Essays in Mechanism Design

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Abstract

This dissertation contains three essays on mechanism design. The common goal of these essays is to assist in the solution of different resource allocation problems where asymmetric information creates obstacles to the efficient allocation of resources. In each essay, we present a mechanism that satisfactorily solves the resource allocation problem and study some of its properties.

In our first essay, ”Combinatorial Assignment under Dichotomous Preferences”, we present a class of problems akin to time scheduling without a pre-existing time grid, and propose a mechanism that is efficient, strategy-proof and envy-free. The mechanism works without money or transfers of any kind. We also specify some situations where the computations required by the mechanism can be carried out efficiently and delineate how the mechanism can fail if one of the assumptions about agents preferences does not hold.

Our second essay, ”Monitoring Costs and the Management of Common-Pool Resources”, studies what can happen to an existing mechanism — the individual tradable quotas (ITQ) mechanism, also known as the cap-and-trade mechanism — when quota enforcement is imperfect and costly. This study is done in the context of a fishery, where the open-access to a common-pool resource (the fish) creates a well-known commons problem. Because quota enforcement is imperfect and costly, the classic result stating that ITQs lead to an efficient harvest of the fish stock is no longer true. We propose an adequate analogue of that statement, and prove that it holds as long as quota violation fines depend only on the absolute magnitude of the violations. Our result implies in particular that violation fines should not be based on the ratio of violations to quota held. We also provide an extensive analysis of the set of equilibria and temporary equilibria, including extensive comparative statics. Finally, we provide a first step in understanding the preferences of fishers

over different levels of monitoring and the total allowable catch (the cap), two design variables that must be set “correctly” if the fishery is to succeed. This analysis is significant because, among other reasons, it highlights how the initial quota endowment can affect preferences which in turn may impact the chosen the level of monitoring and the cap.

Our third essay, ”Vessel Buyback”, coauthored with John O. Ledyard, presents an auction design that can be used to buy back excess capital in overcapitalized industries. It is common for mismanaged fisheries to find themselves in a state where, there are too many boats and too little fish. One way out of the situation is to implement an ITQ program, but that is sometimes not politically feasible. Another solution that may be easier to implement is for someone to buy back the excess capital. The problem with traditional buyback solutions is that they often require significant subsidies from an outside source, typically the government. To avoid this financing problem, note that the retirement of the excess capital benefits the vessels that stay active, and thus we can auction the right to stay in the fishery and use the revenue from this auction to compensate those leaving the industry. Our essay proposes an auction design along those lines that is ex-post efficient and self-financing. If there are enough gains to be made by reducing the capacity of the fleet, our auction design also guarantees that all vessel owners have something to gain by participating in the auction, ensuring that our design is politically feasible. We also provide and analysis of our assumptions and results in the context of a specific fishery model for convenience.

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Chapter 1

Summary

The three essays in this dissertation are part of an exploding literature that applies the tools of mechanism design theory to the creation of institutions that can solve particular resource allocation problems. See Roth (2002) for an overview of some high-profile examples.

We can trace the modern roots of mechanism design theory back to papers writen in the 1960's by Leonid Hurwicz, William Vickrey, David Gale and Lloyd Shapley. Hurwicz (1960) introduced the idea of incentive compatibility (which was expanded in Hurwicz (1972)); Vickrey (1961) set the tone for modern research in auction theory including the statement and proof of the first revenue equivalence theorem; Gale and Shapley (1962) introduced the stable marriage problem and an elegant solution for it, the deferred acceptance algorithm, kickstarting the work on matching theory.

All these papers discuss mechanisms, which can be thought of institutions or "black-boxes" that take messages from economic agents as input and output allocations. To evaluate the performance of such mechanisms, we must make assumptions about the behavior of economic agents and compare the mechanism outcomes under that behavior with some pre-established performance standard. In other words, a mechanism is a solution to a problem of implementing a certain *performance standard* in a certain *environment* under a certain *behavior*.[1]

Let us illustrate these concepts with a classic example. Given the problem of allocating one object to n agents, we might want to allocate the object to the agent that values it the most; that

[1] See Hurwicz (1960), Hurwicz (1972), Mount and Reiter (1974), and Reiter (1977) for a thorough explanation.

is the description of our performance standard. To specify "the agent that values it the most" we need a way to compare utilities, so we restrict ourselves to environments with quasilinear utility functions that are linear in money. Finally, we would like the mechanism to be straightforward and nonmanipulable, that is, we would like our mechanism to induce a game where the agents have a dominant strategy that will lead to the agent with the highest value getting the object. That is what we mean by implementing the performance standard above in dominant-strategy behavior. Given these three components/requirements of the problem (environment, performance standard and behavior), the question is then whether there exists a mechanism that solves the problem. Vickrey (1961) answered this problem with a simple and powerful mechanism: the second-price auction. Vickrey's auction is defined by the following rules:

- Agents submit bids.
- Auctioneer assigns the the object to the highest bidder (breaking ties randomly).
- The winner (the highest bidder) pays the auctioneer an amount equal to the second highest bid.

It is not hard to verify that theses rules create a game to be played by the bidders, and that each bidder has a unique dominant strategy which is to bid his true value for the object.

Competitive markets can also be seen as mechanisms and Hurwicz's early papers contain extensive discussions about that. An important feature of competitive market mechanisms is that they have the potential to allocate resources in a more-or-less decentralized way, using only the price system to coordinate the decisions of different agents. This decentralization property theoretically opens the door for the Pareto-efficient allocation of a large number of resources to a large number of agents in a informationally efficient way. Indeed, it has been shown by Mount and Reiter (1974) and more recently by Nisan and Segal (2006) that to verify that an allocation of goods is efficient we need at least as many numbers as the number of prices that would exist in a competitive market.

Contrary to what the previous examples suggest, mechanisms do not have to involve money or transactions of any sort. Obvious examples are the multitude of voting mechanisms. For another example, imagine a parent of two children fighting for a slice of cake. That parent could do much worse than using the following time-honored solution: let one child cut the cake and the other one pick a slice, leaving the remaining slice for the child that cut the cake. Under mild assumptions, this is a simple mechanism without money that guarantees not only Pareto efficiency but also no-envy (both part of the performance standard) under Nash-equilibrium (or max-min) behavior. Indeed, it is easy to see that assuming the cake is homogenous and that more cake is always desirable, the only Nash equilibrium is for the first child to cut the cake exactly in half. See Brams and Taylor (1995) for more on that literature.

This quick overview of some classic results and examples puts us in the right place to explore the essays in this dissertation. Indeed, each one of the three essays in this dissertation touches on points discussed in the examples above.

In the essay "Combinatorial Assignment under Dichotomous Preferences", we present a mechanism that solves problems of the following type: imagine you have a chunk of time that you want to to assign to different individuals that would like to have an interval of that chunk. How do you make sure that you assign the largest number possible of individuals in an envy-free way? We answer that question in environments where agents care about obtaining an acceptable interval but do not care which particular interval they get. In this environment, our mechanism implements the performance standard under dominant-strategy behavior. A difficulty arises because, in contrast to the cake-cutting problem discussed above, the resources in this problem are not divisible, which makes it difficult to ensure fairness. However, we show how one can use randomization to allocate *lotteries* over the resource thereby restoring the possibility of a fair allocation. Furthermore, we show that the computations for this mechanism can be carried out efficiently. Note that the mechanism works in problems where the set of resources is less structured, but without computational efficiency guarantees.

In the essay "Monitoring Costs and the Management of Common-Pool Resources", we turn our attention to the solution of a commons problem with a market-based mechanism. It is well known that many common-pool resources become overused or degraded over time because of the incentives and dynamics of an open-access regime. Dales (1968) realized that in the context of water pollution, and proposed a market-based solution: assign usage quotas for the resources and let the agents in the economy buy and sell those quotas in a competitive market. In doing so, the market should lead to an efficient allocation of the quota in a decentralized way. This is sometimes known as a program of individual tradable quotas (ITQs). In our essaey, we lay down a model of a fishery and analyze the outcomes of an ITQ program when quota enforcement is costly and imperfect. In this setting, decisions about enforcement level should not be dissociated from other design decisions — like the total quota available or its initial distribution. To support those design decisions, we provide an extensive analysis of ITQ equilibria and full comparative statics for steady-state equilibria. To the best of our knowledge, this is the first time this analysis is carried out. We also provide an extension of the full-compliance result that states that an ITQ program leads to an efficient use of the fish stock. Relaxing the assumption of full compliance, we present a principal-agent model where the principal is a fishery owner and the agents are the fishermen. The principal chooses how to allocate quota among the fishermen and how much to invest in monitoring to set the enforcement level. Agents choose how much fish to catch in face of their quota and the enforcement level. We show that, while the first-best outcome is not incentive-compatible, second-best outcomes can be implemented by an ITQ program if, and generically only if the expected violation fines depend on catch and quota only through absolute violations. (as opposed to violations as a proportion of quota held, for example). Finally, we establish sufficient conditions for fishermen's preferences over small changes in enforcement to be single-peaked. We emphasize that even though the distribution of quota endowments does not affect the attained ITQ equilibrium directly, it may affect outcomes indirectly if fishermen can influence the process that sets the cap or enforcement levels — with or without quota trading.

In the essay "Vessel Buyback" — coauthored with John O. Ledyard — we turn our attention to rights-based policies as in the previous essay, but now using an auction to allocate permits for staying in the fishery. The problem is the following: in an open fishery, the competition for the limited fish stock can lead fishermen to a "race for fish" with suboptimally high capital investments. Buyback programs to reduce excess capacity in national fisheries have been oft-used, but seldom successful. Where they have successfully reduced excess capacity, the programs have come at a high cost, almost always in the form of governmental subsidies to buy out the excess capacity. These subsidies may even exceed the full gain in social surplus from the fishery that is, after all, the main purpose of the programs. While, in principle, the presence of excess capacity implies there are Pareto-improving allocations of fishing rights, which involve the removal of the highest cost or least efficient vessel capacity from the industry, the difficulty is in identifying the least efficient vessels and providing the incentives for their owners to be voluntarily bought out by the owners of vessels remaining in the fishery. Our essay explores, from a mechanism-design approach, the possibilities for and limits of buyback programs - specifically auctions - that are entirely self-financed. Our main result delimits conditions on the fishery (in terms of how quickly aggregate profits in the fishery increase as excess-capacity is removed) that allow an efficient, revenue-neutral (i.e. requiring no outside subsidies) auction design that will also satisfy voluntary participation (i.e. all a priori identified vessel owners will choose to participate in the auction.)

Chapter 2

Combinatorial Assignment under Dichotomous Preferences

2.1 Introduction

It is the same story every end of term: students' schedules for extra activities change, and cello teacher Vanessa Sullivan has to change her teaching schedule accordingly. Finding a schedule that somehow satisfies all students is a frustrating process that involves a large number of emails and stress, and in the end, nobody really knows how good the chosen schedule is.

Companies, government agencies and different organizations face the same kind of problem every day: how to schedule the use of a scarce shared resource. High-tech equipment, a modern conference room or a competent specialist are often expensive, or hard to find. In many cases, it is only sensible for an organization to acquire a limited amount of such a resource and divide it among its workers via time-sharing. Deciding how to assign time with the resource is the problem. The usual solution is analogous to the cello teacher's solution, with similar shortcomings: the parts involved communicate somehow and try to reach an agreement. Such negotiations can be very time consuming, and there is no guarantee that in the end an optimal solution will be found.

In principle, time shares of use of the resource can be assigned in any way. However, it is often the case that only a specific finite set of shares are assignable, be it for technological or institutional constraints. In those cases, time is an imperfectly divisible resource. If the set of shares is finite, then this scheduling is not fundamentally different from the combinatorial allocation problem, where bundles composed of a finite number of objects must be assigned to a finite number of agents. In this paper, we study all such problems under the umbrella of "imperfectly divisible resources."

When a resource is perfectly divisible, it is often called *cake*, and the problem of assigning shares of a cake to agents that have additive and continuous preferences (a non-atomic measure on the over some sigma-algebra of subsets of the cake) has been extensively studied. These additivity and continuity assumptions have led to the proof of existence of mechanisms with very strong efficiency, incentive and fairness properties. In the cake-cutting literature, actual procedures for such mechanisms can be found.

On the other hand, by not assuming that the resource is perfectly divisible, but keeping the assumption of additive preferences, Kojima (2009) has shown that for the combinatorial allocation problem, there is *no mechanism* that is ordinally efficient, envy free and weakly strategy-proof. In this paper, we instead obtain a positive result, by assuming *dichotomous preferences*, an idea inspired by Bogomolnaia and Moulin (2004), which obtain very strong results for the problem of assigning at most 1 object among a finite set of objects to a finite set of agents. The main difference between this paper and Bogomolnaia and Moulin (2004) is that agents may be assigned multiple objects, or, more abstractly, that agents may have preferences over shares/bundles that are not jointly feasible.

In the following, we examine how previous literature relates to our main findings: the characterization of efficient assignments, strategy-proof assignments, and a mechanism that is efficient, strategy-proof and envy free. We provide an example that shows that unlike in the case of Bogomolnaia and Moulin (2004), the existence of a Lorenz-dominant random assignment is not guaranteed. We analyze the running times of computing efficient assignments, and argue that even though worst-case scenarios are not efficiently computable, there are a number of reasons to believe that many real-world applications will yield efficiently computable cases. We end by analyzing what would happen if a mechanism designer used a mechanism that is tailored for dichotomous preferences with agents that have more general preferences. In these mechanisms, agents are only asked to report which shares are better than nothing, and we show that in that case there are preferences for which agents have an incentive to misreport.

2.2 Related Literature

Economists, mathematicians, computer scientists and social scientists in general have long been interested in a family of problems that can be classified as assignment problems: problems where agents have to be given "one share" of a resource that is not privately owned by them. In the following section we present earlier attempts and results on different variants of the assignment problem.

2.2.1 Concepts to Classify the Literature

The problem that we presented in the introduction was related to the literature of three different problems, usually referred to as the assignment problem, which I will refer to as the *house assignment problem*[1] and the *cake-cutting problem*. The first refers to problems such as assignment of houses or jobs, and the second to problems such as assignment of time or land, when those are perfectly divisible.

There are four types of concepts that we will use to organize the literature around the assignment problem: first, the performance criteria to distinguish between "good" and "bad" assignments, namely Pareto efficiency, "fairness" and incentives to truth-telling; second, the divisibility of the resources; third, the continuity of preferences; and fourth, whether or not compensatory transfers are allowed.

2.2.2 Linear Programming and the Optimal Assignment Problem

A typical problem in the early literature was the *personnel assignment* problem: if there are n jobs and n workers *whose productivity at each of the n jobs is known*, what is the match of jobs and workers that maximizes the firm's profits, and how can we compute it? This problem illustrates the focus

[1] Also known as the one-sided one-to-one matching problem

on some measure of efficiency, such as profits, and thus this family of problems was called the *optimal assignment* problem. Computational aspects of the problem were emphasized, with no concerns about fairness or incentives. Nonetheless, the problem was still of considerable economic interest. Problems like the optimal assignment of production facilities to different locations, and whether or not that can be achieved by a price system Koopmans and Beckman (1957) do not depend on any incentive-compatibility constraints, and do not call for fairness concerns.

The literature on the optimal assignment problem started in the early fifties as an application of nascent linear programming methods. The techniques used relied heavily on graph-theoretic arguments, initially developed by Hungarian mathematicians in the thirties. König-Hall's marriage theorem, the Birkhoff-von Neumann decomposition theorem, and Dantzig's simplex method were at the foundation of the results obtained at that time. See Kuhn (1955) for a list of early references, Berge (2001) and Papadimitriou and Steiglitz (1998) for textbook treatments.

As with the literature on the optimal assignment problem, we will be concerned with maximizing some measure of efficiency. However, we differ in a few key points: unlike houses, the resources we are assigning are divisible, and; we will be concerned with issues of fairness, efficiency, and incentive compatibility. Finally, while we try to present constructive solutions as much as possible, we will not attempt to describe and prove theorems about the performance of algorithms to implement our solutions; our focus will be on the properties of the solutions. We wait until the conclusion section to point out references that are relevant for computing some of our proposed solutions.

2.2.3 Fair Ways to Cut a Cake

The problem is how to fairly allocate shares of a divisible and heterogeneous good, usually incarnated in the metaphor of a cake; one can cut a cake however one pleases, but different people might like different parts of the cake.

It is hopeless to try to summarize the literature on fair division in a few pages, so we will highlight some contributions that point to different directions in the literature and that are in some way related to our approach. The problem of defining fairness goes back to Plato; see the first chapters in Moulin (2004) for a concise and modern overview. Whenever it is not crucial, we will refer to "fairness" without specifying precisely which notion of fairness we are talking about.

Following the work of Steinhaus (1948) , mathematicians have been drawn to the problem of fair division. That interest developed into what is often called the *cake-cutting* literature. See Barbanel (2005), Brams and Taylor (1996), and Robertson and Webb (1998) for textbook treatments. A classic performance requirement for a solution is that each agent is assigned at least his/her "fair share". Another trademark of this literature is the assumption that utilities are measures, that is, additive set functions. The focus is on fairness and efficiency, not incentives. See Berliant, Thomson, and Dunz (1992) for an axiomatic treatment that includes incentives and uses Bewley's Bewley (1972) classic result on the existence of general equilibrium in infinite dimensional economies to prove the existence of a group-envy free and efficient allocation.

As with the cake-cutting problem, we are interested in the division of a heterogeneous and divisible good and we also care about fairness, in particular envy freeness. However, we differ from that literature because we do not assume preferences are continuous, , which connects us to the next branch of the literature.

2.2.4 House Assignment Problems and Extensions

The classic house exchange market was introduced by Shapley and Scarf Shapley and Scarf (1974) in 1974: each agent has one house, and can only have one house. Additionally, agents have preferences over houses. How can we find an efficient allocation? Given the ownership structure, is the core of this game nonempty? If so, how can we find it?

The canonical answer was given by Gale's ***top trading cycles (TTC)*** algorithm.[2] Assuming preferences are strict, pick an arbitrary agent and let him point to his favorite house; the owner in that house in turn points to his favorite house, so on and so forth. Because the number of houses/agents is finite, this problem will eventually come to a cycle. Let the agents in this cycle

[2]First presented in Shapley and Scarf's Shapley and Scarf (1974) paper, but attributed to Gale.

trade their houses according to their preferences. Remove these agents from the economy. Restart the procedure with the remaining agents. Cycles will be formed, and trades will occur until there are no more agents left, which signals the end of the procedure. The assignment obtained in the end is efficient, and it is the unique element in the core Roth and Postlewaite (1977). Moreover, TTC provides the basis for the following *strategy-proof* direct mechanism Roth (1982): assign each house to one agent; ask for the agents' preferences; apply the TTC procedure to obtain the final assignment.[3]

A related mechanism is the ***serial dictatorship***, or ***priority mechanism***, introduced by Satterthwaite and Sonnenschein Satterthwaite and Sonnenschein (1981): fix a strict priority order among the agents; ask for the preferences; give the agent at the top of the priority ranking his top choice, the second agent in the priority ranking his favorite object among the remaining ones, etc. This direct mechanism is strategy-proof, and if preferences are strict, it is also efficient.

It is clear that for every outcome of a TTC procedure, there exists a priority ranking of the agents such that the outcome of the corresponding priority mechanism is the same as the one given by the TTC. In this sense, TTC and priority mechanisms are "equivalent".[4]

Priority mechanisms or TTC procedures have great efficiency and incentives properties, but they can yield severely unfair outcomes. This is not a fault of the mechanism, but an inherent property of deterministic mechanisms for assignment problems with non-transferable utility. Think of the setting where all preferences are the same; any allocation is efficient, and someone will face the worst possible outcome.

One way to restore fairness, at least *exa ante*, is to allow for *random assignments*. For example, a *random priority* mechanism associates with each preference profile a probability distribution over priority mechanisms and assigns houses according to a mechanism drawn from this distribution; a TTC with random endowments assigns house-endowments randomly and then applies a TTC procedure. A random assignment can also be viewed as a mapping from agents to lotteries over

[3] Roth's Roth (1982) result does not require preferences to be strict, and then the mechanism has to be modified by requiring that ties in the preferences be broken by a fixed rule.

[4] It is a trivial fact that, for a given outcome of a priority mechanism, there is always an initial assignment and TTC procedure that leads to the same allocation; just make the initial assignment equal to the outcome of the priority assignment.

houses.

Abdulkadiroglu and Sönmez Abdulkadiroglu and Sonmez (1998) prove that, in the strict preference domain[5] random priority assignments are "equivalent" to TTC with random endowments. If the probabilities are chosen uniformly, both mechanisms are strategy-proof, *ex post* efficient, and fair in the sense of equal treatment of equals.

However, random priority assignments are still unambiguously undesirable in the following ways. First, there is no guarantee that for every preference profile there will be no envy among agents with respect to the lotteries they are assigned. Second, if agents have von Neumann-Morgenstern expected utilities, the outcome of these two mechanisms may be *ex-ante* inefficient, as conjectured by David Gale and proved by Zhou (1990). Third, Bogomolnaia and Moulin (2001) provide an example that shows that the outcome of a random priority mechanism may be first-order stochastically dominated by another assignment for all agents in the economy.

To address this problem, Bogomolnaia and Moulin Bogomolnaia and Moulin (2001) propose the concept of *ordinal efficiency* and the *probabilistic serial* mechanism that implements ordinally efficient assignments. An assignment is ***ordinally efficient*** when it is not first-order stochastically dominated by another assignment for all agents in the economy.[6] The ***probabilistic serial*** mechanism lets each agent "eat", with equal speed, shares of one unit of their favorite object; as soon as one object is fully eaten, the agents that were eating that object move to their second-best object, and the process is repeated. After all objects have been eaten, each agent will have eaten shares of different objects. The Birkhoff-von Neumann theorem then guarantees that there exists a random assignment that gives to each agent a probability of getting a certain object equal to the share he has eaten, and there are constructive procedures to accomplish this. The resulting random assignment is ordinally efficient and envy free; however, it is only weakly strategy-proof, that is, an agent cannot obtain an allocation that first-order stochastically dominates the probabilistic serial allocation by misreporting his preferences.

We want solutions to the cake assignment problem satisfying strong incentive, fairness, and

[5] Their results allow for indifferences, but the gist of their results is captured in the strict preference case.

