ON THE CONCENTRATION OF SPACE CHARGE IN THE VICINITY OF AN INSULATING SURFACE.

Thesis by

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The purpose of this investigation is the determination of the order of magnitude of the highest densities of space charge experimentally producible in the immediate vicinity of an insulating surface. The first section of the paper is a theoretical analysis of the problem while the second section describes some simple experiments undertaken in order to obtain the necessary data to get a numerical answer for some particular insulating surfaces.

1. We will first solve the problem of determining the statistically steady distribution of an electron atmosphere between a plane infinite emitting cathode and a plane parallel anode which reflects all electrons which strike it. La~~n~~ue* has discussed, in a general way, the statistically steady distributions of electrons between parallel plates. He integrates the equation $\frac{d^2V}{dx^2} = -4\pi\gamma e^{-\frac{\epsilon V}{kT}}$ generally, leaving the integration constants undetermined by the boundary conditions. The resultant solutions involve the integration constants in a complicated manner in which their physical significance is not clear. However, for our special problem, the actual solution involving the boundary values is very simple and can be obtained as follows:

Measure V, the potential, from the point where the potential gradient $\frac{dV}{dx}$ is equal to $\left(\frac{8\pi\gamma}{b}\right)^{1/2}$, if such a point exists. Here $b = \frac{\epsilon}{kT}$ where ϵ is the ionic charge, k is Boltzmann's constant, and T is the absolute temperature; γ is the charge density when $V=0$. Then the first integral of the equation

$$\frac{d^2V}{dx^2} = -4\pi\gamma e^{-\frac{\epsilon V}{kT}} \qquad (1)$$

which was obtained by eliminating the charge density between Poisson's equation and Boltzmann's distribution law is

$$\frac{dV}{dx} = \left(\frac{8\pi\gamma}{b}\right)^{1/2} e^{-\frac{bV}{2}} \qquad (2)$$

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* Jahrbuch der Elektronik 15 (1918) 205

The justification for assuming that the potential gradient can take the above value at some point is that that assumption leads to the desired solution satisfying the proper boundary conditions.

The final integral of (1) is then

$$x + c = \left(\frac{1}{2\pi\gamma b}\right)^{1/2} e^{\frac{bV}{2}} \tag{3}$$

where c is an integration constant. If the distance is measured from the point where $V=0$ then $c = \frac{1}{(2\pi\gamma b)^{1/2}}$. Expressing V as a function of x gives the solution

$$V = \frac{2}{b} \log\left[(2\pi\gamma b)^{1/2} x + 1\right] \tag{4}$$

Now suppose there is a surface at x_1 of potential V_1 which freely emits positive ions, and that there is a surface at x_2 of potential V_2 which perfectly reflects all positive ions striking it without discharging them. Then the distance d between the surfaces is $x_1 - x_2$ and the potential difference M between the surfaces is $V_1 - V_2$. From the solution (4)

$$M = \frac{2}{b} \log \frac{(2\pi\gamma b)^{1/2} x_1 + 1}{(2\pi\gamma b)^{1/2}(x_1 - d) + 1}$$

or solving for x_1

$$x_1 = \frac{\left(e^{\frac{bM}{2}} - 1\right) - (2\pi\gamma b)^{1/2} d\, e^{\frac{bM}{2}}}{(2\pi\gamma b)^{1/2}\left(1 - e^{\frac{bM}{2}}\right)} \tag{5}$$

Let us take new variables $z = x - x_2$ and $\phi = V - V_2$ so that z is the distance from the reflecting surface and ϕ is the potential relative to that surface. Then (4) becomes

$$\phi = \frac{2}{b} \log\left[\left(\frac{e^{\frac{bM}{2}} - 1}{d}\right) z + 1\right] \tag{6}$$

which gives $\phi = 0$ at $z = 0$, and $\phi = M$ at $z = d$. Evidently this distribution will hold for electrons when the electronic charge and the potential difference M are both regarded as positive.

By differentiating (6) we get the potential gradient as a function of the distance

$$\frac{d\phi}{dz} = \frac{2}{bd} \frac{e^{\frac{bM}{2}} - 1}{\left(\frac{e^{\frac{bM}{2}} - 1}{d}\right) z + 1} \tag{7}$$

The potential gradient at the electron reflecting surfacei is

$$\frac{d\phi}{dz}(0) = \frac{2}{bd}\left(e^{\frac{bM}{2}} - 1\right) \tag{8}$$

and at the emitting electrode it is

$$\frac{d\phi}{dz}(d) = \frac{2}{bd}\left(1 - e^{-\frac{bM}{2}}\right) \tag{9}$$

The density of charge will be given by

$$\rho = \frac{1}{2\pi b d^2} \frac{\left(e^{\frac{bM}{2}} - 1\right)^2}{\left(\left(\frac{e^{\frac{bM}{2}} - 1}{d}\right) z + 1\right)^2} \tag{10}$$

from which it follows that the charge density at the reflecting surface is

$$\rho(0) = \frac{\left(e^{\frac{bM}{2}} - 1\right)^2}{2\pi b d^2} = \frac{b}{8\pi}\left(\frac{d\phi}{dz}(0)\right)^2 \tag{11}$$

and that the charge density at the surface of the emitting electrode is

$$\rho(d) = \rho(0)\, e^{-bM} \tag{12}$$

The total charge per unit area of the electrode surface is

$$Q = \int_0^d \rho\, dz = \frac{1}{\pi b d}\left(\cosh \frac{bM}{2} - 1\right) \tag{13}$$

The electrical capacity per unit area between the reflecting surface and the emitting surface is

$$C = \frac{dQ}{dM} = \frac{1}{2\pi d} \sinh \frac{bM}{2} \tag{14}$$

It is seen that the capacity increases very rapidly with the potential and decreases with the temperature.

