Integral finite surgeries on knots in S^3

Thesis by

Liling Gu

In Partial Fulfillment of the Requirements
for the Degree of
Doctor of Philosophy

California Institute of Technology

Pasadena, California

2014

(Defended May 29, 1014)

Acknowledgements

First and foremost, let me thank my advisor, Yi Ni, a kind and patient mentor, for his countably infinite ideas and intuitive suggestions, on this topic and on topology in general.

I owe also a great debt of gratitude to Danny Calegari. Danny has taught many Caltech graduate students, me included, most of what we know about algebraic topology and hyperbolic geometry.

I would like to thank the physics department for kindly letting me transfer to the mathematics department, and the mathematics department for accepting me as a graduate student.

I would like to thank my friends and fellow graduate students in mathematics, with whom I spent many late nights doing problem sets and studying for qualifying exams, among them Nakul Dawra, Foo Yee Yeo, Dapeng Zhang, Brian Skinner, Daiqi Linghu, and Branimir Ćaćić. Thanks to Nakul Dawra in particular, for having been a wonderful officemate, friend, and colleague.

I would like to thank John Berge for kindly sending me his unpublished papers, and Xingru Zhang for pointing out that I may use Dean's twisted torus knots.

Finally, I would like to thank my roommates Liuyan Zhao and Yu Zhao, who helped me to live a diverse and balanced life.

To my parents

Abstract

Using the correction terms in Heegaard Floer homology, we prove that if a knot in S^3 admits a positive integral **T**-, **O**-, or **I**-type surgery, it must have the same knot Floer homology as one of the knots given in our complete list, and the resulting manifold is orientation-preservingly homeomorphic to the p-surgery on the corresponding knot.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

In the early 1960s, Wallace [28] and Lickorish [11] proved independently that any closed, orientable, connected 3-manifold can be obtained by performing Dehn surgery on a framed link in the 3-sphere. One natural question is which manifolds can be obtained by some surgery on a knot. In this paper we consider the manifolds with finite noncyclic fundamental groups. By Perelman's resolution of the Geometrization Conjecture [22, 23, 24], the manifolds with finite fundamental group are spherical space forms. They fall into five classes: lens spaces; prism manifolds; **T**-, **O**-, and **I**-type spherical space forms.

Berge [1] constructed a list of knots which yield lens space surgeries, and he conjectured that it is complete. Greene [9] proved that if a p-surgery along a knot produces a lens space $L(p, q)$, there exists a p-surgery along a Berge knot with the same knot Floer homology groups as $L(p, q)$. In this paper we focus on knots with finite noncyclic surgeries. According to Thurston [27], a knot is either a torus knot, a hyperbolic knot, or a satellite knot. Moser [14] classified all finite surgeries on torus knots, and Bleiler and Hodgson [3] classified all finite surgeries on cables of torus knots. Boyer and Zhang [4] showed that a satellite knot with a finite noncyclic surgery must be a cable of some torus knot. There is also some progress about hyperbolic knots. Doig [7] proved that there are only finitely many spherical space forms which come from a p/q-surgery on S^3 for a fixed integer p. From now on we will only consider Dehn surgeries on hyperbolic knots.

Suppose M is a 3-manifold with torus boundary, α is a slope on ∂M. Let $M(\alpha)$ be the Dehn filling along α. If M is hyperbolic, Thurston's Hyperbolic Dehn Surgery

Theorem says that at most finitely many of the fillings are nonhyperbolic. These surgeries are called *exceptional surgeries*. In [4], Boyer and Zhang showed that if M is hyperbolic, $M(\alpha)$ has a finite fundamental group and $M(\beta)$ has a cyclic fundamental group, then $|\Delta(\alpha, \beta)| \leq 2$. In particular, if the p/q-surgery on a hyperbolic knot $K \subset S^3$, denoted $S^3_K(p/q)$, has a finite fundamental group, then $|q| \leq 2$. For $|q| = 2$, Li and Ni [10] proved that K has the same knot Floer homology as either $T(5, 2)$ or a cable of a torus knot (which must be $T(3, 2)$ or $T(5, 2)$). From now on we will only consider integral surgeries on hyperbolic knots.

Taking the mirror image of a knot K if necessary, we may assume $p > 0$. We consider here all **T**-, **O**-, and **I**-type spherical space forms. In this paper, all manifolds are oriented. If Y is an oriented manifold, then $-Y$ denotes the same manifold with the opposite orientation. Let $\mathbb{T}$ be the exterior of the right-hand trefoil, then $\mathbb{T}(p/q)$ is the manifold obtained by p/q-surgery on the right-hand trefoil. It is well-known that any **T**-, **O**-, or **I**-type manifold is homeomorphic to some $\pm\mathbb{T}(p/q)$ (see Lemma 4.0.4).

Our main result is the following theorem:

Theorem 1.0.1. *Suppose that K is a knot in S^3, and that the p-surgery on K is a* **T**-*,* **O**-*, or* **I**-*type spherical space form for some integer $p > 0$, then K has the same knot Floer homology as one of the knots $\widetilde{K}$ in table 7.1 or table 7.2, and the resulting manifold is orientation-preservingly homeomorphic to the p-surgery on the corresponding knot $\widetilde{K}$.*

Chapter 2 contains some preliminaries on 3-manifolds. Chapter 3 contains a brief summary of Heegaard Floer Homology, Knot Floer Homology and correction terms. After setting up the background, Chapter 4 briefly explains the proof strategy of Theorem 1.0.1 which is carried out in the following two chapters. Chapter 5 proves that there is no surgeries on knots which yield **T**-, **O**-, or **I**-type spherical space forms when the surgery coefficient is large enough, and gives a list of candidate coefficients. For each candidate coefficient, Chapter 6 then gives a knot which yields a spherical space form. Finally, Chapter 7 summarizes all the results and draws all the hyperbolic knots with **T**-, **O**-, or **I**-type surgeries.

Chapter 2

Preliminaries on 3-Manifolds

2.1 Dehn Surgery

Dehn surgery is probably the most common way of constructing 3-manifolds. Let $L = L_1 \cup ... \cup L_n$ be an oriented link embedded in an oriented 3-manifold Y. A Dehn surgery removes disjoint open neighborhoods N_i of L_i and glues back the disjoint union of n solid tori, which gives a 3-manifold $(M - (N_1 \cup ... \cup N_n)) \cup_\phi (\underset{n}{\cup} S^1 \times D^2)$, where ϕ is a union of homeomorphisms, each of which take a meridian curve of $\partial(S^1 \times D^2)$ onto a specified curve J_i in ∂N_i. By Wallace [28] and Lickorish [11], each closed, orientable, connected 3-manifold can be obtained by surgery on a link in S^3. We will only consider $Y = S^3$ here. For each link component L_i, choose a meridian μ_i and a longitude λ_i with linking number 1. Every simple closed curve J_i on ∂N_i is isotopic to $p_i\mu_i + q_i\lambda_i$, and the surgeried manifold is denoted by $S^3_L(p_1/q_1, ..., p_n/q_n)$.

2.2 Seifert-Fibered Space

A Seifert-fibered space is a 3-manifold fibered by circles, where any fiber has a neighborhood which is a disjoint union of circles that forms a standard fibered torus. A standard fibered torus corresponding to a pair of coprime integers (a, b) with $a > 0$ is the surface bundle of the automorphism of a disk given by rotation by an angle of $2\pi b/a$. If $a = 1$ the middle fiber is called regular, while if $a > 1$ the middle fiber is called exceptional, and a is called the multiplicity of the fiber. A compact Seifert-fibered space has finitely many exceptional fibers with multiplicities $a_1, ..., a_p$.

The set of fibers forms a 2-dimensional orbifold, and is called base orbifold of the Seifert-fibered space. Denote a 2-orbifold which is topologically X with p cone points of indices $a_1, ..., a_p$ by $X(a_1, ..., a_p)$.

By Perelman's resolution of the Geometrization Conjecture [22, 23, 24], the manifolds with finite fundamental group are spherical space forms. They fall into five classes, those with cyclic π_1 and those with finite π_1 based on the four isometries of a sphere:

Theorem 2.2.1. *(Seifert [26]). If Y^3 is closed, oriented and Seifert-fibered with finite but noncyclic fundamental group, then it is one of:*

1. *a* **D***-type spherical space form, or a prism manifold with base orbifold* $S^2(2, 2, n)$,
2. *a* **T***-type spherical space form, or a tetrahedral manifold with base orbifold* $S^2(2, 3, 3)$,
3. *a* **O***-type spherical space form, or a octahedral manifold with base orbifold* $S^2(2, 3, 4)$,
4. *a* **I***-type spherical space form, or a icosahedral manifold with base orbifold* $S^2(2, 3, 5)$.

Chapter 3

Heegaard Floer Homology

3.1 Heegaard Floer Homology

Heegaard Floer homology was introduced by Ozsváth and Szabó [17]. Given a closed oriented 3-manifold Y and a Spin^c structure $\mathfrak{s} \in \text{Spin}^c(Y)$, one can define the Heegaard Floer homology $\widehat{HF}(Y, \mathfrak{s}), HF^+(Y, \mathfrak{s})$,..., which are invariants of $(Y, \mathfrak{s})$.

3.1.1 Heegaard Diagrams

For a closed oriented three-manifold Y, we can associate a pointed Heegaard diagram $(\Sigma_g, \alpha_1, ..., \alpha_g, \beta_1, ..., \beta_g, z)$ to it, and $Y = U_0 \cup_{\Sigma_g} U_1$. Σ_g is an oriented closed surface of genus g and is the boundary of handlebodies U_0 and U_1, $\alpha = (\alpha_1, ..., \alpha_g)$, $\beta = (\beta_1, ..., \beta_2)$ are attaching curves for U_0 and U_1, and $z \in \Sigma_g - \alpha - \beta$. Define $\text{Sym}^g(\Sigma_g) = \Sigma_g \times \cdots \times \Sigma_g / S_g$, where S_g is the symmetric group on g letters. The attaching curves induce two g-dimensional tori in $\text{Sym}^g(\Sigma_g)$,

$$\mathbb{T}_\alpha = \alpha_1 \times \cdots \times \alpha_g \text{ and } \mathbb{T}_\beta = \beta_1 \times \cdots \times \beta_g.$$

The two tori intersect transversally in finitely many points in $\text{Sym}^g(\Sigma_g)$. For a pair of intersection points $\mathbf{x}, \mathbf{y} \in \mathbb{T}_\alpha \cap \mathbb{T}_\beta$, let $\pi_2(\mathbf{x}, \mathbf{y})$ denote the set of homotopy classes of Whitney disks connecting $\mathbf{x}$ and $\mathbf{y}$, where a Whitney disk is a continous map $u : \mathbb{D} \subset \mathbb{C} \to \text{Sym}^g(\Sigma_g)$ with $u(-i) = \mathbf{x}, u(i) = \mathbf{y}, u(\partial\mathbb{D}|_{\text{Re}(z)\geq 0}) \subset \mathbb{T}_\alpha, u(\partial\mathbb{D}|_{\text{Re}(z)\leq 0}) \subset \mathbb{T}_\beta$.

For any point $w \in \Sigma_g - \alpha - \beta$, let $n_w : \pi_2(\mathbf{x}, \mathbf{y}) \to \mathbb{Z}$ denote the algebraic intersection number $n_w(\phi) = \#\phi^{-1}(\{w\} \times \text{Sym}^{g-1}(\Sigma_g))$.

A complex structure on Σ induces a complex structure on $\text{Sym}^g(\Sigma_g)$. For a given homotopy class $\phi \in \pi_2(\mathbf{x}, \mathbf{y})$, let $\mathcal{M}(\phi)$ denote the moduli space of holomorphic representatives of ϕ. There is an $\mathbb{R}$ action on $\mathcal{M}(\phi)$ that corresponds to the group of complex automorphisms of the unit disk that preserves i and $-i$. Define $\widehat{\mathcal{M}}(\phi) = \mathcal{M}(\phi)/\mathbb{R}$. The moduli space $\mathcal{M}(\phi)$ has an expected dimension called the Maslov index $\mu(\phi)$.

3.1.2 Spin^c Structures

Define Spin^c structures over Y, denoted by Spin$^c(Y)$, as nowhere vanishing vector fields modulo equivalent relation where two vector fields are homologous if they are homotopic outside a ball. Let f be a Morse function on Y compatible with the attching curves. Then each $\mathbf{x} \in \mathbb{T}_\alpha \cap \mathbb{T}_\beta$ determines a g-tuple of trajectories for ∇f connetcting index one critical points to index two critical points. Similarly z gives a trajectory connecting the index zero critical point to the index three critical point. Deleting tubular neighborhoods of these $g+1$ trajectories from Y, ∇f defines a nonvanishing vector field. Since each trajectory connects critical points of different parities, the vector field can be extended over Y. Denote the homology class of the vector field constructed this way by $s_z(\mathbf{x})$. This defines a map $s_z : \mathbb{T}_\alpha \cap \mathbb{T}_\beta \to \text{Spin}^c(Y)$. There is a one-to-one correspondence between Spin$^c(Y)$ and $H^2(Y, \mathbb{Z})$.

3.1.3 Floer Chain Complexes

Choose a Spin^c structure $\mathfrak{s} \in \text{Spin}^c(Y)$. Let $\widehat{CF}(\alpha, \beta, \mathfrak{s})$ be the free Abelian group generated by the points $\mathbf{x} \in \mathbb{T}_\alpha \cap \mathbb{T}_\beta$ with $s_z(\mathbf{x}) = \mathfrak{s}$. The group can be endowed with a relative grading $\text{gr}(\mathbf{x}, \mathbf{y}) = \mu(\phi) - 2n_z(\phi)$, where $\phi \in \pi_2(\mathbf{x}, \mathbf{y})$. The differential is definied by

$$\partial \mathbf{x} = \sum_{\mathbf{y} \in \mathbb{T}_\alpha \cap \mathbb{T}_\beta} \sum_{\phi \in \pi_2(\mathbf{x}, \mathbf{y}), \mu(\phi)=1, n_z(\phi)=0} \#(\widehat{\mathcal{M}}(\phi)) \cdot \mathbf{y}$$

By examing the Gromov compactification of $\widehat{\mathcal{M}}(\phi)$ for $n_z(\phi) = 0$ and $\mu(\phi) = 2$, it is proved in [17] that $\partial^2 = 0$. So $(\widehat{CF}(\alpha, \beta, \mathfrak{s}), \partial)$ is a chain complex. Denote the homology groups of this chain complex by $\widehat{HF}(\alpha, \beta, \mathfrak{s})$. Moreover, it is proved in [17] that the homology groups are isomorphic for different pointed Heegaard diagrams of Y, so it is an invariant of $(Y, \mathfrak{s})$ and is denoted by $\widehat{HF}(Y, \mathfrak{s}) = \widehat{HF}(\alpha, \beta, \mathfrak{s})$.