[6] Alternatively, an assignment is ordinally efficient when there exists a utility profile such that the assignment is Pareto efficient for that utility profile McLennan (2002). See Manea (2008) for a constructive proof.

efficiency properties, such as the ones obtained by random priority or by the probabilistic serial mechanism in the house assignment problem. However, the divisional structure of the resource we are interested in is different: a cake is divisible, houses are not. We will see in examples that this feature completely changes the problem. In particular, efficient allocations in the housing problem always have someone obtaining his top choice; this will be no longer true in the cake assignment problem.

2.2.5 There and Back Again

Unfortunately, none of the mechanisms mentioned above work well in the case of the combinatorial assignment problem. Random priority mechanisms are known to be inefficient in the preferences of preferences with large indifference sets, even in the house assignment problem. In the case of strict preferences but in a combinatorial allocation problem, because one agent's allocation can block two agents' allocations, random priority can lead to allocations that allocate something to a small number of agents, which might be undesirable. And not even weak-strategy-proofness is possible Kojima (2009) if we also require the random assignments to be ordinally efficient and envy free.

The negative results above have a common feature: they try to solve the time assignment problem in a relatively large preference domain. [7]. In face of that difficulty, we will restrict our focus to preferences where, for each agent, the set of assignable shares is partitioned in a set of acceptable time slots and a set of unacceptable time slots. That is, there are only two indifference sets: the acceptable shares, and the non-acceptable ones. As we focus on mechanisms that satisfy voluntary participation, these preferences are equivalent to dichotomous preferences.

In the dichotomous preference domain, Bogomolnaia and Moulin Bogomolnaia and Moulin (2004) obtain very strong results for the house assignment problem: their *egalitarian solution* is efficient, group-strategy proof, envy free, and Lorenz dominant among all efficient assignments. Their solution is not directly applicable to our problem. The reason is the following: in the house

[7] Kojima's result about the impossibility of envy free weak strategy-proofness assumes additive preferences.

assignment problem, a dichotomous preference profile can be represented by a bipartite graph connecting agents to their acceptable houses. Efficient assignments are those that correspond to maximal matchings of this graph. *All* such matchings correspond to feasible assignments; in our case, where desired resources may overlap, some maximal matchings are not feasible.

On a final note, we must mention some other directions in the literature that attack similar problems, but with techniques or assumptions very removed from ours. The seminal paper of Hylland and Zeckhouser Hylland and Zeckhauser (1979) introduced the house assignment problem in the economics literature from the point of view of mechanism design. However, they worked with cardinal preferences, while we only work with properties that hold for all cardinal representation. Also in the domain of cardinal preferences, Ledyard, Noussair and PorterLedyard, Noussair, and Porter (1996) present a framework for dealing with a time allocation problem with capacities (like a many-to-one matching problem) in NASA's Deep Space Network. They recommend the use of an ascending-bid auction, the Adaptive User Selection Mechanism (AUSM) with tokens. They conduct some experiments to evaluate the performance of AUSM with tokens, AUSM with money and a random mechanism (the sequential dictator algorithm) and conclude that the AUSM with tokens is a better mechanism for situations with high level of conflict. We focus on mechanisms that do not depend on any sort of transfer, and thus the AUSM mechanism cannot be applied to our problem.

2.3 The Model

Consider the problem of assigning shares of a divisible resource to agents that only care whether or not they get an "acceptable" share or not. As we will focus on mechanisms that satisfy voluntary participation, there is no need to distinguish between the case where agents find unacceptable shares strictly worse than the empty set (which means getting nothing from the mechanism) and the case where they are indifferent between the empty set and unacceptable shares. Such preferences are characterized by one indifference set, the set of acceptable shares, and are called

dichotomous preferences.[8]

An instance of this problem is characterized by:

- a set X of ***resources***.

- a finite collection $\mathcal{F}$ of subsets of X, including the empty set $\emptyset$, that denote the assignable ***shares*** of the resource;

- a set $N = \{1, 2, \ldots, n\}$ of ***agents***;

- for each agent $i \in N$, an ***acceptable set*** $A(\succcurlyeq_i) \subset \mathcal{F}$.

We denote the set of all dichotomous preferences over $\mathcal{F}$ by `2Pref`, and we write $\succcurlyeq_i \in$ `2Pref` to indicate the preference relation of agent i that has A_i as the acceptable set. A preference profile $(\succcurlyeq_1, \ldots, \succcurlyeq_n)$ is denoted $\succcurlyeq$.

A ***deterministic assignment*** is a mapping $\mu : N \to \mathcal{F}$ subject to a feasibility restriction. This feasibility condition depends on the problem. In the case of the cello teacher, it would be

$$i \neq j \Longrightarrow \mu(i) \bigcap \mu(j) = \emptyset. \tag{2.1}$$

In the case of a set of resources that can be used by 5 people at the same time, like a room with 5 machines, the feasibility condition would be:

$$\sum_{i \in N} \mathbb{1}_{\mu(i)}(x) \leqslant 5 \qquad \forall\, x \in X, \tag{2.2}$$

where $\mathbb{1}_{\mu(i)} : X \to \{0, 1\}$ is an indicator function, assuming the value 1 when $x \in \mu(i)$ and the value 0 otherwise.

Note that if for every B, C in $\mathcal{F}$ we have $B \cap C = \emptyset$, then we are back to the classic *housing assignment problem* Shapley and Scarf (1974), Hylland and Zeckhauser (1979). We reserve the symbols μ and η for deterministic assignments, whose set we denote $\mathcal{M}$.

[8] As defined, for example, in Bogomolnaia and Moulin (2004).

We say a deterministic assignment ***respects preferences*** $\succcurlyeq_i$ if $\mu(i) \neq \emptyset \implies \mu(i) \in A_i$. We denote $r(\succcurlyeq_i) \subseteq \mathcal{M}$ the set of all deterministic assignments that respect $\succcurlyeq_i$, and we define $r(\succcurlyeq) = \bigcap_{i \in N} r(\succcurlyeq_i)$ the set of all deterministic assignments that respect all the preferences in $\succcurlyeq$.

We need to define some more notation. For any function f, and binary relation R in the codomain of f, we write $[fRz]$ to indicate those elements x in the domain of f such that $f(x)Rz$, where z is in the codomain of f. For example, $[P > 0]$ is the support of the probability distribution P and $[\mu \neq \emptyset]$ is the set of all ***assigned agents***, while $[\mu = \emptyset]$ is the set of ***unassigned agents***.

A solution to an *instance* of our assignment problem is a ***random assignment***: a probability distribution over deterministic assignments, that is, an element of $\triangle\mathcal{M}$. A solution to our assignment problem is a ***direct mechanism*** $g : \texttt{2Pref}^n \to \triangle\mathcal{M}$, which maps every preference profile $\succcurlyeq$ to a random assignment $g(\succcurlyeq) \in \triangle\mathcal{M}$. We want a solution to satisfy some minimum performance requirements, namely *efficiency*, *voluntary participation*, *strategy-proofness* and *fairness*. Voluntary participation is obtained by focusing on mechanisms that only put positive probability on assignments μ that respect preferences, that is, $g(\succcurlyeq)(\mu) > 0 \implies \mu \in r(\succcurlyeq)$ for all $\succcurlyeq \in \texttt{2Pref}^n$. We define and characterize the other concepts in the next sections.

2.4 Efficiency

We say that a deterministic assignment η ***Pareto dominates*** another assignment μ when $\eta(i) \succcurlyeq_i \mu(i)$ for every $i \in N$ and $\eta(i) \succ_i \mu(i)$ for at least one $i \in N$. We say that μ is Pareto efficient, or simply an ***efficient assignment***, if there is no other assignment that Pareto dominates μ.

With dichotomous preferences, efficient assignments are in some sense maximal. Define the preorder $\supseteq$ on the set of assignments $\mathcal{M}$ as follows:

$$\mu \supseteq \eta \Leftrightarrow [\mu \neq \emptyset] \supseteq [\eta \neq \emptyset],$$

in which case we say μ ***contains*** or ***includes*** η. It should be clear that the $\supseteq$ symbol on the left side is the one we are defining, and the one on the right side refers to the usual set inclusion.

The relations $\supsetneq$, $\subseteq$ and $\subsetneq$ on $\mathcal{M}$ are defined as one would expect. The following lemma is an immediate conclusion of these definitions.

Lemma 1. *Given two deterministic assignments $\mu, \eta \in r(\succcurlyeq)$, we say μ Pareto dominates η if and only if $\mu \supsetneq \eta$. An assignment $\mu \in r(\succcurlyeq)$ is efficient if and only if it is $\supseteq$-maximal in $r(\succcurlyeq)$: for every assignment $\eta \in r(\succcurlyeq)$, $\mu \supseteq \eta$ whenever $\eta \supseteq \mu$.*

Define $f : \mathcal{M} \to \mathbb{R}$ to be $\supseteq$-***-monotonic*** if $f(\mu) \geqslant f(\eta)$ whenever $\mu \supseteq \eta$.

Corollary 1. *If an assignment $\mu \in r(\succcurlyeq)$ maximizes a function f which is $\supseteq$-monotonic, then μ must be efficient.*

We say that a random assignment ψ ***Pareto dominates*** a random assignment ϕ when the probability of getting something acceptable in ψ is at least as great as in ϕ for every agent $i \in N$, and strictly greater for some agent $i \in N$. Our definition is motivated by the fact that for any expected utility representation of a binary preference, the utility of a lottery ψ is greater than the utility of a lottery ϕ if and only if the probability of getting something acceptable in ψ is greater or equal than the probability of getting something acceptable in ϕ. Therefore, we use the probability of obtaining an acceptable share as the canonical definition of the utility of a random assignment.[9]

We say that ψ is an ***efficient random assignment*** if there is no other random assignment that Pareto dominates ψ. We say that g is an ***efficient mechanism*** if $g(\succcurlyeq)$ is an efficient random assignment for every preference profile $\succcurlyeq \in \texttt{2Pref}^n$.

It is easy to see that a random assignment $\phi \in \triangle\mathcal{M}$ is ***utilitarian efficient***—that is, it maximizes the sum of the utilities—if and only if its support is composed of deterministic assignments that respect preferences and that assign the largest possible number of agents.

2.5 Incentives to Truth-Telling

To keep the notation simple, we will extend an agent's preference relation over the set of shares $\mathcal{F}$ to the set of all random assignments $\triangle\mathcal{M}$ in the only natural way: for $\psi, \phi \in \triangle\mathcal{M}$, we say

[9] Our definition of efficiency is then the standard notion of *ex ante* efficiency for random assignments setting the utility of an acceptable share as 1 and the utility of an unacceptable share as 0.

$\psi \succcurlyeq_i \phi$ if and only if the probability that that i gets an acceptable share under ψ is at least as large as the probability that i gets an acceptable share under ϕ. Similarly, we say $g(\succcurlyeq'_i, \succcurlyeq_{-i}) \succcurlyeq_i g(\succcurlyeq''_i, \succcurlyeq_{-i})$ when $g(\succcurlyeq'_i, \succcurlyeq_{-i})(r(\succcurlyeq_i)) \geqslant g(\succcurlyeq''_i, \succcurlyeq_{-i})(r(\succcurlyeq_i))$. Remember that $g(\succcurlyeq) \in \triangle\mathcal{M}$ is a random assignment, that is, $g(\succcurlyeq)(\mu)$ is the probability under $g(\succcurlyeq)$ that the assignment μ will be chosen.

We say that a mechanism is ***strategy-proof*** if for every agent there is never incentive for unilateral manipulation:

$$g(\succcurlyeq) \succcurlyeq_i g(\succcurlyeq'_i, \succcurlyeq_{-i}) \quad \forall \succcurlyeq \in \texttt{2Pref}^n, \forall \succcurlyeq'_i \in \texttt{2Pref}.$$

In the following, we will need the following notion: given preferences $\succcurlyeq_i$ and $\succcurlyeq'_i$, we say that $\succcurlyeq_i$ is ***less flexible than*** $\succcurlyeq'_i$ when $A(\succcurlyeq_i) \subseteq A(\succcurlyeq'_i)$. Alternatively, we say that $\succcurlyeq'_i$ is ***more flexible than*** $\succcurlyeq_i$.

Given $\succcurlyeq_i$ and $\succcurlyeq'_i$, we also define the ***join*** $\succcurlyeq_i \vee \succcurlyeq'_i$ as the preference for which $A(\succcurlyeq_i \vee \succcurlyeq'_i) = A(\succcurlyeq_i) \cup A(\succcurlyeq'_i)$. We also define the ***meet*** $\succcurlyeq_i \wedge \succcurlyeq'_i$ of these two preferences as the preference for which $A(\succcurlyeq_i \wedge \succcurlyeq'_i) = A(\succcurlyeq_i) \cap A(\succcurlyeq'_i)$. It follows that $\succcurlyeq_i \vee \succcurlyeq'_i$ is more flexible than both $\succcurlyeq_i$ and $\succcurlyeq'_i$, which are both more flexible than $\succcurlyeq_i \wedge \succcurlyeq'_i$.

Proposition 1. *A mechanism $g : \texttt{2Pref}^n \to \triangle\mathcal{M}$ is strategy-proof if and only if g is* ***monotonic*** *and* ***sub-additive*** *in the following sense: for all $\succcurlyeq_{-i}$ and for every $\succcurlyeq^+_i$ more flexible than $\succcurlyeq_i$ we have, respectively,*

$$g(\succcurlyeq^+_i, \succcurlyeq_{-i})(r(\succcurlyeq^+_i)) \geqslant g(\succcurlyeq_i, \succcurlyeq_{-i})(r(\succcurlyeq_i)),$$

and

$$g(\succcurlyeq^+_i, \succcurlyeq_{-i})(r(\succcurlyeq_i)) \leqslant g(\succcurlyeq_i, \succcurlyeq_{-i})(r(\succcurlyeq_i))$$

Proof. Until the end of the proof, fix the preferences of all agents but i at $\succcurlyeq_{-i}$. Suppose that a mechanism g is monotonic and sub-additive. Let i's true preferences be $\succcurlyeq_i$. Consider an alternative report $\succcurlyeq'_i$ for i. If $A(\succcurlyeq_i) \cap A(\succcurlyeq'_i) = \emptyset$, then i is better off reporting his true preferences, as g respects preferences. Now, there are two cases. First, if $A(\succcurlyeq'_i) \subsetneq A(\succcurlyeq_i)$, then by monotonicity

i is better off reporting the truth. Second, if $A(\succcurlyeq_i') \not\subseteq A(\succcurlyeq_i)$, then by sub-additivity i is better off reporting $\succcurlyeq_i \wedge \succcurlyeq_i'$. But then, we are back to the first case, and i is better off saying the truth. Therefore, proves that a monotonic and sub-additive mechanism is strategy-proof.

To prove the converse, suppose that a mechanism g is not monotonic, that is, there are $\succcurlyeq_i$ and a more flexible preference $\succcurlyeq_i^+$ such that

$$g(\succcurlyeq_i^+, \succcurlyeq_{-i})(r(\succcurlyeq_i^+)) < g(\succcurlyeq_i, \succcurlyeq_{-i})(r(\succcurlyeq_i)).$$

Then if i's true preference were $\succcurlyeq_i^+$, he would have an incentive to report $\succcurlyeq_i$ instead, and thus g cannot be strategy-proof. Now suppose that the mechanism g is not subadditive, that is, there are $\succcurlyeq_i$ and a more flexible preference $\succcurlyeq_i^+$ such that

$$g(\succcurlyeq_i^+, \succcurlyeq_{-i})(r(\succcurlyeq_i)) > g(\succcurlyeq_i, \succcurlyeq_{-i})(r(\succcurlyeq_i))$$

Then if i's true preference were $\succcurlyeq_i$, he would have an incentive to report $\succcurlyeq_i^+$ instead, and thus g cannot be strategy-proof. It follows that a strategy-proof mechanism must be monotonic and sub-additive, and this completes the proof. □

In other words, to check strategy-proofness, we just have to insure that an agent will never profit from two types of deviations: reporting a larger acceptable set (discouraged by subadditivity) or reporting a smaller acceptable set (discouraged by monotonicity).

Consider a mechanism such that, for every agent with some fixed preference it is always the case that, for every possible report he could send, he is better off both dropping unwanted shares from his report, and adding desirable shares to his report. It is clear that such a mechanism is strategy-proof. However, it is not the case that all strategy-proof mechanisms have that property. While it is true that, when facing a strategy-proof mechanism, an agent should drop unwanted shares from his report (due to subadditivity), it is not the case that he should always add desirable shares, as shown in the following example.

Example 1. Let $X = \{a, b, c\}$ be a set of objects and $\mathcal{F} = 2^X$ the set of assignable shares. Let $N = \{1, 2\}$ be the set of agents, and let g be the following mechanism: if it is possible to give both agents something acceptable under the reported preferences, the mechanism does so. Otherwise, the mechanism gives player 1 an acceptable share, unless player 1 only accepts the whole set X. We divide this remaining case in two, and we only give the winning probabilities of player 2; player 1 gets X with the complementary probability.

If player 2 wants only one object, he gets it with probability .5. If he accepts two shares with one object each (he may accept other shares, but they must have more objects), then he gets a given acceptable object with probability .3. If he accepts three shares with one object each, then he gets b with probability .46 and a or b with probability .12 each. If player 2 only accepts shares with 2 or more elements, then gets nothing and player 1 gets X with probability 1.

It is easy to check that this mechanism is monotonic and subadditive, and therefore strategy-proof. It is also efficient. Now suppose that player 1 reports X as his only acceptable share, and player 2 accepts $\{a\}$ or $\{c\}$ and reports that he accepts $\{a\}$ or $\{b\}$. With that report, he would get something acceptable with probability .3. Now, if he added $\{c\}$ to the report as an acceptable share, he would get something acceptable with probability .24. Thus, it is not in his best interest to add $\{c\}$ to his report, even though it is a desirable share.

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2.6 Fairness

A mechanism g induces individual lotteries $\zeta_{g(\succcurlyeq)} : N \to \triangle\mathcal{F}$. A mechanism is ***envy free*** when for all $\succcurlyeq \in \texttt{2Pref}^n$ and all $i, j \in N$ we have $\zeta_{g(\succcurlyeq)}(i) \succcurlyeq_i \zeta_{g(\succcurlyeq)}(j)$.

First, let us define the concept of no envy and Lorenz dominance. The output of a mechanism is a random assignment $g(\succcurlyeq)$, and associated to this random assignment are individual lotteries $\zeta_{g(\succcurlyeq)} : N \to \triangle\mathcal{F}$ for $i \in N$. We say that a mechanism is ***envy free*** when for every preference profile $\succcurlyeq$ every agent $i \in N$ prefers his lottery $\zeta_{g(\succcurlyeq)}(i)$ to any lottery $\zeta_{g(\succcurlyeq)}(j)$ of another agent

$j \in N$. For every vector $x \in \mathbb{R}^n$, let $\overrightarrow{x} = x_{(1)}, x_{(2)}, \ldots, x_{(n)}$ be a permutation of the coordinates of x where $x_{(i)} < x(i+1)$. Given utility profiles $u, v \in \mathbb{R}^n$ we say that u ***Lorenz dominates*** v when $\sum_{i=1}^{k}(\overrightarrow{u}_i - \overrightarrow{v}_i) >= 0$ for all $k \in \{1, \ldots, n\}$.

Kojima (2009) shows that there is no mechanism that is ordinally efficient, envy free and weakly strategy-proof for the problem of randomly assigning arbitrary bundles of a finite number of objects when preferences are additive. On the other hand, (Bogomolnaia and Moulin (2004)) show that the problem of randomly assigning a finite number of objects (not bundles) to agents that have dichotomous preferences admits a mechanism that is group-strategy-proof, envy free and always yield a random assignment that is Lorenz dominant in the class of all efficient random assignments.

The results we will present falls between the aforementioned results in the literature. By requiring preferences to be dichotomous, we open the door for mechanisms that are not only efficient and envy free, but also strategy-proof. However, as the example below shows, it is not always possible to obtain Lorenz dominant random assignments.

Example 2. Let $N = \{1, 2, 3, 4, 5\}$ and suppose we obtain the following utilitarian efficient support $uEff(\succcurlyeq)$:

$$a_1 = (1, 1, 1, 0, 0) \tag{2.3}$$

$$a_2 = (0, 0, 1, 1, 1) \tag{2.4}$$

$$a_3 = (0, 1, 0, 1, 1) \tag{2.5}$$

Note that for a random assignment to be Lorenz-dominant in the class of all efficient assignments, it has to assign the largest number of people, that is, it has to be utilitarian efficient.

Let a random assignment with support in $uEff(\succcurlyeq)$ be represented by a triple $p = (p_1, p_2, p_3)$ where p_i is the probability that deterministic assignment a_i will be selected. If there was a Lorenz-dominant assignment p for these preferences, then this random assignment would have to maximize any social welfare function $u \mapsto \sum_{i=1}^{n} f(u_i)$ where f is strictly increasing and strictly concave (see Olkin and Marshall (1980). However, if we select p so as to maximize the Nash social welfare function and the Rawlsian welfare function (that maximizes the utility of the lowest-utility agent in society), we obtain different results. Therefore there cannot be a Lorenz-dominant assignment in this example.

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Before we proceed, we need to define some more notation. We say that two assignments μ and η in $\mathcal{M}$ are ***equivalent*** when $[\mu \neq \emptyset] = [\eta \neq \emptyset]$, that is, when they assign the same set of agents. For every set of assignments $M \subseteq \mathcal{M}$ let $\widetilde{M}$ be the set of the corresponding equivalence classes of assignments in M. Note that the utility profiles of the agents are the same across assignments in the same equivalence class. Thus, when we say we will choose an element from $\widetilde{M}$ at random, we mean that we choose an equivalence class at random, and then choose an arbitrary assignment in it for implementation purposes.

Proposition 2. *The mechanism where $g(\succcurlyeq)$ is the uniform distribution over $\widetilde{uEff}(\succcurlyeq)$ is utilitarian efficient, strategy-proof and envy free.*

Proof. Fix arbitrary representatives for each equivalence class in $\widetilde{uEff}(\succcurlyeq)$. We already argued that g is utilitarian efficient and strategy-proof. It remains to show that g is envy free. Let i and j be two different agents in N, and suppose that i weakly prefers j's random lottery to his. Precisely, suppose

$$\zeta_{g(\succcurlyeq)(j)} \succcurlyeq_i \zeta_{g(\succcurlyeq)(i)}. \tag{2.6}$$

We will show that in fact, i must be indifferent between the two lotteries.

