Markets and Microstructure

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Most of all to my advisers, Matt, Ben, Jakša, and John. To Harold for getting me into this mess. To my sister, Ceci, and my brother Kerry.

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Abstract

This document contains three papers examining the microstructure of financial interaction in development and market settings. I first examine the industrial organization of financial exchanges, specifically limit order markets. In this section, I perform a case study of Google stock surrounding a surprising earnings announcement in the 3rd quarter of 2009, uncovering parameters that describe information flows and liquidity provision. I then explore the disbursement process for community-driven development projects. This section is game theoretic in nature, using a novel three-player ultimatum structure. I finally develop econometric tools to simulate equilibrium and identify equilibrium models in limit order markets.

In chapter two, I estimate an equilibrium model using limit order data, finding parameters that describe information and liquidity preferences for trading. As a case study, I estimate the model for Google stock surrounding an unexpected good-news earnings announcement in the 3rd quarter of 2009. I find a substantial decrease in asymmetric information prior to the earnings announcement. I also simulate counterfactual dealer markets and find empirical evidence that limit order markets perform more efficiently than do their dealer market counterparts.

In chapter three, I examine Community-Driven Development. Community-Driven Development is considered a tool empowering communities to develop their own aid projects. While evidence has been mixed as to the effectiveness of CDD in achieving disbursement to intended beneficiaries, the literature maintains that local elites generally take control of most programs. I present a three player ultimatum game which describes a potential decentralized aid procurement process. Players successively split a dollar in aid money, and the final player–the targeted community member–decides between whistle blowing or not. Despite the elite capture present in my model, I find conditions under which money reaches

targeted recipients. My results describe a perverse possibility in the decentralized aid process which could make detection of elite capture more difficult than previously considered. These processes may reconcile recent empirical work claiming effectiveness of the decentralized aid process with case studies which claim otherwise.

In chapter four, I develop in more depth the empirical and computational means to estimate model parameters in the case study in chapter two. I describe the liquidity supplier problem and equilibrium among those suppliers. I then outline the analytical forms for computing certainty-equivalent utilities for the informed trader. Following this, I describe a recursive algorithm which facilitates computing equilibrium in supply curves. Finally, I outline implementation of the Method of Simulated Moments in this context, focusing on Indirect Inference and formulating the pseudo model.

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Chapter 1

Introduction

Since the 1990s, there have been many changes to equities markets and to development aid practices. Historically, equities markets were once centralized, relying on specialists in the New York Stock Exchange, and market makers on NASDAQ, to intermediate the market. Similarly, the World Bank and related donors have carried out aid projects using top-down strategies. Projects were developed, funded, and implemented under donor or World Bank direction. However, mainstay exchanges have fractured, with liquidity going to other exchanges, internalizers, and dark pools (Duffie (2012)), while exchanges are computer-driven more than ever. Community-Driven Development now accounts for a large portion of aid, offering project development and implementation choices to in-need communities themselves (Lessmann and Markwardt (2010)). The 1990s saw decentralization take hold in many development practices and nearly all financial markets. In this thesis, I develop microeconomic and econometric theory to explain interesting patterns in data concerning Community-Driven Development and modern exchanges. In chapter 2, I quantify information asymmetry and efficiency in modern limit order markets. In chapter 3, I set up a three-person ultimatum game and find equilibrium that explains interesting outcomes in decentralized development. In chapter 4, I develop the computational and econometric tools which are necessary for estimating equilibrium in limit order markets.

1.1 Chapter 2: Beyond the Bid-Ask Spread

In this chapter I explore the microstructure of limit order markets empirically. Empirical market microstructure has remained primarily in reduced form estimation. Contrasting with previous studies, I fit parameters using an equilibrium model and explore the industrial organization of these markets. Through a novel inference method, as outlined in Chapter 4, I quantify anticipated information flow and efficiency in a market for a single asset. Information flow is in terms of new information about the asset's underlying value. Efficiency is characterized as the level of hedging need of traders that the market meets.

I empoy my methodology with a case study involving Google stock surrounding an earnings announcement in the third quarter of 2009. I evaluate typical asymmetry measures, specifically depth of book and bid-ask spread. To better communicate my own estimation results, I develop a volume-insensitive measure of information flow into the market, the *Informedness Ratio*. The Informedness Ratio is defined as the ratio of volatility of the asset's true value from the perspective of the market and the perspective of some better-informed trader. A higher number relates to higher asymmetry, which is found at the beginning and the end of the trading day. As a robustness check, the interday levels of the Informedness Ratio appear to fall following the earnings announcement, similar to the bid-ask spread.

I also look at efficiency in the market for Google surrounding the earnings announcement. Employing the structural nature of the model, I can vary model parameters and simulate data to match it. I vary the number of effective liquidity suppliers in the market at any one point of time–essentially changing the market structure. From this simulated data, I compute certainty-equivalent utilites for traders and better-informed traders to illustrate, as one would expect, that more competition in the market results in higher efficiency. The model fits for various numbers of effective liquidity providers is also found, and I find that the model fits markedly better with a larger number of liquidity providers than two. Two liquidity providers may illustrate a duopolist setting, and so we argue that the market for Google behaves as if there is more competition among those *making the market*.

1.2 Chapter 3: Corruption and CDD

I use microeconomic theory to further our understanding of corruption and development. Development on the whole is understood to attract corruption. Community-Driven Development, a newer aid implementation, was designed to reduce corruption, and empower communities to develop their own aid projects. I argue that the way we think about corruption in development, and specifically Community-Driven Development, may not be accurate. Classically, considering corruption in a Becker and Stigler (1974) framework, a third party to corruption serves as deterrent factor. However, such third parties are difficult to characterize in many aid settings, and incentives facing those perpetrating graft lead corrupt individuals to choose projects which may best hide their corrupt dealings (Shleifer and Vishny (1993)).

In a tradition of three-person ultimatum games, I present a new three player ultimatum game which describes a potential decentralized aid procurement process. Players successively split a dollar in aid money, and the final player–the targeted community member–decides between whistle blowing or not. I feel as though this setting may more accurately describe the settings as faced by project implementers as well as village members. The key feature exploited here is that of a less engaged monitoring entity. The comparative statics I find lend some light to possible directions for theoretical as well as empirical research into the economics of decentralized aid and corruption in a decentralized setting.

1.3 Chapter 4: Econometrics and Computation

In my concluding chapter, I develop in more depth the empirical and computational means to estimate model parameters in the case study in Chapter 2. While the setting I apply these techniques to is specifically market microstructure and in particular, limit order markets, the applications for such methods are wider in scope. Competition in supply curves is a notion explored in auctions and other parts of industrial organization. Similarly, the econometric tools used here are standard to empirical industrial organization and structural estimation, however the way I implement them is different than previously explored.

I set up the model I use for estimating equilibrium trading in limit order markets. While this analysis leaves out many of the details of the competing liquidity suppliers problem in a discrete-price order book, the equilibrium concepts are standard. I first describe the liquidity supplier problem, incorporating the discrete-price first-order and boundary conditions for the informed active trader. I also find the equilibrium conditions among these suppliers.

I also develop computational tools to find efficiency in the market for an asset. These utilities are used when analyzing simulated data from varying numbers of effective liquidity suppliers in my paper. However, the structural methodology allows for any number of parametric changes, from trader risk-aversion to the volatility of the asset's underlying value.

The most important methodological piece to analysis of limit order markets is the method of computing equilibrium in supply curves in a discrete-price setting. Following Baruch (2008), it is possible to find a partial differential equation which characterizes equilibrium supply in a continuous-price setting. However, in discrete prices, the equilibrium problem becomes more difficult, and analytially intractable. To circumnavigate ths difficulty, I develop a recursive algorithm which facilitates computation of equilibrium supply curves. The recursive algorithm employs some key features of limit order markets–any market with competing suppliers, for that matter–and allows for implementation of standard dynamic programming numerical methods.

The particular nature of limit order data lends itself well to time-series analysis, but the limited depth in order books (ticks beyond the bid-ask spread), and the decreasing importance of this depth, reduces the applicability of some standard panel asymptotics. Because the analytical forms of my equilibrium supply curves are not available, I turn to the method of simulated moments. Beyond that, the method of simulated moments requires the use of a pseudo model to avoid the intractable equilibrium supply curves, and produce moments. I use Indirect Inference to match the pseudo model from data to the pseudo model from true parameters, and fit using iteration. However, standard Indirect Inference is not an option because each instance of the order book lacks panel characteristics necessary for asymptotic results. I therefore employ the time-series nature of the data, assuming that a single equilibrium is played over a short period of time, and use in-fill asymptotics to achieve consistency of my estimator. I then use several instances of the order book to achieve over-identifying restrictions. By limiting the number of ticks of the order book that are considered at any point in time, I better represent the true nature of the order book.

Chapter 2

Beyond the Bid-Ask Spread: Estimating an Equilibrium Model of Limit Order Markets

2.1 Introduction

In this paper, I study equilibrium trading in limit order markets. Limit order markets are widely used in financial exchanges around the world, including New York Stock Exchange, NASDAQ, Stockholm Stock Exchange, and Paris Bourse, among others. A limit order market allows for direct interaction between traders, without a market-making dealer. Traders choose between market orders–which execute against existing orders in the book–and limit orders–which enter the book with a limit number of shares and limit price and await execution with a market order. Despite widespread use, empirical studies of limit order markets have been hampered by data availability and complexity of existing models. This paper pushes forward the empirical analysis of these markets by specifying and estimating a structural econometric model of equilibrium trading in a limit order market. As a case study, I estimate this model on the limit order book for Google stock surrounding a surprisingly good earnings announcement in the 3rd quarter of 2009.

I develop an econometric framework based on work of Bernhardt and Hughson (1997) and Biais, Martimort, and Rochet (2003), in which the authors study a model of imperfect competition among liquidity suppliers under adverse selection. My estimation methodology incorporates an insight by Baruch (2008), a paper subsumed by Back and Baruch (2012), that liquidity suppliers' choice of supply schedules can be recursively characterized as a dynamic programming problem. This insight is key to my econometric procedure, allowing me to apply tools and methodologies developed for estimating dynamic optimization models to the explicitly non-dynamic (static) model of equilibrium in limit order markets. To my knowledge, this is the first paper performing structural econometric analysis in such a setting.

I use this econometric framework to address two important questions. First, is information transmitted evenly and quickly across the market? I present a measure of information asymmetry, the Informedness Ratio. A higher ratio means that insiders have more information than the market, with a lower ratio meaning asymmetry is less. The informedness ratio is high a day before the examined earnings announcement, settling at a lower level the day of the announcement, and continuing to fall the day after. The fall in the informedness ratio the day of the announcement indicates information asymmetry and uncertainty fall before the announcement is made. Second, what are the welfare implications of the switch from dealer to limit order markets, pervasive in real-world financial markets? I simulate counterfactual dealer markets using real data, and show that limit order markets perform as well as, if not much better than, one would expect dealer markets to perform.

I address an important question in the microstructure literature involving pervasive change in modern exchanges to a limit order-based system. Historically in most markets, trade was handled by a single monopolist dealer. If a trader wanted to buy or sell a certain number of shares, a dealer would quote terms, and the trade could be executed. Now most exchanges use a limit order book. Traders have the option of placing market orders, which execute against existing shares in the book, or placing a limit order, with a limit price and quantity, which enters the book and waits for a market order to execute against it. I use counterfactual examples to assess the change in market performance moving from two dealers to the imperfect competition found in limit order markets. Arguments could be made that limit order markets reduce restrictions on offer schedules, improving terms for trade, increasing welfare.[1] However, a dealer could bring a level of expertise to an asset, allowing a dealer to facilitate the market in times that decentralized traders would be unwilling to make a market for an asset. Using my model, I simulate counterfactual dealer markets from the data. Not surprisingly, more liquidity suppliers leads to higher welfare. As a robustness check, I find the model fits better with five liquidity suppliers than it does with two or ten–when traditional markets typically had one, but no more than two, dealers per asset. This means that the market behaves effectively as if it has five dealers. More dealers unambiguously means higher welfare, hence I conclude that limit order markets are more efficient than a counterfactual dealer market would be.

I touch on several different literatures related to this work in section 2.2. Section 2.3 explains the limit order book as a dynamic program and the risk-averse active trader setting used here. Section 2.4 discusses methods of simulation and estimation. I discuss my results on earnings announcements in section 2.5. In section 2.6, I perform robustness checks of the model specification and conduct an analysis of the move from dealer markets to limit order markets. Section 2.7 concludes.

2.2 Literature Review

I take an equilibrium model of imperfect competition in a common value setting from Bernhardt and Hughson (1997), which is further analyzed for equilibrium in Biais, Martimort, and Rochet (2003)[2] and apply it in an empirical setting. Their analysis involves strategic risk-neutral liquidity suppliers competing in schedules for the business of a risk-averse agent who is privately informed about the value of the asset and about hedging needs in a common value setting. Biais et al. extend previous analyses concerning what creates the bid-ask spread, patterns in trading volumes, and oligopolistic incentives.[3]

[1]See Biais, Foucault, and Salanié (1998).

[2]Madhavan (1992) uses the same risk-averse trader setting, but assumes competitive liquidity suppliers. For a recent model on imperfect competition in a contrasting *private* value setting, see Vives (2011).

[3]See Kyle (1985) for the seminal batch-trading model, Glosten (1989) for the monopolist dealer problem, Glosten (1994) for an early treatment of limit order markets and the limiting case as the number of traders

Baruch (2008) makes a key analytical insight, by applying dynamic programming methods to analyze the same model. From an econometric point of view, this insight proves useful in making these models amenable to estimation. While dynamic programming methods are inherently valuable to a variety of problems in structural analysis, these methods are often employed in explicitly dynamic models, where agents make intertemporal decisions. Using Baruch's intuition, I can solve for equilibrium price schedules using dynamic programming methods in price. Dynamic programming has become a crucial component of the structural econometrics tool box, beginning with Miller (1984), Wolpin (1984), Pakes (1986), and Rust (1987), and is used to approach a variety of problems. While my model is not explicitly dynamic, I use similar intuition in finding the equilibrium in a limit order market. By simulating supply curves from the data, I try to match parameters in the true model to actual data. This process is the principal of Indirect Inference, as in Gouriéroux, Monfort, and Renault (1993) and Smith (1993).[4]

While theory is concerned with microstructure and its implications on liquidity provision and information transmission, the related empirical literature focuses on reduced-form analysis of optimal order placement and dynamics between limit orders and market orders.[5] Biais, Hillion, and Spatt (1995)[6] study the early Paris Bourse computerized limit order exchange. They find that thin books elicit more limit orders whereas market depth results in market orders, or immediate trades.[7] Dufour and Engle (2000) use a model to assess the role of waiting time between transactions in the process of price formation. They find

grows, and Bernhardt and Hughson (1997) for the duopoly case. For related and important models in which nature chooses whether the trader is an informed trader or a liquidity-motivated trader, see Copeland and Galai (1983) and Glosten and Milgrom (1985). Some work has sought to reconcile the two-sided limit order problem, as in Roşu (2009). For a survey of the theoretical literature, see O'Hara (1998), or more recently, Vives (2010).

[4]Goettler and Gordon (2011) is a recent application of Indirect Inference in structural estimation.

[5]For a survey of empirical work, see Hasbrouck (2007).

[6]Ranaldo (2004) studies similar questions, but using ordered probit to empirically investigate order submission strategies. The author finds patient traders become more aggressive when their own side of the book is thicker, when the spread is wider, and when volatility is momentarily high. For an experimental study of liquidity in an electronic limit order market, see Bloomfield, O'Hara, and Saar (2005).

[7]This finding is consistent with modern theoretical and numerical simulation results. See Foucault, Kadan, and Kandel (2005), Goettler, Parlour, and Rajan (2005), and Pagnotta (2010).

that as the speed of trades increases, price adjustment also increases, hence an increased presence of informed traders. Ahn, Bae, and Chan (2001) study the relationship between market depth and transitory volatility. They find evidence that limit order traders enter the market, placing orders where liquidity is needed. This finding is in support of the notion that a limit order market is in equilibrium at any one point in time, with liquidity suppliers waiting to fill gaps in the market.

The literature of structural empirical work on limit order markets is smaller. Sandås (2001) present economic restrictions on price schedules offered in a competitive setting. He finds that there is insufficient depth in limit order books relative to theoretical predictions. Hollifield, Miller, and Sandås (2004)[8] examine optimal order placement and find order submission is a montone function for a trader's valuation of the asset. However, their model assumes traders trade only one unit of the asset, and the authors reject their private value trading restrictions for the order placements of traders with moderate private values. Hollifield, Miller, Sandås, and Slive (2006) study the gains from trade in a limit order market, comparing efficiency in a perfectly liquid market and a market with a monopolist to the actual gains from trade. Kelley and Tetlock (2012) estimate a model of strategic trader behavior that incorporates endogenously informed traders and discretionary liquidity traders. They show that these discretionary traders make up most trading volume, but that from 2001 to 2010, informed trading increasingly contributes to volume and stock price discovery. Their analysis exploits variation in trading and volatility correlated with time of day and public news arrival under a linear pricing equilibrium. I build on the structural estimation literature by estimating equilibrium trading in a limit order model with endogenous liquidity provision.

[8]For an earlier investigation in optimal order strategies, see Harris and Hasbrouck (1996).

2.3 Model

In limit order markets, traders choose between placing market orders and limit orders. Market orders execute against orders in the book at the best price posted, what is called *walking the book*, or *taking liquidity.* Limit orders specify a limit price and quantity, where unexecuted portions of a limit order enter the book, what is called *supplying liquidity.* The limit order market allows traders to interact directly. Posting liquidity to the book and taking it have a timing component however. A market order executes with an order that came before it. The active trader making the market order may have information newer to the market than information liquidity suppliers had when offering shares. If this information asymmetry fully characterized trade, there would be no market (see Milgrom and Stokey (1982), Grossman and Stiglitz (1980)). Instead, active investors have incentives to trade beyond inside information. An active investor may wish to hedge a position, reducing exposure to an asset. This balance between opposing motivations is a key tension in information-based models of limit order markets.

There are n risk-neutral uninformed liquidity suppliers and a single informed active trader trading a single asset. Liquidity suppliers submit limit orders and the active trader observes all bids and offers and submits a marketable order. The asset is then liquidated at $v = \alpha + \epsilon$, where α, distributed normally, is the signal the informed active trader receives, and ϵ is noise. $I \sim N(\mu_I, \sigma_I^2)$ is the informed active trader's inventory of the asset. Latent supply, S_0 is used for estimation and is described in more detail later. Table 2.1 provides model paramters and definitions.

Without loss of generality, I focus my discussion on the offer side of the book. Liquidity suppliers could be viewed as institutional traders, for example, Goldman Sachs, etc. Active traders could be employees at the traded company who have access to information relevant to the company's stock performance that is unavailable to liquidity suppliers. However, active traders do not only have information motivations for trading, but also non-information hedging needs based on their current holding of the stock. Liquidity suppliers do not know

Table 2.1: Model Parameters

n	number of liquidity suppliers
σ_α	standard deviation of informed trader's signal
σ_ϵ	standard deviation of noise of informed trader's signal
μ_I	mean of the distribution of the informed trader's inventory of the asset
σ_I	standard deviation of the distribution of the informed trader's inventory
σ_{S_0}	standard deviation of latent supply

whether a market order posted by an active trader arises from information or liquidity motivations. This information asymmetry between the active trader and liquidity suppliers is a source of adverse selection in the model. Generally, hedging-motivated trades would be profitable for the liquidity suppliers, but information-based trades would not. Thus, liquidity suppliers face adverse selection in deciding how many shares to supply at any one price. Higher information asymmetry will make liquidity suppliers wary, and they will respond by posting fewer shares. Less information asymmetry means more liquidity will be available and more hedging needs will be met.

