ELECTROMAGNETIC WAVE PROPAGATION

AND

RADIATION IN CHIRAL MEDIA

Thesis by

Sassan Bassiri

In Partial Fulfillment of the Requirements

for the Degree of

Doctor of Philosophy

California Institute of Technology

Pasadena, California

1987

(Submitted February 4, 1987)

ACKNOWLEDGEMENTS

I would like to express my deep appreciation and indebtedness to my advisor, Professor Charles Herach Papas, for not only giving me the opportunity to become one of his students but also for his continued guidance, encouragement, and support throughout my graduate years at Caltech. His suggestion of the topic of this thesis, his insightful comments, and his unceasing interest have made this work possible.

I would also like to thank Dr. Nader Engheta for many helpful discussions and suggestions regarding the contents of this thesis. In addition, I would like to thank Dr. Charles Elachi for his support and interest in this work.

ABSTRACT

Propagation and radiation of electromagnetic waves in a lossless, reciprocal, chiral medium is studied in this thesis. Such a medium is described electromagnetically by the constitutive relations $\mathbf{D} = \epsilon\mathbf{E} + i\gamma\mathbf{B}$ and $\mathbf{H} = i\gamma\mathbf{E} + (1/\mu)\mathbf{B}$. The constants ϵ, μ, γ are real and have values that are fixed by the size, shape, and the spatial distribution of the elements that collectively compose the medium. The plane wave propagation in an unbounded chiral medium is considered. The propagation constants are obtained and the polarization properties of electromagnetic waves in such a medium are discussed in detail. The problem of reflection from, and transmission through a semi-infinite chiral medium is solved by obtaining the Fresnel equations. The conditions for the total internal reflection of the incident wave from the interface, and the existance of the Brewster angle are obtained. The effects of the chirality on the polarization and intensity of the reflected wave from the chiral half-space are discussed and illustrated by employing the Stokes parameters. The propagation of electromagnetic waves through an infinite slab of chiral medium is formulated for oblique incidence and solved analytically for the case of normal incidence. The radiation emitted by an oscillating dipole in an unbounded, lossless, chiral medium is calculated. From the constitutive relations and from the time-harmonic Maxwell equations $\nabla \times \mathbf{E} = i\omega\mathbf{B}$ and $\nabla \times \mathbf{H} = \mathbf{J} - i\omega\mathbf{D}$, it is seen that the wave equation for such a medium is given by $\nabla \times \nabla \times \mathbf{E} - \omega^2\mu\epsilon\mathbf{E} - 2\omega\mu\gamma\nabla \times \mathbf{E} = i\omega\mu\mathbf{J}$ where the source term $\mathbf{J}$ is the current density of the oscillating dipole and where $\mathbf{E}$ is the electric vector of the radiated field. The desired solution of this wave equation is found by

the dyadic Green's function method, that is, by first constructing the dyadic Green's function $\mathbf{\Gamma}$ and then evaluating the expression $\mathbf{E} = i\omega\mu \int \mathbf{\Gamma}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{J}(\mathbf{r}')\, dV'$. The dyadic Green's function $\mathbf{\Gamma}$ and the components of the radiated electric field $\mathbf{E}$ are obtained in closed form. The components of the radiated $\mathbf{B}$, $\mathbf{D}$, and $\mathbf{H}$ fields can be derived from knowledge of $\mathbf{E}$ by using the Maxwell equation $\mathbf{B} = (1/i\omega)\nabla \times \mathbf{E}$ and the constitutive relations. The wave impedance of the medium and the radiation resistance of the dipole are also obtained. The effects of the chiral medium on the polarization and intensity of the dipole radiation are discussed.

TABLE OF CONTENTS

CHAPTER I

INTRODUCTION

This report is a theoretical investigation of electromagnetic wave propagation and radiation in chiral media. The motivation for this study, beside its theoretical importance, is provided by its applicability to the vegetation layer problem in remote sensing. Since certain types of terrestrial vegetation layers can be thought of as chiral media, the analytical results of this work provide the necessary tools for analyzing the experimental data.

A chiral medium is a macroscopically continuous medium composed of equivalent chiral objects, uniformly distributed and randomly oriented. A chiral object is a three dimensional body that cannot be brought into congruence with its mirror image by translation and rotation. An object of this sort has the property of handedness and must be either left-handed or right-handed. An object that is not chiral is said to be achiral. Thus all objects are either chiral or achiral. Some chiral objects occur naturally in two versions related to each other as a chiral object and its mirror image. Objects so related are said to be enantiomorphs of each other. If a chiral object is left-handed, its enantiomorph is right-handed, and vice versa. An example of a chiral object is the wire helix; other simple examples are the Möbius strip and the irregular tetrahedron.

When a linearly polarized wave falls normally on a slab of chiral medium two waves are generated in the medium; one is a left-circularly polarized wave and the other, a right-circularly polarized wave of different phase velocity. Behind the slab the two waves combine to yield a linearly polarized wave whose plane of polarization is rotated with respect to the plane of polarization of the incident wave. The amount of rotation depends on the distance travelled in the medium, and this implies that the optical activity occurs not at the surfaces of the slab but throughout the medium. Optical activity in a chiral medium differs from the phenomenon of Faraday rotation in, say, a magnetically biased plasma, by the fact that the former is independent of the direction of propagation whereas the latter is not. The optical activity is invariant under time reversal and the Faraday rotation is invariant under spatial inversion.

The phenomenon of optical activity was first discovered by D. F. Arago [1] in 1811. He found that crystals of quartz rotate the plane of polarization of linearly polarized light which is transmitted in the direction of its optical axis. The experiments of J. B. Biot [2]–[4], dating from 1812 to 1838, on plates of quartz established (i) the dependence of optical activity on the thickness of the plate, (ii) unequal rotation of the plane of polarization of light of different wavelenghts, (iii) the absence of any optical activity when two plates of quartz of the same thickness but opposite handedness were used. In 1815 Biot [5] discovered that optical activity is not restricted to crystalline solids but appears as well in other media such as oils of turpentine, laurel and aqueous solutions of tartaric acid. It was A. Fresnel [6] in 1822 who showed that a ray of light travelling along the axis of a crystal of quartz is resolved into two circularly polarized rays of opposite handedness which travel with unequal phase velocities. He argued that the difference in the two wave velocities is the cause of optical activity. In 1848 L. Pasteur [7] postulated that molecules are three-dimensional figures and that the optical activity of a medium is caused by the chirality of its molecules. More recently, in 1920 and 1922, K. F. Lindman [8], [9] and W. H. Pickering [10] in 1945, devised a macroscopic model for the phenomenon by using microwaves instead of light, and wire spirals instead of chiral molecules.

They illustrated the molecular process responsible for optical activity using this model. Many other experiments were performed, and a very thorough account of them is contained in a book by T. M. Lowry [11].

By the end of the nineteenth century exprimental and empirical facts on optical activity were well established, and physicists had started to develop theories in order to explain the interaction of electromagnetic waves with chiral media. About 1915, M. Born [12], C. W. Oseen [13], and Gray [14] put forward independently and almost simultaneously the explanation of optical activity for a particular molecular model. The molecular model used was that of a spatial distribution of coupled oscillators. W. Kuhn [15] also contributed greatly to the problem by considering the most simple case of the coupled oscillator model to show optical activity. In 1937 E. U. Condon, Altar, and H. Eyring [16] showed that it is possible to explain optical activity by considering a single oscillator moving in a dissymmetric field. A detailed account of these microscopic theories is contained in Condon's paper [17]. Recently, the macroscopic treatment of the subject, which is the theoretical counterpart of Lindman's experiments, was given by C. H. Papas, D. L. Jaggard, and A. R. Mickelson [18]. In their report, the interaction of electromagnetic waves with a collection of randomly oriented short metallic helices of the same handedness were studied, and the optical activity in such media was placed in evidence. The present work is a systematic treatment of Maxwell's equations subject to the constitutive relations for chiral media.

This thesis is divided into five chapters. Chapter II contains the derivation of the constitutive relations for lossless, reciprocal chiral media.

In chapter III, after discussing the plane wave propagation in unbounded chiral media, the interface problem is considered in two parts. First, the problem of reflection from, and transmission through semi-infinite chiral media is solved by obtaining the Fresnel equations. Also the conditions for total internal reflection, and the Brewster angle are obtained. Then the problem of electromagnetic wave propagation through an infinite slab of chiral medium is formulated for oblique incidene, and solved analytically for the case of normal incidence.

Chapter IV contains the derivation of the dyadic Green's function for chiral media. Dipole radiation is then considered and the components of the radiated electric field, the wave impedance of the medium and the radiation resistance of the dipole are obtained. The effects of the chiral medium on the polarization and intensity of the dipole radiation are also discussed. Chapter V concludes this work with a review of the results obtained and the practical implications of these results.

CHAPTER II

CONSTITUTIVE RELATIONS

2.1 Introduction

To recapitulate, a chiral medium is a macroscopically continuous medium composed of equivalent chiral objects, uniformly distributed and randomly oriented. By definition a chiral object does not posses either a center or a plane of symmetry. The simplest of these objects is the idealized wire helix. It consists of a circular loop of wire whose two ends are extended perpendicular to the plane of the loop in opposite directions. According to [18], when an incident electromagnetic wave falls on the helix it induces both electric and magnetic dipole moments. These dipole moments are directed parallel to the axis of the helix. The incident electric field induces currents in the straight portion of the helix, and by continuity must also flow in the circular portion of the helix. The current in the straight portion contributes to the electric dipole moment of the object and the current in the circular portion contributes to its magnetic dipole moment. In a complementary manner, the incident magnetic field induces currents in the circular portion and by continuity in the straight portion. Thus, also the magnetic field contributes to the electric and magnetic dipole moments of the object. From the heuristic argument above, the constitutive relations for the chiral media, for time-harmonic fields ($e^{-i\omega t}$) must have the form

$$\mathbf{P} = \chi_e \epsilon_o \mathbf{E} + \gamma_e \mathbf{B} \tag{2.1}$$

$$\mathbf{M} = -\gamma_m \mathbf{E} + \chi_m (1/\mu_o) \mathbf{B} \, , \tag{2.2}$$

where **P** and **M** are the polarization and magnetization of the medium, and χ_e, χ_m are the electric and magnetic self-susceptibilities, and γ_e, γ_m are the cross-susceptibilities. The permittivity and permeability of the free space are denoted by ϵ_o, μ_o. The polarization vectors [19] are defined by the equations

$$\mathbf{P} = \mathbf{D} - \epsilon_o \mathbf{E} \tag{2.3}$$

$$\mathbf{M} = (1/\mu_o)\mathbf{B} - \mathbf{H} \,. \tag{2.4}$$

Substituting for **P** and **M** in (2.3) and (2.4) from (2.1) and (2.2) it can be shown that

$$\mathbf{D} = \epsilon \mathbf{E} + \gamma_e \mathbf{B} \tag{2.5}$$

$$\mathbf{H} = \gamma_m \mathbf{E} + (1/\mu)\mathbf{B} \tag{2.6}$$

where $\epsilon = \epsilon_o(1 + \chi_e)$ and $\mu = \mu_o/(1 - \chi_m)$, and $\epsilon, \mu, \gamma_e, \gamma_m$ can be complex quantities.

2.2 Lossless Chiral Media

The source free Maxwell's equations for time-harmonic fields are

$$\nabla \times \mathbf{E} = i\omega \mathbf{B} \tag{2.7}$$

$$\nabla \times \mathbf{H} = -i\omega \mathbf{D} \,. \tag{2.8}$$

Upon scalarly multiplying equation (2.7) by $\mathbf{H}^*$, and scalarly multiplying the complex conjugate of equation (2.8) by **E** and then subtracting the resulting equations from one another, it is found that

$$\mathbf{H}^* \cdot \nabla \times \mathbf{E} - \mathbf{E} \cdot \nabla \times \mathbf{H}^* = i\omega(\mathbf{H}^* \cdot \mathbf{B} - \mathbf{E} \cdot \mathbf{D}^*) \,. \tag{2.9}$$

Using the identity

$$\nabla \cdot (\mathbf{U} \times \mathbf{V}) = \mathbf{V} \cdot (\nabla \times \mathbf{U}) - \mathbf{U} \cdot (\nabla \times \mathbf{V}) \,, \tag{2.10}$$

and equation (2.9) it can be shown that

$$\nabla \cdot (\mathbf{E} \times \mathbf{H}^*) = i\omega(\mathbf{H}^* \cdot \mathbf{B} - \mathbf{E} \cdot \mathbf{D}^*) . \tag{2.11}$$

The complex Poynting vector **S** [20] is defined by the equation

$$\mathbf{S} = \frac{1}{2} \mathbf{E} \times \mathbf{H}^* . \tag{2.12}$$

Using (2.12) in (2.11), one obtains

$$-\nabla \cdot \mathbf{S} = \frac{i\omega}{2}(\mathbf{E} \cdot \mathbf{D}^* - \mathbf{H}^* \cdot \mathbf{B}) . \tag{2.13}$$

If the medium is lossless, then the total time-averaged power entering an infinitesmal volume around a point must be zero, *i. e.*, $-Re(\nabla \cdot \mathbf{S}) = 0$. Hence

$$Re\{\frac{i\omega}{2}(\mathbf{E} \cdot \mathbf{D}^* - \mathbf{H}^* \cdot \mathbf{B})\} = 0 . \tag{2.14}$$

Using the constitutive relations (2.5) and (2.6) in (2.14), it follows that when ϵ and μ are real and $\gamma_e = -\gamma_m^*$, then the chiral medium is lossless.

2.3 Reciprocal Chiral Media

Let $\mathbf{E}_1$, $\mathbf{H}_1$ be the electromagnetic field radiated by a current $\mathbf{J}_1$ occupying a finite volume V_1 and let $\mathbf{E}_2$, $\mathbf{H}_2$ be the electromagnetic field radiated by a current $\mathbf{J}_2$ occupying another finite volume V_2. The two source currents oscillate monochromatically at the same frequency and the medium occupying the space outside of V_1 and V_2 is chiral and may be inhomogeneous. Clearly $\mathbf{E}_1$, $\mathbf{H}_1$ are related to $\mathbf{J}_1$, and $\mathbf{E}_2$, $\mathbf{H}_2$ are related to $\mathbf{J}_2$ by the equations $\nabla \times \mathbf{H}_1 = \mathbf{J}_1 - i\omega\mathbf{D}_1$ and $\nabla \times \mathbf{H}_2 = \mathbf{J}_2 - i\omega\mathbf{D}_2$. Upon scalar multiplication of the first equation by $\mathbf{E}_2$ and the second one by $\mathbf{E}_1$, and subtraction of the resulting equations, one obtains

$$\begin{aligned} \mathbf{E}_2 \cdot \nabla \times \mathbf{H}_1 - \mathbf{E}_1 \cdot \nabla \times \mathbf{H}_2 = \mathbf{E}_2 \cdot \mathbf{J}_1 - \mathbf{E}_1 \cdot \mathbf{J}_2 \\ + i\omega(\mathbf{E}_1 \cdot \mathbf{D}_2 - \mathbf{E}_2 \cdot \mathbf{D}_1) . \end{aligned} \tag{2.15}$$

Scalarly multiplying $\nabla\times\mathbf{E}_1 = i\omega\mathbf{B}_1$ and $\nabla\times\mathbf{E}_2 = i\omega\mathbf{B}_2$ by $\mathbf{H}_2$ and $\mathbf{H}_1$, respectively and subtracting the resulting equations, it is found that

$$\mathbf{H}_2 \cdot \nabla \times \mathbf{E}_1 - \mathbf{H}_1 \cdot \nabla \times \mathbf{E}_2 = i\omega(\mathbf{B}_1 \cdot \mathbf{H}_2 - \mathbf{B}_2 \cdot \mathbf{H}_1) \, . \tag{2.16}$$

By first adding (2.15) and (2.16), and then applying identity (2.10) to the sum, it can be shown that

$$\begin{aligned}\nabla \cdot (\mathbf{E}_1 \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_1) &= \mathbf{E}_2 \cdot \mathbf{J}_1 - \mathbf{E}_1 \cdot \mathbf{J}_2 \\ &\quad + i\omega[(\mathbf{E}_1 \cdot \mathbf{D}_2 - \mathbf{E}_2 \cdot \mathbf{D}_1) + (\mathbf{B}_1 \cdot \mathbf{H}_2 - \mathbf{B}_2 \cdot \mathbf{H}_1)] \, .\end{aligned} \tag{2.17}$$

