a}$$

$$Br(D^0 \to K^-\pi^+) = (3.77 \pm 0.35) \times 10^{-2} \quad , \tag{2.14b}$$

$$Br(D^+ \to \overline{K}^0\pi^+) = (3.2 \pm 0.6) \times 10^{-2} \quad , \tag{2.14c}$$

$$Br(D^0 \to \overline{K}^0\eta^0) = (1.5 \pm 0.7) \times 10^{-2} \quad , \tag{2.14d}$$

and

$$Br(D^0 \to \overline{K}^0\pi^0) = (3.2 \pm 0.6) \times 10^{-2} \quad . \tag{2.14e}$$

The ratio of the latter two rates is $\frac{\Gamma(D^0 \to K^-\pi^+)}{\Gamma(D^0 \to \overline{K}^0\pi^0)} \sim 1.2 \pm 0.3$ which is to be compared with the $SU(3)_f$ prediction assuming sextet dominance of 2. The error on this result is sufficiently large not to allow a definite conclusion to be drawn although it hints strongly that, in fact, the sextet component of the Hamiltonian is not dominating the decay amplitude.

There are several relations amongst the Cabibbo-suppressed decay rates, they are [1.8]

$$\Gamma(D^0 \to \pi^+\pi^-) = \Gamma(D^0 \to K^+K^-) \quad , \tag{2.15a}$$

$$\Gamma(D^0 \to \eta^0\eta^0) = \Gamma(D_s^+ \to \pi^0 K^+) \quad , \tag{2.15b}$$

$$\Gamma(D^0 \to \pi^0\pi^0) = \frac{3}{2}\Gamma(D^0 \to \eta^0\pi^0) \quad , \tag{2.15c}$$

$$\Gamma(D^+ \to K^+\overline{K}^0) = \Gamma(D_s^+ \to \pi^+ K^0) \quad . \tag{2.15d}$$

These are all due to the full $SU(3)_f$ symmetry. Two of the decay modes appearing in these relations have been observed experimentally. It is found that

$$Br(D^0 \to \pi^+\pi^-) = (1.3 \pm 0.4) \times 10^{-3} \quad , \tag{2.16a}$$

and

$$Br(D^0 \to K^+K^-) = (4.5 \pm 0.8) \times 10^{-3} \quad . \tag{2.16b}$$

The ratio of the decay rates is then $\frac{\Gamma(D^0 \to K^+K^-)}{\Gamma(D^0 \to \pi^+\pi^-)} = 3.5 \pm 1.2$ which is to be compared with the $SU(3)_f$ prediction of unity. There seems to be a large descrepancy between these two numbers but they still are within 2σ of each other. Amongst the relations that arise from assuming that the sextet component of the Hamiltonian dominates the decays is

$$\Gamma(D^+ \to \overline{K}^0 K^+) = \Gamma(D^0 \to \pi^+\pi^-) \quad . \tag{2.17}$$

Experimentally, it is found that

$$Br(D^+ \to \overline{K}^0 K^+) = (8.4 \pm 2.4) \times 10^{-3} \quad . \tag{2.18}$$

Conversion from branching fractions to decay rates is done by multiplying the branching fraction by the total width, which is equivalent to dividing by the lifetime. Therefore, the experimentally observed ratio of decay rates is $\frac{\Gamma(D^+ \to \overline{K}^0 K^+)}{\Gamma(D^0 \to \pi^+\pi^-)} = 2.7 \pm 1.1$ which is to be compared with the sextet dominated $SU(3)_f$ prediction of unity.

There are several relations between Cabibbo-allowed and Cabibbo-suppressed decay rates. One of the relations is

$$s_1^2\Gamma(D^0 \to K^-\pi^+) = \Gamma(D^0 \to \pi^+\pi^-) = \Gamma(D^0 \to K^+K^-) \quad , \tag{2.19}$$

and the Cabibbo-suppressed decay mode has been observed to have a branching fraction

$$Br(D^0 \to K^-\pi^+) = (1.3 \pm 0.4) \times 10^{-3} \quad . \tag{2.20}$$

The ratio $\frac{s_1^2\Gamma(D^0\to K^-\pi^+)}{\Gamma(D^0\to\pi^+\pi^-)} = 1.4 \pm 0.4$ is compatible with the $SU(3)_f$ prediction of 1 but is only marginally compatible with the prediction for the ratio to a K^+K^- final state, (as was discussed above). There are relations involving the doubly-Cabibbo-suppressed rates but they are too small to be experimentally observable at this time.

The results for the decays $D \to MM$ cannot be straightforwardly carried over to the decays $D \to VV$, where V is the lowest lying vector meson octet

$$V = \begin{pmatrix} \rho^0/\sqrt{2} + V_8/\sqrt{6} & \rho^+ & K^{*+} \\ \rho^- & -\rho^0/\sqrt{2} + V_8/\sqrt{6} & K^{*0} \\ K^{*-} & \overline{K}^{*0} & -\sqrt{2/3}V_8 \end{pmatrix} \quad . \tag{2.21}$$

The octet state $|V_8\rangle = (1/\sqrt{6})(|u\overline{u}\rangle + |d\overline{d}\rangle - 2|s\overline{s}\rangle)$ mixes with the singlet state $|V_1\rangle = (1/\sqrt{3})(|u\overline{u}\rangle + |d\overline{d}\rangle + |s\overline{s}\rangle)$ because of the near degeneracy in the masses. The mass eigenstates $|\phi\rangle = |s\overline{s}\rangle$ and $|\omega\rangle = (1/\sqrt{2})(|u\overline{u}\rangle + |d\overline{d}\rangle)$ are linear combinations of these two states. Explicitly, the states can be written as

$$|V_8\rangle = \frac{1}{\sqrt{3}}(|\omega\rangle - \sqrt{2}|\phi\rangle) \quad , \tag{2.22a}$$

and

$$|V_1\rangle = \frac{1}{\sqrt{3}}(\sqrt{2}|\omega\rangle + |\phi\rangle) \quad . \tag{2.22b}$$

In addition to the three reduced matrix elements contributing to this process (found from $D \to MM$ by substituting $M \to V$) there are two from the contraction formed with the singlet V_1. The terms are

$$H^1_{\text{eff}} = e.D_a H^{ab}_c(\overline{15})V^c_b V_1 + f.D_a H^{ab}_c(6)V^c_b V_1 \quad . \tag{2.23}$$

Only two of the decays $D \to VV$ have been studied experimentally [1.8, 2.3]. It is seen that

$$Br(D^0 \to \rho^0 \overline{K}^{*0}) = (1.2 \pm 1.2) \times 10^{-3} \quad , \tag{2.24a}$$

$$Br(D_s^+ \to \overline{K}^{*0} K^{*+}) = (4.6 \pm 2.4) \times 10^{-2} \quad . \tag{2.24b}$$

The effective Hamiltonian describing the decay $D \to VM$ is written in terms of 10 reduced matrix elements, it is

$$\begin{aligned}H_{\text{eff}} &= a.D_a H^{ab}_c(\overline{15})M^d_b V^c_d + b.D_a H^{ab}_c(\overline{15})V^d_b M^c_d + c.D_a M^a_b H^{bc}_d(\overline{15})V^d_c \\ &+ d.D_a V^a_b H^{bc}_d(\overline{15})M^d_c + e.D_a H^{ab}_c(6)M^d_b V^c_d + f.D_a H^{ab}_c(6)V^d_b M^c_d \\ &+ g.D_a M^a_b H^{bc}_d(6)V^d_c + h.D_a V^a_b H^{bc}_d(6)M^d_c \\ &+ s.D_a H^{ab}_c(\overline{15})M^c_b V_1 + t.D_a H^{ab}_c(6)M^c_b V_1 \quad .\end{aligned} \tag{2.25}$$

Among the Cabibbo-allowed decays, tabulated in Table 2.2, there is only one relation and it is due to isospin,

$$\Gamma(D_s^+ \to \rho^+\pi^0) = \Gamma(D_s^+ \to \rho^0\pi^+) \quad . \tag{2.26}$$

There are three more relations when sextet dominance is assumed, they are

$$\Gamma(D^0 \to \rho^+ K^-) = \Gamma(D_s^+ \to \overline{K}^{*0} K^+) \quad , \tag{2.27a}$$

$$\Gamma(D^0 \to \pi^+ K^{*-}) = \Gamma(D_s^+ \to \overline{K}^0 K^{*+}) \quad , \tag{2.27b}$$

$$\Gamma(D^+ \to \pi^+ \overline{K}^{*0}) = \Gamma(D^+ \to \rho^+ \overline{K}^0) \quad . \tag{2.27c}$$

The branching fraction for ten of these decay modes have been measured to be [1.8,

$$Br(D_s^+ \to \phi\pi^+) = (2.0 \pm 0.4) \times 10^{-2} \quad , \tag{2.28a}$$

$$Br(D^0 \to \rho^+ K^-) = (8.2 \pm 1.2) \times 10^{-2} \quad , \tag{2.28b}$$

$$Br(D_s^+ \to \overline{K}^{*0} K^+) = (1.7 \pm 0.3) \times 10^{-2} \quad , \tag{2.28c}$$

$$Br(D^+ \to \overline{K}^{*0} \pi^+) = (1.7 \pm 0.8) \times 10^{-2} \quad , \tag{2.28d}$$

$$Br(D^+ \to \rho^+ \overline{K}^0) = (6.6 \pm 1.7) \times 10^{-2} \quad , \tag{2.28e}$$

$$Br(D^0 \to \overline{K}^0 \phi) = (0.99 \pm 0.24) \times 10^{-2} \quad , \tag{2.28f}$$

$$Br(D^0 \to \overline{K}^0 \omega) = (3.2 \pm 1.5) \times 10^{-2} \quad , \tag{2.28g}$$

$$Br(D^0 \to \overline{K}^0 \rho^0) = (0.75 \pm 0.5) \times 10^{-2} \quad , \tag{2.28h}$$

$$Br(D^0 \to K^{*-} \pi^+) = (5.2 \pm 1.5) \times 10^{-2} \quad , \tag{2.28i}$$

$$Br(D^0 \to \overline{K}^{*0} \pi^0) = (2.6 \pm 0.8) \times 10^{-2} \quad . \tag{2.28j}$$

From these we find that $\frac{\Gamma(D^0 \to \rho^+ K^-)}{\Gamma(D_s^+ \to \overline{K}^{*0} K^+)} \sim 5 \pm 1$ which should be compared with the $SU(3)_f$ prediction of 1. Conversely, $\frac{\Gamma(D^+ \to \overline{K}^{*0} \pi^+)}{\Gamma(D^+ \to \rho^+ \overline{K}^0)} \sim 4 \pm 2$ which is 2σ away from the $SU(3)_f$ prediction of 1. The results for the Cabibbo-suppressed and doubly-Cabibbo-suppressed decays are not tabulated. There are several relations between the *Cabibbo*-suppressed decays, they are

$$\Gamma(D^0 \to \rho^+\pi^-) = \Gamma(D^0 \to K^{*+}K^-) \quad , \tag{2.29a}$$

$$\Gamma(D^0 \to \overline{K}^{*0}K^0) = \Gamma(D^0 \to K^{*0}\overline{K}^0) \quad , \tag{2.29b}$$

$$\Gamma(D^0 \to \rho^-\pi^+) = \Gamma(D^0 \to K^{*-}K^+) \quad , \tag{2.29c}$$

$$\Gamma(D^+ \to \overline{K}^{*0}K^+) = \Gamma(D_s^+ \to K^{*0}\pi^+) \quad . \tag{2.29d}$$

Experimentally, several of these decay modes have been studied, and it is found that

$$Br(D^0 \to K^{*-}K^+) = (0.8 \pm 0.5) \times 10^{-2} \quad , \tag{2.30a}$$

$$Br(D^+ \to \pi^+\phi) = (1 \pm 0.2) \times 10^{-2} \quad , \tag{2.30b}$$

$$Br(D^+ \to \rho^0\pi^+) = (0.2 \pm 0.1)) \times 10^{-2} \quad , \tag{2.30c}$$

$$Br(D^+ \to \overline{K}^{*0}K^+) = (0.56 \pm 0.20) \times 10^{-2} \quad . \tag{2.30d}$$

2.3 Semileptonic D-Meson Decays to Three-Body Final States

The Hamiltonian for the semileptonic decay of D-mesons is given in Eqs. (2.8). There is only one singlet in the tensor product $3 \otimes \overline{3} \otimes 8$ and hence there is only one reduced matrix element determining the semileptonic decay rates. The effective

Hamiltonian for the system is

$$H_{\text{eff}} = \alpha . D_a M^a_b H^b(\overline{3}) \quad . \tag{2.31}$$

From this Hamiltonian we find the following relations

$$s_1^2 \Gamma(D^0 \to K^- e^+ \nu_e) = s_1^2 \Gamma(D^+ \to \overline{K}^0 e^+ \nu_e) = \frac{3}{2} s_1^2 \Gamma(D_s^+ \to \eta^0 e^+ \nu_e)$$

$$= \Gamma(D^0 \to \pi^- e^+ \nu_e) = 6\Gamma(D^+ \to \eta^0 e^+ \nu_e) = 2\Gamma(D^+ \to \pi^0 e^+ \nu_e)$$

$$= \Gamma(D_s^+ \to K^0 e^+ \nu_e) \quad . \tag{2.32}$$

These relations are easily converted to the semileptonic decay to a vector meson and leptons as was done before. Experimentally, the branching fractions obtained to date [1.8] are,

$$Br(D^0 \to K^- e^+ \nu_e) = (4.1 \pm 0.8) \times 10^{-2} \quad , \tag{2.33a}$$

$$Br(D^0 \to \pi^- e^+ \nu_e) \sim 0.4 \times 10^{-2} \quad , \tag{2.33b}$$

$$Br(D^+ \to \overline{K}^0 e^+ \nu_e) = (5.4 \pm 2.4) \times 10^{-2} \quad . \tag{2.33c}$$

This data seems to be in agreement with the $SU(3)_f$ predictions. The branching ratio of the D^+ decay to a muonic final state is approximately twice that expected by universality and seems not to satisfy the prediction. This deviation will probably vanish with better data.

2.4 Large N_c Limit and Factorisation of Amplitudes

A model of hadronic interactions that has been extensively studied is one in which the number of colors N_c is treated as an expansion parameter (or rather $1/N_c$)[2.4, 2.5]. Of the many interesting results of this model in the limit of large N_c is the factorization of weak amplitudes. Let us consider the decay $D \to \pi\pi$. The leading *planar diagram* with no gluonic corrections is shown in Figure 1a. There are counting rules that apply to these graphs. For each external particle there is a $\frac{1}{\sqrt{N_c}}$ that appears in the amplitude. For each loop there is a factor of N_c that arise from the possible colors circulating in the loop and for each fermion-gluon vertex there is a factor of $\frac{g}{\sqrt{N_c}}$. A gluon is represented by two parallel line (colors) that break the fermion line at the vertex. The N_c counting for Figure 1a gives rise to an amplitude

$$\mathcal{A} \propto \left(\frac{1}{\sqrt{N_c}}\right)^3 N_c^2 = N_c^{\frac{1}{2}} \quad . \tag{2.34}$$

If we examine the diagram in Figure 1b where the gluon is transferred between quarks in the same meson we find that the amplitude for this diagram is

$$\mathcal{A} \propto \left(\frac{1}{\sqrt{N_c}}\right)^3 \left(\frac{g}{\sqrt{N_c}}\right)^2 N_c^3 = g^2 N_c^{\frac{1}{2}} \quad , \tag{2.35}$$

which is the same order in N_c as the leading graph and hence survives in the limit as $N_c \to \infty$. However when we perform the same power counting on the graph shown in Figure 1c. which does not allow the amplitudes to factorise we find that

$$\mathcal{A} \propto \left(\frac{1}{\sqrt{N_c}}\right)^3 \left(\frac{g}{\sqrt{N_c}}\right)^2 N_c = g^2 N_c^{-\frac{3}{2}} \quad . \tag{2.36}$$

This diagram vanishes in the limit $N_c \to \infty$ and hence we find that indeed as the number of colours becomes large the weak amplitudes factorise. Therefore, in this limit,

$$\langle \pi\pi | (c\overline{d})_{V-A} (\overline{u}d)_{V-A} | D \rangle = \langle \pi | (c\overline{d})_{V-A} | D \rangle \langle \pi | (\overline{u}d)_{V-A} | 0 \rangle \quad , \tag{2.37}$$

where $(\overline{c}d)_{V-A} = \overline{c}^{\alpha} \gamma^{\mu} (1 - \gamma_5) d_{\alpha}$. The pion to vacuum matrix element is parametrized as

$$\langle \pi(p) | (\overline{u}d)_{V-A} | 0 \rangle = f_{\pi} p^{\mu} \quad , \tag{2.38}$$

where $f_{\pi} = 0.94 M_{\pi}$ is determined experimentally from the semileptonic π decay and therefore we find that the amplitude for the process $D \to \pi\pi$ is proportional to f_{π}.

We can phenomenologically modify our $SU(3)_f$ predictions for the Cabibbo-suppressed decays from $\frac{\Gamma(D^0 \to K^+K^-)}{\Gamma(D^0 \to \pi^+\pi^-)} = 1$ to

$$\frac{\Gamma(D^0 \to K^+K^-)}{\Gamma(D^0 \to \pi^+\pi^-)} = \left(\frac{f_k}{f_\pi}\right)^2 = 1.7 \quad . \tag{2.39}$$

The leading $1/N_c$ corrections to each of these amplitudes cancels, thereby making this prediction valid up to $1/N_c^2$ [2.6].

However, in phenomenologically correcting for $SU(3)_f$ violation in the decay amplitudes we must be consistent and include the phase space corrections arising from the mass difference between the kaon and pion. This gives rise to an additional multiplicative factor of ~ 0.7. Hence the modified $SU(3)_f$ prediction for this ratio becomes 1.2, which is still significantly different from the experimentally observed value.

2.5 Concluding Remarks on Chapter 2

It is interesting to briefly discuss some of the models employed to try and understand the weak hadronic decays of D-mesons. The large N_c limit and amplitude factorization provides a starting point for elucidating some of the underlying dymnamics. The effective Hamiltonian for the system in the large N_c limit under the assumption of factorization is [2.7]

$$H_{\text{eff}}^{\text{BSW}} \propto \left[a_1(\overline{u}d)_{(V-A)}(\overline{s}c)_{(V-A)} + a_2(\overline{s}d)_{(V-A)}(\overline{u}c)_{(V-A)}\right] \quad , \tag{2.40}$$

where $a_1 = c_1 + \frac{1}{N_c}c_2$ and $a_2 = c_2 + \frac{1}{N_c}c_1$ at the quark level. The $\frac{1}{N_c}$ arises from the color mismatch when the hadron is "reformed" after the weak interaction. Since the color currents in the hadron are not understood this factor of $\frac{1}{N_c}$ is replaced by an arbitrary parameter ζ at the hadronic level. It is important to realize that the assumption that this factor is the same for all decay modes has no firm theoretical basis. A good discussion of this can be found in ref [2.8]. When the decay rates for the known modes are fit with this form for the interaction a value of $\zeta \sim 0$ is found which indicates that the naive quark estimate of $1/3$ is too large. To some extent this *parametrisation* of the weak Hamiltonian reproduces the pattern of lifetimes of the D-mesons.

Another approach to understanding D-meson decays is in the context of the quark-diagram scheme [2.9]. In this scheme each topologically different quark line diagram that can contribute to a decay mode of the D-mesons is assigned an unknown amplitude. $SU(3)_f$ breaking effects are incorporated by assigning a different amplitude to diagrams that have strange quarks appearing in the interaction vertices from those with up or down quarks. Further, final state interactions (FSI) are accounted for by introducing phase shifts for each isospin partial wave in the amplitude. These phases are generally complex as there are resonances at energies near the charm mass. An example where these FSI are important is in understanding the descrepancy between the theoretical and experimental branching ratios for $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$. If there were a resonance in the K^+K^- system (and not the $\pi^+\pi^-$ system) with mass close to the mass of the D^0 then this would be a natural explanation for the difference. (It should be noted that FSI can only give rise to deviations from the predictions of $SU(3)_f$ if they violate the symmetry. If the resonance was not close to the mass (depending on the width of the resonance) of the D^0 then the apparent violation would be significantly less.) These amplitudes and phase shifts are then fit to the available data on the branching fraction for D-meson decays and the relative importance of each type of quark line diagram can be found along with the size of the FSI.

Final state interactions need to be incorporated into our $SU(3)_f$ predictions. The amplitudes at the weak vertex can be decomposed into isospin partial waves as was discussed in the previous paragraph [2.9]. The FSI will give rise to a phase shift for each partial wave. One can then fit these amplitudes and phase shifts to the D-meson decay data, remove the phase shifts and then compare these amplitudes with the predictions of $SU(3)_f$ [2.9,2.10]. Therefore it is not a useless exercise to construct the predictions for various decay processes, even when the energy of the decay lies in the resonance region, as these effects can be removed from the data.

Comparison between the predictions of sextet dominance in the nonleptonic *decay of D-mesons* and the experimental results indicate that in fact the sextet is not dominating the weak decay of the D-mesons. This is despite the fact that short distance QCD gives rise to an enhancement of the sextet component over the $\overline{15}$ by a factor of 2.5 in the effective weak Hamiltonian.

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Table 2.1. Rates for the decays $D \to MM$ in terms of the three reduced matrix elements a, b and c.

Process	Rate
$D^0 \to K^-\pi^+$	$\|a+b-c\|^2$
$D^0 \to \overline{K}^0\eta^0$	$\frac{1}{6}\|-a+b+c\|^2$
$D^0 \to \overline{K}^0\pi^0$	$\frac{1}{2}\|-a+b+c\|^2$
$D^+ \to \overline{K}^0\pi^+$	$4\|b\|^2$
$D_s^+ \to \pi^+\eta^0$	$\frac{2}{3}\|a-b+c\|^2$
$D_s^+ \to K^+\overline{K}^0$	$\|a+b+c\|^2$
$D^0 \to \pi^+\pi^-$	$s_1^2\|a+b-c\|^2$
$D^0 \to \eta^0\eta^0$	$s_1^2\frac{1}{2}\|a+b+c\|^2$
$D^0 \to \pi^0\pi^0$	$s_1^2\frac{1}{2}\|a-b-c\|^2$
$D^0 \to \eta^0\pi^0$	$s_1^2\frac{1}{3}\|-a+b+c\|^2$
$D^0 \to K^+K^-$	$s_1^2\|-a-b+c\|^2$
$D^+ \to \eta^0\pi^+$	$s_1^2\frac{2}{3}\|a+2b+c\|^2$
$D^+ \to K^+\overline{K}^0$	$s_1^2\|a-b+c\|^2$
$D^+ \to \pi^+\pi^0$	$s_1^2 2\|b\|^2$
$D_s^+ \to \eta^0K^+$	$s_1^2\frac{1}{6}\|a+5b+c\|^2$
$D_s^+ \to \pi^0K^+$	$s_1^2\frac{1}{2}\|-a-b-c\|^2$
$D_s^+ \to \pi^+K^0$	$s_1^2\|-a+b-c\|^2$
$D^0 \to \pi^-K^+$	$s_1^4\|-a-b+c\|^2$
$D^0 \to \eta^0K^0$	$s_1^4\frac{1}{6}\|a-b-c\|^2$
$D^0 \to \pi^0K^0$	$s_1^4\frac{1}{2}\|a-b-c\|^2$
$D^+ \to \eta^0K^+$	$s_1^4\frac{1}{6}\|a-b+c\|^2$
$D^+ \to \pi^0K^+$	$s_1^4\frac{1}{2}\|-a+b-c\|^2$
$D^+ \to \pi^+K^0$	$s_1^4\|-a-b-c\|^2$
$D_s^+ \to K^+K^0$	$s_1^4 4\|-b\|^2$

Table 2.2. Rates for the Cabibbo-Allowed decays $D \to VM$ in terms of the reduced matrix elements $a, b, c, d, e, f, g, h, s$ and t.

Process	Rate
$D^0 \to K^- \rho^+$	$\lvert a + c - e + g \rvert^2$
$D^0 \to \overline{K}^0 \rho^0$	$\frac{1}{2}\lvert -a + d + e - h \rvert^2$
$D^0 \to \overline{K}^0 \omega$	$\frac{1}{18}\lvert a - 2b + d - e + 2f - h + \sqrt{12}s - \sqrt{12}t \rvert^2$
$D^0 \to \overline{K}^0 \phi$	$\frac{1}{9}\lvert -a + 2b - d + e - 2f + h + 3s - 3t \rvert^2$
$D^0 \to \eta^0 \overline{K}^{*0}$	$\frac{1}{6}\lvert -2a + b + c + 2e - f - g \rvert^2$
$D^0 \to \pi^+ \overline{K}^{*-}$	$\lvert b + d - f + h \rvert^2$
$D^0 \to \pi^0 \overline{K}^{*0}$	$\frac{1}{2}\lvert -b + c + f - g \rvert^2$
$D^+ \to \pi^+ \overline{K}^{*0}$	$\lvert c + d - g + h \rvert^2$
$D^+ \to \overline{K}^0 \rho^+$	$\lvert c + d + g - h \rvert^2$
$D_s^+ \to \eta^0 \rho^+$	$\frac{1}{6}\lvert a + b - 2c + e + f - 2g \rvert^2$
$D_s^+ \to \pi^0 \rho^+$	$\frac{1}{2}\lvert a - b + e - f \rvert^2$
$D_s^+ \to \pi^+ \rho^0$	$\frac{1}{2}\lvert -a + b - e + f \rvert^2$
$D_s^+ \to \pi^+ \omega$	$\frac{1}{18}\lvert a + b - 2d + e + f - 2h + \sqrt{12}s + \sqrt{12}t \rvert^2$
$D_s^+ \to \pi^+ \phi$	$\frac{1}{9}\lvert -a - b + 2d - e - f + 2h + \sqrt{3}s + \sqrt{3}t \rvert^2$
$D_s^+ \to K^+ \overline{K}^{*0}$	$\lvert a + c + e - g \rvert^2$
$D_s^+ \to K^{*+} \overline{K}^0$	$\lvert b + d + f - h \rvert^2$

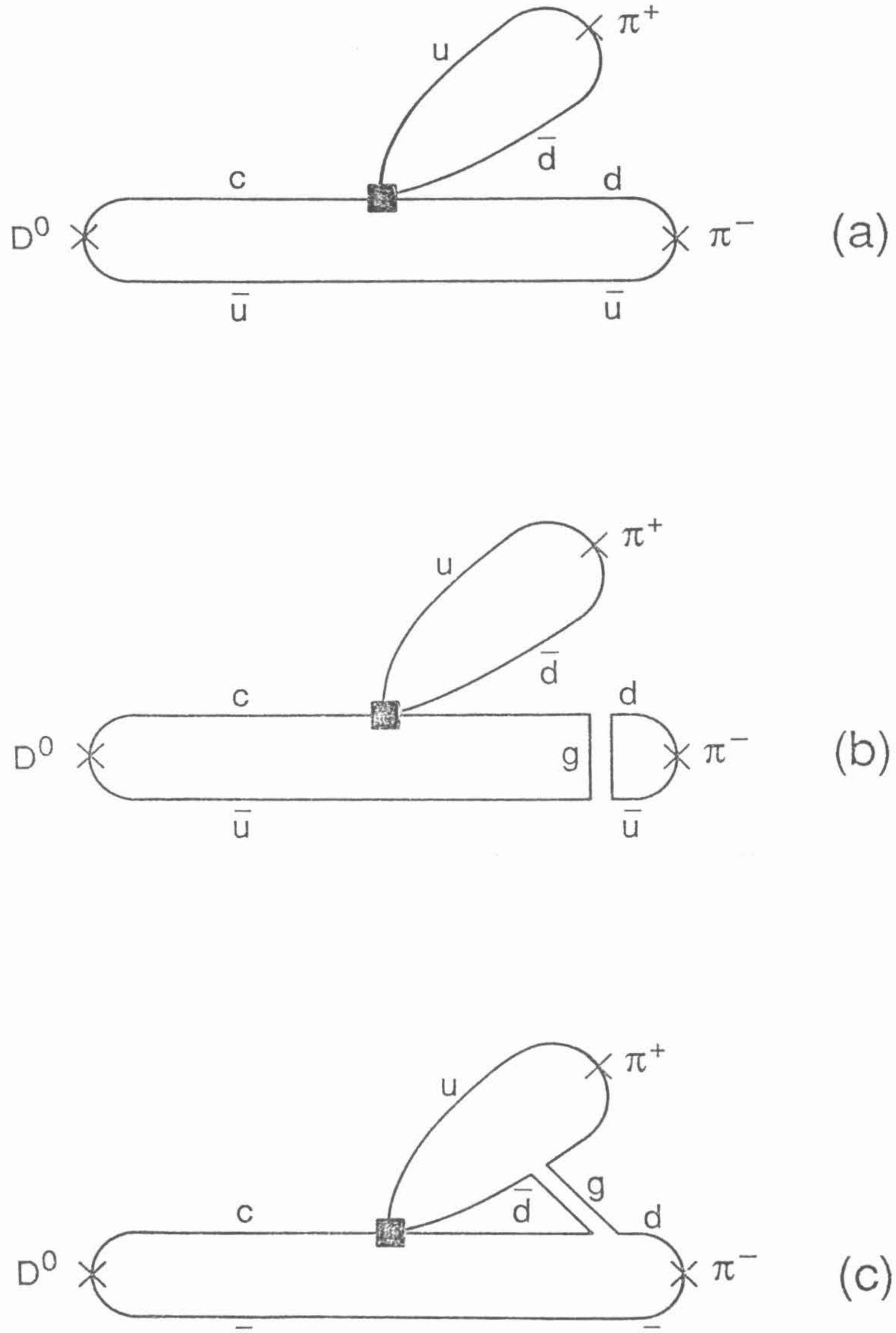
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FIGURE 1

CHAPTER 3. CHARMED BARYON DECAYS

Measurements of the branching fractions for many exclusive decay modes of charmed baryons are starting to be made. Although the weak decays of the Λ_c^+ , Ξ_{c1}^+ , Ξ_{c1}^0 and Ω_c^0 have been observed, only the decays of the Λ_c^+ have been studied in any detail. The large event sample of B-meson decays that will be collected in the near future will allow the study of the decay modes of all the charmed baryons. Charmed baryons belong to one of two representations of flavor SU(3), a $\overline{3}$ or a 6. The Λ_c^+ , Ξ_{c1}^+ and Ξ_{c1}^0 constitute the $\overline{3}$ and the Ω_c^0 , Ξ_{c2}^+ , Ξ_{c2}^0 , Σ_c^{++} , Σ_c^+ and Σ_c^0 comprise the 6. Five of the six members of the 6 decay strongly or electromagnetically, for example $\Sigma_c^{++} \to \Lambda_c^+\pi^+$, $\Xi_{c2}^+ \to \Xi_{c1}^+\gamma$. Only the Ω_c^0 of the 6 and the members of the $\overline{3}$ decay weakly. The lifetime of these four weakly decaying particles have been measured to be

$$\tau_{\Lambda_c^+} = 0.196 \pm 0.016\text{ps.} \quad , \quad \tau_{\Xi_{c1}^+} = 0.57 \pm 0.14\text{ps} \quad ,$$

$$\tau_{\Xi_{c1}^0} = 0.082 \pm 0.06\text{ps.} \quad \text{and} \quad \tau_{\Omega_c^0} = 0.79 \pm 0.34\text{ps} \quad . \tag{3.1}$$

The masses of these particles have been measured to be

$$M_{\Lambda_c^+} = 2284.9 \pm 1.5\text{MeV/c}^2 \quad , \quad M_{\Xi_{c1}^+} = 2467 \pm 3\text{MeV/c}^2 \quad , \quad M_{\Xi_{c1}^0} = 2472 \pm 3\text{MeV/c}^2$$

$$M_{\Sigma_c^{++}} = 2452.2 \pm 1.7\text{MeV/c}^2 \quad \text{and} \quad M_{\Omega_c^0} = 2740 \pm 20\text{MeV/c}^2 \quad . \tag{3.2}$$

3.1 $SU(3)_f$ Representation of the Charmed Baryons

The lowest lying charmed baryons fall into two representations of $SU(3)_f$. If the charmed baryon state vector is antisymmetric under interchange of the two light quark flavors then it is in the $\overline{3}$ representation and if the charmed baryon state vector is symmetric under the interchange of the two light quark flavors then it is in the 6 representation. In the non-relativistic quark potential model the spin-flavor state vectors for the lowest lying $J^\pi = \frac{1}{2}^+$ charmed baryons in the $\overline{3}$ representation of $SU(3)_f$ are

$$|\Lambda_c^+ \frac{1}{2}\rangle = \frac{1}{2}(|c\uparrow u\uparrow d\downarrow\rangle - |c\uparrow u\downarrow d\uparrow\rangle - |c\uparrow d\uparrow u\downarrow\rangle + |c\uparrow d\downarrow u\uparrow\rangle) \quad , \qquad (3.3a)$$

$$|\Xi_{c1}^+ \frac{1}{2}\rangle = \frac{1}{2}(|c\uparrow u\uparrow s\downarrow\rangle - |c\uparrow u\downarrow s\uparrow\rangle - |c\uparrow s\uparrow u\downarrow\rangle + |c\uparrow s\downarrow u\uparrow\rangle) \quad , \qquad (3.3b)$$

$$|\Xi_{c1}^0\rangle = \frac{1}{2}(|c\uparrow d\uparrow s\downarrow\rangle - |c\uparrow d\downarrow s\uparrow\rangle - |c\uparrow s\uparrow d\downarrow\rangle + |c\uparrow s\downarrow d\uparrow\rangle) \quad . \qquad (3.3c)$$

The spin-flavor state vectors for the lowest lying charmed baryons in the 6 representation of $SU(3)_f$ are

$$|\Sigma_c^{++} \frac{1}{2}\rangle = \sqrt{\frac{1}{6}}(2|c\downarrow u\uparrow u\uparrow\rangle - |c\uparrow u\uparrow u\downarrow\rangle - |c\uparrow u\downarrow u\uparrow\rangle) \quad , \qquad (3.4a)$$

$$|\Sigma_c^{+} \frac{1}{2}\rangle = \sqrt{\frac{1}{12}}(2|c\downarrow u\uparrow d\uparrow\rangle - |c\uparrow u\uparrow d\downarrow\rangle - |c\uparrow u\downarrow d\uparrow\rangle$$
$$+ 2|c\downarrow d\uparrow u\uparrow\rangle - |c\uparrow d\uparrow u\downarrow\rangle - |c\uparrow d\downarrow u\uparrow\rangle) \quad , \qquad (3.4b)$$

$$|\Sigma_c^{0} \frac{1}{2}\rangle = \sqrt{\frac{1}{6}}(2|c\downarrow d\uparrow d\uparrow\rangle - |c\uparrow d\uparrow d\downarrow\rangle - |c\uparrow d\downarrow d\uparrow\rangle) \quad , \qquad (3.4c)$$

$$|\Xi_{c2}^{+} \frac{1}{2}\rangle = \sqrt{\frac{1}{12}}(2|c\downarrow u\uparrow s\uparrow\rangle - |c\uparrow u\uparrow s\downarrow\rangle - |c\uparrow u\downarrow s\uparrow\rangle$$
$$+ 2|c\downarrow s\uparrow u\uparrow\rangle - |c\uparrow s\uparrow u\downarrow\rangle - |c\uparrow s\downarrow u\uparrow\rangle) \quad , \qquad (3.4d)$$

$$|\Xi_{c2}^{0} \frac{1}{2}\rangle = \sqrt{\frac{1}{12}}(2|c\downarrow d\uparrow s\uparrow\rangle - |c\uparrow d\uparrow s\downarrow\rangle - |c\uparrow d\downarrow s\uparrow\rangle$$
$$+ 2|c\downarrow s\uparrow d\uparrow\rangle - |c\uparrow s\uparrow d\downarrow\rangle - |c\uparrow s\downarrow d\uparrow\rangle) \quad , \qquad (3.4e)$$

$$|\Omega_c^{0} \frac{1}{2}\rangle = \sqrt{\frac{1}{6}}(2|c\downarrow s\uparrow s\uparrow\rangle - |c\uparrow s\downarrow s\uparrow\rangle - |c\uparrow s\uparrow s\downarrow\rangle) \quad . \qquad (3.4f)$$

It is the hyperfine interaction that gives rise to the mass difference between the Σ_c and Λ_c baryons and between the Ξ_{c1} and Ξ_{c2} baryons. Since the state vectors for the lowest lying charmed baryons in the $\overline{3}$ representation are antisymmetric under interchange of the light quark spins while the state vectors for the lowest lying charmed baryons in the 6 representation are symmetric under interchange of the light quark spins the hyperfine interaction causes the Σ_c to be heavier than the Λ_c and the Ξ_{c2} to be heavier than the Ξ_{c1}.