Let the reflecting surface be an insulating medium, of dielectric constant κ

and thickness t, covering the surface of a conducting electrode at potential W. In order to find the distribution of space charge between the dielectric and the emitting surface we must determine M, the potential at the surface of the dielectric, in terms of W and then the distribution will be given as before.

The capacity per unit area of the dielectric is $C_D = \frac{\kappa}{4\pi t}$. The capacity of the space between the emitting surface and the dielectric is given by (14). The total capacity C_T is $1/(1/C + 1/C_D)$ which is

$$C_T \equiv \frac{dQ}{dW} = \frac{\kappa \sinh\frac{bM}{2}}{4\pi t \sinh\frac{bM}{2} + 2\pi \kappa d} \tag{17}$$

that is

$$\frac{1}{\pi b d}\frac{d}{dW}\left(\cosh\frac{bM}{2} - 1\right) \equiv \frac{\sinh\frac{bM}{2}}{\pi b d}\frac{d}{dW}\left(\frac{bM}{2}\right) = \frac{\kappa \sinh\frac{bM}{2}}{4\pi t \sinh\frac{bM}{2} + 2\pi \kappa d}$$

Integrating gives

$$W = \frac{4t}{\kappa}\left[\cosh\frac{bM}{2} - 1\right] + b\,d\,M \tag{18}$$

which gives the total potential exactly in terms of M. If M is greater than about .3 volts at ordinary temperatures and t is of an order greater than, roughly, 10^{-9} cm. and d is not extremely large, then to a close degree of approximation

$$W = \frac{2te^{\frac{bM}{2}}}{\kappa} \qquad \text{or} \qquad M = \frac{2}{b}\log\frac{\kappa W}{2t} \tag{19}$$

Also, under these conditions formulas (6), (7), (8), and (11) can be approximately written respectively as

$$\phi = \frac{2}{b}\log\left[1 + \frac{\kappa W z}{2td}\right] \tag{20}$$

$$\frac{d\phi}{dz} = \frac{2\kappa}{b}\,\frac{W}{2td + \kappa W z} \tag{21}$$

$$\frac{d\phi}{dz}(0) = \frac{2kW}{tbd} \tag{22}$$

and

$$\rho(0) = \frac{k^2 W^2}{4\pi t^2 b^2 d^2} \tag{23}$$

When W is expressed in volts as V and $n(0)$ is the particle density at the surface of the insulator then

$$n(0) = 1.5 \times 10^{-10} \frac{k^2 T^2 V^2}{t^2 d^2} \tag{24}$$

It is seen that the charge density that can be reached just before the dielectric breaks down is proportional to the square of the dielectric strength of the insulating film as practically the whole drop V is across the film. The best insulators in bulk have a dielectric strength of about 3 x 10^6 volts per cm. For such an insulator such as thin mica with $\overset{k}{\cancel{K}} = 5$, $T = 300^\circ K$, and the emitting surface placed 1 mm. from the mica we get for the maximum particle density 1.35 x 10^{11} electrons per cm.^{3} It is known that thin insulating films have a greater dielectric strength than the bulk material. The following experiment gives some additional support to this observation from which it follows that the above maximum density can be somewhat increased without decreasing d below 1 mm. or increasing the temperature.

2. Experimental. The breakdown voltage of an aluminum oxide film on an aluminum plate was measured experimentally by a very simple arrangement. The apparatus consisted of a large evacuable bulb about 10 cm in diameter containing a hot filament over which the aluminum plate hung from an insulated wire suspension. The suspension was fastened to a winding cock which enabled the adjustment of the distance of the plate from the filament while the bulb was evacuated. The plate could be made positive with respect to the filament up to 150 volts. A microameter was placed in the plate circuit and the current which it indicated was plotted as a function of the plate potential.

The plot showed that the current increased linearly with the potential until a potential between 25 and 30 volts was reached, when the current suddenly increased greatly in an erratic manner. The average value for the samples tested was 27.4 volts. The slight linear increase was a leakage current as it was the same whether the filament was lit or not. Any affect of the distance of the plate from the filament was not noticeable in the measurements of the breakdown voltages.

If the Al_2O_3 film is assumed to be 10^{-6} cm. in thickness, then its dielectric strength is about 2.5×10^7 volts per cm. Taking ~~K~~ K to have the low value of 5 we get $n(0) = 2.2 \times 10^{13}$. At least this density was obtained experimentally. Under the optimum conditions it would seem that this value might be increased a thousandfold but not much more than that. These concentrations are far below those necessary in order to make the electrons behave as a degenerate Fermi gas so that the use of classical statistics in the derivation of the formulas of this paper seems justified.

The order of magnitude of the dielectric strength of the film seems to indicate that the nat~~i~~ure of the breakdown of the film is that of the field pulling electrons out of the Al_2O_3 molecules and that no ionization by collision takes place.

An unsuccessful attempt was made to see whether a layer of electrons concentrate close to an aluminum electrode 10 cms. in length would show any absorption of visible light. The experiment was preliminary and does not ~~prove that such an effect does not exist.~~ disprove the existence of such an effect.