# STUDIES OF THE EQUATION OF STATE AND ELASTICITY OF MANTLE MINERALS

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### **DEDICATION**

This thesis is dedicated to my late father, Thomas L. Hamecher, who inspired my love and appreciation of the natural world, and my passion and curiosity for understanding how things work.

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#### ABSTRACT

Because the Earth's upper mantle is inaccessible to us, in order to understand the chemical and physical processes that occur in the Earth's interior we must rely on both experimental work and computational modeling. This thesis addresses both of these geochemical methods. In the first chapter, I develop an internally consistent comprehensive molar volume model for spinels in the oxide system FeO-MgO-Fe<sub>2</sub>O<sub>3</sub>-Cr<sub>2</sub>O<sub>3</sub>-Al<sub>2</sub>O<sub>3</sub>-TiO<sub>2</sub>. The model is compared to the current MELTS spinel model with a demonstration of the impact of the model difference on the estimated spinel-garnet lherzolite transition pressure. In the second chapter, I calibrate a molar volume model for cubic garnets in the system SiO<sub>2</sub>-Al<sub>2</sub>O<sub>3</sub>-TiO<sub>2</sub>-Fe<sub>2</sub>O<sub>3</sub>-Cr<sub>2</sub>O<sub>3</sub>-FeO-MnO-MgO-CaO-Na<sub>2</sub>O. I use the method of singular value analysis to calibrate excess volume of mixing parameters for the garnet model. The implications the model has for the density of the lithospheric mantle are explored. In the third chapter, I discuss the nuclear inelastic X-ray scattering (NRIXS) method, and present analysis of three orthopyroxene samples with different Fe contents. Longitudinal and shear wave velocities, elastic parameters, and other thermodynamic information are extracted from the raw NRIXS data.

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#### **INTRODUCTION**

Thermodynamic modeling of mineral solid solutions facilitates calculations of phase equilibrium in the Earth's mantle. Mantle melting models include constraints from phase equilibria experiments, calorimetry, and crystal chemistry to indirectly explore mantle processes, as well as the physical properties of solid and liquid phases as a function of pressure, temperature, and composition. MELTS (Ghiorso and Sack 1995; Ghiorso et al. 2002; Asimow et al. 2004) is a software package that models mantle melting. The xMELTS model (Ghiorso et al. 2007) extends the capabilities of MELTS to pressures and temperatures commensurate with those at the base of the transition zone in the Earth's mantle. Future work on xMELTS will incorporate a comprehensive thermodynamic model for garnet and pyroxene solid solutions. The goal is to create a model that reproduces phase relations for temperature and pressure conditions at depths from the shallow crust to the top of the lower mantle, for all compositions of natural magma and coexisting solids.

Garnet is a particularly important phase involved in partial melting of the upper mantle because it controls partitioning of major and minor elements at pressures greater than 3 GPa. Chromium is a minor but significant component of mantle rocks because its presence increases the spinel stability field relative to the garnet and pyroxene stability fields at high pressures. Models incorporating Cr into garnet solid solutions have been missing from MELTS, requiring that simulations only be performed for Cr-free bulk compositions and preventing the modeling of the spinel-garnet phase transition. This type of model is needed to provide constraints on the energetics of mixing in garnet, and to extend the model to conditions found at the base of the transition zone. Additional minor elements (e.g., Ti and Na) should also be included in a thermodynamic model for mantle garnets and pyroxenes.

Internally consistent molar volume models are required before the full activitycomposition models for garnet and pyroxene can be recalibrated. Because most constraints on the activity of garnet and pyroxene at high-*P* are derived from experiments with coexisting spinel, we must be confident in the ability of our spinel model to realistically reproduce thermodynamic behavior over all applicable compositions. Additionally, producing a spinel molar volume model calibrated with recent in situ high-*P*, *T* diffraction data is crucial to our ability to accurately model the spinel–garnet transition in Earth's upper mantle. A new garnet volume model calibrated with recent in situ high-*P*, *T* diffraction data is also critical for accurately modeling key mineralogical transitions in the mantle, e.g., the spinel–garnet transition and the mantle transition zone. Above 5 GPa a majorite component is an essential part of any thermodynamic model of mantle garnets, which to be useful must accurately predict garnet stability with respect to spinel, pyroxene, perovskites, and melt.

Mineral physics experiments give us insight into the elastic behavior of mantle minerals. Determinations of sound velocities of mantle minerals, such as orthopyroxene, allow us to map chemical and thermal properties of the Earth's interior to seismic observations. Ultrasonic and Brillouin scattering measurements often disagree with each other and offer a limited amount of information about the material being studied. Newer synchrotron-based spectroscopic methods, like nuclear resonant inelastic X-ray scattering (NRIXS), allow us to nondestructively probe the properties of very small samples of material, and offer the added benefit of providing additional thermodynamic information about the material.

In this thesis I explore the physical and chemical properties of three upper mantle minerals—spinel, garnet, and orthopyroxene. In the first chapter, I develop a comprehensive model of molar volume for spinels in the oxide system FeO-MgO-Fe<sub>2</sub>O<sub>3</sub>-Cr<sub>2</sub>O<sub>3</sub>-Al<sub>2</sub>O<sub>3</sub>-TiO<sub>2</sub>. In the second chapter, I calibrate a model of molar volume for cubic garnets in the system SiO<sub>2</sub>-Al<sub>2</sub>O<sub>3</sub>-TiO<sub>2</sub>-Fe<sub>2</sub>O<sub>3</sub>-Cr<sub>2</sub>O<sub>3</sub>-FeO-MnO-MgO-CaO-Na<sub>2</sub>O. In the final chapter, I analyze NRIXS data for three samples of orthopyroxene with variable amounts of Fe, and determine longitudinal and shear wave velocity and additional thermodynamic parameters.

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### THE MOLAR VOLUME OF FeO-MgO-Fe<sub>2</sub>O<sub>3</sub>-Cr<sub>2</sub>O<sub>3</sub>-Al<sub>2</sub>O<sub>3</sub>-TiO<sub>2</sub> SPINELS

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#### ABSTRACT

We define and calibrate a new model of molar volume as a function of pressure, temperature, ordering state, and composition for spinels in the supersystem  $(Mg,Fe^{2+})(Al,Cr,Fe^{3+})_2O_4 - (Mg,Fe^{2+})_2TiO_4$ . We use 832 X-ray and neutron diffraction measurements performed on spinels at ambient and in situ high-P, T conditions to calibrate end-member equations of state and an excess volume model for this system. The effect on molar volume of cation ordering over the octahedral and tetrahedral sites is captured with linear dependence on  $Mg^{2+}$ ,  $Al^{3+}$ , and  $Fe^{3+}$  site occupancy terms. We allow standard state volumes and coefficients of thermal expansion of the end members to vary within their uncertainties during extraction of the mixing properties, in order to achieve the best fit. Published equations of state of the various spinel end members are analyzed to obtain optimal values of the bulk modulus and its pressure derivative, for each explicit end member. For any spinel composition in the supersystem, the model molar volume is obtained by adding excess volume and cation order-dependent terms to a linear combination of the five end-member volumes, estimated at pressure and temperature using the high-T Vinet equation of state. The preferred model has a total of nine excess volume and order-dependent parameters and fits nearly all experiments to within 0.02 J/bar/mol, or better than 0.5% in volume. The model is compared to the current MELTS spinel model with a demonstration of the impact of the model difference on the estimated spinel-garnet lherzolite transition pressure.

#### **INTRODUCTION**

Spinel-group minerals are commonly found in igneous and metamorphic rocks in the Earth's crust and upper mantle and are frequently used as petrogenetic indicators (Buddington and Lindsley 1964; Sack 1982; Dick and Bullen 1984; Ghiorso and Sack 1991; Ghiorso and Evans 2008). Spinel is a significant reference phase for high-pressure thermodynamic solution models of melts and of other solid phases, because it contains several key components of upper mantle assemblages at appreciable concentrations and because high-quality activity-composition models for spinels have been constructed (Sack 1982; Nell and Wood 1989; Sack and Ghiorso 1991a, b; Kessel et al. 2003). Beyond their petrologic significance, natural and synthetic spinel-group phases have numerous applications in the material sciences (e.g., Taberna et al. 2006; Yang et al. 2007).

The prevalence of spinels can be partly explained by the variety of cations of different valence that can be accommodated within the structure. While the spinel structure is quite simple, its solid solution behavior is complex. Cubic spinels (space group Fd3m) have the stoichiometry AB<sub>2</sub>O<sub>4</sub>, where A and B are cations with, most often, 2+ and 3+ charge, respectively, although substitution toward end members where A is 4+ and B is 2+ can occur. Additional complexity arises due to the existence of two distinct cation coordination environments and the ability of a wide array of cations to distribute themselves over the octahedral and tetrahedral crystallographic sites. "Normal" spinels are defined as having the A ion in the tetrahedral site and both B ions in the two identical octahedral sites. Perfectly "inverse" spinels have one B ion per formula unit occupying the tetrahedral site, with one A and one B ion residing in the octahedral sites. For many

choices of A and B, spinel solid solutions can adopt ordering states at any point along this continuum. A number of models have described the extent of such ordering in spinels (Callen et al. 1956; O'Neill and Navrotsky 1983, 1984; Sack and Ghiorso 1991a) in terms of the energetics of the cation ordering reactions.

Molar volume is an important thermodynamic quantity at high pressures, both when using spinels to infer petrogenetic information from high-pressure rocks and particularly when using spinels from high-pressure experiments to define chemical potentials in coexisting phases. Because pressures of interest in spinel-bearing experiments range up to at least 3 GPa ( $3 \times 10^4$  bar), differences in volume of only 0.03 J/bar/mol (i.e., 0.3 cm<sup>3</sup>/mol, or  $\leq 1\%$  of typical spinel molar volumes) yield differences of 1 kJ/mol in chemical potentials, which is often the accuracy level sought in calibrations of solid- and liquid-solution models. A model of spinel volumes with the necessary accuracy needs to account not only for equations of state of pure end members and considerable deviation from ideal mixing of compositions, but also for significant effects of ordering state on the volume (O'Neill and Navrotsky 1983; Hazen and Navrotsky 1996). Differences in ionic size, charge, and/or coordination environment can contribute to non-ideal behavior (O'Neill and Navrotsky 1984). Pressure, in addition to temperature and composition, can strongly affect cation-ordering state, which in turn affects physical properties, including the elastic moduli (Hazen and Navrotsky 1996). The complexities due to cation ordering over distinct crystallographic sites, along with the wide range of stable compositions of spinels, create difficulties in modeling their thermodynamic behavior, including molar volume (Sack and Ghiorso 1991a). Generally, studies are restricted to subsystems of spinels, e.g., along a solid-solution binary. While

limiting the system of interest usually allows one to recover the data used in calibration, discrepancies exist between various end-member models; i.e., a particular spinel end member may be assigned different model volumes in fits to adjacent subsystems. Hence currently available models are inadequate for modeling of volumes over the full compositional range of spinels formed in the Earth's upper mantle. It is necessary to devise a comprehensive model applicable to the entire chemical system of the upper mantle.

In this work, we present a model of molar volumes of stoichiometric spinels containing the oxide components FeO-MgO-Fe<sub>2</sub>O<sub>3</sub>-Cr<sub>2</sub>O<sub>3</sub>-Al<sub>2</sub>O<sub>3</sub>-TiO<sub>2</sub>; the cations in this system account for at least 98% of the compositional range of natural spinels (Sack 1982). Our chosen set of independent compositional model end members is spinel sensu stricto (MgAl<sub>2</sub>O<sub>4</sub>), hercynite (FeAl<sub>2</sub>O<sub>4</sub>), magnetite (Fe<sub>3</sub>O<sub>4</sub>), chromite (FeCr<sub>2</sub>O<sub>4</sub>), and ulvöspinel (Fe<sub>2</sub>TiO<sub>4</sub>). Dependent end members in this system—magnesiochromite  $(MgCr_2O_4)$ , magnesioferrite  $(MgFe_2O_4)$ , and gandilite  $(Mg_2TiO_4)$ —are formed from linear combinations of the independent end members. As written, these are *compositional* end members only, and the order in which the cations are written should not be taken to imply an ordering state over tetrahedral and octahedral sites. Within the MELTS software, end-member thermodynamic quantities are calculated for the standard state structural arrangement, and mixing properties are referenced to the end-member values. For calibration of the volume, we are therefore obliged to use end members with the ordering states adopted by Sack and Ghiorso (1991a, b), i.e., normally ordered FeCr<sub>2</sub>O<sub>4</sub> and Fe<sub>2</sub>TiO<sub>4</sub>, almost perfectly normal MgAl<sub>2</sub>O<sub>4</sub> and FeAl<sub>2</sub>O<sub>4</sub>, and near-perfect inversely ordered Fe<sub>3</sub>O<sub>4</sub>. Note that this constraint did not apply to Sack and Ghiorso's formulation

of the activity-composition models, as the adopted end-member heat capacity functions (taken from the internally consistent database of Berman 1988) were independent of ordering state. In fact, the Sack and Ghiorso (1991a, b) model was calibrated using perfectly inverse ordered components; normally ordered and standard state values for the compositional components were inferred from the fitted parameters.

Our primary motivation in this work is to develop a comprehensive spinel molar volume model for use in calibration of activity-composition models of garnet and pyroxene solid solutions. The thermodynamic models, along with a new silicate liquid equation of state (Ghiorso 2004a, 2004b, 2004c; Ghiorso and Kress 2004), will be incorporated into the next generation MELTS (Ghiorso and Sack 1995; Ghiorso et al. 2002; Asimow et al. 2004) model, xMELTS (Ghiorso et al. 2007). The new solid solution models will include some minor components, including Ti<sup>4+</sup> and Cr<sup>3+</sup>. Because most constraints on the activity of garnets and pyroxenes at high-*P* are derived from experiments with coexisting spinels, we must be confident in the ability of our spinel model to realistically reproduce thermodynamic behavior over the entire applicable range of compositions. Additionally, producing a spinel molar volume model calibrated with recent in situ high-P, T X-ray and neutron diffraction data is crucial to our ability to accurately model the spinel–garnet transition in Earth's upper mantle. For example, we recently calibrated Cr-Al exchange equilibria for garnet and spinel (Hamecher et al. 2009). When this new calibration is used with the current MELTS model, a region of garnet-spinel coexistence in lherzolites is predicted with width in pressure comparable to experimental constraints. The transition occurs, however, at the unexpectedly low pressure of  $\sim 1.7$  GPa, though this is not entirely due to the introduction of Cr to the

system, as discussed below. The improved model of spinel molar volume presented here will enable coupled recalibration of the garnet and pyroxene models to match both the absolute pressure and width of this key transition in mantle lithology.

In this chapter, we first discuss previous models of spinel molar volume, with attention to the ranges of composition they cover and inconsistencies among the models. We then present the X-ray and neutron diffraction and ultrasonic data used in our calibration, and the formulation of our model in terms of the components and ordering variables of the spinel solid solution model of Sack and Ghiorso (1991a, b). The calibration strategy we used to estimate the parameters of the model and assess goodness of fit to the data is outlined. Finally, we compare the final model to models from the literature and present an estimate of the magnitude of the impact of the new model on MELTS calculations and other high-pressure thermodynamic inferences.

#### **PREVIOUS MODELS**

Several models for the molar volume of spinel group minerals have been proposed. However, most of these models are restricted to binary subsystems. Furthermore, there are discrepancies between models with corresponding end members. In this section we present examples of previous models of spinel molar volume.

The current molar volume model for spinels in MELTS and pMELTS covers the same compositional range as our proposed model. There is no mention of volume in Sack and Ghiorso (1991a) or Sack and Ghiorso (1991b) and, indeed, initially the entire system was assumed to have zero volume of mixing. However, before being put to use for calculation or calibration of other phases, the model was modified to include asymmetric

excess volume of mixing terms along the Fe<sub>3</sub>O<sub>4</sub>–Fe<sub>2</sub>TiO<sub>4</sub> join (Ghiorso 1990; Ghiorso and Sack 1991): an excess model of the form  $-0.1250X^2_{Fe3O4}X_{Fe2TiO4} +$ 

 $0.1018X_{Fe3O4}X^{2}_{Fe2TiO4}$  J/bar/mol was fitted to the molar volume data of Lindsley (1965). This excess term was used in calibration of subsequent models including the pyroxene family (Sack and Ghiorso 1994b), the MELTS liquid model (Ghiorso and Sack 1995), and the pMELTS liquid model (Ghiorso et al. 2002) and is present in all currently supported versions of the MELTS code. As part of the provisional xMELTS liquid model calibration, the spinel volume model was extended to higher pressures by fitting the formula for the high-T Vinet equation of state (see Eq. 6 below) for MgAl<sub>2</sub>O<sub>4</sub> and Fe<sub>3</sub>O<sub>4</sub> to the Berman (1988) polynomial equation of state (see Ghiorso 2004b). Since standard state volume data for other spinel components are not given in Berman (1988), P- and Tcoefficients for  $MgAl_2O_4$  or  $Fe_3O_4$  were assigned to the remaining end members. There is now an opportunity to simultaneously optimize both standard state and mixing terms in order to form an internally consistent general spinel-system volume model for use in future calibration and calculation efforts. Our recalibration includes refinement of the asymmetric terms along the  $Fe_3O_4$ – $Fe_2TiO_4$  join alongside consideration of all other possible binary excess terms in the composition space.

Oka et al. (1984) fit their molar volume data along the MgAl<sub>2</sub>O<sub>4</sub>–MgCr<sub>2</sub>O<sub>4</sub> binary to an asymmetric regular solution model  $W_{AlAlCr}(1 - X_{Cr})^2 X_{Cr} + W_{AlCrCr}(1 - X_{Cr}) X_{Cr}^2$ (where  $X_{Cr}$  is mole fraction MgCr<sub>2</sub>O<sub>4</sub> for this binary). For their best-characterized data, synthesized at 1250 °C, they obtained excess volume parameters  $W_{AlAlCr} = 0.0524(91)$ and  $W_{AlCrCr} = 0.0040(92)$  J/bar/mol. On the other hand, average parameters for fits to three different sets of synthesis temperatures are  $W_{AlAlCr} = 0.0504$  and  $W_{AlCrCr} =$  0.0182 J/bar/mol (uncertainties not stated). Both versions of the model show positive deviation from ideality along the entire binary.

Doroshev et al. (1997) also investigated the molar volume of the MgAl<sub>2</sub>O<sub>4</sub>– MgCr<sub>2</sub>O<sub>4</sub> subsystem. Phase equilibria experiments containing Cr-rich garnets were performed, and the multiphase products were analyzed by electron microprobe and X-ray diffraction. Doroshev et al. also adopted an asymmetric regular-solution excess volume model, with the largest deviation from ideality in the Al-rich part of the join. Contrary to the previous study, however, Doroshev et al. found negative deviation from ideality in the Cr-rich region. The excess volume parameters are  $W_{AlAlCr} = 0.0722(90)$  and  $W_{AlCrCr} = -0.0483(75)$  J/bar/mol. The authors attribute the difference between the models to a more thorough characterization of the Cr-rich samples.

Brey et al. (1999) performed similar experiments to Doroshev et al. (1997), but in the larger system FeO-MgO-Al<sub>2</sub>O<sub>3</sub>-SiO<sub>2</sub>-Cr<sub>2</sub>O<sub>3</sub>. The explicit spinel end members in the Brey et al. model are MgAl<sub>2</sub>O<sub>4</sub>, FeCr<sub>2</sub>O<sub>4</sub>, and MgCr<sub>2</sub>O<sub>4</sub>. After considering a possible non-zero volume of reaction for the reciprocal (cross-site) reaction, Brey et al. discard this term. They retain an excess mixing volume due to exchange of Mg<sup>2+</sup> and Fe<sup>2+</sup> cations, fit by a symmetric model with the parameter  $W_{\text{FeMg}} = -0.020(7)$  J/bar/mol. Also, like Doroshev et al., Brey et al. use an asymmetric excess volume model for the Cr<sup>3+</sup> -Al<sup>3+</sup> exchange with parameters (recast into common form with above models)  $W_{\text{AlCrCr}} =$ 0.034(18) and  $W_{\text{AlCrCr}} = -0.014(12)$  J/bar/mol. This fit retains a negative deviation from ideality for Cr-rich compositions, but this result is only marginally significant, and does not fit the pure Fe-free data of Doroshev et al. and Oka et al. (1984) especially well. Mattioli et al. (1987) derived a model of the volume of the ternary spinel

system MgAl<sub>2</sub>O<sub>4</sub>-Fe<sub>3</sub>O<sub>4</sub>- $\gamma$ Fe<sub>8/3</sub>O<sub>4</sub>. The volumes of the Fe<sub>3</sub>O<sub>4</sub>- $\gamma$ Fe<sub>8/3</sub>O<sub>4</sub> and MgAl<sub>2</sub>O<sub>4</sub>- $\gamma$ Fe<sub>8/3</sub>O<sub>4</sub> edges of this ternary system are treated as ideal. The MgAl<sub>2</sub>O<sub>4</sub>-Fe<sub>3</sub>O<sub>4</sub> join is modeled as an asymmetric regular solution model, of the same form discussed above, with excess volume parameters  $W_{mt-mt-sp} = 0.075(17)$  and  $W_{mt-sp-sp} = 0.18(5)$  J/bar/mol, where mt = Fe<sub>3</sub>O<sub>4</sub> and sp = MgAl<sub>2</sub>O<sub>4</sub>. In our final model, we find that an asymmetric excess volume term along the MgAl<sub>2</sub>O<sub>4</sub>-Fe<sub>3</sub>O<sub>4</sub> binary is not justified by the data at ambient pressure, and thus treat this join as symmetric.

Three choices of solid solution model for spinel, each one based on the thermodynamic dataset of Holland and Powell (1998), are included in the modeling package THERMOCALC (e.g., Powell et al. 1998; currently hosted at http://www.metamorph.geo.uni-mainz.de/thermocalc/). None of the models incorporate any excess volume terms. The latest version of the thermodynamic database (Holland and Powell 2011) uses a modified equation of state that may be more easily extrapolated to very high pressures than the Murnaghan equation of state used in Holland and Powell (1998). Updated solid solution models have yet to be released. The Perple X modeling package (e.g., Connolly 2009) offers the potential to adopt a wide variety of solid solution models, including several of those mentioned above (see http://www.perplex.ethz.ch/). All of the spinel models treat the MgAl<sub>2</sub>O<sub>4</sub>–FeAl<sub>2</sub>O<sub>4</sub> join as ideal in volume. Only one of the available thermodynamic databases includes a chromebearing component for spinel (Klemme et al. 2009). The corresponding solution model for MgO-FeO-Al<sub>2</sub>O<sub>3</sub>-Cr<sub>2</sub>O<sub>3</sub> spinels adopts the formulation of Oka et al. (1984) for asymmetric excess volume due to  $Al^{3+}$ -Cr<sup>3+</sup> exchange.

#### DATA SOURCES

The American Mineralogist Crystal Structure Database (AMCSD) (Downs and Hall-Wallace 2003) provides a comprehensive collection of published X-ray and neutron diffraction refinements of cell volume and site occupancy; we fit the entire database of spinels with Fd3m space group symmetry in the system FeO-MgO-Fe<sub>2</sub>O<sub>3</sub>-Cr<sub>2</sub>O<sub>3</sub>-Al<sub>2</sub>O<sub>3</sub>-TiO<sub>2</sub> (Figs. 1, 2). The very oldest studies (Bragg 1915; Passerini 1930; Verwey and Heilmann 1947) and all other spinels in the database — e.g., tetragonal spinels, franklinites, trevorites, and maghemites — were excluded. In addition, we found these data sources for refined site occupancy data not in the AMCSD: Carbonin et al. (1996), Della Giusta et al. (1996), Princivalle et al. (1999), and Levy et al. (2004). Furthermore, a few sources are available that provide cell parameter based on powder XRD and electron microprobe analysis of an experimental charge (Doroshev et al. 1997; Brey et al. 1999; Girnis et al. 2003). We used these data by assuming the specimen was quenched from an equilibrium ordering state at experimental conditions and applying the ordering model built into MELTS (Sack and Ghiorso 1991a, b). This approximation introduces two potential sources of error: (a) the temperature recorded by the true ordering state may be lower than the experimental one and (b) there is an inherent uncertainty associated with the MELTS ordering model (e.g., a consequence of the simplistic spinel volume model in MELTS is that the predicted ordering state is independent of pressure). Both errors are probably systematic, and the restricted compositional and *P*-*T* ranges mean that the data are likely to be affected to a similar degree. Hence, although the measurements were given slightly less weight during the regression than those with accompanying siteoccupancy data, their inclusion is reasonable and the model fit is not substantially affected by absence of measured site occupancies.

Our final model is calibrated using a total of 832 experiments. The compositional and *P-T* coverage of the data is summarized in Table 1. All references for data used in model calibration are listed in Supplementary Material 1. Experimental conditions, observed cell parameters, and cation site occupancies are given in Supplementary Material 2. There are a number of compositional gaps in the volume calibration database. Some are due to immiscibility (Barnes and Roeder 2001; Pascal et al. 2011) and some correspond to spinels found in lunar samples too precious for analysis by XRD (Haggerty 1971). Still others are filled by studies of binary spinels for which site occupancies were not characterized (Golla-Schindler et al. 2005; Mattioli et al. 1987; Muan et al. 1972; Robbins et al. 1971; Wechsler et al. 1984; Woodland et al. 2009; Zhao et al. 1998). Unlike the experimental studies discussed above, none of these measurements could be incorporated into the calibration data set. Either the equilibration temperature could not be estimated sufficiently accurately or the measured volume trend disagreed with a reliable data source for one or other pure phase end member, even when ordering state could not be a factor. Hence inclusion of these studies would have had a detrimental effect on the calibrated standard state volumes. Nevertheless, the data were useful for comparison and (once allowance was made for any discrepancies in the end-member volume contributions) provided a valuable independent test of the newly calibrated model.

We assume that all  $Ti^{4+}$  (Wechsler et al. 1984; Sack and Ghiorso 1991a; Bosi et al. 2009) and all  $Cr^{3+}$  (Dunitz and Orgel 1957; Sack and Ghiorso 1991b) cations occupy

the octahedral site in spinels. This choice is supported by the calibration data set: of the data considered, only one  $MgCr_2O_4$  (Tabira and Withers 1999), one  $Fe_2TiO_4$  (Stout and Bayliss 1980), and one  $Mg_2TiO_4$  (O'Neill et al. 2003) are reported as having 2% or more of  $Ti^{4+}$  or  $Cr^{3+}$  partitioned onto the tetrahedral site. Thus, we assigned any reported tetrahedral  $Ti^{4+}$  and  $Cr^{3+}$  to the octahedral site, moving  $Mg^{2+}$  as needed to compensate for site occupancy and charge. For example:

$${}^{[4]}X_{Mg^{2*}} = {}^{[4]}X'_{Mg^{2*}} + {}^{[4]}X'_{Cr^{3*}}$$

$${}^{[6]}X_{Mg^{2*}} = {}^{[6]}X'_{Mg^{2*}} - \frac{1}{2}{}^{[4]}X'_{Cr^{3*}}$$

$${}^{[6]}X_{Cr^{3*}} = {}^{[6]}X'_{Cr^{3*}} + \frac{1}{2}{}^{[4]}X'_{Cr^{3*}}, \qquad (1)$$

where left superscript [4] denotes the tetrahedral site, [6] denotes the octahedral site, and X' represents the proportion of the indicated cation before the adjustment is made. The same relationships hold for the substitution of Ti<sup>4+</sup> by replacing Cr<sup>3+</sup> with Ti<sup>4+</sup> in Eq. 1. Elements not included in our system were projected onto major cations of similar size and charge: Mn<sup>2+</sup> and Mn<sup>3+</sup> were assigned to Fe<sup>2+</sup> and Fe<sup>3+</sup>, respectively; the divalent cations Zn<sup>2+</sup>, Ni<sup>2+</sup>, and Co<sup>2+</sup> were distributed proportionally over Mg<sup>2+</sup> and Fe<sup>2+</sup>; V<sup>3+</sup> was projected onto Fe<sup>3+</sup>; and Si<sup>4+</sup> was cast into Ti<sup>4+</sup>.

To limit the data used to fit the model to those spinels that are applicable to terrestrial or lunar mafic and ultramafic rocks, we developed a set of exclusion criteria. All experiments that contained vacancies on either crystallographic site were excluded. Likewise, experiments that contained  $Ca^{2+}$  were not included in the calibration, because even very small amounts of octahedral  $Ca^{2+}$  produced systematic errors in the model. We established a 5% site occupancy threshold for the other minor elements, i.e., we rejected experiments that reported greater than 5% occupancy in either site of  $Mn^{2+}$ ,  $Mn^{3+}$ ,  $Ni^{2+}$ ,

 $Zn^{2+}$ ,  $Co^{2+}$ ,  $V^{3+}$ , or Si<sup>4+</sup>. We noted no evidence that Jahn-Teller distortion affects the volume at the level of Mn<sup>3+</sup> substitution allowed (Ishii et al. 1972). In order that the projected composition of all calibrated spinels conform to the stoichiometry of the chosen model end members, within a reasonable tolerance, we filtered out any site occupancy data with reported site totals differing from unity by more than ±0.01 and any data with total cation charge greater than +8.03 or less than +7.97 per formula unit.

One drawback with the chosen data set is the lack of reliable error estimates for the observed and modeled molar volumes, which makes it difficult to weight the calibration data in any meaningful way. Errors on lattice parameters are estimated when data are curated in the AMCSD and suggest that the measurement precision values reported in the original sources can be misleadingly small. Room temperature fluctuations have a negligible effect on the total error, naturally, but interlaboratory differences can be significant for in situ experiments at high P and T; several of the high-T and high-P measurements have no reported bounds. Uncertainties associated with site occupancy determinations are almost never quantified but must vary depending on the type of sample (synthetic or natural), composition (end-member, binary, etc.), and measurement procedure. O'Neill and Dollase (1994) compared the effect of different refinement procedures on the final site occupancy distributions and several authors (e.g., Mattioli et al. 1987; O'Neill et al. 1992) have studied the effect of stoichiometry, or lack thereof, on molar volume. Given that it was impossible to come up with an automated strategy for assigning error estimates, we instead weighted all data equally but paid more attention to end-member and binary compositions when deciding between conflicting data or identifying outliers. This approach was later modified slightly to use weighted

nonlinear least squares, where all data were assigned the same nominal standard error except those requiring the MELTS ordering model (Doroshev et al. 1997; Brey et al. 1999; Girnis et al. 2003) which were given a doubled value. Reasons and details are given above and in the appendix (Supplementary Material 3).

### **MODEL FORMULATION**

For fitting molar volume data to our model, the data must first be recast in a consistent manner into the representation of composition and ordering state we adopted. Most of the data we use are given in the AMCSD in the form of cation mole fractions on the tetrahedral and octahedral sites. For this kind of data, we first transformed the molar cation proportions into the following set of linearly independent compositional variables:

$$X_{\rm sp} = {}^{[4]}X_{\rm Mg^{2+}} + 2{}^{[6]}X_{\rm Mg^{2+}}$$
(2a)

$$X_{\rm ch} = {}^{[6]}X_{\rm Cr^{3+}}$$
(2b)

$$X_{\rm uv} = 2^{[6]} X_{\rm Ti^{4+}}$$
(2c)

$$X_{\rm mt} = \frac{1}{2} \left( {}^{[4]}X_{\rm Fe^{3+}} + 2{}^{[6]}X_{\rm Fe^{3+}} \right), \tag{2d}$$

plus the dependent closure variable

$$X_{\rm hc} = 1 - X_{\rm sp} - X_{\rm ch} - X_{\rm uv} - X_{\rm mt}$$
(2e)

where  $sp = MgAl_2O_4$ ,  $ch = FeCr_2O_4$ ,  $uv = Fe_2TiO_4$ ,  $mt = Fe_3O_4$ , and  $hc = FeAl_2O_4$ . The cation ordering variables are

$$s_0 = {}^{[4]}X_{Mg^{2+}} - 2{}^{[6]}X_{Mg^{2+}}$$
(3a)

$$s_{1} = \frac{1}{2} \left( 2^{[6]} X_{AI^{3+}} - {}^{[4]} X_{AI^{3+}} \right)$$
(3b)

$$s_2 = \frac{1}{2} \left( 2^{[6]} X_{\text{Fe}^{3+}} - {}^{[4]} X_{\text{Fe}^{3+}} \right). \tag{3c}$$

Sack and Ghiorso (1991a) tabulate the relationships between these compositional and ordering parameters for various end-member and binary spinels. See the spinel volume web tool (http://magmasource.caltech.edu/calculator/) described below for more general bounds that the composition of a spinel implies for the range of possible values of the order parameters. Note that Sack and Ghiorso (1991a, b) originally included an order parameter for Cr that was later abandoned and that they numbered the order parameters 1 through 4; their  $s_4$  corresponds to our  $s_2$ . Although general dependence of volume on order state was considered, in practice only MgAl<sub>2</sub>O<sub>4</sub>-rich, MgFe<sub>2</sub>O<sub>4</sub>-rich, and FeAl<sub>2</sub>O<sub>4</sub>-rich compositions show order dependence of volumes at fixed composition.

Taking into account the site occupancy restrictions:

$$1 = {}^{[4]}X_{Fe^{2+}} + {}^{[4]}X_{Mg^{2+}} + {}^{[4]}X_{Al^{3+}} + {}^{[4]}X_{Fe^{3+}}$$
(4a)

$$1 = {}^{[6]}X_{Fe^{2+}} + {}^{[6]}X_{Mg^{2+}} + {}^{[6]}X_{AI^{3+}} + {}^{[6]}X_{Fe^{3+}} + {}^{[6]}X_{TI^{4+}} + {}^{[6]}X_{Cr^{3+}},$$
(4b)

expressions mapping mole fractions of cations into our chosen compositional and ordering variables may be readily derived.

The previously mentioned experiments of Doroshev et al. (1997), Brey et al. (1999), and Girnis et al. (2003) were quenched with an unknown ordering state. In order to use this data, we converted the reported oxide wt% into moles of cations. Note that the  $Fe^{2+}/Fe^{3+}$  ratio is calculated based on stoichiometry for this type of data, i.e., the total anion charge is -8, so the total charge of the cations must equal +8. We assumed the

experimental *P*, *T*-conditions represented conditions corresponding to equilibrium ordering state, and applied the MELTS ordering model (Sack and Ghiorso 1991b). Once we recast the electron microprobe data into the site occupancy model, we are able to calculate Eqs. 2a–e, and proceed as above. Measurements not included in the calibration dataset but used for comparison (e.g., Golla-Schindler et al. 2005) were treated in a similar way; reported synthesis conditions or annealing temperatures were used if available and, where necessary, forward models were repeated for a range of plausible equilibrium temperatures.

Standard state end-member properties included in our model are molar volume at reference pressure  $P_0 = 1$  bar and reference temperature  $T_0 = 298.15$  K ( $V^0$ ), coefficient of thermal expansion ( $\alpha$ ), isothermal bulk modulus ( $K_{0T}$ ), and the pressure derivative of the bulk modulus ( $K^2$ ). The general expression for the molar volume of a crystalline solid is  $V = V_{ideal} + V_{excess}$ , where excess volume of mixing is determined by an appropriate mixing model. The expression for  $V_{ideal}$  as a function of pressure, temperature, and composition is

$$V_{\text{ideal}} = \sum_{i} X_{i} V_{i} (P, T)$$

$$, \qquad (5)$$

where i = [sp, ch, uv, mt, hc].  $V_i(P, T)$  is found by using Newton's method to search along the high-*T* Vinet equation of state,

$$P = 3K_{\text{oT},i} \left(\frac{V_i}{V_i^{\text{o}}}\right)^{-\frac{1}{2}} \left[1 - \left(\frac{V_i}{V_i^{\text{o}}}\right)^{\frac{1}{2}}\right] \exp\left\{\frac{3}{2}\left(K_i' - 1\right)\left[1 - \left(\frac{V_i}{V_i^{\text{o}}}\right)^{\frac{1}{2}}\right]\right\} + \alpha_i K_{\text{oT},i} \left(T - T_{\text{o}}\right)$$
(6)

The Vinet formalism is applied to each end-member composition at the P and T of interest, and the resulting volumes are mixed to construct the ideal term. Initial estimates

of  $V^{\circ}$ ,  $\alpha$ ,  $K_{oT}$ , and K' for each end member came from the provisional xMELTS spinel volume model (Ghiorso 2004b) except that  $K_{oT}$  values were taken from ultrasonic studies where available.

During model calibration, we considered excess volume terms of the symmetric regular solution form  $W_{ij}X_iX_j$ , asymmetric regular solution form  $W_{ij}X_iX_j$ +  $dW_{ij}X_iX_j(X_i - X_j)$ , and special terms dependent on ordering parameters. We allowed terms to depend on *P* and *T* as a way to encode non-ideal mixing of compressibility or thermal expansion without resorting to use of explicitly *P*, *T*-dependent expressions for  $K_{oT}$ ,  $K^{\circ}$ , or  $\alpha$ . Ultimately, however, we found that such *P*- and *T*-dependent parameters were not justified by the data.

Although it can be used in any context where a model of spinel molar volume is required, the present model is designed to be compatible with the activity-composition and ordering model of Sack and Ghiorso (1991a, b), which accounts for the standard state, exchange, reciprocal, and excess energies among the same set of independent and dependent end members adopted here and computes equilibrium site occupancy by Gibbs energy minimization. In formulating an extension to describe molar volumes we begin of course by adopting the Sack and Ghiorso set of independent end members to define the standard state contribution to the volume as a function of pressure and temperature. However, in modeling the excess and ordering volumes we have a choice. We might adopt a parameter set strictly parallel to the parameters of the Sack and Ghiorso enthalpy model, or we might formulate a new model guided by the volume data available. The former approach has some theoretical justification in that non-ideal enthalpy and volume of solutions both arise from the same microscopic effect, namely mismatch of ionic radii of substituting cations. However, the Sack and Ghiorso enthalpy model has 32 parameters, which turns out to be many more than are needed to describe the non-ideal component of volume behavior. The formulation of the enthalpy model may not yield a stable minimum set of parameters when fit to the volume data in a conservative way. In practice, the volume data set can be fit with a much smaller set of parameters with a minimum of parameter correlation if we choose a different formulation for excess volumes and volumes of ordering. Since our interest is in creating the most useful and reliable model for use in macroscopic thermodynamic applications, we have therefore adopted a new formulation for the non-ideal parts of our volume model, not parallel to the formulation of the enthalpy and ordering model, even though the result may provide less insight into the microscopic origin of the excess volumes.

#### **MODEL CALIBRATION SUMMARY**

A full description of the strategy for calibration, including parameters deemed significant or negligible and data used or excluded, is provided in the appendix (Supplementary Material 3). In summary, the standard state end-member properties, ordering terms, and contributions to selected symmetric regular solution terms were first calibrated to ambient, high-*T*, and high-*P* data for all end members, including the dependent end members MgCr<sub>2</sub>O<sub>4</sub>, MgFe<sub>2</sub>O<sub>4</sub>, and Mg<sub>2</sub>TiO<sub>4</sub>. The provisional regular solution terms accounted for the volume of reaction for the formation of the dependent end members by Fe-Mg exchange and were constructed in a way that was consistent with the data for binary and ternary spinels. Then the remaining ambient pressure and temperature data were used to calibrate additional contributions to  $V_{excess}$  as a function of composition. Finally, the small number of high-T and high-P data for intermediate compositions were checked to see if they required any revisions to the model, which they did not.

We have developed a calibration scheme that is able to directly query a MySQL database containing phase equilibrium, site occupancy, and volume data. The calibration scheme is written in MATLAB<sup>TM</sup> and uses the MATLAB-MySQL interface written by Robert Almgren (http://www.mathworks.com/matlabcentral/fileexchange/8663-mysql-database-connector). For the database, we adapted the schema from the Library of Experimental Phase Relations (LEPR) (Hirschmann et al. 2008) to incorporate the cell parameters and site occupancy data, with suitable metadata, and made a number of internal changes that reflect the different ways in which the two databases are updated and accessed. The MATLAB-generated MySQL queries allow us to test the effect of including or excluding a particular data source or type of experiment (e.g., heated in situ vs. annealed and quenched) with minimal effort and without the need for intermediate files.

Three terms describe the observed linear volume dependence on each of the cation ordering variables  $s_0$ ,  $s_1$ , and  $s_2$  (Fig. 3). The adopted standard states for MgAl<sub>2</sub>O<sub>4</sub> and FeAl<sub>2</sub>O<sub>4</sub> are sufficiently close to normally ordered (e.g.,  $s_1 = 0.99998$  for FeAl<sub>2</sub>O<sub>4</sub>) that we assume them to be perfectly so. In order for the  $s_0$ -dependent term to vanish at Mg-free compositions, we used the form  $(s_0 - 1)/2$ , multiplied by  $X_{sp}$ , as this represents total Mg cations per formula unit. Likewise, we multiplied the  $s_1$ -dependent term by total Al, given by  $2(X_{sp} + X_{hc})$ . End member Fe<sub>3</sub>O<sub>4</sub> is approximated as perfectly inversely ordered and the  $s_2$  term is multiplied by total Fe<sup>3+</sup>, which is simply  $2X_{mt}$  (see Eq. 7 below). The resulting ordering-composition cross-terms account for nearly all the excess

volume along several key binaries: notably Fe<sub>3</sub>O<sub>4</sub>–Fe<sub>2</sub>TiO<sub>4</sub> (Bosi et al. 2009) and MgAl<sub>2</sub>O<sub>4</sub>–FeAl<sub>2</sub>O<sub>4</sub> (Andreozzi and Lucchesi 2002).

That no additional excess volume terms were required on the MgAl<sub>2</sub>O<sub>4</sub>-FeAl<sub>2</sub>O<sub>4</sub> join was particularly fortuitous as it meant that the  $Cr^{3+}$ -, Fe<sup>3+</sup>-, and Ti<sup>4+</sup>-bearing spinel subsystems could be considered separately when choosing between candidate parameters. This process is described in more detail in the appendix. Briefly, we chose the minimum number of parameters that could reasonably describe the ordering-adjusted volume surface for each reciprocal square (e.g., MgAl<sub>2</sub>O<sub>4</sub>-MgCr<sub>2</sub>O<sub>4</sub>-FeCr<sub>2</sub>O<sub>4</sub>-FeAl<sub>2</sub>O<sub>4</sub>) before moving our attention to other joins, such as Fe<sub>3</sub>O<sub>4</sub>-Fe<sub>2</sub>TiO<sub>4</sub> and Fe<sub>3</sub>O<sub>4</sub>-FeCr<sub>2</sub>O<sub>4</sub>, and to multicomponent spinels.

When calibrating the independent and dependent end-member data we were careful not to activate excess volume terms along joins, such as FeCr<sub>2</sub>O<sub>4</sub>–MgCr<sub>2</sub>O<sub>4</sub> (Lenaz et al. 2004), which showed little or no deviation from ideality once any ordering effects were subtracted. This strategy meant that certain combinations of solution parameters (e.g.,  $W_{mt-sp} - W_{mt-hc}$ ) were constrained by the end-member data. In the second stage calibration, end-member properties and ordering parameters were fixed; instead we adjusted the existing excess volume terms (i.e.,  $W_{sp-ch}$ ,  $W_{hc-ch}$ ,  $W_{mt-sp}$ , and  $W_{mt-hc}$ ) and introduced two new terms (d $W_{sp-ch}$  and  $W_{ch-mt}$ ) in a way that did not disturb the fit for the dependent end members.

We used the Bayesian information criterion (BIC) (Schwarz 1978), along with analysis of the reduced  $\chi^2$  statistic, to measure how efficiently our parameterized model predicts the data. Since we do not have information on measurement error for site occupancies, we are required to estimate the error in observed volume for calculating reduced  $\chi^2$ . The BIC assigns a penalty term that is based on the complexity, or number of parameters, of the model. The formula is BIC =  $-2 \ln(L) + k \ln(n)$ , where  $\ln(L)$  is the optimized log-likelihood function associated with a particular model, *k* is the number of model parameters, and *n* is the number of observations associated with  $\ln(L)$ . By applying the BIC iteratively to the excess volume parameters and analyzing reduced  $\chi^2$ , for example, all excess terms involving  $X_{uv}$  were found to be insignificant.

In all stages of the model, each of the five explicit end members has a pure component equation of state described by four parameters:  $V^{\circ}$ ,  $\alpha$ ,  $K_{oT}$ , and K' (Tables 2–4). The additional parameters of the preferred final model include six excess terms along compositional binaries and three order-dependent terms. The values of the excess volume and order-dependent parameters are given in Table 5. The full expression of the model is:

$$V(X,T,P,s) = \sum_{i} X_{i}V_{i}(T,P) + W_{hc-ch}X_{hc}X_{ch} + W_{ch-mt}X_{ch}X_{mt} + W_{sp-ch}X_{sp}X_{ch} + dW_{sp-ch}X_{sp}X_{ch} (X_{sp} - X_{ch}) + W_{mt-hc}X_{mt}X_{hc} + W_{mt-sp}X_{mt}X_{sp} + \frac{(s_{0}-1)}{2}W_{s0}X_{sp} + 2(s_{1}-1)W_{s1}(X_{sp} + X_{hc}) + 2s_{2}W_{s2}X_{mt}$$
(7).

This model fits virtually all of the data to within 0.02 J/bar/mol, or better than 0.5% in volume (Figs. 4, 5), with a few exceptions for the studies of Antao et al. (2005a, 2005b), Finger et al. (1986), Haavik et al. (2000), and Méducin et al. (2004) (see appendix for exclusion criteria for these studies). The mean of the absolute values of the residuals for calibrated data is 0.0038 J/bar/mol and the root mean squared error is 0.0053 J/bar/mol. The goodness-of-fit of the model is displayed in Fig. 4, where observed molar volume is

plotted first against the volume model with asymmetric excess on the Fe<sub>3</sub>O<sub>4</sub>–Fe<sub>2</sub>TiO<sub>4</sub> join used in current versions of MELTS (Fig. 4a), then against an ideal volume model using the refined end-member equation of state coefficients from Tables 2–4 (Fig. 4b), and finally against the model volume from the full form of Eq. 7 (Fig. 4c). When compared with the ideal model (Fig. 4b), data for dependent end members MgCr<sub>2</sub>O<sub>4</sub> and MgFe<sub>2</sub>O<sub>4</sub> show parallel trends with fixed offsets from the equiline that correspond to the quantities  $2(W_{sp-ch} - W_{hc-ch})$  and  $2(W_{mt-sp} - W_{mt-hc})$  respectively. The residuals in the final model are uncorrelated with observed molar volume and are nearly all ≤0.02 J/bar/mol for data accepted into the calibration set (Fig. 5). It is difficult to derive a more quantitative assessment of goodness-of-fit given uncertain knowledge of errors on composition, site occupancy, and molar volume in the calibration data.