$SU(3)_f$ violations in the hyperfine interaction give rise to mixing between the Ξ_{c1} and Ξ_{c2} states. Physical mass eigenstates will be the following linear combination of these two states

$$\sqrt{\frac{1}{2}}\left[\cos\theta|\Xi_{c1}\rangle + \sin\theta|\Xi_{c2}\rangle\right] \quad \text{and} \quad \sqrt{\frac{1}{2}}\left[\cos\theta|\Xi_{c2}\rangle - \cos\theta|\Xi_{c1}\rangle\right] \quad . \tag{3.5a, b}$$

The two-body hyperfine interaction has the form

$$H_{\text{hf}} = 4A\sum_{j=u,d,s}\frac{\mathbf{s_c}\cdot\mathbf{s_j}}{m_c m_j} + 4A'\sum_{i,j=u,d,s}\frac{\mathbf{s_i}\cdot\mathbf{s_j}}{m_i m_j} \quad , \tag{3.6}$$

where A and A' are determined by the spatial part of the charmed baryon state vectors (which are taken to be $SU(3)_f$ symmetric). The term proportional to A' dominates the hyperfine interaction since it is not suppressed by the heavy charm quark mass. Using Eqs. (3.3,3.4,3.5,3.6) we find that

$$\langle\Xi_{c1}|H_{\text{hf}}|\Xi_{c1}\rangle = -\frac{3A'}{m_u m_s} \quad , \tag{3.7a}$$

$$\langle\Xi_{c2}|H_{\text{hf}}|\Xi_{c2}\rangle = \frac{A'}{m_u m_s} - \frac{2A}{m_c m_u} - \frac{2A}{m_c m_s} \quad , \tag{3.7b}$$

$$\langle\Xi_{c2}|H_{\text{hf}}|\Xi_{c1}\rangle = -\frac{\sqrt{3}A}{m_c}\left(\frac{m_s - m_u}{m_s m_u}\right) \quad , \tag{3.7c}$$

which gives a small mixing angle

$$\theta = -\frac{\sqrt{3}}{4}\left(\frac{A}{A'}\right)\left(\frac{m_s - m_u}{m_c}\right) \quad . \tag{3.8}$$

It is easy to understand why the term proportional to A' does not contribute to the mixing between the Ξ_{c1} and Ξ_{c2} states. In the computation of the Ξ_{c1}, Ξ_{c2} mass matrix the quark masses in this term occur as an overall factor and hence it has the same effect as an $SU(3)_f$ conserving interaction where the two light quarks are taken to be degenerate with mass $\sqrt{m_s m_u}$. Therefore, despite the near degeneracy of the $\overline{3}$ and 6 representations, $SU(3)_f$ violations in the hyperfine interaction do not give rise to significant mixing between the multiplets.

From the measured masses of the Σ_c and Λ_c baryons the hyperfine mass splitting between Ξ_{c1} and Ξ_{c2} can be determined. Using

$$\langle\Lambda_c|H_{\rm hf}|\Lambda_c\rangle = -\frac{3A'}{m_u^2} \quad , \quad \langle\Sigma_c|H_{\rm hf}|\Sigma_c\rangle = \frac{A'}{m_u^2} - \frac{4A}{m_u m_c} \quad , \tag{3.9a, b}$$

we find that (with $m_u = 330\text{MeV}$ and $m_s = 550\text{MeV}$) the measured value of $m_{\Sigma_c} - m_{\Lambda_c}$ imply

$$m_{\Xi_{c2}} - m_{\Xi_{c1}} \approx 100 MeV \quad . \tag{3.10}$$

The lowest lying $J^\pi = \frac{3}{2}^+$ charmed baryons are also in the 6 representation of $SU(3)_f$. Their spin-flavor state vectors are

$$|\Sigma_c^{*++}\frac{3}{2}\rangle = |c\uparrow u\uparrow u\uparrow\rangle \quad , \tag{3.11a}$$

$$|\Sigma_c^{*+}\frac{3}{2}\rangle = \sqrt{\frac{1}{2}}(|c\uparrow u\uparrow d\uparrow\rangle + |c\uparrow d\uparrow u\uparrow\rangle) \quad , \tag{3.11b}$$

$$|\Sigma_c^{*0}\frac{3}{2}\rangle = |c\uparrow d\uparrow d\uparrow\rangle \quad , \tag{3.11c}$$

$$|\Xi_c^{*+}\frac{3}{2}\rangle = \sqrt{\frac{1}{2}}(|c\uparrow u\uparrow s\uparrow\rangle + |c\uparrow s\uparrow u\uparrow\rangle) \quad , \tag{3.11d}$$

$$|\Xi_c^{*0}\frac{3}{2}\rangle = \sqrt{\frac{1}{2}}(|c\uparrow d\uparrow s\uparrow\rangle + |c\uparrow s\uparrow d\uparrow\rangle) \quad , \tag{3.11e}$$

$$|\Omega_c^{*0}\frac{3}{2}\rangle = |c\uparrow s\uparrow s\uparrow\rangle \quad . \tag{3.11f}$$

These states are split in mass from the lowest lying $J^\pi = \frac{1}{2}^+$ charmed baryons in the

6 representation by the part of the hyperfine interaction proportional to A. Explicitly,

$$\langle \Sigma_c^* | H_{\rm hf} | \Sigma_c^* \rangle - \langle \Sigma_c | H_{\rm hf} | \Sigma_c \rangle = \frac{6A}{m_u m_c} \quad , \tag{3.12}$$

which implies that

$$m_{\Sigma_c^*} - m_{\Sigma_c} = \frac{3}{2}\left(\frac{A}{A'}\right)\left(\frac{m_u}{m_c}\right)(M_{\Sigma_c} - M_{\Lambda_c}) \quad . \tag{3.13}$$

Since $m_c \gg m_u$ and m_s, it is expected that A will be somewhat larger then A'.

The remainder of this chapter is divided into two sections, nonleptonic and semileptonic decays. In the first section we examine the flavor SU(3) predictions for the decay of charmed baryons in the $\overline{3}$ and 6 representation to $\frac{1}{2}^+$ or $\frac{3}{2}^+$ uncharmed baryons and one or two mesons. The second section deals with the semileptonic decay of charmed baryons in both representations to $\frac{1}{2}^+$ or $\frac{3}{2}^+$ uncharmed baryons, a (l^+, ν_l) lepton pair and zero or one meson. The matrix elements for the decay processes are tabulated in terms of unknown reduced matrix elements.

3.2 Nonleptonic Decay Of Charmed Baryons To Two Body Final States

First we examine the process $T \to hM$ where T denotes the $\overline{3}$ repesentation of charmed baryons, h is the lowest-lying $\frac{1}{2}^+$ baryon octet and M is the lowest-lying pseudoscalar meson octet. The Hamiltonian for the decay of charmed baryons is the same as that for the decay of charmed mesons, which is given in section 2.1 . Some of the $SU(3)_f$ predictions for this set of decays have been considered before in ref. [2.2]. The effective Hamiltonian for the process is given by

$$\begin{aligned} H_{\rm eff} = {} & aH_a^{bc}(\overline{15})T_b\overline{h}_c^d M_d^a + bH_a^{bc}(\overline{15})T_b M_c^d \overline{h}_d^a + cH_a^{bc}(\overline{15})\overline{h}_b^d M_c^a T_d \\ & + dH_a^{bc}(\overline{15})M_b^d \overline{h}_c^a T_d + eH_{ab}(6)T^{ac}\overline{h}_c^d M_d^b \\ & + fH_{ab}(6)T^{ac}M_c^d\overline{h}_d^b + gH_{ab}(6)\overline{h}_c^a M_d^b T^{cd} \ , \end{aligned} \tag{3.14}$$

where a, b, c, d, e, f, g are unknown reduced matrix elements and T^a is the charmed baryon anti-triplet

$$T_a = (\Xi_{c1}^0, -\Xi_{c1}^+, \Lambda_c^+) \quad , \quad T^{ab} = \epsilon^{abc} T_c \ . \tag{3.15}$$

The square of the matrix elements for Cabibbo-allowed processes are shown in Table 3.1. We see that there is only one relation between the matrix elements of Cabibbo-allowed decays,

$$|M(\Lambda_c^+ \to \Sigma^0 \pi^+)|^2 = |M(\Lambda_c^+ \to \Sigma^+ \pi^0)|^2 \ , \tag{3.16}$$

and this is a result of the SU(2) isospin subgroup of $SU(3)_f$.

There are several relations between squares of matrix elements for Cabibbo-suppressed decays, as can be seen from table 3.2. They are

$$|M(\Xi_{c1}^0 \to \Sigma^- \pi^+)|^2 = |M(\Xi_{c1}^0 \to \Xi^- K^+)|^2 \ , \tag{3.17a}$$

$$|M(\Xi_{c1}^0 \to n\overline{K}^0)|^2 = |M(\Xi_{c1}^0 \to \Xi^0 K^0)|^2 \ , \tag{3.17b}$$

$$|M(\Xi_{c1}^0 \to \Sigma^+ \pi^-)|^2 = |M(\Xi_{c1}^0 \to pK^-)|^2 \ , \tag{3.17c}$$

$$|M(\Xi_{c1}^+ \to p\overline{K}^0)|^2 = |M(\Lambda_c^+ \to \Sigma^+ K^0)|^2 \ , \tag{3.17d}$$

$$|M(\Xi_{c1}^+ \to \Xi^0 K^+)|^2 = |M(\Lambda_c^+ \to n\pi^+)|^2 \ , \tag{3.17e}$$

$$|M(\Xi_{c1}^0 \to \Lambda^0 \eta^0)|^2 = |M(\Xi_{c1}^0 \to \Sigma^0 \pi^0)|^2 \ , \tag{3.17f}$$

which are all a consequence of the full $SU(3)_f$ symmetry. From Table 3.3 we see that there are no relations between squared matrix elements of doubly-Cabibbo-suppressed decays. However, there are relations between the squares of Cabibbo-allowed, suppressed and doubly suppressed matrix elements. They are

$$|M(\Xi_{c1}^0 \rightarrow \Xi^- K^+)|^2 = s_1^2 |M(\Xi_{c1}^0 \rightarrow \Xi^- \pi^+)|^2 \ , \tag{3.18a}$$

$$|M(\Xi_{c1}^0 \rightarrow \Sigma^+ \pi^-)|^2 = s_1^2 |M(\Xi_{c1}^0 \rightarrow \Sigma^+ K^-)|^2 \ , \tag{3.18b}$$

$$|M(\Xi_{c1}^+ \rightarrow \Sigma^+ K^0)|^2 = s_1^4 |M(\Lambda_c^+ \rightarrow p\overline{K}^0)|^2 \ , \tag{3.18c}$$

$$|M(\Xi_{c1}^+ \rightarrow n\pi^+)|^2 = s_1^4 |M(\Lambda_c^+ \rightarrow \Xi^0 K^+)|^2 \ , \tag{3.18d}$$

$$|M(\Xi_{c1}^0 \rightarrow \Sigma^- K^+)|^2 = s_1^4 |M(\Xi_{c1}^0 \rightarrow \Xi^- \pi^+)|^2 \ , \tag{3.18e}$$

$$|M(\Xi_{c1}^0 \rightarrow p\pi^-)|^2 = s_1^4 |M(\Xi_{c1}^0 \rightarrow \Sigma^+ K^-)|^2 \ , \tag{3.18f}$$

$$|M(\Lambda_c^+ \rightarrow nK^+)|^2 = s_1^4 |M(\Xi_{c1}^+ \rightarrow \Xi^0 \pi^+)|^2 \ , \tag{3.18g}$$

$$|M(\Lambda_c^+ \rightarrow pK^0)|^2 = s_1^4 |M(\Xi_{c1}^+ \rightarrow \Sigma^+ \overline{K}^0)|^2 \ , \tag{3.18h}$$

The sum of the masses of the products from charmed baryon decay is not always negligible compared to the energy release. Therefore, $SU(3)_f$ relations between decay rates, derived from relations between the square of matrix elements, have significant phase space corrections. The exception to this is when a relation is due to isospin, where the difference between the sum of the final state masses is small. To find the relation between the decay rates from the square of the matrix elements we can use the expression

$$d\Gamma(a \rightarrow bc) = \frac{1}{32\pi^2} |M(a \rightarrow bc)|^2 \frac{|\mathbf{p_b}|}{m_a^2} d\Omega \ , \tag{3.19}$$

where m_a is the mass of the decaying particle, $\mathbf{p_b}$ is the momentum of one of the final state particles, $d\Omega$ is its solid angle and $M(a \rightarrow bc)$ is the matrix element for the decay $a \rightarrow bc$. There is also an additional factor of $|\mathbf{p_b}|^{2l}$ occurring in the matrix element for decays with final state angular momentum l. Any mass dependence in the matrix element is not corrected for as this is due to explicit SU(3) violation and not a kinematical effect. Consequently we find that, for instance,

$$\frac{\Gamma_l(\Xi_{c1}^0 \to \Sigma^- \pi^+)}{\Gamma_l(\Xi_{c1}^0 \to \Xi^- K^+)} = \left(\frac{\left(1 - (\frac{M_{\Sigma^-} + M_{\pi^+}}{M_{\Xi_{c1}^0}})^2\right)\left(1 - (\frac{M_{\Sigma^-} - M_{\pi^+}}{M_{\Xi_{c1}^0}})^2\right)}{\left(1 - (\frac{M_{\Xi^-} + M_{K^+}}{M_{\Xi_{c1}^0}})^2\right)\left(1 - (\frac{M_{\Xi^-} - M_{K^+}}{M_{\Xi_{c1}^0}})^2\right)} \right)^{l+\frac{1}{2}} , \quad (3.20)$$

where l is the angular momentum of the decay channel and Γ_l is its contribution to the rate. For this process both $l = 0$ and $l = 1$ partial waves can contribute. The angular distribution of the decay products from a polarized charmed baryon can be decomposed to yield the relative magnitude of the $l = 0$ and $l = 1$ partial waves, to which the phase space corrections can be applied accordingly. If, however, the angular distribution information is not available (which is probably the case), then the best estimate of the phase space correction is to say that it lies somewhere in the range between its value for $l = 0$ and $l = 1$. Thus the flavor SU(3) prediction for the above process is $1.2\Gamma(\Xi_{c1}^0 \to \Xi^- K^+) < \Gamma(\Xi_{c1}^0 \to \Sigma^- \pi^+) < 1.7\Gamma(\Xi_{c1}^0 \to \Xi^- K^+)$.

Two decay modes of the type under discussion here have been observed so far. The first $Br(\Lambda_c^+ \to \Lambda^0 \pi^+) = (0.63 \pm 0.25) \times 10^{-2}$ [1.6] does not appear in any of the relations. The second observed is $Br(\Lambda_c^+ \to p\overline{K}^0) = (2.0 \pm 0.4) \times 10^{-2}$ [3.1, 3.2, 3.3], from which we predict that $Br(\Xi_{c1}^+ \to \Sigma^+ K^0) \sim 4 \times 10^{-5}$. These expressions can easily be carried over to decays involving baryons and vector mesons, $T \to BV$. An example of this is

$$|M(\Xi_{c1}^0 \to \Sigma^- \rho^+)|^2 = |M(\Xi_{c1}^0 \to \Xi^- K^{*+}(892))|^2 \quad . \quad (3.21)$$

Two of the branching ratios for final states containing a vector meson have been measured. The first is $Br(\Lambda_c^+ \to p\overline{K}^{*0}(892)) = (5.6 \pm 3) \times 10^{-3}$ [3.2, 3.4], from which we predict that $Br(\Xi_{c1}^+ \to \Sigma^+ K^{*0}(892)) \sim 1 \times 10^{-5}$. The second is $Br(\Lambda_c^+ \to p\phi) = (2 \pm 1) \times 10^{-3}$ which does not appear in any of the relations.

Next we look at the process $T \to h^* M$ where h^* is the decuplet of $\frac{3}{2}^+$ baryon resonances with elements

$$h^{*111} = \Delta^{++}, h^{*112} = \frac{1}{\sqrt{3}}\Delta^{+}, h^{*113} = \frac{1}{\sqrt{3}}\Sigma^{*+}, h^{*122} = \frac{1}{\sqrt{3}}\Delta^{0}, h^{*133} = \frac{1}{\sqrt{3}}\Xi^{*0}$$

$$h^{*123} = \frac{1}{\sqrt{6}}\Sigma^{*0}, h^{*222} = \Delta^{-}, h^{*223} = \frac{1}{\sqrt{3}}\Sigma^{*-}, h^{*233} = \frac{1}{\sqrt{3}}\Xi^{*-}, h^{*333} = \Omega^{-} \quad . \quad (3.22)$$

The effective Hamiltonian for the process is

$$H_{\text{eff}} = \alpha \overline{h}^*_{abc} T^{ad} H^{bc}_e(\overline{15}) M^e_d + \beta \overline{h}^*_{abc} T^{ad} H^{be}_d(\overline{15}) M^c_e$$

$$+ \gamma \overline{h}^*_{abc} H^{ab}_d(\overline{15}) M^c_e T^{de} + \delta \overline{h}^*_{abc} H^{ag}_e(6) T^{be} M^c_g \; , \qquad (3.23)$$

where α, β, γ and δ are unknown reduced matrix elements. The rates for Cabibbo-allowed decay processes in terms of these reduced matrix elements are given in Table 3.4. There are four relations between Cabibbo-allowed decay rates. They are

$$|M(\Lambda_c^+ \to \Sigma^{*+}\pi^0)|^2 = |M(\Lambda_c^+ \to \Sigma^{*0}\pi^+)|^2 \; , \qquad (3.24a)$$

$$|M(\Lambda_c^+ \to \Delta^{++}K^-)|^2 = 3|M(\Lambda_c^+ \to \Delta^{+}\overline{K}^0)|^2 \; , \qquad (3.24b)$$

which are due to isospin and

$$|M(\Xi_{c1}^+ \to \Sigma^{*+}\overline{K}^0)|^2 = |M(\Xi_{c1}^+ \to \Xi^{*0}\pi^+)|^2 \; , \qquad (3.24c)$$

$$|M(\Xi_{c1}^0 \to \Omega^{-}K^+)|^2 = 3|M(\Xi_{c1}^0 \to \Xi^{*-}\pi^+)|^2 \; , \qquad (3.24d)$$

which are due to the full SU(3) symmetry. Again, phase space correction factors must be applied to these equalities giving, for example

$$\Gamma(\Xi_{c1}^0 \to \Omega^- K^+) \sim \Gamma(\Xi_{c1}^0 \to \Xi^{*-}\pi^+) \quad , \qquad (3.25)$$

a modification of $(0.69)^3$ due to the differing final state masses and the fact that the decay is P-wave (neglecting possible D-wave contributions).

There are several relations between the squares of Cabibbo-suppressed matrix elements, as seen in Table 3.5. They are

$$|M(\Lambda_c^+ \to \Delta^0\pi^+)|^2 = |M(\Xi_{c1}^+ \to \Xi^{*0}K^+)|^2 , \tag{3.26a}$$

$$|M(\Lambda_c^+ \to \Sigma^{*+}K^0)|^2 = |M(\Xi_{c1}^+ \to \Delta^+\overline{K}^0)|^2 , \tag{3.26b}$$

$$|M(\Lambda_c^+ \to \Sigma^{*0}K^+)|^2 = |M(\Xi_{c1}^+ \to \Sigma^{*0}\pi^+)|^2 , \tag{3.26c}$$

$$|M(\Lambda_c^+ \to \Delta^{++}\pi^-)|^2 = |M(\Xi_{c1}^+ \to \Delta^{++}K^-)|^2 , \tag{3.26d}$$

$$|M(\Xi_{c1}^0 \to \Sigma^{*-}\pi^+)|^2 = |M(\Xi_{c1}^0 \to \Xi^{*-}K^+)|^2 , \tag{3.26e}$$

$$|M(\Xi_{c1}^0 \to \Delta^0\overline{K}^0)|^2 = |M(\Xi_{c1}^0 \to \Xi^{*0}K^0)|^2 , \tag{3.26f}$$

$$|M(\Xi_{c1}^0 \to \Sigma^{*0}\pi^0)|^2 = 3|M(\Xi_{c1}^0 \to \Sigma^{*0}\eta^0)|^2 , \tag{3.26g}$$

$$|M(\Xi_{c1}^0 \to \Sigma^{*+}\pi^-)|^2 = |M(\Xi_{c1}^0 \to \Delta^+K^-)|^2 , \tag{3.26h}$$

which are all full $SU(3)_f$ relations.

There are also relations between doubly-Cabibbo-suppressed matrix elements as seen from Table 3.6. They are

$$|M(\Lambda_c^+ \to \Delta^+K^0)|^2 = |M(\Lambda_c^+ \to \Delta^0K^+)|^2 , \tag{3.27a}$$

$$|M(\Xi_{c1}^+ \to \Delta^{++}\pi^-)|^2 = 3|M(\Xi_{c1}^+ \to \Sigma^{*+}K^0)|^2 , \tag{3.27b}$$

$$|M(\Xi_{c1}^0 \to \Delta^-\pi^+)|^2 = 3|M(\Xi_{c1}^0 \to \Sigma^{*-}K^+)|^2 , \tag{3.27c}$$

$$|M(\Xi_{c1}^+ \to \Delta^+\eta^0)|^2 = |M(\Xi_{c1}^0 \to \Delta^0\eta^0)|^2 . \tag{3.27d}$$

Several relations between Cabibbo-allowed,-suppressed and doubly-suppressed decay modes are found. They are

$$|M(\Lambda_c^+ \to \Delta^0\pi^+)|^2 = 2s_1^2|M(\Lambda_c^+ \to \Sigma^{*+}\pi^0)|^2 , \tag{3.28a}$$

$$|M(\Lambda_c^+ \to \Delta^{++}\pi^-)|^2 = s_1^2|M(\Lambda_c^+ \to \Delta^{++}K^-)|^2 , \tag{3.28b}$$

$$2|M(\Xi_{c1}^+ \to \Sigma^{*+}\pi^0)|^2 = s_1^2|M(\Lambda_c^+ \to \Xi^{*0}K^+)|^2 , \tag{3.28c}$$

$$|M(\Xi_{c1}^0 \to \Delta^0\overline{K}^0)|^2 = 2s_1^2|M(\Xi_{c1}^0 \to \Sigma^{*0}\overline{K}^0)|^2 , \tag{3.28d}$$

$$|M(\Xi_{c1}^0 \to \Sigma^{*+}\pi^-)|^2 = s_1^2|M(\Xi_{c1}^0 \to \Sigma^{*+}K^-)|^2 , \tag{3.28e}$$

$$|M(\Xi_{c1}^0 \to \Sigma^{*-}\pi^+)|^2 = 4s_1^2|M(\Xi_{c1}^0 \to \Xi^{*-}\pi^+)|^2 , \tag{3.28f}$$

$$|M(\Xi_{c1}^+ \to \Sigma^{*0}K^+)|^2 = s_1^4|M(\Lambda_c^+ \to \Sigma^{*+}\pi^0)|^2 , \tag{3.28g}$$

$$|M(\Xi_{c1}^+ \to \Delta^{++}\pi^-)|^2 = s_1^4|M(\Lambda_c^+ \to \Delta^{++}K^-)|^2 , \tag{3.28h}$$

$$|M(\Xi_{c1}^+ \to \Delta^0\pi^+)|^2 = s_1^4|M(\Lambda_c^+ \to \Xi^{*0}K^+)|^2 , \tag{3.28i}$$

$$|M(\Lambda_c^+ \to \Delta^+K^0)|^2 = s_1^4|M(\Xi_{c1}^+ \to \Sigma^{*+}\overline{K}^0)|^2 , \tag{3.28j}$$

$$|M(\Xi_{c1}^0 \to \Sigma^{*0}K^0)|^2 = s_1^4|M(\Xi_{c1}^0 \to \Sigma^{*0}\overline{K}^0)|^2 , \tag{3.28k}$$

$$|M(\Xi_{c1}^0 \to \Delta^+\pi^-)|^2 = s_1^4|M(\Xi_{c1}^0 \to \Sigma^{*+}K^-)|^2 , \tag{3.28l}$$

$$|M(\Xi_{c1}^0 \to \Delta^-\pi^+)|^2 = s_1^4|M(\Xi_{c1}^0 \to \Omega^-K^+)|^2 , \tag{3.28m}$$

One of these decay modes has been observed with a branching ratio of [3.4]

$$Br(\Lambda_c^+ \to \Delta^{++}K^-) = (6.0 \pm 2.0) \times 10^{-3} , \tag{3.29}$$

and hence we can predict, neglecting possible D-wave contributions, that $Br(\Lambda_c^+ \to \Delta^{++}\pi^-) \sim (3.8 \pm 2.0) \times 10^{-4}$ ($\sim (4.8 \pm 2.5) \times 10^{-4}$ for a purely D-wave process) and that $Br(\Xi_{c1}^+ \to \Delta^{++}\pi^-) \sim 2.3 \times 10^{-5}$ ($\sim 3.8 \times 10^{-5}$ for a purely D-wave process).

Next we look at the two-body decays $S \to hM$, where S denotes the 6 representation of charmed baryons. The element $S_{33} = \Omega_c^0$ is the only member of the 6 that decays weakly, the $\Sigma_c^{++,+,0}$ decay strongly to the Λ_c^+ in the $\overline{3}$ (e.g. $\Sigma_c^{++} \to \Lambda_c^+ \pi^+$) and the $\Xi_{c2}^{+,0}$ decay electromagnetically (e.g. $\Xi_{c2}^0 \to \Xi_{c1}^0 \gamma$). By inspection of the Ω_c^0 flavor wavefunction we see that the only Cabibbo-allowed final state is $\Xi^0 \overline{K}^0$. We therefore look for relations between Cabibbo-allowed and Cabibbo-suppressed decay rates. The effective Hamiltonian for the process is

$$
\begin{aligned}
H_{\text{eff}} = {} & a\epsilon_{abf}\overline{h}_c^f H_d^{ac}(\overline{15})S^{be}M_e^d + b\epsilon_{abf}\overline{h}_c^f H_d^{ae}(\overline{15})S^{bc}M_e^d + c\epsilon_{abf}\overline{h}_c^f H_d^{ae}(\overline{15})S^{bd}M_e^c \\
& + d\epsilon_{abf}\overline{h}_c^f H_d^{ac}(\overline{15})S^{de}M_e^b + e\epsilon_{abf}\overline{h}_c^f H_d^{ae}(\overline{15})S^{cd}M_e^b + f(\overline{h}_a^b M_b^a)(H_{cd}(6)S^{cd}) \\
& + g\overline{h}_a^b M_b^c H_{cd}(6)S^{ad} + hM_a^b\overline{h}_b^c H_{cd}(6)S^{ad} + k\overline{h}_a^b M_c^d H_{bd}(6)S^{ac} \\
& + l\epsilon_{abf}\overline{h}_c^f H_d^{ce}(\overline{15})M_e^a S^{bd} \qquad . \qquad (3.30)
\end{aligned}
$$

The squared matrix elements resulting from this effective Hamiltonian are found in Table 3.7. We see that there are no relations between any of the decay rates involving Cabibbo-allowed, -suppressed or doubly-suppressed decays.

If we look at the isospin structure of the doubly-Cabibbo-suppressed sextet component of the Hamiltonian, $\mathcal{O}_6^{(ds)} = \frac{1}{2}\left[(c\bar{d})(s\bar{u}) - (c\bar{u})(s\bar{d})\right]$, we see that it is an $I=0$ operator, whereas $\mathcal{O}_{\overline{15}}^{(ds)}$ is an $I=1$ operator. Since Ω_c^0 has $I=0$, we expect that the decay to $\Lambda^0\eta^0$ proceeds via $\mathcal{O}_6$ only, and similarly for the $\Sigma^0\pi^0$ final state, since the weak Hamiltonian does not have an $I=2$ component. Therefore, by measuring the relative rate for an $I=1$ decay, for example the $\Sigma^0\eta^0$ or $\Lambda^0\pi^0$ final state, compared to an $I=0$ decay, an estimate of the relative contributions from $\mathcal{O}_6$ and $\mathcal{O}_{\overline{15}}$ can be made. This will not be a strong test of the perturbative QCD prediction since there could be cancellations between the reduced matrix elements contributing to the decays, but it will give a rough estimate of the relative contributions. Unfortunately, since these are doubly-Cabibbo-suppressed decays, they will probably be the last to be measured and hence their predictive power is somewhat limited.

Consider now the process $S \to h^* M$. The only two possible Cabibbo-allowed final states are $\Omega^- \pi^+$ and $\Xi^{*0}\overline{K}^0$. The effective Hamiltonian for the decay is

$$H_{\text{eff}} = \alpha \overline{h}^*_{abc} H^{ab}_d(\overline{15}) S^{ce} M^d_e + \beta \overline{h}^*_{abc} H^{ae}_d(\overline{15}) S^{bc} M^d_e + \gamma \overline{h}^*_{abc} H^{ae}_d(\overline{15}) S^{bd} M^c_e$$

$$+ \delta \overline{h}^*_{abc} H^{ab}_d(\overline{15}) S^{de} M^c_e + \lambda \epsilon^{adf} \overline{h}^*_{abc} H_{de}(6) M^b_f S^{ce} + \eta \epsilon^{adf} \overline{h}^*_{abc} H_{de}(6) M^e_f S^{bc} \quad . \tag{3.31}$$

The resulting squared matrix elements are shown in Table 3.8. We see that there are no relations between any of the Cabibbo-allowed or Cabibbo-suppressed decay modes. However, there are two relations involving doubly-Cabibbo-suppressed processes, they are

$$|M(\Omega^0_c \to \Delta^+ K^-)|^2 = |M(\Omega^0_c \to \Delta^0 \overline{K}^0|^2 \ , \tag{3.32a}$$

$$|M(\Omega^0_c \to \Xi^{*-} K^+)|^2 = s_1^2 \frac{1}{3} |M(\Omega^0_c \to \Omega^- K^+)|^2 \ . \tag{3.32b}$$

3.3 Nonleptonic Decay Of Charmed Baryons To Three Body Final States

In this section we will be considering decays of charmed baryons to final states containing a baryon (either in the lowest lying $\frac{1}{2}^+$ octet or the $\frac{3}{2}^+$ decuplet) and two octet mesons M. As far as $SU(3)_f$ is concerned the two meson octets are identical and consequently the Hamiltonian must be symmetrized if the mesons are in a relatively even angular momentum state or antisymmetrized if they are in a relatively odd angular momentum state. When the Hamiltonian is expanded in terms of the individual particle operators and matrix elements are taken there are symmetry factors that must be included. This is demonstrated most simply by an example. Consider the Hamiltonian

$$H_{\text{eff}} = \pi^+ \pi^- + \pi^0 \pi^0 \ , \tag{3.33}$$

of which matrix elements can be formed to yield

$$\langle 0 | H_{\text{eff}} | \pi^+ \pi^- \rangle = 1 \ , \tag{3.34a}$$

and

$$\langle 0|H_{\rm eff}|\pi^0\pi^0\rangle = 2 \ , \tag{3.34b}$$

due to the two possible ways of annihilating the two neutral pions. When we form a rate from these matrix elements there is an additional factor of $\frac{1}{2}$ multiplying the $\pi^0\pi^0$ phase space integrals from Bose statistics. In the tables this factor of $\frac{1}{2}$ has been omitted and so to obtain rate relations from the squared matrix elements a factor of $\frac{1}{2}$ must be included for processes involving identical particles. Also, in obtaining rate relations from the matrix elements, phase space correction factors must be included just as for the two-body decay modes. However since these factors depend upon the momentum configuration of the final state we will not calculate them in this work. Any processes that are not energetically allowed are not included in the tables.

The first three-body decay process examined is $T \to hMM$ for which there are nineteen reduced matrix elements. The operator $\mathcal{O}_{\overline{15}}$ contributes eleven reduced matrix elements and $\mathcal{O}_6$ contributes eight. The Hamiltonian for the process is

$$\begin{aligned} H_{\rm eff} = {} & A_f T_a \overline{h}^a_b H^{bc}_d(\overline{15}) M^d_e M^e_c + B_f T_a \overline{h}^a_b H^{cd}_e(\overline{15}) M^e_c M^b_d + C_f T_a \overline{h}^c_b H^{ad}_c(\overline{15}) M^b_e M^e_d \\ & + D_f T_a \overline{h}^c_b H^{ab}_d(\overline{15}) M^e_c M^d_e + E_f (T_a \overline{h}^c_b H^{ab}_c(\overline{15}))(M^e_f M^f_e) + F_f T_a \overline{h}^c_b H^{ae}_d(\overline{15}) M^b_c M^d_e \\ & + G_f T_a \overline{h}^c_b H^{ae}_d(\overline{15}) M^d_c M^b_e + I_f T_a \overline{h}^c_b H^{bd}_c(\overline{15}) M^a_e M^e_d + J_f T_a \overline{h}^c_e H^{bd}_c(\overline{15}) M^a_b M^e_d \\ & + K_f T_a \overline{h}^c_b H^{be}_d(\overline{15}) M^a_c M^d_e + L_f T_a \overline{h}^c_b H^{be}_d(\overline{15}) M^a_e M^d_c + A_s (T^{ab} \overline{h}^c_a H_{bc}(6))(M^d_e M^e_d) \\ & + B_s T^{ab} \overline{h}^c_a H_{cd}(6) M^e_b M^d_e + C_s T^{ab} \overline{h}^c_a H_{bd}(6) M^e_c M^d_e + D_s T^{ab} \overline{h}^c_a H_{de}(6) M^d_b M^e_c \\ & + F_s T^{ab} \overline{h}^d_c H_{ad}(6) M^e_b M^c_e + I_s T^{ab} \overline{h}^c_d H_{ae}(6) M^d_b M^e_c + J_s T^{ab} \overline{h}^d_c H_{ae}(6) M^e_b M^c_d \\ & + K_s T^{ab} \overline{h}^d_c H_{de}(6) M^c_a M^e_b \quad . \end{aligned} \tag{3.35}$$

The matrix elements resulting from this Hamiltonian are NOT tabulated in this paper as there are ~ 121 possible decay modes for either angular momentum state.

Despite this large number of operators there are still relations between some matrix elements for various decay modes. The relations between Cabibbo-allowed decays are all due to isospin, examples of which are

$$|M(\Xi_{c1}^0 \to \Xi^- \pi^0 \pi^+)_{(L=0,2,\ldots)}|^2 = |M(\Xi_{c1}^+ \to \Xi^0 \pi^0 \pi^+)_{(L=0,2,\ldots)}|^2 \ , \tag{3.36a}$$

$$|M(\Lambda_c^+ \to \Sigma^0 \pi^0 \pi^+)_{(L=1,3,\ldots)}|^2 = |M(\Lambda_c^+ \to \Sigma^+ \pi^+ \pi^-)_{(L=1,3,\ldots)}|^2 \ , \tag{3.36b}$$

for even and odd angular momentum channels respectively, and

$$|M(\Lambda_c^+ \to \Sigma^0 \eta^0 \pi^+)|^2 = |M(\Lambda_c^+ \to \Sigma^+ \eta^0 \pi^0)|^2 \ , \tag{3.36c}$$

$$|M(\Lambda_c^+ \to \Sigma^0 \pi^0 \pi^+)|^2 \geq \frac{1}{4}|M(\Lambda_c^+ \to \Sigma^- \pi^+ \pi^+)|^2 \ , \tag{3.36d}$$

which are independent of the relative angular momentum between the mesons. The inequality arises from the fact that processes involving identical mesons in the final state can only proceed through even angular momentum channels. There is a relation between a Cabibbo-allowed process and a Cabibbo-suppressed process that holds only for odd relative angular momentum states which is

$$|M(\Xi_{c1}^+ \to \Xi^- K^+ \pi^+)_{(L=1,3,\ldots)}|^2 = s_1^2 |M(\Lambda_c^+ \to \Xi^- K^+ \pi^+)_{(L=1,3,\ldots)}|^2 \quad . \tag{3.36e}$$

More interesting are the Cabibbo-suppressed decays where there are relations due to the full SU(3) symmetry which are independent of the relative angular momentum between the mesons. They are

$$|M(\Lambda_c^+ \to \Sigma^+ K^+ \pi^-)|^2 = |M(\Xi_{c1}^+ \to p K^- \pi^+)|^2 \ , \tag{3.37a}$$

$$|M(\Lambda_c^+ \to p \overline{K}^0 K^0)|^2 = |M(\Xi_{c1}^+ \to \Sigma^+ \overline{K}^0 K^0)|^2 \ , \tag{3.37b}$$

$$|M(\Lambda_c^+ \to n \overline{K}^0 K^+)|^2 = |M(\Xi_{c1}^+ \to \Xi^0 K^0 \pi^+)|^2 \ , \tag{3.37c}$$

$$|M(\Xi_{c1}^+ \to \Sigma^- \pi^+ \pi^+)|^2 \leq \frac{s_1^2}{2} |M(\Lambda_c^+ \to \Xi^- K^+ \pi^+)|^2 \ , \tag{3.37d}$$

$$|M(\Xi_{c1}^0 \to p \overline{K}^0 \pi^-)|^2 = |M(\Xi_{c1}^0 \to \Sigma^+ K^- K^0)|^2 \ , \tag{3.37e}$$

$$|M(\Xi_{c1}^0 \to n K^- \pi^+)|^2 = |M(\Xi_{c1}^0 \to \Xi^0 K^+ \pi^-)|^2 \ , \tag{3.37f}$$

$$|M(\Xi_{c1}^0 \to \Sigma^- \overline{K}^0 K^+)|^2 = |M(\Xi_{c1}^0 \to \Xi^- K^0 \pi^+)|^2 \ , \tag{3.37g}$$

$$|M(\Xi_{c1}^0 \to \Sigma^0 \overline{K}^0 K^0)|^2 = |M(\Xi_{c1}^0 \to \Lambda^0 \overline{K}^0 K^0)|^2 \ . \tag{3.37h}$$

Experimentally, branching ratios for some of these processes have been measured, $Br(\Lambda_c^+ \to pK^-\pi^+) = (2.6 \pm 0.9) \times 10^{-2}$ [3.5] and $Br(\Lambda_c^+ \to \Sigma^+\pi^+\pi^-) = (10 \pm 8) \times 10^{-2}$ [3.6], of which the latter appears in an isospin relation between Cabibbo-allowed decays.