Similarly, we can define $HF^\infty(Y,\mathfrak{s})$. Let $CF^\infty(\alpha,\beta,\mathfrak{s})$ be the free Abelian group generated by pairs $[\mathbf{x},i]$ where $\mathbf{x}\in\mathbb{T}_\alpha\cap\mathbb{T}_\beta$, i is an integer, and $s_z(\mathbf{x})=\mathfrak{s}$. The group can be endowed with a relative grading $\mathrm{gr}([\mathbf{x},i],[\mathbf{y},j])=\mu(\phi)-2n_z(\phi)+2i-2j$, where $\phi\in\pi_2(\mathbf{x},\mathbf{y})$. The differential is definied by

$$\partial[\mathbf{x},i]=\sum_{\mathbf{y}\in\mathbb{T}_\alpha\cap\mathbb{T}_\beta}\ \sum_{\phi\in\pi_2(\mathbf{x},\mathbf{y}),\mu(\phi)=1}\#(\widehat{\mathcal{M}}(\phi))\cdot[\mathbf{y},i-n_z(\phi)]$$

There is an isomorphism U on $CF^\infty(\alpha,\beta,\mathfrak{s})$ given by $U[\mathbf{x},i]=[\mathbf{x},i-1]$.

Let $CF^-(\alpha,\beta,\mathfrak{s})$ be the subgroup of $CF^\infty(\alpha,\beta,\mathfrak{s})$ which is generated by $[\mathbf{x},i]$ where $i<0$, and let $CF^+(\alpha,\beta,\mathfrak{s})$ be the quotient group. The short eaxat sequence of chain complexes

$$0\longrightarrow CF^-(\alpha,\beta,\mathfrak{s})\xrightarrow{\imath}CF^\infty(\alpha,\beta,\mathfrak{s})\xrightarrow{\pi}CF^-(\alpha,\beta,\mathfrak{s})\longrightarrow 0$$

induces a long exact sequence

$$\cdots\longrightarrow HF^-(Y,\mathfrak{s})\xrightarrow{\imath}HF^\infty(Y,\mathfrak{s})\xrightarrow{\pi}HF^+(Y,\mathfrak{s})\longrightarrow\cdots.$$

Similarly,

$$0\longrightarrow \widehat{CF}(\alpha,\beta,\mathfrak{s})\xrightarrow{\hat\imath}CF^+(\alpha,\beta,\mathfrak{s})\xrightarrow{U}CF^+(\alpha,\beta,\mathfrak{s})\longrightarrow 0$$

induces

$$\cdots\longrightarrow \widehat{HF}(Y,\mathfrak{s})\xrightarrow{\imath}HF^+(Y,\mathfrak{s})\xrightarrow{\pi}HF^+(Y,\mathfrak{s})\longrightarrow\cdots.$$

Moreover, a cobordism between two 3-manifolds induces functorial maps between long exact sequences of Heegaard Floer homologies of the two 3-manifolds. More precisely, let $HF^\circ(Y,\mathfrak{s})$ denote one type of Heegaard Floer homology, and denote the map induced by cobordism W from Y_1 to Y_2 by $F^\circ_{W,\mathfrak{s}}:HF^\circ(Y_1,\mathfrak{s}_1)\to HF^\circ(Y_2,\mathfrak{s}_1)$, where $\mathfrak{s}_i=\mathfrak{s}|Y_i$ for $i=1,2$. We have the following commutative diagrams:

$$\begin{array}{ccccccccc}
\cdots & \longrightarrow & HF^-(Y_1,\mathfrak{s}_1) & \xrightarrow{\imath} & HF^\infty(Y_1,\mathfrak{s}_1) & \xrightarrow{\pi} & HF^+(Y_1,\mathfrak{s}_1) & \longrightarrow & \cdots\\
 & & \downarrow{\scriptstyle F^-_{W,\mathfrak{s}}} & & \downarrow{\scriptstyle F^\infty_{W,\mathfrak{s}}} & & \downarrow{\scriptstyle F^+_{W,\mathfrak{s}}} & & \\
\cdots & \longrightarrow & HF^-(Y_2,\mathfrak{s}_2) & \xrightarrow{\imath} & HF^\infty(Y_2,\mathfrak{s}_2) & \xrightarrow{\pi} & HF^+(Y_2,\mathfrak{s}_2) & \longrightarrow & \cdots
\end{array}$$

and

$$\begin{array}{ccccccccc}
\cdots & \longrightarrow & \widehat{HF}(Y_1,\mathfrak{s}_1) & \xrightarrow{\imath} & HF^+(Y_1,\mathfrak{s}_1) & \xrightarrow{\pi} & HF^+(Y_1,\mathfrak{s}_1) & \longrightarrow & \cdots\\
 & & \downarrow{\scriptstyle \widehat{F}_{W,\mathfrak{s}}} & & \downarrow{\scriptstyle F^+_{W,\mathfrak{s}}} & & \downarrow{\scriptstyle F^+_{W,\mathfrak{s}}} & & \\
\cdots & \longrightarrow & \widehat{HF}(Y_2,\mathfrak{s}_2) & \xrightarrow{\imath} & HF^+(Y_2,\mathfrak{s}_2) & \xrightarrow{\pi} & HF^+(Y_2,\mathfrak{s}_2) & \longrightarrow & \cdots
\end{array}$$

3.1.4 Absolute $\mathbb{Q}$-grading

When $\mathfrak{s}$ is torsion, there is an absolute $\mathbb{Q}$-grading which lifts the relative $\mathbb{Z}$-grading on $\widehat{HF}(Y,\mathfrak{s})$. It is uniquely characterized by the following properties:

1. $\hat{\imath}$, $\imath$ and π preserve the absolute grading,

2. $\widehat{HF}(S^3, 0)$ is supported in absolute grading zero,

3. if W is a cobordism from Y_1 to Y_2, and $\xi \in HF^\infty(Y_1, \mathfrak{s}_1)$, then

$$\mathrm{gr}(F^\infty_{W,\mathfrak{s}}(\xi)) - \mathrm{gr}(\xi) = \frac{c_1(\mathfrak{s})^2 - 2\chi(W) - 3\sigma(W)}{4}$$

   where $\mathfrak{s}_\mathfrak{i} = \mathfrak{s}|Y_i$ for $i = 1, 2$.

3.2 Knot Floer Homology

In this section, we restrict ourselves to knots in S^3. Given a Heegaard diagram $(\Sigma_g, \alpha_1, ..., \alpha_g, \beta_1, ..., \beta_g)$ for S^3 with two basepoints w and z, this data specifies a knot in S^3. Connect w and z by a unique curve a (up to homotopy) in $\Sigma_g - \alpha$ and also by a curve b in $\Sigma_g - \beta$. Pushing a and b into U_0 and U_1 respectively, we obtain a knot K in S^3. Conversely, every knot can be represented by a two-pointed Heegaard diagram. For a knot $K \subset S^3$, we can thicken a projection of $K \in S^2 (= \mathbb{R}^2 + \infty)$ to get a handlebody U_0, and take U_1be the complement of U_0. Mark a point x on the boundary of the unbounded region, $U_0 \cap S^2$ minus the outermost curve are α attaching curves. Each double point gives rise to a β curve, these β curves together with a meridian μ near x are β attaching curves. Choose two points on the two sides of μ to be w and z. This gives rise to a two-pointed Heegaard diagram representing K.

Let K be a knot in S^3 and $(\Sigma_g, \alpha_1, ..., \alpha_g, \beta_1, ..., \beta_g, w, z)$ be a compatible two-pointed Heegaard diagram. Let $C(K)$ be the free Abelian group generated by the points $\mathbf{x} \in \mathbb{T}_\alpha \cap \mathbb{T}_\beta$. The differential is definied by

$$\partial_K \mathbf{x} = \sum_{\mathbf{y} \in \mathbb{T}_\alpha \cap \mathbb{T}_\beta} \sum_{\phi \in \pi_2(\mathbf{x},\mathbf{y}), \mu(\phi)=1, n_z(\phi)=n_w(\phi)=0} \#(\widehat{\mathcal{M}}(\phi)) \cdot \mathbf{y}$$

$(\widehat{C}(K), \partial_K)$ is a chain complex. Its homology $\widehat{HFK}(K)$ is independent of the two-pointed Heegaard diagram representing K.

The group can be endowed with two absolute gradings: Maslov grading and Alexander grading. The Maslov grading is the absolute $\mathbb{Q}$-grading coming from the Maslov grading induced by w, since $(\Sigma_g, \alpha_1, ..., \alpha_g, \beta_1, ..., \beta_g, w)$ is S^3. The Alexander grading is the unique function satisfying $A(\mathbf{x}) - A(\mathbf{y}) = n_z(\phi) - n_w(\phi)$ where $\phi \in \pi_2(\mathbf{x}, \mathbf{y})$ and $\#\{\mathbf{x} \in \mathbb{T}_\alpha \cap \mathbb{T}_\beta | A(\mathbf{x}) = i\} \equiv \#\{\mathbf{x} \in \mathbb{T}_\alpha \cap \mathbb{T}_\beta | A(\mathbf{x}) = -i\} \pmod 2$. We write

$$\widehat{HFK}(K) = \oplus_{i,j} \widehat{HFK}_i(K, j),$$

where i is Maslov grading and j is Alexander grading, induced from the Chain complexes.

3.3 Correction Terms

When Y is a rational homology sphere, Ozsváth and Szabó [16] defined a *correction term* $d(Y, \mathfrak{s}) \in \mathbb{Q}$, which is the minimal grading of any non-torsion element in the image of $HF^\infty(Y, \mathfrak{s})$ in $HF^+(Y, \mathfrak{s})$.

The correction terms have the following symmetries:

$$d(Y, \mathfrak{s}) = d(Y, J\mathfrak{s}), \quad d(-Y, \mathfrak{s}) = -d(Y, \mathfrak{s}), \tag{3.1}$$

where $J : \mathrm{Spin}^c(Y) \to \mathrm{Spin}^c(Y)$ is the conjugation.

Suppose that Y is an integral homology sphere, $K \subset Y$ is a knot. Let $Y_K(p/q)$ be the manifold obtained by p/q-surgery on K. Ozsváth and Szabó defined a natural identification $\sigma : \mathbb{Z}/p\mathbb{Z} \to \mathrm{Spin}^c(Y_K(p/q))$ [16, 21]. For simplicity, we often use an integer i to denote the Spin^c structure $\sigma([i])$, when $[i] \in \mathbb{Z}/p\mathbb{Z}$ is the congruence class of i modulo p.

A rational homology sphere Y is an *L-space* if $\mathrm{rank}\widehat{HF}(Y) = |H_1(Y)|$. Examples of L-spaces include spherical space forms. For a knot $K \subset S^3$ that admits an intergral L-space surgery, $\widehat{HFK}$ is determined explicitly from the Alexander polynomial of K by the following theorem.

Theorem 3.3.1. *(Ozsváth and Szabó [20]) If a knot $K \subset S^3$ admits an intergral L-space surgery, then there is an increasing sequence of Alexander gradings $n_{-k} < \cdots < n_k$ with the property that $n_i = -n_{-i}$, $\widehat{HFK}(K, j) = 0$ unless $j = n_i$ for some i, in which case $\widehat{HFK}(K, j) \cong \mathbb{Z}$ and it is supported entirely in Maslov grading δ_i, where*

$$\delta_i = \begin{cases} 0 & \text{if } i = k \\ \delta_{i+1} - 2(n_{i+1} - n_i) + 1 & \text{if } k - i \text{ is odd} \\ \delta_{i+1} - 1 & \text{if } k - i > 0 \text{ is even} \end{cases}$$

The information about the Heegaard Floer homology of an L-space is completely encoded in its correction terms.

Let $L(p, q)$ be the lens space obtained by p/q-surgery on the unknot. The correction terms for lens spaces can be computed inductively as follows:

$$d(S^3, 0) = 0,$$

$$d(-L(p, q), i) = \frac{1}{4} - \frac{(2i + 1 - p - q)^2}{4pq} - d(-L(q, r), j) \tag{3.2}$$

where $0 \leq i < p + q$, r and j are the reductions of p and i modulo q, respectively.

For example, using the recursive formula (3.2), we can get

$$d(L(3, q), i) = \begin{cases} (\frac{1}{2}, -\frac{1}{6}, -\frac{1}{6}) & q = 1, i = 0, 1, 2 \\ (\frac{1}{6}, \frac{1}{6}, -\frac{1}{2}) & q = 2, i = 0, 1, 2 \end{cases} \tag{3.3}$$

$$d(L(4, q), i) = \begin{cases} (\frac{3}{4}, 0, -\frac{1}{4}, 0) & q = 1, i = 0, 1, 2, 3 \\ (0, \frac{1}{4}, 0, -\frac{3}{4}) & q = 3, i = 0, 1, 2, 3 \end{cases} \tag{3.4}$$

$$d(L(5, q), i) = \begin{cases} (1, \frac{1}{5}, -\frac{1}{5}, -\frac{1}{5}, \frac{1}{5}) & q = 1, i = 0, 1, 2, 3, 4 \\ (\frac{2}{5}, \frac{2}{5}, -\frac{2}{5}, 0, -\frac{2}{5}) & q = 2, i = 0, 1, 2, 3, 4 \\ (\frac{2}{5}, 0, \frac{2}{5}, -\frac{2}{5}, -\frac{2}{5}) & q = 3, i = 0, 1, 2, 3, 4 \\ (-\frac{1}{5}, \frac{1}{5}, \frac{1}{5}, -\frac{1}{5}, -1) & q = 4, i = 0, 1, 2, 3, 4. \end{cases} \tag{3.5}$$

From [21], if we know the knot Floer homology, then we can compute the Heegaard Floer homology of all the surgeries on K. In particular, if the p/q-surgery on $K \subset S^3$ is an L-space surgery, where $p, q > 0$, then the correction terms of $S^3_K(p/q)$ can be computed from the Alexander polynomial of K as follows.

