

Bibliography

- R. Abraham and J. E. Marsden. *Foundations of Mechanics*. Addison-Wesley, second edition, 1978. (with the assistance of Tudor Ratiu and Richard Cushman). 1, 9
- R. Abraham, J. E. Marsden, and T. S. Ratiu. *Manifolds, Tensor Analysis and Applications*, volume 75 of *Applied Mathematical Sciences*. Springer-Verlag, second edition, 1988. 75, 89, 91, 94, 101, 102
- D. H. Adams. R-torsion and linking numbers from simplicial abelian gauge theories. arXiv, hep-th/9612009, 1996. 76, 77, 88
- R. Almeida and P. Molino. Suites d’Atiyah et feuilletages transversalement complets. *C. R. Acad. Sci. Paris Sér. I Math.*, 300(1):13–15, 1985. 148
- D. N. Arnold. Differential complexes and numerical stability. In *Proceedings of the International Congress of Mathematicians, Vol. I (Beijing, 2002)*, pages 137–157, Beijing, 2002. Higher Ed. Press. 138
- V. I. Arnold. *Mathematical Methods of Classical Mechanics*, volume 60 of *Graduate Texts in Mathematics*. Springer-Verlag, 1989. Translated from the 1974 Russian original by K. Vogtmann and A. Weinstein. 1
- M. F. Atiyah. Complex analytic connections in fibre bundles. *Trans. Amer. Math. Soc.*, 85:181–207, 1957. 148
- M. J. Baines. *Moving Finite Elements*. Numerical Mathematics and Scientific Computation. Oxford University Press, 1995. 213
- M. Berry. Anticipations of the geometric phase. *Phys. Today*, pages 34–40, December 1990. 140
- A. M. Bloch. *Nonholonomic Mechanics and Control*. Interdisciplinary Applied Mathematics. Springer-Verlag, 2003. 191
- A. I. Bobenko, B. Lorbeer, and Y. B. Suris. Integrable discretizations of the Euler top. *J. Math. Phys.*, 39(12):6668–6683, 1998. 9

- A. I. Bobenko and Y. B. Suris. Discrete Lagrangian reduction, discrete Euler–Poincaré equations, and semidirect products. *Lett. Math. Phys.*, 49(1):79–93, 1999. 9, 72
- A. Bossavit. *Computational Electromagnetism*. Electromagnetism. Academic Press, 1998. Variational formulations, complementarity, edge elements. 4
- A. Bossavit. Generalized finite differences in computational electromagnetics. *Progress in Electromagnetics Research, PIER*, 32:45–64, 2001. 77
- A. Bossavit. Applied differential geometry (a compendium). URL <http://www.icm.edu.pl/edukacja/mat/Compendium.php>. (preprint), 2002a. 77
- A. Bossavit. Extrusion, contraction: Their discretization via Whitney forms. (preprint), 2002b. 77, 104, 105
- A. Bossavit. On “generalized finite differences”: Discretization of electromagnetic problems. (preprint), 2002c. 76, 77
- A. Cannas da Silva and A. Weinstein. *Geometric Models for Noncommutative Algebras*, volume 10 of *Berkeley Mathematics Lecture Notes*. American Mathematical Society, 1999. 129
- J. R. Cardoso and F. Silva Leite. Theoretical and numerical considerations about Padé approximants for the matrix logarithm. *Linear Algebra Appl.*, 330(1-3):31–42, 2001. 181
- É. Cartan. *Geometry of Riemannian spaces*. Math. Sci. Press, Brookline, CA, 1983. (translated from French). 188
- É. Cartan. *Riemannian Geometry in an Orthogonal Frame*. World Scientific, 2001. (translated from French). 188
- M. Castrillón-López. Discrete variational problems on forms. (in preparation), 2003. 123, 135
- E. Celledoni and A. Iserles. Approximating the exponential from a Lie algebra to a Lie group. *Math. Comp.*, 69(232):1457–1480, 2000. 181
- E. Celledoni and A. Iserles. Methods for the approximation of the matrix exponential in a lie-algebraic setting. *IMA J. Num. Anal.*, 21(2):463–488, 2001. 181
- H. Cendra, D. D. Holm, J. E. Marsden, and T. S. Ratiu. Lagrangian reduction, the Euler–Poincaré equations, and semidirect products. In *Geometry of Differential Equations*, volume 186 of *American Mathematical Society Translations Series 2*, pages 1–25. American Mathematical Society, 1998. 72