There are twenty-nine reduced matrix elements contributing to the process $S \to BMM$, twenty of which are from $\mathcal{O}_{\overline{15}}$ and the remaining nine from $\mathcal{O}_6$. Only two relations are found,

$$|M(\Omega_c^0 \to \Sigma^+ \overline{K}^0 K^-)|^2 \geq 2|M(\Omega_c^0 \to \Sigma^0 \overline{K}^0 \overline{K}^0)|^2 \ , \text{ and} \tag{3.38a}$$

$$|M(\Omega_c^0 \to p \overline{K}^0 K^-)|^2 \geq \frac{1}{4} |M(\Omega_c^0 \to n \overline{K}^0 \overline{K}^0)|^2 \ . \tag{3.38b}$$

The first is between Cabibbo-allowed decays and the second between Cabibbo- suppressed decays. They are both consequences of isospin. The matrix elements for the various decay modes are not tabulated.

We consider now the process $T \to h^* MM$. There are twelve reduced matrix elements contributing to the decays. The effective Hamiltonian for the process is given by

$$H_{\rm eff} = A_f \overline{h}^*_{abc} H^{bc}_d(\overline{15}) M^d_e M^a_g T^{eg} + B_f \overline{h}^*_{abc} H^{ab}_d(\overline{15}) M^d_e M^e_g T^{cg}$$

$$+ C_f \overline{h}^*_{abc} H^{ga}_d(\overline{15}) M^b_e M^c_g T^{de} + D_f (\overline{h}^*_{abc} H^{bc}_d(\overline{15}) T^{ad})(M^e_g M^g_e)$$

$$+ E_f \overline{h}^*_{abc} H^{eb}_d(\overline{15}) M^d_e M^c_g T^{ag} + F_f \overline{h}^*_{abc} H^{gb}_d(\overline{15}) M^d_e M^c_g T^{ae}$$

$$+ G_f \overline{h}^*_{abc} H^{eg}_d(\overline{15}) M^b_e M^c_g T^{ad} + I_f \overline{h}^*_{abc} H^{ab}_d(\overline{15}) M^g_e M^c_g T^{de}$$

$$+ C_s \overline{h}^*_{abc} H^{ga}_d(6) M^b_e M^c_g T^{de} + E_s \overline{h}^*_{abc} H^{eb}_d(6) M^d_e M^c_g T^{ag}$$

$$+ F_s \overline{h}^*_{abc} H^{gb}_d(6) M^d_e M^c_g T^{ae} + G_s \overline{h}^*_{abc} H^{eg}_d(6) M^b_e M^c_g T^{ad} \ . \qquad (3.39)$$

The results for Cabibbo-allowed decays with the mesons in an even (odd) angular momentum state are shown in Table 3.9 (Table 3.1). Cabibbo-suppressed decays with the mesons in an even (odd) angular momentum state are shown in Table 3.11 (Table 3.12). We see that there are many relations between squared matrix elements for various processes. For Cabibbo-allowed decays, we find that there are relations between matrix elements when the mesons in an even or odd angular momentum state. They are

$$|M(\Xi^+_{c1} \to \Xi^{*0}\pi^0\pi^+)_{(L=0.2...)}|^2 = \frac{1}{8}|M(\Xi^+_{c1} \to \Xi^{*-}\pi^+\pi^+)_{(L=0,2,...)}|^2$$

$$= \frac{1}{6}|M(\Xi^+_{c1} \to \Omega^- K^+\pi^+)_{(L=0,2,...)}|^2 = |M(\Xi^0_{c1} \to \Xi^{*-}\pi^0\pi^+)_{(L=0,2,...)}|^2 \ , \qquad (3.40a)$$

$$|M(\Xi^0_{c1} \to \Delta^+\overline{K}^0 K^-)_{(L=1,3,...)}|^2 = \frac{1}{3}|M(\Xi^+_{c1} \to \Delta^{++}\overline{K}^0 K^-)_{(L=1,3,...)}|^2 \ , \qquad (3.40b)$$

$$|M(\Lambda^+_c \to \Xi^{*-} K^+\pi^+)_{(L=1,3,...)}|^2 = \frac{1}{3}|M(\Xi^+_{c1} \to \Omega^- K^+\pi^+)_{(L=1,3,...)}|^2 \ . \qquad (3.40c)$$

There are also many relations that are independent of the relative angular momentum between the mesons, they are

$$|M(\Lambda_c^+ \to \Delta^{++} K^- \pi^0)|^2 = \frac{3}{2}|M(\Lambda_c^+ \to \Delta^+ K^- \pi^+)|^2 \ , \tag{3.40d}$$

$$|M(\Lambda_c^+ \to \Sigma^{*+} \eta^0 \pi^0)|^2 = |M(\Lambda_c^+ \to \Sigma^{*0} \eta^0 \pi^+)|^2 \ , \tag{3.40e}$$

$$|M(\Lambda_c^+ \to \Sigma^{*0} \pi^0 \pi^+)|^2 \geq \frac{1}{4}|M(\Lambda_c^+ \to \Sigma^{*-} \pi^+ \pi^+)|^2 \ , \tag{3.40f}$$

$$|M(\Xi_{c1}^+ \to \Delta^{++} \overline{K}^0 K^-)|^2 \geq \frac{3}{4}|M(\Xi_c^+ \to \Delta^+ \overline{K}^0 \overline{K}^0)|^2 \ , \tag{3.40g}$$

$$|M(\Xi_{c1}^+ \to \Xi^{*0} \pi^0 \pi^+)|^2 = \frac{1}{6}|M(\Xi_{c1}^+ \to \Omega^- K^+ \pi^+)|^2 \geq \frac{1}{8}|M(\Xi_{c1}^+ \to \Xi^{*-} \pi^+ \pi^+)|^2 \ . \tag{3.40h}$$

Turning now to the Cabibbo-suppressed decays, again there are many relations between squared matrix elements. All except one of the relations between the squared matrix elements are independent of the relative angular momentum between the mesons. The relations are

$$|M(\Xi_{c1}^0 \to \Delta^0 K^- \pi^+)|^2 = |M(\Xi_{c1}^0 \to \Xi^{*0} K^+ \pi^-)|^2 \ , \tag{3.41a}$$

$$|M(\Lambda_c^+ \to \Sigma^{*+} K^+ \pi^-)|^2 = |M(\Xi_{c1}^+ \to \Delta^+ K^- \pi^+)|^2 \ , \tag{3.41b}$$

$$|M(\Lambda_c^+ \to \Sigma^{*-} K^+ \pi^+)|^2 = |M(\Xi_{c1}^+ \to \Xi^{*-} K^+ \pi^+)|^2 \ , \tag{3.41c}$$

$$|M(\Lambda_c^+ \to \Delta^+ \pi^0 \eta^0)_{(L=1,3,\ldots)}|^2 = |M(\Xi_{c1}^+ \to \Sigma^{*+} \pi^0 \eta^0)_{(L=1,3,\ldots)}|^2 \ , \tag{3.41d}$$

Also, there is a large number of relations between Cabibbo-allowed and Cabibbo-suppressed squared matrix elements. The relations between matrix elements when the mesons are either in a relatively even or odd angular momentum state are

$$|M(\Lambda_c^+ \to \Delta^{++}\pi^-\pi^0)_{(L=0,2,\ldots)}|^2 = \frac{3}{2}s_1^2|M(\Lambda_c^+ \to \Delta^0\overline{K}^0\pi^+)_{(L=0,2,\ldots)}|^2 \ , \qquad (3.42a)$$

$$|M(\Xi_{c1}^0 \to \Xi^{*0}K^0\pi^0)_{(L=0,2,\ldots)}|^2 = 2s_1^2|M(\Xi_{c1}^0 \to \Sigma^{*0}\overline{K}^0\pi^0)_{(L=0,2,\ldots)}|^2 \ , \qquad (3.42b)$$

$$|M(\Xi_{c1}^0 \to \Delta^{++}K^-\pi^-)_{(L=0,2,\ldots)}|^2 = s_1^2|M(\Xi_{c1}^0 \to \Delta^{++}K^-K^-)_{(L=0,2,\ldots)}|^2 \ . \qquad (3.42c)$$

There are also a few relations that are independent of the relative angular momentum between the mesons, they are

$$|M(\Lambda_c^+ \to \Delta^-\pi^+\pi^+)|^2 \leq 12s_1^2|M(\Lambda_c^+ \to \Sigma^{*0}\pi^0\pi^+)|^2 \ , \qquad (3.42d)$$

$$|M(\Xi_{c1}^0 \to \Delta^-\overline{K}^0\pi^+)|^2 = 3s_1^2|M(\Xi_{c1}^0 \to \Sigma^{*-}\pi^+\overline{K}^0)|^2 \ , \qquad (3.42e)$$

$$|M(\Xi_{c1}^0 \to \Delta^0K^-\pi^+)|^2 = 2s_1^2|M(\Xi_{c1}^0 \to \Sigma^{*0}\pi^+K^-)|^2 \ . \qquad (3.42f)$$

Through a cancellation within each operator comprising the Hamiltonian we find that the Cabibbo-suppressed decay $\Xi_{c1}^0 \to \Sigma^{*0}\eta^0\pi^0$ proceeds entirely through even angular momentum channels.

There are sixteen reduced matrix elements contributing to the process $S \to h^*MM$, ten are from $\mathcal{O}_{\overline{15}}$ and the remaining six are from $\mathcal{O}_6$. We find that there is only one relation between matrix elements and it is between Cabibbo-allowed decays with the mesons in an even angular momentum state ($|M(\Omega_c^0 \to \Sigma^{*+}\overline{K}^0K^-)|^2 = 2|M(\Omega_c^0 \to \Sigma^{*0}\overline{K}^0\overline{K}^0)|^2$). There are no relations between decays with the mesons in a relatively odd angular momentum state. Consequently the only relation is

$$|M(\Omega_c^0 \to \Sigma^{*+}\overline{K}^0K^-)|^2 \geq 2|M(\Omega_c^0 \to \Sigma^{*0}\overline{K}^0\overline{K}^0)|^2 \qquad , \qquad (3.43)$$

and this is due to isospin. The matrix elements for these decays are not tabulated.

3.4 Semileptonic Decay Of Charmed Baryons To Three Body Final States

For the process $T \to hl^+\nu_l$ there is only one SU(3) singlet possible from $3 \otimes \overline{3} \otimes 8$ and consequently only one reduced matrix element. Thus, the effective Hamiltonian for semileptonic decay can be written

$$H_{\text{eff}} = \alpha H^a(\overline{3}) T_b \overline{h}^b_a \overline{l^+} \overline{\nu}_l \quad , \tag{3.44}$$

where the weak Hamiltonian is given in Eq. (2.8 a,b). All the matrix elements are related and we find that

$$|M(\Xi^0_{c1} \to \Xi^- l^+ \nu_l)|^2 = |M(\Xi^+_{c1} \to \Xi^0 l^+ \nu_l)|^2 = \frac{3}{2}|M(\Lambda^+_c \to \Lambda^0 l^+ \nu_l)|^2$$

$$= \frac{1}{s_1^2}|M(\Xi^0_{c1} \to \Sigma^- l^+ \nu_l)|^2 = \frac{6}{s_1^2}|M(\Xi^+_{c1} \to \Lambda^0 l^+ \nu_l)|^2$$

$$= \frac{2}{s_1^2}|M(\Xi^+_{c1} \to \Sigma^0 l^+ \nu_l)|^2 = \frac{1}{s_1^2}|M(\Lambda^+_c \to n l^+ \nu_l)|^2 \quad . \tag{3.45}$$

Experimentally, only a few inclusive branching ratios have been measured [3.7], they are

$$Br(\Lambda^+_c \to \Lambda^0 e^+ X) = (1.1 \pm 0.8) \times 10^{-2} \ , \tag{3.46a}$$

$$Br(\Lambda^+_c \to p e^+ X) = (1.8 \pm 0.9) \times 10^{-2} \ , \tag{3.46b}$$

$$Br(\Lambda^+_c \to e^+ X) = (4.5 \pm 1.7) \times 10^{-2} \ , \tag{3.46c}$$

where X denotes unidentified hadrons and ν_e.

The only SU(3) singlets that can be constructed for the process $T \to h^* l^+ \nu_l$ are

$$\epsilon^{bcd} H^a(\overline{3}) T_d \overline{h}^*_{abc} \text{ and } \epsilon^{abc} \overline{h}^*_{abc} H^d(\overline{3}) T_d \ , \tag{3.47}$$

both of which vanish since h^* is totally symmetric on its three indices. Hence we would not expect to see any lone decuplet resonances produced in the semileptonic decay of the charmed baryons in the $\overline{3}$ representation.

Turning now to the 6 representation and the process $S \to hl^+\nu_l$, we see that only one non-zero SU(3) singlet can be formed from the available tensors, giving the effective Hamiltonian

$$H_{\text{eff}} = \beta\epsilon_{abf}H^f(\overline{3})S^{ac}\overline{h}^b_c\overline{l^+}\overline{\nu}_l \quad . \tag{3.48}$$

It is obvious from the flavor wavefunction of the Ω_c^0 that it cannot Cabibbo-allowed decay to a member of the baryon octet and that it will only decay via a Cabibbo-suppressed mode to $\Xi^- l^+\nu_l$, consequently there are no relations possible. This, however, is not the case for the process $S \to h^* l^+\nu_l$ where both Cabibbo-allowed and suppressed decays are possible. The effective Hamiltonian for the process is

$$H_{\text{eff}} = \gamma H^a(\overline{3})S^{bc}\overline{h}^*_{abc}\overline{l^+}\overline{\nu}_l \ , \tag{3.49}$$

from which we find that

$$|M(\Omega_c^0 \to \Omega^- l^+\nu_l)|^2 = \frac{3}{s_1^2}|M(\Omega_c^0 \to \Xi^{*-} l^+\nu_l|^2 \quad . \tag{3.50}$$

3.5 Semileptonic Decay Of Charmed Baryons To Four Body Final States

Returning to the $\overline{3}$ representation of charmed baryons and looking at the decays $T \to hMl^+\nu_l$, we find that there are three reduced matrix elements that can contribute to the decay process. The effective Hamiltonian for such decays is

$$H_{\text{eff}} = a\left(T_a H^a(\overline{3})\right)\left(\overline{h}^d_c M^c_d\right)\overline{l^+}\overline{\nu}_l + bT_a\overline{h}^a_b M^b_c H^c(\overline{3})\overline{l^+}\overline{\nu}_l + cT_a M^a_b \overline{h}^b_c H^c(\overline{3})\overline{l^+}\overline{\nu}_l \quad . \tag{3.51}$$

There are many relations between the squared matrix elements for various decay modes, as shown in Table 3.13 for Cabibbo-allowed decays and in Table 3.14 for

Cabibbo-suppressed decays. Many are due to isospin, for instance

$$|M(\Lambda_c^+ \to \Sigma^0\pi^0 l^+\nu_l)|^2 = |M(\Lambda_c^+ \to \Sigma^+\pi^- l^+\nu_l|^2 = |M(\Lambda_c^+ \to \Sigma^-\pi^+ l^+\nu_l)|^2 \ , \quad (3.52)$$

but some are due to the full SU(3) symmetry, for example

$$|M(\Xi_{c1}^0 \to \Lambda^0\pi^- l^+\nu_l)|^2 = |M(\Xi_{c1}^0 \to \Sigma^-\eta^0 l^+\nu_l)|^2 \quad . \quad (3.53)$$

There is only one non-zero matrix element that can be constructed for the decays $T \to h^* M l^+\nu_l$, and so the matrix elements for all the decay modes are related. The effective Hamiltonian is

$$H_{\text{eff}} = \alpha\epsilon^{abc}\overline{h}^*_{ade} T_b M_c^d H^e(\overline{3})\overline{l^+}\overline{\nu}_l \ , \quad (3.54)$$

where α is the unknown reduced matrix element. The relative squared matrix elements for Cabibbo-allowed (suppressed) decays can be found in Table 3.15 (3.16). Phase space correction factors must be applied as in the previous cases. By coincidence, the Cabibbo-allowed processes with the largest matrix elements are those that will be modified the most by these corrections.

Turning now to the 6 representation and the decay process $S \to BMl^+\nu_l$, we find that there are three reduced matrix elements that can contribute and so the effective Hamiltonian for the process is

$$H_{\text{eff}} = \alpha\epsilon_{acf}S^{ab}H^f(\overline{3})\overline{h}_b^d M_d^c \overline{l^+}\overline{\nu}_l + \beta\epsilon_{acf}S^{ab}H^f(\overline{3})M_b^d\overline{h}_d^c\overline{l^+}\overline{\nu}_l$$

$$+\gamma\epsilon_{cdf}S^{ab}H^f(\overline{3})\overline{h}_a^c M_b^d\overline{l^+}\overline{\nu}_l \quad . \quad (3.55)$$

The results of which are shown in Table 3.17, from which we see that the only relations between matrix elements are those due to isospin, such that

$$|M(\Omega_c^0 \to \Xi^- \overline{K}^0 l^+ \nu_l)|^2 = |M(\Omega_c^0 \to \Xi^0 K^- l^+ \nu_l)|^2 , \tag{3.56a}$$

$$2|M(\Omega_c^0 \to \Xi^- \pi^0 l^+ \nu_l)|^2 = |M(\Omega_c^0 \to \Xi^0 \pi^- l^+ \nu_l)|^2 , \tag{3.56b}$$

$$2|M(\Omega_c^0 \to \Sigma^0 K^- l^+ \nu_l)|^2 = |M(\Omega_c^0 \to \Sigma^- \overline{K}^0 l^+ \nu_l)|^2 \quad . \tag{3.56c}$$

Two reduced matrix elements contribute to the process $S \to h^* l^+ \nu_l$ for which the effective Hamiltonian is

$$H_{\text{eff}} = \delta \overline{h}^*_{abc} S^{ab} M^c_d H^d(\overline{3}) \overline{l^+} \overline{\nu}_l + \zeta \overline{h}^*_{abc} S^{ad} H^b(\overline{3}) M^c_d \overline{l^+} \overline{\nu}_l \quad , \tag{3.57}$$

the matrix elements of which are shown in Table 3.18. We see that there are relations not only due to isospin but some due to the full $SU(3)_f$ symmetry. We find that

$$2|M(\Omega_c^0 \to \Xi^{*0} K^- l^+ \nu_l)|^2 = 2|M(\Omega_c^0 \to \Xi^{*-} \overline{K}^0 l^+ \nu_l)|^2 = |M\Omega_c^0 \to \Omega^- \eta^0 l^+ \nu_l)|^2 , \tag{3.58a}$$

$$3|M(\Omega_c^0 \to \Xi^{*0} \pi^- l^+ \nu_l)|^2 = 6|M(\Omega_c^0 \to \Xi^{*-} \pi^0 l^+ \nu_l)|^2 = |M(\Omega_c^0 \to \Omega^- K^0 l^+ \nu_l)|^2 , \tag{3.58b}$$

and also one purely isospin relation

$$2|M(\Omega_c^0 \to \Sigma^{*0} K^- l^+ \nu_l)|^2 = |M(\Omega_c^0 \to \Sigma^{*-} \overline{K}^0 l^+ \nu_l)|^2 \quad . \tag{3.58c}$$

3.6 Concluding Remarks On Chapter 3.

We have examined the predictions of flavor $SU(3)_f$ for the weak nonleptonic and semileptonic decay of charmed baryons both in the $\overline{3}$ and 6 representation of SU(3). The matrix elements for Cabibbo-allowed, -suppressed and doubly-suppressed decay modes were parameterized in terms of reduced matrix elements which have been tabulated explicitly. At the present time only a few decay modes (Cabibbo-allowed) have been experimentally observed; in the future when a larger event sample has been collected the relations derived in this work can be tested and/or used to reveal some of the underlying dynamics responsible for charmed baryon decay.

The predictive power of the $SU(3)_f$ invariance is, in some cases, somewhat limited due to phase space correction factors that must be included. However, these uncertainties can be eliminated by experimentally determining the relative contributions from different angular momentum channels.

If the sextet component of the Hamiltonian dominates nonleptonic decay processes, as hinted at by perturbative QCD, then this will be directly observable by the absence of $I = 1$ final states in the doubly-Cabibbo-suppressed decay of the Ω_c^0. Sextet dominance will also give rise to new relations between decay rates. These new relations between two-body decay modes have been considered previously in ref. [2.2, 3.8, 3.9] and can be derived from this work for all nonleptonic processes by neglecting the contribution from $\mathcal{O}_{\overline{15}}$.

An interesting prediction of $SU(3)_f$ is that the $\overline{3}$ cannot semileptonically decay to an $h^* l^+ \nu_l$ final state because a non-zero $SU(3)_f$ invariant cannot be constructed. Also, all the matrix elements for the semileptonic decay of the $\overline{3}$ to $Bl^+\nu_l$ final states are related. This is also true for the decays of the $\overline{3}$ to $h^* M l^+ \nu_l$ final states.

Final state interactions (FSI) will be important for these decays. We discussed their inclusion in the decay rates for D-mesons and the same arguements apply in this case. Consider the final state $\Lambda^0\pi^+$, this proceeds entirely in a $I = 1$ partial wave and therfore Σ^{*+} resonances can be excited in the final state. For hyperon decays, this effect will be very small as the energy of the decay is substantially below the mass of the first Σ^{*+} resonance. However, in the decay of charmed baryons the energy release lies in the regime where these resonances could give rise to substantial phase shifts. As was done for D-mesons [2.8] these phase shifts can be fit to the data and removed from the $SU(3)_f$ amplitudes before a comparison is made between theory and experiment. It is possible that the deviation from the predictions of $SU(3)_f$ for charmed baryon decays will be less significant than for D-meson decays as $SU(3)_f$ is a better symmetry for baryons than for mesons.

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Table 3.1. Squared matrix elements for Cabibbo-allowed decays $T \to hM$ in terms of the reduced matrix elements a, b, c, d, e, f and g.

Process	Squared Matrix Element
$\Lambda_c^+ \to \Lambda^0\pi^+$	$\frac{1}{6}\|a+b-2c-2e-2f-2g\|^2$
$\Lambda_c^+ \to \Sigma^0\pi^+$	$\frac{1}{2}\|a-b-2e+2f+2g\|^2$
$\Lambda_c^+ \to \Sigma^+\pi^0$	$\frac{1}{2}\|-a+b+2e-2f-2g\|^2$
$\Lambda_c^+ \to \Sigma^+\eta^0$	$\frac{1}{6}\|a+b-2d-2e-2f+2g\|^2$
$\Lambda_c^+ \to p\overline{K}^0$	$\|a+c-2e\|^2$
$\Lambda_c^+ \to \Xi^0 K^+$	$\|b+d-2f\|^2$
$\Xi_{c1}^0 \to \Xi^-\pi^+$	$\|a+c+2e\|^2$
$\Xi_{c1}^0 \to \Xi^0\pi^0$	$\frac{1}{2}\|-a+d-2e+2g\|^2$
$\Xi_{c1}^0 \to \Xi^0\eta^0$	$\frac{1}{6}\|a-2b+d+2e-4f-2g\|^2$
$\Xi_{c1}^0 \to \Lambda^0\overline{K}^0$	$\frac{1}{6}\|-2a+b+c-4e+2f+2g\|^2$
$\Xi_{c1}^0 \to \Sigma^+K^-$	$\|b+d+2f\|^2$
$\Xi_{c1}^0 \to \Sigma^0\overline{K}^0$	$\frac{1}{2}\|-b+c-2f-2g\|^2$
$\Xi_{c1}^+ \to \Xi^0\pi^+$	$\|-c-d-2g\|^2$
$\Xi_{c1}^+ \to \Sigma^+\overline{K}^0$	$\|-c-d+2g\|^2$

Table 3.2. Squared matrix elements for Cabibbo-suppressed decays $T \to hM$ in terms of the reduced matrix elements a, b, c, d, e, f and g.

Process	Squared Matrix Element (Modulo s_1^2)
$\Lambda_c^+ \to \Lambda^0 K^+$	$\frac{1}{6}\lvert -a+2b+2c+3d+2e-4f+2g\rvert^2$
$\Lambda_c^+ \to \Sigma^0 K^+$	$\frac{1}{2}\lvert -a-d+2e-2g\rvert^2$
$\Lambda_c^+ \to \Sigma^+ K^0$	$\lvert -a+d+2e-2g\rvert^2$
$\Lambda_c^+ \to p\eta^0$	$\frac{1}{6}\lvert 2a-b+3c+2d-4e+2f-2g\rvert^2$
$\Lambda_c^+ \to p\pi^0$	$\frac{1}{2}\lvert -b-c+2f+2g\rvert^2$
$\Lambda_c^+ \to n\pi^+$	$\lvert -b+c+2f+2g\rvert^2$
$\Xi_{c1}^0 \to \Sigma^-\pi^+$	$\lvert a+c+2e\rvert^2$
$\Xi_{c1}^0 \to \Lambda^0\pi^0$	$\frac{1}{12}\lvert -a-b-c+3d-2e-2f+4g\rvert^2$
$\Xi_{c1}^0 \to \Sigma^0\pi^0$	$\frac{1}{4}\lvert a+b-c-d+2e+2f\rvert^2$
$\Xi_{c1}^0 \to \Lambda^0\eta^0$	$\frac{1}{4}\lvert -a-b+c+d-2e-2f\rvert^2$
$\Xi_{c1}^0 \to \Sigma^0\eta^0$	$\frac{1}{12}\lvert -a-b+3c-d-2e-2f-4g\rvert^2$
$\Xi_{c1}^0 \to n\overline{K}^0$	$\lvert a-b+2e-2f-2g\rvert^2$
$\Xi_{c1}^0 \to \Xi^- K^+$	$\lvert -a-c-2e\rvert^2$
$\Xi_{c1}^0 \to \Xi^0 K^0$	$\lvert -a+b-2e+2f+2g\rvert^2$
$\Xi_{c1}^0 \to \Sigma^+\pi^-$	$\lvert b+d+2f\rvert^2$
$\Xi_{c1}^0 \to pK^-$	$\lvert -b-d-2f\rvert^2$
$\Xi_{c1}^+ \to \Lambda^0\pi^+$	$\frac{1}{6}\lvert -a-b-c-3d+2e+2f-4g\rvert^2$
$\Xi_{c1}^+ \to \Sigma^0\pi^+$	$\frac{1}{2}\lvert -a+b+c+d+2e-2f\rvert^2$
$\Xi_{c1}^+ \to \Sigma^+\pi^0$	$\frac{1}{2}\lvert a-b+c+d-2e+2f\rvert^2$
$\Xi_{c1}^+ \to \Sigma^+\eta^0$	$\frac{1}{6}\lvert -a-b-3c-d+2e+2f+4g\rvert^2$
$\Xi_{c1}^+ \to p\overline{K}^0$	$\lvert -a+d+2e-2g\rvert^2$
$\Xi_{c1}^+ \to \Xi^0 K^+$	$\lvert -b+c+2f+2g\rvert^2$

Table 3.3. Squared matrix elements for the doubly-Cabibbo- suppressed decays $T \to hM$ in terms of the reduced matrix elements a, b, c, d, e, f and g.

Process	Squared Matrix Element (Modulo s_1^4)
$\Lambda_c^+ \to pK^0$	$\lvert c+d-2g\rvert^2$
$\Lambda_c^+ \to nK^+$	$\lvert c+d+2g\rvert^2$
$\Xi_{c1}^0 \to \Sigma^- K^+$	$\lvert a+c+2e\rvert^2$
$\Xi_{c1}^0 \to \Lambda^0 K^0$	$\frac{1}{6}\lvert a-2b+c+2e+4f-4g\rvert^2$
$\Xi_{c1}^0 \to \Sigma^0 K^0$	$\frac{1}{2}\lvert -a+c-2e\rvert^2$
$\Xi_{c1}^0 \to n\eta^0$	$\frac{1}{6}\lvert -2a+b+d-4e+2f+4g\rvert^2$
$\Xi_{c1}^0 \to p\pi^-$	$\lvert b+d+2f\rvert^2$
$\Xi_{c1}^0 \to n\pi^0$	$\frac{1}{2}\lvert -b+d-2f\rvert^2$
$\Xi_{c1}^+ \to \Lambda^0 K^+$	$\frac{1}{6}\lvert -a+2b-c+2e-4f-4g\rvert^2$
$\Xi_{c1}^+ \to \Sigma^0 K^+$	$\frac{1}{2}\lvert -a+c+2e\rvert^2$
$\Xi_{c1}^+ \to \Sigma^+ K^0$	$\lvert -a-c+2e\rvert^2$
$\Xi_{c1}^+ \to p\eta^0$	$\frac{1}{6}\lvert 2a-b-d-4e+2f+4g\rvert^2$
$\Xi_{c1}^+ \to p\pi^0$	$\frac{1}{2}\lvert -b+d+2f\rvert^2$
$\Xi_{c1}^+ \to n\pi^+$	$\lvert -b-d+2f\rvert^2$

Table 3.4. Squared matrix elements for Cabibbo-allowed decays $T \to h^* M$ in terms of the reduced matrix elements α, β, γ and δ.

Process	Squared Matrix Element
$\Lambda_c^+ \to \Sigma^{*+}\pi^0$	$\frac{1}{6}\lvert -2\alpha+\beta-2\gamma-\delta\rvert^2$
$\Lambda_c^+ \to \Sigma^{*+}\eta^0$	$\frac{1}{18}\lvert 2\alpha-\beta-2\gamma-3\delta\rvert^2$
$\Lambda_c^+ \to \Sigma^{*0}\pi^+$	$\frac{1}{6}\lvert -2\alpha+\beta-2\gamma-\delta\rvert^2$
$\Lambda_c^+ \to \Delta^{++}K^-$	$\lvert\beta+\delta\rvert^2$
$\Lambda_c^+ \to \Delta^+\overline{K}^0$	$\frac{1}{3}\lvert\beta+\delta\rvert^2$
$\Lambda_c^+ \to \Xi^{*0}K^+$	$\frac{1}{3}\lvert\beta-2\gamma-\delta\rvert^2$
$\Xi_{c1}^+ \to \Sigma^{*+}\overline{K}^0$	$\frac{4}{3}\lvert\alpha\rvert^2$
$\Xi_{c1}^+ \to \Xi^{*0}\pi^+$	$\frac{4}{3}\lvert\alpha\rvert^2$
$\Xi_{c1}^0 \to \Sigma^{*0}\overline{K}^0$	$\frac{1}{6}\lvert 2\alpha-\beta+2\gamma-\delta\rvert^2$
$\Xi_{c1}^0 \to \Xi^{*0}\pi^0$	$\frac{1}{6}\lvert 2\alpha-\beta+\delta\rvert^2$
$\Xi_{c1}^0 \to \Xi^{*0}\eta^0$	$\frac{1}{18}\lvert -2\alpha+\beta-4\gamma+3\delta\rvert^2$
$\Xi_{c1}^0 \to \Sigma^{*+}K^-$	$\frac{1}{3}\lvert -\beta+2\gamma-\delta\rvert^2$
$\Xi_{c1}^0 \to \Xi^{*-}\pi^+$	$\frac{1}{3}\lvert -\beta+\delta\rvert^2$
$\Xi_{c1}^0 \to \Omega^-K^+$	$\lvert -\beta+\delta\rvert^2$

Table 3.5. Squared matrix elements for Cabibbo-suppressed decays $T \to h^* M$ in terms of the reduced matrix elements α, β, γ and δ.

Process	Squared Matrix Element (Modulo s_1^2)
$\Lambda_c^+ \to \Delta^+ \pi^0$	$\frac{2}{3}\lvert -\alpha - \gamma - \delta\rvert^2$
$\Lambda_c^+ \to \Delta^+ \eta^0$	$\frac{2}{9}\lvert \alpha + \beta - \gamma\rvert^2$
$\Lambda_c^+ \to \Delta^0 \pi^+$	$\frac{1}{3}\lvert -2\alpha + \beta - 2\gamma - \delta\rvert^2$
$\Lambda_c^+ \to \Sigma^{*+} K^0$	$\frac{1}{3}\lvert -2\alpha + \beta + \delta\rvert^2$
$\Lambda_c^+ \to \Sigma^{*0} K^+$	$\frac{1}{6}\lvert 2\alpha + \beta - 2\gamma - \delta\rvert^2$
$\Lambda_c^+ \to \Delta^{++} \pi^-$	$\lvert \beta + \delta\rvert^2$
$\Xi_{c1}^+ \to \Delta^+ \overline{K}^0$	$\frac{1}{3}\lvert 2\alpha - \beta - \delta\rvert^2$
$\Xi_{c1}^+ \to \Sigma^{*0} \pi^+$	$\frac{1}{6}\lvert -2\alpha - \beta + 2\gamma + \delta\rvert^2$
$\Xi_{c1}^+ \to \Sigma^{*+} \eta^0$	$\frac{1}{18}\lvert 4\alpha + \beta + 2\gamma + 3\delta\rvert^2$
$\Xi_{c1}^+ \to \Xi^{*0} K^+$	$\frac{1}{3}\lvert 2\alpha - \beta + 2\gamma + \delta\rvert^2$
$\Xi_{c1}^+ \to \Delta^{++} K^-$	$\lvert -\beta - \delta\rvert^2$
$\Xi_{c1}^+ \to \Sigma^{*+} \pi^0$	$\frac{1}{6}\lvert -\beta + 2\gamma + \delta\rvert^2$
$\Xi_{c1}^0 \to \Delta^0 \overline{K}^0$	$\frac{1}{3}\lvert 2\alpha - \beta + 2\gamma - \delta\rvert^2$
$\Xi_{c1}^0 \to \Sigma^{*0} \pi^0$	$\frac{1}{12}\lvert 2\alpha - \beta - 2\gamma + 3\delta\rvert^2$
$\Xi_{c1}^0 \to \Sigma^{*0} \eta^0$	$\frac{1}{36}\lvert 2\alpha - \beta - 2\gamma + 3\delta\rvert^2$
$\Xi_{c1}^0 \to \Xi^{*0} K^0$	$\frac{1}{3}\lvert 2\alpha - \beta + 2\gamma - \delta\rvert^2$
$\Xi_{c1}^0 \to \Sigma^{*+} \pi^-$	$\frac{1}{3}\lvert -\beta + 2\gamma - \delta\rvert^2$
$\Xi_{c1}^0 \to \Sigma^{*-} \pi^+$	$\frac{4}{3}\lvert -\beta + \delta\rvert^2$
$\Xi_{c1}^0 \to \Xi^{*-} K^+$	$\frac{4}{3}\lvert -\beta + \delta\rvert^2$
$\Xi_{c1}^0 \to \Delta^+ K^-$	$\frac{1}{3}\lvert -\beta + 2\gamma - \delta\rvert^2$

Table 3.6. Squared matrix elements for the doubly-Cabibbo- suppressed decays $T \to h^* M$ in terms of the reduced matrix α, β, γ and δ.