Obtaining confidence bounds on the derived parameters is a separate issue from goodness-of-fit and can be addressed using bootstrap estimation (Efron 1982). Uncertainty bounds on each fitted parameter and a full correlation matrix for the parameter set derived from 50,000 bootstrap iterations are given in Supplementary Material 4 and 5. All parameters were varied, except those taken from ultrasonic studies or other sources (e.g., the Levy et al. (2003) fit of  $K_{oT}$  and K' for MgAl<sub>2</sub>O<sub>4</sub>). The end-member properties are generally independent of one another; those trade-offs that exist are understandable given that the high-*T* properties of Fe<sup>3+</sup>-, and Ti<sup>4+</sup>-bearing spinels were constrained by data for the dependent end members MgFe<sub>2</sub>O<sub>4</sub> and Mg<sub>2</sub>TiO<sub>4</sub> respectively. Likewise, there is some correlation between  $V^{\circ}$  and ordering parameters for certain end members. As expected (see the appendix), many of the purely compositional excess parameters are strongly correlated (e.g.,  $W_{sp-ch}$  with  $W_{hc-ch}$ , and  $W_{mt-sp}$  with  $W_{mt-hc}$ )
and are well-defined only in a joint sense. These observations emphasize that the calibrated parameters should only be used in the context of the full model derived here and that they may not be optimal descriptions of subsystems if ideal and excess terms are separated. As noted above, the calibration data vary in coverage and quality, and full estimates of measurement uncertainties are sparse, which means confidence bounds calculated using random sampling are probably unrealistically wide. Large uncertainties in the fit parameters do not necessarily feed into large uncertainties in the volumes calculated using the calibrated model, once the high degree of correlation between parameters is accounted for properly (Powell and Holland 1985). However, as the Vinet equation must be solved iteratively at each stage of the calculation in this case, such sophisticated propagation of uncertainties is neither straightforward nor likely to be that informative. Instead, for comparison, we obtained a more conservative confidence bound for each parameter, again using the bootstrapping technique but holding all other parameters at their optimal values; these uncertainties are reported in Tables 2, 3, and 5.

A few aspects of the model remain underconstrained by data. There are neither ultrasonic velocities nor in situ high-*P* volume data for Ti-bearing compositions. We are forced to assume that the  $K_{oT}$  and K' for Fe<sub>2</sub>TiO<sub>4</sub> are equal to those of Fe<sub>3</sub>O<sub>4</sub> (Table 4). Given the low compressibility of spinels and the small Ti<sup>4+</sup> concentrations in most spinels (and in particular in all the spinels that will be used for calibration of models of coexisting phases like garnet and pyroxene), this assumption is unlikely to have any significant effect on free energies within the pressure range of the spinel stability field. In addition, the value of  $\alpha$  for FeCr<sub>2</sub>O<sub>4</sub> was not varied during the calibration process. However, when the final model is used to calculate the effective  $\alpha$  for a chromian spinel with composition MgAl<sub>0.8</sub>Cr<sub>1.2</sub>O<sub>4</sub>, the value agrees with the one obtained from the lower temperature measurements (i.e., the ones made below the blocking temperature) of Levy and Artioli (1998, see fig. 3a) to within  $5.4 \times 10^{-7}$  K<sup>-1</sup>. In the absence of high-*T* structural refinements for FeCr<sub>2</sub>O<sub>4</sub>, we believe that the value of  $\alpha$  adopted is the best estimate currently available.

# DISCUSSION

#### **Model comparison**

A comparison between our model and the model of Brev et al. (1999) is shown in Fig. 6. The gray surface in Fig. 6a shows the excess volume of the Brey et al. model in the MgAl<sub>2</sub>O<sub>4</sub>-MgCr<sub>2</sub>O<sub>4</sub>-FeCr<sub>2</sub>O<sub>4</sub>-FeAl<sub>2</sub>O<sub>4</sub> reciprocal square, i.e., the end-member contributions have been subtracted from the volume surface. The data plotted have also had the ideal end-member contributions to their volumes subtracted. The surface in Fig. 6b shows our proposed model in the same composition space as the Brey et al. model. Here, contributions from the end members and from our model ordering terms have been subtracted from model and data. That the data appear smoother in Fig. 6b than in Fig. 6a shows that there is a distinct ordering effect on the volume that cannot be mapped into a purely compositional term (cf. the Fe<sub>3</sub>O<sub>4</sub>–Fe<sub>2</sub>TiO<sub>4</sub> join discussed below). Both models give similar fits to the data of Brey et al. and Doroshev et al. (1997) (diamonds), but the molar volume of FeCr<sub>2</sub>O<sub>4</sub> is more tightly constrained in our model due to the availability of data from Lenaz et al. (2004). Unlike Brey et al., we explicitly account for the dependent end member (MgCr<sub>2</sub>O<sub>4</sub> in our model; FeAl<sub>2</sub>O<sub>4</sub> in theirs) so our model predicts a warped volume surface. Finally, by making the asymmetrical excess volume term a

function of  $X_{sp}$  and  $X_{ch}$ , rather than  $X_{Cr}$  and  $X_{Al}$ , our model is able to provide a much better fit to the rest of the data in the composition space (circles in Fig. 6).

The surface in Fig. 7a shows our model for spinels in MgAl<sub>2</sub>O<sub>4</sub>-MgFe<sub>2</sub>O<sub>4</sub>-Fe<sub>3</sub>O<sub>4</sub>-FeAl<sub>2</sub>O<sub>4</sub> space; again, the end-member and ordering contributions have been subtracted from the model surface and the data. The model along the MgAl<sub>2</sub>O<sub>4</sub>–Fe<sub>3</sub>O<sub>4</sub> binary is plotted in Fig. 7b, along with the model of Mattioli et al. (1987); the magnitudes of the positive, symmetric excess terms of the two models along this join are similar, and differences of up to ~0.004 J/bar are due to different standard state volumes for Fe<sub>3</sub>O<sub>4</sub>. The ordering state of Fe<sub>3</sub>O<sub>4</sub> is particularly hard to quench, since it involves only electron exchange rather than cation mobility; hence in Fig. 7b we show curves with and without the modeled volume contribution due to ordering. A significant part of the asymmetry observed by Mattioli et al. on this join is apparently attributable to the compositiondependent closure temperature of the ordering reaction. Once these systematic differences in order state and measured volumes for compositions approaching pure Fe<sub>3</sub>O<sub>4</sub> are accounted for, the data do not seem to justify an asymmetric excess volume term like the one used by Mattioli et al. The calibration data of Nakatsuka et al. (2004) (MgAl<sub>2</sub>O<sub>4</sub>–MgFe<sub>2</sub>O<sub>4</sub>) and the comparison data of Golla-Schindler et al. (2005) (FeAl<sub>2</sub>O<sub>4</sub>–Fe<sub>3</sub>O<sub>4</sub>) also support the simpler symmetric formalism (Fig. 7a).

The current spinel volume model formulation in MELTS and pMELTS includes asymmetric excess volume terms for the  $Fe_3O_4$ – $Fe_2TiO_4$  binary (Sack and Ghiorso 1991a), whereas in our model the asymmetry comes solely from the ordering contribution to the volume (Fig. 8). The asymmetry in the volume variation is subtle, though well resolved by the data. The strong preference of Ti<sup>4+</sup> for the octahedral site limits the configurational freedom along this join but the Mössbauer spectroscopic measurements of Bosi et al. (2009) indicate a sigmoidal variation in  $s_2$  with  $X_{uv}$ . Deviations from the Akimoto (1954) ordering model (equivalent to setting  $s_2 = 0$ ) have the same sense as the deviations from Vegard's Law, so it is not surprising that introducing an ordering dependence to the volume model may reduce the need for asymmetric interaction parameters. However, inasmuch as the ordering terms were calibrated only with data for pure MgFe<sub>2</sub>O<sub>4</sub> ( $s_0$  and  $s_2$ ), MgAl<sub>2</sub>O<sub>4</sub> ( $s_0$  and  $s_1$ ), and FeAl<sub>2</sub>O<sub>4</sub> ( $s_1$ ), the result that excess volume along the Fe<sub>3</sub>O<sub>4</sub>–Fe<sub>2</sub>TiO<sub>4</sub> binary may be completely explained in this way is unexpected. Although we cannot properly constrain the nature of Ti<sup>4+</sup> mixing in spinel, the notion that same-site substitution is nearly ideal for volume (i.e., excess volumes of mixing are due to ordering amongst the other cations) is supported by the few data available within the Ti-bearing subsystem.

The MELTS ordering model of Sack and Ghiorso (1991a, b) predicts a cation distribution that is independent of pressure because the accompanying volume model is ideal. As shown here, ordering effects can successfully explain all of the observed symmetric excess volumes of mixing on the MgAl<sub>2</sub>O<sub>4</sub>–FeAl<sub>2</sub>O<sub>4</sub> join (Andreozzi and Lucchesi 2002), and the MgAl<sub>2</sub>O<sub>4</sub>–Mg<sub>2</sub>TiO<sub>4</sub> and FeAl<sub>2</sub>O<sub>4</sub>–Fe<sub>2</sub>TiO<sub>4</sub> binaries (Muan et al. 1972), as well as the aforementioned variation along Fe<sub>3</sub>O<sub>4</sub>–Fe<sub>2</sub>TiO<sub>4</sub> (Bosi et al. 2009). The ordering parameters in our model also capture most of the mixing behavior along the Fe<sub>3</sub>O<sub>4</sub>–FeCr<sub>2</sub>O<sub>4</sub> join (Robbins et al. 1971; Woodland et al. 2009) (Fig. 9). In our model the excess volume term along this join is symmetric and constrained by data elsewhere in the composition space; introduction of the *s*<sub>2</sub> ordering term accounts for the asymmetry in the data (which were not included in the calibration). Furthermore, when the MELTS

ordering model (Sack and Ghiorso 1991a, b) is updated to use our volume expression in the self-consistent calculation of ordering state by Gibbs energy minimization, it successfully predicts the high-P, T results of Antao et al. (2005b) for MgFe<sub>2</sub>O<sub>4</sub> (Fig. 10). Note that the Antao et al. study was not included in the calibration data set (see appendix) but the observed variation in volume with P and T is nevertheless consistent with the model presented here. The general role of order-disorder reactions in mineral volumes and compressibilities has been discussed by Hazen and Navrotsky (1996) and illustrated for the highly order-sensitive spinel CoFe<sub>2</sub>O<sub>4</sub> by O'Neill and Navrotsky (1983). Spinel molar volume is a specific function of the tetrahedral (A-O) and octahedral (B-O) bond distances. The volume is about twice as sensitive to the B-O distance as it is to the A-O distance. Given differences in the ionic radii of cations in different coordination environments and valence states it is therefore not surprising that spinel volumes are strongly order-dependent. Hazen and Navrotsky argue that such behavior is most pronounced when there are changes in ionic charge and coordination number, such that order-disorder reactions in minerals like olivine and orthopyroxene (mostly involving Fe<sup>2+</sup> and Mg on crystallographically distinct octahedral sites) should show smaller effects than those documented here for spinel. The pyroxene model of Sack and Ghiorso (1994a) includes an ordering contribution to the volume.

## **Applications**

Klemme (2004) presented experimental reversals for the garnet–spinel transition reaction  $MgCr_2O_4 + 4 MgSiO_3 = Mg_3Cr_2Si_3O_{12} + Mg_2SiO_4$ . Klemme (2004) and Klemme et al. (2009) used the experimental brackets to extract enthalpy of formation and standard state entropy for the garnet end member knorringite (Mg<sub>3</sub>Cr<sub>2</sub>Si<sub>3</sub>O<sub>12</sub>). Klemme's (2004) values were designed to be consistent with the thermodynamic data set of Holland and Powell (1990), whereas Klemme et al. (2009) used a later version (Holland and Powell 1998). When Hamecher et al. (2009) repeated the exercise using the provisional xMELTS thermodynamic data set (based on Berman 1988, but updated to use the Vinet equation of state) the recovered standard state entropy of Mg<sub>3</sub>Cr<sub>2</sub>Si<sub>3</sub>O<sub>12</sub> was significantly lower than the values given by Klemme and co-workers. We tested the effect of our newly calibrated spinel model and found only a small difference with the result from Hamecher et al. (2009). We also used the expressions and thermodynamic data of Holland and Powell (1990, 1998) and retrieved essentially the same values as Klemme et al. (2009). We could reproduce the results of Klemme (2004) but only if we used the volume parameters taken from Klemme et al. (2009) for both Mg<sub>3</sub>Cr<sub>2</sub>Si<sub>3</sub>O<sub>12</sub> and MgCr<sub>2</sub>O<sub>4</sub>, instead of those from Irifune et al. (1982) and Robie et al. (1979) (see table 3 in Klemme 2004). Hence, while the volume properties of MgCr<sub>2</sub>O<sub>4</sub> spinel clearly influence the location of the spinelgarnet transition in the MgO-Cr<sub>2</sub>O<sub>3</sub>-SiO<sub>2</sub> system, a final volume model for Cr-bearing garnet is required in order to update the Hamecher et al. (2009) thermodynamic data for  $Mg_3Cr_2Si_3O_{12}$ .

In the meantime, we can gauge the effect that the new spinel volume model might have on MELTS calculations by extracting a typical lherzolite spinel composition from pMELTS, and comparing its molar volume before and after recalibration. At 1000 °C and 3 GPa, in the Workman and Hart (2005) depleted mantle composition, pMELTS predicts a spinel of composition  $Fe^{2+}_{0.42}Mg_{0.60}Fe^{3+}_{0.20}Al_{0.38}Cr_{1.40}Ti_{0.01}O_4$ . The molar volume of this spinel at the applied conditions, calculated with the spinel molar volume model built into all versions of MELTS to date, is V(X, T, P, s) = 4.3285 J/bar/mol, whereas the model proposed in this work yields V(X, T, P, s) = 4.3564 J/bar/mol, a difference  $\delta V_{sp} = +0.0279$  J/bar/mol. Although a full internally consistent recalibration of all aspects of the MELTS model is needed to see all the consequences of such a difference, the following simple calculation gives a preliminary sense of the magnitude of possible effects.

The difference in spinel molar volume corresponds to a difference in the model Gibbs free energy of spinel at elevated pressure, and hence in the Gibbs free energy change of any reaction involving spinel. Keeping other quantities constant, the resulting displacement in pressure of an equilibrium boundary can be found by comparing the change in Gibbs energy of reaction to the volume change across the reaction. In this way, we can estimate, for example, what change in modeled pressure of the spinel–garnet transition will result when the current model is assimilated into MELTS. This is of some interest, because the match between experimental determinations of this boundary and MELTS calculations performed between 1995 and 2004 turned out to depend on an error in the implementation of the garnet solid solution model that was applied during model calibration (Berman and Koziol 1991). Since that error was fixed (Smith and Asimow 2005), the model is no longer self-consistent, and the spinel–garnet lherzolite reaction has been calculated at a pressure as much as 0.8 GPa lower than experimental constraints (Hamecher et al. 2009).

For simplicity, consider the spinel–garnet lherzolite reaction in the simple MgO-Al<sub>2</sub>O<sub>3</sub>-SiO<sub>2</sub> (MAS) system:  $MgAl_2O_4 + 2Mg_2Si_2O_6 = Mg_3Al_2Si_3O_{12} + Mg_2SiO_4$ . At equilibrium, the Gibbs free energy of reaction  $(\Delta G_{rxn})$  is zero, and  $\frac{\partial \Delta G_{rxn}}{\partial P}\Big|_{T} = \Delta V_{rxn}$ . We make the approximation:

$$\delta P_T \approx \frac{\delta \Delta G_{rxn}}{\Delta V_{rxn}} = -\frac{\delta G_{sp}}{\Delta V_{rxn}}, \qquad (8)$$

where  $\delta$  refers to the finite change between the two models, and  $\delta\Delta G_{rxn} = -\delta G_{sp}$  since we are not changing the properties of the other reactants or products. We can estimate the difference between models in the Gibbs free energy of spinel by making the approximation:

$$\delta G_{\rm sp} = \int_{P_{\rm o}}^{P} \delta V_{\rm sp} \, dP \approx \delta V_{\rm sp} \left( P - P_{\rm o} \right) \tag{9}$$

which, for  $\delta V_{sp} = +0.0279$  J/bar/mol and P = 3 GPa, gives  $\delta G_{sp} \sim 837$  J/mol.  $\Delta V_{rxn}$  for the MAS reaction at 1000 °C and 3 GPa is -0.8243 J/bar/mol (from the MELTS model, but substituting the new spinel model would make less than a 4% difference in this number). Hence, the approximate displacement in the equilibrium pressure of the reaction between the two models is  $\delta P_T \approx +1015$  bars = +0.1 GPa. This difference is in the right direction to address the error in the current MELTS model, but is not enough to explain the entire discrepancy. Recovering a self-consistent model that matches experimental constraints on the pressure of the spinel-garnet lherzolite reaction will require a full recalibration of the garnet and pyroxene activity-composition models, with the best available molar volume models built into the calibration.

#### Web tool

There may be many applications of a molar volume model for spinel solid solutions beyond the MELTS-based calculations discussed herein. However, the formulation of the present model in terms of MELTS end members and ordering variables may be an impediment to its wide application. Hence we have provided an online tool (http://magmasource.caltech.edu/calculator/) whereby users may input a spinel either as site occupancy data (that is, both composition and ordering state are measured) or as electron probe data with specified P and T (that is, composition is measured but ordering state is not) or as mole fractions of end members (including dependent ones) with P and T. The data are processed in the same manner as the calibration data herein and assigned to MELTS end members. If site occupancy is not given, the updated MELTS ordering model (i.e., based on Sack and Ghiorso (1991a, b) but with the volume model presented here) is used to estimate site occupancy by Gibbs energy minimization. Much like annealed samples within the calibration data set, there is an option to specify one set of *P-T* conditions for the equilibrium ordering state  $(P_S, T_S)$  and another for calculation of the measureable volume  $(P_V, T_V)$ . The web tool automatically converts between the various input options, such as wt% oxides and site occupancy, and displays the results. Finally,  $V_{\text{ideal}}$  (as a function of  $P_V$  and  $T_V$ ),  $V_{\text{excess}}$  (calculated with site occupancy data, or with the MELTS ordering model using  $P_s$  and  $T_s$ ), and  $V_{\text{total}}$  (i.e.,  $V_{\text{ideal}} + V_{\text{excess}}$ ) and, where appropriate, their derivatives with the relevant P and T are returned to the user.

## **Future prospects**

Many of the assumptions made in our model were required due to the lack of published full structural refinements for certain regions of composition, temperature, and pressure space. There is a dearth of data available for the volume of Ti- and Cr-bearing spinels: we found only one in situ high-T study for Ti-bearing spinel,  $Mg_2TiO_4$  (O'Neill et al. 2003), and not a single in situ high-P (at ambient or elevated-T) data set. Likewise, the Martignago et al. (2003) in situ high-*T* study of three natural samples along the MgAl<sub>2</sub>O<sub>4</sub>–MgCr<sub>2</sub>O<sub>4</sub> binary is the only full refinement we currently have for Cr-bearing spinels measured at elevated conditions. As detailed in our discussion of Haavik et al. (2000), Méducin et al. (2004), and Antao et al. (2005a, b) (see the appendix), there are some substantial disagreements in the literature even among compositions that have been more widely studied, e.g.,  $MgAl_2O_4$  and  $Fe_3O_4$ , and some hints that the behavior of spinel at simultaneous high-P, T may by more complex than at high-P or high-T conditions alone. On the other hand, recent systematic measurements of the site occupancy and volume of binary spinels (Andreozzi and Lucchesi 2002; Bosi et al. 2009) allowed us to calibrate the effect of ordering on volume in a way not possible when the MELTS spinel model was originally developed (Sack and Ghiorso 1991a). It is our hope that with the current advances in X-ray and neutron diffraction methods, more high-quality data will become available for these spinel compositions, particularly at simultaneous very high-P and -T, enabling us to improve upon our model assumptions in future calibrations.

# Acknowledgements

We wish to thank Peter Luffi for identifying the garnet solid solution error in the original MELTS code, Ashley Nagle for pointing out the anomalously low spinel-garnet transition pressures obtained when the corrected garnet model is used, and Aaron Wolf for helpful discussions regarding statistical analysis. Comments by Associate Editor Jon Blundy are greatly appreciated, as are the reviews of two anonymous reviewers. This work was supported by the National Science Foundation and the American Recovery and Reinvestment Act through award 0838244.

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Composition	Total Experiments	Ambient- <i>P</i> , T	High- <i>P</i>	P Range (GPa)	High- <i>T</i>	T Range (ºC)
End members						
$MgAl_2O_4$	151	48	21	0.6 - 29	82	132 - 1600
FeCr <sub>2</sub> O <sub>4</sub>	1	1	0	ı	0	ı
Fe <sub>2</sub> TiO <sub>4</sub>	2	2	0	ı	0	·
$Fe_3O_4$	55	6	46	0.02 - 11.11	0	·
FeAl <sub>2</sub> O <sub>4</sub>	40	23	0	ı	17	200 - 1150
$MgCr_2O_4$	9	9	0	ı	0	ı
$MgFe_2O_4$	69	24	28	0.11 - 34.39	17	100 - 1200
$Mg_2TiO_4$	53	2	0	ı	51	70 - 1416
Binaries						
MgAl <sub>2</sub> O <sub>4</sub> -FeAl <sub>2</sub> O <sub>4</sub>	14	14	0	ı	0	ı
FeAl <sub>2</sub> O <sub>4</sub> -Fe <sub>3</sub> O <sub>4</sub>	1	1	0	ı	0	I
$FeTi_2O_4$ - $Fe_3O_4$	18	18	0	ı	0	ı
MgAl <sub>2</sub> O <sub>4</sub> -MgFe <sub>2</sub> O <sub>4</sub>	68	15	4	1 - 4	49	200 - 1050
MgAl <sub>2</sub> O <sub>4</sub> -MgCr <sub>2</sub> O <sub>4</sub>	194	87	34	4.4 - 8.03	73	200 - 1000
FeCr <sub>2</sub> O <sub>4</sub> -MgCr <sub>2</sub> O <sub>4</sub>	13	13	0	ı	0	ı
$Fe_3O_4$ -Mg $Fe_2O_4$	1	1	0	ı	0	
Multi-component	146	146	0	ŗ	0	ı
Total	832	410	133		289	

Table 2.	Optimized standard sta	ate end-member I	nolar volume for giv	en ordering stat	e	
	V° (J/bar/mol)	$1\sigma^*$	Ordering State	[S <sub>0</sub> , S <sub>1</sub> , S <sub>2</sub> ]	a (Å)	p (kg m <sup>-3</sup> )
MgAl <sub>2</sub> O <sub>4</sub>	3.9722	$1.7 \times 10^{-4}$	normal	[1, 1, 0]	8.0808	3582
FeCr <sub>2</sub> O <sub>4</sub>	4.4243	$3.9 \times 10^{-4}$	normal	[0, 0, 0]	8.3765	5059
Fe <sub>2</sub> TiO <sub>4</sub>	4.6873	$3.1 \times 10^{-4}$	normal	[0, 0, 0]	8.5393	4769
$Fe_3O_4$	4.4553	$5.5 \times 10^{-4}$	inverse	[0, 0, 0]	8.3960	5197
$FeAl_2O_4$	4.0871	$2.0 \times 10^{-4}$	normal	[0, 1, 0]	8.1580	4253
* Bootstra	p estimation of s.d. for	- V°, holding other	parameters at optir	nal values		

Table 3. Standard state end-member coefficient of thermal expansion

	$\alpha(10^{-5}) (K^{-1})$	1σ*
MgAl <sub>2</sub> O <sub>4</sub>	2.4413	$3.8 \times 10^{-8}$
FeCr <sub>2</sub> O <sub>4</sub> <sup>a</sup>	2.1691	1
Fe <sub>2</sub> TiO <sub>4</sub>	3.3458	$4.3 \times 10^{-8}$
Fe <sub>3</sub> O <sub>4</sub>	3.3376	$2.1 \times 10^{-7}$
FeAl <sub>2</sub> O <sub>4</sub>	2.6431	$2.7 \times 10^{-8}$
*Bootstrap estin	nation of s.d. for α	, holding other parameters at optimal
values		
<sup>a</sup> Value of $\alpha$ was	held constant (see	text)

members		
	$K_{oT}$ (GPa)	K'
MgAl <sub>2</sub> O <sub>4</sub>	190.8 <sup>a</sup>	6.77 <sup>a</sup>
FeCr <sub>2</sub> O <sub>4</sub>	203.3 <sup>b</sup>	6.5 <sup>c</sup>
Fe <sub>2</sub> TiO <sub>4</sub>	181 <sup>d</sup>	5.5 <sup>d</sup>
Fe <sub>3</sub> O <sub>4</sub>	181 <sup>e</sup>	5.5 <sup>e</sup>
FeAl <sub>2</sub> O <sub>4</sub>	210.3 <sup>f</sup>	5.5 <sup>d</sup>
<sup>a</sup> From Levy et al. (2003)		

Table 4. Bulk moduli and pressure derivatives of end

<sup>b</sup> From Doraiswami (1947)

 $^{\rm c}$  Constrained using data of Fan et al. (2008) (see appendix)  $^{\rm d}$  Assumed to be equal to that of Fe $_3O_4$  (see text)

<sup>e</sup> From Nakagiri et al. (1986)

<sup>f</sup> From Wang and Simmons (1972)

	*	.8 (J/bar/mol)	6	[4	33	5	5	)49	)49	19	teren date -
meters	1σ	0.01	15 0.1	78 0.01	30.06	33 0.02	20 0.02	92 0.000	54 0.00C	35 0.00	n of c d for
. Model para		h 0.010	ιt 0.071	h 0.057	ch 0.048	с 0.083	p 0.102	-0.06	0.026	0.103	ran ectimatic
Table 5.		$W_{hc-cl}$	$W_{ch-m}$	$W_{\sf sp-cl}$	$dW_{\rm sp-c}$	$W_{ m mt-h}$	$W_{ m mt-s}$	$W_{\rm s0}$	$W_{\rm s1}$	$W_{s2}$	* Bootsti

Bootstrap estimation of s.d. for each parameter, holding all other parameters at optimal values

#### FIGURE CAPTIONS

**Fig. 1** Compositional coverage of the data used in model calibration. Diamonds are experiments measured at ambient conditions, circles are experiments measured at high-*T*, squares are data measured at high-*P*. Triangles are data not included in calibration due to lack of measured site occupancies, and are used only for comparison to model results below. **a** Mg# (Mg/[Mg + Fe<sup>2+</sup>]) vs. Ti<sup>4+</sup>/(Ti<sup>4+</sup> + Cr<sup>3+</sup> + Fe<sup>3+</sup> + Al<sup>3+</sup>); **b** Mg# vs.  $Cr^{3+}/(Ti^{4+} + Cr^{3+} + Fe^{3+} + Al^{3+})$ ; **c** Mg# vs. Fe<sup>3+</sup>/(Ti<sup>4+</sup> + Cr<sup>3+</sup> + Fe<sup>3+</sup> + Al<sup>3+</sup>).

**Fig. 2** Compositional coverage of the data used in model calibration. Symbols the same as in Fig. 1. **a**  $Ti^{4+}$  -  $Fe^{3+}$  -  $Al^{3+}$  ternary; **b**  $Cr^{3+}$  -  $Fe^{3+}$  -  $Al^{3+}$  ternary.

**Fig. 3** Order-dependence of observed volume vs. ordering term for **a**  $s_0$ , **b**  $s_1$ , and **c**  $s_2$ . The order-dependence for  $s_0$ , for example, is calculated by subtracting the model  $V_{ideal}$  and all other model terms not dependent on  $s_0$  from the observed volume,  $V_{obs}$ . The linearity of the data shows the simple form of the order-dependence of the data and the slope of the linear trend is proportional to the  $W_{s0}$  parameter (see Eq. 7). The corresponding calculations were done for  $s_1$  and  $s_2$ . The solid lines are the model order-dependences, plotted over the relevant range of ordering states.

**Fig. 4** Model volume vs. measured volume. **a** Model results using spinel volume model currently implemented in MELTS (shown in box); **b** Ideal model volume results after

fitting of end members. Model results from **a** shown grayed out; **c** Final model results. Model results from **a** shown grayed out.

Fig. 5 Model residuals vs. measured volume. Gray diamonds are samples measured at ambient-P, T, open circles are samples measured at high-T in situ, and black squares are samples measured at high-P in situ.

**Fig. 6** Model volumes and data in the MgAl<sub>2</sub>O<sub>4</sub>-MgCr<sub>2</sub>O<sub>4</sub>-FeCr<sub>2</sub>O<sub>4</sub>-FeAl<sub>2</sub>O<sub>4</sub> reciprocal square. End members are labeled at the corners. **a** Gray surface is the model of Brey et al. (1999) with end-member contributions subtracted. Diamonds are the data of Brey et al. (1999) and Doroshev et al. (1997); open circles are the rest of the room-*T*, *P* calibration data set with <5% magnetite or ulvöspinel component; **b** Gray surface is the model presented in this work, with end-member and ordering contributions subtracted. Symbols are the same as in **a**.

**Fig. 7 a** Excess model volume in the MgAl<sub>2</sub>O<sub>4</sub>-MgFe<sub>2</sub>O<sub>4</sub>-Fe<sub>3</sub>O<sub>4</sub>-FeAl<sub>2</sub>O<sub>4</sub> reciprocal square. End members are labeled at the corners. Gray surface is the model presented in this work, with end-member and ordering contributions subtracted. Open circles are the room-*T*, *P* calibration data with < 5% chromite or ulvöspinel component; diamonds are the data of Mattioli et al. (1987), only the most stoichiometric data (samples synthesized at 1400 °C and oxygen fugacity of 10<sup>-4</sup> atm) are plotted; triangles are the data of Golla-Schindler et al. (2005) for an equilibration temperature of 1100 °C (varying this value

within the stated range of annealing temperatures has little effect); squares are the data of Nakatsuka et al. (2004); data from Zhao et al. (1998) (MgFe<sub>2</sub>O<sub>4</sub>-Fe<sub>3</sub>O<sub>4</sub>) are not shown due to minor amounts of Ca in the analyses; **b** Excess model volumes along the MgAl<sub>2</sub>O<sub>4</sub> – Fe<sub>3</sub>O<sub>4</sub> binary. The upper solid line is the model excess volume along this join, including ordering states calculated with the MELTS ordering routine for the Mattioli et al. experimental temperature of 1400 °C. The lower solid line is the excess volume model plotted without additional ordering effects, i.e., the order is perfectly normal at spinel and perfectly inverse at magnetite. The dashed line is the excess volume model of Mattioli et al. (1987); gray circles are the data of Mattioli et al. (1987); open diamonds are calibration data for the magnetite end member, measured at ambient conditions; the optimized end-member contributions from this work have been subtracted from models and data.

**Fig. 8** Excess model volume (black curve) along the  $Fe_3O_4 - Fe_2TiO_4$  binary. The ordering state is calculated using the MELTS ordering routine at 1100 °C; in other respects the volume is calculated for ambient conditions. Open circles are the data of Wechsler et al. (1984) and gray triangles are the data of Bosi et al. (2009). Optimized end-member contributions from this work have been subtracted from the data. The volume along this binary is treated as ideal in the present model; asymmetry along this join is captured in the model by the effect of ordering alone.

**Fig. 9** Model volume (black curve) along the  $FeCr_2O_4 - Fe_3O_4$  binary, plotted with the data of Robbins et al. (1971) (open circles) and Woodland et al. (2009) (gray circles). The model curve is calculated using the MELTS ordering routine and the volume model from this work at ambient conditions. This binary is modeled with a symmetric excess term. The asymmetry of the data, which were not included in the calibration, is captured by the  $s_2$  ordering term.

**Fig. 10** Order variable  $s_0$  vs. temperature for pure MgFe<sub>2</sub>O<sub>4</sub> data. The Levy et al. (2004) and Antao et al. (2005a, 2005b) data were measured at high-*T*, in situ. O'Neill et al. (1992) samples were annealed over a range of temperatures; the annealing *T* is plotted here. The solid lines are calculated using the updated MELTS ordering model at 1 bar, 3 GPa, 5 GPa, and 6 GPa for pure MgFe<sub>2</sub>O<sub>4</sub>.

Fig. 1



Fig. 2



Fig. 3















Fig. 7



Fig. 8



Fig. 9





# SUPPLEMENTARY MATERIAL

Supplement 1. Data sources used in model calibration

	Reference
1	Andreozzi, G.B. and Lucchesi, S. 2002
2	Andreozzi, G.B. and Princivalle, F. 2002
3	Andreozzi, G.B. et al. 2000
4	Andreozzi, G.B. et al. 2001
5	Bosi, F. et al. 2007
6	Bosi, F. et al. 2009
7	Brey, G.P. et al. 1999
8	Carbonin, S. et al. 1996
9	Carbonin, S. et al. 2002
10	Carraro, A. 2003
11	Della Giusta, A. et al. 1996
12	Doroshev, A.M. et al. 1997
13	Finger, L.W. et al. 1986
14	Fleet, M.E. 1981
15	Fleet, M.E. 1984
16	Gatta, G.D. et al. 2007
17	Girnis, A.V. et al. 2003
18	Harrison, R.J. et al. 1998
19	Hill, R.J. 1984
20	Ishii, M. et al. 1982
21	Larsson, L. et al. 1994
22	Lavina, B. et al. 2003
23	Lavina, B. et al. 2005
24	Lavina, B. et al. 2009
25	Lenaz, D. and Princivalle, F. 2005
26	Lenaz, D. et al. 2004
27	Lenaz, D. et al. 2007
28	Lenaz, D. et al. 2009
29	Levy, D. et al. 2003
30	Levy, D. et al. 2004
31	Lucchesi, S. et al. 1998
32	Martignago, F. et al. 2003
33	Martignago, F. et al. 2006
34	Menegazzo, G. and Carbonin, S. 1998
35	Millard, R.L. et al. 1995
36	Nakagiri, N. et al. 1986
37	Nakatsuka, A. et al. 2004
38	Nestola, F. et al. 2007
39	O'Neill, H.St.C. and Dollase, W.A. 1994
40	O'Neill, H.St.C. et al. 1992
41	O'Neill, H.St.C. et al. 2003

- 42 Peterson, R.C. et al. 1991
- 43 Princivalle, F. et al. 1999
- 44 Princivalle, F. et al. 2006
- 45 Redfern, S.A.T. et al. 1999
- 46 Sedler, I.K. et al. 1994
- 47 Tabira, Y. and Withers, R.L. 1999
- 48 Waerenborgh, J.C. et al. 1994
- 49 Wechsler, B.A. and Von Dreele, R.B. 1989
- 50 Wechsler, B.A. et al. 1984
- 51 Yamanaka, T. et al. 2001

	Ті	.00700	0.01500	0.04152	0.00850	0.00850	00750	0.00850	0.01300	0.01501	00600.0	0.01100	0.01800	0.01800	0.01000	0.01150	0.01050	0.01100	00050	00600.0	.01599	0.01750	0.01100	0.01100	,	,	
	cr	0.56250 (	0.54350 (	0.56678 (	0.61100 (	0.62850 (	0.59050 (	0.59350 (	0.59350 (	0.60080 (	0.56272 (	0.57650 (	0.59050 (	0.59450 (	0.53650 (	0.54150 (	0.53100 (	0.53550 (	0.53450 (	0.53250 (	0.53623 (	0.55950 (	0.58450 (	0.56300 (	1.00000	1.00000	59
e	Mg	0.01590	0.02366	0.04163	0.02550	0.01133	0.02399	0.01896	0.00876	0.01176	0.02633	0.02335	0.02715	0.00900	0.01924	0.02611	0.02523	0.02302	0.02358	0.01636	0.03660	0.02736	0.02556	0.02189	ı	ī	
M sit	Fe <sup>3+</sup>	0.03000	0.07150	0.07654	0.08600	0.10250	0.07700	0.08150	0.14600	0.12806	0.03698	0.03400	0.11500	0.10350	0.08650	0.08700	0.10450	0.10250	0.0960.0	0.0960.0	0.08096	0.12400	0.08350	0.08750		ı	
	Fe <sup>2+</sup>	0.02360	0.00734	0.02791		0.00567	0.01251	0.01054	0.01424	0.01925	0.00316	0.01115	0.00585	0.02600	0.01176	0.00639	0.00877	0.00548	0.00842	0.01364	0.01987	0.00164	0.00444	0.00711	,		
	AI	0.36100	0.33900	0.24562	0.26900	0.24350	0.28850	0.28700	0.22450	0.22511	0.36182	0.34400	0.24350	0.24900	0.33600	0.32750	0.32000	0.32250	0.32800	0.33250	0.31034	0.27000	0.29100	0.30950	,	ı	
	Mg	.41522	.46841	42224	45100	44300	43646	43200	.38400	35800	39500	38700	.26281	.28158	39883	38900	41600	42733	42333	44038	.33837	.35711	.40743	42700	40000	35000	
	Fe <sup>3+</sup>	06500 0.	02700 0.	05700 0.	03900 0.	02500 0.	03600 0.	01500 0.	02800 0.	03900 0.	01200 0.	01000 0.	02800 0.	02700 0.	01200 0.	01600 0.	00700 0.	01200 0.	01700 0.	00700 0.	08300 0.	03600 0.	04500 0.	03700 0.	0	0	
T site	Fe <sup>2+</sup>	51878 0.	49959 0.	51976 0.	51000 0.	53200 0.	50754 0.	52600 0.	58700 0.	60300 0.	56700 0.	56700 0.	70719 0.	68342 0.	55917 0.	57000 0.	53800 0.	53767 0.	53067 0.	51862 0.	57863 0.	60589 0.	54757 0.	53600 0.	60000	65000	
	AI	00100 0.	00500 0.	00100 0.	- 0.	- 0.	02000 0.	02700 0.	00100 0.	- 0.	02600 0.	03600 0.	00200 0.	00800 0.	03000 0.	02500 0.	03900 0.	02300 0.	02900 0.	03400 0.	- 0.	00100 0.	- 0.	- 0.	- 0.	- 0.	
	т °C)	25 0.	25 0.	25 0.	25	25	25 0.	25 0.	25 0.	25	25 0.	25 0.	25 0.	25 0.	25 0.	25 0.	25 0.	25 0.	25 0.	25 0.	25	25 0.	25	25	25	25	
	P (GPa)	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	
	<i>V residuals</i> (J/bar/mol)	0.00703	0.00831	0.00683	-0.00632	-0.01347	0.01077	0.00880	-0.01287	-0.00918	0.01261	0.00498	0.00603	0.01282	0.00930	0.00908	0.00997	0.00864	0.00504	0.00953	-0.00198	-0.02300	-0.01383	0.00175	0.00086	0.00407	
	V (J/bar/mol)	4.2789	4.2895	4.3404	4.3036	4.3103	4.3059	4.3049	4.3309	4.3340	4.2777	4.2766	4.3432	4.3462	4.2907	4.2952	4.2955	4.2965	4.2890	4.2887	4.3033	4.3063	4.2932	4.2985	4.3946	4.4014	
	a (Å)	8.2837(1)	8.2905(1)	8.3232(2)	8.2996(3)	8.3039(2)	8.3011(1)	8.3004(4)	8.3171(1)	8.3191(2)	8.2829(1)	8.2822(2)	8.3250(1)	8.3269(1)	8.2913(1)	8.2942(2)	8.2944(2)	8.2950(1)	8.2902(1)	8.2900(2)	8.2994(2)	8.3013(3)	8.2929(2)	8.2963(3)	8.3577(2)	8.3620(1)	
Reference	(see Supplement 1)	25	25	25	25	25	25	25	25	25	25	25	25	25	27	27	27	27	27	27	27	27	27	27	27	27	
	AMCSD ID	6033	6034	6037	6036	6039	6041	6042	6035	6038	6040	6043	6044	6045	7226	7227	7228	7229	7230	7231	7232	7233	7234	7235	7236	7237	

Supplement 2. Data used in model calibration

'	'	'	'	'	'	'	'	'	'	'	'	'	'	'	'	'	'	'	'	'	'	'	'	'	'	'	'
1.00000	1.00000	1.00000	1.00000	1.00000			,	ı																	,		ı
ı	ı	ı	ı	ı	ı	ı		ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	0.11950	0.12500	0.11350	0.10645
	ı	,			,		,	ï		,	,			,	,	,	,	,		,	,	,	,		,	0.00350	0.00750
ı	ı	ı	ı	ı	ı	0.06750	0.06700	0.06700	0.06650	0.06600	0.06250	0.05600	0.06100	0.06600	0.07150	0.07650	0.08300	0.08750	0.09250	0.09150	0.10050	0.10350	0.10950	,	ı	ı	0.00200
ı	ı	·	ı	ı	1.00000	0.93250	0.93300	0.93300	0.93350	0.93400	0.93750	0.94400	0.93900	0.93400	0.92850	0.92350	0.91700	0.91250	0.90750	0.90850	0.89950	0.89650	0.89050	0.88050	0.87500	0.88300	0.88406
0.33000	0.24000	0.13000	00060.0		,	,	,	ı	,	,	,	,	,	,	,	,	,	,	,	,	,	,	,	0.71500	0.64000	0.58800	0.51400
								,								,									,		
0.67000	0.76000	0.87000	0.91000	1.00000	1.00000	0.86500	0.86600	0.86600	0.86700	0.86800	0.87500	0.88800	0.87800	0.86800	0.85700	0.84700	0.83400	0.82500	0.81500	0.80700	0.79900	0.79300	0.78100	0.04600	0.11100	0.18800	0.27200
	·					0.13500	0.13400	0.13400	0.13300	0.13200	0.12500	0.11200	0.12200	0.13200	0.14300	0.15300	0.16600	0.17500	0.18500	0.19300	0.20100	0.20700	0.21900	0.23900	0.24900	0.22400	0.21400
25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	200	400
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0.00152	0.00418	0.00173	0.00316	0.00000	-0.00332	0.00614	0.00240	-0.00016	-0.00075	-0.00124	-0.00203	-0.00310	-0.00268	-0.00224	-0.00183	-0.00118	-0.00044	0.00012	0.00074	0.00096	0.00156	0.00172	0.00201	0.00282	0.00066	0.00013	-0.00209
4.4003	4.4096	4.4156	4.4202	4.4243	4.08375	4.08606	4.10158	4.12169	4.12696	4.13237	4.13787	4.14345	4.14937	4.15537	4.16134	4.16767	4.17399	4.18041	4.18689	4.19341	4.19987	4.20631	4.21264	3.9850	3.9912	4.0014	4.0110
8.3613(2)	8.3672(2)	8.3710(1)	8.3739(2)	8.3765(2)	8.15579(6)	8.14578(3)	8.15612(3)	8.16949(3)	8.17298(3)	8.17657(3)	8.18021(3)	8.18390(3)	8.18781(3)	8.19177(3)	8.19571(3)	8.19988(3)	8.20404(3)	8.20826(3)	8.21252(3)	8.21680(3)	8.22103(3)	8.22525(3)	8.22939(3)	8.0895(3)	8.0937(3)	8.1006(3)	8.1071(3)
27	27	27	27	27	27	27	27	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	19	18	18	18
7238	7239	7240	7241	7242	7243	7244	7245	8912	8913	8914	8915	8916	8917	8918	8903	8904	8905	8906	8907	8068	6068	8910	8911	958	2031	2032	2033

'	'	'	'	'	'	'	'	'	'	'	'	'	'	'	'	'	'	'	'	'	'	'	1	'	'	1	
	ı	ı	ı	ı	ı	ı			ı	ı	ı	ı	ı		ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	
0.10745	0.08896	0.07646	0.05897	0.03798	,		,	ï	,	,	ı	,		ï	ı	,	,	,	ı	ı	,	,	ı	,	,	·	0.06000
0.00800	0.01000	0.01099	0.02049	0.01849	0.04100		,	ı	ŀ	ı	ı	ŀ	,	ı	ı	ı	ı	ŀ	ı	ı	ŀ	ı	ı	ŀ	ı	ı	0.02000
0.00400	0.01749	0.02699	0.03598	0.05697	0.07650	0.07250	0.0960.0	0.09850	0.10200	0.09650	0.05050	0.05200	0.05900	0.06250	0.07200	0.07350	0.07800	0.08700	0.08950	0.09350	0.09000	0.09400	0.09200	0.08950	0.06750	0.07800	0.04000
0.88056	0.88356	0.88556	0.88456	0.88656	0.88250	0.92750	0.90400	0.90150	0.89800	0.90350	0.94950	0.94800	0.94100	0.93750	0.92800	0.92650	0.92200	0.91300	0.91050	0.90650	0.91000	0.90600	0.90800	0.91050	0.93250	0.92200	0.88000
0.43400	0.36600	0.26700	0.18500	0.10100	·			ı	·	ı	ı	·		ı	ı	ı	ı	·	ı	ı	·	ı	ı	·	ı	ı	0.18000
,	0.01600	0.01000	0.01700	0.03400	0.01800	,	,	ı	,	ï	ı	,	,	ı	ı	,	,	,	ı	ı	,	,	ı	,	,	ŀ	0.02000
0.34700	0.42400	0.53100	0.63000	0.71300	0.84900	0.85500	0.80800	0.80300	0.79600	0.80700	0.89900	0.89600	0.88200	0.87500	0.85600	0.85300	0.84400	0.82600	0.82100	0.81300	0.82000	0.81200	0.81600	0.82100	0.86500	0.84400	0.62000
0.21900	0.19400	0.19200	0.16800	0.15200	0.13300	0.14500	0.19200	0.19700	0.20400	0.19300	0.10100	0.10400	0.11800	0.12500	0.14400	0.14700	0.15600	0.17400	0.17900	0.18700	0.18000	0.18800	0.18400	0.17900	0.13500	0.15600	0.18000
450	500	550	600	650	700	750	800	850	006	950	1000	1050	1100	1150	25	25	25	25	25	25	25	25	25	25	25	25	25
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-0.00159	-0.00385	-0.00222	-0.00415	-0.00493	-0.00348	-0.00179	0.00055	0.00051	0.00043	-0.00165	-0.00472	-0.00381	-0.00247	-0.00225	-0.00109	-0.00048	-0.00031	0.00110	0.00076	0.00118	0.00096	0.00094	-0.00123	-0.00149	0.00219	0.00315	-0.00582
4.0204	4.0334	4.0460	4.0610	4.0742	4.0970	4.0776	4.0775	4.0772	4.0767	4.0752	4.0770	4.0778	4.0784	4.0782	4.0784	4.0788	4.0785	4.0790	4.0784	4.0784	4.0785	4.0781	4.0761	4.0761	4.0821	4.0820	4.0595
8.1134(3)	8.1221(3)	8.1306(3)	8.1406(3)	8.1494(4)	8.1646(3)	8.1517(2)	8.1516(2)	8.1514(2)	8.1511(2)	8.1501(2)	8.1513(2)	8.1518(2)	8.1522(2)	8.1521(2)	8.1522(2)	8.1525(2)	8.1523(2)	8.1526(2)	8.1522(2)	8.1522(2)	8.1523(2)	8.1520(2)	8.1507(2)	8.1507(2)	8.1547(3)	8.1546(3)	8.1396(1)
18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	1	1	1	1	1	1	1	1	1	1	1	21	21
2034	2035	2036	2037	2038	2039	2040	2041	2042	2043	2044	2045	2046	2047	2048	2873	2874	2875	2876	2877	2878	2879	2880	2881	2882	2872	6209	6510

