Chapter 4

Numerical Study: Time-Reversed Reciprocal Method and Damage Detection Method for Weld Fracture

A numerical study is performed to gain insight into applying the proposed method of detecting high-frequency dynamic failure in steel moment-resisting frame buildings that is outlined in Chapter 2. The method relies on a prerecorded catalog of impulse response functions for instrumented buildings. Structural failure during a seismic event is detected by screening continuous data for the presence of waveform similarities to each of the cataloged building responses.

The motivation for this method is described by Kohler et al. (2009). Weld fracture is represented as an opening tensile crack, with a step-like time history. Following the body-force equivalent source method in seismology, the weld fracture is approximated as a localized region that experiences very large elastic tensile strains, and a seismic moment tensor (for a point tensile crack) is used to characterize the source. The response of the structure is given by convolution of the forcing term with the spatial derivative of the structure's Green's functions. If the response of the structure is represented as a sum of rays, each traveling with a constant velocity, then the spatial derivative of the Green's functions is equal to the time-derivative of the Green's function, multiplied by a constant. Under this assumption, the response of the structure is thus given by a linear combination of delta functions convolved with the Green's functions. This is a simplification of building behavior, and it must first be determined whether a Green's function can be used to approximate the response to bolt failure.

In the first part of the study, a steel frame's response to two loading cases, an impulse-like force and an opening crack tensile stress (Mode I crack), is computed on a temporal scale of microseconds. Results indicate that the velocity waveform generated by a tensile crack can be approximated by the velocity waveform generated by an impulse-like force load applied at the proper location. These results support the idea of using a nondestructive impulse-like force (e.g. hammer blow) to characterize the building response to high-frequency dynamic failure (e.g. weld fracture). However, the method may not be robust to noise, and may be better suited for damage localization.

In the second part of this numerical study, a time-reversed reciprocal method is applied to the frame. An impulse-like force is applied to a beam column connection in a linear elastic steel frame, and the resulting displacements are then used to determine the absolute time and location of the initially applied force. Time-reversal methods have seen success in other fields as well as for plate-like structures, where the presence of flexural waves renders perfect recovery impossible, but have yet to be applied to full-scale civil structures (Fink, 1992; Tromp et al., 2005; Wang et al., 2004).

4.1 Comparison of Structural Response to Two Different Source Conditions

The responses of a steel frame to two different loading cases, an impulse-like force and an opening crack tensile stress, both shown in Figure 4.1, are compared to determine whether the waveform generated by the nondestructive source can be used to approximate the waveform generated by the structurally damaging source. First, the response of a two-story one-bay steel frame to an impulse-like force applied to a beam-column connection is computed. A cross section of the three-dimensional steel model is shown in Figure 4.3. Each beam and

