Part I

The Quad-Ridged Flared Horn

Chapter 2

Key Requirements of Radio Telescope Feeds

Almost all of today's radio telescopes operating above 0.5 GHz use reflector antennas consisting of one or more mirror(s), where the primary mirror is very large in terms of wavelength because astronomical signals are extremely weak and a large antenna collecting area (equivalently, large antenna gain) is needed to detect them. This chapter begins with a brief overview of radio telescope reflector antennas followed by a discussion of the metrics used to quantify a reflector antenna's performance, namely aperture efficiency and system noise temperature. The figure of merit most commonly used in radio astronomy is then discussed. The chapter concludes with an explanation of the key requirements of radio telescope feed antennas.

2.1 Reflector Antenna Optics

Reflector antennas come in variety of configurations depending on mirror type, number of mirrors, optical geometry, etc. The most common mirror types are planar, spherical, conical, paraboloid, hyperboloid and ellipsoid with the last three being the dominant types in very high gain applications. Further, reflectors can be symmetric or offset; shaped or unshaped; can have one, two or more mirrors which may be arranged in Gregorian, Cassegrain, ring-focus, beam-waveguide, etc., configurations. In shaped systems, there are at least two mirrors whose surfaces are synthesized to achieve particular amplitude and phase distributions on the reflector aperture. The design of reflector antennas is usually performed first using geometrical optics (GO) and refined through physical optics (PO), physical theory of diffraction (PTD) or geometrical theory of diffraction (GTD) [25].

The most common reflector antenna configurations in radio astronomy are symmetric and offset dual-reflector antennas in Cassegrain or Gregorian geometries which are depicted in Figure 2.1. The front-fed reflector is also sometimes used, especially for low-frequency radio astronomy and is shown in the same figure as well. In all of these configurations, the primary mirror is a paraboloid and