Substituting for $\mathbf{D}_p$ and $\mathbf{H}_p$, where $p = 1, 2$, from the constitutive relations

$$\mathbf{D}_p = \epsilon \mathbf{E}_p + \gamma_e \mathbf{B}_p \tag{2.18}$$

$$\mathbf{H}_p = \gamma_m \mathbf{E}_p + (1/\mu) \mathbf{B}_p \tag{2.19}$$

in the right hand side of (2.17), one obtains

$$\begin{aligned}\nabla \cdot (\mathbf{E}_1 \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_1) &= \mathbf{E}_2 \cdot \mathbf{J}_1 - \mathbf{E}_1 \cdot \mathbf{J}_2 \\ &\quad + i\omega(\gamma_e - \gamma_m)(\mathbf{E}_1 \cdot \mathbf{B}_2 - \mathbf{E}_2 \cdot \mathbf{B}_1) \, .\end{aligned} \tag{2.20}$$

Integrating equation (2.20) throughout all space and converting the left side of the resulting equation to a surface integral using the divergence theorem, the following is obtained

$$\begin{aligned}\int_S (\mathbf{E}_1 \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_1) \cdot \mathbf{n} \, dS &= \int_{V_1} \mathbf{E}_2 \cdot \mathbf{J}_1 \, dV_1 - \int_{V_2} \mathbf{E}_1 \cdot \mathbf{J}_2 \, dV_2 \\ &\quad + i\omega(\gamma_e - \gamma_m) \int_V (\mathbf{E}_1 \cdot \mathbf{B}_2 - \mathbf{E}_2 \cdot \mathbf{B}_1) \, dV\end{aligned} \tag{2.21}$$

where V is the volume containing the sources, S is the surface enclosing V and $\mathbf{n}$ is the unit vector directed along the outward normal to S. It is assumed that sources are confined within a sphere of finite radius. Now, suppose that S is the surface of a sphere which is concentric with the sphere enclosing the sources but whose radius R tends to infinity. The quantities $\mathbf{E}_1 \times \mathbf{H}_2$ and $\mathbf{E}_2 \times \mathbf{H}_1$ will eventually tend to zero as R^{-2} while the area of the surface increases as R^2. Hence

$$\int_S (\mathbf{E}_1 \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_1) \cdot \mathbf{n} \, dS$$

will remain bounded as radius R tends to infinity. However, as the radius increases the fields over any finite part of S must approach more and more nearly a configuration appropriate to outwardly directed plane waves. In a straightforward manner, the plane wave solutions of Maxwell's equations subject to constitutive relations (2.5) and (2.6) can be obtained. Substituting the fields associated with these plane wave solutions into the left hand side of equation (2.21), and using the usual definition of reciprocity [20], which is

$$\int_{V_1} \mathbf{E}_2 \cdot \mathbf{J}_1 \, dV_1 = \int_{V_2} \mathbf{E}_1 \cdot \mathbf{J}_2 \, dV_2 \,, \tag{2.22}$$

it can be shown that when $\gamma_e = \gamma_m = i\gamma$ (γ real) the chiral media is reciprocal.

2.4 Conclusions

From the preceeding discussions, it can be deduced that in the case of a chiral medium composed of lossless, reciprocal, short wire helices, all of the same handedness, the constitutive relations for time-harmonic fields have the form

$$\mathbf{D} = \epsilon \mathbf{E} + i\gamma \mathbf{B} \tag{2.23}$$

$$\mathbf{H} = i\gamma \mathbf{E} + (1/\mu)\mathbf{B} \tag{2.24}$$

where ϵ, μ, γ are real quantities. Moreover, it has been conjectured [18] that (2.23) and (2.24) apply not only to chiral media composed of helicies but also to lossless, reciprocal, isotropic, chiral media composed of chiral objects of arbitrary shape.

Since **D** and **E** are polar vectors and **B** and **H** are axial vectors, it follows that ϵ and μ are true scalars and γ is a pseudo-scalar.

This means that when the axes of a right-handed Cartesian coordinate system are reversed to form a left-handed cartesian coordinate system, γ changes in sign whereas ϵ and μ remain unchanged. Thus the handedness of the medium is manifested by the quantity γ. When $\gamma > 0$, the medium is right-handed and the sense of polarization is right-handed; when $\gamma < 0$, the medium is left-handed and the sense of polarization is left-handed; and when $\gamma = 0$, the medium is simple and there is no optical activity.

Upon substituting **B** by $(1/i\omega)\nabla \times \mathbf{E}$ in (2.25), it is evident that the value of **D** at any given point in space depends not only on the value of **E** at that particular point but also on the behavior of **E** in the vicinity of this point; that is to say, **D** depends also on the derivatives of **E** [21]. This non-local spatial relationship between **D** and **E** is called spatial dispersion. Therefore, the medium described by the constitutive relations (2.23), (2.24) is a spatially dispersive, isotropic, lossless, reciprocal, chiral medium.

CHAPTER III

PLANE WAVES IN CHIRAL MEDIA

3.1 Introduction

In this chapter the plane wave solutions of Maxwell's equations in a source-free homogeneous chiral medium will be examined. Plane waves are not only good approximations to waves at large distances from their sources, but they can also be used to represent complicated waves by utilizing the Fourier integral. Moreover, the concepts of plane wave propagation, reflection, and refraction have many applications in optics, remote sensing and other wave propagation problems.

3.2 Unbounded Chiral Media

Maxwell's equations for source-free regions are

$$\nabla \times \mathbf{E} = i\omega \mathbf{B} \tag{3.1}$$

$$\nabla \times \mathbf{H} = -i\omega \mathbf{D} \tag{3.2}$$

$$\nabla \cdot \mathbf{D} = 0 \tag{3.3}$$

$$\nabla \cdot \mathbf{B} = 0\,. \tag{3.4}$$

Upon substituting in equation (3.2) for **H** and **D** from the constitutive relations

$$\mathbf{D} = \epsilon \mathbf{E} + i\gamma \mathbf{B} \tag{3.5}$$

$$\mathbf{H} = i\gamma \mathbf{E} + (1/\mu)\mathbf{B}\,, \tag{3.6}$$

and then taking the curl of the resulting equation and recalling equation (3.1), the following differential equation for the electric field is obtained

$$\nabla \times \nabla \times \mathbf{E} - k^2\mathbf{E} - 2\omega\mu\gamma\nabla \times \mathbf{E} = 0\,, \tag{3.7}$$

where $k^2 = \omega^2\mu\epsilon$ and ϵ, μ are the permittivity and permeability of the chiral medium, respectively. Using the vector identity $\nabla \times \nabla \times \mathbf{E} = \nabla\nabla \cdot \mathbf{E} - \nabla^2\mathbf{E}$ and noting that $\nabla \cdot \mathbf{E} = 0$, equation (3.7) is reduced to

$$\nabla^2\mathbf{E} + k^2\mathbf{E} + 2\omega\mu\gamma\nabla \times \mathbf{E} = 0\,. \tag{3.8}$$

Since the chiral medium is isotropic, there is no preferred direction of propagation. Without loss of generality, it is assumed that a plane monochromatic wave is propagating along the positive z-axis of a Cartesian coordinate system (x, y, z). The unit vectors of this coordinate system are denoted by $\mathbf{e_x}$, $\mathbf{e_y}$, and $\mathbf{e_z}$. By definition the electric vector of such a plane monochromatic wave has the form

$$\mathbf{E} = \mathbf{E}_o e^{ihz} = (E_{ox}\mathbf{e_x} + E_{oy}\mathbf{e_y} + E_{oz}\mathbf{e_z})e^{ihz} \tag{3.9}$$

where $\mathbf{E}_o$ is a complex-constant amplitude vector, h is the propagation constant, and z is measured along the z-axis. Using equation (3.9) and the fact that $\nabla \cdot \mathbf{E} = 0$, it can be shown that $E_{oz} = 0$. Upon substituting equation (3.9) into (3.8) and noting that $E_{oz} = 0$, the following two homogeneous equations with the two unknowns E_{ox} and E_{oy}, are obtained

$$(k^2 - h^2)E_{ox} - \alpha\, h\, E_{oy} = 0 \tag{3.10}$$

$$\alpha\, h\, E_{ox} + (k^2 - h^2)E_{oy} = 0 \tag{3.11}$$

where $\alpha = 2i\omega\mu\gamma$. This system of equations has a non-trivial solution if and only if

$$(k^2 - h^2)^2 + \alpha^2 h^2 = 0\,. \tag{3.12}$$

The values of h that satisfy (3.12) are the possible propagation constants. They are

$$h = \pm\omega\mu\gamma \pm \sqrt{\omega^2\mu^2\gamma^2 + k^2}\,. \tag{3.13}$$

Since the plane waves are assumed to propagate along the positive z-axis, the permissible values of h are

$$h_1 = \omega\mu\gamma + \sqrt{\omega^2\mu^2\gamma^2 + k^2} \tag{3.14}$$

$$h_2 = -\omega\mu\gamma + \sqrt{\omega^2\mu^2\gamma^2 + k^2}\,. \tag{3.15}$$

Therefore, there are two modes of propagation, one being a wave of phase velocity ω/h_1, and the other a wave of phase velocity ω/h_2. For $\gamma > 0$ the former is the slower mode whereas for $\gamma < 0$ the latter is the slower mode. The electric fields corresponding to these modes are

$$\mathbf{E}_1 = E_{o1}(\mathbf{e_x} + i\mathbf{e_y})e^{ih_1 z} \tag{3.16}$$

which is a right-circularly polarized wave, and

$$\mathbf{E}_2 = E_{o2}(\mathbf{e_x} - i\mathbf{e_y})e^{ih_2 z} \tag{3.17}$$

which is a left-circularly polarized wave. The total electric field $\mathbf{E}$, which is given by

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = \left\{E_{o1}(\mathbf{e_x} + i\mathbf{e_y})e^{ih_1 z} + E_{o2}(\mathbf{e_x} - i\mathbf{e_y})e^{ih_2 z}\right\}, \tag{3.18}$$

is an elliptically polarized wave. The complex-constant amplitudes E_{o1}, E_{o2} can be written as $E_{o1} = \rho_1 e^{i\varphi_1}$ and $E_{o2} = \rho_2 e^{i\varphi_2}$, where their moduli and phases ρ_j and φ_j $(j = 1, 2)$ are real numbers. The length of semi-major axis of the polarization ellipse is $(\rho_1 + \rho_2)$, and that of the semi-minor axis is $|\rho_1 - \rho_2|$. The orientation angle ψ of the ellipse is given by

$$\psi_E = \frac{(h_2 - h_1)}{2} z + \frac{(\varphi_2 - \varphi_1)}{2} = -\omega\mu\gamma\, z + \frac{\varphi_2 - \varphi_1}{2}\,. \tag{3.19}$$

The magnetic field is obtained by substituting for $\mathbf{E}$ from (3.18) into

$$\mathbf{H} = i\gamma\mathbf{E} + (1/\mu)\mathbf{B} = i\gamma\mathbf{E} + (1/i\omega\mu)\nabla \times \mathbf{E}\,, \tag{3.20}$$

and it is found to be

$$\mathbf{H} = -i\sqrt{\gamma^2 + (\epsilon/\mu)}\,\left\{E_{o1}(\mathbf{e_x} + i\mathbf{e_y})e^{ih_1 z} - E_{o2}(\mathbf{e_x} - i\mathbf{e_y})e^{ih_2 z}\right\}. \tag{3.21}$$

The orientation angle ψ for the magnetic field is given by

$$\psi_H = \frac{(h_2 - h_1)}{2}z + \frac{(\varphi_2 - \varphi_1)}{2} - \frac{\pi}{2} = -\omega\mu\gamma\, z + \frac{\varphi_2 - \varphi_1}{2} - \frac{\pi}{2}\,. \tag{3.22}$$

The polarization ellipse of $\mathbf{H}$ is similar and oriented perpendicularly to the polarization ellipse of $\mathbf{E}$. That is, the polarization ellipses of $\mathbf{E}$ and $\mathbf{H}$ are perpendicular to each other and have the same sense of rotation. The sense of rotation being that of the circularly polarized wave of the larger modulus.

Using the notation $\mathbf{E}(t) = Re(\mathbf{E}e^{-i\omega t})$ and $\mathbf{H}(t) = Re(\mathbf{H}e^{-i\omega t})$, it is seen from (3.18) and (3.21) that at any point (x, y, z) and at any time t the vectors $\mathbf{E}(t)$ and $\mathbf{H}(t)$ are perpendicular to each other and to the direction of propagation. Hence, the monochromatic plane waves in a chiral medium are transverse electromagnetic waves. Specifically,

$$\mathbf{e}_z \times \mathbf{E}(t) = \frac{1}{\sqrt{\gamma^2 + (\epsilon/\mu)}}\mathbf{H}(t)\,. \tag{3.23}$$

It follows from this relation that the wave impedance Z of the medium is given by

$$Z = \sqrt{\frac{\mu}{\epsilon}}\frac{1}{\sqrt{1 + (\mu/\epsilon)\gamma^2}}\,. \tag{3.24}$$

Therefore, the wave impedance decreases as the medium becomes more chiral.

The time-averaged power density $\mathbf{P}$ [20] carried by the plane waves, is given by

$$\mathbf{P} = Re\{\mathbf{S}\} = \frac{1}{2}Re\{\mathbf{E} \times \mathbf{H}^*\}\,. \tag{3.25}$$

Upon substituting into (3.25) for **E** and **H**, from (3.18) and (3.21), it is found that

$$\mathbf{P} = \sqrt{\gamma^2 + (\epsilon/\mu)} \left\{ |E_{o1}|^2 + |E_{o2}|^2 \right\} \mathbf{e}_z \,. \tag{3.26}$$

This shows that as the medium becomes more chiral, the time-averaged power density increases.

3.3 The Laws of Reflection and Refraction

When a plane wave falls upon a boundary between a dielectric and a chiral medium it splits into two transmitted waves proceeding into the chiral medium, and a reflected wave propagating back into the dielectric. To mathematically formulate the problem of reflection from and transmission through a semi-infinite chiral medium, a Cartesian coordinate system (x, y, z) is introduced. As shown in figure 3.1, the xy-plane is the plane of interface of a homogeneous dielectric with permittivity ϵ_1 and permeability μ_1, and a homogeneous chiral medium described by the constitutive relations (3.5) and (3.6).

It is assumed that a monochromatic plane wave falls obliquely upon the interface. The complex-constant amplitude vectors of the incident, reflected, and transmitted plane waves always lie on planes perpendicular to the directions of their propagation. Therefore, it is always possible to decompose any one of these amplitudes into a component normal to the plane of incidence, and a second component lying in the plane of incidence (xz-plane), as shown in figure 3.1. The plane of incidence is the plane containing the normal to the interface and the wave vector of the incident wave.

