A STUDY OF AN ARTIFICIAL TELEPHONE LINE

Thesis by

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In Partial Fulfilment of the Requirements
for the Degree of Master of Science

California Institute of Technology

Pasadena, California

1928

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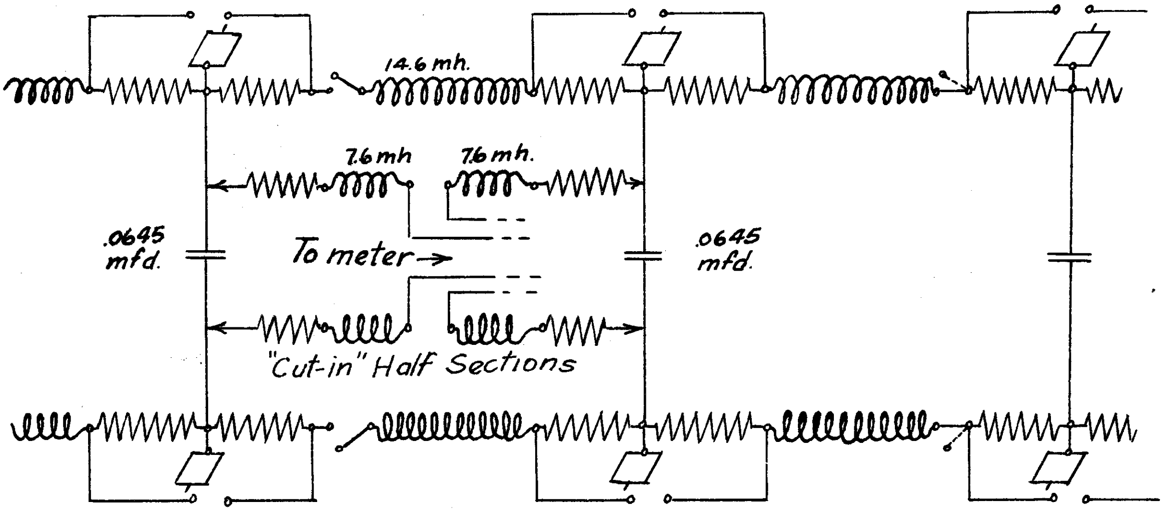
The purpose of this study was to perform tests on an artificial telephone line to illustrate the fundamental principles of telephone transmission line phenomona. The artificial line and the equipment used with it was given to the California Institute of Technology through the generosity of the companies associated with the American Telephone and Telegraph Company. It was designed primarily to be operated as a telephone line although resistors are provided which can be short-circuited to make the line equivalent to a power circuit. The tests described in this study were taken only when the line was connected as a telephone line. The tests included measurements of the distribution of current, voltage, and impedance at a number of frequencies in the voice range with different terminal conditions of the line. The effect of loading was studied by the use of shunt resistors. Resonance, reflection, and the Ferannti effects were also observed and measured. The value of an artificial line lies in approximating or actually reproducing the conditions in a real line, so this line was investigated to see how nearly it behaved as a real line.

Since many artificial lines such as this have been built and tested, their properties are well known and there are probably little of new phenomona to be observed in a line such as this one. The chief value therefore lies in checking transmission line calculations, in being able to observe phenomona that could not be conveniently observed on real transmission lines, and for testing new apparatus.

The apparatus used for testing were, except for a plate voltage supply set, built by the Western Electric Company, and are completely described in the manuals for the apparatus. However, it is worth while to give a brief description of the apparatus here for the manuals cover it in more detail than is required for only a general idea.

The artificial line was assembled by other students previous to this study in an arrangement suggested by the Telephone Company. The line consists of twenty-five "T" sections, each section containing a 14.6 millihenry inductance coil on each side of the line, a shunt condenser of 0.0645 microfarad capacity, and a resistance on each side to bring the loop resistance of the section to approximately 83 ohms. Each section is equivalent to 7.88 miles of No. 12 NBS gauge open-wire non-loaded line, the total equivalent loop length thus being 197 miles. Since the inductance represented in the adjacent halves of two sections is combined into one coil in order to reduce the number of coils required and to save space, a "cut-in" unit made up of two half sections is provided to tap the line at points at which measurements are to be made. The line might have been assembled as a pi (π) line but there is practically no advantage to be gained by this connection for then the condensers would have to be disconnected and be replaced by two half sized condensers when measurments were made. With a pi circuit all the coils can be used thus making the equivalent length of line one section longer but extra condensers are then required. In "Laboratory Experiments in High Frequency Electrical Transmission" by M. P. Weinbach, an arrangement is shown in which the parts can be switched from one connection to the other quite easily. A diagram of a part of the line showing the cut-in section is given below.



One of the difficult problems in connection with the use of alternating current lines in the past has been that of conveniently and quickly measuring the small voltages and currents which are used, particularly in artificial telephone lines. This problem has been satisfactorily solved by the development of the vacuum tube voltmeter-milliammeter. The Western Electric type D-79017 Voltmeter-Milliammeter supplied with the artificial line is a very carefully designed and a flexible instrument for this purpose. Its voltage range is from 3.16 to .001 r.m.s. volts and the current range is from 31.6 to .02 r.m.s.

Milliamperes for frequencies between 100 and 5,000 cycles. Between 200 and 3,000 cycles the accuracy of measurement is about 2 per cent. For frequencies outside of this range the accuracy is not as great but should normally be better than 5 per cent.

The principle of the circuit is based on a comparison method of measuring. The unknown voltage is connected across a high impedance transformer (or for current measurement across a low resistance shunt) and the voltage is then fed into a three stage impedance coupled amplifier whose amplification is variable over a wide range by means of cutout switches and potentiometers in the grid circuits. This amplifier is connected to a detector tube, which has a sensitive d.c. milliammeter in its plate circuit. The amplifier is first adjusted to give a convenient deflection on the meter. Then the input of the amplifier is switched to a potentiometer which has across it a known voltage at the test frequency. When this potentiometer is adjusted to give the same deflection on the meter as before, the voltage applied to the amplifier is the same. The scale of the potentiometer is so graduated that by means of switches and a resistance network, it reads directly in volts and milliamperes or in convenient multiples of them. Voltage is supplied to the voltmeter-milliammeter by the oscillator and is adjusted to the proper value for the calibrated potentiometer by means of the d.c. meter and a sensitive thermo-couple. Since the applied voltage is 3.16 volts, this can be done with the ordinary thermo-couples that are available. A sending circuit which is adjusted to send one milli-watt of testing power into a 600 ohm circuit is also connected to the calibrated circuit. This system of measuring obviously overcomes the disadvantages of errors due to variation of battery voltage, tube characteristics, etc. It is, however, necessary to check the voltage on the calibrated circuit for each reading to get good accuracy. This is particularly true when there are variations in the voltage of the oscillator or if the load on the sending circuit varies. The whole apparatus is substantially assembled in a strong box and weighs about 135 pounds without batteries so that it can be readily moved about.

In order to be certain of the accuracy of the voltmeter-milliammeter its calibration and accuracy was checked before and during tests. The thermocouple calibration was checked using a Weston Standard Laboratory Voltmeter by the method described in the A. T. & T. Manual D 79017, Article 4.12, which was sent with the meter. Only a slight readjustment was found necessary. The accuracy of the voltmeter-milliammeter was checked by the method described in article 4.4 of the same manual.

A Western Electric type 8-A Oscillator supplies testing power. It is capable of supplying from 25 to 35 milliamperes into a 600 ohm load in a frequency range of from 100 to 50,000 cycles. The frequency variations due to temperature changes are less than one per cent and a nearly pure sine wave is generated up to the maximum out put. It is satisfactory for most purposes without the use of a "C" battery but the wave form is still further improved if a "C" battery is used. The oscillating circuit is a conventional vacuum tube generator and is made of inductance coils and condensers which can be cut in and out of the circuit by means of dial switches to vary the frequency. The calibration chart furnished with the oscillator gives the dial settings only for frequencies twenty cycles apart. For some work it is desirable to get nearly the exact frequency so graphs, which are now in the manual for the oscillator, were prepared. It was found that on the lower frequencies the variations in condenser settings (which are divided in tenths) could be plotted against frequency for a given coil setting. This is not quite as accurate as determining each setting by the use of an impedance bridge as described in the oscillator manual but the fact that the curves were smooth and continuous indicated that the settings taken from the curve are nearly as accurate as those taken from the chart. The oscillator, like the voltmeter-milliammeter, is substantially assembled in a strong box so that it is easily portable.

The oscillator requires twenty-four volts for the filament supply and 130 volts for the plate voltage supply. Since the plate current is about 60 milliamperes, the life of dry batteries would be short, so a plate supply rectifier to operate from the a.c. lighting circuit was built by Mr. Ingersoll. Considerable difficulty was encountered this year in using it however, for while it would no doubt be quite satisfactory as a plate voltage supply for a radio set, certain characteristics of the voltmeter-milliammeter make its use less satisfactory than the use of batteries. Due to the fact that slow variations of the a.c. voltage cannot be filtered out, the d.c. voltage varied. These were sufficient to make readings on the voltmeter-milliameter impossible. However, by supplying the voltmeter-milliammeter with a battery and the oscillator only with the rectified voltage readings could be taken. The varying supply voltage, of course, varied the voltage output of the oscillator which in turn produced variations on the meter and changed its calibration. To partially compensate for this the rectifier was rewired to give better voltage regulation and to permit "C" voltage for the oscillator to be taken from it. An increase in voltage then made the grid more negative and tended to lower the oscillator output voltage, and vice versa. A storage battery would have given better results and the dry battery now necessary could have been dispensed with.

Another piece of apparatus furnished with the artificial line is a Western Electric 1-B Impedance Bridge. It is a standard type bridge for measuring the resistance and reactance of telephone apparatus over the voice frequency range (approximately 100 to 3000 cycles). It is used in connection with the oscillator and a receiver. The bridge has a range of from + 560 to -560 millihenries, the "+" sign representing inductive reactance and the "-" sign capacitive reactance.

The theory of transmission lines has been quite completely developed and is presented in several textbooks. Practically all of these texts use the hyperbolic method of calculation so it is not necessary to repeat the development of the formulae here. The notation and equations given in "Principles of Transmission in Telephony" by M. P. Weinbach will be used in the discussion of the results. The equations required will be introduced as required.

RESULTS

The graphs which follow are referred to by plate numbers and are grouped together. Following each plate there is a data sheet from which the graph was plotted. The graphs showing the distribution of current, voltage, and impedance against line length are plotted in per cent of the sending end values since this makes the curves easier to interpret and the scales are arbitrary anyhow. It happens that each small division on the graph paper is one eighth of a section length and since this is 7.88 miles, it is easy to read approximate distances in miles. Distances are plotted in section units,however, since it would be bothersome to plot them exactly into miles.

Plate I shows the distribution of current, voltage and impedance along a line terminating in a load equal to its characteristic impedance, at a frequency of 796.4 cycles ($2\pi f = 5,000$). This frequency has been chosen by the telephone companies as a good average value. If a line is of infinite length, removing some finite length of line, say one unit, should not change its characteristic impedance. That is, for finite distances from the sending end the characteristic impedance does not vary. Thus if a line of finite length terminates in an impedance equal to its characteristic impedance it should behave like a infinite line. The current and voltage will then decay exponentially according equations (6) and (7) given in Weinbach.[1]

$$E = E_s \epsilon^{-pS_s} \qquad (6)$$

$$I = I_s \epsilon^{-pS_s} \qquad (7)$$

The data sheet gives the calculated values as well as the measured values. Due to the fact that the load used was not exactly equal to the characteristic impedance these values do not check as closely as they might. The characteristic impedance was capacitive and it was not possible to get the exact value desired, however it was within ten per cent so the error due to this should be small. Since the impedance remains constant along the line the phase angle between current and voltage does not change. This as well as the exponential decay is also shown on plate II, which shows a polar diagram for this condition. The length of the vector represents the real part of the voltage or current and the angular displacement the imaginary part. That the attentuation increases linearly with distance, is shown by the curve on plate I which gives the loss in transmission units at each section along the line. The transmission unit as chosen by the American Telephone and Telegraph Company is defined by the equation,

$$\text{Loss in T.U.'s} = 10 \log_{10} \frac{E_sI_s}{E_rI_r}$$

This gives the loss in units which are nearly equal in value to the former unit of miles of standard cable and are more convenient to use. The condition that a line act like one

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[1] "Principles of Telephony in Transmission" by M.P.Weinbach.

of infinite length is that there are no reflections and thus no phase shift between the current and voltage. Since the characteristic impedance varies with frequency, it is not possible to have a line of finite length act as an infinite line at all frequencies, but it will be seen later from plates 15 and 16 that the value calculated for 796.4 cycles is not much different from the others except for frequencies below about 500 cycles.

