TURBULENT FLOW IN A NUCLEAR HEAT-EXCHANGER

Thesis by

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ABSTRACT

A gas-cooled, cylindrical, nuclear reactor is used as the basis in the theoretical derivation of the coolant's enthalpy increase and pressure loss across the reactor. Turbulent flow is assumed in the coolant passages of the reactor, and the Reynolds analogy is used as the basis for correlating the heat transfer coefficient and the friction coefficient.

The general equations are derived and two examples, a nuclear-hydrogen rocket and a ramjet, are given to demonstrate applications of the general results.

SUMMARY

The development of nuclear reactors has led to a new class of heat transfer problems. Heat is produced within a solid fuel at a rate that is almost independent of the coolant flow rate, at least for gas-cooled reactors. The power density varies with position in general, although it can be made nearly uniform with the introduction of appropriate reflectors and with variation of fuel concentration. However, these measures are not desirable, particularly if there are weight restrictions for the reflector and cost restrictions on fuel elements.

Aside from considerations of weight and cost, the most important characteristic of a reactor is the ratio of pumping work to the increase of thermodynamic availability of the coolant passing through the reactor. For any single coolant passage through the reactor, this ratio can be determined readily from the Reynolds analogy, for any specified power distribution along the length of the channel, and a given coolant flow rate through the channel or a given pressure drop over the reactor. The limitation on the solid fuel temperature will fix the minimum pressure drop or flow rate. If the power density is the same for all channels, the results are simply summed over the reactor cross-section.

When the power density varies from channel to channel, there is no simple way of calculating pressure drop or temperature rise through the reactor. The pressure drop for all channels must be the same, but the flow rate and temperature rise will vary from channel to channel. Apparently the calculations of temperature and flow rate distributions over the reactor have been done by numerical computation in the past. An iterative procedure is developed in this thesis for

determination of the overall reactor performance in analytical form. The sole restriction on the procedure is that the Mach number of the flow entering any coolant channel be appreciably less than unity. This restriction is not a serious one because the pressure drop resulting from higher entering Mach numbers would be generally unacceptable.

The principal objective of this thesis is the development of the method of calculating reactor performance for any specified power density distribution. In final form the method gives the pressure drop, the temperature distribution and mass flow rate distribution in the coolant leaving the reactor, as well as maximum wall temperatures, where porosity, coolant channel geometry, and power density distribution are prescribed. The temperature of the coolant leaving the reactor can be made uniform over the reactor cross-section by introducing appropriate restrictions in the individual coolant channels. This procedure increases the average temperature rise through the reactor for a given maximum wall temperature, but does so at the cost of increased pressure drop. Whether restriction of the coolant flow channels in this way is advantageous or not will depend upon the particular application. The method of analysis developed here can be used to determine the optimum restriction of the flow channels for any specific example.

To illustrate the application of the method for calculating reactor performance, the nuclear rocket and nuclear ramjet were chosen as examples. The specific impulse for the rocket is found to be higher when the coolant passages are restricted to give a uniform coolant outlet temperature than for no restriction. The thrust coefficient of the ramjet is not improved appreciably by restricting the flow rate.

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SYMBOLS

A	Coolant passage cross-sectional area
a	Radius of reactor
C_f	Friction coefficient
C_h	Heat transfer coefficient (Stanton number)
c_p	Specific heat at constant pressure
$f(\xi)$	Axial power distribution function
$g(\eta)$	Radial power distribution function
k	Conductivity
ℓ	Coolant passage length
$\dot{\mathcal{M}}$	Total mass flow rate through reactor
$\dot{m}$	Mass flow rate per unit area
P	Coolant passage wetted perimeter
p	Gas pressure (static)
q	Power density in reactor; heat produced per unit volume
q_o	Maximum power density in reactor
R	Gas constant
T	Gas temperature (static, averaged over coolant passage cross-section)
u	Gas velocity (averaged over coolant passage cross-section)
α	Porosity; void volume/total volume
β	$C_h \frac{P\ell}{A}$, heat transfer parameter
β_f	$\frac{C_f}{2} \frac{P\ell}{A}$, friction parameter
η	r/a, dimensionless radius from centerline of reactor
μ	Viscosity
ν	Kinematic viscosity
ξ	x/ℓ, dimensionless distance from coolant passage entrance

SYMBOLS

ρ	Gas density (averaged over coolant passage cross-section)
σ	Prandtl number

Subscripts:

o	Entrance to coolant passage
1	Exit from coolant passage
w	Wall of coolant passage
T	Total

I. INTRODUCTION

The power distribution in a nuclear reactor is proportional to the neutron density which, in a bare cylindrical reactor, is highest in the center and decreases both axially and radially away from the center. When the reactor is used as a source of heat, the variation in power density distribution results in an uneven heat supply to the coolant.

The neutron density can be made more uniform by enclosing the reactor with a reflector which causes some of the leaking neutrons to diffuse back into the core, or by varying the fuel loading so that the fuel concentration is higher towards the edges than in the center. However, both of these methods have disadvantages. The reflector material is usually quite bulky and relatively heavy. As a result, in several applications of nuclear reactors as a source of power, specifically, for airborne propulsive applications, it is necessary to restrict the amount of reflector material that might normally be used. Fuel concentration variation is undesirable since reactors are usually constructed by assembling many small lamina, or rods, to obtain the final configuration. For manufacturing ease and interchangeability, it is more desirable to have uniformity of the fuel elements.

The uneven power density distribution of the nuclear reactor results in the coolant having temperature gradients in the exhaust if the mass flow in each of the coolant channels is the same. The coolant in the channel through the center of the core will be heated the most, whereas the flow in the outer channels will be much cooler due to the lower heat generation at the reactor extremities. Flow of this type is usually undesirable, since the surface temperature of the channel walls in the outer channels is much lower than the maximum allowable, and thus the overall efficiency of the power plant is reduced. In addition, the temperature gradients of the exhaust of this type flow lead to mixing losses in the nozzle.

Theoretically, these exhaust gas temperature gradients can be eliminated by restricting the mass flow in the outer channels so that the mass flow in every channel is proportional to the heat release in that channel. The result is that the outer channels, where the heat release is less, are cooled to a lesser degree so that each channel operates at its maximum allowable wall surface temperature, and the temperature of the exhaust gases from the reactor is uniform. This mass flow restriction can be accomplished by varying the flow channel cross-sectional areas continuously across the reactor core so that for the same overall pressure drop across the channel the mass flow is smaller. The same effect can be obtained by installing orifices in the channels to induce a pressure drop so that again the mass flow is proportional to the heat release of the particular channel. Of these two methods, the former is preferable since it utilizes all the available pressure drop to improve the heat transfer coefficient. However, it would not usually be practical to assemble a reactor in this manner, and so the orificing technique is considered to be more practical. Elimination of the exit temperature gradients and increasing the wall surface temperatures of all channels to the maximum allowable by either of these methods increases the overall pressure drop across the reactor and reduces the overall mass flow, in addition to increasing the average temperature rise per unit mass flow. Because of the increased pressure losses and the reduction in mass flow, the overall performance of the reactor must be evaluated to determine the performance gain, if any, by orificing to ascertain if the increased design and assembly complications are warranted.

W. B. Hall (1) has worked out an example of the restricted flow reactor configuration. The basic reactor assumptions made by Hall and those of this thesis are similar. However, the analysis here is more general, including allowance for higher pressure drop and higher velocities in the coolant channels, and a procedure for finding overall performance when the coolant outlet temperature varies radially. These improvements in the analysis are needed for application to reactors for propulsion.

In order to keep the general discussion of heat transfer and pressure drop in gas-cooled reactors for propulsive applications within reasonable limits, simplifying assumptions are required. For this report, the reactor was taken as cylindrical, with radial symmetry in fuel loading and coolant passage distribution. The coolant passages were assumed to be of uniform size, and the cross-sectional area was assumed constant over the length of the reactor. The power density of the reactor was assumed to vary axially and radially according to simple algebraic laws. Since the analysis was made primarily to investigate the result of using a reactor as a type of heat exchanger, or source of heat, reactor physics as such was not considered, and no account was taken of the power distribution shift during operation, etc. The porosity of the reactor was not assumed constant in the derivation of the general equations, but was assumed constant in specific examples worked out to demonstrate the use of the results.

The evaluation of the overall performance of a propulsion system involves determining the overall enthalpy rise of the working fluid and the losses of the system. For a reactor of given design, the controlling factor in operating power is normally some maximum allowable temperature in the system. This limit may be set by the phase stability of the fuel elements, or the moderator, by the allowable thermal stresses in the fuel or some other part of the system, by the influence of temperature on corrosion, or some other particular thermal effect. For a given design and this limiting temperature, the temperature distribution in the reactor core can be determined, and from this, the maximum allowable coolant wall temperature subject to the limiting temperature of the particular configuration. Since this report was intended to be quite general, the controlling factor for reactor operation was selected as the maximum allowable coolant wall temperature which can be determined when given a particular configuration with type of fuel, concentration, etc. Thus the problem to be solved involves calculating the overall enthalpy rise and pressure losses for a given maximum wall temperature and given initial conditions.