Process	Squared Matrix Element (Modulo s_1^4)
$\Lambda_c^+ \to \Delta^+ K^0$	$\frac{4}{3}\|\alpha\|^2$
$\Lambda_c^+ \to \Delta^0 K^+$	$\frac{4}{3}\|\alpha\|^2$
$\Xi_{c1}^+ \to \Delta^+ \eta^0$	$\frac{2}{9}\|-2\alpha+\beta-\gamma\|^2$
$\Xi_{c1}^+ \to \Sigma^{*0} K^+$	$\frac{1}{6}\|-2\alpha+\beta-2\gamma-\delta\|^2$
$\Xi_{c1}^+ \to \Delta^{++} \pi^-$	$\|\beta+\delta\|^2$
$\Xi_{c1}^+ \to \Sigma^{*+} K^0$	$\frac{1}{3}\|\beta+\delta\|^2$
$\Xi_{c1}^+ \to \Delta^0 \pi^+$	$\frac{1}{3}\|\beta-2\gamma-\delta\|^2$
$\Xi_{c1}^+ \to \Delta^+ \pi^0$	$\frac{2}{3}\|\gamma+\delta\|^2$
$\Xi_{c1}^0 \to \Delta^0 \eta^0$	$\frac{2}{9}\|-2\alpha+\beta-\gamma\|^2$
$\Xi_{c1}^0 \to \Sigma^{*0} K^0$	$\frac{1}{6}\|-2\alpha+\beta-2\gamma+\delta\|^2$
$\Xi_{c1}^0 \to \Delta^+ \pi^-$	$\frac{1}{3}\|\beta-2\gamma+\delta\|^2$
$\Xi_{c1}^0 \to \Delta^- \pi^+$	$\|\beta-\delta\|^2$
$\Xi_{c1}^0 \to \Sigma^{*-} K^+$	$\frac{1}{3}\|\beta-\delta\|^2$
$\Xi_{c1}^0 \to \Delta^0 \pi^0$	$\frac{2}{3}\|\gamma-\delta\|^2$

Table 3.7. Squared matrix elements for the decays $S \to hM$ in terms of the reduced matrix elements a, b, c, d, e, f, g, h and k.

Process	Squared Matrix Element
$\Omega_c^0 \to \Xi^0 \overline{K}^0$	$\lvert -a-b+2k\rvert^2$
$\Omega_c^0 \to \Sigma^0 \overline{K}^0$	$s_1^2 \frac{1}{2} \lvert 2a-c+d+2h-l\rvert^2$
$\Omega_c^0 \to \Xi^0 \eta^0$	$s_1^2 \frac{1}{6} \lvert -2a-3b-2c-2d-3e-2g+4h+4k+l\rvert^2$
$\Omega_c^0 \to \Xi^0 \pi^0$	$s_1^2 \frac{1}{2} \lvert b-e+2g+l\rvert^2$
$\Omega_c^0 \to \Xi^- \pi^+$	$s_1^2 \lvert b+e-2g-l\rvert^2$
$\Omega_c^0 \to \Sigma^+ K^-$	$s_1^2 \lvert c-d-2h+l\rvert^2$
$\Omega_c^0 \to \Lambda^0 \overline{K}^0$	$s_1^2 \frac{1}{6} \lvert c+3d+2e+4g-2h+4k-l\rvert^2$
$\Omega_c^0 \to \Sigma^0 \eta^0$	$s_1^4 \frac{1}{3} \lvert -2a+c-2d+l\rvert^2$
$\Omega_c^0 \to \Xi^0 K^0$	$s_1^4 \lvert -b-c-2f-e-2g\rvert^2$
$\Omega_c^0 \to \Xi^- K^+$	$s_1^4 \lvert b+c-2f+e-2g\rvert^2$
$\Omega_c^0 \to \Lambda^0 \pi^0$	$s_1^4 \frac{1}{3} \lvert c+2e-l\rvert^2$
$\Omega_c^0 \to \Sigma^- \pi^+$	$s_1^4 \lvert c-2f+l\rvert^2$
$\Omega_c^0 \to \Sigma^+ \pi^-$	$s_1^4 \lvert c+2f+l\rvert^2$
$\Omega_c^0 \to n \overline{K}^0$	$s_1^4 \lvert d-2f-2h\rvert^2$
$\Omega_c^0 \to p K^-$	$s_1^4 \lvert d+2f+2h\rvert^2$
$\Omega_c^0 \to \Lambda^0 \eta^0$	$s_1^4 \frac{4}{9} \lvert 3f+2g+2h+2k\rvert^2$
$\Omega_c^0 \to \Sigma^0 \pi^0$	$s_1^4 4 \lvert f\rvert^2$

Table 3.8. Squared matrix elements for the decays $S \to h^* M$ in terms of the reduced matrix elements $\alpha, \beta, \gamma, \delta, \lambda$ and η.

Process	Squared Matrix Element
$\Omega_c^0 \to \Xi^{*0}\overline{K}^0$	$\frac{1}{3}\|2\alpha+\beta+2\eta\|^2$
$\Omega_c^0 \to \Omega^-\pi^+$	$\|\beta-2\eta\|^2$
$\Omega_c^0 \to \Sigma^{*0}\overline{K}^0$	$s_1^2\frac{1}{6}\|2\alpha-\gamma-2\delta-2\lambda\|^2$
$\Omega_c^0 \to \Xi^{*0}\eta^0$	$s_1^2\frac{1}{18}\|4\alpha+3\beta+\gamma+4\delta+6\lambda+6\eta\|^2$
$\Omega_c^0 \to \Xi^{*0}\pi^0$	$s_1^2\frac{1}{6}\|-\beta-\gamma+2\lambda-2\eta\|^2$
$\Omega_c^0 \to \Xi^{*-}\pi^+$	$s_1^2\frac{1}{3}\|\beta-\gamma+2\lambda-2\eta\|^2$
$\Omega_c^0 \to \Omega^-K^+$	$s_1^2\|-\beta-\gamma+2\lambda+2\eta\|^2$
$\Omega_c^0 \to \Sigma^{*+}K^-$	$s_1^2\frac{1}{3}\|\gamma+2\delta+2\lambda\|^2$
$\Omega_c^0 \to \Sigma^{*0}\eta^0$	$s_1^4\frac{1}{9}\|-2\alpha+\gamma-2\delta\|^2$
$\Omega_c^0 \to \Xi^{*0}K^0$	$s_1^4\frac{1}{3}\|\beta+\gamma+2\lambda+2\eta\|^2$
$\Omega_c^0 \to \Xi^{*-}K^+$	$s_1^4\frac{1}{3}\|\beta+\gamma-2\lambda-2\eta\|^2$
$\Omega_c^0 \to \Sigma^{*+}\pi^-$	$s_1^4\frac{1}{3}\|\gamma+2\lambda\|^2$
$\Omega_c^0 \to \Sigma^{*-}\pi^+$	$s_1^4\frac{1}{3}\|\gamma-2\lambda\|^2$
$\Omega_c^0 \to \Delta^+K^-$	$s_1^4\frac{4}{3}\|\delta\|^2$
$\Omega_c^0 \to \Delta^0\overline{K}^0$	$s_1^4\frac{4}{3}\|\delta\|^2$
$\Omega_c^0 \to \Sigma^{*0}\pi^0$	$s_1^4\frac{4}{3}\|\lambda\|^2$

Table 3.9. Squared matrix elements for the Cabibbo-allowed decays $T \to h^* MM$ where the mesons are in a relatively even angular momentum state in terms of the reduced matrix elements $A_f, B_f, C_f, D_f, E_f, F_f, G_f, I_f, C_s, E_s, F_s$ and G_s.

Process	Squared Matrix Element
$\Lambda_c^+ \to \Sigma^{*+}\pi^0\pi^0$	$\frac{1}{3}\|2A_f + 2B_f - C_f + 4D_f - F_f - 2I_f + C_s + F_s\|^2$
$\Lambda_c^+ \to \Sigma^{*+}\pi^0\eta^0$	$\frac{1}{9}\|-2B_f + F_f - 2G_f - 2I_f + 2C_s + F_s\|^2$
$\Lambda_c^+ \to \Sigma^{*+}\pi^-\pi^+$	$\frac{1}{3}\|2A_f + 2B_f + 4D_f + E_f - 2I_f - E_s\|^2$
$\Lambda_c^+ \to \Sigma^{*0}\pi^0\pi^+$	$\frac{1}{3}\|-C_f - E_f - F_f + C_s + E_s + F_s\|^2$
$\Lambda_c^+ \to \Sigma^{*0}\pi^+\eta^0$	$\frac{1}{9}\|-2B_f + F_f - 2G_f - 2I_f + 2C_s + F_s\|^2$
$\Lambda_c^+ \to \Xi^{*0}K^+\pi^0$	$\frac{2}{3}\|A_f - C_f - \frac{1}{2}F_f - I_f + C_s + \frac{1}{2}F_s\|^2$
$\Lambda_c^+ \to \Xi^{*0}K^0\pi^+$	$\frac{1}{3}\|2A_f + E_f - 2I_f - E_s\|^2$
$\Lambda_c^+ \to \Delta^{++}K^-\pi^0$	$\frac{1}{2}\|-C_f - F_f + 2G_f - C_s - F_s\|^2$
$\Lambda_c^+ \to \Delta^+K^-\pi^+$	$\frac{1}{3}\|-C_f - F_f + 2G_f - C_s - F_s\|^2$
$\Lambda_c^+ \to \Delta^+\overline{K}^0\pi^0$	$\frac{1}{6}\|-C_f - 2E_f - F_f + 2G_f - C_s - 2E_s - F_s\|^2$
$\Lambda_c^+ \to \Delta^0\overline{K}^0\pi^+$	$\frac{1}{3}\|-C_f - E_f - F_f + 2G_f - C_s - E_s - F_s\|^2$
$\Lambda_c^+ \to \Sigma^{*-}\pi^+\pi^+$	$\frac{4}{3}\|-C_f - E_f - F_f + C_s + E_s + F_s\|^2$
$\Lambda_c^+ \to \Xi^{*-}\pi^+K^+$	$\frac{1}{3}\|-2C_f - E_f - F_f + 2C_s + E_s + F_s\|^2$
$\Lambda_c^+ \to \Delta^{++}\pi^-\overline{K}^0$	$\|E_f + E_s\|^2$
$\Xi_{c1}^+ \to \Sigma^{*+}\pi^0\overline{K}^0$	$\frac{1}{6}\|-2A_f - 2B_f - E_f + F_f - E_s - F_s\|^2$
$\Xi_{c1}^+ \to \Sigma^{*+}\eta^0\overline{K}^0$	$\frac{1}{18}\|-2A_f - 2B_f - 3E_f - F_f - 3E_s - 3F_s\|^2$
$\Xi_{c1}^+ \to \Sigma^{*+}K^-\pi^+$	$\frac{1}{3}\|2A_f + 2B_f + E_f - F_f - E_s - F_s\|^2$
$\Xi_{c1}^+ \to \Sigma^{*0}\pi^+\overline{K}^0$	$\frac{2}{3}\|E_s + F_s\|^2$
$\Xi_{c1}^+ \to \Xi^{*0}\eta^0\pi^+$	$\frac{1}{18}\|-4A_f - 4B_f - 3E_f + F_f + 3E_s + 3F_s\|^2$
$\Xi_{c1}^+ \to \Delta^{++}K^-\overline{K}^0$	$\|E_f + F_f + E_s + F_s\|^2$
$\Xi_{c1}^+ \to \Delta^+\overline{K}^0\overline{K}^0$	$\frac{4}{3}\|E_f + F_f + E_s + F_s\|^2$
$\Xi_{c1}^+ \to \Xi^{*0}\pi^0\pi^+$	$\frac{1}{6}\|-E_f - F_f + E_s + F_s\|^2$

Table 3.9. Continued

Process	Squared Matrix Element
$\Xi_{c1}^{+} \to \Xi^{*-}\pi^{+}\pi^{+}$	$\frac{4}{3}\lvert -E_f - F_f + E_s + F_s\rvert^2$
$\Xi_{c1}^{+} \to \Omega^{-}K^{+}\pi^{+}$	$\lvert -E_f - F_f + E_s + F_s\rvert^2$
$\Xi_{c1}^{0} \to \Sigma^{*+}\pi^{-}\overline{K}^{0}$	$\frac{1}{3}\lvert -2A_f - E_f + 2I_f - E_s\rvert^2$
$\Xi_{c1}^{0} \to \Sigma^{*+}K^{-}\pi^{0}$	$\frac{1}{6}\lvert -2A_f + C_f + F_f - 2G_f + 2I_f - C_s + F_s\rvert^2$
$\Xi_{c1}^{0} \to \Sigma^{*+}K^{-}\eta^{0}$	$\frac{1}{18}\lvert 2A_f - 3C_f - F_f - 2G_f - 2I_f - 5C_s - F_s\rvert^2$
$\Xi_{c1}^{0} \to \Sigma^{*0}\overline{K}^{0}\pi^{0}$	$\frac{1}{12}\lvert -2B_f + C_f + E_f + 2F_f - 2G_f - 2I_f - C_s + E_s\rvert^2$
$\Xi_{c1}^{0} \to \Sigma^{*0}\overline{K}^{0}\eta^{0}$	$\frac{1}{4}\lvert -\frac{2}{3}B_f - C_f - E_f - \frac{2}{3}F_f - \frac{2}{3}G_f$ $-\frac{2}{3}I_f - \frac{5}{3}C_s - E_s - \frac{4}{3}F_s\rvert^2$
$\Xi_{c1}^{0} \to \Xi^{*0}\eta^{0}\pi^{0}$	$\frac{1}{9}\lvert 2A_f + 2B_f - C_f - F_f + 2G_f + C_s - F_s\rvert^2$
$\Xi_{c1}^{0} \to \Delta^{++}K^{-}K^{-}$	$4\lvert C_f + C_s\rvert^2$
$\Xi_{c1}^{0} \to \Delta^{+}K^{-}\overline{K}^{0}$	$\frac{1}{3}\lvert 2C_f + E_f + F_f + 2C_s + E_s + F_s\rvert^2$
$\Xi_{c1}^{0} \to \Sigma^{*-}\pi^{+}\overline{K}^{0}$	$\frac{1}{3}\lvert C_f + E_f + F_f - 2G_f - C_s - E_s - F_s\rvert^2$
$\Xi_{c1}^{0} \to \Xi^{*-}\pi^{+}\eta^{0}$	$\frac{1}{18}\lvert -2C_f - 3E_f - F_f + 4G_f + 2C_s + 3E_s + F_s\rvert^2$
$\Xi_{c1}^{0} \to \Sigma^{*0}\pi^{+}K^{-}$	$\frac{1}{6}\lvert 2B_f + C_f + E_f - 2G_f + 2I_f - C_s - E_s\rvert^2$
$\Xi_{c1}^{0} \to \Delta^{0}\overline{K}^{0}\overline{K}^{0}$	$\frac{4}{3}\lvert C_f + E_f + F_f + C_s + E_s + F_s\rvert^2$
$\Xi_{c1}^{0} \to \Xi^{*0}\pi^{+}\pi^{-}$	$\frac{1}{3}\lvert -2B_f - 4D_f - E_f + E_s\rvert^2$
$\Xi_{c1}^{0} \to \Xi^{*-}\pi^{0}\pi^{+}$	$\frac{1}{6}\lvert E_f + F_f - E_s - F_s\rvert^2$
$\Xi_{c1}^{0} \to \Omega^{-}K^{0}\pi^{+}$	$\lvert -E_f + E_s\rvert^2$
$\Xi_{c1}^{0} \to \Xi^{*0}\pi^{0}\pi^{0}$	$\frac{1}{3}\lvert -2B_f - 4D_f + F_f - F_s\rvert^2$
$\Xi_{c1}^{0} \to \Omega^{-}K^{+}\pi^{0}$	$\frac{1}{2}\lvert F_f - F_s\rvert^2$

Table 3.10. Squared matrix elements for the Cabibbo-allowed decays $T \to h^* MM$ where the two mesons are in a relatively odd angular momentum state in terms of reduced matrix elements $A'_f, B'_f, C'_f, D'_f, E'_f, F'_f, G'_f, I'_f, C'_s, E'_s, F'_s$ and G'_s.

Process	Squared Matrix Element
$\Lambda_c^+ \to \Sigma^{*+}\pi^0\eta^0$	$\frac{1}{9}\|2A'_f + C'_f + C'_s + 2F'_s + 2G'_s\|^2$
$\Lambda_c^+ \to \Sigma^{*+}\pi^-\pi^+$	$\frac{1}{3}\|-2A'_f - 2B'_f - E'_f + 2I'_f + E'_s\|^2$
$\Lambda_c^+ \to \Sigma^{*0}\pi^0\pi^+$	$\frac{1}{3}\|2A'_f + 2B'_f + E'_f - 2I'_f - E'_s\|^2$
$\Lambda_c^+ \to \Sigma^{*0}\pi^+\eta^0$	$\frac{1}{9}\|2A'_f + C'_f + C'_s + 2F'_s + 2G'_s\|^2$
$\Lambda_c^+ \to \Xi^{*0}K^+\pi^0$	$\frac{1}{6}\|2A'_f - F'_f - 2I'_f + F'_s\|^2$
$\Lambda_c^+ \to \Xi^{*0}K^0\pi^+$	$\frac{1}{3}\|2A'_f + E'_f - 2I'_f - E'_s\|^2$
$\Lambda_c^+ \to \Delta^{++}K^-\pi^0$	$\frac{1}{2}\|C'_f + F'_f + C'_s + F'_s + 2G'_s\|^2$
$\Lambda_c^+ \to \Delta^+K^-\pi^+$	$\frac{1}{3}\|C'_f + F'_f + C'_s + F'_s + 2G'_s\|^2$
$\Lambda_c^+ \to \Delta^+\overline{K}^0\pi^0$	$\frac{1}{6}\|-C'_f + 2E'_f - F'_f - C'_s + 2E'_s - F'_s - 2G'_s\|^2$
$\Lambda_c^+ \to \Delta^0\overline{K}^0\pi^+$	$\frac{1}{3}\|-C'_f + E'_f - F'_f - C'_s + E'_s - F'_s - 2G'_s\|^2$
$\Lambda_c^+ \to \Delta^{++}\pi^-\overline{K}^0$	$\|E'_f + E'_s\|^2$
$\Lambda_c^+ \to \Xi^{*-}K^+\pi^+$	$\frac{1}{3}\|-E'_f - F'_f + E'_s + F'_s\|^2$
$\Xi_{c1}^+ \to \Sigma^{*+}\pi^0\overline{K}^0$	$\frac{1}{6}\|-2A'_f + 2B'_f - E'_f + F'_f - E'_s - F'_s\|^2$
$\Xi_{c1}^+ \to \Sigma^{*+}\eta^0\overline{K}^0$	$\frac{1}{18}\|2A'_f + 6B'_f + 3E'_f + F'_f + 3E'_s + 3F'_s\|^2$
$\Xi_{c1}^+ \to \Sigma^{*+}K^-\pi^+$	$\frac{1}{3}\|2A'_f + 2B'_f + E'_f - F'_f - E'_s - F'_s\|^2$
$\Xi_{c1}^+ \to \Sigma^{*0}\pi^+\overline{K}^0$	$\frac{2}{3}\|-2A'_f - E'_f + F'_f\|^2$
$\Xi_{c1}^+ \to \Xi^{*0}\eta^0\pi^+$	$\frac{1}{18}\|-4A'_f - 3E'_f + F'_f + 3E'_s + 3F'_s\|^2$
$\Xi_{c1}^+ \to \Delta^{++}K^-\overline{K}^0$	$\|E'_f + F'_f + E'_s + F'_s\|^2$
$\Xi_{c1}^+ \to \Xi^{*0}\pi^0\pi^+$	$\frac{1}{6}\|-4B'_f - E'_f - F'_f + E'_s + F'_s\|^2$
$\Xi_{c1}^+ \to \Omega^-K^+\pi^+$	$\|-E'_f - F'_f + E'_s + F'_s\|^2$
$\Xi_{c1}^0 \to \Sigma^{*+}\pi^-\overline{K}^0$	$\frac{1}{3}\|-2A'_f - E'_f + 2I'_f - E'_s\|^2$
$\Xi_{c1}^0 \to \Sigma^{*+}K^-\pi^0$	$\frac{1}{6}\|-2A'_f - C'_f + F'_f - 2I'_f + C'_s + F'_s + 2G'_s\|^2$

Table 3.10. Continued.

Process	Squared Matrix Element
$\Xi_{c1}^0 \to \Sigma^{*+}K^-\eta^0$	$\frac{1}{18}\|2A'_f - C'_f - F'_f - 6I'_f + C'_s - F'_s + 2G'_s\|^2$
$\Xi_{c1}^0 \to \Sigma^{*0}\overline{K}^0\pi^0$	$\frac{1}{12}\|4A'_f + 2B'_f + C'_f + E'_f - 2I'_f$ $-C'_s + E'_s - 2F'_s - 2G'_s\|^2$
$\Xi_{c1}^0 \to \Sigma^{*0}\eta^0\overline{K}^0$	$\frac{1}{36}\|4A'_f + 6B'_f - C'_f + 3E'_f - 6I'_f$ $+C'_s - 3E'_s + 2F'_s + 2G'_s\|^2$
$\Xi_{c1}^0 \to \Xi^{*0}\eta^0\pi^0$	$\frac{1}{9}\|2A'_f + C'_f - C'_s - 2F'_s - 2G'_s\|^2$
$\Xi_{c1}^0 \to \Sigma^{*-}\pi^+\overline{K}^0$	$\frac{1}{3}\|C'_f - E'_f + F'_f - C'_s + E'_s - F'_s - 2G'_s\|^2$
$\Xi_{c1}^0 \to \Xi^{*-}\eta^0\pi^+$	$\frac{1}{18}\| - 2C'_f + 3E'_f - F'_f + 2C'_s - 3E'_s + F'_s + 4G'_s\|^2$
$\Xi_{c1}^0 \to \Delta^+K^-\overline{K}^0$	$\frac{1}{3}\|E'_f + F'_f + E'_s + F'_s\|^2$
$\Xi_{c1}^0 \to \Sigma^{*0}K^-\pi^+$	$\frac{1}{6}\| - 2B'_f + C'_f - E'_f + 2I'_f - C'_s + E'_s - 2G'_s\|^2$
$\Xi_{c1}^0 \to \Xi^{*0}\pi^-\pi^+$	$\frac{1}{3}\| - 2B'_f - E'_f + E'_s\|^2$
$\Xi_{c1}^0 \to \Xi^{*-}\pi^0\pi^+$	$\frac{1}{6}\|E'_f - F'_f - E'_s + F'_s\|^2$
$\Xi_{c1}^0 \to \Omega^-K^0\pi^+$	$\| - E'_f + E'_s\|^2$
$\Xi_{c1}^0 \to \Omega^-K^+\pi^0$	$\frac{1}{2}\|F'_f - F'_s\|^2$

Table 3.11. Squared matrix elements for the Cabibbo-suppressed decays $T \to h^* MM$ where the mesons are in a relatively even angular momentum state in terms of reduced matrix elements $A_f, B_f, C_f, D_f, E_f, F_F, G_f, I_f, C_s, E_s, F_s$ and G_s.

Process	Squared Matrix Element (Modulo s_1^2)
$\Lambda_c^+ \to \Delta^+ \pi^0 \pi^0$	$\frac{4}{3}\|A_f + B_f + 2D_f + E_f - G_f - I_f + C_s + E_s + F_s\|^2$
$\Lambda_c^+ \to \Delta^+ \pi^0 \eta^0$	$\frac{1}{9}\|2B_f + C_f + 3E_f + F_f + 2I_f - C_s + 3E_s + F_s\|^2$
$\Lambda_c^+ \to \Delta^+ \pi^+ \pi^-$	$\frac{1}{3}\|2A_f + 2B_f - C_f + 4D_f + E_f - F_f$ $+2G_f - 2I_f - C_s - E_s - F_s\|^2$
$\Lambda_c^+ \to \Delta^+ K^+ K^-$	$\frac{1}{3}\|4D_f + F_f - 2I_f + F_s\|$
$\Lambda_c^+ \to \Delta^+ \overline{K}^0 K^0$	$\frac{1}{3}\|2B_f + 4D_f - F_f - F_s\|$
$\Lambda_c^+ \to \Delta^0 \pi^0 \pi^+$	$\frac{1}{6}\|-C_f - E_f - F_f - 2G_f + 3C_s + 3E_s + 3F_s\|$
$\Lambda_c^+ \to \Delta^0 \pi^+ \eta^0$	$\frac{1}{18}\|-4B_f - 3C_f - 3E_f - F_f + 2G_f$ $-4I_f + C_s - 3E_s - F_s\|$
$\Lambda_c^+ \to \Delta^0 \overline{K}^0 K^+$	$\frac{1}{3}\|-2B_f + F_f - 2I_f + F_s\|$
$\Lambda_c^+ \to \Sigma^{*0} K^+ \pi^0$	$\frac{1}{12}\|4A_f + 2B_f - C_f + 3E_f - 2G_f - 2I_f + 3C_s - E_s\|^2$
$\Lambda_c^+ \to \Sigma^{*0} K^0 \pi^+$	$\frac{1}{6}\|4A_f + 2B_f - C_f + E_f - 2F_f + 2G_f - 2I_f - C_s - E_s\|^2$
$\Lambda_c^+ \to \Sigma^{*+} K^+ \pi^-$	$\frac{1}{3}\|-2A_f - 2B_f - C_f - E_f + 2G_f - C_s + E_s\|^2$
$\Lambda_c^+ \to \Sigma^{*+} K^0 \pi^0$	$\frac{1}{6}\|2A_f + 2B_f - C_f - E_f - 2F_f + 2G_f - C_s - E_s\|^2$
$\Lambda_c^+ \to \Delta^{++} \pi^- \pi^0$	$\frac{1}{2}\|-C_f - E_f - F_f + 2G_f - C_s - E_s - F_s\|^2$
$\Lambda_c^+ \to \Delta^{++} K^- K^0$	$\|F_f + F_s\|^2$
$\Lambda_c^+ \to \Delta^{++} \eta^0 \pi^-$	$\frac{1}{6}\|-C_f + 3E_f + F_f + 2G_f - C_s + 3E_s + F_s\|^2$
$\Lambda_c^+ \to \Delta^- \pi^+ \pi^+$	$4\|-C_f - E_f - F_f + C_s + E_s + F_s\|^2$
$\Lambda_c \to \Sigma^{*-} K^+ \pi^+$	$\frac{4}{3}\|C_f - C_s\|^2$
$\Xi_{c1}^+ \to \Delta^+ \overline{K}^0 \pi^0$	$\frac{1}{6}\|-2A_f - 2B_f + C_f - E_f - 2G_f + C_s - E_s - 2F_s\|^2$
$\Xi_{c1}^+ \to \Delta^+ \overline{K}^0 \eta^0$	$\frac{1}{18}\|-2A_f - 2B_f + C_f + 3E_f + 4F_f$ $-2G_f + C_s + 3E_s + 2F_s\|^2$

Table 3.11. Continued

Process	Squared Matrix Element (Modulo s_1^2)
$\Xi_{c1}^+ \to \Delta^+ K^- \pi^+$	$\frac{1}{3}\|2A_f + 2B_f + C_f + E_f - 2G_f + C_s - E_s\|^2$
$\Xi_{c1}^+ \to \Delta^0 \overline{K}^0 \pi^+$	$\frac{1}{3}\|C_f + E_f + F_f - 2G_f + C_s - E_s - F_s\|^2$
$\Xi_{c1}^+ \to \Sigma^{*0} \overline{K}^0 K^+$	$\frac{1}{6}\| - 4A_f - 2B_f + C_f - E_f + 2F_f$ $-2G_f + 2I_f + C_s + E_s\|^2$
$\Xi_{c1}^+ \to \Sigma^{*0} \eta^0 \pi^+$	$\frac{1}{9}\| - 4A_f - 2B_f - 3E_f + 2G_f + 2I_f - 2C_s - F_s\|^2$
$\Xi_{c1}^+ \to \Sigma^{*+} K^- K^+$	$\frac{1}{3}\| - 2A_f - 2B_f + C_f - 4D_f - E_f + F_f$ $-2G_f + 2I_f + C_s + E_s + F_s\|^2$
$\Xi_{c1}^+ \to \Sigma^{*+} \eta^0 \pi^0$	$\frac{1}{9}\| - 2A_f + F_f + 2G_f + 2I_f - 2C_s - F_s\|^2$
$\Xi_{c1}^+ \to \Delta^{++} K^- \pi^0$	$\frac{1}{2}\|C_f - E_f - 2G_f + C_s - E_s\|^2$
$\Xi_{c1}^+ \to \Delta^{++} K^- \eta^0$	$\frac{1}{6}\|C_f + 3E_f + 2F_f - 2G_f + C_s + 3E_s + 2F_s\|^2$
$\Xi_{c1}^+ \to \Sigma^{*+} \pi^0 \pi^0$	$\frac{1}{3}\|C_f - 4D_f + E_f + 2I_f - C_s + E_s\|^2$
$\Xi_{c1}^+ \to \Sigma^{*0} \pi^0 \pi^+$	$\frac{1}{3}\|C_f - C_s + E_s + F_s\|^2$
$\Xi_{c1}^+ \to \Xi^{*0} K^+ \pi^0$	$\frac{1}{6}\|2B_f + 2C_f + 2E_f + F_f + 2I_f - 2C_s - F_s\|^2$
$\Xi_{c1}^+ \to \Sigma^{*-} \pi^+ \pi^+$	$\frac{4}{3}\|C_f - E_f - F_f - C_s + E_s + F_s\|^2$
$\Xi_{c1}^+ \to \Xi^{*-} K^+ \pi^+$	$\frac{4}{3}\|C_f - C_s\|^2$
$\Xi_{c1}^+ \to \Delta^{++} \overline{K}^0 \pi^-$	$\|F_f + F_s\|^2$
$\Xi_{c1}^+ \to \Sigma^{*+} \overline{K}^0 K^0$	$\frac{1}{3}\| - 2B_f - 4D_f + F_f + F_s\|^2$
$\Xi_{c1}^+ \to \Sigma^{*+} \pi^- \pi^+$	$\frac{1}{3}\| - 4D_f - F_f - F_s + 2I_f\|^2$
$\Xi_{c1}^+ \to \Xi^{*0} K^0 \pi^+$	$\frac{1}{3}\|2B_f - F_f + 2I_f - F_s\|^2$
$\Xi_{c1}^0 \to \Delta^+ \overline{K}^0 \pi^-$	$\frac{1}{3}\| - 2A_f + 2C_f + F_f + 2I_f + 2C_s + F_s\|^2$
$\Xi_{c1}^0 \to \Delta^+ K^- \pi^0$	$\frac{1}{6}\| - 2A_f - C_f - E_f - 2G_f + 2I_f - 3C_s - E_s\|^2$
$\Xi_{c1}^0 \to \Delta^+ K^- \eta^0$	$\frac{1}{18}\|2A_f + 3C_f + 3E_f + 2F_f - 2G_f$ $-2I_f + C_s + 3E_s + 2F_s\|^2$
$\Xi_{c1}^0 \to \Delta^0 \overline{K}^0 \pi^0$	$\frac{1}{6}\| - 2B_f - C_f - E_f - 2G_f - 2I_f - 3C_s - E_s - 2F_s\|^2$

Table 3.11. Continued

Process	Squared Matrix Element (Modulo s_1^2)
$\Xi_{c1}^0 \to \Delta^0 \overline{K}^0 \eta^0$	$\frac{1}{18}\|-2B_f+3C_f+3E_f+4F_f-2G_f$ $-2I_f+C_s+3E_s+2F_s\|^2$
$\Xi_{c1}^0 \to \Sigma^{*0} \overline{K}^0 K^0$	$\frac{2}{3}\|-2A_f-2B_f+C_f-4D_f+F_f+2I_f+C_s+F_s\|^2$
$\Xi_{c1}^0 \to \Sigma^{*0}\pi^0\eta^0$	$\frac{1}{18}\|4A_f+2B_f+C_f+3E_f+2F_f$ $2G_f-2I_f+3C_s+3E_s\|^2$
$\Xi_{c1}^0 \to \Sigma^{*+}K^-K^0$	$\frac{1}{3}\|-2A_f+2C_f+F_f+2I_f+2C_s+F_s\|^2$
$\Xi_{c1}^0 \to \Sigma^{*+}\pi^-\eta^0$	$\frac{1}{18}\|-4A_f-3C_f-3E_f-F_f-2G_f$ $+4I_f-5C_s-3E_s-F_s\|^2$
$\Xi_{c1}^0 \to \Delta^{++}K^-\pi^-$	$4\|C_f+C_s\|^2$
$\Xi_{c1}^0 \to \Sigma^{*+}\pi^-\pi^0$	$\frac{1}{6}\|C_f+E_f+F_f-2G_f-C_s+E_s+F_s\|^2$
$\Xi_{c1}^0 \to \Delta^0 K^-\pi^+$	$\frac{1}{3}\|2B_f+C_f+E_f-2G_f+2I_f-C_s-E_s\|^2$
$\Xi_{c1}^0 \to \Sigma^{*0}K^-K^+$	$\frac{1}{6}\|-2B_f+C_f-8D_f-E_f-2G_f+2I_f-C_s+E_s\|^2$
$\Xi_{c1}^0 \to \Sigma^{*0}\pi^0\pi^0$	$\frac{1}{6}\|-2B_f-C_f-8D_f-E_f+2G_f$ $+2I_f+C_s-E_s-2F_s\|^2$
$\Xi_{c1}^0 \to \Sigma^{*0}\pi^-\pi^+$	$\frac{1}{6}\|-2B_f+C_f-8D_f-E_f-2G_f+2I_f-C_s+E_s\|^2$
$\Xi_{c1}^0 \to \Delta^- \overline{K}^0 \pi^+$	$\|C_f+E_f+F_f-2G_f-C_s-E_s-F_s\|^2$
$\Xi_{c1}^0 \to \Sigma^{*-} \overline{K}^0 K^+$	$\frac{1}{3}\|C_f-E_f+F_f-2G_f-C_s+E_s-F_s\|^2$
$\Xi_{c1}^0 \to \Sigma^{*-}\pi^+\eta^0$	$\frac{1}{18}\|-C_f-3E_f+F_f+2G_f+C_s+3E_s-F_s\|^2$
$\Xi_{c1}^0 \to \Sigma^{*-}\pi^0\pi^+$	$\frac{1}{6}\|-C_f+E_f+F_f+2G_f+C_s-E_s-F_s\|^2$
$\Xi_{c1}^0 \to \Xi^{*0}K^+\pi^-$	$\frac{1}{3}\|2B_f+C_f+E_f-2G_f+2I_f-C_s-E_s\|^2$
$\Xi_{c1}^0 \to \Xi^{*0}K^0\pi^0$	$\frac{1}{6}\|-2B_f+C_f+E_f+2F_f-2G_f-2I_f-C_s+E_s\|^2$
$\Xi_{c1}^0 \to \Xi^{*-}K^+\pi^0$	$\frac{1}{6}\|-C_f-E_f+F_f+2G_f+C_s+E_s-F_s\|^2$
$\Xi_{c1}^0 \to \Xi^{*-}K^0\pi^+$	$\frac{1}{3}\|C_f-E_f+F_f-2G_f-C_s+E_s-F_s\|^2$

Table 3.12. Squared matrix elements for the Cabibbo-suppressed decays $T \to h^* MM$ where the mesons are in a relatively odd angular momentum state in terms of reduced matrix elements $A'_f, B'_f, C'_f, D'_f, E'_f, F'_f, G'_f, I'_f, C'_s, E'_s, F'_s$ and G'_s.