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	ı	ı	,	0.00200	0.01200	0.00750	0.00250	ı	ı	ı	ı			,	,	ı	ı		ı	ı			ı	ı	ı	ı	ı	
,	ı	ı		0.48650	0.57450	0.45800	0.60100	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99600	0.99800	1.00000	1.00000	1.00000	1.00000	1.00000					,	ı	ı	62
0 05500	0.05000	0.04500	0.04500	0.02741	0.01151	0.04248	0.01500			,						,	,				0.44950	0.45000	0.44200	0.44100	0.43450	0.42500	0.41600	
00200	0.02500	0.01000	0.02000	0.00850	0.07350	0.04750	0.03700	ı	ı	ı	ı			0.00400	0.00200	ı	ı		ı	ı	0.55050	0.55000	0.55800	0.55900	0.56550	0.57500	0.58400	
0.04500	0.04500	0.05500	0.04500	0.00609	0.02349	0.00052	0.00750			,						,	,								,	ŀ		
0 88000	0.88000	0.89000	0.89000	0.46950	0.30500	0.44400	0.33700			,																		
0 18000	0.19192	0.21000	0.20000	0.53744	0.52318	0.56683	0.54170	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.98400	0.93200	0.87000	0.80000	0.68000	0.67000	0.63000	0.10100	0.10000	0.11600	0.11800	0.13100	0.15000	0.16800	
0.02000	0.01010	0.03000	0.02000	0.03000	0.02400	0.01700	0.00100														00668.0	00006.0	0.88400	0.88200	0.86900	0.85000	0.83200	
0,62000	0.61616	0.60000	0.61000	0.39856	0.43482	0.36017	0.41630							0.01600	0.06800	0.13000	0.20000	0.32000	0.33000	0.37000								
0 18000	0.18182	0.16000	0.17000	0.03400	0.01800	0.05600	0.04100	ı	ı	ı	ı			,	,	ı	ı		ı	ı			ı	ı	ı	ı	ı	
<u>כ</u> ר	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	
0 0001	0.001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	
-0 00774	-0.00797	-0.00635	-0.00920	-0.00397	0.00275	0.00153	-0.00163	0.00105	0.00120	0.00073	-0.00115	-0.00084	-0.00099	-0.00047	-0.00119	-0.00020	0.00137	0.00143	0.00521	-0.00127	-0.01171	-0.01141	-0.01028	-0.01056	-0.00949	-0.00925	-0.00856	
4 0583	4.0577	4.0571	4.0571	4.2065	4.2874	4.2197	4.2579	4.3575	4.3577	4.3572	4.3553	4.3556	4.3555	4.3573	4.3592	4.3635	4.3691	4.3765	4.3809	4.3770	4.4307	4.4308	4.4342	4.4342	4.4370	4.4399	4.4431	
8 1388(1)	8.1384(1)	8.1380(1)	8.1380(1)	8.2367(5)	8.2892(1)	8.2453(2)	8.2701(2)	8.3341(2)	8.3342(2)	8.3339(2)	8.3327(4)	8.3329(1)	8.3328(5)	8.3340(1)	8.3352(1)	8.3379(3)	8.3415(2)	8.3462(1)	8.3490(1)	8.3465(1)	8.3805(3)	8.3806(3)	8.3827(3)	8.3827(3)	8.3845(3)	8.3863(3)	8.3883(3)	
10	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	48	48	23	23	23	23	23	39	39	39	39	
5511	6512	6513	6514	6515	6516	6517	6518	6519	6520	6521	6522	6523	6524	6525	6526	6527	7893	7897	8938	8939	8940	8941	8942	7767	7768	7769	7771	

	63																											
0.45400			0.09200	0.45400		ı		1.00000		23	0.0001	0.00008	4.6678	8.5274(5)	37	ø												
0.42250	,	ı	0.12300	0.45450		ı	0.06400	0.93600		23	0.0001	0.00466	4.6589	8.5220(4)	37	a												
0.40020			0.13407	0.46573		·	0.13100	0.86900		23	0.0001	0.00188	4.6457	8.5139(5)	37	8863												
0.37800	ı	ı	0.13400	0.48800	I	I	0.22100	0.77900	ı	23	0.0001	0.00317	4.6358	8.5079(4)	37	8862												
0.37550	ı	ı	0.14350	0.48100	I	I	0.21100	0.78900	ı	23	0.0001	0.00107	4.6326	8.5059(5)	37	8861												
0.37600	·	I	0.15450	0.46950	ı	ı	0.18700	0.81300	ı	25	0.0001	-0.00148	4.6314	8.5052(5)	40	1505												
0.34500	ı	ı	0.18700	0.46800	·	ı	0.24500	0.75500	·	25	0.0001	0.00010	4.6189	8.4975(4)	40	1504												
0.35650		ı	0.13750	0.50600	ı	ı	0.30000	0.70000	·	25	0.0001	-0.00237	4.6184	8.4972(5)	40	1503												
0.32200	·	I	0.17600	0.50200	ı	ı	0.36000	0.64000	ı	25	0.0001	-0.00185	4.6026	8.4875(4)	40	1502												
0.29250	,	ı	0.19000	0.50350	0.01400	ı	0.42100	0.57900	,	25	0.0001	-0.00645	4.5768	8.4716(4)	40	1501												
ı	·	0.41000	0.48000	ı	0.11000	0.19000	0.81000	ı	ı	25	0.0001	-0.01410	4.3702	8.3422(5)	40	1500												
ı	·	0.36000	0.43000	ı	0.21000	0.28283	0.66667	ı	0.05051	25	0.0001	0.00717	4.3436	8.3252(4)	40	1499												
ı	,	0.31000	0.34000	ı	0.35000	0.38000	0.53000	ı	00060.0	25	0.0001	0.00875	4.2726	8.2796(5)	40	1498												
ı		0.35500	0.64500	ı		0.29000	0.71000	ı		25	0.0001	-0.00706	4.4614	8.3998(3)	40	1497												
ı		0.35950	0.64050	ı		0.28100	0.71900	ı		25	0.0001	-0.00550	4.4617	8.4000(3)	40	1496												
ı		0.36050	0.63950	ı		0.27900	0.72100	ı		25	0.0001	-0.00555	4.4614	8.3998(3)	40	1495												
ı		0.36250	0.63750	ı		0.27500	0.72500	ı		25	0.0001	-0.00675	4.4596	8.3987(3)	40	1494												
		0.36600	0.63400	·		0.26800	0.73200	ı		25	0.0001	-0.00674	4.4587	8.3981(3)	40	1493												
		0.37450	0.62550	·		0.25100	0.74900	ı		25	0.0001	-0.00615	4.4569	8.3970(3)	40	1492												
ı		0.37300	0.62700	ı		0.25400	0.74600	ı		25	0.0001	-0.00656	4.4569	8.3970(3)	40	1491												
,		0.37850	0.62150	·		0.24300	0.75700	ı	,	25	0.0001	-0.00680	4.4552	8.3959(3)	40	1490												
,		0.37950	0.62050	·		0.24100	0.75900	ı	,	25	0.0001	-0.00652	4.4552	8.3959(3)	40	1489												
ı		0.38600	0.61400	ı		0.22800	0.77200	ı		25	0.0001	-0.00680	4.4531	8.3946(3)	40	1488												
,		0.39100	00609.0	·		0.21800	0.78200	ı	,	25	0.0001	-0.00733	4.4512	8.3934(3)	40	1487												
ı		0.39700	0.60300	ı		0.20600	0.79400	ı		25	0.0001	-0.00886	4.4480	8.3914(3)	40	1486												
ı		0.40000	0.60000	ı		0.20000	0.80000	ı		25	0.0001	-0.00804	4.4480	8.3914(3)	40	1485												
		0.40550	0.59450	·		0.18900	0.81100	ı		25	0.0001	-0.00891	4.4456	8.3899(3)	40	1484												
		0.41300	0.58700			0.17400	0.82600	ı		25	0.0001	-0.00938	4.4431	8.3883(3)	47	8365												
	0.47250	0.50000	0.50000	·	0.04800	0.04900	0.09300	0.12100	0.17700	0.23100	,			,	,	,	ı	,	,	ı	,	,	ı	,	-	-		
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	ı	·	,	ı	·	,	,	·	·	,	·	,	,	·	,	·	ı	,	,	ı	,	ı	ı	,	0.01500	0.01500	0.01500	64
ı	ı	0.50000	0.50000	ı	·	,	,	·	·		·	,	,	·	,	·	0.01300	0.16000	0.06000	0.10900	0.07150	0.13707	0.14150	0.13600	0.07150	0.07100	0.07200	
0.0/000	0.05500	ı	ı	0.50000	0.43750	0.43950	0.36550	0.33900	0.26500	0.23050	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50700	ı	ı	ī	ı	0.05403	0.03950	0.02450	ı	ī	ı	
0.46500	0.47250	,	,	0.50000	0.51450	0.51150	0.54150	0.54000	0.54300	0.53850	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.48000	,	,	ï	,	,	ı	,	,	ı	ı	
·				ı					0.01500								ı	0.84000	0.94000	0.89100	0.92850	0.80890	0.81900	0.83950	0.91350	0.91400	0.91300	
	·	1.00000	1.00000	ı		,	,					,	,		,		0.01300	0.68000	0.88000	0.78200	0.85700	0.72527	0.73173	0.72306	0.85700	0.85800	0.85600	
	ı	ı	,	1.00000	0.93300	0.92400	0.89700	0.83800	0.73300	0.61500	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.98700	ı	ı	ı	ı	0.09291	0.06106	0.03625	ı	ı	ı	
1.0000	1.00000	,	,	·	0.06700	0.07600	0.10300	0.16200	0.26700	0.38500	,	,	,	,	,	,	·	,	,	ı	,	,	·	,	,	·	ı	
	ı	ı	,	ı	ı	,	,	ı	ı	ı	ı	,	,	ı	,	ı	ı	0.32000	0.12000	0.21800	0.14300	0.18182	0.20721	0.24068	0.14300	0.14200	0.14400	
23	23	23	23	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	
0.00113	0.00051	0.00185	-0.00496	0.00111	0.00021	0.00352	0.00035	0.00273	-0.00191	-0.00095	-0.00176	0.00159	-0.00319	-0.00085	-0.00303	0.00143	0.00146	0.00042	-0.00310	0.00039	0.00172	-0.00102	0.00321	-0.00586	0.00356	0.00358	0.00355	
4.6732	4.6757	4.52867	4.5219	4.4564	4.4724	4.4768	4.4848	4.5016	4.5174	4.5530	4.4536	4.4569	4.4521	4.4537	4.4523	4.4567	4.4577	3.97737	3.97059	3.97627	3.97627	4.0234	4.0127	3.9912	3.9841	3.9841	3.9841	
8.5307(4)	8.5322(4)	8.44183(3)	8.4376(5)	8.3967(3)	8.4067(5)	8.4095(5)	8.4145(5)	8.4250(5)	8.4348(5)	8.4569(4)	8.3949(3)	8.3970(1)	8.3940(10)	8.3950(5)	8.3941(7)	8.3969(8)	8.3975(7)	8.08435(7)	8.07975(5)	8.08360(6)	8.08360(6)	8.1154(3)	8.1082(6)	8.0937(5)	8.0889(2)	8.0889(2)	8.0889(2)	
37	37	37	37	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	46	35	49	50	13	
a	a	a	a	822	823	824	825	826	827	828	829	830	831	832	833	815	816	817	818	819	820	821	568	.746	832	953	389	

'	T	1	1	ľ	'	'	'	1	1	'	1	'	1	'	'	'	'	'	'	1	1	1	1		1	T	I	
0.01500	0.01500	0.01500	0.01500	0.01500	0.01500	0.01500	0.01500	0.01500	0.01500	0.01500	0.01500	0.01500	0.01500	0.01500	0.02850	0.02850	0.02850	0.02850	0.02850	0.02850	0.02850	0.02850	0.02850	0.12200	0.12200	0.12200	0.12200	65
0.07200	0.07400	0.07550	0.07800	0.07950	0.08350	0.13550	0.12900	0.12850	0.11300	0.10150	0.10150	0.10000	0.10000	0.09900	0.06750	0.13200	0.13150	0.12750	0.12600	0.12100	0.11950	0.11900	0.11750	0.05650	0.05750	0.05750	0.06350	
	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	·	·	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	
	ı	ı	ï	,	,	,	,	ï	,	,	ï			,	,	ï	ı	ï	ï	ï	ï	ı	ı	·	ı	ı	,	
0.91300	0.91100	0.90950	0.90700	0.90550	0.90150	0.84950	0.85600	0.85650	0.87200	0.88350	0.88350	0.88500	0.88500	0.88600	0.90400	0.83950	0.84000	0.84400	0.84550	0.85050	0.85200	0.85250	0.85400	0.82150	0.82050	0.82050	0.81450	
0.85600	0.85200	0.84900	0.84400	0.84100	0.83300	0.72900	0.74200	0.74300	0.77400	0.79700	0.79700	0.80000	0.80000	0.80200	0.86500	0.73600	0.73700	0.74500	0.74800	0.75800	0.76100	0.76200	0.76500	0.88700	0.88500	0.88500	0.87300	
	ı	ı	ï	,	,			ï			ï					ī	ı	ī	ī	ī	ī	ı	ı	·	ı	ı	ı	
ı	I	I	ı	ı	ī	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	I	ı	ı	ı	ı	I	I	ī	I	ī	ī	
0.14400	0.14800	0.15100	0.15600	0.15900	0.16700	0.27100	0.25800	0.25700	0.22600	0.20300	0.20300	0.20000	0.20000	0.19800	0.13500	0.26400	0.26300	0.25500	0.25200	0.24200	0.23900	0.23800	0.23500	0.11300	0.11500	0.11500	0.12700	
25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	
1	2	ε	4	0.0001	0.63	1.55	2.09	2.76	3.67	4.44	1.42	2	2.62	3.14	3.67	0.74	1.51	2.52	0.76	1.02	1.14	1.21	1.45	1.71	1.83	1.99	2.03	
0.00340	0.00348	0.00328	0.00334	0.00299	0.00300	-0.00217	-0.00239	-0.00223	0.00022	0.00135	0.00165	0.00155	0.00155	0.00159	0.00409	-0.00131	-0.00130	-0.00100	-0.00080	0.00071	0.00091	0.00093	0.00098	0.00485	0.00421	0.00406	0.00364	
3.9839	3.9841	3.9839	3.9841	3.9838	3.9839	3.9805	3.9801	3.9803	3.9822	3.9829	3.9832	3.9831	3.9831	3.9831	3.9899	3.9868	3.9868	3.9869	3.9870	3.9884	3.9885	3.9885	3.9885	4.0272	4.0267	4.0265	4.0264	
8.0888(2)	8.0889(1)	8.0888(2)	8.0889(2)	8.0887(1)	8.0888(1)	8.0865(1)	8.0862(1)	8.0863(1)	8.0876(2)	8.0881(1)	8.0883(2)	8.0882(2)	8.0882(2)	8.0882(1)	8.0928(1)	8.0907(1)	8.0907(1)	8.0908(1)	8.0909(1)	8.0918(1)	8.0919(1)	8.0919(1)	8.0919(1)	8.1180(1)	8.1176(1)	8.1175(1)	8.1174(1)	
13	13	13	13	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	
7390	7391	7392	7393	7418	7419	7420	7421	7422	7423	7424	q	q	q	q	q	q	q	q	q	q	q	q	q	q	q	q	q	

ı	ı	ı	ı	ı	ı	·	ï	ı	ï	ı	ı	·	·	ı	ï	ï	ï	ı	ı	ı	ï	ı	ı	ı	00600.0	0.00400	0.00750
0.12200	0.12200	0.12200	0.12200	0.12200	0.12200	0.12200	0.12200	0.12200	0.12200	0.12200	0.12200	0.12200	0.12200	0.12200	0.12200	0.12200	0.12200	0.12200	ï	ï	0.00100	0.00100	0.00100	0.00100	0.41650	0.32050	0.36550
0.06450	0.07150	0.07450	0.07250	0.07450	0.07500	0.10600	0.10150	0.09500	0.07850	0.07350	0.07100	0.07100	0.07200	0.10750	0.10750	0.10550	0.09300	0.08850	0.11694	0.10450	0.07250	0.07250	0.13550	0.13550	0.05484	0.04914	0.06111
·	ı	ı	ı	ı	ı		,	ı	,	ı	ı			ı	,	,	,	·	0.00300	0.01250	,	ı	ı	ı	0.04450	0.02800	0.00600
,	ı	ŀ	ŀ	ı	ŀ		,	ŀ	,	ŀ	ŀ			ı	,	,	,	,	0.00150	0.01100	,	ŀ	ı	ŀ	0.00666	0.01586	0.00939
0.81350	0.80650	0.80350	0.80550	0.80350	0.80300	0.77200	0.77650	0.78300	0.79950	0.80450	0.80700	0.80700	0.80600	0.77050	0.77050	0.77250	0.78500	0.78950	0.87856	0.87200	0.92650	0.92650	0.86350	0.86350	0.46850	0.58250	0.55050
0.87100	0.85700	0.85100	0.85500	0.85100	0.85000	0.78800	0.79700	0.81000	0.84300	0.85300	0.85800	0.85800	0.85600	0.78500	0.78500	0.78900	0.81400	0.82300	0.76000	0.75624	0.85485	0.85485	0.72800	0.72800	0.57700	0.65074	0.56064
,	ı	,	,	,	,	,	,	,	,	,	,	,	,	,	,	,	,	,	0.00300	0.00300	,	,	ı	,	0.05500	0.03200	0.08200
ı	ī	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	0.00300	0.01299	ı	ı	ī	ı	0.31700	0.22826	0.31236
0.12900	0.14300	0.14900	0.14500	0.14900	0.15000	0.21200	0.20300	0.19000	0.15700	0.14700	0.14200	0.14200	0.14400	0.21500	0.21500	0.21100	0.18600	0.17700	0.23400	0.22777	0.14515	0.14515	0.27200	0.27200	0.05100	0.08900	0.04500
25	25	25	25	25	25	25	25	25	25	25	25	25	25	20	20	20	20	20	20	20	20	20	20	20	20	20	25
2.11	2.23	2.31	2.36	2.53	2.84	3.08	3.31	4.06	0.72	0.65	0.02	0.0001	0.0001	0.0001	4.99	9.21	0.0001	1.66	2.98	3.05	5.69	7.54	9.06	9.65	9.82	11.11	0.0001
0.00374	0.00327	0.00313	0.00308	0.00269	0.00311	0.00006	0.00012	0.00101	0.00295	0.00377	0.00374	0.00359	0.00355	0.00044	0.00044	-0.00036	0.00021	0.00041	0.00444	0.01000	0.00896	0.01976	0.00113	0.00851	-0.00780	-0.00634	-0.00803
4.0265	4.0264	4.0264	4.0262	4.0259	4.0264	4.0247	4.0246	4.0252	4.0264	4.0269	4.0268	4.0267	4.0267	4.0252	4.0252	4.0243	4.0243	4.0243	3.9832	3.9953	3.9839	3.9947	3.9782	3.9856	4.2022	4.1425	4.1709
8.1175(1)	8.1174(1)	8.1174(1)	8.1173(1)	8.1171(1)	8.1174(1)	8.1163(1)	8.1162(2)	8.1166(1)	8.1174(1)	8.1178(1)	8.1177(1)	8.1176(1)	8.1176(1)	8.1166(1)	8.1166(1)	8.1160(1)	8.1160(1)	8.1160(1)	8.0883(3)	8.0965(5)	8.0888(1)	8.0961(2)	8.0849(1)	8.0899(2)	8.2339(2)	8.1947(1)	8.2134(2)
36	36	36	36	36	36	36	36	36	36	36	36	36	51	16	16	16	16	16	16	16	16	16	16	16	16	16	14
q	q	q	q	q	q	q	q	q	q	q	q	q	8502	9110	9110	9111	q	q	q	q	q	q	q	q	q	q	9740

0.00550	0.00150	0.00100	0.00060	0.00050	,	,	0.00010	0.00075		0.00420	0.00745	0.00940	0.00045		ı	,	0.00055	0.00055	0.00235	0.00105	0.00310	0.00450	0.00625	ı	ı		·
0.32600	0.09000	0.19050	0.00010	0.00020	0.00020	0.00005	0.00010	0.00005					0.00030	0.00035	0.00045		·	,	0.00025	·	0.00015	,	0.00035	,	·	0.01250	0.03102
0.07337	0.06050	0.05000	0.12424	0.12049	0.12404	0.12084	0.12070	0.13071	0.12566	0.14017	0.13813	0.13726	0.12640	0.12524	0.12154	0.11846	0.12523	0.12754	0.12894	0.12966	0.10753	0.10835	0.11596	0.11800	0.13600	0.06550	0.06403
0.02450	0.00400	0.01150	0.03041	0.02756	0.02281	0.00805	0.00780	0.02231	0.01096	0.01706	0.01561	0.01456	0.02531	0.02736	0.03026	0.03627	0.03246	0.02296	0.02581	0.00675	0.01145	0.01516	0.01776	ı	0.05550	ı	·
0.00413	ı	ı	0.00010	ı	0.00390	,	·	ı	ı	ı	ı	ı	ı	0.00315	ı	ı	0.00135	ı	ı	ı	ı	ı	ı	ı	ı	ı	·
0.56650	0.84400	0.74700	0.84455	0.85126	0.84905	0.87105	0.87130	0.84618	0.86338	0.83857	0.83880	0.83878	0.84754	0.84390	0.84775	0.84527	0.84041	0.84895	0.84265	0.86253	0.87776	0.87199	0.85967	0.88200	0.80850	0.92200	0.90495
0.58000	0.66426	0.66295	0.63928	0.62690	0.66267	0.66280	0.66688	0.66843	0.68719	0.68274	0.69246	0.68536	0.62519	0.63536	0.61367	0.61091	0.58726	0.66441	0.65995	0.72635	0.78799	0.79048	0.72414	0.76400	0.72800	0.86787	0.86900
0.01800	0.01700	0.00700	0.05691	0.04560	0.06357	0.04181	0.03211	0.05681	0.04193	0.04911	0.05131	0.04821	0.05171	0.06025	0.05212	0.05363	0.05531	0.06191	0.05441	0.04950	0.03720	0.04070	0.05531	ı	0.09100	ı	ı
0.27100	0.21574	0.23705	0.11209	0.13228	0.08175	0.09455	0.09067	0.07022	0.06116	0.04391	0.04459	0.05759	0.12186	0.10753	0.14293	0.15144	0.15999	0.08054	0.08602	0.01502	0.00211	0.00050	0.05471	ı	ı	ı	
0.13100	0.10300	0.09300	0.19172	0.19522	0.19201	0.20084	0.21034	0.20454	0.20973	0.22424	0.21164	0.20884	0.20124	0.19686	0.19128	0.18401	0.19744	0.19314	0.19962	0.20912	0.17270	0.16832	0.16583	0.23600	0.18100	0.13213	0.13100
25	25	20	20	009	650	700	700	750	800	800	850	006	006	1000	1000	26	25	410	434	493	564	630	969	762	827	891	955
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-0.01582	-0.00540	-0.00641	-0.00174	-0.00171	-0.00034	-0.00076	-0.00072	-0.00129	-0.00011	-0.00029	-0.00104	-0.00180	-0.00263	-0.00125	-0.00145	-0.00364	-0.00294	-0.00030	0.00050	0.00183	0.00690	0.00567	0.00043	0.00300	0.00186	0.00682	0.00493
4.1360	4.0386	4.0770	4.0176	4.0158	4.0136	4.0021	3.9992	4.0096	3.9999	4.0066	4.0090	4.0105	4.0143	4.0170	4.0198	4.0214	4.0228	4.0131	4.0158	3.9973	4.0007	4.0030	4.0119	3.9791	4.0264	3.9862	3.9916
8.1904(3)	8.1256(1)	8.1513(3)	8.1115(2)	8.1103(3)	8.1088(3)	8.1011(3)	8.0991(3)	8.1061(3)	8.0996(2)	8.1041(3)	8.1057(3)	8.1067(3)	8.1093(4)	8.1111(3)	8.1130(4)	8.1141(3)	8.1150(4)	8.1085(3)	8.1103(3)	8.0978(3)	8.1001(4)	8.1017(4)	8.1077(3)	8.0855(2)	8.1174(3)	8.0903(8)	8.0940(4)
15	15	42	42	42	42	42	42	42	42	42	42	42	42	42	42	45	45	45	45	45	45	45	45	45	45	45	45
9994	9995	1398	1399	1400	1401	1402	1403	1404	1405	1406	1407	1408	1409	1410	1411	2106	2144	2107	2108	2109	2110	2111	2112	2113	2114	2115	2116

0.00150 0.00115 0.00150 0.00260 0.00260 - - - -	0.03900 0.03910 0.03900 0.06359 0.03050 0.03050 0.03050 0.03050 -	0.04859 0.11002 0.11251 0.09792 0.06500 0.06500 0.1650 0.13000 0.13000 0.13000 0.12150 0.12200 0.11600	- - - 0.00100 0.00100 0.00100 0.00100 0.00100	0.01940 0.02427 0.02778 0.01841 - - - - - -	0.89151 0.82547 0.81921 0.81749 0.90500 0.90600 0.90600 0.86200 0.883850 0.86100 0.87850 0.87850 0.87850 0.87800 0.87800	0.57068 0.57068 0.56790 0.63762 0.877000 0.87500 0.78700 0.78700 0.78700 0.78500 0.75500 0.75600 0.75600	0.07688 0.08170 0.07699 0.05389 - - - - - - -	0.17220 0.16342 0.15462 0.13613 - - - - -	0.05599 0.18420 0.20048 0.17237 0.13000 0.12500 0.21300 0.21500 0.21500 0.24400 0.24400 0.23200	815 751 688 688 625 561 497 497 437 243 243 191 132 200	0.0001 0.0001 0.0001 0.0001 0.0001 0.0001 0.0001 0.0001 0.0001 0.0001	0.00208 -0.00290 -0.00290 0.00503 0.00503 0.00542 0.00542 0.00542 0.00178 0.00158 0.00198 0.00198	4.0380 4.0351 4.0329 4.03290 3.9919 3.9895 3.9893 3.9782 3.9782 3.9769		8.1252(4) 8.1233(5) 8.1218(3) 8.1218(3) 8.1218(3) 8.1259(3) 8.1259(3) 8.0944(5) 8.0944(5) 8.0924(6) 8.0924(6) 8.0929(6) 8.0849(8) 8.0849(6) 8.0849(6)	45       81252(4)         45       81252(5)         45       81218(3)         45       81218(3)         45       81259(3)         45       80942(6)         45       8.0942(6)         45       8.0924(6)         45       8.0926(6)         45       8.0926(6)         45       8.0926(6)         45       8.0926(6)         45       8.0926(6)         45       8.0926(6)         45       8.0849(6)         45       8.0849(6)         45       8.0849(10)
	0.03050	0.06250 0.10650	0.00100 0.00100		0.90600 0.86200	0.87500 0.78700			0.12500 0.21300	497 437		0.0001	0.00542 0.0001 0.00117 0.0001	3.9922         0.00542         0.0001           3.9896         0.00117         0.0001	8.0944(5)         3.9922         0.00542         0.0001           8.0926(6)         3.9896         0.00117         0.0001	45         8.0944(5)         3.9922         0.00542         0.0001           45         8.0926(6)         3.9896         0.00117         0.0001
0.0026(	0.06359 0.03050	0.09792 0.06500	- 0.00100	0.01841 -	0.81749 0.90350	0.63762 0.87000	0.05389	0.13613 -	0.17237 0.13000	625 561		0.0001	0.00237 0.0001 0.00503 0.0001	4.0390         0.00237         0.0001           3.9919         0.00503         0.0001	8.1259(3) 4.0390 0.00237 0.0001 8.0942(6) 3.9919 0.00503 0.0001	45         8.1259(3)         4.0390         0.00237         0.0001           45         8.0942(6)         3.9919         0.00503         0.0001
0.00150	0.03900	0.11251		0.02778 0.02778	0.81921	0.56790	0.07699	0.15462 0.15462	0.20048 0.20048	1 <i>c</i> /		0.0001	-0.00290 0.0001 -0.00290 0.0001	4.0329 -0.00290 0.0001 4.0329 -0.0001	8.1218(3) 4.0329 -0.00290 0.0001 8.1218(3) 4.0329 -0.00290 0.0001	45 8.1238(3) 4.0329 -0.00290 0.0001 45 8.1218(3) 4.0329 -0.00290 0.0001
0.00150	0.03900	0.04859	ı	0.01940	0.89151	0.69493	0.07688	0.17220	0.05599	815		0.0001	0.00208 0.0001	4.0380 0.00208 0.0001	8.1252(4) 4.0380 0.00208 0.0001 e.1252(4) 4.0380 0.0001	45 8.1252(4) 4.0380 0.00208 0.0001
		0.01250	0.00250		0.98500	1.00000			·	879		0.001	0.01199 0.0001	3.9854 0.01199 0.0001	8.0898(9) 3.9854 0.01199 0.0001	45 8.0898(9) 3.9854 0.01199 0.0001
,	ı	ï	ı	ī	1.00000	1.00000	ı	ı	ı	941		0.0001	-0.00037 0.0001	3.9718 -0.00037 0.0001	8.0806(5) 3.9718 -0.00037 0.0001	45 8.0806(5) 3.9718 -0.00037 0.0001
0.0035(	0.39950	0.04262	0.00700	0.00788	0.53950	0.65717	0.03300	0.24983	0.06000	1005		0.0001	0.00122 0.0001	4.1701 0.00122 0.0001	8.2129(9) 4.1701 0.00122 0.0001	45 8.2129(9) 4.1701 0.00122 0.0001
0.00350	0.39150	0.04997	0.01350	0.00053	0.54100	0.66100	0.03200	0.24800	0.05900	1132		0.0001	0.00421 0.0001	4.1718 0.00421 0.0001	8.2140(20) 4.1718 0.00421 0.0001	45 8.2140(20) 4.1718 0.00421 0.0001
0.00500	0.34700	0.04872	0.04500	0.00628	0.54800	0.58700	0.03900	0.31800	0.05600	1195		0.0001	0.00600 0.0001	4.1811 0.00600 0.0001	8.2201(8) 4.1811 0.00600 0.0001	45 8.2201(8) 4.1811 0.00600 0.0001
0.00350	0.27300	0.05757	0.02800	0.01193	0.62600	0.59700	0.04600	0.27400	0.08300	1259		0.0001	0.00343 0.0001	4.1414 0.00343 0.0001	8.1940(10) 4.1414 0.00343 0.0001	45 8.1940(10) 4.1414 0.00343 0.0001
0.0015(	0.20500	0.06517	0.01150	0.00683	0.71000	0.66655	0.03100	0.19145	0.11100	1322		0.0001	-0.00601 0.0001	4.0859 -0.00601 0.0001	8.1572(4) 4.0859 -0.00601 0.0001	45 8.1572(4) 4.0859 -0.00601 0.0001
0.0020(	0.11600	0.08543	0.03000	0.00157	0.76500	0.63227	0.03700	0.20173	0.12900	1389		0.0001	0.00143 0.0001	4.0706 0.00143 0.0001	8.1470(9) 4.0706 0.00143 0.0001	45 8.1470(9) 4.0706 0.00143 0.0001
0.0005(	0.10800	0.07698	0.01300	0.01152	0.79000	0.64936	0.04200	0.17764	0.13100	1371		0.0001	0.00210 0.0001	4.0592 0.00210 0.0001	8.1394(5) 4.0592 0.00210 0.0001	45 8.1394(5) 4.0592 0.00210 0.0001
0.00050	0.10250	0.08723	0.00700	0.00627	0.79650	0.63900	0.03800	0.17900	0.14400	1333		0.0001	0.00278 0.0001	4.0538 0.00278 0.0001	8.1358(6) 4.0538 0.00278 0.0001	45 8.1358(6) 4.0538 0.00278 0.0001
,	0.11606	0.05403	ı	ı	0.82991	0.88600		ı	0.11400	1271		0.0001	0.00574 0.0001	4.0261 0.00574 0.0001	8.1172(3) 4.0261 0.00574 0.0001	45 8.1172(3) 4.0261 0.00574 0.0001
ï	0.08296	0.06097	ı	I	0.85607	0.87800	,	ı	0.12200	1208		0.0001	0.00687 0.0001	4.0140 0.00687 0.0001	8.1091(8) 4.0140 0.00687 0.0001	45 8:1091(8) 4.0140 0.00687 0.0001
	0.06003 0.06653	0.06253		1 1	0.87744 0.87344	0.87387 0.87800		1 1	0.12613	1082 1145		0.0001	0.00534 0.0001 0.00638 0.0001	4.0035 0.00534 0.0001 4.0070 0.00638 0.0001	8.1020(10) 4.0035 0.00534 0.0001 8.1044(5) 4.0070 0.00638 0.0001	45 8.1020(10) 4.0035 0.00534 0.0001 45 8.1044(5) 4.0070 0.00638 0.0001
	0.04350	0.06200	ı	ı	0.89450	0.87400	ı	ı	0.12600	1018		0.0001	0.00612 0.0001	3.9977 0.00612 0.0001	8.0981(7) 3.9977 0.00612 0.0001	45 8.0981(7) 3.9977 0.00612 0.0001

ı	ı	ı	ı	,	,	,	,	0.50000	ı	ı	ı	ı	ı	ı	0.00300	ı	0.50000	0.30303	,	ı	ı	,	ı	ı	ı	ı	ı	
0.03300	0.03300	0.06153	0.06153	0.08100	0.08100	,	,	ı	ı	ı	ı	,	,	,	0.40550	0.01500	,	0.02020	,	ı	ı	,	ı	ı	ı	ı	ı	69
0.06250	0.06250	0.06053	0.06053	0.06350	0.06350	0.06957	0.06957	ı	0.11000	0.14000	0.13000	0.19000	0.22000	0.20000	0.04571	0.07150	ı	0.02546	0.01250	0.01250	0.01250	ı	ı	ı	ı	I	ı	
·	ı	ı	ı		·	0.03654	0.03654	ı	ı	0.01000	0.03000	0.06000	0.13000	0.16000	00600.0	·	·	·	0.00250	0.00250	0.00250	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	
	ı	ı	ı	,	,	,	,	0.50000	ı	ı	ı	,	,	,	0.00779	,	0.50000	0.60586	,	ı	ı	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	
0.90450	0.90450	0.87794	0.87794	0.85550	0.85550	0.89389	0.89389		0.89000	0.85000	0.84000	0.75000	0.65000	0.64000	0.52900	0.91350		0.04545	0.98500	0.98500	0.98500				,		ı	
0.88000	0.88000	0.88498	0.88498	0.87498	0.87498	0.86016	0.86016		0.78000	0.73000	0.73000	0.62000	0.55446	0.61000	0.65300	0.85700			1.00000	1.00000	1.00000				,	·	ı	
ı	ī	ī	ı	ı	·	0.00704	0.00704	ı	ı	0.01000	0.03000	0.11000	0.20792	0.26000	0.02900	·	·	0.65000	·	ı	ı	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	
0.00100	0.00100	0.00202	0.00202	0.00202	0.00202	0.00101	0.00101	1.00000	ı	ı	ı	ı	ı	ı	0.24900	ı	1.00000	0.35000	ı	ı	ı		ı	ı	ı	ı	ı	
0.11900	0.11900	0.11300	0.11300	0.12300	0.12300	0.13179	0.13179	ı	0.22000	0.26000	0.24000	0.27000	0.23762	0.13000	0.0690.0	0.14300	ı	·	ı	ı	ı	·	ı	ı	ı	ı	ı	
400	500	600	700	800	006	1000	1100	1200	1300	1400	1500	1600	25	25	25	200	400	450	500	550	600	650	700	800	006	1000	1050	
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	
0.00042	0.00087	0.00445	0.00430	0.00584	0.00658	0.00667	0.00637	0.00763	0.00434	0.00611	0.00191	-0.00154	-0.00453	0.00952	0.00153	0.00356	-0.00735	0.01440	0.01374	0.01462	0.00731	-0.00026	0.00219	0.00138	0.00093	0.00260	0.00220	
3.9878	3.9882	4.0033	4.0032	4.0124	4.0131	3.9989	3.9986	4.6949	3.9800	3.9896	3.9995	4.0319	4.0865	4.1228	4.1717	3.9841	4.6799	3.9674	3.9471	3.9291	3.9035	4.4397	4.4204	4.4071	4.3915	4.3730	4.3561	
8.0914(11)	8.0917(10)	8.1019(10)	8.1018(10)	8.1080(12)	8.1085(8)	8.0989(10)	8.0987(6)	8.5439(0)	8.0861(3)	8.0926(3)	8.0993(4)	8.1211(4)	8.1576(3)	8.1817(7)	8.2139(8)	8.0889(2)	8.5348(2)	8.0776(5)	8.0638(1)	8.0515(6)	8.0340(10)	8.3862(2)	8.3740(2)	8.3656(2)	8.3557(2)	8.3440(2)	8.3332(2)	
45	45	45	45	45	45	45	45	45	45	45	45	45	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	
2146	2147	2148	2149	2150	2151	2152	2153	2154	2155	2156	2157	2158	4018	4031	4045	4019	4020	4021	4022	4023	4024	4025	4026	4027	4028	4029	4030	

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	·	0.11500	0.10000	0.12500	0.12500	0.13500	0.13500	0.13500	0.14500	0.16500	0.15500	0.18000	0.17000	0.09800	0.08900	0.09450	0.09200	0.08450	0.07300	0.08000	0.09950	0.11100	0.10300	0.11700	0.11800	0.12450	0.13550
0.50000	0.50000	ı	ı	ı	ı	,	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı
0.50000	0.50000	ı	ı	ı	ı	,	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı
		0.88500	00006.0	0.87500	0.87500	0.86500	0.86500	0.86500	0.85500	0.83500	0.84500	0.82000	0.83000	0.90200	0.91100	0.90550	0.90800	0.91550	0.92700	0.92000	0.90050	0.88900	0.89700	0.88300	0.88200	0.87550	0.86450
		0.77000	0.80000	0.75000	0.75000	0.73000	0.73000	0.73000	0.71000	0.67000	0.69000	0.64000	0.66000	0.80400	0.82200	0.81100	0.81600	0.83100	0.85400	0.84000	0.80100	0.77800	0.79400	0.76600	0.76400	0.75100	0.72900
1.00000	1.00000	ı	·	·	·	,	ı	ı	·	·	·	·	·	ı	ı	ı	ı	ı	ı	ı	·	ı	·	·	ı	ı	
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·	ı	0.23000	0.20000	0.25000	0.25000	0.27000	0.27000	0.27000	0.29000	0.33000	0.31000	0.36000	0.34000	0.19600	0.17800	0.18900	0.18400	0.16900	0.14600	0.16000	0.19900	0.22200	0.20600	0.23400	0.23600	0.24900	0.27100
200	400	450	500	550	600	650	700	800	006	1000	1050	200	400	450	500	550	600	650	700	800	006	1000	1050	25	25	25	25
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-0.00657	-0.01120	0.00266	-0.00222	0.00203	-0.00315	-0.00365	0.00103	-0.00367	-0.00407	-0.00115	-0.00453	-0.00154	-0.00600	0.00125	0.00105	0.00128	0.00083	0.00151	0.00237	0.00171	-0.00012	-0.00119	-0.00150	-0.00229	-0.00344	-0.00303	-0.00333
4.3351	4.2475	4.03769	4.03779	4.04840	4.04322	4.04864	4.05898	4.05429	4.05994	4.06930	4.06560	4.08120	4.07641	4.01544	4.01746	4.02412	4.03121	4.03882	4.04659	4.05356	4.05976	4.06645	4.07338	4.08054	4.08715	4.09549	4.10340
8.3198(8)	8.2634(8)	8.12501(5)	8.12508(5)	8.13219(5)	8.12872(4)	8.13235(5)	8.13927(6)	8.13613(5)	8.13991(5)	8.14616(6)	8.14369(5)	8.15409(4)	8.15090(5)	8.11006(6)	8.11142(6)	8.11590(5)	8.12066(6)	8.12577(6)	8.13098(6)	8.13564(6)	8.13979(6)	8.14426(6)	8.14888(6)	8.15365(6)	8.15805(6)	8.16360(6)	8.16885(6)
33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	44	44	44	44
4032	4033	4034	4035	4036	4037	4038	4039	4040	4041	4042	4043	4046	4047	4048	4049	4050	4051	4052	4053	4054	4055	4056	4057	4059	4060	4061	4062

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0.13400	0.14300	0.14700	0.14900	0.14800	0.13750	0.13850	0.13950	0.12250	0.10700	0.10350	0.10100	0.09950	0.09800	0.07700	0.06600	0.06350	0.06300	0.07000	0.06650	0.06950	0.06850	0.07150	0.06850	0.07750	0.07250	0.06550	0.06850	
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0.86600	0.85700	0.85300	0.85100	0.85200	0.86250	0.86150	0.86050	0.87750	0.89300	0.89650	0.89900	0.90050	0.90200	0.92300	0.93400	0.93650	0.93700	0.93000	0.93350	0.93050	0.93150	0.92850	0.93150	0.92250	0.92750	0.93450	0.93150	
0.73200	0.71400	0.70600	0.70200	0.70400	0.72500	0.72300	0.72100	0.75500	0.78600	0.79300	0.79800	0.80100	0.80400	0.84600	0.86800	0.87300	0.87400	0.86000	0.86700	0.86100	0.86300	0.85700	0.86300	0.84500	0.85500	0.86900	0.86300	
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0.26800	0.28600	0.29400	0.29800	0.29600	0.27500	0.27700	0.27900	0.24500	0.21400	0.20700	0.20200	0.19900	0.19600	0.15400	0.13200	0.12700	0.12600	0.14000	0.13300	0.13900	0.13700	0.14300	0.13700	0.15500	0.14500	0.13100	0.13700	
25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	
-0.00346	-0.00361	-0.00286	-0.00211	-0.00173	-0.00257	-0.00272	-0.00297	0.00013	-0.00544	-0.00223	-0.00212	-0.00177	-0.00135	-0.00051	-0.00013	0.00003	0.00007	0.0000	0.00329	0.00001	0.00031	0.00303	0.00362	0.00301	0.00267	0.00341	0.00347	
4.11119	4.11932	4.12515	4.12833	4.12710	4.11876	4.11056	4.10227	4.09698	4.07551	4.07101	4.06380	4.05674	4.04985	4.04295	4.03603	4.02915	4.02231	4.01620	4.00886	3.99609	3.99111	3.98807	3.99533	4.01548	4.02553	4.03682	4.04800	
8.17402(6)	8.17940(6)	8.18326(6)	8.18536(6)	8.18455(6)	8.17903(6)	8.17360(6)	8.16810(6)	8.16459(6)	8.15030(6)	8.14730(6)	8.14249(6)	8.13777(6)	8.13316(6)	8.12854(6)	8.12390(6)	8.11928(6)	8.11468(6)	8.11057(6)	8.10563(6)	8.09701(5)	8.09365(5)	8.09159(6)	8.09650(6)	8.11009(6)	8.11685(6)	8.12443(6)	8.13192(6)	
44	44	44	44	44	44	44	44	44	44	44	44	44	44	44	44	44	44	44	44	44	44	44	44	44	44	44	44	
4063	4064	4065	4066	4067	4068	4069	4070	4071	4072	4073	4074	4075	4076	4077	4078	4079	4080	4081	4082	4083	4084	4085	4086	4087	4088	4089	4090	

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0.08150	0.09700	0.10250	0.11500	0.11800	0.12750	0.14900	0.16800	0.17350	0.07250	0.07250	0.07250	0.07250	0.13550	0.13550	0.13550	0.13550	0.13957	0.13900	0.13750	0.13857	0.13707	0.13550	0.13350	0.14600	0.15100	0.15850	0.16450
	,	,	,		·	,			,	,		,	,			,	0.05053	0.05400	0.05500	0.05603	0.05303	0.06050	0.05950	0.05350	0.05600	0.05750	0.05150
	,	,							,	,						,	,		,								
0.91850	0.90300	0.89750	0.88500	0.88200	0.87250	0.85100	0.83200	0.82650	0.92650	0.92650	0.92650	0.92650	0.86350	0.86350	0.86350	0.86350	0.80990	0.80700	0.80750	0.80540	0.80990	0.80400	0.80700	0.80050	0.79300	0.78400	0.78400
0.83700	0.80600	0.79500	0.77000	0.76400	0.74500	0.70200	0.66400	0.65300	0.85485	0.85485	0.85485	0.85485	0.72800	0.72800	0.72800	0.72800	0.71900	0.72172	0.72400	0.72200	0.72500	0.72827	0.73200	0.70929	0.69930	0.68432	0.67300
,	,	,	,	,	ı	,	,	,	,	,	,	,	,	,	,	,	0.09800	0.09109	00060.0	0.08800	0.09300	0.07892	0.08100	0.09391	0.08891	0.08691	0.09800
	,	,							,	,						,	,		,								
0.16300	0.19400	0.20500	0.23000	0.23600	0.25500	0.29800	0.33600	0.34700	0.14515	0.14515	0.14515	0.14515	0.27200	0.27200	0.27200	0.27200	0.18300	0.18719	0.18600	0.19000	0.18200	0.19281	0.18700	0.19680	0.21179	0.22877	0.22900
25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.44	2.92	7.34
0.00261	0.00161	0.00128	0.00061	0.00020	-0.00007	0.01565	0.00600	0.00371	0.01449	0.00848	0.01246	0.01261	0.00313	0.00012	0.00057	0.00376	-0.00372	-0.00342	-0.00322	-0.00217	-0.00191	-0.00072	-0.00014	-0.00133	-0.00145	-0.00211	-0.00404
4.05882	4.06984	4.08148	4.09330	4.10536	4.11809	4.14754	4.15190	4.16355	3.9804	3.9260	3.8536	3.8428	3.9711	3.9197	3.8438	3.8360	4.0386	4.0599	4.0658	4.0724	4.0781	4.0847	4.0910	4.0965	4.1083	4.1202	4.1310
8.13916(6)	8.14652(6)	8.15428(6)	8.16214(6)	8.17015(6)	8.17859(6)	8.19804(6)	8.20091(6)	8.20857(6)	8.0864(4)	8.0494(4)	7.9996(3)	7.9921(5)	8.0801(3)	8.0451(3)	7.9928(3)	7.9874(3)	8.1256(2)	8.1399(4)	8.1438(4)	8.1482(3)	8.1520(5)	8.1564(5)	8.1606(5)	8.1643(5)	8.1721(6)	8.1800(7)	8.1871(7)
44	44	44	44	44	44	44	44	44	44	44	44	44	44	44	44	44	44	44	2	5	38	38	38	38	38	38	38
1091	4092	4093	4094	4095	4096	4097	4098	4099	4100	4101	4102	4103	4104	4105	4106	4107	4108	e	4260	4261	4464	4465	4470	4471	4466	4467	4468