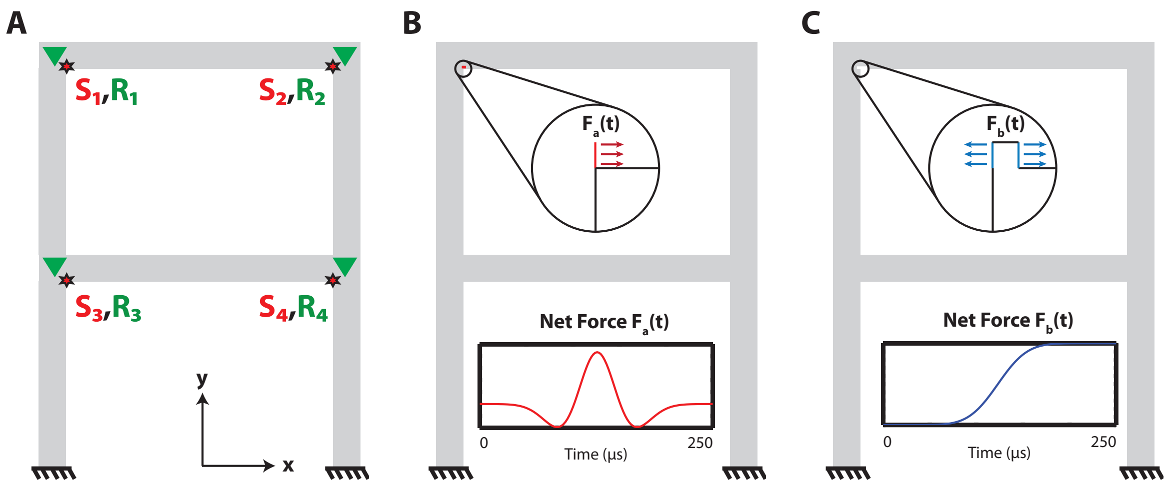

Figure 4.1: **Numerical Setup. A.** Receiver and source locations on the frame. **B.** Impulse-like force using Ricker wavelet force-time history applied to top left beam-column connection in an unnotched frame, and **C**. Opening crack tensile stress using error function force-time history applied to the same connection in a notched frame.

column has a square cross section of length 0.5 m. The model parameters are governed by linear elastic material properties of A36 structural steel ($E = 200GPa$, $\mu = 80GPa$, $\rho = 7850kg/m^3$), which correspond to shear and compression wave velocities of $c_s = 3.2km/s$ and $c_p = 5.6km/s$. The hex 8 mesh elements have a discretization length of 2.5 cm; the total time is 4 ms with a time step of 2 μ s. CUBIT is used for mesh generation, PyLith for physics code, and ParaView for visualization (Aagaard et al., 2008, 2013). A square notch is introduced in the opening crack tensile stress case, to simulate crack initiation at the beam-column connection. The square notch has a length of 0.05 m, consistent with the dimensions used for the unnotched frame.

Resulting displacements and velocities are recorded at four receivers located along the central cross section of the frame, shown in Figure 4.1. The simulation is repeated at each of the four source locations for both a force impulse and a tensile crack.

As seen in Figure 4.2, the displacement records generated by using the nondestructive source differ significantly from the displacement records generated by using the structurally damaging sources, primarily due to the static offset created across the notch. The two sets of velocity records are more similar to each other than are the two sets of displacement records. The similarities between force impulse and tensile crack velocity waveforms underscore the fact that, for traveling elastic waves, the strains are proportional to their associated particle velocities, regardless of the source mechanism. Thus, the template is created using the velocity record.

4.1.1 Stacked Cross-Correlation Values

A stacked cross-correlation method is used to determine the similarity of velocity waveforms generated by a force impulse at source location S_k and a tensile crack at source location S_l. The summarized method follows:

1. A set of velocities $\{v_1^k(t), v_2^k(t), v_3^k(t), v_4^k(t)\}$ are recorded for a force impulse applied at source location S_k, where $v_i^k(t)$ is the three-component velocity vector recorded at the i^{th} receiver. Due to symmetry and the fact that the receivers are located along the central cross section of the frame, the z-component of the velocity vector is zero.

2. The set of envelopes $\{e_1(t), e_2(t), e_3(t), e_4(t)\}$ is computed using the velocities. The magnitude of each 3-component envelope is passed through a low-pass filter to produce a set of scalar functions of time $e_1^k(t), e_2^k(t), e_3^k(t), e_4^k(t)$. These records are archived as our template signals for damage caused at the k^{th} source location. The duration of our template is time T.

3. Similarly, a set of scalar filtered envelopes for a tensile crack at the lth source location Sl are computed: $\{\tilde{e}_1(t), \tilde{e}_2(t), \tilde{e}_3(t), \tilde{e}_4(t)\}$.

4. For simplicity, pad each record $\{\tilde{e}_1(t), \tilde{e}_2(t), \tilde{e}_3(t), \tilde{e}_4(t)\}$ with a duration of T zeros at both the beginning and end, and compute the cross-correlation value for each receiver