The high power requirements of today's rockets and ramjets and the expanding technology in the nuclear reactor field make the possibility of using the gas-cooled reactor as a source of propulsive power for these vehicles a distinct reality. Applications for rocket engines hold promise, since the effective exhaust velocity is proportional to the inverse of the square root of the molecular weight, which suggests the possibility of using low molecular weight gases such as hydrogen for the working fluid. For ramjet applications, the nuclear reactor offers the possibility of relatively unlimited range or endurance at low as well as high altitudes.

In this thesis, the general equations are derived; and two examples, a rocket using hydrogen as the working fluid and a ramjet, are given. The results of these examples show a distinct advantage in orificing to increase the specific impulse of a rocket, but the advantages for a ramjet are so slight that it is doubtful if the additional complexity in reactor construction arising from orificing is warranted.

II. ANALYSIS AND GENERAL SOLUTION

2.1 Reynolds Analogy

In most nuclear reactors the flow of the coolant through the cooling passages is turbulent. For this reason the motion of the fluid cannot be defined exactly, so exact analytical solution of flow and heat transfer problems is impossible. However, using the equations of motion, where the laminar transport properties are replaced by effective values which account for the effects of turbulence and which must be determined experimentally, we are able to solve these problems.

The Reynolds analogy, first postulated by Osborne Reynolds in 1890 and verified by experiment, states that momentum and energy are transferred in the same way in turbulent shear flow. This analogy can be expressed quantitatively by the equation:

$$\frac{q_o}{\rho c_p \, dT/dy} = \frac{\tau_o}{\rho \, du/dy} \tag{2.1}$$

where q_o is the heat flow rate per unit area normal to the surface and τ_o is the shear stress at the wall. For fully developed turbulent flow, the velocity profile normal to the surface is nearly flat; hence, integrating between the wall where u is zero and a station where u and T can be considered to be mean values

$$\frac{q_o}{\rho c_p (\bar{T} - T_w) \bar{u}} = \frac{\tau_o}{\rho \bar{u}^2} \tag{2.2}$$

or

$$C_h = \frac{1}{2} C_f \tag{2.3}$$

where C_h and C_f are the heat transfer coefficient (Stanton number) and friction coefficient respectively.

If we further write the equations of momentum and energy, averaged with respect to time, for mean flow parallel to a surface we find:

$$\tau_o = -\rho \overline{u'v'} + \mu \, du/dy \tag{2.4}$$

$$q_o = -\rho c_p \overline{T'v'} + k \, dT/dy \tag{2.5}$$

where μ is the viscosity and k is the conductivity, and upon re-arrangement to better show the similarity:

$$\tau_o = \rho\left(-\frac{\overline{u'v'}}{du/dy} + \nu\right)\frac{du}{dy}$$
$$q_o = \rho c_p\left(-\frac{\overline{T'v'}}{dT/dy} + \frac{\nu}{\sigma}\right)\frac{dT}{dy} \tag{2.6}$$

where $\nu = \mu/\rho$ is the kinematic viscosity and $\sigma = \frac{\mu c_p}{k}$ is the Prandtl number. Thus, it can be seen that if the Reynolds analogy holds for the turbulent properties, and if $\sigma = 1$, Equation 2.3 holds throughout turbulent and laminar regions.

For most gases $\sigma \hookrightarrow$ 0.7 - 1.0, so the Reynolds analogy can be assumed to hold reasonably well. The analysis and examples below are restricted to gas-cooled reactors with turbulent flow in the coolant channels.

2.2 Temperature Distribution

Due to non-uniformity of the heat generation in the core of the reactor, the average temperature rise of the coolant through the reactor core must be determined to find the overall enthalpy rise through the reactor. The temperature rise in the individual channels must be found first. Integration of each coolant channel temperature rise over the reactor cross-section gives the total enthalpy rise.

The heat balance for a length of coolant passage, $dx = \ell d\xi$, at a given radial station, η, is given by:

$$\dot{m} c_p A \, dT = q_o A \ell f(\xi) \frac{g(\eta)}{\alpha} d\xi \qquad (2.7)$$

where $\dot{m}$ is the mass flow rate per unit area, c_p is the specific heat at constant pressure, A is the cross-sectional area of the coolant passage, T is the static temperature of the coolant, q_o is the power density at the point of maximum heat generation in the reactor core (usually at the center of the core), $\xi = \frac{x}{\ell}$ is the dimensionless coolant passage length measured from the channel entrance, $\eta = \frac{r}{a}$ is the dimensionless radius of the core measured from the center, $f(\xi)$ is the power distribution function in the axial direction, $g(\eta)$ is the function in the radial direction, and α is the porosity, or void fraction, defined as the ratio of the total coolant passage cross-sectional area to the total cross-sectional area. In general, α could be a function of η.

Rewrite Equation 2.7 as:

$$\frac{dT}{d\xi} = \lambda T_o f(\xi) \qquad (2.7a)$$

where $\lambda = \frac{q_o l}{\rho u c_p T_o} \frac{g(\eta)}{\alpha}$ is a dimensionless parameter. Since $\dot{m}$ is constant for a given coolant passage, Equation 2. 7a can be integrated to:

$$T = T_o \left\{ 1 + \lambda(\eta) \int_0^{\xi} f(\xi) d\xi \right\} \tag{2.8}$$

The heat balance between the wall and the coolant is given by the relation:

$$\dot{m} c_p A dT = C_h \rho u c_p (T_w - T) P l d\xi \tag{2.9}$$

where C_h is the Stanton number and depends chiefly upon the Reynolds number for gases with Prandtl number close to unity; ρ is the coolant density, u is the velocity of the coolant, T is the channel surface temperature, and P is the wetted perimeter.

An empirical relation used in flow through pipes of circular cross-section is:

$$C_h = \frac{0.023}{R_e^{0.2} \sigma^{0.6}} \tag{2.10}$$

where R_e is the Reynolds number. The Reynolds number is defined as:

$$R_e = 4 \frac{\rho u A}{\mu P} \tag{2.11}$$

where $4A/P = D_h$, the hydraulic diameter. Since R_e for a constant area channel varies only with μ, which in turn depends only on the temperature and is approximately proportional to the three-fourths power of the absolute temperature, the variation of C_h (and of C_f, the friction coefficient) along the passage will be quite small, particularly for turbulent flow.

Since $\dot{m} = \rho u$, Equation 2.9 can be written as:

$$\frac{dT}{d\xi} = \beta (T_w - T) \tag{2.12}$$

where $\beta = C_h \frac{P}{A}$, is constant, and is a dimensionless parameter proportional to the heat transfer coefficient, C_h. Hence, the wall temperature is given by:

$$T_w = T + \frac{1}{\beta} \frac{dT}{d\xi} \tag{2.13}$$

or, from Equation 2.8:

$$T_w = T_o + \lambda T_o \left\{ \frac{1}{\beta} f(\xi) + \int_o^\xi f(\xi)\, d\xi \right\} \tag{2.14}$$

The maximum wall temperature is given by the condition, $\frac{dTw}{d\xi} = o$, i.e.,

$$\frac{d}{d\xi}\left[f(\xi)\right] + \beta f(\xi) = o \tag{2.14a}$$

From this equation, the value of ξ for the maximum wall temperature can be found, and substituting into Equation 2.14 T_{w_m}, the maximum wall temperature, can be obtained.

2.3 Pressure Distribution

The coolant will undergo a pressure drop as it flows through the channels. This pressure drop arises from two factors; the acceleration of the coolant as it passes through the channels, and the friction drag opposing the flow of the coolant through the passages. The acceleration of the flow results from decreasing density of the coolant, which in turn results from increasing temperature and decreasing pressure. The friction drag cannot be avoided since the Reynolds analogy holds.

The pressure drop along an individual channel can be determined by applying the equations of momentum and continuity. The momentum balance for an element of length of the coolant passage is given by:

$$\frac{dp}{d\xi} + \rho u \frac{du}{d\xi} + \frac{1}{2} C_f \rho u^2 \frac{P\ell}{A} = 0 \qquad (2.15)$$

where p is the static pressure of the coolant and C_f is an empirical friction coefficient. From the continuity and perfect gas relations:

$$\frac{du}{d\xi} = \dot{m} \frac{d}{d\xi}\left(\frac{1}{\rho}\right) = \dot{m} R \frac{d}{d\xi}\left(\frac{T}{p}\right) = \rho_o u_o R \left[\frac{1}{p}\frac{dT}{d\xi} - \frac{T}{p^2}\frac{dp}{d\xi}\right] \qquad (2.16)$$

where R is the gas constant and subscripts, o, refer to the entrance station of the coolant channel. Substituting Equation 2.16 into Equation 2.15 and rearranging:

$$\left[p - \frac{p_o^2 u_o^2}{R T_o^2}\frac{T}{p}\right]\frac{dp}{d\xi} + \frac{p_o^2 u_o^2}{R T_o^2}\left[\frac{dT}{d\xi} + \beta_f T\right] = 0 \qquad (2.17)$$

where $\beta_f = \frac{C_f}{2}\frac{P}{A}$, a dimensionless parameter proportional to C_f, the friction coefficient.