Plate 3 shows the current, voltage, and impedance distribution at 796.4 cycles with the end of the line open. The current here falls to zero at the end of the line and is reflected back along the line producing standing waves instead of the normal exponential decay. The rapid decrease of current at the end of the line generates a higher voltage there due to the rapid decrease in flux associated with the current. The voltage and current curves clearly show that both quantities decay exponentially with the distance from the sending end and also vary as sine and cosine functions of the distance. By observing the distances between two successive intersections of the current and voltage, the wave length at this frequency can be easily determined. It is, according to the curves 218 miles, by calculation it is 220 miles. The impedance given by the ratio of volts to amperes, varies along the line because of the phase shift, becoming infinite at the end of the line. The attentuation in transmission units is plotted against line length. It is interesting to observe that the rate of attenuation is about the same in the first half of the line as it is for the infinite line given in plate I. This is because lines that are long electrically show little effect of the terminal conditions on the sending end impedance, particularly when the line losses are great. The values of current and voltage were also calculated for this value of frequency and terminal conditions and show a good agreement with the measured values. There are occasional points that are clearly in error and these may have been due to errors in reading or to poor connections. These errors could be eliminated by taking the measurements over and in fact this was done for many of the tests when there were too many errors in the curve. The voltage and current distributions always come out as smooth curves unless there is some kind of discontinuity in the line. In this case it is very easy to see where this discontinuity is from the graph and it can then be located on the line. This is not a practical method, however, for use on actual line since it would be very difficult to measure the current and voltage distribution along a real line. On most of the curves it can be seen that every other value of current is slightly in error. This is due in some way to the method of using the half sections for measurements. Since they are reversed every time it appeared that this might be due to the fact that they are not symetrical but measurements of resistance showed that this was not true. It may be due to the fact that at every other section the coils of the section are connected at one side to the sending end and the other times to the receiving end and may draw a small charging current due to their distributed capacity. This error is small, however, and is not very objectionable.

It is quite tedious to make calculations for the current and voltage at all points along the line so after it was found that these values checked well with the measured, there would be very little purpose of making any more. A polar diagram of the open circuited line is also given on plate 2.

Plate 4 which gives the curves for the same conditions as plate 3, except that the end is short circuited, can be seen to be the same in general character but the voltage and current curves are now interchanged and the impedance curve for the short circuit condition is the reciprocal of that for the open circuited line. This is indicated by the equations for the current and voltage along the line under the two different conditions and the two quantities are here also interchanged. This shows then that there is no particular purpose in measuring the distribution under each of these conditions when one of them is known. In the case of the short circuited line the voltage wave is reflected.

Plate 5 shows the curves for the distribution of current voltage and impedance at 100 cycles. Due to the fact that the wave length of this line is long (1725 miles) and the damping is great,there is little reflection. One hundred cycles is below the frequency used for ordinary telephone conversation but it is desirable to transmit this frequency when high quality such as for radio broadcasting is desired.

Plate 6 shows the distribution of current, voltage and impedance at 220 cycles, which shows more pronounced reflection than at 100 cycles. This is true because here the wave length is about 1016 miles, thus a quarter wave length is equal to 256 miles and at this wave length the reflection on open circuit would be greatest if the line length were 256 miles. Since it is near this length the reflection is large.

Plate 7 shows the distribution of current, voltage and impedance at 1000 cycles. Measurements of the distances between the intersections of the current and voltage waves gives a wave length of 178 miles, which is less than the length of line. It is evident that at every odd quarter wave length the voltage is a maximum and the current is a minimum, while the reverse is true at distances equal to even quarter wave lengths. Plate 8 also shows the distribution at 1000 cycles but with the voltages and currents plotted in volts and milliamperes.

The purpose of this is for comparison with the preceding curve to show the advantage of the scales used there.

Plate 9 shows the distribution of current, voltage and impedance with the line short circuited, at a frequency of 3,000 cycles. The wave length as measured from the graph is about 57 miles. The phenomona mentioned for the other curves are also evident here. This is considered the upper limit of frequency necessary for good intelligibility in conversation, but for good quality a cut off frequency of about 5,000 cycles may be desirable.

Plate 10 shows the distribution of current, voltage and impedance at 5,000 cycles which is about the limit at which a good distribution curve can be drawn from measurements. This is because the length of each section is nearly equal an eighth of a wave length and the curves to a certain extent have to be estimated. For measurements taken only at the end of the line this limitation is not present, but this value is also at the limit of the range of the vacuum tube voltmeter-milliammeter, and practically, except for carrier current frequencies which go much higher, no measurements need to be made. The Calculated cut-off frequency of the line is 7,300 cycles so measurements could not be carried much farther on this line. This cut-off frequency is based on a resonance phenomona and because of resistance in the circuit it is not sharp. However the correction for the readings taken at 5,000 cycles is wery small and can be neglected, hence the line can be considered equivalent to an actual line up to this frequency. This is discussed in detail in "Artificial Electric Lines" by Kennely and also in "Transmission Circuits for Telephonic Communication" by Johnson.

Plate 11 which shows the distribution of current, voltage and impedance at 215 cycles, for the line open circuited, the frequency at which the line length is equal one fourth the natural wave length. This is indicated by the fact that the current and voltage waves intersect at almost the midpoint of the line. The value for the resonant frequency was calculated according to the exact formula given on page 50 of "Principles of Transmission in Telephony" by Weinbach. The measured value checks with the calculated value. The value as given by the approximate formula is 236 cycles which is close enough if the line resistance is low, that is, if the line is assumed to be composed only of inductance and capacitance. The graph shown in plate 12 which gives the distribution at 860 cycles at which the natural frequency should be equal to the line length. However, it does not check as well as the 215 distribution curve. The curve indicates that the frequency should be a little lower.

Plate 13 shows a very interesting effect which is encountered in lines which are short electrically, namely, the Ferannti effect. This is the effect in which the voltage is higher at the receiving end of the line than at the sending end. The maximum value occurs when the line length is equal to one fourth of the natural wave length. However, if the attenuation is too great no Ferranti effect will be observed above thedefinite length of line, when the attenuation constant is less than 56.09 per cent of the wave length constant. This is shown very well by the curves. By looking at plate 5 showing the distribution on open circuit at 100 cycles it is seen that since the receiver voltage is less than the sending voltage there will not be a Ferranti effect for the full length of line. However, if a little lower frequency were used a Ferannti effect might be observed, for the attenuation decreases with decrease of frequency although the Ferranti effect also decreases. Another interesting effect can be obtained by short circuiting the line at various lengths, when curves will be obtained which correspond to the Ferannti effect for voltages.

One of the distorting effects on unloaded lines is the unequal attenuation of the different frequencies. If the line is loaded the maximum amount, all frequencies are attenuated alike. However, lines are seldom loaded this much. The theory at first published by Heaviside, states that for a distortionless line the product of the line resistance and shunt capacity must be equal to the product of the line inductance and the shunt conductance. In the ordinary telephone line this result is attained through increasing the line inductance by lumped loading in the form of low resistance inductance coils. The same result could be obtained by decreasing the decreasing shunt capacitance or by increasing the shunt conductance.

The effect of loading was studied by inserting shunt resistors across the line at the section junctions. It happens that telephone lines are normally loaded at points 7.88 miles apart so it is quite easy to imitate the effect of regular loading by putting shunt resistors at the middle of each section, since the inductance in an artificial line is already in lumps. This was accomplished by soldering clips on the condensers to hold the resistors, which were the ordinary grid leaks such as are used in radio sets. These had to be measured and sorted to get resistors which came within 5% of an average value.

Calculations show that in the case of the line being studied the shunt resistors should be 25,000 ohms each at each section for distortionless transmission. This is about a thousandth of the leakage resistance of a real line. In order to approximate an actual loading condition,resistors having 65,700 ohms resistance were used. It would, however, be quite easy to test other values of resistors in the same clips. The sending end receiving volt-amperes were measured for a range of frequencies from 100 to 2,600 cycles with the line terminating in a load equal to the characteristic impedance at 796.4 cycles, with the resistors on the line and without the resistors. Using this data the attenuation in transmission units was calculated and plotted against frequency. The results of this test were disappointing as the only conclusion evident was that the attenuation is greater with the resistors. The line is rather short to require loading for correcting attenuation variations and since the transmission unit is a logarithm the variations appear small on the graph. The distribution of current and voltage along the line was measured with the resistors on the line but the percent variations were so near those without the resistors that there is no need to plot it.

Plates 15 and 16 show the results of measuring the characteristic line impedance at open circuit and short circuit by using the impedance bridge. The resistance and inductance were measured over a range of frequencies from 300 to 4,200 cycles. From this data the short circuit and open circuit impedance could be calculated. Weinbach gives the formula for characteristic impedance as,

$$Z_o \epsilon^{j\alpha} = \sqrt{Z_{sc}\epsilon^{j\theta_1} \times Z_{oc}\epsilon^{j\theta_2}}$$

The characteristic phase angle α is also plotted. It is seen that Z_o represents the mean of the open circuit and the short circuit impedance and also that above about 800 cycles its value is nearly constant. The line presents capacitative reactance at all frequencies but values plotted in millihenries since these were measured on the bridge. Their values can, however, easily be converted into microfarads.

CONCLUSIONS

The measured values as shown on the plates show a good agreement with the calculated values and show that this artificial line, for purposes of tests, is equivalent to a real line up to 5000 cycles. The data on the use of shunt resistors for decreasing distortion is not conclusive for the line was not long enough to have much distortion. The theory indicates that this method would produce the desired results, but at the expense of a large decrease in efficiency.

This report represents only a small portion of the possible number of tests which can be performed with this apparatus but it illustrates the main points in the theory of electric wave transmission over lines. To take up every point in connection with the theory would require much more time than was available. A considerable amount of time was spent in becoming familiar with the manipulation of the apparatus and in getting it to operate satisfactorily.

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The writer wishes to express his appreciation for the assistance given him by Mr. Hossack and Mr. Biddle during the second term. He also wishes to thank Prof. Sorensen and Mr. Maxstadt for suggestions and criticisms.

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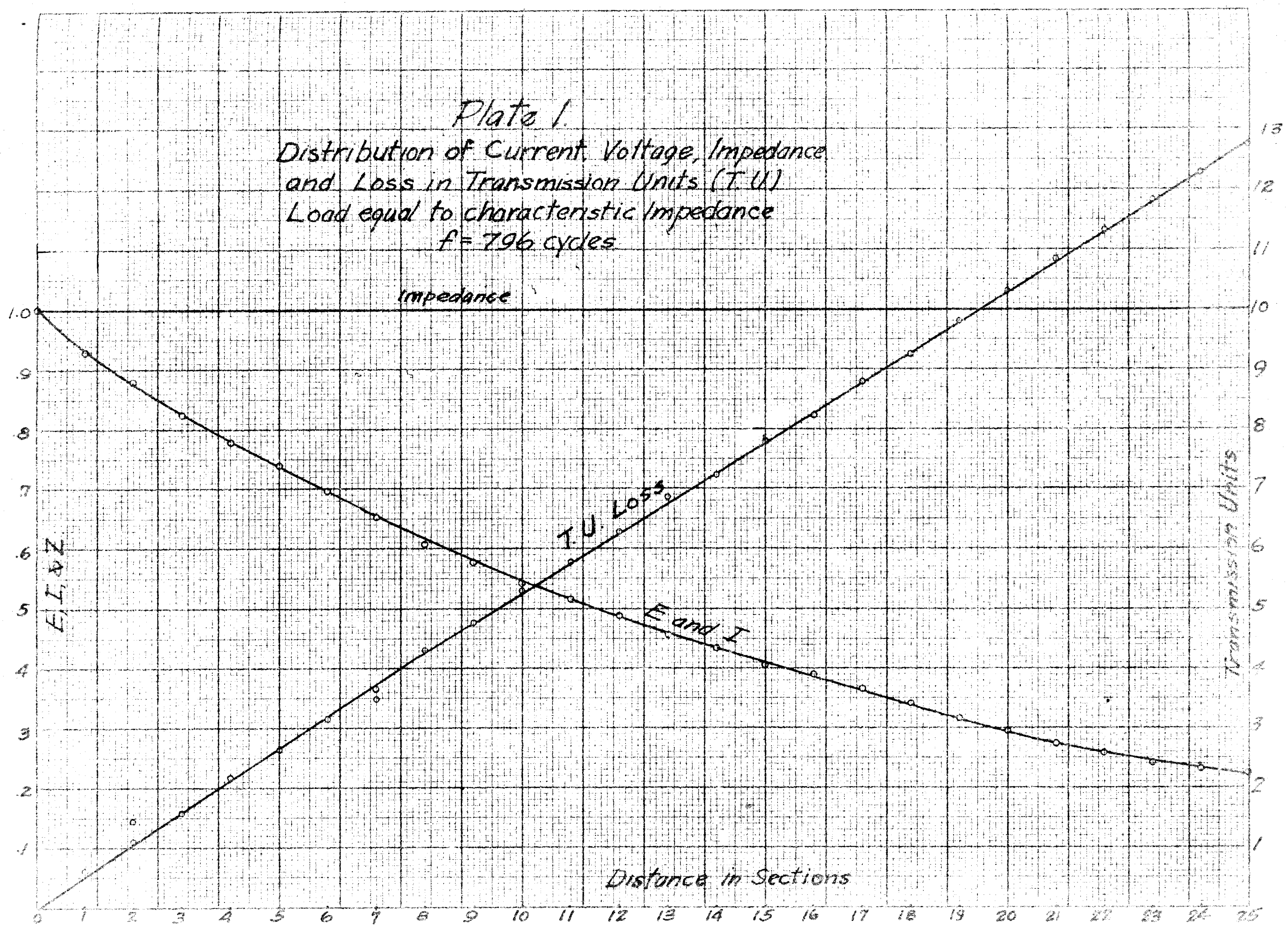


Plate I.

Distribution of Current, Voltage, Impedance and Loss in Transmission Units (T.U.)