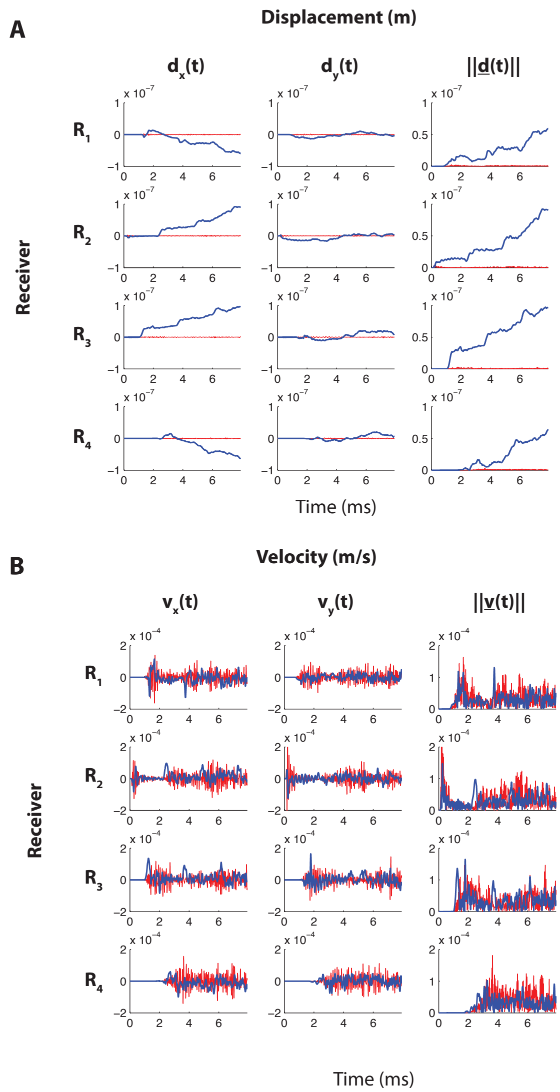

Figure 4.2: **High-Frequency Seismograms A.** Each type of source (impulsive force - red and tensile stress - blue) is applied at source location S_1. Displacements differ significantly between the two cases. **B.** Velocities provide a better agreement, and polarity differences are improved by taking the absolute value or magnitude of the record. Due to symmetry, $d_z = v_z = 0$.

Force Impulse Source Location	Tensile Crack Source Location S_1	S_2	S_3	S_4
S_1	**0.92**	0.78	0.79	0.84
S_2	0.78	**0.92**	0.84	0.79
S_3	0.77	0.92	**0.91**	0.81
S_4	0.88	0.77	0.81	**0.91**

Table 4.1: **Numerical Steel Frame: Maximum Stacked Cross-Correlation Values.** The response (velocity) of the frame to a hammer blow and to an opening crack are computed, and the envelopes are calculated. The cross-correlations are computed and stacked over the number of receivers, and the maximum values of the stacked cross-correlations are shown here.

location as given by

$$C_i^{kl} = \frac{\max_{t\in(-T,2T)} \int_0^T e_i^k(\tau)\tilde{e}_i^l(t+\tau)\, d\tau}{\sqrt{\int_0^T (e_i^k(\tau))^2\, d\tau \int_0^T (\tilde{e}_i^l(t+\tau))^2\, d\tau}}. \tag{4.1}$$

5. Compute the stacked cross-correlation value by summing over all four receiver locations to obtain

$$C^{kl} = \frac{1}{4}\sum_{i=1}^{4} C_i^{kl}. \tag{4.2}$$

The maximum value of C^{kl} occurs near time t = 0, and is recorded in the k^{th} row and l^{th} column in Table 4.1 below. Correlation values are highest when the location of the tensile crack and the location of the force impulse are the same. However, as the off-diagonal values are also close to unity, the method may not be robust to noise, and may be better suited for damage localization.

4.2 A Time-Reversed Reciprocal Method

A time-reversed reciprocal method is applied to demonstrate that the location of a nondestructive impulse-like force can be determined by using the numerically computed displacement records to time-reverse and retransmit the signal. First, the response of a two-story one-bay steel frame to an impulse-like force applied to a beam-column connection is computed. A cross section of the three-dimensional steel model is shown in Figure 4.3. Each