2.3.1 Notation

Liquidity suppliers play a static game. To accommodate the data, I take prices as discrete. p_{ask} is the minimum price at which liquidity is offered and p_{max} is the maximum, hence prices are $p_{ask} \equiv p_1, \ldots, p_M \equiv p_{max}$, and are taken as given. A strategy for the i^{th} liquidity supplier is $S^i : \{p_1, \ldots, p_M\} \to \mathcal{R}$ where $S^i_r \equiv S^i(p_r)$ represents the total number of shares offered by i through price p_r. The limit order book changes rapidly within a short period of time. At the lowest price, p_1, I assume there is some additional, *latent supply*, S_0, which represents impatient trades, hidden orders, or spillover from the buy side of the market. While this does not change implications of the model, it is important for econometric implementation, as it generates variation in the limit order book across trading episodes.[9] Because the set

[9]For a detailed discussion of latent supply, see the equilibrium solution section of the appendix, 4.4.

of prices is discrete, the function $s_r^i \equiv S_r^i - S_{r-1}^i$ is well defined. I also define $s_r^{-i} \equiv \sum_{j \neq i} s_r^j$, $S_r \equiv \sum_{i=1,\ldots,n} S_r^i + S_0$, and $s_r \equiv \sum_{i=1,\ldots,n} s_r^i$.

In this common value environment, profitability of a limit order is dependent on probability of execution and the expected value of selling the asset conditional on execution. If marketable, the active trader's bid walks up the book, picking off offers until it reaches its limit price or is filled. Private information observed by the active trader leads to adverse selection. Profitability of a liquidity supplier's offer at a price p_r is a function of that price, liquidity offered up to that price by all liquidity suppliers, liquidity offered at that price by the agent, and liquidity offered at that price by all other liquidity suppliers.

2.3.2 Dynamic Programming Characterization of the Limit Order Book

Because a single trader's order walks up the book, I can treat the limit order book, which consists in equilibrium, of the liquidity suppliers optimally chosen supply schedules, as the solution to a dynamic programming problem in price. This is a key insight of Baruch (2008). While profitability of shares offered at a lower price affects profitability of shares offered at a higher price, profitability of shares offered at lower prices are unaffected by shares offered at higher prices, hence at the price p_r, I define the value function,

$$V_L(p_r, S_{r-1}) = \max_{s_m^i,\, m=r,\ldots,M} \sum_{m=r}^{M} u_L(p_m, S_{m-1}, s_m^i, s_m^{-i}) \tag{2.1}$$

such that

$$S_{m+1} = S_m + s_m^i + s_m^{-i} \tag{2.2}$$

Here $u_L(p_r, S_{r-1}, s_r^i, s_r^{-i})$ represents profitability to the liquidity supplier.[10] The state variable is S_{r-1}, the total volume supplied at prices lower than the current price, p_r. The

[10] The form of profitability is described later in section 2.3.4 and appendix 4.2.

corresponding Bellman equation is,

$$V_L(p_r, q_r) = \max_{s_r^i}[u_L(p_r, S_{r-1}, s_r^i, s_r^{-i}) + V_L(p_{r+1}, S_{r-1} + s_r^i + s_r^{-i})] \tag{2.3}$$

This recursively characterizes the optimal supply schedule $\{s_1^{i*}, s_2^{i*}, \ldots, s_M^{i*}\}$. At maximal price p_M, the Bellman equation simplifies to

$$V_L(p_M, S_{M-1}) = \max_{s_M^i} u_L(p_M, S_{M-1}, s_M^i, s_M^{-i}) \tag{2.4}$$

For all values of S_{M-1}, I can solve for s_M^{i*} at the maximal price. Plugging in this strategy to the Bellman equation at p_{M-1}, the strategy s_{M-1}^{i*} satisfies,

$$V_L(p_{M-1}, S_{M-2}) = \max_{s_{M-1}^i}[u_L(p_{M-1}, S_{M-2}, s_{M-1}^i, s_{M-1}^{-i}) + V_L(p_M, S_{M-2} + s_{M-1}^i + s_{M-1}^{-i} + s_M^{i*} + s_M^{-i*})] \tag{2.5}$$

Given the finiteness of prices, this dynamic programming problem can be solved for $s^{i*} \equiv \{s_1^{i*}, s_2^{i*}, \ldots, , s_M^{i*}\}$ by backward induction, starting at the highest price p_M.

2.3.3 Risk Averse Active Trader

In this paper I use a setting with a risk averse, *informed* trader facing risk neutral, *uninformed* liquidity suppliers, found in many papers, namely Glosten (1989) for the monopolist dealer setting, Madhavan (1992) for a competitive setting, Bernhardt and Hughson (1997) for duopoly, and Biais, Martimort, and Rochet (2003) for the further oligopoly setting. In this setting, active traders placing market orders bring new information to the market and are informed. Liquidity suppliers, however, have orders already in the book when a market order is placed, and are uninformed. As in Copeland and Galai (1983), active traders coming to the market–after limit orders are already posted–bring information to the market that liquidity suppliers do not yet have.

The active trader sees a signal, α, where $v = \alpha + \epsilon$ and ϵ is normally distributed with mean 0 and standard deviation σ_ϵ. In addition, the active trader has inventory of the asset I and knows this. The active trader uses this information to decide an optimal order, q, maximizing the expectation of the utility of wealth, where

$$u_A(q|I, \alpha) \equiv -e^{-\gamma W}, \tag{2.6}$$

$$W \equiv \underbrace{(q+I)v}_{\text{value of shares owned}} \underbrace{-\sum_{r=0}^{M} \max\{0, \min\{q, S_r\} - S_{r-1}\} p_r}_{\text{payment to suppliers}}, [11] \tag{2.7}$$

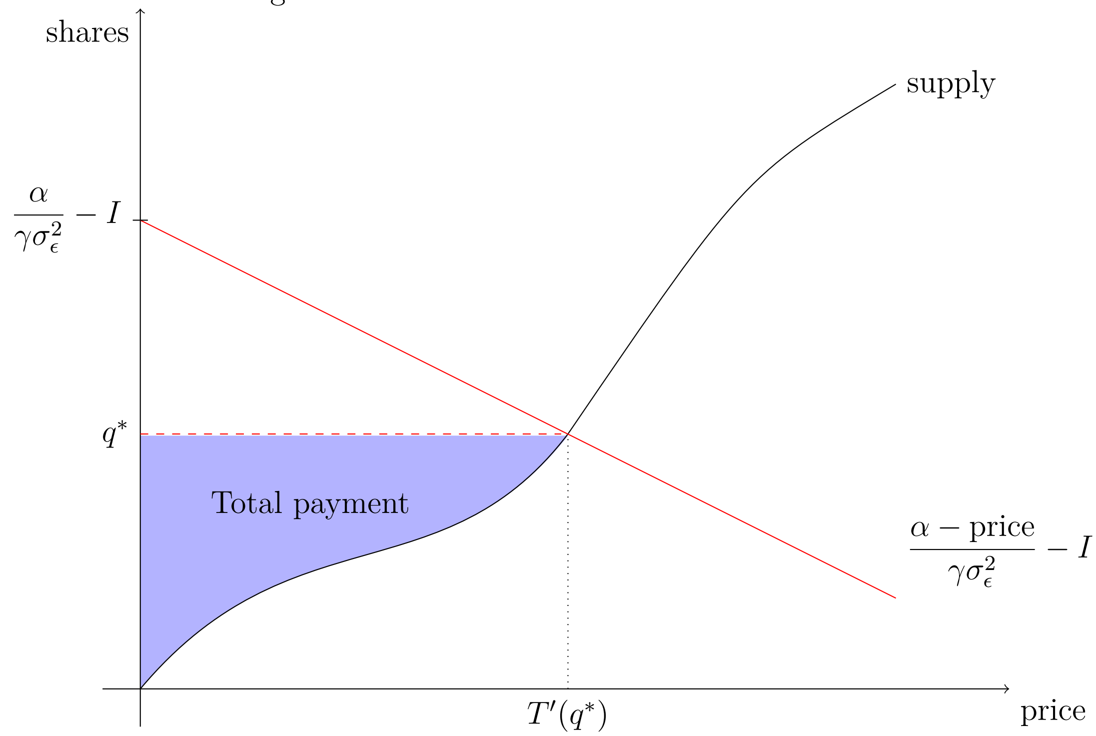
and γ is the coefficient of risk aversion. The active trader does not observe v directly, but receives a noisy signal α. A large α means that the underlying asset's value is likely high, and the active trader's order will walk the book picking off stale asks. Conversely, if I is large and positive, the active trader wants to sell, and when I is large and negative, the active trader wants to buy. The active trader is incentivized by α and I, so a high α (news) may mean buying even when I (exposure) is also high. The uninformed risk neutral liquidity suppliers know the distribution of $\alpha \sim N(\mu_\alpha, \sigma_\alpha)$ and $I \sim N(\mu_I, \sigma_I)$, but a liquidity supplier can only infer a posterior distribution on (α, I) from trades.

For illustration purposes, I first consider the case where prices are continuous. A supplier posts an offer to sell at a price and an informed active trader's buy order walks the book, picking off these shares. Following rational expectations, the seller must exhibit no regret in selling shares to the profile of traders that bought them. In Figure 2.1, a supply schedule generates a purchase choice of q^* shares. The optimal choice by the active trader is the intersection of supply and incentives to buy shares, $\frac{\alpha - \text{price}}{\gamma \sigma_\epsilon^2} - I$. Given that $\gamma \sigma_\epsilon^2$ is common knowledge, the supplier knows the slope of this curve, and can back out the intercept, revealing the profile of active traders (α, I). The liquidity supplier considers the profile of

[11] For ease of notation, and following the definition of latent supply, $p_0 = p_1$.

traders who would buy at a given price and quantity and responds with the optimal number of shares.[12]

Figure 2.1: Inference and Continuous Solution



To put it more formally, liquidity suppliers compete in transfer schedules $T(\cdot) : \mathcal{R}^+ \to \mathcal{R}^+$ giving the payment for any quantity, q, demanded by an active trader. As a result, the active trader chooses q maximizing $E[-e^{-\gamma W}]$, from (2.6). This is equivalent to maximizing $E[W|\alpha, I] + \frac{\gamma}{2}V[W|\alpha, I]$. Variance of holdings is $(q+I)^2\sigma_\epsilon^2$. From transfer schedules this is equal to, $(q+I)\alpha - T(q) - \frac{\gamma}{2}(q+I)^2\sigma_\epsilon^2$. This yields an interior solution for supply schedules where $q^* = \frac{\alpha - T'(q^*)}{\gamma\sigma_\epsilon^2} - I$. Here the supply curve in Figure 2.1 is $T'(q)$, and $T'(q^*)$ is simply the price at which the last portion of q^* is supplied.

In typical data, supply schedules are not continuous functions of price, but rather step functions. This introduces complications to the model, as I show here.[13] Solving

[12] This reasoning is similar to strategies conditional on winning the prize in auctions (see Milgrom and Weber (1982)), and vote pivotality in juries (see Feddersen and Pesendorfer (1998)). The liquidity supplier must offer enough shares to take advantage of the hedging needs of traders reaching that far in the book while balancing information asymmetry faced at that point as well.

[13] A treatment of price discreteness for bidding in multiunit auctions is found in Kastl (2006), McAdams

for equilibrium q, the best response quantity demanded by an active trader when utility is $u(q|\alpha, I) \equiv -e^{-\gamma W}$, is again equivalent to the active trader choosing a q maximizing $E[W|\alpha, I] + \frac{\gamma}{2}V[W|\alpha, I]$. Wealth is given by equation 2.7, and the trader maximizes,

$$\underbrace{(q+I)\alpha}_{\text{expected value of shares}} \underbrace{-\sum_{r=0}^{M} \max\{0, \min\{q, S_r\} - S_{r-1}\}p_r}_{\text{payment to suppliers}} - \underbrace{\frac{\gamma}{2}(q+I)^2\sigma_\epsilon^2}_{\text{volatility penalty}}. \tag{2.8}$$

In equation 2.8, v in wealth as in equation 2.7 is replaced by α, because σ_ϵ is expectation 0. However, σ_ϵ shows up in the risk aversion variance of wealth penalty. Because of the discrete nature of these supply curves, either the active trader reaches a first order condition, at some point between S_r and S_{r+1} for some r, or not, at S_{r+1} for some r. In the first case, $\alpha - p_m - \frac{\gamma}{2}\sigma_\epsilon^2(2q + 2I) = 0$, where p_r is s.t. $S_r < q < S_{r+1}$. In the second, the first order condition is not reached, and $q = S_{r+1}$ s.t. $\alpha - p_r - \frac{\gamma}{2}\sigma_\epsilon^2(2q + 2I) > 0$ and $\alpha - p_{r+1} - \frac{\gamma}{2}\sigma_\epsilon^2(2q + 2I) < 0$, hence the active trader has exhausted all gains from trade at price p_r, but would lose money trading at p_{r+1} for the next available liquidity in the book. The two cases are as follows,

Case 1: interior solution, where p_r is s.t. $S_{r-1} < q < S_r$

$$q = \frac{\alpha - p_r}{\gamma\sigma_\epsilon^2} - I \tag{2.9}$$

As can be seen, given a supply schedule and an active trader places an order, liquidity suppliers can infer a statistic, $\dfrac{\alpha}{\gamma\sigma_\epsilon^2} - I$, for α and I which contains all the information the liquidity suppliers can infer about α and I.

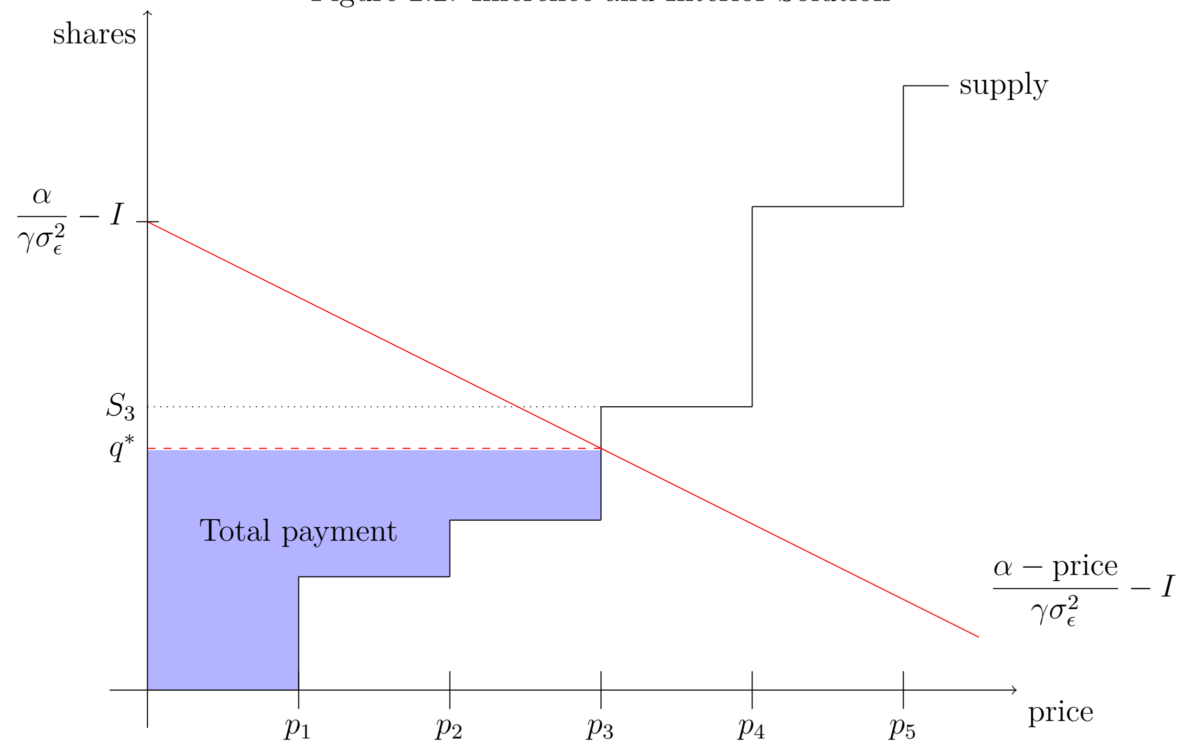
Case 2: boundary solution, $q = S_r$

$$\frac{\alpha - p_{r+1}}{\gamma\sigma_\epsilon^2} - I < q < \frac{\alpha - p_r}{\gamma\sigma_\epsilon^2} - I \tag{2.10}$$

However, if $q^* = S_r$ for some r, then inference is not as clear for the liquidity suppliers.

(2008), and Hortaçsu and Kastl (2012).

Figure 2.2: Inference and Interior Solution



Now, $\frac{\alpha}{\gamma\sigma_\epsilon^2} - I$ can only be narrowed down to a range of profiles.

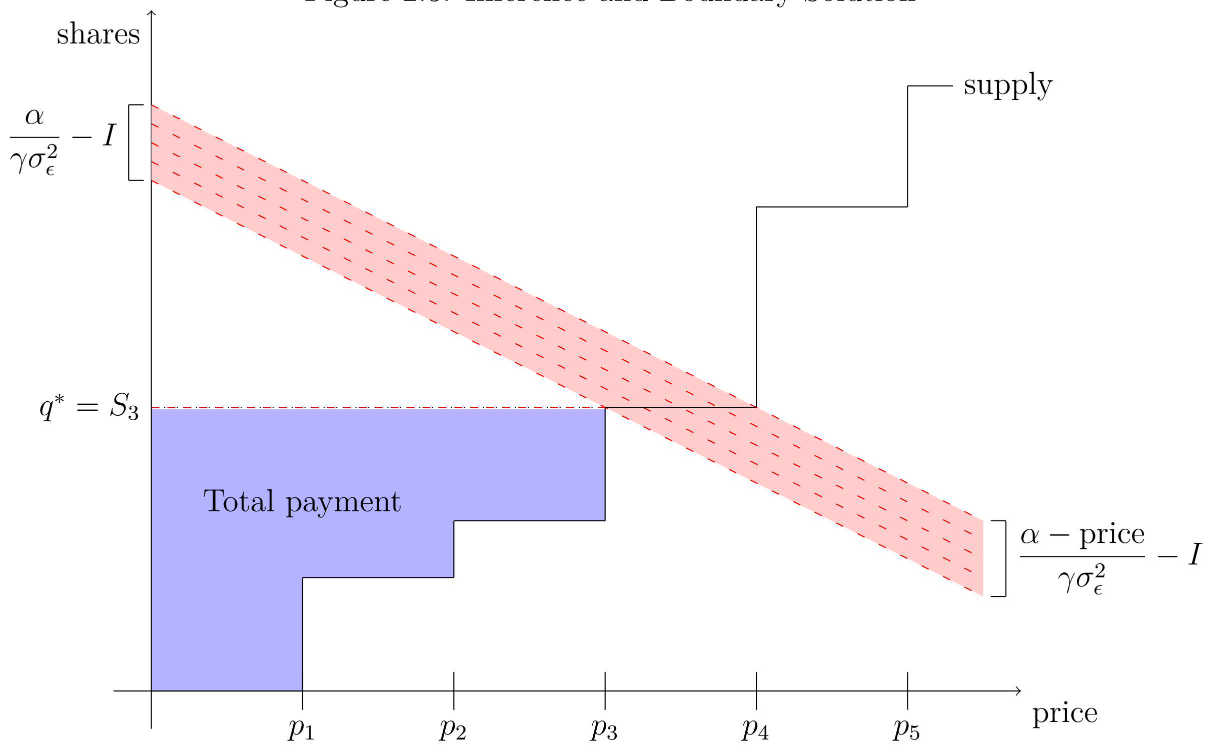
2.3.4 Liquidity Suppliers and the Optimal Supply Problem

I assume that offers at a given price are executed proportionally.[14] This means that at price p_r, the bid walks through the orders at a rate $\frac{s_r^i}{s_r^i + s_r^{-i}}$ for each supplier i until the order moves up the book to the next price. With these solutions I compute the utility of offering shares for the liquidity supplier at each price p_r, which is expected profitability given liquidity suppliers are risk-neutral. Either $q > S_{r-1} + s_r^i + s_r^{-i}$ or $S_{r-1} < q \leq S_{r-1} + s_r^i + s_r^{-i}$.

$$u_L(p_r, S_{r-1}, s_r^i, s_r^{-i}) =$$
$$E_{\alpha,\epsilon,I}\left[s_r^i \mathbf{1}_{\{S_{r-1}+s_r^i+s_r^{-i} \leq q\}}(p_r - v) + \frac{s_r^i}{s_r^i + s_r^{-i}}(q - S_{r-1})\mathbf{1}_{\{S_{r-1}<q<S_{r-1}+s_r^i+s_r^{-i}\}}(p_r - v)\right] \quad (2.11)$$

[14]Because the model used here is not explicitly dynamic, assuming that some orders are executed after others would negate non-trivial symmetric equilibrium and likely would make the number of equilibria infinite.