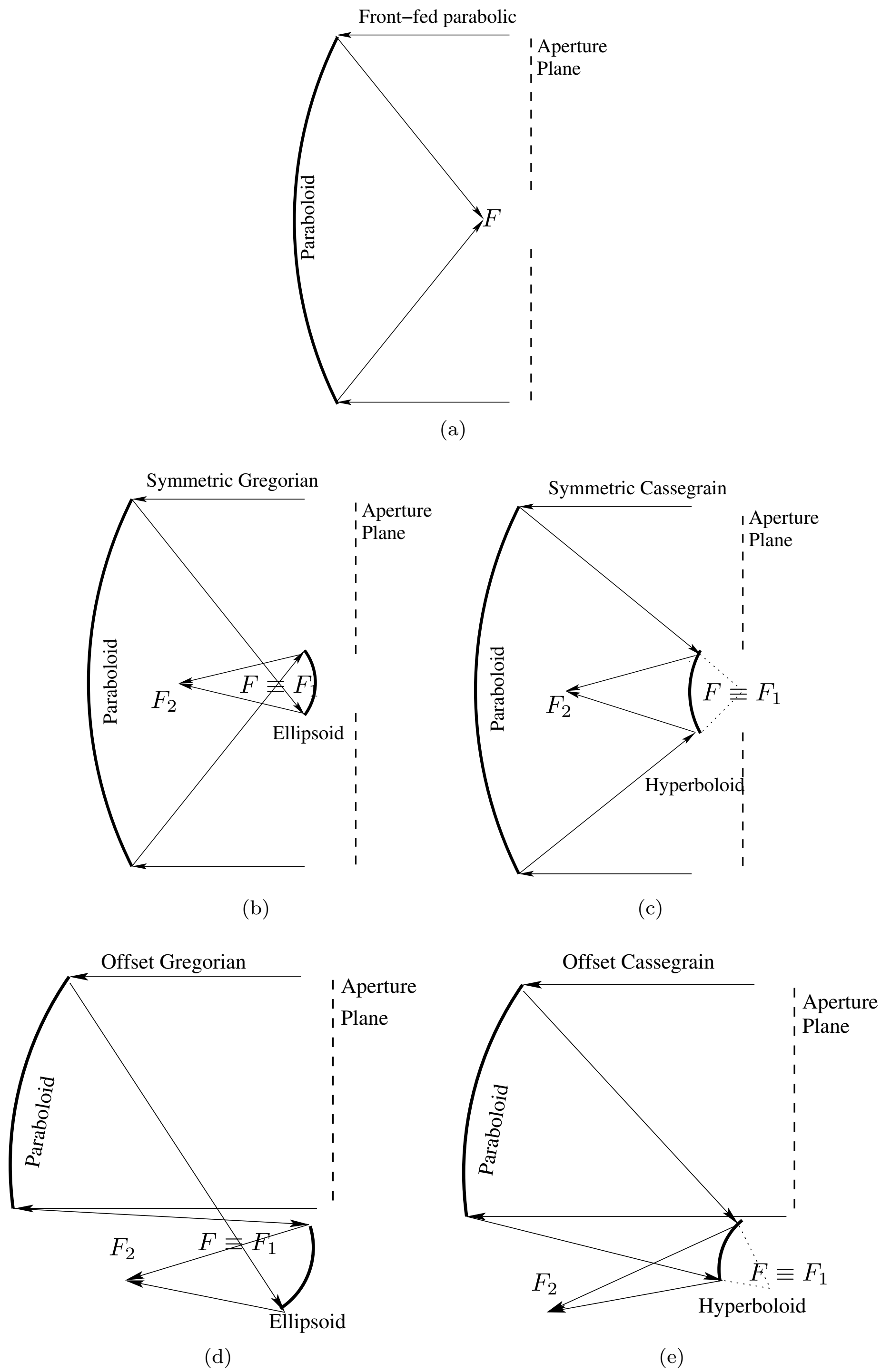


Figure 2.1: The most common reflector antenna optical configurations in radio astronomy

focuses plane waves from distant sources onto a single focal point F.

The Gregorian configuration comprises an ellipsoidal secondary mirror in addition to the parabolic primary. The focal point of the parabola, F, is collocated with one of the foci of the ellipsoid F_1; the feed antenna is located at the other focal point of the ellipsoid, F_2. The Cassegrain reflector antenna uses a hyperboloidal secondary mirror instead of an ellipsoid. The focal point of the primary is collocated with the hyperboloid's focal point behind the secondary mirror F_1 and the feed antenna is located at the other focal point F_2. An important disadvantage of the symmetric configurations as compared to their offset counterparts of Figure 2.1 is the aperture blockage due to the secondary mirror (or the feed in the case of the front-fed parabola) which reduces the aperture efficiency as explained in the next section.

Some of the parameters of interest in the design of a reflector antenna include gain, spillover, first and far-out sidelobe levels, cross-polarization level. All of these are strongly dependent on the feed antenna performance and are most commonly quantified via aperture efficiency discussed in detail next.

2.2 Aperture Efficiency

Reflector antennas, just like horn and lens antennas, are aperture antennas which are characterized by a planar aperture, perpendicular to the direction of maximum radiation, through which the majority of the radiation passes [25]. Apertures of the typical reflector configurations are depicted in Figure 2.1. In all of these cases, the aperture is circular, and reflector analysis using GO reduces to tracing rays from the feed antenna to the aperture and taking the Fourier transform of aperture fields to obtain far-field radiation patterns [11].

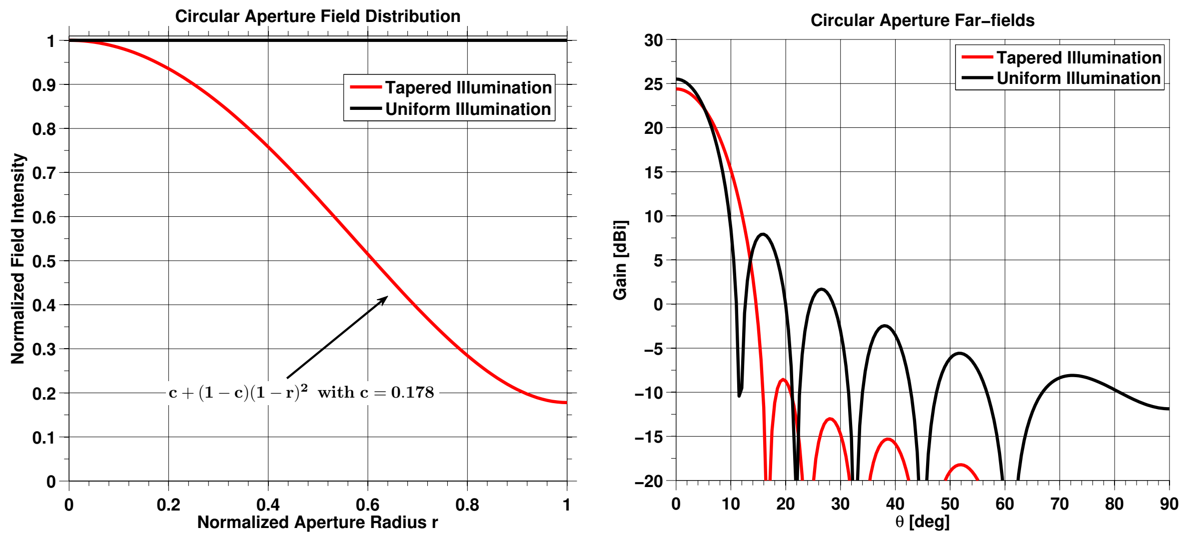
One of the canonical examples of an aperture antenna (and one that is relevant to reflector antennas) is a circular aperture of radius a in an infinite ground plane which can be analyzed in closed form under uniform illumination and yields the well-known Airy disc far-field pattern[1] [26]