Suppose

$$\Delta_K(T) = a_0 + \sum_{i>0} a_i(T^i + T^{-i}).$$

Define a sequence of integers

$$t_i = \sum_{j=1}^{\infty} j a_{i+j}, \quad i \geq 0,$$

then a_i can be recovered from t_i by

$$a_i = t_{i-1} - 2t_i + t_{i+1}, \quad \text{for } i > 0. \tag{3.6}$$

If K admits an L-space surgery, then one can prove [21, 25]

$$t_s \geq 0, \quad t_s \geq t_{s+1} \geq t_s - 1, \quad t_{g(K)} = 0. \tag{3.7}$$

Moreover, the following proposition from Ozsváth and Szabó [21] and Rasmussen [25] holds.

Proposition 3.3.2. *Suppose the p/q-surgery on $K \subset S^3$ is an L-space surgery, where $p, q > 0$. Then for any $0 \leq i < p$ we have*

$$d(S^3_K(p/q), i) = d(L(p,q), i) - 2\max\{t_{\lfloor \frac{i}{q} \rfloor}, t_{\lfloor \frac{p+q-1-i}{q} \rfloor}\}.$$

Chapter 4

Proof Strategy

Recall that $\phi : \mathbb{Z}/p\mathbb{Z} \to \text{Spin}^c(Y_K(p/q))$ is the natural identification defined by Ozsváth and Szabó, $\mathbb{T}$ is the exterior of the right-hand trefoil, and $\mathbb{T}(p/q)$ is the manifold obtained by p/q-surgery on the right-hand trefoil. The following two lemmas are from [10].

Lemma 4.0.3. *Suppose i is an integer satisfying $0 \leq i < p+q$, then $J(\sigma([i]))$ is represented by $p+q-1-i$.*

Lemma 4.0.4. *Any $\mathbf{T}$-type manifold is homeomorphic to $\pm\mathbb{T}(\frac{6q\pm3}{q})$ for some positive integer q with $(q,3)=1$. Any $\mathbf{O}$-type manifold is homeomorphic to $\pm\mathbb{T}(\frac{6q\pm4}{q})$ for some positive integer q with $(q,2)=1$. Any $\mathbf{I}$-type manifold is homeomorphic to $\pm\mathbb{T}(\frac{6q\pm5}{q})$ for some positive integer q with $(q,5)=1$.*

Let $p, q > 0$ be coprime integers. Using Proposition 3.3.2, we get

$$d(\mathbb{T}(p/q), i) = d(L(p,q), i) - 2\chi_{[0,q)}(i), \tag{4.1}$$

where $\chi_{[0,q)}(i) = \begin{cases} 1 & \text{when } 0 \leq i < q \\ 0 & \text{when } q \leq i < p. \end{cases}$

Suppose $S^3_K(p)$ is a spherical space form, then by Proposition 3.3.2,

$$\begin{aligned} d(S^3_K(p), i) &= d(L(p,1), i) - 2\max\{t_i, t_{p-i}\} \\ &= \frac{(2i-p)^2 - p}{4p} - 2t_{\min(i,p-i)}. \end{aligned}$$

If $S^3_K(p) \cong \varepsilon\mathbb{T}(p/q)$, $\varepsilon = \pm 1$, then the two sets

$$\{d(S^3_K(p), i) | i \in \mathbb{Z}/p\mathbb{Z}\}, \quad \{\varepsilon d(\mathbb{T}(p/q), i) | i \in \mathbb{Z}/p\mathbb{Z}\}$$

are equal. However, the two parametrizations of Spin^c structures may differ by an affine isomorphism of $\mathbb{Z}/p\mathbb{Z}$. More precisely, there exists an affine isomorphism $\phi : \mathbb{Z}/p\mathbb{Z} \to \mathbb{Z}/p\mathbb{Z}$, such that

$$d(S^3_K(p), i) = \varepsilon d(\mathbb{T}(p/q), \phi(i)).$$

For any integers a, b, define $\phi_{a,b} : \mathbb{Z}/p\mathbb{Z} \to \mathbb{Z}/p\mathbb{Z}$ by

$$\phi_{a,b}(i) = ai + b \mod p.$$

Lemma 4.0.5. *There are at most two values for b, $b_j = \frac{jp+q-1}{2}$, $j = 0, 1$.*

Proof. The affine isomorphism ϕ commutes with J, i.e., $\phi J_p = J_{\frac{p}{q}}\phi$. Using lemma 4.0.3, we get the desired values for b. Note that b_0 or b_1 may be a half-integer, in this case we discard it. □

Note $\phi_{a,b}(i) = \phi_{p-a,b}(p-i)$, By (3.1) and Lemma 4.0.3,

$$d(\mathbb{T}(p/q), \phi_{a,b}(i)) = d(\mathbb{T}(p/q), \phi_{p-a,b}(p-i)) = d(\mathbb{T}(p/q), \phi_{p-a,b}(i)).$$

So we may assume

$$0 < a < \frac{p}{2}, \quad (p, a) = 1. \tag{4.2}$$

Then we may assume

$$d(S^3_K(p), i) = \varepsilon d(\mathbb{T}(p/q), \phi_{a,b_j}(i)), \quad \text{for some } a, \text{ any } i \in \mathbb{Z}/p\mathbb{Z}, \text{ and } j = 0 \text{ or } 1.$$

Let

$$\Delta^{\varepsilon}_{a,b_j}(i) = d(L(p,1), i) - \varepsilon d(\mathbb{T}(p/q), \phi_{a,b_j}(i)). \tag{4.3}$$

By Proposition 3.3.2, we should have

$$\Delta^{\varepsilon}_{a,b_j}(i) = 2t_{\min(i,p-i)} \tag{4.4}$$

if $S^3_K(p) \cong \varepsilon\mathbb{T}(p/q)$ and ϕ_{a,b_j} identifies their Spin^c structures.

In order to prove Theorem 1.0.1, we will compute the correction terms of the **T**-, **O**-, and **I**-type manifolds using (4.1). For all a satisfying (4.2), we compute the sequences $\Delta^{\varepsilon}_{a,b_j}(i)$. Then we check whether they satisfy (4.4) for some $\{t_s\}$ as in (3.7). We will show that (4.4) cannot be satisfied when p is sufficiently large. For small p, a direct computation yields all the p/q's. By a standard argument in Heegaard Floer homology [20], we can get the knot Floer homology of the corresponding knots, which should be the knot Floer homology of either a (p,q)-torus knot ($(p,q) = (2,3),(2,5),(3,4),(3,5)$), a cable knot or some hyperbolic knot. We will list torus knots and cables of torus knots separately for completeness, one may also consult Moser [14] and Bleiler and Hodgson [3].

Chapter 5

Obstructions When p Is Large

In this chapter, we will assume that $S^3_K(p) \cong \varepsilon\mathbb{T}(p/q)$, and

$$p = 6q + \zeta r, r \in \{3, 4, 5\}, \varepsilon, \zeta \in \{-1, 1\}.$$

We will prove that this cannot happen when p is sufficiently large:

Proposition 5.0.6. *If $p > 310r(36r+1)^2$, then $S^3_K(p) \ncong \varepsilon\mathbb{T}(p/q)$, where $p = 6q + \zeta r, r \in \{3, 4, 5\}$.*

Let $s \in \{0, 1, ..., r-1\}$ be the reduction of q modulo r. For any integer n, let $\theta(n) \in \{0, 1\}$ be the reduction of n modulo 2, and let $\bar{\theta}(n) = 1 - \theta(n)$.

Lemma 5.0.7. *For $0 \leq i < q$,*

$$d(L(q, \frac{1-\zeta}{2}q + \zeta r), i) = \zeta\left(\frac{(2i+1-q-\zeta r)^2}{4qr} - \frac{1}{4} - d(L(r,s), i \mod r)\right). \quad (5.1)$$

Proof. For $0 \leq i < q$, using (3.2), we have

$$\begin{aligned}
d(L(q,r),i) &= \frac{(2i+1-q-r)^2}{4qr} - \frac{1}{4} - d(r,s,i \mod r) \\
d(L(q,q-r),i) &= \frac{(2i+1-2q+r)^2}{4q(q-r)} - \frac{1}{4} - d(L(q-r,r),i) \\
&= \frac{(2i+1-2q+r)^2}{4q(q-r)} - \frac{(2i+1-q)^2}{4r(q-r)} + d(L(r,s),i \mod r) \\
&= -\left(\frac{(2i+1-q+r)^2}{4qr} - \frac{1}{4} - d(L(r,s),i \mod r)\right).
\end{aligned}$$

□

Recall $\phi_{a,b} : \mathbb{Z}/p\mathbb{Z} \to \mathbb{Z}/p\mathbb{Z}$ is defined by

$$\phi_{a,b}(i) = ai + b \mod p.$$

Lemma 5.0.8. *When $p > 52$, there is at most one value for b in $\{b_0, b_1\}$.*

Proof. For a **T**- or **I**-type p-surgery on a knot, by lemma 4.0.4, $S^3_K(p) \cong \varepsilon\mathbb{T}(p/q)$, where $p = 6q + \zeta r$, $r = 3$ or 5. Here p is odd, $\frac{p}{2}$ is a half integer. By lemma 4.0.5, if q is odd, $b = \frac{q-1}{2}$; if q is even, $b = \frac{p+q-1}{2}$. We may write $b = \frac{\bar{\theta}(q)p+q-1}{2}$.

For an **O**-type p-surgery on a knot, by lemma 4.0.4, $S^3_K(p) \cong \varepsilon\mathbb{T}(p/q)$, where $p = 6q+\zeta r$, $r = 4$. Note here p is even, $(p, q) = (p, a) = 1$, q, a are odd, so $q = 4l + s$, where $s = 1, 3$. By lemma 4.0.5, $b_j = \frac{jp+q-1}{2}$, $j = 0, 1$, and both of them are integers. Denote $\phi_{a,j}(i) = ai + b_j$.

More specifically, $p = 6q + \zeta r$, $r = 4$, $S^3_K(p) \cong \varepsilon\mathbb{T}(p/q)$, $q = 4l + s$, $\zeta, \varepsilon \in \{1, -1\}$, $s \in \{1, 3\}$. For $\phi_{a,j}$, $\phi_{a,j}(0) = \frac{jp+q-1}{2}$, $\phi_{a,j}(\frac{p}{2}) = \frac{(1-j)p+q-1}{2}$.

Using (3.2) and (5.1), we get

$$\begin{aligned}
&d(L(p,1),0) - d(L(p,1),\frac{p}{2}) = \frac{p}{4}\\
&d(L(p,q),\frac{q-1}{2}) - d(L(p,q),\frac{p+q-1}{2})\\
&= \frac{p^2}{4pq} - d(L(q,\frac{1-\zeta}{2}q+\zeta r),\frac{q-1}{2}) + d(L(q,\frac{1-\zeta}{2}q+\zeta r),\frac{q+\zeta r-1}{2})\\
&= \frac{p^2}{4pq} - \zeta[\frac{r^2}{4qr} - d(L(4,s),2l+\frac{s-1}{2} \mod 4) + d(L(4,s),2l+2+\frac{s-1}{2} \mod 4)]\\
&= \frac{3}{2} + \zeta(-1)^l.
\end{aligned}$$

Here we require $q - r > r$ and $\frac{q+r-1}{2} < q$, it suffices to take $p > 52 = 6 * 2r + r$.

Using Proposition 3.3.2, (4.1) and (4.4), we get

$$\Delta^\varepsilon_{a,b_j}(0) - \Delta^\varepsilon_{a,b_j}(\frac{p}{2}) = \frac{p}{4} \mp \varepsilon(\frac{3}{2} + \zeta(-1)^l - 2) = 6l + \zeta \mp \varepsilon\zeta(-1)^l + \frac{3}{2}(s \mp \varepsilon) \pm 2\varepsilon.$$

The parity of $\Delta^\varepsilon_{a,b_j}(0) - \Delta^\varepsilon_{a,b_j}(\frac{p}{2})$ depends only on the parity of $\frac{3}{2}(s \mp \varepsilon)$, and by

(4.4), it should be even, so we get

$$b = \begin{cases} \frac{q-1}{2} & \text{if } s=1, \varepsilon=1 \quad \text{or} \quad s=3, \varepsilon=-1 \\ \frac{p+q-1}{2} & \text{if } s=1, \varepsilon=-1 \quad \text{or} \quad s=3, \varepsilon=1 \end{cases}.$$

We can write $b = \frac{\bar{\theta}(\frac{s+\varepsilon}{2})p+q-1}{2}$ for $p > 52$. □

Because of Lemma 5.0.8, we can treat **T**-, **O**-, and **I**-type manifolds uniformly. Let

$$\theta = \theta(q, \varepsilon) = \begin{cases} \bar{\theta}(q) & \text{if } r = 3, 5 \\ \bar{\theta}(\frac{s+\varepsilon}{2}) & \text{if } r = 4, q = 4l + s \end{cases},$$

then $b = \frac{\theta p+q-1}{2}$, we may denote $\phi_{a,b}$ by $\phi_{a,\theta}$.

Lemma 5.0.9. *Assume that $S^3_K(p) \cong \varepsilon\mathbb{T}(p/q)$. Let $m \in \{0, 1, 2, 3\}$ satisfy that*

$$0 \le a - mq + \frac{\theta\zeta r + q - 1}{2} < q,$$

then

$$|a - \frac{mp}{6}| < \sqrt{\frac{11rp}{6}}.$$

Proof. By (3.7), $\Delta^{\varepsilon}_{a,\theta}(0) - \Delta^{\varepsilon}_{a,\theta}(1) = 0$ or 2. Let $h = \begin{cases} 0 & \text{if } 0 \le \frac{\theta p+q-1}{2} + a < p \\ 1 & \text{if } \frac{\theta p+q-1}{2} + a \ge p \end{cases}$.

$$\begin{aligned} &\Delta^{\varepsilon}_{a,\theta}(0) - \Delta^{\varepsilon}_{a,\theta}(1) \qquad (5.2) \\ =&d(L(p,1),0) - \varepsilon[d(L(p,q), \frac{\theta p+q-1}{2}) - 2\chi_{[0,q)}(\frac{\theta p+q-1}{2})] \\ &- d(L(p,1),1) + \varepsilon[d(L(p,q), \frac{\theta p+q-1}{2} + a) - 2\chi_{[0,q)}(\frac{\theta p+q-1}{2} + a - hp)] \\ =&2\varepsilon\{\chi_{[0,q)}(\frac{\theta p+q-1}{2}) - \chi_{[0,q)}(\frac{\theta p+q-1}{2} + a - hp)\} + \frac{p^2}{4p} - \frac{(p-2)^2}{4p} \\ &- \varepsilon\{\frac{[(\theta-1)p]^2 - pq}{4pq} - d(L(q, \frac{1-\zeta}{2}q + \zeta r), \frac{\theta\zeta r + q - 1}{2}) \\ &- \frac{[2a + (\theta-1)p]^2 - pq}{4pq} + d(L(q, \frac{1-\zeta}{2}q + \zeta r), \frac{\theta\zeta r + q - 1}{2} + a - mq)\} \end{aligned}$$

Let $i = \frac{\theta\zeta r+q-1}{2} \mod r, j = \frac{\theta\zeta r+q-1}{2} + a - mq \mod r$.

