Spinal Cord Injury Therapy through Active Learning

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To Caroline. We made it!
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Abstract

Therapy employing epidural electrostimulation holds great potential for improving therapy for patients with spinal cord injury (SCI) (Harkema et al., 2011). Further promising results from combined therapies using electrostimulation have also been recently obtained (e.g., van den Brand et al., 2012). The devices being developed to deliver the stimulation are highly flexible, capable of delivering any individual stimulus among a combinatorially large set of stimuli (Gad et al., 2013). While this extreme flexibility is very useful for ensuring that the device can deliver an appropriate stimulus, the challenge of choosing good stimuli is quite substantial, even for expert human experimenters. To develop a fully implantable, autonomous device which can provide useful therapy, it is necessary to design an algorithmic method for choosing the stimulus parameters. Such a method can be used in a clinical setting, by caregivers who are not experts in the neurostimulator’s use, and to allow the system to adapt autonomously between visits to the clinic. To create such an algorithm, this dissertation pursues the general class of active learning algorithms that includes Gaussian Process Upper Confidence Bound (GP-UCB, Srinivas et al., 2010), developing the Gaussian Process Batch Upper Confidence Bound (GP-BUCB, Desautels et al., 2012) and Gaussian Process Adaptive Upper Confidence Bound (GP-AUCB) algorithms. This dissertation develops new theoretical bounds for the performance of these and similar algorithms, empirically assesses these algorithms against a number of competitors in simulation, and applies a variant of the GP-BUCB algorithm in closed-loop to control SCI therapy via epidural electrostimulation in four live rats. The algorithm was tasked with maximizing the amplitude of evoked potentials in the rats’ left tibialis anterior muscle. These experiments show that the algorithm is capable of directing these experiments sensibly, finding effective stimuli in all four animals. Further, in direct competition with an expert human experimenter, the algorithm produced superior performance in terms of average reward and comparable or superior performance in terms of maximum reward. These results indicate that variants of GP-BUCB may be suitable for autonomously directing SCI therapy.
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