From the boundary conditions, that is, the continuity of tangential electric field and tangential magnetic field at the interface, it can be shown that

$$\mathbf{k}_i \times \mathbf{e}_z = \mathbf{k}_r \times \mathbf{e}_z = \mathbf{h}_1 \times \mathbf{e}_z = \mathbf{h}_2 \times \mathbf{e}_z \tag{3.27}$$

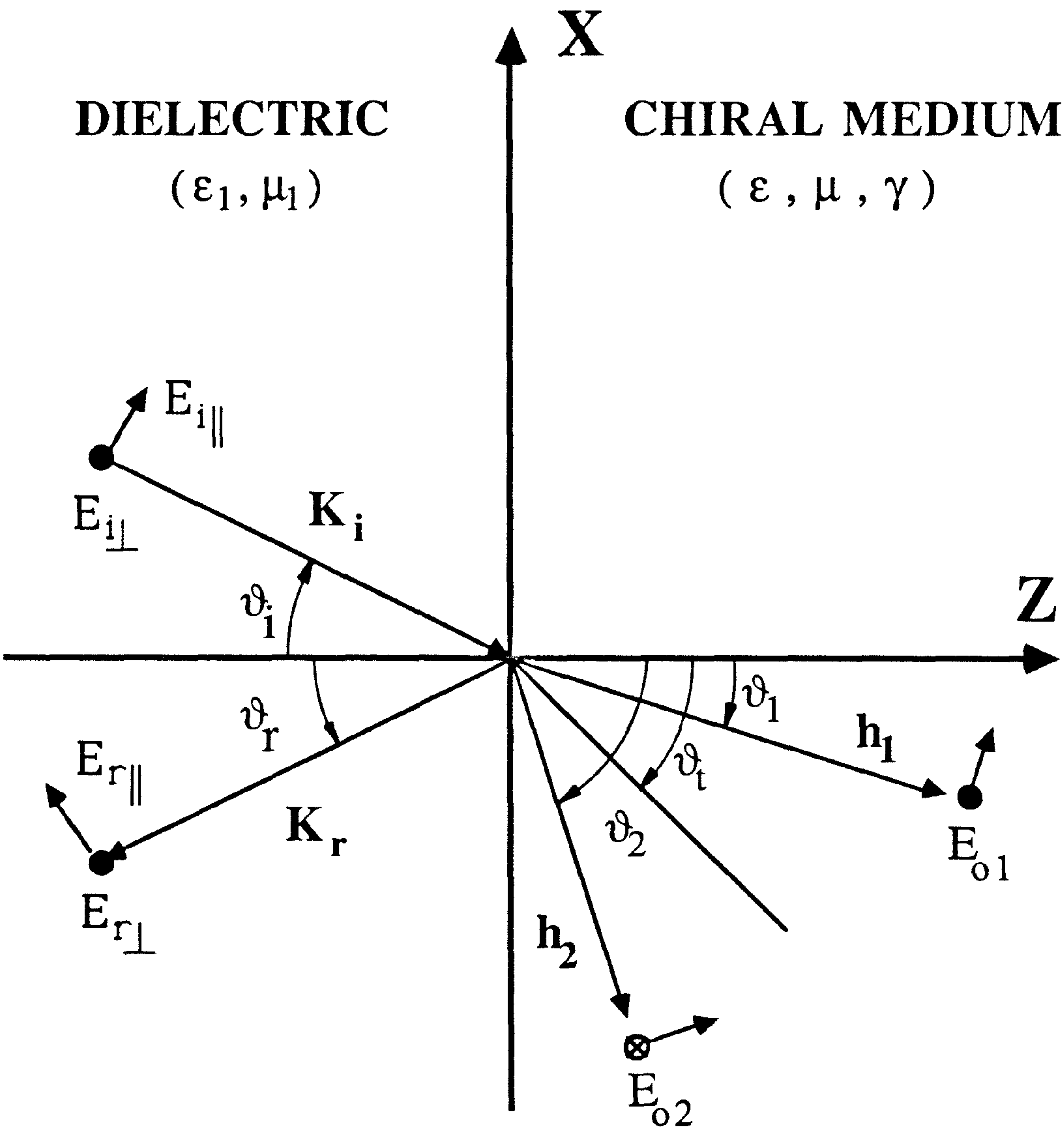

Figure 3.1 *Orientation of the wave vectors of the incident, the reflected, and the transmitted waves at an oblique incidence on a semi-infinite chiral medium.*

where $\mathbf{k}_i, \mathbf{k}_r, \mathbf{h}_1$, and $\mathbf{h}_2$ are the wave vectors of the incident, reflected, and the two transmitted waves, respectively [22]. Upon taking the magnitude of the vector equation (3.27), it is found that

$$k_i \sin\theta_i = k_r \sin\theta_r = h_1 \sin\theta_1 = h_2 \sin\theta_2 \tag{3.28}$$

since $k_i = k_r$, then $\theta_i = \theta_r$. Using equation (3.28), the angles θ_1 and θ_2 corresponding to the two transmitted waves (see fig. 3.1) are found to be

$$\theta_1 = \arcsin\left(\frac{k_i \sin\theta_i}{h_1}\right) \tag{3.29}$$

$$\theta_2 = \arcsin\left(\frac{k_i \sin\theta_i}{h_2}\right) \tag{3.30}$$

where θ_i is the angle of incidence, $k_i = \omega\sqrt{\mu_1\epsilon_1}$, and h_1, h_2 are given by (3.14), (3.15). When $\gamma = 0$, the angle of the transmitted wave is given by

$$\theta_t = \arcsin\left(\frac{k_i \sin\theta_i}{k}\right) \tag{3.31}$$

where θ_t is the refraction angle of a dielectric-dielectric inteface. Therefore, as $\gamma \to 0$, angles θ_1 and θ_2 approach θ_t . As $\gamma \to \infty$, angles θ_1 and θ_2 approach 0 and $\frac{\pi}{2}$, respectively. That is, the h_2-wave will become evanescent and only the h_1-wave will propagate and its direction of propagation is along the positive z-axis. As $\gamma \to -\infty$, angles θ_1 and θ_2 approach $\frac{\pi}{2}$ and 0, respectively. In this case the h_1-wave will become evanescent, and the h_2-wave will propagate along the positive z-axis.

3.4 Total Internal Reflection

In general, there can be two propagating transmitted waves inside the chiral medium, namely the h_1-wave and the h_2-wave. When neither of these waves propagate inside the chiral medium, the phenomena of total internal reflection occur. By letting θ_1 and θ_2 be equal to $\frac{\pi}{2}$ in equations (3.29) and (3.30), the critical angles of incidence θ_{c1} and θ_{c2} can be found. They are

$$\theta_{c1} = \arcsin\left(\frac{\mu\gamma + \sqrt{\mu^2\gamma^2 + \mu\epsilon}}{\sqrt{\mu_1\epsilon_1}}\right) \tag{3.32}$$

$$\theta_{c2} = \arcsin\left(\frac{-\mu\gamma + \sqrt{\mu^2\gamma^2 + \mu\epsilon}}{\sqrt{\mu_1\epsilon_1}}\right). \tag{3.33}$$

When $\gamma > 0$ and $h_2 < h_1 < k_i$, then θ_{c1} is always greater than θ_{c2}. There are three possibilities:

(i) If $\theta_i < \theta_{c2} < \theta_{c1}$, then both h_1 and h_2-waves will propagate. Their direction of propagation is given by (3.29) and (3.30).

(ii) If $\theta_{c2} \leq \theta_i < \theta_{c1}$, then only h_1-wave will propagate and h_2-wave will become evanescent. The direction of propagation for h_1-wave is given by equation (3.29).

(iii) If $\theta_{c2} < \theta_{c1} \leq \theta_i$, then neither of the waves will propagate and there will be total internal reflection of the two waves into the dielectric.

When $\gamma < 0$ and $h_1 < h_2 < k_i$, then θ_{c1} is always less than θ_{c2}. There are three possibilities:

(i) If $\theta_i < \theta_{c1} < \theta_{c2}$, then both h_1 and h_2-waves will propagate. The direction of propagation of these waves is given by (3.29) and (3.30).

(ii) If $\theta_{c1} \leq \theta_i < \theta_{c2}$, then only h_2-wave will propagate and h_1-wave will become evanescent. The direction of propagation for h_2-wave is given by equation (3.30).

(iii) If $\theta_{c1} < \theta_{c2} \leq \theta_i$, then none of the waves will propagate and the phenomena of total internal reflection occur.

Hence, depending on the incidence angle, one, both or none of the transmitted waves will propagate inside the chiral medium.

3.5 The Fresnel Equations

In order to study the power carried by the reflected and the transmitted waves, and also the polarization properties of these waves, it is necessary to determine the complex-constant amplitude vectors associated with these waves. This is done by matching the fields at the interface using the boundary conditions [20]

$$(\mathbf{E}_{oi} + \mathbf{E}_{or}) \times \mathbf{e}_z = (\mathbf{E}_{o1} + \mathbf{E}_{o2}) \times \mathbf{e}_z \tag{3.34}$$

$$(\mathbf{H}_{oi} + \mathbf{H}_{or}) \times \mathbf{e}_z = (\mathbf{H}_{o1} + \mathbf{H}_{o2}) \times \mathbf{e}_z \, , \tag{3.35}$$

where $\mathbf{E}_{oi}$, $\mathbf{E}_{or}$ are the complex-constant amplitudes of the incident and the reflected electric fields, respectively. Similarly, $\mathbf{E}_{o1}$ and $\mathbf{E}_{o2}$ are the amplitudes of the electric fields associated with the right-circularly and the left-circularly polarized transmitted waves, respectively. The incident electric and magnetic fields can be written as (see fig. 3.1)

$$\mathbf{E}_i = \mathbf{E}_{oi} e^{ik_i(z\cos\theta_i - x\sin\theta_i)} \tag{3.36}$$

$$\mathbf{H}_i = \mathbf{H}_{oi} e^{ik_i(z\cos\theta_i - x\sin\theta_i)} \tag{3.37}$$

where

$$\mathbf{E}_{oi} = E_{i\perp}\mathbf{e}_y + E_{i\parallel}(\cos\theta_i \mathbf{e}_x + \sin\theta_i \mathbf{e}_z) \tag{3.38}$$

$$\mathbf{H}_{oi} = \eta_1^{-1}\left[E_{i\parallel}\mathbf{e}_y - E_{i\perp}(\cos\theta_i \mathbf{e}_x + \sin\theta_i \mathbf{e}_z)\right] , \tag{3.39}$$

and $\eta_1 = \sqrt{\mu_1/\epsilon_1}$. The reflected fields may be written as (see fig. 3.1)

$$\mathbf{E}_r = \mathbf{E}_{or} e^{-ik_i(z\cos\theta_i + x\sin\theta_i)} \tag{3.40}$$

$$\mathbf{H}_r = \mathbf{H}_{or} e^{-ik_i(z\cos\theta_i + x\sin\theta_i)} \tag{3.41}$$

where

$$\mathbf{E}_{or} = E_{r\perp}\mathbf{e_y} + E_{r\|}(\cos\theta_i\mathbf{e_x} - \sin\theta_i\mathbf{e_z}) \tag{3.42}$$

$$\mathbf{H}_{or} = \eta_1^{-1}\left[-E_{r\|}\mathbf{e_y} + E_{r\perp}(\cos\theta_i\mathbf{e_x} - \sin\theta_i\mathbf{e_z})\right] . \tag{3.43}$$

The subscript "$\perp$"refers to the amplitude of the field component perpendicular to the plane of incidence, and the subscript "$\|$ "refers to the amplitude of the field component which lies in the plane of incidence. Since the two transmitted waves are circularly polarized, they can be written as

$$\mathbf{E}_t = \mathbf{E}_{o1}e^{ih_1(z\cos\theta_1 - x\sin\theta_1)} + \mathbf{E}_{o2}e^{ih_2(z\cos\theta_2 - x\sin\theta_2)} \tag{3.44}$$

$$\mathbf{H}_t = \mathbf{H}_{o1}e^{ih_1(z\cos\theta_1 - x\sin\theta_1)} + \mathbf{H}_{o2}e^{ih_2(z\cos\theta_2 - x\sin\theta_2)} \tag{3.45}$$

where

$$\mathbf{E}_{o1} = E_{o1}(\cos\theta_1\mathbf{e_x} + \sin\theta_1\mathbf{e_z} + i\mathbf{e_y}) \tag{3.46}$$

$$\mathbf{H}_{o1} = -iZ^{-1}E_{o1}(\cos\theta_1\mathbf{e_x} + \sin\theta_1\mathbf{e_z} + i\mathbf{e_y}) , \tag{3.47}$$

and

$$\mathbf{E}_{o2} = E_{o2}(\cos\theta_2\mathbf{e_x} + \sin\theta_2\mathbf{e_z} - i\mathbf{e_y}) \tag{3.48}$$

$$\mathbf{H}_{o2} = iZ^{-1}E_{o2}(\cos\theta_2\mathbf{e_x} + \sin\theta_2\mathbf{e_z} - i\mathbf{e_y}) , \tag{3.49}$$

and the wave impedance of the medium Z, is defined by equation (3.24).

It is assumed that the amplitude, polarization, direction of propagation, and frequency of the incident field is known. To find the complex-constant amplitude vectors of the reflected and transmitted waves, the boundary conditions at the interface must be applied to the x- and y-components of the electric and magnetic fields. Upon substituting into (3.34) and (3.35) from (3.36)–(3.48), a system of four non-homogeneous equations with the four unknowns $E_{r\perp}$, $E_{r\|}$, E_{o1}, and E_{o2}

is obtained. The expressions for these quantities in terms of the components of the incident wave can be written as

$$\begin{pmatrix} E_{r\perp} \\ E_{r\|} \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} \begin{pmatrix} E_{i\perp} \\ E_{i\|} \end{pmatrix} \tag{3.50}$$

where the 2×2 matrix is the reflection coefficient matrix, and its entries are

$$R_{11} = \frac{\cos\theta_i(1-g^2)(\cos\theta_1+\cos\theta_2) + 2g(\cos^2\theta_i - \cos\theta_1\cos\theta_2)}{\cos\theta_i(1+g^2)(\cos\theta_1+\cos\theta_2) + 2g(\cos^2\theta_i + \cos\theta_1\cos\theta_2)} \tag{3.51}$$

$$R_{12} = \frac{-2ig\cos\theta_i(\cos\theta_1 - \cos\theta_2)}{\cos\theta_i(1+g^2)(\cos\theta_1+\cos\theta_2) + 2g(\cos^2\theta_i + \cos\theta_1\cos\theta_2)} \tag{3.52}$$

$$R_{21} = \frac{-2ig\cos\theta_i(\cos\theta_1 - \cos\theta_2)}{\cos\theta_i(1+g^2)(\cos\theta_1+\cos\theta_2) + 2g(\cos^2\theta_i + \cos\theta_1\cos\theta_2)} \tag{3.53}$$

$$R_{22} = \frac{\cos\theta_i(1-g^2)(\cos\theta_1+\cos\theta_2) - 2g(\cos^2\theta_i - \cos\theta_1\cos\theta_2)}{\cos\theta_i(1+g^2)(\cos\theta_1+\cos\theta_2) + 2g(\cos^2\theta_i + \cos\theta_1\cos\theta_2)} \tag{3.54}$$

where $g = \eta_1 Z^{-1} = \sqrt{(\mu_1/\epsilon_1)\gamma^2 + (\mu_1\epsilon/\epsilon_1\mu)}$. Comparing (3.52) with (3.53) it is evident that the cross-reflection coefficient R_{12} and R_{21} are equal. This is a consequence of the reciprocity principle. When the incident wave falls normally on the interface, that is, $\theta_i = 0$, expressions (3.51)–(3.54) reduce to

$$R_{11} = R_{22} = \frac{1-g}{1+g} \tag{3.55}$$

$$R_{12} = R_{21} = 0\,. \tag{3.56}$$

Similarly, the results for the amplitudes of the transmitted wave can be expressed as

$$\begin{pmatrix} E_{o1} \\ E_{o2} \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} E_{i\perp} \\ E_{i\|} \end{pmatrix} \tag{3.57}$$

where the 2×2 matrix is the transmission coefficient matrix, and its entries are

$$T_{11} = \frac{-2i\cos\theta_i(g\cos\theta_i + \cos\theta_2)}{\cos\theta_i(1+g^2)(\cos\theta_1 + \cos\theta_2) + 2g(\cos^2\theta_i + \cos\theta_1\cos\theta_2)} \tag{3.58}$$

$$T_{12} = \frac{2\cos\theta_i(\cos\theta_i + g\cos\theta_2)}{\cos\theta_i(1+g^2)(\cos\theta_1 + \cos\theta_2) + 2g(\cos^2\theta_i + \cos\theta_1\cos\theta_2)} \tag{3.59}$$

$$T_{21} = \frac{2i\cos\theta_i(g\cos\theta_i + \cos\theta_1)}{\cos\theta_i(1+g^2)(\cos\theta_1 + \cos\theta_2) + 2g(\cos^2\theta_i + \cos\theta_1\cos\theta_2)} \tag{3.60}$$

$$T_{22} = \frac{2\cos\theta_i(\cos\theta_i + g\cos\theta_1)}{\cos\theta_i(1+g^2)(\cos\theta_1 + \cos\theta_2) + 2g(\cos^2\theta_i + \cos\theta_1\cos\theta_2)} \cdot \tag{3.61}$$

When $\theta_i = 0$, the incident wave is normal to the interface, and the expressions (3.58)–(3.61) reduce to

$$T_{11} = -iT_{22} = \frac{-i}{1+g} \tag{3.62}$$

$$T_{12} = iT_{21} = \frac{1}{1+g} \cdot \tag{3.63}$$

3.6 The Brewster Angle

Under a certain condition a monochromatic plane wave upon reflection from a chiral medium becomes a linearly polarized wave. The angle of incidence which satisfies this condition is called the Brewster angle.

