## Chapter 2

# **Measurement Techniques**

### 2.1 Overview

Several measurements were performed on this prototype array in order to demonstrate its functionality and characterize it fully. Under dark conditions, when the detector is not exposed to light, the temperature dependence of the resonant frequency and quality factor for several pixels was measured. In addition the noise equivalent power (NEP) was obtained as a function of both temperature and readout power. Under illumination, the response of a pixel to a small change in illumination was measured, as well as the NEP and time constant associated with the time taken for the signal to return to baseline upon removal of the radiation source.

#### 2.2 Measurement setup

See Figure 2.1. The basic measurement is a change in amplitude and phase of a tone of a given frequency after interacting with the pixels of the array. A frequency tone is generated by the microwave source and split, with one branch sent straight to a mixer and the other to a transmission line coupled to each pixel, where absorption from a resonance may occur. The two branches are recombined by the mixer, and the two orthogonal components, which can be thought of as the real and imaginary components of the signal, are low pass filtered before being read in by a data acquisition card and saved to file.

Measurements were taken in several distinct runs. For each, the array was first cooled to 4 K via thermal coupling to a liquid helium bath. To achieve temperatures as low as  $\sim 1.5$  K, we pumped on the helium to lower its boiling point. By varying the pumping strength we could stabilize the temperature anywhere in this 1.5 - 4 K range. The data collection software was written in LabVIEW (Fig. 2.2) and served to automate the data collection process as each independent variable was varied.



Figure 2.1: Measurement setup. A frequency tone generated by a microwave source is split, with one component traversing the transmission line near the detector pixels, and recombined by a mixer to retrieve orthogonal components of the signal, which are used to calculate amplitude and phase information.



Figure 2.2: Sample control panel and instrument programming diagram written to control the measurement electronics.

### 2.3 Fitting for the resonant frequency and quality factor

Sweeping over a range of frequencies, one can reconstruct the resonance profile for a given pixel. Using the standard scattering matrix description of the circuit, the transmitted raw data obeys the relation [6]:

$$S_{21} \equiv I + jQ = a(f)e^{j(\omega\tau + \varphi)} \left(1 - \frac{Q_r}{Q_c}\frac{e^{j\phi}}{1 + 2jQ_rx}\right),\tag{2.1}$$

where the fractional frequency shift x is given by

$$x = \frac{f - f_{resonance}}{f_{resonance}}.$$
(2.2)

In equation (2.1), a(f) represents the frequency-dependent amplitude of the signal if the resonance were absent (the baseline transmission) and  $e^{j(\omega\tau+\varphi)}$  is the cable delay term with  $\omega = 2\pi f$ ,  $\tau$  being the phase shift at f for the cable length present (the cable delay) and  $\varphi$  being the phase at zero frequency.  $Q_r = (Q_i^{-1} + Q_c^{-1})^{-1}$  is the total quality factor with  $Q_i$  being the internal quality factor and  $Q_c$  being the coupling quality factor, and  $\phi$  is an arbitrary phase due to wire bonding.  $S_{21}$  is the fraction of power transmitted. Note that Q, the imaginary part of  $S_{21}$ , is distinct from the various quality factors  $Q_r$ ,  $Q_c$  and  $Q_i$ .

The first steps in solving for the resonance parameters  $f_r$ ,  $Q_i$  and  $Q_c$  are to remove the leading factors: the baseline transmission and the cable delay term. By adequately sampling the frequencies near, but not in, the resonance, the cable delay term  $\omega \tau + \varphi$  may be fit to a line and the baseline transmission a(f) may be fit to a high degree polynomial (Fig. 2.3 (a), (b)).

Next, the resonance is fit to a circle in the IQ plane (Fig. 2.3 (c)) according to the equation

$$S_{21} = 1 - \frac{Q_r}{Q_c} \frac{e^{j\phi}}{1 + 2jQ_r x}$$

from which the resonant frequency and quality factors which produce the best fit are found.

While the details differ, this basic procedure and electronic setup was followed for all of the measurements I will describe.



Figure 2.3: Steps in the calibration process. (a) The uncalibrated data traces out a circle centered at the origin for frequencies far from resonance. The resonance is a smaller circle tangent to the large one. (b) Fitting the smooth variation in frequency of the baseline to subtract it from the resonance. (c) The calibrated data, after removing the cable delay and normalizing. The plotted points are equally spaced in frequency. Points at frequencies far from resonance cluster around (1,0). (d) The amplitude and (e) the phase of the calibrated transmitted signal, with the plotted points equally spaced in frequency.