Process	Squared Matrix Element (Modulo s_1^2)
$\Lambda_c^+ \to \Delta^+\pi^0\eta^0$	$\frac{1}{9}\lvert 2A'_f - C'_f + 3E'_f - F'_f - C'_s + 3E'_s + F'_s - 2G'_s\rvert^2$
$\Lambda_c^+ \to \Delta^+\pi^+\pi^-$	$\frac{1}{3}\lvert 2A'_f + 2B'_f - C'_f + E'_f - F'_f - 2I'_f\rvert$ $-C'_s - E'_s - F'_s - 2G'_s\rvert^2$
$\Lambda_c^+ \to \Delta^+K^+K^-$	$\frac{1}{3}\lvert F'_f - 2I'_f + F'_s\rvert$
$\Lambda_c^+ \to \Delta^+\overline{K}^0K^0$	$\frac{1}{3}\lvert 2B'_f + F'_f + F'_s\rvert$
$\Lambda_c^+ \to \Delta^0\pi^0\pi^+$	$\frac{1}{6}\lvert 4A'_f + 4B'_f - C'_f + 3E'_f - F'_f - 4I'_f$ $-C'_s - E'_s - F'_s - 2G'_s\rvert$
$\Lambda_c^+ \to \Delta^0\pi^+\eta^0$	$\frac{1}{18}\lvert 4A'_f - C'_f + 3E'_f - 3F'_f - C'_s + 3E'_s + F'_s - 2G'_s$
$\Lambda_c^+ \to \Delta^0\overline{K}^0K^+$	$\frac{1}{3}\lvert -2B'_f - F'_f + 2I'_f - F'_s\rvert$
$\Lambda_c^+ \to \Sigma^{*0}K^+\pi^0$	$\frac{1}{12}\lvert 2B'_f + C'_f + E'_f + 2F'_f + 2I'_f$ $+C'_s - 3E'_s - 2F'_s + 2G'_s\rvert^2$
$\Lambda_c^+ \to \Sigma^{*0}K^0\pi^+$	$\frac{1}{6}\lvert 2B'_f + C'_f - E'_f + 2I'_f + C'_s + E'_s + 2F'_s + 2G'_s\rvert^2$
$\Lambda_c^+ \to \Sigma^{*+}K^+\pi^-$	$\frac{1}{3}\lvert 2A'_f + 2B'_f + C'_f + E'_f + C'_s - E'_s + 2G'_s\rvert^2$
$\Lambda_c^+ \to \Sigma^{*+}K^0\pi^0$	$\frac{1}{6}\lvert 2A'_f + 2B'_f + C'_f + E'_f + C'_s + E'_s + 2F'_s + 2G'_s\rvert^2$
$\Lambda_c^+ \to \Delta^{++}\pi^-\pi^0$	$\frac{1}{2}\lvert C'_f + E'_f + F'_f + C'_s + E'_s + F'_s + 2G'_s\rvert^2$
$\Lambda_c^+ \to \Delta^{++}K^-K^0$	$\lvert F'_f + F'_s\rvert^2$
$\Lambda_c^+ \to \Delta^{++}\eta^0\pi^-$	$\frac{1}{6}\lvert C'_f - 3E'_f - F'_f + C'_s - 3E'_s - F'_s + 2G'_s\rvert^2$
$\Lambda_c^+ \to \Sigma^{*-}K^+\pi^+$	$\frac{4}{3}\lvert E'_f + F'_f - E'_s - F'_s\rvert^2$
$\Xi_{c1}^+ \to \Delta^+\overline{K}^0\pi^0$	$\frac{1}{6}\lvert 2A'_f - 2B'_f + C'_f - E'_f + C'_s - E'_s + 2F'_s + 2G'_s\rvert^2$
$\Xi_{c1}^+ \to \Delta^+\overline{K}^0\eta^0$	$\frac{1}{18}\lvert -2A'_f - 6B'_f - C'_f - 3E'_f$ $-C'_s - 3E'_s - 2F'_s - 2G'_s\rvert^2$

Table 3.12. Continued

Process	Squared Matrix Element (Modulo s_1^2)
$\Xi_{c1}^{+} \to \Delta^{+}K^{-}\pi^{+}$	$\frac{1}{3}\lvert -2A'_f - 2B'_f - C'_f - E'_f - C'_s + E'_s - 2G'_s\rvert^2$
$\Xi_{c1}^{+} \to \Delta^{0}\overline{K}^{0}\pi^{+}$	$\frac{1}{3}\lvert 4A'_f + C'_f + E'_f - F'_f + C'_s - E'_s + F'_s + 2G'_s\rvert^2$
$\Xi_{c1}^{+} \to \Sigma^{*0}\overline{K}^{0}K^{+}$	$\frac{1}{6}\lvert 2B'_f + C'_f - E'_f + 2I'_f + C'_s + E'_s + 2F'_s + 2G'_s\rvert$
$\Xi_{c1}^{+} \to \Sigma^{*0}\eta^{0}\pi^{+}$	$\frac{1}{9}\lvert -C'_f - 2F'_f - C'_s + 3E'_s + F'_s - 2G'_s\rvert^2$
$\Xi_{c1}^{+} \to \Sigma^{*+}K^{-}K^{+}$	$\frac{1}{3}\lvert 2A'_f + 2B'_f - C'_f + E'_f - F'_f$ $-2I'_f - C'_s - E'_s - F'_s - 2G'_s\rvert^2$
$\Xi_{c1}^{+} \to \Sigma^{*+}\eta^{0}\pi^{0}$	$\frac{1}{9}\lvert 2A'_f - C'_f + 3E'_f - F'_f - C'_s + 3E'_s + F'_s - 2G'_s\rvert^2$
	$-6I'_f - 2C'_s - F'_s - 4G'_s\rvert^2$
$\Xi_{c1}^{+} \to \Delta^{++}K^{-}\pi^{0}$	$\frac{1}{2}\lvert -C'_f + E'_f - C'_s + E'_s - 2G'_s\rvert^2$
$\Xi_{c1}^{+} \to \Delta^{++}K^{-}\eta^{0}$	$\frac{1}{6}\lvert -C'_f - 3E'_f - 2F'_f - C'_s - 3E'_s - 2F'_s - 2G'_s\rvert^2$
$\Xi_{c1}^{+} \to \Sigma^{*0}\pi^{0}\pi^{+}$	$\frac{1}{12}\lvert -4B'_f - 2E'_f - 4I'_f + 2F'_s\rvert^2$
$\Xi_{c1}^{+} \to \Xi^{*0}K^{+}\pi^{0}$	$\frac{1}{6}\lvert 2B'_f + F'_f - 2I'_f - 2E'_s - F'_s\rvert^2$
$\Xi_{c1}^{+} \to \Xi^{*-}K^{+}\pi^{+}$	$\frac{4}{3}\lvert E'_f + F'_f - E'_s - F'_s\rvert^2$
$\Xi_{c1}^{+} \to \Delta^{++}\overline{K}^{0}\pi^{-}$	$\lvert F'_f + F'_s\rvert^2$
$\Xi_{c1}^{+} \to \Sigma^{*+}\overline{K}^{0}K^{0}$	$\frac{1}{3}\lvert 2B'_f + F'_f + F'_s\rvert$
$\Xi_{c1}^{+} \to \Sigma^{*+}\pi^{-}\pi^{+}$	$\frac{1}{3}\lvert F'_f - 2I'_f + F'_s\rvert^2$
$\Xi_{c1}^{+} \to \Xi^{*0}K^{0}\pi^{+}$	$\frac{1}{3}\lvert 2B'_f + F'_f - 2I'_f + F'_s\rvert^2$
$\Xi_{c1}^{0} \to \Delta^{+}\overline{K}^{0}\pi^{-}$	$\frac{1}{3}\lvert 2A'_f - F'_f - 2I'_f - F'_s\rvert^2$
$\Xi_{c1}^{0} \to \Delta^{+}K^{-}\pi^{0}$	$\frac{1}{6}\lvert 2A'_f + C'_f + E'_f + 2I'_f - C'_s + E'_s - 2G'_s\rvert^2$
$\Xi_{c1}^{0} \to \Delta^{+}K^{-}\eta^{0}$	$\frac{1}{18}\lvert -2A'_f + C'_f - 3E'_f - 2F'_f$ $+6I'_f - C'_s - 3E'_s - 2F'_s - 2G'_s\rvert^2$
$\Xi_{c1}^{0} \to \Delta^{0}\overline{K}^{0}\pi^{0}$	$\frac{1}{6}\lvert 4A'_f + 2B'_f + C'_f + E'_f - 2I'_f - C'_s + E'_s - 2F'_s - 2G'_s\rvert^2$

Table 3.12. Continued

Process	Squared Matrix Element (Modulo s_1^2)
$\Xi_{c1}^0 \to \Delta^0 \overline{K}^0 \eta^0$	$\frac{1}{18}\|-4A'_f - 6B'_f + C'_f - 3E'_f + 6I'_f$ $-C'_s - 3E'_s - 2F'_s - 2G'_s\|^2$
$\Xi_{c1}^0 \to \Sigma^{*+} K^- K^0$	$\frac{1}{3}\|2A'_f - F'_f - 2I'_f - F'_s\|^2$
$\Xi_{c1}^0 \to \Sigma^{*+} \pi^- \eta^0$	$\frac{1}{18}\|4A'_f + C'_f + 3E'_f + F'_f$ $-C'_s + 3E'_s + F'_s - 2G'_s\|^2$
$\Xi_{c1}^0 \to \Sigma^{*+} \pi^- \pi^0$	$\frac{1}{6}\|-C'_f + E'_f + F'_f - 4I'_f + C'_s + E'_s + F'_s + 2G'_s\|^2$
$\Xi_{c1}^0 \to \Delta^0 K^- \pi^+$	$\frac{1}{3}\|2B'_f - C'_f + E'_f - 2I'_f + C'_s - E'_s + 2G'_s\|^2$
$\Xi_{c1}^0 \to \Sigma^{*0} K^- K^+$	$\frac{1}{6}\|-2B'_f - C'_f - E'_f - 2I'_f + C'_s + E'_s + 2G'_s\|^2$
$\Xi_{c1}^0 \to \Sigma^{*0} \pi^- \pi^+$	$\frac{1}{6}\|-2B'_f - C'_f - E'_f - 2I'_f + C'_s + E'_s + 2G'_s\|^2$
$\Xi_{c1}^0 \to \Delta^- \overline{K}^0 \pi^+$	$\|-C'_f + E'_f - F'_f + C'_s - E'_s + F'_s + 2G'_s\|^2$
$\Xi_{c1}^0 \to \Sigma^{*-} \overline{K}^0 K^+$	$\frac{1}{3}\|-C'_f - E'_f - F'_f + C'_s + E'_s + F'_s + 2G'_s\|^2$
$\Xi_{c1}^0 \to \Sigma^{*-} \pi^+ \eta^0$	$\frac{1}{18}\|C'_f - 3E'_f - F'_f - C'_s + 3E'_s + F'_s - 2G'_s\|^2$
$\Xi_{c1}^0 \to \Sigma^{*-} \pi^0 \pi^+$	$\frac{1}{6}\|C'_f + E'_f - F'_f - C'_s - E'_s + F'_s - 2G'_s\|^2$
$\Xi_{c1}^0 \to \Xi^{*0} K^+ \pi^-$	$\frac{1}{3}\|2B'_f - C'_f + E'_f - 2I'_f + C'_s - E'_s + 2G'_s\|^2$
$\Xi_{c1}^0 \to \Xi^{*0} K^0 \pi^0$	$\frac{1}{6}\|-2B'_f + C'_f - E'_f + 2I'_f - C'_s - E'_s - 2F'_s - 2G'_s\|^2$
$\Xi_{c1}^0 \to \Xi^{*-} K^+ \pi^0$	$\frac{1}{6}\|C'_f - E'_f - F'_f - C'_s + E'_s + F'_s - 2G'_s\|^2$
$\Xi_{c1}^0 \to \Xi^{*-} K^0 \pi^+$	$\frac{1}{3}\|-C'_f - E'_f - F'_f + C'_s + E'_s + F'_s + 2G'_s\|^2$

Table 3.13. Squared matrix elements for the Cabibbo-allowed decays $T \to BMl^+\nu_l$ in terms of the reduced matix elements a, b and c.

Process	Squared Matrix Element
$\Lambda_c^+ \to \Lambda^0\eta^0 l^+\nu_l$	$\frac{1}{9}\|3a+2b+2c\|^2$
$\Lambda_c^+ \to \Sigma^0\pi^0 l^+\nu_l$	$\|a\|^2$
$\Lambda_c^+ \to \Sigma^+\pi^- l^+\nu_l$	$\|a\|^2$
$\Lambda_c^+ \to \Sigma^-\pi^+ l^+\nu_l$	$\|a\|^2$
$\Lambda_c^+ \to pK^- l^+\nu_l$	$\|a+b\|^2$
$\Lambda_c^+ \to n\overline{K}^0 l^+\nu_l$	$\|a+b\|^2$
$\Lambda_c^+ \to \Xi^- K^+ l^+\nu_l$	$\|a+c\|^2$
$\Lambda_c^+ \to \Xi^0 K^0 l^+\nu_l$	$\|a+c\|^2$
$\Xi_{c1}^+ \to \Sigma^+ K^- l^+\nu_l$	$\|b\|^2$
$\Xi_{c1}^+ \to \Lambda^0\overline{K}^0 l^+\nu_l$	$\frac{1}{6}\|b-2c\|^2$
$\Xi_{c1}^+ \to \Sigma^0\overline{K}^0 l^+\nu_l$	$\frac{1}{2}\|b\|^2$
$\Xi_{c1}^+ \to \Xi^0\eta^0 l^+\nu_l$	$\frac{1}{6}\|2b-c\|^2$
$\Xi_{c1}^+ \to \Xi^-\pi^+ l^+\nu_l$	$\|c\|^2$
$\Xi_{c1}^+ \to \Xi^0\pi^0 l^+\nu_l$	$\frac{1}{2}\|c\|^2$
$\Xi_{c1}^0 \to \Lambda^0 K^- l^+\nu_l$	$\frac{1}{6}\|b-2c\|^2$
$\Xi_{c1}^0 \to \Sigma^0 K^- l^+\nu_l$	$\frac{1}{2}\|b\|^2$
$\Xi_{c1}^0 \to \Sigma^-\overline{K}^0 l^+\nu_l$	$\|b\|^2$
$\Xi_{c1}^0 \to \Xi^-\eta^0 l^+\nu_l$	$\frac{1}{6}\|2b-c\|^2$
$\Xi_{c1}^0 \to \Xi^-\pi^0 l^+\nu_l$	$\frac{1}{2}\|c\|^2$
$\Xi_{c1}^0 \to \Xi^0\pi^- l^+\nu_l$	$\|c\|^2$

Table 3.14. Squared matrix elements for the Cabibbo-suppressed decays $T \to BMl^+\nu_l$ in terms of the reduced matrix elements a, b and c.

Process	Squared Matrix Element (Modulo s_1^2)
$\Lambda_c^+ \to p\pi^- l^+\nu_l$	$\|b\|^2$
$\Lambda_c^+ \to n\eta^0 l^+\nu_l$	$\frac{1}{6}\|b-2c\|^2$
$\Lambda_c^+ \to n\pi^0 l^+\nu_l$	$\frac{1}{2}\|b\|^2$
$\Lambda_c^+ \to \Lambda^0 K^0 l^+\nu_l$	$\frac{1}{6}\|2b-c\|^2$
$\Lambda_c^+ \to \Sigma^- K^+ l^+\nu_l$	$\|c\|^2$
$\Lambda_c^+ \to \Sigma^0 K^0 l^+\nu_l$	$\frac{1}{2}\|c\|^2$
$\Xi_{c1}^+ \to \Lambda^0\eta^0 l^+\nu_l$	$\frac{1}{36}\|6a+b+c\|^2$
$\Xi_{c1}^+ \to \Sigma^0\pi^0 l^+\nu_l$	$\frac{1}{4}\|2a+b+c\|^2$
$\Xi_{c1}^+ \to \Sigma^+\pi^- l^+\nu_l$	$\|a+b\|^2$
$\Xi_{c1}^+ \to \Sigma^-\pi^+ l^+\nu_l$	$\|a+c\|^2$
$\Xi_{c1}^+ \to pK^- l^+\nu_l$	$\|a\|^2$
$\Xi_{c1}^+ \to n\overline{K}^0 l^+\nu_l$	$\|a+c\|^2$
$\Xi_{c1}^+ \to \Xi^- K^+ l^+\nu_l$	$\|a\|^2$
$\Xi_{c1}^+ \to \Xi^0 K^0 l^+\nu_l$	$\|a+b\|^2$
$\Xi_{c1}^+ \to \Sigma^0\eta^0 l^+\nu_l$	$\frac{1}{12}\|b+c\|^2$
$\Xi_{c1}^+ \to \Lambda^0\pi^0 l^+\nu_l$	$\frac{1}{12}\|b+c\|^2$
$\Xi_{c1}^0 \to \Lambda^0\pi^- l^+\nu_l$	$\frac{1}{6}\|b+c\|^2$
$\Xi_{c1}^0 \to \Sigma^0\pi^- l^+\nu_l$	$\frac{1}{2}\|b-c\|^2$
$\Xi_{c1}^0 \to \Sigma^-\eta^0 l^+\nu_l$	$\frac{1}{6}\|b+c\|^2$
$\Xi_{c1}^0 \to \Sigma^-\pi^0 l^+\nu_l$	$\frac{1}{2}\|b-c\|^2$
$\Xi_{c1}^0 \to \Xi^- K^0 l^+\nu_l$	$\|b\|^2$
$\Xi_{c1}^0 \to nK^- l^+\nu_l$	$\|c\|^2$

Table 3.15. Squared matrix elements for the Cabibbo-allowed decays $T \rightarrow h^* M l^+ \nu_l$ normalized with respect to the process $\Xi_{c1}^0 \rightarrow \Sigma^{*0} K^- l^+ \nu_l$.

Process	Squared Matrix Element
$\Lambda_c^+ \rightarrow \Sigma^{*0} \pi^0 l^+ \nu_l$	2
$\Lambda_c^+ \rightarrow \Sigma^{*-} \pi^+ l^+ \nu_l$	2
$\Lambda_c^+ \rightarrow \Xi^{*-} K^+ l^+ \nu_l$	2
$\Lambda_c^+ \rightarrow \Sigma^{*+} \pi^- l^+ \nu_l$	2
$\Lambda_c^+ \rightarrow \Xi^{*0} K^0 l^+ \nu_l$	2
$\Xi_{c1}^+ \rightarrow \Sigma^{*+} K^- l^+ \nu_l$	2
$\Xi_{c1}^+ \rightarrow \Sigma^{*0} \overline{K}^0 l^+ \nu_l$	1
$\Xi_{c1}^+ \rightarrow \Xi^{*0} \eta^0 l^+ \nu_l$	3
$\Xi_{c1}^+ \rightarrow \Xi^{*0} \pi^0 l^+ \nu_l$	1
$\Xi_{c1}^+ \rightarrow \Xi^{*-} \pi^+ l^+ \nu_l$	2
$\Xi_{c1}^+ \rightarrow \Omega^- K^+ l^+ \nu_l$	6
$\Xi_{c1}^0 \rightarrow \Xi^{*0} \pi^- l^+ \nu_l$	2
$\Xi_{c1}^0 \rightarrow \Xi^{*-} \pi^0 l^+ \nu_l$	1
$\Xi_{c1}^0 \rightarrow \Xi^{*-} \eta^0 l^+ \nu_l$	3
$\Xi_{c1}^0 \rightarrow \Omega^- K^0 l^+ \nu_l$	6
$\Xi_{c1}^0 \rightarrow \Sigma^{*0} K^- l^+ \nu_l$	1
$\Xi_{c1}^0 \rightarrow \Sigma^{*-} \overline{K}^0 l^+ \nu_l$	2

Table 3.16. Squared matrix elements for the Cabibbo-suppressed decays $T \rightarrow h^* M l^+ \nu_l$ normalized with respect to the process $\Xi^0_{c1} \rightarrow \Sigma^{*-}\pi^0 l^+ \nu_l$.

Process	Squared Matrix Element (Modulo s_1^2)
$\Lambda_c^+ \rightarrow \Delta^+\pi^- l^+\nu_l$	2
$\Lambda_c^+ \rightarrow \Delta^0\pi^0 l^+\nu_l$	4
$\Lambda_c^+ \rightarrow \Sigma^{*0} K^0 l^+\nu_l$	1
$\Lambda_c^+ \rightarrow \Delta^-\pi^+ l^+\nu_l$	6
$\Lambda_c^+ \rightarrow \Sigma^{*-} K^+ l^+\nu_l$	2
$\Xi_{c1}^+ \rightarrow \Delta^+ K^- l^+\nu_l$	2
$\Xi_{c1}^+ \rightarrow \Delta^0 \overline{K}^0 l^+\nu_l$	2
$\Xi_{c1}^+ \rightarrow \Sigma^{*0}\eta^0 l^+\nu_l$	3
$\Xi_{c1}^+ \rightarrow \Sigma^{*0}\pi^0 l^+\nu_l$	$\frac{1}{2}$
$\Xi_{c1}^+ \rightarrow \Sigma^{*-}\pi^+ l^+\nu_l$	2
$\Xi_{c1}^+ \rightarrow \Xi^{*-} K^+ l^+\nu_l$	2
$\Xi_{c1}^0 \rightarrow \Delta^0 K^- l^+\nu_l$	2
$\Xi_{c1}^0 \rightarrow \Delta^- \overline{K}^0 l^+\nu_l$	6
$\Xi_{c1}^0 \rightarrow \Sigma^{*-}\pi^0 l^+\nu_l$	1
$\Xi_{c1}^0 \rightarrow \Sigma^{*-}\eta^0 l^+\nu_l$	3
$\Xi_{c1}^0 \rightarrow \Sigma^{*0}\pi^- l^+\nu_l$	1
$\Xi_{c1}^0 \rightarrow \Xi^{*-} K^0 l^+\nu_l$	2

Table 3.17. Squared matrix elements for the decays $S \to BMl^+\nu_l$ in terms of the reduced matrix elements α, β and γ.

Process	Squared Matrix Element
$\Omega_c^0 \to \Xi^- \overline{K}^0 l^+ \nu_l$	$\|\gamma\|^2$
$\Omega_c^0 \to \Xi^0 K^- l^+ \nu_l$	$\|\gamma\|^2$
$\Omega_c^0 \to \Xi^- \pi^0 l^+ \nu_l$	$s_1^2 \frac{1}{2} \|\alpha\|^2$
$\Omega_c^0 \to \Xi^- \eta^0 l^+ \nu_l$	$s_1^2 \frac{1}{6} \|\alpha - 2\beta + 2\gamma\|^2$
$\Omega_c^0 \to \Xi^0 \pi^- l^+ \nu_l$	$s_1^2 \|\alpha\|^2$
$\Omega_c^0 \to \Lambda^0 K^- l^+ \nu_l$	$s_1^2 \frac{1}{6} \|-2\alpha + \beta - 2\gamma\|^2$
$\Omega_c^0 \to \Sigma^0 K^- l^+ \nu_l$	$s_1^2 \frac{1}{2} \|\beta\|^2$
$\Omega_c^0 \to \Sigma^- \overline{K}^0 l^+ \nu_l$	$s_1^2 \|\beta\|^2$

Table 3.18. Squared matrix elements for the decays $S \to h^* M l^+ \nu_l$ in terms of the reduced matrix elements δ and ζ.

Process	Squared Matrix Element
$\Omega_c^0 \to \Xi^{*0} K^- l^+ \nu_l$	$\frac{1}{3} \|\delta + \zeta\|^2$
$\Omega_c^0 \to \Xi^{*-} \overline{K}^0 l^+ \nu_l$	$\frac{1}{3} \|\delta + \zeta\|^2$
$\Omega_c^0 \to \Omega^- \eta^0 l^+ \nu_l$	$\frac{2}{3} \|\delta + \zeta\|^2$
$\Omega_c^0 \to \Xi^{*0} \pi^- l^+ \nu_l$	$s_1^2 \frac{1}{3} \|\delta\|^2$
$\Omega_c^0 \to \Xi^{*-} \eta^0 l^+ \nu_l$	$s_1^2 \frac{1}{18} \|\delta - 2\zeta\|^2$
$\Omega_c^0 \to \Xi^{*-} \pi^0 l^+ \nu_l$	$s_1^2 \frac{1}{6} \|\delta\|^2$
$\Omega_c^0 \to \Omega^- K^0 l^+ \nu_l$	$s_1^2 \|\delta\|^2$
$\Omega_c^0 \to \Sigma^{*0} K^- l^+ \nu_l$	$s_1^2 \frac{1}{6} \|\zeta\|^2$
$\Omega_c^0 \to \Sigma^{*-} \overline{K}^0 l^+ \nu_l$	$s_1^2 \frac{1}{3} \|\zeta\|^2$

CHAPTER 4. B-MESON DECAYS TO CHARMED BARYONS

B–mesons, for the first time, offer an opportunity to study the weak nonleptonic decays of mesons to baryons. Experimentally, it is known that [1.6] Br ($B \to \bar{p}$ + anything) $\geq 2 \times 10^{-2}$ and Br ($B \to \bar{\Lambda}$ + anything) $\geq 1 \times 10^{-2}$. It is likely that, in the future, branching ratios will be measured for many of the exclusive B–meson decays to baryons. In this chapter we explore the implications of the approximate $SU(3)_f$ flavor symmetry of the strong interactions for the decays of B mesons to the lowest–lying charmed baryons. Some of the relations implied by $SU(3)_f$ may provide insight into the nature of various competing dynamical effects that can occur in nonleptonic B decays.

The effective Hamiltonian for $\Delta b = -1$ $\Delta c = +1$ nonleptonic B–meson decays has the flavor quantum numbers of the operator $(b\bar{c})(u\bar{d})$ for the Cabibbo-allowed decays and of the operator $(b\bar{c})(u\bar{s})$ for the Cabibbo-suppressed decays. These are two different components of the same octet representation of $SU(3)_f$. Using this transformation property we derive, in Section 4.1, relations between two body B meson decays to a charmed baryon plus an antibaryon. The consequences of $SU(3)_f$ for some of the two body B–meson decays to charmed baryons in the $\bar{3}$ representation have been considered in ref. [4.1]. The effective Hamiltonian for $\Delta b = -1$, $\Delta c = -1$ weak nonleptonic decays has the flavor quantum numbers of the operator $(b\bar{u})(c\bar{s})$. Note that these decays vanish if s_3 is zero. This operator transforms as $3 \oplus \bar{6}$ under flavor $SU(3)_f$. Using this transformation property we derive, in Section 4.2, relations between two–body B meson decays to a baryon and a charmed antibaryon. Section 4.3 contains concluding remarks, which include some $SU(3)_f$ relations for weak nonleptonic decays of bottom baryons.

4.1. Two Body B Decays to Charmed Baryons

The B meson fields come in three types, B^-, B^0 and B_s^0 which transform under $SU(3)_f$ as a $\bar{3}$ representation with components B_i:

$$B_1 = B^- \quad , \quad B_2 = B^0 \text{ and } B_3 = B_s^0 \quad . \tag{4.1}$$

We begin by considering two body decays of the B–mesons to charmed baryons in the lowest–lying antitriplet representation and antibaryons in the charge conjugate octet of nucleons and hyperons. As far as group theory factors are concerned, we can take for the effective Hamiltonian for nonleptonic decays, $B \to T\bar{h}$,

$$\begin{aligned} H_{\text{eff}} = & \alpha \overline{T}^l H^p_q h^q_\ell B_p \; + \; \beta \overline{T}^l H^q_\ell h^p_q B_p \\ & + \; \gamma \overline{T}^l B_\ell \left(H^p_q h^q_p \right) \quad . \end{aligned} \tag{4.2}$$

where h^i_j is given in Eq. (1.28), T^l is given in Eq. (3.15) and H^i_j are components of the matrix

$$H \; = \; \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ -s_1 & 0 & 0 \end{pmatrix} \quad . \tag{4.3}$$

Table 4.1 presents the predictions for $B \to T\bar{h}$ decays which follow from the effective Hamiltonian in Eq. (4.2). There is a simple relation amongst the Cabibbo-allowed decays

$$\Gamma(B^0 \to \Xi^0_{c_1}\bar{\Sigma}^0) \; + \; 3\Gamma(B^0 \to \Xi^0_{c_1}\bar{\Lambda}) \; = \; \Gamma(B^0 \to \Lambda^+_c\bar{p}) \; + \; \Gamma(B^0 \to \Xi^0_{c_1}\bar{\Xi}^0) \quad . \tag{4.4}$$

Li and Wu [4.1] have also considered the predictions of SU(3) for the Cabibbo-allowed $B \to T\bar{h}$ decays. These results agree with theirs. There are several simple relations between Cabibbo-allowed and Cabibbo-suppressed decays. For example,

$$\Gamma(B^- \to \Xi^0_{c_1}\bar{p}) = s_1^2\Gamma(B^- \to \Xi^0_{c_1}\bar{\Sigma}^-) \ , \tag{4.5a}$$

$$\Gamma(B^0 \to \Xi^0_{c_1}\bar{n}) = s_1^2\Gamma(B^0_s \to \Xi^0_{c_1}\bar{\Xi}^0) \ , \tag{4.5b}$$

and

$$\Gamma(B^0 \to \Xi^+_{c_1}\bar{p}) = s_1^2\Gamma(B^0_s \to \Lambda^+_c\bar{\Sigma}^-) \ . \tag{4.5c}$$

Next we consider two–body decays of the B–mesons to the lowest–lying charmed baryons in the 6 representation and antibaryons in the charge conjugate of the octet of *nucleons* and hyperons. The sextuplet of charmed baryon fields is denoted by a two index symmetric tensor S^{ij} with components

$$S^{11} = \Sigma_c^{++} \ , \ S^{12} = \frac{1}{\sqrt{2}}\Sigma_c^{+} \ , \ S^{22} = \Sigma_c^{0} \ , \ S^{13} = \frac{1}{\sqrt{2}} \ \Xi_{c2}^{+} \ ,$$
$$S^{23} = \frac{1}{\sqrt{2}}\Xi_{c_2}^{0} \text{ and } S^{33} = \Omega_c^0 \ . \tag{4.6}$$

As far as group theory factors are concerned we can use, as the effective Hamiltonian for weak nonleptonic $B \to S\bar{h}$ decays,

$$H_{\text{eff}} = a\epsilon^{ijp} \ B_p \ H_j^k \ \bar{S}_{k\ell} \ h_i^\ell \ + \ b \ \epsilon^{ijp} \ B_p \ H_j^k \ h_k^\ell \ \bar{S}_{i\ell}$$
$$+ c \ \epsilon^{ijp} \ B_p \ H_\ell^k h_j^\ell \ \bar{S}_{ik} \ \ , \tag{4.7}$$

with H_j^k given by Eq. (4.3). The decay rates for Cabibbo allowed decays, which follow from Eq. (4.7), are presented in Table 4.2. There are several simple relations amongst the Cabibbo allowed decays. For example

$$\Gamma(B^- \to \Xi_{c_2}^0 \bar{\Sigma}^-) = \frac{1}{2}\Gamma(B^- \to \Sigma_c^0 \bar{p}) \ \ , \tag{4.8a}$$
$$\Gamma(B^0 \to \Xi_{c_2}^+ \bar{\Sigma}^-) = \frac{1}{2}\Gamma(B^0 \to \Omega_c^0 \bar{\Xi}^0) \ \ . \tag{4.8b}$$

Table 4.3 presents the results that follow from the effective Hamiltonian in Eq. (4.7) for the Cabibbo-suppressed decays. In addition to there being several relations between the Cabibbo-suppressed decays, there are also some simple relations between the Cabibbo-allowed and the Cabibbo-suppressed decays. For example, inspection of Tables 4.2 and 4.3 reveals that

$$\Gamma(B^- \to \Omega_c^0 \bar{\Sigma}^-) = s_1^2 \Gamma(B^- \to \Sigma_c^0 \bar{p}) \ \ , \tag{4.9a}$$
$$\Gamma(B^0 \to \Xi_{c_2}^0 \bar{n}) = s_1^2 \Gamma(B_s^0 \to \Xi_{c_2}^0 \bar{\Xi}^0) \ \ . \tag{4.9b}$$

The results of Tables 4.2 and 4.3 trivially generalize to decays $B \to S^* \bar{h}$, where S^* denotes a member of the lowest-lying 6 of $J^P = 3/2^+$ charmed baryons.

There is only one singlet representation in the product $3 \otimes 10 \otimes \bar{3} \otimes 8$ (i.e., $\bar{T}_{pq} h^{*q\ell j} H^p_\ell B_j$) so all the decays $B \to T\bar{h}^*$ are related by $SU(3)_f$ symmetry. Table 4.4 presents the relative rates for these decays normalized to the rate for the decay $B^- \to \Lambda_c^+ \bar{\Delta}^{--}$.

For the decays $B \to S\bar{h}^*$ we can take, as far as group theory factors are concerned, the following effective Hamiltonian

$$H_{\text{eff}} = \eta_1 h^{*pq\ell} \bar{S}_{pq} H^j_\ell B_j + \eta_2 h^{*pq\ell} \bar{S}_{pk} H^k_q B_\ell \quad . \tag{4.10}$$

The term proportional to η_1 only effects Cabibbo allowed B^0 decays and Cabibbo suppressed B^0_s decays. So all the Cabibbo allowed B^- and B^0_s decays and the Cabibbo suppressed B^- and B^0 decays are related by $SU(3)_f$ flavor. Tables 4.5 and 4.6 present rates, which follow from the effective Hamiltonian in Eq. (4.10), for the Cabibbo allowed and Cabibbo suppressed $B \to S\bar{h}^*$ decays. These results generalize straightforwardly to decays $B \to S^*\bar{h}^*$, where S^* denotes a member of the lowest–lying 6 of $J^P = 3/2^+$ charmed baryons.

4.2. Two Body B–Decays to Charmed Anti–Baryons

The $\Delta b = -1$, $\Delta c = -1$ part of the effective Hamiltonian for weak nonleptonic B–decays arises from the $b \to u$ weak coupling and has the flavor quantum numbers of the operator $(b\bar{u})(c\bar{s})$. This operator transforms as $\bar{6} \oplus 3$ with respect to flavor SU(3). Decomposing it into irreducible operators

$$(b\bar{u})(c\bar{s}) = O_{(\bar{6})} + O_{(3)} \quad , \tag{4.11}$$

where

$$O_{(\bar{6})} = \frac{1}{2}\Big[(b\bar{u})(c\bar{s}) + (b\bar{s})(c\bar{u})\Big] \quad , \tag{4.12a}$$

transforms as a $\bar{6}$ and

$$O_{(3)} = \frac{1}{2}\Big[(b\bar{u})(c\bar{s}) - (b\bar{s})(c\bar{u})\Big] \quad , \tag{4.12b}$$

transforms as a 3. There is a small dynamical enhancement of the coefficient of $O_{(3)}$, in the effective weak Hamiltonian, over that of $O_{(\bar{6})}$ coming from perturbative strong interaction corrections which take into account effects coming from loop momenta p in the range $M_W > p > m_b$. It is equal to [2.1]

$$\left[\frac{\alpha_s(m_b)}{\alpha_s(M_W)}\right]^{18/23} \simeq 1.5 \quad . \tag{4.13}$$

For the two-body decays $B \to \bar{T}h$ we can use, as far as group theory factors are concerned, the following effective Hamiltonian.

$$\begin{aligned} H_{\text{eff}} = & \eta_{(3)} B_i \bar{h}^i_j T_k H(3)^{jk} + \eta'_{(3)} B_i \bar{h}^k_j T_k H(3)^{ij} \\ & + \eta_{(\bar{6})} B_i \bar{h}^i_j T_k H(\bar{6})^{jk} + \eta'_{(\bar{6})} B_i T_k \bar{h}^k_j H(\bar{6})^{ij} \quad . \end{aligned} \tag{4.14}$$

In Eq. (4.14) $H(3)$ is a two index antisymmetric tensor that takes into account the transformation properties of $O_{(3)}$. It has non-zero components

$$H(3)^{13} = 1 \quad , \quad H(3)^{31} = -1 \quad . \tag{4.15}$$

$H(\bar{6})$ is a two index symmetric tensor that takes into account the transformation properties of $O_{(\bar{6})}$. It has non-zero components

$$H^{13}(\bar{6}) = 1 \quad , \quad H^{31}(\bar{6}) = 1 \quad . \tag{4.16}$$

Table 4.7 presents the predictions for $B \to \bar{T}h$ decays that follow from the effective Hamiltonian in Eq. (4.14). The Hamiltonian is $I = 1/2$ and it follows that the relations

$$\Gamma(B^- \to \bar{\Lambda}_c^+ \Sigma^0) = \frac{1}{2}\Gamma(B^0 \to \bar{\Lambda}_c^+ \Sigma^+) \quad , \tag{4.17a}$$

$$\Gamma(B_s^0 \to \bar{\Xi}_{c_1}^0 \Sigma^0) = \frac{1}{2}\Gamma(B_s^0 \to \bar{\Xi}_{c_1}^- \Sigma^+) \quad , \tag{4.17b}$$

are consequences of isospin invariance. There are no simple $SU(3)_f$ relations that are not consequences of isospin. However if, for dynamical reasons, matrix elements of $O_{(3)}$ dominate over those of $O_{(\bar{6})}$, then there would be additional relations. For example, either 3 or $\bar{6}$ dominance implies that

$$\Gamma(B^0 \to \bar{\Lambda}_c^+ \Sigma^+) = \Gamma(B^0 \to \bar{\Xi}_{c_1}^0 \Xi^0) \quad . \tag{4.18}$$

Next we consider decays of the type $B \to \bar{S}h$. As far as group theory factors are concerned, we can take as the effective Hamiltonian for these decays

$$\begin{aligned} H_{\text{eff}} = &F_{(3)} \epsilon_{kpq} B_i \bar{h}^i_j S^{jk} H(3)^{pq} + G_{(3)} \epsilon_{kpq} B_i \bar{h}^k_j S^{ij} H(3)^{pq} \\ &+ F_{(\bar{6})} \epsilon_{ijk} B_\ell \bar{h}^k_p S^{ip} H(\bar{6})^{j\ell} + G_{(\bar{6})} \epsilon_{ijk} B_\ell S^{i\ell} H(\bar{6})^{jp} \bar{h}^k_p \quad . \end{aligned} \tag{4.19}$$

Rates for $B \to \bar{S}h$ decays that follow from this effective Hamiltonian are presented in Table 4.8. Most of the simple relations are consequences of isospin. These are

$$\Gamma(B^- \to \bar{\Sigma}_c^- \Lambda^0) = \frac{1}{2}\Gamma(B^0 \to \bar{\Sigma}_c^0 \Lambda^0) \quad , \tag{4.20a}$$

$$\Gamma(B^0 \to \bar{\Sigma}_c^0 \Sigma^0) = \Gamma(B^0 \to \bar{\Sigma}_c^- \Sigma^+) \quad , \tag{4.20b}$$

$$\Gamma(B_s^0 \to \bar{\Sigma}_c^- p) = \frac{1}{2}\Gamma(B_s^0 \to \bar{\Sigma}_c^0 n) \quad , \tag{4.20c}$$

$$\Gamma(B_s^0 \to \bar{\Xi}_{c_2}^0 \Sigma^0) = \frac{1}{2}\Gamma(B_s^0 \to \bar{\Xi}_{c_2}^- \Sigma^+) \quad . \tag{4.20d}$$

There is a simple relation that is not a consequence of isospin.

$$\Gamma(B^- \to \bar{\Xi}_{c_2}^0 \Xi^-) = \frac{1}{2}\Gamma(B^- \to \bar{\Sigma}_c^0 \Sigma^-) \quad . \tag{4.21}$$

The results in Table 4.7 generalize straightforwardly to $B \to \bar{S}^* h$ decays, where S^* is a member of the lowest–lying $J^P = 3/2^+$ multiplet of charmed baryons.