The plane containing the electric field vector and the direction of propagation is called the plane of polarization. For a linearly polarized wave, the angle between the plane of polarization and the plane of incidence is called the azimuthal angle. This angle lies in the range $-\pi/2$ and $\pi/2$. It is defined to be positive, whenever the sense of rotation of the plane of polarization toward the plane of incidence, and the direction of wave propagation form a right-handed screw. Let α_i and α_r be the azimuthal angles of the incident and the reflected waves respectively. From (3.38), (3.42), (3.46), and (3.48) it is evident that

$$\tan\alpha_i = \frac{E_{i\perp}}{E_{i\|}} \tag{3.64}$$

$$\tan\alpha_r = \frac{E_{r\perp}}{E_{r\|}}\,. \tag{3.65}$$

The amplitudes of the perpendicular and the parallel components of the incident and the reflected waves are related by the matrix equation (3.50). Using (3.64), (3.65) and (3.50), it can be shown that

$$\tan\alpha_r = \frac{R_{12} + R_{11}\tan\alpha_i}{R_{22} + R_{21}\tan\alpha_i}\,. \tag{3.66}$$

If the incident wave falls on the interface at Brewster's angle θ_B, then the reflected wave must be linearly polarized. Therfore, α_r must be a constant for all α_i, at this angle [23]. Upon differentiating equation (3.66) with respect to α_i, the following condition is obtained

$$R_{11}R_{22} - R_{12}R_{21} = 0\,. \tag{3.67}$$

Under this condition equation (3.66) becomes

$$\tan\alpha_r = \frac{R_{12}}{R_{22}} = \frac{R_{11}}{R_{21}}\,. \tag{3.68}$$

Upon substituting in (3.67) from (3.51)–(3.54), the following equation is obtained

$$(1-g^2)^2\cos^2\theta_i(\cos\theta_1 + \cos\theta_2)^2 = 4g^2(\cos^2\theta_i - \cos^2\theta_1)(\cos^2\theta_i - \cos^2\theta_2) \tag{3.69}$$

where θ_1 and θ_2 can be written in term of θ_i, using equations (3.29) and (3.30). The incidence angle that satisfies the transcendental equation (3.69) is the Brewster angle, and it can be solved for, by using standard numerical techniques.

3.7 The Stokes Parameters

A plane monochromatic TEM wave of the form

$$\mathbf{E} = (E_\perp\mathbf{e}_\perp + E_\|\mathbf{e}_\|)e^{i\mathbf{k}\cdot\mathbf{r}}e^{-i\omega t} \tag{3.70}$$

is by its very nature, elliptically polarized. The Stokes parameters [20], which are defined by

$$S_0 = E_\perp E_\perp^* + E_\| E_\|^* \tag{3.71}$$

$$S_1 = E_\perp E_\perp^* - E_\| E_\|^* \tag{3.72}$$

$$S_2 = 2Re\{E_\perp E_\|^*\} \tag{3.73}$$

$$S_3 = 2Im\{E_\perp E_\|^*\}\,, \tag{3.74}$$

describe completely the state of polarization of the wave. The power carried by the wave is proportional to S_0, and the orientation angle ψ of the polarization ellipse, shown in figure 3.2, is given by

$$\tan 2\psi = \frac{S_2}{S_1}\,, \qquad (0 \le \psi \le \pi) \tag{3.75}$$

and the ellipticity angle χ, is given by (see fig. 3.2)

$$\tan\chi = \frac{(S_3/S_0)}{1+\sqrt{1-(S_3/S_0)^2}}\,. \qquad (-\pi/4 \le \chi \le \pi/4) \tag{3.76}$$

The numerical value of $\tan\chi$ yields the reciprocal of the axial ratio a_o/b_o of the ellipse, where a_o and b_o are the semi-major and semi-minor axes of the ellipse, respectively. The sign of χ differentiates the two senses of polarization, *e.g.*, for left-handed polarization $0 < \chi \le \pi/4$ and for right-handed polarization $-\pi/4 \le \chi < 0$.

The power carried by the reflected wave can be found by substituting into (3.71), the expressions for $E_{r\perp}$ and $E_{r\|}$. These expresions can be found using equations (3.50)–(3.54). Since the analytical expression for the power involves many parameters and is not very informative, only the graphs of the reflected power versus the angle of incidence are given. If the incident electric field is in the plane of incidence, then the normalized reflected power is denoted by $P_\|$, and if it is perpendicular to the plane of incidence, then the normalized reflected power is

POLARIZATION ELLIPSE

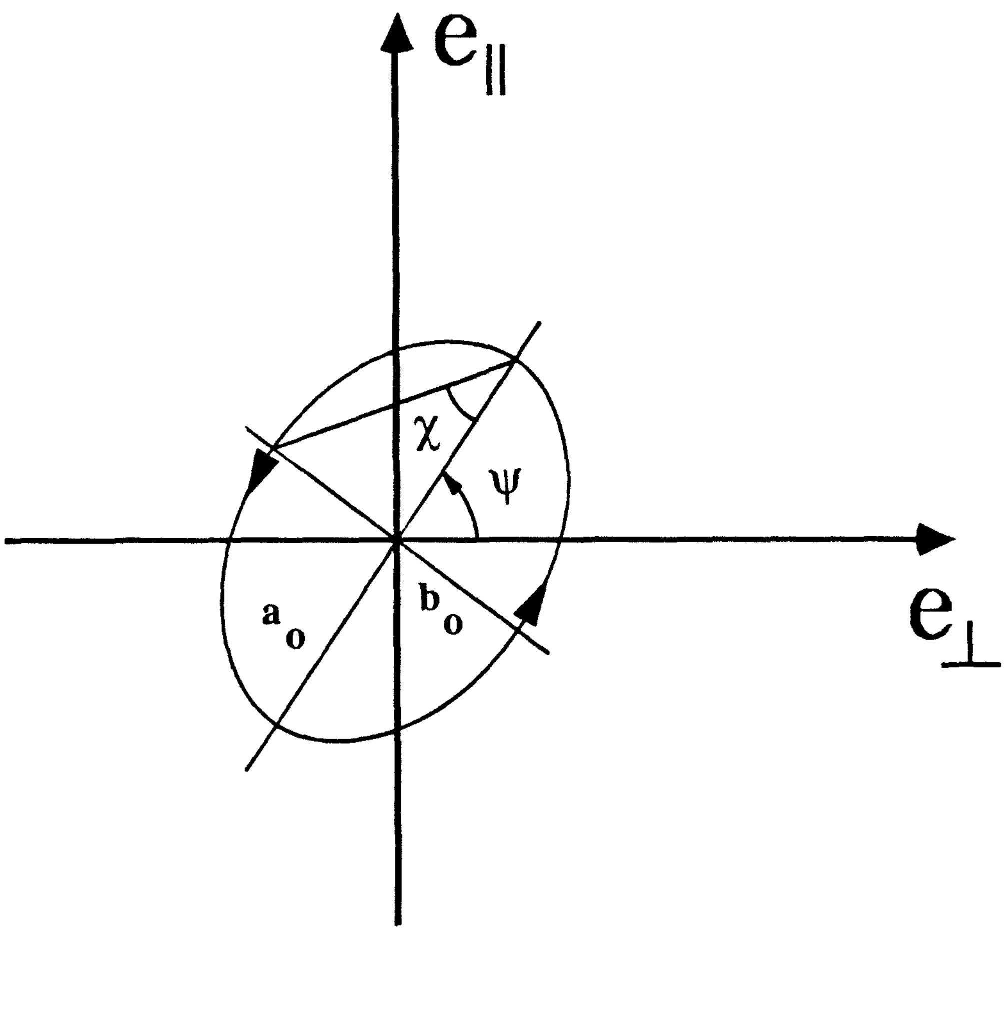

Figure 3.2 *Polarization ellipse for right-handed polarized wave having orientation angle* ψ *and ellipticity angle* χ.

denoted by $P_\perp$. The normalization is with respect to the power carried by the corresponding component of the incident wave. In figure 3.3 normalized power is graphed versus the angle of incidence, as γ varies, for two different cases (a) and (b). The following conclusions can be drawn from the figure.

(a) $\epsilon < \epsilon_1$, the dielectric is denser than the chiral medium. In this case, there is a Brewster angle for small values of γ, the power is almost totally reflected when γ is large and the incident electric field is in the plane of incidence, and as γ increases the reflected power increases.

(b) $\epsilon > \epsilon_1$, the chiral medium is denser than the dielectric. In this case, there is a Brewster angle when the incident electric field is in the plane of incidence. As γ increases the Brewster's angle disappears, and $P_\parallel$ becomes almost equal to unity, that is, most of the incident power is reflected back into the dielectric.

Polarization characteristics of the reflected wave can be obtained from (3.75) and (3.76). Upon substituting the expressions for $E_{r\perp}$ and $E_{r\parallel}$ in (7.73), it may be shown that $S_2 = 0$. Hence

$$\psi = \frac{1}{2}\arctan\left(\frac{S_2}{S_1}\right) = 0\,, \tag{3.77}$$

that is, the major and minor axes of the polarization ellipse are along $\mathbf{e}_\perp$ and $\mathbf{e}_\parallel$, respectively. The fatness (b_o/a_o) of the ellipse can be found from (3.76), and is plotted versus the angle of incidence, for different values of γ. If the incident electric field is in the plane of incidence, then the ellipse fatness is denoted by $(b_o/a_o)_\parallel$, and if the incident electric field is perpendicular to the plane of incidence, then the ellipse fatness is denoted by $(b_o/a_o)_\perp$. Figure 3.4 shows the ellipse fatness, for the two separate cases (a) and (b). The following conclusions can be drawn from the figure.

(a) $\epsilon < \epsilon_1$, the dielectric is denser than the chiral medium. In this case, for small

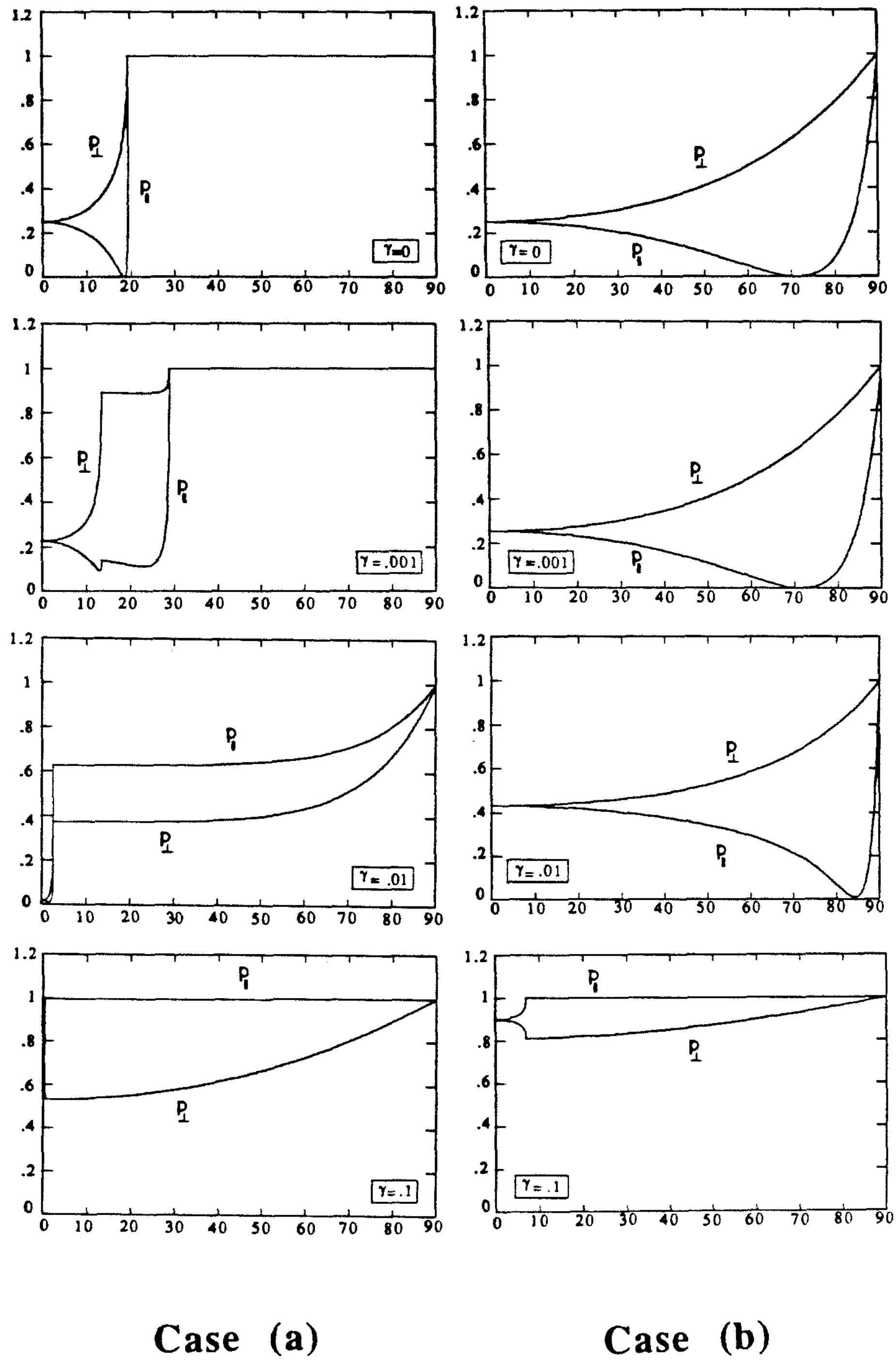

Figure 3.3 *Normalized reflected power as a function of the incident angle, θ_i. For case (a), $\epsilon_1 = 9\epsilon_o$, $\epsilon = \epsilon_o$, and for case (b), $\epsilon_1 = \epsilon_o$, $\epsilon = 9\epsilon_o$. The values of γ are shown on each plot. For both cases it is assumed that $\mu_1 = \mu = \mu_o$.*

γ, there is a right-handed circularly polarized reflected wave if the incident electric field is polarized perpendicular to the plane of incidence. The fatnesses $(b_o/a_o)_{||}$ and $(b_o/a_o)_{\perp}$ of the left-handed and right-handed elliptically polarized reflected waves, respectively, get thinner as γ increases.

(b) $\epsilon > \epsilon_1$, the chiral medium is denser than the dielectric. In this case, there is a change of handedness as θ_i passes through the Brewster angle. As γ increases the fatnesses $(b_o/a_o)_{||}$ and $(b_o/a_o)_{\perp}$ become almost equal to zero, that is, the polarization ellipse becomes very thin. Therefore, in this case, the reflected wave is almost linearly polarized.