Since $0 \le \frac{\theta\zeta r+q-1}{2} + a - mq < q$, we use (5.1), the right-hand side of (5.2) becomes

$$
\begin{aligned}
&2\varepsilon\left(\chi_{[0,q)}(\frac{\theta p+q-1}{2}) - \chi_{[0,q)}(\frac{\theta p+q-1}{2}+a-hp)\right) + \frac{p-1}{p} + \varepsilon\frac{a[a+(\theta-1)p]}{pq} + \\
&\varepsilon\zeta\left(\frac{[(\theta-1)\zeta r]^2-qr}{4qr} - d(L(r,s),i) - \frac{[2a-2mq+(\theta-1)\zeta r]^2-qr}{4qr} + d(L(r,s),j)\right) \\
&= C + \varepsilon\frac{a[a+(\theta-1)p]}{pq} - \varepsilon\zeta\frac{[a-mq+(\theta-1)\zeta r](a-mq)}{qr} \\
&= -\frac{6\varepsilon\zeta}{pr}(a-\frac{mp}{6})^2 - \varepsilon m(1-\theta) + \frac{\varepsilon m^2}{6} + C,
\end{aligned}
$$

where

$$
\begin{aligned}
C =& 2\varepsilon\{\chi_{[0,q)}(\frac{\theta p+q-1}{2}) - \chi_{[0,q)}(\frac{\theta p+q-1}{2}+a-hp)\} \\
&+ \varepsilon\zeta[d(L(r,s),j) - d(L(r,s),i)] + \frac{p-1}{p}.
\end{aligned}
$$

Using (2),(3),(4), $|C| \le \frac{6}{5} + 2 + 1 < \frac{9}{2}$.

Moreover, $|-\varepsilon m(1-\theta) + \frac{\varepsilon m^2}{6}| \le m + \frac{m^2}{6} \le 3 + \frac{3}{2} = \frac{9}{2}$. So we get

$$
|\frac{6}{pr}(a-\frac{mp}{6})^2| < 2 + \frac{9}{2} + \frac{9}{2} = 11,
$$

so our conclusion holds. □

Lemma 5.0.10. *Suppose $p > 767$. Let k be an integer satisfying*

$$
0 \le k < \frac{1}{6}\left(\frac{\sqrt{6}}{13\sqrt{11r}}\sqrt{p} - 1\right). \tag{5.3}
$$

Let

$$
i_k = \frac{\theta\zeta r+q-1}{2} + 6ka - kmp \mod r, \, j_k = \frac{\theta\zeta r+q-1}{2} + (6k+1)a - kmp - mq \mod r.
$$

Then $\Delta^{\varepsilon}_{a,\theta}(6k)-\Delta^{\varepsilon}_{a,\theta}(6k+1)=Ak+B+C_k,$ *where*

$$\begin{aligned}
A &= -\frac{72\varepsilon\zeta}{pr}(a-\frac{mp}{6})^2-\frac{12}{p},\\
B &= \varepsilon\left(-\frac{6\zeta}{pr}(a-\frac{mp}{6})^2-m(1-\theta)+\frac{m^2}{6}\right)\\
&\quad +2\varepsilon\{2\varepsilon\{\chi_{[0,q)}(3\theta q)-\chi_{[0,q)}((3\theta+m-6h)q)\}+\frac{p-1}{p},\\
C_k &= \varepsilon\zeta[d(L(r,s),j_k)-d(L(r,s),i_k)].
\end{aligned}$$

and

$$h=\begin{cases} 0 & \textit{if } 0\le 3\theta+m<6 \\ 1 & \textit{if } 3\theta+m=6 \end{cases}.$$

Proof. Using(4.3), we get

$$\begin{aligned}
&\Delta^{\varepsilon}_{a,\theta}(6k)-\Delta^{\varepsilon}_{a,\theta}(6k+1) \qquad (5.4)\\
=&d(L(p,1),6k)-d(L(p,1),6k+1)-\varepsilon[d(L(p,q),\frac{\theta p+q-1}{2}+6ka-kmp)\\
&-2\chi_{[0,q)}(\frac{\theta p+q-1}{2}+6ka-kmp)]+\varepsilon[d(L(p,q),\frac{\theta p+q-1}{2}+(6k+1)a-kmp)\\
&-2\chi_{[0,q)}(\frac{\theta p+q-1}{2}+(6k+1)a-(km+h)p)]\\
=&2\varepsilon\{\chi_{[0,q)}(\frac{\theta p+q-1}{2}+6ka-kmp)-\chi_{[0,q)}(\frac{\theta p+q-1}{2}+(6k+1)a-(km+h)p)\}\\
&+\frac{(p-12k)^2}{4p}-\frac{[p-2(6k+1)]^2}{4p}\\
&-\varepsilon\{\frac{[12ka-(2km+1-\theta)p]^2}{4pq}-\frac{[(12k+2)a-(2km+1-\theta)p]^2}{4pq}\\
&-d(L(q,\frac{1-\zeta}{2}q+\zeta r),\frac{\theta\zeta r+q-1}{2}+6ka-kmp)\\
&+d(L(q,\frac{1-\zeta}{2}q+\zeta r),\frac{\theta\zeta r+q-1}{2}+(6k+1)a-kmp-mq)\}.
\end{aligned}$$

We require

$$\begin{aligned}
0\le&\frac{\theta\zeta r+q-1}{2}+6ka-kmp<q,\\
0\le&\frac{\theta\zeta r+q-1}{2}+(6k+1)a-kmp-mq<q.
\end{aligned}$$

It suffices that

$$k < \frac{1}{6}\left(\frac{q-9}{2}\sqrt{\frac{6}{11rp}} - 1\right).$$

This implies

$$3\theta q \le \frac{\theta p + q - 1}{2} + 6ka - kmp < (3\theta+1)q, \tag{5.5}$$

$$(3\theta+m)q \le \frac{\theta p + q - 1}{2} + (6k+1)a - kmp < (3\theta+m+1)q. \tag{5.6}$$

When $m = 3$, $\theta = 1$, (5.6) becomes

$$6q \le \frac{\theta p + q - 1}{2} + (6k+1)a - kmp < 7q.$$

Here we require

$$p \le \frac{\theta p + q - 1}{2} + (6k+1)a - kmp < p + q.$$

We know $a < \frac{p}{2}$, so

$$\frac{\theta p + q - 1}{2} + (6k+1)a - kmp < \frac{\theta p + q - 1}{2} + \frac{p}{2} < p + q.$$

Moreover, we know when $m = 3$, by Lemma 5.0.9, $a > \frac{p}{2} - \sqrt{\frac{11rp}{6}}$.
If

$$k \le \frac{1}{6}\left(\frac{q-1}{2}\sqrt{\frac{6}{11rp}} - 1\right),$$

then

$$\frac{\theta p + q - 1}{2} + (6k+1)a - kmp \ge p.$$

When $p > 767$,

$$\frac{1}{6}\left(\frac{p}{13}\sqrt{\frac{6}{11rp}} - 1\right) < \frac{1}{6}\left(\frac{q-9}{2}\sqrt{\frac{6}{11rp}} - 1\right).$$

Using (5.1), (5.5) and (5.6), the right-hand side of (5.4) becomes

$$
\begin{aligned}
&2\varepsilon\{\chi_{[0,q)}(3\theta q)-\chi_{[0,q)}((3\theta+m-6h)q)\}+\frac{p-(12k+1)}{p}\\
&+\varepsilon\frac{a((12k+1)a-2kmp+(\theta-1)p)}{pq}\\
&+\varepsilon\zeta\{\frac{[12ka-2kmp+(\theta-1)\zeta r]^2-qr}{4qr}-d(L(r,s),i_k)\\
&-\frac{[2(6k+1)a-2kmp-2mq+(\theta-1)\zeta r]^2-qr}{4qr}+d(L(r,s),j_k)\}.
\end{aligned}
$$

This simplifies to be

$$
\begin{aligned}
&2\varepsilon\{\chi_{[0,q)}(3\theta q)-\chi_{[0,q)}((3\theta+m-6h)q)\}+\varepsilon\zeta[d(L(r,s),j_k)-d(L(r,s),i_k)]\\
&+\frac{p-(12k+1)}{p}+\varepsilon\frac{a((12k+1)a-2kmp+(\theta-1)p)}{pq}\\
&-\varepsilon\zeta\frac{((12k+1)a-2kmp-mq+(\theta-1)\zeta r)(a-mq)}{qr}\\
=&-\frac{6(12k+1)\varepsilon\zeta}{pr}(a-\frac{mp}{6})^2-\varepsilon m(1-\theta)+\frac{\varepsilon m^2}{6}\\
&+2\varepsilon\{\chi_{[0,q)}(3\theta q)-\chi_{[0,q)}((3\theta+m-6h)q)\}\\
&+\varepsilon\zeta[d(L(r,s),j_k)-d(L(r,s),i_k)]+\frac{p-(12k+1)}{p}\\
=&Ak+B+C_k.
\end{aligned}
$$

□

Proof of Proposition 4.0.8. If $S^3_K(p)\cong\varepsilon\mathbb{T}(p/q)$, then (4.4) holds, so

$$\Delta^{\varepsilon}_{a,\theta}(6k)-\Delta^{\varepsilon}_{a,\theta}(6k+1)=0 \text{ or } 2 \tag{5.7}$$

for all k satisfying (5.3). If $p>310r(36r+1)^2$, then

$$6\cdot 6r+1<\frac{\sqrt{6}}{13\sqrt{11r}}\sqrt{p}$$

hence $k=6r$ satisfies (5.3).

Let A,B,C_k be as in Lemma 4.4. If $A\neq 0$, then $Ak+B+C$ is equal to 0 or 2 for at most two values of k for any given C. Given p,q,a,ε,ζ, as k varies, C_k can take at most $3r$ values. It follows that $Ak + B + C_k$ cannot be 0 or 2 for $k = 0, 1, ..., 6r$. As a consequence, if $p > 310r(36r + 1)^2$, then (5.7) does not hold.

The only case we need to consider is that $A = 0$. In this case $\varepsilon\zeta = -1$.

$$A = \frac{12}{p}\left(\frac{6}{r}(a - \frac{mp}{6})^2 - 1\right) = 0$$

We get $|a - \frac{mp}{6}| = \sqrt{\frac{r}{6}}$, which is an irrational number. This contradicts that a is an integer and $\frac{mp}{6}$ is a rational number. □

Since we get an upper bound for p, an easy computer search will yield all possible p/q's. They are 1/1, 2/1, 3/1, 7/2, 9/1, 9/2, 10/1, 10/1, 11/1, 13/3, 13/3, 14/3, 17/2, 17/2, 19/4, 21/4, 22/3, 23/3, 27/4, 27/5, 29/4, 29/4, 37/7, 37/7, 38/7, 43/8, 46/7, 47/7, 49/9, 50/9, 51/8, 58/9, 59/9, 62/11, 69/11, 70/11, 81/13, 81/14, 83/13, 86/15, 91/16, 93/16, 94/15, 99/16, 99/17, 101/16, 106/17, 106/17, 110/19, 110/19, 113/18, 113/18, 119/19, 131/21, 133/23, 137/22, 137/22, 143/23, 146/25, 154/25, 157/27, 157/27, 163/28, 211/36, 221/36. Here if p_i/q_i appears twice, this means they correspond to candidate knots with different Heegaard Floer Homologies.

Chapter 6

Constructions When p Is Small

6.1 Dean's Construction

Let H be a genus two handlebody, and α a simple closed curve lying on ∂H. Denote by $H(\alpha)$ the manifold obtained by adding a 2-handle to H along the curve α. We say that α is primitive with respect to H if $H(\alpha)$ is a solid torus. We say that α is (m, n) Seifert-fibered if $H(\alpha)$ is a Seifert-fibered manifold over the disk with two exceptional fibers of multiplicities m and n. Equivalently, we say that α is primitive with respect to H if it is part of a basis for the free group $\pi_1(H)$, and α is (m, n) Seifert-fibered if it represents a word w in the free group $\pi_1(H) =< x, y >$, such that $< x, y|w >\cong< a, b|a^m b^n >$ for some nonzero integers m and n.

Let K be a knot contained in a genus two Heegaard surface Σ for S^3, that is, $S^3 = H \cup_\Sigma H'$, where H and H' are genus two handlebodies. Let r be the surface slope of K with respect to Σ. Dean [6] proved that $S^3_K(r) = H(r) \cup_{\Sigma'} H'(r)$. If K is primitive/primitive, then $S^3_K(r)$ is a lens space. If K is primitive/Seifert-fibered, then $S^3_K(r)$ is a Seifert-fibered manifold over the sphere with at most three exceptional fibers or a connected sum of two lens spaces.

Let $\Delta(r_1, r_2)$ be the minimal geometric intersection number between two slopes r_1 and r_2.

Theorem 6.1.1. *(Boyer and Zhang [4]) Let $K \subset S^3$ be a hyperbolic knot. If r_1 is a finite surgery slope and r_2 is a cyclic surgery slope of K, then $\Delta(r_1, r_2) \leq 2$.*

This motivates the following results:

Proposition 6.1.2. *Only 11 hyperbolic Berge knots have* **T**-*,* **O**-*, and* **I**-*type surgeries. More precisely, let $K(p, q; \omega)$ be the Berge knot corresponding to homology class ω in $L(p, q)$. They are $K(18, 5; 5)$, $K(39, 16; 16)$, $K(45, 19; 8)$, $K(46, 19; 11)$, $K(68, 19; 5)$, $K(71, 27; 11)$, $K(82, 23; 5)$, $K(93, 26; 5)$, $K(107, 30; 5)$, $K(118, 33; 5)$, $K(132, 37; 5)$.*

Proof. Berge knots have lens space surgeries, we can compute their Heegaard Floer Homology using Proposition 3.3.2 and compare them with the list from finite surgeries, we get 11 candidates. We draw the link diagram in SnapPy and compute the fundamental group of the Dehn filling with the finite surgery coefficient. If what we get is a $(2, 3, n)$-type group, then we have verified that the knot has indeed a required finite surgery. Below is a table of the candidates.