- H. Cendra, J. E. Marsden, and T. S. Ratiu. Lagrangian reduction by stages. *Mem. Amer. Math. Soc.*, 152(722), 2001. 3, 9, 150, 168, 176, 186
- D. Chang and J. E. Marsden. Geometric derivation of the Delaunay variables and geometric phases. *Nonlinearity*, 16(2):1257–1275, 2003. 57
- Z. Chen and T. Y. Hou. A mixed multiscale finite element method for elliptic problems with oscillating coefficients. *Math. Comp.*, 72(242):541–576 (electronic), 2003. 197, 215
- J. Cortés. *Geometric, Control and Numerical Aspects of Nonholonomic Systems*, volume 1793 of *Lecture Notes in Mathematics*. Springer-Verlag, 2002. 191
- M. Desbrun, A. N. Hirani, M. Leok, and J. E. Marsden. Discrete exterior calculus. (in preparation), 2003a. 3, 72
- M. Desbrun, A. N. Hirani, M. Leok, and J. E. Marsden. Discrete Poincaré lemma. *Appl. Numer. Math.*, 2003b. (submitted). 108
- R. A. DeVore. Nonlinear approximation. In *Acta Numerica*, volume 7, pages 51–150. Cambridge University Press, 1998. 197, 213
- A. A. Dezin. *Multidimensional Analysis and Discrete Models*. CRC Press, 1995. 77, 88
- W. E and B. Engquist. The heterogeneous multiscale methods. *Commun. Math. Sci.*, 1(1):87–132, 2003. 229
- M. Eastwood. A complex from linear elasticity. In *The Proceedings of the 19th Winter School “Geometry and Physics” (Srní, 1999)*, number 63 in *Rend. Circ. Mat. Palermo (2) Suppl.*, pages 23–29, 2000. 138
- Y. R. Efendiev, T. Y. Hou, and X. -H. Wu. Convergence of a nonconforming multiscale finite element method. *SIAM J. Numer. Anal.*, 37(3):888–910 (electronic), 2000. 197, 215
- R. Forman. Discrete Morse theory and the cohomology ring. *Trans. Amer. Math. Soc.*, 354(12):5063–5085 (electronic), 2002. 76
- P. Goldreich and A. Toomre. Some remarks on polar wandering. *J. Geophys. Res.*, 10:2555–2567, 1969. 143
- E. Grinspun, P. Krysl, and P. Schröder. CHARMS: A simple framework for adaptive simulation. *ACM Transactions on Graphics (SIGGRAPH)*, 21(21):281–290, July 2002. 229
- P. W. Gross and P. R. Kotiuga. Data structures for geometric and topological aspects of finite element algorithms. *Progress in Electromagnetics Research, PIER*, 32:151–169, 2001. 77

- E. Hairer, C. Lubich, and G. Wanner. *Geometric Numerical Integration*, volume 31 of *Springer Series in Computational Mathematics*. Springer-Verlag, 2002. Structure-preserving algorithms for ordinary differential equations. 1
- E. Hairer, S. P. Nørsett, and G. Wanner. *Solving Ordinary Differential Equations I : Nonstiff problems*, volume 8 of *Springer Series in Computational Mathematics*. Springer-Verlag, second edition, 1993. 36
- E. Hairer and G. Wanner. *Solving Ordinary Differential Equations II : Stiff and differential-algebraic problems*, volume 14 of *Springer Series in Computational Mathematics*. Springer-Verlag, second edition, 1996. 36
- A. Hatcher. *Algebraic Topology*. Cambridge University Press, 2001. 94, 97
- N. J. Higham. Evaluating Padé approximants of the matrix logarithm. *SIAM J. Matrix Anal. Appl.*, 22(4):1126–1135 (electronic), 2001. 181
- R. Hiptmair. Canonical construction of finite elements. *Math. Comp.*, 68(228):1325–1346, 1999. 4, 77, 88
- R. Hiptmair. Discrete Hodge-operators: An algebraic perspective. *Progress in Electromagnetics Research, PIER*, 32:247–269, 2001a. 77
- R. Hiptmair. Higher order Whitney forms. *Progress in Electromagnetics Research, PIER*, 32:271–299, 2001b. 77
- R. Hiptmair. Finite elements in computational electromagnetism. In *Acta Numerica*, volume 11, pages 237–339. Cambridge University Press, 2002. 77, 92
- A. N. Hirani. *Discrete Exterior Calculus*. PhD thesis, California Institute of Technology, 2003. 94, 105
- D. D. Holm, J. E. Marsden, and T. S. Ratiu. Euler–Poincaré models of ideal fluids with nonlinear dispersion. *Phys. Rev. Lett.*, 349:4173–4177, 1998. 9, 72
- T. Y. Hou and X. -H. Wu. A multiscale finite element method for PDEs with oscillatory coefficients. In *Numerical treatment of multi-scale problems (Kiel, 1997)*, volume 70 of *Notes Numer. Fluid Mech.*, pages 58–69. Vieweg, 1999. 197, 215, 229
- T. W. Hungerford. *Algebra*, volume 73 of *Graduate Texts in Mathematics*. Springer-Verlag, 1974. 232, 233, 234