3.8 The Infinite Chiral Slab

In this section the problem of the plane wave propagation through an infinite chiral slab of thickness d is considered. The slab (ϵ, μ, γ) is confined between two infinitely extended planes, $z = 0$ and $z = d$ and lies between two dielectrics with the same constitutive parameters (ϵ_1, μ_1), as shown in figure 3.5. A plane wave is incident at an angle θ_i on the chiral slab, from the dielectric which borders the slab at $z = 0$. The aim of the following analysis is to find the amplitudes of the reflected, and the transmitted waves outside the slab. The incident electric and magnetic fields can be written as (see fig. 3.5)

$$\mathbf{E}_i = \mathbf{E}_{oi} e^{ik_i(z\cos\theta_i - x\sin\theta_i)} \tag{3.78}$$

$$\mathbf{H}_i = \mathbf{H}_{oi} e^{ik_i(z\cos\theta_i - x\sin\theta_i)} \tag{3.79}$$

where

$$\mathbf{E}_{oi} = E_{i\perp}\mathbf{e_y} + E_{i||}(\cos\theta_i\mathbf{e_x} + \sin\theta_i\mathbf{e_z}) \tag{3.80}$$

$$\mathbf{H}_{oi} = \eta_1^{-1}\left[E_{i||}\mathbf{e_y} - E_{i\perp}(\cos\theta_i\mathbf{e_x} + \sin\theta_i\mathbf{e_z})\right], \tag{3.81}$$

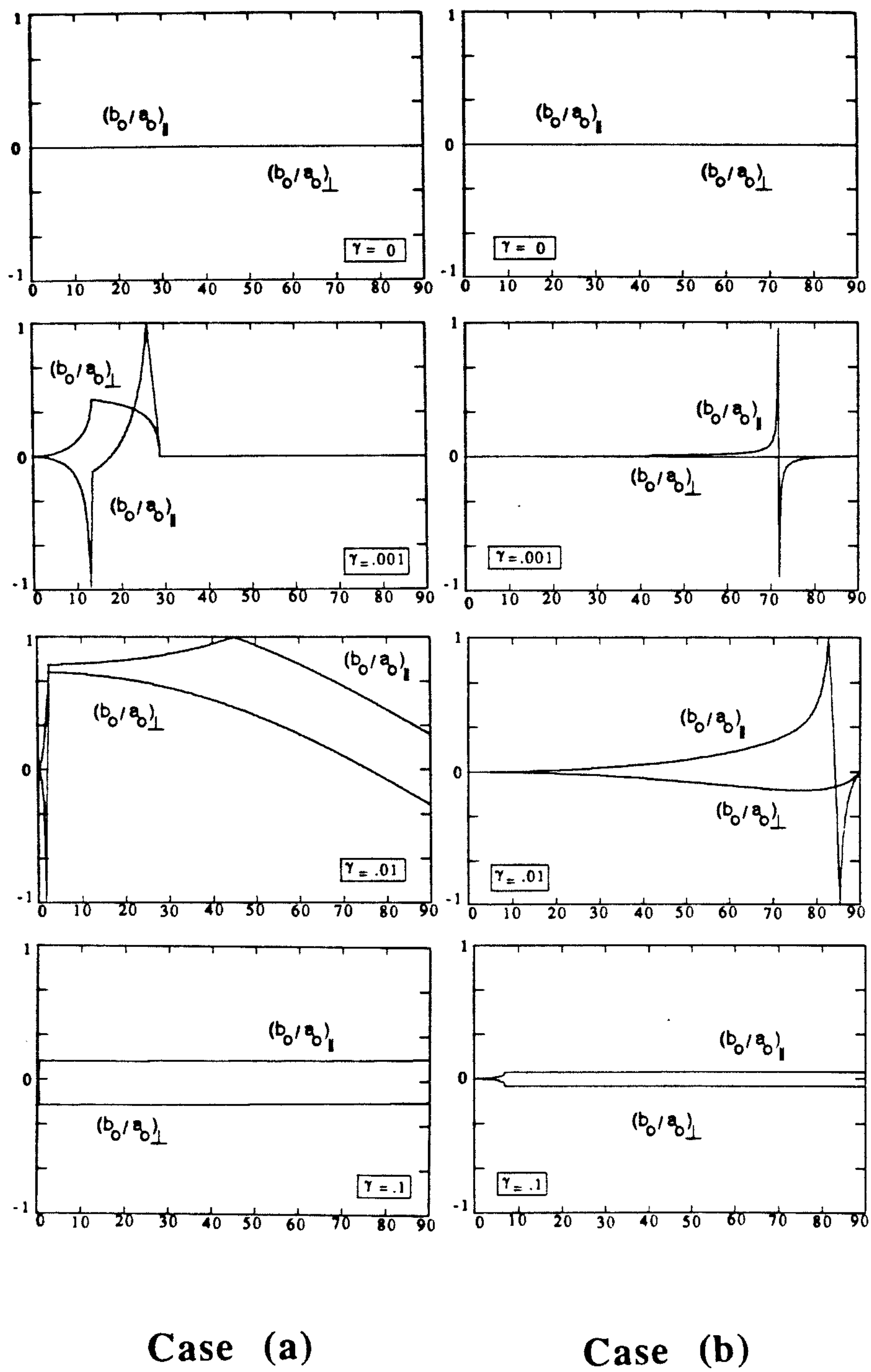

Figure 3.4 *Ellipse fatness of the reflected wave as a function of the incident angle, θ_i. For case (a), $\epsilon_1 = 9\epsilon_o$, $\epsilon = \epsilon_o$, and for case (b), $\epsilon_1 = \epsilon_o$, $\epsilon = 9\epsilon_o$. The values of γ are shown on each plot. For both cases it is assumed that $\mu_1 = \mu = \mu_o$.*

and $\eta_1 = \sqrt{\mu_1/\epsilon_1}$. The reflected fields may be written as (see Fig. 3.5)

$$\mathbf{E}_r = \mathbf{E}_{or} e^{-ik_i(z\cos\theta_i + x\sin\theta_i)} \tag{3.82}$$

$$\mathbf{H}_r = \mathbf{H}_{or} e^{-ik_i(z\cos\theta_i + x\sin\theta_i)} \tag{3.83}$$

where

$$\mathbf{E}_{or} = E_{r\perp}\mathbf{e_y} + E_{r\|}(\cos\theta_i\mathbf{e_x} - \sin\theta_i\mathbf{e_z}) \tag{3.84}$$

$$\mathbf{H}_{or} = \eta_1^{-1}\left[-E_{r\|}\mathbf{e_y} + E_{r\perp}(\cos\theta_i\mathbf{e_x} - \sin\theta_i\mathbf{e_z})\right] . \tag{3.85}$$

As before, the subscript "⊥"refers to the amplitude of the field component perpendicular to the plane of incidence, and the subscript "∥ " refers to the amplitude of the field component which lies in the plane of incidence. In the chiral slab, it is assumed that, there will be four total waves, two propagating toward the interface $z = d$, and the other two propagating toward the interface $z = 0$ (see fig. 3.5). The electric and magnetic fields of the two waves propagating inside the chiral medium toward the interface $z = d$, can be written as

$$\mathbf{E}_c^+ = \mathbf{E}_{o1}^+ e^{ih_1(z\cos\theta_1 - x\sin\theta_1)} + \mathbf{E}_{o2}^+ e^{ih_2(z\cos\theta_2 - x\sin\theta_2)} \tag{3.86}$$

$$\mathbf{H}_c^+ = \mathbf{H}_{o1}^+ e^{ih_1(z\cos\theta_1 - x\sin\theta_1)} + \mathbf{H}_{o2}^+ e^{ih_2(z\cos\theta_2 - x\sin\theta_2)} \tag{3.87}$$

where

$$\mathbf{E}_{o1}^+ = E_{o1}^+(\cos\theta_1\mathbf{e_x} + \sin\theta_1\mathbf{e_z} + i\mathbf{e_y}) \tag{3.88}$$

$$\mathbf{H}_{o1}^+ = -iZ^{-1}E_{o1}^+(\cos\theta_1\mathbf{e_x} + \sin\theta_1\mathbf{e_z} + i\mathbf{e_y}) , \tag{3.89}$$

and

$$\mathbf{E}_{o2}^+ = E_{o2}^+(\cos\theta_2\mathbf{e_x} + \sin\theta_2\mathbf{e_z} - i\mathbf{e_y}) \tag{3.90}$$

$$\mathbf{H}_{o2}^+ = iZ^{-1}E_{o2}^+(\cos\theta_2\mathbf{e_x} + \sin\theta_2\mathbf{e_z} - i\mathbf{e_y}) . \tag{3.91}$$

The electric and magnetic fields of the other two total waves propagating inside the chiral medium toward the interface $z = 0$, may be written as

$$\mathbf{E}_c^- = \mathbf{E}_{o1}^- e^{-ih_1(z\cos\theta_1 + x\sin\theta_1)} + \mathbf{E}_{o2}^- e^{-ih_2(z\cos\theta_2 + x\sin\theta_2)} \tag{3.92}$$

$$\mathbf{H}_c^- = \mathbf{H}_{o1}^- e^{-ih_1(z\cos\theta_1 + x\sin\theta_1)} + \mathbf{H}_{o2}^- e^{-ih_2(z\cos\theta_2 + x\sin\theta_2)} \tag{3.93}$$

where

$$\mathbf{E}_{o1}^- = E_{o1}^-(\sin\theta_1\mathbf{e_z} - \cos\theta_1\mathbf{e_x} + i\mathbf{e_y}) \tag{3.94}$$

$$\mathbf{H}_{o1}^- = -iZ^{-1}E_{o1}^-(\sin\theta_1\mathbf{e_z} - \cos\theta_1\mathbf{e_x} + i\mathbf{e_y})\,, \tag{3.95}$$

and

$$\mathbf{E}_{o2}^- = E_{o2}^-(\sin\theta_2\mathbf{e_z} - \cos\theta_2\mathbf{e_x} - i\mathbf{e_y}) \tag{3.96}$$

$$\mathbf{H}_{o2}^- = iZ^{-1}E_{o2}^-(\sin\theta_2\mathbf{e_z} - \cos\theta_2\mathbf{e_x} - i\mathbf{e_y})\,, \tag{3.97}$$

and the wave impedance Z, of the chiral medium, is defined by equation (3.24). Outside the slab, in the dielectric which borders the slab at $z = d$, the total transmitted wave, can be written as

$$\mathbf{E}_t = \mathbf{E}_{ot} e^{ik_t(z\cos\theta_t - x\sin\theta_t)} \tag{3.98}$$

$$\mathbf{H}_t = \mathbf{H}_{ot} e^{ik_t(z\cos\theta_t - x\sin\theta_t)} \tag{3.99}$$

where

$$\mathbf{E}_{ot} = E_{t\perp}\mathbf{e_y} + E_{t\|}(\cos\theta_t\mathbf{e_x} + \sin\theta_t\mathbf{e_z}) \tag{3.100}$$

$$\mathbf{H}_{ot} = \eta_1^{-1}\left[E_{t\|}\mathbf{e_y} - E_{t\perp}(\cos\theta_t\mathbf{e_x} + \sin\theta_t\mathbf{e_z})\right] \tag{3.101}$$

where $k_t = k_i$ and $\theta_t = \theta_i$.

To find the complex-constant amplitude vectors of the reflected and transmitted waves in the two dielectrics, and those of the waves inside the slab, the boundary

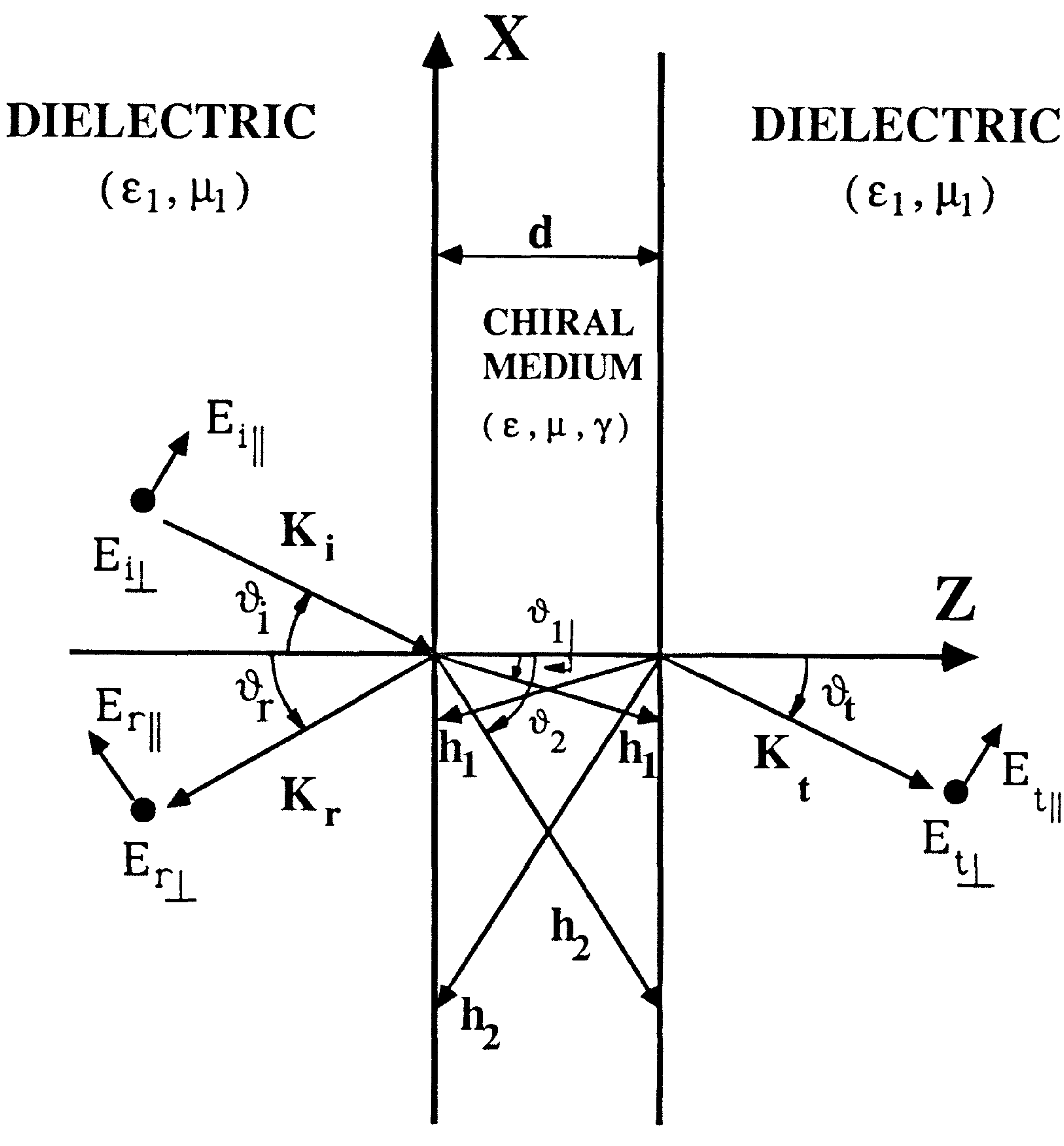

Figure 3.5 *Oblique incidence on an infinite slab of chiral medium. The dielectrics occupying the regions $z < 0$ and $z > d$ have the same constitutive parameters.*

conditions at the two interfaces $z = 0$ and $z = d$ must be applied to the x- and y-components of the electric and magnetic fields. Upon doing this, a system of eight non-homogeneous equations with the eight unknowns $E_{r\perp}$, $E_{r\parallel}$, E_{o1}^{+}, E_{o2}^{+}, E_{o1}^{-}, E_{o2}^{-}, $E_{t\perp}$, and $E_{t\parallel}$ is obtained. This system of equations can be written in the following matrix form

$$\begin{pmatrix} E_{r\perp} \\ E_{r\parallel} \\ E_{o1}^{+} \\ E_{o2}^{+} \\ E_{o1}^{-} \\ E_{o2}^{-} \\ E_{t\perp} \\ E_{t\parallel} \end{pmatrix} = \mathbf{Q}^{-1} \cdot \begin{pmatrix} E_{i\parallel} \\ E_{i\perp} \\ E_{i\perp} \\ E_{i\parallel} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{3.102}$$

where $\mathbf{Q}$ is the following matrix

$$\mathbf{Q} = \begin{pmatrix} 0 & -1 & R_1 & R_2 & -R_1 & -R_2 & 0 & 0 \\ -1 & 0 & i & -i & i & -i & 0 & 0 \\ 1 & 0 & igR_1 & -igR_2 & -igR_1 & igR_2 & 0 & 0 \\ 0 & 1 & g & g & g & g & 0 & 0 \\ 0 & 0 & R_1 e^{i\delta_1} & R_2 e^{i\delta_2} & -R_1 e^{-i\delta_1} & -R_2 e^{-i\delta_2} & 0 & -e^{i\delta_i} \\ 0 & 0 & ie^{i\delta_1} & -ie^{i\delta_2} & ie^{-i\delta_1} & -ie^{-i\delta_2} & -e^{i\delta_i} & 0 \\ 0 & 0 & -igR_1 e^{i\delta_1} & igR_2 e^{i\delta_2} & igR_1 e^{-i\delta_1} & -igR_2 e^{-i\delta_2} & e^{i\delta_i} & 0 \\ 0 & 0 & ge^{i\delta_1} & ge^{i\delta_2} & ge^{-i\delta_1} & ge^{-i\delta_2} & 0 & -e^{i\delta_i} \end{pmatrix}$$

where $R_1 = \cos\theta_1/\cos\theta_i$, $R_2 = \cos\theta_2/\cos\theta_i$, $\delta_1 = h_1 d\cos\theta_1$, $\delta_2 = h_2 d\cos\theta_2$, and $\delta_i = k_i d\cos\theta_i$.