Table 6.1: Candidates of Berge knots with **T**-, **O**-, and **I**-type surgeries

p	q	ω	finite surgery coefficient p'
18	5	5	17
39	16	16	38
45	19	8	46
46	17	11	47
68	19	5	69
71	21	11	70
82	23	5	81
93	25	5	94
107	25	5	106
118	25	5	119
132	25	5	131

Here we use a point of view of dual Berge knots in the corresponding lens space, as they have the same knot complement as Berge knots in S^3, the only thing is to figure out the corresponding coefficients. The computation is as follows.

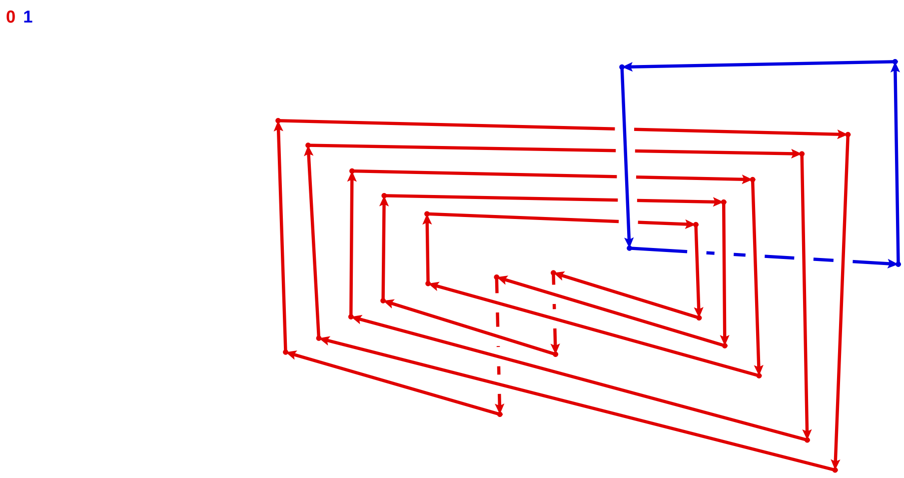

Figure 6.1: Link for $K(107, 30; 5)$

We would like to draw a link L with two components as above. When we perform on the trivial component (1-component) p/q-surgery, we get $L(p, q)$. The other component then becomes the dual Berge knot in $L(p, q)$. We would like to choose the orientations of the two components consistently, and one choice is shown above. Denote the longitude and meridian of 0-component and 1-component by λ and μ, and l and m, respectively. In homology, we have $\omega\mu = l$ and $\lambda = \omega m$. After performing p/q-surgery on 1-component, we have $pm + ql = 0$ in homology. The resultant 0-component K' is the dual Berge knot, and its longitude and meridian are denoted by λ' and μ' respectively. We would like to write λ' and μ' in terms of λ and μ.

By performing p-surgery on K' we get $L(p, q)$, and by performing ∞-surgery on K' we get S^3. Note we have $q\omega^2\mu + p\lambda = q\omega l + p\omega m = 0$, this means $\lambda' = (q\omega^2, p)$. Note that $L(p, q_1) \cong L(p, q_2)$ if and only if $q_1q_2 \equiv \pm 1 \pmod p$, so there is an indeterminacy in q. Take $K(107, 25; 5)$ for example. We have $\lambda' = (25 * 5^2, 107)$ and $107\mu' + \lambda' = (\pm 1, 0)$, there is no integer solutions. As $30 * 25 \equiv 1 \pmod{107}$, we take $q = 30$ and denote the knot by $K(107, 30; 5)$. We have $\lambda' = (30*5^2, 107)$ and $107\mu' + \lambda' = (\pm 1, 0)$, so we get $\mu' = (-7, -1)$, $107\mu' + \lambda' = (1, 0)$ and $106\mu' + \lambda' = (8, 1)$.

Now we have $S^3_L(1/0, 107/30) \cong L(107, 30)$ and $S^3_L(7/1, 107/30) \cong S^3$, we hope to get a spherical space form by performing p'-surgery on K'. By SnapPy, $S^3_L(8/1, 107/30)$ has fundamental group of the form $< a, b | abab^{-2}, ab(b^3a^2)^{77}b^4ab((a^{-2}b^{-3})^3a^{-1})^{21}a^{-1}b >$.

The first relator is $(ab)^2 = b^3$, which is in the center. Mod it out, the other relator becomes $a^4 = 1$. By upper central series theory, this group is finite and is of type $(2, 3, 4)$, which means this dehn filling is indeed the required finite surgery.

Table 6.2: Berge knots with **T**-, **O**-, and **I**-type surgeries

knot	p'	$Z(\pi_1(S^3_K(p')))$	$\pi_1(S^3_K(p'))/Z(\pi_1(S^3_K(p')))$
$K(18,5;5)$	17	$\langle (ab)^2\rangle$	$\langle a,b \vert (ab)^2, b^3, a^5\rangle$
$K(39,16;16)$	38	$\langle a^4\rangle$	$\langle a,b \vert b^2, (a^{-1}b)^3, a^4\rangle$
$K(45,19;8)$	46	$\langle (ab)^2\rangle$	$\langle a,b \vert (ab)^2, a^3, b^4\rangle$
$K(46,17;11)$	47	$\langle b^3\rangle$	$\langle (b^2a^{-2})^2, b^3, a^5\rangle$
$K(68,19;5)$	69	$\langle (ab)^2\rangle$	$\langle a,b \vert (ab)^2, b^3, a^3\rangle$
$K(71,27;11)$	70	$\langle a^3\rangle$	$\langle a,b \vert b^2, a^3, (ba^2)^4\rangle$
$K(82,23;5)$	81	$\langle (ab)^2\rangle$	$\langle a,b \vert (ab)^2, b^3, a^3\rangle$
$K(93,26;5)$	94	$\langle (ab)^2\rangle$	$\langle a,b \vert (ab)^2, b^3, a^4\rangle$
$K(107,30;5)$	106	$\langle (ab)^2\rangle$	$\langle a,b \vert (ab)^2, b^3, a^4\rangle$
$K(118,33;5)$	119	$\langle (ab)^2\rangle$	$\langle a,b \vert (ab)^2, b^3, a^5\rangle$
$K(132,37;5)$	131	$\langle (ab)^2\rangle$	$\langle a,b \vert (ab)^2, b^3, a^5\rangle$

□

Many examples of primitive/Seifert-fibered knots can be found among the twisted torus knots, which Dean defined in [6]. Let V be a standardly embedded solid torus in B^3, the 3-ball. ∂B^3 intersects with ∂V in a 2-disk D. Let $T(p,q)$ be a (p,q) torus knot on ∂V, and $T(p,q)$ intersects r times with D. Remove $T(p,q) \cap D$ from $T(p,q)$ we get a torus tangle, denoted by $T(p,q)_r$, on a once-punctured torus $\partial V \backslash D$. Similarly, we can get $T(m,n)_1$ on a once-punctured torus $\partial V' \backslash D'$. Take r parallel copies of $T(m,n)_1$, denote by $rT(m,n)_1$, on $\partial V' \backslash D'$. Glue $\partial V \backslash D$ and $\partial V' \backslash D'$ along the boundary, we get a knot $T(p,q)_r + rT(m,n)_1$. The knot obtained in this manner is called a twisted torus knot, and is denoted by $K(p,q,r,m,n)$, where $(p,q) = (m,n) = 1$, and $0 < r \leq p+q$.

Not all twisted torus knots are primitive/Seifert-fibered. One necessary condition for the knot to be primitive is $m = 1$ if $p \neq 1$. From now on, we will denote $K(p,q,r,1,n)$ by $K(p,q,r,n)$. Dean [6] gave two special classes of twist torus knots which are primitive/Seifert-fibered:

Theorem 6.1.3. *(Dean [6])*

1. $(pq+(p-kq)^2)$*-surgery on* $K(p,q,p-kq,1)$ *produces a Seifert-fibered manifold with base orbifold* $S^2(\mu_1,\mu_2,\mu_3) = S^2(p+q-kq, p-kq, k)$ *with* $(\mu_1,\mu_2) = 1$ *and* $|\mu_1 - \mu_2| > 1$.

2. $(pq-(p-kq)^2)$*-surgery on* $K(p,q,p-kq,-1)$ *produces a Seifert-fibered manifold with base orbifold* $S^2(\mu_1,\mu_2,\mu_3) = S^2(-p+q+kq, p-kq, k)$.

Miyazaki and Motegi [12] gave another special class of twist torus knots which are primitive/Seifert-fibered:

Proposition 6.1.4. *(Miyazaki and Motegi [12])* $(pq+(p+q)^2n)$*-surgery on* $K(p,q,p+q,n)$ *produces a Seifert-fibered manifold with base orbifold* $S^2(|p|,q,|n|)$.

6.2 Montesinos Construction

An n-tangle (B^3,t) is a pair that consists of a 3-ball B^3 and n disjoint arcs t properly embedded in B^3, and possibly some simple closed curves. (B^3,t) is a trivial tangle if there is an homeomorphism of pairs from (B^3,t) to the tangle $(D^2\times I, \{x_1,...,x_n\}\times I)$ where D^2 is the unit disk in $\mathbb{R}^2$ and $x_1,...,x_n$ are distinct points in the interior of D^2. A rational tangle is a trivial 2-tangle. There is a natural one to one correspondence between rational tangles and $\mathbb{Q}\cup\{1/0\}$. Denote by $R(p/q)$ the rational tangle determined by $p/q \in \mathbb{Q}\cup\{1/0\}$.

A Montesinos link, denoted by $K(e;\beta_1/\alpha_1,\ldots,\beta_n/\alpha_n)$, is composed of n rational tangles. The double branched cover of S^3, with branched set a Montesinos link of type $(e;\beta_1/\alpha_1,...,\beta_n/\alpha_n)$, is a Seifert-fibred space over $S^2(\alpha_1,...,\alpha_n)$, and that every surgery on a strongly invertible link yields a manifold which is a double branched cover of S^3 branched over a link in S^3 [13]. We first try to find a strongly invertible link $L\subset S^3$, such that some surgery on L yields a manifold which is a double cover of S^3, branched over a Montesinos link of type $(e,\beta_1/\alpha_1,\beta_2/\alpha_2,\beta_3/\alpha_3)$ with $(\alpha_1,\alpha_2,\alpha_3) = (2,3,m)$ or $(2,2,n)$, where $m=3,4,5$ and $n\geq 2$. On the other hand, the surgery on L may be transferred, using Kirby calculus, to a surgery on a hyperbolic knot $K\subset S^3$.

6.3 Berge's Construction

Berge and Kang [2] claimed that they produced a complete list of the hyperbolic primitive/Seifert-fibered knots in S^3. If their list is indeed complete, all knots in our list can be found in their list. Most of the work here was done before their paper, and we have taken four knots from their list.

Chapter 7

Results

I summarize all results below with a table of torus knots and satellite knots and a table of hyperbolic knots. Let $T(p,q)$ be the (p,q) torus knot, and let $[p_1,q_1;p_2,q_2]$ denote the (p_1,q_1)-cable of $T(p_2,q_2)$. A knot K is said to be a (p,q)-cable of a knot k if K is a simple loop in the boundary of neighborhood $N(k)$ wrapping p times in the meridional direction and q times in the longitudinal direction. I also include pictures of all the hyperbolic knots using Mathematica package KnotTheory`.

Table 7.1: Torus knots and satellite knots with **T-**, **O-**, and **I-**type surgeries

p	knot	p	knot	p	knot	p	knot	p	knot
1	$T(3,2)$	2	$T(3,2)$	3	$T(3,2)$	7	$T(5,2)$	9	$T(3,2)$
10	$T(3,2)$	10	$T(4,3)$	11	$T(3,2)$	13	$T(5,3)$	13	$T(5,2)$
14	$T(4,3)$	17	$T(5,3)$	19	$[9,2;3,2]$	21	$[11,2;3,2]$	27	$[13,2;3,2]$
29	$[15,2;3,2]$	37	$[19,2;5,2]$	43	$[21,2;5,2]$	49	$[16,3;3,2]$	50	$[17,3;3,2]$
59	$[20,3;3,2]$	91	$[23,4;3,2]$	93	$[23,4;3,2]$	99	$[25,4;3,2]$	101	$[25,4;3,2]$
106	$[35,3;4,3]$	110	$[37,3;4,3]$	133	$[44,3;5,3]$	137	$[46,3;5,3]$	146	$[29,5;3,2]$
154	$[31,5;3,2]$	157	$[39,4;5,2]$	163	$[41,4;5,2]$	211	$[35,6;3,2]$	221	$[37,6;3,2]$

Table 7.2: Hyperbolic knots with **T**-, **O**-, and **I**-type surgeries

p	knot
17	Pretzel knot $P(-2,3,7)$
22	Pretzel knot $P(-2,3,9)$
23	Pretzel knot $P(-2,3,9)$
29	mirror image of $K(1,1,0)$ from section 4 of [8]
37	$K(11,3,2,1)$ from [6]
38	Berge knot $K(39,16;16)$
46	Berge knot $K(45,19;8)$
47	K_3^* from [4]
51	$K_2^\sharp$ from [4]
58	mirror image of the P/SF$_\mathrm{d}$ KIST IV knot with $(n,p,\epsilon,J_1,J_2)=(-2,1,1,-4,1)$ from [2]
62	P/SF$_\mathrm{d}$ KIST III knot with $(h,k,h',k',J)=(-5,-3,-2,-1,1)$ from [2]
69	mirror image of $K(2,3,5,-3)$ from [12]
70	Berge knot $K(71,27;11)$
81	$K(2,3,5,3)$ from [12]
83	P/SF$_\mathrm{d}$ KIST V knot with $(n,p,\epsilon,J_1,J_2)=(1,-2,-1,2,2)$ from [2]
86	mirror image of $K(3,4,7,-2)$ from [12]
94	mirror image of $K(2,3,5,-4)$ from [12]
106	$K(2,3,5,4)$ from [12]
110	$K(3,4,7,2)$ from [12]
113	mirror image of $K(3,5,8,-2)$ from [12]
113	mirror image of the P/SF$_\mathrm{d}$ KIST V knot with $(n,p,\epsilon,J_1,J_2)=(-3,-2,-1,2,2)$ from [2]
119	mirror image of $K(2,3,5,-5)$ from [12]
131	$K(2,3,5,5)$ from [12]
137	mirror image of $K(2,5,7,-3)$ from [12]
143	$K(3,5,8,2)$ from [12]
157	$K(2,5,7,3)$ from [12]