- C. L. Hwang and L. T. Fan. A discrete version of Pontryagin's maximum principle. *Operations Res.*, 15:139–146, 1967. 8
- J. M. Hyman and M. Shashkov. Adjoint operators for the natural discretizations of the divergence, gradient and curl on logically rectangular grids. *Appl. Numer. Math.*, 25(4):413–442, 1997a. 4
- J. M. Hyman and M. Shashkov. Natural discretizations for the divergence, gradient, and curl on logically rectangular grids. *Comput. Math. Appl.*, 33(4):81–104, 1997b. 4
- A. Iserles. On the numerical quadrature of highly-oscillating integrals I: Fourier transforms. Technical Report 2003/NA05, DAMTP, Cambridge, 2003a. (to appear in *IMA J. Num. Anal.*). 220
- A. Iserles. On the numerical quadrature of highly-oscillating integrals II: Irregular oscillators. Technical Report 2003/NA09, DAMTP, Cambridge, 2003b. 220
- A. Iserles. On the method of Neumann series for highly oscillatory equations. Technical Report 2004/NA02, DAMTP, Cambridge, 2004. 220
- A. Iserles, H. Munthe-Kaas, S. P. Nørsett, and A. Zanna. Lie-group methods. In *Acta Numerica*, volume 9, pages 215–365. Cambridge University Press, 2000. 203
- A. Iserles and S. P. Nørsett. Efficient quadrature of highly oscillatory integrals using derivatives. Technical Report 2004/NA03, DAMTP, Cambridge, 2004. 220
- J. D. Jackson. *Classical Electrodynamics*. Wiley, third edition, 1998. 125
- S. M. Jalnapurkar, M. Leok, J. E. Marsden, and M. West. Discrete Routh reduction. *J. FoCM*, 2003. (submitted). 2
- S. M. Jalnapurkar and J. E. Marsden. Reduction of Hamilton's variational principle. *Dynam. Stabil. Syst.*, 15(3):287–318, 2000. 9, 20, 22, 46, 72
- S. M. Jalnapurkar and J. E. Marsden. Variational discretization of Hamiltonian systems. (in preparation), 2003. 8
- B. W. Jordan and E. Polak. Theory of a class of discrete optimal control systems. *J. Electron. Control*, 17:697–711, 1964. 8
- J. V. José and E. J. Saletan. *Classical Dynamics: A Contemporary Approach*. Cambridge University Press, 1998. 222
- T. Kaczynski, K. Mischaikow, and M. Mrozek. *Computational homology*, volume 157 of *Applied Mathematical Sciences*. Springer-Verlag, 2004. 76

- C. Kane, J. E. Marsden, and M. Ortiz. Symplectic-energy-momentum preserving variational integrators. *J. Math. Phys.*, 40(7):3353–3371, 1999. 1, 194, 202, 210, 212, 213
- C. Kane, J. E. Marsden, M. Ortiz, and M. West. Variational integrators and the Newmark algorithm for conservative and dissipative mechanical systems. *Int. J. Numer. Meth. Eng.*, 49(10):1295–1325, 2000. 1, 8, 51
- S. Kobayashi and K. Nomizu. *Foundations of Differential Geometry*, volume 1. Wiley, 1963. 145, 148, 149
- P. Krysl, A. Trivedi, and B. Zhu. Object-oriented hierarchical mesh refinement with CHARMS. *Int. J. Numer. Meth. Eng.*, 2003. (to appear). 229
- S. Lall and M. West. Discrete variational mechanics and duality. (in preparation), 2003. 201
- B. Leimkuhler and S. Reich. *Simulating Hamiltonian Dynamics*, volume 14 of *Cambridge Monographs on Applied and Computational Mathematics*. Cambridge University Press, 2004. 1
- M. Leok. A mathematical model of true polar wander. Caltech SURF Report, 1998. 143
- M. Leok. Discrete Routh reduction and discrete exterior calculus. Caltech Control and Dynamical Systems, Ph.D. Candidacy Report, 2002. 3
- M. Leok. Generalized Galerkin variational integrators: Lie group, multiscale, and pseudospectral methods. (in preparation), 2004. 3
- M. Leok, J. E. Marsden, and A. Weinstein. A discrete theory of connections on principal bundles. (in preparation), 2003. 3, 72, 86
- A. Lew, J. E. Marsden, M. Ortiz, and M. West. Asynchronous variational integrators. *Arch. Ration. Mech. An.*, 167(2):85–146, 2003. 1, 74, 194
- A. Lew, J. E. Marsden, M. Ortiz, and M. West. Variational time integrators. *Int. J. Numer. Meth. Eng.*, 2004. (to appear). 1, 212
- K. C. H. Mackenzie. Lie algebroids and Lie pseudoalgebras. *Bull. London Math. Soc.*, 27(2):97–147, 1995. 148
- S. Maeda. Lagrangian formulation of discrete systems and concept of difference space. *Math. Japonica*, 27(3):345–356, 1981. 8
- E. L. Mansfield and P. E. Hydon. On a variational complex for difference equations. In *The Geometrical Study of Differential Equations (Washington, DC, 2000)*, volume 285 of *Contemporary Mathematics*, pages 121–129. American Mathematical Society, 2001. 77