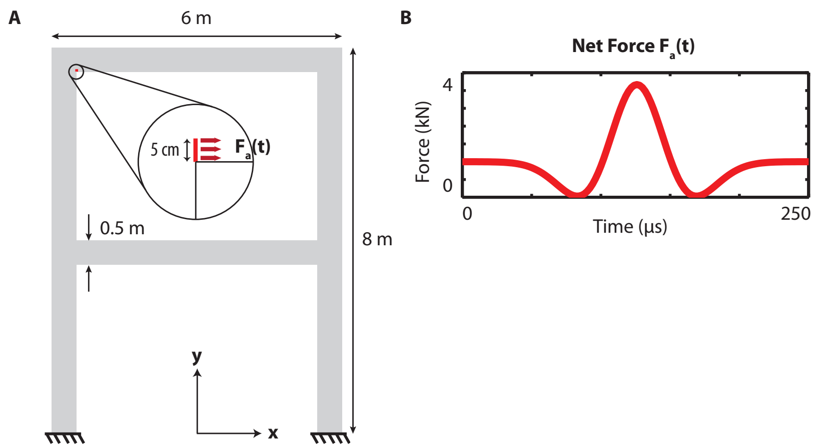

Figure 4.3: **Numerical Setup for Steel Frame.A.** Impulse-like force is applied at beam-column connection in a direction parallel to the x-axis. **B.** Ricker wavelet is used for time-force history.

beam and column has a square cross section of length 0.5 m. The model parameters are governed by linear elastic material properties of A36 structural steel ($E = 200GPa$, $\mu = 80GPa$, $\rho = 7850kg/m^3$), which correspond to shear and compression wave velocities of $c_s = 3.2km/s$ and $c_p = 5.6km/s$. The hex 8 mesh elements have a discretization length of 2.5 cm; the total time is 4 ms with a time step of 2 μs.

4.2.1 Forward Simulation

The response of the steel frame to an impulse-like force applied to a beam-column connection is computed. As can be seen in Figure 4.3, the force is applied along the positive x-axis to the close-up section of the connection. The total force is distributed proportionally over nodes according to the amount of surface area contained by each node. The force-time history is a Ricker wavelet. Waves propagate away from the location of the source, reflecting off the edges of the frame, as shown in Figure 4.4 below. Resulting displacements are recorded at the twelve receiver locations approximately evenly spaced along the central cross section of the frame. A representative sample of displacements is provided in Figure 4.5.

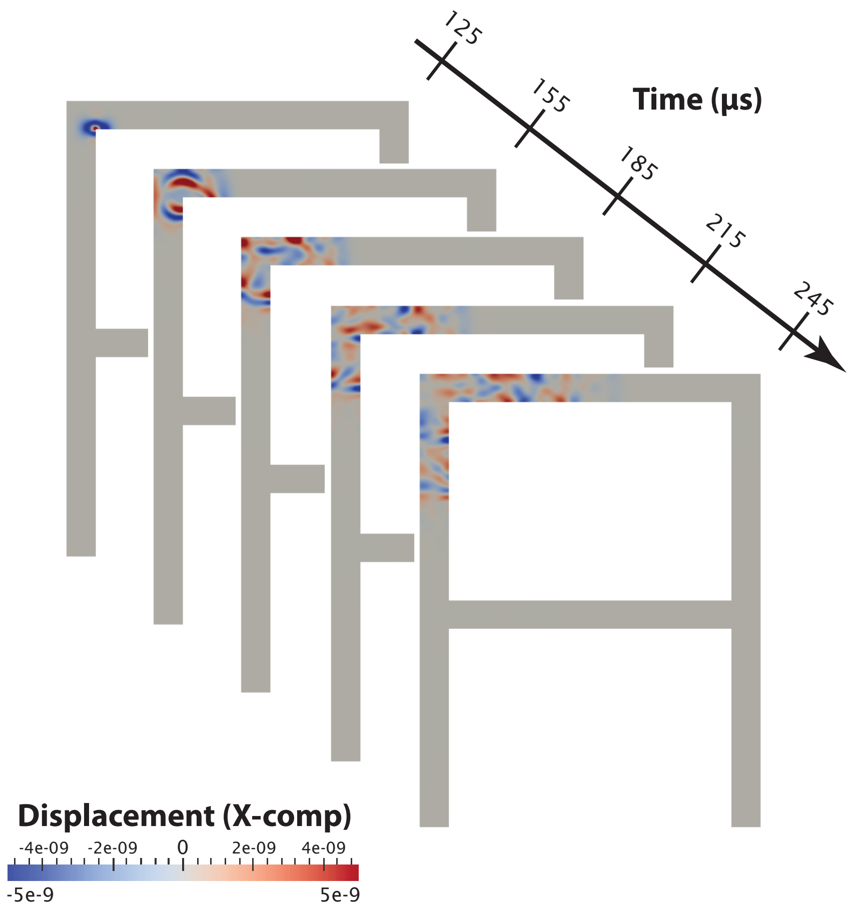

Figure 4.4: **Forward Simulation: Response of Steel Frame to Hammer Blow.** The response of the frame to a hammer blow applied at the upper left corner is computed.