Load equal to characteristic Impedance

f = 796 cycles

Data Sheet for Plates 1 & 2

Distribution of Current, Voltage and Impedance along a Line Terminated in a Load equal to its Characteristic Impedance, f = 796.4 cycles

S_s	E_s volts	I_s m amp	E_o/E_s	I_o/I_s	aS_s		vS_s degrees	T.U. Loss
0	.850	1.170	1.000	1.000	0	1.000	- 0	0
1	.788	1.12	.928	.948	.0588	.943	-12.9	.60
2	.750	1.04	.883	.880	.1176	.890	-25.5	1.10
3	.705	.995	.827	.845	.1768	.838	-38.5	1.58
4	.661	.925	.777	.783	.2355	.791	-51.5	2.17
5	.622	.880	.732	.745	.2945	.745	-64.4	2.64
6	.588	.825	.696	.694	.3535	.703	-77.0	3.14
7	.550	.815	.651	.670	.4125	.662	-90.0	3.48
8	.520	.715	.616	.606	.472	.624	-103.0	4.30
9	.490	.682	.580	.577	.551	.588	-116.0	4.76
10	.462	.640	.546	.542	.589	.555	-128.6	5.30
11	..455	.608	.515	.516	.647	.524	-141.8	5.78
12	.409	.575	.484	.487	.706	.494	-154.4	6.28
13	.382	.540	.452	.451	.765	.465	-167.3	6.86
14	.370	.513	.438	.435	.824	.438	-180.0	7.22
15	.347	.480	.411	.406	.882	.414	-193.2	7.78
16	.327	.460	.387	.389	.942	.393	-206.0	8.25
17	.307	.430	.363	.364	1.001	.367	-219.0	8.80
18	.297	.402	.35[illegible]	.340	1.060	.346	-231.5	9.24
19	.280	.373	.332	.316	1.118	.327	-244.8	9.81
20	.254	.322	.301	.273	1.236	.291	-270.4	10.31
21	.254	.322	.301	.273	1.236	.291	-270.4	10.87
22	.243	.303	.285	.257	1.295	.273	-281.5	11.32
23	.229	.285	.271	.241	1.354	.258	-296.3	11.85
24	.215	.275	.254	.233	1.413	.243	-309.2	12.28
25	.187	.260	.222	.220	1.472	.229	-322.2	13.13

$p = \sqrt{zy} = .2325\ \epsilon^{j\,75^\circ 22'}$

$a = .0588$ hyp. rad.

$v = .2248$ " "

$\lambda = \frac{2\pi}{v} = 220$ miles

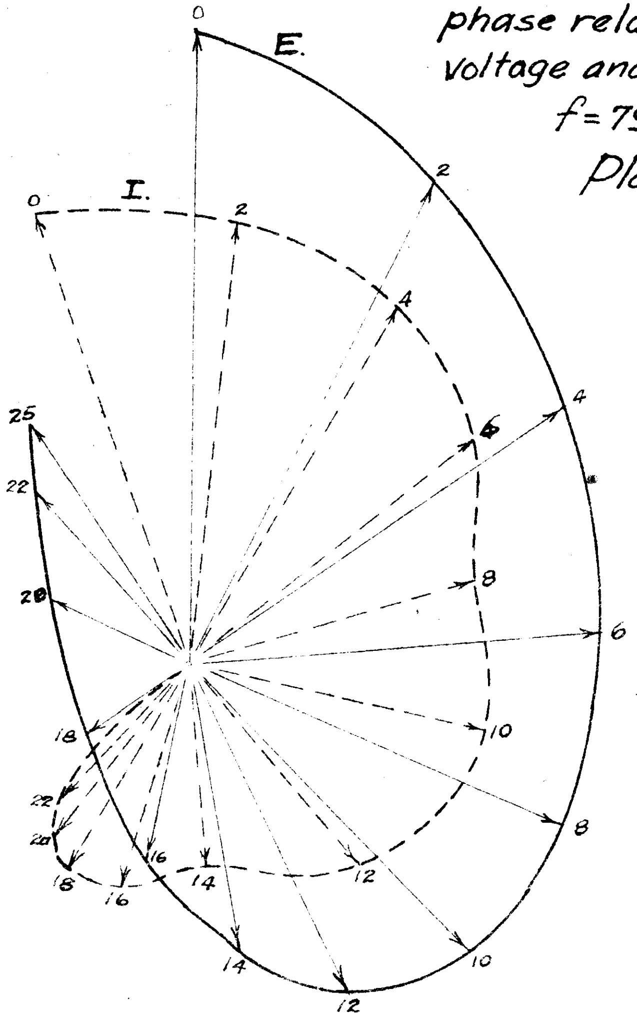
$V = \frac{2\pi f}{v} = 175{,}000$ miles per second

$Z_0 = 720.5\ \epsilon^{-j\,14.5^\circ}$

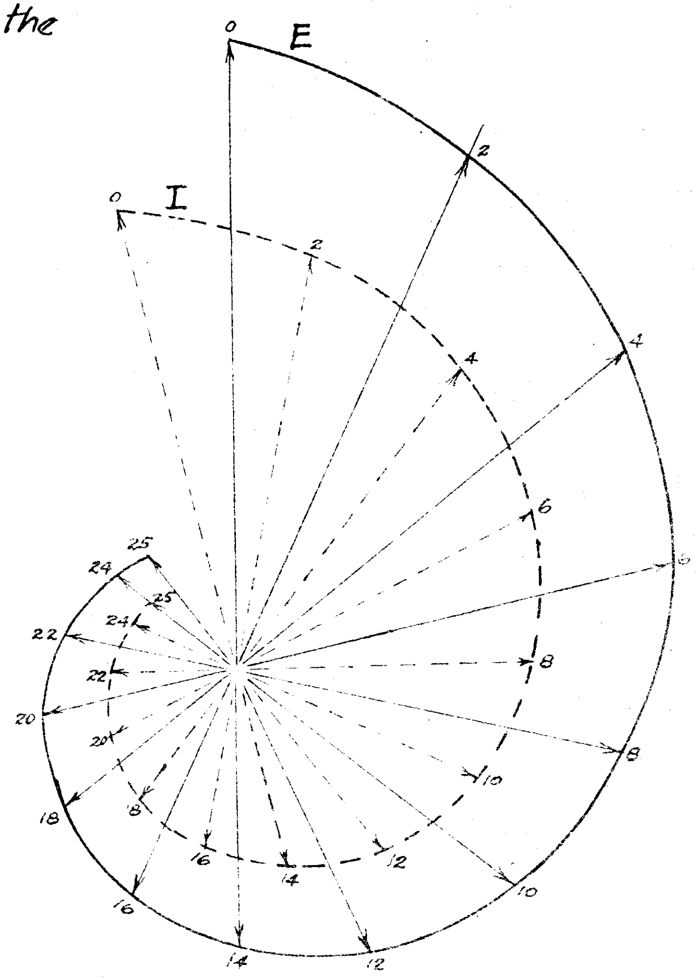
Polar Diagrams showing the phase relation between the voltage and current

f = 796.4 cycles

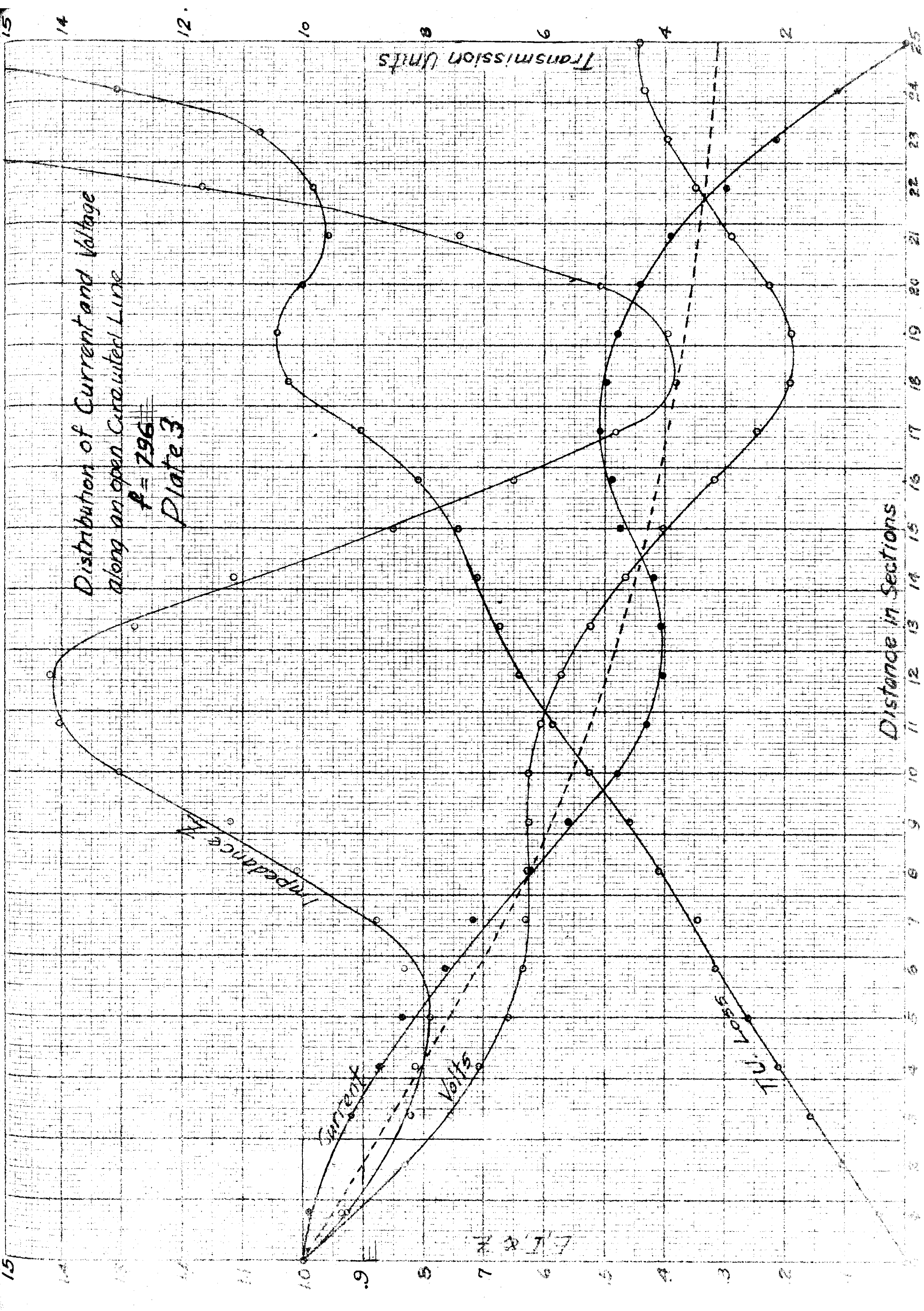
Plate 2



End Open Circuited



End terminated in characteristic Impedance (Infinite Line)



Distribution of Current and Voltage along an open Circuited Line

f = 796~

Plate 3

Data sheet for Plates 2 & 3

The Distribution of Current, Voltage, and Impedance along an Open Circuited Line at 796.4 cycles

						(calculated)		Phase	T.U.
S_s	E_s	I_s	E_0/E_s	I_0/I_s	E/I	E_s	I_s	Angle	Loss
0	.862	1.130	1.000	1.000	1.000	1.000	1.000	20.5	0
1	.791	1.120	.927	.990	.936	.926	.970	21.5	.41
2	.716	1.070	.831	.947	.831	.858	.937	22.7	1.04
3	.651	1.040	.755	.920	.821	.771	.907	25.9	1.57
4	.611	.985	.709	.872	.814	.715	.869	30.6	2.09
5	.547	.862	.635	.765	.848	.656	.756	35.2	3.15
5	.568	.945	.659	.836	.788	.687	.818	30.6	2.58
7	.543	.810	.630	.717	.878	.642	.691	39.2	3.45
8	.540	.702	.627	.621	1.010	.639	.618	40.9	4.10
9	.538	.631	.625	.558	1.12	.644	.549	41.6	4.58
10	.537	.542	.624	.479	1.30	.632	.477	34.8	5.24
11	.520	.480	.603	.430	1.40	.616	.428	26.0	5.86
12	.492	.453	.571	.401	1.42	.583	.405	15.1	6.41
13	.450	.460	.522	.407	1.28	.535	.411	3.6	6.72
14	.402	.473	.466	.418	1.115	.472	.432	-4.2	7.10
15	.346	.522	.401	.473	.850	.401	.464	-7.7	7.42
16	.273	.552	.317	.488	.650	.322	.491	-5.8	8.10
17	.212	.572	.246	.507	.465	.244	.507	6.9	9.05
18	.164	.563	.190	.498	.381	.193	.505	26.0	10.22
19	.163	.540	.189	.478	.391	.191	.480	55.2	10.43
20	.194	.498	.225	.442	.510	.238	.442	77.1	10.01
21	.248	.431	.288	.390	.740	.301	.380	89.5	9.60
22	.300	.336	.348	.298	1.168	.363	.301	95.4	9.85
23	.342	.241	.397	.213	1.865	.412	.210	109.1	10.72
24	.375	.127	.435	.112	3.87	.443	.107	100.8	13.10
25	.381	.000	.442	.0	∞	.454	.0	29.0	∞

$R = 81.86$ ohm

$X = L\omega \doteq .0292 \times 5000 = 146.0$ ohms

$C\omega = .0645 \times 10^{-6} \times 5000 = .3225 \times 10^{-3}$ mho.