The second term in the coefficient of $dp/d\xi$ in Equation 2.17 is a result of the acceleration due to the pressure drop in the channel. In many applications this term can be neglected and then Equation 2.17 can be integrated exactly; however, in flow through a reactor for propulsion applications, the pressure drop may be quite large, so this term must be retained. The term containing $dT/d\xi$ is a result of acceleration due to the heat input to the coolant as it flows through the passage, and will be quite large for nuclear reactors. Equation 2.17 is an exact equation for the pressure drop, but it cannot be integrated in closed form. However, the equation can be integrated by an iteration process if the Mach number is low.

2.3.1 Iteration Procedure

Introduce a dimensionless parameter, ϵ, defined as:

$$\epsilon = (\dot{m})^2 \frac{RT_o}{p_o^2} = (\rho_o u_o)^2 \frac{RT_o}{(\rho_o RT_o)^2} = \frac{u_o^2}{RT_o} \ll 1$$

The subscripts, o, refer to the channel entrance, and since the product RT_o is proportional to the square of the local speed of sound for perfect gases, ϵ is seen to be γM^2, and thus proportional to the Mach number at the channel entrance. It will be assumed that $\frac{u_1^2}{RT_1} = \left(\frac{p_o}{p_1}\right)^2 \frac{T_1}{T_o}\epsilon \ll 1$ as well.

Equation 2.17 becomes, upon substitution of ϵ,

$$\left[\frac{p}{p_o} - \epsilon \frac{T/T_o}{p/p_o}\right] \frac{d}{d\xi}\left(\frac{p}{p_o}\right) + \epsilon \frac{1}{T_o}\left(\frac{dT}{d\xi} + \beta_f T\right) = 0 \tag{2.18}$$

Let p be expanded in a power series in ϵ:

$$p = p_o + \epsilon p^{(1)} + \epsilon^2 p^{(2)} + \ldots\ldots \tag{2.19}$$

Substituting this series into Equation 2.18, we obtain:

$$\left[1 + \epsilon\left(\frac{p^{(1)}}{p_o} - \frac{T}{T_o}\right) + \ldots\right]\left[\frac{d}{d\xi}\left(\frac{p^{(1)}}{p_o}\right) + \epsilon \frac{d}{d\xi}\left(\frac{p^{(2)}}{p_o}\right) + \ldots\right] \tag{2.20}$$

$$+ \frac{1}{T_o}\left(\frac{dT}{d\xi} + \beta_f T\right) = 0$$

and, upon collecting powers of ϵ, the differential equation for $p^{(1)}$ is found as:

$$\frac{d}{d\xi}\left(\frac{p^{(1)}}{p_o}\right) + \frac{1}{T_o}\left(\frac{dT}{d\xi} + \beta_f T\right) = 0 \tag{2.21}$$

and integrating Equation 2. 21,

$$\frac{p^{(1)}}{p_o} = -\frac{1}{T_o}\int_0^{\xi}\left(\frac{dT}{d\xi} + \beta_f T\right) d\xi \tag{2. 22}$$

For the second approximation, the equation for $p^{(2)}$ is:

$$\frac{d}{d\xi}\left(\frac{p^{(2)}}{p_o}\right) + \left(\frac{p^{(1)}}{p_o} - \frac{T}{T_o}\right)\frac{d}{d\xi}\left(\frac{p^{(1)}}{p_o}\right) = 0 \tag{2. 23}$$

and integrating Equation 2. 23:

$$\frac{p^{(2)}}{p_o} = -\int_0^{\xi}\left(\frac{p^{(1)}}{p_o} - \frac{T}{T_o}\right)\frac{d}{d\xi}\left(\frac{p^{(1)}}{p_o}\right) d\xi \tag{2. 24}$$

Define the static pressure loss as:

$$\Delta p = p_o - \left(p_o + \epsilon p_1^{(1)} + \epsilon^2 p_1^{(2)} + \cdots\cdots\right)$$

where $p_1^{(1)}$ is the first approximation for p at station 1, the reactor exit, and $p_1^{(2)}$ is the second approximation at this station. Therefore, the pressure loss ratio may be expressed as:

$$\begin{aligned}\frac{\Delta p}{p_o} &= -\epsilon\frac{p_1^{(1)}}{p_o} - \epsilon^2\frac{p_1^{(2)}}{p_o} - \cdots\cdots \\ &= +\epsilon\frac{1}{T_o}\int_0^1\left(\frac{dT}{d\xi} + \beta_f T\right) d\xi \\ &\quad + \epsilon^2\int_0^1\frac{1}{T_o^2}\left(\frac{dT}{d\xi} + \beta_f T\right)\left\{\int_0^{\xi}\left(\frac{dT}{d\xi'} + \beta_f T\right)d\xi' + T(\xi)\right\} d\xi \\ &\quad + \cdots\cdots\end{aligned} \tag{2. 25}$$

or

$$\frac{\Delta p}{p_o} = \epsilon F_1 + \epsilon^2 F_2 + \cdots\cdots \tag{2. 26}$$

These functions, F_1 and F_2, may be expressed in a more convenient form by integration and rearrangement to the following:

$$F_1 = \beta_f + \lambda \left[\int_0^1 f(\xi)\, d\xi + \beta_f \int_0^1 (1-\xi)\, f(\xi)\, d\xi \right] \tag{2.27}$$

$$F_2 = \frac{1}{T_0^2} T_1 (T_1 - T_0) + \beta_f \frac{T_1}{T_0^2} \left[T_1 - \int_0^1 \xi \frac{dT}{d\xi}\, d\xi \right]$$
$$+ \frac{\beta_f}{T_0^2} \int_0^1 T (T - T_0)\, d\xi + \frac{\beta_f^2}{T_0^2} \int_0^1 T \left\{ \xi T - \int_0^\xi \xi' \frac{dT}{d\xi'}\, d\xi' \right\} d\xi \tag{2.28}$$

2.4 First Order Approximation

The function, $f(\xi)$, has been defined as the axial power density distribution. Hence, F_1 may be written as:

$$F_1 = \beta_f + \lambda \left[B_0 + \beta_f (B_0 - B_1) \right] \tag{2.29}$$

where B_0 and B_1 are defined as:

$$B_0 = \int_0^1 f(\xi)\, d\xi = \text{const.} \tag{2.30}$$

$$B_1 = \int_0^1 \xi\, f(\xi)\, d\xi = \text{const.} \tag{2.31}$$

If we further require that $f(\xi)$ be symmetrical about the center of the reactor, $\xi = \frac{1}{2}$, integration of B_1 shows that $B_1 = \frac{1}{2} B_0$. Therefore, F_1, under these assumptions becomes:

$$F_1 = \beta_f + \lambda B_0 (1 + \beta_f/2) \tag{2.32}$$

In order to integrate Equation 2.22 to find the first order approximation to the static pressure loss across the reactor, Equation

2.7 can be rewritten as:

$$\frac{dT}{d\xi} = \lambda T_o f(\xi) = K T_o \frac{g(\eta)}{\alpha} \frac{1}{\sqrt{\epsilon}} f(\xi) \qquad (2.33)$$

where K is defined as: $K = \dfrac{g_o \, l(\gamma - 1)}{\gamma \, p_o \sqrt{R T_o}}$

and the parameter, λ, becomes:

$$\lambda = K \frac{g(\eta)}{\alpha} \frac{1}{\sqrt{\epsilon}}$$

The exit temperature of the working fluid as a function of radial position can be determined by integrating Equation 2.33, since the integration is in the axial direction only:

$$T_1(\eta) - T_o = K T_o \frac{g(\eta)}{\alpha} \frac{1}{\sqrt{\epsilon}} B_o \qquad (2.34)$$

Using these results, Equation 2.26 for the first approximation to the pressure loss can be expressed in either of the following equations:

$$\frac{\Delta p}{p_o} = \epsilon F_1 + \cdots$$

$$= \epsilon \beta_f + K \frac{g(\eta)}{\alpha} B_o (1 + \beta_f/_2) \sqrt{\epsilon} + \cdots \qquad (2.35)$$

or, alternatively, using the relation of Equation 2.34:

$$\frac{\Delta p}{p_o} = \epsilon \beta_f + \frac{T_1(\eta) - T_o}{T_o} (1 + \beta_f/_2) \epsilon + \cdots \qquad (2.36)$$

Neither of the two Equations 2.35 and 2.36 is in a convenient form for integration. However, without changing the value of the equation, we may add and subtract the term, $\epsilon \dfrac{\bar{T}_1 - T_o}{T_o} (1 + \beta_f/_2)$, from the right side of the equation. $\dfrac{\bar{T}_1 - T_o}{T_o}$ is at present unknown and is to be determined. Performing this operation, Equation 2.35 becomes:

$$\frac{\Delta p}{p_o} = \epsilon \left[\beta_f + \frac{\bar{T}_1 - T_o}{T_o} (1 + \beta_f/_2)\right] + (1 + \beta_f/_2) \left[K B_o \frac{g}{\alpha} \sqrt{\epsilon} - \frac{\bar{T}_1 - T_o}{T_o} \epsilon\right] \qquad (2.37)$$

For a given reactor under given flow conditions, $\frac{\overline{T}_1 - T_0}{T_0}$ will be a constant. Further, when integrated over the radial area of the reactor, T_1 and $\overline{T}_1$ will not differ to a great degree, so the second term on the right side of Equation 2.37 will now be very small when compared to either of the other two terms.