Condition (2.6) holds if and only if there is a set of shares S acceptable to i such that the probability that j obtains a share from S under $g(\succcurlyeq)$ is greater or equal than the probability that i obtains an acceptable share under $g(\succcurlyeq)$. Let M_j be the set of assignments in $\widetilde{uEff}(\succcurlyeq)$ such that j gets a share from S and i is assigned the empty set. For every assignment $\mu \in M_j$, we can construct an assignment $\eta \in \mathcal{M}$ where $\eta(l) = \mu(l)$ for every $l \in N \setminus \{i, j\}$ and $\eta(i) = \mu(j)$. Denote the set of such assignments η by M_i. By construction, all assignments in M_i respect preferences.

Because each $\mu \in M_j$ is in a different equivalence class of $\widetilde{uEff}(\succcurlyeq)$ and assigns the same number of people to acceptable shares, there must be an agent in each $[\mu \neq \emptyset]$ that is assigned the empty set in the other elements of M_j. It follows that each $\eta \in M_i$ is also in a different equivalence class in $\widetilde{uEff}(\succcurlyeq)$. Because the mechanism is utilitarian efficient and the number of agents in M_i is the same as the number of agents in M_j, it follows that all assignments in M_i are in $uEff(\succcurlyeq)$. Finally, as i reports his true preference and the mechanism chooses assignments from $\widetilde{uEff}(\succcurlyeq)$ with uniform probability, it must be the case that $g(\succcurlyeq)(M_i) = g(\succcurlyeq)(M_j)$. Therefore, as $M_i \subset r(\succcurlyeq_i)$, it must be the case that

$$\zeta_{g(\succcurlyeq)(i)} \succcurlyeq_i \zeta_{g(\succcurlyeq)(j)},$$

which proves that g is envy free.

□

2.7 Computational Aspects

A mechanism can be applied in a real-life problem only if we can carry out the computations that it requires in a "reasonable amount of time". Here we show that any utilitarian efficient mechanism in our setting requires computations that in the worst case, run in time that is at least exponential in the number of agents. Worst-case analysis is the usual way of comparing running time of algorithms in computer science. However, we will also point out that many real-life applications of such mechanisms should lead to running times that are at worst polynomial in the number of agents. In what follows, we consider the feasibility constraint on deterministic assignments μ to be $\mu_i \cap \mu_j = \emptyset$ for all $i \neq j$ in N.

We will show the problem of computing an efficient assignment is equivalent to the problem of finding an inclusion-maximal independent set in a graph. An ***independent set*** in a graph is a subset of vertices such that no two vertices are connected by an edge. An inclusion-maximal independent set is an independent set that is not strictly contained in another independent set

Proposition 3. *For every assignment problem $AP = (N, X, \mathcal{F}, \succcurlyeq)$ with $\succcurlyeq \in \mathtt{2Pref}^n$, we can construct* *an undirected graph $G = (V, E)$ where every efficient assignment in AP corresponds to one and only one inclusion-maximal independent set in G and vice-versa. Conversely, for every undirected graph $G = (V, E)$, we can construct an assignment problem $AP = (N, X, \mathcal{F}, \succcurlyeq)$ with $\succcurlyeq \in$ 2Pref^n such that every inclusion-maximal independent set in G corresponds to one and only one efficient assignment in AP and vice-versa.*

Proof. Let $AP = (N, X, \mathcal{F}, \succcurlyeq)$ with $\succcurlyeq \in \texttt{2Pref}^n$ be an assignment problem. For every agent $i \in N$, define $Z_i = \{i\} \times A(\succcurlyeq_i)$, and let $V = \cup_{i \in N} Z_i$. Now, let E be the set of all pairs $(a, b) \in V$, $a = (i_a, r_a)$, $b = (i_b, r_b)$, such that either the individuals i_a and i_b coincide, or the desired shares r_a and r_b are not jointly feasible, that is, $r_a \cap r_b \neq \emptyset$. It is easy to see that there is a bijection between the feasible assignments of AP and the independent sets of $G = (V, E)$. Furthermore, it is easy to check that the efficient assignments in AP correspond exactly to the inclusion-maximal independent sets of G.

Conversely, let $G = (V, E)$ be a undirected graph. Define the following assignment problem $AP = (N, X, \mathcal{F}, \succcurlyeq$ with $\succcurlyeq \in \texttt{2Pref}^n$: let $N = V$, $X = E$, $\mathcal{F} = 2^X$ and let the acceptable set $A(\succcurlyeq_i)$ of every agent $i \in N$ contain only one share: the set of all edges for which i is one of the vertices. Then there is a bijection between independent sets of G and feasible assignments of AP and the inclusion-maximal independent sets of G correspond exactly to the efficient assignments of AP. □

The proof of proposition 3 gives us a recipe for computing efficient assignments: cast the problem as an independent set problem and compute the desired independent sets. Software for computing independent sets (or cliques, which are the dual problem) is readily available[10]. Computing an inclusion-maximal independent set can be done efficiently; computing all such sets is an NP-hard problem.

A maximum independent set (or cardinality-maximal) is an independent set that has no less vertices than any other independent set. It is well known (see Kleinberg and Éva Tardos (2005))

[10]For a useful but incomplete list, check Skiena (2008), or the companion website http://www.cs.sunysb.edu/~algorith/.

that the problem of finding a maximum independent set or listing all inclusion-maximal independent sets is an NP-hard problem, that is, the worst-case running time is at least exponential in the number of vertices. Using the reduction provided in the proof of proposition 3, we conclude that to compute utilitarian efficient assignments (those that assign the largest possible number of agents) is equivalent to computing cardinality-maximal independent sets.

Corollary 2. *Computing a utilitarian efficient assignment is NP-hard: the worst-case running time is at least exponential in the number of agents.*

To sum up: finding merely efficient (inclusion maximal) assignments is computationally easy; finding utilitarian efficient (cardinality-maximial) assignments is hard. We remark that the computational difficulties presented above have nothing to do with strategic issues: the problem is solely that of computing efficient and utilitarian efficient assignments, even if agents report truthfully.

However, the worst case scenarios necessary to obtain exponential running times might not reflect many real-world problems. For example, problems of time scheduling are often just a problem of assigning time *intervals*; for the graphs generated by these problems—called interval graphs—it is easy to find maximum-cardinality independent sets. Independent sets can be computed in polynomial time for many other special families of graphs, like planar graphs. Additionally, for some families of "sparse" graphs, cliques[11] (the duals of independent sets) can be computed in polynomial time. See Chiba and Nishizeki (1985) for details.

2.8 Robustness

So far, we have shown how to assign shares of a imperfectly divisible resource to agents that have dichotomous preferences in a way that is efficient (even utilitarian efficient), strategy-proof and envy free. Such mechanisms are not very complicated from a communication point of view, requiring agents only to report a subset of acceptable shares. Even so, as we argued in the previous section, the computations necessary to run the mechanism are potentially very lengthy. Therefore,

[11] A clique in a graph is a subset of vertices such that every two vertices are connected by an edge.

it is reasonable to want to use mechanisms that are no more complicated than the mechanism of proposition 2 for this kind of problem. This is especially the case if the mechanism designer only cares about assigning the largest number of agents to some acceptable share (imagine a company selling uniformly priced time slots for some service).

In real-world applications, it may be naive to assume that agents only care about obtaining an acceptable share or not. Even though that can be the only thing that the designer cares about, it may well be the case that agents have strict preferences between acceptable shares. It would be a very positive result if the designer could implement his objective of assigning the largest number of agents to some acceptable share by asking only each agent's acceptable shares (those that are better than nothing). In the next proposition, we show that this is impossible: indeed, any strategy-proof mechanism for our assignment problem with dichotomous preferences may give incentives for some agents to report an acceptable set that is smaller than the true one. To make things simpler, in the following we assume that $\mathcal{F}$ is a finite algebra of subsets of X.

It is easy to see now that efficiency and strategy-proofness (in the strong sense that it has to holds for all possible utility representations of non-dichotomous preferences) are not always compatible if we only ask agents to report their acceptable sets. A simple example is the following: suppose we have a monotonic and additive mechanism g and all agents but agent 1 report the empty set as the only acceptable set. Then efficiency requires that whenever agent 1 accepts something, he must be given that with probability one. In particular, if $A, B \in \mathcal{F}$ are disjoint, and 1 reports reports A as the only acceptable set, then efficiency demands that 1 gets A with probability 1; if 1 reports B as the only acceptable set, then efficiency requires that 1 gets B with probability one. However, if 1 reports exactly A and B as acceptable, then additivity requires that 1 obtains A or B with probability 2, which is absurd. It is easy to see that such problems would occur in less artificial settings. This remark yields the following corollary.

Proposition 4. *Let $g : \mathtt{2Pref}^n \to \triangle\mathcal{M}$ be a mechanism that is strategy-proof in $\mathtt{2Pref}$. If an agent $i \in N$ has preferences represented by any utility function $u_i : \mathcal{F} \to \mathbb{R}$, then it is in i's best interest to report a subset of $\{z \in \mathcal{F} : u_i(z) > u_i(\emptyset)\}$ as his acceptable set. For some u_i, it may be a best reply to report a* *strict subset of* $\{z \in \mathcal{F} : u_i(z) > u_i(\emptyset)\}$.

2.9 Conclusion

In this paper we present alternatives for a mechanism designer that wants to simplify the complex process of assigning shares of a divisible resource by allowing agents to report only a set of acceptable shares.

Assuming preferences are dichotomous, we show that strategy-proof assignment mechanisms are characterized by a monotonicity and a sub-additivity condition, which translate into the property that no agent would like to report a larger or a smaller acceptable set. We note that by requiring preferences to be dichotomous, we avoid impossibilities given by Kojima (2009) for more general preferences, and we provide a strategy-proof, efficient and envy free mechanism. However we show that, unlike the case of assignment of individual goods, the existence of a Lorenz-dominant assignment is not guaranteed.

We also provide an analysis of how easy it is to run such mechanisms "in the real world", by examining their computational complexity and the incentive compatibility of these mechanisms when agents have general preferences. Drawing on theorems about the computational complexity of finding independent sets/cliques in graphs, we show that finding utilitarian efficient assignments in arbitrary problems is "hard"(NP-hard), but that this worst-case scenario might not reflect the difficulties of a real-world application. Drawing on the same literature, we remark that an efficient assignment, not necessarily utilitarian efficient, can be computed in polynomial time. We also show that agents with general utility functions may want to report an acceptable set that is significantly smaller than $\{A \in \mathcal{F} : u(A) > u(\emptyset)\}$, and that there is no way to guarantee strategy-proofness and efficiency when agents have strict preferences among acceptable shares (those better than nothing).

Chapter 3

Monitoring Costs and the Management of Common-Pool Resources

3.1 Introduction

Since the seminal papers of Dales (1968), Montgomery (1972) (in the context of pollution) and Moloney and Pearse (1979) (in the context of fisheries), individual tradable quotas (ITQs) have become a very popular tool in the management of common-pool resources (CPRs), attracting a lot of attention not only in academia, but also in government and industry.[1] To have an idea of the impact that an improvement on the management of such programs can have in fisheries worldwide, we point out that, according to Bonzon, McIlwain, Strauss, and Van Leuvan (2010)), one out of every five coastal countries are using some form of catch-share regime to manage over 850 species of fish, and the adoption of ITQ programs continues to increase.

ITQs became so popular not only because they can prevent the collapse of a natural resource, but also because they allow the resource stock to be exploited efficiently without requiring centralized knowledge of information about individual agents' private information. The argument is by now standard and featured in standard textbooks:[2] the cap will avoid over-exploitation[3] of the

[1] See Freeman and Kolstad (2007), Grafton, Arnason, Bjorndal, Campbell, Campbell, Clark, Connor, Dupont, Hannesson, Hilborn, Kirkley, (2006), and Bonzon, McIlwain, Strauss, and Van Leuvan (2010).

[2] See for example Perman, Ma, Common, Maddison, and Mcgilvray (2012), Clark (2010), Tietenberg and Lewis (2008), and Conrad (2010).

[3] Of course, this depends on the growth rate of the resource and on the discount factor of the quota-holder, as exemplified by the case of the Antarctic blue whale fishery Clark (1973). For an argument that this should not be a problem in general, see Grafton, Kompas, and Hilborn (2007).

resource, while the quota market will allow the more efficient producers to buy the production rights from the less efficient producers and produce the target output at a lower cost.[4] There are other theoretical arguments in favor of ITQs, but those are beyond the scope of this paper.[5]

Beyond theoretical predictions, there is strong evidence that cap-and-trade programs often perform well in practice: Costello, Gaines, and Lynham (2008) compiled a global database on 11,135 fisheries from 1950 to 2003 and concluded that the fraction of ITQ-managed fisheries that collapsed was about half that of non-ITQ fisheries.[6] ITQs also avoids the race for fish and the consequent rent dissipation. See Knapp and Murphy (2010) for an experimental argument and Bonzon, McIlwain, Strauss, and Van Leuvan (2010) for a large number of references on the effects of ITQs.

One point where both academics and managers agree is that no cap-and-trade program can work without adequate monitoring and quota enforcement. For example, Copes (1986) explains how quota violations led to the abandonment of the cap-and-trade program at the Bay of Fundy herring fishery.[7] However, little theoretical work has been done to understand how costly and imperfect enforcement affect outcomes. The work that is closest to ours was done by Malik (1990), in the simpler context of air pollution, and Hatcher (2005) and Chavez and Salgado (2005) in the context of fishing.[8] However, none of those models take into account the stock dynamics of the resource or technological differences among fishermen. Without the first, steady-state analysis is not possible, and without the second, we cannot examine how preferences for the cap or enforcement levels may differ across economic agents.

Our point of departure is a classic result in the cap-and-trade literature with perfect and costless monitoring: the outcomes of an optimal.[9] command-and-control policy can be achieved by setting a total output cap and a market for quotas. That is the well-known ***efficiency of ITQ mar-***

[4] That ITQ programs stand on those two pillars—the cap and the market— is the reason why such programs are also known as *cap-and-trade* policies.

[5] See the aforementioned textbooks or Bonzon, McIlwain, Strauss, and Van Leuvan (2010).

[6] Their definition of collapse is taken from Worm, Barbier, Beaumont, Duffy, Folke, Halpern, Jackson, Lotze, Micheli, Palumbi, Sala, Selkoe, Stac (2006): a fishery collapses in year t if the harvest in that year is less than 10% of the maximum recorded harvest up to that year.

[7] He also discusses other hurdles that can get in the way of the well functioning of a cap-and-trade program.

[8] See Montero (2004) for some related empirical work.

[9] In the sense of maximizing the total industry profit.

kets An important question is whether or not there is a similar result in the case of imperfect enforcement. If violations were perfectly and costlessly observable, the problem would be easy: set fines for violation high enough and nobody will violate their quota in equilibrium, which we know is the first-best outcome. In reality it is costly to observe quota violations, and therefore it may not be socially optimal to have zero quota violations. This new optimum is what we call the *second-best* outcome, a concept we make precise in Section 3.4. In the simpler setting of air pollution, Malik (1990) showed that for the attainment of a second best, it is necessary that expected violations depend only on absolute violations instead of, say, violations as a proportion of quota, or magnitude of the catch. We provide a similar and more complete characterization in the more complicated case of renewable resources: it is sufficient and *generically* necessary that expected fines depend only on absolute violations for a second best to be attainable by an ITQ program. While building this result, we uncover the fact that optimal cap and enforcement levels imply a positive amount of quota violations.

We go on to build, from the bottom up, the equilibrium notion we will focus on: the stable, steady-state equilibrium. The most important concept we need before the aforementioned equilibrium is that of a *temporary equilibrium given a stock level s*, which is simply a competitive market equilibrium when the stock level is s. We present full comparative statics for both types of equilibria, in particular how the steady-state equilibrium changes with respect to changes in the cap and enforcement levels. Along the way, we show that ITQ equilibria have the following *all-or-nothing* property: either nobody violates their quota, or everyone violates their quota.

Note that while Chavez and Salgado (2005) provide *some* comparative statics for what we call *temporary equilibrium*, they do not have a model for the replenishment of the natural resource, and thus cannot present *steady-state* comparative statics. As we will see, steady-state analysis is a more delicate matter than temporary equilibrium analysis because we may have multiple equilibria, and an equilibrium may be degenerate or unstable. The work of Hatcher (2005) also does not touch on steady-state issues and makes the decision to comply with or violate quota exogenous, thereby leading to a result that is different from ours: that if expected violation fines are a function of absolute violations *as a fraction of quota held*, then the quota price in a compliant market may be lower than in an otherwise non-compliant market.

Finally, with imperfect and costly enforcement, the question of how much enforcement becomes central, as well as the complementary effects of raising enforcement or lowering the cap. Monitoring affects all fishermen irrespective of who paid for it, and therefore fishermen may have an incentive to free-ride on the contributions of others. Indeed, it is not hard to show that letting enforcement be paid solely by voluntary contributions will lead to no enforcement at all. There is a more basic problem however: even if no fisherman has to pay for monitoring costs, one fisherman might want more monitoring, while another might want less. The reason is simple: all else equal, buyers in the quota market want a low quota price, and sellers want a high quota price. Intuitively (and shown in Theorem 7) a higher level of monitoring leads to higher quota prices. At an even more fundamental level, agents may disagree on the level of monitoring even when no quota trade is allowed. That is because increased monitoring leads on one hand to higher stock levels and thus lower costs for fishermen, but also to higher violation fines for the same fishermen. For some fishermen, the cost decrease might offset the steeper violation fines; for others, it might not. In light of the ambiguity outlined above it is hard to obtain strong conclusions about collective preferences over monitoring. We conclude the paper with a first-step in that direction with a local result (theorem 8), saying that under certain conditions —including no quota trade, or no wealth effects in the quota market— if a given boat wants slightly more monitoring, then all larger boats will want slightly more monitoring.

We now proceed to the model in Section 3.2. There we will define the primitives of the model and our core Assumptions. In Section 3.3 we define our notion of ITQ equilibrium. Section 3.4 presents the single-owner problem and some of the properties of a solution. In that Section we establish our first main result: the solution to the single-owner problem can be implemented by an ITQ program if and generically only if expected violation fines depend only on absolute violations. We go on to perform a detailed analysis of equilibria in Section 3.5. We split the analysis in three layers: individual optimal behavior, temporary equilibrium (a competitive equilibrium given a fixed stock level). and steady-state equilibrium. This conceptual organization allows us to analyze existence, multiplicity, regularity and stability of equilibria in a convenient and intuitive way. In this Section we also establish our second main result: in any temporary equilibrium either nobody violates their quota or everyone does. Restricting ourselves to regular, stable equilibria, we then show how equilibrium points change when the monitoring expenditure M and the $\mathcal{TAC}$ level suffer small changes. We close the paper with a brief analysis of a fishery with larger boats and smaller boats (multiple types). We provide some comparisons between the equilibrium decisions of different fishermen, and explain how endowments may affect equilibria if fishermen have any influence on the design variables that determine the enforcement levels or the cap. Precisely, we show that if endowments are such that changes in the design variables cause no changes in wealth, then support for more monitoring will come from larger boats, if from anyone at all. A brief conclusion follows, where we provide final remarks on our results and directions for future research.

3.2 The Model

Let $\mathcal{I}$ be a set of $n \in \mathbb{N}$ fishermen. Each fisherman produces an amount $y_i \geqslant 0$ of fish (referred to i's ***production***, ***output*** or ***catch***) using a ***technology*** $\theta_i \in \Theta$. The ***cost*** $c(y_i, s, \theta_i) \in \mathbb{R}$ incurred by agent i depends not only on his catch y_i and technology θ_i but also on the ***stock of fish*** $s \geqslant 0$. The fishermen can sell their output in a competitive market where the ***price of fish*** is $p > 0$. Their objective is to choose a level output y_i such that their profits are maximized.

The stock of fish is governed by the law

$$s' = g(s) - Y(s) \tag{3.1}$$

where $g : \mathbb{R}_+ \to \mathbb{R}$ is a growth function and $Y(s)$ is the ***total catch***, that is, the sum of each fisherman's i' catch $y_i(s)$ when the stock of fish is s. We will restrict attention to ***steady-state*** outcomes, that is, those where $s' = 0$.

Our goal is to study the performance of this industry under a ***cap-and-trade*** program —also known as a program of ***individual tradable quotas*** (ITQs)— when monitoring is imperfect. To that end, we will introduce a regulator in the model. She[10] has three ***regulatory instruments***: the cap, the initial quota endowments, and the level of monitoring (we will also call those ***design variables***). In more detail, the regulator sets a ***total allowable catch*** $\mathcal{TAC} \geqslant 0$, and allocates ***initial endowments of quota*** $\omega_i \geqslant 0$ to each fisherman i such that $\sum_i \omega_i = \mathcal{TAC}$. She also determines the *monitoring* expenditure $M \geqslant 0$. The fishermen then observe the regulator's decisions, the stock level s and the quota price q and make their choices: how much quota to buy and how many fish to catch. The regulator's instruments $(M, \mathcal{TAC})$ affect the fishermen via a quota market where the ***price of quota*** will be denoted $q \geqslant 0$ and through ***quota violation fines*** $\phi(M, y_i, w_i) \geqslant 0$ charged to agent i for catching y_i while holding quota w_i. In that setting, a fisherman's ***profit*** is given by

$$\pi_i = py_i - c(y_i, s, \theta_i) - q(w_i - \omega_i) - \phi(M, y_i, w_i). \tag{3.2}$$

3.2.1 Notation, Conventions, and Assumptions

Unless otherwise noted, we write $y = (y_1, \ldots, y_n)$, $w = (w_1, \ldots, w_n)$, $\omega = (\omega_1, \ldots, \omega_n)$, and capital letters denote aggregates, so $Y = \sum_{i \in \mathcal{I}} y_i$. When we make a statement involving y_i, w_i, θ_i without specifying which agent i we are talking about, it is to be understood that the statement is true for any fixed $i \in \mathcal{I}$ that is consistent with the given context.

We denote the partial derivative of a function $f : \mathbb{R}^n \to \mathbb{R}$ with respect to it's i-th argument as $D_i f$, and we define $D_{ij} f \equiv D_i(D_j f)$. When we want to differentiate a mathematical expression h with respect to a certain variable x we write $D_x h$. *Example:* we use $D_1 c$ for marginal cost, and $D_M \pi_i^*$ for the derivative of a value function π_i^* with respect to the parameter M.

When defining monitoring functions ϕ, it is useful to have a shorthand notation for the positive part map which we will write as $[x]^+ = \max\{0, x\}$.