Since the analytical solution of this system of eight non-homogeneous equations, leads to very involved expressions for the field amplitudes, it is, therefore, best to use numerical techniques to invert the matrix equation (3.102). However, it is very interesting and important to obtain the analytical solution of this system when a linearly polarized incident wave falls normally upon the interface. Without loss of generality, it is assumed that the incident electric field is directed along the positive

x-axis. This field can be written as

$$\mathbf{E}_i = E_i \, \mathbf{e_x} \, e^{ik_i z} \, . \tag{3.103}$$

By setting θ_i and $E_{i\perp}$ to zero in the matrix equation (3.102), and then solving the resulting matrix equation, the reflected and transmitted electric fields can be found. The reflected field can be written as

$$\mathbf{E}_r = E_r \, \mathbf{e_x} \, e^{-ik_i z} \tag{3.104}$$

where

$$E_r = E_i \, \Big(\frac{1+g}{1-g}\Big) \, \frac{\big[1 - e^{i(\delta_1+\delta_2)}\big]}{\Big[\big[(1+g)/(1-g)\big]^2 - e^{i(\delta_1+\delta_2)}\Big]} \, . \tag{3.105}$$

Therefore, the polarization of the reflected wave is the same as that of the incident wave, that is, the chiral slab behaves as an ordinary dielectric as far as the reflected wave is concerned. The transmitted wave can be written as

$$\mathbf{E}_t = E_t \, \Big(\mathbf{e_x} + \tan\big(\frac{\delta_2 - \delta_1}{2}\big)\mathbf{e_y}\Big) \, e^{ik_i z} \tag{3.106}$$

where

$$E_t = E_i \, \frac{2g}{(1-g)^2} \, \frac{\big[e^{i(\delta_1-\delta_i)} + e^{i(\delta_2-\delta_i)}\big]}{\Big[\big[(1+g)/(1-g)\big]^2 - e^{i(\delta_1+\delta_2)}\Big]} \, . \tag{3.107}$$

Since $\tan\big[(\delta_2 - \delta_1)/2\big]$ is real, the transmitted wave is linearly polarized, and the ratio of its x- and y-components

$$\frac{E_{ty}}{E_{tx}} = \tan\big(\frac{\delta_2 - \delta_1}{2}\big) = \tan(-\omega\mu\gamma d) \tag{3.108}$$

shows that, the plane of polarization of the transmitted wave is rotated by an angle of $-\omega\mu\gamma d$ with respect to the positive x-axis. If γ is positive, then the rotation is toward the negative y-axis; and if γ is negative, then the rotation is toward the positive y-axis.

3.9 Conclusions

In this chapter three cases of plane wave propagation in an unbounded, semi-infinite and an infinite slab of, chiral medium were discussed in detail. By solving the source-free Maxwell equations in an unbounded chiral medium, the possible wave vectors were obtained, and the polarization properties of the electric and magnetic fields were placed in evidence. The problem of reflection from, and transmission through semi-infinite chiral medium was solved by obtaining the Fresnel equations. Also the conditions for total internal reflection, and the Brewster angle were obtained. By using the Stokes parameters, the power carried by, and the polarization of, the reflected wave was studied. The problem of electromagnetic wave propagation through an infinite slab of chiral medium was formulated for oblique incidence, and was solved analytically for the case of normal incidence. It was shown that, in this case, the plane of polarization of the transmitted wave was rotated with respect to that of the incident wave, and the plane of polarization of the reflected wave was unchanged with respect to that of the incident wave.

CHAPTER IV

RADIATION IN CHIRAL MEDIA

4.1 Introduction

In this chapter the basic problem of determining the electromagnetic fields generated by a given distribution of sources in a chiral medium will be considered. The method that will be used, is the Green's function method. In this method the electric field is written as a volume integral directly in terms of the source current distribution. These two vectors are related via a dyadic called dyadic Green's function. Knowledge of this dyadic enables one to find the fields corresponding to any distribution of currents, on the condition that the volume integration over the source region is doable.

4.2 Dyadic Green's Function

The Maxwell equations in a region with an external source can be written as

$$\nabla \times \mathbf{E} = i\omega\mathbf{B} \tag{4.1}$$

$$\nabla \times \mathbf{H} = \mathbf{J} - i\omega\mathbf{D}\,. \tag{4.2}$$

It is desired to calculate the radiation emitted by a monochromatic dipole antenna surrounded by an unbounded chiral medium [24]. Proceeding from the constitutive relations

$$\mathbf{D} = \epsilon\mathbf{E} + i\gamma\mathbf{B} \tag{4.3}$$

$$\mathbf{H} = i\gamma\mathbf{E} + (1/\mu)\mathbf{B}\,, \tag{4.4}$$

and the Maxwell equations, the differential equation for the emitted electric field $\mathbf{E}$ is found to be

$$\nabla \times \nabla \times \mathbf{E} - k^2\mathbf{E} - 2\omega\mu\gamma\nabla \times \mathbf{E} = i\omega\mu\mathbf{J} \tag{4.5}$$

where $\mathbf{J}$ denotes the current density of the antenna and $k^2 = \omega^2\mu\epsilon$. Since this equation is linear, the desired solution can be expressed as the volume integral

$$\mathbf{E}(\mathbf{r}) = i\omega\mu \int \mathbf{\Gamma}(\mathbf{r},\mathbf{r}') \cdot \mathbf{J}(\mathbf{r}')dV' \tag{4.6}$$

where $\mathbf{\Gamma}(\mathbf{r},\mathbf{r}')$ is the dyadic Green's function. Thus, to carry out the calculations, $\mathbf{\Gamma}$ must first be determined and then integral (4.6) be evaluated.

To find the Green's function a procedure that was used in the case of simple media and in the case of magnetically biased plasma [20] will be followed. Upon substituting (4.6) into (4.5) it is seen that $\mathbf{\Gamma}$ must satisfy the differential equation

$$(\nabla^2 + k^2)\mathbf{\Gamma}(\mathbf{r},\mathbf{r}') + 2\omega\mu\gamma\nabla \times \mathbf{\Gamma}(\mathbf{r},\mathbf{r}') = -(\mathbf{u} + \frac{1}{k^2}\nabla\nabla)\delta(\mathbf{r}-\mathbf{r}') \tag{4.7}$$

where $\mathbf{u}$ is the unit dyadic and $\delta(\mathbf{r}-\mathbf{r}')$ is the Dirac delta function. to solve this equation, it is assumed that $\mathbf{\Gamma}(\mathbf{r},\mathbf{r}')$ can be written as a Fourier integral, viz.

$$\mathbf{\Gamma}(\mathbf{r},\mathbf{r}') = \frac{1}{8\pi^3}\int_{-\infty}^{\infty} \mathbf{\Lambda}(\mathbf{p})e^{i\mathbf{p}\cdot(\mathbf{r}-\mathbf{r}')}d\mathbf{p} \tag{4.8}$$

where $\mathbf{p}$ is the position vector and $d\mathbf{p}$ the volume element in p-space, and where $\mathbf{\Lambda}(\mathbf{p})$ is a dyadic function of $\mathbf{p}$. Also the Dirac delta function can be expressed as

$$\delta(\mathbf{r}-\mathbf{r}') = \frac{1}{8\pi^3}\int_{-\infty}^{\infty} e^{i\mathbf{p}\cdot(\mathbf{r}-\mathbf{r}')}d\mathbf{p}\,. \tag{4.9}$$

It follows from (4.7), (4.8), and (4.9) that the dyadic $\mathbf{\Lambda}(\mathbf{p})$ must satisfy the algebraic equation

$$(k^2 - p^2)\mathbf{\Lambda} + 2i\omega\mu\gamma\,\mathbf{p} \times \mathbf{\Lambda} = -(\mathbf{u} - \frac{1}{k^2}\mathbf{p}\mathbf{p})\,. \tag{4.10}$$

By dyadic algebra it is found from (4.10) that

$$\mathbf{A} = \frac{k^2 - p^2}{(k^2 - p^2)^2 + \alpha^2 p^2}\left(\frac{1}{k^2}\mathbf{pp} - \mathbf{u}\right) + \frac{1}{(k^2 - p^2)^2 + \alpha^2 p^2}\left(\alpha \mathbf{p} \times \mathbf{u} - \frac{\alpha^2}{k^2}\mathbf{pp}\right) \tag{4.11}$$

where $\alpha = 2i\omega\mu\gamma$ and $p^2 = \mathbf{p} \cdot \mathbf{p}$. Upon substituting (4.11) into (4.8) the following expression is obtained for the dyadic Green's function in terms of three-dimensional integrals in p-space:

$$\begin{aligned}\mathbf{\Gamma}(\mathbf{r}, \mathbf{r}') = &-\frac{1}{8\pi^3}\left[\left(\mathbf{u} + \frac{1}{k^2}\nabla\nabla\right)\int_{-\infty}^{\infty} \frac{k^2 - p^2}{(k^2 - p^2)^2 + \alpha^2 p^2} e^{i\mathbf{p}\cdot(\mathbf{r}-\mathbf{r}')} d\mathbf{p}\right] \\ &+ \frac{1}{8\pi^3}\left[\frac{\alpha^2}{k^2}\nabla\nabla \int_{-\infty}^{\infty} \frac{1}{(k^2 - p^2)^2 + \alpha^2 p^2} e^{i\mathbf{p}\cdot(\mathbf{r}-\mathbf{r}')} d\mathbf{p}\right] \\ &- \frac{1}{8\pi^3}\left[i\alpha\nabla \times \left(\mathbf{u}\int_{-\infty}^{\infty} \frac{1}{(k^2 - p^2)^2 + \alpha^2 p^2} e^{i\mathbf{p}\cdot(\mathbf{r}-\mathbf{r}')} d\mathbf{p}\right)\right].\end{aligned} \tag{4.12}$$

To reduce the three-dimensional integrals to one-dimensional integrals, the spherical coordinates (p, η, ξ) in p-space with polar axis along $\mathbf{R}(= \mathbf{r} - \mathbf{r}')$ are introduced. In these coordinates each of the integrals in (4.12) has the form

$$I = \int_{-\infty}^{\infty} f(p) e^{i\mathbf{p}\cdot\mathbf{R}} d\mathbf{p} \tag{4.13}$$

where $f(p)$ is an even function of P. That is,

$$I = \int_0^{\infty} p^2 f(p) dp \int_0^{\pi} e^{ipR\cos\eta} \sin\eta \, d\eta \int_0^{2\pi} d\xi \tag{4.14}$$

where $R = \sqrt{\mathbf{R}\cdot\mathbf{R}}$. Clearly, this three-dimensional integral reduces to the one-dimensional integral

$$I = \frac{4\pi}{R}\int_0^{\infty} p f(p) \sin pR \, dp$$

which, since $f(p)$ is an even function, can be written as

$$\begin{aligned} I &= \frac{2\pi}{R}\int_{-\infty}^{\infty} p f(p) \sin pR \, dp \\ &= \frac{2\pi}{iR}\int_{-\infty}^{\infty} p f(p) e^{ipR} \, dp \, . \end{aligned} \tag{4.15}$$

From (4.13) and (4.15) it follows that

$$\int_{-\infty}^{\infty} f(p) e^{i\mathbf{p}\cdot\mathbf{R}} d\mathbf{p} = \frac{2\pi}{iR} \int_{-\infty}^{\infty} p f(p) e^{ipR} dp \,. \tag{4.16}$$

Using (4.16) equation (4.12) can be written as

$$\begin{aligned} \mathbf{\Gamma}(\mathbf{r},\mathbf{r}') = &-\frac{1}{4\pi^2 iR}\left(\mathbf{u} + \frac{1}{k^2}\nabla\nabla\right) \int_{-\infty}^{\infty} \frac{p(k^2-p^2)}{(k^2-p^2)^2+\alpha^2 p^2} e^{ipR} dp \\ &+ \frac{1}{4\pi^2 iR}\frac{\alpha^2}{k^2}\nabla\nabla \int_{-\infty}^{\infty} \frac{p}{(k^2-p^2)^2+\alpha^2 p^2} e^{ipR} dp \\ &- \frac{\alpha}{4\pi^2 R}\nabla \times \left(\mathbf{u}\int_{-\infty}^{\infty} \frac{p}{(k^2-p^2)^2+\alpha^2 p^2} e^{ipR} dp\right). \end{aligned} \tag{4.17}$$

The integrals in this expression are one-dimensional and can be evaluated by contour integration.

4.3 Radiation Condition

To evaluate the integrals in (4.17) by contour integration (theorem of residues) it is noted that $p = \pm h_1, p = \pm h_2$ are the roots of

$$(k^2 - p^2)^2 + \alpha^2 p^2 = 0 \,. \tag{4.18}$$

Since $\alpha = 2i\omega\mu\gamma$, it is seen from (4.18) that

$$\begin{aligned} h_1 &= \omega\mu\gamma + \sqrt{\omega^2\mu^2\gamma^2 + k^2} \\ h_2 &= -\omega\mu\gamma + \sqrt{\omega^2\mu^2\gamma^2 + k^2} \end{aligned} \,. \tag{4.19}$$

Accordingly, the poles of the integrands in (4.17) lie along the real axis of the complex p-plane at $\pm h_1$, and $\pm h_2$. Expressions (4.19) show that for $\gamma > 0$, $h_1 > k > h_2 > 0$ and that for $\gamma < 0$, $h_2 > k > h_1 > 0$.

The path of integration for $\gamma > 0$ is shown in figure 4.1. The contour integration consists of an integration along the real axis of tha complex p-plane, with upward

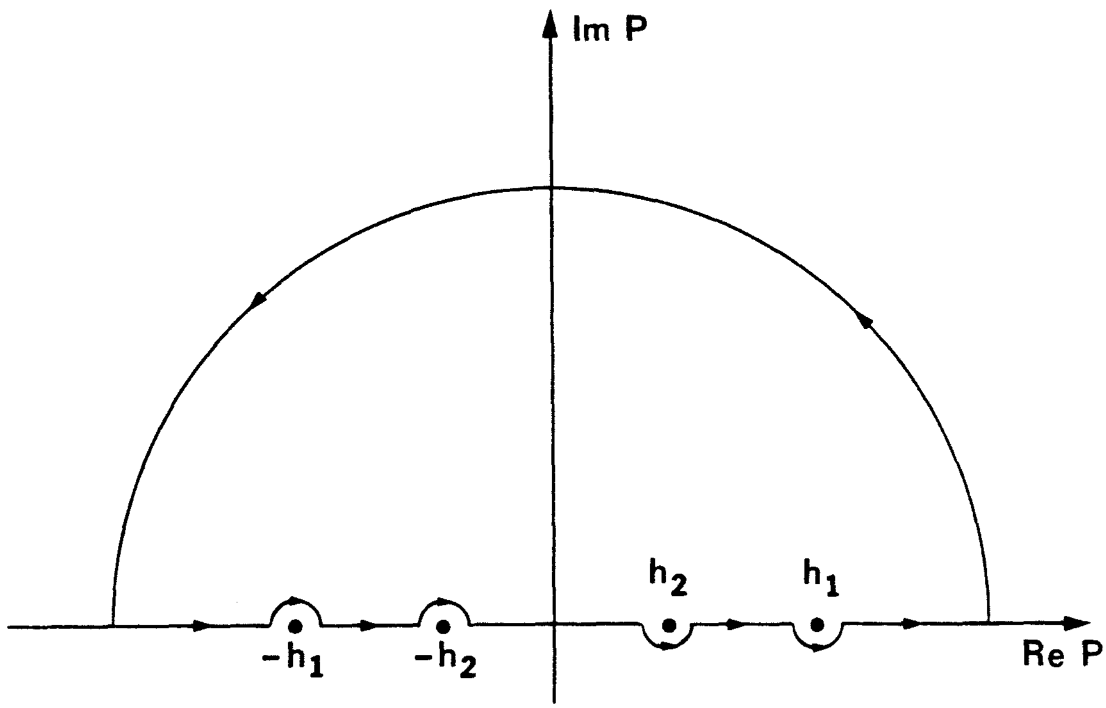

Figure 4.1 *Path of integration in complex p-plane for right-handed ($\gamma > 0$) chiral medium.*

indentation at the poles $p = -h_2$, $p = -h_1$ and downward indentation at the poles $p = h_2$, $p = h_1$, and of an integration along the semicircle at infinity. The factor $\exp(ipR)$ makes the contribution of the integration along the semicircle disappear. The value of $\mathbf{\Gamma}$, as given by (4.17), is undetermined unless the manner of going around the poles is specified. Here, the chosen path yields a description of $\mathbf{\Gamma}$ in terms of only outgoing waves and thus makes $\mathbf{\Gamma}$ satisfy the physically required radiation condition.

