Magnetic Trapping of Atomic Tritium for Neutrino Mass Measurement

Thesis by

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In Partial Fulfillment of the Requirements

for the Degree of

Bachelor of Science

California Institute of Technology

Pasadena, California

2014

(Submitted May 9, 2014)

Abstract

Improved measurement of the neutrino mass via β decay spectroscopy requires the development of new energy measurement techniques and a new β decay source. A promising proposal is to measure the β energy by the frequency of the cyclotron radiation emitted in a magnetic field and to use a high purity atomic tritium source. This thesis examines the feasibility of using a magnetic trap to create and maintain such a source. We demonstrate that the loss rate due to β decay heating is not a limiting factor for the design. We also calculate the loss rate due to evaporative cooling and propose that the tritium can be cooled sufficiently during trap loading as to render this negligible. We further demonstrate a design for the magnetic field which produces a highly uniform field over a large fraction of the trap volume as needed for cyclotron frequency spectroscopy while still providing effective trapping.

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Chapter 1

Introduction

1.1 Background

According to the classic Standard Model, neutrinos are massless particles. However, experiment has shown otherwise. Numerous experiments have observed neutrino flavor oscillations and massive neutrinos are central to the theoretical interpretation of these results. The Standard Model must be modified to explain these results. The most widely used extension is the see-saw mechanism which requires the addition of heavy right-handed neutrinos [1].

Unfortunately, while neutrino oscillation experiments have informed us that neutrinos are massive, they do not tell us how massive. The Standard Model extensions giving neutrinos mass make varying predictions. In certain models, the three neutrino masses are nearly equal (quasi-degenerate) while in others, they cover a large range (hierarchical) as is the case for the charged leptons and quarks. Thus, measurement of the neutrino masses can eliminate some of these beyond the Standard Model theories from consideration [2].

Neutrino mass also has an impact on cosmology. Massive neutrinos contribute to the energy density of the universe and thus affect a variety of cosmological observables such as the CMB anisotropy and large scale structure [3]. These observations in combination with a particular cosmological model can be used to measure or set limits on the neutrino mass. Conversely, if the neutrino mass was measured by other means, these observations could be used to test existing cosmological models.

Given the potential implications to particle physics and cosmology, numerous experiments have attempted to measure neutrino mass. Of primary importance are neutrinoless double beta decay searches, cosmology observations, and beta decay spectroscopy. Each of these is sensitive to the neutrino mass through different physics processes.

In experimental cosmology, the direct observable related to neutrino mass is the contribution of neutrinos to the energy density of the universe. In combination with theoretical calculations of the number density of neutrinos (the same for all neutrino flavors) from cosmology models, this is equivalent to a measurement of the total mass of the three neutrinos. Depending on exactly which data sets are included, current cosmology fits give upper bounds of $\sum_i m_i < 0.28 - 0.76$ eV (95% CL) [4]. In combination with measurements of the squared mass differences from oscillation experiments, this is directly convertible into limits on the masses of the individual neutrinos.

The objective of neutrinoless double beta ($0\nu\beta\beta$) decay experiments is to search for the theoretically predicted, but yet unobserved, process of neutrinoless double beta decay. This is sensitive to the neutrino masses as the decay rate is proportional to $|m_{\beta\beta}|^2$ where the effective mass $m_{\beta\beta}$ is given by

$$m_{\beta\beta} = \sum_i U_{ei}^2 m_i \tag{1.1}$$

with U_{ei} are the complex neutrino mixing parameters and m_i are the neutrino masses [5]. Thus, measurement of the half life of $0\nu\beta\beta$ decay serves as a measurement of the effective mass. So far, experiments have not observed the decay and have set upper limits on $|m_{\beta\beta}|$. A recent result by the CUORICINO experiment is $|m_{\beta\beta}| < 0.3 - 0.71$ eV (90% CL) [6] (the range is due to systematics arising from nuclear matrix element calculations).

An important caveat to this limit is that the process of neutrinoless double beta decay is possible only if the neutrino is a Majorana particle [1, 7]. Since it is presently unknown whether the neutrino is a Majorana or Dirac particle, the lack of observed decays could be interpreted as evidence that the neutrino is a Dirac particle rather than that $m_{\beta\beta}$ is small. Another problem with using neutrinoless double beta decay experiments to measure the neutrino mass is that a measurement of $m_{\beta\beta}$ cannot be directly converted into a measurement of the neutrino masses as it also depends on the Majorana phases of the neutrino mixing matrix which are unknown. In fact, since the coefficients of m_i in the definition of $m_{\beta\beta}$ are complex, cancellation can occur so that $|m_{\beta\beta}|$ is smaller than any of the m_i. Thus, it is challenging to obtain useful information about the neutrino mass from $0\nu\beta\beta$ decay experiments.

While $0\nu\beta\beta$ decay experiments may be searching for a nonexistent decay, beta decay spectroscopy experiments are based upon thoroughly understood physics and serve as a

direct measurement of neutrino mass. When an atom undergoes β decay, energy is released which is available as kinetic energy to the electron, neutrino, and daughter nuclide. Since the decay is a three body process, these kinetic energies have a continuous spectrum rather than defined values as would be the case for a two body decay. The probability distribution for the electron kinetic energy has been calculated [8, 9]. The electron energy distribution for beta decay of tritium is shown in Figure 1.1. This spectrum is representative of that for β decay in general, though, there is a slight dependence on the atomic number of the β emitter. This energy distribution is sensitive to the effective electron neutrino mass

$$m_{\nu_e} = \sqrt{\sum_i |U_{ei}|^2 \, m_i^2} \tag{1.2}$$

with both the maximum value, termed the endpoint energy, and shape of the distribution depending on the electron neutrino mass. However, this effect is only significant within a few m_{ν_e} of the endpoint energy. Thus, observation of the spectrum within this region yields a measurement of the neutrino mass. An example of how the spectrum is altered is shown in Figure 1.2. It is worth noting that the branching ratio to this endpoint is quite small. For tritium, only $2 \cdot 10^{-13}$ of all decays occur within the last 1 eV. Another benefit of this technique over $0\nu\beta\beta$ decay is that in combination with current oscillation results for the mixing matrix parameters and the squared mass differences, a measurement for the electron neutrino mass determines the masses of all three neutrinos so long as the neutrino mass hierarchy is known. The mass hierarchy refers to the fact that present oscillation results have determined the magnitude, but not the sign of the atmospheric squared mass difference and so the ordering of the masses m_i is unknown. With the mass hierarchy undetermined, each of the possibilities, combined with the electron neutrino mass, yields values for the neutrino masses. However, the neutrino mass hierarchy is likely to be resolved by the NOνA experiment or other oscillation experiments in the near future, so this need not be a concern for a planned beta decay spectroscopy experiment. The best current results from beta decay spectroscopy are from the Mainz and Troitsk experiments which found $m_{\nu_e} < 2.3$ eV (95% CL) [10] and $m_{\nu_e} < 2.05$ eV (95% CL) [11], respectively. The KATRIN experiment, which is of the same general design and is currently under construction, will be able to measure a mass as low as 0.35 eV or set a limit of $m_{\nu_e} < 0.2$ eV (90% CL) in the absence of a signal [2].

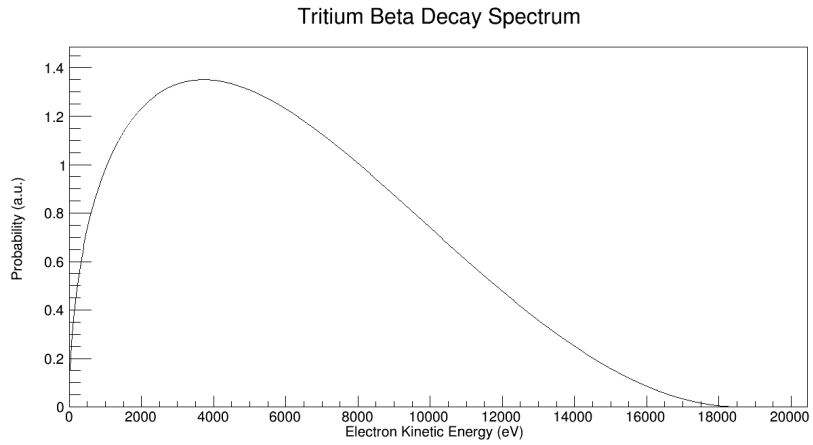

Figure 1.1: Kinetic energy spectrum for electrons from β decay of tritium.

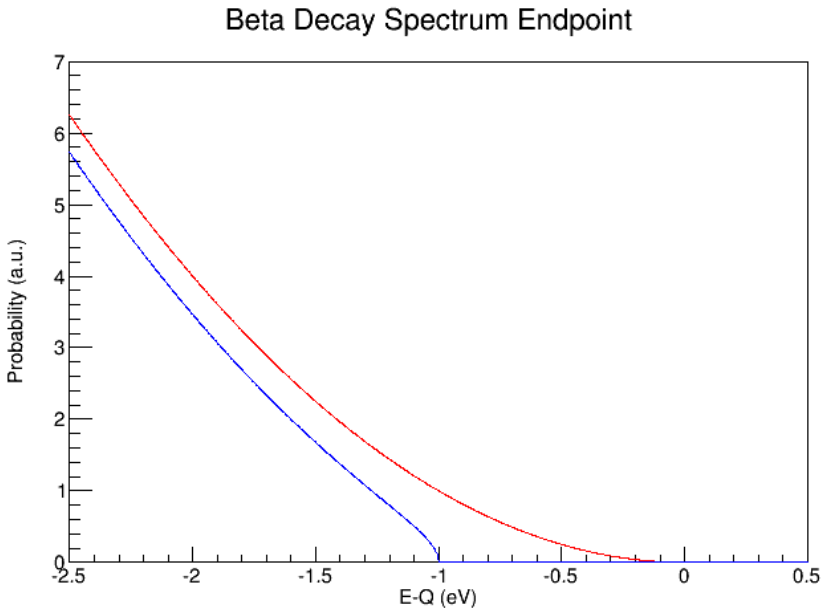

Figure 1.2: The electron kinetic energy spectrum near the endpoint energy (Q) for $m_{\nu_e} = 0$ eV (red) and $m_{\nu_e} = 1$ eV (blue).