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0.16500	0.14000	0.14207	0.13950	0.14100	0.13907	0.14107	0.13150	0.13900	0.14900	0.15400	0.16150	0.16950	0.13413	0.13950	0.13757	0.13707	0.13800	0.12300	0.12256	0.12800	0.13750	0.14950	0.15942	0.16200	0.06700	0.06400	0.06300
0.05850	0.04050	0.03852	0.04200	0.04150	0.04102	0.04152	0.04150	0.04050	0.03900	0.04250	0.04050	0.04000	0.02002	0.02550	0.02551	0.02701	0.02250	0.02050	0.02251	0.02150	0.02000	0.02450	0.02449	0.02250	0.00100	0.00100	0.00100
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0.77650	0.81950	0.81941	0.81850	0.81750	0.81991	0.81741	0.82700	0.82050	0.81200	0.80350	0.79800	0.79050	0.84585	0.83500	0.83692	0.83592	0.83950	0.85650	0.85493	0.85050	0.84250	0.82600	0.81609	0.81550	0.90150	0.90450	0.90550
0.67200	0.73273	0.72873	0.73473	0.73100	0.73400	0.73173	0.75000	0.73600	0.71772	0.70729	0.69300	0.67800	0.72581	0.71573	0.71903	0.72004	0.71903	0.75025	0.75126	0.74119	0.72205	0.69889	0.67907	0.67505	0.86600	0.87200	0.87400
0.08400	0.06206	0.06507	0.05806	0.05900	0.06000	0.05906	0.06000	0.06000	0.06206	0.05594	0.05900	0.06000	0.03327	0.02823	0.02820	0.02618	0.02920	0.03323	0.03323	0.03424	0.03525	0.03726	0.03521	0.04024	ı	ı	,
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0.24400	0.20521	0.20621	0.20721	0.21000	0.20600	0.20921	0.19000	0.20400	0.22022	0.23676	0.24800	0.26200	0.24093	0.25605	0.25277	0.25378	0.25176	0.21652	0.21551	0.22457	0.24270	0.26385	0.28571	0.28471	0.13400	0.12800	0.12600
25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25
8.03	0.44	2.92	7.34	8.03	0.0001	0.535	1.136	1.642	2.484	3.023	4.153	4.752	5.961	6.584	6.975	7.253	7.374	7.427	0.0001	0.535	1.136	1.642	2.484	3.023	4.153	4.752	5.961
-0.00590	-0.00012	0.00029	0.00080	0.00165	0.00195	0.00267	0.00312	0.00392	0.00370	0.00349	0.00247	0.00185	-0.00549	-0.00680	-0.00613	-0.00614	-0.00497	-0.00291	-0.00252	-0.00260	-0.00245	-0.00517	-0.00630	-0.00904	0.00287	0.00427	0.00534
4.1351	4.0278	4.0493	4.0548	4.0613	4.0671	4.0734	4.0794	4.0857	4.0973	4.1092	4.1205	4.1264	4.0063	4.0271	4.0331	4.0386	4.0439	4.0511	4.0580	4.0635	4.0751	4.0872	4.0979	4.1018	4.0070	4.0287	4.0350
8.1898(7)	8.1184(6)	8.1328(5)	8.1365(7)	8.1408(7)	8.1447(5)	8.1489(7)	8.1529(7)	8.1571(9)	8.1648(9)	8.1727(9)	8.1802(9)	8.1841(9)	8.1039(3)	8.1179(6)	8.1219(6)	8.1256(7)	8.1292(6)	8.1340(7)	8.1386(8)	8.1423(8)	8.1500(8)	8.1581(7)	8.1652(8)	8.1678(8)	8.1044(8)	8.1190(7)	8.1232(7)
38	38	38	38	38	38	38	38	38	38	38	38	38	38	38	38	38	38	38	38	38	38	38	38	38	38	38	38
4469	4472	4473	4474	4475	q	q	q	q	q	q	q	q	q	q	q	q	q	q	q	q	q	q	q	q	q	q	q

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0.03050	0.03050	0.03050	0.03050	0.03050	0.03050	0.03050	0.03050	0.03050	0.03050	0.03050	0.03050	0.03050	0.03050	0.03050	0.03050	0.03050	0.03050	0.03050	0.03050	0.03050	0.03050	0.03050	0.03050	0.03050	0.03050	0.03050	0.03050
0.06250	0.06300	0.06350	0.06050	0.06250	0.06850	0.10200	0.12350	0.14400	0.12900	0.12900	0.11750	0.10900	0.10750	0.10550	0.10750	0.13300	0.13100	0.13000	0.12950	0.13100	0.12200	0.11950	0.12700	0.13500	0.12500	0.11600	0.11250
0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100
		·	ı	I		ı				I	·	ı		I		I	I		ı	I		I	I	I	I	I	ı
0.90600	0.90550	0.90500	0.90800	0.90600	0.90000	0.86650	0.84500	0.82450	0.83950	0.83950	0.85100	0.85950	0.86100	0.86300	0.86100	0.83550	0.83750	0.83850	0.83900	0.83750	0.84650	0.84900	0.84150	0.83350	0.84350	0.85250	0.85600
0.87500	0.87400	0.87300	0.87900	0.87500	0.86300	0.79600	0.75300	0.71200	0.74200	0.74200	0.76500	0.78200	0.78500	0.78900	0.78500	0.73400	0.73800	0.74000	0.74100	0.73800	0.75600	0.76100	0.74600	0.73000	0.75000	0.76800	0.77500
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ī		ı	,	ı	ī	,			ī	ı	ı	,	ī	ı	ī	ı	ı	ī	,	ı	ī	ı	ı	ı	ı	ı	
0.12500	0.12600	0.12700	0.12100	0.12500	0.13700	0.20400	0.24700	0.28800	0.25800	0.25800	0.23500	0.21800	0.21500	0.21100	0.21500	0.26600	0.26200	0.26000	0.25900	0.26200	0.24400	0.23900	0.25400	0.27000	0.25000	0.23200	0.22500
25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25
6.584	6.975	7.253	7.374	7.427	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0.00507	0.00586	0.00576	0.00837	0.00752	0.00851	0.00816	0.00741	0.00660	0.00631	0.00631	0.00610	0.00526	0.00426	0.00174	0.00023	0.00271	-0.00225	-0.00167	-0.00011	-0.00014	0.00031	0.00333	0.00308	0.00257	0.00195	0.00256	0.00200
4.0295	4.0408	4.0354	4.0486	4.0424	4.0544	4.0662	4.0775	4.0889	4.0766	4.0766	4.0647	4.0526	4.0408	4.0277	4.0059	4.0788	4.0042	4.0252	4.0372	4.0426	4.0481	4.0620	4.0733	4.0845	4.0721	4.0611	4.0495
8.1195(7)	8.1271(6)	8.1235(6)	8.1323(7)	8.1282(6)	8.1362(6)	8.1441(7)	8.1516(8)	8.1592(8)	8.1510(8)	8.1510(8)	8.1431(7)	8.1350(6)	8.1271(6)	8.1183(8)	8.1036(8)	8.1525(8)	8.1025(8)	8.1166(5)	8.1247(6)	8.1283(6)	8.1320(9)	8.1413(6)	8.1488(7)	8.1563(9)	8.1480(8)	8.1407(8)	8.1329(7)
38	38	38	38	38	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	4	4
q	q	q	q	q	:730	:731	732	;733	734	;735	3736	737	738	3739	740	741	742	743	744	;745	:746	747	;748	;749	;750	861	863

'	1	'	1	1	'	1	1	'	1	1	1	'	1	1	'	1	'	1	1	'	1	1	1	1	'	'	1
0.03050	0.03050	0.03050	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	0.03300	0.03300	0.03300	0.03300	0.03300	0.03300
0.10750	0.10650	0.10650	0.12250	0.12200	0.12500	0.12750	0.12550	0.12500	0.11450	0.13050	0.14150	0.15050	0.14450	0.13600	0.13350	0.12250	0.11600	0.11600	0.11550	0.11300	0.11700	0.05950	0.07000	0.06200	0.07000	0.06900	0.06600
0.00100	0.00100	0.00100	ı	ı	ı	ı	ı	ı	ı	ı	ī	ı	ı	ī	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı
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0.86100	0.86200	0.86200	0.87750	0.87800	0.87500	0.87250	0.87450	0.87500	0.88550	0.86950	0.85850	0.84950	0.85550	0.86400	0.86650	0.87750	0.88400	0.88400	0.88450	0.88700	0.88300	0.90750	0.89700	0.90500	0.89700	0.89800	0.90100
0.78500	0.78700	0.78700	0.75500	0.75600	0.75000	0.74500	0.74900	0.75000	0.77100	0.73900	0.71700	0.69900	0.71100	0.72800	0.73300	0.75500	0.76800	0.76800	0.76900	0.77400	0.76600	0.88600	0.86500	0.88100	0.86500	0.86700	0.87300
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	ï	ı	·	·	ı	·	·	ı	·	·	ï	ı	·	ï	ı	·	ı	·	·	ı	·	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100
0.21500	0.21300	0.21300	0.24500	0.24400	0.25000	0.25500	0.25100	0.25000	0.22900	0.26100	0.28300	0.30100	0.28900	0.27200	0.26700	0.24500	0.23200	0.23200	0.23100	0.22600	0.23400	0.11300	0.13400	0.11800	0.13400	0.13200	0.12600
25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0.00083	0.00007	-0.00048	-0.00036	0.00023	0.00103	0.00050	0.00086	0.00247	0.00448	0.00332	0.00312	0.00162	0.00138	0.00134	0.00082	0.00123	0.00008	-0.00061	0.00981	0.00923	-0.00137	-0.00126	0.00040	0.00111	0.00025	0.00288	0.00246
4.0374	4.0261	4.0051	3.9931	4.0142	4.0256	4.0146	4.0308	4.0378	4.0505	4.0611	4.0728	4.0775	4.0712	4.0593	4.0587	4.0475	4.0351	4.0290	4.0235	4.0125	3.9919	4.0032	4.0256	4.0313	4.0255	4.0386	4.0328
8.1248(8)	8.1172(8)	8.1031(7)	8.0950(8)	8.1092(9)	8.1169(8)	8.1095(9)	8.1204(9)	8.1251(9)	8.1336(9)	8.1407(9)	8.1485(9)	8.1516(9)	8.1474(8)	8.1395(9)	8.1391(10)	8.1316(12)	8.1233(9)	8.1192(8)	8.1155(10)	8.1081(10)	8.0942(9)	8.1018(11)	8.1169(10)	8.1207(10)	8.1168(10)	8.1256(8)	8.1217(8)
22	22	22	22	22	22	22	10	10	10	10	10	10	10	10	10	10	20	34	34	34	34	6	6	6	6	6	6
6982	6983	6984	6985	6986	6987	6988	7002	7003	7004	7005	7006	7007	7008	2009	7010	7011	7349	8151	8152	8153	8158	8661	8666	8682	8684	8699	8700

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0.03300	0.03300	0.03300	0.03300	0.03300	0.03300	0.03300	0.06153	0.06153	0.06153	0.06153	0.06153	0.06153	0.06153	0.06153	0.06153	0.06153	0.06153	0.06153	0.06153	0.08100	0.08100	0.08100	0.08100	0.08100	0.08100	0.08100	0.08100
0.06250	0.06400	0.06750	0.11450	0.13150	0.14450	0.14950	0.06253	0.06753	0.06503	0.06753	0.06253	0.06603	0.06403	0.06703	0.07554	0.10505	0.12206	0.13557	0.13757	0.06200	0.05550	0.05750	0.05550	0.05650	0.05150	0.05250	0.05750
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0.90450	0.90300	0.89950	0.85250	0.83550	0.82250	0.81750	0.87594	0.87094	0.87344	0.87094	0.87594	0.87244	0.87444	0.87144	0.86293	0.83342	0.81641	0.80290	0.80090	0.85700	0.86350	0.86150	0.86350	0.86250	0.86750	0.86650	0.86150
0.88000	0.87700	0.87000	0.77600	0.74200	0.71600	0.70600	0.88098	0.87098	0.87598	0.87098	0.88098	0.87398	0.87798	0.87198	0.85498	0.79598	0.76198	0.73498	0.73098	0.87798	0.89098	0.88698	0.89098	0.88898	0.89898	0.89698	0.88698
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0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00202	0.00202	0.00202	0.00202	0.00202	0.00202	0.00202	0.00202	0.00202	0.00202	0.00202	0.00202	0.00202	0.00202	0.00202	0.00202	0.00202	0.00202	0.00202	0.00202	0.00202
0.11900	0.12200	0.12900	0.22300	0.25700	0.28300	0.29300	0.11700	0.12700	0.12200	0.12700	0.11700	0.12400	0.12000	0.12600	0.14300	0.20200	0.23600	0.26300	0.26700	0.12000	0.10700	0.11100	0.10700	0.10900	00660.0	0.10100	0.11100
25	25	200	400	450	400	500	450	550	500	600	700	800	006	800	800	700	009	500	400	200	850	200	400	500	550	600	700
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0.00404	0.00412	0.00542	0.00449	0.00466	0.00383	0.00545	0.00193	0.00330	0.00446	0.00360	0.00422	0.00397	0.00497	0.00546	0.00599	0.00550	0.00492	0.00516	0.00469	0.00368	0.00395	0.00435	0.00380	0.00602	0.00520	0.00762	0.00760
4.0448	4.0504	4.0572	4.0635	4.0755	4.0866	4.1002	4.0177	4.0396	4.0459	4.0399	4.0508	4.0454	4.0569	4.0629	4.0692	4.0754	4.0866	4.0988	4.1101	4.0272	4.0475	4.0532	4.0474	4.0601	4.0538	4.0668	4.0724
8.1298(9)	8.1335(10)	8.1381(7)	8.1423(10)	8.1503(11)	8.1577(11)	8.1667(14)	8.1116(10)	8.1263(10)	8.1305(7)	8.1265(9)	8.1338(9)	8.1302(8)	8.1379(9)	8.1419(11)	8.1461(9)	8.1502(9)	8.1577(8)	8.1658(12)	8.1733(4)	8.1180(11)	8.1316(10)	8.1354(11)	8.1315(14)	8.1400(14)	8.1358(13)	8.1445(6)	8.1482(8)
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
:703	:721	662	663	664	665	667	668	669	670	671	672	673	674	675	676	677	678	679	680	681	1683	685	686	687	688	689	0699

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0.08100	0.08100	0.08100	0.08100	0.08100	,	·	,	ı	,	ı	ı	,	ı	ı	ı	,	ı	0.87500	0.86100	0.86000	0.85100	0.85000	0.78078	0.27200	,	ı	·	
0.06650	0.09550	0.10400	0.11300	0.12300	0.06910	0.06653	0.07157	0.06653	0.06907	0.06657	0.07357	0.08704	0.10705	0.13157	0.14114	0.15023	0.15716		0.00700	0.01300	0.01064	0.01482	0.02681	0.05067	0.04300	0.04500	0.04749	
ı	ı	ı	ı	ı	0.03405	0.03352	0.03453	0.03352	0.03253	0.03353	0.02853	0.02801	0.02501	0.02151	0.02202	0.02404	0.02503	0.01700	0.02800	0.02100	0.02100	0.01600	0.06907	ı	0.02800	0.01500	0.02000	
ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ī	ı		ı	0.01100	0.01100	0.00400	0.00236	0.00318	0.00722	0.00633	0.04400	0.06200	0.04951	
0.85250	0.82350	0.81500	0.80600	0.79600	0.89685	0.89995	0.89389	0.89995	0.89840	0.89990	0.89790	0.88494	0.86793	0.84692	0.83684	0.82574	0.81782	0.09500	0.09200	0.09600	0.11500	0.11400	0.09810	0.66900	0.88300	0.87500	0.88100	
0.86898	0.81098	0.79398	0.77598	0.75598	0.85901	0.86646	0.85628	0.86646	0.86030	0.86606	0.85211	0.82613	0.78794	0.73969	0.71988	0.70080	0.68875	0.56757	0.58759	0.56356	0.59100	0.51952	0.52252	0.64800	0.06061	0.06061	0.07071	
ı	ı	ı	ı	ı	0.01309	0.01205	0.01106	0.01205	0.01508	0.01410	0.02314	0.02412	0.03015	0.03719	0.03614	0.03414	0.03213	0.02202	ı	ı	0.01200	0.03704	0.01502	ı	0.02020	0.02020	0.02020	
0.00202	0.00202	0.00202	0.00202	0.00202	0.00101	0.00101	0.00101	0.00101	0.00101	0.00101	0.00101	0.00101	0.00101	0.00101	0.00101	0.00101	0.00101	0.41041	0.37337	0.41041	0.38100	0.44344	0.43944	0.24200	0.77778	0.73737	0.74747	
0.12900	0.18700	0.20400	0.22200	0.24200	0.12689	0.12048	0.13166	0.12048	0.12362	0.11883	0.12374	0.14874	0.18090	0.22211	0.24297	0.26406	0.27811	,	0.03904	0.02603	0.01600	,	0.02302	0.11000	0.14141	0.18182	0.16162	
800	006	800	700	600	500	400	200	200	400	500	400	550	600	700	800	006	950	006	800	800	700	600	550	400	300	200	25	
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	
0.00826	0.00820	0.00964	0.00857	0.00631	0.00385	0.00557	0.00611	0.00617	0.00612	0.00577	0.00629	0.00686	0.00587	0.00339	0.00330	0.00283	0.00184	0.00128	-0.00078	-0.00402	0.00358	-0.00069	-0.00059	-0.00120	-0.00796	-0.00336	-0.00858	
4.0788	4.0854	4.0984	4.1091	4.1189	4.0142	4.0360	4.0423	4.0366	4.0478	4.0421	4.0539	4.0605	4.0661	4.0704	4.0821	4.0941	4.1053	4.35535	4.3454	4.3457	4.3431	4.3500	4.3652	4.10738	4.0776	4.0757	4.0712	
8.1525(6)	8.1569(7)	8.1655(5)	8.1726(11)	8.1791(9)	8.1092(6)	8.1239(6)	8.1281(7)	8.1243(6)	8.1318(5)	8.1280(6)	8.1359(6)	8.1403(7)	8.1440(7)	8.1469(7)	8.1547(8)	8.1627(9)	8.1701(10)	8.33273(8)	8.3264(3)	8.3266(3)	8.3249(2)	8.3293(3)	8.3390(2)	8.17149(7)	8.1517(6)	8.1504(5)	8.1474(6)	
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	32	
8691	8692	8693	8694	8695	8696	8697	8698	8701	8702	8704	8705	8706	8707	8708	8709	8710	8711	8712	8713	8714	8715	8716	8717	8718	8719	8720	8765	

70         31         8.445(6)         4.0001         -4.0033         0.0010         2         1.111         0.7773         0.1010         0.9753         0.1040      <	I	I	>>>++->		I		~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~	I			222	10000	110000		1+10+00.0	1	10
7703281.46(6)4.061-0.003180.010020.11117.7730.11200.101000.1040711785328.046(1)3.9770.013230.01610.001120.10100.1040710.01437100.01437100.0143710.01437100.0143710.01437100.0143710.014371000	ı	,	0.11600	,	,	0 88400	0.76800	ı	,	0.23200	600	0 0001	0 00174	3 9777	R 0846(1)	37	789
770316146164.0631-0.003180.0001250.11110.177380.101010.192700.103600.104671.0.103000.103000.103000.103000.103000.103000.103000.103000.103000.103000.103000.103000.103000.103000.013000.103000.01	ı	ı	0.11600		,	0.88400	0.76700	ı		0.23300	550	0.0001	0.00143	3.9774	8.0844(1)	32	788
7702181,440(6)4.06310.001310.010310.11110.77780.01010.101010.012320.01840.01970.014470.00.0000780228.084(1)3.9770.001420.0013250.11100.75000.11000.19700.114500.114500.0<10	ı	·	0.12200	ı	·	0.87800	0.75500	ī	·	0.24500	450	0.0001	0.00109	3.9773	8.0843(1)	32	787
770328.146(6)4.0611-0.03330.0001320.11100.77300.01360.01360.014671.00.01300.014671.0780328.084(1)37770.01320.0013370.010137700.11300.11400.14300.1<400	ı		0.12500	ı	ı	0.87500	0.75000	ı	·	0.25000	500	0.0001	0.00056	3.9769	8.0840(1)	32	786
770318.1460(6)4.06910.001310.0011250.11110.177780.10010.103710.00470.04370.04370.0430 <td></td> <td></td> <td>0.13200</td> <td></td> <td></td> <td>0.86800</td> <td>0.73600</td> <td>ı</td> <td></td> <td>0.26400</td> <td>400</td> <td>0.0001</td> <td>-0.00026</td> <td>3.9763</td> <td>8.0836(1)</td> <td>32</td> <td>784</td>			0.13200			0.86800	0.73600	ı		0.26400	400	0.0001	-0.00026	3.9763	8.0836(1)	32	784
7703281460(6)4061-0003180.00150.11110.77780.10100.10360.00970.004770.00.1<0780328.084(1)37740.01120.0101370.01120.01010.11000.13000.11000.10300.01000.10000.010000.01000.01000.01000.010000.010000.010000.010000.010000.01000<		•	0.13400			0.86600	0.73300	ı		0.26700	450	0.0001	-0.00061	3.9760	8.0834(1)	32	783
770318.446(6)4.0631-0.03330.0031250.11110.77730.10120.03230.10360.6445711785328.084(1)3.774-0.01320.0011280.11000.37700.13000.13000.146311785328.084(1)3.976-0.01320.0011280.220010.75000.88500.885010.11500.90001785328.084(1)3.976-0.01320.0011280.220010.75000.88500.885010.115010.0000810328.084(1)3.9760.00030.0001250.220010.75000.88500.885011118113.9760.00030.0001250.220010.75000.88500.885011118128.083(1)3.9760.0001200.220010.75000.88500.885011118133.9750.0001200.001250.220010.75000.8850111118143.9750.00170.001200.220020.75000.88500111118153.9750.00170.001200.220020.75000.8850011118163.		•	0.13600			0.86400	0.72800	ı		0.27200	400	0.0001	-0.00054	3.9761	8.0835(1)	32	782
7702181.460(6)4.061-0003180.0001250.11110.77780.10100.952.20.013690.04870.0470780328.1451(5)4.067-001320.00140.001280.10000.11000.91700.913000.013600.048700785328.084(1)3.7760.00140.0011280.20100.75000.885900.114500.913000.1145000785328.084(1)3.7760.001610.0011280.2200270.773000.885900.913000.114500000780328.083(1)3.9760.0001290.2200270.773000.885000.885000.113000000781328.083(1)3.9760.0001200.2000270.738000.8850000.13100000780328.083(1)3.9760.0001200.2000270.738000.8850000.1310000781328.083(1)3.9760.0011200.2000270.738000.8850000.1310000000000000000000000000000000000000 <td>·</td> <td></td> <td>0.12700</td> <td></td> <td></td> <td>0.87300</td> <td>0.74600</td> <td>ı</td> <td></td> <td>0.25400</td> <td>200</td> <td>0.0001</td> <td>0.00020</td> <td>3.9766</td> <td>8.0838(1)</td> <td>32</td> <td>781</td>	·		0.12700			0.87300	0.74600	ı		0.25400	200	0.0001	0.00020	3.9766	8.0838(1)	32	781
770         21         8.1460(6)         4.0691         -0.0013         0.00101         25         0.11101         0.10101         0.10321         0.01037         0.01437         0.1           780         32         8.1451(5)         4.0677         -0.01322         0.0011         28         0.1100         0.88550         0.0149         0.1480         7         0.01400         7           781         32         8.0441(1)         3.9774         -0.01322         0.0011         28         0.1100         0.88550         0.11460         7         0.01400         7         0.01400         7         0.01440         7         0.01400         7         0.01460         7         0.01440         7         0.01440         7         0.01440         7         0.01440         7         0.01460         7         0.01460         7         0.01460         7         0.01460         7         0.01460         7         0.01460         7         0.01460         7         0.01460         7         0.01460         7         0.01460         7         0.01460         7         0         0.01460         7         0         0.01460         7         0         0.01460         7         0         0.01460		•	0.12800			0.87200	0.74500	ı		0.25500	1000	0.0001	0.00002	3.9764	8.0837(2)	32	779
7703281460(6)4.061-0.003180.0001250.11110.717380.010100.92520.018940.010970.0447736328.4451(5)4.0677-0.013220.0011280.10100.75000.91700.91700.013900.0480-0.00400735328.084(1)3.97760.001430.0011280.23000.50.775000.88530-0.114500.0-0.00400736328.084(1)3.97760.000770.0011250.230020.750000.885000.88500-0.114500.01450766328.083(1)3.9760.000770.0011250.220020.750000.88500280.11400-0.114500.114500.01450766328.083(1)3.9760.000770.0011250.220020.750000.88500-0.114500.114500.114500.114500.145000.145000.114500.114500.114500.114500.114500.114500.114500.114500.114500.114500.11450 <t< td=""><td></td><td>•</td><td>0.12800</td><td></td><td></td><td>0.87200</td><td>0.74400</td><td>ı</td><td></td><td>0.25600</td><td>006</td><td>0.0001</td><td>0.00016</td><td>3.9766</td><td>8.0838(2)</td><td>32</td><td>778</td></t<>		•	0.12800			0.87200	0.74400	ı		0.25600	006	0.0001	0.00016	3.9766	8.0838(2)	32	778
770328.1460(6)4.0631-0.003180.0001250.11110.77780.101010.101010.292520.018940.001970.0487730328.0441(1)39774-0.013220.0011280.11000.750000.11000.91700.018000.0480-0.00400735328.0841(1)397740.001430.0011280.230000.750000.88590-0.018000.04800-0.00400-0.00400735328.0841(1)39760.00050.0001250.230001.10000.885000.885000.113000.11300-0.11300-0.01480760328.0841(1)39760.00050.0001250.230001.10000.885000.88690-0.11300-0.11300761328.0831(1)39760.0001250.230001.10.740000.885000.88690-0.113000.135000.135000.135000.135000.135000.135000.135000.135000.135000.135000.135000.135000.135000.135000.135000.135000.135000.13500- <td></td> <td>•</td> <td>0.13000</td> <td></td> <td></td> <td>0.87000</td> <td>0.74000</td> <td>ı</td> <td></td> <td>0.26000</td> <td>800</td> <td>0.0001</td> <td>-0.00020</td> <td>3.9763</td> <td>8.0836(4)</td> <td>32</td> <td>777</td>		•	0.13000			0.87000	0.74000	ı		0.26000	800	0.0001	-0.00020	3.9763	8.0836(4)	32	777
770328.146(6)4.06310.003180.0011250.11110.77780.010100.023220.013970.014970.0449711780328.144(1)3.97740.013220.0011280.11000.76000.013000.013000.0480010000400785328.0844(1)3.97740.01430.00140.0011280.210000.76000.013000.013000.048000.0480000785328.0844(1)3.97760.001410.0011280.221000.75000.885500.885500.114500.0145000800328.0844(1)3.97760.000170.0001250.22000200.750000.885500.885500.11450000810328.0834(1)3.97760.000170.0011250.2200020.750000.885500.885500.1145020.1145028113.97760.000170.0001250.2200020.770000.885500.885500.1145020.114502281281846(1)3.97760.000170.0001200.2000020.770000.885500.875000.1145020.11450228133.97760.001770.0011200.200020.770000.885500.875000.1145020.1145022 </td <td>·</td> <td></td> <td>0.13000</td> <td>·</td> <td>·</td> <td>0.87000</td> <td>0.74000</td> <td>ı</td> <td></td> <td>0.26000</td> <td>700</td> <td>0.0001</td> <td>-0.00034</td> <td>3.9761</td> <td>8.0835(2)</td> <td>32</td> <td>776</td>	·		0.13000	·	·	0.87000	0.74000	ı		0.26000	700	0.0001	-0.00034	3.9761	8.0835(2)	32	776
770328.146(6)4.0631-0.003180.0010250.11110.777780.010100.012010.013000.014870.010370.0448700.044870780328.1451(5)4.0677-0.013220.0013280.11000.275000.11000.31500.018070.0480700.00000785328.0844(1)3.97740.001490.0014280.22900280.110000.88500.013000.018000.0480700.000007958.0844(1)3.97760.001610.001610.0011250.22100270.71000.88500.013070.115000.116000.116000.018000.0180007958.0841(1)3.97760.001610.001610.0011250.22100270.750000.887000.887000.115000.115000.115000.115000.018000.018000.018008103.2760.001610.00170.0011250.22100270.750000.887000.887000.115000.	ı		0.13600	ı	·	0.86400	0.72900	ī	·	0.27100	650	0.0001	-0.00053	3.9761	8.0835(1)	32	775
770328.1460(6)4.0691-0.003180.0001250.11110.777780.10100.101010.925220.010840.010470.4648720.00400780328.0441(1)3.97740.001320.001310.80.11000.750000.11000.917000.018000.018000.04807770.00400785328.0844(1)3.97740.001410.001410.0011280.2290070.77000.885500.917000.01800	ı		0.13800	ı	·	0.86200	0.72500	ī	·	0.27500	600	0.0001	-0.00104	3.9757	8.0832(3)	32	774
77032 $8.1460(6)$ $4.0691$ $-0.00318$ $0.0001$ 25 $0.1111$ $0.77778$ $0.01010$ $0.26200$ $0.01807$ $0.01807$ $0.0487$ $0.0487$ $0.0400$ 78032 $8.1451(5)$ $4.0677$ $-0.01322$ $0.00149$ $0.0011$ 28 $0.11000$ $0.25200$ $0.11000$ $0.29770$ $0.01300$ $0.0480$ $0.0480$ $0.00400$ 78032 $8.084(1)$ $3.9776$ $0.00149$ $0.0011$ 28 $0.22900$ $0.76000$ $0.28700$ $0.38750$ $0.11300$ $0.1450$ $0.00400$ 78032 $8.084(1)$ $3.9776$ $0.00161$ $0.0011$ 28 $0.22900$ $0.76000$ $0.38700$ $0.88400$ $0.11450$ $0.11450$ $0.00400$ 780 $32$ $8.0840(1)$ $3.9769$ $0.00161$ $0.0011$ 25 $0.24000$ $0.76000$ $0.88400$ $0.88400$ $0.11450$ $0.11450$ $0.00100$ 810 $32$ $8.083(1)$ $3.9769$ $0.00017$ $20$ $0.24000$ $20$ $0.76000$ $0.88400$ $0.88400$ $0.11250$ $0.11450$ $0.00100$ 810 $32$ $8.083(1)$ $3.9769$ $0.00017$ $20$ $0.22000$ $20$ $0.75000$ $0.88400$ $0.88400$ $0.11250$ $0.11350$ $0.01300$ $0.01300$ 810 $32000$ $32083(1)$ $3.9769$ $0.00017$ $2001$ $2001$ $2000$ $20700$ $0.88400$ $0.88400$ $0.14500$ $0.14500$ $0.14500$ $0.14500$ $0.1450$	I		0.13600	ı	·	0.86400	0.72800	ī	·	0.27200	550	0.0001	-0.00098	3.9757	8.0832(2)	32	773
77032 $8.1460(6)$ $4.0691$ $-0.00318$ $0.0011$ 25 $0.1111$ $0.7778$ $0.01010$ $0.29222$ $0.0197$ $0.0497$ $0.0487$ $ -$ 78032 $8.1451(5)$ $4.0677$ $-0.01322$ $0.0011$ 28 $0.11000$ $0.7600$ $0.01700$ $0.01300$ $0.0180$ $0.0480$ $  0.00400$ 78532 $8.0844(1)$ $3.776$ $0.00149$ $0.0011$ 28 $0.22900$ $ 0.77100$ $0.88500$ $0.88450$ $0.01490$ $0.04800$ $  0.00400$ 78532 $8.0841(1)$ $3.776$ $0.00149$ $0.0001$ 28 $0.22900$ $ 0.77100$ $0.88500$ $0.88450$ $0.11450$ $  -$ 79532 $8.0841(1)$ $3.776$ $0.00161$ $0.0001$ 28 $0.24000$ $  0.77100$ $0.88500$ $0.88500$ $  -$ <td>ı</td> <td></td> <td>0.09000</td> <td>ı</td> <td>,</td> <td>0.91000</td> <td>0.82000</td> <td>ī</td> <td>,</td> <td>0.18000</td> <td>450</td> <td>0.0001</td> <td>0.00406</td> <td>3.9792</td> <td>8.0856(1)</td> <td>32</td> <td>772</td>	ı		0.09000	ı	,	0.91000	0.82000	ī	,	0.18000	450	0.0001	0.00406	3.9792	8.0856(1)	32	772
770328.1460(6)4.0691-0.003180.0001250.11110.777780.010100.101010.925220.010970.004970.04487780328.1451(5)4.0677-0.013220.001490.0011280.110000.110000.917000.013800.048070.048070.048000.00400785328.0841(1)3.97740.001490.00141280.22900-0.110000.917000.013800.018000.048070.00400785328.0841(1)3.97760.001610.00161280.22900-0.771000.885500.114500.00400785328.0841(1)3.97760.001610.00161250.23100-0.771000.885500.88450-0.114500.00400780328.0841(1)3.97760.001610.0011250.23100-0.776000.885000.88500-0.115000.00400810328.0840(1)3.97690.000710.0011250.225000.775000.875000.87500-0.11500	I		0.10400	ı	·	0.89600	0.79200	ī	·	0.20800	500	0.0001	0.00287	3.9785	8.0851(1)	32	771
770         32         8.1460(6)         4.0691         -0.0318         0.0001         25         0.1111         0.7778         0.01010         0.10101         0.01097         0.0487         -         -         -         -         -         -         -         -         -         -         -         -         -         -         0.01312         0.00131         25         0.1110         0.1700         0.10100         0.01300         0.01480         0.00149         0.0001         28         0.11000         0.01300         0.01480         0.0001         28         0.11000         0.11000         0.11450         0.000400           755         32         8.0841(1)         3.9776         0.00161         0.0011         28         0.23100         7.7100         0.13800         0.11450         7         0.00400           750         32         8.0841(1)         3.9776         0.00161         0.0011         25         0.23100         7.7600         0.88550         7         0.11550         7         0.11450         7         0.000400           750         8.0841(1)         3.9766         0.00012         25         0.25000         7         0.77000         0.88550         7         0.11450<	I		0.11500	ı	·	0.88500	0.77000	ī	·	0.23000	400	0.0001	0.00177	3.9777	8.0846(1)	32	769
770328.1460(6)4.0691-0.003180.0001250.11110.777780.010100.101010.925220.018940.010970.04807780328.1451(5)4.0677-0.013220.0011280.110000.760000.270000.110000.917000.018040.018070.0480720.00400785328.0844(1)3.97740.001490.00161280.2290020.771000.8855020.1145020.00400785328.0841(1)3.97760.001610.00161280.2290020.771000.8855020.1145020.00400780328.0841(1)3.97760.001610.0011250.2310020.760000.8845020.11450220.20000810328.0838(1)3.97690.000560.0011250.2400020.760000.88450220.1145022810328.0838(1)3.97690.000560.0001250.2400020.750000.8850020.115502228113.97660.000770.0001250.2500020.750000.88500220.115502228128.0836(1)3.97690.00071200.26200220.738000.86500220.1155022 <td>ı</td> <td></td> <td>0.13300</td> <td>ı</td> <td>,</td> <td>0.86700</td> <td>0.73400</td> <td>ī</td> <td>,</td> <td>0.26600</td> <td>450</td> <td>0.0001</td> <td>-0.00015</td> <td>3.9764</td> <td>8.0837(1)</td> <td>32</td> <td>768</td>	ı		0.13300	ı	,	0.86700	0.73400	ī	,	0.26600	450	0.0001	-0.00015	3.9764	8.0837(1)	32	768
770328.1460(6)4.0691-0.003180.0001250.11110.77780.010100.125220.018940.010970.04487780328.1451(5)4.0677-0.013220.0011280.110000.760000.017000.925220.018940.018970.044800.00400785328.0844(1)3.97740.001490.0011280.229000.771000.885500.01450-0.00400795328.0841(1)3.97760.001610.0001250.231000.760000.885500.114500.00400705328.0841(1)3.97760.001610.0001250.240000.760000.885500.115500.00400810328.0841(1)3.97660.000560.0001250.240000.760000.885000.11550810328.0831(1)3.97660.000560.0001250.250000.760000.8550000.1250000.12500000000100-	ı	,	0.14500	ı	·	0.85500	0.71000	ī	·	0.29000	400	0.0001	-0.00172	3.9752	8.0829(1)	32	767
770         32         8.1460(5)         4.0691         -0.00318         0.0001         25         0.1111         0.7778         0.01010         0.92522         0.01894         0.01097         0.04807         -         -         -           780         32         8.1451(5)         4.0677         -0.01322         0.0001         28         0.11000         0.91700         0.91700         0.01800         0.04800         -         0.00400           785         32         8.044(1)         3.9774         0.00149         0.0001         28         0.11000         0.91700         0.91700         0.01800         0.04800         -         0.01400           795         32         8.084(1)         3.9776         0.00149         0.0001         25         0.23100         -         -         0.76000         0.88550         -         0.11450         -         -         0.00400           795         32         8.0841(1)         3.9776         0.00161         0.0001         25         0.23100         -         0.76000         0.88550         -         0         0.11550         -         -         -         0.11550         -         -         -         0.11550         -         -	I		0.13500	ı	·	0.86500	0.73000	ī	·	0.27000	200	0.0001	-0.00051	3.9761	8.0835(1)	32	766
770         32         8.1460(5)         4.0691         -0.00318         0.0001         25         0.1111         0.7778         0.01010         0.92522         0.01894         0.01097         0.04877         -         -         -           780         32         8.1451(5)         4.0677         -0.01322         0.01001         28         0.11000         0.76000         0.02000         0.11000         0.91700         0.01800         0.04800         -         0.00400           785         32         8.084(1)         3.9774         0.00149         0.0001         28         0.22900         -         -         0.77100         0.88550         -         0.11450         -         -         0.000400           795         32         8.0841(1)         3.9776         0.00161         0.0001         25         0.23100         -         0.76600         0.88550         -         -         0.11450         -         -         -         -         -         -         0.11450         -         -         0.00400           700         32         8.0841(1)         3.9776         0.00161         25         0.24000         -         0.76000         0.88450         -         0.11450         <	I		0.13100	ı	·	0.86900	0.73800	ī	·	0.26200	25	0.0001	0.00007	3.9766	8.0838(1)	32	815
770         32         8.1460(6)         4.0691         -0.00318         0.0001         25         0.1111         0.7778         0.01010         0.92522         0.01894         0.01097         0.04877         -         -         -         -         -         -         -         -         -         0.04007         2         0.01111         0.77778         0.01010         0.92522         0.01894         0.01097         0.04800         -         0.00400           780         32         8.1451(5)         4.0677         -0.01322         0.0001         28         0.11000         0.91700         0.01300         0.04800         -         0.00400           785         32         8.0844(1)         3.9774         0.00149         0.0001         28         0.22900         -         0.77100         0.88550         -         0.11450         -         0.00400           795         32         8.0845(1)         3.9776         0.00161         0.0001         25         0.23100         -         0.76900         0.88550         -         0.11550         -         -         -         0.11550         -         -         -         -         -         0.11550         -         -         -	·		0.12500	·	·	0.87500	0.75000	ı		0.25000	25	0.0001	0.00056	3.9769	8.0840(1)	32	810
770       32       8.1460(6)       4.0691       -0.00318       0.0001       25       0.11111       0.77778       0.010101       0.92522       0.01097       0.04877       -       0.0400       -       -       0.01322       0.01322       0.01300       0.01800       0.04800       -       0.01100       0.91700       0.01300       0.04800       -       0.000400         785       32       8.0844(1)       3.9774       0.00149       0.0001       28       0.22900       -       -       0.77100       0.88550       -       -       0.11450       -       -       -       0.000400         775       32       8.0845(1)       3.9776       0.00161       28       0.23100       -       -       0.71100       0.88450       -       0.11450       -       -       -       0.11450       -       -       -       -       -       -       -       0.11450       -       -       -       -	·		0.12000	·	·	0.88000	0.76000	ı		0.24000	25	0.0001	0.00087	3.9770	8.0841(1)	32	800
770     32     8.1460(6)     4.0691     -0.00318     0.0001     25     0.11111     0.77778     0.01010     0.92522     0.01097     0.04487     -     -     -       778     32     8.1451(5)     4.0677     -0.01322     0.0001     28     0.11000     0.76000     0.02000     0.11000     0.91700     0.01300     0.04800     -     0.00400       785     32     8.0844(1)     3.9774     0.00149     0.0001     28     0.22900     -     -     0.77100     0.88550     -     0.11450     -     -     0.01450     -     0.01450     -     -     0.012000     -     -     0.711450     -     0.11450     -     -     -     0.01450     -     -     0.01450     -     -     0.01450     -     -     0.01450     -     -     0.01450     -     -     0.01450     -     -     0.01450     -     -     0.01450     -     -     0.01450     -     -     0.01450     -     -     0.01450     -     -     0.01450     -     0.01450     -     -     0.01450     -     -     0.01450     -     -     0.01450     -     -     0.01450     -     -	I		0.11550	ı	·	0.88450	0.76900	ī	·	0.23100	25	0.0001	0.00161	3.9776	8.0845(1)	32	795
770 32 8.1460(6) 4.0691 -0.00318 0.0001 25 0.11111 0.77778 0.01010 0.10101 0.92522 0.01894 0.01097 0.0487 5780 32 8.1451(5) 4.0677 -0.01322 0.0001 28 0.11000 0.76000 0.02000 0.11000 0.91700 0.01300 0.01800 0.04800 - 0.00400	I		0.11450	ı	·	0.88550	0.77100	ī	·	0.22900	28	0.0001	0.00149	3.9774	8.0844(1)	32	3785
5770 32 8.1460(6) 4.0691 -0.00318 0.0001 25 0.11111 0.77778 0.01010 0.10101 0.92522 0.01894 0.01097 0.04487 -	0.00400		0.04800	0.01800	0.01300	0.91700	0.11000	0.02000	0.76000	0.11000	28	0.0001	-0.01322	4.0677	8.1451(5)	32	3780
	ı		0.04487	0.01097	0.01894	0.92522	0.10101	0.01010	0.77778	0.11111	25	0.0001	-0.00318	4.0691	8.1460(6)	32	3770

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0.11600	0.11500	0.13600	0.13600	0.12400	0.11900	0.10600	0.10600	0.10500	0.10400	0.10500	0.10500	0.10500	0.10500	0.10500	0.10500	0.10500	0.10500	0.10500	0.10500	0.10500	0.10500	0.10500	0.10500	0.10500	0.10500	0.10500	0.10500
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0.88400	0.88500	0.86400	0.86400	0.87600	0.88100	0.89400	0.89400	0.89500	0.89600	0.89500	0.89500	0.89500	0.89500	0.89500	0.89500	0.89500	0.89500	0.89500	0.89500	0.89500	0.89500	0.89500	0.89500	0.89500	0.89500	0.89500	0.89500
0.76900	0.77000	0.72800	0.72900	0.75300	0.76200	0.78700	0.78700	0.79000	0.79200	0.79000	0.79000	0.79000	0.79000	0.79000	0.79000	0.79000	0.79000	0.79000	0.79000	0.79000	0.79000	0.79000	0.79000	0.79000	0.79000	0.79000	0.79000
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ī	ī	ī	ı	ī	ī	ı	ī	ī	ī	I	ī	ı	ı	ī	ī	I	I	ī	I	ī	ī	I	I	ī	I	I	ı
0.23100	0.23000	0.27200	0.27100	0.24700	0.23800	0.21300	0.21300	0.21000	0.20800	0.21000	0.21000	0.21000	0.21000	0.21000	0.21000	0.21000	0.21000	0.21000	0.21000	0.21000	0.21000	0.21000	0.21000	0.21000	0.21000	0.21000	0.21000
650	700	800	006	1000	200	400	450	400	500	450	550	600	650	700	800	006	1000	200	400	450	400	500	450	550	009	650	700
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0.00160	0.00177	-0.00098	-0.00112	0.00030	0.00075	0.00235	0.00220	0.00239	0.00272	-0.00249	-0.00322	-0.00213	-0.00408	-0.00112	-0.00003	-0.00452	-0.00322	-0.00564	-0.00162	-0.00230	-0.00084	-0.00258	-0.00190	-0.00266	-0.00167	-0.00313	-0.00038
3.9776	3.9777	3.9757	3.9755	3.9766	3.9769	3.9780	3.9779	3.9780	3.9783	3.97315	3.96008	3.95111	3.93735	3.91373	3.88956	3.87633	3.86227	3.83842	3.82811	3.81195	3.79838	3.78201	3.76427	3.74303	3.72172	3.70258	3.68701
8.0845(1)	8.0846(1)	8.0832(2)	8.0831(3)	8.0838(3)	8.0840(3)	8.0848(3)	8.0847(2)	8.0848(3)	8.0850(1)	8.08149(1)	8.07262(1)	8.06652(2)	8.05714(2)	8.04100(2)	8.02441(7)	8.01530(5)	8.00560(7)	7.98909(9)	7.98193(7)	7.97068(7)	7.96121(8)	7.94976(9)	7.93731(9)	7.92235(11)	7.90729(11)	7.89371(14)	7.88263(20)
32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32
8790	8791	8792	8793	8794	8796	8797	8798	8799	8801	8802	8803	8804	8805	8806	8807	8808	8809	8811	8812	8813	8814	8816	8817	8818	8819	8820	8821

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0.5000		0.50000	I	ı	ı	1.00000	I	ı	ı	25	0.0001		-0.00492	4.5517 -0.00492	8.4561(1) 4.5517 -0.00492	3 8.4561(1) 4.5517 -0.00492
0.5000		0.50000			1 1	1.00000				25 25	0.0001		-0.00045 0.00642	4.6867 -0.00045 4.5434 0.00642	8.5389(1) 4.6867 -0.00045 8.4510(1) 4.5434 0.00642	3 8.5389(1) 4.6867 -0.00045 3 8.4510(1) 4.5434 0.00642
0.5000		0.50000	·			1.00000			·	25	0.0001		-0.00023	4.6808 -0.00023	8.5353(1) 4.6808 -0.00023	3 8.5353(1) 4.6808 -0.00023
0.5000	,	0.50000	ī		I	1.00000	ı	ı	ı	25	0.0001		-0.00014	4.6683 -0.00014	8.5277(1) 4.6683 -0.00014	3 8.5277(1) 4.6683 -0.00014
0.5000	,	0.50000		,	ı	1.00000			ı	25	0.0001		-0.00036	4.6570 -0.00036	8.5208(1) 4.6570 -0.00036	3 8.5208(1) 4.6570 -0.00036
0.5000	,	0.50000	ı		ı	1.00000		ı	·	25	0.0001		-0.00045	4.6458 -0.00045	8.5140(1) 4.6458 -0.00045	3 8.5140(1) 4.6458 -0.00045
0.5000	ı	0.50000	ī		ı	1.00000	ī	ı	ı	25	0.0001		-0.00075	4.6349 -0.00075	8.5073(1) 4.6349 -0.00075	3 8.5073(1) 4.6349 -0.00075
0.5000		0.50000			ı	1.00000			ı	25	0.0001	0	-0.00059	4.6242 -0.00059 (	8.5008(1) 4.6242 -0.00059	3 8.5008(1) 4.6242 -0.00059
0.5000	,	0.50000		,	ı	1.00000			ı	25	0.0001		-0.00111	4.6133 -0.00111	8.4941(1) 4.6133 -0.00111	24 8.4941(1) 4.6133 -0.00111
0.5000		0.50000			·	1.00000			ı	25	0.0001		-0.00116	4.6029 -0.00116	8.4877(1) 4.6029 -0.00116	24 8.4877(1) 4.6029 -0.00116
0.5000		0.50000			·	1.00000			ı	25	0.0001		0.00061	4.5930 0.00061	8.4816(1) 4.5930 0.00061	24 8.4816(1) 4.5930 0.00061
0.5000	,	0.50000	ī	,	ı	1.00000	ī	ı	ı	25	0.0001		-0.00021	4.5834 -0.00021	8.4757(1) 4.5834 -0.00021	24 8.4757(1) 4.5834 -0.00021
0.5000		0.50000				1.00000			ı	25	0.0001		-0.00013	4.5738 -0.00013	8.4698(1) 4.5738 -0.00013	24 8.4698(1) 4.5738 -0.00013
0.500		0.50000				1.00000			ı	25	0.0001		0.00050	4.5641 0.00050	8.4638(1) 4.5641 0.00050	28 8.4638(1) 4.5641 0.00050
0.500		0.50000				1.00000			ı	25	0.0001		0.00066	4.5544 0.00066	8.4578(1) 4.5544 0.00066	28 8.4578(1) 4.5544 0.00066
0.500	,	0.50000			ı	1.00000		ı	ı	25	0.0001		0.00085	4.5451 0.00085	8.4520(1) 4.5451 0.00085	28 8.4520(1) 4.5451 0.00085
0.500		0.50000				1.00000			ı	25	0.0001		0.00103	4.5376 0.00103	8.4474(1) 4.5376 0.00103	28 8.4474(1) 4.5376 0.00103
0.500(	,	0.50000		,	ı	1.00000			ı	25	0.0001		0.00182	4.5347 0.00182	8.4456(1) 4.5347 0.00182	28 8.4456(1) 4.5347 0.00182
0.5000	ı	0.50000	ī		ı	1.00000	ī	ı	ı	25	0.0001		0.00118	4.5368 0.00118	8.4469(1) 4.5368 0.00118	28 8.4469(1) 4.5368 0.00118
'	,	0.10500			0.89500	0.79000		ı	0.21000	25	0.0001		-0.00202	3.57219 -0.00202	7.79994(26) 3.57219 -0.00202	28 7.79994(26) 3.57219 -0.00202
·	,	0.10500			0.89500	0.79000		ı	0.21000	1000	0.0001		-0.00228	3.61123 -0.00228	7.82825(24) 3.61123 -0.00228	32 7.82825(24) 3.61123 -0.00228
I	,	0.10500	ī		0.89500	0.79000	ı	ī	0.21000	006	0.0001		-0.00328	3.63703 -0.00328	7.84685(27) 3.63703 -0.00328	32 7.84685(27) 3.63703 -0.00328
		0.10500	ī	ı	0.89500	0.79000	ī	ı	0.21000	800	0001	Ö	-0.00513 0.	3.66223 -0.00513 0.	7.86493(18) 3.66223 -0.00513 0.	32 7.86493(18) 3.66223 -0.00513 0.