For convenience, rewrite Equation 2.37 as:

$$\frac{\Delta p}{p_0} = E\left(\sqrt{\epsilon}\right)^2 + \left[C\sqrt{\epsilon} - D\left(\sqrt{\epsilon}\right)^2\right] \tag{2.38}$$

or, rearranging in terms of $\sqrt{\epsilon}$:

$$\left(\sqrt{\epsilon}\right)^2 + \left[\frac{C}{E}\sqrt{\epsilon} - \frac{D}{E}\left(\sqrt{\epsilon}\right)^2\right] - \frac{\Delta p/p_0}{E} = 0 \tag{2.39}$$

where:

$$\left.\begin{aligned} E &= \beta_f + \frac{\overline{T}_1 - T_0}{T_0}\left(1 + \beta_f/2\right) \\ C &= \left(1 + \beta_f/2\right) K B_0 \frac{g(\eta)}{\alpha} \\ D &= \left(1 + \beta_f/2\right)\frac{\overline{T}_1 - T_0}{T_0} = E - \beta_f \end{aligned}\right\} \tag{2.40}$$

Since the middle term in Equation 2.39 has been purposely made very small in comparison with the other two terms, this equation can now be solved by successive approximations. Write Equation 2.39 as:

$$\lim_{\delta \to 1}\left[\left(\sqrt{\epsilon}\right)^2 + \delta\left[\frac{C}{E}\sqrt{\epsilon} - \frac{D}{E}\left(\sqrt{\epsilon}\right)^2\right] - \frac{\Delta p/p_0}{E}\right] = 0 \tag{2.41}$$

and assume a solution of the form:

$$\sqrt{\epsilon} = \lim_{\delta \to 1}\left[a_0\left(1 + b_1\delta + b_2\delta^2 + \ldots\ldots\right)\right] \tag{2.42}$$

After substituting this assumed solution into Equation 2.41, collecting similar powers of δ, and taking the limit as $\delta \to 1$, we obtain:

$$\sqrt{\epsilon} = \sqrt{\frac{\Delta P/P_o}{E}}\left\{1 - \frac{1}{2}\left[\frac{C}{\sqrt{E\Delta P/P_o}} - \frac{D}{E}\right]\right. \tag{2.43}$$

$$\left. + \frac{1}{8}\left[\frac{C^2}{E\,\Delta P/P_o} - \frac{5\,C\,D}{E\sqrt{E\Delta P/P_o}} + 3\left(\frac{D}{E}\right)^2\right] + \cdots\right\}$$

It is more compact to leave Equation 2.43 in terms of C, D, and E, since they are defined in Equation 2.40. For integration in the radial direction, it can be seen that C contains the variable of integration, $g(\eta)$. For the first approximation, use the first term (which does not contain C or D); similarly, for the second approximation to $\sqrt{\epsilon}$ terms linear in C and D only are required. Higher order requires higher powers.

2.5 Average Temperature Rise Through Reactor

The total enthalpy rate of increase, ΔQ, in the exhausting gases is:

$$\begin{aligned} \Delta\dot{Q} &= \dot{m}\, c_p(\bar{T}_1 - T_o) \\ &= c_p(\bar{T}_1 - T_o)\, 2\pi a^2 \int_0^1 \alpha\, \dot{m}\, \eta\, d\eta \\ &= c_p(\bar{T}_1 - T_o)\, 2\pi a^2 p_o \sqrt{RT_o} \int_0^1 \alpha \sqrt{\epsilon}\, \eta\, d\eta \end{aligned} \tag{2.44}$$

where $\dot{m}$ is defined as the total mass flow rate found by integrating the individual coolant mass flow rates over the cross-section of the reactor, the integrated average value of C_p is used, and a is the radius of the reactor.

However, $\Delta\dot{Q}$ may be found directly as:

$$\Delta \dot{Q} = 2\pi a^2 c_p \int_0^1 \dot{m}(\eta)\left[T_1(\eta) - T_0\right]\eta\, d\eta$$

$$= 2\pi a^2 q_0 \ell B_0 \int_0^1 g(\eta)\,\eta\, d\eta \qquad (2.45)$$

Therefore, by equating Equations 2.44 and 2.45 we may solve for the average temperature rise through the reactor:

$$\bar{T}_1 - T_0 = \frac{q_0 \ell B_0}{c_p \rho_0 \sqrt{RT_0}} \; \frac{\int_0 g(\eta)\,\eta\, d\eta}{\int_0^1 \alpha(\eta)\sqrt{\epsilon(\eta)}\,\eta\, d\eta}$$

$$= K T_0 B_0 \frac{\int_0^1 g(\eta)\,\eta\, d\eta}{\int_0^1 \alpha(\eta)\sqrt{\epsilon(\eta)}\,\eta\, d\eta} \qquad (2.46)$$

The exit temperature at the centerline of the reactor is found by putting η equal to zero in Equation 2.34:

$$T_1(0) - T_0 = K T_0 \frac{g(0)}{\alpha} \frac{1}{\sqrt{\epsilon(0)}} B_0 \qquad (2.47)$$

$$= \lambda(0)\, T_0\, B_0 \qquad (2.47a)$$

Hence, dividing Equation 2.47 by 2.46, the ratio between the centerline exit temperature of the reactor and the average exit temperature is:

$$\frac{T_1(0) - T_0}{\bar{T}_1 - T_0} = \frac{g(0)}{\alpha\sqrt{\epsilon(0)}} \; \frac{\int_0^1 \alpha\sqrt{\epsilon}\,\eta\, d\eta}{\int_0^1 g(\eta)\,\eta\, d\eta} \qquad (2.48)$$

It is not convenient to express the relation between the maximum wall temperature and the average temperature increase across the reactor in general. However, if $f(\xi)$ and $g(\eta)$ are known functions, this relation can be found quite readily as will be shown in the following examples.

III. EXAMPLES

3.1 Reactor Calculation Using Specific Power Distribution Functions

Thus far in this thesis the general equations for determining the overall temperature rise and pressure drop through the reactor have been obtained while leaving the specific power distribution as general functions. An example will now be given showing how the results obtained in the general derivation might be applied to a given reactor.

The general solutions for the neutron density distribution in a cylindrical nuclear reactor are given in terms of trigonometric and Bessel functions. However, for convenience, a parabolic neutron density was assumed since the integrated area under a parabola differs but slightly from the more generally accepted solutions of Bessel functions or trigonometric functions. For this example, assume a right-circular cylindrical reactor having parabolic normalized power distribution characteristics as:

$$f(\xi) = 1 - 4\theta\left(\xi - \tfrac{1}{2}\right)^2 \tag{3.1}$$

$$g(\eta) = 1 - \phi\eta^2 \tag{3.2}$$

Further, assume the porosity is uniform across the reactor. The power distribution parameters to be perturbated will be θ and ϕ, which determine the relative "flatness" of the axial and radial power input of the reactor. These parameters are primarily a function of the amount of reflector material used in the reactor design if homogeneous fuel loading is assumed.

With these functions, B_o is found from Equation 2.30 to be:

$$B_o = \int_0^1 f(\xi)\,d\xi = 1 - \theta/3 \qquad (3.3)$$

Using Equation 2.48 and working out the integrals indicated to the second approximation in $\sqrt{\epsilon}$ and with:

$$g(0) = 1$$

$$\sqrt{\epsilon(0)} = \sqrt{\Delta p/p_o}\left\{\frac{(1-\phi/2)\beta_f + \frac{\bar{\bar{T}}_1 - T_o}{T_o}(1+\beta_f/2)(1-\frac{3}{4}\phi)}{(1-\phi/2)\left[\beta_f + \frac{\bar{\bar{T}}_1 - T_o}{T_o}(1+\beta_f/2)\right]^{3/2}}\right\} \qquad (3.4)$$

we find, after simplification, that:

$$\frac{T_1(0) - T_o}{\bar{\bar{T}}_1 - T_o} = \frac{1}{1-\phi/2}\left\{\frac{\beta_f + \frac{\bar{\bar{T}}_1 - T_o}{T_o}(1+\beta_f/2)}{\beta_f + \frac{1}{2}\frac{\bar{\bar{T}}_1 - T_o}{T_o}(1+\beta_f/2)\left(3 - \frac{1}{1-\phi/2}\right)}\right\} \qquad (3.5)$$

To find the maximum wall temperature, which will occur on the centerline of the reactor, use Equation 2.14a to determine the value of ξ to put in Equation 2.14. From Equation 2.14a, T_{w_m} occurs when

$$\xi = \frac{1}{2} - \frac{1}{\beta} + \sqrt{\frac{1}{\beta^2} + \frac{1}{4\theta}}$$

and hence, from Equation 2.14:

$$\frac{T_{w_m} - T_o}{T_o} = \lambda(0)\left[\left(\frac{8\theta}{3\beta^2} + \frac{2}{3}\right)\left(\sqrt{\frac{1}{\beta^2} + \frac{1}{4\theta}} - \frac{1}{\beta}\right) + \frac{2}{3\beta} + \frac{1}{2} - \frac{\theta}{6}\right] \qquad (3.6)$$

and using Equation 2.47a:

$$\frac{T_{w_m} - T_o}{T_1(0) - T_o} = \frac{1}{1-\theta/3}\left[\left(\frac{8\theta}{3\beta^2} + \frac{2}{3}\right)\left(\sqrt{\frac{1}{\beta^2} + \frac{1}{4\theta}} - \frac{1}{\beta}\right) + \frac{2}{3\beta} + \frac{1}{2} - \frac{\theta}{6}\right] \qquad (3.7)$$