1.2 Experiment Motivation

Given the three different types of neutrino mass measurements and the results which they have produced so far, what neutrino mass experiment should the physics community devote its efforts to next? We suggest that a beta decay spectroscopy experiment with a sensitivity of $m_{\nu_e} = 50$ meV would be extremely useful to both particle physics and cosmology. Most importantly, it would either measure the neutrino mass or resolve the neutrino mass hierarchy. It also has secondary impacts on cosmology and the question of whether neutrinos are Majorana or Dirac fermion when combined with results from neutrinoless double beta decay and cosmology measurement of the neutrino mass.

In order to motivate such an experiment, it is necessary to first argue that there is a substantial probability that $m_{\nu_e} \leq 350$ meV. If m_{ν_e} were larger than this value, KATRIN will measure the neutrino mass and there is not an immediate need for an improved neutrino mass experiment. However, current cosmology results set limits on the total neutrino mass as low as 0.28 eV (depending on which data sets are included) [4]. As the total neutrino mass is obviously strictly greater than the electron neutrino mass, this would imply $m_{\nu_e} \leq$ 280 meV which is below the 3σ discovery threshold for KATRIN. Total neutrino mass measurements from cosmology are indirect results which rely on the variety of assumptions that the ΛCDM model implies, so there is the possibility that these limits would shift based upon changes in the cosmological model. However, such modifications are not generally degenerate with neutrino mass measurements [3]. Thus, the limits from cosmology are reasonably robust.

If we are designing a more sensitive beta decay spectroscopy experiment, for what sensitivity should we aim? In order to guarantee a discovery, we should aim for the lower limits on the electron neutrino mass established by oscillation experiments. These limits require either $m_{\nu_e} \geq 9$ meV for the normal mass hierarchy or $m_{\nu_e} \geq 50$ meV for the inverted mass hierarchy. Given the multiple challenges to reaching a sensitivity of 50 meV which we will discuss later in this thesis and the possibility that the mass hierarchy is inverted and so further sensitivity is unnecessary, we leave a sensitivity of 9 meV for future consideration. If we learn from other experiments that the hierarchy is inverted, then this proposed experiment can be built with the expectation that it will produce a measurement rather than limit.

A direct measurement of the neutrino mass via beta decay spectroscopy is also necessary for a conclusive interpretation of current and future neutrinoless double beta decay experiments. If the neutrino mass is known, then upper and lower limits can be set on $|m_{\beta\beta}|$ by taking the most extreme values for the Majorana phases. If neutrinoless double beta decay experiments exclude this region, then that would be conclusive evidence that neutrinos are Dirac particles. If neutrinos are indeed Majorana particles and the possible region has not yet been experimentally excluded, then this would guide future neutrinoless double beta decay experiments as to what sensitivity is required to finally settle the issue. As for cosmology, comparison between cosmological and direct measurements of the total neutrino mass would be a significant test of the ΛCDM model. To achieve either of these goals requires that we know the neutrino mass more precisely through direct experiments than the indirect techniques of cosmology and neutrinoless double beta decay. Thus, beta decay spectroscopy measurements of the neutrino mass are a significant physics goal in the next decade.

1.3 Experimental Requirements

Given the stated objective of designing a beta decay spectroscopy experiment with a sensitivity of $m_{\nu_e} = 50$ meV, we first should consider whether the improvement is sensitivity is possible while using the same beta decay isotope and spectroscopy technique as used in previous beta decay experiments. Unfortunately, the answer to both of these questions is no, so the next generation of beta decay spectroscopy experiments must be radically different than the previous (Mainz, Troitsk) and current (KATRIN) generations.

Let us first consider what beta decay isotope should be used. The list of candidates is dramatically shortened by applying three requirements: low decay energy, appropriate decay half-life, and narrow final state spectrum of decay product. The first requirement is due to the strong dependence of the endpoint branching ratio on the decay energy. The fraction of beta decays with electron energy within ΔE of the endpoint is proportional to $(\Delta E/Q)^3$ where Q is the endpoint energy. Thus, use of a β emitter with endpoint energy of large Q, say 1 MeV, would yield an extremely small branching ratio to the endpoint. A β emitter with small Q therefore greatly improves experiment statistics.

The second requirement is motivated by a combination of practical concerns and nec-

essary statistics. The half-life must not be so long that only a very small portion of the sample decays else an experiment will not be able to gain sufficient statistics near the end-point energy. However, practical concerns for producing and shipping the source prevent use of a decay isotope with an extremely short half-life.

The third requirement of a narrow final state spectrum has been a relatively minor concern in previous experiments, but becomes a major factor in the choice of decay isotope at our desired sensitivity. For certain decay sources, the product of the decay may have a variety of excited states which could be excited by the decay. This will leave less energy available to the electron and will disturb the energy spectrum. If the final state spectrum is either sufficiently narrow or well-known, this can be accounted for, but even a small uncertainty in the width of the spectrum can have a significant effect on the experimental sensitivity.

The Mainz, Troitsk, and KATRIN experiments have all used a molecular tritium source. Tritium is a strong candidate for beta decay spectroscopy as it has a decay energy of only 18.6 keV and a half-life of 12.3 yr. While there are a small number of isotopes with lower decay energies, they generally have significantly longer half-lifes. For instance, ^{187}Re has a decay energy of 2.47 keV, but a half-life of $4.3 \cdot 10^{10}$ yr. While such elements are of interest to beta decay calorimetry experiments, the current neutrino mass limits from these experiments are far higher than those from Mainz and Troitsk and we do not consider the possible use of such experiments to reach the desired sensitivity in this thesis. The Milano experiment used ^{187}Re microcalorimeters and set a limit of $m_{\nu_e} < 15$ eV (90% CL) [12]. Thus, molecular tritium satisfies the decay energy and half-life requirements. However, it fails the the third requirement. The $(^3\mathrm{HeT})^+$ daughter molecule has numerous rotovibrational states which can be excited. The nuclear recoil of the decay results in a mean excitation energy of 1.7 eV and a width of 0.36 eV. While this is small enough that it was not a major factor for Mainz and Troitsk, it is one of the leading systematic uncertainties for KATRIN. Even if all other systematic and statistical uncertainties of KATRIN were set to zero, the resulting sensitivity would still not reach the desired level.

Given the desirable characteristics of tritium as the beta decay isotope and problems with molecular tritium, it would be ideal if we could use an atomic tritium source. This solves the final state spectrum issue entirely as $^3\mathrm{He}^+$ has a final state spectrum of negligible width (due only to hyperfine structure). However, it introduces certain other challenges

which must be addressed. Foremost among them is the task of producing and maintaining an atomic tritium source with a very small molecular tritium contamination. This is necessary as the endpoint energy of beta decay from molecular tritium is 8.1 eV higher than the endpoint energy of beta decay from atomic tritium [2]. Since the fraction of total decays within energy ΔE of the endpoint is proportional to ΔE^3, a small molecular tritium contamination will result in a significant background near the atomic tritium endpoint. In order to have a signal-background ratio of ~ 1, the mass fraction of molecular tritium must be $\sim (\Delta E/\Delta Q)^3 < 10^{-6}$ for ΔE of order the desired sensitivity, 50 meV. In order to maintain such a low concentration of molecular tritium, recombination of atomic tritium to molecular tritium must somehow be prevented.

Assuming an atomic tritium source, we should also ask whether the spectroscopy technique used by Mainz, Troitsk and KATRIN will be sufficient. The MAC-E-Filter (Magnetic Adiabatic Collimation combined with an Electrostatic Filter) spectrometer used by these experiments functions by aligning the momentum of all decay electrons into a given direction using a magnetic field and then passing the electrons through a retarding electric field [2]. Figure 1.3 provides a visual of the spectrometer design and operating principle. This field will stop all electrons with energy along the field direction less than the electric field strength. Thus, a detector placed after the electric field will count the number of electrons with greater energy. Thus, the spectrometer measures the integrated energy spectrum above the adjustable electric field strength.

It is important to discuss further how the spectrometer aligns the electrons into a given direction as this limits the spectrometer energy resolution. In a slowly spatially varying magnetic field, the magnetic moment is an invariant, so E_T/B is conserved where E_T is the energy of the momentum perpendicular to the magnetic field. Thus, by producing the decay electrons in a region of high magnetic field and placing the retarding electric field in a region of much lower field, we can limit E_T at the detecting plane to

$$E_T = \frac{B_{min}}{B_{max}} Q \tag{1.3}$$

Since the retarding electric field does not act on the momentum perpendicular to the field,

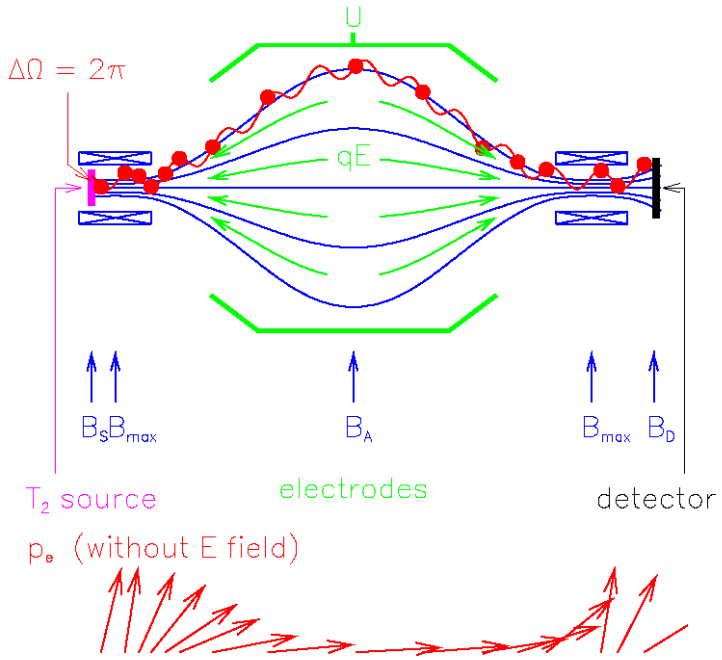

Figure 1.3: The experimental setup of a MAC-E-Filter spectrometer and the rotation of the electron momentum that results. Figure reprinted from [2].

this sets the energy resolution. Thus,

$$\Delta E = \frac{B_{min}}{B_{max}} Q \tag{1.4}$$

But, by conservation of magnetic flux, we have $B_{max} A_S = B_{min} A_D$ where A_S and A_D are the cross sectional area of the source and spectrometer at the location of the electric field, respectively. Thus,

$$\Delta E = \frac{A_S}{A_D} Q \tag{1.5}$$

Thus, to improve the energy resolution, we must increase the area ratio. However, in order to obtain sufficient statistics, we need to increase A_S (since this is proportional to the count rate) and so A_D must by an even larger factor. Using the scaling rule for the count rate within ΔE of the endpoint, A_S is required to grow inverse cubically with the energy resolution and so A_D must grow inverse quartically. The KATRIN spectrometer is 9m in diameter so improving the resolution by only a factor of two would require a radius of 36m. A sensitivity of 50 meV would likely require at least a factor of two improvement in resolution over KATRIN, so 36m is a lower limit on the size of the required spectrometer.