0.5000		0.50000	- 0.50000	- 0.50000			- 1.00000			25 25	3.1	0.00406	4.4253	8.3771(2)	29	2937
0.50000	,	0.50000				1.00000			•	77	i				73	
0.50000						00000				10	1.7	0.00265	1 5491	8 AEAE(1)		2936
	'	0.50000	ı	ı	ı	1.00000	ı	,	ï	25	1.1	0.00232	4.5530	8.4569(1)	29	2935
0.50000	ī	0.50000	ı	ı	I	1.00000	ı	,	ı	25	0.6	0.00189	4.5599	8.4612(1)	29	2934
0.50000	ī	0.50000	ı	ı	,	1.00000	ı	ı	ı	25	0.0001	0.00089	4.5678	8.4661(1)	29	2933
0.50000	,	0.50000	ı	,	,	1.00000	ı	ı	,	25	0.001	0.00058	4.5768	8.4716(1)	2	2847
0.50000	,	0.50000	ı	,	,	1.00000	ı	ı	,	25	0.001	-0.00036	4.5853	8.4769(1)	2	2846
0.50000	ī	0.50000	ı	ı	,	1.00000	ı	ı	ı	25	0.0001	-0.00049	4.5949	8.4828(1)	2	2845
0.50000		0.50000	ı	ı	,	1.00000	ı	ı	ı	25	0.001	-0.00094	4.6044	8.4886(1)	2	2844
0.50000	ī	0.50000	ı	ı	,	1.00000	ı	ı	ı	25	0.0001	-0.00092	4.6143	8.4947(1)	2	2843
0.50000	ī	0.50000	ı	ı	,	1.00000	ı	ı	ı	25	0.0001	-0.00058	4.6247	8.5011(1)	2	2842
0.50000	,	0.50000	ı	,	,	1.00000	ı	ı	,	25	0.001	-0.00042	4.6352	8.5075(1)	2	2841
0.50000	ī	0.50000	ı	ı	,	1.00000	ı	ı	ı	25	0.0001	-0.00150	4.6409	8.5110(1)	2	2840
0.50000	,	0.50000	ı	,	,	1.00000	ı	ı	,	25	0.001	-0.00132	4.6555	8.5199(1)	2	2839
0.50000		0.50000	ı	,		1.00000	·	ı	,	25	0.0001	0.00041	4.6679	8.5275(1)	2	2838
0.50000		0.50000	ı	,		1.00000	·	ı	,	25	0.0001	-0.00141	4.6906	8.5413(1)	2	2837
0.50000		0.50000	ı	,		1.00000	·	ı	,	25	0.0001	0.00126	4.7192	8.5586(1)	2	2836
0.50000		0.50000				1.00000		ı		25	0.0001	0.00123	4.7316	8.5661(1)	2	2835
0.50000		0.50000				1.00000		ı		25	0.0001	0.00097	4.7439	8.5735(1)	2	2834
0.50000		0.50000	ı	,		1.00000	·	ı	,	25	0.0001	0.00257	4.7559	8.5807(1)	2	2833
0.50000		0.50000	ı	,		1.00000	·	ı	,	25	0.0001	0.00082	4.7580	8.5820(1)	2	2832
0.50000		0.50000	,	,	,	1.00000	,	,	,	25	0.0001	0.00250	4.7570	8.5814(1)	2	2831
0.50000		0.50000	ı	,		1.00000	·	ı	,	25	0.0001	0.00151	4.7471	8.5754(1)	2	2830
0.50000		0.50000				1.00000		ı		25	0.0001	0.00063	4.7338	8.5674(1)	2	2829
0.50000		0.50000				1.00000		ı		25	0.0001	0.00057	4.7210	8.5597(1)	2	2828
0.50000		0.50000	ı	,	,	1.00000	,	,	,	25	0.0001	0.00008	4.7081	8.5519(1)	2	2827
0.50000	ı	0.50000	·			1.00000			'	25	0.0001	-0.00068	4.6954	8.5442(1)	2	2826
0.50000	ı	0.50000	ı			1.00000		,	I	25	0.0001	-0.00135	4.6831	8.5367(1)	2	2825

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1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
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ı	ī	ī	ı	ı	ı	ı	ī	ï	ı	ı	ï	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ī	ı	I	ı
25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	06	70	97	152	220	289	360	425	483	559	625
4.5	2	5.9	7.2	8.1	9.1	10.1	11.1	12.4	13.9	15.6	17	18.5	20.2	22.6	25.1	29	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0.00406	0.00220	0.00277	0.00464	0.00143	0.00237	0.00247	0.00271	0.00256	0.00286	0.00262	0.00223	0.00354	0.00378	0.00336	0.00301	0.00247	0.00316	0.00387	0.00390	0.00365	0.00240	0.00381	0.00353	0.00208	0.00461	0.00261	0.00303
4.4118	4.3959	4.3848	4.3751	4.4388	4.4215	4.3984	4.4396	4.4332	4.4307	4.4288	4.4228	4.4180	4.4155	4.4114	4.4101	4.4077	4.4057	4.4046	4.4035	4.3994	4.3912	4.3872	4.3819	4.3641	4.4424	4.4421	4.4579
8.3686(2)	8.3585(2)	8.3515(3)	8.3453(1)	8.3856(1)	8.3747(3)	8.3601(1)	8.3861(11)	8.3821(8)	8.3805(5)	8.3793(3)	8.3755(6)	8.3725(5)	8.3709(7)	8.3683(6)	8.3675(2)	8.3660(6)	8.3647(6)	8.3640(6)	8.3633(8)	8.3607(11)	8.3555(2)	8.3530(6)	8.3496(5)	8.3383(3)	8.3879(3)	8.3877(3)	8.3976(6)
29	29	29	29	29	29	29	29	29	29	29	29	29	29	29	29	29	41	41	41	41	41	41	41	41	41	41	41
2938	2939	2940	2941	2942	2943	2944	2945	2946	2947	2948	2949	2950	2951	2952	2953	2954	3080	3081	3082	3083	3084	3085	3086	3087	3088	3089	3090

	ı		I			ı	ı	ı		·		0.00025	0.00780	0.00035	0.00065	0.00075	0.00035	0.00035	0.00035	0.00035	0.00035	0.00080	0.00080	0.00080	0.00080	0.00080	0.00080	0.00075	
			I	·		ı	,	,	ı	,	ı	0.00010	0.61334	0.00325	0.00155	0.00495	0.00325	0.00325	0.00325	0.00325	0.00325	0.00635	0.00635	0.00635	0.00635	0.00635	0.00635	0.00635	
			I	,					ï		,	0.06081	0.00235	0.05911	0.06082	0.06064	0.05906	0.05966	0.09800	0.11625	0.11730	0.06133	0.06525	0.11193	0.10571	0.10020	0.06440	0.08335	
				0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	06000.0	0.09134	0.02910	0.02661	0.02881	0.02905	0.02930	0.02870	0.02840	0.02860	0.02385	0.01970	0.02520	0.02325	0.02150	0.02450	0.02270	
				0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.00065	0.00780	0.00160	ı	0.00123	0.00165	0.00135	0.00155	0.00490	0.00505	0.00032		0.00263	ı	ı	ı	ı	
			I	,		I			ı		ı	0.93728	0.27737	0.90659	0.91038	0.90363	0.90664	0.90609	0.86816	0.84685	0.84545	0.90735	0.90790	0.85309	0.86389	0.87115	0.90395	0.88684	
	ı		I			ı			ı		ı	0.79881	0.48543	0.65412	0.65484	0.65088	0.65429	0.65208	0.57529	0.53792	0.53778	0.65817	0.65156	0.55355	0.56480	0.57881	0.65113	0.60609	
			T	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	ı	0.00440	ı	0.00110	ı	ı	ı	ı	0.00060	ı	ı	0.00890	ı	0.00400	0.00650	0.00040	0.00620	
	ı		I	,		ı	,	,	ı	,	ı	0.07614	0.51017	0.22345	0.22261	0.22591	0.22328	0.22448	0.22609	0.22038	0.21800	0.21963	0.21943	0.21843	0.22482	0.22199	0.22146	0.22801	
	ı		I	ı		ı	ı	ı	ı	ı	ı	0.12505	ı	0.12242	0.12145	0.12321	0.12244	0.12344	0.19862	0.24110	0.24422	0.12220	0.12011	0.22802	0.20638	0.19270	0.12701	0.15970	
600	75.6	000	070	885	949	1020	1054	100	240	559	850	912	975	1039	1103	1167	1232	1297	1359	1403	1416	1397	1346	1283	1219	1081	944	882	
			1000.0	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	
30000	0.00200	-0.000 n-		-0.00420	-0.00249	-0.00569	-0.00806	-0.0097	-0.01034	-0.01108	-0.00962	0.00461	0.00430	-0.00224	-0.00421	-0.00068	-0.00220	-0.00318	-0.00221	-0.00298	-0.00325	-0.00027	0.0000	-0.00414	-0.00397	-0.00305	-0.00010	-0.00290	
1 1607	4.4002	4.4205 4.4126	01111	4.3808	4.3809	4.3215	4.2823	4.2515	4.2403	4.2364	4.2148	3.9902	4.3069	4.0159	4.0125	4.0186	4.0159	4.0152	4.0162	4.0154	4.0148	4.0165	4.0173	4.0137	4.0145	4.0149	4.0173	4.0161	
0 2070/6/	(c)0/60.0	(2)2660.0		8.3489(2)	8.3490(2)	8.3111(2)	8.2859(2)	8.2660(2)	8.2587(2)	8.2562(2)	8.2421(12)	8.0930(3)	8.3017(5)	8.1104(5)	8.1081(5)	8.1122(4)	8.1104(4)	8.1099(3)	8.1106(4)	8.1100(3)	8.1096(4)	8.1108(4)	8.1113(3)	8.1089(4)	8.1094(3)	8.1097(3)	8.1113(4)	8.1105(3)	
11	4 T 7 T	41 11	;	41	41	41	41	41	41	41	41	41	41	41	41	41	41	41	41	41	41	41	41	41	41	41	41	41	
1000	TENC	3003		3094	3095	3096	3097	3098	3099	3100	3101	3102	3103	3104	3105	3106	3107	3108	3109	3110	3111	3112	3113	3114	3115	3116	3117	3118	

0.00075	0.00080	0.00050	0.00040	0.00040	0.00065	0.00075	0.00120	06000.0	0.00100	0.00050	0.00050	0.00050			·	·	·	ı	,		,	,			,		ı
0.00635	0.00635	0.00950	0.00950	0.00950	0.00155	0.00495	0.00905	0.00900	0.00900	ı	ı	0.00350	ı	·	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100
0.08371	0.08410	0.05808	0.06680	0.05843	0.06111	0.06087	0.06120	0.11670	0.12665	0.07450	0.07150	0.05900	0.06053	0.11800	0.13550	0.13550	0.13550	0.13550	0.13550	0.13550	0.13550	0.13550	0.13550	0.13550	0.13550	0.13550	0.13550
0.02525	0.02410	0.02770	0.02760	0.01155	0.02611	0.02880	0.02470	0.01465	0.02675	0.02450	0.02550	0.02900	0.03352	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı
0.00089	ı	0.00357	·	0.01607	ı	0.00123	ı	0.01849	0.00570	0.00650	0.00700	0.00150	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	·	ı	ı	
0.88305	0.88466	0.90065	0.89569	0.90405	0.91058	0.90340	0.90385	0.84026	0.83090	0.89400	0.89550	0.90650	0.90595	0.88200	0.86350	0.86350	0.86350	0.86350	0.86350	0.86350	0.86350	0.86350	0.86350	0.86350	0.86350	0.86350	0.86350
0.60602	0.60836	0.64374	0.62603	0.64156	0.65444	0.65088	0.64056	0.52638	0.50801	0.77400	0.78100	0.65449	0.87436	0.76500	0.72800	0.72800	0.72800	0.72800	0.72800	0.72800	0.72800	0.72800	0.72800	0.72800	0.72800	0.72800	0.72800
	0.00420	ı	0.00120	0.03360	0.00190	,	0.00110	0.02640	ı	0.04600	0.04500	ı	0.00997	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı
0.22535	0.22475	0.23386	0.24089	0.21024	0.22251	0.22591	0.23924	0.20512	0.22919	0.06400	0.06100	0.22351	0.00301	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı
0.16863	0.16268	0.12240	0.13189	0.11460	0.12115	0.12321	0.11910	0.24210	0.26280	0.11600	0.11300	0.12200	0.11266	0.23500	0.27200	0.27200	0.27200	0.27200	0.27200	0.27200	0.27200	0.27200	0.27200	0.27200	0.27200	0.27200	0.27200
797	756	693	630	567	503	439	375	312	250	198	168	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-0.00338	-0.00173	0.00020	-0.00111	0.00080	-0.00419	-0.00066	-0.00051	-0.00334	-0.00502	0.00379	0.00175	-0.00244	0.00930	0.00301	0.00039	0.00050	0.00053	0.00066	0.00042	0.00043	-0.00003	-0.00036	-0.00098	-0.00119	-0.00144	-0.00112	-0.00119
4.0149	4.0170	4.0217	4.0210	4.0229	4.0125	4.0186	4.0207	4.0185	4.0164	4.0112	4.0090	4.0159	4.0010	3.9791	3.9774	3.9665	3.9545	3.9446	3.9282	3.9181	3.8971	3.8862	3.8649	3.8543	3.8477	3.8435	3.8415
8.1097(4)	8.1111(3)	8.1143(3)	8.1138(3)	8.1151(3)	8.1081(5)	8.1122(4)	8.1136(3)	8.1121(4)	8.1107(4)	8.1072(1)	8.1057(1)	8.1104(4)	8.1003(3)	8.0855(2)	8.0844(1)	8.0770(1)	8.0688(1)	8.0621(1)	8.0509(1)	8.0440(2)	8.0296(3)	8.0221(1)	8.0074(1)	8.0001(1)	7.9955(1)	7.9926(1)	7.9912(1)
41	41	41	41	41	41	41	41	41	41	41	41	8	8	∞	∞	∞	11	11	11	11	11	11	11	11	11	11	11
3119	3120	3121	3122	3123	3124	3125	3126	3127	3128	3129	3130	U	U	C	U	U	U	U	U	U	U	U	U	U	U	U	U

1	ľ	1	'	'	1	'	'	ľ	'	'	1	'	'	1	1	'	1	'	'	'	'	'	'	1	ı	ı	1	
0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	,	,	,	,	,	,			,	,	ı	ı	ı	
0.13550	0.07250	0.07250	0.07250	0.07250	0.07250	0.07250	0.07250	0.07250	0.07250	0.07250	0.07250	0.07250	0.07250	0.07250	0.40100	0.40100	0.40100	0.40100	0.40100	0.40100	0.40100	0.40100	0.40100	0.40100	0.40100	0.40100	0.40100	
	,			,			,	,	,	,		,			0.59900	0.59900	0.59900	0.59900	0.59900	0.59900	0.59900	0.59900	0.59900	0.59900	0.59900	0.59900	0.59900	
																											ı	
0.86350	0.92650	0.92650	0.92650	0.92650	0.92650	0.92650	0.92650	0.92650	0.92650	0.92650	0.92650	0.92650	0.92650	0.92650													·	
0.72800	0.85485	0.85485	0.85485	0.85485	0.85485	0.85485	0.85485	0.85485	0.85485	0.85485	0.85485	0.85485	0.85485	0.85485	0.19800	0.19800	0.19800	0.19800	0.19800	0.19800	0.19800	0.19800	0.19800	0.19800	0.19800	0.19800	0.19800	
	ï	ı	ı	ı	ı	ı	ı	,	ı	ı	ı	ı	ı	ı	0.80200	0.80200	0.80200	0.80200	0.80200	0.80200	0.80200	0.80200	0.80200	0.80200	0.80200	0.80200	0.80200	
	,									,																		
0.27200	0.14515	0.14515	0.14515	0.14515	0.14515	0.14515	0.14515	0.14515	0.14515	0.14515	0.14515	0.14515	0.14515	0.14515	ı		ı							ı	ı	ı	·	
25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.11	0.48	1.21	1.81	2.48	3.06	3.38	4.28	5.02	6.1	7.15	8.14	
-0.00135	0.00940	0.00950	0.00981	0.00993	0.00953	0.00908	0.00919	0.00912	0.00833	0.00841	0.00800	0.00831	0.00781	0.00837	-0.00380	-0.00348	-0.00406	-0.00077	-0.00020	0.00152	0.00772	0.00337	0.00114	0.00369	0.00753	0.00664	0.00595	
3.8405	3.9844	3.9735	3.9617	3.9518	3.9352	3.9247	3.9042	3.8936	3.8721	3.8618	3.8550	3.8508	3.8484	3.8481	4.45196	4.44943	4.43935	4.42433	4.4103	4.3960	4.3888	4.3771	4.3548	4.34138	4.32258	4.30048	4.28045	
7.9905(1)	8.0891(1)	8.0817(1)	8.0737(2)	8.0670(1)	8.0557(1)	8.0485(2)	8.0345(1)	8.0272(1)	8.0124(1)	8.0053(1)	8.0006(1)	7.9977(1)	7.9960(1)	7.9958(1)	8.39389(5)	8.39230(5)	8.38596(6)	8.37649(9)	8.3676(1)	8.3586(2)	8.3540(2)	8.3466(2)	8.3324(1)	8.32381(9)	8.31178(6)	8.29759(7)	8.28469(8)	
11	11	11	11	11	11	11	11	11	11	11	43	43	43	43	30	30	30	30	30	30	30	30	30	30	30	30	30	
U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	J	J	J	

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ī	ı	ı	ī	ī	ī	ī	ī	ı	ı	ī	ı	ī	ī	ı	ī	ı	ı	ı	ı	ı	ī	ı	ī	ı	ı	ı		
0.40100	0.40100	0.40100	0.40100	0.40100	0.40100	0.40100	0.40100	0.40100	0.40100	0.40100	0.40100	0.40100	0.40100	0.40100	0.40100	0.40100	0.40450	0.39800	0.39600	0.39200	0.40500	0.40100	0.41050	0.40150	0.39800	0.38350	0.37750	
0.59900	0.59900	0.59900	0.59900	0.59900	0.59900	0.59900	0.59900	0.59900	0.59900	0.59900	0.59900	0.59900	0.59900	0.59900	0.59900	0.59900	0.59550	0.60200	0.60400	0.60800	0.59500	0.59900	0.58950	0.59850	0.60200	0.61650	0.62250	
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0.19800	0.19800	0.19800	0.19800	0.19800	0.19800	0.19800	0.19800	0.19800	0.19800	0.19800	0.19800	0.19800	0.19800	0.19800	0.19800	0.19800	0.19181	0.20400	0.20800	0.21600	0.19000	0.19800	0.17900	0.19700	0.20400	0.23300	0.24500	
0.80200	0.80200	0.80200	0.80200	0.80200	0.80200	0.80200	0.80200	0.80200	0.80200	0.80200	0.80200	0.80200	0.80200	0.80200	0.80200	0.80200	0.80819	0.79600	0.79200	0.78400	0.81000	0.80200	0.82100	0.80300	0.79600	0.76700	0.75500	
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ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı		
25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	100	200	300	400	502	600	650	700	750	800	850	
8.94	10.35	11.12	12.07	13.59	17.09	18.8	19.48	20	20.7	24.13	25.54	27.53	29.58	32.18	34.39	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	
0.00337	0.00281	0.00360	0.00961	0.00582	0.00996	0.00601	0.01057	0.01065	0.00847	0.00956	0.01343	0.01056	0.01289	0.00998	0.01440	-0.00380	-0.00064	0.00046	0.00335	0.00259	0.00479	0.00431	0.00538	0.00577	0.00321	0.00010	0.00004	
4.26268	4.23626	4.2234	4.2129	4.1837	4.1331	4.1041	4.0990	4.0917	4.0799	4.0356	4.0217	3.9947	3.9732	3.9412	3.9221	4.45196	4.46414	4.48163	4.49960	4.51484	4.52906	4.54506	4.55151	4.56250	4.56914	4.57837	4.58843	
8.27321(8)	8.25608(9)	8.2477(1)	8.2409(1)	8.2218(1)	8.1885(1)	8.1693(1)	8.1659(1)	8.1611(1)	8.1532(1)	8.1236(2)	8.1143(3)	8.0961(4)	8.0815(4)	8.0598(4)	8.0467(4)	8.39389(5)	8.40154(7)	8.41250(7)	8.42373(7)	8.43323(7)	8.44207(7)	8.45200(7)	8.45600(7)	8.46280(7)	8.46690(7)	8.47260(7)	8.47880(7)	
30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	
U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	C	U	

'	,	1	1	'	'	'	'	ľ	'	'	1	'	'	1	'	'	1	1	'	1	1	'	1	1	'	1	i.
ı	ı	,	,	ı	,	0.37133	0.38343	0.35158	0.36389	0.31044	0.32328	0.30412	0.30109	0.35642	0.76088	0.71817	0.74102	0.73841	0.68058	0.69254	0.64152	0.66269	0.65054	0.63896	0.64556	0.60333	0.84954
0.37000	0.37350	0.36000	0.36350	0.34850	0.34700	0.10624	0.10167	0.11673	0.08384	0.12377	0.13294	0.13571	0.14944	0.10698	0.03427	0.05014	0.02574	0.04265	0.06190	0.04681	0.06641	0.07003	0.05583	0.07197	0.08290	0.06820	0.02396
0.63000	0.62650	0.64000	0.63650	0.65150	0.65300	0.00528	0.00688	0.00491	0.00745	0.01055	0.00607	0.00870	0.00987	0.00984	,	,	0.00347		0.00324	0.00204	0.00767	0.00350	0.00257	0.00518	0.00143	0.00939	ı
,	·	ï	,	,	,	0.03076	0.03978	0.03269	0.05406	0.03976	0.03113	0.03778	0.02823	0.05210	0.03054	0.02692	0.03771	0.03271	0.02913	0.03688	0.03674	0.03059	0.04153	0.03729	0.02813	0.04589	0.02357
		,				0.48640	0.46824	0.49408	0.49076	0.51548	0.50658	0.51370	0.51137	0.47466	0.17431	0.20477	0.19206	0.18623	0.22516	0.22173	0.24767	0.23319	0.24953	0.24660	0.24198	0.27319	0.10293
0.26000	0.25300	0.28000	0.27300	0.30300	0.30600	0.65530	0.61988	0.63106	0.57448	0.59339	0.61197	0.58195	0.59536	0.56016	0.62716	0.69197	0.52293	0.63066	0.67954	0.61573	0.63397	0.66674	0.59887	0.62696	0.67097	0.58407	0.62594
0.74000	0.74700	0.72000	0.72700	0.69700	0.69400	0.00588	0.00903	0.00597	0.01167	0.01432	0.00768	0.01209	0.01280	0.01562	·	·	0.00863		0.00457	0.00366	0.01226	0.00520	0.00472	0.00868	0.00211	0.01704	ı
ı	ı	ı	ı	ı	ı	0.07072	0.09722	0.07010	0.14970	0.07954	0.05988	0.07108	0.04930	0.12168	0.24322	0.15392	0.35017	0.21862	0.13841	0.21688	0.15974	0.13201	0.20642	0.15452	0.10697	0.18775	0.27900
ı	ı	ı	ı	ı	,	0.26810	0.27387	0.29287	0.26415	0.31274	0.32046	0.33488	0.34254	0.30254	0.12962	0.15411	0.11828	0.15072	0.17748	0.16373	0.19403	0.19605	0.18999	0.20984	0.21996	0.21113	0.09506
006	950	1000	1050	1100	1200	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-0.00218	-0.00083	-0.00249	-0.00138	-0.00551	-0.00201	-0.00627	-0.02441	-0.01458	-0.02016	-0.01053	-0.00662	-0.00400	-0.00730	-0.00516	-0.01728	-0.00588	-0.01873	-0.03072	-0.00508	-0.00781	-0.01242	-0.00542	-0.00829	-0.01284	-0.01213	-0.01048	-0.01578
4.59688	4.60599	4.61690	4.62604	4.63517	4.65779	4.1311	4.1220	4.1157	4.1248	4.1104	4.1134	4.1127	4.1068	4.1375	4.2710	4.2616	4.2733	4.2485	4.2511	4.2568	4.2355	4.2446	4.2415	4.2321	4.2288	4.2280	4.3061
8.48400(6)	8.48960(8)	8.49630(7)	8.50190(9)	8.50749(6)	8.52131(11)	8.1872(5)	8.1812(5)	8.1770(40)	8.1830(20)	8.1735(3)	8.1755(4)	8.1750(4)	8.1711(4)	8.1914(7)	8.2786(5)	8.2725(4)	8.2801(5)	8.2640(4)	8.2657(4)	8.2694(4)	8.2556(5)	8.2615(3)	8.2595(2)	8.2534(4)	8.2512(4)	8.2507(4)	8.3012(5)
30	30	30	30	30	30	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
U	U	U	U	U	U	q	q	q	q	q	q	q	q	q	q	q	q	q	q	q	q	q	q	q	q	q	σ

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0.80105	0.81465	0.79930	0.76702	0.76115	0.82778	0.81867	0.81532	0.78491	0.79448	0.77959	0.80031	0.80993	0.78956	0.79197	0.73011	0.74603	0.72661	0.81130	0.78393	0.80918	0.74960	0.79435
0.03885	0.02444	0.03575	0.04966	0.03976	0.03538	0.04378	0.03601	0.05388	0.04404	0.04246	0.06967	0.06744	0.07695	0.07637	0.09587	0.09210	0.10140	0.06712	0.07831	0.07212	0.09124	0.08007
	ı	0.00184	0.00182	0.00679	0.00405	0.00227	0.00693	0.00213	0.00343	0.00745	,		,	ŀ		,	ŀ	,	,	ı		
0.02241	0.02822	0.02812	0.02435	0.03240	0.02696	0.02276	0.02939	0.02498	0.03013	0.03470	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000		
0.13769	0.13269	0.13498	0.15715	0.15990	0.10583	0.11251	0.11234	0.13410	0.12792	0.13580	0.13002	0.12263	0.13349	0.13167	0.17402	0.16187	0.17199	0.12158	0.13776	0.11870	0.15916	0.12558
0.70441	0.58641	0.63992	0.69984	0.62292	0.63768	0.69540	0.62059	0.68653	0.63399	0.59782	0.86066	0.86512	0.84610	0.84726	0.80825	0.81581	0.79720	0.86576	0.84339	0.85575	0.81752	0.83985
	ı	0.00370	0.00279	0.01300	0.00912	0.00424	0.01562	0.00390	0.00747	0.01700	,		,	ı		,	ı	,	,	ı		
0.17306	0.30827	0.23234	0.15214	0.23277	0.23764	0.17151	0.24860	0.15575	0.21766	0.24787	,		,	,		,	,	,	,	ï		
0.12253	0.10532	0.12404	0.14523	0.13131	0.11556	0.12885	0.11518	0.15382	0.14088	0.13731	0.13934	0.13488	0.15390	0.15274	0.19175	0.18419	0.20280	0.13424	0.15661	0.14425	0.18248	0.16015
25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-0.00549	-0.00330	-0.01332	-0.00657	-0.00262	-0.00554	-0.00471	0.00085	-0.00772	-0.00915	-0.00333	-0.00744	-0.00687	0.00007	-0.00390	0.00005	-0.00824	-0.00321	0.00910	0.00598	-0.00030	0.01028	-0.00246
4.2929	4.3081	4.2902	4.2805	4.2923	4.3112	4.3028	4.3170	4.2871	4.2943	4.3011	4.2816	4.2856	4.2857	4.2826	4.2647	4.2622	4.2607	4.3021	4.2896	4.2924	4.2820	4.2854
8.2927(4)	8.3025(4)	8.2910(3)	8.2847(5)	8.2923(5)	8.3045(4)	8.2991(3)	8.3082(2)	8.2890(4)	8.2936(4)	8.2980(4)	8.2854(15)	8.2880(13)	8.2881(8)	8.2861(9)	8.2745(9)	8.2729(10)	8.2719(8)	8.2986(5)	8.2906(8)	8.2924(12)	8.2857(12)	8.2879(13)
7	7	7	7	7	7	7	7	7	7	7	17/12 <sup>e</sup>	17/12 <sup>e</sup>	17/12 <sup>e</sup>	17/12 <sup>e</sup>	$17/12^{e}$	17/12 <sup>e</sup>	$17/12^{e}$	17/12 <sup>e</sup>				
q	q	q	q	q	q	q	q	q	q	q	q	q	q	q	q	q	q	q	q	q	q	q

Note: Reported cation site occupancy reported after projection into the model component space (see text)

- <sup>a</sup> Experiments not listed in the AMCSD
- $^{\mathrm{b}}$  Experiments not listed in AMCSD; full structural refinements not reported
  - $^{\rm c}$  References not included in the AMCSD
- <sup>d</sup> Powder XRD/EMPA studies. Site occupancy determined with MELTS ordering model (Sack and Ghiorso 1991a, b)
  - $^{\circ}$  Compositional data from ref. 17; cell parameter from ref. 12

## **Supplement 3. Appendix**

# DATA ISSUES: ADJUSTMENTS AND EXCLUSIONS

#### High-*T*, room-*P* measurements

We allowed  $\alpha$  to vary for four of the independent end members; there are no high-*T* data for FeCr<sub>2</sub>O<sub>4</sub> or MgCr<sub>2</sub>O<sub>4</sub>. It was important for us to use both data from quenched samples and high-*T* in situ data in order to isolate the separate effects of ordering and thermal expansion, particularly above the blocking temperature where ordering state reaches equilibrium on the timescale of an XRD measurement.

Systematic residuals with temperature seen in early calibrations turned out to be the result of a discrepancy between the in situ high-temperature study of Harrison et al. (1998), which defines  $\alpha$  for FeAl<sub>2</sub>O<sub>4</sub>, and the other room-*T* data for FeAl<sub>2</sub>O<sub>4</sub> (Fig. A1a; all appendix figures are located in Supplementary Material 6; captions are located at the end of this appendix). The dependence of  $s_1$  on *T* in the study of Harrison et al. is quite similar to that of Larsson et al. (1994) (Fig. A1b). So, while the Harrison et al. data could be made consistent with other data for FeAl<sub>2</sub>O<sub>4</sub> either by adjusting their volume or by adjusting their site occupancy, the simplest fix is to adjust all the Harrison et al. volumes by a fixed amount. The value of  $\alpha$  for hercynite is quite insensitive to this adjustment to the Harrison et al. data. As the ordering state of most hercynites is close to that of ideal normal spinel but most of the high-*T* data in the calibration data set are for dependent end members that tend toward inverse ordering states (i.e., MgFe<sub>2</sub>O<sub>4</sub> and Mg<sub>2</sub>TiO<sub>4</sub>), the inconsistency has a detrimental effect on other calibrated values of  $V^{\circ}$  and  $\alpha$ . We made the size of the volume adjustment a dependent parameter in the calibration so that the chosen quantity gave the best-resolved values for  $V^{\circ}$ ,  $\alpha$ , and the coefficient for the variation with  $s_1$ ; unless otherwise stated the Harrison et al. data are shown adjusted by +0.0173 J/bar/mol. Figure A2 shows that the resulting model fits all the Harrison et al. data well except the room-T data point that motivated this correction in the first place. Thus a closer fit for the Harrison et al. data might be achieved by using, for example, a multiplication factor rather than a fixed offset to the volume, but a reasonable value of  $\alpha$ (i.e., consistent with the Harrison et al. study) is retrieved either way. We are required to make a compromise among the data sets in order to develop a better predictive volume model for spinel and emphasize that the choice of a fixed volume adjustment in no way reflects on the quality of the Harrison et al. measurements. Shifting all of the other room-T data for  $FeAl_2O_4$  would result in violations of more constraints than making a slight adjustment to the high-T data. Note that, when lattice parameter and temperature error estimates reported by Harrison et al. are propagated, the resulting uncertainty in volume residual is approximately  $\pm 0.006$  J/bar/mol (assuming the uncertainty in  $s_1(T)$  is similar to the difference between Harrison et al. and Larsson et al.); also model residuals for the raw Harrison et al. data, without the adjustment, are well within the  $\pm 0.02$  J/bar/mol range we consider as acceptable.

While performing the high-*T* fitting, we also discovered opposing trends between in situ high-*T* (Antao et al. 2005a) and annealed and quenched sample measurements (O'Neill et al. 1992) for MgFe<sub>2</sub>O<sub>4</sub>. Comparing the Mg<sup>2+</sup> ordering variable ( $s_0$ ) for the two studies (see Fig. 10), the slope of the equilibrium relationship between  $s_0$  and *T* seems well determined. O'Neill et al. appear to have annealed their samples for a sufficiently long time and quenched them rapidly enough to lock in ordering states down to 450 °C. There is, however, an offset of about 0.05 in  $s_0$  at equal temperature between the two studies. One cannot attribute this to quench effects in the O'Neill et al. study because the quenched samples preserve an ordering state corresponding to a temperature 100 °C *higher* than their annealing temperature (when the equilibrium  $s_0(T)$  is defined from the in situ data of Antao et al.). Rather, we suspect the discrepancy is due to differences in experimental methods for determining the ordering state. To isolate the effects of ordering and thermal expansion in MgFe<sub>2</sub>O<sub>4</sub> using these two data sets we needed to adjust the measurements from one of the studies to account for the systematic offset between them. Even when the data were brought into agreement in this way they appeared to require a T-dependent thermal expansion term. Conversely, no offset exists between the O'Neill et al. data and the Levy et al. (2004) high-T measurements, and using data from the latter instead of Antao et al. yields a smooth thermal expansion trend that can be well fit with a constant value of  $\alpha$  (Figs. A3a, b). We therefore adopted the study of Levy et al. (2004) to define the high-T volume of  $MgFe_2O_4$ , and excluded the study of Antao et al. (2005a). The Antao et al. (2005a) data show systematic differences in  $s_0$ when compared to the MELTS ordering model (Fig. 10), which are reflected in the model residuals for volume. However, volume residuals for the whole Antao et al. (2005a) data set still fall within or very close to the  $\pm 0.02$  J/bar/mol range.

## High-P, room-T measurements

We initially fixed  $K_{oT}$  and K' for all five independent end members (see Table 4):  $K_{oT}$  to values estimated from ultrasonic studies and K' to 4. In this case the high-P MgAl<sub>2</sub>O<sub>4</sub> data of Levy et al. (2003) displayed a concave-up trend in volume residuals versus pressure. This was eliminated by adopting the Levy et al.  $K_{oT}$  and K' for the MgAl<sub>2</sub>O<sub>4</sub> end member. Using a similar argument, we tentatively set  $K_{oT}$  and K' for the Fe<sub>3</sub>O<sub>4</sub> end member to the values from Nakagiri et al. (1986); we also set K' = 5.5 for Fe<sub>2</sub>TiO<sub>4</sub>. Finally, holding  $K_{0T}$  fixed for each end member and K' fixed for MgAl<sub>2</sub>O<sub>4</sub>, Fe<sub>3</sub>O<sub>4</sub>, and  $Fe_2TiO_4$ , optimizing the remaining values of K' suggests that K' = 5.5 is the most reasonable and best fitting value for  $FeAl_2O_4$  (*N.B.*,  $FeAl_2O_4 + MgFe_2O_4 = Fe_3O_4 +$ MgAl<sub>2</sub>O<sub>4</sub>). We excluded the Fe<sub>3</sub>O<sub>4</sub> data of Finger et al. (1986) and the MgFe<sub>2</sub>O<sub>4</sub> data of Antao et al. (2005b) during this process; while the measured  $V/V_0$  ratios are consistent with those from other studies, the absolute volumes are systematically lower by ~0.03 J/bar/mol and ~0.05 J/bar/mol respectively. Residuals for MgFe<sub>2</sub>O<sub>4</sub> do show a systematic increase with P and we investigated the effect of various P-dependent terms on the overall fit. However, as the largest discrepancy for MgFe<sub>2</sub>O<sub>4</sub> at high P(~0.014 J/bar/mol) is well within our final cutoff of 0.02 J/bar/mol, P-dependent terms in the model were eventually rejected. The equation of state of a natural Cr-rich spinel was measured up to 26.8 GPa and 628 K by Fan et al. (2008). We used their data to constrain K' for FeCr<sub>2</sub>O<sub>4</sub>, with the optimum value being K' = 6.5.

The in situ high-P, room-T powder X-ray diffraction study by Haavik et al. (2000) measured the cell parameter of Fe<sub>3</sub>O<sub>4</sub> up to much higher pressures than the high-P,

single-crystal experiments of Gatta et al. (2007), Nakagiri et al. (1986), and Reichmann and Jacobsen (2004). The Haavik et al. data agree with the other studies at low pressure, but begin to deviate above ~5 GPa (Fig. A4). Additionally, the calculated  $K_{oT}$  of Haavik et al. for Fe<sub>3</sub>O<sub>4</sub> is significantly larger than that of the single-crystal X-ray diffraction studies, and of previous ultrasonic results (e.g., Doraiswami 1947 and Bhagavantam 1955). Haavik et al. used N<sub>2</sub> as the pressure medium in their diamond anvil cell experiments, and acknowledge that N<sub>2</sub> transitions to a solid phase at ~2.3 GPa and ambient temperature. This suggests that their experiments suffered from non-hydrostatic conditions with increasing pressure. We consider the perturbation that would be applied to the model by fitting the Haavik et al. data to be unacceptably large.

### Measurements at high-P, T

Upon fitting the high-*P*, *T* experiments, we exposed two additional complications within the data. The in situ neutron diffraction experiments at very high-*T* and moderate-*P* by Méducin et al. (2004) are the only simultaneous high-*P*, *T* measurements of end-member MgAl<sub>2</sub>O<sub>4</sub>. These measurements seem to be discrepant with other studies of this composition, especially at elevated conditions (Finger et al. 1986; Levy et al. 2003; Nestola et al. 2007). Likewise, random and possibly systematic errors in the study of Antao et al. (2005b) for MgFe<sub>2</sub>O<sub>4</sub> render the data unusable for the volume calibration, although the observed variation in volume as a function of *P* and *T* is largely captured by the final model.

The results of Méducin et al. (2004) produce model fits that are markedly different from the other data for the MgAl<sub>2</sub>O<sub>4</sub> end member (Figs. A5a, b). As this is the only simultaneous high-P, T study, it is difficult to understand why the observed behavior differs so much from other in situ experiments performed on this composition (Finger et al. 1986; Levy et al. 2003; Nestola et al. 2007). Gradients in pressure and/or temperature in the Paris-Edinburgh (PE) cell employed in the experiments are a possibility and the authors do attribute the larger errors on the structural parameters at elevated conditions to limitations of the PE cell, e.g., smaller sample volume and lower signal-to-noise ratio. However, the observed trends in ordering state and lattice parameter are relatively smooth, suggesting that the combined effect of elevated P and T on ordering and volume in spinel does indeed deviate from that when high-P or high-T conditions are applied separately. In order to model volumes for these and the other MgAl<sub>2</sub>O<sub>4</sub> data used in calibration, we would need to invoke a large cross-term, involving pressure, temperature, or likely both. Unfortunately, there are not yet enough data at simultaneous high-P, T conditions to calibrate such a term and the large deviation caused by fitting the Méducin et al. study would result in an overall less useful model for the parameter space of interest for peridotite calculations or activity-composition model calibrations. Final model volume residuals for the Méducin et al. data increase gently with P and T, from negligible at ambient conditions to just over the 0.02 J/bar/mol cut off at the most elevated conditions. The exception is the data points gathered at 0.4 GPa, which differ by ~0.04 J/bar/mol from the trend defined by the other data (Fig. A5b). This observation leads us to speculate that these two data points may be discrepant (perhaps because the

MgO pressure marker used in the experiments is less suited to lower pressure conditions) rather than the dataset as a whole.

The only other data for simultaneous high-P and high-T conditions come from the experiments of Antao et al. (2005b) for pure MgFe<sub>2</sub>O<sub>4</sub>. As noted in the previous section, absolute volumes measured in this study are ~0.05 J/bar/mol lower than would be expected from comparison with the other data for this composition at ambient and elevated conditions (O'Neill et al. 1992; Antao et al. 2005a; Levy et al. 2004). As with the Méducin et al. (2004) study previously mentioned, this may be an effect of combined high-P and high-T conditions but there are insufficient data to resolve any model parameters. Due to the complexity of the in situ high-P, T experiments, associated uncertainties must be significantly larger than for measurements at ambient, high-T, or high-P conditions. Antao et al (2005b) do not really quantify these errors and, given the difficulty in estimating measurement errors in general, we have no choice but to exclude the study from the calibration dataset. It appears that the data that Antao et al. (2005b) collected during heating are more reliable than those collected during cooling. Volume residuals for these data show a slight increase with increasing P, similar to that observed for the Levy et al. (2004) study, and a very slight decrease with increasing T. These variations are superimposed on a more or less constant negative misfit so that the typical residual,  $V_{obs} - V_{model}$ , is ~ -0.05 J/bar/mol. As the variation in volume with either P or T is well described, despite the offset, the updated MELTS ordering model (Sack and Ghiorso 1991a, b; this work) successfully predicts the ordering states for most of the data (Fig. 10).

### **CALIBRATION STRATEGY NOTES**

Because different kinds of data with different and poorly known errors are used, we adopted a sequential multistep fitting procedure to avoid serious correlation between standard state and mixing parameters. We first fit the volume of all pure spinel end members (including dependent ones) as functions of *P* and *T*. Then taking that model as a starting point, we next fit all the room-*P* and room-*T* data for spinel solid solutions. In the first stage calibration, we used a nonlinear least squares algorithm and simultaneously fit end-member data for ambient, high-*T*, and high-*P* conditions. We used the three ordering-composition cross-terms described in the text. Two of the dependent end members (MgCr<sub>2</sub>O<sub>4</sub> and MgFe<sub>2</sub>O<sub>4</sub>) show a measurable volume of reaction when formed from the appropriate independent end members. Therefore, for example, the excess volume at MgCr<sub>2</sub>O<sub>4</sub> (where  $X_{sp} = 1$ ,  $X_{hc} = -1$ , and  $X_{ch} = 1$ ) must include a non-zero contribution  $W_{sp-ch} - W_{hc-ch} - W_{sp-hc}$ . Foreshadowing calibration of the intermediate composition data, we chose to fix  $W_{sp-hc}$  to zero and fit the quantities  $W_{sp-ch} - W_{hc-ch}$  and  $W_{mt-hc} - W_{mt-sp}$  using the end-member data.