- J. E. Marsden. *Lectures on Mechanics*, volume 174 of *London Mathematical Society Lecture Note Series*. Cambridge University Press, 1992. 9, 11, 12, 13, 59, 66
- J. E. Marsden. Geometric mechanics, stability, and control. In L. Sirovich, editor, *Applied Mathematical Sciences*, volume 100, pages 265–291. Springer-Verlag, 1994. 140, 143
- J. E. Marsden. Geometric foundations of motion and control. In *Motion, Control and Geometry*, pages 3–19. National Academy Press, 1997. 140, 143
- J. E. Marsden, R. Montgomery, and T. S. Ratiu. Reduction, symmetry, and phases in mechanics. *Mem. Amer. Math. Soc.*, 88(436):1–110, 1990. 13, 59, 143
- J. E. Marsden, G. W. Patrick, and S. Shkoller. Multisymplectic geometry, variational integrators, and nonlinear PDEs. *Commun. Math. Phys.*, 199(2):351–395, 1998. 8, 193, 195
- J. E. Marsden, S. Pekarsky, and S. Shkoller. Discrete Euler–Poincaré and Lie–Poisson equations. *Nonlinearity*, 12(6):1647–1662, 1999. 2, 9, 129, 205, 207, 209
- J. E. Marsden, S. Pekarsky, and S. Shkoller. Symmetry reduction of discrete Lagrangian mechanics on Lie groups. *J. Geom. Phys.*, 36(1-2):140–151, 2000a. 2, 9, 129
- J. E. Marsden, S. Pekarsky, S. Shkoller, and M. West. Variational methods, multisymplectic geometry and continuum mechanics. *J. Geom. Phys.*, 38(3-4):253–284, 2001. 193, 195
- J. E. Marsden and T. S. Ratiu. *Introduction to Mechanics and Symmetry*, volume 17 of *Texts in Applied Mathematics*. Springer-Verlag, second edition, 1999. 1, 17, 44, 140
- J. E. Marsden, T. S. Ratiu, and J. Scheurle. Reduction theory and the Lagrange–Routh equations. *J. Math. Phys.*, 41(6):3379–3429, 2000b. 9, 11, 13, 22, 46, 59, 72
- J. E. Marsden and J. Scheurle. Lagrangian reduction and the double spherical pendulum. *Z. Angew. Math. Phys.*, 44(1):17–43, 1993a. 9, 46
- J. E. Marsden and J. Scheurle. The reduced Euler-Lagrange equations. In *Dynamics and Control of Mechanical Systems (Waterloo, ON, 1992)*, volume 1 of *Fields Institute Communications*, pages 139–164. American Mathematical Society, 1993b. 9, 46
- J. E. Marsden and J. M. Wendlandt. Mechanical systems with symmetry, variational principles and integration algorithms. In M. Alber, B. Hu, and J. Rosenthal, editors, *Current and Future Directions in Applied Mathematics*, pages 219–261. Birkhäuser, 1997. 8
- J. E. Marsden and M. West. Discrete mechanics and variational integrators. In *Acta Numerica*, volume 10, pages 317–514. Cambridge University Press, 2001. 1, 8, 14, 37, 38, 194, 203