$z = R + jL \doteq 167.4\,\epsilon^{-j60^\circ 45'}$

$y = -j\omega C = .3225 \times 10^{-3}\,\epsilon^{j90.^\circ}$

$$Z_0 = \sqrt{\frac{z}{y}} = 720.5\,\epsilon^{-j14^\circ 38'}$$

$$p = \sqrt{zy} = .2325\,\epsilon^{j75^\circ 22'}$$

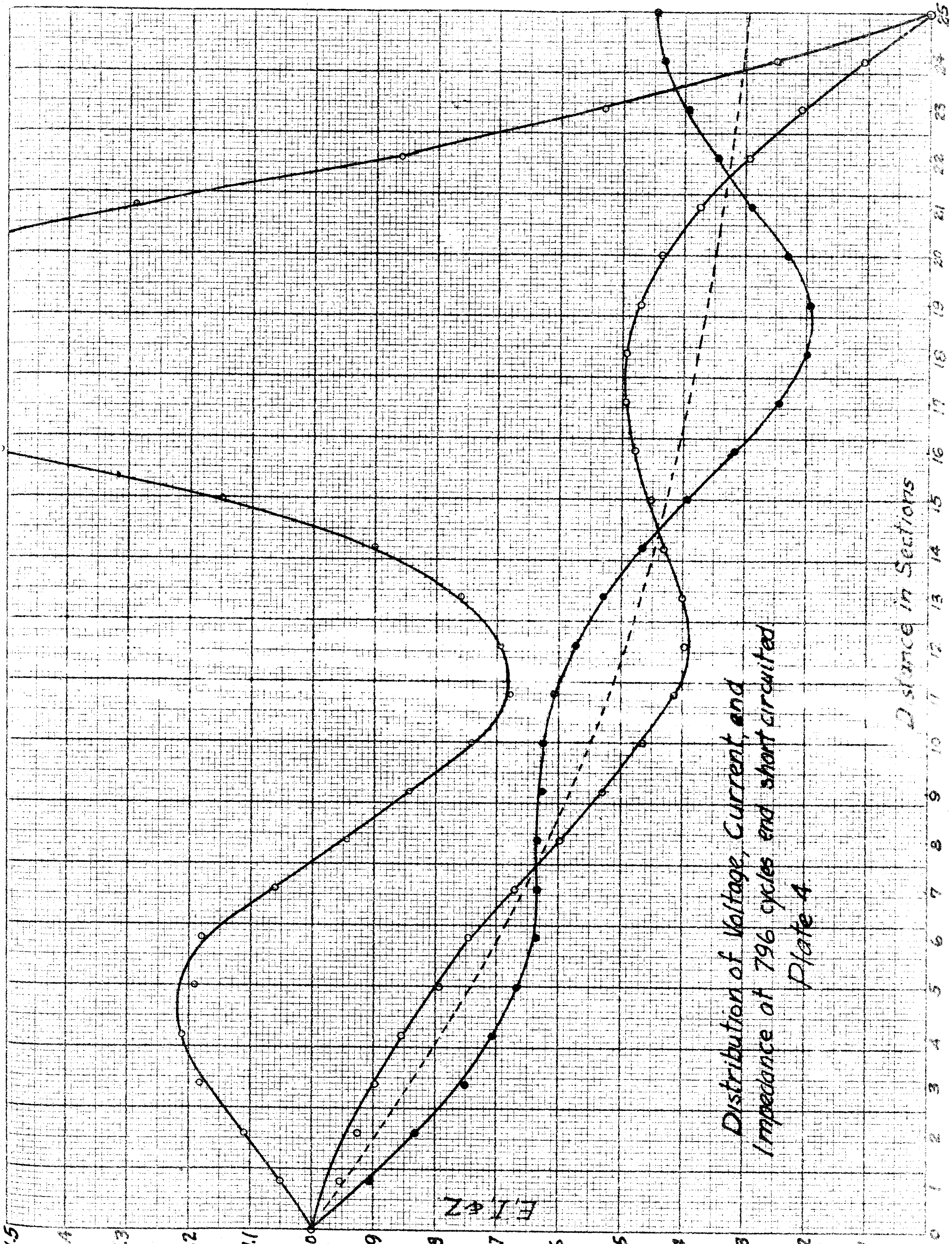
$a = p \cos\delta = .0588$ hyp. rad.

$v = p \sin\delta = .2248$ " "

$$\lambda \doteq \frac{2\pi}{v} = 27.9 \text{ sections} = 220 \text{ miles}$$

$$V = \frac{\omega}{v} = 175,000 \text{ miles per second.}$$

$$e = E_0 \frac{\cosh p S_s}{\cosh p S_0} \qquad i = I_0 \frac{\sinh p S_s}{\sinh p S_0}$$



Distribution of Voltage, Current, and Impedance at 796 cycles and short circuited
Plate 4.

Data Sheet for Plate 4

Distribution of Currant, Voltage, and Impedance, along a Short Circuited Line at 796.4 cycles.

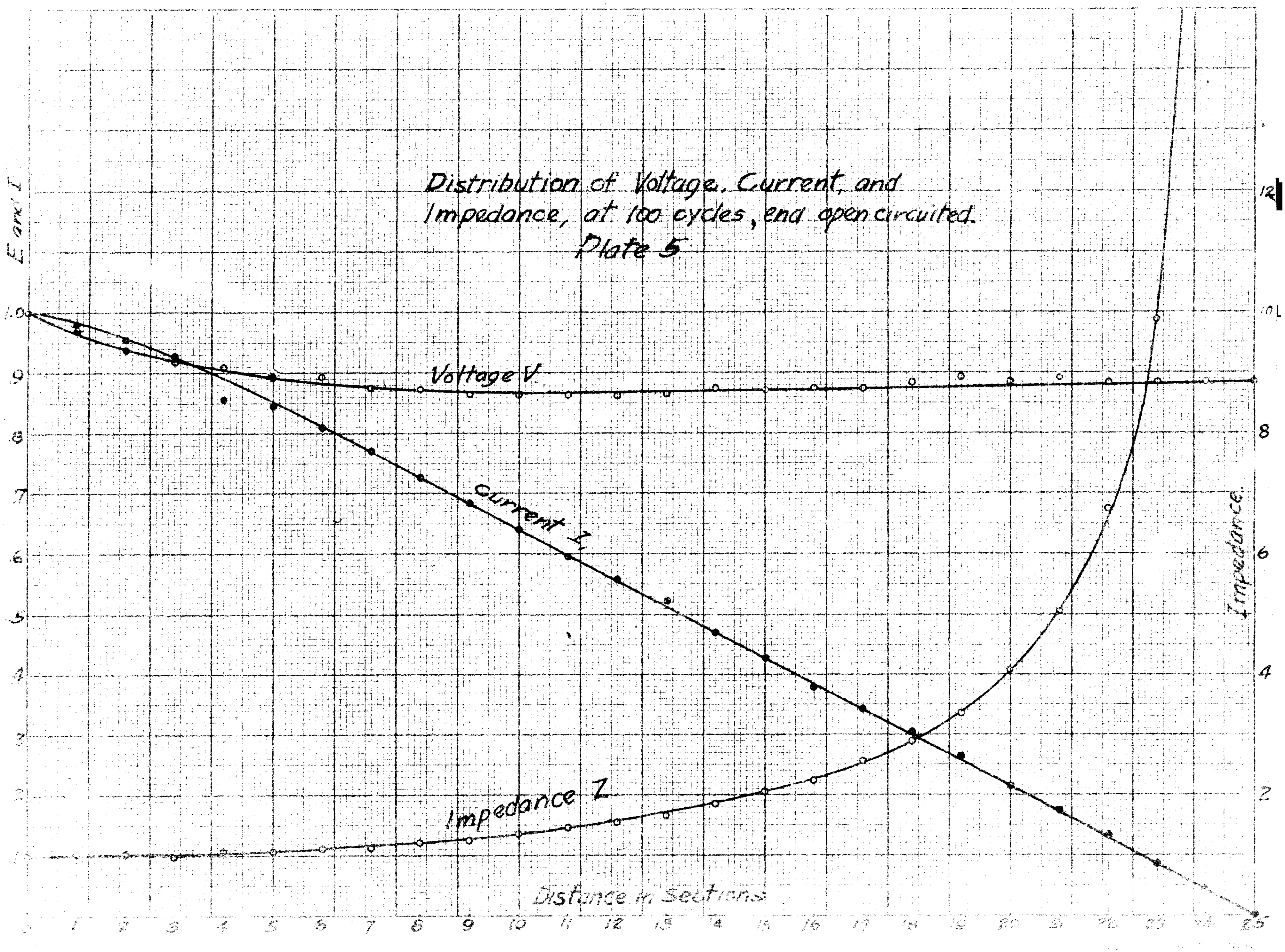
S_s	E_s	I_s	E/E_s	I/I_s	$E/I = Z$
0	.860	1.225	1.000	1.000	1.000
1	.820	1.110	.953	.832	1.050
2	.795	1.020	.924	.751	1.110
3	.765	.918	.889	.707	1.182
4	.737	.865	.856	.668	1.210
5	.682	.818	.793	.668	1.19
6	.642	.778	.746	.634	1.18
7	.577	.778	.671	.634	1.06
8	.515	.768	.598	.635	.943
9	.456	.759	.530	.627	.845
10	.400	.760	.465	.628	.742
11	.355	.745	.412	.608	.678
12	.342	.700	.397	.572	.695
13	.344	.645	.400	.527	.760
14	.362	.572	.421	.467	.900
15	.388	.482	.452	.394	1.147
16	.412	.387	.479	.316	1.515
17	.426	.297	.495	.243	2.04
18	.426	.242	.495	.198	2.50
19	.404	.236	.470	.193	2.44
20	.375	.282	.436	.230	1.895
21	.322	.355	.374	.290	1.290
22	.253	.423	.294	.345	.852
23	.180	.482	.209	.394	.530
24	.092	.528	.107	.432	.250
25	.0	.546	0	.446	0

$$E_s = E_0 \frac{\sinh p S_s}{\sinh p S_{25}}$$

$$I_s = I_0 \frac{\cosh p S_s}{\cosh p S_{25}}$$

$$\lambda = \frac{2\pi}{v} = 220 \text{ miles}$$

From graph λ = 223 miles.



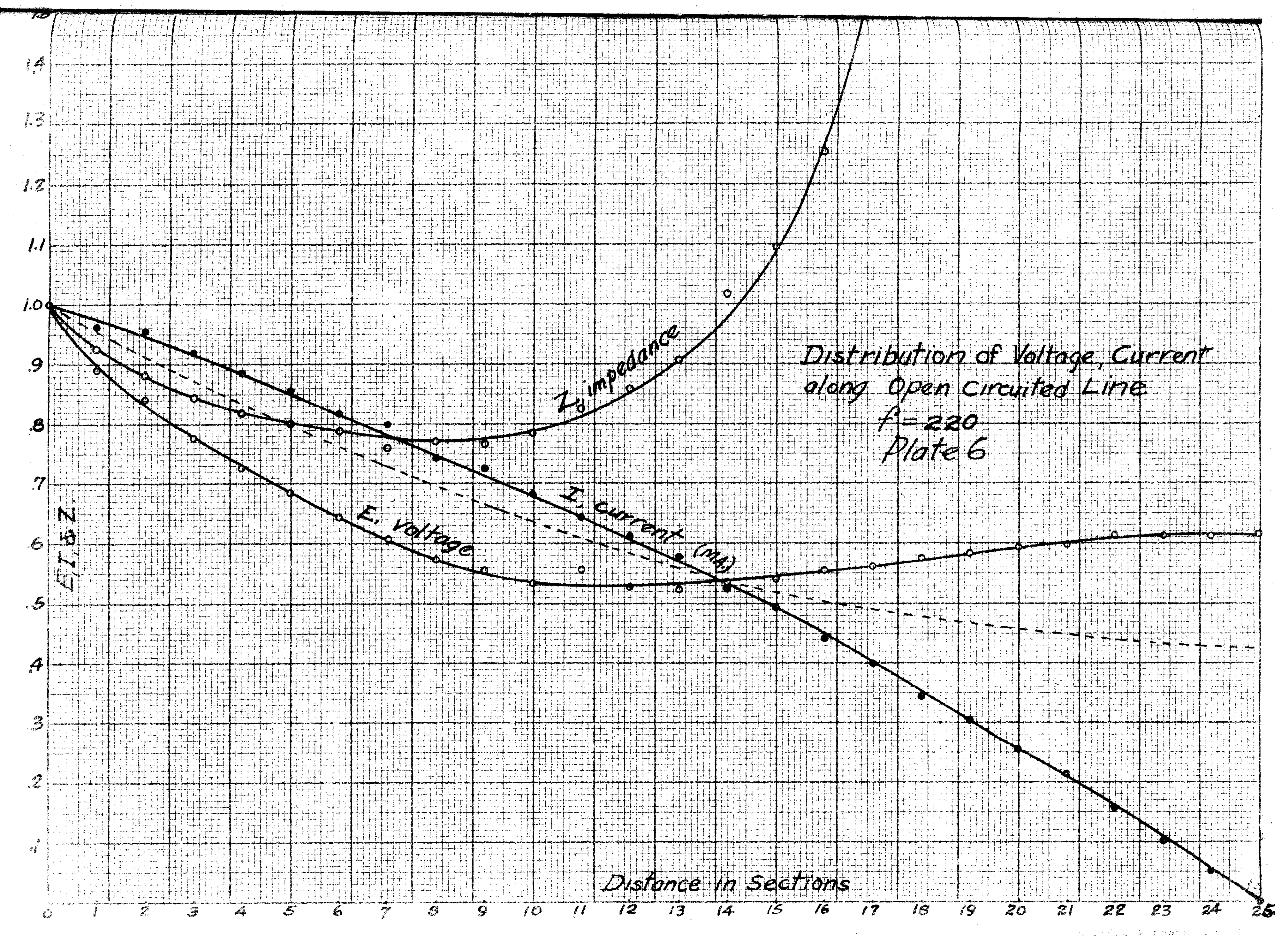
Distribution of Voltage, Current, and Impedance, at 100 cycles, end open circuited.
Plate 5

Data Sheet for Plate 5

The Distribution of Current, Voltage and Impedance Along an Open Circuited Line at 100 cycles

S_s	E_s	I_s	E/E_s	I/I_s	$E/I = Z$
0	1.14	.938	1.000	1.000	1.000
1	1.12	.810	.980	.970	1.010
2	1.09	.880	.954	.938	1.015
3	1.05	.870	.918	.927	.990
4	1.04	.800	.910	.853	1.065
5	1.02	.795	.893	.854	1.058
6	1.02	.752	.893	.801	1.112
7	1.00	.723	.875	.770	1.136
8	.995	.678	.871	.723	1.203
9	.990	.641	.866	.683	1.268
10	.987	.601	.864	.640	1.348
11	.987	.558	.864	.595	1.450
12	.985	.525	.862	.559	1.540
13	.990	.490	.866	.522	1.660
14	1.00	.442	.875	.471	1.856
15	.995	.401	.871	.427	2.035
16	1.000	.364	.875	.388	2.25
17	1.000	.321	.875	.342	2.56
18	1.01	.287	.884	.306	2.89
19	1.02	.250	.892	.266	3.35
20	1.01	.203	.884	.216	4.09
21	1.02	.165	.892	.176	5.06
22	1.02	.124	.892	.132	6.75
23	1.01	.0840	.884	.0895	9.88
24	1.01	.0404	.884	.0426	20.7
25	1.02	.0	.892	0	∞

Wave length 1750 miles.