The  $s_0$ -dependent and  $s_1$ -dependent parameters were fit using end-member data only. However, the modeled ordering effects agree well with volume measurements along the MgAl<sub>2</sub>O<sub>4</sub>-FeAl<sub>2</sub>O<sub>4</sub> join (e.g., the data of Andreozzi and Lucchesi (2002), which are plotted in Fig. A1a as green squares) and validate the  $W_{sp-hc} = 0$  condition imposed during the first stage calibration. As the MgAl<sub>2</sub>O<sub>4</sub>-FeAl<sub>2</sub>O<sub>4</sub> join is common to each of the Cr<sup>3+</sup>-, Fe<sup>3+</sup>-, and Ti<sup>4+</sup>-bearing spinel subsystems this simple result was also convenient. It allowed us to identify those excess parameters that best describe each subsystem separately; these parameters were then fit jointly to the whole solution data set. Our approach is perhaps best illustrated by the MgAl<sub>2</sub>O<sub>4</sub>-FeAl<sub>2</sub>O<sub>4</sub>-MgFe<sub>2</sub>O<sub>4</sub>-Fe<sub>3</sub>O<sub>4</sub> reciprocal square in Fig. 7a where model and data are shown with ideal and ordering contributions subtracted. The convex volume surface in this case requires two regular solution parameters:  $W_{mt-sp}$  and  $W_{mt-hc}$  (Fig. A6). The slope (which is proportional to  $W_{mt-sp} - W_{mt-hc}$ ) and intercept of the ordering adjusted volume variation along the MgFe<sub>2</sub>O<sub>4</sub>-Fe<sub>3</sub>O<sub>4</sub> join are defined by data for the implicit end member MgFe<sub>2</sub>O<sub>4</sub>. The quantity  $W_{mt-sp} + W_{mt-hc}$  (and hence  $W_{mt-hc}$ ) is constrained in the second stage calibration using solution data: for example, the MgAl<sub>2</sub>O<sub>4</sub>-MgFe<sub>2</sub>O<sub>4</sub> binary (Nakatsuka et al. 2004) requires an additional excess volume of ~0.03 J/bar/mol at  $X_{mt} = 0.5$ .

Data coverage in the MgAl<sub>2</sub>O<sub>4</sub>-FeAl<sub>2</sub>O<sub>4</sub>-MgFe<sub>2</sub>O<sub>4</sub>-Fe<sub>3</sub>O<sub>4</sub> reciprocal square is uneven. The data from Mattioli et al. (1987) (MgAl<sub>2</sub>O<sub>4</sub>-Fe<sub>3</sub>O<sub>4</sub>), Golla-Schindler et al. (2005) (FeAl<sub>2</sub>O<sub>4</sub>-Fe<sub>3</sub>O<sub>4</sub>) and Zhao et al. (1998) (MgFe<sub>2</sub>O<sub>4</sub>-Fe<sub>3</sub>O<sub>4</sub>) were not included in the calibration as the site occupancy, in particular the ordering state of Fe<sup>3+</sup>, was not determined. Calibrated data are concentrated along the MgAl<sub>2</sub>O<sub>4</sub>-FeAl<sub>2</sub>O<sub>4</sub> and MgAl<sub>2</sub>O<sub>4</sub>-MgFe<sub>2</sub>O<sub>4</sub> binaries but forward modeling, with  $s_0$ ,  $s_1$ , and  $s_2$  calculated using the MELTS ordering model of Sack and Ghiorso (1991a, b), shows that the volume model matches data on the other joins as well. Zhao et al. (1998) did not anneal the natural spinels they studied but their data agree with the volume model as long as equilibration temperatures were at least 300 °C. In the absence of other data, we conclude that there is no evidence for excess volumes along the MgFe<sub>2</sub>O<sub>4</sub>-Fe<sub>3</sub>O<sub>4</sub> join, other than those due to ordering, and no requirement for symmetric or asymmetric interaction parameters in addition to  $W_{\text{sp-mt}}$  and  $W_{\text{hc-mt}}$ .

 $W_{\rm sp-ch}$  and  $W_{\rm hc-ch}$  were fit in a similar way to  $W_{\rm mt-sp}$  and  $W_{\rm mt-hc}$ , allowing non-ideal interactions between Cr-free and Cr-bearing end members while preserving the more or less linear variation in volume along the FeCr<sub>2</sub>O<sub>4</sub>-MgCr<sub>2</sub>O<sub>4</sub> join (Fig. 6b). In detail there is a slight negative deviation along this join (Lenaz et al. 2004); binary and ternary spinels in the subsystem also show ample evidence for asymmetric ordering-adjusted excess volumes (e.g., Brey et al. 1999). The best fit was obtained by simply allowing the asymmetry along the MgAl<sub>2</sub>O<sub>4</sub>-FeCr<sub>2</sub>O<sub>4</sub> binary with a term of the form: dW<sub>sp-ch</sub> X<sub>sp</sub> X<sub>ch</sub>  $(X_{sp} - X_{ch})$ . Along the FeCr<sub>2</sub>O<sub>4</sub>-MgCr<sub>2</sub>O<sub>4</sub> binary,  $X_{ch} = 1$  and  $X_{sp}$  varies between 0 and 1; therefore a positive value of  $dW_{\rm sp-ch}$  implies a negative volume of mixing between  $FeCr_2O_4$  and  $MgCr_2O_4$ , as is observed. When all data were weighted equally, the value of  $dW_{\rm sp-ch}$  obtained by the least squares algorithm was skewed toward fitting the more MgAl<sub>2</sub>O<sub>4</sub>-rich part of the MgAl<sub>2</sub>O<sub>4</sub>-FeAl<sub>2</sub>O<sub>4</sub>-MgCr<sub>2</sub>O<sub>4</sub>-FeCr<sub>2</sub>O<sub>4</sub> reciprocal square, where data are more densely clustered (refer to Fig. 6b), so that the predicted excess volume along the  $FeCr_2O_4$ -MgCr\_2O\_4 join was overly negative. This problem was solved by applying a weighting factor of 2 to the data of Doroshev et al. (1997), Brey et al. (1999), and Girnis et al. (2003); that is to say, those calibration data that required the MELTS ordering model were assigned a relative standard error twice that of other calibration data. We experimented with weighting factors between 1.5 and 3: the resulting models were very similar and all gave a visually good fit along the FeCr<sub>2</sub>O<sub>4</sub>-MgCr<sub>2</sub>O<sub>4</sub> binary. Chi squared ( $\chi^2$ ) measures of the quality of fit are increased by this choice, but we
consider the resulting model to be better in the sense that it may be more safely extrapolated to high pressures (e.g., in forward MELTS calculations).

Unlike MgFe<sub>2</sub>O<sub>4</sub> and MgCr<sub>2</sub>O<sub>4</sub>, the data indicate little or no volume of reaction for formation of cubic Mg<sub>2</sub>TiO<sub>4</sub> from the independent end members (i.e., Mg<sub>2</sub>TiO<sub>4</sub> +  $2FeAl_2O_4 = Fe_2TiO_4 + 2MgAl_2O_4$ ) which implies that  $W_{sp-uv} \approx W_{he-uv}$ . During the calibration we set both parameters to zero as there was no evidence to suggest otherwise. In particular, we were unable to locate any data near the MgAl<sub>2</sub>O<sub>4</sub>-Fe<sub>2</sub>TiO<sub>4</sub> join itself and, likewise, no data for compositions between Mg<sub>2</sub>TiO<sub>4</sub> and Fe<sub>2</sub>TiO<sub>4</sub>. When the MELTS ordering model (Sack and Ghiorso 1991a, b) is used with the data of Muan et al. (1972) the volume residuals are approximately linear when plotted against  $X_{uv}$ . Along the FeAl<sub>2</sub>O<sub>4</sub>-Fe<sub>2</sub>TiO<sub>4</sub> join the curvature of the volume variation is reproduced closely, but there are significant differences in the measured volume of end member Fe<sub>2</sub>TiO<sub>4</sub>. Measurements along the MgAl<sub>2</sub>O<sub>4</sub>-Mg<sub>2</sub>TiO<sub>4</sub> join are noisier but once again suggest that, within error, the excess volume can be accounted for by ordering effects alone.

Once the Cr-bearing,  $Fe^{3+}$ -bearing, and Ti-bearing spinel volumes were calibrated, we used the available data to investigate the remaining excess volumes of mixing between the three subsystems. Along the Fe<sub>3</sub>O<sub>4</sub>-Fe<sub>2</sub>TiO<sub>4</sub> binary the proportion of Fe<sub>2</sub>TiO<sub>4</sub> controls the value of  $s_2$ . Wechsler et al. (1984) did not characterize the Fe<sup>3+</sup> site occupancy for their samples so initially we assumed that  $s_2 = 0$  (Akimoto 1954). In this case, model residuals for the Wechsler et al. data set show an asymmetric variation with  $X_{uv}$ , seemingly requiring excess volume of mixing terms like those in the current MELTS spinel volume model (Ghiorso 1990; Ghiorso and Sack 1991). However, if  $s_2$  is instead calculated using the MELTS ordering model (Sack and Ghiorso 1991a, b), the volume contribution due to ordering is sufficient such that no excess volume terms, symmetric or asymmetric, are required (see Fig. 8). Inclusion of a  $W_{ch-mt}$  term does improve the overall fit noticeably because many of the natural chromian spinels in the dataset contain significant amounts of Fe<sup>3+</sup> (Lenaz and Princivalle 2005; Lenaz et al. 2007). Lattice parameters for compositions along this binary are available (Robbins et al. 1971; Woodland et al. 2009), however site occupancies were not determined, so we used these data for comparison only. We fit the excess volume on this join with a single symmetric term; the asymmetry in excess volume is described by the *s*<sub>2</sub>-dependent term (see Fig. 9).

Finally, we reexamined the high-*P* and high-*T* datasets to see if any *P* or *T*-dependent compositional terms were justified. Some high-*P* and high-*T* data were not directly calibrated by the model either because the sample used was only nominally pure (e.g., the natural MgAl<sub>2</sub>O<sub>4</sub> crystals used in Carbonin et al. 2002, Finger et al. 1986, and Nestola et al. 2007 all contained small amounts of ZnO) or because they measured the in situ volumes of spinel solid solutions and hence were excluded from both stages of the fitting process (e.g., Martinago et al. 2003; Martinago et al. 2006). The 2nd stage model fits all these data well, without the need to appeal to additional parameters.

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### FIGURE CAPTIONS

**Fig. A1 a** Data along the MgAl<sub>2</sub>O<sub>4</sub> – FeAl<sub>2</sub>O<sub>4</sub> binary. Measured volume vs.  $s_1$ . All data were measured at room-*P*. Circles denote pure FeAl<sub>2</sub>O<sub>4</sub>, diamonds are pure MgAl<sub>2</sub>O<sub>4</sub>, and squares lie on the join between the end members. Grey diamonds are pure MgAl<sub>2</sub>O<sub>4</sub> from various studies in the dataset, measured at room-*P*, *T*; grey squares are binary compositions from various sources; other colors correspond to specific studies called out in the legend. The Harrison et al. (1998) data were measured at high-*T*, in situ; the raw data are plotted, i.e., before adding the volume correction we determined is needed to bring this study into agreement with the rest of the data. Larsson et al. (1994) samples were annealed at a series of temperatures; all others were measured at room-*T*; **b** Order variable  $s_1$  vs. temperature for pure FeAl<sub>2</sub>O<sub>4</sub> data of Harrison et al. (in situ *T*) and Larsson et al. (annealing *T*). The solid line is calculated using the MELTS ordering model for pure FeAl<sub>2</sub>O<sub>4</sub>. Colors are the same as in **a**.

**Fig. A2** Measured volume vs. temperature for the Harrison et al. (1998) (adjusted) and Larsson et al. (1994) data sets (symbols are the same as Fig. A1). The temperature plotted is the in situ *T*, i.e., the Larsson et al. data all plot at 25 °C. Crosses are the modeled volumes for these data. Gray contours show modeled volume for pure FeAl<sub>2</sub>O<sub>4</sub> at constant values of  $s_1$ , printed beneath each line.

**Fig. A3 a** Measured volume vs. temperature for the Levy et al. (2004) (diamonds) and O'Neill et al. (1992) (squares) datasets. The temperature plotted is the in situ *T*, i.e., the

O'Neill et al. data all plot at 25 °C. Crosses are the modeled volumes for these data. Gray contours show modeled volume for pure MgFe<sub>2</sub>O<sub>4</sub> at constant values of  $s_0$ , printed beneath each line; **b** Measured and modeled volumes as a function of  $s_0$  for the O'Neill et al. data set (symbols are the same as **a**) showing excellent agreement in the slope of the two trends.

**Fig. A4** Volume vs. pressure for pure  $Fe_3O_4$  measured at high-*P* in situ. Black line shows model volume at room-*T* for pure  $Fe_3O_4$ .

**Fig. A5 a** Volume vs. pressure for pure MgAl<sub>2</sub>O<sub>4</sub> measured at high-*P* in situ. The data of Méducin et al. (2004) were measured at simultaneous high-*P*, *T* conditions. Black line shows model volume at room-*T* for pure MgAl<sub>2</sub>O<sub>4</sub>; **b** Equilibrium data from Méducin et al. (red diamonds) connected with tie lines to modeled volumes (open diamonds), which lie on lines of model volume at constant *T*, where the temperatures shown are the temperatures at which the measurements were made.

**Fig. A6** Excess model volume in the MgAl<sub>2</sub>O<sub>4</sub>-MgFe<sub>2</sub>O<sub>4</sub>-Fe<sub>3</sub>O<sub>4</sub>-FeAl<sub>2</sub>O<sub>4</sub> reciprocal square. Compare with Fig. 7a.

	1σ <sup>*</sup>	
$V^{o}_{sp}$	0.0010	(J/bar/mol)
$V^{\circ}_{ch}$	1.1	
$V^{o}_{uv}$	0.0072	
$V^{o}_{mt}$	0.00061	
$V^{o}_{hc}$	0.0018	
$W_{hc-ch}$	1.4	
$W_{ ext{ch-mt}}$	0.92	
$W_{ m sp-ch}$	2.5	
$dW_{ m sp-ch}$	1.9	
$W_{\rm mt-hc}$	0.17	
$W_{mt-sp}$	0.17	
$W_{s0}$	0.011	
$W_{s1}$	0.0052	
$W_{s2}$	0.016	
$\alpha_{sp}$	$1.4 \times 10^{-7}$	(K <sup>-1</sup> )
$\alpha_{uv}$	$4.0 \times 10^{-7}$	
$\alpha_{mt}$	$4.7 \times 10^{-7}$	
$\alpha_{hc}$	$1.8 \times 10^{-7}$	

Supplement 4. Unconstrained standard deviation of fit parameters

\*Bootstrap estimation of s.d. for each parameter, varying all other fit parameters

		$W_{hc-ch}$	$W_{\rm ch-mt}$	$W_{\rm sp-ch}$	$W_{\rm mt-hc}$	W <sub>mt-sp</sub>	$dW_{\rm sp-ch}$	W <sub>s0</sub>	$W_{s1}$	$W_{s_2}$	$V_{\rm ch}^{\rm o}$	$V_{\rm hc}^{\circ}$	$V_{\rm mt}$	$V_{\rm sp}^{\rm o}$	$V_{uv}^{o}$	$\alpha_{hc}$	$\alpha_{\text{mt}}$	$\alpha_{sp}$	α
$W_{m,m}$ 0.681128         1.000000         0.776688         0.209311         0.673564         0.006219         0.005660         0.773388         0.001929         0.000138         0.0002312         0.000131         0.001131         0.000131         0.001131         0.000131         0.001131         0.000131         0.001131         0.000131         0.000131         0.000131         0.000131         0.000131         0.000131         0.000131         0.000131         0.000131         0.000131         0.001131         0.000131         0.0001311         0.0001311         0.0001311	$W_{hc-ch}$	1.000000	0.681128	0.999204	-0.142064	-0.142152	-0.998338	0.000293	0.001864	-0.004929	-0.995713	0.002072	0.000969	0.005313	-0.000843	-0.000828	0.008894	-0.006624	0.005691
$W_{\rm spin}$ 0.399204         0.706088         1.000000         0.141474         0.141562         0.997329         0.000125         0.001833         0.001281         0.000283         0.0001291         0.0001291         0.0001291         0.0001294         0.0001291         0.0001294         0.0001294         0.0001294         0.000131         0.0001294         0.0001294         0.0001294         0.000131         0.0001391         0.0001311         0.0001391         0.0001391         0.0001391         0.0001391         0.0001391         0.0001391         0.0001391         0.0001391         0.0001391         0.0001391         0.0001391         0.0001391         0.00013191         0.0001391         0.000	$W_{ch-mt}$	0.681128	1.000000	0.706088	-0.209011	-0.209431	-0.673564	-0.006219	0.005461	-0.006690	-0.737388	0.006082	0.001929	-0.001385	-0.005982	-0.002212	0.012528	-0.002529	0.000233
	$W_{\rm sp-ch}$	0.999204	0.706088	1.000000	-0.141474	-0.141562	-0.997329	0.000125	0.001507	-0.005023	-0.998608	0.001853	0.001042	0.004028	-0.000812	-0.001203	0.009003	-0.005996	0.004968
$W_{m, q_0}$ $-0.14152$ $0.209731$ $0.14152$ $0.20833$ $0.01254$ $0.001054$ $0.001054$ $0.001054$ $0.001054$ $0.001054$ $0.001054$ $0.001054$ $0.001054$ $0.001054$ $0.0013554$ $0.0013554$ $0.0013554$ $0.0013554$ $0.0013554$ $0.0013554$ $0.0013554$ $0.0013564$ $0.0013264$ $0.0013264$ $0.0013264$ $0.0013264$ $0.0013264$ $0.0013264$ $0.0013264$ $0.0013264$ $0.0013264$ $0.0012364$ $0.0012364$ $0.0012364$ $0.0012364$ $0.0012364$ $0.0012364$ $0.0012364$ $0.0012364$ $0.0012364$ $0.0012364$ $0.0012364$ $0.0012364$ $0.0012364$ $0.0012364$ $0.0012364$ $0.0012364$ $0.0012364$ $0.0012364$ $0.00123$	$W_{\rm mt-hc}$	-0.142064	-0.209011	-0.141474	1.000000	0.997770	0.132552	-0.038691	0.038024	0.062472	0.140418	0.034498	0.001181	-0.009634	-0.038440	-0.000141	-0.025189	-0.000336	0.006521
	$W_{\rm mt-sp}$	-0.142152	-0.209431	-0.141562	0.997770	1.000000	0.132413	0.015804	-0.012616	0.008086	0.140414	-0.013168	-0.001296	0.004717	0.015404	-0.001064	-0.007038	-0.001784	-0.000736
$W_{u}$ 0.000233         0.006219         0.00126         0.0138691         0.013804         0.003380         0.003249         0.00128         0.001248         0.023756         0.002269         0.002269         0.002269         0.002269         0.002269         0.002269         0.002269         0.002269         0.002269         0.002269         0.002269         0.002269         0.002269         0.002269         0.0012192         0.002269         0.002269         0.002269         0.0012192         0.0012192         0.0012192         0.0012192         0.001239         0.0012169         0.0012169         0.0012169         0.0012169         0.0012169         0.0012369         0.0012369         0.0012369         0.0012369         0.0012369         0.0012369         0.001239         0.001239         0.0012192         0.001239         0.001239         0.001239         0.001239         0.0012369         0.0012369         0.0012369         0.0012369         0.0012369         0.001239         0.001239         0.001239         0.001239         0.001239         0.001239         0.0012349         0.0012349         0.0012349         0.0013349         0.0013349         0.0013349         0.0013349         0.0013349         0.0013349         0.0002349         0.0002349         0.00013496         0.0002349         0.0002349 <th><math>dW_{\rm sp-ch}</math></th> <th>-0.998338</th> <th>-0.673564</th> <th>-0.997329</th> <th>0.132552</th> <th>0.132413</th> <th>1.000000</th> <th>-0.003829</th> <th>0.002309</th> <th>0.006394</th> <th>0.993561</th> <th>0.001833</th> <th>-0.001092</th> <th>-0.004068</th> <th>-0.002853</th> <th>0.001354</th> <th>-0.008267</th> <th>0.005549</th> <th>-0.004335</th>	$dW_{\rm sp-ch}$	-0.998338	-0.673564	-0.997329	0.132552	0.132413	1.000000	-0.003829	0.002309	0.006394	0.993561	0.001833	-0.001092	-0.004068	-0.002853	0.001354	-0.008267	0.005549	-0.004335
$W_{\rm u}$ 0.001864         0.005561         0.001267         0.003802         0.0012192         0.0012192         0.0012192         0.0012192         0.0012192         0.0012192         0.0012192         0.0012192         0.0012192         0.0012192         0.0012192         0.0012192         0.0012192         0.0012195         0.0012195         0.0012195         0.0012195         0.0012195         0.0012195         0.0012195         0.0012195         0.0012195         0.001381         <	Wso	0.000293	-0.006219	0.000125	-0.038691	0.015804	-0.003829	1.000000	-0.921771	-0.352496	-0.001628	-0.877785	0.013443	0.287766	0.979551	0.002269	0.011201	-0.053375	-0.103861
$ \begin{array}{llllllllllllllllllllllllllllllllllll$	$W_{s1}$	0.001864	0.005461	0.001507	0.038024	-0.012616	0.002309	-0.921771	1.000000	0.316223	0.000467	0.957363	-0.008815	0.091576	-0.932750	-0.012192	0.028469	-0.069653	0.155578
$V_{\rm h}^{\rm ch} = -0.95713 - 0.737388 - 0.998608 - 0.140414 - 0.993561 - 0.001628 - 0.001647 - 0.005776 - 1.000000 - 0.000005 - 0.001169 - 0.003115 - 0.00813 - 0.001336 - 0.00134 - 0.002712 - 0.002072 - 0.002072 - 0.002072 - 0.001232 - 0.001232 - 0.001232 - 0.001232 - 0.001232 - 0.001232 - 0.002501 - 0.005701 - 0.005$	Ws2	-0.004929	-0.006690	-0.005023	0.062472	0.008086	0.006394	-0.352496	0.316223	1.000000	0.005776	0.312230	-0.019166	-0.126589	-0.336569	-0.036115	-0.552354	0.025122	0.005455
$V_{m}^{\rm k} = 0.002072  0.006282  0.001853  0.034498  -0.013168  0.001833  -0.877785  0.957363  0.312230  0.000005  1.000000  -0.012332  0.092235  -0.846647  -0.220755  -0.01169  -0.011332  -0.002361  -0.002501  -0.01231  -0.002486  -0.002501  -0.005071  -0.005051  -0.0050510  -0.0050510  -0.0050510  -0.0050510  -0.0050510  -0.0050510  -0.0000736  -0.000736  -0.000736  -0.000736  -0.000736  -0.0002510  -0.000254  -0.0002550  -0.0002503  -0.0005950  -0.0050701  -0.0050701  -0.0050500  -0.0050500  -0.0050500  -0.0050500  -0.0050500  -0.0050500  -0.0050500  -0.000000  -0.000000  -0.000000  -0.000000  -0.000000  -0.000000  -0.000000  -0.000000  -0.000000  -0.000000  -0.0000000  -0.0000000  -0.000000  -0.000000  -0.0000000  -0.0000000  -0$	ر م	-0.995713	-0.737388	-0.998608	0.140418	0.140414	0.993561	-0.001628	0.000467	0.005776	1.000000	0.000005	-0.001169	-0.003115	-0.000813	0.001336	-0.009354	0.005493	-0.004227
$V_{\rm m}^{\rm m} = 0.000669 = 0.001292 = 0.001181 - 0.001296 - 0.001296 - 0.001296 - 0.001815 - 0.012166 - 0.001169 - 0.012332 1.000000 0.013508 0.009486 0.002501 0.0 0.0 0.00000 0.013532 0.00000 0.013563 0.0002401 - 0.0 0.00000 0.013563 0.00000 0.013753 0.00000 0.013753 0.00000 0.013753 0.00000 0.013753 0.00000 0.013753 0.00000 0.013753 0.000000 0.010000 0.000000 0.0000000 0.0000000 0.000000$	ر ۳	0.002072	0.006082	0.001853	0.034498	-0.013168	0.001833	-0.877785	0.957363	0.312230	0.000005	1.000000	-0.012332	0.092235	-0.846647	-0.220755	-0.060582	-0.047895	-0.020523
$ V_{\rm u}^{\rm u} = 0.005313 - 0.001385 - 0.004028 - 0.004717 - 0.004068 - 0.287766 - 0.091576 - 0.126589 - 0.003115 - 0.092235 - 0.013508 - 1.000000 - 0.187638 - 0.006207 - 0.187638 - 0.005207 - 0.126589 - 0.00313 - 0.02523 - 0.005312 - 0.02523 - 0.005312 - 0.02551 - 0.02551 - 0.02551 - 0.02551 - 0.02551 - 0.02551 - 0.02551 - 0.02551 - 0.02551 - 0.02551 - 0.02551 - 0.02551 - 0.00313 - 0.84647 - 0.009486 - 0.187638 - 1.000000 - 0.397614 - 0.0 - 0.0265 - 0.000813 - 0.005212 - 0.005217 - 0.009514 - 0.0 - 0.00000 - 0.097614 - 0.0 - 0.000000 - 0.000000 - 0.000000 - 0.000000 - 0.000000 - 0.000000 - 0.000000 - 0.000000 - 0.000000 - 0.000000 - 0.000000 - 0.000000 - 0.00000 - 0.00000 - 0.000000 $	$\sqrt[n]{m}$	0.000969	0.001929	0.001042	0.001181	-0.001296	-0.001092	0.013443	-0.008815	-0.019166	-0.001169	-0.012332	1.000000	0.013508	0.009486	0.002501	0.006797	-0.002753	0.004579
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$ \alpha_{\rm hc} = -0.000228 - 0.002212 - 0.001203 - 0.000141 - 0.001064 - 0.001354 - 0.002269 - 0.012192 - 0.0036115 - 0.001336 - 0.220755 - 0.006207 - 0.097614 - 1.000000 - 0.3 \\  \omega_{\rm mt} = -0.008894 - 0.012528 - 0.002003 - 0.025189 - 0.007038 - 0.008267 - 0.011201 - 0.028469 - 0.552354 - 0.000354 - 0.066797 - 0.143395 - 0.052469 - 0.384253 - 1.0 \\  \omega_{\rm p} = -0.006624 - 0.002529 - 0.000336 - 0.001784 - 0.0053375 - 0.069653 - 0.025122 - 0.0005493 - 0.047895 - 0.44395 - 0.052469 - 0.384253 - 1.0 \\  \omega_{\rm p} = -0.006624 - 0.002529 - 0.000336 - 0.001784 - 0.0053375 - 0.069653 - 0.025122 - 0.006249 - 0.002753 - 0.454325 - 0.025006 - 0.053184 - 0.2 \\  \omega_{\rm m} = -0.005691 - 0.000233 - 0.006796 - 0.000736 - 0.00336 - 0.00336 - 0.00336 - 0.00336 - 0.00336 - 0.004335 - 0.005451 - 0.005455 - 0.004227 - 0.002523 - 0.47895 - 0.223374 - 0.220766 - 0.007334 - 0.2 \\  \omega_{\rm m} = -0.005691 - 0.000233 - 0.006521 - 0.000736 - 0.00336 - 0.00336 - 0.00336 - 0.00336 - 0.004335 - 0.004257 - 0.004257 - 0.004257 - 0.002523 - 0.04579 - 0.243304 - 0.74644 - 0.4 \\  \omega_{\rm m} = -0.005691 - 0.000233 - 0.006551 - 0.000736 - 0.00336 - 0.00336 - 0.00336 - 0.004335 - 0.004257 - 0.004257 - 0.002523 - 0.004579 - 0.212560 - 0.243304 - 0.74644 - 0.4 \\  \omega_{\rm m} = -0.005691 - 0.000238 - 0.000736 - 0.000736 - 0.00336 - 0.00336 - 0.004335 - 0.004257 - 0.004227 - 0.020523 - 0.004579 - 0.212560 - 0.243304 - 0.774644 - 0.4 \\  \omega_{\rm m} = -0.004250 - 0.000234 - 0.000736 - 0.000736 - 0.00336 - 0.00336 - 0.005455 - 0.004227 - 0.002523 - 0.004579 - 0.243304 - 0.774644 - 0.4 \\  \omega_{\rm m} = -0.004250 - 0.000730 - 0.000736 - 0.000736 - 0.000736 - 0.000736 - 0.000736 - 0.000736 - 0.000736 - 0.00427 - 0.000223 - 0.004579 - 0.004579 - 0.004579 - 0.004579 - 0.004570 - 0.004570 - 0.000730 - 0.00464 - 0.4 \\  \omega_{\rm m} = -0.0000000000000000000000000000000000$	∿ ∾	-0.000843	-0.005982	-0.000812	-0.038440	0.015404	-0.002853	0.979551	-0.932750	-0.336569	-0.000813	-0.846647	0.009486	0.187638	1.000000	-0.097614	-0.052469	0.025006	-0.243304
$ \alpha_{\rm m} = 0.008894  0.012528  0.009003  -0.025189  -0.007038  -0.007038  -0.008267  0.011201  0.028469  -0.552354  -0.000354  -0.006582  0.006797  0.144395  -0.052469  0.384253  1.0 384253  -0.0066624  -0.002529  -0.006366  -0.007184  0.005549  -0.069653  0.025122  0.005493  -0.007753  -0.47895  -0.0257066  -0.025184  -0.25006  -0.0073184  -0.20753  -0.007753  -0.047895  -0.02753  -0.047895  -0.0253184  -0.2070336  -0.007336  -0.007336  -0.007336  -0.007338  -0.059663  -0.073664  -0.0734644  0.4 3606363  -0.006251  -0.000736  -0.000736  -0.000736  -0.000736  -0.000736  -0.000736  -0.007356  -0.000736  -0.007357  -0.005455  -0.004227  -0.0002523  0.004579  0.212560  -0.243304  -0.746644  0.4 3006456  -0.000736 $	$\alpha_{hc}$	-0.000828	-0.002212	-0.001203	-0.000141	-0.001064	0.001354	0.002269	-0.012192	-0.036115	0.001336	-0.220755	0.002501	-0.006207	-0.097614	1.000000	0.384253	-0.053184	0.774644
$\alpha_{\mu}$ -0.006624 -0.002759 -0.005996 -0.000336 -0.001784 0.005549 -0.053375 -0.069653 0.025122 0.005493 -0.047895 -0.002753 -0.454325 0.025006 -0.053184 -0.2 $\alpha_{\mu}$ 0.005691 0.000233 0.004568 0.006521 -0.000736 -0.074644 0.4 $\alpha_{\mu}$	$\alpha_{\rm mt}$	0.008894	0.012528	0.009003	-0.025189	-0.007038	-0.008267	0.011201	0.028469	-0.552354	-0.009354	-0.060582	0.006797	0.144395	-0.052469	0.384253	1.000000	-0.269828	0.435996
$\alpha_{m}$ 0.005691 0.000233 0.004568 0.006521 -0.000736 -0.004335 -0.103861 0.155578 0.005455 -0.004227 -0.020523 0.004579 0.212560 -0.243304 0.774644 0.4	$\alpha_{sp}$	-0.006624	-0.002529	-0.005996	-0.000336	-0.001784	0.005549	-0.053375	-0.069653	0.025122	0.005493	-0.047895	-0.002753	-0.454325	0.025006	-0.053184	-0.269828	1.000000	-0.586005
	α	0.005691	0.000233	0.004968	0.006521	-0.000736	-0.004335	-0.103861	0.155578	0.005455	-0.004227	-0.020523	0.004579	0.212560	-0.243304	0.774644	0.435996	-0.586005	1.000000

**Supplement 6. Appendix Figures** 



















Chapter 2

# THE MOLAR VOLUME OF CUBIC GARNETS IN THE SYSTEM SiO<sub>2</sub>-Al<sub>2</sub>O<sub>3</sub>-TiO<sub>2</sub>-Fe<sub>2</sub>O<sub>3</sub>-Cr<sub>2</sub>O<sub>3</sub>-FeO-MnO-MgO-CaO-Na<sub>2</sub>O

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## ABSTRACT

We define and calibrate a new model of molar volume as a function of pressure, temperature, and composition for cubic garnets (space group *Ia3d*) in the oxide system listed in the title. We use 766 X-ray diffraction measurements performed on garnets at ambient and in situ high-P, T conditions to calibrate end-member equations of state and an excess volume model for this system. Optimal values of the bulk modulus and its pressure derivative are obtained by analyzing published compression and/or ultrasonic data for the end members for which such studies exist; for other end members, density functional theory results and systematics in cation radii are used. For any cubic garnet in this chemical system, the model molar volume is obtained by adding excess volume terms to a linear combination of nine independent end-member volumes. In the first step of our least squares fitting procedure we calculate volumes of the explicit end members as a function of P and T using the Vinet equation of state, with the Mie-Grüneisen-Debye thermal pressure formalism to model thermal expansion. For each dependent end member for which there are data, we calculate the volume of reaction for formation of the phase from the independent end members,  $\Delta V$ . We then fit the binary and mixed composition data, using singular value analysis to ensure that the calibrated combinations of excess parameters obey the  $\Delta V$  constraints from the first step. The preferred model has 19 excess volume parameters and fits nearly all experiments to within 0.06 J/bar/mol, or approximately 0.5% in volume. The implications our model has for the density of the lithospheric mantle are explored.

## **INTRODUCTION**

Garnet-group minerals are important constituents of upper mantle and high-grade metamorphic crustal rocks. Garnets can accommodate a wide variety of cations, and form extensive solid solutions between end members. Garnets are also stable over a wide pressure range. These qualities, along with their abundance in the Earth's upper mantle, make them excellent indicators of the *P*- and *T*-conditions of crystallization. Because of this, the properties of garnets have been studied exhaustively (see Geiger 2008 for a recent review). However, there remains vast disagreement between different measurements of molar volume of the various garnet end members and their solid solutions.

Garnets have the general formula  $X_3Y_2Z_3O_{12}$ . Cations denoted by X are coordinated by eight nearest-neighbor oxygen atoms (dodecahedral coordination). The Y cations are octahedrally coordinated (six), and the Z cations are tetrahedrally coordinated (four) by oxygen. In cubic garnets (*Ia3d*), all sites of a given coordination are symmetrically equivalent, and the structure can be described as chains of octahedrally coordinated cations linked by non-sharing dodecahedral and tetrahedral polyhedra (Deer et al. 1997). Garnet is a critical phase that controls major and trace element partitioning at pressures above ~3 GPa during partial melting of the upper mantle. Any useful thermodynamic model of mantle phase equilibria must include a treatment of garnet that affords accurate prediction of garnet stability with respect to spinel, pyroxene, and melt.

Molar volume is an important thermodynamic quantity at high pressures, both when using garnets to infer petrogenetic information from high-pressure rocks and particularly when using garnets from high-pressure experiments to define chemical potentials in coexisting phases. A model of garnet volumes with the necessary accuracy needs to account not only for equations of state of pure end members, but also to take into account considerable deviation from ideal mixing of compositions along some binary joins. Generally, studies are restricted to subsystems of garnets, e.g., along a solid solution binary. While limiting the system of interest usually allows one to recover the data used in calibration, discrepancies that exist between various end-member and solid solution data sets and currently available models are inadequate for modeling of volumes over the full compositional range of garnets formed in the Earth's upper mantle. This is especially true for pressures above ~15 GPa, where majoritic garnet becomes a major constituent of the mantle phase assemblage. It is necessary to devise a comprehensive model applicable to the entire chemical system of the upper mantle.

In this work, we present a model of molar volumes of stoichiometric garnets containing the oxide components SiO<sub>2</sub>-Al<sub>2</sub>O<sub>3</sub>-TiO<sub>2</sub>-Fe<sub>2</sub>O<sub>3</sub>-Cr<sub>2</sub>O<sub>3</sub>-FeO-MnO-MgO-CaO-Na<sub>2</sub>O. Our model system contains nine independent end members. The end-member compositions, designations, and abbreviations used in this paper are given in Table 1. Our primary motivation in this work is to develop a comprehensive garnet molar volume model for use in calibration of activity-composition models of garnet and pyroxene solid solutions. The thermodynamic models, along with a new silicate liquid equation of state (Ghiorso 2004a, 2004b, 2004c; Ghiorso and Kress 2004) and a new model of molar volume for spinel (Hamecher et al. 2013), will be incorporated into the next generation MELTS (Ghiorso and Sack 1995; Ghiorso et al. 2002; Asimow et al. 2004) model, xMELTS (Ghiorso et al. 2007). The new solid solution models will include some minor components, including Ti, Na, and Cr. The new garnet and spinel molar volume models will enable coupled recalibration of the garnet and pyroxene thermodynamic models.

In this chapter, we first discuss previous determinations of garnet end-member properties and previous models of garnet molar volume, with attention to the ranges of compositions they cover and inconsistencies among data and models. We then present the X-ray diffraction data used in our calibration, and the formulation of our model in terms of the components in Table 1. The calibration strategy we used to estimate the parameters of the model and assess goodness of fit to the data is outlined. Finally, we compare the final model to some models from the literature and present a test of the effect of our new volume models on density of mantle rocks extracted from three different thermodynamic models.

## PREVIOUS WORK ON GARNET VOLUME

A considerable amount of work has been done in deriving the physical properties of endmember garnets, mostly among the aluminosilicate garnets. Comparatively fewer studies have examined garnets containing Cr, Ti, Na, or a majoritic component. However, even in the heavily sampled regions of garnet composition space, the various published determinations of standard state properties for end members are inconsistent with one another. Considering not only end-member properties but also mixing in solid solutions, numerous models for the molar volume of garnet solutions have been proposed, but most are restricted to single compositional binaries, and there are many discrepancies among the models that do exist. In this section we discuss previous determinations of end-
member volumetric properties and examples of previous volume of mixing models for garnets.

Of the end-member garnets, the aluminosilicate garnets (grossular, pyrope, almandine, and spessartine) and their solid solutions have received by far the most attention. There have been numerous determinations of the cell parameter, *a*, for these garnets over the past 50+ years, but still uncertainty persists regarding their correct values (e.g., Abrahams and Geller 1958; Boyd and England 1959; Gibbs and Smith 1965; Novak and Gibbs 1971; Geiger et al. 1992; Geiger and Armbruster 1997). Thermal expansion for the aluminosilicate garnets and their solid solutions have been determined by Skinner (1956), Meagher (1975), Geiger et al. (1992), Pavese et al. (1995), Artioli et al. (1997), Thieblot et al. (1998), and Rodehorst et al. (2002).

The bulk moduli ( $K_{oT}$ ) for the aluminosilicate garnets have been determined by ultrasonic, Brillouin spectroscopic, and static compression X-ray diffraction methods. There is much disagreement between values determined by the different methods (e.g., Takahashi and Liu 1970; Weaver et al. 1976; Bonczar et al. 1977; Babuska et al. 1978; Sato et al. 1978; Levien et al. 1979; Leitner et al. 1980; Bass 1989; Webb 1989; Leger et al. 1990; Olijnyk et al. 1991; Hazen et al. 1994; Zhang et al. 1998; 1999). We discuss our strategy to constrain the bulk moduli, and their pressure derivatives, in the Model Calibration section below.

There is much less data, if any, available for the rest of our chosen independent set of end members. There are published lattice parameters for garnets synthesized along the pyrope-knorringite binary (Ringwood 1977). Information about the compressibility of uvarovite is reported in Bass (1986) and Leger et al. (1990). Data for Fe<sup>3+</sup>-bearing garnets

are available from Bass (1986), Hazen and Finger (1989), Armbruster and Geiger (1993), Woodland and O'Neill (1993), Woodland and Ross (1994), Zhang et al. (1999), Jiang et al. (2004), Pavese et al. (2001), and Woodland et al. (2009). Ringwood and Major (1971) synthesized and measured the lattice parameters of some high-pressure Tiand Na-bearing garnets, as well as a synthetic sample of cubic majorite. Armbruster et al. (1998) determined cell parameters for natural Ti-bearing andradites. Properties of cubic majorite garnets are given in Bass and Kanzaki (1990), Yeganeh-Haeri et al. (1990), Pacalo et al. (1992), Hazen et al. (1994), and Wang et al. (1998).

The current molar volume model for garnets in MELTS and pMELTS is applicable only in the ternary grossular-pyrope-almandine system. The model adopts the Berman (1990) symmetric excess volume model along the grossular-pyrope join and asymmetric model for the grossular-almandine and pyrope-almandine binaries. Terms involving spessartine are not included in the current MELTS models. Geiger (2000) also models the Ca-Mg-Fe-Mn aluminosilicate system. Pyrope-almandine is modeled with ideal volume of mixing, and all other binaries are modeled with slightly positive, symmetric excess volume parameters. Bosenick et al. (2001; see table 1) fit experimental data along all six binaries in this garnet subsystem with positive symmetric volume mixing terms. However, there is little agreement in the literature on the excess volume along the grossular-pyrope join (e.g., Newton et al. 1977; Haselton and Newton 1980; Wood 1988; Koziol and Newton 1989; Berman 1990; Berman and Aranovich 1996; Du 2012). Bosenick et al. (2001) fit experimental data on this binary with a symmetric excess volume term, though the original authors (Ganguly et al. 1993; Bosenick and Geiger 1997) fit their respective data with asymmetric terms. Figure 1 shows the various models

that have been proposed for the grossular-pyrope binary. The asymmetric models agree in the sense that the larger excess volumes exist at the grossular-rich end of the binary, but the magnitudes of the excess volume models are quite different. Du (2012) suggests that the large variability in shape and magnitude of excess volume at room temperature along the grossular-pyrope join could be due to differences in the way in which the samples were synthesized (e.g., piston cylinder versus multi-anvil).

Ungaretti et al. (1995) model excess volume of ternary grossular-pyropealmandine garnets, using data along the grossular-pyrope (discussed in previous paragraph), grossular-almandine (Cressey et al. 1978; Perkins 1979; Geiger et al. 1987; Koziol 1990), and almandine-pyrope (Armbruster et al. 1992; Geiger and Feenstra 1997) binary joins. Mukhopadhyay et al. (1997) also modeled mixing properties in the Ca-Mg-Fe<sup>2+</sup> ternary using the data of Newton et al. (1977), Wood (1988), Ganguly et al. (1993), Hackler and Wood (1989), and Koziol (1990), but excluded the data of Cressey et al. (1978), based on the observation by Berman (1990), who also modeled this ternary system, that the other data show no evidence for negative volume of mixing along the grossular-almandine join. Other binary models exist for grossular-almandine (Berman 1990; Geiger et al. 1989), pyrope-almandine (Berman 1990), spessartine-grossular (Rodehorst et al. 2002), spessartine-almandine (Geiger and Feenstra 1997), almandineskiagite, and andradite-skiagite (Woodland and Ross 1994).

# DATA SOURCES

The sources of garnet volume data used in model calibration are given in Supplementary Material 1. The American Mineralogist Crystal Structure Database (AMCSD) (Downs and Hall-Wallace 2003) provides a collection of published X-ray diffraction refinements of cell volume and site occupancy; we fit all included garnets with *Ia3d* space group symmetry in the system SiO<sub>2</sub>-Al<sub>2</sub>O<sub>3</sub>-TiO<sub>2</sub>-Fe<sub>2</sub>O<sub>3</sub>-Cr<sub>2</sub>O<sub>3</sub>-FeO-MnO-MgO-CaO-Na<sub>2</sub>O. Additionally, we culled volume determinations for applicable garnet compositions from the literature. Other garnets in the AMCSD and other sources—e.g., hydrogarnets, tetragonal garnets, garnets with cations other than Si in the tetrahedral site—were excluded.

Our final model is calibrated using a total of 766 experiments. The compositional and *P*-*T* coverage of the data is summarized in Table 2. Experimental conditions, observed cell parameters, and composition are given in Supplementary Material 2. The majority of the solid solution data is clustered in the grossular-pyrope-almandine ternary system.

To limit the data used to fit the model to those garnets that are most applicable to the upper mantle, we developed a set of exclusion criteria. All garnets that contained vacancies were excluded. We rejected experiments that contained greater than 5% occupancy by  $Mn^{3+}$  and greater than 0.5% occupancy of  $Sc^{3+}$  on any crystallographic site. In order that the composition of all calibrated garnets conform to the stoichiometry of the chosen model end members, within a reasonable tolerance, we filtered out any site occupancy data with reported site totals differing from unity by more than  $\pm 0.01$  and any data with total cation charge greater than +24.08 or less than +23.92 per formula unit.

One downside to the chosen data set is lack of reliable error estimates for observed volume and composition, which makes it difficult to weight the calibration data in any meaningful way. While many of the unit cell determinations do include error bounds, there is no way to estimate errors in site occupancy, particularly since many of the garnet compositions reported this way are assumed to have ideal stoichiometry. Only a subset of the compositions determined by electron microprobe analysis includes errors. Given that it was impossible to come up with an automated strategy for assigning errors, we used unweighted nonlinear least squares, where all data were assigned the same nominal error.

# **MODEL FORMULATION**

For fitting molar volume data to our model, the data must first be recast in a consistent manner into the representation of composition we adopted. The compositions of the data are reported in one of two ways: (a) cation mole fraction on each of the crystallographic sites or (b) weight percent of the oxides. For site occupancy data, we first transformed the molar cation proportions into the following set of linearly independent variables:

$$r_{0} = {}^{[6]}\text{Al} - {}^{[8]}\text{Ca} - {}^{[8]}\text{Fe}^{2+} - \left(\frac{2}{3}\right)^{[6]}\text{Fe}^{2+} - {}^{[8]}\text{Mn} - \left(\frac{2}{3}\right)^{[6]}\text{Mn}$$
(1)

$$r_1 = {}^{[8]} \mathrm{Fe}^{2+} + \left(\frac{2}{3}\right)^{[6]} \mathrm{Fe}^{2+}$$
(2)

$$r_2 = {}^{[6]}\mathrm{Cr} \tag{3}$$

$$r_3 = 2^{[6]} \mathrm{Si} - 3^{[8]} \mathrm{Na} \tag{4}$$

$$r_4 = {}^{[6]} \mathrm{Fe}^{3+} \tag{5}$$

$$r_5 = \left(\frac{3}{2}\right)^{[8]} \mathrm{Na} \tag{6}$$

$$r_6 = 2^{[6]} \text{Ti}$$
 (7)

$$r_7 = {}^{[8]}Mn + \left(\frac{2}{3}\right){}^{[6]}Mn , \qquad (8)$$

where superscript [8] indicates the dodecahedrally coordinated cation and [6] indicates the octahedrally coordinated cation. Then, we obtained the set of end-member component molar proportions by:

$$X_{\rm grs} = 1 - r_0 - r_1 - r_2 - r_3 - r_4 - r_5 - r_6 - r_7 \tag{9}$$

$$X_{\rm pyr} = r_0 \tag{10}$$

$$X_{\rm alm} = r_{\rm l} \tag{11}$$

$$X_{\rm knr} = r_2 \tag{12}$$

$$X_{\rm maj} = r_3 \tag{13}$$

$$X_{\rm kho} = r_4 \tag{14}$$

$$X_{\rm nag} = r_5 \tag{15}$$

$$X_{\rm mmr} = r_6 \tag{16}$$

$$X_{\rm sps} = r_7 \tag{17}$$

For compositional data reported in oxide weight percent, we first calculated the number of moles of cations per formula unit, then calculated moles of end-member components (given here by m):

$$m_{\rm grs} = \frac{\rm mols \ Ca}{3}$$
(18)

$$m_{\rm pyr} = \frac{\rm mols \ Al}{2} - \frac{\rm mols \ Ca}{3} - \frac{\rm mols \ Fe^{2+}}{3} - \frac{\rm mols \ Mn}{3}$$
(19)

$$m_{\rm alm} = \frac{\rm mols \ Fe^{2+}}{3}$$
(20)

$$m_{\rm knr} = \frac{\rm mols \ Cr}{2} \tag{21}$$

$$m_{\rm maj} = \frac{\text{mols Mg}}{4} - 3\frac{\text{mols Fe}^{3+}}{8} - \text{mols Ti} - 3\frac{\text{mols Cr}}{8} - \frac{\text{mols Na}}{8} - 3\frac{\text{mols Al}}{8} + \frac{\text{mols Ca}}{4} + \frac{\text{mols Fe}^{2+}}{4} + \frac{\text{mols Mn}}{4}$$
(22)

$$m_{\rm kho} = \frac{\rm mols \ Fe^{3+}}{2}$$
(23)

$$m_{\rm nag} = \frac{\rm mols \ Na}{2} \tag{24}$$

$$m_{\rm mmr} = {
m mols Ti}$$
 (25)

$$m_{\rm sps} = \frac{\rm mols \ Mn}{3} \tag{26}$$

Once the moles of components have been calculated, it is then trivial to compute the set of end-member component molar proportions.