This does not appear to be a practical option. Thus, a new spectroscopy technique is needed.

The combination of switching to atomic tritium and developing a new spectroscopy technique will create a beta decay spectroscopy experiment entirely unlike those before it. This will require a significant degree of research and development. However, it opens up the possibility of new avenues which may have better performance and better scaling rules than the spectrometer design used by KATRIN, Mainz, and Troitsk.

1.4 Experiment Outline

Given that the scaling behavior of the KATRIN spectrometer design prevents it from reasonably being used for improved experiments, it is informative to consider how we can design an experiment with an improved scaling rule. In the existing design, the decay source and spectrometer are separated so that the decay electrons must be transported in the course of the experiment. This forces the source strength to scale with the cross sectional area of the source rather than volume as the length of the source cannot be greater than the mean free path of the decay electrons for collisions with the tritium source.

If the spectrometer used a new energy measurement technique so that the beta decay electrons did not have to be transported out of the source, then the source strength would instead scale with volume. Such a technique has been proposed and is currently under investigation by the Project 8 collaboration [13, 14]. The idea for the energy measurement is to take advantage of the fact that an electron with a kinetic energy T in a magnetic field B emits cyclotron radiation of frequency

$$\omega = \frac{eB}{T + m_e c^2} \tag{1.6}$$

Note that the cyclotron frequency does not depend upon orientation of the particle's momentum relative to the magnetic field. Given this relationship, if the cyclotron frequency and magnetic field are each known accurately, then an accurate measurement of the electron kinetic energy can be made.

In order to measure the cyclotron frequency to an accuracy $\Delta\omega$, the cyclotron radiation must be observed for a time $t = \frac{2\pi}{\Delta\omega}$ by Nyquist's theorem. As tritium beta decay electrons

with energies near the endpoint have velocities of $0.26c$, an energy resolution of < 1 eV requires observing the electron for several kilometers. It is infeasible to build an experimental apparatus of such size, so it is necessary to trap the electrons within a smaller region within which they can trace out such a path. However, this trapping must not alter the electron's energy so it must be done using a magnetic field. This can be done using magnetic mirrors. Magnetic mirrors are based upon the same physical principle as the KATRIN spectrometer, but operating in reverse. Given the invariance of the magnetic moment E_T/B, an electron with total energy E, initial transverse energy E_T, and produced in magnetic field of B_{init} cannot later reach a region with magnetic field greater than that given by

$$\frac{E_T}{B_{init}} = \frac{E}{B_{max}} \tag{1.7}$$

As the electron approaches the high field region, the angle between its momentum and the magnetic field increases towards 90°. Finally, at the field value given by the above equation, its momentum is perpendicular to the field and it turns around and heads back in the initial direction. For a fixed magnetic mirror with low field B_l and high field B_h, electrons will be reflected when they obey

$$\frac{E_T}{E} > \frac{B_l}{B_h} \tag{1.8}$$

Further, we have $\frac{E_T}{E} = \sin^2(\theta)$ where θ is the angle of the momentum to the magnetic field. Thus, electrons with

$$\theta > \arcsin(\sqrt{\frac{B_l}{B_h}}) \tag{1.9}$$

are reflected.Therefore, by placing two such magnetic mirrors opposite each other, the electrons will be trapped between the mirrors and can be observed long enough for an energy measurement to be made.

Now, for this magnetic mirror technique to work, the decay electrons must originate in the low magnetic field region between the magnetic mirrors. Thus, the tritium source must be located there. This raises the question of how to trap atomic tritium. Since it is neutral, the magnetic mirror concept does not work. However, atomic tritium has a useful property of which we can take advantage – its magnetic moment. A particle with a magnetic moment μ behaves as if it is moving in a potential $V(r) = \mu|B(r)|$ (under the assumption of adiabatic motion). Thus, a tritium atom with total energy E will be confined to regions

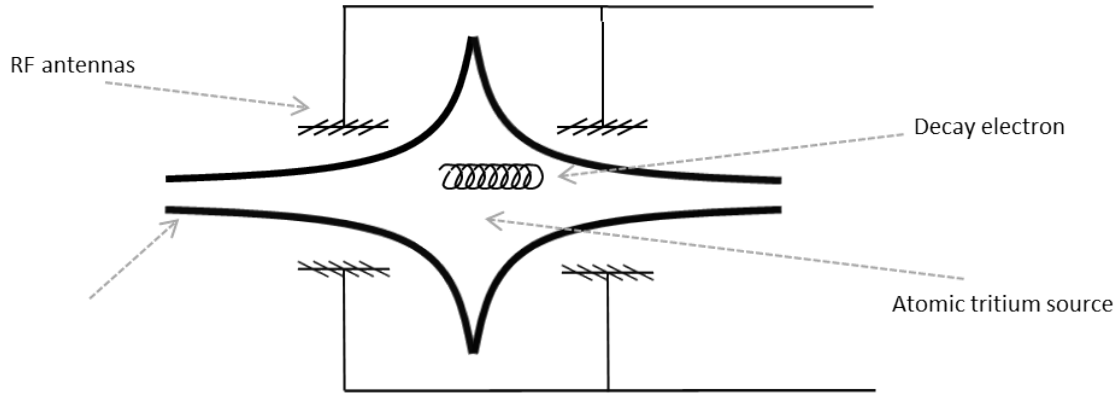

Figure 1.4: A schematic of the experiment. The shape of the magnet coils shown shall be motivated in later chapters. The trap radius and length are not to scale as the length is to be much greater than the radius. The absolute size of the experiment is not yet fixed, but a radius of 10 cm and length of 10 m are indicative of the intended scale. The electron's cyclotron radius is 0.5 mm in 1 T field and is not to scale with other components of the figure.

with $|B| \leq \frac{E}{\mu}$. It should therefore be possible to both contain atomic tritium and trap the beta decay electrons using a magnetic field.

Magnetic trapping also offers a potential solution for our other problem of needing a source of atomic tritium with very low molecular contamination. While atomic tritium has a magnetic moment, molecular tritium does not and therefore will not be trapped. Thus, it can be allowed to diffuse out so that only the atomic tritium remains. Further, for trapped atomic tritium, recombination is negligible since it is a three-body process and the the tritium density is very low due to the long electron mean free path required for the energy measurement. It would only occur at a significant rate on the walls of the apparatus which is prevented by the trapping.

The two ideas of magnetic trapping and measuring the energy via cyclotron radiation fit together very well. The use of a magnetic trap prevents "direct" measurement of the electron since it stays in the trap so a "non-invasive" energy measurement technique is needed. Cyclotron frequency measurement satisfies that requirement. These two concepts serve as the basis for our experimental concept. A sketch of the proposed experiment is shown in Figure 1.4.

1.5 R&D Required

While this experimental concept has potential to allow significant improvements in direct neutrino mass measurements, there are many outstanding questions regarding the design which could prevent the experiment from being realized. These concerns are generally related to the magnetic trapping of tritium. While various materials have been magnetically trapped for the purpose of condensed matter experiments (for example, to produce Bose Einstein condensates), the trapping of tritium introduces potential difficulties due to its radioactivity. Also, the field uniformity needed for the spectroscopy introduces complications for the trap design. There are also questions regarding how to load the trap and experimental requirements on the atomic tritium temperature in the trap, among others.

One immediate concern is the possibility that long term trapping of atomic tritium is not possible due to heating from the beta decay electrons. These electrons represent our experimental signal, but after the observation period during which we measure the cyclotron frequency, they collide with a tritium atom in the source and deposit some of their energy upon that tritium atom. The electron will then repeatedly collide with tritium atoms until it scatters out of the trap. This may result in a large number of energetic and ionized tritium atoms in the source for each decay electron. Given that the beta decay electron has an average kinetic energy of 6 keV and the energy required for a tritium atom to escape the trap is < 1 meV for the expected trap strength of a few T, one could imagine the possibility of several million tritium atoms being lost for every decay electron. This process would occur via the electrons depositing a relatively large amount of energy of a few tritium atoms which would in turn collide with other tritium atoms spreading the resultant energy throughout the whole tritium source.

If this scenario were realized, the trap lifetime would be of order a few seconds (with the exact value depending on trap depth and fraction of beta decay energy deposited in the trap). While it is not clear how long the procedure of loading the trap would take and whether or not the escaped tritium could be captured and reused, this would certainly complicate the experimental design. Thus, it is necessary to determine whether the described situation does in fact occur or if the loss rate is significantly lower. Also, we can investigate whether it is possible to continuously cool the trapped tritium to counteract the heating. This could possibly be done by including a gas such as He which has zero magnetic moment

and thus is not contained in the magnetic trap. This gas would provide thermal contact between the trapped tritium and an external heat sink.

Even if atomic tritium trapping is possible, the magnetic field that provides the trapping may make accurate electron energy measurements impossible. Recall that the energy is given by

$$T + m_e c^2 = \frac{eB}{\omega}. \tag{1.10}$$

Thus, to measure the electron kinetic energy with a resolution of $\Delta E = 1$ eV, we require an energy precision of $\Delta E / m_e c^2 = 2 \cdot 10^{-6}$. Since we are making a measurement of ω and B to calculate T, both ω and B must be known to this precision. Given that we have no way to determine the location of a particular beta decay electron, the magnetic field must be uniform to $\sim$1ppm. This seems rather difficult to reconcile with the concept of a magnetic trap which requires the magnetic field to be significantly larger near the trap boundaries than in the center (by several T). Clearly, if the magnetic field makes this change linearly, then the uniformity condition cannot be satisfied. If, however, the magnetic field strength behaved like a step function (or something with similarly rapid growth, such as an exponential or high degree polynomial), then this issue would be solved. In such a case, the magnetic field magnitude would be constant over a large trap volume and then rise sharply at the boundaries.