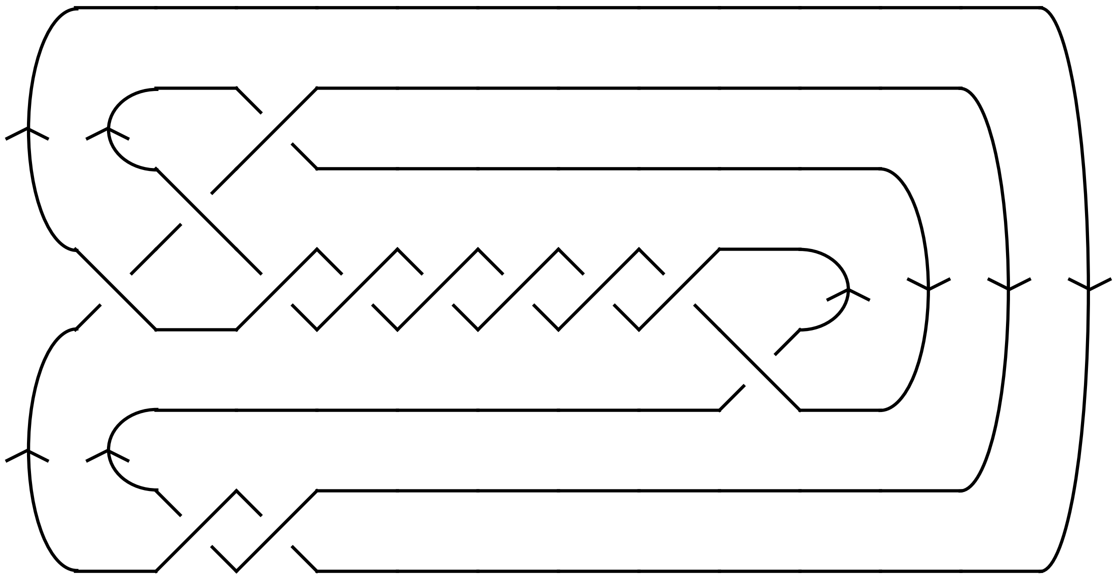

Figure 7.1: Pretzel knot $P(-2, 3, 7)$

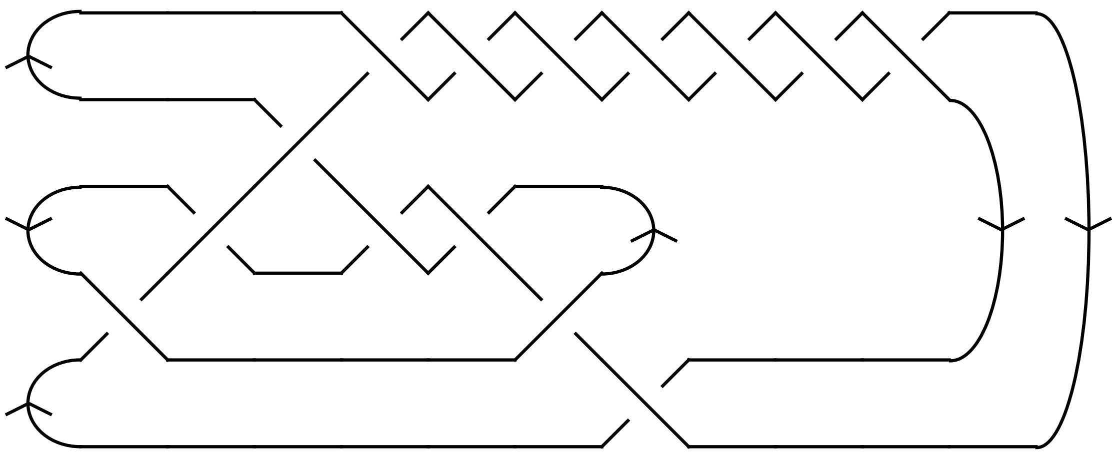

Figure 7.2: Pretzel knot $P(-2, 3, 9)$

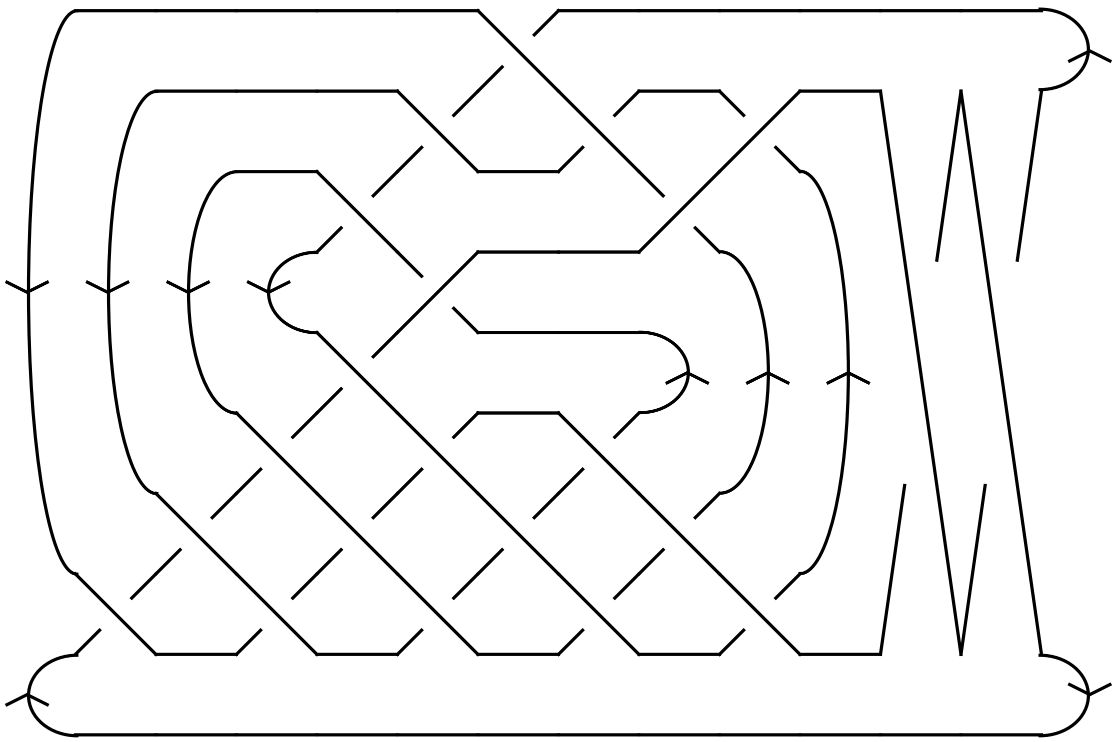

Figure 7.3: Mirror image of $K(1, 1, 0)$ from section 4 of [8]

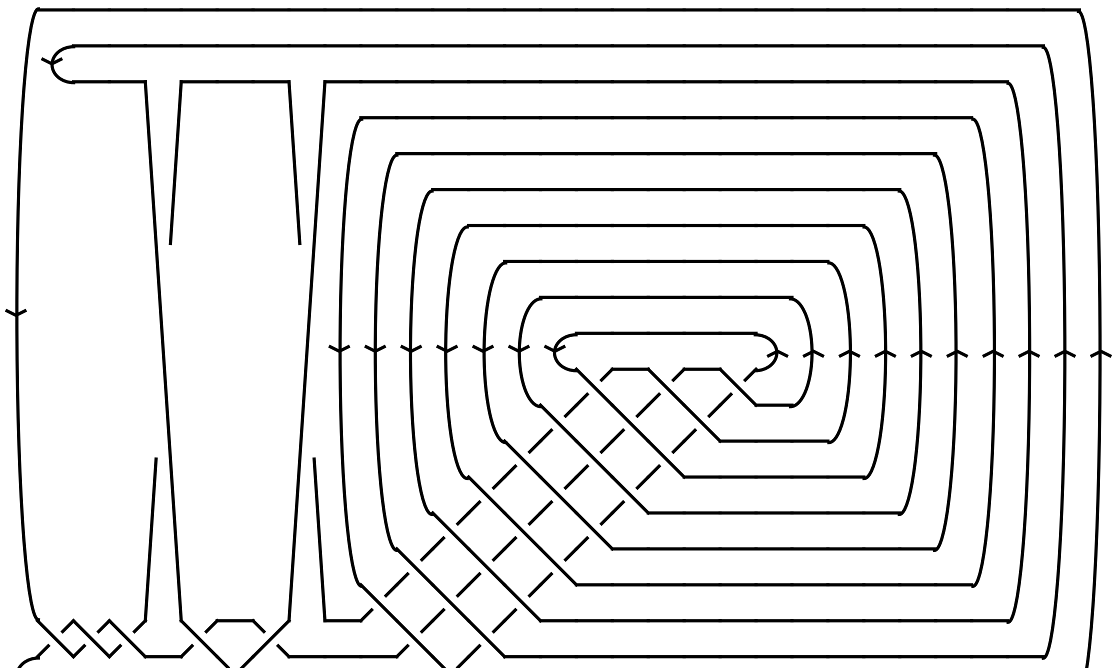

Figure 7.4: Twisted torus knot $K(11, 3, 2, 1)$ from [6]

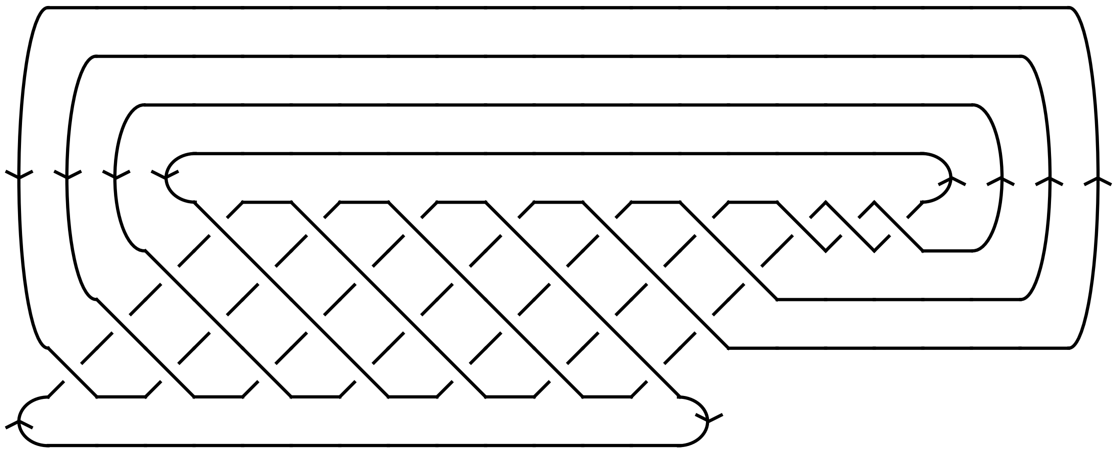

Figure 7.5: Berge knot $K(39, 16; 16)$

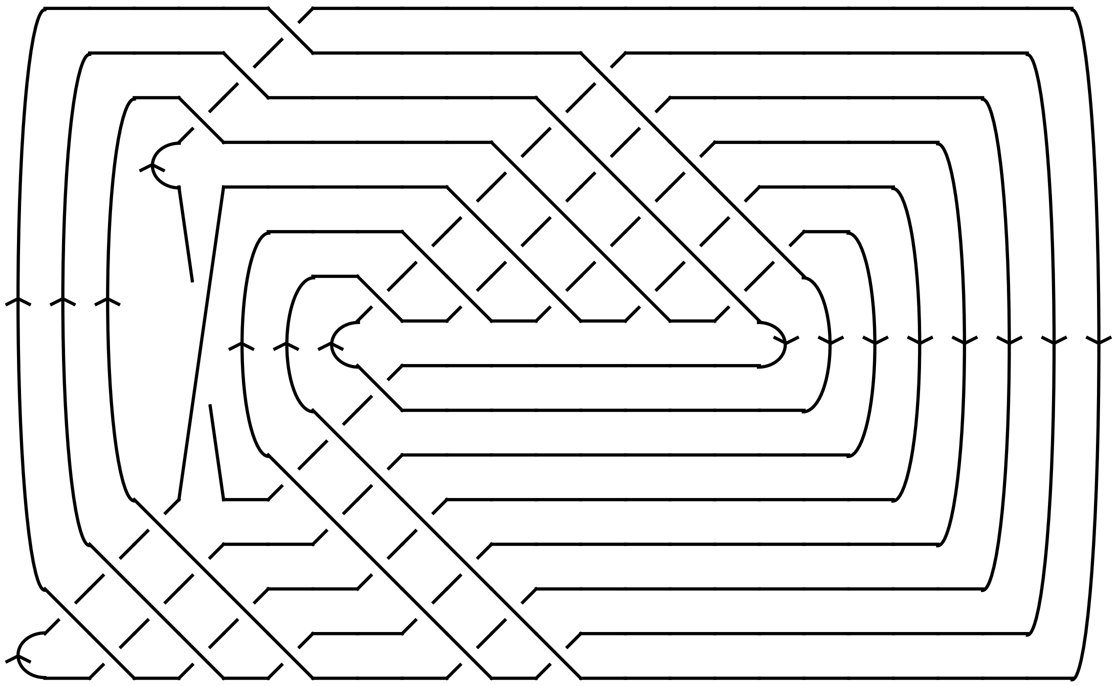

Figure 7.6: Berge knot $K(45, 19; 8)$

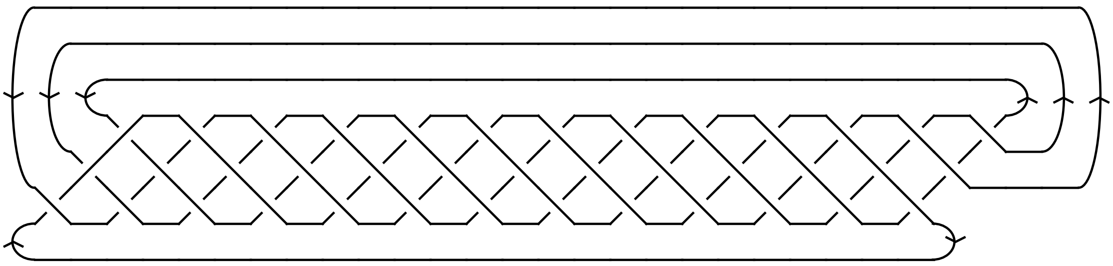

Figure 7.7: K_3^* from [4]