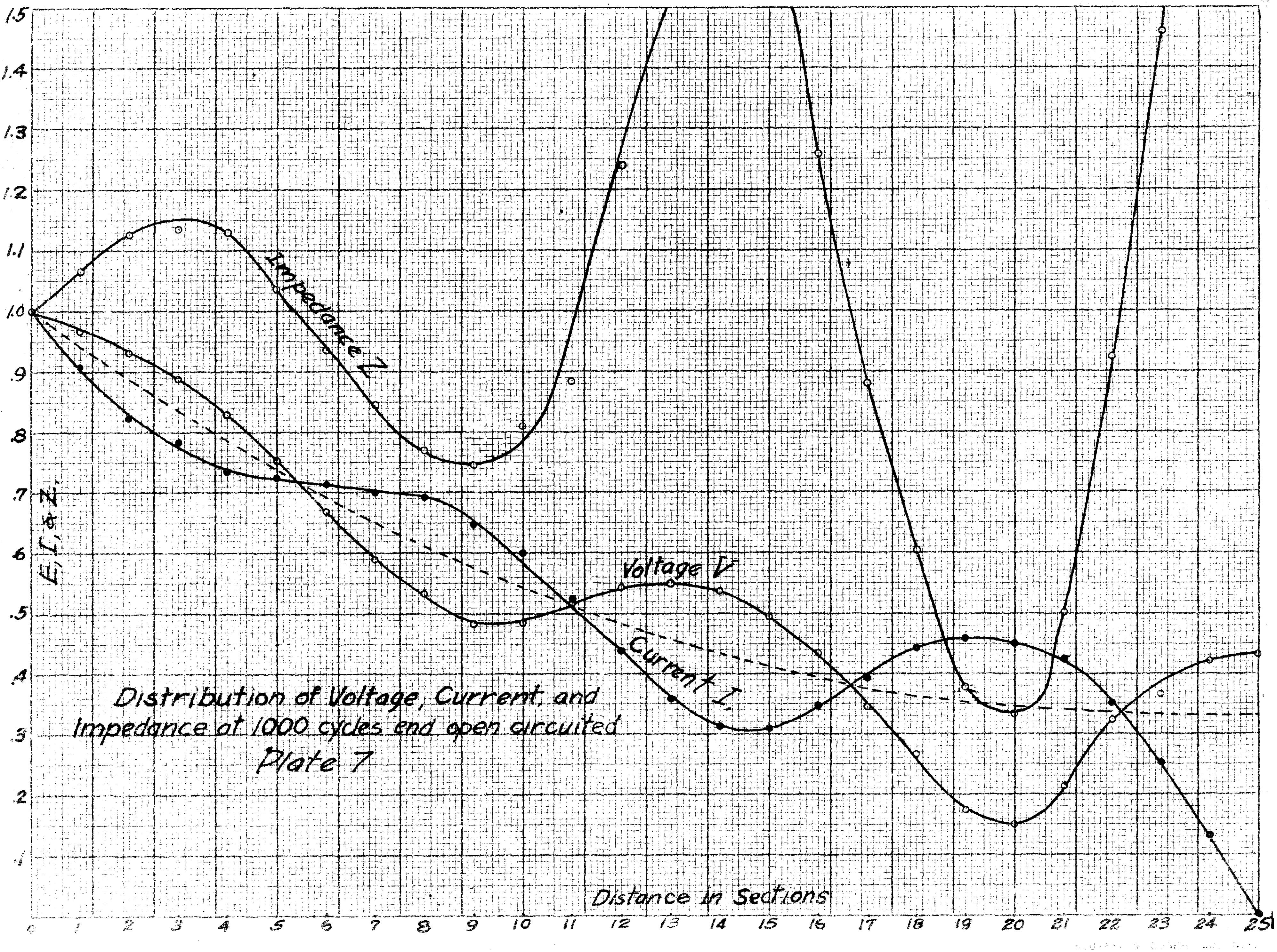


Distribution of Voltage, Current along Open Circuited Line
f = 220
Plate 6

Data Sheet for Plate 6

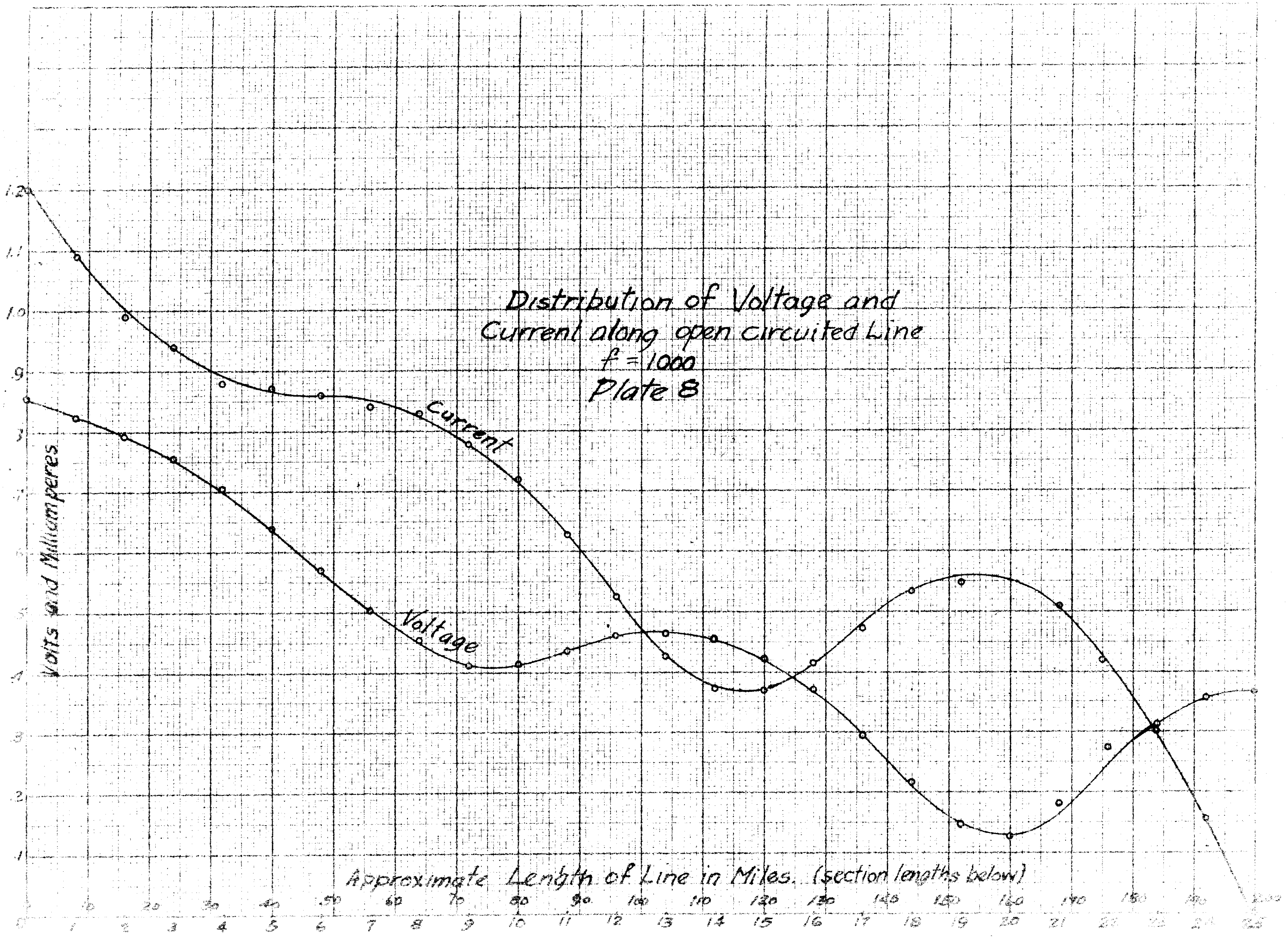
The Distribution of Current Voltage and Impedance along an Open Circuited Line at 220 cycles.

S_s	E_s	I_s	E/E_s	I/I_s	$E/I = Z$
0	.977	1.07	1.000	1.000	1.000
1	.870	1.03	.890	.962	.926
2	.822	1.02	.841	.953	.882
3	.760	.985	.778	.920	.845
4	.710	.950	.727	.887	.820
5	.670	.917	.686	.857	.801
6	.630	.877	.645	.819	.788
7	.593	.857	.607	.800	.760
8	.560	.796	.573	.743	.771
9	.542	.777	.556	.726	.766
10	.522	.730	.535	.682	.785
11	.505	.689	.550	.644	.864
12	.515	.656	.527	.615	.860
13	.512	.620	.524	.578	.907
14	.524	.564	.536	.526	1.018
15	.528	.528	.541	.494	1.095
16	.540	.472	.553	.441	1.252
17	.548	.428	.562	.400	1.405
18	.563	.370	.576	.345	1.668
19	.570	.327	.584	.305	1.915
20	.580	.273	.594	.255	2.33
21	.583	.228	.597	.213	2.70
22	.598	.168	.613	.157	3.91
23	.598	.108	.615	.101	6.07
24	.597	.055	.612	.054	11.90
25	.600	0	.615	0	∞



Distribution of Voltage, Current, and Impedance at 1000 cycles end open circuited

Plate 7



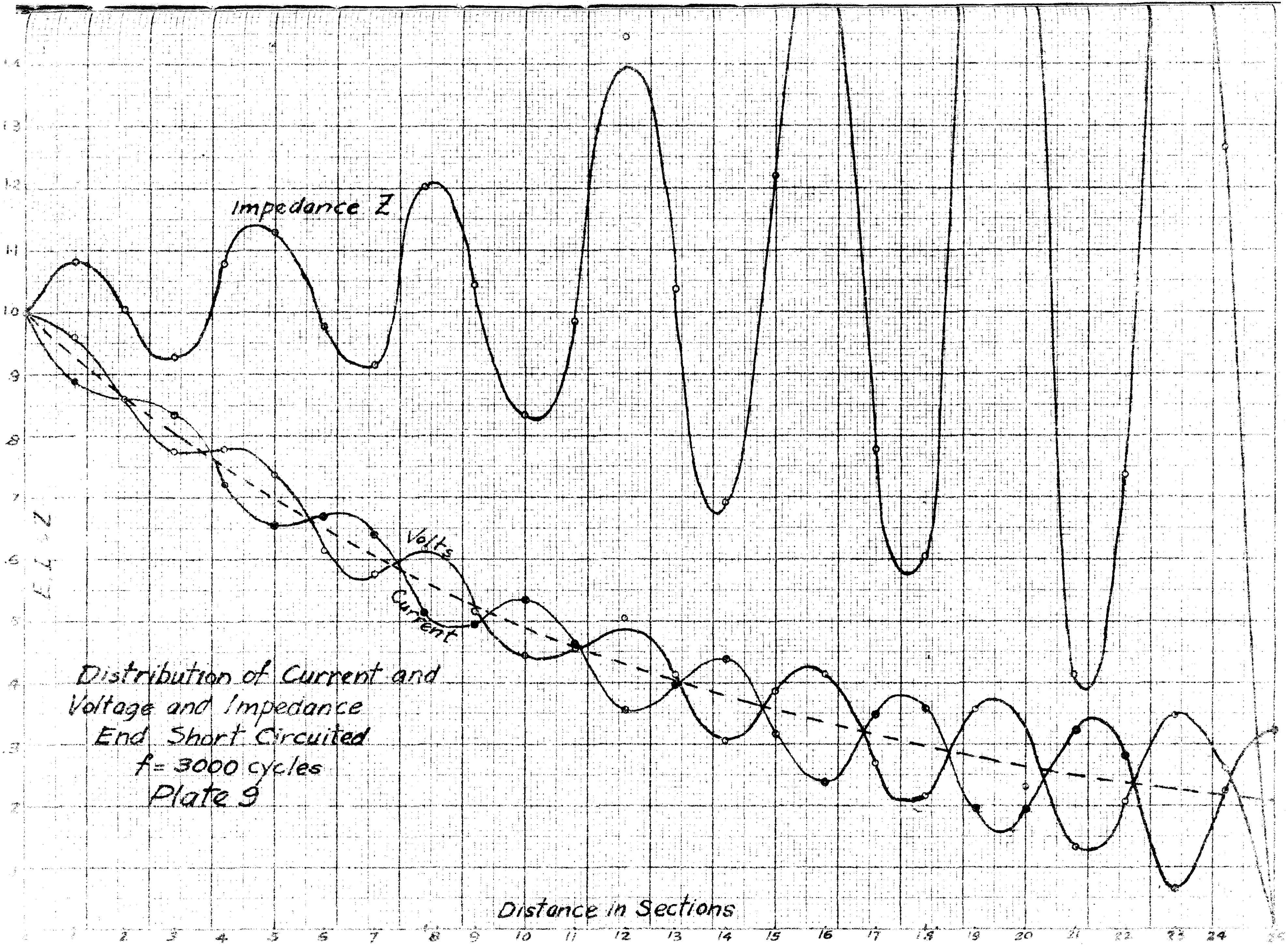
Distribution of Voltage and
Current along open circuited Line
f = 1000
Plate 8