- C. Mattiussi. An analysis of finite volume, finite element, and finite difference methods using some concepts from algebraic topology. *J. Comput. Phys.*, 133(2):289–309, 1997. 77
- C. Mattiussi. The finite volume, finite difference, and finite element methods as numerical methods for physical field problems. *Adv. Imag. Elect. Phys.*, 113:1–146, 2000. 74, 77
- R. McLachlan and M. Perlmutter. Integrators for nonholonomic mechanical systems. (preprint), 2003. 191
- M. Meyer, M. Desbrun, P. Schröder, and A. H. Barr. Discrete differential-geometry operators for triangulated 2-manifolds. *VisMath*, 2002. 92, 103
- R. Montgomery. How much does a rigid body rotate? A Berry’s phase from the eighteenth century. *Amer. J. Phys.*, 59:394–398, 1991. 143
- B. Moritz. *Vector Difference Calculus*. PhD thesis, University of North Dakota, 2000. 77
- B. Moritz and W. A. Schwalm. Triangle lattice green functions for vector fields. *J. Phys. A.*, 34(3):589–602, 2001. 77
- J. Moser and A. P. Veselov. Discrete versions of some classical integrable systems and factorization of matrix polynomials. *Commun. Math. Phys.*, 139(2):217–243, 1991. 8
- S. Müller and M. Ortiz. On the Γ -convergence of discrete dynamics and variational integrators. *J. Nonlinear Sci.*, 2004. (to appear). 1
- J. R. Munkres. *Elements of Algebraic Topology*. Addison-Wesley, 1984. 78, 87, 88
- R. M. Murray, Z. Li, and S. S. Sastry. *A Mathematical Introduction to Robotic Manipulation*. CRC Press, 1994. 235
- R. A. Nicolaides and D. -Q. Wang. Convergence analysis of a covolume scheme for Maxwell’s equations in three dimensions. *Math. Comp.*, 67(223):947–963, 1998. 77
- J. Nocedal and S. J. Wright. *Numerical Optimization*. Springer Series in Operations Research. Springer-Verlag, 1999. 197
- S. P. Novikov. Discrete connections on the triangulated manifolds and difference linear equations. arXiv, math-ph/0303035, 2003. 188
- M. Oliver, M. West, and C. Wulff. Approximate momentum conservation for spatial semidiscretizations of nonlinear wave equations. *Numer. Math.*, 2004. (to appear). 213

- P. J. Olver. Geometric foundations of numerical algorithms and symmetry. *Appl. Algebra Engrg. Comm. Comput.*, 11(5):417–436, 2001. Special issue “Computational geometry for differential equations”. 177
- J. -P. Ortega and T. S. Ratiu. *Hamiltonian Singular Reduction*. Progress in Mathematics. Birkhäuser, 2001. (to appear). 65
- G. W. Patrick. *Two Axially Symmetric Coupled Rigid Bodies: Relative Equilibria, Stability Bifurcations, and a Momentum Preserving Symplectic Integrator*. PhD thesis, University of California at Berkeley, 1991. 71
- S. Pekarsky and M. West. Discrete diffeomorphism groupoids and circulation conserving fluid integrators. (in preparation), 2003. 131
- J. E. Prussing and B. A. Conway. *Orbital Mechanics*. Oxford University Press, 1993. 57
- E. J. Routh. *Stability of a Given State of Motion*. Macmillan, 1877. Reprinted in *Stability of Motion*, A.T. Fuller (ed.), Halsted Press, 1975. 9, 43
- E. J. Routh. *Advanced Rigid Dynamics*. Macmillan, 1884. 9
- A. Sanyal, J. Shen, and N. H. McClamroch. Variational integrators for mechanical systems with cyclic generalized coordinates. (preprint), 2003. 45
- J. M. Sanz-Serna and M. P. Calvo. *Numerical Hamiltonian Problems*, volume 7 of *Applied Mathematics and Mathematical Computation*. Chapman & Hall, 1994. 1
- J. R. Schewchuck. What is a good linear finite element? Interpolation, conditioning, anisotropy and quality measures. URL <http://www.cs.berkeley.edu/~jrs/papers/elemj.ps>. (preprint), 2002. 79
- W. A. Schwalm, B. Moritz, M. Giona, and M. K. Schwalm. Vector difference calculus for physical lattice models. *Phys. Rev. E.*, 59(1, part B):1217–1233, 1999. 77
- S. Sen, S. Sen, J. C. Sexton, and D. H. Adams. Geometric discretization scheme applied to the abelian Chern–Simons theory. *Phys. Rev. E*, 61(3):3174–3185, 2000. 76, 77, 88, 91
- A. Shapere and F. Wilczek. *Geometric Phases in Physics*, volume 5 of *Advanced Series in Mathematical Physics*. World Scientific, 1989. 140
- M. Shashkov. *Conservative finite-difference methods on general grids*. Symbolic and Numeric Computation Series. CRC Press, 1996. 4