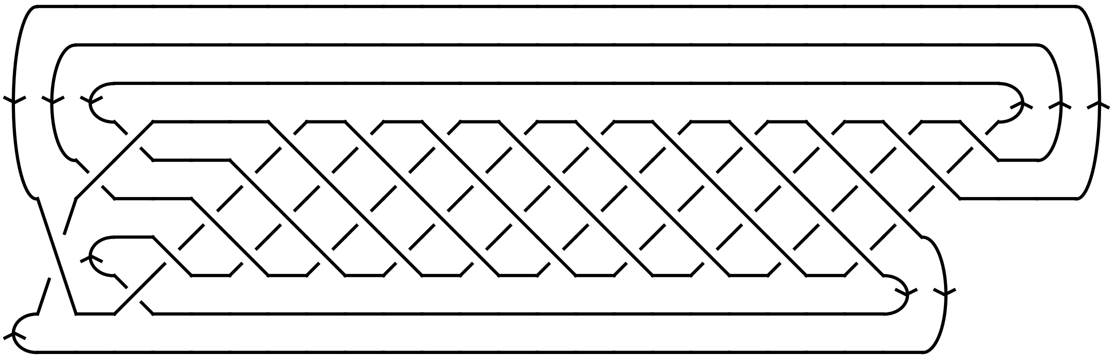

Figure 7.8: $K_2^\sharp$ from [4]

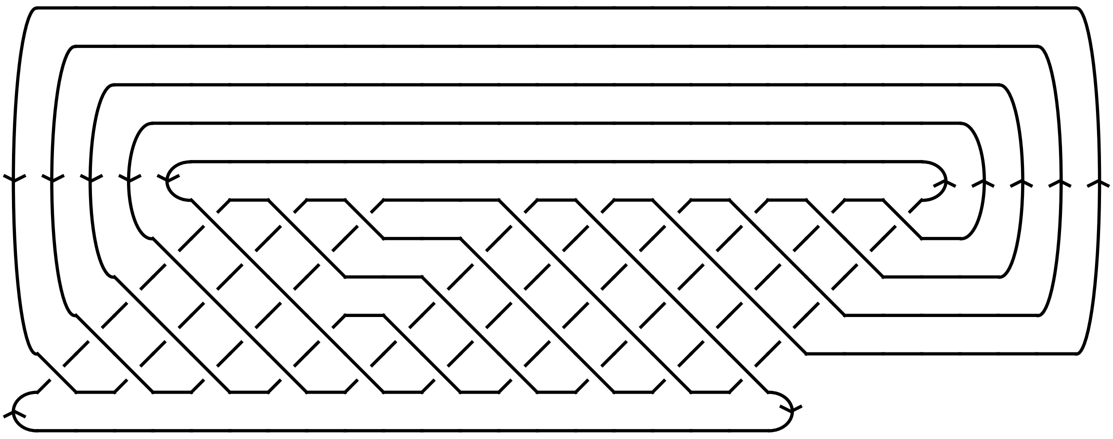

Figure 7.9: Mirror image of the P/SF$_\mathrm{d}$ KIST IV knot with $(n, p, \epsilon, J_1, J_2) = (-2, 1, 1, -4, 1)$ from [2]

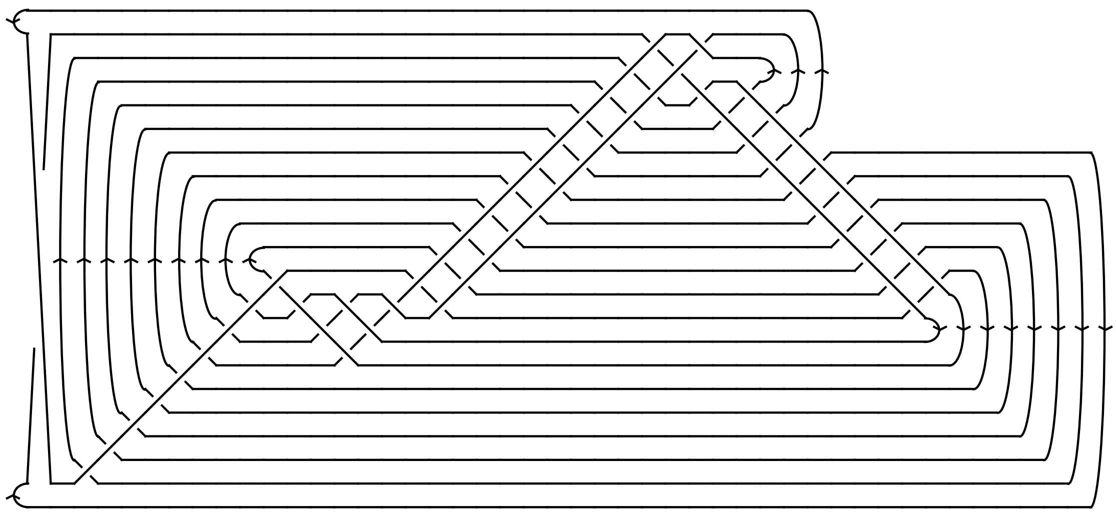

Figure 7.10: P/SF_d KIST III knot with $(h, k, h', k', J) = (-5, -3, -2, -1, 1)$ from [2]

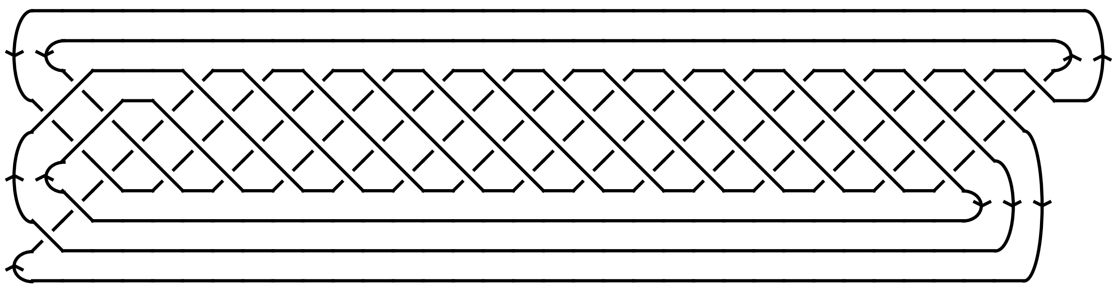

Figure 7.11: Mirror image of twisted torus knot $K(2, 3, 5, -3)$ from [12]

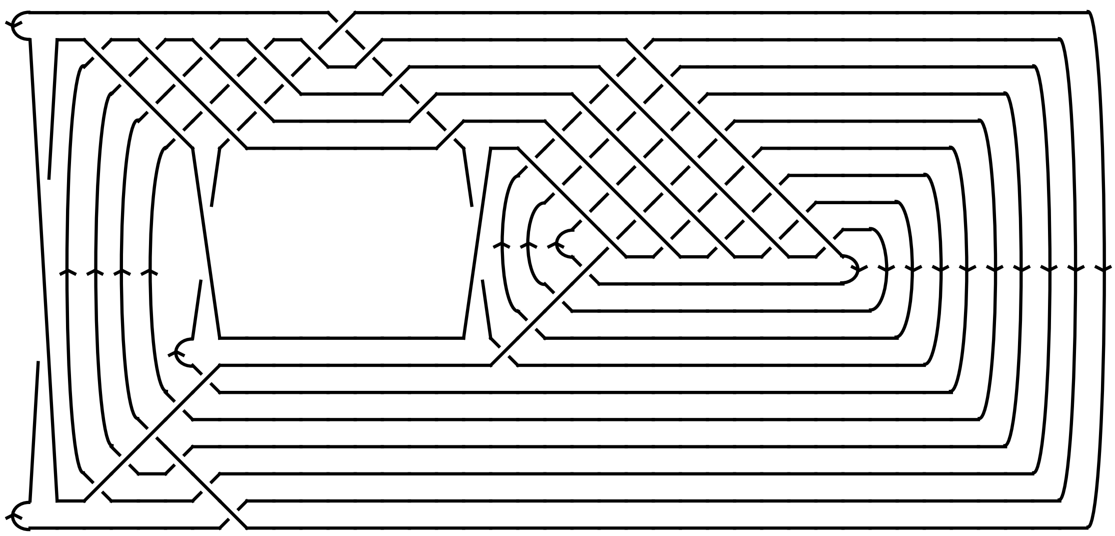

Figure 7.12: Berge knot $K(71, 27; 11)$

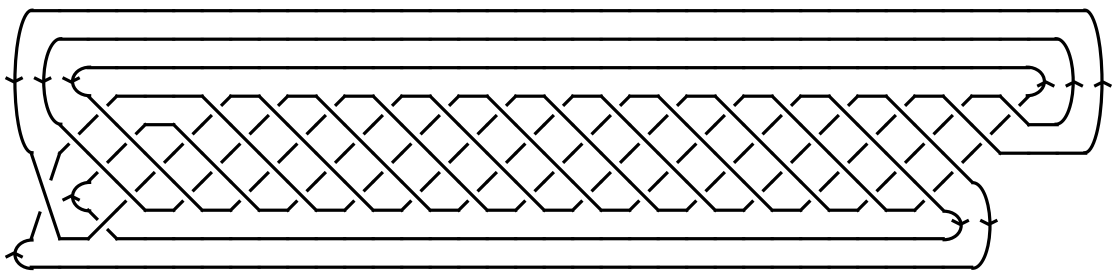

Figure 7.13: Twisted torus knot $K(2, 3, 5, 3)$ from [12]

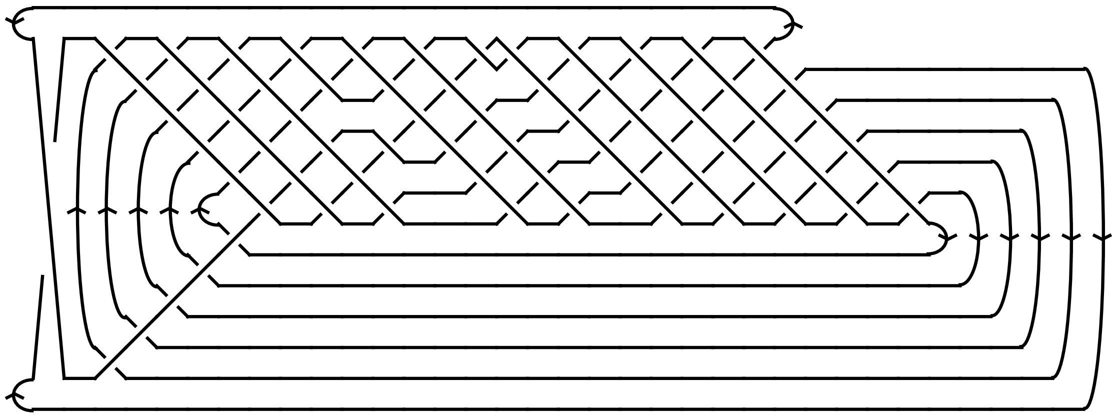

Figure 7.14: P/$\mathrm{SF_d}$ KIST V knot with $(n, p, \epsilon, J_1, J_2) = (1, -2, -1, 2, 2)$ from [2]

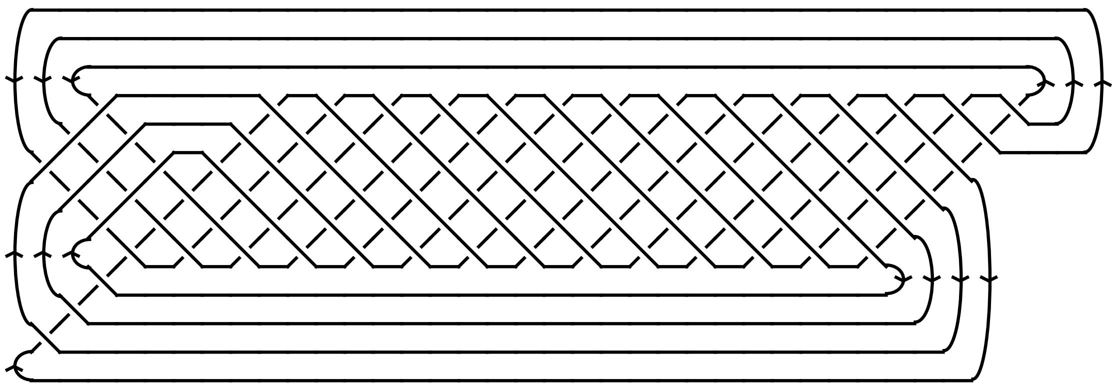

Figure 7.15: Mirror image of twisted torus knot $K(3, 4, 7, -2)$ from [12]

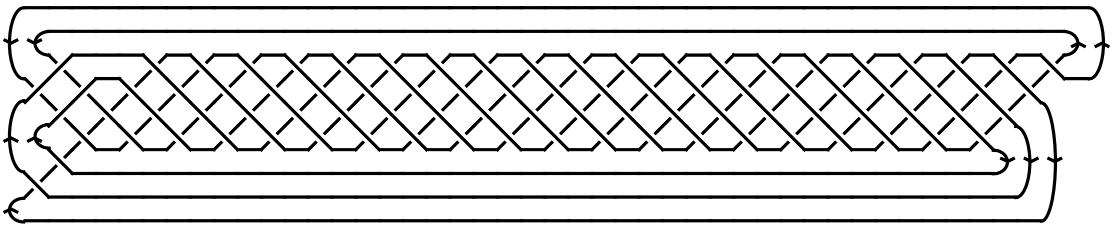

Figure 7.16: Mirror image of twisted torus knot $K(2,3,5,-4)$ from [12]