Figure 2.3: Inference and Boundary Solution



The expected profit is added to the continuation payoff for subsequent prices. For each number of shares offered, there is an expected profit for that price, and an expected payoff for subsequent supply. I consider a symmetric equilibrium for liquidity suppliers when the derivative of this expected profit with the derivative of continuation is 0.[15] I leave further derivation to the appendix, 4.2.

2.4 Estimation

Here I describe my econometric method in which I use a nested fixed point algorithm, as in Rust (1987)[16]. Starting with parameter values of the structural model, I find equilibrium supply curves. Using a pseudo-model[17] (following the terminology in Gouriéroux, Monfort,

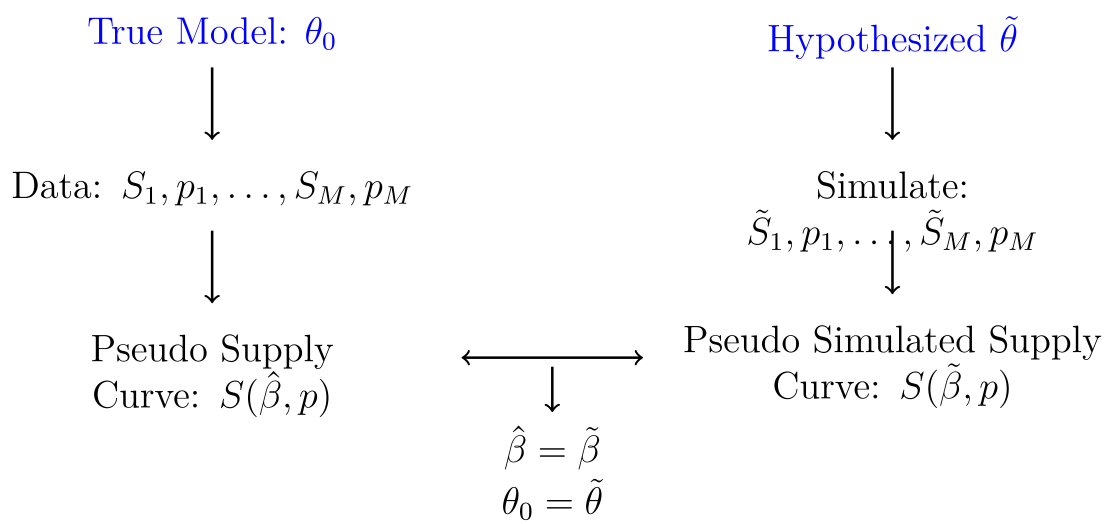
[15]While Back and Baruch (2012) show equilibrium theoretically for only a range of parameter values, I verify the equilibrium numerically.

[16]See Rust (1994) and Rust (1996) for further discussion.

[17]The pseudo-model is referred to as an *approximated model*, an *instrumental model*, and a *statistical model* by the literature. For clarity, I will use the term *pseudo-model* from here on.

and Renault (1993)), I fit the generated supply curves to those found in the data. Choosing new parameter values, I iterate until I achieve a best fit. This is illustrated in Figure 2.4, below. In addition, the following sections describe the individual components of this analysis.

Figure 2.4: Indirect Inference



2.4.1 Solving for Equilibrium

Players compete in supply schedules at each price successively, as liquidity at lower prices affects the profitability of liquidity supplied later. In the optimal Markov strategy, liquidity suppliers' offers of shares at a given price are based soley on the amount of liquidity offered up to that point.[18] Suppliers consider a trade executing against their supply schedule. The market order picks off shares at prices at which liquidity is offered, until the order is met. A liquidity supplier infers what profile of information and liquidity incentives were faced given the trade reached that deep in the book, and decides how many shares to offer at that price. Suppliers determine their strategies starting at the limit of economically meaningful share depth and backward induct to find their optimal supply curves.

For interested readers, solving for equilibrium is described in detail in the appendix, 4.4.

[18]Given the nature of modeling competition in supply, if an optimal strategy exists for this problem, then a Markov optimal strategy exists.

2.4.2 Indirect Inference

For estimating model parameters, I use the simulated method of Indirect Inference, following seminal work in Gouriéroux, Monfort, and Renault (1993), and Smith (1993).[19] I estimate a pseudo-model for data and limit order supply curves simulated from the model. Realizing a best fit of the model means generating estimates as close together as possible. I find the set of parameters such that the pseudo-model's two sets of estimations coincide.[20] Coefficients of the pseudo-model consist of a least squares regression of share quantities on four linear and nonlinear functions of price.

Let $\hat{\beta}$ denote parameters of the pseudo-model estimated from actual data. Analogously, let $\tilde{\beta}^{\lambda}(\theta)$ denote parameters estimated from data simulated from the limit order book under parameters θ (λ superscripts the specific simulation). The Indirect Inference estimator $\hat{\theta}^{\Lambda}$ [21] optimizes the following criterion,

$$\hat{\theta}^{\Lambda} \equiv \arg\min_{\theta \in \Theta} \left[\hat{\beta} - \frac{1}{\Lambda} \sum_{\lambda=1}^{\Lambda} \tilde{\beta}^{\lambda}(\theta) \right]' \Omega \left[\hat{\beta} - \frac{1}{\Lambda} \sum_{\lambda=1}^{\Lambda} \tilde{\beta}^{\lambda}(\theta) \right] \tag{2.12}$$

I explain details of the Indirect Inference method, demonstrate asymptotic normality of the estimator, and also prove the following proposition in the appendix, 4.5,

Proposition 3. Under assumptions 1–6, $\hat{\theta}^{\Lambda}$ is a consistent estimator of θ_0.

[19] See Goettler and Gordon (2011) for another recent use of Indirect Inference for estimating equilibrium of a structural model.

[20] For all estimates, I try different starting values for the parameter search algorithm. In most cases, the outcomes for the model parameters are the same. When different starting values produced different parameter estimates, I chose the model parameters that led to the lowest criterion function values.

[21] Λ signifies the number of simulated supply curves analyzed by the pseudo-model. In this paper, I use 200 simulations for each criterion calculation.

2.5 Empirical Results: A Snapshot of Google, October 14-16, 2009

In this section I estimate the model. I examine the limit order book for Google stock (trading on NASDAQ), using nine episodes bracketing the 2009 third quarter earnings announcement by Google. Each episode includes a series of observations of the limit order book, characterizing equilibrium in the limit order market at the time of the episode. Each set of observations are grouped together to form estimates of the model. I obtain a picture of how information asymmetry evolved before and after the announcement.

2.5.1 Earnings Announcements, Asymmetric Information

Earnings announcements convey meaningful information about a company. Markets price an asset consistently with market expectations, but the true profits and losses a company realizes over the quarter may shift these expectations, causing jumps in asset prices. Trading on this information could be very profitable and insiders trading on information could undermine the market. Leading up to an earnings announcement, a Google employee, for example an upper level manager, who may be a significant Google shareholder, could have information about the announcement not yet known to the market. Uninformed institutional traders offer liquidity to this Google employee, keeping in mind that the employee may be trading based on liquidity preferences, e.g., reducing exposure to the asset, or inside information. If institutional traders anticipate heavy information trading, depth of the book will be shallow. However, if they anticipate liquidity trading, supply schedules will become steeper and more depth will fill the book.

There is an important literature studying returns and markets surrounding earnings announcements. Linnainmaa (2010) shows that using limit orders changes the inferences one can make about trading intentions. Examining several regularly identified investor trading patterns, he shows that most of these observed effects are due to the use of limit orders, and that much of the inferences about investors' trading abilities are due to limit orders' exposure to adverse selection risk. Christophe, Ferri, and Angel (2004) use NASDAQ data to examine short-selling prior to earnings announcements, and find a significant link between abnormal short-sales and post-announcement stock returns.[22] Similarly, Kaniel, Liu, Saar, and Titman (2012) consider large individual investor buys and sells on the New York Stock Exchange, and show corresponding abnormal returns following earnings announcements.[23] While these studies analyze price movements and trading volumes, as indirect evidence of information leakage, I use my structural model to directly quantify changes in information asymmetry surrounding an earnings announcement. As I show below, the implications of my results differ from implications one might draw from analyzing price movements and trading volume.

On Thursday, October 15, 2009, Google closed on NASDAQ at $530, well below the Wednesday close of $535.50. That afternoon, then CEO Eric Schmidt made the 3rd quarter earnings announcement, remarking,

> Google had a strong quarter–I saw 7% year-over-year revenue growth despite the tough economic conditions. While there is a lot of uncertainty about the pace of economic recovery, I believe the worst of the recession is behind us and now feel confident about investing heavily in the future.[24]

This was apparently a positive shock to the market; the stock reopened at $546.50 on Friday, Oct. 16, eventually closing at $550, while market returns were flat these days. Next, I

[22] Ball and Brown (1968) were the first to note the link between abnormal returns and unexpected earnings announcements. Foster, Olsen, and Shevlin (1984) replicated these findings regarding abnormal returns, uncovering similar market inefficiencies. Bernard and Thomas (1989) and Bernard and Thomas (1990) show support of the hypothesis of price response delay, rejecting that capital asset pricing systematically underestimates (overestimates) risk surrounding good (bad) news.

[23] Lee (1992) were early to find patterns regarding strategies of large and small investors. Bartov, Radhakrishnan, and Krinsky (2000), and Bhattacharya (2001) find that investor sophistication is negatively correlated with abnormal returns–investors with less sophistication underestimate the implications of a surprise earnings announcement. Battalio and Mendenhall (2005) show that investors making large trades respond to earnings forecast errors, while investors making small trades respond to a less-sophisticated signal. See Hirshleifer, Myers, Myers, and Teoh (2008) for contrasting analysis.

[24] `http://investor.google.com/pdf/2009Q3_earnings_google.pdf`

examine this through the lens of the structural estimates from my equilibrium limit order model.

2.5.2 Estimation Results

I estimate the model on three days surrounding the announcement. The earnings announcement was made 4:30 p.m. EST, Oct. 15, a half-hour after the market closed, and estimates are taken from Oct. 14, 15, and 16, early, midday, and late during trading hours. I characterize information asymmetries and liquidity motivations for trading at each point in time.

Before turning to my structural estimates, I consider typical measures of information asymmetry; specifically, I look at the bid-ask spread and the amount of shares offered in the order book. Figure 2.5 shows an overall downward trend in bid-ask spreads in this three day period, with a spike in spread in the middle of the day following the announcement, consistent with Lee, Mucklow, and Ready (1993).[25] This hints at an overall reduction in information asymmetry during this period. Similarly, Figure 2.5 demonstrates the amount of shares in the first five ticks of the market are low early in the day, high late.

Next I turn to my structural estimates. Results and definitions of terms can be found in Table 2.2. The level of μ_I changes a great deal over this period of time, being lower (and negative, so greater in absolute magnitude, and more hedging preferences for trade) later in the day than earlier, with a peak (and therefore less liquidity preferences for trade) in the middle of the day. However, σ_α changes rapidly, and beginning Oct.14, σ_ϵ is gradually increasing. As σ_α falls and σ_ϵ rises, the ratio of σ_α to σ_ϵ decreases, hence, asymmetric information drops significantly. Information asymmetry drops from Oct. 14 to Oct. 15, continuing to fall later on Oct. 15–as evidenced by the increased hedging incentive revealed in the market at that time in the day. I see this asymmetry fall again on Oct. 16, with a lower point in the middle of the trading day.

[25] Lee, Mucklow, and Ready (1993) show that liquidity providers are sensitive to changes in information asymmetry risk and use both spreads and depths to actively manage this risk.

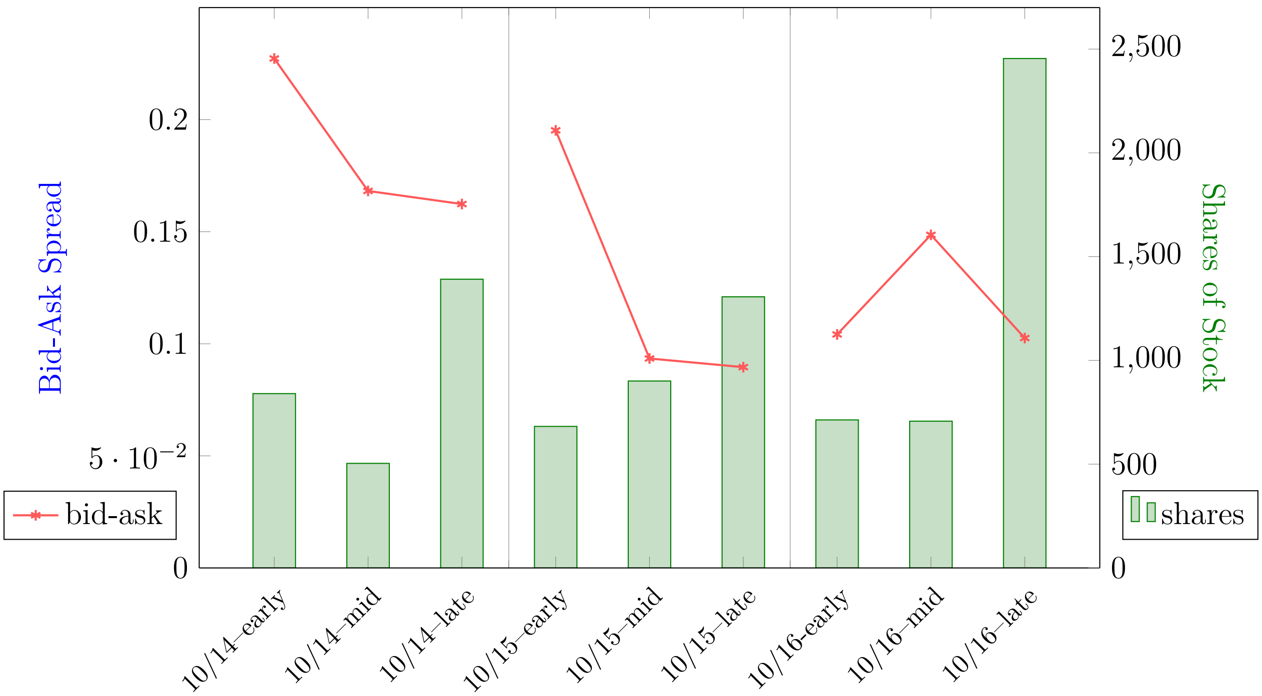
Table 2.2: Earnings Announcements Results

This table shows fitted parameters of the true model for the sell side of the Google limit order book surrounding its 3rd quarter earnings announcement, made 4:30 p.m. EST, October 15, 2009. Parameters are estimated from the first five ticks of the book in a set of three samples, one on each side of a time of day, where samples are separated by 15 minutes. The dates and times chosen are early, midday and late on October 14, 15, and 16. Market open at 9 a.m., each early estimate is taken at 10:00 a.m., EST, unless otherwise specified, midday, 1:00 p.m., and late, 3:30 p.m., market closing at 4 p.m. σ_α is the standard deviation of the informed trader's signal about the asset's true value, and σ_ϵ is the standard deviation of the white noise around the informed trader's signal, both in \$. μ_I and σ_I are the mean and variance of the inventory of the informed trader, in shares of Google stock. σ_{S_0} is the standard deviation of a calibration variable representing hidden orders and impatient sellers, in shares of Google. Bootstrap standard errors in parentheses.

Date-Time	σ_α	$\sigma_\epsilon{}^a$	μ_I	σ_I	σ_{S_0}
10/14 early	0.7260	0.0415	-700.106	199.806	20.927
	(0.0076)	(0.0005)	(2.9226)	(6.8451)	(3.2822)
10/14 mid	0.3367	0.0306	-401.749	111.887	20.005
	(0.0066)	(0.0020)	(5.247)	(3.454)	(0.2524)
10/14 late	0.5720	0.0216	-1598.362	561.992	19.407
	(0.0013)	(0.00004)	(2.352)	(2.667)	(0.2983)
10/15 early	0.4375	0.0254	-947.812	148.192	12.030
	(0.0009)	(0.00009)	(0.5244)	(3.454)	(0.0154)
10/15 mid	0.4274	0.0339	-736.098	230.567	19.861
	(0.0026)	(0.0001)	(2.963)	(3.452)	(0.1252)
10/15 late	0.4571	0.0284	-2169.115	275.685	20.300
	(0.0058)	(0.0002)	(34.372)	(13.408)	(1.1364)
10/16 early[b]	0.6871	0.0481	-912.498	81.159	8.0851
	(0.0006)	(0.00003)	(0.1377)	(0.3464)	(0.0077)
10/16 mid	0.3855	0.0445	-704.534	253.961	20.851
	(0.0177)	(0.0011)	(9.602)	(17.688)	(0.3929)
10/16 late	0.2933	0.0204	-2376.286	725.681	20.496
	(0)	(0)	(0)	(0)	(0)
N[c]	51	–	–	–	–
S[d]	200	–	–	–	–

[a] The coefficient of risk aversion, γ never *stands alone*, meaning that it is estimated directly with σ_ϵ. I assume $\gamma = 1$ and show estimates for σ_ϵ. [b] This time was offset by a total of two (2) fifteen minute periods to 10:30 a.m.. The bid-ask spread was too wide to produce meaningful estimates, and I waited until a point in the day that the market was back in equilibrium. [c] For each set of estimates I take 3 samples. Each sample has a number of instances of the order book. This is the sample size. [d] I use simulations to fit data. I take a certain number of draws of latent supply and find the associated supply curves.

Figure 2.5: Bid-Ask Spread and Depth



2.5.3 Informedness Ratio

To summarize the implications of parameter estimates on the extent of information asymmetry and adverse selection, I introduce the *informedness ratio.* The informedness ratio is defined as the ratio of the standard deviations of the asset value from the liquidity suppliers' perspective and the active trader's perspective. The active trader receives a signal about the asset's true value and the liquidity suppliers do not. Therefore, the ratio is simply the ratio of the standard deviation of the true value from its expectation, and the standard deviation of the white noise the active trader faces in the signal, which is a natural measure of how much less informed the liquidity suppliers are compared to the active trader. The true value of the asset is $v = \alpha + \epsilon$; assuming that the signal α is not correlated with white noise ϵ, the informedness ratio is defined as,

$$\frac{\sigma_v}{\sigma_\epsilon} = \frac{\sqrt{\sigma_\alpha^2 + \sigma_\epsilon^2}}{\sigma_\epsilon} \equiv \text{Informedness Ratio}^{26} \tag{2.13}$$

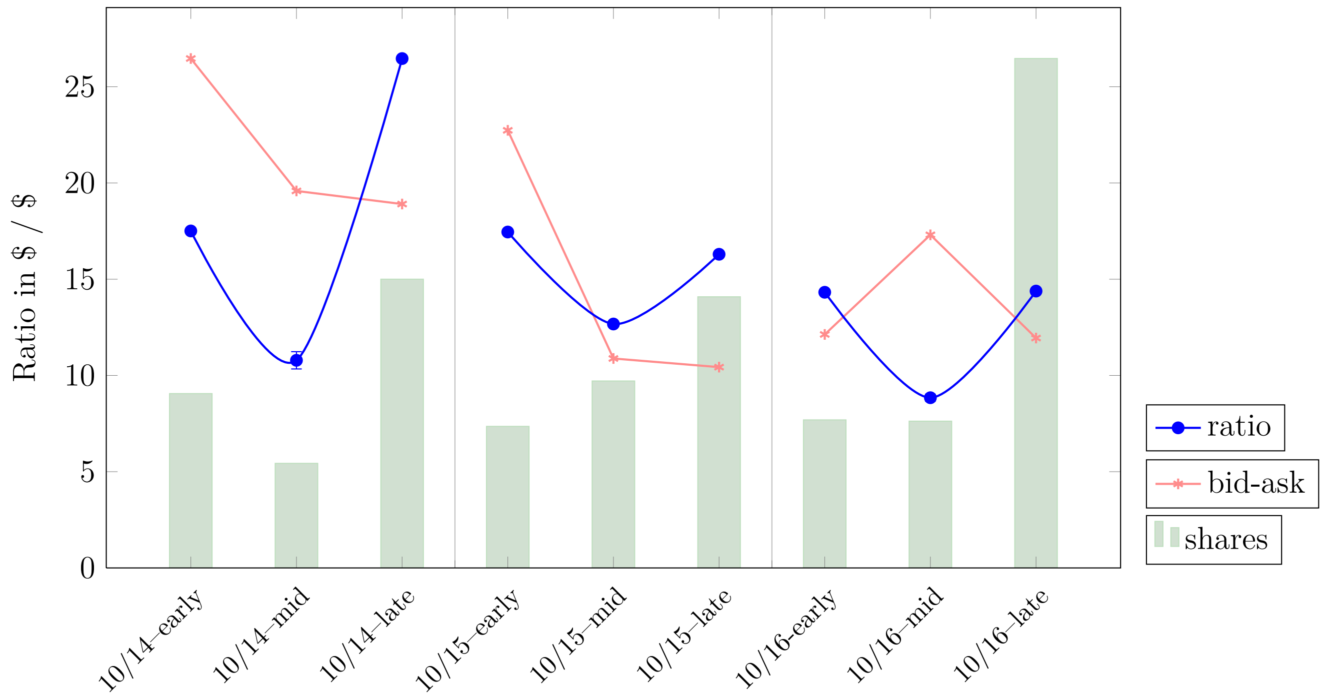
The informedness ratio has no absolute cardinal meaning. However, a higher value may be indicative of large asymmetry in information or a preponderance of information trading, while a lower value indicates more liquidity-motivated trading and hence, less information asymmetry.