- N. Steenrod. *The Topology of Fibre Bundles*. Princeton University Press, 1951. 145
- Y. Suris. Hamiltonian methods of Runge–Kutta type and their variational interpretation. *Mat. Model.*, 2(4):78–87, 1990. 36, 37
- F. L. Teixeira. Geometric aspects of the simplicial discretization of Maxwell’s equations. *Progress in Electromagnetics Research, PIER*, 32:171–188, 2001. 77
- K. Theodoropoulos, Y. -H. Qian, and I. G. Kevrekidis. “Coarse” stability and bifurcation analysis using timesteppers: a reaction diffusion example. *Proc. Natl. Acad. Sci.*, 97(18):9840–9843, 2000. 229
- P. Thoutireddy and M. Ortiz. A variational r -adaptation and shape-optimization method for finite-deformation elasticity. *Int. J. Numer. Meth. Eng.*, 2003. (to appear). 213
- Y. Y. Tong, S. Lombeyda, A. N. Hirani, and M. Desbrun. Discrete multiscale vector field decomposition. *ACM Transactions on Graphics (SIGGRAPH)*, July 2003. 75, 101
- E. Tonti. Finite formulation of electromagnetic field. *IEEE Trans. Mag.*, 38:333–336, 2002. 77
- L. N. Trefethen. *Spectral methods in MATLAB*. Software, Environments, and Tools. Society for Industrial and Applied Mathematics (SIAM), 2000. 222
- V. Vedral. Geometric phases and topological quantum computation. *Int. J. Quantum Inform.*, 1(1):1–23, 2003. 140
- A. P. Veselov. Integrable discrete-time systems and difference operators. *Funct. Anal. Appl.*, 22(2):83–93, 1988. 8
- A. P. Veselov. Integrable Lagrangian correspondences and the factorization of matrix polynomials. *Funct. Anal. Appl.*, 25(2):112–122, 1991. 8
- G. Walsh and S. Sastry. On reorienting linked rigid bodies using internal motions. *IEEE Trans. Robotic Autom.*, 11:139–146, 1993. 143
- A. Weinstein. Lagrangian mechanics and groupoids. In *Mechanics Day (Waterloo, ON, 1992)*, volume 7 of *Fields Institute Communications*, pages 207–231. American Mathematical Society, 1996. 129
- A. Weinstein. Groupoids: unifying internal and external symmetry. A tour through some examples. In *Groupoids in Analysis, Geometry, and Physics (Boulder, CO, 1999)*, volume 282 of *Contemporary Mathematics*, pages 1–19. American Mathematical Society, 2001. 129

- J. M. Wendlandt and J. E. Marsden. Mechanical integrators derived from a discrete variational principle. *Physica D*, 106(3-4):223–246, 1997. 8
- H. Whitney. *Geometric Integration Theory*. Princeton University Press, 1957. 76
- J. Wisdom, S. J. Peale, and F. Mignard. The chaotic rotation of Hyperion. *Icarus*, 58(2):137–152, 1984. 71
- Z. J. Wood. *Computational Topology Algorithms for Discrete 2-Manifolds*. PhD thesis, California Institute of Technology, 2003. 76
- A. Zanna and H. Z. Munthe-Kaas. Generalized polar decompositions for the approximation of the matrix exponential. *SIAM J. Matrix Anal. Appl.*, 23(3):840–862 (electronic), 2001/02. 181

Index

Symbols

$\mathbb{F}L$, *see* Legendre transform
 $\mathbb{F}L_d$, *see* Legendre transform, discrete
 J , *see* momentum map
 J_L , *see* momentum map, Lagrangian
 J_d , *see* momentum map, discrete
 L_d , *see* Lagrangian, discrete
 \hat{R}^μ , *see* Routhian
 $\mathfrak{X}_d(K)$, *see* vector field, primal
 $\mathfrak{X}_d(\star K)$, *see* vector field, dual
 Δ , *see* Laplace–Beltrami
 div , *see* divergence
 $\Omega_d^k(K)$, *see* form, primal
 $\Omega_d^k(\star K)$, *see* form, dual
 $*$, *see* Hodge star
 δ , *see* codifferential
 \flat , *see* flat
 κ , *see* causality sign
 \sharp , *see* sharp
 \star , *see* circumcentric, duality operator
 \wedge , *see* wedge product
 \mathbf{d} , *see* exterior derivative
 \mathbf{i}_X , *see* contraction

A

action
 discrete, 14
 Atiyah sequence, 148
 discrete, *see* discrete Atiyah sequence

augmentation
 example, 114, 117
 one-ring cone, 114

B

boundary, 89
 dual, 91
 example, 90
 bundle
 G -bundle, 145
 adjoint, 148
 base space, 145
 bundle space, 145
 fiber, 145
 fiber bundle, 145
 principal bundle, 145
 projection, 145
 structure group, 145

C

causality sign, 127
 example, 127
 cell
 complex, 84
 example, 84, 85
 chain, 87
 complex, 90
 example, 87
 change of variables, 135

