

# The Short-Timescale Behavior of Glacial Ice

Thesis by  
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This thesis is dedicated to William (Bill) Bing  
and Dr. Jennifer Howes. Without their unending  
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## ABSTRACT

Glaciers are often assumed to deform only at slow (i.e., glacial) rates. However, with the advent of high rate geodetic observations of ice motion, many of the intricacies of glacial deformation on hourly and daily timescales have been observed and quantified. This thesis explores two such short timescale processes: the tidal perturbation of ice stream motion and the catastrophic drainage of supraglacial meltwater lakes. Our investigation into the transmission length-scale of a tidal load represents the first study to explore the daily tidal influence on ice stream motion using three-dimensional models. Our results demonstrate both that the implicit assumptions made in the standard two-dimensional flow-line models are inherently incorrect for many ice streams, and that the anomalously large spatial extent of the tidal influence seen on the motion of some glaciers cannot be explained, as previously thought, through the elastic or viscoelastic transmission of tidal loads through the bulk of the ice stream. We then discuss how the phase delay between a tidal forcing and the ice stream's displacement response can be used to constrain *in situ* viscoelastic properties of glacial ice. Lastly, for the problem of supraglacial lake drainage, we present a methodology for implementing linear viscoelasticity into an existing model for lake drainage. Our work finds that viscoelasticity is a second-order effect when trying to model the deformation of ice in response to a meltwater lake draining to a glacier's bed. The research in this thesis demonstrates that the first-order understanding of the short-timescale behavior of naturally occurring ice is incomplete, and works towards improving our fundamental understanding of ice behavior over the range of hours to days.

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