We will always maintain the following Assumptions on the functions c, g, and ϕ.

[10] Simple convention: fishermen are male, regulator is female.

Assumption 1. We define agent i's technology $\theta_i \in \Theta$ as his boat capacity, and assume that $c(y_i, s, \theta_i) = \infty$ whenever $y_i \geqslant \theta_i$. Furthermore, we assume that where $y_i < \theta_i$, c is twice differentiable, strictly increasing, and strictly convex in output y, and decreasing in stock s. Finally, we assume that marginal costs $D_1 c$ are decreasing in both stock s and boat size θ. The convexity Assumption is standard, and will contribute to making the individual optimization problems (3.4) convex; the Assumption of $D_1 c$ decreasing in θ expresses the heterogeneity among agents, namely that larger boats, with higher fixed cost, have lower marginal costs.

Assumption 2. The growth function $s \mapsto g(s)$ is concave, satisfies $g(0) = 0$, $g(K) = 0$, and $g(s) > 0$ when $s \in (0, K)$, where $K > 0$ is the maximum stock of fish that can be sustained by that environment's resources in a steady state, known as the ***carrying capacity*** of that environment. We also assume that the maximum of g —known as the ***maximum sustainable yield*** (MSY)— is attained at a unique stock level $sMSY > 0$.

Assumption 3. The ***monitoring function*** is defined by

$$\phi^+(M, y_i, w_i) = \max\{0, \phi(M, y_i, w_i)\} \tag{3.3}$$

where $(M, y_i, w_i) \mapsto \phi(M, y_i, w_i)$ is twice continuously differentiable, increasing in M, increasing and convex in y_i, decreasing and strictly convex in w_i. equal to zero whenever $M = 0$ or $y_i = w_i$.

The easiest way to think of ϕ is to assume it has the form

$$\phi(M, y_i, w_i) = \rho(M) v(y_i, w_i)$$

where $\rho(M)$ is the probability of being audited and $v(y_i, w_i)$ is some penalty function for violations. *It should be clear that we are implicitly making the important implicit Assumption that fishermen are risk neutral.*

3.3 Equilibrium with Individual Transferable Quotas

From the regulator's point of view, her choice of $(M, \mathcal{TAC}, \omega)$ leads to an ***outcome*** (y, w, q, s) (all entries being non negative). We assume the fishermen behave *competitively* —that is, they do not take into account the impact of their catch on the stock— and we study equilibrium outcomes.

An (ITQ) ***equilibrium*** is an outcome (y, w, q, s) that satisfies the following three conditions.

1. *Fishermen maximize profits.* Given the regulator's choice of $M, \mathcal{TAC}, \omega$, the price of fish p, the price of quota q and the state s, every fisherman's $i \in \mathcal{I}$ choice of y_i, w_i maximizes profits (3.2), i.e., it solves

$$\underset{y_i, w_i}{\texttt{maximize}} \quad \pi_i \quad \texttt{subject to} \quad y_i \geqslant 0,\ w_i \geqslant 0. \tag{3.4}$$

2. *The quota market clears.* That is,

$$\sum_{i \in \mathcal{I}} w_i = \mathcal{TAC}. \tag{3.5}$$

3. *The fish stock is at a steady state.* In other words, the total amount of fish caught is the same as the amount by which the stock grows:

$$Y = g(s) \tag{3.6}$$

An outcome that satisfies conditions 1 and 2 above is called a ***temporary equilibrium***.

The set of equilibria corresponds to the set of non negative solutions to a system of equations. Recall that the individual problems (3.4) are convex, and all the constraints are affine. Therefore, we can write the Lagrangian of the problem of agent i as

$$\mathcal{L}_i(y_i, \mu_i^y, w_i, \mu_i^w) = py_i - c(y_i, s, \theta_i) - q(w_i - \omega_i) - \phi(M, y_i, w_i) + \mu_i^y y_i + \mu_i^w w_i \tag{3.7}$$

and conclude that the following conditions are necessary and sufficient for an outcome (y, w, q, s) with $y_i \neq w_i$ for all $i \in \mathcal{I}$ (that is the smooth case, we will deal with the nonsmooth case shortly) to be an equilibrium: for every $i \in \mathcal{I}$ there exists $\mu_i^y \geqslant 0$ and $\mu_i^w \geqslant 0$ such that the system of equations below is satisfied. *Notation:* we write c_i for $c(y_i, s, \theta_i)$ and ϕ_i for $\phi(M, y_i, w_i)$.

$$p - D_1 c_i - D_2 \phi_i + \mu_i^y = 0 \qquad i = 1, \ldots, n \tag{3.8}$$

$$-q - D_3 \phi_i + \mu_i^w = 0 \qquad i = 1, \ldots, n \tag{3.9}$$

$$\mu_i^y y_i = 0 \qquad i = 1, \ldots, n \tag{3.10}$$

$$\mu_i^w w_i = 0 \qquad i = 1, \ldots, n \tag{3.11}$$

$$\sum_{i=1}^{n} w_i - \mathcal{TAC} = 0 \tag{3.12}$$

$$\sum_{i=1}^{n} y_i - g(s) = 0 \tag{3.13}$$

Note this is a square system: $4n + 2$ variables and equations. The first $4n$ equations indicate individual optimal behavior; the last two equations say that the quota market clears and the environment is at a steady state, respectively. The first $4n + 1$ equations characterize a temporary equilibrium.

When $y_i = w_i$ for some $i \in \mathcal{I}$, the monitoring function ϕ need not be differentiable and thus the equations in (3.8, 3.9) are replaced by the following conditions (see Lemma 12 in page 81 for more details). For every i such that $y_i = w_i$ there exists $\mu_i^y \geqslant 0$, $\mu_i^w \geqslant 0$, *and* $\alpha_i \in [0, 1]$ such that $y_i \mu_i^y = 0$, $w_i \mu_i^w = 0$, and

$$p - D_1 c_i - \alpha_i D_2 \phi_i + \mu_i^y = 0 \tag{3.14}$$

$$-q - \alpha_i D_3 \phi_i + \mu_i^w = 0 \tag{3.15}$$

where $D_2 \phi_i$ and $D_3 \phi_i$ are the lateral derivatives defined earlier.

Remark 1. First, because fishermen have quasilinear preferences, the regulator's choice of quota endowments $\omega \in \mathbb{R}_+^n$ does not affect the equilibrium outcome, only the equilibrium profits. Second, the regulator's choice of the $\mathcal{TAC}$ enters the agents decision problem only *indirectly*; it affects equilibrium solely through the market-clearing condition (3.5). Third, observe that condition 1 above is equivalent to (y, w) solving the centralized problem

$$\begin{aligned} \underset{y,w}{\texttt{maximize}} \quad & \sum_{i \in \mathcal{I}} \pi_i \\ \texttt{subject to} \quad & y_i \geqslant 0 \quad \forall i \in I \\ & w_i \geqslant 0 \quad \forall i \in I. \end{aligned} \tag{3.16}$$

In other words, the n individual optimization problems in 2 variables are equivalent to one centralized optimization problem in $2n$ variables. Fourth, (y, w, q, s) satisfy conditions (1, 2) above —characterizing temporary equilibria— if and only if (y, w) solve (3.16) with the added constraint $\sum w_i = \mathcal{TAC}$ and q is a Lagrange multiplier for that constraint.

A classic result from the cap-and-trade theory with perfect monitoring is that we can attain optimal outcomes that could be obtained in a centralized way by setting up a cap on total output and a market for output quotas. We will investigate whether a similar result carries over to the case of costly imperfect monitoring in Section 3.4.

3.4 The Single-Owner Problem and ITQ Implementability

Consider a central manager that is charged with running a quota-managed fishery for the fishermen. Suppose the manager solves the following problem.

$$\begin{aligned} \underset{y,w,s,M}{\texttt{maximize}} \quad & \sum_{i=1}^{n} [p y_i - c(y_i, s, \theta_i)] - M \\ \texttt{subject to} \quad & y, w, s, M \geqslant 0 \\ & \sum_{i=1}^{n} y_i \leqslant g(s) \\ (IC) \quad & y_i \in \operatorname{argmax} p y_i - c(y_i, s, \theta_i) - \phi(M, y_i, w_i) \qquad i = 1, \ldots, n \end{aligned} \tag{3.17}$$

This is essentially a moral-hazard problem with imperfect monitoring where the principal is the central manager and the agents are the fishermen.

For simplicity, we will assume only in this Section that the monitoring function ϕ^+ is smooth at zero violations; that is, when $y_i = w_i$, the derivatives $D_2\phi(M, y_i, w_i)$ and $D_3\phi(M, y_i, w_i)$ are uniquely defined and are equal to zero. That makes the problem smoother, and focuses on the interesting case here, which is the case when there are quota violators in the industry. We also assume that ϕ is strictly convex in y_i, which guarantees the existence of certain useful derivatives, notably $D_{w_i}y_i^*$ and $D_M y_i^*$ in (3.19).

Under our Assumptions, for every (w_i, s, M) there exists a single y_i^* that satisfies the IC constraint. We can rewrite the manager's problem (3.17) by changing y_i to this optimal y_i^*, and optimizing only over w, s, M. The Lagrangian would then be

$$L = \sum_i [py_i - c(y_i^*, s, \theta_i)] - M + \sum_i \mu_i^y y_i + \sum_i \mu_i^w w_i + \mu^s \left(g(s) - \sum_i y_i^* \right) + \mu^M M \tag{3.18}$$

In that case, the system of equations characterizing equilibrium (aside from complementary slackness conditions and original constraints) would be given by the fishermen's IC's FOC on y and the manager's FOC on w, s, M. In that order:

$$\begin{aligned} p - D_1 c_i - D_2\phi_i + \mu_i^y &= 0 \qquad i = 1, \ldots, n \\ D_{w_i} y_i^* \left(p - D_1 c_i - \mu^s\right) + \mu_i^w &= 0 \qquad i = 1, \ldots, n \\ \sum_i [D_s y_i^* (p - D_1 c_i - \mu^s)] + \mu^s g'(s) &= 0 \\ \sum_i [D_M y_i^* (p - D_1 c_i - \mu^s)] - 1 + \mu^M &= 0 \end{aligned} \tag{3.19}$$

Remark 2. Note agent i violates his quota if and only if $D_{w_i} y_i^* > 0$ or, equivalently $D_M y_i^* < 0$. It follows that the summation in the last equation in (3.19) (the first-order-condition in M) has positive terms only on violators, and therefore, it cannot be satisfied with $M > 0$ if there are no violators. That shows that in any given solution to the single-owner problem, at least one agent must be a quota violator.

However, it cannot be the case that *all* violators have positive quota; indeed, for any such i we would have $y_i > w_i > 0$, and thus by the second equation (first-order-condition on w_i), it must be the case that $p - D_1 c_i - \mu^s = 0$. Therefore, if all violators had positive quota, the first-order condition on $M > 0$ also would not be satisfied.

We conclude that in the single-owner solution, either $M = 0$ or there must be a quota violator that is given zero quota.

The intuition is clear for the case $n = 1$: the objection function in (3.17) tells us that, all else equal, we want to spend as little money as possible with monitoring. Therefore, it is cheaper to reduce the fisherman's output with a lower $\mathcal{TAC}$ than with a higher M. It follows that we should set $w_1 = 0$.

3.4.1 ITQ Implementability of the Single-Owner problem

We will now show that a quota market can implement the optimal single-owner solution as long as the fines depend on catch y_i and quota w_i only through the absolute violation $y_i - w_i$.

Theorem 1. *Let $(\widetilde{y}, \widetilde{w}, \widetilde{s}, \widetilde{M} > 0)$ be a solution of the single-owner problem. Then there exists $q \geqslant 0$ such that $(\widetilde{y}, \widetilde{w}, q, \widetilde{s})$ is an ITQ equilibrium at monitoring level $\widetilde{M}$ and cap $\sum \widetilde{w}_i$ if $D_2\phi(M, y, w) = -D_3\phi(M, y, w)$ for all $M > 0$, $y > w \geqslant 0$.*

In other words, the single-owner optimal solution is ITQ implementable if the penalty function ϕ depends on y, w only through the absolute violation $y - w$.

Proof. Suppose that $D_2\phi(M, y, w) = -D_3\phi(M, y, w)$ for all $M > 0$, $y > w \geqslant 0$. Let $\widetilde{\mu}^y, \widetilde{\mu}^w \geqslant 0$ be the Lagrange multipliers of the nonnegativity constraints in the single-owner problem (3.17) and

$\widetilde{\mu}^s \geqslant 0$ the multiplier of the output/stock-growth constraint. Define

$$y_i = \widetilde{y}_i$$

$$\mu_i^y = \widetilde{\mu}_i^y$$

$$w_i = \widetilde{w}_i$$

$$\mu_i^w = \widetilde{\mu}^s + D_3\phi(\widetilde{M}, \widetilde{y}_i, \widetilde{w}_i)$$

$$q = \widetilde{\mu}^s$$

$$s = \widetilde{s}$$

A quick check reveals that (y, w, q, s) satisfy the system of equations that characterize an ITQ equilibrium (see p. 36) at monitoring level $\widetilde{M}$ and cap $\sum \widetilde{w}_i$ as long as $w_i, \mu_i^w \geqslant 0$ satisfy the complementary slackness condition $\mu_i^w w_i = 0$. To verify that, remember that $D_3\phi_i = D_2\phi_i$, and thus

$$\mu_i^w = \widetilde{\mu}^s - D_2\phi(\widetilde{M}, \widetilde{y}_i, \widetilde{w}_i)$$

The first-order conditions on w_i for the single-owner problem ((3.19), second equation) guarantee that $\mu^s - D_2\phi(\widetilde{M}, \widetilde{y}_i, \widetilde{w}_i) \geqslant 0$, because $D_{w_i}y_i^*$ is always no non negative and $\widetilde{\mu}_i^w \geqslant 0$. Therefore $\mu_i^w \geqslant 0$. Furthermore, if $w_i > 0$, then $\widetilde{\mu}_i^w = \widetilde{\mu}_i^y = 0$ and $D_{w_i}y_i^* > 0$ at the single-owner solution, which implies by (3.19) (first two equations) that $\widetilde{\mu}^s - D_2\phi(\widetilde{M}, \widetilde{y}_i, \widetilde{w}_i) = 0$. Therefore, $\mu_i^w w_i = 0$, as desired. □

From the point of view of the manager, every situation she faces is characterized by p, c, g, ϕ, θ satisfying our Assumptions; those form the set of possible environments.

The manager chooses $M, \mathcal{TAC}, s, y, w$ to induce an industry-profit-maximizing outcome (y, s, M). While she chooses M directly, $s \in [0, K]$ and $y \in \mathbb{R}_+^n$ must satisfy steady state and incentive compatibility conditions. We say an outcome (y, s, M) is ***ITQ implementable*** if there exists $\widetilde{M}, \mathcal{TAC}$ and an ITQ equilibrium $(\widetilde{y}, \widetilde{w}, \widetilde{q}, \widetilde{s})$ such that $\widetilde{y} = y, \widetilde{s} = s, \widetilde{M} = M$.

We showed in Theorem 1 that for any environment with a monitoring function that depends on

y_i, w_i only through $y_i - w_i$, every solution to the manager problem is ITQ implementable. We now show that this is a rather special feature of that particular monitoring function. In other words, the impossibility of ITQ-implementation is a generic property (see definition in Appendix A.3). The intuition behind the proof is simple: a solution of (3.17) requires that $D_2\phi_i$ be equalized for all violators, while an ITQ equilibrium only guarantees that $D_3\phi_i$ are equalized among quota holders. Intuitively, that should not hold for most monitoring functions ϕ^+. The economic interpretation is the following: the single-owner solution requires that marginal costs be equalized for all violating quota holders; we can see from the first-order condition on y_i that this will happen only if the marginal fine $D_2\phi_i$ is equalized among such agents. However, an *ITQ* equilibrium only equalizes the marginal benefit of quota $-D_3\phi_i$ among those agents. Therefore, if $-D_3\phi = D_2\phi$, then in an ITQ equilibrium, we also equalize marginal costs $D_1 c_i$ among violating quota holders.

Theorem 2. *Fix p, c, g, θ and let Φ be a set of monitoring functions ϕ^+ that are smooth at zero violations, are restricted to some compact domain, and such that any solution $(M, \mathcal{TAC}, s, y, w)$ of the single-owner problem (3.17) hands out strictly positive aggregate quota $\sum w_i$. Given a solution (y, s, M) to the single-owner problem (3.17), let $\Phi' \subset \Phi$ be the subset of monitoring functions for which we can implement (y, s, M) with an ITQ program. Then $\Phi \setminus \Phi'$ is a generic set in Φ.*

Proof. First remember that in this Section we are assuming that ϕ^+ is differentiable at zero violations, and therefore every active fisherman in an ITQ equilibrium is a violator. Clearly, if a solution to the single-owner problem has active fishermen respecting their quota, then that solution is not ITQ implementable. For that reason, we restrict attention to solutions of the single-owner problem where all active fishermen are violators.

If a solution to the single-owner problem is ITQ implementable, then the square system of equations (3.8–3.6) has to be satisfied. In addition, it follows from the second equation in (3.19) that $p - D_1 c_i$ has to be equalized for all violators with positive quota. We will now argue that this is generally impossible. The intuition for that generic impossibility is that we need to satisfy a smooth system with more equations than variables. Let us make that precise.

Fix $k \geqslant 2$ and let $\widetilde{\Phi}$ be the vector space of all C^k functions $(M, y, w) \mapsto f(M, y, w)$ from a compact set[11] to $\mathbb{R}$ such that $D_2 f = D_3 f = 0$ whenever $y = w$. Equipped with the usual norm of uniform convergence of the function and its k derivatives $\widetilde{\Phi}$ is a Banach space. It is also separable because the set of all polynomials on that same domain is dense in $\widetilde{\Phi}$ (as stated by the Stone-Weierstrass Theorem). Now define $\widetilde{\Phi}' \subset \widetilde{\Phi}$ as the set of members of $\widetilde{\Phi}$ (not necessarily members of Φ) where the overdetermined system we just mentioned has a solution. It follows then from the transversality Theorem 7 that $\widetilde{\Phi} \setminus \widetilde{\Phi}'$ is a generic set in $\widetilde{\Phi}$.

It remains to show that $\Phi \setminus \Phi'$ is a generic set in Φ. To that end, consider the set of all C^k strongly convex monitoring functions with a positive parameter, that is, those elements of Φ hose hessian have all eigenvalues greater than some $m > 0$ across their domain. It is not hard to show that this is a dense subset of Φ that is also open in $\widetilde{\Phi}$; it is therefore a generic set in Φ. Because the intersection of two generic sets is also generic, it follows that $\Phi \setminus \widetilde{\Phi}'$ is generic in Φ. From $\Phi' = \Phi \cap \widetilde{\Phi}'$ we conclude that $\Phi \setminus \Phi'$ generic in Φ as desired. □

3.5 The Set of Equilibria

We will now build our equilibrium notion by separating the economic equilibrium and the environmental equilibrium parts. This separation makes the analysis conceptually clearer, and allows us to build the equilibrium in a "bottom-up" way, making clear the interdependence relations between the different variables of the model.

On the economic side, we have two "layers":

- In the bottom layer, we study individual optimal behavior in both fishing activity and trading activity in the quota market. Agents take $p, M, \mathcal{TAC}, q, s$ as given and make their optimal decisions about how much quota w_i to hold and how much fish y_i to catch. This corresponds to the first condition for equilibrium: fishermen maximize profits by solving (3.4). At this level, the concept of *quota demand* and the distinction between *quota violators* and *quota holders* are central.

[11] Realistically, y, w, M are all nonnegative and bounded above, so this technical requirement does not get in the way of realism.

- On top of the previous layer, we impose the *market-clearing* condition (3.5). At this level, outcomes are denominated *temporary equilibria*, and a lot of what we want to know is summarized in the variables that quantify total output Y and quota price q. The parameters $p, M, \mathcal{TAC}, s$ are still exogenous.

On the environmental side, we have the final layer.

- If we impose the steady-state condition (3.6), we obtain the definitive notion of *equilibrium* in this model. At this level, the only free parameters are the price of fish p and the regulator's design variables M and $\mathcal{TAC}$.

The center of our approach lies in the Section on temporary equilibria; once that is well understood, the results about existence, number and stability of equilibria can be inferred after a graphical analysis of the growth curve $s \mapsto g(s)$ and the temporary equilibrium total output curve $Y(s)$.

3.5.1 Competitive Behavior: Fishing and Quota Demand

From a fisherman i's point of view, he takes $M, \mathcal{TAC}, \omega_i, s, q$ as given and his choice of y_i, w_i leads to a certain private outcome (y_i, w_i, q, s) with an associated profit π_i.

Let us define some terminology.

- We define for every $i \in \mathcal{I}, q \geqslant 0$ and $s \geqslant 0$ the ***optimal catch*** $\widetilde{y}_i(q,s)$ and ***optimal quota holdings*** $\widetilde{w}_i(q,s)$ as the $y_i \geqslant 0$ and $w_i \geqslant 0$ that maximize profits π_i as defined in (3.2).

- Given a stock s, note that ***open-access*** behavior is given by fishermen's optimal behavior at $q = 0$. That is $\widetilde{y}_i(0,s)$ is agent i's open-access catch when the stock is s. Analogously, the ***pure-poaching*** catch of agent i at stock s is equal to $\widetilde{y}_i(\infty,s)$.

We say that a fisherman i is ***active*** when $y_i > 0$; we say he is a ***violator*** when $y_i > w_i$.

In Appendix A.2, p. 80, we characterize the agents that will be active and those who will hold positive amounts of quota. Based on Lemma 12 presented there, we can also say something about who will violate quota, and who will not. It is easy to see, for example, if $q > 0$ and $D_3\phi = 0$ when violations $y_i - w_i$ are zero, then any active agents will be violators. This observation applies to the case where the monitoring function ϕ^+ is smooth at zero violations. Theorem 3 below specifies what can happen when this condition fails.

Theorem 3. *Consider arbitrary $M > 0, \mathcal{TAC}, q, s$, and let (y, w) be profit maximizing for all fishermen. Then either no active fisherman violates his quota or every active fisherman violates his quota.*

Proof. We will prove the following equivalent statement: if there is an agent that holds positive quota and does not violate it, then no fisherman violates his quota.

Before we proceed, note that the $\mathcal{TAC}$ is irrelevant here, and any case with $M = 0$, $s = 0$ or $q = 0$ is trivial.