Hence, given a maximum wall temperature limitation in the reactor, the relation (as a function of θ, ϕ, β, and β_f) between the average temperature increase of the gas flowing through the reactor may be found by multiplying Equations 3.7 and 3.5:

$$\frac{T_{wm} - T_0}{\overline{T}_1 - T_0} = \frac{1}{(1-\theta/3)(1-\phi/2)} \left\{ \frac{\beta_f + \frac{\overline{T}_1 - T_0}{T_0}(1+\beta_f/2)}{\beta_f + \frac{1}{2}\frac{\overline{T}_1 - T_0}{T_0}\left(1+\frac{\beta_f}{2}\right)\left(3 - \frac{1}{1-\frac{\phi}{2}}\right)} \right\} \left\{ \left(\frac{8\theta}{3\beta^2} + \frac{2}{3} \right) \left(\sqrt{\frac{1}{\beta^2} + \frac{1}{4\theta}} - \frac{1}{\beta} \right) + \frac{2}{3\beta} + \frac{1}{2} - \frac{\theta}{6} \right\} \tag{3.8}$$

It is simpler to leave Equation 3.8 in this form; however, for given values of the parameters of the equation, it reduces to a simple quadratic to determine $\frac{\overline{T}_1 - T_0}{T_0}$ for a given $\frac{T_{wm} - T_0}{T_0}$.

If all wetted surfaces of the channel are heat transfer surfaces, and from the Reynolds analogy, it is reasonable to assume that $\beta = \beta_f$ for fully developed turbulent flow. Furthermore, it is assumed that $\phi = \theta$ for comparison purposes, though neither of these assumptions are requirements of the analysis. The results obtained from using Equation 3.8 are shown in Table I. This table shows the maximum wall temperature required to obtain a temperature increase of 1000°R and 1500°R for an initial gas temperature of 1000°R for various values of β and ϕ. The table was prepared on the assumption that $\phi = \theta$ and $\beta = \beta_f$. The required temperature increases were selected quite arbitrarily, but they furnish an indication of the trend in maximum wall temperature versus average temperature increase through the reactor. From Table I, it can be seen that the maximum wall temperature required for a given temperature increase of the working gas is markedly increased as the power distribution varies away from a linear relation (ϕ equals zero) or when the coefficient of heat, β , gets very low.

3.1.1 No-orifice Condition

The static pressure loss may be found by using Equations 2.44 and 2.45:

$$\dot{m}\, c_p(\bar{T}_1 - T_o) = \Delta\dot{Q}$$
$$= 2\pi a^2 \rho_o \sqrt{RT_o}\, K T_o B_o \int_0^1 g(\eta)\eta\, d\eta \qquad (3.9)$$

or

$$\frac{\dot{m}}{\pi a^2}\frac{\bar{T}_1 - T_o}{T_o} = \rho_o \sqrt{RT_o}\; K\,(1-\theta/_3)(1-\phi/_2)$$

But, from the simplification procedure (not shown) used in determining Equation 3.5, it was found that

$$K = \frac{\alpha}{(1-\theta/_3)(1-\phi/_2)}\,\frac{\bar{T}_1 - T_o}{T_o}\sqrt{\frac{\Delta p/p_o}{\beta_f + \frac{\bar{T}_1 - T_o}{T_o}(1+\beta_f/_2)}}$$

Therefore, Equation 3.9 can be expressed as:

$$\frac{\Delta p}{p_o} = \left\{\beta_f + \frac{\bar{T}_1 - T_o}{T_o}\left(1 + \beta_f/_2\right)\right\}\left(\frac{\dot{m}}{\alpha\pi a^2}\right)^2 \frac{RT_o}{p_o^2} \qquad (3.10)$$

However, $\frac{\dot{m}}{\alpha\pi a^2}$ is just the mass flow rate, $\rho_o u_o$, per channel, so Equation 3.10 can be reduced further to:

$$\frac{\Delta p}{p_o} = \left\{\beta_f + \frac{\bar{T}_1 - T_o}{T_o}\left(1 + \beta_f/_2\right)\right\}\epsilon \qquad (3.10a)$$

Figure 1 shows the variation in static pressure loss with $\sqrt{\epsilon}$ (which is proportional to the mass flow) for various values of β_f for an assumed temperature rise ratio of unity across the reactor.

3.1.2 Artificial Flow Restriction (Orifices)

For installed orifices scaled so that the mass flow in the outer channels is restricted to the correct value to give a uniform temperature rise across the reactor, the average exit temperature will be identical with the exit temperature from the center channel of the unrestricted reactor. Therefore, from Equation 3.7

$$\frac{T_{w_m} - T_0}{\overline{T}_{1\,res.} - T_0} = \frac{1}{(1-\theta/3)}\left[\left(\frac{8\theta}{3\beta^2}+\frac{2}{3}\right)\left(\sqrt{\frac{1}{\beta^2}+\frac{1}{4\theta}}-\frac{1}{\beta}\right)+\frac{2}{3\beta}+\frac{1}{2}-\frac{\theta}{6}\right] \qquad (3.11)$$

If the orifices restrict the flow correctly, the pressure decrement they provide will make all channel pressure losses identical with the pressure loss in the centerline channel, which is unrestricted. Therefore, the first approximation to the pressure loss ratio can be determined from Equation 3.4 by assigning a value to $\sqrt{\epsilon(0)}$ under given reactor conditions.

The other change that will result from artificially restricting the channel entrances will be the total mass flow. This total mass flow can be determined directly using Equation 3.11. Since the average temperature of the gases is the exit temperature, a constant across the reactor, by using Equation 2.34 we are able to determine $\dot{m}$ if it is recalled that it is a factor in the parameter, $\sqrt{\epsilon}$. Hence, integrating, the total mass flow is:

$$\dot{M} = 2\pi a^2 \int_0^1 \alpha \dot{m}\, \eta\, d\eta = 2\pi a^2 \int_0^1 \frac{q_0 l}{C_p T_0} \frac{(1-\theta/3)}{\frac{\overline{T}_{1\,res.}-T_0}{T_0}}(1-\phi\eta^2)\,\eta\, d\eta$$

$$\dot{M} = \pi a^2 \frac{q_0 l}{C_p T_0} \frac{1}{\frac{\overline{T}_{1\,res.}-T_0}{T_0}}(1-\theta/3)(1-\phi/2) \qquad (3.12)$$

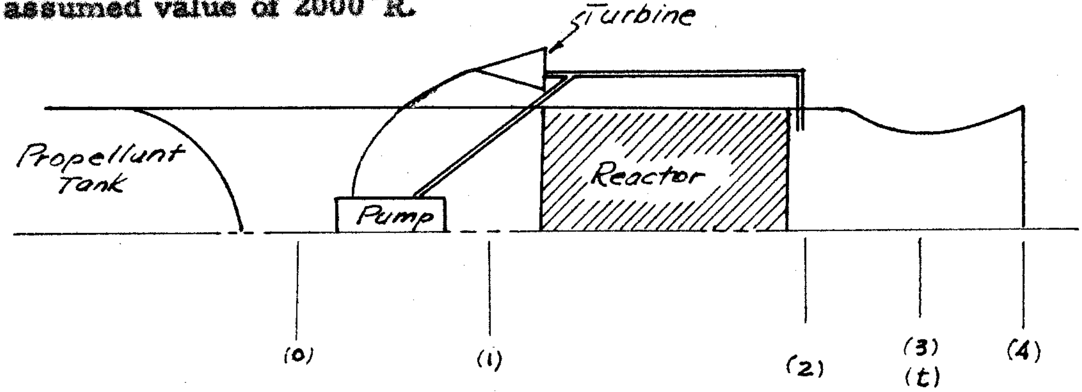
However, since $\overline{T}_{1\,res}$ is the exit temperature of the centerline channel, Equation 3.12 can be rewritten by using Equation 2.47 and the fact that $\dot{m} = \frac{p_0}{RT_0}\sqrt{\epsilon}$:

$$\dot{m} = \alpha \pi a^2 \frac{p_0}{RT_0}\sqrt{\epsilon(0)}\,(1 - \phi/2) \qquad (3.13)$$

3.2 Rocket Example

Using the assumed reactor power distribution characteristics of Equations 3.1 and 3.2, a rocket engine using molecular hydrogen as the working fluid was analyzed. No dissociation effects were considered. The fuel was assumed to be liquified in the propellant tanks; however, due to cooling requirements of the nozzle and reactor reflector, the hydrogen was assumed to have reached a gaseous state before entering the pumps of the "combustion chamber".

Account was taken of the power required to drive the pumps by using a portion of the gas from the reactor exit to drive a turbine which furnished power for the pumps. The temperature of the gases as they were bled from the reactor exit was reduced to acceptable turbine operating temperatures by mixing it with the required amount of coolant hydrogen to lower the turbine gas inlet temperature to an assumed value of 2000°R.