$$\frac{J_1\left(ka\sin\theta\right)}{ka\sin\theta} \tag{2.1}$$

where k is the wavenumber and J_1 is the first-order Bessel function of the first kind. Figure 2.2(a) displays the aperture field distributions with uniform and tapered illuminations, and the resultant far-fields are plotted in part (b) for $a = 3\lambda$. The tapered illumination is designed with an edge taper of -15 dB which represents a reasonable approximation of realistic aperture distributions on radio telescopes.

The first observation from these plots is the small but significant difference in gains between the

[1]Note that the aperture area is assumed to be much larger than λ^2.



(a) (b)

Figure 2.2: (a) Field distribution (with uniform phase), (b) the resultant far-field patterns of a circular aperture in an infinite ground plane under uniform and tapered illuminations. The far-field pattern of the tapered distribution is calculated using the results in [27].

two cases. In fact, it can be shown that uniform illumination always yields the maximum gain[2] for any aperture shape, and in the case of a circular aperture, the maximum gain is [26]

$$G_{max,circ} = \frac{4\pi}{\lambda^2}\left(\pi a^2\right)$$

or more generally, the maximum gain for an arbitrary aperture shape is

$$G_{max} = \frac{4\pi}{\lambda^2} A_{phys}$$

where A_{phys} is the physical area of the aperture. The tapered illumination, on the other hand, yields slightly lower gain and larger beamwidth with the same aperture size. This directly leads to the definition of *effective* aperture area or *effective* area of an antenna: it is the area of a uniformly illuminated aperture that yields the same gain G, namely [25],

$$A_{eff} \equiv \frac{\lambda^2}{4\pi} G \tag{2.2}$$

The aperture efficiency, also known as antenna efficiency, is then defined as the ratio of an

[2]Technically, directivity is the correct term to use here; however, the two are equal for an antenna with no ohmic losses which is the underlying assumption.

antenna's effective area to its physical area [25]

$$\eta \equiv \frac{A_{eff}}{A_{phys}} = \frac{G}{G_{max}} \tag{2.3}$$

which is valid for $A_{phys} \gg \lambda^2$. Under tapered illumination, for instance, the aperture efficiency is approximately 78% because gain is about 1.1 dB lower than that for uniform illumination.

Aperture efficiency of a reflector antenna is determined not only by the aperture field taper. In fact, any power loss in the on-axis direction (i.e., $\theta = 0$) results in lower efficiency. Such loss factors include phase errors in aperture fields, spillover energy, cross-polarization, aperture blockage, scattering from support structures, surface errors, etc. In the design of feed antennas for reflectors, it is common to approximate the aperture efficiency by neglecting losses due to blockage, support structure scattering, and surface errors. This approximate efficiency is then primarily a function of feed antenna performance, and is sometimes called the antenna feed efficiency [28]. Henceforth, the term aperture efficiency is used to refer to this approximate efficiency.

Obtaining a general, closed-form expression for aperture efficiency of a reflector antenna is not possible as it depends on the particular reflector optics and feed antenna radiation pattern. The efficiency can be calculated numerically using GO, PO, GTD or PTD; however, these are computationally expensive severely limiting their use in feed antenna design. Thus, an approximate closed-form expression is sought. To that end, the following radiation pattern is frequently assumed for the feed antenna (x-polarization):

$$E_f \equiv V_\theta(\theta)\cos\phi\hat{\theta} - V_\phi(\theta)\sin\phi\hat{\phi} \tag{2.4}$$