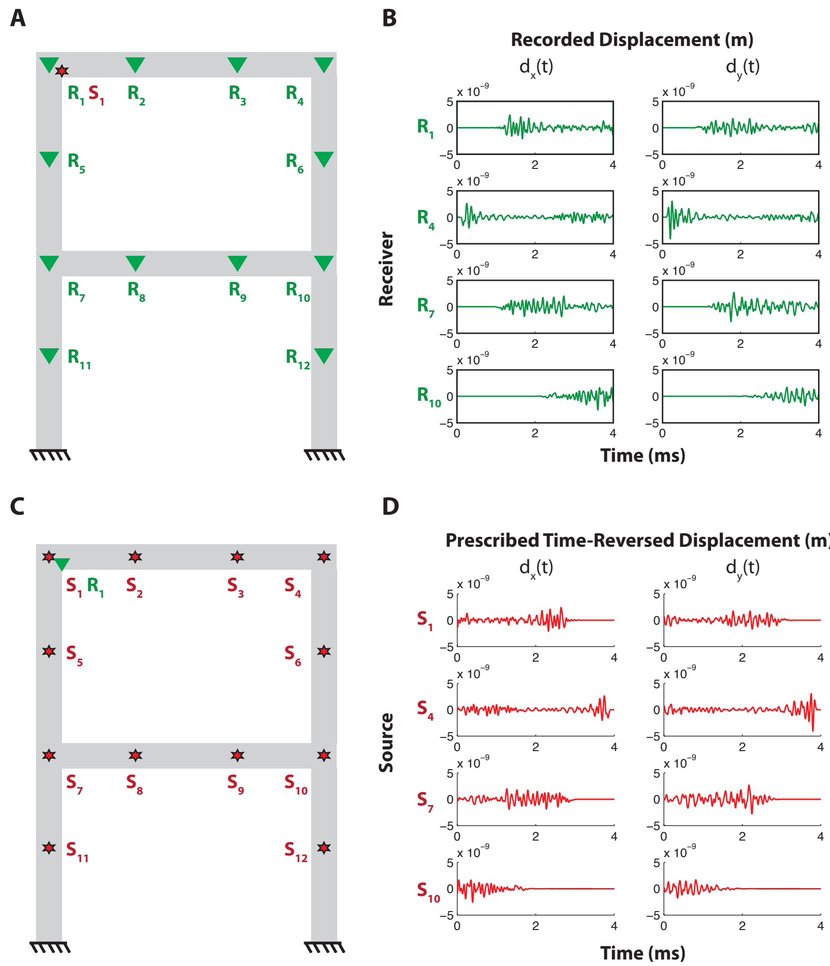

Figure 4.5: **Receivers, Sources, and Displacements A.** Receiver locations and examples of recorded displacements for forward simulation.**B.** Source locations and corresponding prescribed time-reverse displacements for reverse simulation. Due to symmetry, $d_z(t) = 0$.

4.2.2 Reverse Simulation

Following the time-reversed reciprocal method, the receiver and source locations are interchanged, and each of the displacement records is time-reversed and applied at the respective new source location as prescribed Dirichlet conditions, as shown in Figure 4.5 (b) above. The retransmitted signal propagates through the frame, and the waves generated by the twelve new source locations interfere constructively to focus at the original source location S_1, where the nondestructive load was applied, and at the correct time. To simplify timing, the reverse simulation begins at -4 ms and ends at 0 ms, with waves focusing on the beam-column connection at the correct time of -125 μs. Thus, by using a time-reversed reciprocal method, the recorded displacements are used to successfully determine the absolute time and location of the original applied force.