Data Sheet for Plate 7&8

The Distribution of Current, Voltage, and Impendance along an Open Circuited Line at 1,000 cycles

S_s	E_s	I_s	E/E_s	I/I_s	$E/I = Z$
0	.855	1.20	1.000	1.000	1.000
1	.825	1.09	.966	.908	1.065
2	.795	.990	.931	.825	1.127
3	.755	.940	.889	.783	1.136
4	.705	.880	.830	.734	1.130
5	.638	.872	.752	.726	1.035
6	.570	.860	.670	.716	.986
7	.502	.840	.590	.700	.843
8	.453	.830	.533	.692	.770
9	.411	.778	.483	.647	.746
10	.414	.720	.486	.600	.810
11	.438	.628	.515	.524	.983
12	.461	.525	.542	.437	1.24
13	.466	.429	.548	.530	1.035
14	.457	.354	.537	.312	1.72
15	.422	.371	.496	.309	1.605
16	.371	.416	.436	.347	1.254
17	.294	.472	.346	.393	.880
18	.218	.533	.267	.444	.602
19	.148	.552	.174	.460	.378
20	.127	.541	.150	.451	.332
21	.181	.510	.213	.425	.502
22	.275	.420	.323	.350	.923
23	.312	.301	..367	.251	1.460
24	.357	.157	.420	.131	3.21
25	.367	0	.432	0	∞

Wave length = 178 miles



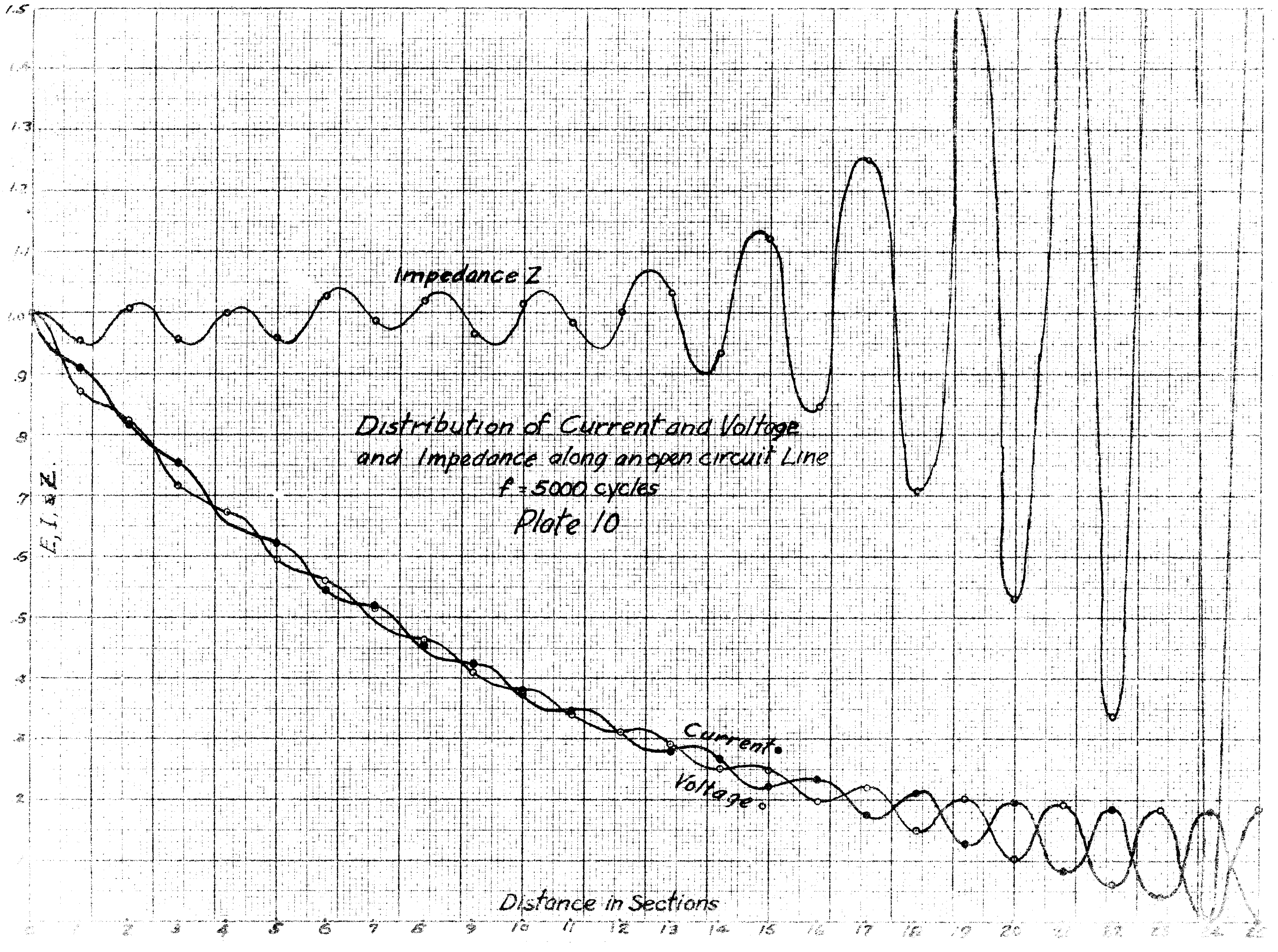
Distribution of Current and
Voltage and Impedance
End Short Circuited
f = 3000 cycles
Plate 9

Data Sheet for Plate 9

The Distribution of Current, Voltage, and Impedance along A Short Citcuited Line at 3,000 Cycyles

S_s	E_s	I_s	E/E_s	I/I_s	$E/I = Z$
0	.770	1.270	1.000	1.000	1.000
1	.740	1.120	.961	.888	1.082
2	.662	1.090	.860	.856	1.005
3	.595	1.060	.773	.834	.928
4	.600	.918	.780	.722	1.080
5	.568	.832	.739	.654	1.130
6	.473	.853	.615	.670	.918
7	.445	.803	.578	.632	.916
8	.475	.652	.617	.513	1.202
9	.428	.628	.517	.495	1.045
10	.343	.677	.445	.535	.835
11	.352	.588	.457	.463	.987
12	.388	.448	.504	.353	1.435
13	.317	.505	.412	.397	1.035
14	.317	.558	.304	.448	.692
15	.298	.403	.387	.317	1.220
16	.319	.302	.414	.238	1.740
17	.208	.443	.270	.348	.777
18	.167	.456	.217	.359	.605
19	.275	.249	.357	.196	1.820
20	.256	.248	.332	.195	1.702
21	.103	.410	.134	.323	.415
22	.160	.359	.208	.282	.738
23	.267	.083	.347	.065	5.340
24	.202	.286	.262	.225	1.165
25	0	.410	0	.323	0

Wave length from graph = 57 miles

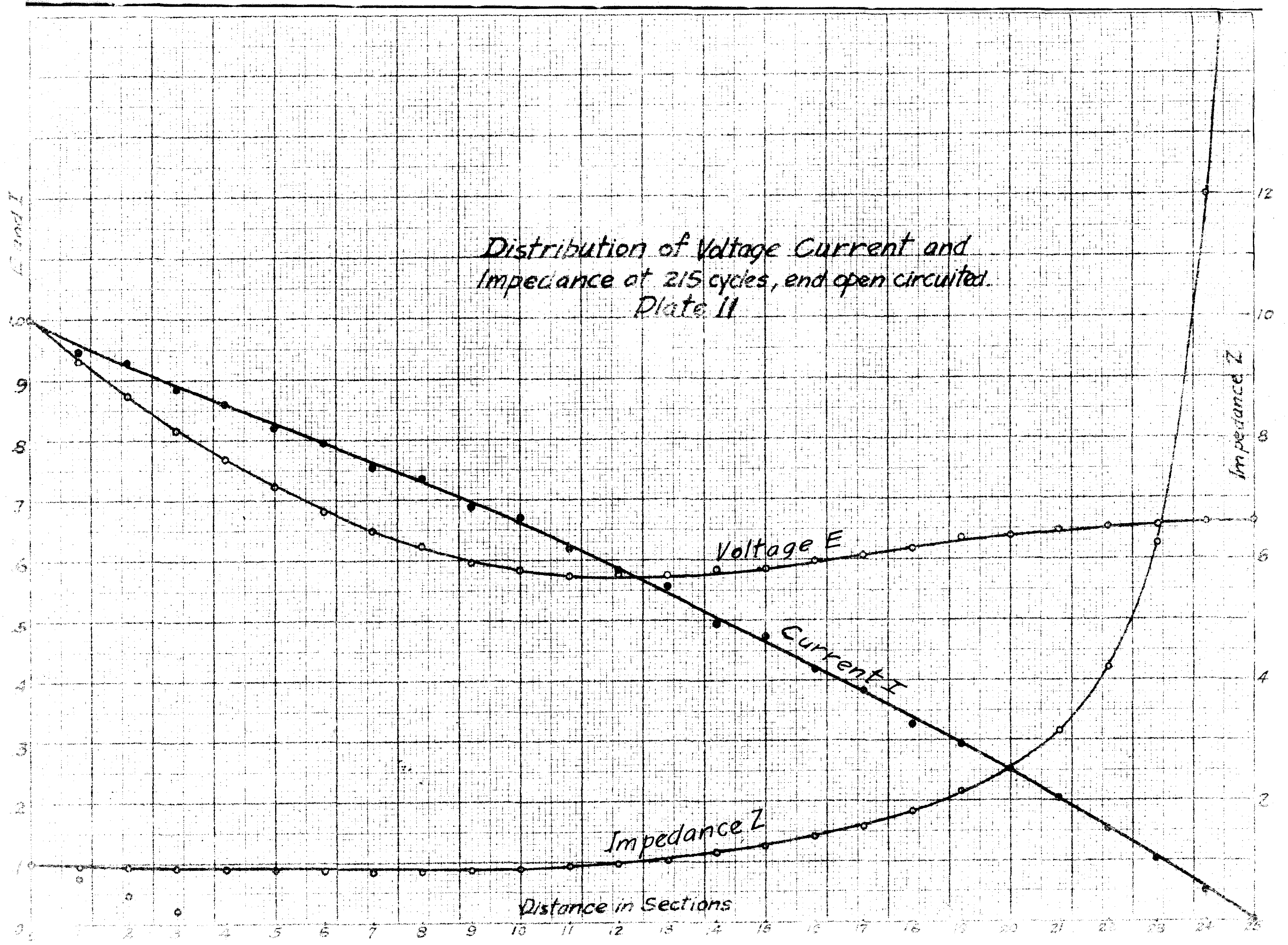


Distribution of Current and Voltage
and Impedance along an open circuit Line
$f = 5000$ cycles
Plate 10

Data Sheet for Plate 10

The Distribution of Current, Voltage, and Impedance along an Open Circuited Line at 5,000 Cycyles

S_s	E_s	I_s	E/E_s	I/I_s	$E/I = Z$
0	.656	1.285	1.000	1.000	1.000
1	.570	1.170	.870	.910	.955
2	.539	1.050	.823	.817	1.007
3	.471	.965	.718	.752	.956
4	.442	.864	.673	.673	1.000
5	.391	.788	.596	.622	.958
6	.368	.702	.561	.546	1.028
7	.336	.663	.513	.520	.987
8	.304	.585	.464	.455	1.018
9	.268	.543	.408	.423	.965
10	.250	.483	.381	.376	1.012
11	.223	.444	.340	.346	.983
12	.203	.398	.310	.310	1.000
13	.191	.359	.291	.279	1.042
14	.164	.344	.250	.268	.935
15	.163	.284	.248	.221	1.121
16	.129	.300	.197	.233	.846
17	.144	.226	.220	.176	1.250
18	.098	.272	.212	.212	.707
19	.132	.166	.201	.129	1.555
20	.068	.252	.104	.196	.531
21	.125	.108	.191	.084	2.28
22	.041	.239	.062	.186	.336
23	.119	.053	.182	.041	4.44
24	.0015	.232	.0023	.181	.003
25	.120	0	.183	0	∞



Distribution of Voltage Current and Impedance at 215 cycles, end open circuited.
Plate II