because: 1) ϕ integration can be performed analytically due to circular symmetry; 2) far-field patterns first-order azimuthal terms maximize gain of a paraboloidal reflector antenna [29] (also see Section 3.5). Then, the approximate aperture efficiency of a symmetric prime-focus or Cassegrain reflector is given by [30]

$$\eta = 2\cot^2\frac{\theta_s}{2}\,\frac{\left|\int_0^{\theta_s} V_{co45}(\theta)\tan\frac{\theta}{2}\,d\theta\right|^2}{\int_0^{\pi}\left[\left|V_{co45}(\theta)\right|^2 + \left|V_{xp45}(\theta)\right|^2\right]\sin\theta\,d\theta} \tag{2.5}$$

where

$$V_{co45}(\theta) \equiv \frac{1}{2}\left[V_\theta(\theta) - V_\phi(\theta)\right] \tag{2.6}$$

$$V_{xp45}(\theta) \equiv \frac{1}{2}\left[V_\theta(\theta) + V_\phi(\theta)\right] \tag{2.7}$$

and θ_s is one half of the subtended angle to the edge of the primary (or secondary) reflector.

In order to further evaluate sources of efficiency loss during feed antenna design, the aperture efficiency expression is frequently factored into "sub-efficiency" terms η_i such that

$$\eta = \prod_i \eta_i. \tag{2.8}$$

Many factorization has been proposed in the literature [28, 31, 32]. In this research, a slightly modified version of the factorization in [30, 32] is used. In particular, the aperture efficiency is factored as,

$$\eta = \eta_{ill}\ \eta_{sp}\ \eta_{\phi}\ \eta_{xp}\ \eta_{BOR1} \tag{2.9}$$

which, from left to right, are illumination, spillover, phase, cross-polarization, and Body-of-Revolution-1 (BOR1) sub-efficiencies. The expressions for these terms are available in [30, 32] and are not reproduced; however, a brief explanation of each follows.

Illumination: Measures how close the realized aperture distribution is to uniform illumination. This is what was referred to as "aperture efficiency" above for the non-uniformly illuminated circular aperture, because all other "sub-efficiency" terms are equal to one in that case. This term is sometimes called taper efficiency.

Spillover: Ratio of power captured by the reflector to total radiated power. Conversely, $(1 - \eta_{sp}) \times 100$ represents the percentage power lost to energy spilling past the reflector. In the circular aperture example above, the assumption was that the aperture fields abruptly fall to zero right at the aperture rim which is impossible to realize. This is a very important term, as explained in the next section, because spillover energy increases antenna noise temperature.

Phase: Measures how uniform the phase distribution is in the reflector aperture. It is the only sub-efficiency that depends on the physical location of the feed with respect to the focal point of the reflector. Linear phase in the aperture would steer the reflector beam and any other phase perturbation results in far-field pattern degradation such as increased sidelobe level and shallow nulls [11]. Feed antennas with significant phase center variation in terms of wavelength in the frequency band of interest yield poor phase efficiency. Conversely, constant phase center implies constant phase efficiency that is approximately equal to one.

Polarization: Measures the peak cross-polarization level in the $\phi = 45°$ plane, namely $\max_\theta \left|V_{xp45}(\theta)\right|$. Most reflector antennas contribute little to no cross-polarization, and the feed cross-polarization performance predominantly determines the overall performance. This efficiency is called the polarization sidelobe efficiency in [30]. (Note that the definition of V_{xp45} in (2.7) is same as Ludwig's 3rd definition [33] only for the so-called Body-of-Revolution-1 antennas)

BOR1: The feed radiation pattern definition in (2.4) only has first-order azimuthal terms, because,