4.3 Conclusions

To numerically test a method for damage detection, a steel frame's response to two loading cases, an impulse-like force and an opening crack tensile stress (Mode I crack), was computed on a temporal scale of microseconds. It was found that the velocity waveform of a tensile crack can be approximated by the velocity waveform of an impulse-like force applied at the same beam-column connection of a steel frame. The results support the use of waveform cross-correlation using a pre-event catalog of impulse response function templates to determine the location and time of occurrence of a subsequent fracture recorded on a network of vibration sensors. However, the damage detection method may not be robust in a real setting, and the method may be better suited for damage localization.

A time-reversed reciprocal method was applied to a two-story one-bay numerical steel frame, as a proof of concept for applying the methodology to a complex structure such as a bridge or building. The signal was not fully recovered, but the location and application time of the impulse-like force were successfully determined. In applying this method to an actual structure, an accurate numerical model would first need to be developed for the structure

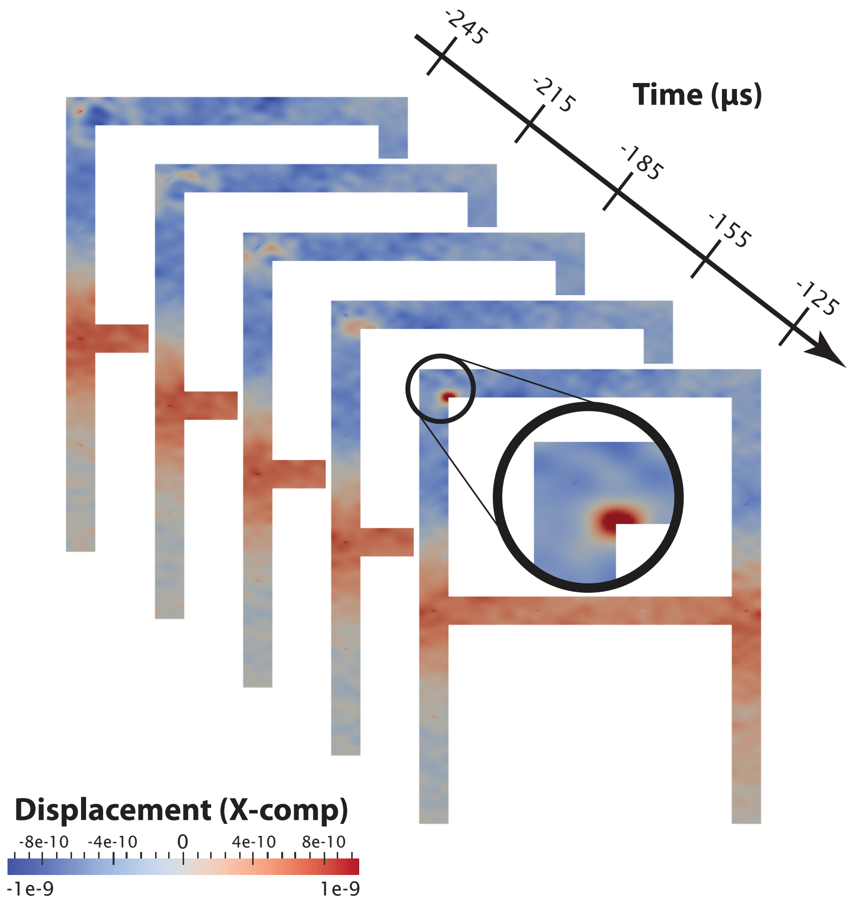

Figure 4.6: **Reverse Simulation: Response of Steel Frame to Prescribed Time-Reversed Displacements.** Waves generated at each of the twelve source locations converge at the correct location at the correct time. There appears to be a low-frequency artifact created whereby the upper story has a net negative displacement and the lower story has a net positive displacement. This might be due to the contribution of waves reflecting off the fixed base of the structure.

in the frequency range of interest, which could be both challenging and computationally-expensive. Once a satisfactory numerical model is obtained, the experimental data, which would contain some elements of noise, would be time-reversed and input to the model at the original receiver locations. It would be interesting to see this method applied to a full-scale experimental structure.