Schematic Drawing of Rocket

For this example, define the following parameters:

$\dot{m}_p$ mass flow rate of pump

$\dot{m}_f$ mass flow rate of nozzle

δ bleed fraction to drive auxiliary turbine, therefore, $\dot{m}_p = \dot{m}_f (1+\delta)$

π_R reactor exit-entry total pressure ratio = p_{2T}/p_{1T}

π_m assumed exit-entry total pressure ratio due to mixing loss from exit temperature gradient = p_{3T}/p_{2T}

π_n nozzle exit-entry total pressure ratio (assumed 1.0)

The thrust, F, of a rocket is given by the equation:

$$F = \dot{m}_f u_4 + A_4 (p_4 - p_a) \tag{3.14}$$

where p_a is the ambient pressure and subscript 4 refers to exit station. The specific impulse, I, is found by the equation:

$$I = \frac{F}{\dot{m}_f g} = \frac{u_4}{g} + \frac{A_4/A_t}{g\, \dot{m}_f/A_t} (p_4 - p_a) \tag{3.15}$$

Using the defined parameters, the nozzle mass flow rate, $\dot{m}_f$, can be found using the continuity equation and the perfect gas relations for a choked throat as:

$$\frac{\dot{m}_f}{A_t} = p_{1T} \pi_R \pi_m \sqrt{\frac{\gamma}{R}} \frac{1}{\sqrt{T_{2T}}} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \tag{3.16}$$

If the expansion ratio, p_{2T}/p_4, is specified, the nozzle-exit ratio can be determined from the following equation if isentropic flow is assumed in the nozzle:

$$\frac{A_4}{A_t} = \frac{\sqrt{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left(\frac{\gamma-1}{2}\right)}}{\left(p_4/p_{4_T}\right)^{1/\gamma}\left\{1-\left(p_4/p_{4_T}\right)^{\frac{\gamma-1}{\gamma}}\right\}^{1/2}} \tag{3.17}$$

where $p_{4T}/p_4 = \pi_m\, p_{2T}/p_4$. Equation 3.17 uniquely defines the area ratio and subsequently the exit Mach number from the relation:

$$\frac{A_4}{A_t} = \frac{1}{M_4}\left\{\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_4^2\right)\right\}^{\frac{\gamma+1}{2(\gamma-1)}} \tag{3.18}$$

Hence, M_4 is now known.

Since M_4 has now been determined, the exit velocity, u_4 , can be found if T_4 is known.

$$T_4 = T_{4_T}\left(1+\frac{\gamma-1}{2}M_4^2\right)^{-1}$$

But,

$$T_{4_T} = T_{2_T} = T_{1_T}\left(1+\frac{T_{2T}-T_{1T}}{T_{1T}}\right)$$
$$= T_{1T}\left[1+\frac{T_2\left(1+\frac{\gamma-1}{2}M_2^2\right)-T_{1T}}{T_{1T}}\right]$$

Therefore;

$$T_{4_T} = T_1\left(1+\frac{\bar{T}_2-T_1}{T_1}\right)\left(1+\frac{\gamma-1}{2}M_2^2\right) \tag{3.19}$$

where $\frac{\bar{T}_2-T_1}{T_1}$ is the static temperature increase found from Equation 3.8, T_1 is assumed equal to T_{1_T} , since the entering Mach number will be very small, and M_2 was determined from Fig. 6, Dailey and Wood (2), modified to account for the friction in the coolant passages.

Knowing T_{4_T} , T_4 is now known, and the exit velocity, u_4, is:

$$u_4 = a_4 M_4 = \sqrt{\gamma R T_4}\, M_4 \tag{3.20}$$

The total pressure ratio across the reactor is found in a similar manner using the perfect gas relations:

$$\pi_R = 1 - \frac{\Delta p_T}{p_1} = \left(1 - \frac{\Delta p}{p_1}\right)\left(1 + \frac{\gamma - 1}{2} M_2^2\right)^{\frac{\gamma}{\gamma - 1}} \tag{3.21}$$

where $\frac{\Delta p}{p_1}$ is the static pressure ratio across the reactor as determined from Equation 3.10a by assigning various values of ϵ. After having determined π_R, the static pressure of the exhaust is given by the relation:

$$p_4 = p_{4_T}\left(1 + \frac{\gamma - 1}{2} M_4^2\right)^{-\frac{\gamma}{\gamma - 1}}$$

$$= p_{1_T}\, \pi_R\, \pi_m \left(1 + \frac{\gamma - 1}{2} M_4^2\right)^{-\frac{\gamma}{\gamma - 1}} \tag{3.22}$$

The total mass flow rate that must be pumped by the pumps is known if ϵ is assigned, since from Equations 3.10 and 3.10a:

$$\frac{\dot m_p}{\pi a^2} = \frac{\alpha}{p_1}\sqrt{\epsilon R T_1} \tag{3.23}$$

In order to pump the required total amount of mass flow, the pumps expend power which was assumed to be supplied by the turbine. Equating the turbine power output to the required pumping power:

$$\frac{p_{1_T}\, \dot m_p}{\rho\, \eta_p} = \frac{c_p\, \Delta T_{turb.}\, \dot m_{turb.}}{\eta_t} \tag{3.24}$$

Therefore, assuming that the mixture of bled gas and coolant gas enters the turbine at 2000°R and exits at the reactor entrance temperature, the amount of bled gas from the reactor exit is given by:

$$\delta \dot m_f = \frac{p_{1_T}\, \dot m_p}{\rho\, c_p}\, \frac{\eta_t}{\eta_p} \tag{3.25}$$

In working out the example, a "rubber" engine was assumed, i.e., the rocket was assumed to be correctly designed for each variation in the parameters. A graphitic reactor is used as the basis for assuming the high maximum wall temperature (Ref. 3) since it was visualized that a rocket engine would not have a long operating life. No account was taken of any possible mass addition to the flow from the reactor material itself.

Assumed conditions

p_1 =	50 atmospheres
p_a =	0.1 atmospheres (53,000 feet)
T_{w_m} =	$5500^\circ R$
$\bar{v}_m$ =	0.90 for no-orifice condition
	1.00 for orifice condition
α =	0.5
T_1 =	$500^\circ R$
p_{2T}/p_4 =	50
$\bar{\gamma}$ =	1.35
$\bar{C}_p$ =	3.8 Btu/lb $^\circ R$

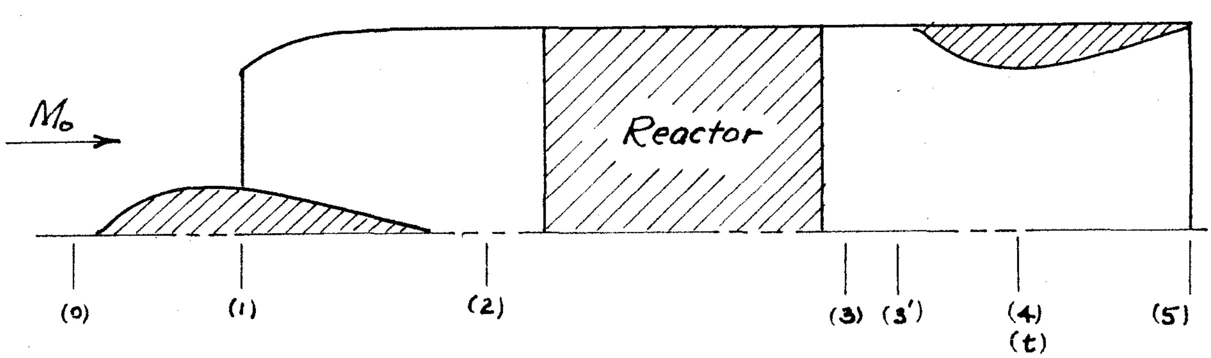
Using these conditions, the exit-throat area ratio is 5.53 and the exit Mach number is 3.17.

Figure 2 shows the specific impulse versus β obtained from the calculations for $\phi = \theta = 0.5$. A linear power distribution (ϕ = zero) is also shown for comparison. No attempt is made to distinguish between the values of mass flow used at each β, since their effect on the specific impulses was negligible, although their values do affect the pressure loss across the reactor. From these results, the specific impulse appears to be primarily a function of the temperature rise imparted to the gas in passing through the reactor. Therefore, to get high specific impulses in a rocket, it would seem preferable to attempt to increase the temperature of the coolant gas by increasing the heat transfer coefficient, although this lowers the allowable mass flow rate to preclude very high pressure losses, or choked flow, in the coolant passages due to the higher heat input to the gases. For this same reason, orificing appears to be advantageous in increasing the specific impulses.

3.3 Ramjet Example

In the ramjet example, the conventional burner is replaced by a nuclear reactor having the power distribution characteristics of Equations 3.1 and 3.2. This analysis was based on standard ICAO conditions at 1000 feet altitude.

In working out the example, a "rubber" engine was assumed and the nozzle was assumed to be designed to expand to atmospheric pressure unless the throat static pressure was lower than ambient. In such a case, no expansion was considered, and the nozzle was assumed to be terminated at the throat.