This thesis will first examine the feasibility of magnetically trapping atomic tritium. We address tritium escape from the trap due to both β decay electrons and evaporative cooling and demonstrate how each of these can be handled. We then propose a magnetic field design which would provide trapping, but not interfere with electron energy measurement via cyclotron frequency. Finally, we propose a research program to implement the proposed experiment and discuss topics which this thesis has not addressed and should be considered in further work.

Chapter 2

Trapping Feasibility

2.1 Collisional Cooling

The β decay of tritium produces energetic electrons of average energy 6 keV. Some fraction of this energy will be deposited in the tritium atoms of the source before the electrons escape. This power input will cause the average energy of the trapped tritium to increase and thereby leads to tritium escaping from the trap. In order to understand at what rate such escape occurs, we first need to understand how this energy is dissipated from the small number of tritium atoms which directly collide with β decay electrons to the tritium source as a whole and possibly some other gas present used to dissipate the heat. The mechanism responsible for this process is collisional cooling.

A particle can be cooled by repeated collisions with particles of lower energy. If we consider a energetic particle of mass M with energy E_a in a volume of particles of mass m with average energy E_b, then, in the average collision, the energy lost by the energetic particle is

$$\Delta E = (E_a - E_b)\frac{2Mm}{(M+m)^2}. \tag{2.1}$$

The energy of the particle thus decays exponentially to the background energy. After n collisions, the average energy is

$$E_a(n) = E_b + (E_a - E_b)\left(\frac{M^2+m^2}{(M+m)^2}\right)^n. \tag{2.2}$$

Here we are interested in cooling tritium atoms which gained kinetic energy from collisions with β decay electrons sufficiently that they remain trapped. From kinematics, we know that the maximum kinetic energy which a tritium atom can gain in a collision with a

β decay electron (which has kinetic energy in the range 0 to 18.6 keV) occurs for a direct backscatter collision and is 13 eV. The strength of the magnetic trap is perhaps at most 5 T, giving a trap height of $\mu_B B = 0.29$ meV. Thus, in order for such a particle to be trapped, it must undergo n collisions for n satisfying

$$0.29 \text{ meV} > 13 \text{ eV} \left(\frac{M^2 + m^2}{(M+m)^2} \right)^n \tag{2.3}$$

before it reaches the wall of the trap and is thus lost. If we first consider only collisions with tritium, then $M = m$ and the number of collisions required is 16. This is a minimum as for $M \neq m$, the required n increases. Thus, the optical depth of the source and background gas used for cooling against tritium atoms must be at least $\sqrt{n} = 4$. This places a requirement on the size of the trap relative to the tritium mean free path which must be satisfied for collisional cooling to be effective.

The tritium mean free path is determined by the density of tritium and the cross section. The density of the tritium gas is set by energy measurement considerations. In order to measure the cyclotron frequency accurately, we must observe it for a sufficiently long time. Using Nyquist's theorem and the equations given previously for the cyclotron frequency, the mean free path of electrons must be at least $l = 3 \text{ km} \left(\frac{B}{1 \text{ T}}\right)^{-1} \left(\frac{\Delta E}{1 \text{ eV}}\right)^{-1}$ with energy resolution ΔE and magnetic field B. Given the cross section for electron scattering on tritium, this would then determine the density. Thus, since the electron mean free path is fixed, the ratio of the cross sections for electron and tritium scattering on tritium determines the tritium mean free path

Values for these cross sections are not known extremely well, but some data is available. For electrons of tritium endpoint energy (18.6 keV), the elastic and ionization cross sections are $1.6 \cdot 10^{-19}$ cm^2 and $9.4 \cdot 10^{-19}$ cm^2, respectively [15, 16]. Useful data on the excitation cross section was not found, so a lower limit on the total electron tritium cross section is $1.1 \cdot 10^{-18}$ cm^2 $= .012 \pi a_0^2$. For the tritium-tritium cross section, we derive a value a value of 50 – 80 πa_0^2 [17] with the uncertainty here is due to the unknown energy distribution of tritium atoms resulting from elastic scattering by β decay electrons. However, given that the ionization cross section dominates the elastic cross section, we should instead be interested in the tritium ion-tritium cross section which we find to be 100–200 πa_0^2 with the uncertainty again based on the unknown energy distribution [18].

Based on these values, the maximum possible cross section ratio is $16 \cdot 10^3$. This gives a mean free path for tritium atoms of $l = 20$ cm $\left(\frac{B}{1\text{ T}}\right)^{-1}\left(\frac{\Delta E}{1\text{ eV}}\right)^{-1}$. Considering $\Delta E = 1$ eV and $B = 1$ T to be reasonable values, the mean free path is 20 cm so the source would need to have radius ~ 80 cm in order for collisional cooling to effective at dissipating the input energy throughout the source. As this is significantly larger than is under consideration at this time due to other design constraints, collisional cooling using only the tritium source does not appear to be an effective technique for the experiment at hand. Based on the previous argument, we see that the only way a background gas could change this result is to have a very large ratio of cross section against tritium versus cross section against electrons was available. Such a gas could allow for collisional cooling over short distance scales while not preventing electrons from having the several kilometer mean free path required.

Since energetic tritium atoms are not brought into thermal equilibrium with the bulk of the source via repeated collisions, we are motivated to study the problem on the individual level rather than on the bulk.

2.2 β Decay Electron Heating

In order to determine the trap loss rate due to the β decay electrons, we have modeled the behavior of the β decay electrons from the time of the decay until trap escape, the effect that the electrons have upon tritium atoms with which they collide, and the subsequent effect of these tritium atoms and their ionized electrons upon the rest of the trapped atomic tritium. Together, these will allow us to calculate the average number of tritium atoms which escape per β decay electron produced.

The first part of this simulation is focused on the behavior of β decay electrons within a magnetic trap while undergoing repeated collisions with tritium atoms. There are three primary components to this simulation: the initial energy distribution of the electrons, the differential cross section for collisions of high energy electrons with tritium, and the model of the trap. The energy spectrum for β decay electrons is

$$\frac{dN}{dT} \propto F(Z,T)p(T + m_e c^2)(Q - T)^2 \tag{2.4}$$

where T is the kinetic energy, p is the momentum, Q is the endpoint energy, and $F(Z,T)$ is the Fermi function. We have taken the neutrino mass to be zero as it has negligible effect for this purpose. The direction of the electron momentum is distributed isotropically.

For the purpose of this simulation, we can use a very simple model of the trap. Given the extremely long mean free path of the electron (as is required for a precise energy measurement), we do not need to model the propogation of the electron within the trap as an electron will make many cycles of the trap between collisions. We thus treat the trap as having an ideal magnetic mirror at each end. If an electron has pitch angle $\theta > \arcsin(\sqrt{\frac{B_l}{B_h}})$, either due to its original direction or a collision, then it will escape when it next reaches one end of the trap. Thus, $\frac{B_l}{B_h}$, hereafter referred to as the mirror ratio, is an adjustable parameter of the simulation.

The final component of the simulation is the modeling of the electron-tritium collisions. As a simplification, we assume that all collisions result in an ionization. This is not an unreasonable approximation as based upon the total cross sections cited previously, 85% of collisions result in ionization. We thus require the differential cross section for ionizing collisions of high energy electrons with tritium atoms. In particular, we need the cross section to be differential in both the scattering angle and the energy loss. While such calculations are generally rather difficult, it is greatly simplified in the limit of high energy collisions (kinetic energy much larger than the binding energy of the atoms). In this case, the differential cross section can be calculated from Bethe theory [19] to be

$$\frac{d^2\sigma}{dEd[\ln(Ka_0)^2]} = 4\pi a_0^2 \frac{R}{T}\frac{R}{E}\frac{df(K,E)}{dE} \tag{2.5}$$

where E is the energy lost by the electron, $\hbar K$ is the momentum transfer, R is the ionization energy (1 rydberg), T is the kinetic energy, and $\frac{df(K,E)}{dE}$ is the generalized oscillator strength. The generalized oscillator strength is in turn given by

$$\begin{aligned}\frac{df(K,E)}{dE} =& \frac{2^7[(Ka_0)^2 + (E/3R)]ER^{-2}}{[(K+\kappa)^2a_0^2+1]^3[(K-\kappa)^2a_0^2+1]^3} \\ &\times \frac{1}{1-\exp(-2\pi/(\kappa a_0))}\exp\left(-\frac{2}{ka_0}\arctan\left[\frac{2\kappa a_0}{(Ka_0)^2-(\kappa a_0)^2+1}\right]\right)\end{aligned} \tag{2.6}$$

where $\kappa a_0 = \sqrt{E/R-1}$. Note that while this is given in terms of the energy loss and momentum transfer, this can be converted to energy loss and scattering angle easily though

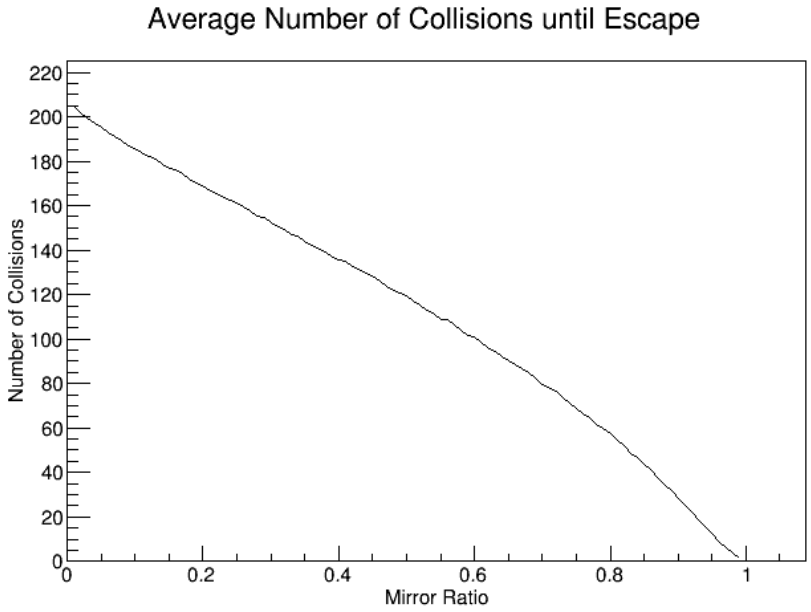

Figure 2.1: The average number of collisions that a β decay electron undergoes before it escaps the trap. An equal number of secondary electrons are produced via the ionization the collision causes.

the resulting formula would be rather complicated.