The informedness ratio from my results is plotted in Figure 2.6. There is a pronounced downward trend in the informedness ratio over the three day period, implying a reduction in information asymmetry. The informedness ratio is rather high late on Oct. 14. It falls steeply by Oct. 15, settling around 17 that day. This shows that the market responded to asymmetry that is reduced the day of the earnings announcement. Whatever information traders were ready to trade on, prior to the earnings announcement, the market is no longer responding to it once the earnings announcement is made. However, this information asymmetry dissipates further, as evidenced by continued decline in the informedness ratio the day after the earnings announcement is made. It reaches 14 and then 8 after being double that the day before.

This plot indicates that the market behaves as if active traders were better informed (relative to uninformed liquidity suppliers) the day before the announcement, and that this informedness dissipated by the day of the announcement. The fall in the informedness ratio between Oct. 14 late and Oct. 15 early contrasts with the trend in the bid-ask spread, which peaked during Oct. 15 early, as shown in Figure 2.5. Overall, the downward trend in the informedness ratio corroborates a similar trend in the bid-ask spread, however, the informedness ratio implies that the decline in information asymmetry began earlier.

[26] The coefficient of risk aversion, γ never *stands alone*, meaning that it is estimated directly with σ_ϵ. I assume $\gamma = 1$ and show estimates for σ_ϵ.

Figure 2.6: Informedness Ratio



2.6 Robustness Checks: Levels of Competition

In limit order data, individual-level order placement goes unobserved. However, the model specifies a number of traders. I estimate the model using $n = 5$, a choice resulting from a specification search over different values of n.

2.6.1 Comparison of Fit

I show that the market behaves as if there are more liquidity suppliers than two and fewer than ten. If the model were to fit well with $n = 5$ liquidity suppliers, and not as well with $n = 2$ or $n = 10$, this would be evidence the market behaves as if there are effectively five liquidity suppliers competing for the business of active traders. Model fit and estimates for $n = 2$, $n = 5$, and $n = 10$ are compared in Table 2.3.

Table 2.3: Robustness Checks

This table shows comparative fitted parameters of the true model for market structures with two, five, and ten liquidity suppliers for the sell side of the Google limit order book surrounding its 3rd quarter earnings announcement, made 4:30 p.m. EST, October 15, 2009. The market for Google opens at 9 a.m., EST, and unless otherwise noted, early refers 10:00 a.m., mid refers to 1:00 p.m., and late refers to estimates being made for data at 3:30 p.m., market closing at 4:00 p.m. σ_α is the standard deviation of the informed trader's signal about the asset's true value, and σ_ϵ is the standard deviation of the white noise around the informed trader's signal, both in \$. μ_I and σ_I are the mean and variance of the inventory of the informed trader, in shares of Google stock, where greater magnitude means more liquidity preferences. σ_{S_0} is the standard deviation of a calibration variable representing hidden orders and impatient sellers, in shares of Google. Fit shows the mean-squared error of the estimates, and a lower number is a better fit. The model fits better with number of liquidity suppliers being five rather than two, hence markets behave as if they have five dealers, as opposed to two. This is evidence that modern limit order markets are more efficient than hypothetical dealer markets–characterized by one, or maybe two dealers–would be. Standard errors in parentheses.

	n	fit	σ_α	σ_ϵ	μ_I	σ_I	σ_{S_0}
10/14 late	2	52,244.2	0.4253	0.0391	-2633.938	814.957	20.607
	5	7651.4	0.5720	0.0216	-1598.362	561.992	19.407
	10	10,863.5	0.2794	0.0227	-527.943	176.344	17.314
10/15 early	2	214.0	0.3754	0.0579	-1454.298	415.548	20.388
	5	6.0	0.4375	0.0254	-947.812	148.192	12.030
	10	55.4	0.6434	0.0698	-285.119	190.829	22.548
10/15 mid	2	6296.4.8	0.3608	0.0436	-1853.380	542.495	20.210
	5	90.0	0.4274	0.0339	-736.098	230.567	19.861
	10	367.5	0.3772	0.0327	-366.799	105.985	20.116
10/15 late	2	95,737.8	0.2632	0.0209	-2193.646	265.143	20.225
	5	83.9	0.4571	0.0284	-2169.115	275.685	20.300
	10	297.5	0.1925	0.0433	-692.216	294.226	20.538
10/16 early[a]	2	9631.1	0.4286	0.0424	-2361.003	683.236	20.436
	5	687.7	0.6871	0.0481	-912.431	80.932	8.004
	10	581.4	0.2484	0.0214	-463.436	137.343	19.778

[a] This time was offset by a total of two (2) fifteen minute periods to 10:30 a.m.. The bid-ask spread was too wide to produce meaningful estimates, and I waited until a point in the day that the market was back in equilibrium.

Table 2.3 contains fitted θs for five dates and times examined earlier. I also include θs for $n = 2$ and $n = 10$ to contrast specification fits. The model fits are better in four of the five dates and times. Market data is fitted by the model consistently better for $n = 5$ over $n = 2$ with only one instance of a fit worse than $n = 10$.

2.6.2 Comparing Limit Order Markets and Dealer Markets

The model I estimate fits better with $n = 5$ than it does with $n = 2$ or $n = 10$. Looking at $n = 2$, corresponds to estimating a dealer market. I investigate the change in welfare moving from dealer to limit order markets. Historically, NASDAQ, New York Stock Exchange, and others, depended on dealers or market makers to facilitate the market for an asset. More recently, modern markets have employed a direct trading scheme in the form of a limit order market. In dealer markets, traders never directly interact. Instead, a trader would make known to a dealer that she was interested in selling shares of a stock. The dealer would post terms for trade, and the trader would sell as much as she wished according to those terms. The dealer would then sell those shares on the other side of the market. This way a trader could always trade and never had to wait for terms to be met as long as the dealer was there to take the other side of any transaction. Limit order markets are centralized, but trade need not go through a single intermediary.

While the move from dealer markets to limit order markets is pervasive, it is not clear they are better in terms of welfare. It may be the case that while limit order markets bypass the middleman, that liquidity is more available under a dealer market scheme. However, it may be that the ability of any trader to play the role of dealer allows for greater liquidity provision.

The difficulty of characterizing the limit order book theoretically makes this welfare question very difficult to analyze. Pagnotta (2010) presents a computational model that nests dealer and limit order markets, allowing for a comparison of utilities. Finding that welfare is not noticeably better by adding a dealer, he deduces that the limit order market does better than its historical counterpart.

The model estimated here also nests dealer and limit order markets, and is amenable to a welfare comparison of the two. Because active traders and liquidity suppliers trade with each other, the model is essentially a dealer market. If liquidity suppliers offer a large number of shares early, then the informed traders they face at higher prices in the book will likely have a strong signal about the price of the asset, decreasing expected profits. In a monopolistic setting, a liquidity supplier bears the entirety of this loss, and so will offer liquidity at higher prices, leading to a large bid-ask spread. However, adding liquidity suppliers increases incentives to offer liquidity at low prices. With a large number of suppliers, there is close to efficient liquidity provision and a small bid-ask spread. I showed earlier that the model fit is better for $n = 5$ than it is for $n = 2$. I demonstrate here that increasing the number of liquidity suppliers in the market raises welfare. Because a large number of dealers is unambiguously good, and the model fits better for $n = 5$, data confirms Pagnotta's results; limit order markets are better for welfare than dealer markets.

2.6.3 Counterfactual Utilities

I calculate profitability for liquidity suppliers and certainty-equivalent utility for the active trader. These values will be useful in considering counterfactual utilities, comparing social welfare in dealer and limit order markets. Interested readers can find relevant calculations in the appendix, 4.3.

I generate counterfactual markets, varying the number of liquidity suppliers $n = 1, \ldots, 50$, ranging from a monopoly to relative competition. Using these simulated markets, I compute expected profits to liquidity suppliers and the certainty-equivalent utility increase for active traders. Table 2.4 presents counterfactual estimates for five dates and times examined in the earnings announcements section. For each example, profits to liquidity suppliers go down as the number of competitors increases. Conversely, certainty-equivalent utility goes up for active traders as the number of liquidity suppliers competing for their business increases.

Comparing profits and utilities in absolute number is misleading because of the nature of the active trader's exponential utility. However, profits to liquidity suppliers are a transfer of utility from the active trader, so relative changes in the sum of utilities matter. Moving from a more monopolistic setting to a more competitive one is better for the market for this asset. Hence, I show with data that limit order markets are more efficient than their dealer market predecessors.

Comparing certainty-equivalent utility gain to the active trader at different points in time, the active trader sees a much lower gain Oct. 14 late than Oct. 16 early, despite volatility being similar, 0.5724 and 0.6888, respectively. At both points in time, active traders are exposed to a large amount of risk in the volatility of the asset. The informedness ratio is high Oct 14 late, 26.463, compared to 14.320 on Oct 16 early, so liquidity suppliers are offering a less favorable supply schedule. The increased hedging active traders can do on Oct 16 early is due to reduced asymmetric information in the market. Similarly, on Oct. 15 late, there is an even greater gain to liquidity suppliers and active traders. Despite volatility being relatively high, 0.4601, the informedness ratio was 16.295, and there was a large amount of depth in the book. Many traders were posting liquidity, leading to steep supply curves, allowing for a large amount of hedging needs to be met.

Following robustness checks, I determined that the model fit data better with $n = 5$ than specifications with fewer or more liquidity suppliers.[27] This number represents the oligopolistic incentives liquidity suppliers face in the limit order market. The limit order market does not operate as a duopoly, nor does it operate under perfect competition. In addition, counterfactual utilities show that competition among liquidity suppliers is unambiguously good for welfare. The model specified with $n = 2$ is essentially a dealer market. Moving from this hypothetical dealer market to $n = 5$, the robust specification for estimating the data, increases counterfactual utilities. Hence, data confirms Pagnotta's result that moving from

[27]While there may be settings in which a value of $n > 5$ would be appropriate, it is clear that n is at least larger than $n = 2$, evidence limit order market structure is more efficient than dealer markets.

Table 2.4: Counterfactual Profits and Utilities

This table shows counterfactual profits to liquidity suppliers and utility to informed active traders across varied market structures. I study here the sell side of the limit order book for Google stock on dates October 14, 15, and 16, surrounding the 3rd quarter earnings announcement by Google in 2009. The market for Google opens at 9 a.m., EST, and unless otherwise noted, early refers 10:00 a.m., mid refers to 1:00 p.m., and late refers to estimates being made for data at 3:30 p.m., market closing at 4:00 p.m. Liquidity suppliers are considered the uninformed dealers while the informed active trader takes liquidity from the limit order book. The number of suppliers is varied from 1 (monopoly) to 50 (relative competition). Total profits are listed for liquidity suppliers and decrease with competition, whereas certainty-equivalent utility as defined for the informed active trader increases with competing liquidity suppliers. Because profits to liquidity suppliers are zero-sum, this is evidence that more competition, and so more liquidity suppliers, in these markets leads to higher levels of efficiency. Bootstrap standard errors are in parentheses.

		Number of Suppliers					
		1	2	5	10	20	50
10/14 late	suppliers	1.525	0.8029	0.3480	0.1887	0.1185	0.0691
	active trader	70.043	80.275	79.311	82.383	84.483	85.287
10/15 early	suppliers	27.034	11.739	4.172	2.357	1.260	0.6383
	active trader	61.843	121.451	141.205	145.073	147.294	148.882
10/15 mid	suppliers	29.129	11.999	3.291	1.556	0.8809	0.4941
	active trader	163.773	198.182	205.612	210.075	211.387	214.921
10/15 late	suppliers	134.179	49.909	25.937	20.708	16.926	14.156
	active trader	1385.844	1527.734	1541.288	1574.801	1596.956	1610.083
10/16 early[a]	suppliers	62.334	24.417	7.341	4.339	2.437	1.245
	active trader	634.427	693.385	707.728	707.093	718.011	723.144

[a] This time was offset by a total of two (2) fifteen minute periods to 10:30 a.m.. The bid-ask spread was too wide to produce meaningful estimates, and I waited until a point in the day that the market was back in equilibrium.

dealer markets to limit order markets increases welfare.

2.7 Conclusion

I develop a method of estimating an equilibrium model of the limit order book using market data.

As an application, I examine the market for Google around the time of an earnings announcement. The informedness ratio, a model-consistent measure of information asymmetry, begins decreasing even before the earnings announcement. This is consistent with the notion that some information related to the announcement was already present in the market before the announcement itself. A second application examines the welfare implications of dealer vs. limit order markets.

My method lends itself to a number of further applications. Initial public offerings involve important market structure changes over time. In addition, mergers would be an interesting phenomenon to investigate. These I leave for future research.

Chapter 3

Corruption and Community-Driven Development

3.1 Introduction

In 2006, donors contributed \$4.5 B–19% of all donor money–to developing countries through projects to be carried out using a bottom-up, decentralized mechanism (Lessmann and Markwardt (2010)). In its idealized form, decentralized aid, or Community-Driven Development, gives target villages the power to implement and make decisions regarding aid projects. Projects are designed to depend on grass roots movements for their development and production, increasing competition between bureaucrats and allowing inter-regional bidding for project funds (see Arikan (2004)). If bureaucrats are more accountable, officials will face difficulty in extracting rents. However, the literature finds that such an aid tool is at times far from ideal in practice. I present a model describing a possible corrupt community-driven development project. I offer a game theoretic representation of how targeted individuals in a community could receive aid while capture is taking place, and what problems may exist in the incentive structure for corrupt government agencies.

For background, I describe an example of a community-driven development aid disbursement process as administered by the World Bank. This example comes from Ensminger (2012).[1] To begin with, the World Bank distributes money to the government of a devel-

[1]For a thorough discussion of Community-Driven Development in all its possible forms, see Dongier,

oping country. The government establishes regional or district offices, reviewing community proposals for aid, while also being responsible for educating villages about possible aid programs and the procedure for obtaining a project.[2] Possible projects are discussed by the villages and a proposal is approved by majority vote. Subsequently the village elects a committee which takes the proposal to the district for approval. Given authorization, project funds are often matched to some degree[3] by the village while the project committee receives control of the account from the district for the remaining money. The committee is to leave a balance in the donor bank account for subsequent projects. At the district and village level these projects undergo announced monitoring by the World Bank and project donors. Results motivate distribution of funds at the project level. Projects often involve building construction, increasing the numbers of livestock, or improving agricultural practices.[4]

Community-Driven Development gives the opportunity to motivated individuals to design and carry out their own aid projects, benefitting their own communities. While this has many positive possible outcomes in terms of efficiency, it leaves open some questions regarding proper implementation. Without a central authority overseeing all stages of the project, it is unclear how much donors can affect elite control, project choice, and aid outcomes. I discuss how typical models of corruption fail to illustrate the way malfeasance could encroach on disbursement to intended beneficiaries.

World Bank and donor monitoring means to empower villages to address their own problems. Monitoring visits are announced and they are infrequent, but this is not any different than a setting such as Becker and Stigler (1974).[5] Success in Becker and Stigler (1974) is

Van Domelen, Ostrom, Ryan, Wakeman, Bebbington, Alkire, Esmail, and Polski (2003).

[2]The review of community proposals takes on several forms, as this is just one. Additionally, many CDD projects require some educating of communities about developing aid projects.

[3]A variety of programs exist, but the core concept is matching funds by the village. In the case study by Ensminger (2012), all projects require that the village raise 30% of project funds on its own, amounting to a total of 130% of donor money (in the form of donor and village funding) withheld for the project.

[4]This list is by no means exhaustive. It is important to note that projects may address varying needs within communities.

[5]In Becker and Stigler's paper, the government or firm decides on an appropriate pension given the probability of detection and the benefits of malfeasance. This reward is given to employees upon successful completion of their tenure.

defined as never being caught in corrupt dealings by a government, firm or other central authority. In a decentralized aid problem, what would the central authority be? While elite capture is by no means strictly pervasive (see Dasgupta and Beard (2007), Rao and Ibanez (2005)), elite control of projects is (Fritzen (2007)). Elites therefore have some control over the project's likelihood of detection for various forms of capture (Shleifer and Vishny (1993)). This disempowers the central authority because project details are not controlled by donors. I thus argue that the Becker-Stigler setting, with a centralized monitoring authority, may not best describe the incentives involved in Community-Driven Development. I model those incentives, showing that project outcomes–some amount of aid money reaching target individuals–are not enough to rule out elite capture.

In their seminal 1974 paper, Becker and Stigler formulated a principal-agent model for addressing corruption. Fundamental to their model is a firm or government capable of detecting and punishing malfeasance.[6] While this framework is effective in the deterrence of graft, petty or otherwise, it depends crucially on a central monitoring authority which may be undermined by the decentralized nature of Community-Driven Development–because decentralization strengthens power for local elites, it might therefore increase corruption and cronyism (see Bardhan (2002) and Bardhan and Mookherjee (2006)). Namely, donor and World Bank monitoring may be ineffective in detecting malfeasance due to choices in procurement project choice and implementation, (see Shleifer and Vishny (1993)).[7]

[6] Becker and Stigler model personal corruption, corruption benefitting the individual directly, as opposed to official corruption, corruption explicitly or implicitly required by the principal (Banfield (1975)). This is an important distinction within decentralized aid. While official corruption may be an important component in a model of Community-Driven Development, that is beyond the scope of this paper. Banfield (1975) also argues that project managers have incentive to reduce detection of malfeasance through coersion. However, Rose-Ackerman (1975) focuses on market structure and bribery of bureaucrats in the government contracting process.

[7] Lui (1986) produces an overlapping generations model in which as corruption becomes more common, it becomes more difficult to detect corruption effectively. Of course, this result hinges on the assumption that it becomes more costly to monitor as the prevalence of corruption goes up. Although, secrecy in government decisions down to the district level likely does increase the costs of effective monitoring. Basu, Bhattacharya, and Mishra (1992) look at corrupt officials being caught and possibly bribing the official punishing them, what is referred to as "recursive corruption." They find that even ineffective policies may play a role in the deterrence of corruption. Alam (1995) looks at the countervailing actions the victims of corruption may take to avoid losses. Alam comments, "It is time to take the victims of corruption more seriously."

There is a literature cautioning against ramping up CDD for these reasons. Platteau and Gaspart (2003) and Platteau (2004) argue that the only way to prevent corruption is an active role played by the purveyors of money.[8] Similarly Lessmann and Markwardt (2010) and Lessmann and Markwardt (2012) argue that decentralized aid works in nations that are heavily centralized already.[9] The Becker and Stigler setting does not effectively describe decentralized aid, and asking questions about malfeasance requires a different theoretical investigation.