Schematic Drawing of Ramjet

For this analysis, define the following parameters:

π_r ratio of free-stream total pressure to static = p_{0T}/p_0

π_d ratio of total pressure across diffuser = p_{2T}/p_{0T}

π_R ratio of total pressure across Reactor = p_{3T}/p_{2T}

π_m ratio of total pressures due to assumed mixing loss in non-restricted case = $p_{3'T}/p_{3T}$ (Assumed 1 for restricted case.)

π_n ratio of nozzle total pressures (assumed 1)

The thrust, F, of a ramjet is given by:

$$F = \dot{m} u_0 \left(\frac{u_s}{u_0} - 1 \right) + A_s p_s \left(1 - p_0/p_s \right) \tag{3.26}$$

Due to the fact that ramjet engines must operate for long periods of time, and in an oxidizing atmosphere, a different type reactor material must be used in a ramjet than in a rocket. Because of this, the materials suitable for ramjet applications can withstand a much lower maximum wall temperature than those for a rocket engine. The heat input to the coolant gases will be much less, and consequently the acceleration of the gases in the coolant passages will be less. Therefore, in this example, it was assumed that the static temperature rise and the static pressure loss across the reactor were approximately equal to their total, respectively.

u_s/u_0 can be found by finding the ratios of exit Mach number to the free stream Mach number and exit total temperature to the initial total temperature.

$$\left(\frac{u_s}{u_0}\right)^2 = \left(\frac{a_s M_s}{a_0 M_0}\right)^2 = \frac{T_{sT}\left(1 + \frac{\gamma-1}{2} M_0^2\right)}{T_{0T}\left(1 + \frac{\gamma-1}{2} M_s^2\right)} \left(\frac{M_s}{M_0}\right)^2 \tag{3.27}$$

But:

$$p_{s_T} = p_s\left(1 + \frac{\gamma-1}{2}M_s^2\right)^{\frac{\gamma}{\gamma-1}} = p_o\,\pi_d\,\pi_R\,\pi_m\left(1 + \frac{\gamma-1}{2}M_o^2\right)^{\frac{\gamma}{\gamma-1}} \tag{3.28}$$

and from this we obtain:

$$\left(\frac{M_s}{M_o}\right)^2 = \left(\frac{p_o}{p_s}\,\pi_d\,\pi_R\,\pi_m\right)^{\frac{\gamma-1}{\gamma}} \frac{1 + \frac{\gamma-1}{2}M_o^2}{\frac{\gamma-1}{2}M_o^2} - \frac{1}{\frac{\gamma-1}{2}M_o^2} \tag{3.29}$$

Also:

$$\frac{T_{s_T}}{T_{o_T}} = \frac{T_{3_T}}{T_{o_T}} = 1 + \frac{\Delta T}{T_{o_T}} \tag{3.30}$$

where $\frac{\Delta T}{T_{o_T}} \doteq \frac{\bar{T}_1 - T_o}{T_o}$ of the general reactor analysis.

Therefore, the exit velocity can be expressed as:

$$\frac{u_s}{u_o} = \sqrt{\left(1 + \frac{\Delta T}{T_{o_T}}\right)\left\{1 + \frac{1}{\frac{\gamma-1}{2}M_o^2}\left[1 - \frac{1}{\left(\frac{p_o}{p_s}\,\pi_d\,\pi_R\,\pi_m\right)^{\frac{\gamma-1}{\gamma}}}\right]\right\}} \tag{3.31}$$

Since it was assumed that the static pressure loss and total pressure loss are approximately equal, π_R is found by using Equation 3.10a to determine $\Delta p/p$ for various values of ϵ.

$$\pi_R \doteq 1 - \Delta p/p_{2_T} \tag{3.32}$$

Knowing π_R, the throat pressure can be found from:

$$p_4/p_o = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \pi_n\,\pi_d\,\pi_R\,\pi_m \tag{3.33}$$

If this ratio is less than unity, no further expansion was assumed; if the ratio was greater than unity, ideal expansion to ambient pressure was assumed.

Figures 3 and 4 show the results obtained for C_F (based on the reactor cross-sectional area), the thrust coefficient, versus Mach number for various values of β at an assumed maximum reactor wall temperature of 2960°R. C_F is defined as the total thrust divided by the free stream dynamic pressure and the cross-sectional area of the reactor. Values of π_d used in the example were:

M_o	π_d
1.5 — 2.5	0.85
2.5 — 3.5	0.75
3.5 — 4.5	0.65

π_m, the assumed mixing loss, was arbitrarily assigned a value of 0.90 for the non-restricted example and unity for the orificed example. Results shown are for an assumed reactor porosity of 0.5.

The curves shown are for $\epsilon = 0.03$, but other mass flow rates give similar curves. The one shown is about optimum, since it was found that further increase in mass flow rate increased the pressure loss across the reactor to a point where the overall thrust is reduced. While it should be stressed that the initial conditions and reactor configuration chosen were arbitrary and this example was worked to demonstrate the methods, still, the trend of the results should be realistic, if not their absolute magnitudes. From these two figures at the arbitrarily chosen configuration, optimum performance is obtained at β of approximately 3-5 at a Mach number of about 3. Figure 4 shows that while C_F at the optimum Mach number is slightly increased by orificing, the vehicle performance drops more sharply away from the optimum Mach number, so it is doubtful that there is much advantage to orificing for a ramjet.

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TABLE I

Maximum Wall Temperature Required to Obtain Given Average Temperature Increase of Coolant Gas (Initial gas temperature assumed to be 1000°R)

	$\Delta T = 1000$			$\Delta T = 1500$	
	$\beta = \beta_f$	$T_1(o)$	T_{w_m}	$T_1(o)$	T_{w_m}
$\phi = 1.0$	20	3430	3440	4870	4880
	10	3460	3470	4930	4940
	5	3520	3625	5040	5250
	1	3860	7000	5600	10,630
$\phi = .75$	20	2790	2818	3770	3820
	10	2801	2868	3810	3910
	5	2828	2981	3840	4080
	1	2950	4760	4030	6850
$\phi = .50$	20	2418	2518	3160	3310
	10	2420	2533	3170	3340
	5	2430	2620	3190	3440
	1	2480	3720	3260	5170
$\phi = .25$	20	2173	2450	2771	3190
	10	2175	2452	2772	3190
	5	2177	2490	2779	3250
	1	2193	3190	2801	4315

TABLE I
(continued)

	$\Delta T = 1000$			$\Delta T = 1500$	
	$\beta = \beta_f$	$T_1(o)$	T_{w_m}	$T_1(o)$	T_{w_m}
$\phi = .0$	20	2000	2000	2500	2500
	10	2000	2000	2500	2500
	5	2000	2000	2500	2500
	1	2000	2000	2500	2500