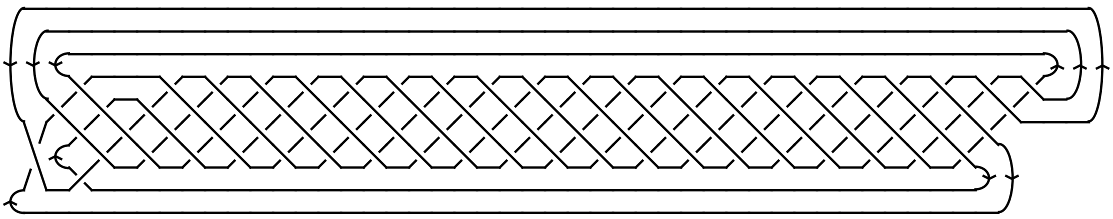

Figure 7.17: Twisted torus knot $K(2,3,5,4)$ from [12]

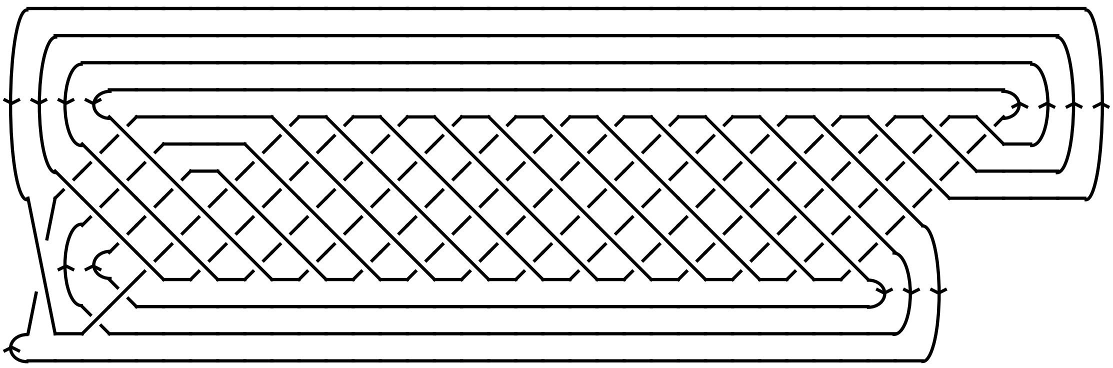

Figure 7.18: Twisted torus knot $K(3,4,7,2)$ from [12]

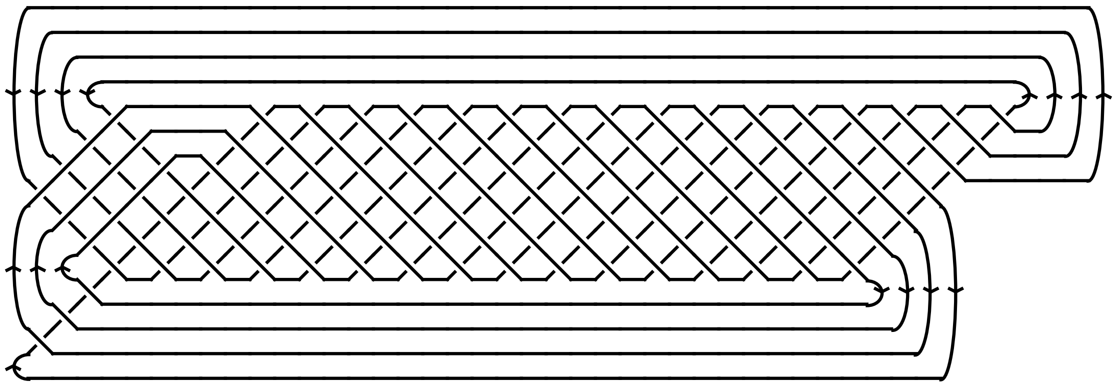

Figure 7.19: Mirror image of twisted torus knot $K(3,5,8,-2)$ from [12]

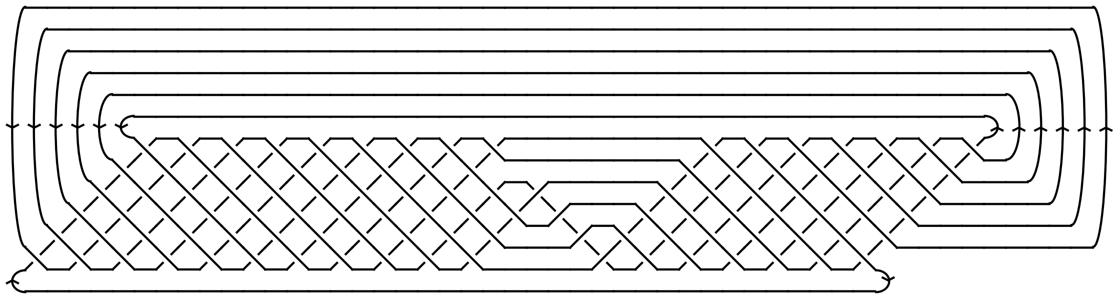

Figure 7.20: Mirror image of the P/SF$_\mathrm{d}$ KIST V knot with $(n, p, \epsilon, J_1, J_2) = (-3, -2, -1, 2, 2)$ from [2]

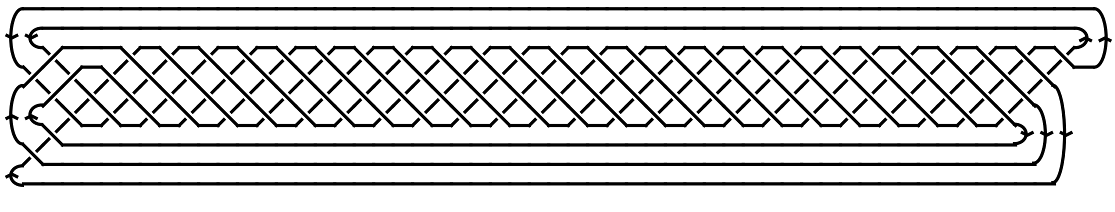

Figure 7.21: Mirror image of twisted torus knot $K(2,3,5,-5)$ from [12]

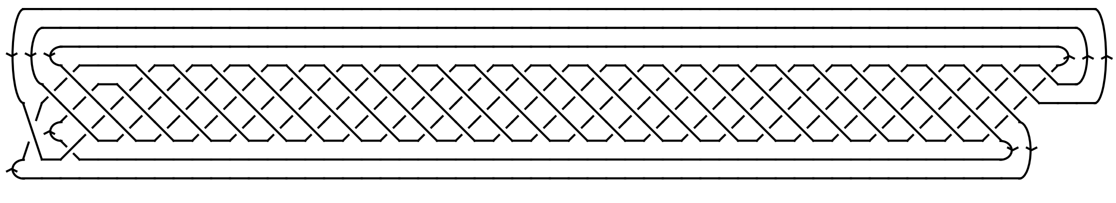

Figure 7.22: Twisted torus knot $K(2,3,5,5)$ from [12]

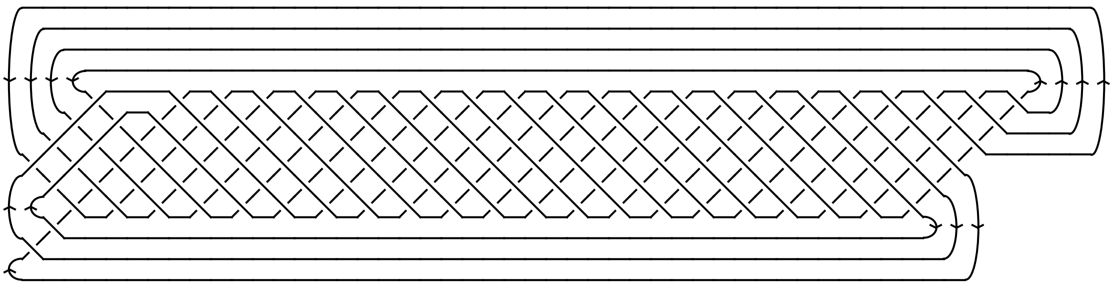

Figure 7.23: Mirror image of twisted torus knot $K(2,5,7,-3)$ from [12]

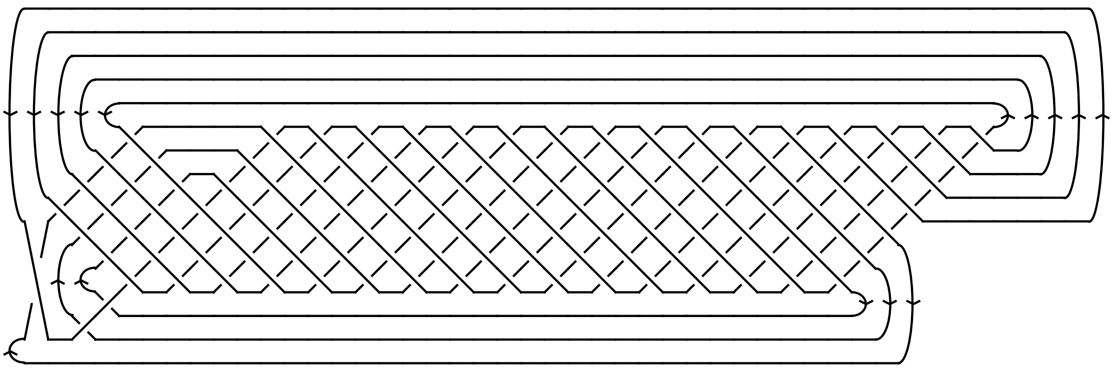

Figure 7.24: Twisted torus knot $K(3,5,8,2)$ from [12]

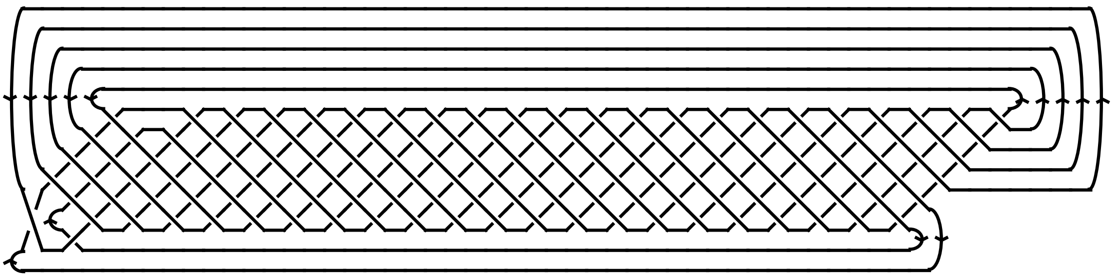

Figure 7.25: Twisted torus knot $K(2,5,7,3)$ from [12]

Bibliography

[1] J. Berge, Some knots with surgeries yielding lens spaces, Unpublished manuscript (1990).

[2] J. Berge, S. Kang, The Hyperbolic P/P, $\mathrm{P/SF_d}$, and $\mathrm{P/SF_m}$ Knots in S^3, Preprint (2013).

[3] S. Bleiler, C. Hodgson, Spherical space forms and Dehn filling, Topology 35 (1996), no. 3, 809–833.

[4] S. Boyer, X. Zhang, Finite Dehn surgery on knots, J. Amer. Math. Soc. 9 (1996), no. 4, 1005–1050.

[5] M. Culler, C. Gordon, J. Luecke, P. Shalen, Dehn surgery on knots, Ann. of Math. (2) 125 (1987), no. 2, 237–300.

[6] J. C. Dean, Small Seifert-fibered Dehn surgery on hyperbolic knots, Algebr. Geom. Topol. 3 (2003), 435–472.

[7] M. I. Doig, Obstructing finite surgery, available at arXiv:1302.6130.

[8] M. Eudave-Muñoz, On hyperbolic knots with Seifert fibered Dehn surgeries, Topology Appl. 121 (2002), 119–141.

[9] J. E. Greene, The lens space realization problem, Ann. of Math. 177 (2013), 449–511.

[10] E. Li, Y. Ni, Half-integral finite surgeries on knots in S^3, available at arXiv:1310.1346.

[11] W. B. R. Lickorish, A representation of orientable combinatorial 3-manifolds, Ann. of Math. (2) 76 (1962), 531–540.

[12] K. Miyazaki, K. Motegi, On primitive/Seifert-fibered constructions, Math. Proc. Camb. Phil. Soc. 138 (2005), 421–435.

[13] J. Montesinos, Surgery on links and double branched coverings of S^3, Annals of Math. Studies 84 (1975), 227–260.

[14] L. Moser, Elementary surgery along a torus knot, Pacific J. Math. 38 (1971), 737–745.

[15] Y. Ni, Knot Floer homology detects fibred knots, Invent. Math. 170 (2007), no. 3, 577–608.

[16] P. Ozsváth, Z. Szabó, Absolutely graded Floer homologies and intersection forms for four-manifolds with boundary, Adv. Math. 173 (2003), no. 2, 179–261.

[17] P. Ozsváth, Z. Szabó, Holomorphic disks and topological invariants for closed three-manifolds, Ann. of Math. (2), 159 (2004), no. 3, 1027–1158.

[18] P. Ozsváth, Z. Szabó, Holomorphic disks and genus bounds, Geom. Topol. 8 (2004), 311–334.

[19] P. Ozsváth, Z. Szabó, Holomorphic disks and knot invariants, Adv. Math. 186 (2004), no. 1, 58–116.

[20] P. Ozsváth, Z. Szabó, On knot Floer homology and lens space surgeries, Topology 44 (2005), no. 6, 1281–1300.

[21] P. Ozsváth, Z. Szabó, Knot Floer homology and rational surgeries, Algebr. Geom. Topol. 11 (2011), 1–68.

[22] G. Perelman, The entropy formula for the Ricci flow and its geometric applications, available at arXiv:math/0211159.

[23] G. Perelman, Ricci flow with surgery on three-manifolds, available at arXiv:math/0303109.

[24] G. Perelman, Finite extinction time for the solutions to the Ricci flow on certain three-manifolds, available at arXiv:math/0307245.

[25] J. Rasmussen, Floer homology and knot complements, PhD Thesis, Harvard University (2003), available at arXiv:math.GT/0306378.

[26] H. Seifert, Topologie Dreidimensionaler Gefaserter Räume, Acta Math. 60 (1933), no. 1, 147–238.

[27] W. P. Thurston, Three-dimensional manifolds, Kleinian groups and hyperbolic geometry, Bull. Amer. Math. Soc. (N.S.) 6 (1982), no. 3, 357–381.

[28] A. H. Wallace, Modifications and cobounding manifolds. IV, J. Math. Mech. 12 (1963), 445–484.