Empirical work has shown that CDD projects have positive outcomes, but it is unclear what this criterion means. Dasgupta and Beard (2007) perform four case studies in Indonesia, but measures of effectiveness are qualitative and include no data on the amount of money reaching targeted individuals. Rao and Ibanez (2005) obtain survey data surrounding aid projects in Jamaica, and find that 80% of members express satisfaction with aid outcomes. It would be difficult to determine the efficiency level to which an 80% satisfaction rate corresponds.[10]

I present a model of the decentralized aid process without a central monitoring authority. In addition, a level of project money is disbursed to targeted individuals, qualitatively in

[8]Specifically, Platteau and Gaspart (2003) consider community-driven development mechanisms and argue that recent goals of ramping up CDD aid need to be tempered with a so-called Leader-Disciplining Mechanism. The authors argue that administrators must be patient and adhere to a sequential disbursement procedure supported by a fraud detection mechanism to reduce capture. Platteau (2004) discusses several methods to achieve disbursement to intended benficiaries. Platteau also highlights multilateral disbursement mechanisms as an effective tool for reducing elite capture.

[9]Lessmann and Markwardt (2010) investigate empirically the effectiveness of keeping corruption at bay. The authors find that preventing elite capture requires central monitoring of bureaucrats' behavior in some form. They argue that a free press is a necessary, but not sufficient, condition for successful implementation of community-driven development. They find that a high degree of freedom of the press results in decentralization counteracting corruption. Lessmann and Markwardt (2012) investigate the effectiveness of community-driven development in centralized vs. decentralized countries and find that such aid programs are effective in centralized nations. However, they find that decentralized aid programs are ineffective or even harmful in decentralized countries.

[10]Dasgupta and Beard (2007) examine community-driven poverty alleviation projects in Indonesia for four neighborhoods with contrasting social orders and elite control. The authors find that elite capture is not prevalent, although elite control of projects is. Fritzen (2007) supports this work by following up in Indonesia with a series of surveys given out to elites and project managers, and finds that projects can be influenced by project-related accountability arrangements. Rao and Ibanez (2005) look at Jamaican social investment fund projects and find that 80 percent of community members express satisfaction with the outcome of projects, but highlight that better educated and better networked individuals dominate the aid process.

agreement with evidence in Dasgupta and Beard (2007) and in Rao and Ibanez (2005). Despite significant elite capture, the levels–as presented in example–are similar to those found in a case study by Ensminger (2012).

3.2 Model

3.2.1 Splitting a Dollar in Aid

I model a corruptible Village-Committee-District relationship with three players. The game involves the splitting of a dollar into three non-negative shares

$$m_D + m_C + m_V = 1. \tag{3.1}$$

The model discussed here presents a new ultimatum game in which players act successively and punishment is determined by the player at the end of the line. This is related to the three-person ultimatum games literature, specifically Oppewal and Tougareva (1992), Riedl and Vyrastekova (2003), and Shupp, Schmitt, and Swope (2006). While the phenomenon of an intermediary between first proposer and final receiver could be represented generally in an "n-person" successive ultimatum game, the most basic representation requires three players. In period 1 the District is in charge of making the first split of the dollar into m_D and $1-m_D$. The Committee subsequently determines the portion of $1 - m_D$ to send to the Village, m_V, and the amount to keep $m_C = 1 - m_D - m_V$. I assume that any precommitment by the Committee and District to some sort of splitting strategy is not directly enforceable. The Village receives utility $w \sim U[0, \alpha]$ for blowing the whistle on the Committee and District, making the decision whether to blow the whistle or not based on the amount of funds it receives.[11] If the Village blows the whistle to the central government (subsequently HQ)[12], then HQ performs an monitoring visit of the District and the District pays c.[13] The District never deals with the village again in subsequent periods, meaning payoffs of zero for all players forever.[14] Each District deals with a certain number of Villages. I assume that there is no outside option to working with one Village. Each District is working to the capacity of its region. If the Village chooses not to blow the whistle, then HQ has no reason to peform a costly monitoring visit and I assume that the District, Committee and Village receive a continuation payoff forever. The continuation payoff is discounted by a respective discount factor $\delta_D, \delta_C, \delta_V$ and is at the same level as the first period split.

I consider singleton actors at each level. In assuming singleton actors, I am considering that dissention at any one of these levels is debilitating. The existence of a District, let alone a Committee, is tenuous. HQ has full authority over each District, and the District can approve or disapprove of any proposal a Committee makes. If individuals at each level do not cooperate, they will falter. I leave deliberation by individual actors for further investigation.

Considering that $0 < \delta_V, \delta_C, \delta_D < 1$, I have that nondegenerate equilibrium payoffs for

[11] Ensminger (2012) highlights the fact that whistle blowing *does* happen in practice. In modeling the aid process, I am agnostic about the direct benefits to Village. There may be a variety of economic reasons for blowing the whistle, from feeling cheated, to achieving a payoff by corrupt individuals (this is described in Ensminger (2012)). The important facets here are that there is whistle blowing and that it is responsive to environment.

[12] Here, HQ is still the corrupt government bureau I discussed above. It is important to note that in maintaining secrecy, each level has something to lose if a level lower down makes malfeasance clear to outsiders. This notion is highlighted in Banfield (1975).

[13] I could model c as the costs of maintaining secrecy or stopping whistle blowing before it gets to a higher level than the District, however I am agnostic about the origins of c. I assume the Village has some way to hurt the District. The punishment could be handed down by the World Bank, by a higher level of government, by individual officials, or it could be a payoff to upset targeted individuals (as I discussed in a footnote above).

[14] This assumption may seem stark. However, the District may have limited means to continue dealing with a community (Committee *and* Village) following a whistle blow. While this may not describe every project, Ensminger (2012) notes that this does happen in practice, that Villages rarely see project money again after blowing the whistle.

the Village, Committee, and District are,

$$m_V^* = \frac{1}{4} + \left(\frac{1-\delta_D}{4}\right)c - \alpha\left(\frac{1-\delta_V}{\delta_V}\right)\left(\frac{1-\delta_D}{2\delta_D}\right) - \alpha\left(\frac{1-\delta_V}{\delta_V}\right)\frac{1}{4}\left(\frac{1-\delta_C}{\delta_C}\right) \tag{3.2}$$

$$m_C^* = \frac{1}{4} + \left(\frac{1-\delta_D}{4}\right)c - \alpha\left(\frac{1-\delta_V}{\delta_V}\right)\left(\frac{1-\delta_D}{2\delta_D}\right) + \alpha\left(\frac{1-\delta_V}{\delta_V}\right)\frac{3}{4}\left(\frac{1-\delta_C}{\delta_C}\right) \tag{3.3}$$

$$m_D^* = \frac{1}{2} - \left(\frac{1-\delta_D}{2}\right)c + \alpha\left(\frac{1-\delta_V}{\delta_V}\right)\left(\frac{1-\delta_D}{\delta_D}\right) - \alpha\left(\frac{1-\delta_V}{\delta_V}\right)\frac{1}{2}\left(\frac{1-\delta_C}{\delta_C}\right) \tag{3.4}$$

Details of these derivations can be found in the Appendix, sections 3.5.1, 3.5.2, and 3.5.3. It is important to note that $\alpha\left(\frac{1-\delta_V}{\delta_V}\right)$ represents the propensity to whistle blow by the Village. A lower value, and the Village is more patient, and has less incentive[15] to whistle blow. A higher value, and the Village is more willing to blow the whistle, upsetting future project implementation.

3.2.2 Equilibrium Existence

I look for the conditions such that m_V^*, m_C^*, m_D^* all to be greater than 0.

Definition 1. *A Positive Payoff Equilibrium is an equilibrium in best responses in which* $m_V^*, m_C^*, m_D^* > 0$.

Theorem 1. –Existence of a Positive Payoff Equilibrium

Given that blowing the whistle benefits the Village $w \sim U[0, \alpha]$, $0 < \alpha \leq 1$, costing the District $c > 0$, and the Village, Committee, and District discount future payoffs according to $0 < \delta_V < 1$, $\frac{c}{2+c} < \delta_C < 1$, $0 < \delta_D < 1$, then there exists a positive payoff equilibrium iff

$$\left[\frac{1+(1-\delta_D)c}{\left(4+\frac{1-\delta_C}{\delta_C}+\frac{2-2\delta_D}{\delta_D}\right)}\right] < \alpha\left(\frac{1-\delta_V}{\delta_V}\right) < \left[\frac{1+(1-\delta_D)c}{\left(\frac{1-\delta_C}{\delta_C}+\frac{2-2\delta_D}{\delta_D}\right)}\right]. \tag{3.5}$$

The proof can be found in the appendix, 3.5.4.

[15]Recall here that benefit, $w \sim U[0, \alpha]$.

The inequalities in (3.5) highlight an important intuition behind my equilibrium result. To receive positive payoffs, a Village needs to be sufficiently patient ($\alpha\frac{1-\delta_V}{\delta_V}$ low enough), but also have enough of a desire to blow the whistle ($\alpha\frac{1-\delta_V}{\delta_V}$ high enough) to hold the other players accountable to a positive payoff equilibrium. If the Village were unable to punish or uninterested in punishing malfeasance, the Committee would fully abuse its power, keeping all the funds it receives from the District. The District would anticipate this and keep all the money to itself. Conversely, if the Village were too ready to report malfeasance, the expected continuation payoffs would be so low as to disincentivize any investment in those future payoffs. The Committee would keep all its money, meaning the District wouldn't hand any down the chain.

For some Village profiles, despite elite capture of a large share of project funds, the Village still receives a positive payoff. While several studies interpret the qualities of Community-Driven Development and the outcomes due to elite capture, the measures of effectiveness are coarse in nature. Dasgupta and Beard (2007) examines four neighborhood projects in Indonesia and find that target individuals receive some level of the intended funds. But it is unclear how much of the money achieves proper disbursement. Fritzen (2007) examines Dasgupta and Beard's work using multi-district surveys of those implementing the aid process, but this lends little, if any, evidence of the exact effectiveness of aid practices. Rao and Ibanez (2005) investigates attitudes of target households in a case study of the Jamaica social investment fund, finding that 80% of the community expresses satisfaction with aid outcomes.[16] This, the authors claim, is evidence of limited elite capture. It may be unclear, however, how a positive response to an aid project relates to levels of disbursement to intended beneficiaries. Ensminger (2012) makes an exhaustive case study of a particular aid project in Kenya, determining that capture is prevalent, despite some money reaching tar-

[16]This level is despite a significant majority of households reporting prior to implementation dissatisfaction with possible aid projects. This is possibly further evidence that an 80% satisfaction rate may be misleading about overall effectiveness of an aid project. Are households happy to receive any money at all? Are aid project participants conditioned to respond positively about a project for fear of losing any future project money? These and other possibilities may arise.

geted individuals (and, it is argued, not in an effective manner). Dasgupta and Beard (2007) and Rao and Ibanez (2005) argue that elite capture is mitigated by a variety of endogenous mechanisms, contrasting heavily with the findings in Ensminger (2012). My model shows that elite capture can exist with seemingly useful aid projects, and that these studies may be studying the same phenomenon: noticeable portions of funds reaching target villages amidst substantive elite capture.

I illustrate this phenomenon through an example. In my model, the propensity for a Village to blow the whistle must be at the right level for a positive payoff equilibrium. Assuming some relatively impatient Village and Committee (community members), a patient District, and a penalty to the District for malfeasance,[17]

$$\delta_V = \delta_C = \frac{1}{2},\ \delta_D = \frac{7}{8},\ c = 1$$

α must be between $\dfrac{63}{296} \sim \dfrac{1}{5}$ and $\dfrac{7}{8}$ (as in (3.5)). Choosing $\alpha = \frac{1}{4}$, it is clear from (3.2), (3.3), and (3.4), that,

$$\begin{aligned} m_V^* &= \frac{39}{224} \sim 18\% \\ m_C^* &= \frac{341}{672} \sim 50\% \\ m_D^* &= \frac{107}{336} \sim 32\% \end{aligned}$$

In the example, the community (Committee and Village together) receive $\dfrac{7.5}{11} \sim 68\%$ of money, while disbursement to targeted individuals remains nearly a fifth of total project funding. These numbers approximate positive disbursement, possibly substantial enough to be consistent with observations in Dasgupta and Beard (2007) and Rao and Ibanez (2005). The numbers do fall in line with break downs found in Ensminger (2012). This model shows how a positive amount of money can reach the Village without a central authority. The out-

[17] Patience is related to project-to-project payoffs. Projects could span several years, if not longer, so it may be reasonable to assume that a Village or Committee discount heavily future project funds.

comes of this model are consistent with the finding of elite capture in Bardhan (2002), and Bardhan and Mookherjee (2006). While this possibility is not debilitating to Community-Driven Development, it shows a reason for substantive donor oversight on aid projects. A call for increased oversight is consistent with cautionary research by Lessmann and Markwardt (2010) and Lessmann and Markwardt (2012) and suggestions for implementation of community driven development found in Platteau and Gaspart (2003) and Platteau (2004). I show through this result that while there are several mitigating factors which may endogenously reduce elite capture, that *success* of an aid project may be too coarse a qualitative assessment, and that the decentralized aid process makes detection of elite capture more difficult than previously considered.

3.3 Relative patience and comparative statics

3.3.1 Village patience and whistle blowing

The Village propensity to whistle blow is described by the term $\alpha\dfrac{1-\delta_V}{\delta_V}$. The higher this term is, the more willing the Village is to blow the whistle, in expectation. One might expect that a higher propensity to blow the whistle means a greater take for the Village (as the Committee and District attempt to keep the Village quiet). However,

$$\frac{\partial}{\partial\alpha\frac{1-\delta_V}{\delta_V}} m_V^* = -\frac{1-\delta_D}{2\delta_D} - \frac{1-\delta_C}{4\delta_C} < 0 \tag{3.6}$$

Counterintuitively, within a *positive payoff equilibrium*, a Village with a higher incentive to blow the whistle is going to command a smaller portion of funds. The intuition is clear. With a lower $\alpha\dfrac{1-\delta_V}{\delta_V}$, the expected overall pie is bigger. That is, if the Village is more patient, each player has more to benefit from in later periods, in expectation. As a Village becomes less patient, the Committee and District are less willing to part with first period funds because investment in continuation payoffs has a lower return.

Considering Committee patience,

$$\frac{\partial}{\partial \delta_C} m_C^* = -3 \frac{\partial}{\partial \delta_C} m_V^* = -\frac{3}{2} \frac{\partial}{\partial \delta_C} m_D^* = -\alpha \left(\frac{1-\delta_V}{\delta_V} \right) \frac{3}{4\delta_C^2} < 0 \tag{3.7}$$

which implies that increased Committee patience yields a lower payoff for the Committee and a higher payoff for the Village and the District. Intuitively, the Village will receive more in equilbrium from the Committee. Anticipating this, the District will keep more of the dollar.

For the District,

$$\frac{\partial}{\partial \delta_D} m_D^* = -2 \frac{\partial}{\partial \delta_D} m_V^* = -2 \frac{\partial}{\partial \delta_D} m_C^* = \frac{c}{2} - \alpha \left(\frac{1-\delta_V}{\delta_V} \right) \frac{1}{\delta_D^2} \tag{3.8}$$

which is > 0 when $\delta_D > \sqrt{\frac{2(1-\delta_V)\alpha}{c\delta_V}}$. In other words, if $\frac{2(1-\delta_V)\alpha}{c\delta_V} < 1$, a patient enough District will give more money to the Committee, which in turn passes more on to the Village, because this would mean a higher continuation payoff.

Naturally,

$$\frac{\partial}{\partial c} m_V^* = \frac{\partial}{\partial c} m_C^* = -\frac{1}{2} \frac{\partial}{\partial c} m_D^* = \frac{1-\delta_D}{4} \tag{3.9}$$

a higher c results in higher payoffs for the Committee and the Village because the District is trying to avoid paying the penalty to HQ.

Comparative statics are Summarized in Table 3.1. As can be seen in the table, a change in propensity to whistle blow has ambiguous effects on payoffs for the Committee and District. I show in the appendix, 3.5.5, that the more patient player gets more in equilibrium.

Within existence bounds, individual patience unambiguously helps the Village and hurts the Committee. The District either suffers or benefits from more patience, depending on their valuation of the continuation payoff. The District has a different problem here because it must anticipate the choice of the Committee. However, one would expect that a more

Table 3.1: PPE Comparative Statics

	PPE Payoffs		
	m_V^*	m_C^*	m_D^*
$\dfrac{\partial}{\partial \alpha \frac{1-\delta_V}{\delta_V}}$	$\downarrow$	?	?
$\dfrac{\partial}{\partial \delta_C}$	$\uparrow$	$\downarrow$	$\uparrow$
$\dfrac{\partial}{\partial \delta_D}$	?	?	?
$\dfrac{\partial}{\partial c}$	$\uparrow$	$\uparrow$	$\downarrow$

impatient party would be paid off more to keep them from, in the Committee's case, taking all the funds, and in the Village's, blowing the whistle. In the Village's case, I see that more patience is bad for their single period payoffs. This is explained by the fact that a higher propensity to blow the whistle means a lower expected continuation payoff and therefore less incentive for those higher on the money chain to pass down first period funds. The way the funds are kept from the Village depends on the relative patience of the District and Committee.

3.4 Conclusion

Community-Driven Development is designed to improve project proposal and implementation through grass roots movement and competition among bureaucrats. A typical investigation of corruption involves a firm or government agency interested in preventing employees or officials from abusing their postions. Here I do not assume such a heirarchy, choosing to model the process in a decentralized manner.

My model involves three players, Village, project Committe, and District, splitting a dollar and acting selfishly. I remain agnostic about the origins of costs targeted individuals can impose on higher powers, but my model shows that there are parameters under which projects could be have positive payoffs. I make the subtle but important point that detection of elite capture could be more complicated than previously considered. Namely, project funds could achieve disbursement to targeted individuals while implementers capture much of the intended aid money. This means that projects could appear to be functioning (as claimed in Dasgupta and Beard (2007), and Rao and Ibanez (2005)) while project implementers capture a significant portion of funds (as presented in Ensminger (2012)).

My model is a start for analyzing this problem at the district level. While my description of the Community-Driven Development aid process may not describe all settings, it gives a feasible example for which coarse measures of project implementation effectiveness (that money reaches targeted individuals at all) are ineffective at detecting malfeasance.

3.5 Appendix

3.5.1 V - Village

$w \sim U[0, \alpha]$, $0 < \alpha \leq 1$, $0 < \delta_V < 1$

I let the Village blow the whistle be represented by $d(m_V, w) = 1$ and not blowing the whistle represented by $d(m_V, w) = 0$. As such,

$$u_V(m_V) = \begin{cases} m_V + \dfrac{\delta_V}{1-\delta_V} m_V & \text{if } \ d(m_V, w) = 0 \\ m_V + w & \text{if } \ d(m_V, w) = 1 \end{cases} \tag{3.10}$$

A mixed strategy is dominated as follows: If m_V is passed on to the Village, the choice of Village share by Committee after $1 - m_D$ decision by District, then let $\sigma_{\text{whistle blow}}(w)$ be the mixed strategy of blowing the whistle. Once the Village realizes w, either $\frac{\delta_V}{1-\delta_V} m_V \geq w$ or $\frac{\delta_V}{1-\delta_V} m_V < w$. If I have the former, the Village gains the most benefit from keeping quiet;

the latter implies the Village does better blowing the whistle. This means any optimal $\sigma_{\text{whistle blow}}(w)$ is degenerate.

$$d(m_V, w) = \begin{cases} 0 & \text{if } \frac{\delta_V}{1-\delta_V} m_V \geq w \\ 1 & \text{if } \frac{\delta_V}{1-\delta_V} m_V < w \end{cases} \tag{3.11}$$

And the probability of the Village staying quiet must then be,[18]

$$P(d(m_V, w) = 0 | m_V) = \frac{\delta_V}{(1-\delta_V)\alpha} m_V \text{ if } m_V \leq \alpha \left(\frac{1-\delta_V}{\delta_V} \right) \tag{3.12}$$

3.5.2 C - Committee

The Committee has discount factor $0 < \delta_C < 1$ and receives $1 - m_D$ from the District and decides how to split it with the Village. Considering (3.12), the utility to the committee for different choices of m_V are,

$$u_C(m_D, m_V, \delta_C) = 1 - m_D - m_V + (1 - m_D - m_V) \frac{\delta_C}{1-\delta_C} \frac{\frac{\delta_V}{1-\delta_V} m_V}{\alpha} \tag{3.13}$$

with F.O.C.

$$\frac{\partial}{\partial m_V} u_C(m_D, m_V, \delta_C) = -1 + \frac{\delta_C \delta_V}{(1-\delta_C)(1-\delta_V)\alpha} (1 - m_D - 2m_V) \tag{3.14}$$

with a necessary negative second derivative,

$$\frac{\partial^2}{\partial m_V^2} u_C(m_D, m_V, \delta_C) = -\frac{2\delta_C \delta_V}{(1-\delta_C)(1-\delta_V)\alpha} < 0 \tag{3.15}$$

[18] It is never beneficial to the Committee to give the Village more than $\alpha \left(\frac{1-\delta_V}{\delta_V} \right)$. It is not subgame perfect to offer the Village anything more. Nash equilibrium would require that giving the Village more than $\alpha \left(\frac{1-\delta_V}{\delta_V} \right)$ somehow hurts the District. If the Village receives $m_V \geq \alpha \left(\frac{1-\delta_V}{\delta_V} \right)$, $d(m_V, w) = 0$ by definition, so there would be no effect of increasing the Village's share.