Given the initial momenta distribution, differential cross section, and condition for the electron to escape the trap, we have implemented a Monte Carlo simulation in ROOT to determine the average effect of a β decay electron. One of the key results of this simulation is the number of collisions which an electron undergoes before it escapes the trap. This depends strongly on the mirror ratio of the magnetic trap which is shown in Figure 2.1. The number of collisions required for escape is significantly greater than would be required for isotropic scattering as most collisions cause only a small scattering of the high energy electrons. We can also determine the distribution of the energy lost in these collisions which is shown in Figure 2.2.

Therefore, within the framework of our model, each β decay electron results in a number of ionized tritium atoms and low energy electrons. The number of such ion electron pairs depends on the mirror ratio of the magnetic trap. In reality, these tritium ions and electrons will be accompanied by some number of ground state and excited tritium atoms from elastic and excitation collisions, but, as discussed above, such particles are of comparatively small number and will quickly escape the trap and thus have no further effect. On the other hand, as charged particles, the tritium ions and electrons are trapped by the magnetic mirrors.

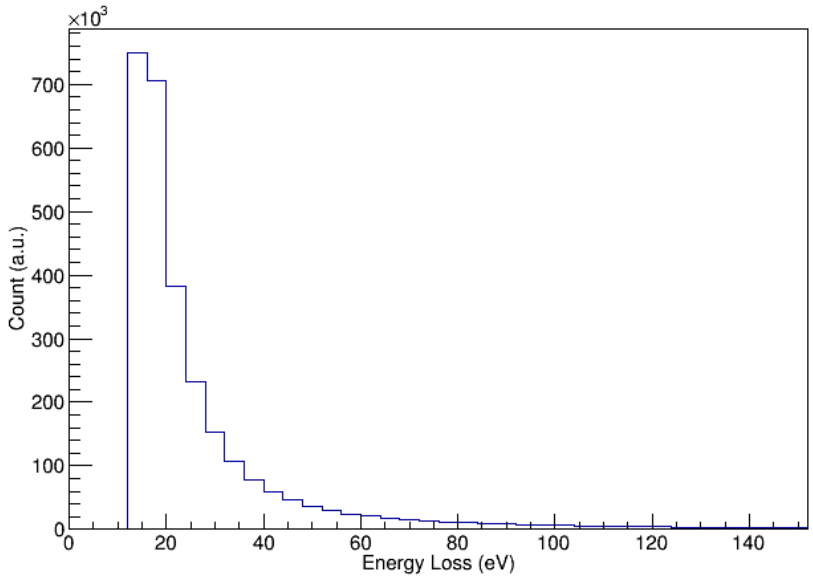

Figure 2.2: β decay electrons lose only a small fraction of their energy in each collision with the probability distribution sharply peaked at just above the energy required for ionization.

Thus, we must consider the behavior of these tritium ions and electrons. A first item to note is that these particles are of relatively low energy. The average energy lost by the β decay electron in a collision is 29 eV and, after subtracting off the energy which goes to ionize the electron, only 16 eV is available as kinetic energy for the tritium ion and electron. Thus, on the average, the tritium ion and electron will only have enough energy to excite or ionize one other tritium atom. Since this sets an upper limit on the possible impact of the tritium ion and electron via inelastic collisions, we instead focus on their behavior in elastic collisions. In fact, we can consider only the electron as for the typical energy loss and momentum transfer, the electron shall acquire almost all the kinetic energy by virtue of its significantly lighter mass.

Theoretical calculations of the differential cross section for such low energy electrons are lacking with only a small number of results for particular energies in the literature in contrast to the analytical formula available for high energy electrons. This lack of detailed data results in some level of approximation being required. Fon, Burke, & Kingston (1978) give the differential cross section for a handful of energy values in the range 0 to 4 rydberg as a table of values for scattering angles from 0 to 180° in 10° increments [20]. Based upon this data which shows relatively little variation in cross section with scattering angle, I am

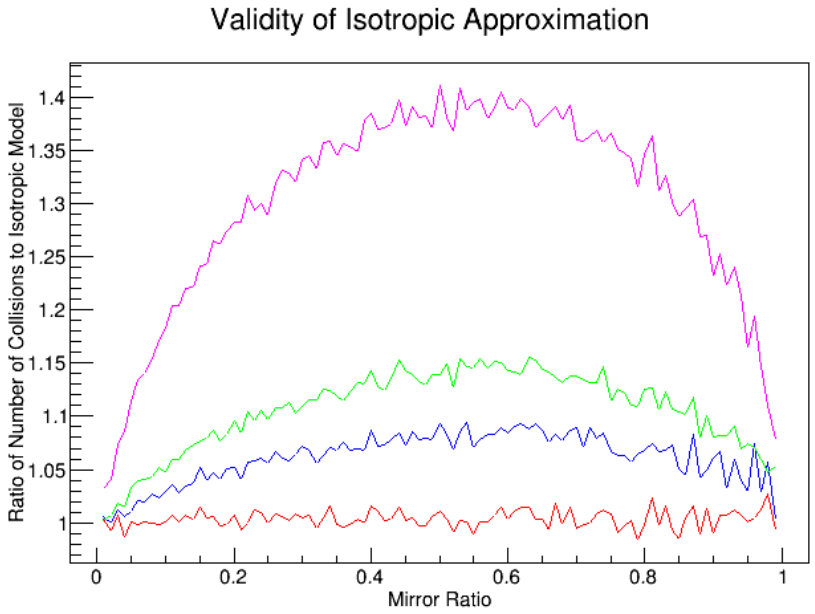

Figure 2.3: The ratio of the number of collisions required for escape to the number that would be required for isotropic scattering for electrons with kinetic energy 1.22 eV (red), 4.90 eV (blue), 8.71 eV (green), and 30.6 eV (purple).

motivated to consider whether it is reasonable to treat the cross section as isotropic. In order to test this suggestion, I calculate the number of scattering events required for an electron to escape the trap for isotropic scattering and the differntial cross sections given by Fon, Burke, & Kingston (1978) across a range of magnetic mirror values using a similar simulation to that performed previously for the β decay electrons. The results are shown in Figure 2.3. We see that the approximation is quite good as the very lowest energies, but deteriorates to up to 40% error at higher energies. Given that our average energy is within the range of energies shown, we shall accept the isotropic approximation while recognizing that it may result in errors in the number of tritium atoms lost by 20-30%.

Then, we combine the results from Figure 2.1 for the average number of ionizations caused by a β decay electron and that the average number of collisions an isotropically scattering electron will undergo before escaping a trap of magnetic mirror ratio $\frac{B_l}{B_h}$ is

$$\frac{\sqrt{1-\frac{B_l}{B_h}}}{1-\sqrt{1-\frac{B_l}{B_h}}} \tag{2.7}$$

to determine the total number of tritium atoms which undergo an elastic collision with one

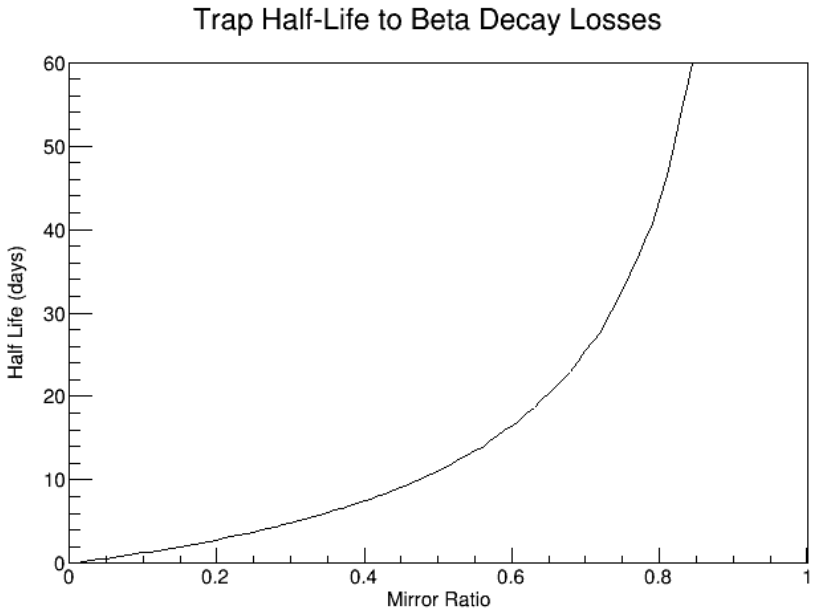

Figure 2.4: The trap half-life if the only loss mechanism is β decay heating as it depends on the magnetic mirror ratio.

of the ionization electrons for each β decay. We should expect a large fraction of these to have sufficient kinetic energy to escape the trap. Rather than attempt to calculate this fraction, we simply assume the worst case scenario of 100% being able to escape. Also, after the elastic collision, these tritium atoms should escape the trap without undergoing any scattering events due to the long mean free path for tritium. Thus, the number of tritium atoms which undergo an elastic collision with one of the ionization electrons is equal to the number of tritium atoms which escape the trap. From this and the half-life of tritium, we can calculate the loss rate from the trap and thereby the half-life of the trap if β decay heating is the only loss mechanism. The result of this calculation is shown in Figure 2.4. These results are encouraging as even for a strong trap with a mirror ratio of 0.2 (which traps 89% of β decay electrons), the half-life is 3 days and the half-life increases rapidly for higher mirror ratios. Thus, the tritium escape due to the β decay electrons occurs slowly enough that atomic tritium can be trapped for several days to months. Note that while the approximations used may result in an error of perhaps 50% in the half-life, this does not change the qualitative conclusions.