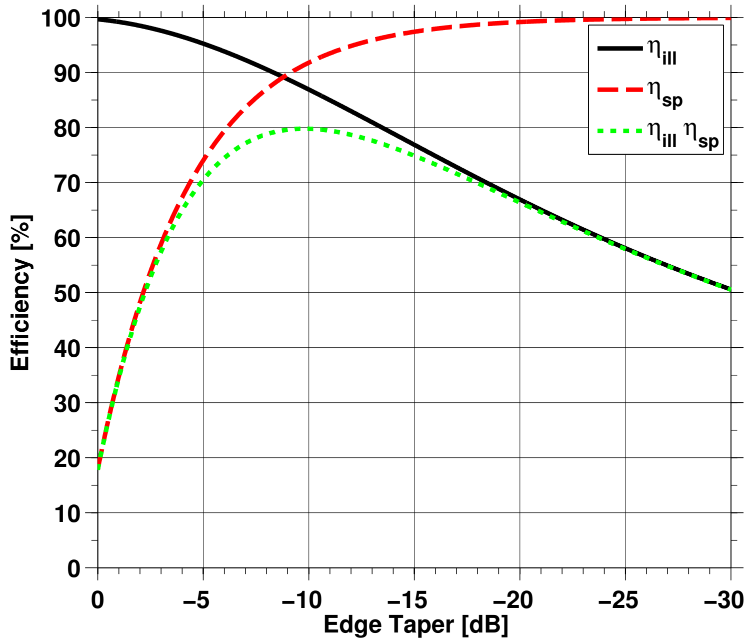


Figure 2.3: Illumination efficiency η_{ill} (black solid), spillover efficiency η_{sp} (red dashed) and $\eta_{ill} \times \eta_{sp}$ (green dotted) calculated using equations in [30]

as alluded to earlier, such far-field patterns give the maximum secondary gain. In other words, higher-order azimuthal terms do not contribute to on-axis gain of the reflector antenna, and thus represent a power loss [29]. BOR1 efficiency quantifies this power loss and is defined as the ratio of power in first-order azimuthal modes to total radiated power. This efficiency is sometimes called azimuth mode efficiency [31].

The illumination and spillover efficiencies calculated using the results in [30] are plotted in Figure 2.3 as a function of feed edge taper[3] which, in combination with Figure 2.2, reveals some key points regarding aperture illumination and spillover. In particular, tapered aperture illumination results in

1. Broader beamwidth;
2. Reduced gain $\Rightarrow$ reduced aperture efficiency;
3. Greatly reduced first sidelobe level;
4. Much reduced spillover energy.

These figures also show the well-known trade-off between illumination and spillover, and demonstrate the theoretical maximum aperture efficiency of a symmetric paraboloidal reflector antenna, 80%, attained at approximately -10 dB feed edge taper. The realized aperture efficiency will necessarily be lower due both to aforementioned losses in the reflector and the feed.

[3]Due to path loss, edge taper at the reflector rim would be slightly less

The approximate aperture efficiency and the sub-efficiencies mentioned above are only used in this research in comparing one quad-ridged horn to another during the design process. When a promising quad-ridged horn design is identified, PO calculations are carried out to evaluate its performance on the telescope.

2.3 Figure of Merit for a Radio Telescope

The primary goal in the design of radio telescopes and deep-space communication reflector antennas is to maximize the gain and minimize system noise temperature, both of which increase telescope's sensitivity. Therefore, the most widely used figure of merit in radio telescope design is given as

$$FoM \equiv \frac{A_{eff}}{T_{sys}} = \eta \frac{A_{phys}}{T_{sys}} \tag{2.10}$$

where T_{sys} is the system noise temperature and the other parameters are defined in the previous section. The system noise temperature is given by[4]

$$T_{sys} \equiv T_{Ant} + T_{LNA} = (1 - \eta_{spill})\, T_n + T_{sky} + T_{LNA} \tag{2.11}$$

where T_{LNA} is the effective input noise temperature of the low-noise amplifier proceeding the feed antenna; T_{sky} represents noise pick-up from the sky; T_n is the effective temperature seen by the spillover energy (e.g., if all spillover energy hit the ground, T_n would be 300K). It is seen from (2.10) that maximizing aperture efficiency is critical, but minimizing spillover is perhaps even more important because both the numerator and the denominator depend on it. For instance, assuming $T_n = 150$ K, $\eta = 0.6$, $\eta_{spill} = 0.9$, and $T_{sky} + T_{LNA} = 10$ K, a 2% increase in spillover efficiency (increases from 90 to 92%) yields more than 14% increase in the figure of merit.

2.4 Requirements of Radio Telescope Feed Antennas

The discussions in the previous two sections bring out the most important requirements of feed antennas to be used in radio astronomy:

1. Constant beamwidth
2. Circularly symmetric radiation pattern (first-order azimuthal terms only)
3. Small, preferably no, phase center variation
4. Low cross-polarization

[4]Assuming gain of the LNA is large enough so that input noise contribution of components following the LNA is negligible

5. Good input return loss to reduce impact on T_{sys}

and these need to be achieved over an octave or larger bandwidth. In addition, the feed should be easily integrable into a cryogenic dewar. Presently, the only antennas that meet and exceed all of these requirements are Pickett-Potter type [34, 35] and corrugated horns, which can achieve an octave bandwidth at most.