The general expression for the molar volume of a crystalline solid is  $V = V_{ideal} + V_{excess}$ , where excess volume of mixing is determined by an appropriate mixing model. The expression for  $V_{ideal}$  as a function of pressure, temperature, and composition is

$$V_{\text{ideal}} = \sum_{i} X_{i} V_{i} (P, T)$$
(27)

where i = [grs, pyr, alm, knr, maj, kho, nag, mmr, sps].  $V_i(P, T)$  is found by using Newton's method to search along the Vinet equation of state, plus a term to account for thermal pressure:

$$P = 3K_{\text{oT},i} \left(\frac{V_i}{V_{\text{o},i}}\right)^{-\frac{1}{2}_3} \left[1 - \left(\frac{V_i}{V_{\text{o},i}}\right)^{\frac{1}{2}_3}\right] \exp\left\{\frac{3}{2}\left(K'_i - 1\right)\left[1 - \left(\frac{V_i}{V_{\text{o},i}}\right)^{\frac{1}{2}_3}\right]\right\} + P_{thermal}$$
(28)

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where  $V_0$  is molar volume at reference pressure  $P_0 = 1$  bar and reference temperature  $T_0 = 298.15$  K,  $K_{0T}$  is the isothermal bulk modulus, and K' is the pressure derivative of the bulk modulus. In our spinel molar volume model (Hamecher et al. 2013) we used a thermal pressure term with a constant coefficient of thermal expansion,  $\alpha$ ,

 $P_{thermal} = \alpha_i K_{oT,i}(T - T_o)$ . However, during model calibration of in situ high-*T* garnet volume data, it became clear that this formalism is not sufficient to explain the thermal expansion of the garnets in our system (Fig. 2). Instead, we adopt the Mie-Grüneisen-Debye thermal pressure formalism (Luo et al. 2003):

$$P_{thermal} = \frac{\gamma_i C_{vm,i}}{V_i} \left[ TD\left(\frac{\theta}{T}\right) - T_o D\left(\frac{\theta_{o,i}}{T_o}\right) \right], \tag{29}$$

where

$$D(x) = \frac{3}{x^3} \int_0^x \frac{y^3 dy}{e^y - 1}$$
(30)

$$\theta = \theta_{o} \exp\left[-\int_{V_{o}}^{V} \frac{\gamma}{v} dv\right], \tag{31}$$

and  $\gamma$  is the *T*-independent Grüneisen parameter,  $C_{vm}$  is the high-temperature limit of isochoric specific heat (close to the Dulong-Petit value of 3nR),  $\theta_0$  is the reference Debye temperature, and D(x) is the Debye function. A common expression for  $\gamma$  is

$$\frac{\gamma}{\gamma_{\rm o}} = \left(\frac{V}{V_{\rm o}}\right)^q \tag{32}$$

where *q* is the logarithmic volume dependence of  $\gamma$  and  $\gamma_0$  is the Grüneisen parameter at the reference volume. The conversion between isochoric specific heat and isobaric specific heat (and, thus, thermal expansion) is given by

$$\frac{C_p}{C_V} = 1 + \alpha \gamma T \tag{33}$$

where  $\alpha$  is the coefficient of thermal expansion,  $\frac{1}{V} \frac{\partial V}{\partial T}\Big|_{P}$ . We used the approximation of

Masyukov and Dmitriev (2007) to compute the Debye function, which returns the same values as numerical integration, but significantly reduces computing time. Values and sources of the standard state end-member properties ( $V_0$ ,  $K_{0T}$ , K',  $\gamma_0$ ,  $C_{vm}$ ,  $\theta_0$ , q) used in the volume model calibration are discussed in the following section.

During model calibration, we considered excess volume terms of the symmetric regular solution form  $W_{ij}X_iX_j$  and asymmetric regular solution form  $W_{ij}X_iX_j + dW_{ij}X_iX_j(X_i - X_j)$ . We considered the possibility of allowing terms to depend on *P* and *T* as a way to encode non-ideal mixing of compressibility or thermal expansion, but ultimately determined that such *P*- and *T*-dependent parameters were not justified by the data. Calibration and significance of excess volume parameters are discussed below.

# **MODEL CALIBRATION**

Because of the different types of data and chemical complexity of our system, the volume model calibration required a multi-step approach. First, we needed to determine the standard state end-member properties discussed in the previous section for each of our independent end members. Since data do not exist for all of our chosen end members, different methods were used to determine these properties based on the information available. The first step of our global fitting routine fits all end-member data, at ambient and high-P or -T conditions, while also obtaining constraints from dependent end

members for fitting the excess volume. We then moved on to calibrating the mixing properties for the binary joins between both the independent end members and the three dependent end members for which there are data (andradite, uvarovite, and skiagite). In this section, we first discuss the determinations of the standard state properties for the end members, and then discuss our strategy for calibrating the excess volume model.

# **End-member standard state properties**

# $K_{oT}$ and K'

Because there is so much disagreement in the literature with regards to the bulk modulus (e.g., between static compression and ultrasonic studies), we needed to come up with a way to use the high-*P* data to constrain  $K_{oT}$  and K' for the independent end members. In some cases, we had to use dependent end-member data to constrain the behavior of the independent end members, due to the availability, or lack thereof, of data for some compositions, e.g., we needed to use andradite data to constrain the khoharite end-member properties. For the end members that have in situ high-*P* data (i.e., grossular, pyrope, almandine, spessartine, and dependent end member andradite), we normalized each XRD study to its own reported  $V_0$ . This eliminates interlaboratory calibration differences and small specimen composition differences, focuses as well as possible on the actual thermal expansion and compression, and should be most comparable to ultrasonic or other elasticity measurements. We then performed a least squares minimization to determine the optimal values of the bulk modulus and its pressure derivative for these compositions. Values of  $K_{oT}$  and K' are given in Table 3.

Zhang et al. (1998; 1999) performed in situ high-*P* XRD on single crystals of all five of the high-*P* compositions listed above. We also included the high-*P* data of Olijnyk et al. (1991) and Pavese et al. (2001) in fitting  $K_{oT}$  and K' for grossular. Our value for the bulk modulus of grossular,  $K_{oT} = 168.0$  GPa, agrees well with previous ultrasonic (Babuska et al. 1978) and Brillouin spectroscopy (Bass 1989) single crystal measurements. The fit value of K' = 6.1 also compares well with determinations by Olijnyk et al. (1991) and Pavese et al. (2001).

We used the compression data of Levien et al. (1979) in addition to Zhang et al. (1998) to fit  $K_{oT}$  and K' of pyrope. There have been several studies on pyrope (see table 4 in Zhang et al. 1998). Previously determined values for  $K_{oT}$  vary from 168.2 to 190 GPa with K' values from 1.8 to 5.45. Our fit of the high-P diffraction data yielded  $K_{oT} = 168.3$  GPa and K' = 4.8. Almandine and spessartine  $K_{oT}$  and K' were fit over the data of Zhang et al. (1999). Our  $K_{oT}$  and K' values for almandine compare well with theirs, and our chosen values for spessartine are consistent with their values given without constraining the value of K' (Zhang et al. 1999; table 3).

There are no high-*P* data for khoharite, so we used the compression data of Zhang et al. (1999) and Pavese et al. (2001) for andradite to constrain  $K_{oT}$  and K' for the ferric iron end member. We ended up adopting the Pavese et al. (2001) andradite value for the bulk modulus of khoharite,  $K_{oT} = 158.5$  GPa, and then adjusted K' down to a best fitting value of 4.7, which allowed us to better fit both the Zhang et al. (1999) and the Pavese et al. (2001) data. These values also agree well with the density functional theory (DFT) results of Milman et al. (2001) for khoharite. They calculated that  $K_{oT} = 163$  GPa and K' = 4.4 for khoharite.

Similarly, high-*P* data for knorringite does not exist, so we used data for another Cr-bearing garnet end member, uvarovite. Bass (1986) measured the elastic properties of a single crystal of synthetic uvarovite using Brillouin spectroscopy. Leger et al. (1990) used the Bass (1986) value for the bulk modulus,  $K_{oT} = 162$  to fit K' = 4.7 with the Birch-Murnaghan equation of state. We have adopted these values for the knorringite end member. Our chosen values fall between calculated values reported by Ottonello et al. (1996) and Milman et al. (2001).

There is no high-P data on pure samples of the three remaining independent end members-cubic majorite, Na-garnet, and Mg-Mg-morimotoite. For majorite we used the values recommended by Saxena (1996) and Fabrichnaya et al. (2004). Hazen et al. (1994) determined  $K_{0T}$  of a synthetic garnet with the formula (Na<sub>1.88</sub>Mg<sub>0.12</sub>)(Mg<sub>0.06</sub>Si<sub>1.94</sub>)Si<sub>3</sub>O<sub>12</sub> to be 191.5 GPa, assuming K' = 4; we have adopted these values for the Na-garnet end member. We performed a DFT calculation of the Ti-bearing end member, Mg-Mgmorimotoite. DFT calculations were also performed for grossular, pyrope, and majorite. To account for the systematic errors that typically occur in volume and bulk modulus estimates from DFT calculations with the LDA functional, we took the ratios of our preferred bulk moduli (from Table 3) to the DFT-determined bulk moduli for grossular, pyrope, and majorite. We then refined the estimate of  $K_{oT}$  for Mg-Mg-morimotoite by scaling the DFT result by the average of the ratios for the other three end members. Comparing the DFT results for grossular, pyrope, and majorite with independent estimates suggests that  $K_{oT}$  is overestimated by the DFT calculation (and  $V_o$  slightly underestimated).  $K_{0T}$  is plotted as a function of  $R_x/R_y$ , where  $R_x$  is the ionic radius of the x-site cation and  $R_v$  is the ionic radius of the y-site cation, for each end member in Fig. 3.

Systematic trends in  $K_{oT}$  versus cation size are evident among Ca-rich garnets (grs, uvr, and), Al-rich garnets (alm, sps, grs), and Mg-rich garnets (pyr, knr, kho, maj, mmr).

# *Thermal pressure parameters* ( $\theta_0$ , $\gamma_0$ , $C_{vm}$ , q)

In order to calibrate the parameters for the Mie-Grüneisen-Debye thermal pressure term, we again were forced to appeal to different methods depending on the data available for the end members. After some trial-and-error, we set q = 1.5 for all end members, which is within the typical range of values reported for garnets (Stixrude and Lithgow-Bertelloni 2005). We then turned to fitting the three remaining parameters— $\theta_0$ ,  $\gamma_0$ , and  $C_{vm}$ . Our preferred set of parameters are given in Table 3.

Skinner (1956) measured thermal expansion for four of our chosen independent end members: grossular, pyrope, almandine, and spessartine. We used these along with additional high-*T* data for grossular (Thieblot et al. 1998; Rodehorst et al. 2002), pyrope (Meagher 1975; Pavese et al. 1995), almandine (Geiger et al. 1992), and spessartine (Rodehorst et al. 2002) to tune the  $\theta_0$ ,  $\gamma_0$ , and  $C_{vm}$  parameters. We found that while volume, *V*, and heat capacity,  $C_p$ , both depend on  $\theta_0$ ,  $\gamma_0$ , and  $C_{vm}$ ,  $\gamma_0$  is better constrained by fitting *V*, but  $\theta_0$  and  $C_{vm}$  are better constrained by fitting  $C_p$ . We iterated between fitting  $\gamma_0$  to the thermal expansion data, then fitting  $\theta_0$  and  $C_{vm}$  to the  $C_p$  expression of Berman (1988) over the relevant temperature range, until we converged on a best-fitting set of parameters for each of these end members.

To calibrate the parameters for khoharite, we again had to look to and radite data, as there are no high-*T* data for the independent  $Fe^{3+}$ -bearing end member. We performed the same type of iterations as above using high-*T* and radite volume data of Skinner

(1956) and the  $C_p$  expression from Ottonello et al. (1996); Berman (1988) does not include andradite. We iteratively fit both the  $C_p$  and  $\alpha$  expressions of Ottonello et al. (1996) for khoharite, which yielded a similar set of parameters as the andradite fit. Ottonello et al. (1996) also gives values for  $C_p$  and  $\alpha$  for knorringite and uvarovite, and fitting over both expressions gives similar values of parameters for the Cr-bearing end members as well. For majorite, we set  $C_{vm} = 3nR$  and  $\theta_0 = 980$  K, then used systematics to estimate the values for Mg-Mg-morimotoite (Figs. 4 and 5). We set the values of  $\theta_0$ and  $C_{vm}$  for Na-garnet equal to those of majorite and  $\gamma_0$  to unity, for convenience, as there is no data available to constrain the parameters for that end member.

 $V_{0}$ 

For end members with good constraints on  $V_0$  from the data set (i.e., grossular, pyrope, almandine, knorringite (via uvarovite), and spessartine), we allowed standard state volumes to vary during the first step of our global least squares fitting routine, discussed below. Khoharite was also fit during that step using an intermediate Fe<sup>3+</sup>-bearing composition to constrain the value; details are given in the next section. Majorite was fixed to the value given by Fabrichnaya et al. (2004), Na-garnet was fixed to the value determined by Hazen et al. (1994) for a Na-bearing garnet with the formula (Na<sub>1.88</sub>Mg<sub>0.12</sub>)(Mg<sub>0.06</sub>Si<sub>1.94</sub>)Si<sub>3</sub>O<sub>12</sub>, and Mg-Mg-morimotoite was fixed to a value determined by our DFT calculation, refined in the same way as  $K_{oT}$  for this end member, as described above. Values of  $V_0$  are given in Table 3 and shown plotted in Fig. 6. Novak and Gibbs (1971) noted the same variations of  $V_0$  with cation radii shown for the "ugrandite" and "pyralspite" garnet series.

#### Volume model calibration

#### End members

Our computational scheme for model calibration using our full data set is detailed in Hamecher et al. (2013). During the first step of least squares fitting we allowed the endmember  $V_0$  to adjust for the five end members listed in the previous section. For each dependent end member for which there are data (andradite, uvarovite, and skiagite), we calculated the volume of reaction for formation of the phase from the independent end members,  $\Delta V$  (e.g.,  $\Delta V_{and} = V_{grs} - V_{pyr} + V_{kho}$ ), to be used as constraints as described below. There is only one data point in the database for dependent end member calderite,  $Mn_3Fe_2Si_3O_{12}$  (Nishizawa et al. 1977). We set  $\Delta V_{calderite}$  to zero (N.B.,  $\Delta V_{calderite} = V_{kho} + V_{sps} - V_{pyr}$ ), which helped constrain  $V_0$  of khoharite and improved the fit of the endmember andradite data.

### Excess volume

We fit the binary and mixed composition data using the method of singular value analysis (SVA) of Lawson and Hanson (1974) to solve the least squares problem. Briefly, we use the  $\Delta V$  constraints from the first step to constrain which combinations of excess parameters, i.e., the *W*s and *dW*s, can be varied without affecting the constrained values. The *W*s are assembled into a matrix and an eigenvector-eigenvalue decomposition is performed on the matrix. The eigenvalues with non-zero values correspond to the set of constrained values. Eigenvectors associated with zero eigenvalues define the linear combination of *W*s that are orthogonal to the constraint set. We can vary this set of linear

combinations without affecting the constrained values. Detailed descriptions of solving least squares problems with eigenvector-eigenvalue decomposition and SVA can be found in Lawson and Hanson (1974) and Press et al. (2007).

The SVA yields a starting set of parameters to vary. We then used a combination of the Bayesian information criteria (BIC) (Schwarz 1978; Hamecher et al. 2013), analysis of the reduced  $\chi^2$  statistic, and visual inspection of excess volume data and models along binary joins to reduce our number of parameters to a minimum set (Table 4). Data and models along select binary joins are shown in Fig. 7.

In the final model, each of the nine explicit end members has a pure component equation of state described by seven parameters:  $V_0$ ,  $K_{0T}$ , K',  $\theta_0$ ,  $\gamma_0$ ,  $C_{vm}$ , q (Table 3). The additional parameters of the preferred final model include symmetric excess terms  $(W_{ij}X_iX_j)$  along nine compositional binaries and asymmetric terms  $(W_{ij}X_iX_j + dW_{ij}X_iX_j(X_i - X_j))$  along five binaries. The values of the excess volume parameters are given in Table 4. This model fits virtually all of the data to within 0.06 J/bar/mol, or better than 0.5% in volume (Fig. 8), with a few exceptions for data points and/or studies that were excluded from the calibration. The mean of the absolute values of the residuals for calibrated data is 0.0113 J/bar/mol and the root mean squared error is 0.0156 J/bar/mol. It is difficult to obtain a more quantitative evaluation of goodness-of-fit given uncertain knowledge of errors on the different types of data used as inputs to the model, and because of our multi-step approach to fitting the data.

We have completed a preliminary bootstrap estimation (Efron 1982) to attempt to derive confidence bounds on the derived parameters. Uncertainty bounds on each fitted parameter and a correlation matrix for the parameter set derived from 3000 bootstrap

iterations are given in Supplementary Material 3 and 4. All the excess volume parameters and non-fixed  $V_0$  values were varied, except for khoharite  $V_0$  which was constrained by  $\Delta V$  for calderite as described above. Many of the compositional excess parameters are strongly correlated, as expected. This observation highlights that the calibrated parameters should be used in the context of the full model derivation and that they may not be optimal descriptions of subsystems if ideal and excess terms are separated. As noted above, the calibration data vary in coverage and quality and full estimates of uncertainties are sparse, which means confidence bounds calculated using random sampling are unrealistically wide. We obtained a more conservative confidence bound for each parameter, again using the bootstrapping technique but holding all other parameters at their optimal values; these uncertainties are reported in Table 4.

#### DISCUSSION

#### **Model comparison**

Figure 9 shows excess volume along the grossular-pyrope join for the various models shown in Fig. 1, our proposed model, and the calibration data along this join; the ideal end-member contributions to the volume have been subtracted from other models and data. As noted above, there is a very large scatter in the data along this binary, especially at the pyrope-rich end, where excess volumes range from approximately -0.03 to +0.025 J/bar/mol. Similar to other models, our model fits this binary with an asymmetric term skewed towards grossular, but with much smaller magnitude. The difference between our model and the others is partly a matter of the data selected to calibrate each model, but is also due to the fact that we are simultaneously fitting the pyrope-knorringite and

grossular-uvarovite binaries (Figs. 7d, e). The total lack of agreement of the numerous studies for the grossular-pyrope binary suggest that there is much more work to be done to understand the effect that Ca-Mg ordering has on molar volume in garnets (Du 2012).

Figure 10 shows excess volume along the grossular-almandine join for the calibration data, the models of Geiger et al. (1989) and Berman (1990), and our proposed model. Our model fit is in general agreement with the Berman (1990) model. There is quite a bit of scatter in the data on this join, as well, with excess volumes ranging from approximately -0.013 to +0.049 J/bar/mol. The Geiger et al. (1989) model is based only on the data of Geiger et al. (1987), whereas the Berman (1990) model also used the data of Koziol (1990). Our model uses both of these data sets in addition to the data of Cressey et al. (1978), and agrees fairly well even with the inclusion of that data.

Rodehorst et al. (2002) modeled the molar volume of the grossular-spessartine binary at room-*T* and fit a symmetric excess volume parameter  $W^V = 0.08(4)$  J/bar/mol, essentially the same value that we obtain,  $W_{grs-sps} = 0.0811$  J/bar/mol (Fig. 7a). Rodehorst et al. (2004) repeated the fitting at temperatures of 290 and 30 K and extracted the mixing parameters  $W^V = 0.058(9)$  J/bar/mol and  $W^V = 0.10(1)$  J/bar/mol, respectively. They observe that the apparent asymmetry of mixing in the data that occurs with change in temperature does not appear to be systematic, and that more study is required to establish whether excess volume along the grossular-spessartine join is dependent on temperature or not.

Berman (1990) models the pyrope-almandine join with a very small asymmetric term slightly skewed towards the almandine-rich end. Geiger et al. (1989) and Geiger and

Feenstra (1997) treat this join as ideal. Our model also adopts ideal molar volume along the pyrope-almandine join as the data do not suggest excess volume of mixing. Geiger and Feenstra (1997) model the almandine-spessartine binary with a symmetric excess volume term  $W^V = 0.024(5)$  J/bar/mol; this join is treated as ideal in our model.

# Enthalpy of formation and standard state entropy of knorringite

As discussed in Hamecher et al. (2013), we would like to examine the effect that our new garnet and spinel volume models have on the enthalpy of formation  $(H^{\circ})$  and standard state entropy  $(S^{\circ})$  of our model Cr-bearing garnet, knorringite. Klemme (2004) presented experimental reversals for the garnet-spinel transition reaction  $MgCr_2O_4 + 4MgSiO_3 =$  $Mg_3Cr_2Si_3O_{12} + Mg_2SiO_4$ . H<sup>o</sup> and S<sup>o</sup> of knorringite were extracted using the experimental brackets from that study, and then again by Klemme et al. (2009). Klemme's (2004) values were designed to be consistent with the thermodynamic dataset of Holland and Powell (1990), whereas Klemme et al. (2009) used a later version (Holland and Powell 1998). When Hamecher et al. (2009) repeated the exercise using the provisional xMELTS thermodynamic dataset (based on Berman (1988), but updated to use the Vinet equation of state) the recovered standard-state entropy of Mg<sub>3</sub>Cr<sub>2</sub>Si<sub>3</sub>O<sub>12</sub> was significantly lower than the values given by Klemme and co-workers. We tested the effect of our newly calibrated garnet and spinel (Hamecher et al. 2013) models, and obtained  $H^{\circ} = -5672$ kJ/mol and  $S^{\circ} = 322.1$  J/mol/K, compared with  $H^{\circ} = -5542$  kJ/mol and  $S^{\circ} = 376.7 \text{ J/mol/K}$  from Klemme (2004) and  $H^{\circ} = -5276 \text{ kJ/mol}$  and  $S^{\circ} = 357.9 \text{ J/mol/K}$ from Klemme et al. (2009).

Much, though perhaps not all, of the difference between our fitted value of  $S^{\circ}$ and that of Klemme (2004) and Klemme et al. (2009) can be attributed to the choice of  $V_{\circ}$ for knorringite. Klemme (2004) claims to use  $V_{\circ}$  for knorringite from Irifune et al. (1982). However, the Perple\_X (Connolly 1990; Connolly and Petrini 2002) data files available on the web (http://www.perplex.ethz.ch/) and Klemme et al. (2009) use  $V_{\circ}$  of pyrope from Holland and Powell (1998) for knorringite standard state molar volume. We tested fitting  $H^{\circ}$  and  $S^{\circ}$  using both Holland and Powell (1990) and Holland and Powell (1998) based code, using the  $V_{\circ}$  of pyrope (11.318 J/bar/mol) and the volume expressions from Perple\_X in each case, and retrieved values for  $S^{\circ}$  close to those given in Klemme (2004) and Klemme et al. (2009), respectively. Our tests suggest that the  $S^{\circ}$  we obtain for knorringite with the xMELTS model is not that dissimilar to what Perple\_X using Holland and Powell (1998) would give had the  $V_{\circ}$  value for knorringite been comparable to that of Irifune et al. (1982) ( $V_{\circ} = 11.738$  J/bar/mol) or of this study

 $(V_{\rm o} = 11.7412 \text{ J/bar/mol}).$ 

#### **Comparison of model densities**

As a test, we compared garnet densities retrieved with our proposed model to densities from other models, to see what effect, if any, our new model has on densities of upper mantle rocks. First, we compared grossular-pyrope-almandine garnet densities with those calculated with the current garnet model in pMELTS (Ghiorso et al. 2002). Calculations with pMELTS are performed in the pressure range of 0 to 3 GPa, applicable to oceanic lithosphere and asthenospheric melts. Second, we compared densities with the model of Klemme et al. (2009), which is applicable to deep continental lithosphere (pressures from 3 to 6 GPa). Finally, we compare model densities with those of the global mantle model of Stixrude and Lithgow-Bertelloni (2011) (calculated for pressures from 0 to 30 GPa).

We used pMELTS to calculate a near-fractional decompression melting path using the DMM composition of Workman and Hart (2005) with 150 ppm H<sub>2</sub>O with total entropy of 256 J/K, which corresponds to a potential temperature of 1344 °C and a solidus of 3.3 GPa. This path and source generates 6 km of oceanic crust, on the slightly low-temperature end of normal ridges. As noted above, pMELTS is currently calibrated only for grossular-pyrope-almandine garnets. To compare the pMELTS residual density to the new model using the proposed garnet model and the spinel model of Hamecher et al. (2013), we calculated the garnet and spinel densities using the new models, and then recomputed the bulk density using the masses and volumes of the model olivine and pyroxenes (Fig. 11a). The difference in density between the models is  $\sim 1 \text{ kg/m}^3$  relative to a change in bulk density of  $\sim 100 \text{ kg/m}^3$  along the melting path. One reason for this small difference is that there is very little difference in the volume models for grossularpyrope-almandine garnets; the molar volume of garnets in this compositional system is well constrained by the current model used by pMELTS. Additionally, there is not much garnet in the residue, and the modal abundance of garnet decreases rapidly with decompression melting.

Although the difference in density between the pMELTS model and our proposed model is small, this analysis provides us with the opportunity to consider the implications for compositional buoyancy of peridotites. The density determined above is a combination of compositional buoyancy, thermal expansion, and compression, and applies along the sub-ridge melting path. To understand the behavior of the peridotites later in the lithosphere, or during subduction, we need to isolate purely compositional buoyancy. We took the series of evolving residual bulk compositions and recomputed all their phase assemblages and properties during progressive depletion at constant P and T: 3.5 GPa and 1400 °C for the garnet stability field and 1.5 GPa and 1200 °C for the spinel stability field. Results for the two models are shown in Fig. 11b and 11c in terms of absolute density and compositional buoyancy (i.e., density minus density of the unmelted source) versus extent of melting. There is not significant difference between the two models, which is not surprising as the grossular-pyrope-almandine volumes are already well constrained in pMELTS. The density difference between fertile source and the most-depleted harzburgite is  $\sim 50 \text{ kg/m}^3$  at 3.5 GPa, but only  $\sim 20 \text{ kg/m}^3$  at 1.5 GPa, where the fertile composition is not crystallizing garnet. The reduced ability to form modal garnet due to Al extraction in depleted peridotites has a large effect on compositional buoyancy (compare Figs. 11b and 11c). Hence, the magnitude of compositional buoyancy in the garnet stability field is more than twice as large as in the spinel stability field.

Figure 12 shows a calculation using the Cr-bearing harzburgite bulk composition B47 of Klemme et al. (2009). We used the mineral mode and composition output from the Klemme et al. model (Klemme, personal communication) to recalculate garnet and spinel densities with our models, and recombined with the other mineral modes and compositions, as described in the pMELTS example. Our retrieved bulk density ends up being between 40 to 80 kg/m<sup>3</sup> lower than the Klemme model. This could be in large part due to the difference in thermodynamic properties being used for knorringite between the

two models, as discussed in the previous section. The predicted difference is large enough to be quite significant to our understanding of Cr-rich lithosphere, e.g., diamondbearing kimberlites.

We repeated this exercise for three additional compositions from Stixrude and Lithgow-Bertelloni (2012; figure 6) using the model output of the HeFESTo code (Stixrude and Lithgow-Bertelloni 2011; Stixrude, personal communication). Figure 13 shows the Stixrude and Lithgow-Bertelloni model for a basalt/eclogite bulk composition and the model with density recomputed using our garnet volume model. We also performed the calculation for harzburgite and pyrolite mantle compositions. Figure 14a shows the difference in density of the basalt and harzburgite compositions for the two models; Fig. 14b shows the difference in density between the basalt and pyrolite compositions. It is interesting to note that the largest difference between the models is at ~20 GPa, in the mantle transition zone where the majorite garnet field begins to grow (e.g., see figure 17 in Stixrude and Lithgow-Bertelloni 2011). This is likely due to the differences in our majorite end-member properties.

Basalt has the greatest abundance of garnet of the three compositions. The largest difference between the models is for the basalt composition at ~20 GPa, where the amount of majorite being crystallized is increasing. The maximum difference in density between the models (~20 kg/m<sup>3</sup>) is small compared to the bulk density of peridotite (~4000 kg/m<sup>3</sup>), but the density difference between the different rock types needs to be considered for compositional buoyancy. Compositional buoyancy of eclogite relative to pyrolite is thought to be driving force for slab pull and plate tectonics. The basaltic part of slabs in our model is less dense, so the model predicts smaller negative compositional

buoyancy for eclogite in the transition zone where garnets are majoritic. These differences in model results have consequences for the overall buoyancy of slabs and the tendency of different parts of slabs to separate (e.g., eclogite descending to the lower mantle and harzburgite remaining in the upper mantle). Results from convection models will depend to a large extent on the differences in compositional buoyancy in the chosen density models.

# Conclusions

Regardless of the fact that garnets have been studied in depth for well over 50 years, disagreement about the end-member and excess volumetric properties of garnets persists. In fact, the most heavily investigated regions of garnet compositions space have the largest scatter in data. More work is needed to understand the volume effects of Ca-Mg ordering in grossular-pyrope garnets. It is our hope that more high-quality volume data will become available for both the aluminosilicate garnets and garnets with substantial cubic majorite component, as well as for samples measured at in situ high-P, -T conditions.

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Table 1. Model components

Name	Symbol	Formula
Independent end membe	ers	
Grossular	grs	$Ca_3Al_2Si_3O_{12}$
Pyrope	pyr	$Mg_{3}Al_{2}Si_{3}O_{12}$
Almandine	alm	$Fe_3Al_2Si_3O_{12}$
Knorringite	knr	$Mg_3Cr_2Si_3O_{12}$
Majorite	maj	Mg <sub>3</sub> (MgSi)Si <sub>3</sub> O <sub>12</sub>
Khoharite	kho	$Mg_3Fe_2Si_3O_{12}$
Na-garnet	nag	$(Na_2Mg)Si_2Si_3O_{12}$
Mg-Mg-morimotoite	mmr	Mg <sub>3</sub> (TiMg)Si <sub>3</sub> O <sub>12</sub>
Spessartine	sps	$Mn_{3}Al_{2}Si_{3}O_{12}$
Dependent end members	s (for which	there are data)
Andradite	and	$Ca_3Fe_2Si_3O_{12}$
Uvarovite	uvr	$Ca_3Cr_2Si_3O_{12}$
Skiagite	ski	$Fe_3Fe_2Si_3O_{12}$

Table 2. Summary t	table of data coverage in	n composition, <i>P</i> ,	, and T			
			High-	<i>P</i> Range		T Range
Composition	Total Experiments	Ambient-P, T	Ρ	(GPa)	High- <i>T</i>	(J <sup>o</sup> )
End members						
grs	63	21	15	2.3 - 11.8	27	57 - 1140
pyr	98	33	43	2.27 - 33.38	22	56 - 758
alm	53	21	18	2.3 - 21.3	14	50 - 581
knr	C	c	0	ı	0	ı
sds	40	11	12	2.3 - 14.6	17	68 - 702
and	35	6	13	2.3 - 14	13	59 - 690
uvr	4	4	0	ı	0	ı
ski	ъ	ß	0	I	0	I
Binaries						
grs-pyr	41	41	0	ı	0	ı
grs-alm	26	26	0	I	0	I
grs-sps	6	6	0	ı	0	I
grs-and	-1	1	0	ı	0	I
grs-uvr	11	11	0	ı	0	I
grs-nct	-1	1	0	I	0	I
grs-csg	1	1	0	ı	0	I
pyr-alm	11	11	0	ı	0	I
				1.241 -		
pyr-maj	94	13	81*	10.253	81*	33 - 890
pyr-knr	6	6	0	ı	0	I
alm-sps	28	28	0	I	0	I

ı	I	I	I	I	ı		ı	80 - 1225			
0	0	0	0	0	0		0	29			
ı	I	0.39 - 4.72	I	I	ı		0.39 - 4.72	5.8 - 19			
0	0	∞	0	0	0		∞	4			
42	1	1	12	38	∞		22	60			
42	1	6	12	38	8		30	93	766	neous high- <i>P</i> , -T	
alm-ski	knr-maj	maj-nag	and-uvr	and-ski	ski-fcg	Ternary	grs-pyr-alm	Multi-component	Total	* Measured at simultar	

	:	0	:		-			Cvm	
	V <sub>o</sub> (J/bar/mol)	a (A)	p (kg/m <sup>-</sup> )	K <sub>oT</sub> (GPa)	K'	γ٥	q	(J/K/mol)	θ <sub>o</sub> (K)
$Ca_3Al_2Si_3O_{12}$	12.5275	11.8504	3596	168.0	6.1	1.038	1.5	502.2	096
$Mg_3Al_2Si_3O_{12}$	11.3207	11.4570	3561	168.3	4.8	1.055	1.5	503.0	959.7
$Fe_3Al_2Si_3O_{12}$	11.5308	11.5274	4317	188.2	3.9	1.065	1.5	507.6	889.5
$Mg_3Cr_2Si_3O_{12}$	11.7412	11.5971	3860	162.0	4.7	1.084	1.5	506.9	946.7
Mg <sub>3</sub> (MgSi)Si <sub>3</sub> O <sub>12</sub>	11.4000	11.4837	3522	154.2	4.0	1.08	1.5	498.87	980
$Mg_3Fe_2Si_3O_{12}$	11.9596	11.6686	3853	158.5	4.7	1.081	1.5	510.2	928.4
$(Na_2Mg)Si_2Si_3O_{12}$	11.1831	11.4104	3601	191.5	4.0	1	1.5	498.87	980
Mg <sub>3</sub> (TiMg)Si <sub>3</sub> O <sub>12</sub>	11.8150	11.6214	3566	148.9	4.5	1.08	1.5	502	970
$Mn_3Al_2Si_3O_{12}$	11.7953	11.6149	4197	184.5	5.1	1.267	1.5	492.4	900.9

Table 3. Optimized standard state end-member properties

## Table 4. Model

## parameters

		1σ <sup>*</sup>	
W <sub>grs-pyr</sub>	0.0487	0.0246	(J/bar/mol)
$W_{\sf grs-alm}$	0.1073	0.0893	
W <sub>grs-knr</sub>	0.1205	0.0493	
$W_{ m grs-maj}$	-0.3035	3.4454	
$W_{ m grs-nag}$	0.1054	1.5421	
W <sub>grs-mmr</sub>	0.1079	0.2851	
W <sub>grs-sps</sub>	0.0811	0.2768	
W <sub>pyr-knr</sub>	0.0641	0.0396	
$W_{ m pyr-maj}$	0.0596	0.0000	
$W_{ m pyr-kho}$	-0.4093	0.0000	
W <sub>pyr-mmr</sub>	-0.2585	1.9250	
$W_{alm ext{-}kho}$	-0.1764	0.0161	
$W_{knr-kho}$	-0.1355	0.2020	
$W_{ m kho-sps}$	-0.1369	11.8864	
<i>dW</i> grs-pyr	-0.0234	0.0130	
$dW_{ m grs-alm}$	-0.0511	0.2782	
$dW_{ m grs-nag}$	-0.1456	21.7412	
$dW_{ m pyr-knr}$	-0.0453	0.0220	
<i>dW</i> <sub>pyr-kho</sub>	0.1362	0.0100	
*Bootstrap est	timation of s.o	d. for each pa	arameter,
holding all oth	ier parameter	's at optimal	values

#### **FIGURE CAPTIONS**

**Fig. 1** Various published models of excess volume along the grossular-pyrope binary. Models shown: Wood (1988) (red); Berman (1990) (magenta); Ganguly et al. (1993) (green); Bosenick and Geiger (1997) (blue); Bosenick et al. (2001) (cyan).

**Fig. 2** High-*T*, room-*P* data of Skinner (1956) for almandine (upper) and spessartine (lower) plotted as residuals in pressure versus temperature. Black squares show the fit when using the constant  $\alpha$  thermal pressure term; blue filled circles show the data fit with the Mie-Grüneisen-Debye thermal pressure formalism.

**Fig. 3** Preferred values of  $K_{oT}$  versus  $R_x/R_y$  for end members, where  $R_x$  is the ionic radius of the x-site cation and  $R_y$  is the ionic radius of the y-site cation. Effective ionic radii from Shannon (1976). Dashed arrows show trends of increasing cation radius among Carich, Mg-rich, and Al-rich end members. Colors highlight systematic relationships for Carich and Mg-rich garnet  $K_{oT}$  with y-site cation radii: Al (red), Cr (green), Fe<sup>3+</sup> (blue), Mg (gray).

Fig. 4 Debye temperature  $(\theta_0)$  versus  $R_x/R_y$  for end members. Colors have same meaning as in Fig. 3. Dashed arrows show systematic relationship for garnets with Mg of Ca on the x-site.

Fig. 5  $C_{vm}$  versus  $R_x/R_y$  for end members. Colors have same meaning as in Fig. 3.

**Fig. 6**  $V_o$  versus  $R_x/R_y$  for end members. Dashed arrows show clear systematic relationships between  $V_o$  and cation radius for the "pyralspite" (pyrope-almandinespessartine) and "ugrandite" (uvarovite-grossular-andradite) garnet series, which was also observed by Novak and Gibbs (1971).

**Fig. 7** Excess volume models (black curves) and excess volume of calibration data (gray filled circles) along the **a** grossular-spessartine; **b** andradite-uvarovite; **c** pyrope-knorringite; **d** uvarovite-grossular binaries.

Fig. 8 Model residuals versus measured volume.

**Fig. 9** Excess volume model along the grossular-pyrope binary from this work (black curve) plotted with models shown in Fig. 1 (muted colors correspond to colors in Fig. 1). Excess volume of calibration data plotted as gray filled circles.

**Fig. 10** Excess volume models along the grossular-almandine binary from this work (solid curve), Geiger et al. (1989) (dotted curve), and Berman (1990) (dashed curve). Excess volume of calibration data plotted as gray filled circles.

**Fig. 11 a** Left-hand axis shows bulk density versus pressure for the DMM bulk composition of Workman and Hart (2005) calculated with current pMELTS model (black

dashed curve) and recalculated with garnet density from this work and spinel density from Hamecher et al. (2013) (solid black line). Right-hand axis shows the difference in bulk density between the two models (red dashed curve); **b** Bulk density and compositional buoyancy versus extent of melt depletion in the spinel stability field (1.5 GPa and 1200 °C); **c** Bulk density and compositional buoyancy versus extent of melt depletion in the garnet stability field (3.5 GPa and 1400 °C).

**Fig. 12** Left-hand axis shows bulk density versus pressure for the B47 bulk composition of Klemme et al. (2009) for the Klemme et al. (2009) model (black dashed curve) and recalculated with garnet density from this work and spinel density from Hamecher et al. (2013) (solid black line). Right-hand axis shows the difference in bulk density between the two models (red dashed curve).

**Fig. 13** Left-hand axis shows bulk density versus pressure for the basalt bulk composition used by Stixrude and Lithgow-Bertelloni (2012; figure 6) for their model computed with HeFESTo (Stixrude and Lithgow-Bertelloni 2011) (black dashed curve) and recalculated with garnet density from this work (solid black line). Right-hand axis shows the difference in bulk density between the two models (red dashed curve).

**Fig. 14 a** Left-hand axis shows the difference in bulk density difference between the basalt and harzburgite compositions used by Stixrude and Lithgow-Bertelloni (2012; figure 6) versus pressure for their model computed with HeFESTo (Stixrude and Lithgow-Bertelloni 2011) (black dashed curve) and recalculated with garnet density from

this work (solid black line). Right-hand axis shows the difference between the two models (red dashed curve); **b** Left-hand axis shows the difference in bulk density difference between the basalt and pyrolite compositions used by Stixrude and Lithgow-Bertelloni (2012; figure 6) versus pressure for their model computed with HeFESTo (Stixrude and Lithgow-Bertelloni 2011) (black dashed curve) and recalculated with garnet density from this work and spinel density from Hamecher et al. (2013) (solid black line). Right-hand axis shows the difference between the two models (red dashed curve).