- circumcenter, 79
 - circumcentric
 - subdivision, 79
 - dual, 79
 - duality operator, 79
 - coboundary, 90
 - cochain, 88
 - complex, 90
 - cocone, 109
 - codifferential, 92
 - complex
 - cell, *see* cell, complex
 - chain, *see* chain, complex
 - cochain, *see* cochain, complex
 - prismal, 126
 - simplicial, *see* simplicial, complex
 - sub, *see* subcomplex
 - cone, 109
 - example, 112, 115
 - generalized, 113
 - geometric, 109
 - logical, 112
 - connection, 147
 - 1-form, 148
 - discrete, *see* discrete connection
 - from discrete connection, 176
 - mechanical, 11
 - principal, 10
 - to discrete connection, 181
 - constraints
 - structured, 74
 - contraction
 - algebraic, 105
 - extrusion, 105
 - curvature
 - Levi-Civita connection, *see* connection, Levi-Civita, curvature
- D**
- diffeomorphism, 131
 - groupoid, 133
 - interpolatory methods, 133
 - non-degenerate, 132
 - one-parameter family, 132
 - discrete Atiyah sequence, 154
 - splitting, 165, 167
 - from discrete connection 1-form, 166
 - from discrete horizontal lift, 167
 - to discrete connection 1-form, 166
 - to discrete horizontal lift, 168
 - discrete connection
 - 1-form, *see* discrete connection 1-form
 - computation, 178
 - derived geometric structures, 172
 - exact, 178
 - extended groupoid composition, 173
 - from continuous connection, 181
 - from discrete connection 1-form, 158
 - from discrete horizontal lift, 162
 - geometric control, 186
 - higher-order tangent bundle, 176
 - horizontal component, 171
 - isomorphism, 168
 - Lagrangian reduction, 185
 - Levi-Civita, 188
 - curvature, 189
 - mechanical, 161, 179
 - from discrete Lagrangian, 182
 - order of approximation, 180
 - relating the representations, 156

- to continuous connection, 176
- to discrete connection 1-form, 156
- to discrete horizontal lift, 161
- vertical component, 171
- discrete connection 1-form
 - from discrete connection, 156
 - from discrete horizontal lift, 165
 - from splitting of Atiyah sequence, 166
 - local representation, 160
 - properties, 157
 - to discrete connection, 158
 - to discrete horizontal lift, 163
 - to splitting of Atiyah sequence, 166
- discrete generator, 152
- discrete horizontal lift, 161
 - from discrete connection, 161
 - from discrete connection 1-form, 163
 - from splitting of Atiyah sequence, 168
 - to discrete connection, 162
 - to discrete connection 1-form, 165
 - to splitting of Atiyah sequence, 167
- discrete mechanics, 14, 121, 194
 - groupoid, 129
 - with symmetry, 17
- discrete Riemannian manifold, 188
- divergence, 101
- dual cell
 - circumcentric, *see* circumcentric, dual
 - orientation, *see* orientation, dual cell
- dynamic problems, 129

E

- Euler–Lagrange
 - discrete operator, 185
 - equations, 9

- discrete, 15
- reduced
 - discrete, 186
- exterior derivative, 90
- extrusion, 104
 - contraction, *see* contraction, extrusion
 - example, 104
 - Lie derivative, *see* Lie derivative, extrusion

F

- face, 78
- fiber product, 146
- Filon quadrature, 220
- flat, 93
- flow, 104
 - example, 107
- form
 - dual, 91
 - primal, 88

G

- generating function, 16
- geometric phase
 - rigid-body, 143
- groupoid
 - composition, 130
 - extended, *see* discrete connection, extended groupoid composition
 - diffeomorphism, *see* groupoid, diffeomorphism
 - discrete mechanics, *see* discrete mechanics, groupoid
 - inverse, 131
 - source, 130
 - target, 130