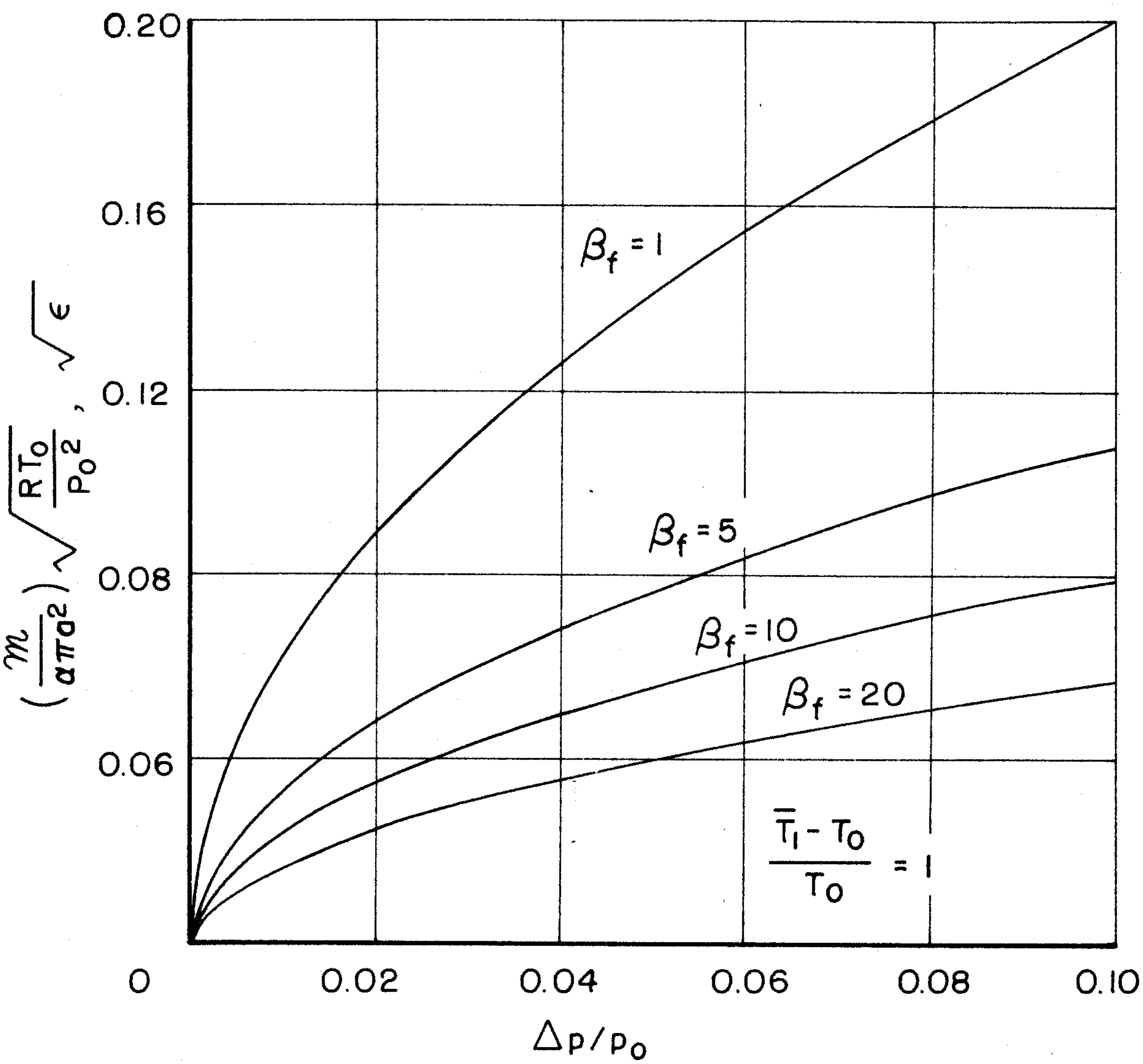

FIG. 1 – STATIC PRESSURE LOSS VS. DIMENSIONLESS MASS FLOW RATE FOR UNRESTRICTED COOLANT PASSAGES

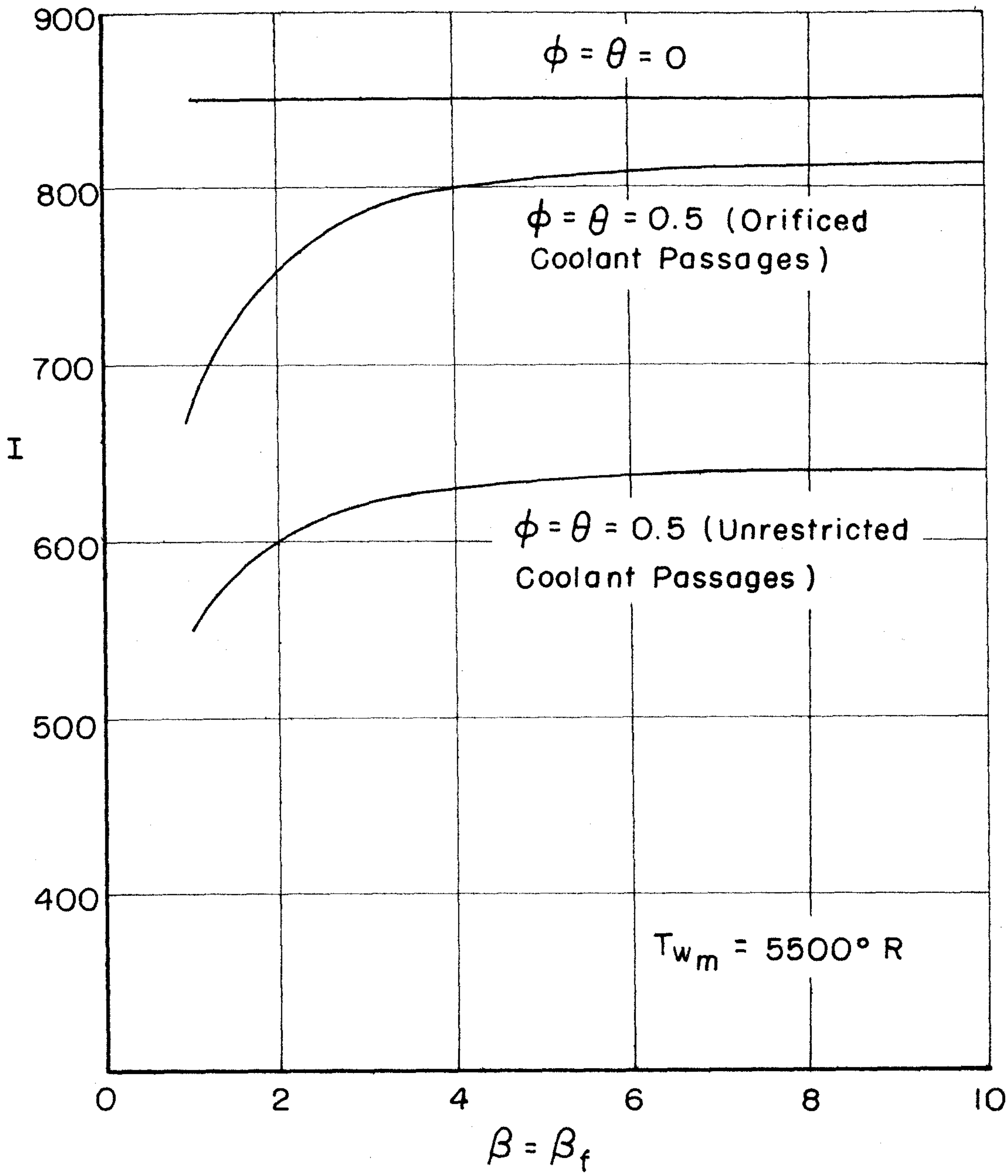

FIG. 2 — SPECIFIC IMPULSE VS. HEAT COEFFICIENT FOR NUCLEAR — HYDROGEN ROCKET EXAMPLE

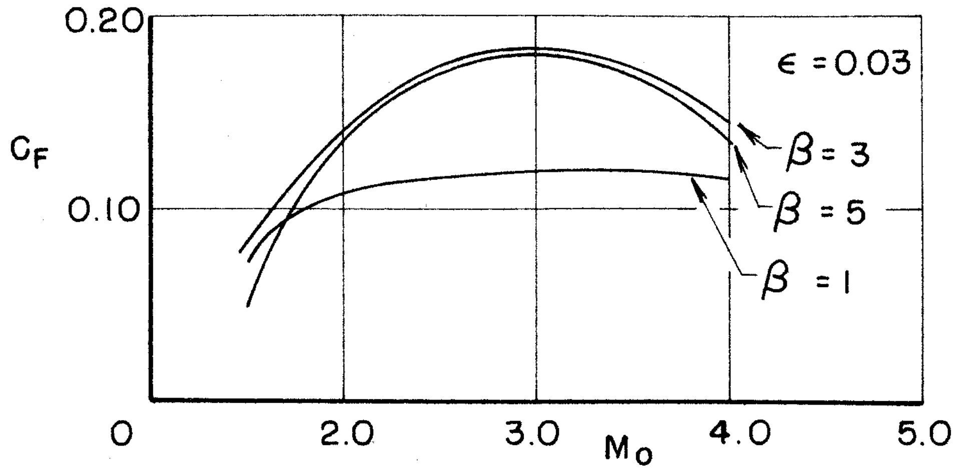

FIG. 3 – RAMJET EXAMPLE (UNRESTRICTED COOLANT PASSAGES) – THRUST COEFFICIENT VS. FREE STREAM MACH NUMBER

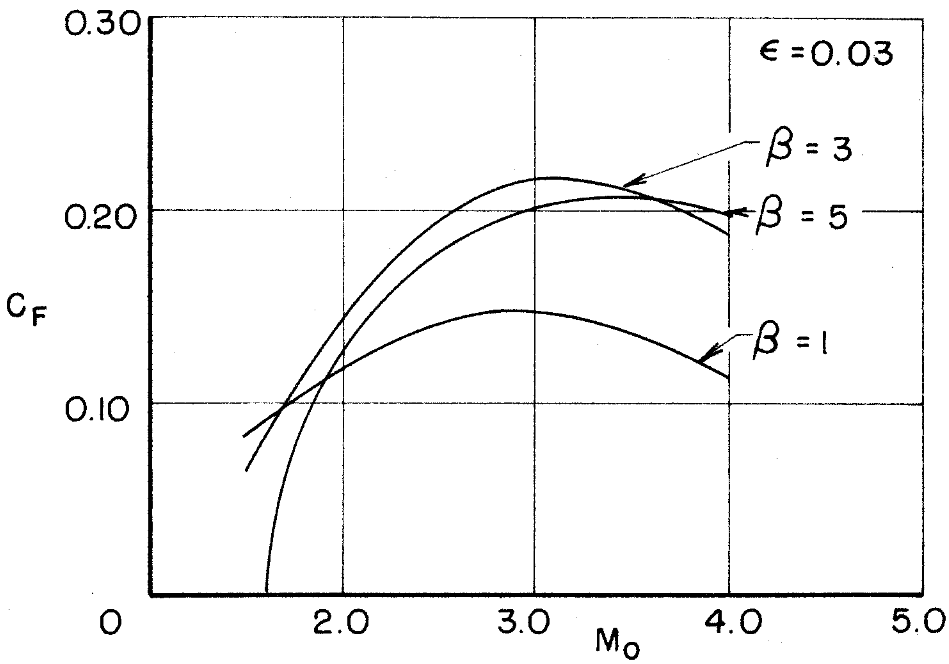

FIG. 4 – RAMJET EXAMPLE (ORIFICED COOLANT PASSAGES) – THRUST COEFFICIENT VS. FREE STREAM MACH NUMBER