Equation (3.14) gives,

$$m_V^* = \frac{1}{2} - \frac{m_D}{2} - \frac{(1-\delta_C)(1-\delta_V)\alpha}{2\delta_C\delta_V} \text{ if } 0 < m_V^* < \frac{1-\delta_V}{\delta_V}\alpha. \tag{3.16}$$

3.5.3 D - District

The District has discount factor $0 < \delta_D < 1$. The District's decision becomes clear when I backward induct from the decisions of the Village and the Committee. Given their best response behavior, the District determines $1-m_D$ factoring in the tradeoff between the payoff of m_D now and in continuation, given the probability that Village chooses $d(m_V, \delta_V) = 0$, and the payment to HQ, $c > 0$, if the Village does blow the whistle. Incorporating (3.16),

$$u_D = m_D + m_D \frac{\delta_D}{1-\delta_D} \frac{\delta_V}{(1-\delta_V)\alpha} m_V^* - \delta_D c \left[1 - \frac{\delta_V}{(1-\delta_V)\alpha} m_V^*\right] \tag{3.17}$$

$$= m_D + m_D \frac{\delta_D}{1-\delta_D} \frac{\delta_V}{(1-\delta_V)\alpha} \left(\frac{1}{2} - \frac{m_D}{2} - \frac{(1-\delta_C)(1-\delta_V)\alpha}{2\delta_C\delta_V}\right) - \delta_D c \left[1 - \frac{\delta_V}{(1-\delta_V)\alpha} \left(\frac{1}{2} - \frac{m_D}{2} - \frac{(1-\delta_C)(1-\delta_V)\alpha}{2\delta_C\delta_V}\right)\right] \tag{3.18}$$

F.O.C.,

$$\frac{\partial}{\partial m_D} u_D = 1 - \delta_D c \frac{\delta_V}{2(1-\delta_V)\alpha} + \left(\frac{\delta_D}{1-\delta_D}\right) \left(\frac{\delta_V}{2(1-\delta_V)\alpha}\right) \left[1 - 2m_D - \left(\frac{(1-\delta_C)(1-\delta_V)\alpha}{\delta_C\delta_V}\right)\right] \tag{3.19}$$

and necessary negative second derivative,

$$\frac{\partial^2}{\partial m_D^2} = -\left(\frac{\delta_D}{1-\delta_D}\right) \left(\frac{\delta_V}{(1-\delta_V)\alpha}\right) < 0 \tag{3.20}$$

$$\implies m_D^* = \frac{1}{2} - \left(\frac{1-\delta_D}{2}\right)c + \alpha\left(\frac{1-\delta_V}{\delta_V}\right)\left(\frac{1-\delta_D}{\delta_D}\right) - \alpha\left(\frac{1-\delta_V}{\delta_V}\right)\frac{1}{2}\left(\frac{1-\delta_C}{\delta_C}\right) \quad (3.21)$$

From (3.1), substituting in m_D^* to (3.16),

$$m_V^* = \frac{1}{4} + \left(\frac{1-\delta_D}{4}\right)c - \alpha\left(\frac{1-\delta_V}{\delta_V}\right)\left(\frac{1-\delta_D}{2\delta_D}\right) - \alpha\left(\frac{1-\delta_V}{\delta_V}\right)\frac{1}{4}\left(\frac{1-\delta_C}{\delta_C}\right) \quad (3.22)$$

$$m_C^* = \frac{1}{4} + \left(\frac{1-\delta_D}{4}\right)c - \alpha\left(\frac{1-\delta_V}{\delta_V}\right)\left(\frac{1-\delta_D}{2\delta_D}\right) + \alpha\left(\frac{1-\delta_V}{\delta_V}\right)\frac{3}{4}\left(\frac{1-\delta_C}{\delta_C}\right) \quad (3.23)$$

3.5.4 Proof of Theorem 1

Theorem 1. *–Existence of a Positive Payoff Equilibrium*

Given that blowing the whistle benefits the Village $w \sim U[0,\alpha]$, $0 < \alpha \leq 1$, costing the District $c > 0$, and the Village, Committee, and District discount future payoffs according to $0 < \delta_V < 1$, $\frac{c}{2+c} < \delta_C < 1$, $0 < \delta_D < 1$, then there exists a positve payoff equilibrium iff

$$\left[\frac{1+(1-\delta_D)c}{\left(4+\frac{1-\delta_C}{\delta_C}+\frac{2-2\delta_D}{\delta_D}\right)}\right] < \alpha\left(\frac{1-\delta_V}{\delta_V}\right) < \left[\frac{1+(1-\delta_D)c}{\left(\frac{1-\delta_C}{\delta_C}+\frac{2-2\delta_D}{\delta_D}\right)}\right]. \quad (3.24)$$

Proof. For a postive payoff equilibrium to exist, I need m_V^*, m_C^*, m_D^* to be equilibrium values chosen, I need m_V^*, m_C^*, $m_D^* > 0$ and importantly,

$$m_V^* < \alpha\left(\frac{1-\delta_V}{\delta_V}\right) \quad (3.25)$$ [19]

m_V^* and m_D^* solve the F.O.C. for the Committee and the District, respectively. $0 < \delta_V, \delta_C <$

[19]Giving any more to the Village would be of no benefit to the Committee. If the Committee is giving the

1 is given, so this ensures that
$\frac{\partial^2}{\partial m_V^2} u_C(m_D, m_V, \delta_C) = -\frac{2\delta_C\delta_V}{(1-\delta_C)(1-\delta_V)\alpha} < 0$, showing m_V^* is the optimum for the Committee.

$0 < \delta_D < 1$ as well, meaning $\frac{\partial^2}{\partial m_D^2} = -\left(\frac{\delta_D}{1-\delta_D}\right)\left(\frac{\delta_V}{(1-\delta_V)\alpha}\right) < 0$

A positive payoff equilibrium exists. From definition 1, equilibrium values from (3.21), (3.22), (3.23), $m_V^*, m_C^*, m_D^* > 0$. Clearly, $m_C^* > m_V^*$, so the condition on m_C^* is not binding.

Considering the condition for the District, (3.21), there are three cases

1. $\delta_D > \frac{2\delta_C}{1+\delta_C} \iff \left(\frac{1-\delta_C}{\delta_C} - 2\frac{1-\delta_D}{\delta_D}\right) > 0$

$$\implies \alpha\left(\frac{1-\delta_V}{\delta_V}\right) < \frac{1-(1-\delta_D)c}{\left(\frac{1-\delta_C}{\delta_C} - 2\frac{1-\delta_D}{\delta_D}\right)} \tag{3.26}$$

2. $\delta_D < \frac{2\delta_C}{1+\delta_C} \iff \left(\frac{1-\delta_C}{\delta_C} - \frac{2-2\delta_D}{\delta_D}\right) < 0$

$$\implies \alpha\left(\frac{1-\delta_V}{\delta_V}\right) > \frac{1-(1-\delta_D)c}{\left(\frac{1-\delta_C}{\delta_C} - \frac{2-2\delta_D}{\delta_D}\right)} \tag{3.27}$$

   However, $c \leq 1 \implies 1-(1-\delta_D)c > 0 \; \forall \delta_D \in (0,1)$, so this inequality becomes trivial as the RHS is negative.

3. $\delta_D = \frac{2\delta_C}{1+\delta_C} \implies \left(\frac{1-\delta_C}{\delta_C} - \frac{2-2\delta_D}{\delta_D}\right) = 0$

   This inequality states $0 < 1-(1-\delta_D)c$, something already implied by $c \leq 1$

Village exactly $\alpha\frac{1-\delta_V}{\delta_V}$, then I have that the Committee is willing to incur no potential for whistle blowing, meaning a degenerate solution on the boundary.

Only case 1 remains relevant. Considering bounds on δ_C,

$$\begin{aligned}
\frac{c}{2+c} < \delta_C &\iff c - \delta_C c < 2\delta_C \\
&\iff \left(\frac{1-\delta_C}{\delta_C}\right) c < 2 \\
&\iff \left(\frac{1-\delta_C}{\delta_C}\right)(1-\delta_D)c < 2\left(\frac{1-\delta_D}{\delta_D}\right) \\
&\iff \frac{1+(1-\delta_D)c}{\left(\frac{1-\delta_C}{\delta_C} + \frac{2-2\delta_D}{\delta_D}\right)} < \frac{1-(1-\delta_D)c}{\left(\frac{1-\delta_C}{\delta_C} - \frac{2-2\delta_D}{\delta_D}\right)} \qquad (3.28)
\end{aligned}$$

It can be shown from the conditions for the Village, from (3.22) and (3.25), respectively,

$$\frac{1}{4} + \frac{1}{4}(1-\delta_D)c > \frac{1}{2}\alpha\left(\frac{1-\delta_V}{\delta_V}\right)\left(\frac{1-\delta_D}{\delta_D}\right) + \frac{1}{4}\left(\frac{1-\delta_V}{\delta_V}\right)\left(\frac{1-\delta_C}{\delta_C}\right) \tag{3.29}$$

$$\frac{1}{4} + \frac{1}{4}(1-\delta_D)c > \alpha\left(\frac{1-\delta_V}{\delta_V}\right) + \frac{1}{2}\alpha\left(\frac{1-\delta_V}{\delta_V}\right)\left(\frac{1-\delta_D}{\delta_D}\right) + \frac{1}{4}\left(\frac{1-\delta_V}{\delta_V}\right)\left(\frac{1-\delta_C}{\delta_C}\right) \tag{3.30}$$

Together, (3.29) and (3.30) are,

$$\left[\frac{1+(1-\delta_D)c}{\left(4+\frac{1-\delta_C}{\delta_C}+\frac{2-2\delta_D}{\delta_D}\right)}\right] < \alpha\left(\frac{1-\delta_V}{\delta_V}\right) < \left[\frac{1+(1-\delta_D)c}{\left(\frac{1-\delta_C}{\delta_C}+\frac{2-2\delta_D}{\delta_D}\right)}\right]. \tag{3.31}$$

This inequality, (3.26), and (3.28) show that the District constraint is non-binding. □

3.5.5 Comparative Statics

It depends on relative patience whether the Committee, the District, or both short a more impatient Village.

$$\frac{\partial}{\partial \alpha \frac{1-\delta_V}{\delta_V}} m_C^* = -\frac{1}{2}\left(\frac{1-\delta_D}{\delta_D}\right) + \frac{3}{4}\left(\frac{1-\delta_C}{\delta_C}\right) \tag{3.32}$$

which is > 0 when $\frac{3}{4}\left(\frac{1-\delta_C}{\delta_C}\right) > \frac{1}{2}\left(\frac{1-\delta_D}{\delta_D}\right)$
$\implies \delta_D > \frac{2\delta_C}{3-\delta_C}$

$$\frac{\partial}{\partial\alpha\frac{1-\delta_V}{\delta_V}} m_D^* = \frac{1-\delta_D}{\delta_D} - \frac{1}{2}\left(\frac{1-\delta_C}{\delta_C}\right) \tag{3.33}$$

which is > 0 when $\frac{1-\delta_D}{\delta_D} > \frac{1}{2}\left(\frac{1-\delta_C}{\delta_C}\right)$
$\implies \delta_D < \frac{2\delta_C}{1+\delta_C}$, which is impossible if $\delta_D > \frac{\delta_C}{(1-\delta_C)c}$, a possibility in case 1 of the equilibrium existence, meaning in such cases, the partial derivative is always negative.

This means that higher $\alpha\frac{1-\delta_V}{\delta_V}$ has a positive effect only when the District is markedly more patient than the Committee. As long as Committee patience is sufficiently lower than that of the District, the District must take it into account when making the $1-m_D$ and m_D split. An impatient Committee means that the District must dispense more funds to keep the Village share at an optimum for reducing costs and increasing continuation payoffs, as the Committee will be more apt to take advantage of an increased $\alpha\frac{1-\delta_V}{\delta_V}$ –investing less in the future and taking more now.

On the other hand, if the District is not sufficiently more patient, the effect goes the other way and the District takes more of the funds in the first place, knowing the Committee will appease the Village. I have a of patience game played by the District and the Committee. With a smaller continuation payoff in expectation, Committee and District may be less willing to invest in the future, resulting in a lower first period payoff for the Village.

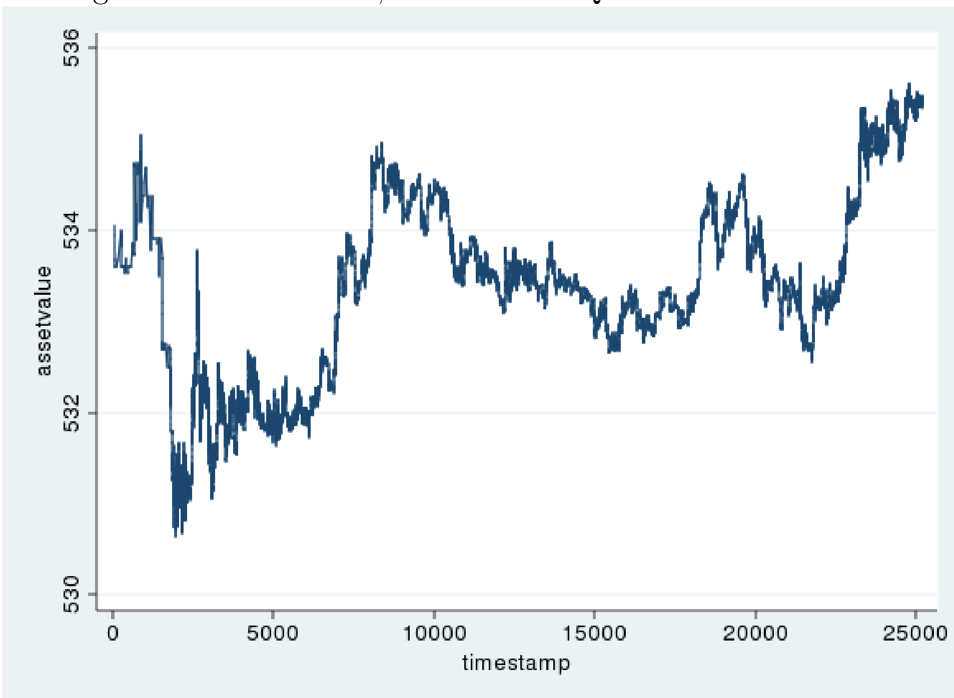
Chapter 4

Econometrics and Computation in Market Microstructure Models

4.1 Introduction

Here I present computational and econometric contributions to study of market microstructure models. These methods were used in the case study of Google surrounding an earnings announcement in Chapter 2. This includes setting up the liquidity supplier problem and equilibrium among those suppliers. Following, I describe the analytical forms for computing certainty-equivalent utilities for the informed trader, which allows one to compute certainty-equivalent utilties. Next, I present my recursive algorithm. The recursive algorithm allows a researcher to compute supply function equilibria between several liquidity suppliers. I conclude with a description of Indirect Inference while outlining what sets my use of the method apart.

Figure 4.1: October 14, 2009 NASDAQ Totalview-ITCH feed



4.1.1 Price Evolution surrounding EA by Google

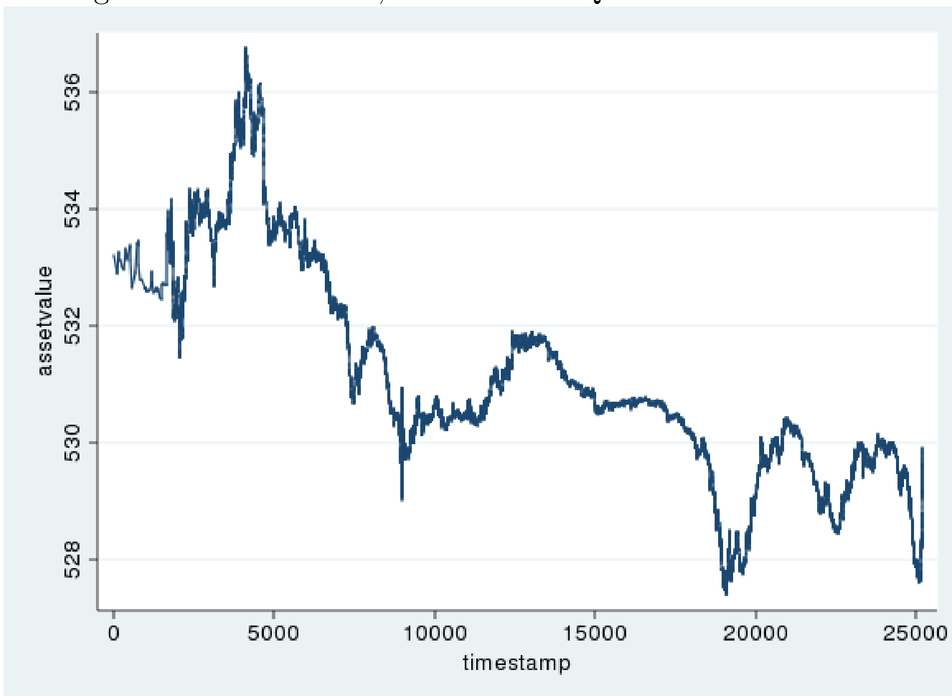
4.2 Liquidity Suppliers and the Optimal Supply Problem

Beginning with the utility of offering shares,

$$u_L(p_r, S_{r-1}, s_r^i, s_r^{-i}) =$$
$$E_{\alpha,\epsilon,I}\left[s_r^i \mathbf{1}_{\{S_{r-1}+s_r^i+s_r^{-i}\leq q\}}(p_r - v) + \frac{s_r^i}{s_r^i + s_r^{-i}}(q - S_{r-1})\mathbf{1}_{\{S_{r-1}<q<S_{r-1}+s_r^i+s_r^{-i}\}}(p_r - v)\right] \quad (4.1)$$

If $S_{r-1}+s_r^i+s_r^{-i} \leq q$, then following the boundary conditions, $\frac{\alpha - p_{r+1}}{\gamma\sigma_\epsilon^2} - I \leq S_{r-1}+s_r^i+s_r^{-i} \leq \frac{\alpha - p_r}{\gamma\sigma_\epsilon^2} - I$, or $S_{r-1}+s_r^i+s_r^{-i} < \frac{\alpha - p_{r+1}}{\gamma\sigma_\epsilon^2} - I$. The condition $S_{r-1} < q < S_{r-1}+s_r^i+s_r^{-i}$, results in an interior solution, so $q = \frac{\alpha - p_r}{\gamma\sigma_\epsilon^2} - I$, so I have that $S_{r-1} < \frac{\alpha - p_r}{\gamma\sigma_\epsilon^2} - I < S_{r-1} + s_r^i + s_r^{-i}$.