Since the product $\bar{3} \otimes \overline{10} \otimes \bar{3} \otimes \bar{3}$ does not contain a singlet, the decays $B \to \bar{T}h^*$ only proceed via the $\bar{6}$ part of the effective Hamiltonian. In addition, the product $\bar{3} \otimes \overline{10} \otimes \bar{3} \otimes 6$ contains only one singlet (i.e., $B_i T^{ij} \bar{h}^*_{jk\ell} H(\bar{6})^{k\ell}$) and so the relative rates for all the $B \to \bar{T}h^*$ are determined by $SU(3)_f$ symmetry. Table 4.9 gives the relative rates. Note that the relations

$$\Gamma(B^- \to \bar{\Lambda}_c^+ \Sigma^{*0}) = \frac{1}{2}\Gamma(B^0 \to \bar{\Lambda}_c^+ \Sigma^{*+}) \ , \tag{4.22a}$$

$$\Gamma(B_s^0 \to \bar{\Xi}_{c_1}^0 \Sigma^{*0}) = \frac{1}{2}\Gamma(B_s^0 \to \bar{\Xi}_{c_1}^- \Sigma^{*+}) \ , \tag{4.22b}$$

are consequences of isospin.

Finally we consider the decays $B \to \bar{S}h^*$. As far as group theory factors are concerned, we can use

$$\begin{aligned} H_{\text{eff}} &= \alpha_{(\bar{6})} B_i H(\bar{6})^{ij} \bar{h}^*_{jk\ell} S^{k\ell} + \beta_{(\bar{6})} B_i H(\bar{6})^{k\ell} \bar{h}^*_{jk\ell} S^{ij} \\ &+ \alpha_{(3)} B_i H(3)^{ij} \bar{h}^*_{jk\ell} S^{k\ell} \ , \end{aligned} \tag{4.23}$$

as the effective Hamiltonian for these decays. Note that the terms proportional to $\alpha_{(3)}$ and $\alpha_{(\bar{6})}$ only cause B^- and B_s^0 decays. It follows that the rates for $B^0 \to \bar{S}h^*$ decays are related by $SU(3)_f$ symmetry. Table 4.10 shows the results that follow from expanding the effective Hamiltonian in Eq. (4.23). There are several simple relations that are not consequences of isospin. For example:

$$\Gamma(B^- \to \bar{\Sigma}_c^- \Sigma^{*0}) = \frac{1}{2}\Gamma(B^- \to \bar{\Xi}_{c_2}^- \Xi^{*0}) \ , \tag{4.24a}$$

$$\Gamma(B^- \to \bar{\Sigma}_c^0 \Sigma^{*-}) = \frac{1}{2}\Gamma(B^- \to \bar{\Xi}_{c_2}^0 \Xi^{*-}) \ , \tag{4.24b}$$

$$\Gamma(B^0 \to \bar{\Sigma}_c^- \Sigma^{*+}) = \Gamma(B^0 \to \bar{\Xi}_{c_2}^0 \Xi^{*0}) \ . \tag{4.24c}$$

The results in Table 4.10 generalize straightforwardly to the decays $B \to \bar{S}^* h^*$.

4.3 Concluding Remarks On Chapter 4

In this chapter we have examined the consequences of the approximate $SU(3)_f$ flavor symmetry of the strong interactions for nonleptonic B meson decays that produce low lying charmed baryons. Many simple $SU(3)_f$ relations were found. For example, all the rates for two body decays, to a $J^P = 1/2^+$ charmed baryon in the lowest–lying $\bar{3}$ representation and a $J^P = 3/2^+$ antibaryon in the $\bar{10}$ representation, are related by the $SU(3)_f$ flavor symmetry of the strong interactions.

The hadronic spectrum contains bottom baryons analogous to charmed baryons. In fact, if we approximate m_b and m_c as very heavy compared with the QCD scale, then the heavy quarks act essentially as static color sources in these baryons [1.9] and the hyperfine splitting between the lowest-lying three and six $J^P = 1/2^+$ multiplets in the charm case is related to that in the bottom case. For example, one expects

$$m_{\Sigma_b} - m_{\Lambda_b} \simeq m_{\Sigma_c} - m_{\Lambda_c} \,. \tag{4.25}$$

If color suppression [4.2] does not diminish the rate, the decay chain, $\Lambda_b^0 \to \Lambda^0 J/\psi \to p\pi^- e^+ e^-$, may prove to be a useful way to detect the Λ_b^0 baryon. There are $SU(3)_f$ relations for weak bottom baryon decays. For example, the decays of the lowest-lying $J^P = 1/2^+$ bottom baryons in the $\bar{3}$ representation to hJ/ψ are related by $SU(3)_f$. Explicitly

$$\Gamma(\Xi_{b_1}^- \to \Xi^- J/\psi) = \Gamma(\Xi_{b_1}^0 \to \Xi^0 J/\psi) = \frac{3}{2}\Gamma(\Lambda_b^0 \to \Lambda^0 J/\psi)\,. \tag{4.26}$$

The decays of the lowest–lying $\bar{3}$ bottom baryons to $h^* J/\psi$ are forbidden by SU(3) flavor because the product $3 \otimes \bar{3} \otimes \bar{10}$ doesn't contain a singlet. Note that the decays of the lowest–lying $J^P = 1/2^+$ bottom baryons in the $\bar{3}$ representation, to $h^* D$ (where D denotes one of the three lowest-lying pseudoscalar D-mesons) are also determined by a single reduced matrix element, because the product $\bar{3} \otimes 3 \otimes 8 \otimes \bar{10}$ only contains one singlet. Normalizing to the rate for $\Lambda_b \to \Delta^- D^+$ we find that:

$$\Gamma(\Lambda_b \to \Sigma^{*-} D_s^+) = \Gamma(\Lambda_b \to \Delta^0 D^0) = \frac{1}{3} \quad , \tag{4.27a}$$

$$2\Gamma(\Xi_{b_1}^0 \to \Sigma^{*0} D^0) = \Gamma(\Xi_{b_1}^0 \to \Sigma^{*-} D^+) = \Gamma(\Xi_{b_1}^0 \to \Xi^{*-} D_s^+) = \frac{1}{3} \quad . \tag{4.27b}$$

References for Chapter 4

[4.1] X. Li and D. Wu, Phys. Lett. **218B** (1989) 357.

[4.2] M. B. Wise, Phys. Lett. **89B** (1980) 229.

Table 4.1. $SU(3)_f$ predictions for decays $B \to T\bar{h}$, where T denotes a member of the lowest–lying SU(3) antitriplet of charmed baryons and h denotes a member of the lowest–lying octet of hyperons and nucleons. Rates are expressed in terms of three reduced matrix elements α, β and γ.

Decay	Rate
$B^- \to \Xi^0_{c_1}\bar{\Sigma}^-$	$\|\beta+\gamma\|^2$
$B^- \to \Xi^0_{c_1}\bar{p}$	$s_1^2\|\beta+\gamma\|^2$
$B^0 \to \Xi^0_{c_1}\bar{\Sigma}^0$	$\frac{1}{2}\|\alpha-\beta\|^2$
$B^0 \to \Xi^0_{c_1}\bar{\Lambda}$	$\frac{1}{6}\|\alpha+\beta\|^2$
$B^0 \to \Xi^+_{c_1}\bar{\Sigma}^-$	$\|\alpha+\gamma\|^2$
$B^0 \to \Lambda_c\bar{p}$	$\|\alpha\|^2$
$B^0 \to \Xi^0_{c_1}\bar{n}$	$s_1^2\|\beta\|^2$
$B^0 \to \Xi^+_{c_1}\bar{p}$	$s_1^2\|\gamma\|^2$
$B^0_s \to \Xi^0_{c_1}\bar{\Xi}^0$	$\|\beta\|^2$
$B^0_s \to \Lambda_c\bar{\Sigma}^-$	$\|\gamma\|^2$
$B^0_s \to \Xi^0_{c_1}\bar{\Sigma}^0$	$\frac{1}{2}s_1^2\|\alpha\|^2$
$B^0_s \to \Xi^0_{c_1}\bar{\Lambda}$	$\frac{1}{6}s_1^2\|\alpha-2\beta\|^2$
$B^0_s \to \Xi^+_{c_1}\bar{\Sigma}^-$	$s_1^2\|\alpha\|^2$
$B^0_s \to \Lambda_c\bar{p}$	$s_1^2\|\alpha+\gamma\|^2$

Table 4.2. $SU(3)_f$ predictions for Cabibbo-allowed decays $B \to S\bar{h}$ where S denotes a member of the lowest–lying $J^P = 1/2^+$ sextuplet of charmed baryons and h denotes a member of the lowest–lying octet of hyperon and nucleons. Rates are expressed in terms of the three reduced matrix elements a, b and c.

Decay	Rate
$B^- \to \Sigma_c^0 \bar{p}$	$\|c\|^2$
$B^- \to \Xi_{c_2}^0 \bar{\Sigma}^-$	$\frac{1}{2}\|c\|^2$
$B^0 \to \Sigma_c^+ \bar{p}$	$\frac{1}{2}\|a - c\|^2$
$B^0 \to \Xi_{c_2}^0 \bar{\Sigma}^0$	$\frac{1}{4}\|b - c\|^2$
$B^0 \to \Xi_{c_2}^0 \bar{\Lambda}$	$\frac{1}{12}\| - 2a + b + c\|^2$
$B^0 \to \Omega_c \bar{\Xi}^0$	$\|b\|^2$
$B^0 \to \Xi_{c_2}^+ \bar{\Sigma}^-$	$\frac{1}{2}\|b\|^2$
$B^0 \to \Sigma_c^0 \bar{n}$	$\|a\|^2$
$B_s^0 \to \Sigma_c^+ \bar{\Sigma}^-$	$\frac{1}{2}\| - a - b + c\|^2$
$B_s^0 \to \Sigma_c^0 \bar{\Sigma}^0$	$\frac{1}{2}\|a + b - c\|^2$
$B_s^0 \to \Sigma_c^0 \bar{\Lambda}$	$\frac{1}{6}\| - a - b - c\|^2$
$B_s^0 \to \Xi_{c_2}^0 \bar{\Xi}^0$	$\frac{1}{2}\|a + b\|^2$

Table 4.3. $SU(3)_f$ predictions for Cabibbo-suppressed decays $B \to S\bar{h}$ where S denotes a member of the lowest–lying $J^P = 1/2^+$ sextuplet of charmed baryons and h denotes a member of the lowest–lying octet of hyperons and nucleons. Rates are expressed in terms of the three reduced matrix elements a, b and c.

Decay	Rate (divided by s_1^2)
$B^- \to \Xi_{c2}^0 \bar{p}$	$\frac{1}{2}\|c\|^2$
$B^- \to \Omega_c \bar{\Sigma}^-$	$\|c\|^2$
$B^0 \to \Omega_c \bar{\Sigma}^0$	$\frac{1}{2}\|c\|^2$
$B^0 \to \Omega_c \bar{\Lambda}$	$\frac{1}{6}\|-2a-2b+c\|^2$
$B^0 \to \Xi_{c2}^+ \bar{p}$	$\frac{1}{2}\|a+b-c\|^2$
$B^0 \to \Xi_{c2}^0 \bar{n}$	$\frac{1}{2}\|a+b\|^2$
$B_s^0 \to \Xi_{c2}^+ \bar{\Sigma}^-$	$\frac{1}{2}\|a-c\|^2$
$B_s^0 \to \Xi_{c2}^0 \bar{\Sigma}^0$	$\frac{1}{4}\|a-c\|^2$
$B_s^0 \to \Xi_{c2}^0 \bar{\Lambda}$	$\frac{1}{12}\|-a+2b-c\|^2$
$B_s^0 \to \Sigma_c^+ \bar{p}$	$\frac{1}{2}\|b\|^2$
$B_s^0 \to \Sigma_c^0 \bar{n}$	$\|b\|^2$
$B_s^0 \to \Omega_c \bar{\Xi}^0$	$\|a\|^2$

Table 4.4. $SU(3)_f$ predictions for Cabibbo decays $B \to T\bar{h}^*$ where T denotes a member of the lowest–lying antitriplet of charmed baryons and h^* denotes a member of the lowest–lying decouplet of $J^P = 3/2^+$ baryons.

Decay	Rate
$B^- \to \Lambda_c \bar{\Delta}^{--}$	1
$B^- \to \Xi^0_{c_1} \bar{\Sigma}^{*-}$	$\frac{1}{3}$
$B^- \to \Xi^+_{c_1} \bar{\Delta}^{--}$	s_1^2
$B^- \to \Xi^0_{c_1} \bar{\Delta}^{-}$	$\frac{1}{3} s_1^2$
$B^0 \to \Lambda_c \bar{\Delta}^{-}$	$\frac{1}{3}$
$B^0 \to \Xi^0_{c_1} \bar{\Sigma}^{*0}$	$\frac{1}{6}$
$B^0 \to \Xi^+_{c_1} \bar{\Delta}^{-}$	$\frac{1}{3} s_1^2$
$B^0 \to \Xi^0_{c_1} \bar{\Delta}^{0}$	$\frac{1}{3} s_1^2$
$B^0_s \to \Lambda_c \bar{\Sigma}^{*-}$	$\frac{1}{3}$
$B^0_s \to \Xi^0_{c_1} \bar{\Xi}^{*0}$	$\frac{1}{3}$
$B^0_s \to \Xi^+_{c_1} \bar{\Sigma}^{*-}$	$\frac{1}{3} s_1^2$
$B^0_s \to \Xi^0_{c_1} \bar{\Sigma}^{*0}$	$\frac{1}{6} s_1^2$

Table 4.5. $SU(3)_f$ predictions for Cabibbo–allowed decays $B \to S\bar{h}^*$ where S denotes a member of the lowest–lying $J^P = 1/2^+$ sextuplet of charmed baryons and h^* denotes a member of the lowest–lying decouplet of $J^P = 3/2^+$ baryons. Rates are expressed in terms of the two reduced matrix elements η_1 and η_2.

Decay	Rate
$B^- \to \Sigma_c^+ \bar{\Delta}^{--}$	$\frac{1}{2}\|\eta_2\|^2$
$B^- \to \Sigma_c^0 \bar{\Delta}^{-}$	$\frac{1}{3}\|\eta_2\|^2$
$B^- \to \Xi_{c_2}^0 \bar{\Sigma}^{*-}$	$\frac{1}{6}\|\eta_2\|^2$
$B^0 \to \Sigma_c^{++} \bar{\Delta}^{--}$	$\|\eta_1\|^2$
$B^0 \to \Sigma_c^+ \bar{\Delta}^{-}$	$\frac{1}{6}\|2\eta_1 + \eta_2\|^2$
$B^0 \to \Xi_{c_2}^+ \bar{\Sigma}^{*-}$	$\frac{2}{3}\|\eta_1\|^2$
$B^0 \to \Xi_{c_2}^0 \bar{\Sigma}^{*0}$	$\frac{1}{12}\|2\eta_1 + \eta_2\|^2$
$B^0 \to \Sigma_c^0 \bar{\Delta}^{0}$	$\frac{1}{3}\|\eta_1 + \eta_2\|^2$
$B^0 \to \Omega_c \bar{\Xi}^{*0}$	$\frac{1}{3}\|\eta_1\|^2$
$B_s^0 \to \Sigma_c^+ \bar{\Sigma}^{*-}$	$\frac{1}{6}\|\eta_2\|^2$
$B_s^0 \to \Sigma_c^0 \bar{\Sigma}^{*0}$	$\frac{1}{6}\|\eta_2\|^2$
$B_s^0 \to \Xi_{c_2}^0 \bar{\Xi}^{*0}$	$\frac{1}{6}\|\eta_2\|^2$

Table 4.6. $SU(3)_f$ predictions for Cabibbo–suppressed decays $B \to S\bar{h}^*$, where S denotes a member of the lowest–lying $J^P = 1/2^+$ sextuplet of charmed baryons and h^* denotes a member of the lowest–lying decouplet of $J^P = 3/2^+$ baryons. Rates are expressed in terms of the two reduced matrix elements η_1 and η_2.

Decay	Rate (divided by s_1^2)
$B^- \to \Xi_{c_2}^+ \bar{\Delta}^{--}$	$\frac{1}{2}\|\eta_2\|^2$
$B^- \to \Xi_{c_2}^0 \bar{\Delta}^-$	$\frac{1}{6}\|\eta_2\|^2$
$B^- \to \Omega_c \bar{\Sigma}^{*-}$	$\frac{1}{3}\|\eta_2\|^2$
$B^0 \to \Xi_{c_2}^+ \bar{\Delta}^-$	$\frac{1}{6}\|\eta_2\|^2$
$B^0 \to \Xi_{c_2}^0 \bar{\Delta}^0$	$\frac{1}{6}\|\eta_2\|^2$
$B^0 \to \Omega_c \bar{\Sigma}^{*0}$	$\frac{1}{6}\|\eta_2\|^2$
$B_s^0 \to \Sigma_c^{++} \bar{\Delta}^{--}$	$\|\eta_1\|^2$
$B_s^0 \to \Sigma_c^+ \bar{\Delta}^-$	$\frac{2}{3}\|\eta_1\|^2$
$B_s^0 \to \Xi_{c_2}^+ \bar{\Sigma}^{*-}$	$\frac{1}{6}\|2\eta_1 + \eta_2\|^2$
$B_s^0 \to \Xi_{c_2}^0 \bar{\Sigma}^{*0}$	$\frac{1}{12}\|2\eta_1 + \eta_2\|^2$
$B_s^0 \to \Sigma_c^0 \bar{\Delta}^0$	$\frac{1}{3}\|\eta_1\|^2$
$B_s^0 \to \Omega_c \bar{\Xi}^{*0}$	$\frac{1}{3}\|\eta_1 + \eta_2\|^2$

Table 4.7. $SU(3)_f$ predictions for decays $B \to \bar{T}h$, where T denotes a member of the lowest–lying antitriplet of charmed baryons and h denotes a member of the lowest–lying octet of hyperons and nucleons. Rates are expressed in terms of the four reduced matrix elements $\eta_{(3)}$, $\eta'_{(3)}$, $\eta_{(\bar{6})}$ and $\eta'_{(\bar{6})}$.

Decay	Rate
$B^- \to \bar{\Lambda}_c \Sigma^0$	$\frac{1}{2}\lvert\eta_{(3)} + \eta_{(\bar{6})}\rvert^2$
$B^- \to \bar{\Lambda}_c \Lambda$	$\frac{1}{6}\lvert\eta_{(3)} - 2\eta'_{(3)} + \eta_{(\bar{6})} - 2\eta'_{(\bar{6})}\rvert^2$
$B^- \to \bar{\Xi}^0_{c_1} \Xi^-$	$\lvert -\eta_{(3)} + \eta'_{(3)} + \eta_{(\bar{6})} + \eta'_{(\bar{6})}\rvert^2$
$B^- \to \bar{\Xi}^-_{c_1} \Xi^0$	$\lvert\eta'_{(3)} + \eta'_{(\bar{6})}\rvert^2$
$B^0 \to \bar{\Lambda}_c \Sigma^+$	$\lvert\eta_{(3)} + \eta_{(\bar{6})}\rvert^2$
$B^0 \to \bar{\Xi}^0_{c_1} \Xi^0$	$\lvert\eta_{(3)} - \eta_{(\bar{6})}\rvert^2$
$B^0_s \to \bar{\Lambda}_c p$	$\lvert\eta_{(3)} - \eta'_{(3)} + \eta_{(\bar{6})} + \eta'_{(\bar{6})}\rvert^2$
$B^0_s \to \bar{\Xi}^0_{c_1} \Lambda$	$\frac{1}{6}\lvert 2\eta_{(3)} - \eta'_{(3)} - 2\eta_{(\bar{6})} + \eta'_{(\bar{6})}\rvert^2$
$B^0_s \to \bar{\Xi}^0_{c_1} \Sigma^0$	$\frac{1}{2}\lvert\eta'_{(3)} - \eta'_{(\bar{6})}\rvert^2$
$B^0_s \to \bar{\Xi}^-_{c_1} \Sigma^+$	$\lvert\eta'_{(3)} - \eta'_{(\bar{6})}\rvert^2$

Table 4.8. $SU(3)_f$ predictions for decays $B \to \bar{S}h$, where S denotes a member of the lowest–lying $SU(3)_f$ sextuplet of charmed baryons and h denotes a member of the lowest–lying $SU(3)_f$ octet of hyperons and nucleons. Rates are expressed in terms of the four reduced matrix elements $F_{(\bar{6})}$, $G_{(\bar{6})}$, $F_{(3)}$ and $G_{(3)}$.

Decay	Rate
$B^- \to \bar{\Sigma}_c^{--}\Sigma^+$	$\lvert F_{(\bar{6})} + G_{(\bar{6})} + 2G_{(3)}\rvert^2$
$B^- \to \bar{\Sigma}_c^-\Sigma^0$	$\lvert F_{(\bar{6})} + \frac{1}{2}G_{(\bar{6})} - F_{(3)} + G_{(3)}\rvert^2$
$B^- \to \bar{\Xi}_{c_2}^-\Xi^0$	$\frac{1}{2}\lvert F_{(\bar{6})} - G_{(\bar{6})} + 2G_{(3)}\rvert^2$
$B^- \to \bar{\Xi}_{c_2}^0\Xi^-$	$\frac{1}{2}\lvert F_{(\bar{6})} - 2F_{(3)}\rvert^2$
$B^- \to \bar{\Sigma}_c^0\Sigma^-$	$\lvert F_{(\bar{6})} - 2F_{(3)}\rvert^2$
$B^- \to \bar{\Sigma}_c^-\Lambda$	$\frac{3}{4}\lvert G_{(\bar{6})} - \frac{2}{3}F_{(3)} - \frac{2}{3}G_{(3)}\rvert^2$
$B^0 \to \bar{\Sigma}_c^0\Sigma^0$	$\frac{1}{2}\lvert G_{(\bar{6})} + 2F_{(3)} + 2G_{(3)}\rvert^2$
$B^0 \to \bar{\Sigma}_c^0\Lambda$	$\frac{3}{2}\lvert G_{(\bar{6})} - \frac{2}{3}F_{(3)} - \frac{2}{3}G_{(3)}\rvert^2$
$B^0 \to \bar{\Sigma}_c^-\Sigma^+$	$\frac{1}{2}\lvert G_{(\bar{6})} + 2F_{(3)} + 2G_{(3)}\rvert^2$
$B^0 \to \bar{\Xi}_{c_2}^0\Xi^0$	$\frac{1}{2}\lvert G_{(\bar{6})} - 2F_{(3)} - 2G_{(3)}\rvert^2$
$B_s^0 \to \bar{\Sigma}_c^- p$	$\frac{1}{2}\lvert F_{(\bar{6})} + 2F_{(3)}\rvert^2$
$B_s^0 \to \bar{\Sigma}_c^0 n$	$\lvert F_{(\bar{6})} + 2F_{(3)}\rvert^2$
$B_s^0 \to \bar{\Xi}_{c_2}^0\Lambda$	$\frac{1}{12}\lvert 3F_{(\bar{6})} + 3G_{(\bar{6})} + 4F_{(3)} - 2G_{(3)}\rvert^2$
$B_s^0 \to \bar{\Xi}_{c_2}^-\Sigma^+$	$\frac{1}{2}\lvert F_{(\bar{6})} - G_{(\bar{6})} - 2G_{(3)}\rvert^2$
$B_s^0 \to \bar{\Xi}_{c_2}^0\Sigma^0$	$\frac{1}{4}\lvert - F_{(\bar{6})} + G_{(\bar{6})} + 2G_{(3)}\rvert^2$
$B_s^0 \to \bar{\Omega}_c\Xi^0$	$\lvert F_{(\bar{6})} + G_{(\bar{6})} - 2G_{(3)}\rvert^2$

Table 4.9. Relative rates for decays $B \to \bar{T}h^*$, where T denotes a member of the lowest-lying $SU(3)_f$ antitriplet of charmed baryons and h^* denotes a member of the lowest-lying $SU(3)_f$ decouplet of $J^P = 3/2^+$ baryons.

Decay	Rate
$B^- \to \bar{\Lambda}_c \Sigma^{*0}$	$\frac{1}{2}$
$B^- \to \bar{\Xi}_{c_1}^- \Xi^{*0}$	1
$B^0 \to \bar{\Lambda}_c \Sigma^{*+}$	1
$B^0 \to \bar{\Xi}_{c_1}^0 \Xi^{*0}$	1
$B_s^0 \to \bar{\Xi}_{c_1}^- \Sigma^{*+}$	1
$B_s^0 \to \bar{\Xi}_{c_1}^0 \Sigma^{*0}$	$\frac{1}{2}$

Table 4.10. Implications of $SU(3)_f$ for decays $B \to \bar{S}h^*$, where S denotes a member of the lowest–lying $SU(3)_f$ sextuplet of charmed baryons and h^* denotes a member of the lowest–lying $J^P = 3/2^+$ $SU(3)_f$ decouplet of baryons.

Decay	Rate
$B^- \to \bar{\Sigma}_c^{--}\Sigma^{*+}$	$\frac{1}{3}\lvert\alpha_{(\bar{6})} + \alpha_{(3)} + 2\beta_{(\bar{6})}\rvert^2$
$B^- \to \bar{\Sigma}_c^{0}\Sigma^{*-}$	$\frac{1}{3}\lvert\alpha_{(\bar{6})} + \alpha_{(3)}\rvert^2$
$B^- \to \bar{\Sigma}_c^{-}\Sigma^{*0}$	$\frac{1}{3}\lvert\alpha_{(\bar{6})} + \alpha_{(3)} + \beta_{(\bar{6})}\rvert^2$
$B^- \to \bar{\Xi}_{c_2}^{-}\Xi^{*0}$	$\frac{2}{3}\lvert\alpha_{(\bar{6})} + \alpha_{(3)} + \beta_{(\bar{6})}\rvert^2$
$B^- \to \bar{\Omega}_c\Omega$	$\lvert\alpha_{(\bar{6})} + \alpha_{(3)}\rvert^2$
$B^- \to \bar{\Xi}_{c_2}^{0}\Xi^{*-}$	$\frac{2}{3}\lvert\alpha_{(\bar{6})} + \alpha_{(3)}\rvert^2$
$B^0 \to \bar{\Sigma}_c^{-}\Sigma^{*+}$	$\frac{2}{3}\lvert\beta_{(\bar{6})}\rvert^2$
$B^0 \to \bar{\Sigma}_c^{0}\Sigma^{*0}$	$\frac{2}{3}\lvert\beta_{(\bar{6})}\rvert^2$
$B^0 \to \bar{\Xi}_{c_2}^{0}\Xi^{*0}$	$\frac{2}{3}\lvert\beta_{(\bar{6})}\rvert^2$
$B_s^0 \to \bar{\Sigma}_c^{--}\Delta^{++}$	$\lvert\alpha_{(\bar{6})} - \alpha_{(3)}\rvert^2$
$B_s^0 \to \bar{\Sigma}_c^{-}\Delta^{+}$	$\frac{2}{3}\lvert\alpha_{(\bar{6})} - \alpha_{(3)}\rvert^2$
$B_s^0 \to \bar{\Sigma}_c^{0}\Delta^{0}$	$\frac{1}{3}\lvert\alpha_{(\bar{6})} - \alpha_{(3)}\rvert^2$
$B_s^0 \to \bar{\Omega}_c\Xi^{*0}$	$\frac{1}{3}\lvert\alpha_{(\bar{6})} - \alpha_{(3)} + 2\beta_{(\bar{6})}\rvert^2$
$B_s^0 \to \bar{\Xi}_{c_2}^{-}\Sigma^{*+}$	$\frac{2}{3}\lvert\alpha_{(\bar{6})} - \alpha_{(3)} + \beta_{(\bar{6})}\rvert^2$
$B_s^0 \to \Xi_{c_2}^{0}\Sigma^{*0}$	$\frac{1}{3}\lvert\alpha_{(\bar{6})} - \alpha_{(3)} + \beta_{(\bar{6})}\rvert^2$

CHAPTER 5. NON-BARYONIC B-MESON DECAYS

In this chapter we investigate the prediction of $SU(3)_f$ for the decay of B-mesons to final states that do not contain baryons. In section 5.1 the B-meson decays to two lower mass mesons are explored. In particular we look at the final states DM, DV, $D\overline{D}$, $J/\psi M$ and MM, where D represents a D-Meson and M denotes a member of the lowest lying pseudoscalar octet of mesons. The three body decay modes are examined in section 5.2. The decay to final states J/ψ, DMM and $D\overline{D}M$ are discussed. In the last section of the chapter, section 5.3, closing remarks concerning non-baryonic B-meson decays are made.

5.1 Two–Body Non-Baryonic B-Meson Decays

We first consider $\Delta b = -1$, $\Delta c = 1$ decays of the type $B \to DM$ where D denotes one of the D–mesons D^0, D^+ and D_s^+. M is one of the eight lowest–lying 0^- mesons $\pi, K, \overline{K}, \eta$. As we discussed in section 4.1 of Chapter 4, the B mesons like the D mesons transform as anti–triplets under $SU(3)_f$. They are written as row vectors with components B_i, as in Eq (4.1). As a reminder we write them again,

$$B \;=\; (B^-, B^0, B^0_s) \quad . \tag{5.1}$$

We are interested in the transition amplitudes $A(B \to DM) = \langle DM|H_{\text{eff}}|B\rangle$. As far as the group theory is concerned we can imagine these amplitudes arising from the effective Hamiltonian

$$H_{\text{eff}} \;=\; a(B_i\bar{D}^i)(M^k_\ell H^\ell_k) \;+\; b(B_i M^i_k H^k_j \bar{D}^j) \;+\; c(B_i H^i_k M^k_j \bar{D}^j) \tag{5.2}$$

with H^ℓ_k given by Eq. (4.3). Expanding out these three terms gives the results for the Cabibbo–allowed decays shown in Table 5.1 and for the Cabibbo–suppressed decays shown in Table 5.2.

There is one $SU(3)_f$ relation amongst the decay amplitudes, $A(B \to DM)$, for the Cabibbo allowed decays

$$|A(B^0 \to D^0\pi^0)|^2 + 3|A(B^0 \to D^0\eta^0)|^2 = |A(B^0 \to K^- D_s^+)|^2 + |A(B_s^0 \to D^0 K^0)|^2 \quad . \tag{5.3}$$

There are several simple relations between the Cabibbo–allowed and the Cabibbo–suppressed decay rates. For example, SU(3) symmetry implies that

$$\frac{A(B^- \to D^0\pi^-)}{A(B^- \to D^0 K^-)} = -1/s_1 \quad . \tag{5.4}$$

The large value of the B–meson mass (compared with the QCD scale) suggests that relative complex phases between the reduced matrix elements a, b and c, which can be generated by final state strong interactions, are small. If this is the case then, up to sign ambiguities, measuring three of the Cabibbo–allowed decays determines a, b and c. At the present time there are measurements of the branching ratios for two of the decays in Table 5.1. Experimentally [5.2, 5.2]:

$$Br(B^- \to D^0\pi^-) = (3.0 \pm 1.4) \times 10^{-3}, \tag{5.5a}$$

$$Br(B^0 \to D^+\pi^-) = (3.6 \pm 1.4) \times 10^{-3}. \tag{5.5b}$$

Although we have focussed on decays of the type $B \to DM$ our results can be trivially taken over for the decays of the form $B \to D^* M$. Also, for decays not involving the η we can use the results in Tables 5.1 and 5.2 for the corresponding decays $B \to DV$ and $B \to D^* V$ where V is one of the $1^- \rho$ or K^* vector mesons. So, for example, generalizations of Eq. (5.4) of the type

$$\frac{A(B^- \to D^{*0}\pi^-)}{A(B^- \to D^{*0} K^-)} = \frac{A(B^- \to D^0\rho^-)}{A(B^- \to D^0 K^{*-})} = \frac{A(B^- \to D^{*0}\rho^-)}{A(B^- \to D^{*0} K^{*-})} = -1/s_1 \tag{5.6}$$

hold. Experimentally [5.1, 5.2, 5.3, 5.4, 5.5]

$$Br(B^0 \to D^*(2010)^+\pi^-) = \left(3.3 \begin{matrix} +1.2 \\ -1.0 \end{matrix}\right) \times 10^{-3}, \tag{5.7a}$$

$$Br(B^0 \to D^*(2010)^+\rho^-) = \left(8\,{}^{+7}_{-4}\right) \times 10^{-2}, \tag{5.7b}$$

$$Br(B^- \to D^*(2010)^0\pi^-) = (3 \pm 4) \times 10^{-3}, \tag{5.7c}$$

$$Br(B^- \to D^0\rho^-) = (2.1 \pm 1.2) \times 10^{-2}, \tag{5.7d}$$

$$Br(B^0 \to D^+\rho^-) = (2.2 \pm 1.5) \times 10^{-2}. \tag{5.7e}$$

Two–body decays of the type $B \to D\bar{D}$ arise from weak Hamiltonians with flavor quantum numbers $(b\bar{c})(c\bar{s})$ for the Cabibbo–allowed decays and $(b\bar{c})(c\bar{d})$ for the Cabibbo–suppressed decays. These Hamiltonians are different components of the same anti–triplet representation. So, as far as group theory is concerned, the decays $B \to D\bar{D}$ can be thought of as arising from an effective Hamiltonian

$$H_{\text{eff}} \;=\; \alpha(B_i H^i)(D_j \bar{D}^j) \;+\; \beta(B_i \bar{D}^i)(H^j D_j) \tag{5.8}$$

where for the Cabibbo allowed decays

$$H \;=\; \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{5.9a}$$

and for the Cabibbo suppressed decays

$$H \;=\; \begin{pmatrix} 0 \\ s_1 \\ 0 \end{pmatrix} \quad . \tag{5.9b}$$

Table 5.3 presents results for decays $B \to D\bar{D}$ which follow from Eq. (5.8). Some of the relations that follow from Table 5.3 are just consequences of SU(2) isospin symmetry. They are, for the Cabibbo–allowed decays [5.6]

$$\Gamma(B^- \to D^0 D_s^-) \;=\; \Gamma(B^0 \to D^+ D_s^-) \quad , \tag{5.10a}$$

$$\Gamma(B_s^0 \to D^0 \bar{D}^0) \;=\; \Gamma(B_s^0 \to D^+ D^-) \quad . \tag{5.10b}$$

For the Cabibbo suppressed decays there are no isospin relations. Table 5.3 indicates that there are several $SU(3)_f$ relations between the Cabibbo–allowed and the

Cabibbo-suppressed $B \to D\bar{D}$ decays (e.g., $\Gamma(B^- \to D^0 D^-) = s_1^2 \Gamma(B^- \to D^0 D_s^-)$). The Cabibbo suppressed $B \to D\bar{D}$ decays also get contributions from terms in the effective weak Hamiltonian that have no charm quarks. For example, the operator $(b\bar{u})(u\bar{d})$ which transforms under SU(3) as $\overline{3} \oplus 6 \oplus \overline{15}$ contributes to the Cabibbo-suppressed $B \to D\bar{D}$ decays and if s_3 is near its experimental limit, this could alter the results in Table 5.3.

The same Hamiltonians which give rise to the decays $B \to D\bar{D}$ also cause the decays $B \to J/\psi M$. Since there is only one way to combine the product of a triplet, antitriplet and octet representations (J/ψ transforms as a singlet) into a singlet, these decays are characterized by a single reduced matrix element. SU(3) predictions for these decays are presented in Table 5.4. The contributions of operators without charm quarks (e.g., $(b\bar{u})(u\bar{d})$) to the Cabibbo-suppressed decays $B \to J/\psi M$ are negligible because they violate the Okubo–Zweig–Iizuka (OZI) rule. For the Cabibbo allowed decays the relation

$$\Gamma(B^0 \to J/\psi \bar{K}^0) = \Gamma(B^- \to J/\psi K^-) \quad , \tag{5.11}$$

is a consequence of isospin invariance. It has previously been noted that a comparison of branching ratios for these modes would determine the ratio of B^0 and B^- lifetimes [5.7]. At the present time it is known that [1.6] $0.4 < \tau_{B^0}/\tau_{B^-} < 2.1$. One of the relevant branching ratios has been measured:[8),14),15)]

$$Br(B^- \to J/\psi K^-) = (8.0 \pm 2.8) \times 10^{-4} \quad . \tag{5.12}$$

For the Cabibbo suppressed decays the relation

$$\Gamma(B^0 \to J/\psi \pi^0) = \frac{1}{2}\, \Gamma(B^- \to J/\psi \pi^-) \quad , \tag{5.13}$$

is also a consequence of isospin invariance.