According to the theorem of residues, the integrals in (4.17) are equal to $2\pi i$ multiplied by the sum of residues at the poles $p = h_1$, $p = h_2$. With the aid of this theorem the following equations are obtained

$$\int_{-\infty}^{\infty} \frac{p(k^2 - p^2)}{(k^2 - p^2)^2 + \alpha^2 p^2} e^{ipR} dp = 2\pi i \left[\frac{(k^2 - {h_1}^2) e^{ih_1 R} - (k^2 - {h_2}^2) e^{ih_2 R}}{2({h_1}^2 - {h_2}^2)} \right] \tag{4.20}$$

$$\int_{-\infty}^{\infty} \frac{p}{(k^2 - p^2)^2 + \alpha^2 p^2} e^{ipR} dp = 2\pi i \left[\frac{e^{ih_1 R} - e^{ih_2 R}}{2({h_1}^2 - {h_2}^2)} \right]. \tag{4.21}$$

Finally, by substituting (4.20) and (4.21) into (4.17) it is found that the desired Green's function is given by

$$\mathbf{\Gamma}(\mathbf{r}, \mathbf{r}') = \frac{1}{{h_2}^2 - {h_1}^2} \Big\{ \Big(\mathbf{u} + \frac{1}{k^2} \nabla\nabla \Big) \Big[(k^2 - {h_1}^2) \frac{e^{ih_1 R}}{4\pi R} - (k^2 - {h_2}^2) \frac{e^{ih_2 R}}{4\pi R} \Big] - \Big(-i\alpha \mathbf{u} \times \nabla + \frac{\alpha^2}{k^2} \nabla\nabla \Big) \Big[\frac{e^{ih_1 R}}{4\pi R} - \frac{e^{ih_2 R}}{4\pi R} \Big] \Big\}. \tag{4.22}$$

This is the solution of (4.7) that satisfies the radiation condition and is valid for all γ (positive and negative). It involves two modes of propagation, one being an outgoing wave of phase velocity ω/h_1, and the other an outgoing wave of phase velocity ω/h_2. For $\gamma > 0$ the former is the slower mode whereas for $\gamma < 0$ the latter is the slower mode.

4.4 Dipole Radiation

Now consider an oscillating dipole antenna, of length ℓ, located in the chiral medium. It is assumed that the antenna lies at the origin of a Cartesian coordinate system (x, y, z) and is directed parallel to the z-axis. It is convenient to introduce a spherical coordinate system (r, θ, ϕ) where $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$, because, as will be seen later, the field of the antenna is composed of spherical waves. See figure 4.2.

The current density of the antenna is assumed to be

$$\mathbf{J} = \mathbf{e}_z I_o \delta(x)\delta(y)\delta(z) = \mathbf{e}_z I\,\ell\,\delta(\mathbf{r}) \tag{4.23}$$

where $\mathbf{e}_z$ is the unit vector in the z-direction and I is the antenna current. Substituting this expression into (4.6) it is seen that the electric field emitted by the antenna can be obtained from

$$\mathbf{E}(\mathbf{r}) = i\omega\mu I_o \boldsymbol{\Gamma}(\mathbf{r}, 0)\cdot\mathbf{e}_z \tag{4.24}$$

where $\boldsymbol{\Gamma}(\mathbf{r}, 0)$ is given by (4.22) with R replaced by r. It follows from (4.22) and (4.24) that the far-zone or radiation field is given by

$$\mathbf{E} = -i\omega\mu I_o \left\{ \frac{{h_1}^2 - k^2}{{h_1}^2 - {h_2}^2}(\mathbf{e}_\theta + i\mathbf{e}_\phi)\sin\theta\frac{e^{ih_1 r}}{4\pi r} + \frac{k^2 - {h_2}^2}{{h_1}^2 - {h_2}^2}(\mathbf{e}_\theta - i\mathbf{e}_\phi)\sin\theta\frac{e^{ih_2 r}}{4\pi R} \right\} \tag{4.25}$$

where $\mathbf{e}_\theta$ is the unit vector in the θ direction and $\mathbf{e}_\phi$ is the unit vector in the ϕ direction.

4.5 Polarization and Wave Impedance

Using (4.25) the electric field vector $\mathbf{E}$ of the dipole antenna's radiation field can be written as

$$\mathbf{E} = \frac{-i\omega\mu I_o \sin\theta}{4\pi r}\left[a(\mathbf{e}_\theta + i\mathbf{e}_\phi)e^{ih_1 r} + b(\mathbf{e}_\theta - i\mathbf{e}_\phi)e^{ih_2 r}\right] \tag{4.26}$$

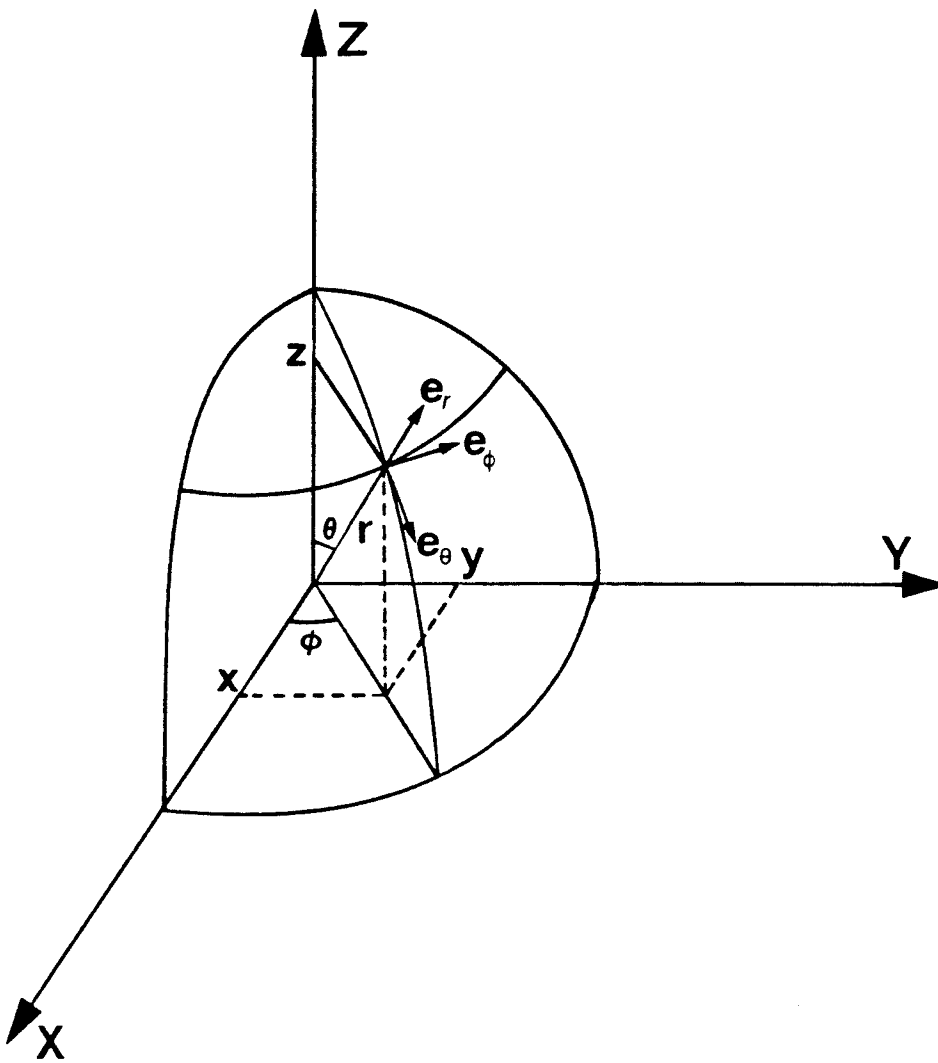

Figure 4.2 *Cartesian and spherical coordinate systems. Dipole antenna (not shown) is located at the origin and is directed parallel to the z-axis.*

where

$$a = \frac{{h_1}^2 - k^2}{{h_1}^2 - {h_2}^2}, \qquad b = \frac{k^2 - {h_2}^2}{{h_1}^2 - {h_2}^2}. \tag{4.27}$$

Taking $(-i\omega\mu I_o \sin\theta/4\pi r)$ as a normalization factor, the quantity in the square brackets is considered to be descriptive of the state of polarization (the polarization ellipse) of the electric field. The first term in the square brackets describes a right-handed circularly polarized wave of amplitude a and phase velocity ω/h_1; and the second term describes a left-handed circularly polarized wave of amplitude b and phase velocity ω/h_2. From (4.19) and (4.27) it is clear that $a = b = 1/2$ when $\gamma = 0$; that $a = 0$, $b = 1$ when $\gamma \to -\infty$; and that $a = 1$, $b = 0$ when $\gamma \to \infty$.

From figure 4.3, which shows a sketch of a and b versus γ, it is seen that $a > b$ when $\gamma > 0$ and that $b > a$ when $\gamma < 0$. This means that the polarization ellipse of $\mathbf{E}$ has a right-handed sense of rotation when $\gamma > 0$ and a left-handed sense of rotation when $\gamma < 0$, and that the twist of the major axis of the ellipse is to the left when $\gamma > 0$ and to the right when $\gamma < 0$. The lengths of the semi-major and semi-minor axes of the ellipse are given by $(a+b)$ and $|a-b|$ respectively. Recalling (4.19) it is seen from (4.27) that

$$\text{length of semi} - \text{major axis} = a + b = 1 \tag{4.28}$$

$$\text{length of semi} - \text{minor axis} = |a - b| = \frac{\omega\mu|\gamma|}{\sqrt{\omega^2\mu^2\gamma^2 + k^2}}. \tag{4.29}$$

As $\gamma \to 0$, the ellipse gets thin and the polarization becomes linear; as $|\gamma| \to \infty$, the ellipse gets thick and the polarization becomes circular. The orientation angle ψ of the ellipse is given by

$$\psi_E = \frac{h_2 - h_1}{2} r = -\omega\mu\gamma r. \tag{4.30}$$

The magnetic vector $\mathbf{H}$ of the dipole's radiation field can be obtained by substituting (4.26) into

$$\mathbf{H} = i\gamma\mathbf{E} + (1/\mu)\mathbf{B} = i\gamma\mathbf{E} + (1/i\omega\mu)\nabla \times \mathbf{E},$$

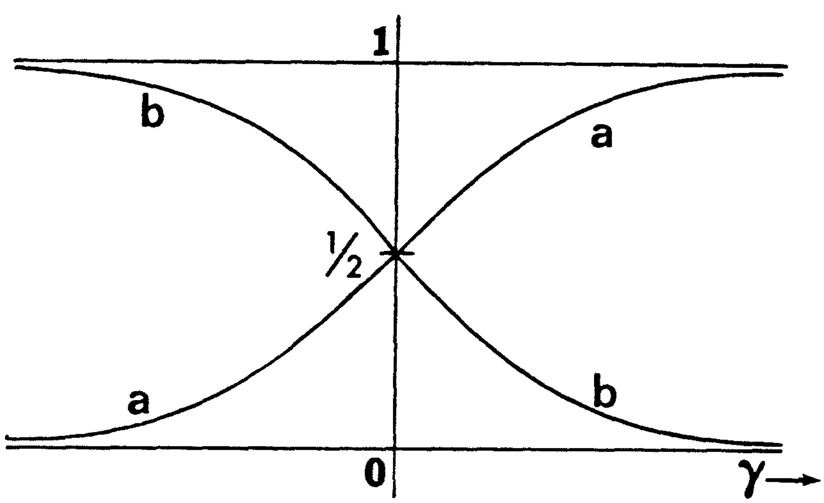

Figure 4.3 *Sketch of amplitudes a and b versus γ, where a is amplitude of right-handed circularly polarized wave and b is amplitude of left-handed circularly polarized wave, and where γ is the measure of chirality.*

and retaining only terms in $1/r$. Thus this field can be written as

$$\mathbf{H} = -i\omega\mu I_o \frac{\sin\theta}{4\pi r}\sqrt{\gamma^2 + (\epsilon/\mu)} \cdot \left\{a(\mathbf{e}_\theta + i\mathbf{e}_\phi)e^{ih_1 r}e^{i3\pi/2} + b(\mathbf{e}_\theta - i\mathbf{e}_\phi)e^{ih_2 r}e^{i\pi/2}\right\}. \tag{4.31}$$

The orientation angle ψ for the magnetic field is given by

$$\psi_H = \frac{h_2 - h_1}{2}r - \frac{\pi}{2} = -\omega\mu\gamma r - \frac{\pi}{2}. \tag{4.32}$$

The polarization ellipse of $\mathbf{H}$ is similar and perpendicularly oriented to the polarization ellipse of $\mathbf{E}$. That is, the polarization ellipses of $\mathbf{E}$ and $\mathbf{H}$ are perpendicular to each other and have the same sense of rotation, as shown in figure 4.4.

Using the notation $\mathbf{E}(t) = Re(\mathbf{E}e^{-i\omega t})$ and $\mathbf{H}(t) = Re(\mathbf{H}e^{-i\omega t})$, it is seen from (4.26) and (4.31) that at any point (r, θ, ϕ) and at any time t the vectors $\mathbf{E}(t)$ and $\mathbf{H}(t)$ are perpendicular to each other and to the direction of propagation. Specifically,

$$\mathbf{e}_r \times \mathbf{E}(t) = \frac{1}{\sqrt{\gamma^2 + (\epsilon/\mu)}}\mathbf{H}(t). \tag{4.33}$$

It follows from this relation that the wave impedance Z of the medium is given by

$$Z = \sqrt{\frac{\mu}{\epsilon}}\frac{1}{\sqrt{1 + (\mu/\epsilon)\gamma^2}}. \tag{4.34}$$

In a straightforward manner one can also find from (4.22) and (4.24) the induction part of $\mathbf{E}$ which decreases with distance as $(1/r^2)$, and the near-zone part of $\mathbf{E}$ which decreases as $(1/r^3)$. The complete electric field can be written as

$$\mathbf{E} = \mathbf{E}_{(1/r)} + \mathbf{E}_{(1/r^2)} + \mathbf{E}_{(1/r^3)} \tag{4.35}$$

where

$$\mathbf{E}_{(1/r)} = \frac{-iZI\ell}{8\pi r}\left\{h_1(\mathbf{e}_\theta + i\mathbf{e}_\phi)e^{ih_1 r} + h_2(\mathbf{e}_\theta - i\mathbf{e}_\phi)e^{ih_2 r}\right\}\sin\theta, \tag{4.36}$$

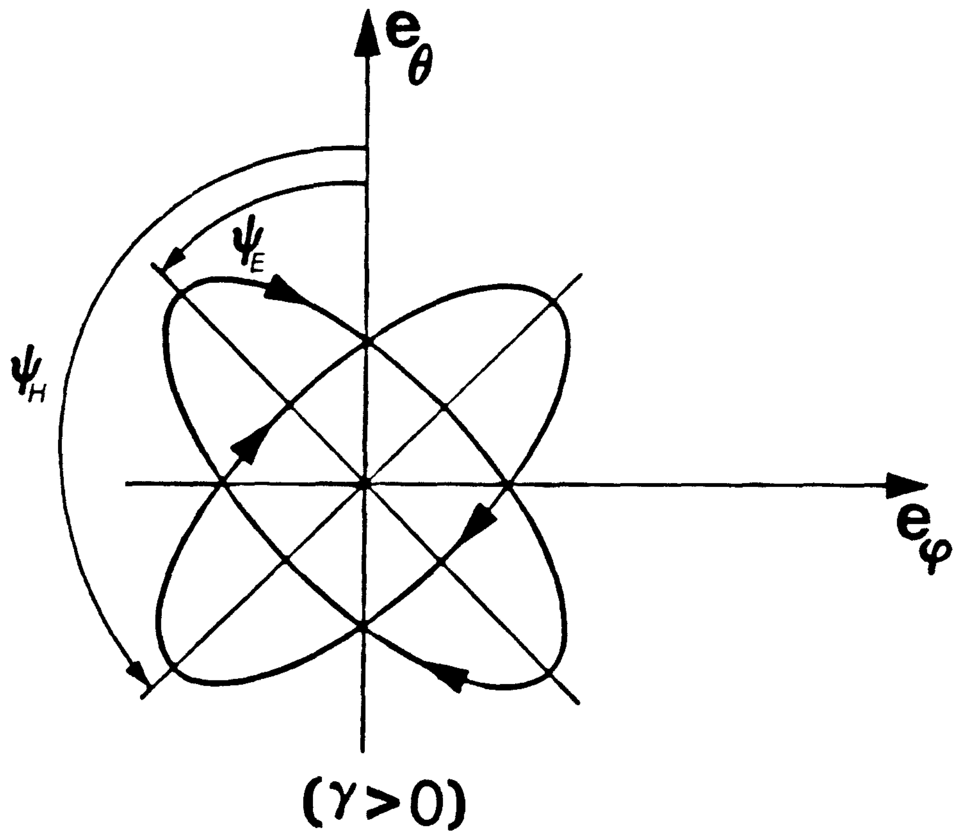

Figure 4.4 *Polarization ellipses for* **E** *and* **H** *fields.*

$$\mathbf{E}_{(1/r^2)} = \frac{ZI\ell}{8\pi r^2}\Big[\big\{(\mathbf{e}_\theta + i\mathbf{e}_\phi)e^{ih_1 r} + (\mathbf{e}_\theta - i\mathbf{e}_\phi)e^{ih_2 r}\big\}\sin\theta + 2\cos\theta\big\{e^{ih_1 r} + e^{ih_2 r}\big\}\mathbf{e}_r\Big] , \tag{4.37}$$

and

$$\mathbf{E}_{(1/r^3)} = \frac{iZI\ell}{8\pi r^3}\Big\{\big(\frac{e^{ih_1 r}}{h_1} + \frac{e^{ih_2 r}}{h_2}\big)(\sin\theta\, \mathbf{e}_\theta + 2\cos\theta\, \mathbf{e}_r)\Big\} . \tag{4.38}$$