2.3 Evaporative Cooling

There are a variety of potential loss mechanisms for a magnetic trap. The majority of these occur for any variety of trapped particle and thus are well understood. Losses due to β decay heating was a novel possibility and thus required the more detailed study described previously. However, the other loss mechanisms are still present and set constraints on the experiment design. Here we discuss evaporative cooling and the constraint this places on the temperature to which the atomic tritium must be cooled.

If a particle contained in a magnetic trap of depth $\mu\Delta B$ has kinetic energy $E > \mu\Delta B$, then it will escape from the trap (unless it first collides with another particle and loses energy). In a trap at temperature T with $k_B T < \mu\Delta B$, the average particle will not have enough energy to do so, but a fraction of particles at the upper end of the energy spectrum will. The loss rate to this process, termed evaporative cooling, is therefore dependent on the ratio $\eta = \frac{\mu B}{k_B T}$. Also, as a particle must gain energy through a collision to have enough energy to escape, the loss rate $\dot{N}_{evap}$ should be proportional to the collision rate $n\sigma\bar{v}$ where n is the density, σ the cross section, and $\bar{v}$ the mean velocity. Then, the loss rate will be given by

$$\dot{N}_{evap} = N n \sigma \bar{v} f_{esc} \tag{2.8}$$

where N is the total number of atoms and f_{esc} is the fraction of collisions which result in an escaped particle. deCarvalho (2003) gives $f_{esc} = \eta e^{-\eta}$ for large η while Fried (1999) makes a more involved calculation using a truncated Boltzmann distribution (since particles with energy above $\mu\Delta B$ have already escaped and do not enter into collision) for the energy of two incoming particles and calculating the fraction of collisions in which one particle has energy above $\mu\Delta B$ afterwards [21, 22]. Fried's results give the same functional dependence, but do differ by a factor of order unity. However, due to how rapidly f_{esc} diminishes with η, this will not materially affect the results and so we use the simplified formula from deCarvalho (2003).

Then, the trap mean lifetime from evaporative cooling is

$$\tau = \frac{1}{n\sigma\bar{v}} \frac{e^{\eta}}{\eta} \tag{2.9}$$

This exponential growth should yield a small value of η at which the loss rate to evaporative

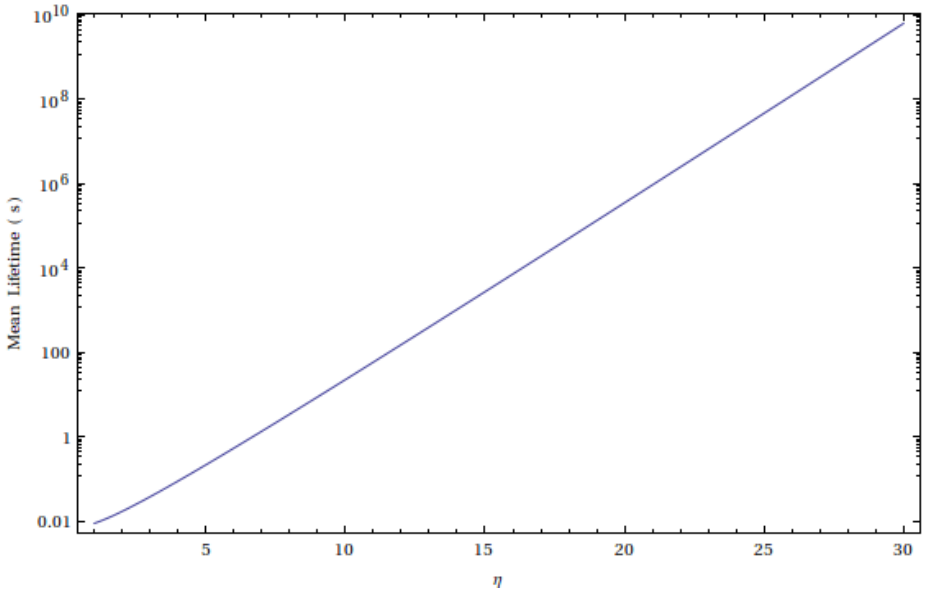

Figure 2.5: The fraction of particle in a Boltzmann distribution with energy above $\eta k_B T$ decreases exponentially with η and so the mean life due to evaporative cooling grows similarly.

cooling becomes negligible. Based upon the cross sections values discussed previously, we have a mean free path of $\frac{1}{n\sigma} \approx 50$ cm for the densities required to obtain an energy resolution of 1 eV in a 1 T magnetic field. For a magnetic trap of height 5 T, Figure 2.5 shows the mean lifetime as a function of η. Thus, $\eta \sim 20 - 25$ gives a mean life of several days to a year rendering evaporative negligible relative to other potential loss mechanisms. This corresponds to a tritium source temperature of 130 – 170 mK for a 5 T trap height.

2.4 Trap Loading

In the previous section, we considered the loss rate of tritium from the magnetic trap due to evaporative cooling. We found that if the trapped tritium is at low enough temperature relative to the trap height, then the loss rate due to evaporative cooling negligible. However, this requires a temperature of $\sim$ 150 mK for a magnetic trap of 5 T height. We consider this to be a reasonable estimate of the maximum trap height feasible based upon the field strengths provided by existing solenoids and the additional challenges which the trap design may impose. Thus, the atomic tritium must be cooled to 150 mK.

This is a key element of the larger question of how to produce, cool, and trap atomic tritium. While we do not address the question of how to produce atomic tritium here, it

is expected that the resultant atomic tritium will be at a temperature of several hundred K [23]. Cooling this to sub-K temperatures is quite challenging. A standard technique for doing so in magnetic trapping experiments is laser cooling, but this is not applicable for hydrogen isotopes [24].

An alternative technique is to use collisional cooling. While this technique was not viable for removing the heat generated by β decay, the experiment requirements during loading are significantly different that during data-taking operations making this feasible. In particular, during trap loading, the density is not constrained as it is during data-taking by the need for β decay electrons to have a long mean free path. Thus, we can use a very high density of some buffer gas (likely either ^{3}He or ^{4}He) so that the energetic tritium atoms have a mean free path much less than the experiment radius and so are cooled before reaching the edge of the trap.

This technique should allow cooling to 200–300 mK using ^{3}He. It will be unable to cool further as from the vapor density-temperature curve for ^{3}He shown in Figure 2.6, the vapor density reaches the minimum density at which the mean free path for tritium is sufficiently short at a temperature of 200–300 mK [24]. As this is not the 150 mK stated previously, we must either use a stronger magnetic trap or accept that we will lose some fraction of our trapped tritium to evaporative cooling after trap loading has been completed. A detailed calculation of what fraction would be lost before the temperature reaches 150 mK has not yet been performed and is a possible subject of future work. However, general scaling rules for evaporative cooling imply that

$$\mathrm{T} \propto \mathrm{N}^{1/\eta} \tag{2.10}$$

for temperature T and number of tritium atoms N under the approximation of η constant.Thus, so long as we start the evaporative cooling at $\eta \gg 1$ which holds here, it should be quite efficient. For instance, if we have $\eta = 10$ initially, then the cooling by a factor of two to $\eta = 20$ causes only a 7% decrease in the amount of tritium in the trap. We also need not be concerned with the duration that this process takes as we should be able to take data while the evaporative cooling is occurring. While this will result in the source strength changing over time, the source strength can easily be measured from the total rate of β decay observed via cyclotron radiation across the entire kinetic energy range.

Another complication is that after the dense buffer gas is used for cooling, it must be

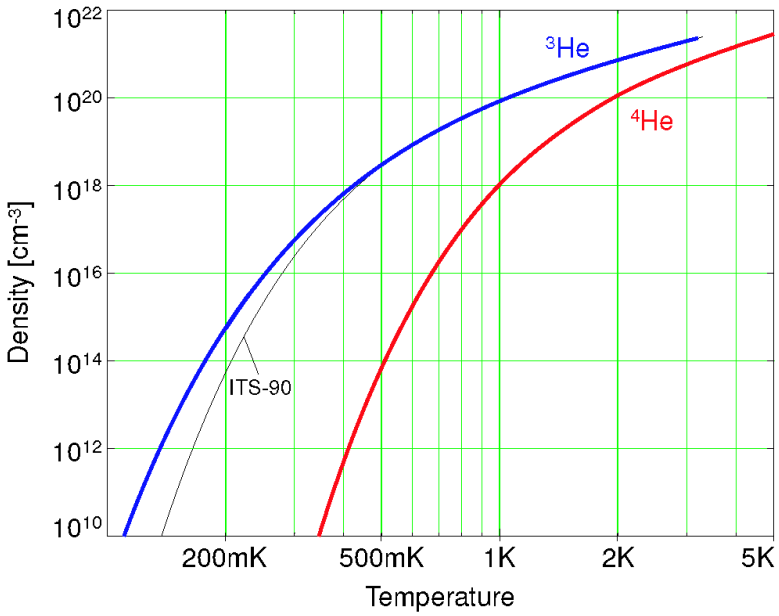

Figure 2.6: The vapor density curve give the maximum density possible at a given temperature. It is shown here for ^{3}He (blue) and ^{4}He (red). The curve labeled ITS-90 is the vapor density curve for ^{3}He under the ITS-90 standard and is not important for our purposes. The trap loading would occur at a density of 10^{14}–10^{15} cm^{-3} in order to give a very short tritium mean free path. Figure reprinted from [25].

removed so that data-taking can begin. However, there is the potential that during the time required for removal, we may lose a large fraction of the tritium from the trap. While the loss rate is small both in the extremes of a high and low buffer gas density, it has a sharp peak for intermediate values of the buffer gas density [24]. The buffer gas must be removed on a time scale shorter than the half-life for this loss rate. This can be achieved by rapidly pumping out the buffer gas through a large valve. This is likely to set limits on the experiment size or shape. These are yet undetermined due to a lack of data regarding the loss rate in this intermediate density regime.