The results of Table 5.4 generalize trivially to other $c\bar{c}$ resonances and also to decays $B \to J/\psi V$, where V is a $1^-\rho$ or K^* meson. There is also some experimental information on these decays [5.2, 5.8]:

$$Br(B^- \to \psi(2S)K^-) = (2.2 \pm 1.7) \times 10^{-3}, \tag{5.14a}$$

$$Br(B^0 \to J/\psi \bar{K}^*(892)^0) = (3.7 \pm 1.3) \times 10^{-3}. \tag{5.14b}$$

We consider next the $SU(3)_f$ relations between the decay amplitudes which can arise from the $b \to uW^-$ weak coupling. For final states without charm the effective Hamiltonian has the flavor quantum numbers of the operator $(b\bar{u})(u\bar{d})$ which transforms as $\overline{15} \oplus 6 \oplus \overline{3}$. Explicitly, the decomposition of $(b\bar{u})(u\bar{d})$ into operators that are in irreducible $SU(3)_f$ representations is

$$(b\bar{u})(u\bar{d}) = \frac{1}{8}O_{(\overline{15})} + \frac{1}{4}O_{(6)} - \frac{1}{8}O_{(\overline{3})} + \frac{3}{8}O'_{(\overline{3})} \tag{5.15}$$

where

$$O_{(\overline{15})} = 3(b\bar{u})(u\bar{d}) + 3(b\bar{d})(u\bar{u}) - 2(b\bar{d})(d\bar{d}) - (b\bar{s})(s\bar{d}) - (b\bar{d})(s\bar{s}) \tag{5.16a}$$

$$O_{(6)} = (b\bar{u})(u\bar{d}) - (b\bar{d})(u\bar{u}) - (b\bar{s})(s\bar{d}) + (b\bar{d})(s\bar{s}) \tag{5.16b}$$

$$O_{(\overline{3})} = (b\bar{d})(u\bar{u}) + (b\bar{d})(d\bar{d}) + (b\bar{d})(s\bar{s}) \tag{5.16c}$$

$$O'_{(\overline{3})} = (b\bar{u})(u\bar{d}) + (b\bar{d})(d\bar{d}) + (b\bar{s})(s\bar{d}) \quad . \tag{5.16d}$$

In Eqs. (5.15) and (5.16) the subscripts on the operators denote the irreducible representation of $SU(3)_f$ to which they belong. As far as group theory factors are concerned we can take as the effective Hamiltonian for B–meson decays $B \to MM$

$$\begin{aligned} H_{\text{eff}} = & A_{(\overline{3})} B_i H(\overline{3})^i (M_\ell^k M_k^\ell) \ + \ C_{(\overline{3})} B_i M_k^i M_j^k H(\overline{3})^j \\ & + A_{(\overline{15})} B_i H(\overline{15})_k^{ij} M_j^\ell M_\ell^k \ + \ C_{(\overline{15})} B_i M_j^i H(\overline{15})_\ell^{jk} M_k^\ell \\ & + A_{(6)} B_i H(6)_k^{ij} M_j^\ell M_\ell^k \quad . \end{aligned} \tag{5.17}$$

In eq. (24) $H(\overline{3})$ is a vector with non–zero component

$$H(\overline{3})^2 = 1 \quad . \tag{5.18a}$$

$H(\overline{15})$ is a traceless three–index tensor that is symmetric on its upper indices and has non–zero components

$$H(\overline{15})^{12}_1 = 3\ ,\ H(\overline{15})^{21}_1 = 3\ ,\ H(\overline{15})^{22}_2 = -2\ ,\ H(\overline{15})^{32}_3 = -1\ ,\ H(\overline{15})^{23}_3 = -1 \quad . \tag{5.18b}$$

Finally, in Eq. (5.17), $H(6)$ is a traceless three–index tensor that is antisymmetric on its upper indices and has non–zero components

$$H(6)^{12}_1 = 1\ ,\ H(6)^{21}_1 = -1\ ,\ H(6)^{32}_3 = -1\ ,\ H(6)^{23}_3 = 1 \quad . \tag{5.18c}$$

The parameters $A_{(\overline{3})}$, $C_{(\overline{3})}$, $A_{(\overline{15})}$, $C_{(\overline{15})}$ and $A_{(6)}$ are the reduced matrix elements in terms of which the $B \to MM$ decay amplitudes are expressed. Note that since

$$B_i H(6)^{ij}_k M^\ell_j M^k_\ell + B_i M^i_\ell H(6)^{\ell j}_k M^k_j = 0 \quad , \tag{5.19}$$

there is only one reduced matrix element, $A_{(6)}$ parametrizing the contribution of the part of the Hamiltonian that transforms as a 6.

Table 5.5 summarizes the $SU(3)_f$ predictions that follow from expanding the effective Hamiltonian in Eq. (5.17). There is only one simple relation

$$\frac{A(B^0_s \to K^0\pi^0)}{A(B^0_s \to K^0\eta^0)} = \sqrt{3} \quad . \tag{5.20}$$

There are no simple isospin relations between the $B \to MM$ decay rates in Table 5.5. The effective Hamiltonian has both $I = 1/2$ and $I = 3/2$ pieces. The $I = 3/2$ piece arises solely from the operator $O_{(\overline{15})}$. In the decays $B \to \pi\pi$ the two–pion final state is a linear combination of $I = 0$ and $I = 2$ states. The $I = 2$ state can only be reached through the $I = 3/2$ part of the effective Hamiltonian while the $I = 0$ state gets contributions from both the $I = 1/2$ and $I = 3/2$ parts. Since the $\pi^0\pi^-$ state is charged it is pure $I = 2$, consequently the rate for $B^- \to \pi^0\pi^-$ originates only from the matrix element of $O_{(\overline{15})}$.

There are contributions to the decays $B \to MM$ listed in Table 5.5, which survive in the limit $s_3 \to 0$ (where the $b \to uW^-$ coupling is absent). They come from penguin–type Feynman diagrams with a charm or top quark in the loop (see Fig. 1). Writing

$$A_{(\overline{3})} = -s_1 s_3 \hat{A}_{(\overline{3})} - e^{i\delta} s_1 s_2 \hat{A}'_{(\overline{3})} \quad , \tag{5.21a}$$

$$C_{(\overline{3})} = -s_1 s_3 \hat{C}_{(\overline{3})} - e^{i\delta} s_1 s_2 \hat{C}'_{(\overline{3})} \quad , \tag{5.21b}$$

it is $\hat{A}'_{(\overline{3})}$ and $\hat{C}'_{(\overline{3})}$ that characterize these contributions, since the penguin–type diagrams only give rise to terms that transform as a $\overline{3}$ in the effective Hamiltonian of Eq. (5.17). The contribution of penguin–type Feynman diagrams is probably suppressed by $[\alpha_s(m_b)/\pi]$, so unless (s_3/s_2) is very small (a prospect that is unlikely if the standard six–quark model is to describe the CP violation observed in kaon decays)[5.10] $\hat{A}'_{(\overline{3})}$ and $\hat{C}'_{(\overline{3})}$ are unimportant for the $B \to MM$ decays in Table 5.5. However, if we examine $B \to MM$ decays that change strangeness by one unit, the situation is quite different. Here the penguin–type diagrams are again suppressed by $\alpha_s(m_b)/\pi$ but they are enhanced (over operators like $(b\bar{u})(u\bar{s})$) by the ratio of weak mixing angles

$$\frac{(s_2^2 + s_3^2 + 2s_2 s_3 c_\delta)^{1/2}}{s_1^2 s_3} \quad . \tag{5.22}$$

These decays may be dominated by the penguin–type diagrams with a charm or top quark in the loop. Assuming this is the case we can use, as far as group theory factors are concerned, the following effective Hamiltonian to describe the $\Delta s = -1$ $B \to MM$ decays

$$H_{\text{eff}} = -(s_2 e^{i\delta} + s_3)\left[\hat{A}'_{(\overline{3})}\left(B_i H(\overline{3})^i\right)\left(M_\ell^k M_k^\ell\right) + \hat{C}'_{(\overline{3})} B_i M_k^i M_j^k H(\overline{3})^j\right] \quad , \tag{5.23}$$

where now the non-zero component of $H(\overline{3})$ is

$$H(\overline{3})^3 = 1 \quad . \tag{5.24}$$

Table 5.6 gives the $SU(3)_f$ predictions that follow from Eqs. (5.23) and (5.24). Note that since $\hat{A}'_{(\overline{3})}$ only effects the B_s^0 decays, the ratios of the various $\Delta s = -1$ $B^0 \to MM$ and $B^- \to MM$ decay rates are determined by $SU(3)_f$.

Our assumption that penguin-type diagrams dominate the $\Delta s = -1$ $B \to MM$ decays implies that the effective Hamiltonian is $I = 0$. Since there is only one way to combine two $I = 1/2$ states into an $I = 1$ state, all the relations between $B \to K\pi$ decays in Table 5.6 are consequences of isospin. Similar isospin relations hold for decays of the type $B \to K\rho$, $B \to K^*\pi$ and $B \to K^*\rho$. The relations

$$\Gamma(B^0 \to \bar{K}^0\eta) = \Gamma(B^- \to K^-\eta) \tag{5.25a}$$

$$\Gamma(B_s^0 \to K^+K^-) = \Gamma(B_s^0 \to K^0\bar{K}^0) \tag{5.25b}$$

$$\Gamma(B_s^0 \to \pi^0\pi^0) = \frac{1}{2}\Gamma(B_s^0 \to \pi^+\pi^-) \tag{5.25c}$$

are also consequences of isospin symmetry. The $(b\bar{u})(u\bar{s})$ operator has both $I = 0$ and $I = 1$ pieces. Verifying some of the above isospin relations would provide strong evidence that penguin-type diagrams dominate the $\Delta s = -1$ $B \to MM$ decays.

The $b \to uW^-$ coupling also causes $\Delta b = -1$, $\Delta c = -1$ decays $B \to \bar{D}M$. To leading order in weak mixing angles the effective Hamiltonian for such decays has the flavor quantum numbers of $(b\bar{u})(c\bar{s})$. Under SU(3) this operator transforms as $3 \oplus \overline{6}$. Explicitly, the decomposition in terms of operators in irreducible representations is

$$(b\bar{u})(c\bar{s}) = O_{(3)} + O_{(\overline{6})} \tag{5.26}$$

where

$$O_{(3)} = \frac{1}{2}\ [(b\bar{u})(c\bar{s}) - (b\bar{s})(c\bar{u})] \tag{5.27a}$$

$$O_{(\overline{6})} = \frac{1}{2}\ [(b\bar{u})(c\bar{s}) + (b\bar{s})(c\bar{u})] \quad . \tag{5.27b}$$

As far as group theory factors are concerned we can take as the effective Hamiltonian for the $\Delta b = -1$, $\Delta c = -1$ decays $B \to \bar{D}M$

$$\begin{aligned} H_{\text{eff}} &= \alpha_{(\overline{6})} D_i H(\overline{6})^{ij} B_k M_j^k + \beta_{(\overline{6})} B_i H(\overline{6})^{ij} D_k M_j^k \\ &+ \alpha_{(3)} D_i H(3)^{ij} B_k M_j^k + \beta_{(3)} B_i H(3)^{ij} D_k M_j^k \quad . \end{aligned} \tag{5.28}$$

Here $H(\overline{6})$ is a two–index symmetric tensor with non–zero components

$$H(\overline{6})^{13} = 1 \quad , \quad H(\overline{6})^{31} = 1 \tag{5.29}$$

and $H(3)$ is a two–index antisymmetric tensor with non–zero components

$$H(3)^{13} = 1 \quad , \quad H(3)^{31} = -1 \quad . \tag{5.30}$$

Table 5.7 shows the results which follow from the effective Hamiltonian in Eq. (5.28). There are two simple relations

$$\Gamma(B_s^0 \to D^- \pi^+) = 2\Gamma(B_s^0 \to \bar{D}^0 \pi^0) \tag{5.31a}$$

$$\Gamma(B^0 \to D_s^- \pi^+) = 2\Gamma(B^- \to D_s^- \pi^0) \tag{5.31b}$$

and they are a consequence of isospin invariance.

There is a small dynamical enchancement of the Wilson coefficient of $O_{(3)}$ over that of $O_{(\overline{6})}$ coming from perturbative QCD corrections. In the effective Hamiltonian for $\Delta b = -1$, $\Delta c = -1$ decays the ratio of Wilson coefficients for $O_{(3)}$ and $O_{(\overline{6})}$ is [2.1] $[\alpha_s(m_b)/\alpha_s(m_W)]^{(18/23)} \approx 1.5$. If either the matrix elements of $O_{(3)}$ or $O_{(\overline{6})}$ dominate the $B \to \bar{D}M$ decays, then Table 5.7 indicates that there would be some $SU(3)_f$ relations. For example, either 3 or $\overline{6}$ dominances implies that

$$|A(B^0 \to \bar{D}^0 \bar{K}^0)| = |A(B^0 \to D_s^- \pi^+)| \quad . \tag{5.32}$$

Of course, generalizations of Table 5.7's results to decays $B \to \bar{D}^* M$, $B \to \bar{D}V$ and $B \to \bar{D}^* V$ hold.

5.2 Three–Body Non-Baryonic B-Meson Decays

The three-body decays, $B \to J/\psi MM$, can have the relative orbital angular momentum, L, of the two M–mesons be either even or odd. For the case L even we can take, as far as group theory factors are concerned,

$$H_{\text{eff}} = [F(B_i H^i)(M_j^k M_k^j) + G(B_i M_\ell^i M_k^\ell H^k)](J/\psi) \quad , \tag{5.33}$$

as the effective Hamiltonian (Lorentz indices are suppressed). In Eq. (5.33)

$$H = \begin{pmatrix} 0 \\ s_1 \\ 1 \end{pmatrix} \quad . \tag{5.34}$$

Table 5.8 presents the results that follow from the effective Hamiltonian in Eq. (5.33). For the case L odd the effective Hamiltonian must be change sign under interchange of the $SU(3)_f$ quantum numbers of the two M mesons. Only the second term in Eq. (5.33) can be antisymmetrized and so the rates for $B \to J/\psi(MM)_{L=1,3\ldots}$ are determined in terms of a single reduced matrix element. Table 5.9 presents the relative $B \to J/\psi MM$ decay rates for odd L. With L odd the amplitudes for $B^- \to J/\psi\pi^-\eta^0$ and $B^0 \to J/\psi\pi^0\eta^0$ vanish by $SU(3)_f$ symmetry and therefore these processes don't appear in Table 5.9.

The Cabibbo allowed $B \to J/\psi MM$ decays arise from an effective Hamiltonian that is an isosinglet. There are several isospin relations among the Cabibbo allowed decays that hold independent of L. They are

$$\Gamma(B^0 \to J/\psi\pi^+K^-) = \Gamma(B^- \to J/\psi\pi^-\bar{K}^0)$$
$$= 2\Gamma(B^0 \to J/\psi\pi^0\bar{K}^0) = 2\Gamma(B^- \to J/\psi\pi^0K^-) \tag{5.35a}$$
$$\Gamma(B^0 \to J/\psi\eta^0\bar{K}^0) = \Gamma(B^- \to J/\psi\eta^0K^-) \tag{5.35b}$$
$$\Gamma(B_s^0 \to J/\psi K^0\bar{K}^0) = \Gamma(B_s^0 \to J/\psi K^+K^-) \tag{5.35c}$$

and

$$\frac{1}{2}\,\Gamma(B_s^0 \to J/\psi\pi^+\pi^-) = \Gamma(B_s^0 \to J/\psi\pi^0\pi^0) \quad . \tag{41d}$$

In the case $B_s^o \to J/\psi\pi\pi$ (Eq. (5.35d)) isospin invariance forces the two pions to be in an even L state. The effective Hamiltonian for Cabibbo–suppressed $B \to J/\psi MM$ decays is $I = 1/2$. Again, there are isospin relations which are L independent. They are

$$\Gamma(B^0 \to J/\psi\pi^0\eta^0) = \frac{1}{2}\,\Gamma(B^- \to J/\psi\pi^-\eta^0) \tag{5.36a}$$

$$\Gamma(B_s^0 \to J/\psi K^0\pi^0) = \frac{1}{2}\,\Gamma(B_s^0 \to J/\psi K^+\pi^-) \quad . \tag{5.36b}$$

Some isospin relations that hold only for L even are

$$\Gamma(B^0 \to J/\psi\pi^0\pi^0) = \frac{1}{2}\,\Gamma(B^0 \to J/\psi(\pi^+\pi^-)_{L=0,2,\ldots}) \tag{5.37a}$$

$$\Gamma(B^0 \to J/\psi(\pi^0\pi^-)_{L=0,2,\ldots}) = 0 \quad . \tag{5.37b}$$

There are also $SU(3)_f$ relations between the Cabibbo–allowed and Cabibbo–suppressed decay amplitudes that are L independent. For example, two such relations are

$$s_1^2|A(B^- \to J/\psi\pi^-\bar{K}^0)|^2 = |A(B^- \to J/\psi K^-K^0)|^2 \tag{5.38a}$$

$$s_1^2|A(B^0 \to J/\psi\pi^+K^-)|^2 = |A(B_s^0 \to J/\psi K^+\pi^-)|^2 \quad . \tag{5.38b}$$

All the L odd $B \to J/\psi MM$ decays are related by $SU(3)_f$ flavor symmetry. However, there is an important source of $SU(3)_f$ violation for resonant MM pairs. There is significant mixing between the lowest–lying $SU(3)_f$ singlet and octet 1^- mesons resulting in the ϕ and w mass eigenstates with flavor quantum numbers $s\bar{s}$ and $\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$, respectively. This occurs not because of an anomalously large $SU(3)_f$ violating mass mixing element, but rather because of the near degeneracy of the $SU(3)_f$ singlet and octet states. Nonetheless, because the decay of the w to $K\bar{K}$ is kinematically forbidden, this mass mixing can result in large violations of our $SU(3)_f$ predictions for decays $B \to J/\psi(MM)_{L=1}$ when the MM pair is resonant.

Next we consider the implications of $SU(3)_f$ symmetry for decays $B \to DMM$. As was noted in section 5.1, the effective Hamiltonian for these decays transforms as an octet under flavor $SU(3)_f$. Again, we shall separately treat the cases where the relative orbital angular momentum L of the MM pair is even and odd. As far as group theory factors are concerned, when L is even we can take as our effective Hamiltonian

$$H_{\rm eff} = aB_i M^i_j M^k_\ell H^j_k \bar{D}^\ell + bB_i M^i_j M^j_k H^k_\ell \bar{D}^\ell + cB_i H^i_j M^j_k M^k_\ell \bar{D}^\ell \\ + d(B_i M^i_j \bar{D}^j)(M^k_\ell H^\ell_k) + e(B_i H^i_j \bar{D}^j)(M^k_\ell M^\ell_k) + f(B_i \bar{D}^i)(M^j_k M^k_\ell H^\ell_j). \quad (5.39)$$

In Eq. (5.39) H^i_j are elements of the 3×3 matrix (upper index labelling rows and lower index labelling columns)

$$H = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ -s_1 & 0 & 0 \end{pmatrix} \quad . \quad (5.40)$$

Table 5.10 presents the results that follow from this effective Hamiltonian for the Cabibbo allowed decays. Under isospin the effective Hamiltonian for the Cabibbo allowed decays is $I = 1$. When two pions possessing a net charge are in an even partial wave they form an $I = 2$ state. This implies the following isospin relation

$$(1/4)\Gamma(B^- \to D^+\pi^-\pi^-) = \Gamma(B^- \to D^0(\pi^-\pi^0)_{L=0,2...}) \\ = \Gamma(B^0 \to D^+(\pi^-\pi^0)_{L=0,2,...}) \quad . \quad (5.41)$$

The first process that appears in Eq. (5.41) has been observed. Experimentally [5.2]

$$Br(B^- \to D^+\pi^-\pi^-) = \left(2.5 \begin{matrix} +4.8 \\ -2.4 \end{matrix}\right) \times 10^{-3} \quad . \quad (5.42)$$

Since the amplitudes with L odd don't interfere with those with L even, we conclude that

$$4\Gamma(B^- \to D^0\pi^0\pi^-) > \Gamma(B^- \to D^+\pi^-\pi^-) \qquad (5.43a)$$
$$4\Gamma(B^0 \to D^+\pi^-\pi^0) > \Gamma(B^- \to D^+\pi^-\pi^-) \quad . \qquad (5.43b)$$

Of course, the results in Table 5.10 generalize to decays involving a D^* instead of a D. Experimentally [5.2, 5.3]

$$Br(B^- \to D^*(2010)^+\pi^-\pi^-) = \left(2.5 \begin{matrix} +1.5 \\ -1.3 \end{matrix}\right) \times 10^{-3} \qquad (5.44a)$$
$$Br(B^0 \to D^*(2010)^+\pi^-\pi^0) = (1.5 \pm 1.1) \times 10^{-2} \qquad (5.44b)$$

which is consistent with the generalization of Eq. (5.43b) and indicates that the $B^0 \to D^{*+}\pi^-\pi^0$ rate is dominated by L odd.

There are also some $SU(3)_f$ relations between the Cabibbo allowed amplitudes with L even. They are

$$|A\left(B^0 \to D_s^+(K^-\pi^0)_{L=0,2,\ldots}\right)|^2 = 3|(B^0 \to D_s^+(K^-\eta^0)_{L=0,2})|^2 \qquad (5.45a)$$
$$|A\left(B^- \to D_s^+(\pi^-K^-)_{L=0,2,\ldots}\right)|^2 = \frac{1}{2}|A(B^- \to D^+\pi^-\pi^-)|^2 \qquad (5.45b)$$
$$|A\left(B_s^0 \to D^+(K^0\pi^-)_{L=0,2,\ldots}\right)|^2 = \frac{1}{2}|A(B^- \to D^+\pi^-\pi^-)|^2 \qquad (5.45c)$$
$$|A\left(B_s^0 \to D^0(K^0\eta^0)_{L=0,2,\ldots}\right)|^2 = \frac{1}{3}|A\left(B_s^0 \to D^0(K^0\pi^0)_{L=0,2\ldots}\right)|^2. \quad (5.45d)$$

For the case L odd, the effective Hamiltonian must be antisymmetric under interchange of the flavor quantum numbers of the M mesons. For example, an antisymmetric version the term proportional to a in Eq. (5.39) is

$$(\partial^\mu B_i)[(\partial_\mu M_j^i)M_\ell^k - M_j^i(\partial_\mu M_\ell^k)]\bar{D}^\ell H_k^j \quad . \qquad (5.46)$$

Only the term proportional to the reduced matrix element e has no antisymmetric analog. So the $B \to DMM$ decay amplitudes with L odd are parametrized by *five reduced* matrix elements which we denote by a', b', c', d', and f'. Table 5.11 presents the implications of the SU(3) flavor symmetry of the strong interactions for the Cabibbo–allowed decays $B \to D(MM)_{L=1,3,\ldots}$. There are several $SU(3)_f$ relations. For example

$$|A\,(B^- \to D^0(\eta\pi^-)_{L=1,3,\ldots})\,|^2 \;=\; \frac{1}{6}|A\,(B^- \to D_s^+(\pi^- K^-)_{L=1,3,\ldots})\,|^2 \quad . \qquad (5.47)$$

Tables 5.12 and 5.13 present the predictions of SU(3) flavor symmetry for the Cabibbo–suppressed $B \to DMM$ decays with L even and L odd, respectively. Since the Hamiltonian for the Cabibbo–suppressed decays is part of the same octet as the Hamiltonian for the Cabibbo–allowed decays, we can express the Cabibbo–suppressed decay rates in terms of the same reduced matrix elements as were used for the Cabibbo–allowed decays. An inspection of tables 5.10, 5.11, 5.12, and 5.13 reveals that there are simple $SU(3)_f$ relations between the Cabibbo–allowed decays and the Cabibbo–suppressed decays which hold independent of the value of L. They are

$$|A(B^0 \to D^+\pi^-\bar{K}^0)|^2 = s_1^2|A(B_s^0 \to D_s^+K^-K^0)|^2 \qquad (5.48a)$$

$$|A(B^0 \to D_s^+\bar{K}^0K^-)|^2 = s_1^2|A(B_s^0 \to D^+K^0\pi^-)|^2 \qquad (5.48b)$$

$$|A(B^- \to D_s^+K^-K^-)|^2 = s_1^2|A(B^- \to D^+\pi^-\pi^-)|^2 \qquad (5.48c)$$

$$|A(B^- \to D^0\pi^-\bar{K}^0)|^2 = s_1^2|A(B^- \to D^0K^-K^0)|^2 \qquad (5.48d)$$

$$2|A(B_s^0 \to D_s^+\pi^0K^-)|^2 = s_1^2|A(B^0 \to D^+K^-K^0)|^2 \qquad (5.48e)$$

$$|A(B_s^0 \to D^0\pi^+\pi^-)|^2 = s_1^2|A(B^0 \to D^0K^-K^+)|^2 \qquad (5.48f)$$

$$|A(B_s^0 \to D^0\bar{K}^0K^0)|^2 = s_1^2|A(B^0 \to D^0\bar{K}^0K^0)|^2 \qquad (5.48g)$$

$$|A(B_s^0 \to D^0K^+K^-)|^2 = s_1^2|A(B^0 \to D^0\pi^+\pi^-)|^2 \qquad (5.48h)$$

$$|A(B_s^0 \to D^+K^0K^-)|^2 = s_1^2|A(B^0 \to D_s^+\bar{K}^0\pi^-)|^2 \quad . \qquad (5.48i)$$

There are important sources of $SU(3)_f$ violation in the decays $B \to DMM$ which can occur when two of the final state particles arise from the decay of a resonance. In addition to the consequences of the mixing of the SU(3) singlet and SU(3) octet 1^-

vector mesons mentioned earlier, large SU(3) violations can arise because the D^* can decay to $D\pi$ while the D_s^* is kinematically forbidden from decaying to $D_s\pi$ or DK.

Finally, we consider the three–body decays $B \to D\bar{D}M$. As far as group theory factors are concerned we can take as the effective Hamiltonian for these processes

$$\begin{aligned} H_{\text{eff}} &= \eta_1(B_iH^i)(D_kM^k_\ell\bar{D}^\ell) + \eta_2(B_i\bar{D}^i)(D_kM^k_\ell H^\ell) \\ &+ \eta_3(B_iM^i_\ell\bar{D}^\ell)(D_kH^k) + \eta_4(B_iM^i_\ell H^\ell)(\bar{D}^kD_k) \end{aligned} \tag{5.49a}$$

where

$$H = \begin{pmatrix} 0 \\ s_1 \\ 1 \end{pmatrix} \quad . \tag{5.49b}$$

Tables 5.14 and 5.15 present the predictions that follow from the Hamiltonian for Cabibbo–allowed and Cabibbo–suppressed decays, respectively. Cautionary remarks, similar to those given in the case of $B \to DMM$ decays, concerning possible large $SU(3)_f$ violations induced from resonance effects also apply here.

The effective Hamiltonian for Cabibbo–suppressed decays given by Eq.(5.49) neglects the contribution of operators like $(b\bar{u})(u\bar{d})$, which arise from the $b \to uW^-$ coupling and transform as $\overline{3} \oplus 6 \oplus \overline{15}$. Since the $\overline{15}$ representation contains an $I = 3/2$ piece, isospin relations for the Cabibbo–suppressed modes which follow from the $I = 1/2$ Hamiltonian in Eq. (5.49) are useful for testing the dominance of the operators with charm quarks. Table 5.15 contains two isospin relations

$$\Gamma(B^- \to D_s^- D_s^+ \pi^-) = 2\Gamma(B^0 \to D_s^- D_s^+ \pi^0) \tag{5.50a}$$

$$\Gamma(B_s^0 \to D_s^+ \bar{D}^0 \pi^-) = 2\Gamma(B_s^0 \to D_s^+ D^- \pi^0) \quad . \tag{5.50b}$$

Fig. 2 shows quark line diagrams which illustrate how the two operators $(b\bar{c})(c\bar{d})$ and $(b\bar{u})(u\bar{d})$ can contribute to the decay $B_s^0 \to D_s^+ \bar{D}^0 \pi^-$.

For the Cabibbo–allowed decays, the effective Hamiltonian transforms as an isosinglet. The following relations in Table 5.14 follow from isospin symmetry:

$$\Gamma(B^- \to D^0 D^- \bar{K}^o) = \Gamma(B^0 \to D^+ \bar{D}^0 K^-) \tag{5.51a}$$

$$\Gamma(B^0 \to D^0 \bar{D}^0 \bar{K}^0) = \Gamma(B^- \to D^+ D^- K^-) \tag{5.51b}$$

$$\begin{aligned} 2\Gamma(B^- \to D_s^- D^0 \pi^0) &= \Gamma(B^0 \to D^0 D_s^- \pi^+) = \Gamma(B^- \to D_s^- D^+ \pi^-) \\ &= 2\Gamma(B^0 \to D^+ D_s^- \pi^0) \end{aligned} \tag{5.51c}$$

$$\Gamma(B^- \to D_s^+ D_s^- K^-) = \Gamma(B^0 \to D_s^+ D_s^- \bar{K}^0) \tag{5.51d}$$

$$\begin{aligned} \Gamma(B_s^0 \to \bar{D}^0 D^+ \pi^-) &= \Gamma(B_s^0 \to D^- D^0 \pi^+) = 2\Gamma(B_s^0 \to D^0 \bar{D}^0 \pi^0) \\ &= 2\Gamma(B_s^0 \to D^+ D^- \pi^0) \end{aligned} \tag{5.51e}$$

$$\Gamma(B_s^0 \to D_s^- D^0 K^+) = \Gamma(B_s^0 \to D_s^- D^+ K^0) \tag{5.51f}$$

$$\Gamma(B_s^0 \to D^- D_s^+ \bar{K}^0) = \Gamma(B_s^0 \to \bar{D}^0 D_s^+ K^-) \tag{5.51g}$$

$$\Gamma(B_s^0 \to D^+ D^- \eta^0) = \Gamma(B_s^0 \to D^0 \bar{D}^0 \eta^0) \quad . \tag{5.51h}$$

Comparison of Tables 5.14 and 5.15 reveals that there are many simple $SU(3)_f$ relations between the Cabibbo–allowed and the Cabibbo–suppressed $B \to D\bar{D}M$ decays. Some of them are

$$|A(B^- \to D^0 D_s^- K^0)|^2 = s_1^2 |A(B^- \to D^0 D^- \bar{K}^0)|^2 \tag{5.52a}$$

$$|A(B^- \to D_s^+ D^- K^-)|^2 = s_1^2 |A(B^- \to D_s^- D^+ \pi^-)|^2 \tag{5.52b}$$

$$|A(B^- \to D_s^+ D_s^- \pi^-)|^2 = s_1^2 |A(B^0 \to D^0 \bar{D}^0 \bar{K}^0)|^2 \tag{5.52c}$$

$$|A(B^- \to D^+ D^- \pi^-)|^2 = s_1^2 |A(B^- \to D_s^+ D_s^- K^-)|^2 \quad . \tag{5.52d}$$

5.3 Concluding Remarks On Chapter 5

In this chapter we have used the transformation properties of the weak Hamiltonian for nonleptonic B–meson decays to derive $SU(3)_f$ relations amongst many of the possible two– and three–body B–meson decays. Since the SU(2) isospin symmetry works much better than the full SU(3) emphasise has been placed on the predictions that follow from isospin. The isospin relations provide useful tools for discerning the importance of various competing effects that can occur in nonleptonic B–meson decays.

As we have discussed previously, it is possible to include, in a phenomenological fashion, some $SU(3)_f$ breaking effects and hence improve upon the results of this chapter. For example, in section 5.1 it was noted that generalizing the predictions for $B \to DM$ to $B \to DV$ where V is one of the low–lying 1^- mesons is not straightforward because of mixing of the $SU(3)_f$ octet state $|V_8\rangle = \frac{1}{\sqrt{6}}(|u\bar{u}\rangle + |d\bar{d}\rangle - 2\,|s\bar{s}\rangle)$ with the $SU(3)_f$ singlet $|V_1\rangle = \frac{1}{\sqrt{3}}(|u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle)$. This was discussed in Chapter 2.

As far as group theory is concerned we can take as our effective Hamiltonian for the decays $B \to DV$

$$H_{\rm eff} = a'(B_i\bar{D}^i)(V_\ell^k H_k^\ell) + b'(B_i V_k^i H_j^k \bar{D}^j) + c'(B_i H_k^i V_j^k \bar{D}^j) + e'(B_i H_j^i \bar{D}^j)V_1, \quad (5.53)$$

where H_k^ℓ given by Eq. (4.3). The amplitude $A(B^0 \to D^0\phi)$ is expected to be very small since the decay $B^0 \to D^0\phi$ is forbidden by the Okubo–Zweig–Iizuka (OZI) rule. Setting this amplitude to zero implies the relation

$$e' = \frac{b' + c'}{\sqrt{3}} \quad (5.54)$$

between reduced matrix elements. So the $B \to DV$ decay amplitudes are expressible in terms of the three reduced matrix elements, a', b' and c'. Using these expressions we find that the generalization of Eq. (5.3) is

$$\begin{aligned} |A(B^0 \to D^0\rho^0)|^2 + |A(B^0 \to D^0 w)|^2 &= |A(B^0 \to D_s^+ K^{*-})|^2 \\ &+ |A(B_s^0 \to D^0 K^{*0})|^2 \quad . \end{aligned} \quad (5.55)$$

As we discussed in Chapter 1, in the large N_c limit matrix elements for nonleptonic B–decays factorize. This provides a pattern of $SU(3)_f$ breaking that might be used to improve some of our results. For example, factorization suggests that

$$\frac{A(B^- \to D^0 K^-)}{A(B^- \to D^0 \pi^-)} = -\left(\frac{f_K}{f_\pi}\right) s_1 \quad , \tag{5.56}$$

would be an improvement over Eq. (5.4).

In this chapter we have focussed on $SU(3)_f$ predictions for nonleptonic B–meson decays to final states with mesons. It is also possible to consider $SU(3)_f$ predictions for B–meson decays to final states involving uncharmed baryons. For example, there may be $SU(3)_f$ relations between the Cabibbo allowed and the Cabibbo suppressed decays $B \to DN\bar{N}$, where N denotes a member of the lowest–lying baryon octet (consisting of the nucleons and hyperons). It is also possible to consider the consequences of SU(3) flavor symmetry for semileptonic B–meson decays. For example, since the effective Hamiltonian for $\Delta c = 1$ $B \to De\bar{\nu}_e$ decays is an SU(3) singlet, it follows that

$$\Gamma(B^0 \to D^+ e\bar{\nu}_e) = \Gamma(B^- \to D^0 e\bar{\nu}_e) = \Gamma(B_s^0 \to D_s^+ e\bar{\nu}_e) \quad . \tag{5.57}$$

The first equality in Eq. (5.57) follows from isospin. For semileptonic decays $B \to Me\bar{\nu}_e$ that don't change charm, the effective Hamiltonian transforms as an antitriplet. Since there is only one way to combine the product of a triplet, an antitriplet and an octet into a singlet, these decays are also related by $SU(3)_f$ flavor symmetry. In this case

$$\begin{aligned}\Gamma(B^0 \to \pi^+ e\bar{\nu}_e) &= 2\Gamma(B^- \to \pi^0 e\bar{\nu}_e) = \Gamma(B_s^0 \to K^+ e\bar{\nu}_e) \\ &= 6\Gamma(B^- \to \eta^0 e\bar{\nu}_e) \quad .\end{aligned} \tag{5.58}$$

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Table 5.1. Rates for Cabibbo–allowed decays $B \to DM$ in terms of the three reduced matrix elements a, b and c.

Process	Rate (divided by s_1^2)
$B^0 \to D^+\pi^-$	$\|a+c\|^2$
$B^0 \to D^0\pi^0$	$\frac{1}{2}\|b-c\|^2$
$B^0 \to D^0\eta^0$	$\frac{1}{6}\|b+c\|^2$
$B^0 \to D_s^+K^-$	$\|c\|^2$
$B^- \to D^0\pi^-$	$\|a+b\|^2$
$B_s^0 \to D^0K^0$	$\|b\|^2$
$B_s^0 \to D_s^+\pi^-$	$\|a\|^2$

Table 5.2. Rates for Cabibbo–suppressed decays $B \to DM$ in terms of the three reduced matrix elements a, b and c.