Let i be the non violating quota holder. According to Lemma 12 in Appendix A.2, his optimal w_i satisfies

$$-q - \alpha_i D_3\phi_i = 0. \tag{3.20}$$

for some $\alpha_i \in [0, 1]$.

Let j be another fisherman. Suppose, by means of contradiction, that j violates his quota. His optimal w_j satisfies

$$-q - D_3\phi_j + \mu_j^w = 0 \tag{3.21}$$

for some $\mu_j^w \geqslant 0$. Subtracting (3.21) from (3.20), we obtain

$$\mu_j^w = D_3\phi_j - \alpha_i D_3\phi_i$$

Because j violates his quota while i does not, and because we assume that ϕ is strictly convex in quota holdings when violations are positive, (see Assumption 3 in page 34) we must have $D_3\phi_i > D_3\phi_j$ (remember $D_3\phi$ is always non positive). It follows then from $\alpha_i \leqslant 1$ that $\mu_j^w < 0$, which is a contradiction. □

Remark 3. Note that in the proof of Theorem 3 above, it was crucial to assume that the monitoring function ϕ was *strictly* convex in w_j. If ϕ was simply convex in w_j, we would only be able to conclude that $\alpha_i = 1$ and $\mu_j^w = 0$ which does not contradict our knowledge that $\mu_j^w \geqslant 0$.

The following Lemmas are a preparation for Section 3.5.2 where we study temporary equilibria. They help us establish monotonicity properties of the excess demand function so that quota prices can be uniquely defined in temporary equilibria.

Lemma 2. *Fix $M, \mathcal{TAC}, s, i$ and $q > 0$. Then for every $y_i \geqslant 0$ there is a unique w_i^* that maximizes profits. The implicit mapping $y_i \mapsto w_i^*(y_i)$ is nondecreasing and continuous, differentiable where $w_i^* \neq y_i$ with $w_i^*(y_i) \leqslant y_i$. Furthermore, there is $s_i \geqslant 0$ such that $w_i^*(s) = 0$ if and only if $s \leqslant s_i$. Where $s > s_i$, the map $s \mapsto w_i^*(s)$ is strictly increasing.*

Proof. Existence of w_i^* is a consequence of $\lim_{w_i \to \infty} \pi = -\infty$ and Weierstrass' Theorem; uniqueness follows from strict concavity of the profit function in w_i. Continuity of the implicit mapping is an implication of the maximum Theorem. The monotonicity properties follow from the increasing differences of the profit function in (w_i, y_i). Differentiability is a consequence of the invertibility of the hessian of the profit function —see Lemma 8 in Appendix A.1. □

Lemma 3. *Fix $M, \mathcal{TAC}, i$. Then for every $q > 0, s \geqslant 0$ there exists a unique pair $\widetilde{y}_i, \widetilde{w}_i$ that maximizes profits; it must be the case that $\widetilde{w}_i \leqslant \widetilde{y}_i$. The maps $(q,s) \mapsto \widetilde{y}_i(q,s)$ and $(q,s) \mapsto \widetilde{w}_i(q,s)$ are continuous, continuously differentiable where $q, s > 0$ and $\widetilde{y}_i > \widetilde{w}_i$ and strictly decreasing in q once $\widetilde{w}_i, \widetilde{y}_i > 0$. The map $\widetilde{y}_i$ is strictly increasing in s once $\widetilde{y}_i > 0$, and $\widetilde{w}_i$ is strictly increasing in s once $\widetilde{w}_i > 0$.*

Proof. Note that the profit function $\pi_i(y_i, w_i) = py_i - c(y_i, s, \theta_i) - q(w_i - \omega_i)\phi(M, y_i, w_i)$ is continuous and defined over a closed domain. It is also *coercive*, that is, for all $i \in \mathcal{I}$ and all $t < 0$ there exists $r > 0$ such that if $\|(y_i, w_i)\| > r$ then $\pi_i < t$. We can then bound the domain and appeal to Weierstrass' Theorem to conclude that the set of (y_i, w_i) that maximize π_i is not empty.

Let us now show that the set of profit-maximizing (y_i, w_i) is a singleton. First, take the case where there exists a maximizer such that $w_i > 0$. We know from Lemma 2 that associated to that w_i there is a single profit-maximizing y_i, and that for any other possible maximizer $(\widetilde{y}_i, \widetilde{w}_i)$ we must have $(y_i - \widetilde{y}_i)(w_i - \widetilde{w}_i) > 0$. Suppose $\widetilde{y}_i > y_i$. As we assumed that $D_{11}c > 0$, we have

$D_1c(\tilde{y}_i, s, \theta_i) > D_1c(y_i, s, \theta_i)$. The first-order conditions on output are $p = D_1c + D_2\phi$. Therefore, we must have $D_2\phi(M, \tilde{y}_i, \tilde{w}_i) < D_2\phi(M, y_i, w_i)$. But that implies that $D_3\phi$ also moved, violating the first-order condition on quota holdings $D_3\phi = q$. The argument for the case $\tilde{y}_i < y_i$ is analogous. That proves that the set of maximizers is a singleton.

The monotonicity results come from the fact that the profit function π_i has increasing differences in (y_i, s) (strict when $y_i, s > 0$), in $(w_i, -q)$ (strict when $w_i, q > 0$, and in (y_i, w_i).

That $\tilde{y}_i$ and $\tilde{w}_i$ are continuous functions of q, s follows from the maximum Theorem.

Because the hessian (A.1) of the profit function is always invertible when $y_i, w_i > 0$ and varies smoothly with $q, s > 0$ it follows from the implicit function Theorem that $\tilde{y}_i, \tilde{w}_i$ are smoooth functions of $q > 0$ and $s > 0$ when $\tilde{y}_i > 0, \tilde{w}_i > 0$. □

Given $M, \mathcal{TAC}$, define the ***excess demand*** function as

$$z(q, s) = \left(\sum_{i \in \mathcal{I}} \tilde{w}_i(q, s) \right) - \mathcal{TAC} \tag{3.22}$$

but only for strictly positive values of q. We have then the following Corollary from Lemma 3.

Corollary 3. *The excess demand function in* (3.22) *is non increasing in* $q > 0$. *Furthermore, if* $z(q, s) = 0$ *with* $q > 0$, *then there exists an open interval around* q *where* $z(\cdot, s)$ *is strictly decreasing.*

Remark 4. We studied $\tilde{y}_i(q, s)$ and $\tilde{w}_i(q, s)$ when $q > 0$. Matters are simpler when $q = 0$. In this case a pair (y_i, w_i) maximizes profits if and only if y_i maximizes operational profits $py_i - c(y_i, s, \theta_i)$, and w_i is high enough that $\phi(M, y_i, w_i) = 0$. As we assumed c strictly convex in y_i, it follows that there is only one profit-maximizing y_i.

3.5.2 Temporary Equilibria

We show in Theorem 4 that a temporary equilibrium always exists at every stock level and that it is unique when the $\mathcal{TAC}$ is lower than the open access catch at that stock level. In Theorem 5, we show that the equilibrium maps $s \mapsto q(s)$ and $s \mapsto Y(s)$ are continuous and nondecreasing.

Theorem 4. *Fix $M, \mathcal{TAC}, s > 0$ arbitrarily. There exists $w \in \mathbb{R}_+^n$ and unique $y \in \mathbb{R}_+^n, q \geqslant 0$ such that (y, w, q, s) is a temporary equilibrium. If $q > 0$, then w is also unique. Furthermore, $q > 0$ if and only if $\sum_i \widetilde{y}_i(0, s) > \mathcal{TAC}$.*

In more detail: if $\sum_i \widetilde{y}_i(0, s) > \mathcal{TAC}$ then there exists unique $q > 0, y, w \in \mathbb{R}_+^n$ such that (y, w, q, s) is a temporary equilibrium. If $\sum_i \widetilde{y}_i(0, s) < \mathcal{TAC}$, then (y, w, q, s) is an equilibrium if and only if $q = 0$, $y = \widetilde{y}(0, s)$, $\sum_i w_i = \mathcal{TAC}$, and $w_i \geqslant y_i$ for all i. Finally, if $\sum_i \widetilde{y}_i(0, s) = \mathcal{TAC}$, then (y, w, q, s) is an equilibrium if and only if $y = w = \widetilde{y}(0, s)$ and $q = 0$.

Proof.

Case 1. First, suppose $\mathcal{TAC} - \sum \widetilde{y}_i(0, s) < 0$. We claim excess demand of quota will be positive if $q = 0$. Indeed, if y_i, w_i solve (3.4), then $q = 0$ implies $w_i \geqslant y_i$, and thus $z(q = 0, s) > 0$. Therefore, we can restrict our search to temporary equilibria with positive quota price. We know that $z(q, s) \to -\mathcal{TAC}$ as $q \to \infty$, and thus by the intermediate value Theorem there will be $q > 0$ such that $z(q, s) = 0$. It follows from Corollary 3 that there is only one such q. Furthermore, in an equilibrium with $q > 0$, we must have $w_i \leqslant y_i$ for all i. Therefore, profits π_i are strictly concave in (y_i, w_i) thus proving the uniqueness of the temporary equilibrium at s.

Case 2. In case $\mathcal{TAC} - \sum \widetilde{y}_i(0, s) > 0$, there can be no temporary equilibrium with $q > 0$. To see that, notice that at positive price $w_i \leqslant y_i$ at a solution of the fishermen problems (3.4). Therefore, excess quota demand will be negative if $q > 0$. We will now show that there is a temporary equilibrium with $q = 0$. Set $q = 0$ and let

$$S = \{(y, w) \in \mathbb{R}_+^n \times \mathbb{R}_+^n : (y, w) \text{ solve } (3.16)\}$$

As for all $(y, w) \in S$ and any $w' \geqslant y$ we have $(y, w') \in S$ we conclude that there are points (y, w)in S such that $\sum w_i = \mathcal{TAC}$ because $\sum \widetilde{y}_i(q = 0, s) \leqslant \mathcal{TAC}$. Let (y, w) be any such points. By construction, the tuple (y, w, q, s) satisfies all the conditions for a temporary equilibrium. Furthermore, we cannot have any $w_i < y_i$ in a temporary equilibrium with $q = 0$ because fisherman i could increase his profit by buying more quota for free.

Case 3. Finally, $\mathcal{TAC} - \sum \tilde{y}_i(0, s)$ may be zero. In that case, a natural equilibrium candidate is (y, w, q, s) with $q = 0$, $y = w = \tilde{y}(0, s)$. We can verify that (y, w, q, s) is indeed a temporary equilibrium by checking that there exists $\mu_i^y, \mu_i^w \geqslant 0$ for all $i \in \mathcal{I}$ such that conditions (3.14–3.15) are satisfied with $\alpha_i = 0$ for all i. In fact, we can show that this is the only temporary equilibrium. To see that, suppose (y, w, q, s) is a temporary equilibrium. Because $\mathcal{TAC} > 0$, some fisherman i must have positive quota $w_i > 0$. As $y_i = w_i$, his choice of y_i must be an interior maximizer of operational profits $py_i - c(y_i, s, \theta_i)$ It follows from first-order conditions that, $p = D_1 c(y_i, s, \theta_i)$. Therefore, condition (3.14) holds only if $\alpha_i = 0$. It follows from (3.15) that $q = 0$, and therefore $y = w = \tilde{y}(0, s)$.

□

Lemma 4. *Given $M, \mathcal{TAC}$ and under our simplifying Assumptions, the temporary equilibrium maps $s \mapsto y(s)$, $s \mapsto w(s)$ are continuous. The temporary equilibrium map $s \mapsto q(s)$ is continuous on all points but those s^* where $Y(s^*) = \mathcal{TAC}$; where that happens, $q(s^*)$ might be a set, but if marginal violations $D_2\phi$ are zero when violations $y_i - w_i$ are zero, then $q(s^*)$ is a point, and q is continuous at s^*.*

Proof. Fix M and $\mathcal{TAC}$ arbitrarily. The tuple (y, w, q, s) is a temporary equilibrium if and only if y, w solve

$$\begin{aligned} \underset{y,w}{\texttt{maximize}} \quad & \sum_{i \in \mathcal{I}} py_i - c(y_i, s, \theta_i) - \phi(M, y_i, w_i) \\ \texttt{subject to} \quad & y_i \geqslant 0 \quad \forall i \in I \\ & w_i \geqslant 0 \quad \forall i \in I \\ & \mathcal{TAC} - \sum_{i \in \mathcal{I}} w_i = 0 \end{aligned} \tag{3.23}$$

and q is equal to the multiplier of the last constraint. It follows from the maximum Theorem that the temporary equilibrium maps $s \mapsto y(s)$ and $s \mapsto w(s)$ are continuous functions.

Let us now prove that $s \mapsto q(s)$ is continuous. Define the set

$$V(\tilde{s}) = \{i \in \mathcal{I} : y_i(\tilde{s}) > w_i(\tilde{s})\}$$

as the set of *violators* and

$$H(\widetilde{s}) = \{i \in \mathcal{I} : w_i(\widetilde{s}) > 0\}$$

as the set of *quota holders*. Suppose that $Y(\widetilde{s}) > \mathcal{TAC}$. This Assumption guarantees that $V(\widetilde{s}) \neq \emptyset$ and the constraint $\mathcal{TAC} - \sum w_i = 0$ in the problem above guarantees that $H(\widetilde{s}) \neq \emptyset$. Theorem 3 guarantees that $\emptyset \neq H(\widetilde{s}) \subset V(\widetilde{s})$. Therefore we can pick a fisherman j in $V(\widetilde{s}) \cap H(\widetilde{s})$.

Because of the continuity of the maps $y(s), w(s)$ and the fact that $q(s) > 0$ whenever $Y(s) > \mathcal{TAC}$ we can find an open interval U containing $\widetilde{s}$ such that for all $s \in U$

$$j \in V(s) \cap H(s) \text{ and } q(s) > 0$$

Therefore, using the optimality condition on w_i in the problem above, we conclude that for all $s \in U$

$$q(s) = -D_3\phi(M, y_i(s), w_i(s))$$

It follows from the continuity of $D_3\phi$ that $s \mapsto q(s)$ is a continuous function on U, as desired. From here, the cases where $Y(s) \leqslant \mathcal{TAC}$ are straightforward. □

Theorem 5. *Given $M, \mathcal{TAC} > 0$ the temporary equilibrium maps $s \mapsto q(s)$ (aside from, possibly, a point where $(Y(s) = \mathcal{TAC})$) $s \mapsto Y(s)$ are non decreasing, $Y(s) > \mathcal{TAC}$ implies $q(s) > 0$, and $Y(s) < \mathcal{TAC}$ implies $q(s) = 0$. Furthermore, where $q, Y > 0$, those maps are strictly increasing in s. If $D_2\phi = 0$ when violations $y_i - w_i$ are zero, then $Y(s) = \mathcal{TAC}$ implies $q(s) = 0$.*

Proof. Remember that $q(s)$ and $Y(s)$ are the quota price and total output associated with the temporary equilibrium at s. Note that as s increases, marginal costs $D_1c(y_i, s, \theta_i)$ (for fixed y_i) go down, and thus the marginal benefit of violating the constraint $\mathcal{TAC} - \sum w_i \geqslant 0$ cannot go down. Therefore, $q(s)$ cannot be decreasing in s. Indeed, as we assumed that $D_{21}c(y_i, s, \theta_i) < 0$ for all s, it follows that the marginal benefit of violating the constraint *goes up* once there is any benefit at all. By the same token, $Y(s)$ cannot be decreasing in s and is strictly increasing once $Y(s) > 0$.

Note that for all s, the temporary equilibrium $y_i(s)$ is given by the optimal catch $\widetilde{y}_i(q(s), s)$.

Now, fix s such that $Y(s) > \mathcal{TAC}$. As $Y(s) = \sum_i \widetilde{y}_i(q(s), s) \geqslant \sum_i \widetilde{y}_i(0, s)$, we must have $\sum_i \widetilde{y}_i(0, s) > \mathcal{TAC}$. It follows from Theorem 4 that $q(s) > 0$. We now prove the converse via its contrapositive. Fix s satisfying $Y(s) < \mathcal{TAC}$. Then a solution of problem (3.23) is also a solution of the same problem without the constraint $\sum_i w_i = \mathcal{TAC}$. Therefore, 0 has to be a multiplier of that constraint, and thus a temporary equilibrium quota price q at s. As we already showed in Theorem 4 that for every $s \geqslant 0$ and $\mathcal{TAC} > 0$ there is at most one temporary equilibrium $q(s)$, we conclude that $q(s)$ has to be zero. □

Corollary 4. *Fix $M, \mathcal{TAC} > 0$. There exists at most one point $\bar{s} \geqslant 0$ such that $Y(\bar{s}) = \mathcal{TAC}$. If such a point exists, then*

$$s < \bar{s} \Longrightarrow q(s) = 0$$

$$s > \bar{s} \Longrightarrow q(s) > 0$$

Lemma 5. *The temporary equilibrium variables y, w, q are smooth functions of $p, M, \mathcal{TAC}, q, s$ as long as $y_i > w_i > 0$ for all agents i.*

Proof. Follows from the implicit function Theorem. See Lemma 10 in Appendix A.1. □

Lemma 6. *Fix $\bar{s}, \overline{M}, \overline{\mathcal{TAC}} > 0$ and an associated temporary equilibrium $(\bar{y}, \bar{w}, \bar{q}, \bar{s})$ such that every active fisherman violates his quota. There exits an open neighborhood $V \times W$ of $(\overline{M}, \overline{\mathcal{TAC}})$ where the temporary equilibrium map $M \mapsto Y(M, \overline{\mathcal{TAC}}, \bar{s})$ is decreasing and the temporary equilibrium map $TAC \mapsto Y(\overline{M}, \mathcal{TAC}, \bar{s})$ is increasing.*

Proof. We already showed in Lemma 5 that those maps are differentiable. The monotonicities are very intuitive, and can be verified by computing the partial derivatives. □

3.5.3 Steady-State Equilibria

An outcome (y, w, q, s) is an equilibrium when it is a temporary equilibrium and $Y = g(s)$. Thus, with the results from Section 3.5.2 we can now visualize the set of equilibria by making a superimposed plot of the maps $s \mapsto g(s)$ and $s \mapsto Y(s)$ where $Y(s)$ is the total output of the temporary equilibrium at s. We say an outcome $(y, w, q, s)$is a ***high-stock equilibrium*** if $s \geqslant sMSY$ where $sMSY = \text{argmax}\, g(s)$ is the stock associated with the maximum sustainable yield. We say that the outcome is a ***low-stock equilibrium*** if $s < sMSY$.

Remark 5. For fixed $\mathcal{TAC}, M$, an equilibrium with stock at least $s \geqslant sMSY$ exists if and only if $Y(s) < g(s)$. If such an equilibrium exists, then it is the only high-stock equilibrium. It follows that if the pure-poaching catch $\widetilde{Y}(\infty, sMSY)$ is larger than the maximum sustainable yield MSY, then a high-stock equilibrium is attainable only if the monitoring expenditure is raised. In other words, if closing the fishery cannot bring the total catch to levels below MSY, then a high-stock equilibrium will not be attained without more investment in monitoring.

Remark 6. Given a fixed monitoring expenditure M, we can use the Theorems in Section 3.5.2 to classify possible instances of our model in three categories:

- *High cost of fishing the $\mathcal{TAC}$.* In this case, agents can never sustain a low-stock equilibrium with positive stock, that is, $Y(s) < g(s)$ for all $s \in (0, sMSY)$. Here, we have one and only one equilibrium besides the one at $s = 0$, and it presents a high stock level.

- *Low cost of fishing the $\mathcal{TAC}$.* In this case, the agents are productive enough to sustain a low-stock equilibrium, that is, there exists $s \in (0, sMSY)$ such that $Y(s) = g(s)$. While one might think that a high-stock equilibrium should exist too, that need not be the case, as shown in figure 3.1.

- *Extremely low cost of fishing the $\mathcal{TAC}$.* In this case fishermen always want to fish more than the environment can provide, that is, $Y(s) > g(s)$ for all $s > 0$. The only equilibrium is total stock collapse at $s = 0$.

See figure 3.1 for some illustrations.[12]

[12] Both x and y axis are measured in tons of fish. The parabola in green is the stock-growth function $g(s)$, and the other curve in red is the temporary equilibrium total output function $Y(s)$. In the top left, the agents are "too productive" and the only equilibrium is stock collapse. In the top right, there is a low-stock, unstable equilibrium and a high-stock stable equilibrium. In the bottom left, the agents are "not very productive" in the sense that there is a stable high-stock equilibrium and no low-stock equilibrium besides collapse. In the bottom right, there is a minimum stock level for positive production, and that allows the emergence of a *stable* low-stock equilibrium. All plots were generated by numerically computing the temporary equilibrium at $s = 0, 1, 2, \ldots, 100$ for different instances of the model.

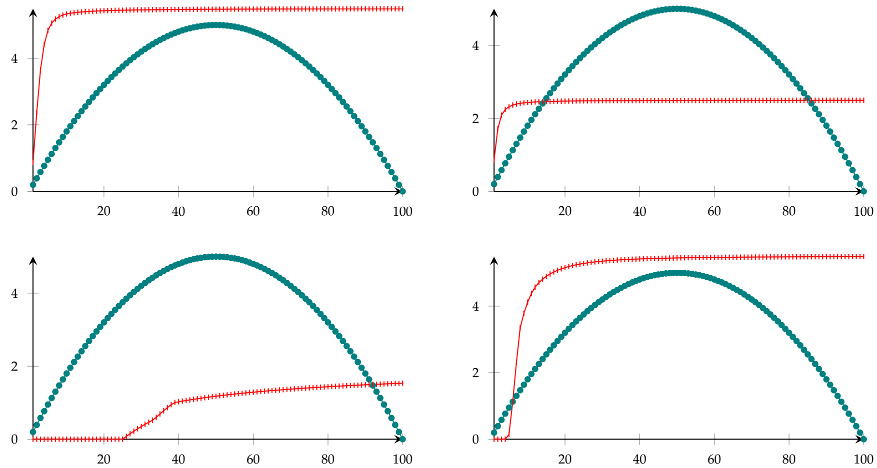


Figure 3.1: Temporary equilibrium graphs

We say an equilibrium $(\overline{y}, \overline{w}, \overline{q}, \overline{s})$ is ***degenerate*** when the temporary equilibrium curve $s \mapsto Y(s)$ is tangent to the stock-growth curve $s \mapsto g(s)$ at $\overline{s}$. An equilibrium is ***regular*** when it is not degenerate. From now on, we will focus on regular equilibria because they are the only outcomes from this model that we could, in principle, actually observe. We will make that precise now.