Chapter 3

Magnetic Trap Design

In the previous chapter, we used very simple models of the magnetic trap in order to evaluate trapping feasibility. The ability of the trap to contain electrons depended only on the ratio between the trap center and and trap boundary magnetic field magnitude which determined a range of electron pitch angles which would be trapped. Obviously, a realistic trap will not be characterized by only two field magnitudes. The minimum trapped pitch angle would vary depending on the location in the trap at which the β decay or last electron collision occurred. However, an average of the trapping pitch angle will characterize the behavior of the magnetic trap as it applies to electrons so the approximation was reasonable

However, the actual design of the trap is significantly more complicated. The magnetic field must be uniform to 1ppm or better in the experimental region so that the electron energy can be calculated precisely from the cyclotron frequency. However, the depth of the trap in temperature units is $\frac{\mu_B \Delta B}{k_B}$. Since the tritium in the trap must have a temperature lower than the trap depth by a factor of 20–25 to slow the rate of evaporative cooling, in order for atomic tritium to be trappable even at a low temperature of ~ 150 mK, we require $\Delta B \sim 5$ T. Clearly there is a dramatic difference in that the magnetic field can vary by only a few μT in the experimental region and by a few T over the entire trap volume. One way to resolve this difference is to have the experimental region be only a small portion of total trap volume. While tritium will be present throughout the trap, the cyclotron frequency is given by $\frac{eB}{T+mc^2}$ so β decay electrons with energy near the endpoint, but outside the experimental region (so at higher magnetic field) will have the same cyclotron frequency as lower energy electrons in the experimental region. Thus, the experiment will be unable to determine that these are endpoint energy electrons and so they will not contribute to the statistics of the experiment. Thus, the effective source strength will scale with the volume

of the experimental region, not the whole trap. Consider a simple trap design, such as a uniform field combined with a magnetic quadrupole where the magnetic field magnitude is given by $B = \sqrt{B_0^2 + B_\perp^2 r^2/r_0^2}$ where r is the distance from the central axis of the trap and r_0 is the radius of the trap. Then, the magnetic field is only within 1ppm of B_0 when $(\frac{B_\perp r}{B_0 r_0})^2 \lesssim 10^{-6}$. Since we expect $B_\perp \gtrsim B_0$, the experimental region is given by $r \lesssim 10^{-3} r_0$. Thus, the volume of the experimental region is $\sim 10^{-6}$ of the total trap volume. The resulting low source strength and low statistics would prevent the successful use of this experiment for measuring the neutrino mass. Thus, we need to design a magnetic trap with a high experimental-volume/trap-volume ratio.

3.1 Two Dimensional Trapping

If we only needed to trap the particle in two dimensions, this could be easily achieved using magnetic multipole fields. Consider the $2m$-pole magnetic field

$$\vec{B}_{2m}(r,\theta) = \frac{B_\perp}{r_0^{m-1}} \left(r^{m-1} \sin(m\theta - \phi)\hat{r} + r^{m-1} \cos(m\theta - \phi)\hat{\theta} \right) \tag{3.1}$$

It is easy to verify that $\nabla \times \vec{B} = 0$ and $\nabla \cdot \vec{B} = 0$, so this magnetic field can occur in free space (a requirement for our experiment's magnetic field as there cannot be any current wires within the trap). Now, if this is combined with a uniform field $\vec{B}_{solenoid} = B_0 \hat{z}$, then

$$|\vec{B}_{2m} + \vec{B}_{solenoid}| = B_0 \sqrt{1 + \frac{B_\perp^2}{B_0^2} \left(\frac{r}{r_0} \right)^{2m-2}} \tag{3.2}$$

In order for this to differ from B_0 by at most a fraction η, we must have $\frac{B_\perp^2}{2B_0^2} \left(\frac{r}{r_0} \right)^{2m-2} < \eta$ or equivalently,

$$\left(\frac{r}{r_0} \right)^2 < \left(\frac{2\eta B_0}{B_\perp} \right)^{\frac{1}{m-1}} \tag{3.3}$$

The left hand side of this inequality is the fraction of the trap cross sectional area which is within the experimental region. Thus, it is the experimental-volume/trap-volume ratio. Note that as m increase, the right hand side approaches 1. For instance, if we take $\eta = 10^{-6}$ and $B_\perp = 2B_0$, then the below table gives a few examples of the volume ratio for different values of m.

m	Volume Ratio
2	10^{-6}
4	0.01
6	0.06
10	0.22
20	0.48
50	0.75

Therefore, by combining a uniform field and a sufficiently high multipole field, particles can be magnetically trapped in the radial direction while the experimental volume is only slightly smaller than the total trap volume. However, magnetically trapping particles into a finite volume rather than an infinite cylinder will be a more challenging task.

3.2 Three Dimensional Trapping

The most general magnetic field possible in free space is given by $\vec{B} = \nabla\Phi$ for Φ satisfying Laplace's equation. Given our success with radial trapping above, we consider Laplace's equation in cylindrical coordinates. Using separation of variables, we find that

$$\Phi_{m\lambda}(r,\theta,z) = J_m(\lambda r)(A\cos(m\theta) + B\sin(m\theta))(C\cosh(\lambda z) + D\sinh(\lambda z)) \tag{3.4}$$

is a solution for integer m and arbitrary λ, A, B, C, and D. Here $J_m(\lambda r)$ is the Bessel function of the first kind (and the Bessel function of the second kind in not considered as we are only interested in solutions that are finite everywhere). The general solution to Laplace's equation is the linear combination of all such solutions.

However, these basis functions are themselves of interest. In the limit of $\lambda r \ll 4m$, $J_m(\lambda r) \propto (\lambda r)^m$. Thus, in the same limit, $\vec{B}_{m\lambda} = \nabla\Phi_{m\lambda}$ has components proportional to r^{m-1} and r^m. This is the same high degree polynomial growth which we found in the two dimensional case to give high experimental volume-trapping volume ratios which is promising. Now consider the total field magnitude when this field is combined with a uniform field $\vec{B}_{solenoid} = B_0\hat{z}$ (since we need a nonzero field in the trap to measure the

electron energy). In the limit $\lambda r \ll 4m$, we get

$$\begin{aligned} |\vec{B}_{m\lambda} + \vec{B}_{solenoid}| &= \sqrt{B_0^2 + 2B_0 B_{m\lambda,z} + |\vec{B}_{m\lambda}|^2} \\ &\approxeq B_0 + B_{m\lambda,z} \\ &\approxeq B_0 + (\lambda r)^m \left(A\cos(m\theta) + B\sin(m\theta)\right)\left(C\sinh(\lambda z) + D\cosh(\lambda z)\right) \end{aligned} \tag{3.5}$$

For fixed z and θ, the field grows as r^m in this limit. Thus, by the same reasoning as applied to the two dimensional case above, by taking m large enough, we can make the experimental volume-trap volume as close to 1 as desired.

Before we conclude that this field is a satisfactory trapping field, we must ensure that it actually does form a trap. If there is some path a particle can follow out of the trap while staying at the central field B_0 or less, then we do not have a trap. If we look at the field $\vec{B}_{m\lambda} + \vec{B}_{solenoid}$, we see that particles are trapped against escape in the radial direction since it grows polynomially in r. They are also trapped against escape in the axial direction by the term $r^m \cosh(\lambda z)$ in the magnetic field with one exception. At $r = 0$, this term is zero. Indeed, the magnetic field is constant along the z-axis at all orders in the expansion. Thus, a particle could escape from this trap by traveling up the axis. At first glance, this might be considered a fatal flaw of using this field for trapping, but consider the size of the "hole" in the trap that this causes. In the limit of $\lambda z \gg m$, the radius r at which the magnetic field reaches a specific value obeys

$$r \propto \exp^{-\frac{\lambda z}{m}} \tag{3.6}$$

Thus, the area through which the particles could escape from the trap decreases exponentially in λz and the escape rate should behave similarly.

Thus, our desired trap has radius R and length Z obeying $R \ll \frac{m}{\lambda} \ll Z$. The first inequality ensures radial trapping and a high experimental region-trap volume ratio while the second ensures axial trapping. Note that while this places a constraint on the ratio R/Z, we are still free to set the absolute scale of the experiment as desired by the value of λ. In order to better illustrate the appearance of such a trap, Figure 3.1 shows a cross section is θ of the magnetic field contours for such a trap with $m = 20$. While the contours appear somewhat different for alternative value of θ, this plot is representative. The contours shown are for $B = (1+10^{-6})B_0$, $B = (1+10^{-3})B_0$, and $B = 2B_0$. The volume with radius smaller than the $B = (1 + 10^{-6})B_0$ contour corresponds to the experimental volume and we see

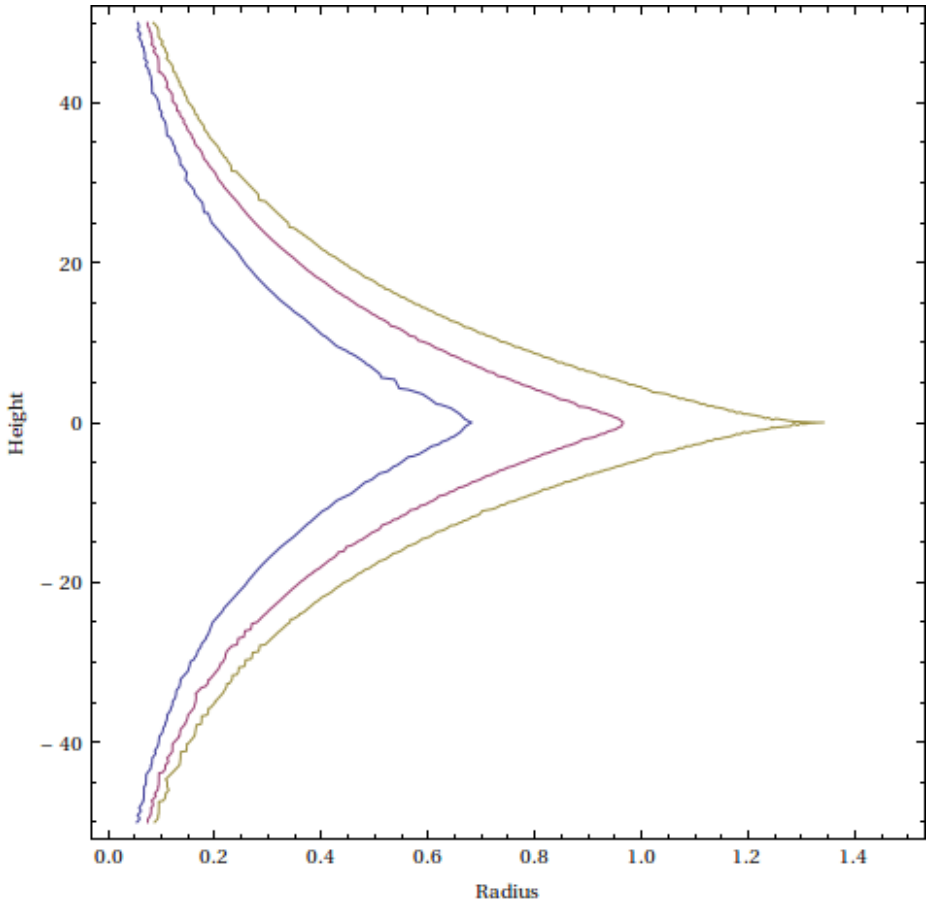

Figure 3.1: Contours of the magnetic field for a trap with $m = 20$, $A = 1$, $B = 0$, $C = 10^{-4}B_0/J_m(1)$, $D = 10^{-3}B_0/J_m(1)$ in the $\theta = 0$ cross section. The contours shown are $B = (1 + 10^{-6})B_0$ (blue), $B = (1 + 10^{-3})B_0$ (purple), and $B = 2B_0$ (yellow). Units are arbitrary, but the same for both radius and height.

that it is a substantial fraction of the volume inside the $B = 2B_0$ contour.