Figure 4.2: October 15, 2009 NASDAQ Totalview-ITCH feed



Because ϵ is not correlated with I or α, I replace v with α. I can see that (4.1) is equal to,

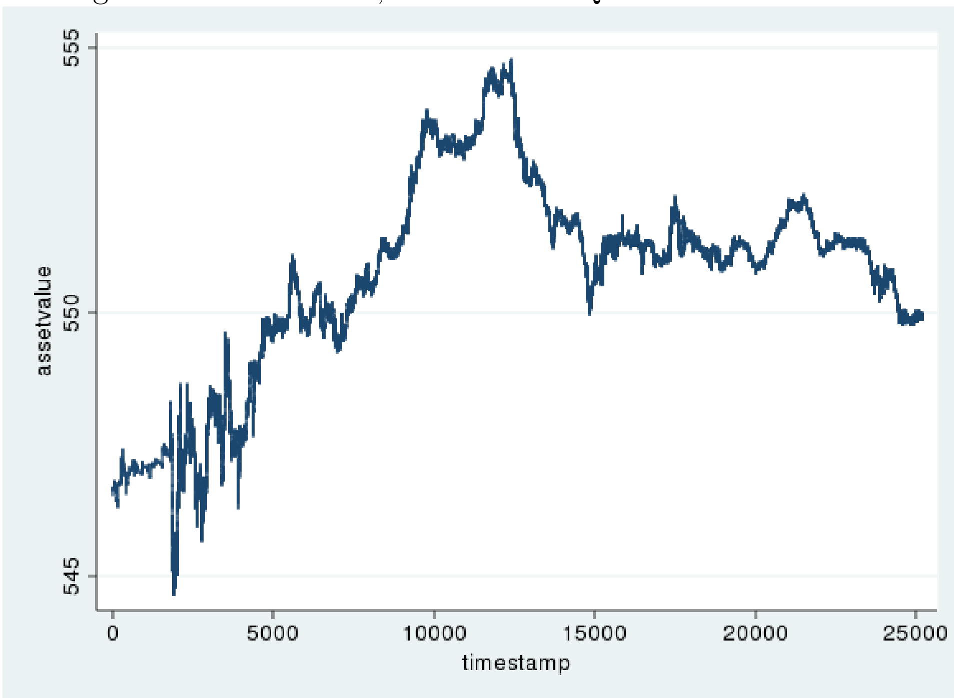
$$
\begin{aligned}
& s_r^i E\mathbf{1}_{\left\{S_{r-1}+s_r^i+s_r^{-i}\leq\frac{\alpha-p_r}{\gamma\sigma_\epsilon^2}-I\right\}}(p_r-\alpha)+ \\
& \qquad \frac{s_r^i}{s_r^i+s_r^{-i}}E\mathbf{1}_{\left\{S_{r-1}<\frac{\alpha-p_r}{\gamma\sigma_\epsilon^2}-I<S_{r-1}+s_r^i+s_r^{-i}\right\}}(p_r-\alpha)\left(\frac{\alpha-p_r}{\gamma\sigma_\epsilon^2}-I-S_{r-1}\right)
\end{aligned}
\tag{4.2}
$$

If I let $\phi_\alpha(x)=\frac{1}{\sqrt{2\pi}\sigma_\alpha}e^{-\frac{1}{2}\left(\frac{x-\mu_\alpha}{\sigma_\alpha}\right)^2}$ and $\phi_I(x)=\frac{1}{\sqrt{2\pi}\sigma_I}e^{-\frac{1}{2}\left(\frac{x-\mu_I}{\sigma_I}\right)^2}$ be the *pdf*s of α and I, then (4.2) is,

$$
\begin{aligned}
& s_r^i\int_{-\infty}^{\infty}\int_{\gamma\sigma_\epsilon^2(S_{r-1}+s_r^i+s_r^{-i}+I)+p_r}^{\infty}(p_r-\alpha)\phi_I(I)\phi_\alpha(\alpha)d\alpha dI+ \\
& \quad \frac{s_r^i}{s_r^i+s_r^{-i}}\int_{-\infty}^{\infty}\int_{\gamma\sigma_\epsilon^2(S_{r-1}+I)+p_r}^{\gamma\sigma_\epsilon^2(S_{r-1}+s_r^i+s_r^{-i}+I)+p_r}\left[\frac{\alpha-p_r}{\gamma\sigma_\epsilon^2}-I-S_{r-1}\right](p_r-\alpha)\phi_I(I)\phi_\alpha(\alpha)d\alpha dI
\end{aligned}
\tag{4.3}
$$

And to find the optimal liquidity provision at each price p_r, a liquidity supplier chooses s_r^i such that (4.3) is maximized. I find a first order condition, and the derivative of (4.3) with

Figure 4.3: October 16, 2009 NASDAQ Totalview-ITCH feed



respect to s_r^i is,

$$\int_{-\infty}^{\infty}\int_{\gamma\sigma_\epsilon^2(S_{r-1}+s_r^i+s_r^{-i}+I)+p_r}^{\infty}(p_r-\alpha)\phi_I(I)\phi_\alpha(\alpha)d\alpha dI$$
$$+\gamma\sigma_\epsilon^2 s_r^i\int_{-\infty}^{\infty}\gamma\sigma_\epsilon^2(S_{r-1}+s_r^i+s_r^{-i}+I)\phi_\alpha(\gamma\sigma_\epsilon^2(S_{r-1}+s_r^i+s_r^{-i}+I)+p_r)\phi_I(I)dI$$
$$+\frac{s_r^{-i}}{(s_r^i+s_r^{-i})^2}\int_{-\infty}^{\infty}\int_{\gamma\sigma_\epsilon^2(S_{r-1}+I)+p_r}^{\gamma\sigma_\epsilon^2(S_{r-1}+s_r^i+s_r^{-i}+I)+p_r}\left[\frac{\alpha-p_r}{\gamma\sigma_\epsilon^2}-I-S_{r-1}\right](p_r-\alpha)\phi_I(I)\phi_\alpha(\alpha)d\alpha dI$$
$$-\gamma\sigma_\epsilon^2\frac{s_r^i}{s_r^i+s_r^{-i}}\int_{-\infty}^{\infty}\left[s_r^i+s_r^{-i}\right]\gamma\sigma_\epsilon^2(S_{r-1}+s_r^i+s_r^{-i}+I)\phi_\alpha(\gamma\sigma_\epsilon^2(S_{r-1}+s_r^i+s_r^{-i}+I)+p_r)\phi_I(I)dI \tag{4.4}$$

Which reduces to,

$$\int_{-\infty}^{\infty}\int_{\gamma\sigma_\epsilon^2(S_{r-1}+s_r^i+s_r^{-i}+I)+p_r}^{\infty}(p_r-\alpha)\phi_I(I)\phi_\alpha(\alpha)d\alpha dI$$
$$+\frac{s_r^{-i}}{(s_r^i+s_r^{-i})^2}\int_{-\infty}^{\infty}\phi_I(I)\int_{\gamma\sigma_\epsilon^2(S_{r-1}+I)+p_r}^{\gamma\sigma_\epsilon^2(S_{r-1}+s_r^i+s_r^{-i}+I)+p_r}\left[\frac{\alpha-p_r}{\gamma\sigma_\epsilon^2}-I-S_{r-1}\right](p_r-\alpha)\phi_\alpha(\alpha)d\alpha dI \tag{4.5}$$

and that makes my second derivative

$$\gamma\sigma_\epsilon^2 \int_{-\infty}^{\infty} \gamma\sigma_\epsilon^2 (S_{r-1} + s_r^i + s_r^{-i} + I)\phi_\alpha(\gamma\sigma_\epsilon^2(S_{r-1} + s_r^i + s_r^{-i} + I) + p_r)\phi_I(I)dI$$
$$+ \frac{2s_r^{-i}}{(s_r^i + s_r^{-i})^2} \int_{-\infty}^{\infty} \gamma\sigma_\epsilon^2(S_{r-1} + s_r^i + s_r^{-i} + I)\phi_I(I)\phi_\alpha(\gamma\sigma_\epsilon^2(S_{r-1} + s_r^i + s_r^{-i} + I) + p_r)dI \quad (4.6)$$

4.3 Counterfactual Utilities

I look at certainty-equivalent utility for the active trader, summed up over the different ranges of q, the number of shares the active trader chooses to buy. From these utilities I subtract utility from inventory holdings, as a baseline.

Different types of traders are categorized into the same cases as they are in other calculations, but I compute the optimal choice over orders here. I consider a supply schedule $S = \{S_1, \ldots, S_M\}$, and $P = \{p_1, \ldots, p_M\}$ given.

Case 1: p_r is s.t. $S_{r-1} < q < S_r$

$$q = \frac{\alpha - p_r}{\gamma\sigma_\epsilon^2} - I \quad (4.7)$$

Case 2: $q = S_r$

$$\frac{\alpha - p_{r+1}}{\gamma\sigma_\epsilon^2} - I < q < \frac{\alpha - p_r}{\gamma\sigma_\epsilon^2} - I \quad (4.8)$$

I also consider that the active trader is trying to maximize $-e^{-\gamma W}$ where W is wealth, from equation (2.7). This is equivalent to maximizing $E[W|\alpha, I] + \frac{\gamma}{2}V[W|\alpha, I] = (q+I)\alpha - \sum_{r=1}^{M} \max\{0, \min\{q, S_r\} - S_{r-1}\}p_r - \frac{\gamma}{2}(q+I)^2\sigma_\epsilon^2$. To find the average certainty-equivalent utility change for active traders, I must integrate over the spaces of α and I, taking into account latent supply S_0. Presented are the equations for M and all other $r = 1, \ldots, M-1$

simultaneously. Calculating the counterfactual utility involves taking,

$$(q+I)\alpha - \sum_{r=1}^{M} \max\{0, \min\{q, S_r\} - S_{r-1}\} p_r - \frac{\gamma}{2}(q+I)^2\sigma_\epsilon^2 \tag{4.9}$$

and summing over the the range of q.

Expected utility for those traders choosing to take liquidity at price p_M is,

$$E_{\alpha,I}\left\{\mathbf{1}_{\{S_M \le \frac{\alpha-p_M}{\gamma\sigma_\epsilon^2} - I\}}\left[\alpha(S_M+I) - \frac{\gamma\sigma_\epsilon^2}{2}(S_M+I)^2 - \sum_{k=1}^{M} s_k p_k - p_1 S_0\right]\right.$$
$$+\mathbf{1}_{\{S_{M-1} < \frac{\alpha-p_M}{\gamma\sigma_\epsilon^2} - I < S_M\}}$$
$$\left.\times\left[\alpha\left(\frac{\alpha-p_M}{\gamma\sigma_\epsilon^2}\right) - \frac{\gamma\sigma_\epsilon^2}{2}\left(\frac{\alpha-p_M}{\gamma\sigma_\epsilon^2}\right)^2 - \sum_{k=1}^{M-1} s_k p_k - p_1 S_0 - p_M\left(\frac{\alpha-p_M}{\gamma\sigma_\epsilon^2} - I - S_{M-1}\right)\right]\right\} \tag{4.10}$$

and for traders taking more liquidity than at price p_r,

$$E_{\alpha,I}\left\{\mathbf{1}_{\{\frac{\alpha-p_r}{\gamma\sigma_\epsilon^2} - I \le S_r \le \frac{\alpha-p_r}{\gamma\sigma_\epsilon^2} - I\}}\left[\alpha(S_r+I) - \frac{\gamma\sigma_\epsilon^2}{2}(S_r+I)^2 - \sum_{k=1}^{r} s_k p_k - p_1 S_0\right]\right.$$
$$+\mathbf{1}_{\{S_{r-1} < \frac{\alpha-p_r}{\gamma\sigma_\epsilon^2} - I < S_r\}}$$
$$\left.\times\left[\alpha\left(\frac{\alpha-p_r}{\gamma\sigma_\epsilon^2}\right) - \frac{\gamma\sigma_\epsilon^2}{2}\left(\frac{\alpha-p_r}{\gamma\sigma_\epsilon^2}\right)^2 - \sum_{k=1}^{r-1} s_k p_k - p_1 S_0 - p_r\left(\frac{\alpha-p_r}{\gamma\sigma_\epsilon^2} - I - S_{r-1}\right)\right]\right\} \tag{4.11}$$

I sum up the above for p_M and $p_1, \ldots, p_{M-1}$, respectively, then I add utility from latent supply,

$$E_{\alpha,I}\left\{\mathbf{1}_{\{0 \le \frac{\alpha-p_1}{\gamma\sigma_\epsilon^2} - I \le S_0\}}\left[\alpha\left(\frac{\alpha-p_1}{\gamma\sigma_\epsilon^2}\right) - \frac{\gamma\sigma_\epsilon^2}{2}\left(\frac{\alpha-p_1}{\gamma\sigma_\epsilon^2}\right)^2 - p_1\left(\frac{\alpha-p_1}{\gamma\sigma_\epsilon^2} - I\right)\right]\right\} \tag{4.12}$$

while subtracting utility of original endowment,

$$E_{\alpha,I}\left\{\mathbf{1}_{\{0 \le \frac{\alpha-p_1}{\gamma\sigma_\epsilon^2} - I\}}\left[\alpha I - \frac{\gamma\sigma_\epsilon^2}{2} I^2\right]\right\} \tag{4.13}$$

Whereas disregarding utility from latent supply means ignoring (4.12) and (4.13) and subtracting

$$E_{\alpha,I}\left\{\mathbf{1}_{\{S_0\le\frac{\alpha-p_1}{\gamma\sigma_\epsilon^2}-I\}}\left[\alpha(S_0+I)-\frac{\gamma\sigma_\epsilon^2}{2}(S_0+I)^2-p_1S_0\right]\right\}. \tag{4.14}$$

These give us respectively, in integral form,

$$\begin{aligned}&\int_{\gamma\sigma_\epsilon^2(S_M+I)+p_M}^{\infty}\int_{-\infty}^{\infty}\left[\alpha(S_M+I)-\frac{\gamma\sigma_\epsilon^2}{2}(S_M+I)^2-\sum_{k=1}^{M}s_kp_k-p_1S_0\right]\phi_\alpha(\alpha)\phi_I(I)dId\alpha\\&+\int_{\gamma\sigma_\epsilon^2(S_{M-1}+I)+p_M}^{\gamma\sigma_\epsilon^2(S_M+I)+p_M}\int_{-\infty}^{\infty}\Bigg[\alpha\left(\frac{\alpha-p_M}{\gamma\sigma_\epsilon^2}\right)-\frac{\gamma\sigma_\epsilon^2}{2}\left(\frac{\alpha-p_M}{\gamma\sigma_\epsilon^2}\right)^2\\&\qquad-\sum_{k=1}^{M-1}s_kp_k-p_1S_0-p_M\left(\frac{\alpha-p_M}{\gamma\sigma_\epsilon^2}-I-S_{M-1}\right)\Bigg]\phi_\alpha(\alpha)\phi_I(I)dId\alpha\end{aligned} \tag{4.15}$$

$$\begin{aligned}&\int_{\gamma\sigma_\epsilon^2(S_r+I)+p_r}^{\gamma\sigma_\epsilon^2(S_r+I)+p_{r+1}}\int_{-\infty}^{\infty}\left[\alpha(S_r+I)-\frac{\gamma\sigma_\epsilon^2}{2}(S_r+I)^2-\sum_{k=1}^{r}s_kp_k-p_1S_0\right]\phi_\alpha(\alpha)\phi_I(I)dId\alpha\\&+\int_{\gamma\sigma_\epsilon^2(S_{M-1}+I)+p_r}^{\gamma\sigma_\epsilon^2(S_r+I)+p_r}\int_{-\infty}^{\infty}\Bigg[\alpha\left(\frac{\alpha-p_r}{\gamma\sigma_\epsilon^2}\right)-\frac{\gamma\sigma_\epsilon^2}{2}\left(\frac{\alpha-p_r}{\gamma\sigma_\epsilon^2}\right)^2\\&\qquad-\sum_{k=1}^{M-1}s_kp_k-p_1S_0-p_r\left(\frac{\alpha-p_r}{\gamma\sigma_\epsilon^2}-I-S_{r-1}\right)\Bigg]\phi_\alpha(\alpha)\phi_I(I)dId\alpha\end{aligned} \tag{4.16}$$

$$\int_{\gamma\sigma_\epsilon^2I+p_1}^{\gamma\sigma_\epsilon^2(S_0+I)+p_1}\int_{-\infty}^{\infty}\left[\alpha\left(\frac{\alpha-p_1}{\gamma\sigma_\epsilon^2}\right)-\frac{\gamma\sigma_\epsilon^2}{2}\left(\frac{\alpha-p_1}{\gamma\sigma_\epsilon^2}\right)^2-p_1\left(\frac{\alpha-p_1}{\gamma\sigma_\epsilon^2}-I\right)\right]\phi_\alpha(\alpha)\phi_I(I)dId\alpha \tag{4.17}$$

$$\int_{\gamma\sigma_\epsilon^2I+p_1}^{\infty}\int_{-\infty}^{\infty}\left[\alpha I-\frac{\gamma\sigma_\epsilon^2}{2}I^2\right]\phi_\alpha(\alpha)\phi_I(I)dId\alpha \tag{4.18}$$

$$\int_{\gamma\sigma_\epsilon^2(S_0+I)+p_1}^{\infty}\int_{-\infty}^{\infty}\left[\alpha(S_0+I)-\frac{\gamma\sigma_\epsilon^2}{2}(S_0+I)^2-p_1S_0\right]\phi_\alpha(\alpha)\phi_I(I)dId\alpha. \tag{4.19}$$

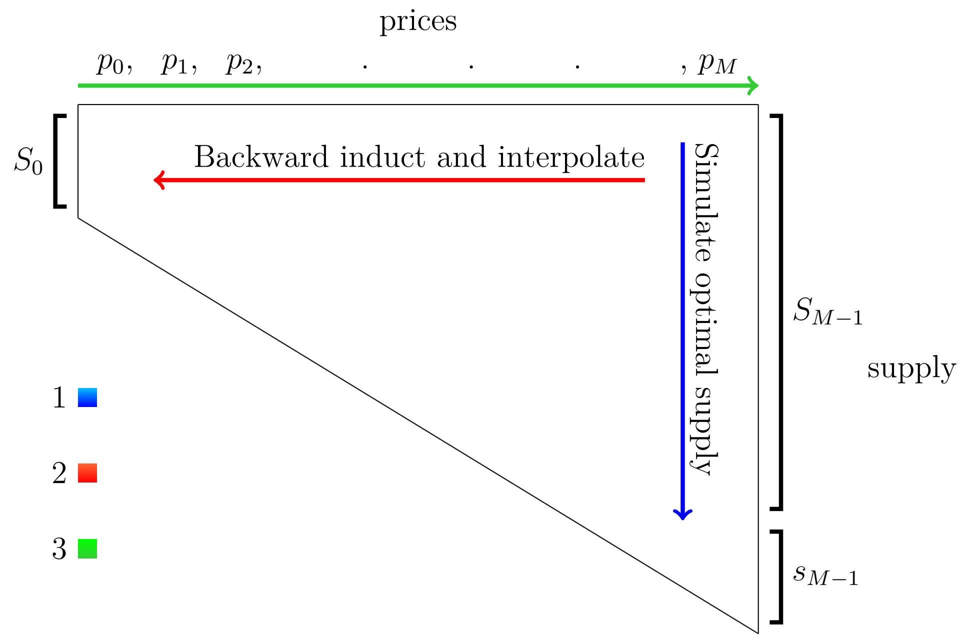
4.4 Solving for Equilibrium, detail

For an n player symmetric equilibrium, I start at the highest price, p_M and work backward. Given existing supply S_{M-1}, the optimal supply solves the F.O.C. given in (4.5) where $r = M$, and $s_M^{-i} = (n-1)s_M^i$. While it is possible that a F.O.C. will not be reached, my computational methods avoid this. If the algorithm reaches the boundary at 0, naturally the liquidity supplier offers 0 shares. If, however, the boundary is at the computed supply matrix maximum, then the matrix is expanded and I restart the entire process. This ensures that liquidity is determined endogenously in the model. I call optimal supply for individual i at M, s_M^{i*}. Finding this value for all possible S_{M-1} and generating the value function at that price and quantity, $V(p_M, S_{M-1}) \equiv u(p_M, S_{M-1}, s_M^{i*}, (n-1)s_M^{i*})$, I can then solve for s_{M-1}^{i*} using (4.5) added to the derivative of $V(p_{M-1}, S_{M-2}+n\cdot s_{M-1}^i)$ for all S_{M-2}, generating $V(p_{M-1}, S_{M-2}) \equiv u(p_{M-1}, S_{M-2}, s_{M-1}^{i*}, (n-1)s_{M-1}^{i*}) + V(p_M, S_{M-2}+n\cdot s_{M-1}^{i*}(q), M)$. Working backward from $M-2$ to 1, I find a symmetric equilibrium discrete price supply function for all i. The function is in the form of a supply matrix.