A necessary and sufficient condition for regularity of an equilibrium with violators is that the Jacobian of the equilibrium system (3.8–3.6) (depicted in page 75) be invertible. The implicit function Theorem allows us to write regular equilibria locally as a continuously differentiable function of the parameters $p, M, \mathcal{TAC}$.

It follows that, starting from a regular equilibrium, if we perturb the parameters $p, M, \mathcal{TAC}$ "just a little bit", we will obtain a new equilibrium that is also regular. This is graphically intuitive. Perturbing the parameters $p, M, \mathcal{TAC}$ entails perturbing the temporary equilibrium curve $s \mapsto Y(s)$; it should not be surprising then that if a curve $s \mapsto Y(s)$ at first crosses the growth curve $s \mapsto g(s)$ then a perturbation of Y should also cross g.

Degenerate equilibria on the other hand do not have that property. Intuitively, if the temporary equilibrium catch curve $s \mapsto Y(s)$ is tangent to the growth curve $s \mapsto g(s)$ at $\overline{s}$, then a small perturbation of the parameters can lead to either the curves crossing each other, or not intersecting at all.

Real-world measurements of Y, s, g and other variables always contain some error. In that sense, it is virtually impossible to observe a degenerate equilibrium. Theorem 6 formalizes the discussion up to here.

Theorem 6. *Almost all equilibria are regular. More formally, consider an open set $\Gamma \subset \mathbb{R}^3$ of tuples $(p, M, \mathcal{TAC})$ for which equilibrium exists. Let $\Gamma' \subset \Gamma$ be the set of such tuples where the associated equilibria are regular. Then $\Gamma \setminus \Gamma'$ is a nowhere dense set of measure zero.*

Proof. Locally, this is a consequence of the implicit function Theorem and Lemma 11 in Appendix A.1. We can then globalize the result because Γ is separable and the countable union of sets of measure zero has measure zero. Nowhere denseness is a local property, so we do not have to worry about globalizing that. □

The arguments presented here deal with the problem of *determinacy of equilibrium*. See Debreu (1970) for the start of this literature in economic theory and Shannon (2008) for a comprehensive survey that includes pointers on how to extend these arguments to the nonsmooth case.

3.5.4 Stability of Equilibrium under Myopic Dynamics

Here we analyze what happens to equilibria when we slightly perturb the stock level. The dynamics are determined by

$$s' = g(s) - Y(s) \tag{3.24}$$

where $Y(s)$ is the temporary equilibrium total output at stock s, so $Y(s) = \sum_i y_i(s)$ where each $y_i(s)$ is in a solution to (3.4).

We say a regular equilibrium (y, w, q, s) is (locally) ***stable*** when it is robust to small perturbations in the stock level s. A necessary and sufficient condition for that is $g'(s) < Y'(s)$. In other words, at a stable equilibrium, the temporary equilibrium curve Y cuts the stock-growth curve g from below. It follows that high-stock equilibria are always locally stable, while low-stock equilibria may not be.

Remark 7. Stable equilibria are the only outcomes of this model that could, in principle, be "credible" outcomes in a steady state. Indeed, while we do not incorporate stock shocks into our model, in reality they do exist; the only equilibria that can persist under such shocks are stable ones. It is reassuring then to know that high-stock equilibria are always stable. However, as figure 3.1 shows, very inefficient, low-stock equilibria can also be stable. If this model is a suitable approximation of reality, then this last observation suggests that, if a high-stock equilibrium is desired, then fisheries with severely depleted stocks should cease activities for some time to allow the stock to recover before a cap-and-trade system is put in place.

3.6 Comparative Statics: Varying the Design Variables

We now investigate how changing the monitoring level $M \geqslant 0$, the $\mathcal{TAC} \geqslant 0$ and quota endowments $\omega \in \mathbb{R}^n_+$ affects equilibrium.

Theorem 7. *Fix $(p, M, \mathcal{TAC}) \in \Gamma'$ as defined in Theorem 6. Fix an associated regular, stable equilibrium (y, w, q, s) with positive violations. The equilibrium variables Y, q and s have the following local monotonicity properties:*

- *The stock s is increasing in the monitoring level M.*
- *All other parameters equal, q varies in the same direction as s.*
- *At a high-stock equilibrium, Y is decreasing in M.*
- *At low-stock equilibria, Y is increasing in M.*
- *Lowering the $\mathcal{TAC}$ changes Y, q, s in the same direction as raising M.*

Proof. From what we discussed in Section 3.5.3, we know that regular equilibria (y, w, q, s) are locally continuously differentiable functions of M and $\mathcal{TAC}$. Therefore, we can pin down the monotonicity results we want by analyzing the appropriate derivatives.

Making explicit the role of M in the steady-state equation (3.6), we obtain

$$g(s(M, \mathcal{TAC})) - Y(s(M, \mathcal{TAC}), M, \mathcal{TAC}) = 0$$

differentiating that with respect to M we obtain

$$D_M s = \frac{Y_M}{g' - Y_s}$$

where Y_s, Y_m are *partial* derivatives of the *temporary equilibrium* map $(s, M, \mathcal{TAC}) \mapsto Y(s, M, \mathcal{TAC})$ and g' is the derivative of the growth function $s \mapsto g(s)$. As $Y_M < 0$ (remember, this is a temporary equilibrium map, so it just measures the direct effect of M on total catch) it follows that the sign of $D_M s$ is the sign of $Y_s - g'$. Remember that Y_s is the slope of the temporary equilibrium map $s \mapsto Y(s)$, and thus at stable equilibria, $Y_s - g'$ must be strictly positive. It follows that $D_M s > 0$, as desired.

Remember from Theorem 5 that the direct effect of s on q is positive and because q is the Lagrange multiplier of the market-clearing constraint in (3.23) it must be the case that the direct effect of M on q is positive. Differentiating the equilibrium map $(M, \mathcal{TAC}) \mapsto q(s(M, \mathcal{TAC}), M, \mathcal{TAC})$ with respect to M we conclude that $D_M q = q_s D_M s + q_M$. We conclude from what we just discussed and the previous paragraph that $D_M q > 0$.

The sign of $D_{\mathcal{TAC}} s$ and $D_{\mathcal{TAC}} q$ can be obtained analogously, and it should be clear by now why their signs are opposite to the signs of the corresponding M-derivatives.

The monotonicity on Y is graphically intuitive: raising M lowers the temporary equilibrium curve $s \mapsto Y(s)$; therefore, it will intercept the graph of g at lower points.

The effects of the $\mathcal{TAC}$ are opposite because raising it pushes the temporary equilibrium curve $s \mapsto Y(s)$ up. □

3.7 Larger Agents vs. Smaller Agents

In order to compare equilibrium outcomes across agents, we will need to make further Assumptions.

Assumption 4. The type space Θ is an interval of real numbers. Given agents $i, j \in \mathcal{I}$, we say i is ***larger than*** j or that j is ***smaller than*** i when i has a higher fixed cost and a lower marginal cost than j; we indicate that by setting their types $\theta_i > \theta_j$. Given our Assumptions, the geometrical meaning is that the marginal cost curve $y_i \mapsto D_1 c(y_i, s, \theta_i)$ of a given agent i is always below the marginal cost curve of agents smaller than i.

Assumption 5. The larger agents bear the brunt of the externality: for all $y, s \geqslant 0$ we have $D_2 c(y, s, \theta)$ decreasing in θ. Remember $D_2 c$ is always negative. The intuition is that, holding the catch fixed, with a big boat costs rise sharply if the stock decreases, or equivalently, costs fall quickly if the stock increases. One can imagine that with a very low stock, both a big boat and a small boat would need to spend more or less the same number of hours in the water, while with high stock the big boat can catch a lot of fish very quickly on the same spot, spending less time in the water than the small boat. The cost increases or decreases are then explainedby the fact that a big boat has higher operational costs per hour.

Lemma 7. *In equilibrium, larger agents produce more and buy more quota. Precisely, if $\theta_i > \theta_j$, then in equilibrium $w_i \geqslant w_j$ and $y_i \geqslant y_j$. Furthermore, if $w_i > 0$, then these inequalities are strict.*

Proof. Let $\theta_i > \theta_j$ as in the Theorem statement. Note that π has strictly increasing differences in (y_i, θ_i) and (weakly) increasing differences in (w_i, θ_i) and (y_i, w_i). Therefore, if y_i, y_j maximizes the profits of fishermen i, j, we must have $y_i \geqslant y_j$. Finally, by Lemma 2, it follows that $w_i \geqslant w_j$. □

3.7.1 Varying the Design Variables

The message of Theorem 8 below is the following: if wealth effects are absent (identical net trades of quota across agents) or larger for larger agents, then larger agents benefit marginally from monitoring more than smaller agents. For this result, we will need to assume that ϕ depends on y_i, w_i only through the absolute violation $y_i - w_i$).

Theorem 8. *Fix an interior equilibrium. Let $\theta_i > \theta_j$ and suppose wealth effects are absent or larger for the larger agent, that is, $\omega_i - w_i \geqslant \omega_j - w_j$. Then $D_M \pi_i > D_M \pi_j$.*

Proof. Suppose $\omega_i - w_i = \omega_j - w_j$ for simplicity. It follows from Lemma 7 that $y_i > y_j$. As

$D_M\pi_i = D_M q(\omega_i - w_i) - D_M s D_2 c(y_i, s, \theta_i) - D_1\phi(M, y_i, w_i),$

$$\begin{aligned} D_M\pi_i - D_M\pi_j &= D_M s \left(D_2 c(y_j, s, \theta_j) - D_2 c(y_i, s, \theta_i)\right) \\ &= D_M s \left(D_2 c(y_j, s, \theta_j) - D_2 c(y_j, s, \theta_i) + D_2 c(y_j, s, \theta_i) - D_2 c(y_i, s, \theta_i)\right) \end{aligned}$$

It follows from our Assumptions that both the first difference and the second difference in the parentheses above are positive. Therefore, as $D_M s > 0$ by Theorem 7, $D_M\pi_i - D_M\pi_j > 0$, as desired. It should be clear now that if $\omega_i - w_i > \omega_j - w_j$ then the gap $D_M\pi_i - D_M\pi_j > 0$ is further widened. □

Theorem 9. *Fix an interior equilibrium. Let $\theta_i > \theta_j$ and suppose wealth effects are absent or larger for the larger agent, that is, $\omega_i - w_i \geqslant \omega_j - w_j$. Then $D_{\mathcal{TAC}}\pi_i < D_{\mathcal{TAC}}\pi_j$.*

Proof. Analogous to the proof of Theorem 8, but note that $D_{\mathcal{TAC}} s < 0$ according to Theorem 7. □

3.8 Final Remarks

In this paper we studied individual tradable quota (ITQ) programs for the exploitation of renewable resources when monitoring is imperfect and costly. We examined how the welfare, political and stability properties of equilibria change as we vary the intensity and technology of enforcement or the level of the cap. We also investigated the properties of the second-best solution to the fishery problem and of the multiple steady-state equilibria that may arise in an ITQ program.

We learned that the optimal single-owner choice of enforcement and cap is associated with positive quota violations and that in an ITQ equilibrium either nobody or everyone violates quota. We also learned that if expected fines ϕ depend on the catch y_i and quota held w_i only through the absolute violation $y_i - w_i$, then the second-best outcome is ITQ immplementable, and, if quota holdings are “right”, then larger boats are more likely to want more enforcement and a lower cap.

Finally, we saw how the initial allocation of quota —while incapable of affecting equilibria directly— can *indirectly* affect outcomes if fishermen can influence the choice of the cap and enforcement levels.

We believe these results provide a foundation for what is the next step in this research agenda: a good mechanism that regulators could use to set the $\mathcal{TAC}$, its distribution among fishermen, and enforcement levels.

It is natural to expect that in most institutional setups, the expenditure on monitoring is largely influenced (if not paid for) by the members of the industry, while the $\mathcal{TAC}$ is insulated from such influence. It is therefore crucial to know, given a certain status quo, which fishermen will support more monitoring, and which will support less monitoring; that knowledge will tell us which outcomes are feasible from a political point of view. While we provide the first steps in that direction, significant work remains ahead: crucially, we do not know when the optimal monitoring level (in the sense of (3.17)) is supported by a majority of fishermen, a topic left for future research. We believe any new developments in that direction could be of great value to regulators and managers of tradable quota programs.

Chapter 4

Vessel Buyback

4.1 Introduction

When access to a fishery is unrestricted, there is an incentive for fishers to invest in capital so they can catch more fish before other fishers. This incentive leads over time to low fish stocks and excessive investment in capital, a combination that has led the World Bank and the UN (see Willman, Kelleher, Arnason, and Franz (2009)) to report that 50 billion dollars are lost every year across marine fisheries worldwide. To reduce these losses, the total fishing effort must be reduced and part of the excess capital must be retired. Buyback policies (*buybacks* from now on) accomplish that by setting up a market where the excess capital can be sold.

Buybacks have multiple purposes, among them:

- modernizing the fleet;
- addressing compensational and distributional concerns;
- creating an opportunity for a transition to stronger rights-based policies;
- presenting an alternative when stronger rights-based policies (like individual tradable quotas) are not feasible.

However, a buyback also present some challenges, the most fundamental of them being that it does not address the main failures of open-access fisheries: it does not restrict access and it does not assign clear property rights.

In practice (see references in Clark, Munro, and Sumaila (2005)), buybacks have required enormous amounts of money in the form of subsidies, as much as one third of all subsidies to fisheries (Munro and Sumaila (2001)). As explained by Clark, Munro, and Sumaila (2005), the losses from such subsidies can be amplified if fishers expect buybacks to occur; in those cases even more money will be spent and the stock of fish will be lower than if there were no buybacks. For a recent survey on the effects and practices of buybacks, see Squires (2010).

This paper addresses the problem of excessive subsidy in buyback auctions using the tools of mechanism design theory. We first cast the problem in the language of that theory, and then explain when certain desirable properties of these policies are simultaneously attainable.

4.2 The Model and Preliminary Results

4.2.1 The Fishery

We model the fishery in a standard manner. There is a set $\mathcal{I} = \{1, \ldots, n\}$ of vessel owners (henceforth ***fishers***). Each fisher i has a cost-relevant, private characteristic $\theta_i \in \Theta = [\underline{\theta}, \overline{\theta}] \subset \Re_+$ (henceforth i's ***type***). Each fisher pays a cost $c(y_i, s, \theta_i, k_i)$ to fish an amount $y_i \in \Re_+$ of fish (referred to as the ***catch***) when the ***stock*** of fish is $s \in \Re_+$ and the vessel's ***capacity*** is $k_i \in \Re_+$. It is assumed that c has continuous second derivatives, is convex in y_i, increasing in y_i and k_i, and decreasing in s and θ_i.

We assume the market for fish is competitive and, thus, the ***profits*** of fisher i are given by

$$\pi(y_i, s, \theta_i, k_i) = py_i - c(y_i, s, \theta_i, k_i). \tag{4.1}$$

The fisher will choose y_i to maximize profits in each period. Let $y_i(s, \theta_i, k_i) = \arg\max_{y_i} \pi(y_i, s, \theta_i, k_i)$ and $\pi^*(s, \theta_i, k_i) = \max_{y_i} \pi(y_i, s, \theta_i, k_i)$.

The stock of fish s obeys a ***growth function***

$$s_{t+1} - s_t = g(s_t) - \sum_{i=1}^{n} y_i(s_t, \theta_i, k_i). \tag{4.2}$$

The function g is initially increasing in s and single-peaked. We say s is a ***steady state***[1] when $0 = g(s) - \sum_i y_i(s, \theta_i, k_i)$

4.2.2 Buyback Policies

In this paper we are interested in the use of buyback programs to transform a low profit, poorly managed fishery into a high-profit, well-managed fishery. The idea is simple. To make the analysis transparent, we compare two steady states stocks, s_0 and s_1, where $s_0 < s_1$. At the initial stock levels, s_0, all fishers in $\mathcal{I}$ are active and $g(s_0) = \sum_{i=1}^{n} y_i(s_0, \theta_i, k_i)$. Since $\partial c/\partial s < 0$, if the total catch does not change and if we can increase the stock from s_0 to s_1, then the profits of each boat would also increase. But, without further control, as s increases, y also increases which drives the stock back down towards s_0. So unless y is controlled, per period profits remain around $\pi^*(s_0, \theta_i, k_i)$.

Buyback policies are intended to control the catch so that increases in profits can be maintained. In simple terms, a buyback policy identifies a set of boats $W \subset \mathcal{I}$ which will remain in the industry. The rest of the boats leave the industry after being bought out. The aggregate catch of those who stay is $y^W(s) = \sum_{i \in W} y_i(s, \theta_i, k_i) < y(s)$. This leads to a new steady state, $s_1 > s_0$, satisfying $g(s_1) = y^W(s_1)$. For the boats in W, profits are higher. Further, if the more efficient boats are chosen, then this increase in profits will be high enough to allow the winners to compensate the losers without any subsidization by outside sources.

More formally, the **outcome** of a buyback policy is (W, t) where W is the list of fishers remaining in the fishery, and t_i is the amount that fisher i pays (or receives if $t_i < 0$.). A buyback policy is an outcome for each possible value of $\theta = (\theta_1, ..., \theta_n)$. That is, it is a function

[1]Note that this does not take into account the possible entry or exit of boats as a function of profits. We will come to that later

$(W(\theta), t(\theta)) : \Theta \rightarrow (2^{\mathcal{I}}, \Re^n)$.

In this paper we are interested in identifying a buyback policy with outcomes satisfying some desirable properties. We are going to assume that a target stock K^* is chosen prior to our analysis.[2] Given a target stock, a desirable buyback policy will be self-financing (requiring no subsidy and generating no surplus), select the more efficient boats to stay in the fishery, and to leave all fishers better off than they would have been with no policy. Whether a policy does depends on the profits of the fishers both in the current fishery and in the fishery after the policy has been implemented.

The key part of the environment is the expectations of the fishers for profits in the fishery both before and after the buyback policy is implemented. We let $u_0(\theta_i, k_i)$ be fisher i's expected present discounted value of being in the fishery if there is no buyback policy and the current situation continues. In the initial situation, all boats are in the fishery and so, in the steady state, $g(s_0) = \sum_{i=1}^{n} y_i(s_0, \theta_i, k_i)$. So $u_0^i(\theta_i, k_i)$ is proportional to $\pi^*(s_0, \theta_i, k_i)$. We let $u^i(\theta_i, k_i)$ be fisher i's expected present discounted value of being in the fishery if the buyback policy is implemented and fisher i is in W. This value, of course, depends on the composition of the boats in W which is unknown until after θ is revealed and the policy is implemented. But the key fact the fisher would like to know is the stock in the new steady state, s_1 where $g(s_1) = \sum_{i \in W} y_i(s_1, \theta_i, k_i)$. We assume for purposes of this paper, that the fisher ignores the effect of the composition of W and just estimates s_1. Then i can estimate $\pi^*(s_1, \theta_i, k_i)$. $u^i(\theta_i, k_i)$ will be proportional to $\pi^*(s_1, \theta_i, k_i)$.

Knowing the values the fishers have for $u_0^i(\theta_i, k_i)$ and $u^i(\theta_i, k_i)$ we can formally define some desirable properties that a buyback policy should have.

- (Efficient Selection) Given K^*, the boats that stay are the most efficient ones;

[2] It is possible to expand the analysis to make the choice of K^* endogenous, but we leave that for future work.

$$W(\theta) \in \arg\max_{W \in \mathcal{I}} \sum_{i \in W} u^i(\theta_i, k_i) \quad \text{subject to} \quad \sum_{i \in W} k_i \leq K^*. \tag{4.3}$$

- (Self-Financing) Any revenue from the policy should go back to the fishers; any loss from the policy should be paid by the fishers;

$$\sum_{i \in \mathcal{I}} t_i(\theta) = 0. \tag{4.4}$$

- (Voluntary Participation) All fishers should be better off at the policy outcome than they would have been without any policy;

$$u^i(\theta_i, k_i) - t^i(\theta) \geq u_0^i(\theta_i, k_i), \forall i = 1, ..., n. \tag{4.5}$$

And we want all of this to be implemented in a way that is consistent with the fishers' incentives.

4.2.3 Policy Design

Mechanism design is a well-developed theory we can use to identify conditions under which a satisfactory buyback policy may exist. In this Section, we explore what can be learned about good buyback policies from that theory. We begin with the basics: environments, the revelation principle, and incentive compatibility. We then go over some results.

The Environment In the current situation, fisher i has expected present discounted profits of $u_0^i(k_i, \theta^i)$ if i continues to fish. A buyback policy has been announced with a target of K^* capacity to remain in the fishery. Fisher i has expected discounted profits of $u^i(k_i, \theta^i)$ if the policy is implemented and fisher i continues to fish. Types θ^i are private information and are independently distributed with densities f_i on Θ. Everything except θ is common knowledge. We assume fishers are risk neutral. We define θ_{-i} as the list of types of all agents other than i and define $f_{-i}(\theta_{-i}) = \prod_{j \neq i} f_j(\theta_j)$.

The Revelation Principle Given K^*, a buyback policy selects a set of fishers, W, to stay in the fishery and a set of transfers, t. Since Θ is private information, any process that implements a buyback policy on the basis of information collected from the fishers, can be viewed as a Bayesian game. For example, if an auction is held then fishers provide information in the form of their bids and (W, t) is determined by the auction rules and the equilibrium strategies of the fishers. This process produces a buyback policy $(W, t) : \Theta \to (2^{\mathcal{I}}, \Re^n)$. The **Revelation Principle** is that any buyback policy attainable through the Bayesian Equilibrium of some process is also attainable through a Direct Revelation Process that is Incentive Compatible.