While the above argument indicates that the loss rate to escape along the axis can be made negligible by making the trap sufficiently long, it does not determine how long the trap needs to be. To do this, the constant of proportionality in front of this exponential must be determined via simulation. This has yet to be completed and is an objective for continuation of this work.

Another matter that needs to be addressed is how precisely the field can be constructed. Any inhomogeneities in the magnet construction will result in the presence of other terms in the general expansion for the magnetic field in a vacuum than the main term. Most concerning is the possibility of terms with scalar potential $\Phi_{n\lambda'}$ with $n < m$. This term will grow as a lower polynomial power and thus may cause the field to not meet the uniformity

requirement for the desired energy measurement. Engineering constraints will set the ultimate field uniformity feasible. Given that the desired uniformity is achievable but difficult for a uniform magnetic field over the volumes for which we are interested, this is likely to set the ultimate limit in the field uniformity over the experimental region.

3.3 Majorana Loss

In addition to the loss mechanisms discussed in the previous chapter, atomic tritium can also be lost if it undergoes a Majorana spin flip. When a spin flip occurs, the magnetic field minimum of the trap switches from being a minimum in the associated potential to a maximum. Thus, such atoms are no longer trapped and escape. Note that these tritium atoms can be regarded as escaping immediately without undergoing any collisions due to the long mean free path for tritium-tritium collisions.

Majorana spin flips occur only when the magnetic field experienced by the tritium atom changes rapidly relative to the Larmor frequency $\omega = \frac{\mu_B B}{\hbar}$. In the opposite case, when the Larmor frequency is much greater than the frequency at which the magnetic field changes, the spin of the tritium atom will precess in order to keep the spin aligned with the magnetic field thereby preventing spin flips. In many magnetic trapping experiments, spin flips occurs predominantly in the vicinity of the trap center as the experiments use a magnetic quadrupole or similar trap in which the magnetic field goes to zero there so the field magnitude and direction both vary rapidly. Any particles whose trajectories take them near the trap center have a chance of undergoing a spin flip.

The situation is quite different for our trap. First, the minimum field in the trap we consider is not zero as it is this central field which results in cyclotron radiation from the β electrons. Also, this field is very uniform as required by the cyclotron spectroscopy technique. Majorana spin flips do not occur in a uniform magnetic field and so we expect that our trap will not experience loss from the central region of the trap. Instead, losses occur near the boundary of the trap both due to rapid changes in magnitude and direction. At the trap boundary where the non-uniform component dominates the uniform solenoid field, the direction of the magnetic field changes with θ with period $\frac{2\pi}{m}$. Thus, for large m, a particle moving circumferentially could experience a 180° rotation in its local magnetic field in a small fraction of the trap size. This suggests that we might expect the loss rate

to increase with m as it is a proxy for rate of change in the magnetic field. However, another factor may counter this. For large m, the experimental-volume/trap-volume ratio approaches 1 leading to a smaller volume on the boundary of the trap in which spin flips occur decreasing the overall rate.

Due to these and other factors which contribute to the rate of Majorana spin flips, some of which depend sensitively on the trap geometry, calculation of the Majorana loss rate is a problem for computer simulation. Generally, such a simulation would operate by propagating the tritium atoms through space under the influence of the potential resulting from the interaction of the magnetic moment and magnetic field classically while calculating the rate of spin flips at each time step quantum mechanically. Then, by integrating over time and a large number of particles, we could determine the Majorana loss rate.

We have taken initial steps towards such a calculation by writing an extension to the Kassiopeia simulation program. This program was developed by the KATRIN project for the purpose of simulating β decay electrons in KATRIN. This extension adds atomic tritium as an available particle, introduces the spin degree of freedom to Kassiopeia, and provides the equations of motion including the spin-orbit interaction. However, much work remains in designing the simulation, implementing the magnetic field in Kassiopeia and resolving an issue regarding the time step required which has consequences for the simulation running time among other issues. Thus, this remains a subject for future work.

Chapter 4

Proposed Experimental Program and Future Work

Measuring the neutrino mass from the β decay of atomic tritium contained within a magnetic trap has potential as the next generation of direct neutrino mass experiments, but also introduces several new experimental challenges which must be addressed for the experiment to be feasible. This thesis has addressed many of the questions relevant to trapping atomic tritium for the neutrino mass measurement focusing particularly on the overall trap design and trap loss mechanisms. However, a large amount of laboratory work is necessary to turn this into a working experiment. This R&D can be divided into discrete components which each provide a particular piece of the experiment.

The first component of the program is to confirm that we can observe the cyclotron radiation from a single electron. This thesis does not attempt to address this subject instead operating under the assumption that this is feasible, but the viability of the entire technique depends upon successful development of this capability. This is a difficult experimental goal as the power from a tritium endpoint electron is only

$$P = 1.17 \text{ fW} \left(\frac{B}{1 \text{ T}} \right)^2 \sin^2(\theta) \tag{4.1}$$

where θ is the angle of the electron's momentum relative to the field. The noise temperature must be reduced so that this power is discernible from noise. Note that it is not an option to take the magnetic field to be extremely large to make the power easily detectable as the total energy radiated during the observation period must be less than the energy resolution,

$Pt < \Delta E$. This implies

$$B < 7.25 \text{ T} \left(\frac{\Delta E}{1 \text{ eV}} \right)^2 . \tag{4.2}$$

For this maximum magnetic field, the power is $P = 61.4 \text{ fW} \left(\frac{\Delta E}{1 \text{ eV}} \right)^4$. The noise power is given by

$$\begin{aligned} P_{noise} &= k_B T \Delta f \\ &= 5.3 \text{ aW} \left(\frac{\Delta E}{1 \text{ eV}} \right)^3 \left(\frac{T}{1 \text{ K}} \right) \end{aligned} \tag{4.3}$$

The signal-to-noise ratio is thus proportional to ΔE and inversely proportional to T. From the above, one can derive the noise temperature required to achieve a specified energy resolution and signal-to-noise ratio. This stage of the experimental program is currently underway with a prototype in operation at the University of Washington by the Project 8 collaboration. They aim to reach a noise temperature of 50 K.

Once the ability to detect the cyclotron radiation from a β decay electron has been demonstrated, two R&D projects should proceed in parallel. The first would focus on improving the resolution of the frequency-based energy measurement and applying the technique to molecular tritium. Ideally, this project would be able to set an independent limit on the neutrino mass that, while not competitive with the results of Mainz or Troitsk, would demonstrate the technique. A reasonable objective would be $m_{\nu_e} \lessapprox 20$ eV as the field uniformity required for this sensitivity ($\sim$1 part per thousand) is easily achievable. Also, since the number of events within ΔE of the endpoint scales as ΔE^3, sufficient statistics should be attainable with a small scale experiment.

The second would conduct the R&D needed for the transition to atomic tritium. This includes construction of a magnetic trap, possibly following the design described herein, and demonstrating the ability to cool and load the tritium in the trap. Once these steps have been completed successfully, the final component of the research program would be to combine them into one experiment. Independent development of the trapping and frequency-based energy measurement should reduce the chance of complications at this point.

This structured program is motivated by awareness of the potential challenges which could emerge. If we are unable to successfully observe the cyclotron radiation from a single electron, development of a magnetic trap is futile so it should follow sequentially. On the

other hand, if difficulties arise in designing a magnetic trap which meets all requirements, there is still independent reason to develop the the frequency-based energy measurement. One potential application is for the PTOLEMY experiment [26]. This proposed experiment aims to detect relic neutrinos via neutrino capture by atomic tritium. As the rate for this process is extremely low, a large background rejection factor is required. The combination of detecting an electron via both its cyclotron radiation and a calorimeter would provide a high background rejection factor due to the low rate of coincidence background events.

Within the more narrow range of this thesis, there is also additional work to be done. As discussed in the previous chapter, Monte Carlo simulation is required to determine the loss rate due to Majorana spin flips. We have completed initial work towards such a simulation, but much work remains. Simulation is also needed to understand the fraction of atomic tritium lost in the process of removing the ^{3}He buffer gas after cooling is completed. This simulation must model the general increase in loss rate at intermediate values of the mean free path and the "wind" force which results from the net movement of the buffer gas out of the trap that depends on the trap geometry and buffer gas removal procedure.

It would also be desirable to determine how effective the procedure of magnetic trapping is at reducing the molecular tritium contamination. While a magnetic trap allows for separation of atomic and molecular tritium in principle, in practice, there will be some residual molecular tritium. We would expect it will diffuse out of the experimental volume along with the buffer gas leaving a low concentration though we should also consider that some may freeze onto the walls of the experiment. Some aspects of this can be determined by direct calculation, but others require simulation, particularly understanding tritium frozen out on the walls.

Another avenue for further study is to consider the magnetic trap design in more detail. While we have addressed the fundamental physics concerns for designing the trap, several engineering questions remain. This includes calculation of the perturbation to the field arising from the finite size of the magnet system as the field of a finite solenoid is not precisely uniform and the same may occur for the trapping field. There will also be perturbations to the field from imperfections in construction that need to be modeled.

In line with the experimental program suggested above, some of this work should be conducted now while other elements, particularly related to the trap design, can wait until the second stage of the research program.

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