4.6 Radiated Power

From Maxwell's equations and the divergence theorem, the poynting theorem can be written as

$$\frac{1}{2}Re\int \mathbf{n}\cdot(\mathbf{E}\times\mathbf{H}^*)\,dS = -\frac{1}{2}Re\int \mathbf{J}^*\cdot\mathbf{E}\,dV + \frac{1}{2}Re\Big[i\omega\int(\mathbf{B}\cdot\mathbf{H}^* - \mathbf{D}^*\cdot\mathbf{E})\,dV\Big] . \tag{4.39}$$

The volume integration are taken throughout a region boundedby a far-zone sphere centered on the antenna. The surface integration is taken over the far-zone sphere whose outward normal vector is $\mathbf{n}$. From the constitutive relations (4.3) and (4.4) it follows that the integrand $\mathbf{B}\cdot\mathbf{H}^* - \mathbf{D}^*\cdot\mathbf{E}$ is purely real and hence the second term on the right is zero. The first term on the right side, by definition, is the time-average radiated power input. Consequently the time-average radiated power P is expressed by the quantity on the left side. That is,

$$P = \frac{1}{2}Re\int \mathbf{n}\cdot(\mathbf{E}\times\mathbf{H}^*)\,dS \tag{4.40}$$

substituting (4.26) and (4.31) into (4.40) it is found that the radiated power is given by

$$P = \frac{\omega^2\mu^2 I^2\ell^2}{12\pi}\;\frac{2\gamma^2 + (\epsilon/\mu)}{\sqrt{\gamma^2 + (\epsilon/\mu)}} . \tag{4.41}$$

Defining the radiation resistance R by $P = (1/2)I^2 R$, it is seen from (4.41) that

$$R = \frac{\omega^2\mu^2\ell^2}{6\pi}\;\frac{2\gamma^2 + (\epsilon/\mu)}{\sqrt{\gamma^2 + (\epsilon/\mu)}} . \tag{4.42}$$

This shows that the radiation resistance increases as γ or as the medium becomes more chiral.

4.7 Conclusions

In this chapter the dyadic Green's function for a source of finite spatial extend in an unbounded chiral medium described by the constitutive relations (4.3) and (4.4) was found to be given by (4.22). The Green's function involves two spherical waves $\exp[(ih_1R)]/4\pi R$ and $\exp[(ih_2R)]/4\pi R$ which place in evidence that the medium supports double-mode propagation when $\gamma \neq 0$ and single-mode propagation when $\gamma = 0$.

For a right-handed ($\gamma > 0$) medium the former wave is "slow" and the latter wave is "fast"; and for a left-handed ($\gamma < 0$) medium the converse is true. These waves are considered slow and fast in comparison to the velocity they have when $\gamma = 0$, *i.e.*, when the medium is achiral and the double-mode propagation is reduced to single-mode propagation.

From (4.22) it is also seen that the Green's function is reciprocal; that is, $\mathbf{\Gamma}(\mathbf{r}, \mathbf{r}') = [\mathbf{\Gamma}(\mathbf{r}', \mathbf{r})]^T$ where the subscript T denotes the transpose of the dyadic. This reciprocity implies that the transmitting and receiving patterns of an antenna in the medium would be the same.

In the case of dipole antenna, it was found that the chirality of the ambient medium decreases the wave impedance of the medium, increases the radiation resistance of the dipole, but has no effect on the directivity of the radiation. The dominant effect of the chirality is the change it produces in the state of polarization of the dipole's field. However, although the directivity of a dipole is not changed by chirality, it cannot be concluded that the directivity of an antenna that is not small compared to the wavelenghts $2\pi/h_1$ and $2\pi/h_2$ is not changed by chirality.

For example, if a two-element array is placed in a chiral medium, the directivity of the array would change with the chirality of the medium.

Due to mode-multiplicity, it appears that chirality could play an important role in the design of sizable antennas in general and antenna arrays in particular.

CHAPTER V

CONCLUSIONS

This thesis describes a theoretical investigation of electromagnetic wave propagation and radiation in chiral media. The source-free Maxwell equations were solved subject to the constitutive relations for a lossless, reciprocal, unbounded chiral medium, and the plane wave solutions were obtained. It was shown that the medium supports double-mode propagation and that the plane of polarization of the electric field rotates as the wave propagates. It was also shown that the time-averaged power density carried by the plane waves increases as the medium becomes more chiral. The problem of reflection from and transmission through a semi-infinite chiral medium was solved by obtaining the Fresnel equations. It was found that, as a consequence of the reciprocity principle, the cross-reflection coefficients are equal. It was also shown that the phenomena of total internal reflection can occur and that the Brewster angle can exist. By means of the Stokes parameters the polarization and intensity of the reflected wave were discussed. It was found that the reflected power increases as the chirality of the medium increases and that when the chiral medium is denser than the dielectric, the reflected wave is almost always linearly polarized. The plane wave propagation through an infinite chiral slab was formulated for oblique incidence and solved analytically for the case of normal incidence. It was shown that for a linearly polarized wave normally incident on the slab, the plane of polarization of the reflected wave was unchanged with respect to that of the incident wave, and the plane of polarization of the transmitted wave was rotated with respect to that of the incident wave.

In chapter IV the dyadic Green's function for a finite source in an unbounded lossless, reciprocal, chiral medium was obtained, and it was found to be reciprocal. This reciprocity implies the transmitting and receiving patterns of an antenna in the medium would be the same. In the case of dipole antenna, it was found that as the chirality of the medium increases the wave impedance of the medium decreases, and the radiation resistance of the dipole increases, but the directivity of radiation remains unchanged. The dominant effect of the chirality is the change it produces in the state of polarization of the dipole's field.

However, although the directivity of a dipole is not changed by chirality, it cannot be concluded that the directivity of an antenna that is not small compared to the wavelenghts $2\pi/h_1$ and $2\pi/h_2$ is not changed by chirality. For example, if a two-element array is placed in a chiral medium, the directivity of the array would change with the chirality of the medium. Due to mode-multiplicity, it appears that chirality could play an important role in the design of sizable antennas in general and antenna arrays in particular.

One of the important areas of applied electromagnetics that can benefit from the above results is remote sensing. Since certain types of vegetation layers can be thought of as chiral media the analytical results of this thesis can be applied to the exprimental data in order to identify these vegetation layers. Another application could be the determination of the constitutive parameters (ϵ, μ, γ) of any unkown chiral medium, for example, the medium may be a chiral chemical compound or a random collection of wire helicies.

Undertaking a program of study, in order to find a set of rules that relates the chirality of a macroscopic medium to the geometrical characteristics and dimensions of the composing chiral objects, would be of great theoretical and practical importance. Since it will then be possible to design and construct new devices using artificial chiral media. In one such study [18], and as far as the authur knows the only one, the chirality of a collection of identical short wire helicies were found in terms of the geometrical dimensions of one the helicies (see addendum A). A very interesting and important chiral medium, that can be used to continue this course of study, is a medium composed of equivalent irregular tetrahedrons.

ADDENDUM A

A Lower Bound on Chirality

In this addendum, a lower bound on the magnitude of the chirality γ, of a medium composed of randomly oriented equivalent short wire helicies will be found. A typical short wire helix is shown in figure A.1.

From [18], it can be shown that the parameters χ_e, χ_m, γ_e, and γ_m, present in equations (2.1) and (2.2), are given by

$$\chi_e = N\epsilon_o^{-1}\ell^2 C \tag{A.1}$$

$$\chi_m = -N\mu_o(\pi a^2/2)^2 L^{-1} \tag{A.2}$$

$$\gamma_e = \gamma_m = iN(\ell/2)(\pi a^2)\omega C \tag{A.3}$$

where N is the number of short wire helicies per unit volume, C and L are respectively the capacitance and inductance of the body, and 2ℓ and $2a$ represent the length and the width of the short helix (see fig. A.1). The constraint $LC = \omega^{-2}$ is placed upon the inductance and capacitance of the helix.

The constitutive relations for a reciprocal chiral medium composed of equivalent lossless, short wire helicies, all of the same handedness was shown to have the form (see chapter II)

$$\mathbf{D} = \epsilon\mathbf{E} + i\gamma\mathbf{B} \tag{A.4}$$

$$\mathbf{H} = i\gamma\mathbf{E} + (1/\mu)\mathbf{B} \tag{A.5}$$

From the results of section 2.1 and (A.1)–(A.3) it can be shown that

$$\epsilon = \epsilon_o + N\ell^2 C \tag{A.6}$$

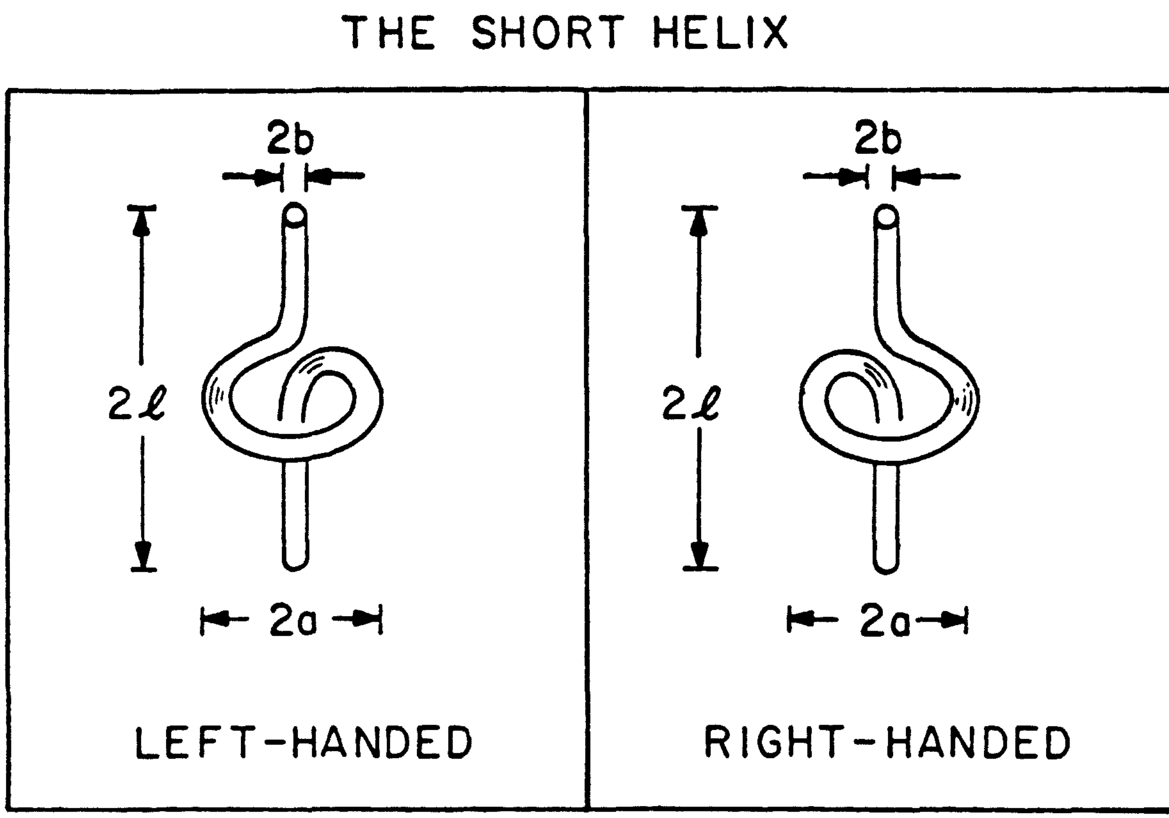

Figure A.1 *Idealized short wire helix. The plane of the loop is perpendicular to the axis of the straight portion of the wire.*

$$1/\mu = (1/\mu_o) + N(\pi a^2/2)^2 L^{-1} \qquad (A.7)$$

$$\gamma = N(\ell/2)(\pi a^2)\omega C\,. \qquad (A.8)$$

A lower bound on the capacitance C is given by [18]

$$C \geq \epsilon_o (4\pi)^{2/3} (3V_h)^{1/3} \qquad (A.9)$$

where $V_h (= 2\pi b^2(\ell + \pi a))$ is the volume occupied by the short wire helix and b is the wire radius.

Substituting for C from (A.9) into (A.8), the following lower bound on the magnitude of γ can be obtained

$$|\gamma| \geq 4^{1/6} \pi^{5/3} \sqrt{\frac{\epsilon_o}{\mu_o}} N \lambda^{-1} (2\pi \ell a^2)(3V_h)^{1/3}\,. \qquad (A.10)$$

This bound is proportional to the product of the third root of the volume V_h of the wire helix and the cylindrical volume $(= 2\ell\pi a^2)$ containing the helix.

As a numerical example, one can assume that: $a = 1\,[cm]$, $b = .2\,[cm]$, $\ell = 1.35\,[cm]$, $\lambda = 10\,[cm]$, and $N = .3\,[\#/cm^3]$. From (A.10) the lower bound is found to be $|\gamma| \geq .01\,[\Omega^{-1}]$.

REFERENCES

[1] D. F. Arago, *Mém. Inst.*, **1**, 93 (1811)

[2] J. B. Biot, *Mém. Inst.*, **1**, 1 (1812)

[3] J. B. Biot, *Mém. Acad. Sci.*, **2**, 41 (1817)

[4] J. B. Biot, *Mém. Acad. Sci.*, **13**, 93 (1838)

[5] J. B. Biot, *Bull. Soc. Philomat.*, 190 (1815)

[6] A. Fresnel, *Oeuvres*, **1**, 738 (1822)

[7] L. Pasteur, *Ann. de Chim. et de Phy.*, **24**, 442 (1848)

[8] K. F. Lindman, *Ann. d. Phy.*, **63**, 621 (1920)

[9] K. F. Lindman, *Ann. d. Phy.*, **69**, 270 (1922)

[10] W. H. Pickering, Experiment performed at Caltech (1945)

[11] T. M. Lowry, *Optical Rotatory Power*, Dover Edition, New York (1964)

[12] M. Born, *Physik. Zeits.*, **16**, 251 (1915)

[13] C. W. Oseen, *Ann. d. Physik*, **48**, 1 (1915)

[14] Gray, *Phys. rev.*, **7**, 472 (1916)

[15] W. Kuhn, *Zeits. f. Physik. Chemie*, **B4**, 14 (1929)

[16] E. U. Condon, Altar and H. Eyring, *J. Chem. Phys.*, **5**, 753 (1937)

[17] E. U. Condon, Theories of Optical Rotatory power, *Rev. of Modern Phys.*, **9**, 432 (1937)

[18] D. L. Jaggard, A. R. Mickelson and C. H. Papas, On Electromagnetic Waves in Chiral Media, *Appl. Phys.*, **18**, 211 (1978)

[19] J. A. Stratton, *Electromagnetic Theory,* McGraw-Hill, New York (1941)

[20] C. H. Papas, *Theory of Electromagnetic Wave Propagation,* McGraw-Hill, New York (1965)

[21] A. Sommerfeld, *Optics,* Academic Press, New York (1954)

[22] M. Born, E. Wolf, *Principles of Optics,* Pergamon Press, New York (1980)

[23] H. C. Chen, *Theory of Electromagnetic Waves,* McGraw-Hill, New York (1983)

[24] S. Bassiri, N. Engheta, and C. H. Papas, Dyadic Green's Function and Dipole Radiation in Chiral Media, *Alta Frequenza,* **LV - N. 2,** 83 (1986)