Incentive Compatibility A Direct Revelation Process asks fishers to report their value of θ^i and then chooses $(W(\theta), t(\theta))$. A Direct Revelation Process is Incentive Compatible if reporting truthfully is a Bayesian Equilibrium. Let $q_i(\theta)$ be the probability that $i \in W(\theta)$. The interim probability that i will win (i.e., $i \in W$) if they report θ_i, and others report truthfully is $Q_i(\theta_i) = \int_{\Theta^{n-1}} q_i(\theta)\ f_{-i}(\theta_{-i})\ d\theta_{-i}$. The expected payment of i will be $T_i(\theta_i) = \int_{\Theta^{n-1}} t_i(\theta)\ f_{-i}(\theta_{-i})\ d\theta_{-i}$. The interim utility of i from reporting θ_i when their true type is $\hat{\theta}_i$ is therefore $v^i(\theta_i, \hat{\theta}_i, k_i) = u^i(\hat{\theta}_i, k_i)Q_i(\theta_i) - T_i(\theta_i)$. The policy, $(W(\cdot), t(\cdot))$ is Incentive Compatible (IC) if $\hat{\theta}_i \in \arg\max_{\Theta_i} v^i(\theta_i, \hat{\theta}_i, k_i)$.

Desirable Properties We gather up some well-known properties of direct revelation mechanisms that are relevant for buyback policies. Let $V_i(\theta_i, k_i) = \max_{\hat{\theta}_i} u^i(\theta_i, k_i)Q_i(\hat{\theta}_i) - T_i(\hat{\theta}_i)$. A policy (W, t) satisfies

Incentive compatibility (IC) iff $V(\theta, k) = V_0 + \int_{\underline{\theta}}^{\theta} u_\theta(s)Q(s)ds$,

Self-financing (SF) iff $\sum t^i(\theta) = 0$,

Efficient selection (ES) iff $W(\theta) \in \arg\max_{A \subset \mathcal{I}} \sum_{i \in A} u^i(\theta^i, k_i)$ subject to $\sum_{i \in A} k_i \leq K^*$, and

Voluntary participation (VP) iff $V^i(\theta^i) \geq u_0^i(\theta^i)$ for all θ^i.

Some Standard Results There are well-known results that can help us to understand whether buyback policies with desirable properties can be found.

- There are policies (W, t) that satisfy (IC), (ES) and (SF).

  The direct revelation mechanism of d'Aspremont and Gérard-Varet (1979), sometimes called the expected externality payment mechanism, can be easily modified to fit our situation and that is (IC), (ES), and (SF).

- In general, there may be no direct revelation polices (W, t) that simultaneously satisfy (IC), (ES), (SF), and (VP).

  This follows from the work of Myerson and Satterthwaite (1983). See also Ledyard and Palfrey (2007).

- To see whether the environment is such that we can have a policy satisfying (SF), (IC), (VP), and (ES) is straight-forward if complex.

First recognize[3] that given W there are t such that (W, t) satisfies (SF), (IC) and (VP) iff

$$\sum \int (u^i - \frac{1 - F^i}{f^i} u^i_\theta) Q^i dF^i \geq \sum \max_\theta [u^i_0(\theta) - \int_{\underline{\theta}}^{\theta} u^i_\theta(s) Q^i(s) ds]. \tag{4.6}$$

Let $W^*(\theta) \in \arg\max_{A \subset \mathcal{I}} \sum_{i \in A} u^i(\theta_i, k_i)$ subject to $\sum_{i \in A} k_i \leq K^*$. Compute $Q^i(\theta^i)$. Then plug this into (4.6). If the functions u_0 and u are such that the inequality holds then for this fishery, then it is possible to design a buyback policy that satisfies (SF), (IC), (VP), and (ES). Unfortunately that computation is complex and not very informative, so we do not do that here.

4.2.4 A Weak Sufficient Condition

If we are willing to relax (ES) a little it is possible to use (4.6) to provide a set of simple sufficient conditions on the fishery so that there is a buyback policy that satisfies (SF), (IC), and (VP), and is almost (ES). Suppose that the environment satisfies a regularity condition[4] that

$$\frac{d\left\{ u^i - \frac{1-F^i}{f^i} u^i_\theta \right\}}{d\theta} > 0 \tag{4.7}$$

Let $\bar{\theta}_i$ solve $u^i(\bar{\theta}_i) - \frac{1-F^i(\bar{\theta}_i)}{f^i(\bar{\theta}_i)} u^i_\theta(\bar{\theta}_i) = 0$ and let $R^i = u(\bar{\theta}_i, k_i)$. Think of R^i as a reserve price for which fisher i has to pay to be considered for continued fishing. Let $W^{**} \in \arg\max_{A \subset \mathcal{I}} \sum_{i \in A} u^i(\theta_i, k_i)$ subject to $\sum_{i \in A} k_i \leq K^*$ and $i \notin A$ if $u^i(\theta_i, k_i) < R_i$. W^{**} selects boats according to essentially the same criteria as (ES) but does not consider boats with values below their reservation value. Aggregate profits will be less under W^{**} that under (ES) unless the private type of all boats is such that they all above their reservation prices.

[3]See Section B.1.

[4]This is a standard monotonicity condition used in auction theory that ensures there is no pooling in the optimal solution.

Now note that $\sum \int (u(\theta_i, k_i) - \frac{1-F^i}{f^i} u_\theta(\theta_i, k_i) Q^{**}(\theta_i) dF^i \geq \sum \int (u^i - \frac{1-F^i}{f^i} u^i_\theta) dF^i \geq \sum_i u(\underline{\theta}, k_i)$.[5] The latter is true since $\int udF - \int 1 d\theta - \int F d\theta = \int udF - (u(\overline{\theta}) - u(\underline{\theta})) + (u(\overline{\theta}) - \int udF)$. It is, therefore, sufficient for (4.6) to hold that $\sum_i u(\underline{\theta}, k_i) \geq \sum \max_\theta [u^i_0(\theta) - \int_{\underline{\theta}}^{\theta} u^i_\theta(s) Q^i(s) ds]$. Now notice that, since $u_\theta > 0$ and $du_0/d\theta > 0$, $\max_\theta [u^i_0(\theta) - \int_0^\theta u^i_\theta(s) Q^i(s) ds] < u_0(\overline{\theta}^i, k_i)$. Combining all of this we have the following

Theorem 10. *If the regularity condition $d[u - \frac{1-F}{f} u_\theta]/d\theta > 0$ holds, and if $u(\underline{\theta}^i, k_i) \geq u_0(\overline{\theta}^i, k_i)$, then there is a payment scheme t^{**} such that (W^{**}, t^{**}) satisfies (SF), (IC), and (VP).*

In words, if the regularity condition is true and if the payoff to the worst type in fishery after capacity is limited is at least as great as the payoff to the best type in the current fishery, then there is a buyback policy that is self-financing, satisfies voluntary participation, and provides efficient selection subject to a reserve policy.

4.2.5 Second Best Selection

To get a sense for how strong the conditions in Theorem 10 are, consider the following problem: Choose a policy (W, t) to maximize aggregate profits subject to (IC), (SF), and (VP1), where (VP1) requires that $V(\theta_i, k_i) \geq u^i_0(\overline{\theta})$. Note that (VP1) is stronger than (VP) in the sense that any policy that satisfies (VP1) will also satisfy (VP). More formally, using the (IC) condition and (4.6), choose (W, t) to solve

$$\max_{q \in [0,1]^n} \sum \int V_0(k_i) + \int_{\underline{\theta}}^{\theta} u_\theta Q(x) dx dF(x) \tag{4.8}$$

subject to

$$\sum V_0(k_i) = \sum \int (u^i - \frac{1-F^i}{f^i} u^i_\theta) Q^i dF^i \geq \sum u_0(\theta_1, k_i) \tag{4.9}$$

$$\sum_{i \in A} k_i q_i(\theta) \leq K^* \tag{4.10}$$

Actually we are now choosing the probability that any fisher i will be a winner rather than choosing W by slightly changing the efficient selection problem. Earlier we chose W by maximizing over

[5] $\underline{\theta}$ is the lowest possible type.

$q \in \{0,1\}$. Here we choose $q \in [0,1]$. This helps us avoid some messy issues raised by discreteness. With this change, however, there may be one fisher for whom $0 < q_i < 1$. The interpretation is that there is a random draw to determine whether they are in or out.[6]

Substituting the left-hand side of (4.9) into (4.8) and letting λ be the Lagrange multiplier for the right-hand side of (4.9), we can rewrite part of this, given λ, as

$$\max \sum \int [(1+\lambda)u^i - \lambda \frac{1-F^i}{f^i} u^i_\theta] Q dF \tag{4.11}$$

subject to

$$\sum_{i \in A} k_i q_i(\theta) \leq K^* \tag{4.12}$$

This is equivalent to, for each θ, where $\delta = \lambda/(1+\lambda)$,

$$\max_{q \in [0,1]^n} \sum [u(\theta^i, k_i) - \delta \frac{1-F}{f} u_\theta(\theta^i, k_i)] q_i(\theta) \tag{4.13}$$

subject to

$$\sum_i k_i q_i(\theta) \leq K^*. \tag{4.14}$$

Note that if the (VP) constraint is not binding, $\lambda = 0$ and $\delta = 0$, and this yields (ES). As the (VP) constraint binds tighter, λ increases to ∞ and δ increases to 1. When (VP) binds as tightly as it can, we have (W^{**}, t^{**}) as the solution as in Theorem 10. That is, (W^{**}, t^{**}) is the worst one needs to do to satisfy (VP), (IC), and (SF). One can do better by setting lower reserve prices. But to choose the right one requires solving the problem (4.8), which is not particularly easy in practice.

We take up a simpler approach in the next Section.

[6] An alternative interpretation is that this boat gets to use only q_i of its capacity. This interpretation makes sense if there are constant returns to scale in capacity; that is, $u^i(\theta_i, k_i) = \rho^i(\theta_i)k_i$. We are not sure whether this is a reasonable assumption or not.

4.3 A Simple, Sealed-Bid, Buyback Auction Design

In the previous Section, we saw that under some very plausible conditions, it was possible to design a direct revelation buyback policy that satisfied incentive compatibility (IC), self-financing (SF), and voluntary participation (VP). We had to back off a bit from efficient selection (ES) to get this because of the use of a reserve price. In this Section we push a little harder to see whether we can get (IC), (SF), (VP), and (ES). We do this by considering a standard auction format, the sealed-bid auction, and modifying it slightly to fit our situation. With an additional condition on the environment, we can accomplish our goal.

Consider the buyback auction described by the following steps, where K^* is the capacity desired after the auction:

1. Ask the participants to submit bids $b_1, \ldots, b_n$. A bid is loosely interpreted to be a fisher's (per capacity) willingness to pay to stay in the industry; that is, it is related to $u^i(\theta_i, k_i)/k_i$.

2. The bidders are ranked from 1 to n according to b_i. Then as many winners as possible are chosen in order from 1 to n while keeping the aggregate capacity of the winners less than or equal to $K*$. We allow the possibility that only part of a boat will win; that is, $q_i \in [0, 1]$.

3. Collect from each winner, i, a fee equal to the highest losing bid times $q_i k_i$.

4. Distribute the sum of the payments collected in the previous step equally among all participants, proportionally to k_i .

More formally the simple, sealed-bid, buyback auction is described as follows.

1. (Bidding Rule) Each fisher i submits a bid b_i.

2. (Winners Determination) Let q^* solve

$$\max \sum_{i=1}^{n} q_i k_i b_i \tag{4.15}$$

subject to

$$\sum_{i=1}^{n} q_i k_i \leq K^* \tag{4.16}$$

$$q_i \in [0, 1]. \tag{4.17}$$

Notice that there may be one i for whom $0 < q_i^* < 1$. We interpret q_i^* as the probability that $i \in W$.

3. (Payment Rule) Let $P^* = \max_{i \in L} b^i$ and let $C^* = \sum_{i \in \mathcal{I}} P^* q_i^* k_i$. $t^i = P q_i^* k_i - C^* \frac{k_i}{\sum_{j=1}^{n} k^j}$.

Note that the first three steps constitute a standard Vickrey-Clarke-Groves (VCG) auction. If we stopped there, it would be a dominant strategy for all agents to bid their true value of staying in the fishery, and that would meet our efficiency requirement. However, that auction would neither be revenue-neutral nor satisfy voluntary participation. The last step of the auction tweaks the VCG auction to ensure revenue-neutrality. The improvement comes at a cost in terms of complexity for the bidders: it is no longer a dominant strategy for agents to bid their true valuations.

The open question at this point is: if bidders play a Bayes equilibrium, under what conditions do (ES) and (VP) hold for this simple auction.

4.3.1 Efficient Selection

An Bayes equilibrium for the auction is a function $\beta^i(\theta^i) \to \Re_+$. If β is symmetric, $\beta^i(\theta^i) = \beta(\theta^i)$, and increasing, $\hat{\theta}^i > \theta^i$ implies that $\beta(\hat{\theta}^i) > \beta(\theta^i)$, then the Winners Determination insures that (ES) will hold; those fishers with the highest per capacity profits will be chosen to remain in the fishery. But to find an equilibrium that is symmetric and increasing requires some additional assumptions. To get the increasing part, we need our familiar monotonicity condition. To get the symmetry part we need two things: constant returns to scale and sufficiency of θ. Constant returns to scale means $u^i(\theta_i, k_i) = \rho^i(\theta_i)k_i$. Sufficiency means $\rho^i(x) = \rho(x), \forall i$. That is, θ captures all of the differences between fishers.

Theorem 11. *If $u^i(\theta_i, k_i) = \rho(\theta_i)k_i, \forall i$, and $d[u - \frac{1-F}{f}u_\theta]d\theta \geq 0$,then there exists a symmetric and nondecreasing Bayes-Nash equilibrium for the auction above such that ties happen with probability zero.*

Proof. This result is a consequence of results established by Araujo and de Castro (2009). In that paper, they use the Kakutani-Glicksberg-Fan fixed-point theorem to show that the best-reply map on the space of nondecreasing bid functions[7] to show that there exists a nondecreasing, pure-strategy equilibrium. Modifying the proof of Lemma 3 (on p. 43) in a standard way[8] we obtain a *symmetric* monotone pure-strategy equilibrium.

To see that ties happen with probability zero, remember that bidders weigh the expected value from obtaining the object plus the expected value of raising the rebate against the expected loss of paying more than the object's value to the bidder, and that this weighing is done in an additive way. Therefore, if θ_j is best-replying, these potential gains and losses must offset each other. If $\theta_i > \theta_j$, then his value for the object is higher and expected loss is also lower, so he cannot offset the potential gains and losses from bidding with the same bid as θ_j. It follows that a symmetric equilibrium must not have ties with positive probability. □

Corollary 5. *There exists a Bayes-Nash equilibrium for the simple, sealed-bid, buyback auction that satisfies (ES) and (SF) with probability one.*

[7] They actually work on the equivalence classes of such functions that are equal almost everywhere.

[8] Instead of using the best reply correspondence Γ_i, use a correspondence that maps a nondecreasing bid-function β_i to the bid functions β_i^* that are a best reply to all other $n-1$ agents playing according to β_i. The rest of the argument follows through without change, and we obtain a correspondence Υ_i (that is the restriction of Γ_i to the space of nondecreasing bid functions) that is upper-semicontinuous and convex-valued. It follows from the Kakutani-Glicksberg-Fan theorem that Υ_i has a fixed point that is a symmetric, monotone pure-strategy equilibrium.

4.3.2 Voluntary Participation

For the simple, sealed-bid, buyback auction to satisfy (VP) we need the following to be true:

$$u^i(\theta^i, k_i) - P^* k_i + \frac{k_i}{\sum_j k_j} P^* K^* \geq u_0^i(\theta^i, k_i), \quad \forall i \in W \tag{4.18}$$

$$\frac{k_i}{\sum_j k_j} P^* K^* \geq u_0^i(\theta^i, k_i), \quad \forall i \in L \tag{4.19}$$

For ease of analysis we consider the case of a competitive auction. Since we set the price P^* to be the value of the first rejected bid, the only time a bidder can have an effect on the price is if they are that first rejected bidder. If n is large then the probability of that is small and $\beta^i(\theta^i)$ will be near $u^i(K^*, \theta^i)/k_i$. If in addition a bidder bids behaviorally by ignoring that probability, then they will bid their true value. That is, $\beta^i(\theta^i) = u^i(K^*, \theta^i)/k_i$.

Assumption The auction is competitive and bidders are behavioral; $\beta^i(\theta^i) = u^i(K^*, \theta^i)/k_i$.

Under this assumption, in order for (VP) to hold it must be true that $P^* \frac{K^*}{K} \geq \frac{u_0^i(\theta^i, k_i}{k_i}), \forall i$, where $K = \sum_{i \in \mathcal{I}} k_i$. It is easy to see that this will certainly be true if $\frac{K^*}{K} \min_i \frac{u^i(\underline{\theta}, k_i)}{k_i} \geq \max_i \frac{u_0^i(\overline{\theta}, k_i)}{k_i}$. We thus have

Theorem 12. *Suppose θ^i is sufficient, there are constant returns to scale, and the regularity condition $d[u - \frac{1-F}{f} u_\theta]/d\theta > 0$ holds.*

If $\min_i u(\underline{\theta}^i, k_i) \geq \frac{K}{K^*} \max_i u_0(\overline{\theta}^i, k_i)$, then the simple, sealed-bid buyback auction satisfies (SF), (IC), and (VP).

Notice that this is stronger that what was needed in Section 4.2.3. In that Section we could consider asymmetric payments to the various fishers. Here we are restricting ourselves to a symmetric payments, differentiated only by whether one is a winner or loser. Therefore we need to insure that the surplus in the fishery after contraction is sufficiently larger to accommodate this.

4.4 Conclusion

We have presented an auction design that is self-financing and has an ex post efficient selection. We also present conditions that guarantee that profit-maximizing fishers will voluntarily agree to participate in the auction. Because we interpret capital simply as vessel capacity, this model cannot address all of the goals and concerns related to buyback policies (see Squires (2010)). However, this restriction allows us to focus on a very important issue that have been plaguing buybacks: the large losses due to excessive subsidies (see Clark, Munro, and Sumaila (2005)).

Appendices

Appendix A

Appendix to Chapter 3

A.1 Invertibility of the Jacobian of Various Equilibrium Subsystems

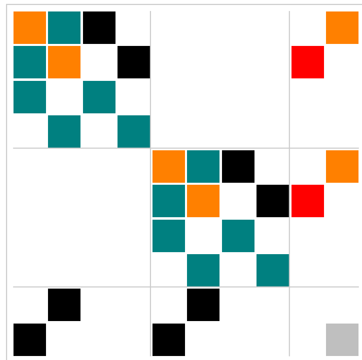
The smoothness of the temporary equilibrium maps (and later, the equilibrium maps) depend on the invertibility of the Jacobian of the subsystem relevant to temporary equilibrium (for equilibrium, we will need to look at the whole system) The Jacobian of the equilibrium system (3.8–3.13) in the case $n = 2$ and $y_1 \neq w_1, y_2 \neq w_2$ is displayed in figure A.1.

Figure A.1: Jacobian of the equilibrium system, n is 2.

	y_1	w_1	μ_1^y	μ_1^w	y_2	w_2	μ_2^y	μ_2^w	q	s
y_1	$-D_{11}c_1 - D_{22}\phi_1$	$-D_{32}\phi_1$	1	0	0	0	0	0	0	$-D_{21}c_1$
w_1	$-D_{23}\phi_1$	$-D_{33}\phi_1$	0	1	0	0	0	0	-1	0
μ_1^y	μ_1^y	0	y_1	0	0	0	0	0	0	0
μ_1^w	0	μ_1^w	0	w_1	0	0	0	0	0	0
y_2	0	0	0	0	$-D_{11}c_2 - D_{22}\phi_2$	$-D_{32}\phi_2$	1	0	0	$-D_{21}c_2$
w_2	0	0	0	0	$-D_{23}\phi_2$	$-D_{33}\phi_2$	0	1	-1	0
μ_2^y	0	0	0	0	μ_2^y	0	y_2	0	0	0
μ_2^w	0	0	0	0	0	μ_2^w	0	w_2	0	0
q	0	1	0	0	0	1	0	0	0	0
s	1	0	0	0	1	0	0	0	0	$-Dg$

We partitioned the Jacobian matrix in blocks and labeled each column by its corresponding variable, and we assigned to each equation its key related variable. For example, y_1 is associated with the first-order conditions on output of the individual optimization problem for agent 1, μ_1^y with the complementary slackness condition of the constraint $y_1 \geqslant 0$ and q with the market-clearing condition. The dependency relationships between variables in the equilibrium system is illustrated by the sparsity pattern of the matrix in figure A.1. It may be easier to analyze it in figure A.2. The block relevant for temporary equilibrium is the top-left square block of size $4n+1$.

Figure A.2: Sparsity pattern of the Jacobian of the equilibrium system, n is 2



.

The top-left square block of size $4n$ corresponds to individual optimal behavior.

Lemma 8. *The hessian of the profit function* $(y_i, w_i) \mapsto \pi_i(y_i, w_i)$ *is invertible whenever* $y_i > w_i > 0$.