Process	Rate
$B^0 \to D^+K^-$	$s_1^2\|a\|^2$
$B^0 \to D^0\bar{K}^0$	$s_1^2\|b\|^2$
$B^- \to D^0K^-$	$s_1^2\|a+b\|^2$
$B_s^0 \to D^0\pi^0$	$s_1^2\frac{1}{2}\|c\|^2$
$B_s^0 \to D^0\eta^0$	$s_1^2\frac{1}{6}\|c-2b\|^2$
$B_s^0 \to D_s^+K^-$	$s_1^2\|a+c\|^2$
$B_s^0 \to D^+\pi^-$	$s_1^2\|c\|^2$

Table 5.3. Rates for B-meson decays of the type $B \to D\bar{D}$ in terms of the two reduced matrix elements α and β.

Process	Rate
$B^0 \to D^+ D_s^-$	$\|\beta\|^2$
$B^0 \to D^0 \bar{D}^0$	$s_1^2 \|\alpha\|^2$
$B^0 \to D^+ D^-$	$s_1^2 \|\alpha + \beta\|^2$
$B^0 \to D_s^+ D_s^-$	$s_1^2 \|\alpha\|^2$
$B^- \to D^0 D_s^-$	$\|\beta\|^2$
$B^- \to D^0 D^-$	$s_1^2 \|\beta\|^2$
$B_s^0 \to D^0 \bar{D}^0$	$\|\alpha\|^2$
$B_s^0 \to D^+ D^-$	$\|\alpha\|^2$
$B_s^0 \to D_s^+ D_s^-$	$\|\alpha + \beta\|^2$
$B_s^0 \to D_s^+ D^-$	$s_1^2 \|\beta\|^2$

Table 5.4. $SU(3)_f$ predictions for rates for $B \to J/\psi M$ normalized to the decay rate for $B^- \to J/\psi K^-$.

Process	Rate
$B^0 \to J/\psi \bar{K}^0$	1
$B^0 \to J/\psi \pi^0$	$s_1^2/2$
$B^0 \to J/\psi \eta^0$	$s_1^2/6$
$B^- \to J/\psi K^-$	1
$B^- \to J/\psi \pi^-$	s_1^2
$B_s^0 \to J/\psi \eta^0$	$2/3$
$B_s^0 \to J/\psi K^0$	s_1^2

Table 5.5. $SU(3)_f$ predictions for decays $B \to MM$ that do not change strangeness.

Process	Rate
$B^0 \to \pi^+\pi^-$	$\|2A_{(\bar{3})} + C_{(\bar{3})} + A_{(\overline{15})} + 3C_{(\overline{15})} - A_{(6)}\|^2$
$B^0 \to \pi^0\pi^0$	$\frac{1}{2}\|2A_{(\bar{3})} + C_{(\bar{3})} + A_{(\overline{15})} - 5C_{(\overline{15})} - A_{(6)}\|^2$
$B^0 \to \eta^0\eta^0$	$\frac{1}{2}\|2A_{(\bar{3})} + \frac{1}{3}C_{(\bar{3})} - A_{(\overline{15})} + C_{(\overline{15})} + A_{(6)}\|^2$
$B^0 \to \pi^0\eta^0$	$\frac{1}{3}\| - C_{(\bar{3})} + 5A_{(\overline{15})} + C_{(\overline{15})} - A_{(6)}\|^2$
$B^0 \to K^0\bar{K}^0$	$\|2A_{(\bar{3})} + C_{(\bar{3})} - 3A_{(\overline{15})} - C_{(\overline{15})} + A_{(6)}\|^2$
$B^0 \to K^+K^-$	$\|2A_{(\bar{3})} + 2A_{(\overline{15})}\|^2$
$B^- \to \pi^0\pi^-$	$32\|C_{(\overline{15})}\|^2$
$B^- \to \eta^0\pi^-$	$\frac{1}{6}\|2C_{(\bar{3})} + 6A_{(\overline{15})} + 6C_{(\overline{15})} + 2A_{(6)}\|^2$
$B^- \to K^0K^-$	$\|C_{(\bar{3})} + 3A_{(\overline{15})} - C_{(\overline{15})} + A_{(6)}\|^2$
$B_s^0 \to K^+\pi^-$	$\|C_{(\bar{3})} - A_{(\overline{15})} + 3C_{(\overline{15})} - A_{(6)}\|^2$
$B_s^0 \to K^0\pi^0$	$\frac{1}{2}\| - C_{(\bar{3})} + A_{(\overline{15})} + 5C_{(\overline{15})} + A_{(6)}\|^2$
$B_s^0 \to K^0\eta^0$	$\frac{1}{6}\| - C_{(\bar{3})} + A_{(\overline{15})} + 5C_{(\overline{15})} + A_{(6)}\|^2$

Table 5.6. $SU(3)_f$ predictions for $B \to MM$ decays that change strangeness. Here it is assumed that penguin–type diagrams with a charm quark or top quark in the loop dominate. Entries in the second column should be multiplied by $|s_2 e^{i\delta} + s_3|^2$ if compared with Table 5.5 using Eq. (5.21).

Process	Rate (divided by $\lvert s_2 e^{i\delta} + s_3\rvert^2$)
$B^0 \to \pi^+ K^-$	$\lvert\hat{C}'_{(\bar{3})}\rvert^2$
$B^0 \to \pi^0 \bar{K}^0$	$\frac{1}{2}\lvert\hat{C}'_{(\bar{3})}\rvert^2$
$B^0 \to \bar{K}^0 \eta^0$	$\frac{1}{6}\lvert\hat{C}'_{(\bar{3})}\rvert^2$
$B^- \to K^- \pi^0$	$\frac{1}{2}\lvert\hat{C}'_{(\bar{3})}\rvert^2$
$B^- \to K^- \eta^0$	$\frac{1}{6}\lvert\hat{C}'_{(\bar{3})}\rvert^2$
$B^- \to \bar{K}^0 \pi^-$	$\lvert\hat{C}'_{(\bar{3})}\rvert^2$
$B_s^0 \to K^+ K^-$	$\lvert 2\hat{A}'_{(\bar{3})} + \hat{C}'_{(\bar{3})}\rvert^2$
$B_s^0 \to K^0 \bar{K}^0$	$\lvert 2\hat{A}'_{(\bar{3})} + \hat{C}'_{(\bar{3})}\rvert^2$
$B_s^0 \to \pi^0 \pi^0$	$2\lvert\hat{A}'_{(\bar{3})}\rvert^2$
$B_s^0 \to \eta^0 \eta^0$	$2\lvert\hat{A}'_{(\bar{3})} + \frac{2}{3}\hat{C}'_{(\bar{3})}\rvert^2$
$B_s^0 \to \pi^+ \pi^-$	$4\lvert\hat{A}'_{(\bar{3})}\rvert^2$

Table 5.7. $SU(3)_f$ predictions for decays $B \to \bar{D}M$.

Process	Rate
$B^- \to \bar{D}^0 K^-$	$\lvert\alpha_{(\bar{6})} + \alpha_{(3)} + \beta_{(\bar{6})} + \beta_{(3)}\rvert^2$
$B^- \to D_s^- \eta^0$	$\frac{1}{6}\lvert\alpha_{(\bar{6})} - \alpha_{(3)} - 2\beta_{(\bar{6})} - 2\beta_{(3)}\rvert^2$
$B^- \to D^- \bar{K}^0$	$\lvert\beta_{(\bar{6})} + \beta_{(3)}\rvert^2$
$B^- \to D_s^- \pi^0$	$\frac{1}{2}\lvert\alpha_{(\bar{6})} - \alpha_{(3)}\rvert^2$
$B^0 \to \bar{D}^0 \bar{K}^0$	$\lvert\alpha_{(\bar{6})} + \alpha_{(3)}\rvert^2$
$B^0 \to D_s^- \pi^+$	$\lvert\alpha_{(\bar{6})} - \alpha_{(3)}\rvert^2$
$B_s^0 \to \bar{D}^0 \pi^0$	$\frac{1}{2}\lvert\beta_{(\bar{6})} - \beta_{(3)}\rvert^2$
$B_s^0 \to D_s^- K^+$	$\lvert\alpha_{(\bar{6})} - \alpha_{(3)} + \beta_{(\bar{6})} - \beta_{(3)}\rvert^2$
$B_s^0 \to \bar{D}^0 \eta^0$	$\frac{1}{6}\lvert - 2\alpha_{(\bar{6})} - 2\alpha_{(3)} + \beta_{(\bar{6})} - \beta_{(3)}\rvert^2$
$B_s^0 \to D^- \pi^+$	$\lvert\beta_{(\bar{6})} - \beta_{(3)}\rvert^2$

Table 5.8. $SU(3)_f$ predictions for decays $B \to J/\psi MM$, when the relative angular momentum of the two pseudoscalar mesons is even.

Process	Rate
$B^0 \to J/\psi\pi^+K^-$	$\|G\|^2$
$B^0 \to J/\psi\pi^0\bar{K}^0$	$\frac{1}{2}\|G\|^2$
$B^0 \to J/\psi\eta\bar{K}^0$	$\frac{1}{6}\|G\|^2$
$B^0 \to J/\psi\pi^+\pi^-$	$s_1^2\|2F+G\|^2$
$B^0 \to J/\psi\pi^0\pi^0$	$s_1^2\|2F+G\|^2$
$B^0 \to J/\psi\eta^0\eta^0$	$\frac{1}{2}s_1^2\|2F+\frac{1}{3}G\|^2$
$B^0 \to J/\psi\pi^0\eta^0$	$\frac{1}{3}s_1^2\|G\|^2$
$B^0 \to J/\psi K^+K^-$	$s_1^2\|2F\|^2$
$B^0 \to J/\psi K^0\bar{K}^0$	$s_1^2\|2F+G\|^2$
$B^- \to J/\psi\pi^0K^-$	$\frac{1}{2}\|G\|^2$
$B^- \to J/\psi\eta^0K^-$	$\frac{1}{6}\|G\|^2$
$B^- \to J/\psi\pi^-\bar{K}^0$	$\|G\|^2$
$B^- \to J/\psi\pi^-\eta^0$	$\frac{2}{3}s_1^2\|G\|^2$
$B^- \to J/\psi K^-K^0$	$s_1^2\|G\|^2$
$B_s^0 \to J/\psi\pi^0\pi^0$	$\frac{1}{2}\|2F\|^2$
$B_s^0 \to J/\psi\eta^0\eta^0$	$\frac{1}{2}\|2F+\frac{4}{3}G\|^2$
$B_s^0 \to J/\psi K^0\bar{K}^0$	$\|2F+G\|^2$
$B_s^0 \to J/\psi K^+K^-$	$\|2F+G\|^2$
$B_s^0 \to J/\psi\pi^+\pi^-$	$\|2F\|^2$
$B_s^0 \to J/\psi K^+\pi^-$	$s_1^2\|G\|^2$
$B_s^0 \to J/\psi K^0\pi^0$	$\frac{1}{2}s_1^2\|G\|^2$
$B_s^0 \to J/\psi K^0\eta^0$	$\frac{1}{6}s_1^2\|G\|^2$

Table 5.9. $SU(3)_f$ predictions for decays $B \to J/\psi MM$, when the relative orbital angular momentum of the two pseudoscalar mesons is odd. Rates are normalized to that for $B^0 \to J/\psi\pi^+K^-$.

Process	Rate
$B^0 \to J/\psi\pi^+K^-$	1
$B^0 \to J/\psi\pi^0\bar{K}^0$	$\frac{1}{2}$
$B^0 \to J/\psi\eta^0\bar{K}^0$	$\frac{3}{2}$
$B^0 \to J/\psi\pi^+\pi^-$	s_1^2
$B^0 \to J/\psi K^0\bar{K}^0$	s_1^2
$B^- \to J/\psi\pi^0K^-$	$\frac{1}{2}$
$B^- \to J/\psi\pi^-\bar{K}^0$	1
$B^- \to J/\psi\eta^0K^-$	$\frac{3}{2}$
$B^- \to J/\psi\pi^0\pi^-$	$2s_1^2$
$B^- \to J/\psi K^0K^-$	s_1^2
$B_s^0 \to J/\psi K^+K^-$	1
$B_s^0 \to J/\psi K^0\bar{K}^0$	1
$B_s^0 \to J/\psi K^+\pi^-$	s_1^2
$B_s^0 \to J/\psi K^0\eta^0$	$\frac{3}{2}s_1^2$
$B_s^0 \to J/\psi K^0\pi^0$	$\frac{1}{2}s_1^2$

Table 5.10. Implications of $SU(3)_f$ symmetry for Cabibbo–allowed decays $B \to DMM$, where the relative angular momentum of the M mesons is even.

Process	Rate
$B^0 \to D^0\pi^0\pi^0$	$\frac{1}{2}\|2e + c + b - a\|^2$
$B^0 \to D^0\eta^0\eta^0$	$\frac{1}{2}\|2e + \frac{1}{3}c + \frac{1}{3}b + \frac{1}{3}a\|^2$
$B^0 \to D^0\eta^0\pi^0$	$\frac{1}{3}\|c - b\|^2$
$B^0 \to D^0\pi^+\pi^-$	$\|2e + d + c + b\|^2$
$B^0 \to D^0K^0\bar{K}^0$	$\|2e + b\|^2$
$B^0 \to D^0K^+K^-$	$\|2e + c\|^2$
$B^0 \to D^+\eta^0\pi^-$	$\frac{1}{6}\|2f + d + 2c + a\|^2$
$B^0 \to D^+\pi^-\pi^0$	$\frac{1}{2}\|d + a\|^2$
$B^0 \to D^+K^-K^0$	$\|f + c\|^2$
$B^0 \to D_s^+\bar{K}^0\pi^-$	$\|d + c\|^2$
$B^0 \to D_s^+K^-\pi^0$	$\frac{1}{2}\|c - a\|^2$
$B^0 \to D_s^+K^-\eta^0$	$\frac{1}{6}\|c - a\|^2$
$B^- \to D^0\pi^0\pi^-$	$\frac{1}{2}\|d + a\|^2$
$B^- \to D^0\eta^0\pi^-$	$\frac{1}{6}\|2f + d + 2b + a\|^2$
$B^- \to D^0K^-K^0$	$\|f + b\|^2$
$B^- \to D^+\pi^-\pi^-$	$\frac{1}{2}\|2d + 2a\|^2$
$B^- \to D_s^+\pi^-K^-$	$\|d + a\|^2$
$B_s^0 \to D^0K^+\pi^-$	$\|d + b\|^2$
$B_s^0 \to D^0K^0\pi^0$	$\frac{1}{2}\|b - a\|^2$
$B_s^0 \to D^0K^0\eta^0$	$\frac{1}{6}\|b - a\|^2$
$B_s^0 \to D^+K^0\pi^-$	$\|d + a\|^2$
$B_s^0 \to D_s^+\eta^0\pi^-$	$\frac{2}{3}\|f - d\|^2$
$B_s^0 \to D_s^+K^-K^0$	$\|f + a\|^2$

Table 5.11. Implications of $SU(3)_f$ symmetry for Cabibbo–allowed decays $B \to DMM$, where the relative orbital angular momentum of the M mesons is odd. Rates are expressed in terms of five reduced matrix elements; a', b', c', d' and f'.

Process	Rate
$B^0 \to D^0\eta^0\pi^0$	$\frac{1}{3}\|a'\|^2$
$B^0 \to D^0\pi^+\pi^-$	$\|d' - c' + b'\|^2$
$B^0 \to D^0K^0\bar{K}^0$	$\|b'\|^2$
$B^0 \to D^0K^+K^-$	$\|c'\|^2$
$B^0 \to D^+\eta^0\pi^-$	$\frac{1}{6}\|d' + a'\|^2$
$B^0 \to D^+\pi^-\pi^0$	$\frac{1}{2}\|-2f' + d' - 2c' + a'\|^2$
$B^0 \to D^+K^-K^0$	$\|f' + c'\|^2$
$B^0 \to D_s^+\bar{K}^0\pi^-$	$\|d' - c'\|^2$
$B^0 \to D_s^+K^-\pi^0$	$\frac{1}{2}\|-c' + a'\|^2$
$B^0 \to D_s^+K^-\eta^0$	$\frac{1}{6}\|3c' + a'\|^2$
$B^- \to D^0\pi^0\pi^-$	$\frac{1}{2}\|2f' + d' + 2b' - a'\|^2$
$B^- \to D^0\eta^0\pi^-$	$\frac{1}{6}\|d' - a'\|^2$
$B^- \to D^0K^-K^0$	$\|f' + b'\|^2$
$B^- \to D_s^+\pi^-K^-$	$\|d' - a'\|^2$
$B_s^0 \to D^0K^+\pi^-$	$\|d' + b'\|^2$
$B_s^0 \to D^0K^0\pi^0$	$\frac{1}{2}\|-b' + a'\|^2$
$B^0 \to D^0K^0\eta^0$	$\frac{1}{6}\|3b' + a'\|^2$
$B_s^0 \to D^+K^0\pi^-$	$\|d' + a'\|^2$
$B_s^0 \to D_s^+\eta^0\pi^-$	$\frac{2}{3}\|d'\|^2$
$B_s^0 \to D_s^+K^-K^0$	$\|f' - a'\|^2$
$B_s^0 \to D_s^+\pi^0\pi^-$	$2\|f'\|^2$

Table 5.12. Implications of $SU(3)_f$ symmetry for Cabibbo–suppressed $B \to DMM$ decays when the relative angular momentum of the M mesons is even.

Process	Rate (divided by s_1^2)
$B^0 \to D^+\pi^0K^-$	$\frac{1}{2}\|f-d\|^2$
$B^0 \to D^+\pi^-\bar{K}^0$	$\|f+a\|^2$
$B^0 \to D^+\eta^0K^-$	$\frac{1}{6}\|-f+d\|^2$
$B^0 \to D^0\pi^+K^-$	$\|d+b\|^2$
$B^0 \to D_s^+\bar{K}^0K^-$	$\|d+a\|^2$
$B^0 \to D^0\pi^0\bar{K}^0$	$\frac{1}{2}\|-b+a\|^2$
$B^0 \to D^0\eta^0\bar{K}^0$	$\frac{1}{6}\|-b+a\|^2$
$B^- \to D^0\pi^0K^-$	$\frac{1}{2}\|f+d+b+a\|^2$
$B^- \to D^0\pi^-\bar{K}^0$	$\|f+b\|^2$
$B^- \to D^0\eta^0K^-$	$\frac{1}{6}\|-f+d-b+a\|^2$
$B^- \to D^+\pi^-K^-$	$\|d+a\|^2$
$B^- \to D_s^+K^-K^-$	$2\|d+a\|^2$
$B_s^0 \to D_s^+\pi^0K^-$	$\frac{1}{2}\|f+c\|^2$
$B_s^0 \to D_s^+\pi^-\bar{K}^0$	$\|f+c\|^2$
$B_s^0 \to D_s^+\eta^0K^-$	$\frac{1}{6}\|f+2d+c+2a\|^2$
$B_s^0 \to D^0\pi^0\pi^0$	$\frac{1}{2}\|2e+c\|^2$
$B_s^0 \to D^0\eta^0\eta^0$	$\frac{1}{2}\|2e+c/3+4b/3-2a/3\|^2$
$B_s^0 \to D^0\pi^+\pi^-$	$\|2e+c\|^2$
$B_s^0 \to D^0\bar{K}^0K^0$	$\|2e+b\|^2$
$B_s^0 \to D^0K^+K^-$	$\|2e+d+c+b\|^2$
$B_s^0 \to D^+K^0K^-$	$\|d+c\|^2$
$B_s^0 \to D^0\eta^0\pi^0$	$\frac{1}{3}\|-a+c\|^2$
$B_s^0 \to D^+\eta^0\pi^-$	$\frac{2}{3}\|c-a\|^2$

Table 5.13. Implications of $SU(3)_f$ symmetry for Cabibbo–suppressed decays $B \to DMM$ where the relative orbital angular momentum of the M mesons is odd. Rates are expressed in terms of the same five reduced matrix elements (a', b', c', d' and f') as the Cabibbo–allowed case.

Process	Rate (divided by s_1^2)
$B^0 \to D^+\pi^0K^-$	$\frac{1}{2}\lvert f' - d'\rvert^2$
$B^0 \to D^+\pi^-\bar{K}^0$	$\lvert f' - a'\rvert^2$
$B^0 \to D^+\eta^0K^-$	$\frac{1}{6}\lvert 3f' + d'\rvert^2$
$B^0 \to D^0\pi^+K^-$	$\lvert d' + b'\rvert^2$
$B^0 \to D_s^+\bar{K}^0K^-$	$\lvert d' + a'\rvert^2$
$B^0 \to D^0\pi^0\bar{K}^0$	$\frac{1}{2}\lvert b' + a'\rvert^2$
$B^0 \to D^0\eta^0\bar{K}^0$	$\frac{1}{6}\lvert 3b' - a'\rvert^2$
$B^- \to D^0\pi^0K^-$	$\frac{1}{2}\lvert f' + d' + b' - a'\rvert^2$
$B^- \to D^0\pi^-\bar{K}^0$	$\lvert f' + b'\rvert^2$
$B^- \to D^0\eta^0K^-$	$\frac{1}{6}\lvert 3f' + d' + 3b' - a'\rvert^2$
$B^- \to D^+\pi^-K^-$	$\lvert d' - a'\rvert^2$
$B_s^0 \to D_s^+\pi^0K^-$	$\frac{1}{2}\lvert f' + c'\rvert^2$
$B_s^0 \to D_s^+\pi^-\bar{K}^0$	$\lvert f' + c'\rvert^2$
$B_s^0 \to D_s^+\eta^0K^-$	$\frac{1}{6}\lvert 3f' - 2d' + 3c' - 2a'\rvert^2$
$B_s^0 \to D^0\pi^+\pi^-$	$\lvert c'\rvert^2$
$B_s^0 \to D^0\bar{K}^0K^0$	$\lvert b'\rvert^2$
$B_s^0 \to D^0K^+K^-$	$\lvert d' - c' + b'\rvert^2$
$B_s^0 \to D^+K^0K^-$	$\lvert d' - c'\rvert^2$
$B_s^0 \to D^0\eta^0\pi^0$	$\frac{1}{3}\lvert a'\rvert^2$
$B_s^0 \to D^+\pi^0\pi^-$	$2\lvert c'\rvert^2$
$B_s^0 \to D^+\eta^0\pi^-$	$\frac{2}{3}\lvert a'\rvert^2$

Table 5.14. Implications of flavor $SU(3)_f$ for Cabibbo–allowed decays $B \to D\bar{D}M$ assuming the effective Hamiltonian transforms as a $\overline{3}$.

Process	Rate
$B^- \to D^0 \bar{D}^0 K^-$	$\|\eta_2 + \eta_4\|^2$
$B^- \to D^+ D^- K^-$	$\|\eta_4\|^2$
$B^- \to D^0 D^- \bar{K}^0$	$\|\eta_2\|^2$
$B^- \to D^0 D_s^- \eta^0$	$\frac{1}{6}\|-2\eta_2 + \eta_3\|^2$
$B^- \to D^0 D_s^- \pi^0$	$\frac{1}{2}\|\eta_3\|^2$
$B^- \to D_s^+ D_s^- K^-$	$\|\eta_3 + \eta_4\|^2$
$B^- \to D_s^- D^+ \pi^-$	$\|\eta_3\|^2$
$B^0 \to \bar{D}^0 D^+ K^-$	$\|\eta_2\|^2$
$B^0 \to D^+ D^- \bar{K}^0$	$\|\eta_2 + \eta_4\|^2$
$B^0 \to D^+ D_s^- \eta^0$	$\frac{1}{6}\|-2\eta_2 + \eta_3\|^2$
$B^0 \to D^0 D_s^- \pi^+$	$\|\eta_3\|^2$
$B^0 \to D^+ D_s^- \pi^0$	$\frac{1}{2}\|\eta_3\|^2$
$B^0 \to D_s^+ D_s^- \bar{K}^0$	$\|\eta_3 + \eta_4\|^2$

Continued . . .

Table 5.14. Continued.

Process	Rate
$B^0 \to D^0 \bar{D}^0 \bar{K}^0$	$\|\eta_4\|^2$
$B_s^0 \to D^0 \bar{D}^0 \pi^0$	$\frac{1}{2}\|\eta_1\|^2$
$B_s^0 \to D^0 \bar{D}^0 \eta^0$	$\frac{1}{6}\|\eta_1 - 2\eta_4\|^2$
$B_s^0 \to \bar{D}^0 D^+ \pi^-$	$\|\eta_1\|^2$
$B_s^0 \to \bar{D}^0 D_s^+ K^-$	$\|\eta_1 + \eta_2\|^2$
$B_s^0 \to D^- D^0 \pi^+$	$\|\eta_1\|^2$
$B_s^0 \to D^+ D^- \pi^0$	$\frac{1}{2}\|\eta_1\|^2$
$B_s^0 \to D^+ D^- \eta^0$	$\frac{1}{6}\|\eta_1 - 2\eta_4\|^2$
$B_s^0 \to D^- D_s^+ \bar{K}^0$	$\|\eta_1 + \eta_2\|^2$
$B_s^0 \to D_s^- D^0 K^+$	$\|\eta_1 + \eta_3\|^2$
$B_s^0 \to D_s^- D^+ K^0$	$\|\eta_1 + \eta_3\|^2$
$B_s^0 \to D_s^+ D_s^- \eta^0$	$\frac{2}{3}\|\eta_1 + \eta_2 + \eta_3 + \eta_4\|^2$

Table 5.15. Implications of flavor $SU(3)_f$ for Cabibbo–suppressed decays $B \to D\bar{D}M$. The effective Hamiltonian is assumed to transform as a $\overline{3}$. Entries in the second column should be multiplied by s_1^2 when comparing with the results of Table XIV.

Process	Rate (divided by s_1^2)
$B^- \to D^0 \bar{D}^0 \pi^-$	$\|\eta_2 + \eta_4\|^2$
$B^- \to D^0 D^- \pi^0$	$\frac{1}{2}\|-\eta_2 + \eta_3\|^2$
$B^- \to D^+ D^- \pi^-$	$\|\eta_3 + \eta_4\|^2$
$B^- \to D^0 D^- \eta^0$	$\frac{1}{6}\|\eta_2 + \eta_3\|^2$
$B^- \to D^0 D_s^- K^0$	$\|\eta_2\|^2$
$B^- \to D_s^+ D^- K^-$	$\|\eta_3\|^2$
$B^- \to D_s^+ D_s^- \pi^-$	$\|\eta_4\|^2$
$B^0 \to D^0 \bar{D}^0 \pi^0$	$\frac{1}{2}\|\eta_1 - \eta_4\|^2$
$B^0 \to D^0 \bar{D}^0 \eta^0$	$\frac{1}{6}\|\eta_1 + \eta_4\|^2$
$B^0 \to D^- D^0 \pi^+$	$\|\eta_1 + \eta_3\|^2$
$B^0 \to \bar{D}^0 D_s^+ K^-$	$\|\eta_1\|^2$
$B^0 \to D^+ \bar{D}^0 \pi^-$	$\|\eta_1 + \eta_2\|^2$

Continued . . .

Table 5.15. Continued.

Process	Rate (divided by s_1^2)
$B^0 \to D^+D^-\pi^0$	$\frac{1}{2}\|\eta_1+\eta_2+\eta_3+\eta_4\|^2$
$B^0 \to D^+D^-\eta^0$	$\frac{1}{6}\|\eta_1+\eta_2+\eta_3+\eta_4\|^2$
$B^0 \to D^-D_s^+\bar{K}^0$	$\|\eta_1+\eta_3\|^2$
$B^0 \to D_s^-D^0K^+$	$\|\eta_1\|^2$
$B^0 \to D_s^-D^+K^0$	$\|\eta_1+\eta_2\|^2$
$B^0 \to D_s^+D_s^-\pi^0$	$\frac{1}{2}\|\eta_4\|^2$
$B^0 \to D_s^+D_s^-\eta^0$	$\frac{1}{6}\|-2\eta_1+\eta_4\|^2$
$B_s^0 \to D_s^+\bar{D}^0\pi^-$	$\|\eta_2\|^2$
$B_s^0 \to D_s^+D^-\pi^0$	$\frac{1}{2}\|\eta_2\|^2$
$B_s^0 \to D_s^+D^-\eta^0$	$\frac{1}{6}\|\eta_2-2\eta_3\|^2$
$B_s^0 \to D_s^+D_s^-K^0$	$\|\eta_2+\eta_4\|^2$
$B_s^0 \to D^0D^-K^+$	$\|\eta_3\|^2$
$B_s^0 \to D^+D^-K^0$	$\|\eta_3+\eta_4\|^2$
$B_s^0 \to D^0\bar{D}^0K^0$	$\|\eta_4\|^2$

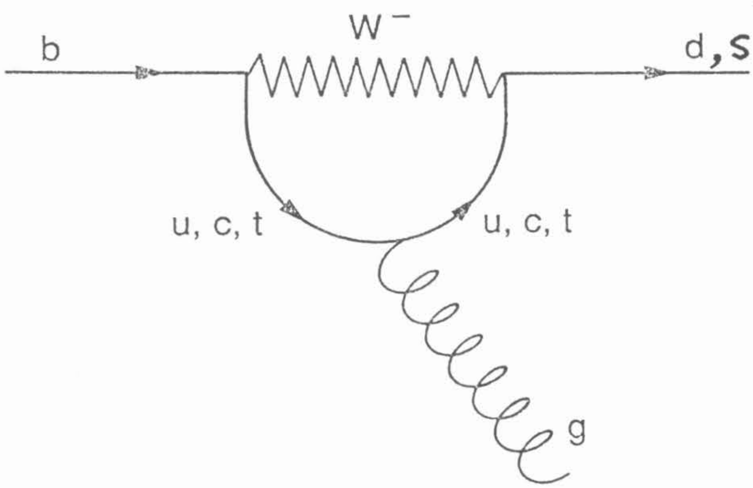

FIGURE 5.1

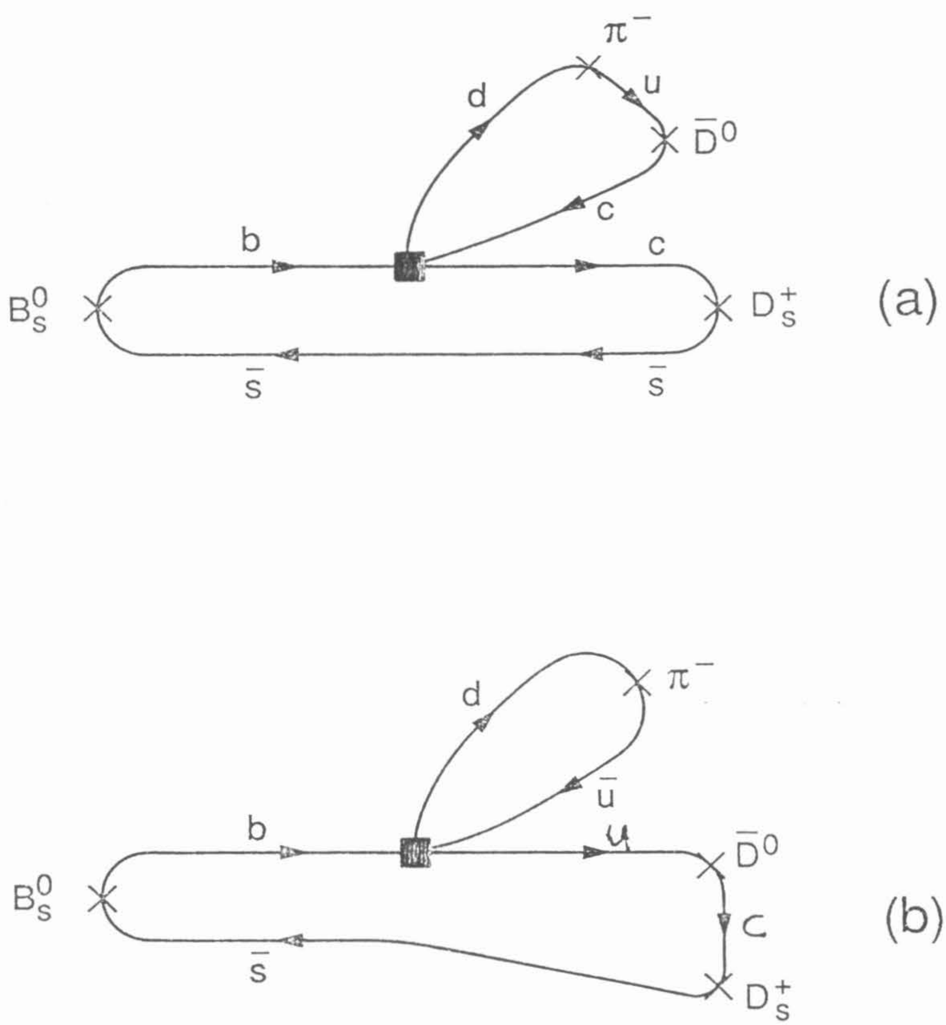

FIGURE 5.2

ADDENDUM

In deriving our results for charmed baryon, B-meson and D-meson decays we neglected the mixing between the pseudoscalar mesons in the octet and the SU(3) singlet. The isosinglet octet meson, the η^0, mixes slightly with the isosinglet SU(3) singlet meson, the η'^0. The mixing angle between the mass eigestates and the SU(3) eigenstates is approximately 13^o. We will again assume that the SU(3) eigenstates are the mass eigenstates and derive relations between the decay rates to final states containing an η'^0 meson.

Let us begin by considering the process $D \to M\eta'$, where M is a pseudoscalar octet meson. The effective Hamiltonian for the process is

$$H_{\text{eff}} = \alpha . D_a H_c^{ab}(\overline{15}) M_b^c + \beta . D_a H_c^{ab}(6) M_b^c \tag{6.1}$$

From this effective Hamiltonian we find that there are the following relations

$$\Gamma(D^0 \to \overline{K}^0 \eta'^0) = \frac{2}{3s_1^2}\Gamma(D^0 \to \eta^0 \eta'^0) = \frac{2}{s_1^2}\Gamma(D^0 \to \pi^0 \eta'^0) = \frac{1}{s_1^4}\Gamma(D^0 \to K^0 \eta'^0) \tag{6.2a}$$

$$\Gamma(D_s^+ \to \pi^+ \eta'^0) = \frac{1}{s_1^2}\Gamma(D^+ \to \pi^+ \eta'^0) = \frac{1}{s_1^2}\Gamma(D_s^+ \to K^+ \eta'^0) = \frac{1}{s_1^4}\Gamma(D^+ \to K^+ \eta'^0) \tag{6.2b}$$

We can also derive relations for B-meson decays. In the case of $B \to D\eta'$ we find that only one SU(3) singlet can be formed, consequently

$$\Gamma(B^0 \to D^0 \eta'^0) = \frac{1}{s_1^2}\Gamma(B_s^0 \to D^0 \eta'^0) \tag{6.3}$$

The same is true for the decays $B \to J/\psi\eta'$ where we find

$$\Gamma(B_s^0 \to J/\psi \eta'^0) = \frac{1}{s_1^2}\Gamma(B^0 \to J/\psi \eta'^0) \tag{6.4}$$

There are two reduced matrix elements that contribute to the decay $B \to \overline{D}\eta'$, and there are no relations between the decay modes. However, if either the $\overline{3}$ or the

6 components of the Hamiltonian dominates the decay then

$$\Gamma(B^- \to \overline{D}_s^- \eta'^0) = \Gamma(B_s^0 \to \overline{D}^0 \eta'^0) \tag{6.5}$$

Finally, we consider the decays $B \to M\eta'$ which are induced by the $b \to u$ coupling. There are three reduced matrix elements that contribute to the decay process and the effective Hamiltonian is

$$H_{\text{eff}} = A_{\overline{3}}.B_a M_b^a H^b(\overline{3})\eta' + A_{\overline{15}}.B_a H_c^{ab}(\overline{15}) M_b^c \eta' + A_6.B_a H_c^{ab}(6) M_b^c \eta' \tag{6.6}$$

There are no relations between the Cabibbo-allowed decay rates. The decay rates for the modes are

$$B^- \to \pi^- \eta'^0 \qquad |A_{\overline{3}} + 3A_{\overline{15}} + A_6|^2 \tag{6.7a}$$

$$B_d^0 \to \pi^0 \eta'^0 \qquad \frac{1}{2}|-A_{\overline{3}} + 5A_{\overline{15}} - A_6|^2 \tag{6.7b}$$

$$B_d^0 \to \eta^0 \eta'^0 \qquad \frac{1}{6}|A_{\overline{3}} + 3A_{\overline{15}} - 3A_6|^2 \tag{6.7c}$$

$$B_s^0 \to K^0 \eta'^0 \qquad |A_{\overline{3}} - A_{\overline{15}} - A_6|^2 \tag{6.7d}$$

As we previously discussed, the Cabibbo–suppressed decay could be dominated by the penguin diagrams. If the penguin diagrams do dominate the decay process then the following relations are found

$$\Gamma(B^- \to K^- \eta'^0) = \Gamma(B_d^0 \to \overline{K}^0 \eta'^0) = \Gamma(B_s^0 \to \eta^0 \eta'^0) \tag{6.8}$$