Constructing a supply curve from a supply matrix involves interpolation. I begin with latent supply S_0 and compute optimal response in symmetric equilibrium at p_1. Moving forward, taking into account supply S_1, the optimal response at p_2 is computed. This process continues successively to p_M. My interpolation tool is a piecewise cubic hermite interpolating polynomial.

From the lowest ask price, I assume a level of latent supply, S_0, present in the book before the best ask, just below the best ask. This is to accommodate a variety of things. First of all, the model is made for one side of the market. While a story about institutional and active traders is fitting, it is restrictive. In response, I might see some spillover from the buy side of the market with active traders posting quickly executable liquidity. Second, there may be some impatient liquidity suppliers who do not play as part of an equilibrium that looks like what I consider here. Third, there are hidden orders present in the market at many points in time. While it is difficult or impossible to recreate these hidden orders as econometrician, I can allow for some liquidity to be hidden in the form of latent supply. I calculate the optimal supply schedule at each latent supply and interpolating according to calculated optimal values walking up the book. This brings us to the architecture of the supply matrix. Because of interpolation methods, I must have a bound on total aggregate supply. In addition, I need a delimeter over this space to give us a finite number of operations. In my calculations, I define limiting supply endogenously in my procedure so as to always reach a F.O.C. I call this $\bar{S}$, representing the maximum liquidity that can be offered by a single trader at each of the M prices, $n \cdot \bar{S}$ for all. The procedure is illustrated in figure 4.4.

Figure 4.4: Supply Matrix Procedure



Selection of lowest offer price and highest offer price is not arbitrary. The lowest price is determined by the bid-ask spread. The highest offer price is a point at which offering shares is no longer profitably meaningful, as discussed above. One might consider a market setting in which offering liquidity at any high price, no matter how high, would be profitable.

However, those outcomes are improbable and economically irrelevant.

4.5 Indirect Inference Method, Proofs, and Asymptotic Normality

Here I outline in more detail my implementation of Indirect Inference in finding parameter estimates.

Over the course of the trading week, and even within the trading day, the composition of the market for a single asset changes. An individual instance of an order book tells us about a particular point in time. However, executions, limit orders, and cancellations can all change the shape of the order book. The same parameters could describe books with very different shapes, depending on the prices traders choose to offer liquidity. Hence, several different instances of the order book around the same point in time look different, but tell the same economic story. Additionally, placing or removing a single order does not immediately dissipate market conditions. I consider the order book from time T_i to T_j. I want to find $\theta = \{\sigma_\alpha, \sigma_\epsilon, \mu_I, \sigma_I, \sigma_{S_0}\}$–refer to Table 2.1 for parameter definitions–which describes equilibrium supply schedules over that period of time.

I look at a period of time (T_i, T_j), where $\triangle \equiv T_j - T_i$. I take K order book samples $t_1, t_2, \ldots, t_k, \ldots, t_K$, each of size L, where $t_k \equiv T_i + \frac{\triangle}{K+1}k$. Keeping in mind that total depth is M, a single supply curve is $(S_k^\ell, P_k^\ell) \equiv ([S_{k,1}^\ell, S_{k,2}^\ell, \ldots, S_{k,M}^\ell], [p_{k,1}^\ell, p_{k,2}^\ell, \ldots, p_{k,M}^\ell])$ for each sample, $k = 1, \ldots, K$ and all instances of the order book within each sample, $\ell = 1, \ldots, L$.

I introduce a pseudo-model for supply,

$$S_{k,m}^\ell = g(p_{k,m}^\ell) + \nu_{k,m}^\ell, \ \nu_{k,m}^\ell \sim IID(0, \sigma_\nu^2) \tag{4.20}$$

and I assume

$$S^\ell_{k,m} = \beta_1 + \beta_2 p^\ell_{k,m} + \beta_3 p^{\ell\,2}_{k,m} + \beta_4 log(p^\ell_{k,m}) + \nu^\ell_{k,m} + \eta^\ell_{k,m},\ \eta^\ell_{k,m} \sim IID(0, \sigma^2_\eta) \tag{4.21}$$

so, given

A 1. $E(\eta^\ell_{k,m} | \nu^\ell_{k,m}, p^\ell_{k,m}) = 0$

$$S^\ell_{k,m} \simeq \beta_1 + \beta_2 p^\ell_{k,m} + \beta_3 p^{\ell\,2}_{k,m} + \beta_4 log(p^\ell_{k,m}) + \eta^\ell_{k,m} \tag{4.22}$$

or in matrix notation

$$\mathcal{S}^L_k = X^L_k \beta_k + \eta^L_k,\ \eta^L_k \sim IID(0, \sigma^2_\eta I) \tag{4.23}$$

where $\mathcal{S}^L_k \equiv [S^1_{k,1}, S^1_{k,2}, \ldots, S^1_{k,M}, S^2_{k,1}, \ldots, S^L_{k,M}]'$, $X^L_k \equiv [\mathbf{1}\ \ P^L_k\ \ P^{L^2}_k\ \ log(P^L_k)]$, $P^L_k \equiv [p^1_{k,1}, p^1_{k,2}, \ldots, p^1_{k,M}, p^2_{k,1}, \ldots, p^L_{k,M}]'$, and $\eta^L_k \equiv [\eta^1_{k,1}, \eta^1_{k,2}, \ldots, \eta^1_{k,M}, \eta^2_{k,1}, \ldots, \eta^L_{k,M}]'$. My criterion is that of reducing the sum of squared residuals,

$$\max_{\beta \in B} C_{k,L}(\mathcal{S}^L_k, P^L_k, \beta) \equiv \max_{\beta \in B} - \sum_{\ell=1}^{L} \sum_{m=1}^{M} (s^\ell_{k,m} - x^\ell_{k,m}\beta)^2. \tag{4.24}$$

I assume this criterion tends asymptotically to a limit,

A 2. $\lim_{L\to\infty} C_{k,L}(\mathcal{S}^L_k, P_k, \beta) = C_{k,\infty}(Q_0, P_{k,\infty}, \theta_0, \beta)$

I assume that the limit criterion is continuous in β and has a unique maximum,

A 3. $\beta_{k,\infty} \equiv \arg\max_{\beta \in B} C_{k,\infty}(Q_0, P_{k,\infty}, \theta_0, \beta)$

I use the ordinary least squares estimator, $\hat{\beta}(\mathcal{S}^L_k, P^L_k) = [X^{L\prime}_k X^L_k]^{-1} X^{L\prime}_k \mathcal{S}^L_k$. Because prices are bounded, and each element of X^L_k is a continuous function of price, $\exists$ vector c s.t. $E\, x^\ell_{k,m} {x^\ell_{k,m}}' = c$, $\forall \ell$, m. Here $x^\ell_{k,m} \equiv [1\ \ p^\ell_{k,m}\ \ p^{\ell\,2}_{k,m}\ \ log(p^\ell_{k,m})]$. By the law of large numbers, $\frac{1}{L}(X^{L\prime}_k X^L_k) \to c$ and so, $\operatorname*{plim}_{L\to\infty} \frac{1}{L}(X^{L\prime}_k X^{L\prime}_k) = S_{X^{L\prime}_k X^L_k}$ exists.

From A 1, $E(\eta^{\ell}_{k,m}|x^{\ell}_{k,m}) = \mathbf{0}$, and therefore,

$$\underset{L\to\infty}{\text{plim}}\, \frac{1}{L} X_k^{L\prime} \eta_k^L = \underset{L\to\infty}{\text{plim}}\, \frac{1}{L} \sum_{\ell=1}^{L} \sum_{m=1}^{M} x^{\ell\prime}_{k,m} \eta^{\ell}_{k,m} = \mathbf{0} \tag{4.25}$$

This gives us that $\hat{\beta}_k = \hat{\beta}(\mathcal{S}_k^L, P_k^L)$ is a consistent estimator of $\beta_{k,\infty}$.

I introduce the *binding function* from Gouriéroux, Monfort, and Renault (1993),

$$b(Q, P_{k,\infty}, \theta) \equiv \arg\max_{\beta\in B} C_\infty(Q, P_{k,\infty}, \theta, \beta) \tag{4.26}$$

The binding function links the auxiliary parameters of the pseudo-model with the parameters of the true model. I have

$$\beta_{k,\infty} = b(Q_0, P_{k,\infty}, \theta_0) \tag{4.27}$$

The binding function is analytically intractable. I find it numerically, however. For a θ, $\bar{P}_k^L \equiv \frac{1}{L}\sum_{\ell=1}^{L} P_k^{\ell}$, $\bar{X}_k \equiv [\mathbf{1}\ \ \bar{P}_k^L\ \ \bar{P}_k^{L\,2}\ \ log(\bar{P}_k^L)]$ given, I consider Λ supply curves, $\tilde{S}^{\lambda}(\theta, \bar{P}_k^L)$, $\lambda = 1, \ldots, \Lambda$, generated from drawings of latent supply S_0. Latent supply represents noisy liquidity supply which may be some spillover from the other side of the order book, a few impatient suppliers, or hidden orders. S_0 is drawn from a truncated normal distribution with mean calibrated to the empirical supply curves, and variance $\sigma^2_{S_0}$. From the supply matrix determined by θ and each draw of S_0^{λ}, $\theta = 1, \ldots, \Lambda$, I interpolate to find the respective supply curve, $\tilde{\mathcal{S}}^{\lambda}(\theta, \bar{P}_k^L)$. For each curve I can find the respective estimates solving,

$$\max_{\beta\in B} C_{k,L}(\tilde{S}^{\lambda}(\theta, \bar{P}_k^L), \bar{P}_k^L, \beta) \tag{4.28}$$

for which I consider the OLS estimate

$$\tilde{\beta}^{\lambda}(\theta, \bar{P}_k^L) = [\bar{X}_k^{L\prime} \bar{X}_k^L]^{-1} \bar{X}_k^{L\prime} \tilde{\mathcal{S}}^{\lambda}(\theta, \bar{P}_k^L). \tag{4.29}$$

I assume that the vector $\bar{P}_k^L$ reaches asymptotically the population prices from sample k.

A 4. $\lim_{L\to\infty} \bar{P}_k^L = P_{k,\infty}$

And so, I define $\lim_{L\to\infty} \bar{X}_k^L = X_{k,\infty}$. This means that as L tends to infinity, this solution tends toward $\tilde{\beta}^\lambda(\theta, P_{k,\infty}) = [X'_{k,\infty} X_{k,\infty}]^{-1} X'_{k,\infty} \tilde{\mathcal{S}}^\lambda(\theta, P_{k,\infty})$, which solves $\max_{\beta\in B} C_\infty(Q, P_{k,\infty}, \theta, \beta)$. Therefore,

$$\lim_{L\to\infty} \tilde{\beta}^\lambda(\theta, \bar{P}_k^L) = b(Q, P_{k,\infty}, \theta) \tag{4.30}$$

and is therefore a consistent functional estimator of the binding function. I define the following,

$$b_K(\theta) \equiv [b(Q, P_{1,\infty}, \theta); b(Q, P_{2,\infty}, \theta), \ldots, b(Q, P_{K,\infty}, \theta)] \tag{4.31}$$

$$\hat{\beta}_K^L \equiv [\hat{\beta}(\mathcal{S}_1^L, P_1^L); \hat{\beta}(\mathcal{S}_2^L, P_2^L); \ldots; \hat{\beta}(\mathcal{S}_K^L, P_K^L)] \tag{4.32}$$

$$\tilde{\beta}_K^{\lambda,L}(\theta) \equiv [\tilde{\beta}^\lambda(\theta, \bar{P}_1^L); \tilde{\beta}^\lambda(\theta, \bar{P}_2^L); \ldots; \tilde{\beta}^\lambda(\theta, \bar{P}_K^L)] \tag{4.33}$$

And make the assumptions

A 5. $\bar{P}_k^L \neq \bar{P}_j^L$, $\forall\ j \neq k$

A 6. $\bar{X}_k^{L\prime} \dfrac{\partial \tilde{S}^\lambda}{\partial \theta'}(\theta_0, \bar{P}_k^L)$ has full column rank, $\forall\ k = 1, \ldots, K$.

Claim 2. *For number of samples* $K \geq \dfrac{|\theta|}{|\beta|}$, $\dfrac{\partial b_K}{\partial \theta'}(\theta_0)$ *is of full column rank.*

Proof. $\bar{X}_1^{L\prime} \dfrac{\partial \tilde{S}^\lambda}{\partial \theta'}(\theta_0, \bar{P}_1^L)$ has full column rank. It is therefore clear that $\tilde{\beta}^\lambda(\theta_0, \bar{P}_1^L) = [\bar{X}_1^{L\prime} \bar{X}_1^L]^{-1} \bar{X}_1^{L\prime} \tilde{\mathcal{S}}^\lambda(\theta_0, \bar{P}_1^L)$ has full column rank. Because $\lim_{L\to\infty} \tilde{\beta}^\lambda(\theta_0, \bar{P}_1^L) = b(Q, P_{1,\infty}, \theta_0)$, $\dfrac{\partial b}{\partial \theta'}(Q, P_{1,\infty}, \theta_0)$ is also of full column rank. The same is true for $\dfrac{\partial b}{\partial \theta'}(Q, P_{k,\infty}, \theta_0)$ for $k = 2, \ldots, K$. Each matrix of derivatives is independent of all others because of assumption 5. I stack up binding functions on top of each other, and because $K \geq \dfrac{|\theta|}{|\beta|}$, I get a matrix of full column rank. This matrix is contained in $\dfrac{\partial b_K}{\partial \theta'}(\theta_0)$, so it is of full column rank.

□

Using both pseudo-model objects, (4.32) and (4.33), the Indirect Inference estimator, $\hat{\theta}_K^{\Lambda,L}$ optimizes the following criterion

$$\hat{\theta}_K^{\Lambda,L} \equiv \min_{\theta\in\Theta} \left[\hat{\beta}_K^L - \frac{1}{\Lambda}\sum_{\lambda=1}^{\Lambda} \tilde{\beta}_K^{\lambda,L}(\theta)\right]' \Omega \left[\hat{\beta}_K^L - \frac{1}{\Lambda}\sum_{\lambda=1}^{\Lambda} \tilde{\beta}_K^{\lambda,L}(\theta)\right] \tag{4.34}$$

Proposition 3. *Under assumptions 1–6, $\hat{\theta}_K^{\Lambda,L}$ is a consistent estimator of θ_0*

Proof. The F.O.C. are

$$\left[\frac{1}{\Lambda}\sum_{\lambda=1}^{\Lambda} \frac{\partial\tilde{\beta}_K^{\lambda,L\prime}}{\partial\theta}(\hat{\theta}_K^{\Lambda,L})\right] \Omega \left[\hat{\beta}_K^L - \frac{1}{\Lambda}\sum_{\lambda=1}^{\Lambda} \tilde{\beta}^{\lambda,L}(\hat{\theta}_K^{\Lambda,L})\right] = \mathbf{0} \tag{4.35}$$

$$\left[\frac{1}{\Lambda}\sum_{\lambda=1}^{\Lambda} \frac{\partial\tilde{\beta}_K^{\lambda,L\prime}}{\partial\theta}(\hat{\theta}_K^{\Lambda,L})\right] \Omega \left[\hat{\beta}_K^L - \frac{1}{\Lambda}\sum_{\lambda=1}^{\Lambda} \tilde{\beta}^{\lambda,L}(\theta_0) - \frac{1}{\Lambda}\sum_{\lambda=1}^{\Lambda} \frac{\partial\tilde{\beta}_K^{\lambda,L}}{\partial\theta'}(\theta_0)[\hat{\theta}_K^{\Lambda,L} - \theta_0]\right] \simeq \mathbf{0} \tag{4.36}$$

$$\sqrt{L}(\hat{\theta}_K^{\Lambda,L} - \theta_0) \simeq \left[\frac{\partial b_K'}{\partial\theta}(\theta_0)\Omega\frac{\partial b_K}{\partial\theta'}(\theta_0)\right]^{-1} \frac{\partial b_K'}{\partial\theta}(\theta_0)\Omega\sqrt{L}\left[\hat{\beta}_K^L - \frac{1}{\Lambda}\sum_{\lambda=1}^{\Lambda} \tilde{\beta}^{\lambda,L}(\theta_0)\right] \tag{4.37}$$

Because $\hat{\beta}_K^L$ and $\frac{1}{\Lambda}\sum_{\lambda=1}^{\Lambda} \tilde{\beta}^{\lambda,L}(\hat{\theta}_K^{\Lambda,L})$ are both consistent estimators of $\beta_{k,\infty}$, the estimator is consistent. □

I also show asymptotic normality of the estimator.

A 7. $\forall$ k and λ, $\operatorname*{plim}_{L\to\infty} \frac{\partial^2 C_{k,L}}{\partial\beta\,\partial\beta'}(\tilde{S}^\lambda(\theta, \bar{P}_k^L), \bar{P}_k^L, \beta_0) = \frac{\partial^2 C_{k,\infty}}{\partial\beta\,\partial\beta'}(Q_0, P_{k,\infty}, \theta_0, \beta_0)$

Asymptotically, by assumption 7 and viewing $\mathcal{S}_k^L$ and P_k^L as simulations, so $\forall$ k,

$$\frac{\partial C_{k,L}}{\partial\beta}(\mathcal{S}_k^L, P_k^L, \hat{\beta}(\mathcal{S}_k^L, P_k^L)) = 0 \tag{4.38}$$

$$\sqrt{L}\frac{\partial C_{k,L}}{\partial\beta}(\mathcal{S}_k^L, P_k^L, \beta_{k,\infty}) + \frac{\partial^2 C_{k,L}}{\partial\beta\,\partial\beta'}(\mathcal{S}_k^L, P_k^L, \beta_{k,\infty})\sqrt{L}(\hat{\beta}(\mathcal{S}_k^L, P_k^L) - \beta_{k,\infty}) \simeq 0 \tag{4.39}$$

$$\sqrt{L}(\hat{\beta}(\mathcal{S}_k^L, P_k^L) - \beta_{k,\infty}) \simeq -\frac{\partial^2 C_{k,\infty}}{\partial\beta\,\partial\beta'}(Q_0, P_{k,\infty}, \theta_0, \beta_{k,\infty})\sqrt{L}\frac{\partial C_{k,L}}{\partial\beta}(\mathcal{S}_k^L, P_k^L, \beta_{k,\infty}) \tag{4.40}$$

By a similar argument,

$$\sqrt{L}(\tilde{\beta}^{\lambda}(\theta, \bar{P}_k^L) - \beta_{k,\infty}) \simeq -\frac{\partial^2 C_{k,\infty}}{\partial\beta\,\partial\beta'}(Q_0, P_{k,\infty}, \theta_0, \beta_{k,\infty})\sqrt{L}\frac{\partial C_{k,L}}{\partial\beta}(\tilde{S}^{\lambda}(\theta, \bar{P}_k^L), \bar{P}_k^L, \beta_{k,\infty}) \tag{4.41}$$

By 4.40 and 4.41, I have that,

$$\begin{aligned}\sqrt{L}\left[\hat{\beta}_k^L(\mathcal{S}_k^L, P_k^L) - \frac{1}{\Lambda}\sum_{\lambda=1}^{\Lambda}\tilde{\beta}^{\lambda,L}(\theta_0, \bar{P}_k^L)\right] &\simeq -\frac{\partial^2 C_{k,\infty}}{\partial\beta\,\partial\beta'}(Q_0, P_{k,\infty}, \theta_0, \beta_{k,\infty}) \\ &\times\left[\sqrt{L}\frac{\partial C_{k,L}}{\partial\beta}(\mathcal{S}_k^L, P_k^L, \beta_{k,\infty}) - \sqrt{L}\frac{1}{\Lambda}\sum_{\lambda=1}^{\Lambda}\frac{\partial C_{k,L}}{\partial\beta}(\tilde{S}^{\lambda}(\theta, \bar{P}_k^L), \bar{P}_k^L, \beta_{k,\infty})\right]\end{aligned} \tag{4.42}$$

Therefore, this difference is asymptotically normal with zero mean. Therefore the difference in stacks,

$$\sqrt{L}\left[\hat{\beta}_K^L - \frac{1}{\Lambda}\sum_{\lambda=1}^{\Lambda}\tilde{\beta}^{\lambda,L}(\theta_0)\right] \tag{4.43}$$

is also asymptotically normal with zero mean.

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