visualizing, 131

H

Hamiltonian, 10

vector field, 10

Harmonic functions, 123

harmonic functions

action functional, 123

Euler–Lagrange, 123

highly-oscillatory integral, 220

Hodge star, 91

Lorentizan space, 127

holonomy, 140

horizontal

component, 153

discrete, *see* discrete connection, horizontal component

directions, 147

lift, 149

discrete, *see* discrete horizontal lift

space, 11

discrete, 153

I

inner product, 122

integrator, 17

L

Lagrange 1-forms

discrete, 15

Lagrange 2-form

discrete, 15

Lagrange map

discrete, 15

push-forward, 16

Lagrange–Poincaré

discrete operator, 185

Lagrangian, 9

discrete, 14, 194

approximate, 17

exact, 16, 194

group-regular, 20

polynomials and quadrature, 37

symplectic Runge–Kutta, 36

double spherical pendulum, 64

satellite, 57

Laplace–Beltrami, 102

Laplace–deRham, 103

lattice theory, 74

Legendre transform, 9

discrete, 16

reduced Routh, 13

Lie derivative

algebraic, 108

extrusion, 107

M

manifolds

flat, 89

non-flat, 85, 89, 137

Maxwell equations, 124

action functional, 125

Euler–Lagrange, 125

metric

local, 86, 190

momentum map, 10

discrete, 17

Lagrangian, 10

momentum shift, 12, 13

multigrid, 136

multiscale

- shape function, 215, 218
- variational integrator, *see* variational integrator, multiscale
- multisymplectic
 - configuration bundle, 195
 - first jet bundle, 195
 - first jet extension, 195
 - geometry, 195
 - variational integrator, *see* variational integrator, multisymplectic
- N**
- natural charts, 205
- Noether's theorem, 10
 - discrete, 18
- norm, 123
 - Lorentzian, 127
- O**
- orientation
 - dual cell, 81
 - dual of dual, 81
 - example, 81, 82
- P**
- pairing
 - natural, 89
- phase space
 - discrete, 14
- Poincaré lemma, 108
 - counterexample, 120
- polytope, 78
- pull-back
 - form, 135
 - vector field, 134
- push-forward
 - form, 135
 - vector field, 134
- R**
- reduction
 - contangent bundle, 42
 - continuous, 9
 - cotangent bundle, 12, 42
 - discrete, 14, 22
 - Euler–Poincaré, 207
 - Hamiltonian, 12
 - Lagrange–Poincaré, 185
 - Lagrangian, 11
 - reconstruction, 13
 - discrete, 19
 - relating discrete and continuous, 34
 - relating Lagrangian and Hamiltonian, 12
 - Routh, 11, 42
 - symplectic Runge–Kutta, *see* symplectic Runge–Kutta, reduction
- remeshing, 136
- Routh equations, 12
 - discrete, 26
 - computational considerations, 69
 - constrained, 46, 47, 56
 - forced, 46, 51, 52, 56
 - preservation of symplectic form, 27
 - double spherical pendulum, 63
 - satellite, 57
- Routhian, 12
 - double spherical pendulum, 67
 - satellite, 59
- S**
- Schrödinger equation, 222
 - action functional, 222

- discrete action, 225
- Euler–Lagrange, 226
- shape space, 10
- sharp, 93
- simplex, 78
 - example, 84, 85
 - examples, 78
- simplicial
 - complex, 78
 - triangulation, 78
- skeleton, 78
- star-shaped
 - generalized, 114
 - logically, 112
 - trivially, 109
- Stokes’ theorem
 - generalized, 90
- subcomplex, 78
- support volume, 83
 - example, 84, 85
- symplectic Runge–Kutta, 35
 - reduction, 38, 42
- symplectic structure
 - canonical, 10
 - reduced, 12
 - discrete, 27

T

- trajectory
 - discrete, 14

V

- variational integrator
 - discrete action, 202
 - Euler–Poincaré, 207
 - function space, 197

- Galerkin, 203
 - generalized, 197
 - higher-order, 198
- Lie group, 202
- multiscale, 215
 - planar pendulum with a stiff spring, 216
- multisymplectic, 196, 199
- numerical quadrature, 197
- pseudospectral, 222
 - Schrödinger equation, 223
- spatio-temporally adaptive, 212
- symplectic-energy-momentum, 200
- variational principle
 - discrete, 194
 - Hamilton’s, 11
 - Lagrange–d’Alembert, 51
 - reduced, 11
- variational problems, 74
 - harmonic functions, *see* harmonic functions
 - Maxwell equations, *see* Maxwell equations
 - Schrödinger equation, *see* Schrödinger equation
- vector field
 - dual, 92
 - primal, 92
- velocity
 - material, 132
 - spatial, 133
- vertex, 78
- vertical
 - component, 154
 - discrete, *see* discrete connection, vertical component
 - directions, 147
 - space, 11

discrete, 152

W

wedge product, 94
dual-dual, 95
anti-commutative, 95
associativity, 98
convergence, 100
example, 94
Leibniz rule, 97
natural, 135
naturality, 135
primal-dual, 122
primal-primal, 94
Whitney sum, 146

Vita

Melvin Leok will join the mathematics department of the University of Michigan, Ann Arbor, in September 2004, as a T.H. Hildebrandt Research Assistant Professor. He received his B.S. *with honors* and M.S. in Mathematics in 2000, and his Ph.D. in Control and Dynamical Systems with a minor in Applied and Computational Mathematics under the direction of Jerrold Marsden in 2004, all from the California Institute of Technology. His primary research interests are in computational geometric mechanics, discrete geometry, and structure-preserving numerical schemes, and particularly how these subjects relate to systems with symmetry and multiscale systems. He was the recipient of the SIAM Student Paper Prize, and the Leslie Fox Prize (second prize) in Numerical Analysis, both in 2003, for his work on *Foundations of Computational Geometric Mechanics*. While a doctoral student at Caltech, he held a Poincaré Fellowship (2000–2004), a Josephine de Kármán Fellowship (2003–2004), an International Fellowship from the Agency for Science, Technology, and Research (2002–2004), a Tau Beta Pi Fellowship (2000–2001), and a Tan Kah Kee Foundation Postgraduate Scholarship (2000). As a Caltech undergraduate, he received the Loke Cheng-Kim Foundation Scholarship (1996–2000), the Carnation Scholarship (1998–2000), the Herbert J. Ryser Scholarship (1999), the E.T. Bell Undergraduate Mathematics Research Prize (1999), and the Jack E. Froehlich Memorial Award (1999).