Data Sheet for Plate 11

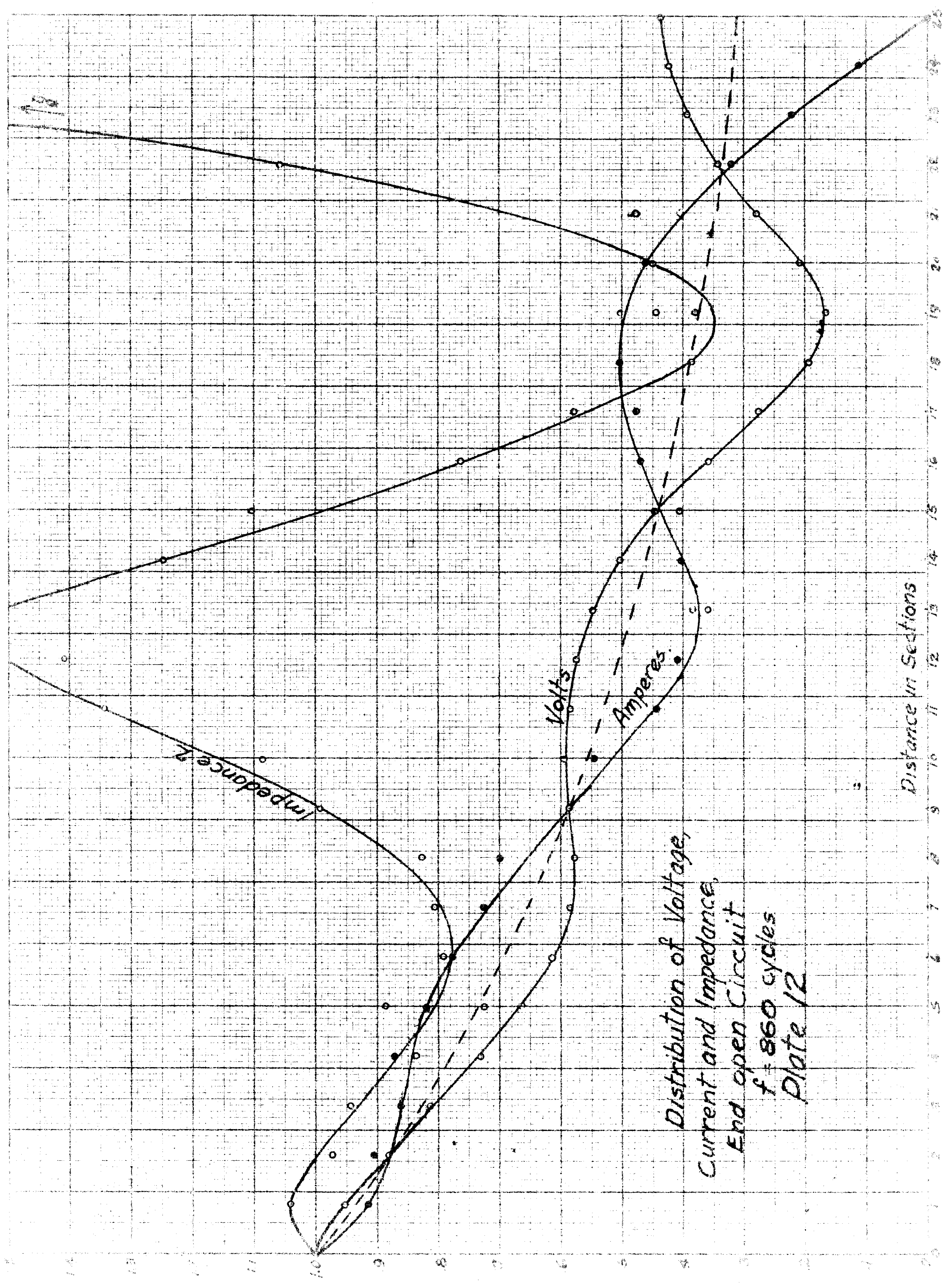
The distribution of Current, Voltage, and Impedance along
a
an Open Circuited Line f = 215 Cycles (1/4 wave length)

S_s	E_s	I_s	E/E_s	I/I_s	$E/I = Z$
0	.935	1.080	1.000	1.000	1.000
1	.875	1.020	.931	.948	.975
2	.820	1.000	.872	.950	.947
3	.768	.950	.817	.884	.922
4	.722	.926	.768	.860	.894
5	.678	.880	.722	.819	.874
6	.681	.795	.661	.795	.862
7	.610	.812	.649	.755	.855
8	.585	.790	.622	.735	.852
9	.560	.740	.596	.688	.864
10	.549	.720	.584	.670	.867
11	.541	.665	.575	.618	.920
12	.540	.627	.574	.583	.975
13	.539	.598	.574	.557	1.050
14	.546	.532	.582	.494	1.150
15	.550	.508	.585	.572	1.25
16	.561	.450	.597	.418	1.41
17	.570	.412	.606	.382	1.58
18	.581	.352	.618	.327	1.82
19	.595	.314	.633	.292	2.16
20	.598	.259	.636	.241	2.54
21	.608	.219	.647	.205	3.15
22	.612	.163	.651	.151	4.20
23	.615	.110	.654	.102	6.23
24	.620	.0525	.659	.049	12.0
25	.622	0	.661	0	∞

$$f = \frac{(2n - 1)}{4S_2} \sqrt{\frac{(2n - 1)^2\pi^2 + 4S_2RG}{S_2(RC + LG)^2 + (2n - 1)^2\pi^2 LC}}$$

f = 860 cycles

$f/4$ = 215 cycles

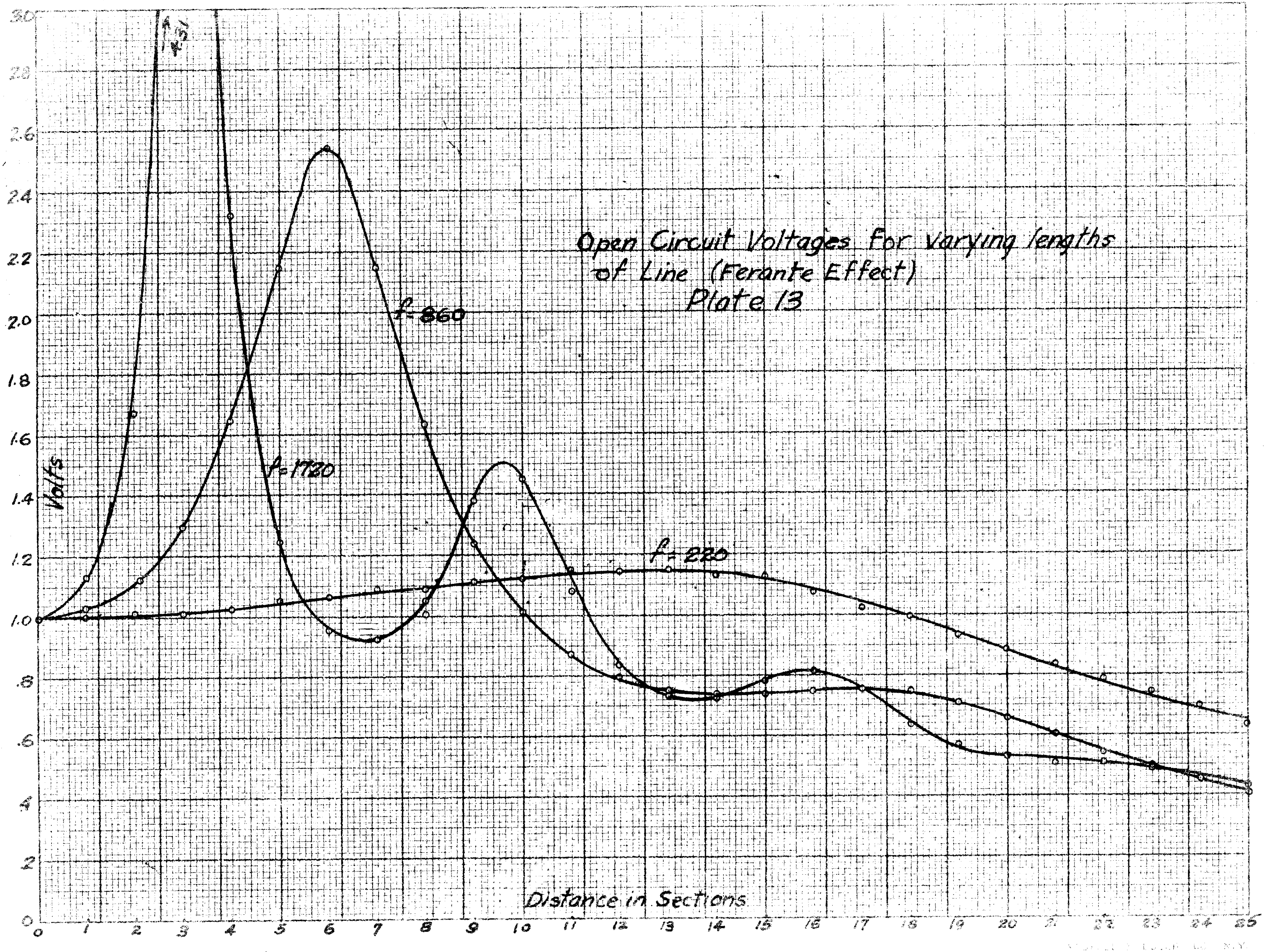


Distribution of Voltage, Current and Impedance, End open Circuit
f = 860 cycles
Plate 12

Data Sheet for Plate 12

The Distribution of Current, Voltage and Impedance along an Open Circuited Line at 860 Cycles.

S_s	E_s	I_s	E/E_s	I/I_s	$E/I = Z$
0	.862	1.17	1.000	1.000	1.000
1	.820	1.07	.952	.915	1.040
2	.758	1.06	.880	.905	.972
3	.700	1.01	.812	.862	.942
4	.630	1.02	.731	.872	.838
5	.626	.96	.726	.820	.885
6	.528	.91	.614	.777	.791
7	.505	.85	.586	.726	.807
8	.498	.82	.578	.700	.827
9	.506	.69	.587	.590	.995
10	.513	.64	.595	.546	1.088
11	.505	.51	.586	.436	1.345
12	.497	.48	.577	.410	1.410
13	.473	.42	.549	.359	1.530
14	.432	.47	.502	.402	1.250
15	.387	.475	.448	.406	1.104
16	.309	.550	.359	.470	.765
17	.238	.558	.276	.477	.578
18	.168	.590	.195	.504	.887
19	.145	.510	.166	.456	.380
20	.180	.540	.209	.462	.453
21	.241	.468	.280	.585	.478
22	.295	.377	.342	.322	1.060
23	.340	.258	.393	.221	1.778
24	.366	.129	.425	.110	3.84
25	.378	0	.438	0	∞



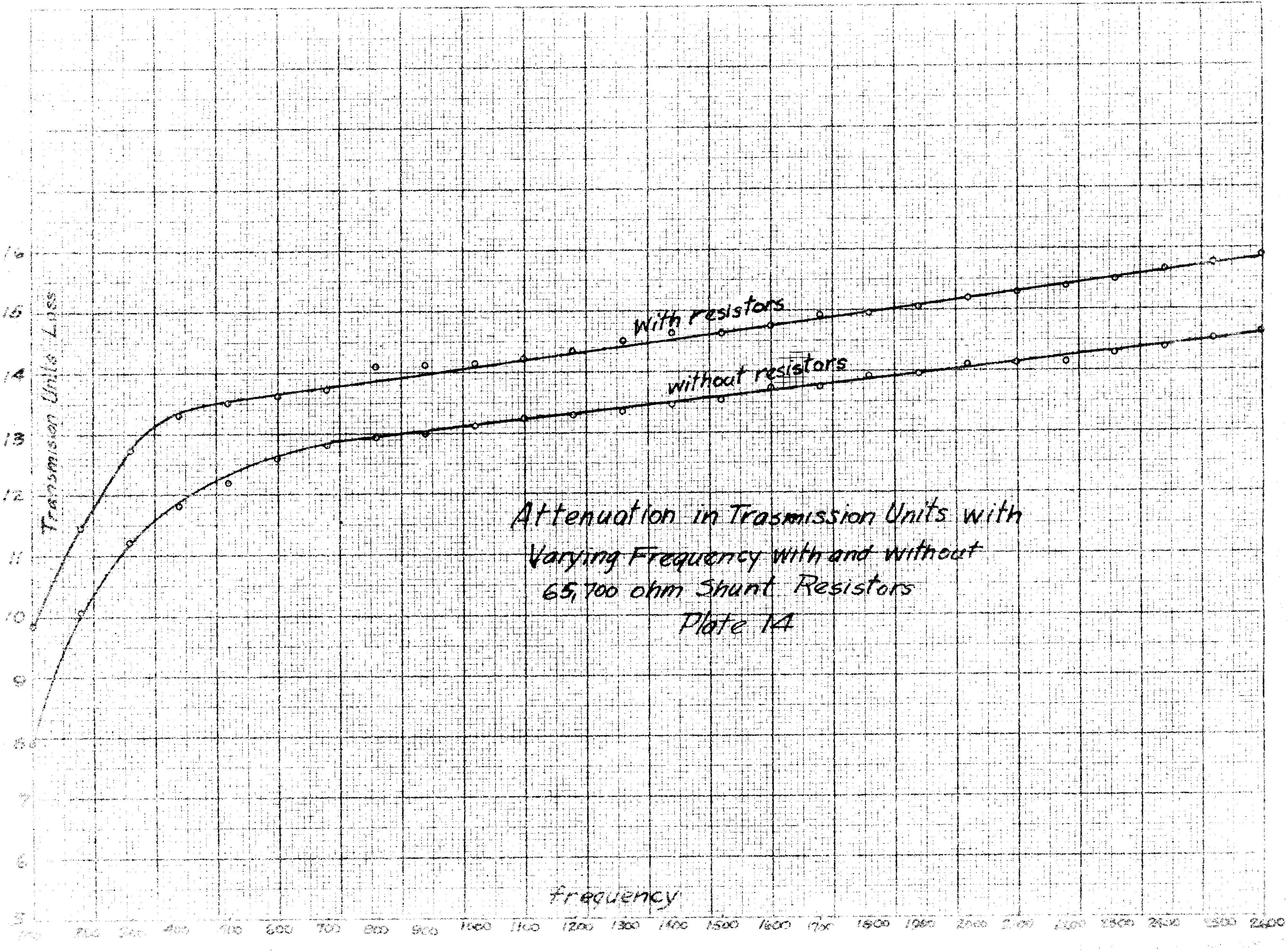
Open Circuit Voltages for Varying lengths of Line (Ferante Effect)
Plate 13