Proof. The hessian of the profit function is given by

$$H_{\pi_i} = \begin{bmatrix} -D_{11}c_i - D_{22}\phi_i & -D_{23}\phi_i \\ -D_{32}\phi_i & -D_{33}\phi_i \end{bmatrix} \tag{A.1}$$

Its determinant where $y_i > w_i > 0$ is

$$\det H_{\pi_i} = (D_{11}c_i + D_{22}\phi_i)D_{33}\phi_i - (D_{23}\phi_i D_{32}\phi_i)$$

As $D_{11}c_i > 0$ when $y_i > 0$, it follows that where $y_i > w_i > 0$

$$\det H_{\pi_i} > D_{22}\phi_i D_{33}\phi_i - (D_{23}\phi_i D_{32}\phi_i) \tag{A.2}$$

The right-hand side of (A.2) is the determinant of the hessian of ϕ_i as a function of (y_i, w_i). The convexity of ϕ_i in (y_i, w_i) implies that the hessian on the right-hand side of (A.2) is positive semidefinite, and thus its determinant is non negative. We conclude that $\det H_{\pi_i} > 0$ whenever $y_i > w_i > 0$, as desired. □

Lemma 9. *Take $p, M, \mathcal{TAC}, s, q$ arbitrarily and let (y_i, w_i) maximize the profits of fisherman i. Then the 4×4 block corresponding to individual optimal behavior in the Jacobian above is invertible if and only if either $y_i > w_i > 0$ or $\mu_i^y, \mu_i^w > 0$.*

Proof. We can write one such 4×4 diagonal block as a block matrix

$$J_i = \begin{bmatrix} H_{\pi_i} & I \\ \operatorname{diag}(\mu_i^y, \mu_i^w) & \operatorname{diag}(y_i, w_i) \end{bmatrix} \tag{A.3}$$

where each block is 2×2. The matrix H_{π_i} is the hessian of the profit function $(y_i, w_i) \mapsto \pi_i(y_i, w_i)$

$$\begin{bmatrix} -D_{11}c_i - D_{22}\phi_i & -D_{32}\phi_i \\ -D_{23}\phi_i & -D_{33}\phi_i \end{bmatrix}$$

Note that the two bottom blocks of J_i are diagonal matrices and therefore they commute (i.e., their matrix product is the same, irrespective of the order of multiplication). We can thus write the determinant of J_i as

$$\det J_i = \det(H_{\pi_i} \operatorname{diag}(y_i, w_i) - I \operatorname{diag}(\mu_i^y, \mu_i^w))$$

Expanding the first product

$$H_{\pi_i} \operatorname{diag}(y_i, w_i) = \begin{bmatrix} y_i(-D_{11}c_i - D_{22}\phi_i) & w_i(-D_{32}\phi_i) \\ y_i(-D_{23}\phi_i) & w_i(-D_{33}\phi_i) \end{bmatrix}$$

and thus

$$\det J_i = \det \begin{bmatrix} y_i(-D_{11}c_i - D_{22}\phi_i) - \mu_i^y & w_i(-D_{32}\phi_i) \\ y_i(-D_{23}\phi_i) & w_i(-D_{33}\phi_i) - \mu_i^w \end{bmatrix}$$

It can now be verified by simple substitutions that the determinant of the 2×2 matrix above is zero if and only if $y_i, w_i > 0$ or $\mu_i^y, \mu_i^w > 0$, which completes the proof. □

Corollary 6. *Consider $p, M, \mathcal{TAC}$ and (y, w, q, s) such that y_i, w_i maximizes profits for every fisherman i. The Jacobian of the individual optimality subsystem (3.8–3.11) is invertible if and only if for all agents $i \in \mathcal{I}$ either $y_i, w_i > 0$ or $\mu_i^y, \mu_i^w > 0$.*

Proof. The block corresponding to individual optimal behavior is a block-diagonal matrix with n 4×4 diagonal elements coming from the four equations and four variables involved in the first-order conditions of problem (3.4). Therefore, this whole $4n \times 4n$ block is invertible if and only if each of the diagonal blocks is invertible. The conditions for invertibility are proved in the preceding Lemma. □

Lemma 10. *The Jacobian of the top-left square block of size $4n + 1$ of the Jacobian in (A.1), referring to the temporary equilibrium equations, is invertible whenever $y_i > w_i > 0$ for all $i \in \mathcal{I}$.*

Proof. We can write the temporary equilibrium block as the following block matrix:

$$J = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \tag{A.4}$$

Block A corresponds to the top-left square block of size $4n$ corresponding to the individual optimality conditions. B is $4n \times 1$ C is $1 \times 4n$, and D is 1×1.

As we already know from Corollary 6 that A is invertible, we can write the determinant of the matrix J in (A.4) as

$$\det(J) = \det(A)\det(D - CA^{-1}B)$$

Because $D = 0$, the invertibility of J hinges on the invertibility of $CA^{-1}B$.

Note that because of the block-diagonal structure of A, we can restrict ourselves without loss of generality to the case $n = 1$ displayed in (A.5).

$$\begin{bmatrix} y_1 & -D_{11}c_1 - D_{22}\phi_1 & -D_{32}\phi_1 & 1 & 0 & 0 & -D_{21}c_1 \\ w_1 & -D_{23}\phi_1 & -D_{33}\phi_1 & 0 & 1 & -1 & 0 \\ \mu_1^y & \mu_1^y & 0 & y_1 & 0 & 0 & 0 \\ \mu_1^w & 0 & \mu_1^w & 0 & w_1 & 0 & 0 \\ q & 0 & 1 & 0 & 0 & 0 & 0 \\ s & 1 & 0 & 0 & 0 & 0 & -Dg \end{bmatrix} \tag{A.5}$$

In that case, $CA^{-1}B$ is a 1×1 matrix, and some calculation shows that its value is

$$\frac{D_{11}c_i + D_{22}\phi_i}{\det H_{\pi_i}}$$

which is always strictly negative by Assumption. Therefore, J is invertible, as desired. □

Lemma 11. *Let $\Gamma \subset \mathbb{R}^3$ be the set of tuples $p, M, \mathcal{TAC}$ where agents violate quota in equilibrium and the Jacobian of the equilibrium system* (A.1) *is not invertible. Then Γ is a nowhere dense set of measure zero.*

Proof. Graphically, invertibility of the Jacobian corresponds to the temporary equilibrium curve $s \mapsto Y(s)$ being *transversal* to the stock growth curve $s \mapsto g(s)$. It may be the case that those curves are tangent, and therefore we cannot prove that the Jacobian (A.1) of the equilibrium system will be invertible for *all* $(p, M, \mathcal{TAC})$. However, we can use the transversality Theorem from Appendix A.3 to show that this will happen only in a nowhere dense set of measure zero.

To that end, substitute the column corresponding to the derivatives with respect to s with a column with derivatives with respect to the $\mathcal{TAC}$ to obtain the following square matrix (again, the case $n = 1$ is sufficient)

$$\begin{bmatrix} y_1 & -D_{11}c_1 - D_{22}\phi_1 & -D_{32}\phi_1 & 1 & 0 & 0 & 0 \\ w_1 & -D_{23}\phi_1 & -D_{33}\phi_1 & 0 & 1 & -1 & 0 \\ \mu_1^y & \mu_1^y & 0 & y_1 & 0 & 0 & 0 \\ \mu_1^w & 0 & \mu_1^w & 0 & w_1 & 0 & 0 \\ q & 0 & 1 & 0 & 0 & 0 & 0 \\ s & 1 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \tag{A.6}$$

Again, partitioning the matrix in (A.6) in blocks A, B, C, D as in (A.4), with A being the $4n \times 4n$ individual optimality part (which is invertible), we can write its determinant as

$$\det(A)\det(D - CA^{-1}B)$$

Because we know A is invertible, the invertibility of the Jacobian matrix above hinges on the invertibility of $D - CA^{-1}B$. We can show that

$$D - CA^{-1}B = \begin{bmatrix} \frac{-D_{11}c_i - D_{22}\phi_i}{\det H_{\pi_i}} & -1 \\ \frac{-D_{32}\phi_i}{\det H_{\pi_i}} & 0 \end{bmatrix} \tag{A.7}$$

It is clear that the determinant of the matrix in (A.7) is not zero. Therefore, the matrix in (A.6) is invertible.

The result follows then from the transversality Theorem. See Theorem 14 in Appendix A.3. □

A.2 Nonsmooth Optimality Conditions when Violations Are Zero

Suppose q, s are such that there are optimal y_i, w_i for agent i where $y_i = w_i$. At this point, the map $(y_i, w_i) \mapsto \phi^+(M, y_i, w_i)$ need not be differentiable. The first-order conditions at this point are

$$0 \in \partial_{y_i, w_i} L \tag{A.8}$$

where $\partial_{y_i,w_i} L$ is the subgradient of the Lagrangian

$$L = py_i - c(y_i, s, \theta_i) - q(w_i - \omega_i) - \phi^+(M, y_i, w_i) + \mu_i^y y_i + \mu_i^w w_i$$

So (A.8) translates into

$$\begin{aligned} p - D_1 c_i + \mu_i^y &= \eta \\ -q + \mu_i^w &= \nu \end{aligned} \tag{A.9}$$

for some $(\eta, \nu) \in \partial_{y_i,w_i}\phi^+$. As $\phi^+ = \max\{\phi, 0\}$ and ϕ is convex and differentiable, we can write the subgradient $\partial_{y_i,w_i}\phi^+$ as the convex combination of $\nabla_{y_i,w_i}\phi$ and $(0,0)$. Therefore, we can state the following.

Lemma 12. *Fix $M, \mathcal{TAC}, q, s$. If $y_i, w_i \geqslant 0$ with $y_i = w_i$ maximizes profits for i, then there exists $\alpha_i \in [0,1]$, $\mu_i^y \geqslant 0$, and $\mu_i^w \geqslant 0$ such that $y_i \mu_i^y = 0$, $w_i \mu_i^w = 0$ and*

$$\begin{aligned} p - D_1 c(y_i, s, \theta_i) + \mu_i^y &= \alpha_i D_2 \phi(M, y_i, w_i) \\ -q + \mu_i^w &= \alpha_i D_3 \phi(M, y_i, w_i) \end{aligned}$$

We can now examine who will have positive production and who will hold positive amounts of quota.

Lemma 13. *Fix $M, \mathcal{TAC}, q, s$. If $p > D_1 c(0, s, \theta_i)$ for some fishermen i, and (y_i, w_i) maximizes profits, then $y_i > 0$. Conversely, if there is a profit-maximizing (y_i, w_i) with $y_i > 0$, then $p \geqslant D_1 c(0, s, \theta_i)$, with strict inequality if $q > 0$.*

Proof. Straight out of the first order conditions. See Lemma 12 at page 81. □

Lemma 14. *Fix $M, \mathcal{TAC}, q, s, y_i > 0$. If $q < D_3\phi(M, y_i, 0)$ for some fishermen i, and (y_i, w_i) maximizes profits, then $w_i > 0$. Conversely, if there is a profit-maximizing (y_i, w_i) with $w_i > 0$, then $q < -D_3\phi(M, y_i, 0)$.*

Proof. Follows from the first-order conditions and the fact that ϕ is strictily convex in w_i. □

A.3 Transversality Theory

Some of our results rely on a collection of propositions loosely referred to as "the transversality Theorem(s)". These results formalize and generalize the heuristic analysis of the solution set of a system of nonlinear equations through "counting equations and unknowns" by stating that smooth curves and surfaces (more generally, manifolds) are generally transversal. It is thus necessary to be precise about the definitions of "smooth", "manifold", "generic" and "transversal". These concepts have had useful applications in economic theory for a long time (see Mas-Colell (1985)). We provide a brief introduction to these topics in this Appendix. See Guillemin and Pollack (2010) or Hirsch (1976) for a textbook treatment of the subject. The Theorems listed here were taken from Aubin and Ekeland (2006).

Let $X \subset \mathbb{R}^k$ and $Y \subset \mathbb{R}^l$ be arbitrary. We say a map $f : X \to Y$ is ***smooth map*** of class C^r if for each $x \in X$ there is an open set $U \subset X$ about x and a map $F : U \to \mathbb{R}^l$ of class C^r, $r \geqslant 1$ such that F coincides with f on $U \cap X$. We call f a ***diffeomorphism*** if it is a smooth bijection with a smooth inverse. *Examples of smooth maps and diffeomorphisms*: the identity map is always smooth, but the map $x \mapsto x^3$ of $(-1, 1)$ on itself is not a diffeomorphism; it is smooth with a continuous inverse, but the inverse $y \mapsto y^{1/3}$ is not differentiable at $y = 0$.

A set $X \subset \mathbb{R}^n$ is a m-dimensional ***smooth manifold*** if every point $x \in X$ has a neighborhood in X that is diffeomorphic to an open subset of $\mathbb{R}^m$. *Examples of smooth manifolds*: any singleton is a 0-dimensional smooth manifold; any open set in $\mathbb{R}^k$ is a k-dimensional manifold; the graph of any smooth function $f : \mathbb{R}^k \to \mathbb{R}$ is a smooth manifold of dimension $k - 1$.

Let x be an element of an m-dimensional manifold $M \subset \mathbb{R}^n$ Let $U \subset \mathbb{R}^m$ be an open set containing x, and $g : U \to M$ the smooth parametrization of a neighborhood of x. The ***tangent space*** at x relative to M, denoted by $T_x M$, is the image of the linear operator $Dg(x) : \mathbb{R}^m \to \mathbb{R}^n$. One can prove that the tangent space does not depend on the choice of parametrization g. *Examples of tangent spaces*: for every $x \in \mathbb{R}^n$ we have $T_x \mathbb{R}^n = \mathbb{R}^n$; relative to the 2-dimensional unit-sphere in $\mathbb{R}^3$, the tangent space at any point is $\mathbb{R}^2$. One can show that the tangent space has the same dimension as the manifold it is tangent to.

Consider two smooth manifolds $M \subset \mathbb{R}^k$ and $N \subset \mathbb{R}^l$ and a smooth map $f : M \to N$ with $f(x) = y$. The ***derivative*** $Df(x) : T_xM \to T_yN$ is defined as follows. Since f is smooth, there exists an open set about x and a smooth map $F : W \to \mathbb{R}^l$ that coincides with f on $W \cap M$. For all $v \in T_xM$ define the $Df(x) \cdot v$ to be equal to the directional derivative $DF(x) \cdot v$. One can prove that the derivative of f at x does not depend on the choice extension F.

Let us now move on to the notion of *genericity*. Let X be a complete metric space. A G_δ subset of X is defined as the intersection of a countable family of open subsets of X. We say $Y \subset X$ is ***generic set*** if it contains a dense G_δ of X. We say a statement $P(x)$ about points $x \in X$ is a ***generic property*** if the set $\{x \in X : P(x)$ is true $\}$ is generic.

Finally, let us define the notion of *transversality*. It is a generalization of the notion of *regularity*. Let $f : X \to Y$ be a smooth map between smooth manifolds and Z be a submanifold of Y such that $Z \cap f(X) \neq \emptyset$. It may or may not be the case that $f^{-1}(Z)$ is a smooth submanifold of X. A sufficient condition for that is that f be *transversal* to Z in a sense that we explain now in increasing level of generality.

- Let $X = Y = \mathbb{R}^n$, $Z = \{0\}$. In this case, $f(x) \in Z$ represents a *square system of nonlinear equations*. We say f is transversal to Z if for all $x \in f^{-1}(Z)$ the derivative $Df(x)$ is *invertible*. The *inverse function Theorem* guarantees that $f^{-1}(Z)$ is a set of isolated points, or, in other words, it is a manifold of dimension 0 (equivalently, with the same codimension of Z in Y: n).

- Let $X = \mathbb{R}^n, Y = \mathbb{R}^p$, $Z = \{0\}$. In this case, $f(x) \in Z$ represents a *nonlinear system of equations* We say f is transversal to Z if for all $x \in f^{-1}(Z)$ the derivative $Df(x)$ is *surjective* (synonym: onto). This is the same as saying that 0 is a ***regular value*** of f. The *implicit function Theorem* guarantees that $f^{-1}(Z)$ is a smooth manifold of dimension $n - p$ (equivalently, with the codimension of Z in Y: p).

- Let $X = \mathbb{R}^n, Y = \mathbb{R}^p$, and Z some m-dimensional submanifold of Y. In this case, $f(x) \in Z$

represents a *system of nonlinear inclusions*. We say f is transversal to Z if for all $x \in X$:

$$Im(Df(x)) + T_{f(x)}(Z) = \mathbb{R}^p$$

The *implicit function Theorem* will guarantee that $f^{-1}(Z)$ is a smooth manifold of dimension $n - p + m$ (equivalently, with the codimension of Z in Y: $p - m$).

- General case, where X, Y and $Z \subset Y$ are manifolds. In this case, $f(x) \in Z$ represents a *system of nonlinear inclusions*. We say f is ***transversal to*** Z if for all $x \in X$:

$$Im(Df(x)) + T_{f(x)}(Z) = T_{f(x)}Y$$

  The *implicit function Theorem* guarantees that $f^{-1}(Z)$ is a smooth manifold with codimension equal to the codimension of Z in Y.

in the nonlinear system $f(x) \in Z$. The dimension of $f^{-1}(Z)$ is the formalization of the idea of "number of degrees of freedom" in the nonlinear system $f(x) \in Z$.

We can now state versions of the transversality Theorem that are sufficiently general for our needs. In the following, U is an open subset of $\mathbb{R}^n$, Λ is a separable Banach space and Z is a C^∞ submanifold of $\mathbb{R}^p$ with codimension q.

Theorem 13 (Transversality Theorem 1)**.** *Let $f : U \times \Lambda \to \mathbb{R}^p$ be a smooth map of class C^r, $r \geqslant 1$. If f is transversal to Z and $r \geqslant \max\{1, n - q + 1\}$ then*

$$L = \{\lambda \in \Lambda : x \mapsto f(x, \lambda) \text{ is transversal to } Z\}$$

is a generic set in Λ.

The fact that V is infinite dimensional allows us to pick the function itself as a parameter.

Corollary 7 (Transversality Theorem 2)**.** *The property*

$$P(f) = \{f : U \to \mathbb{R}^p \text{ is transversal to } Z\}$$

is generic in $C^r(U;\mathbb{R})$*.*

In particular, this Corollary implies that "for most" (that is, generic) smooth, square, nonlinear systems of equations, the solution set is a set of isolated points (a 0-dimensional manifold). More generally, if this square system of equations has k (exogenous) parameters then "for most" (that is, generic) smooth, nonlinear systems of equations, the solution set as a function of the exogenous parameters is a manifold of dimension k.

Let us quickly relate the transversality Theorem to the heuristic analysis of the solution set via "numbers of variables vs. numbers of equations" arguments that is familiar from linear algebra. The dimension of the solution set $f^{-1}(Z)$ is the formal concept of "degrees of freedom"; The codimension of the solution set $f^{-1}(Z)$ "typically" is the number of equations. The transversality Theorem formalizes, generalizes and generically validates the following heuristic *local* analysis of systems of nonlinear equations:

- degrees of freedom = number of variables - number of equations;
- positive degrees of freedom imply multiple solutions;
- negative degrees of freedom imply no solutions (because the empty set is the only manifold of negative dimension);
- zero degrees of freedom imply a unique solution.

If we confine ourselves to a finite-dimensional set of parameters Λ, we can strengthen the conclusion of the transversality Theorem.

Theorem 14 (Transversality Theorem 3)**.** *Let* $M \subset \mathbb{R}^n$, $\Lambda \subset \mathbb{R}^l$, *and* $Z \subset \mathbb{R}^p$ *be smooth manifolds. Let*

$f : M \times \Lambda \to \mathbb{R}^p$ be a smooth map. If f is transversal to Z then

$$L = \{\lambda \in \Lambda : x \mapsto f(x,\lambda) \textit{ is transversal to } Z\}$$

is a generic set in Λ, and $\Lambda \setminus L$ has measure zero.

A.4 Assumptions on the Monitoring Function

We now make some additional Assumptions based on the interpretation of $\phi(M, y_i, w_i)$ as the expected fine for violation. First, we assume ϕ to be jointly convex in (y_i, w_i), but not necessarily strictly so, as that rules out interesting violation measures like $v(y_i, w_i) = (y_i - w_i)/(1 + w_i)$. Second, we also assume that $|D_2\phi| \leqslant |D_3\phi|$ That means that given any change in catch y_i, there is a (weakly) smaller change in quota holdings w_i that changes the total fine at least as much as the change in y_i. Third, we assume that for all $(y_i, w_i) \neq (y_i, \widetilde{y}_i)$ we have

$$\left(D_2\phi(M, y_i, w_i) - D_2\phi(M, \widetilde{y}_i, \widetilde{w}_i)\right)\left(D_3\phi(M, \widetilde{y}_i, \widetilde{w}_i) - D_3\phi(M, y_i, w_i)\right) > 0$$

That simply means that if the marginal fines $D_2\phi$ go up, then so should the marginal fine savings from buying quota $-D_3\phi$, and vice-versa. Finally, note that it follows from our Assumptions that if $D_2\phi$ and $D_3\phi$ exist at a point where $y_i = w_i$, then $D_2\phi = D_3\phi$ at that point. This equality may or may not hold at other points depending on the violation measure (for example, it will always hold if violations are measured absolutely).

Note that it may well be the case that ϕ^+ is not differentiable when violations are exactly zero, that is, when $y_i = w_i$. Example: $\phi(M, y_i, w_i) = \rho(M)((y_i - w_i) + (y_i - w_i)^2)$. Allowing this type of nonsmoothness at zero violations makes the analysis of first-order conditions a little more complex but it enriches the model in a way that we believe is significant: it makes it possible for respecting one's quota to be an optimal action.

Appendix B

Appendix to Chapter 4

B.1 Auxiliary Results

From (IC) and (BB): $T = uQ - V_0 - \int_0^\theta u_\theta(s)Q(s)ds$. Given Q, let $A = \int_0^\theta u_\theta(s)Q(s)ds - u(\theta)Q(\theta)$. Let $t^i = -A^i + \alpha^i + \frac{1}{N-1}\sum_{j\neq i} A^j$. Then $T^i = -A^i + \alpha^i + \frac{1}{N-1}\sum_{j\neq i} \bar{A}^j$. Thus, $V_0 = -\alpha^i - \frac{1}{N-1}\sum_{j\neq i} \bar{A}^j$. Now $\sum t^i = 0$ if and only if $\sum \bar{T}^i = 0$ iff $\sum \alpha^i = 0$ iff $\sum V_0^i + \sum \bar{A}^i = 0$ iff $\sum V_0^i + \sum \int [\int_0^\theta u_\theta^i(s)Q^i(s)ds - u^i(\theta)Q^i(\theta)]dF^i = 0$ iff (integrating by parts) $\sum V_0^i - \sum \int (u^i - \frac{1-F^i}{f^i}u_\theta)Q^i dF^i = 0$ iff $\sum \int (u^i - \frac{1-F^i}{f^i}u_\theta)Q^i dF^i = \sum V_0^i$.

From (IC) and (VP): Given Q, $V_0 \geq u_0(\theta) - \int_0^\theta u_\theta(s)Q(s)ds, \forall\theta$. For standard models in which $u_0(\theta) = u_0$, a constant, and $u_\theta > 0$ then this requires $V_0 \geq u_0$. In our case, however, $du_0/d\theta > 0$, so the standard approach does not work. An alternative that would work if $du_0/d\theta \geq u_\theta \forall\theta$, is to note then that we would need $V_0 \geq u_0(\theta_1) - \int_{\theta_0}^{\theta_1} u_\theta Q ds$. But for fishing it is probably true that $du_0/d\theta \leq u_\theta, \forall\theta$. Therefore the most we can say is that $V_0 \geq \max_\theta[u_0(\theta) - \int_0^\theta u_\theta(s)Q(s)ds]$.

All Together Therefore we have the following result: Given Q there are t such that (Q, T) satisfies

(BB), (IC) and (VP) iff

$$\sum \int (u^i - \frac{1-F^i}{f^i}u_\theta)Q^i dF^i \geq \sum \max_\theta [u_0(\theta) - \int_0^\theta u_\theta(s)Q(s)ds]. \tag{B.1}$$

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