Data Sheet for Plate 13

The Ferannti Effect at three freqeuncies

E_r/E_s

S_s	f= 220	f= 860	f = 1720
0	1.000	1.000	1.000
1	1.005	1.032	1.132
2	1.012	1.121	1.675
3	1.012	1.297	4.310
4	1.025	1.645	2.320
5	1.054	2.150	1.245
6	1.062	2.540	.955
7	1.090	2.150	.922
8	1.087	1.632	1.050
9	1.112	1.238	1.38
10	1.120	1.021	1.45
11	1.151	.873	1.08
12	1.142	.795	.835
13	1.150	.754	.731
14	1.128	.740	.723
15	1.123	.739	.782
16	1.074	.748	.816
17	1.021	.753	.752
18	.995	.745	.637
19	.934	.708	.568
20	.885	.635	.532
21	.835	.601	.509
22	.788	.542	.509
23	.742	.495	.490
24	.695	.452	.450
25	.633	.433	.406



Attenuation in Trasmission Units with
Varying Frequency with and without
65,700 ohm Shunt Resistors
Plate 14

Data for Plate 14

The Attentuation in Transmission Units with Frqeuency for the Line with and without 65,700 ohm Shunt Resistors

frequency	T.U. without	T.U. with
100	7.90	9.84
200	10.07	11.46
300	11.22	12.72
400	11.81	13.13
500	12.20	13.51
600	12.60	13.63
700	12.81	13.72
800	12.95	14.11
900	13.00	14.12
1000	13.12	14.14
1100	13.24	14.22
1200	13.30	14.35
1300	13.35	14.51
1400	13.47	14.62
1500	13.54	14.63
1600	13.73	14.76
1700	13.76	14.92
1800	13.94	14.96
1900	13.96	15.05
2000	14.10	15.19
2100	14.13	15.28
2200	14.15	15.37
2300	14.29	15.47
2400	14.38	15.64
2500	14.51	15.77
2600	14.62	15.88

$$\text{T.U. Loss} = 10 \log_{10} \frac{E_s I_s}{E_r I_r}$$

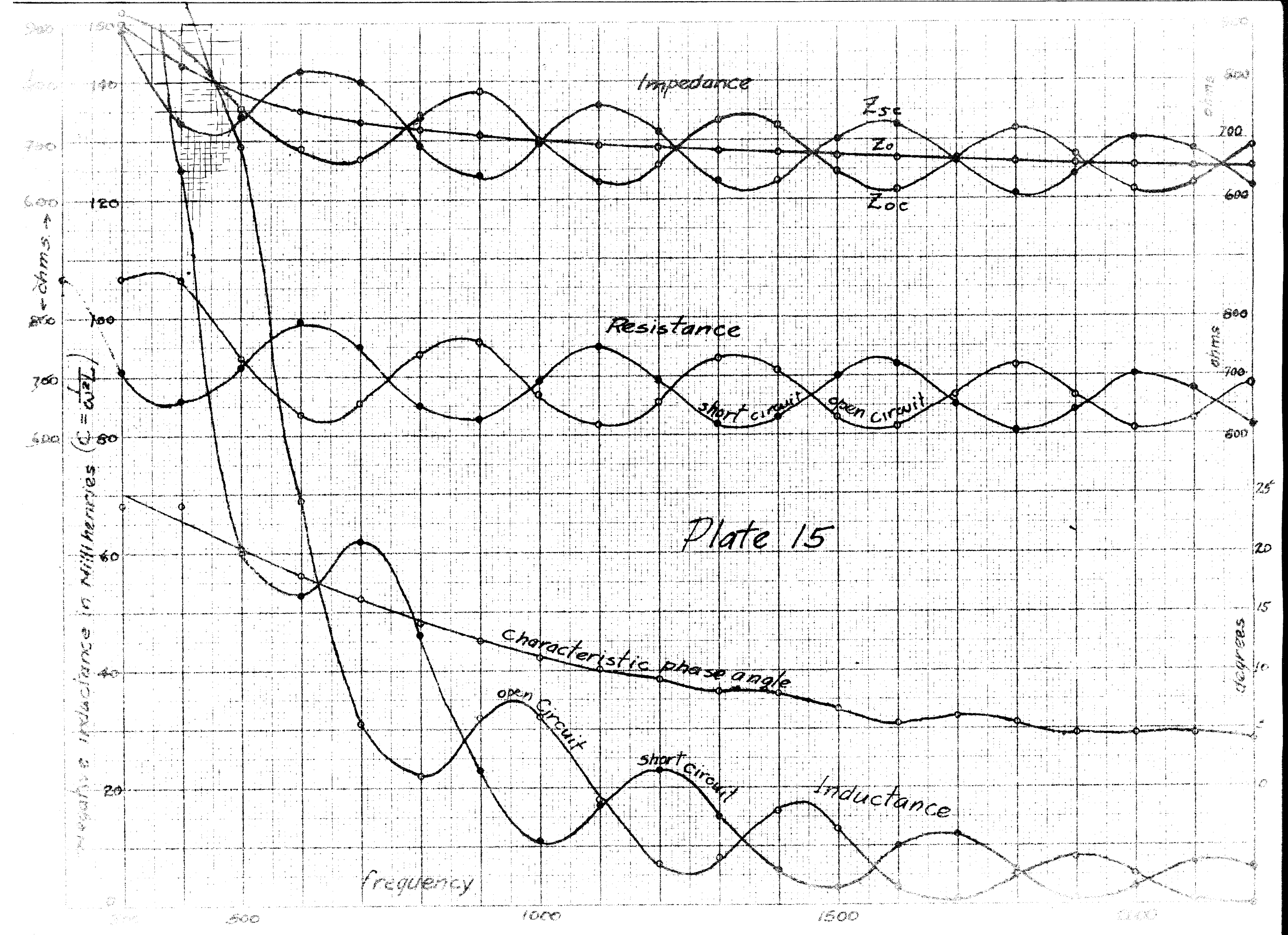
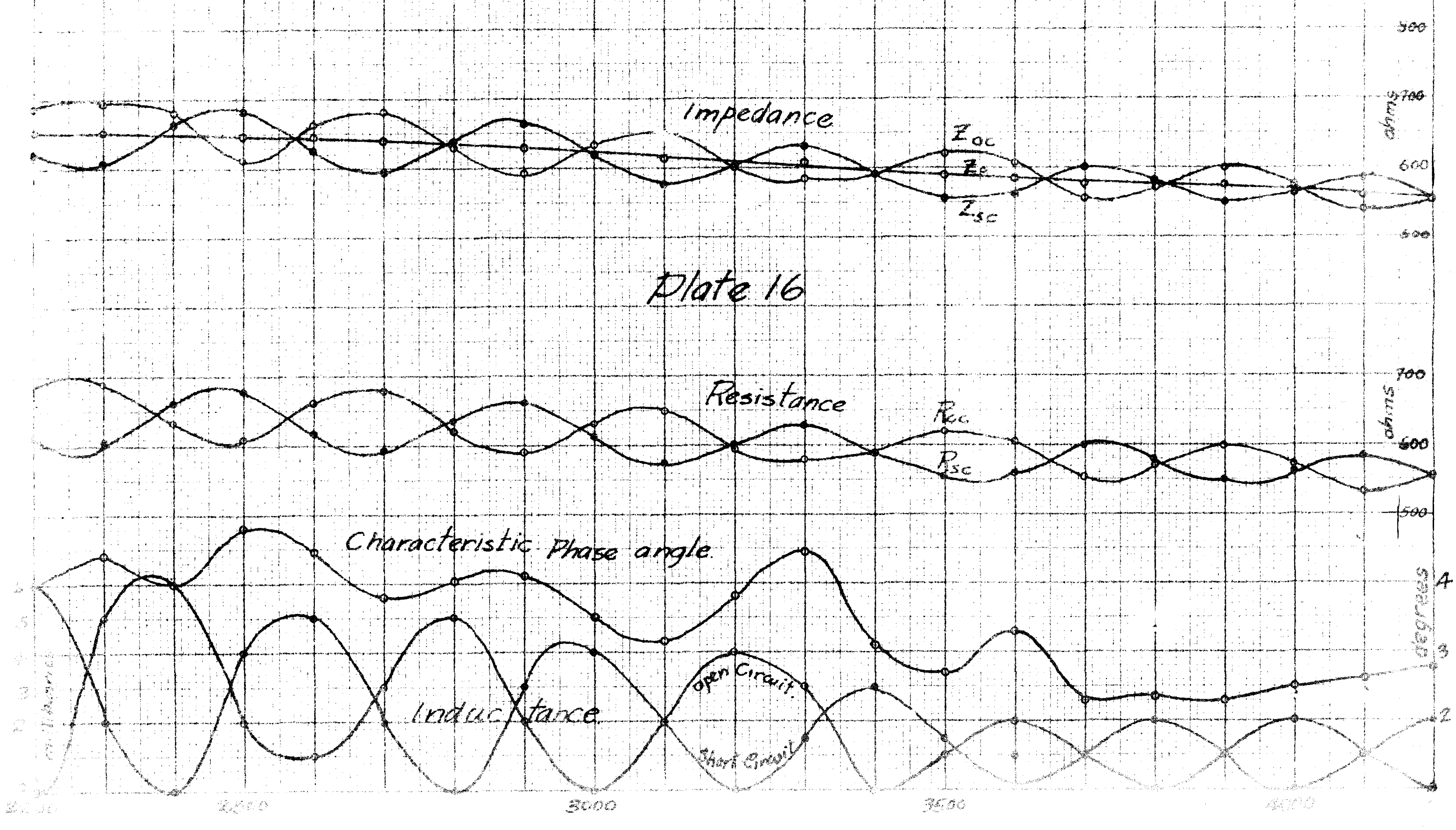

Impedance
Zsc
Zo
Zoc
Resistance
short circuit
open circuit
Characteristic phase angle
open Circuit
short Circuit
Inductance
Frequency
Negative inductance in Millihenries (C = 1/ω²L)
← ohms →
ohms
ohms
degrees
900
800
700
600
800
700
600
1500
140
120
100
80
60
40
20
0
500
1000
1500
800
700
600
800
700
600
25
20
15
10
5
0

Plate 15

Impedance
Plate 16
Z_{oc}
Z_o
Z_{sc}
500
700
ohms
600
500
Resistance
R_{oc}
R_{sc}
700
ohms
600
500
Characteristic Phase angle.
degrees
4
3
2
Inductance
open Circuit.
Short Circuit
millihenries
6
5
4
3
2
1
2000
2500
3000
3500
4000

Data for Plates 15 & 16

The Varaition of Open Circuit and Short Circuit Impedance Resistance, Reactance, and Phase Angle with Frequency.

open circuit			short circuit					
Z ohm	R ohm	L -mh	Z ohm	R ohm	L -mh	Z_0 ohm	θ degree	f cycle
922	868	164	887.5	710	282	900	24.1	300
941	867	146	731	660	125	829	24.2	400
756	732	119	741	717	60	748	20.8	500
687	636	69	819	794	53	750	18.2	600
669	655	31	798	750	62	731	16.0	700
745	737	22	691	651	46	717	14.1	800
782	758	32	641	628	23	708	12.5	900
700	670	32	695	692	11	697	11.2	1000
630	618	18	759	750	17	691	10.1	1100
658	656	7	715	694	23	686	9.3	1200
733	730	8	631	619	15	680	8.2	1300
725	711	16	632	630	6	677	8.0	1400
644	631	13	700	700	3	672	6.7	1500
615	614	3	726	719	10	668	5.4	1600
669	669	1	665	652	12	666	6.1	1700
719	717	5	609	605	6	662	5.5	1800
673	666	8	642	642	1	657	4.6	1900
613	610	5	702	701	3	666	4.6	2000
626	626	1	683	677	7	654	4.5	2100
686	686	0	620	614	6	652	3.9	2200
694	690	5	607	606	2	650	4.4	2300
639	633	0	663	663	0	651	4.1	2400
612	610	2	682	679	4	646	4.8	2500
662	662	1	625	619	5	643	4.5	2600
682	680	3	595	594	2	637	3.8	2700
627	621	5	636	636	0	632	4.0	2800
593	592	2	664	662	3	627	4.1	2900
632	632	0	618	613	4	625	3.5	3000
651	650	2	577	576	2	613	3.2	3100
600	595	4	602	602	0	601	3.0	3200
584	581	3	631	630	1.5	607	4.4	3300
590	590	0	590	587	3	590	3.1	3400
620	620	1	556	555	1.5	588	2.7	3500
606	605	2	561	560	1	583	3.3	3600
555	555	1	600	600	1	577	2.3	3700
571	571	0	582	580	2	576	2.35	3800
600	600	1	550	550	1	575	2.4	3900
575	573	2	565	565	0	569	2.5	4000
535	535	1	565	565	1	566	2.6	4100
555	555	0	553	550	2	554	2.7	4200