# FIGURES



Fig. 2











Fig. 7



Fig. 8















## Supplement 1. Data sources used in model calibration\*

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	500															
Reference #										cat	ions per f	ormula u	nit			
(see Supplement 1)	AMCSD ID <sup>a</sup>	a (Å)	V (J/bar/mol)	V residuals (J/bar/mol)	P (GPa)	τ (°C)	Si	ï	AI	Fe <sup>3+</sup>	ŗ	Fe <sup>2+</sup>	ЧИ М	Mg	Са	Na
1		11.457(1)	11.321	0.0000	0.0001	25	3.000	0.000	2.000	0.000	0.000	0.000	0.000	3.000	0.000	0.000
1		11.526(1)	11.526	-0.00434	0.0001	25	3.000	0.000	2.000	0.000	0.000	3.000	0.000	0.000	0.000	0.000
1		11.461(1)	11.333	-0.01086	0.0001	25	3.177	0.000	1.646	0.000	0.000	0.000	0.000	3.177	0.000	0.000
1	ı	11.459(1)	11.327	-0.00245	0.0001	25	3.062	0.000	1.876	0.000	0.000	0.000	0.000	3.062	0.000	0.000
1	·	11.536(1)	11.557	0.00483	0.0001	25	3.152	0.000	1.696	0.000	0.000	3.152	0.000	0.000	0.000	0.000
2	8734	11.997(1)	12.999	0.00071	0.0001	20	3.000	0.000	0.000	0.000	2.000	0.000	0.000	0.000	3.000	0.000
S		11.521(1)	11.511	-0.02113	0.0001	25	3.001	0.000	1.993	0.000	0.000	3.009	0.000	0.000	0.000	0.000
4	6471	12.063(1)	13.214	0.00718	0.0001	20	3.000	0.000	0.000	2.000	0.000	0.000	0.000	0.000	3.000	0.000
4	6472	12.068(1)	13.230	0.01024	0.0001	77	3.000	0.000	0.000	2.000	0.000	0.000	0.000	0.000	3.000	0.000
4	6473	12.081(1)	13.273	0.01117	0.0001	227	3.000	0.000	0.000	2.000	0.000	0.000	0.000	0.000	3.000	0.000
4	6474	11.476(1)	11.377	0.00755	0.0001	227	3.000	0.000	2.000	0.000	0.000	0.000	0.000	3.000	0.000	0.000
5	1468	11.516(2)	11.497	0.00867	0.0001	20	3.000	0.000	2.000	0.000	0.000	2.400	0.000	0.600	0.000	0.000
5	1469	11.525(1)	11.523	-0.00639	0.0001	20	3.000	0.000	2.000	0.000	0.000	3.000	0.000	0.000	0.000	0.000
5	1465	11.452(1)	11.306	-0.01383	0.0001	20	3.000	0.000	2.000	0.000	0.000	0.000	0.000	3.000	0.000	0.000
5	1466	11.473(2)	11.368	0.00645	0.0001	20	3.000	0.000	2.000	0.000	0.000	0.600	0.000	2.400	0.000	0.000
5	1467	11.485(2)	11.404	0.00013	0.0001	20	3.000	0.000	2.000	0.000	0.000	1.200	0.000	1.800	0.000	0.000
9	6759	11.764(1)	12.255	0.02529	0.0001	20	2.982	0.014	1.702	0.272	0.000	0.729	069.0	0.006	1.587	0.000
7	ı	12.054(2)	13.184	0.00103	0.0001	25	3.000	0.000	0.080	1.920	0.000	0.000	0.000	600.0	2.991	0.000
7	ı	11.997	12.998	-0.00132	0.0001	25	3.000	0.000	0.000	0.000	2.000	0.000	0.000	0.000	3.000	0.000
8	ı	11.848(7)	12.520	-0.01100	0.0001	25	3.000	0.000	1.984	0.016	0.000	0.000	0.006	0.000	2.994	0.000
8		11.617(3)	11.802	0.01798	0.0001	25	3.000	0.000	2.000	0.000	0.000	0.150	2.844	0.000	0.006	0.000
6		11.456(1)	11.318	-0.03867	0.0001	25	3.164	0.000	1.790	0.000	0.000	0.000	0.000	2.860	0.000	0.198
10	ı	11.639(1)	11.869	-0.01428	0.0001	25	3.000	0.000	2.000	0.000	0.000	2.000	0.000	0.000	1.000	0.000
																198

Supplement 2. Data used in model calibration

0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	8 199
0.000	0.311	0.780	1.526	2.241	2.749	3.000	0.482	0.462	0.458	0.459	0.463	0.447	0.472	0.444	0.488	0.428	0.793	0.761	0.768	0.761	0.000	0.000	0.270	0.600	006.0	006.0
3.000	2.757	2.152	1.455	0.730	0.294	0.000	2.563	2.522	2.579	2.562	2.559	2.579	2.562	2.574	2.553	2.603	2.263	2.277	2.213	2.246	3.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.002	0.002	0.001	0.001	0.001	0.000	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.000	3.000	2.730	2.400	2.100	2.100
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2.000	1.972	1.968	1.977	1.989	1.976	2.000	1.995	2.025	2.014	1.991	1.990	2.006	1.994	1.984	1.987	1.982	1.970	2.009	1.992	2.009	2.000	2.000	2.000	2.000	2.000	2.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.000	2.985	3.057	3.026	3.022	2.996	3.000	2.980	2.989	2.970	2.995	2.996	2.982	2.988	3.003	2.989	2.997	2.994	2.974	3.015	2.989	3.000	3.000	3.000	3.000	3.000	3.000
22	22	22	22	22	22	22	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-0.00444	-0.00294	-0.02567	0.00405	0.02214	0.00136	0.00236	-0.00247	-0.00293	-0.00178	-0.00540	-0.00650	-0.00434	-0.00835	0.00048	-0.02348	-0.01268	-0.02221	0.01673	0.00122	0.00256	-0.00296	-0.00434	-0.01820	0.01966	0.01159	0.02076
11.3157	11.4463	11.6222	11.9547	12.2632	12.4195	12.5293	11.5154	11.5070	11.5058	11.5040	11.5046	11.4992	11.5058	11.5043	11.4974	11.4845	11.6225	11.6475	11.6397	11.6354	11.318	11.526	11.608	11.762	11.860	11.869
11.4553(3)	11.4992(4)	11.5578(4)	11.6670(5)	11.7665(4)	11.8163(5)	11.8510(5)	11.5223(10)	11.5195(11)	11.5191(5)	11.5185(5)	11.5187(6)	11.5169(4)	11.5191(5)	11.5186(4)	11.5163(6)	11.5120(7)	11.5579(15)	11.5662(6)	11.5636(9)	11.5622(6)	11.456(2)	11.526(1)	11.553(1)	11.604(1)	11.636(1)	11.639(1)
ı	ı								ı	ı	ı				ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	ı	1
11	11	11	11	11	11	11	12	12	12	12	12	12	12	12	12	12	12	12	12	12	13	14	14	14	14	14

0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	8 200
1.500	1.500	1.860	1.860	2.100	2.460	2.940	3.000	0.000	0.000	0.000	3.000	1.500	2.106	2.391	3.000	0.000	0.588	0.774	0.219	0.402	1.152	1.608	3.000	3.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	3.000	0.000	1.500	0.888	0.609	0.000	3.000	2.412	2.226	2.781	2.598	1.848	1.392	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	3.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	3.000	0.000
1.500	1.500	1.140	1.140	006.0	0.540	090.0	0.000	0.000	3.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	3.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.000	2.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	0.000	0.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000
25	25	25	25	25	25	25	25	25	25	25	25	25	20	20	20	20	20	20	20	20	20	20	20	277	20	25
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0.01281	0.00353	0.01291	0.01291	0.02192	-0.00055	-0.00989	-0.00458	0.01937	0.01383	0.01483	0.00494	-0.01231	0.01375	0.01906	0.00432	-0.00020	0.01285	0.01168	0.00398	0.00092	0.00400	0.01558	-0.00995	0.00660	0.01360	0.00256
12.069	12.060	12.190	12.190	12.277	12.368	12.501	12.523	12.262	11.992	11.336	12.532	11.924	12.1945	12.3108	12.5309	11.3195	11.5746	11.6499	11.4137	11.4860	11.7974	11.9947	12.517	12.596	11.808	11.5334
11.704(1)	11.701(1)	11.743(1)	11.743(1)	11.771(1)	11.800(1)	11.842(1)	11.849(1)	11.766(2)	11.679(2)	11.462(3)	11.852(3)	11.657(2)	11.7445(3)	11.7817(3)	11.8515(2)	11.4566(2)	11.5420(4)	11.5670(2)	11.4883(2)	11.5125(4)	11.6156(4)	11.6800(2)	11.847(1)	11.872(1)	11.619(1)	11.5283(6)
	ı	,		,	·			,		,			1567	1568	1570	1564	1565	1566			ı	,	1924	1925	1927	·
14	14	14	14	14	14	14	14	15	15	16	16	16	17	17	17	17	17	17	17	17	17	17	18	18	18	19

0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	8.201
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.210	0.459	0.780	1.479
0.000	0.243	0.300	0.447	0.381	0.360	0.762	0.753	0.741	0.753	1.557	1.527	1.542	1.629	2.127	2.178	2.478	2.355	2.394	2.745	3.000	3.000	0.000	0.000	0.000	0.000	0.000
3.000	2.757	2.700	2.553	2.619	2.640	2.238	2.247	2.259	2.247	1.443	1.473	1.458	1.371	0.873	0.822	0.522	0.645	0.606	0.255	0.000	0.000	2.991	2.799	2.529	2.220	1.521
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0:030	0.010	0.040	0.030	0.050
2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	1.970	1.980	1.980	1.980	1.960
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	2.991	3.009	2.991	2.991	3.000
25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-0.00134	0.00697	0.00105	-0.00499	0.00563	0.00087	0.00758	0.00415	0.00370	0.00204	0.00453	0.00446	0.00828	0.00182	0.00307	0.00221	0.00827	0.00149	0.00898	0.00816	-0.00306	0.00060	-0.00206	-0.00182	-0.00761	-0.00212	-0.00916
11.5295	11.5592	11.5583	11.5652	11.5700	11.5634	11.6056	11.6013	11.5998	11.5992	11.6726	11.6699	11.6750	11.6762	11.7214	11.7250	11.7575	11.7399	11.7508	11.7809	11.7922	11.7959	11.5358	11.5166	11.4995	11.4800	11.4263
11.5270(7)	11.5369(12)	11.5366(8)	11.5389(5)	11.5405(4)	11.5383(3)	11.5523(3)	11.5509(3)	11.5504(2)	11.5502(4)	11.5745(3)	11.5736(2)	11.5753(4)	11.5757(2)	11.5906(9)	11.5918(2)	11.6025(6)	11.5967(3)	11.6003(4)	11.6102(5)	11.6139(3)	11.6151(6)	11.5291(3)	11.5227(2)	11.5170(2)	11.5105(2)	11.4925(3)
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19	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19

0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	8.202
0.000	0.000	0.000	0.000	0.000	0.138	0.321	0.291	0.426	0.795	1.218	1.485	1.974	2.220	2.703	3.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1.839	2.259	2.739	2.991	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.270	0.516	0.843	1.251	1.533	1.851	2.301	2.679	3.000	3.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1.170	0.741	0.270	0.000	3.000	2.862	2.679	2.709	2.574	2.205	1.782	1.515	1.026	0.780	0.297	0.000	3.000	2.730	2.484	2.157	1.749	1.467	1.149	0.699	0.321	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.010	0.020	0.010	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1.970	1.980	1.980	1.980	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.009	3.000	3.009	3.021	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000
25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-0.00583	-0.00449	-0.00711	-0.00444	-0.00434	-0.00178	-0.00906	-0.00149	0.00223	0.02099	-0.01356	-0.00372	-0.02146	0.00230	0.00809	0.00176	-0.00734	-0.00042	-0.00117	0.00079	-0.00052	0.00133	-0.01215	-0.00443	-0.00765	-0.00889	0.00297
11.3980	11.3703	11.3332	11.3163	11.526	11.578	11.635	11.632	11.683	11.832	11.945	12.047	12.193	12.296	12.450	12.529	11.523	11.511	11.494	11.473	11.443	11.425	11.389	11.365	11.336	11.312	11.324
11.4830(2)	11.4737(2)	11.4612(2)	11.4555(3)	11.526(1)	11.543(1)	11.562(2)	11.561(1)	11.578(1)	11.627(2)	11.664(2)	11.697(2)	11.744(2)	11.777(2)	11.826(1)	11.851(1)	11.525(1)	11.521(1)	11.515(1)	11.508(1)	11.498(1)	11.492(1)	11.480(1)	11.472(1)	11.462(1)	11.454(1)	11.458(1)
ı	ı	ı	ı	ı	,	·	ı		,	ı	ı	·	,	·	ı	ı	ı	ı	ı	ı	ı	ı	,	·	ı	,
19	19	19	19	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	21

0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.881	1.881	1.881	1.881	1.881	1.881	1.881	1.881	1.881	0.000	0.000	0.000	0.000	8203
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	3.000	1.194	2.970	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.060	0.060	0.060	0.060	0.060
3.000	3.000	3.000	0.000	0.000	0.000	0.000	3.000	3.000	0.000	1.806	0.000	3.000	1.179	1.179	1.179	1.179	1.179	1.179	1.179	1.179	1.179	2.841	2.841	2.841	2.841	2.841
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	3.000	3.000	3.000	3.000	0.000	0.000	0.000	0.000	0.030	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.099	0.099	0.099	0.099	660.0
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.000	2.000	2.000	2.000	2.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	4.940	4.940	4.940	4.940	4.940	4.940	4.940	4.940	4.940	3.000	3.000	3.000	3.000	3.000
25	25	25	20	20	147	227	20	25	25	25	20	20	25	25	25	25	25	25	25	25	25	25	25	25	25	25
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.39	0.99	1.62	2.23	2.42	3.24	4.1	4.72	0.0001	0.39	0.99	1.62	2.23
0.00297	0.0000	0.0000	-0.00639	-0.00639	0.00529	0.00968	0.00691	-0.00889	0.00081	0.02366	-0.00471	-0.00198	-0.01429	-0.01654	-0.01678	-0.01753	-0.00881	-0.00928	-0.01063	-0.01235	-0.01303	0.01139	0.00684	0.00949	0.01247	0.00786
11.324	11.321	11.321	11.523	11.523	11.563	11.587	11.327	11.312	12.528	11.835	12.513	11.318	11.1823	11.1570	11.1218	11.0849	11.0592	11.0481	11.0015	10.9533	10.9197	11.3637	11.3332	11.2964	11.2589	11.2158
11.458(1)	11.457(1)	11.457(1)	11.525(1)	11.525(1)	11.538(1)	11.546(1)	11.459	11.4540(5)	11.8507(3)	11.628(1)	11.846(1)	11.456(1)	11.4101(11)	11.4015(8)	11.3895(12)	11.3769(8)	11.3681(10)	11.3643(12)	11.3483(12)	11.3317(11)	11.3201(10)	11.4715(17)	11.4612(8)	11.4488(15)	11.4361(9)	11.4215(22)
·	,	·	2690	7691	7692	2693	142			·	634	629		·		·		·	,	ı		·	,	·	·	
21	21	21	22	22	22	22	23	24	24	24	25	25	26	26	26	26	26	26	26	26	26	26	26	26	26	26

0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	8.204
0.060	0.060	0.060	0.060	0.000	0.000	0.000	0.000	0.330	0.330	0.570	0.330	0.480	0.600	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	3.000
2.841	2.841	2.841	2.841	3.741	3.498	3.249	3.000	2.670	2.670	2.430	2.670	2.520	2.400	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.600	0.600	1.200	1.200	1.800	1.800	2.400	2.400	3.000	3.000	0.000
0.099	0.099	0.099	0.099	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	3.000	3.000	2.400	2.400	1.800	1.800	1.200	1.200	0.600	0.600	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2.000	2.000	2.000	2.000	0.518	1.004	1.502	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.000	3.000	3.000	3.000	3.741	3.498	3.249	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000
25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25
2.42	3.24	4.1	4.72	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0.01674	0.01258	0.01029	0.01083	-0.01526	-0.01285	-0.00892	-0.00180	-0.00181	-0.01674	-0.00477	0.00118	-0.00072	-0.00810	0.00166	-0.00734	-0.00916	-0.00013	-0.01384	0.00125	-0.00629	-0.00932	-0.01068	-0.01068	-0.00275	-0.00884	0.00518
11.2129	11.1585	11.1048	11.0691	11.3756	11.3623	11.3427	11.3189	11.455	11.440	11.551	11.458	11.517	11.560	11.532	11.523	11.575	11.584	11.623	11.638	11.683	11.680	11.732	11.732	11.793	11.786	13.005
11.4205(14)	11.4020(11)	11.3837(8)	11.3715(14)	11.4755(2)	11.4710(3)	11.4644(4)	11.4564(4)	11.502(5)	11.497(5)	11.534(5)	11.503(5)	11.523(5)	11.537(5)	11.528	11.525	11.542	11.545	11.558	11.563	11.578	11.577	11.594	11.594	11.614	11.612	11.999(2)
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26	26	26	26	27	27	27	27	28	28	28	28	28	28	29	29	29	29	29	29	29	29	29	29	29	29	30

0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	8.205
3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	3.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2.000	1.900	1.800	1.600	1.400	1.200	1.000	0.800	0.600	0.400	0.200	0.100	0.000	0.000	0.050	0.100	0.200	0.400	0.600	0.800	1.000	1.200	1.400	1.600	1.800	1.900	0.108
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.000	1.950	1.900	1.800	1.600	1.400	1.200	1.000	0.800	0.600	0.400	0.200	0.100	0.000
0.000	0.100	0.200	0.400	0.600	0.800	1.000	1.200	1.400	1.600	1.800	1.900	2.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.892
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000
25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-0.00457	0.00702	0.00479	0.02729	0.01171	0.00395	-0.00197	-0.00231	0.00023	0.00935	-0.00941	0.00075	-0.00141	-0.00706	-0.01061	-0.00437	-0.00191	-0.00091	0.00249	-0.00151	0.00016	0.00422	0.00088	-0.00006	0.00463	0.00014	0.00459
12.995	12.979	12.949	12.920	12.856	12.801	12.749	12.705	12.663	12.628	12.564	12.551	12.526	13.201	13.191	13.191	13.181	13.158	13.138	13.112	13.093	13.076	13.053	13.034	13.021	13.008	11.353
11.996(2)	11.991(2)	11.982(3)	11.973(3)	11.953(2)	11.936(2)	11.920(2)	11.906(2)	11.893(2)	11.882(2)	11.862(2)	11.858(3)	11.850(2)	12.059(3)	12.056(4)	12.056(2)	12.053(3)	12.046(2)	12.040(2)	12.032(3)	12.026(3)	12.021(3)	12.014(2)	12.008(2)	12.004(4)	12.000(3)	11.468(1)
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31	31	31	31	31	31	31	31	31	31	31	31	31	32	32	32	32	32	32	32	32	32	32	32	32	32	33

0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	8.206
0.000	0.000	0.000	0.000	0.000	0.000	0:030	2.906	0.151	3.263	0.000	0.000	2.906	2.906	2.906	2.906	1.983	1.311	0.732	0.138	0.147	0.564	0.876	0.132	0.300	0.879	1.161
3.000	3.000	3.000	3.000	3.000	3.000	0.000	0.073	0.586	0.008	3.160	0.000	0.073	0.073	0.073	0.073	0.000	0.000	0.000	0.000	0.345	0.360	0.291	0.564	0.546	0.417	0.417
0.000	0.000	0.000	0.000	0.000	0.000	1.380	0.005	0.093	0.004	0.000	0.000	0.005	0.005	0.005	0.005	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	1.560	0.227	2.207	2.137	0.000	3.000	0.227	0.227	0.227	0.227	1.017	1.689	2.268	2.862	2.508	2.076	1.833	2.304	2.154	1.704	1.422
0.236	0.350	0.500	0.784	1.410	2.000	0.000	0.000	0.000	0.000	1.680	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1.764	1.650	1.500	1.216	0.590	0.000	2.000	1.848	2.012	0.045	0.000	0.000	1.848	1.848	1.848	1.848	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.022	0.001	0.000	0.000	0.000	0.022	0.022	0.022	0.022	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.000	3.000	3.000	3.000	3.000	3.000	3.000	2.987	2.972	3.260	3.160	3.000	2.987	2.987	2.987	2.987	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000
25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	5.8	15.7	17.9	19	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-0.00348	0.00504	-0.00540	0.00233	0.01120	-0.00344	0.00047	0.07335	-0.00162	-0.00102	0.04356	-0.04286	0.04947	-0.01444	-0.03456	-0.04171	0.00365	0.00351	-0.01473	0.00123	-0.00200	-0.00336	-0.00232	0.00056	-0.00292	-0.00354	0.00258
11.377	11.413	11.437	11.505	11.638	11.738	11.665	12.641	11.557	13.1924	11.7302	12.087	12.227	11.647	11.529	11.476	12.221	11.995	11.774	11.581	11.557	11.701	11.817	11.538	11.596	11.808	11.912
11.476(1)	11.488(1)	11.496(1)	11.519(1)	11.563(2)	11.596(1)	11.572(1)	11.886	11.536	12.0565	11.5935(1)	11.710(50)	11.755(25)	11.566(18)	11.527(6)	11.509(23)	11.753(3)	11.680(1)	11.608(1)	11.544(2)	11.536(1)	11.584(1)	11.622(2)	11.530(1)	11.549(3)	11.619(3)	11.653(2)
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33	33	33	33	33	34	35	36	36	37	38	39	40	40	40	40	41	41	41	41	42	42	42	42	42	42	42

0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	8.207
1.506	0.138	0.576	0.912	1.467	0.099	0.609	1.200	0.141	0.582	0.903	1.158	3.000	3.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.961	2.961	2.961	0.000	0.000	0.000
0.300	0.825	0.696	0.573	0.423	1.230	1.032	0.774	1.617	1.344	1.182	1.047	0.000	0.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	0.000	0.000	0.000	3.000	3.000	3.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.039	0.039	0.039	0.000	0.000	0.000
1.194	2.040	1.728	1.512	1.110	1.671	1.359	1.026	1.242	1.074	0.915	0.795	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.050	0.050	0.050	0.000	0.000	0.000
2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	1.950	1.950	1.950	2.000	2.000	2.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000
25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	365	675	25	350	550
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	2.27	3.07	3.57	4.32	4.96	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-0.01003	-0.00730	-0.00510	-0.01932	0.00944	0.00195	-0.01374	-0.00242	-0.00946	-0.01185	-0.00131	0.01751	-0.00141	0.00811	-0.01777	-0.00296	0.00239	0.00472	0.00562	0.00623	0.00547	-0.02000	0.00030	0.01967	-0.00296	0.01391	0.00209
12.026	11.514	11.680	11.793	12.022	11.482	11.659	11.893	11.458	11.629	11.762	11.878	12.526	12.536	11.303	11.318	11.176	11.129	11.100	11.056	11.018	12.513	12.621	12.740	11.318	11.419	11.470
11.690(3)	11.522(2)	11.577(1)	11.614(2)	11.689(2)	11.511(3)	11.570(2)	11.647(2)	11.503(4)	11.560(1)	11.604(3)	11.642(2)	11.850(2)	11.853(2)	11.451	11.456(2)	11.408(2)	11.392(1)	11.382(1)	11.367(1)	11.354(1)	11.846(2)	11.880(2)	11.917(4)	11.456(2)	11.490	11.507
ı																					450	451	452	446	447	448
42	42	42	42	42	42	42	42	42	42	42	42	43	43	44	45	45	45	45	45	45	46	46	46	46	46	46
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	8.208
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0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.324	0.549	0.852	1.410	2.388	2.712	3.000	3.000	0.000	0.129	2.970	1.341	2.961	0.000	060.0	0.081	2.991	1.452	0.072	0.000
3.000	3.000	3.050	3.130	3.238	3.000	3.000	2.676	2.451	2.148	1.590	0.612	0.288	0.000	0.000	0.000	0.270	0.020	060.0	0.000	3.000	2.631	0.000	0.000	0.828	2.697	3.030
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	3.000	0.010	0.010	0.810	0.039	0.000	0.009	2.580	0.009	0.018	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.589	0.000	0.759	0.000	0.000	0.270	0.339	0.000	0.702	0.228	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.570	0.000	1.730	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.000	2.000	0.000	1.990	0.000	0.050	0.000	060.0	0.010	0.050	0.009	0.000	0.000
2.000	2.000	1.900	1.740	1.524	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	0.000	0.000	1.980	0.010	1.990	1.950	2.000	1.340	1.990	0.210	1.982	2.000	1.970
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.010	0.000	0.000	0.000	0.000	0.010	0.009	0.000	0.000
3.000	3.000	3.050	3.130	3.238	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	2.991
750	20	25	25	25	25	25	25	25	25	25	25	25	25	25	25	20	20	20	20	20	20	20	20	25	25	25
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0.00369	0.00187	0.02169	0.03788	0.07289	0.0000	0.0000	-0.03515	0.01891	0.00887	0.02736	0.00141	-0.00524	-0.00458	-0.00377	0.00018	-0.01584	0.00212	-0.03762	-0.02220	0.00691	0.00211	-0.00235	0.02596	0.02248	0.05233	-0.01848
11.538	11.3216	11.349	11.3756	11.4233	11.321	11.321	11.419	11.566	11.680	11.927	12.293	12.412	12.523	13.204	12.434	11.541	13.197	12.026	12.510	11.327	11.526	11.786	12.969	12.001	11.419	11.321
11.530	11.4573(9)	11.467(1)	11.4755(9)	11.4915(8)	11.457(1)	11.457(1)	11.490(1)	11.539(1)	11.577(1)	11.658(1)	11.776(1)	11.814(1)	11.849(1)	12.060	11.821	11.531(1)	12.058(1)	11.690(1)	11.845(1)	11.459(1)	11.526(1)	11.612(1)	11.988(1)	11.682(3)	11.490(3)	11.457(2)
449	2417	2255	2256	2257					ı	ı	ı	ı			ı	240	246	242	243	238	239	241	244	ı	ı	ı
46	47	48	48	48	49	49	49	49	49	49	49	49	49	50	50	51	51	51	51	51	51	51	51	52	53	53

0.000	0.000	0.000	0.000	0.000	1.869	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	8.209
3.000	3.000	3.000	3.000	3.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.240	0.510	0.510	2.139	2.889	2.901	1.389	3.000	0.081	0.081	2.970	2.970	2.970	2.970	3.000
0.000	0.000	0.000	0.000	0.000	1.179	3.380	3.480	3.750	3.000	3.000	3.000	1.119	1.020	0.921	060.0	0.000	090.0	1.140	0.000	0.000	0.000	0.020	0.020	0.020	0.020	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.039	0.039	0.039	0.039	0.039	0.009	0.009	0.000	2.580	2.580	0.010	0.010	0.010	0.010	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.590	1.431	1.560	0.729	0.189	0:030	0.459	0.000	0.339	0.339	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.010	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.120	0.760	0.190	0.010	2.000	0.010	0.010	1.990	1.990	1.990	1.990	0.662
2.000	2.000	2.000	2.000	2.000	0.000	1.240	1.040	0.500	2.000	2.000	2.000	1.990	2.000	1.980	1.870	1.000	1.810	1.990	0.000	1.990	1.990	0.010	0.010	0.010	0.010	1.338
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.010	0.000	0.010	0.010	0.120	0.010	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.000	3.000	3.000	3.000	3.000	4.940	3.380	3.480	3.750	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000
25	25	25	25	25	25	25	25	25	27	500	700	20	20	20	20	20	20	20	20	25	350	25	350	575	850	20
3.8	6.8	10.1	8.6	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-0.00628	-0.01914	-0.03111	-0.02253	0.00176	-0.01676	-0.01154	-0.00396	-0.00739	-0.00810	0.03588	0.03534	0.00788	0.00240	0.01666	0.03110	0.04708	0.04321	0.00987	06600.0-	-0.00039	-0.02749	0.00430	0.00670	-0.00178	-0.01969	0.00933
12.258	12.063	11.869	11.958	12.529	11.1778	11.353	11.3697	11.3840	11.3130	11.4872	11.5529	11.547	11.644	11.662	12.327	12.901	12.593	11.942	13.1967	11.789	11.854	13.201	13.296	13.366	13.452	12.756
11.765(14)	11.702(4)	11.639(10)	11.668(7)	11.851(1)	11.4086(20)	11.468(1)	11.4735(5)	11.4783(3)	11.4544(2)	11.5129(5)	11.5348(5)	11.533	11.565	11.571	11.787	11.967	11.871	11.663	12.0578(2)	11.613(1)	11.634(2)	12.059(1)	12.088(2)	12.109(2)	12.135(2)	11.922(1)
ı	ı		·		ı		ı	ı	1741	1742	1743	7901	7902	2062	7905	2062	7067	7904	4182	ı	ı	ı	ı	ı	ı	10465
54	54	54	54	54	55	56	56	56	57	57	57	58	58	58	58	58	58	58	59	60	60	60	60	60	60	61

0.999	2.001	0.999	2.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	8210
2.001	0.999	2.001	0.999	3.000	0.000	0.000	0.000	0.000	0.000	3.000	3.000	3.000	3.000	3.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.285	0.438	0.618
0.000	0.000	0.000	0.000	1.000	3.000	3.000	3.000	3.000	3.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	2.715	2.562	2.382
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	1.000	1.500	1.800	2.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1.000	0.000	1.000	0.000	0.000	2.000	1.000	0.500	0.200	0.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000
0.000	0.000	1.000	2.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4.000	5.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000
25	25	25	25	25	25	25	25	25	25	25	75	125	175	225	25	75	125	175	225	275	325	375	25	25	25	25
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0.01111	-0.00659	-0.01073	0.00664	-0.00638	0.00890	0.01252	0.01815	0.00576	0.00871	-0.00141	-0.00229	-0.00748	-0.00405	0.00503	0.00030	0.00671	-0.00330	-0.01119	-0.01678	-0.01686	-0.00216	-0.00625	0.00182	-0.00175	0.00065	-0.00130
12.103	11.575	12.823	12.9040	13.286	11.330	11.560	11.662	11.707	11.750	12.526	12.536	12.542	12.558	12.580	11.796	11.814	11.817	11.823	11.832	11.847	11.878	11.890	11.7971	11.8701	11.9129	11.9581
11.715(5)	11.542(2)	11.943(2)	11.9680(2)	12.085(2)	11.460(1)	11.537(2)	11.571(2)	11.586(2)	11.600(1)	11.850(1)	11.853(1)	11.855(1)	11.860(1)	11.867(1)	11.615(1)	11.621(1)	11.622(1)	11.624(1)	11.627(1)	11.632(1)	11.642(1)	11.646(1)	11.6155(3)	11.6394(5)	11.6534(4)	11.6681(7)
ı	ı	ı	·	ı	ı	ı	·		·	2771	2772	2773	2774	2775	2781	2782	2783	2784	2785	2786	2787	2788	ı	ı	ı	ı
62	62	62	62	62	63	63	63	63	63	64	64	64	64	64	64	64	64	64	64	64	64	64	64	64	64	64

0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	8 211
0.723	1.164	1.485	1.776	2.208	2.688	3.000	0.000	2.967	3.194	3.055	3.031	2.875	0.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	3.000	0.000	0.000	0.000	0.000	0.000	3.810	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2.277	1.836	1.515	1.224	0.792	0.312	0.000	0.000	0.011	0.015	0.070	0.011	0.040	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.719	0.717	0.459	0.888	0.554	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	1.244	1.196	1.567	1.090	1.440	0.400	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.008	0.027	0.009	0.010	0.030	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.031	2.935	2.910	2.985	3.016	3.790	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000
25	25	25	25	25	25	25	25	25	25	25	25	25	25	19	21.4	25	57.2	94.1	108.8	185.5	277.7	387.9	389.9	480.1	589.3	707.7
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0.00012	-0.00167	-0.00186	0.00263	0.00568	-0.00308	-0.00077	0.02077	-0.00786	-0.02050	-0.01483	0.04255	0.04796	-0.00725	0.00070	0.00056	0.00081	0.00124	0.00168	0.00145	0.00150	0.00163	0.00137	0.00141	0.00113	-0.00091	0.00029
11.9867	12.0970	12.1761	12.2510	12.3556	12.4558	12.5267	11.341	12.733	12.769	12.650	12.881	12.701	11.3953	12.5271	12.5274	12.5283	12.5353	12.5439	12.5470	12.5661	12.5912	12.6231	12.6237	12.6511	12.6841	12.7247
11.6774(4)	11.7131(5)	11.7386(9)	11.7626(3)	11.7960(1)	11.8278(7)	11.8502(4)	11.464(1)	11.915(2)	11.926(2)	11.889(2)	11.961(2)	11.905(2)	11.4821(25)	11.8503(10)	11.8504(10)	11.8507(10)	11.8529(10)	11.8556(10)	11.8566(10)	11.8626(10)	11.8705(10)	11.8805(10)	11.8807(10)	11.8893(10)	11.8996(10)	11.9123(10)
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64	64	64	64	64	64	64	65	66	66	66	66	66	67	68	68	68	68	68	68	68	68	68	68	68	68	68

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3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
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0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000
21.4	25	59	109.1	149.3	208.9	304.2	349	427.1	501.8	589.2	629.9	069	21.5	25	50.5	108.6	146.9	205.4	243.8	321	361.6	401	455.5	489.2	543.2	581.3
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-0.04203	-0.04184	-0.04019	-0.03811	-0.03756	-0.03672	-0.03596	-0.03633	-0.03679	-0.03761	-0.03751	-0.03587	-0.03773	-0.00487	-0.00494	-0.00518	-0.00566	-0.00610	-0.00653	-0.00636	-0.00595	-0.00555	-0.00530	-0.00528	-0.00375	-0.00432	-0.00434
13.1649	13.1659	13.1754	13.1901	13.2016	13.2197	13.2500	13.2641	13.2898	13.3149	13.3463	13.3629	13.3834	11.5253	11.5259	11.5307	11.5427	11.5511	11.5649	11.5749	11.5956	11.6071	11.6182	11.6336	11.6448	11.6599	11.6711
12.0481(10)	12.0484(10)	12.0513(10)	12.0558(10)	12.0593(10)	12.0648(10)	12.0740(10)	12.0783(10)	12.0861(10)	12.0937(10)	12.1032(10)	12.1082(10)	12.1144(10)	11.5256(10)	11.5258(10)	11.5274(10)	11.5314(10)	11.5342(10)	11.5388(10)	11.5421(10)	11.5490(10)	11.5528(10)	11.5565(10)	11.5616(10)	11.5653(10)	11.5703(10)	11.5740(10)
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68	68	68	68	68	68	68	68	68	68	68	68	68	68	68	68	68	68	68	68	68	68	68	68	68	68	68

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0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000
3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
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18.7	19.3	25	68.5	129.7	232.5	299.3	394.2	443.5	501	557.7	623	702	11	25	27.7	56	89.6	122.3	153.9	195.5	247.1	270.5	289	328.1	395.9	441
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0.01693	0.01771	0.01767	0.01809	0.01740	0.01763	0.01984	0.01649	0.01365	0.01683	0.01835	0.01944	0.01801	0.00508	0.00593	0.00599	0.00606	0.00747	0.00803	0.00818	0.00893	0.00793	0.00879	0.00856	0.00818	0.00719	0.00788
11.8108	11.8117	11.8129	11.8236	11.8389	11.8688	11.8918	11.9194	11.9332	11.9562	11.9778	12.0024	12.0302	11.3231	11.3266	11.3272	11.3332	11.3421	11.3504	11.3584	11.3700	11.3831	11.3905	11.3956	11.4066	11.4260	11.4406
11.6200(10)	11.6203(10)	11.6207(10)	11.6242(10)	11.6292(10)	11.6390(10)	11.6465(10)	11.6555(10)	11.6600(10)	11.6675(10)	11.6745(10)	11.6825(10)	11.6915(10)	11.4578(10)	11.4590(10)	11.4592(10)	11.4612(10)	11.4642(10)	11.4670(10)	11.4697(10)	11.4736(10)	11.4780(10)	11.4805(10)	11.4822(10)	11.4859(10)	11.4924(10)	11.4973(10)
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0.000	0.000	0.000	0.000	0.000	0.093	0.000	0.093	0.266	0.000	0.000	2.971	2.971	2.971	2.971	2.971	2.971	2.971	2.971	2.971	2.971	2.971	2.971	2.971	0.009	0.009	600.0
3.000	3.000	3.000	3.000	3.000	0.598	0.000	0.598	1.789	3.000	3.000	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	2.826	2.826	2.826
0.000	0.000	0.000	0.000	0.000	0.074	0.000	0.074	0.016	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	1.997	3.000	1.997	0.911	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.039	0.039	0.039
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.008	0.000	2.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.195	0.000	0.195	0.078	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2.000	2.000	2.000	2.000	2.000	1.998	2.000	1.998	1.839	2.000	0.000	1.980	1.980	1.980	1.980	1.980	1.980	1.980	1.980	1.980	1.980	1.980	1.980	1.980	1.961	1.961	1.961
0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.001	0.017	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.000	3.000	3.000	3.000	3.000	2.974	3.000	2.974	3.048	3.000	3.000	3.029	3.029	3.029	3.029	3.029	3.029	3.029	3.029	3.029	3.029	3.029	3.029	3.029	3.092	3.092	3.092
524.2	597.1	629.5	698.5	758	25	23	23	23	23	25	27	132	218	334	451	553	702	823	917	968	1035	1091	1140	27	80	149
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0.00448	0.00698	0.00368	0.00343	0.00241	0.01487	0.01405	-0.00875	-0.00728	-0.00849	-0.00526	0.00279	0.02083	0.02901	0.01524	0.02182	0.03499	0.03121	0.02375	0.01689	0.02115	0.00789	0.00576	0.00601	0.00052	0.01298	0.01012
11.4636	11.4899	11.4974	11.5205	11.5400	11.541	11.544	11.517	11.511	11.312	11.7359	12.5299	12.5712	12.6014	12.6202	12.6620	12.7074	12.7529	12.7873	12.8140	12.8369	12.8485	12.8675	12.8866	11.3258	11.3495	11.3632
11.5050(10)	11.5138(10)	11.5163(10)	11.5240(10)	11.5305(10)	11.531(2)	11.532(17)	11.523(17)	11.521(17)	11.454(17)	11.5954(5)	11.8512(6)	11.8642(24)	11.8737(24)	11.8796(13)	11.8927(13)	11.9069(24)	11.9211(24)	11.9318(13)	11.9401(13)	11.9472(13)	11.9508(13)	11.9567(13)	11.9626(13)	11.4587(5)	11.4667(23)	11.4713(23)
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68	68	68	68	68	69	70	70	70	70	71	72	72	72	72	72	72	72	72	72	72	72	72	72	72	72	72

0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	8215
0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.009	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0.000	0.000	0.000	0.000	0.000
2.826	2.826	2.826	2.826	2.826	2.826	2.826	2.826	2.826	2.826	2.826	2.826	2.826	0.337	0.337	0.337	0.337	0.337	0.337	0.337	0.337	0.337	3.380	3.380	3.380	3.380	3.380
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.119	0.119	0.119	0.119	0.119	0.119	0.119	0.119	0.119	0.000	0.000	0.000	0.000	0.000
0.039	0.039	0.039	0.039	0.039	0.039	0.039	0.039	0.039	0.039	0.039	0.039	0.039	2.494	2.494	2.494	2.494	2.494	2.494	2.494	2.494	2.494	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1.961	1.961	1.961	1.961	1.961	1.961	1.961	1.961	1.961	1.961	1.961	1.961	1.961	1.981	1.981	1.981	1.981	1.981	1.981	1.981	1.981	1.981	1.240	1.240	1.240	1.240	1.240
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.000	0.000	0.000	0.000	0.000
3.092	3.092	3.092	3.092	3.092	3.092	3.092	3.092	3.092	3.092	3.092	3.092	3.092	2.990	2.990	2.990	2.990	2.990	2.990	2.990	2.990	2.990	3.380	3.380	3.380	3.380	3.380
216	325	446	554	717	818	915	963	1028	1084	1132	1180	1225	27	124	223	393	496	641	759	872	950	302	505	602	969	503
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	5.686	5.439	5.406	5.217	4.898
-0.00060	-0.00079	0.00370	0.00336	-0.00644	-0.01395	-0.00744	-0.01442	-0.01107	-0.02161	-0.01940	-0.02044	0.00342	-0.01156	-0.00819	-0.00138	-0.02003	-0.00825	-0.01727	-0.00509	0.00413	0.00648	0.00538	-0.00391	0.03476	0.01341	0.00627
11.3700	11.4006	11.4418	11.4759	11.5205	11.5481	11.5893	11.5998	11.6273	11.6378	11.6584	11.6759	11.7174	11.5463	11.5706	11.6020	11.6297	11.6714	11.7062	11.7554	11.8010	11.8291	11.065	11.123	11.191	11.208	11.167
11.4736(13)	11.4839(13)	11.4977(13)	11.5091(13)	11.5240(13)	11.5332(13)	11.5469(13)	11.5504(13)	11.5595(13)	11.5630(13)	11.5698(13)	11.5756(13)	11.5893(69)	11.5326(4)	11.5407(12)	11.5511(12)	11.5603(23)	11.5741(23)	11.5856(35)	11.6018(35)	11.6168(23)	11.6260(23)	11.370(1)	11.390(1)	11.413(1)	11.419(2)	11.405(1)
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0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380
400	301	201	101	28	705	499	298	28	500	698	890	799	700	599	498	400	300	199	29	27	30	404	599	796	599	399
4.757	4.595	4.445	4.277	4.195	7.357	7.02	6.715	6.258	8.297	8.614	8.78	8.628	8.471	8.321	8.155	7.947	7.789	7.608	7.278	0.0001	0.0001	10.253	10.13	9.864	9.388	9.079
0.00468	-0.00325	0.00014	-0.01733	-0.01716	0.01422	0.01308	0.00382	-0.00492	0.03573	0.03327	0.03620	0.03096	0.02722	0.01816	0.01056	0.01076	0.00477	-0.00362	-0.01062	-0.00006	-0.00663	-0.00559	0.00574	0.02000	0.01018	0.00583
11.147	11.123	11.112	11.082	11.074	11.076	11.041	11.001	10.963	10.989	11.018	11.062	11.041	11.021	10.995	10.972	10.960	10.940	10.919	10.899	11.365	11.359	10.816	10.879	10.957	10.925	10.891
11.398(1)	11.390(1)	11.386(2)	11.376(2)	11.373(2)	11.374(2)	11.362(2)	11.348(2)	11.335(2)	11.344(2)	11.354(1)	11.369(2)	11.362(2)	11.355(2)	11.346(2)	11.338(1)	11.334(2)	11.327(1)	11.320(2)	11.313(1)	11.472(1)	11.470(1)	11.284(1)	11.306(1)	11.333(1)	11.322(1)	11.310(1)
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0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380
199	34	33	33	802	601	400	200	34	800	601	400	199	34	802	600	401	199	34	801	600	399	200	34	800	600	400
8.785	8.567	8.39	7.911	9.194	8.809	8.439	8.112	7.845	8.688	8.305	7.931	7.575	7.295	8.034	7.629	7.269	6.925	6.663	7.231	6.814	6.483	6.142	5.883	6.274	5.83	5.468
0.00151	-0.00358	-0.00731	-0.01347	0.02007	0.01124	0.00129	-0.00431	-0.00862	0.01971	0.01381	0.00404	0.00030	-0.01045	0.02350	0.01475	0.00878	0.00070	-0.00579	0.02110	0.01208	0.00911	0.00115	-0.00739	0.02245	0.01234	0.00529
10.859	10.836	10.841	10.862	10.998	10.960	10.922	10.891	10.870	11.027	10.992	10.954	10.925	10.899	11.071	11.033	10.998	10.963	10.940	11.117	11.079	11.044	11.009	10.983	11.179	11.141	11.103
11.299(1)	11.291(1)	11.293(1)	11.300(1)	11.347(1)	11.334(1)	11.321(1)	11.310(1)	11.303(1)	11.357(1)	11.345(1)	11.332(1)	11.322(1)	11.313(1)	11.372(1)	11.359(1)	11.347(1)	11.335(1)	11.327(1)	11.388(1)	11.375(1)	11.363(1)	11.351(1)	11.342(1)	11.409(1)	11.396(1)	11.383(1)
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0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.276	0.381
3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.000	2.724	2.619
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	1.240	2.000	2.000	2.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.380	3.000	3.000	3.000
200	34	801	600	400	200	34	800	32	801	601	399	198	33	32	803	599	400	200	33	600	399	200	33	25	25	25
5.153	4.922	5.037	4.587	4.221	3.882	3.681	4.141	2.763	4.006	3.63	3.376	2.93	2.696	2.345	3.225	2.825	2.49	2.265	2.046	2.149	1.753	1.45	1.241	0.0001	0.0001	0.0001
0.0009	-0.00615	0.02365	0.01169	0.00274	-0.00857	-0.00907	0.02831	-0.00806	0.02759	0.02141	0.01855	-0.00161	-0.00667	-0.00562	0.02273	0.01714	0.00609	0.00423	-0.00470	0.01383	0.00893	0.00249	-0.00503	0.00593	0.01445	0.00410
11.068	11.041	11.262	11.220	11.179	11.138	11.114	11.327	11.173	11.336	11.294	11.250	11.205	11.179	11.203	11.386	11.344	11.297	11.256	11.223	11.389	11.350	11.309	11.276	11.327	11.449	11.482
11.371(1)	11.362(1)	11.437(1)	11.423(1)	11.409(1)	11.395(1)	11.387(1)	11.459(1)	11.407(1)	11.462(1)	11.448(1)	11.433(1)	11.418(1)	11.409(1)	11.417(1)	11.479(1)	11.465(1)	11.449(1)	11.435(1)	11.424(1)	11.480(1)	11.467(1)	11.453(1)	11.442(1)	11.459(1)	11.500	11.511
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0.453	0.600	2.931	3.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2.547	2.400	0.069	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.040	0.180	0.240	0.240	0.340	0.200	0.060	0.260	0.220	0.240	0.260	0.180	0.260	0.360	0.880	1.400	1.580	1.500	1.520	1.540	2.000
2.000	2.000	2.000	2.000	2.000	2.000	1.960	1.820	1.760	1.760	1.660	1.800	1.940	1.740	1.780	1.760	1.740	1.820	1.740	1.640	1.120	0.600	0.420	0.500	0.480	0.460	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000
25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0.01040	-0.00810	0.01524	0.00176	0.01067	0.00527	0.00817	-0.01304	0.00508	0.00538	0.00704	-0.00454	0.00424	0.00800	0.00542	0.00719	0.00348	0.00199	-0.00195	0.01723	-0.00303	-0.02391	-0.01001	-0.01833	0.00108	0.00176	0.01483
11.517	11.560	12.517	12.529	11.5415	11.5361	11.5484	11.5625	11.5968	11.5971	11.6270	11.5764	11.5493	11.6053	11.5917	11.5989	11.6007	11.5776	11.5953	11.6430	11.7873	11.9378	12.0073	11.9747	12.0002	12.0070	12.1451
11.523	11.537	11.847	11.851	11.5310(2)	11.5292(7)	11.5333(2)	11.5380(2)	11.5494(2)	11.5495(2)	11.5594(2)	11.5426(5)	11.5336(5)	11.5522(6)	11.5477(7)	11.5501(2)	11.5507(10)	11.5430(4)	11.5489(2)	11.5647(5)	11.6123(8)	11.6615(4)	11.6841(7)	11.6735(4)	11.6818(3)	11.6840(4)	11.7286(10)
74	74	74	74	75	75	75	75	75	75	75	75	75	75	75	75	75	75	75	75	75	75	75	75	75	75	75

0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	8 220
0.000	0.000	0.000	0.000	0.000	0.000	2.460	2.460	2.520	2.490	1.080	006.0	0.510	1.200	0.510	0.000	0.330	0.630	1.080	0.000	1.560	1.920	3.000	3.000	2.820	2.790	2.760
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.000	3.000	3.000	3.000	3.000	3.000	0.540	0.540	0.480	0.510	1.920	2.100	2.490	1.800	2.490	3.000	2.670	2.370	1.920	3.000	1.440	1.080	0.000	0.000	0.180	0.210	0.240
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2.000	2.000	1.960	0.260	0.660	1.180	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	0.300	2.000	2.000	2.000	1.800	2.000	2.000	2.000	2.000	2.000	2.000	2.000
0.000	0.000	0.040	1.740	1.340	0.820	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.700	0.000	0.000	0.000	0.200	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000
25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
0.01079	0.00676	0.01433	0.00438	0.00405	-0.00917	0.00316	-0.01703	-0.01586	0.00992	-0.00011	0.00608	-0.00410	-0.00654	0.03799	0.00401	0.00490	-0.00442	0.00274	-0.00010	0.00972	-0.00635	0.02581	-0.00509	0.00406	-0.01348	-0.00870
12.1410	12.1370	12.1351	11.6016	11.7223	11.8811	13.0410	13.0208	13.0417	13.0576	12.5448	12.4827	12.3230	12.5836	12.3651	11.6125	12.2625	12.3688	12.5477	12.0799	12.7334	12.8469	13.2335	13.2026	13.1570	13.1302	13.1256
11.7273(4)	11.726(20)	11.7254(4)	11.5510(1)	11.5909(2)	11.6430(12)	12.0102(6)	12.0040(5)	12.0104(4)	12.0153(6)	11.8559(8)	11.8363(3)	11.7856(6)	11.8681(17)	11.7990(9)	11.5546(4)	11.7663(2)	11.8002(5)	11.8568(7)	11.7076(7)	11.9150(4)	11.9503(8)	12.0690(5)	12.0596(2)	12.0457(15)	12.0375(1)	12.0361(7)
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75	75	75	75	75	75	76	76	76	76	76	76	76	76	76	77	77	77	77	77	77	77	77	77	77	77	77

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2.640	2.490	2.460	2.250	2.190	2.160	2.010	1.980	1.800	1.770	1.590	1.410	1.350	1.110	1.080	1.080	1.080	0.780	0.660	0.600	0.330	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.360	0.510	0.540	0.750	0.810	0.840	066.0	1.020	1.200	1.230	1.410	1.590	1.650	1.890	1.920	1.920	1.920	2.220	2.340	2.400	2.670	3.000	3.000	3.000	3.000	3.000	3.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	0.000	0.000	0.040	0.040	0.140	0.160
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.000	2.000	1.960	1.960	1.860	1.840
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000
25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-0.00537	0.01253	-0.01540	-0.02246	-0.00330	-0.00857	-0.01799	-0.00522	-0.00264	-0.02148	-0.00125	-0.01154	0.00082	0.00539	0.00655	-0.00583	-0.00424	0.00167	-0.00872	-0.01771	0.01116	0.00316	0.00316	-0.00205	0.00186	0.00493	-0.00177
13.0909	13.0603	13.0225	12.9452	12.9439	12.9283	12.8669	12.8691	12.8079	12.7783	12.7334	12.6569	12.6470	12.5617	12.5515	12.5391	12.5407	12.4325	12.3761	12.3440	12.2688	11.5340	11.5340	11.5382	11.5421	11.5700	11.5685
12.0255(8)	12.0161(7)	12.0045(6)	11.9807(7)	11.9803(6)	11.9755(1)	11.9565(7)	11.9572(5)	11.9382(3)	11.9290(9)	11.9150(7)	11.8911(6)	11.8880(5)	11.8612(6)	11.8580(5)	11.8541(7)	11.8546(4)	11.8204(6)	11.8025(6)	11.7923(9)	11.7683(7)	11.5285(2)	11.5285(2)	11.5299(5)	11.5312(8)	11.5405(2)	11.5400(6)
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77	77	77	77	77	77	77	77	77	77	77	77	77	77	77	77	77	77	77	77	77	77	77	77	77	77	77

0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	8.222
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	3.000	3.580	0.000	0.000	3.000	3.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	0.000	0.000	3.000	3.180	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.920	1.820	1.540	1.540	1.260	0.920	0.940	0.440	0.000	0.000	0.000	0.000	0.000	0.000
0.160	0.180	0.420	0.600	0.580	0.780	0.820	1.000	1.040	1.580	1.640	1.980	2.000	0.080	0.180	0.460	0.460	0.740	1.080	1.060	1.560	0.000	0.000	0.000	0.000	0.000	0.000
1.840	1.820	1.580	1.400	1.420	1.220	1.180	1.000	0.960	0.420	0.360	0.020	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.000	0.840	2.000	1.640	2.000	2.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.580	3.000	3.180	3.000	3.000
25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	20	20
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	3.48
-0.00478	-0.00462	-0.02344	0.00139	-0.01045	0.00295	-0.00202	0.00271	-0.00758	-0.01001	-0.00864	0.01454	0.01048	-0.00318	-0.00240	-0.00625	-0.00841	0.00204	-0.00127	0.00956	-0.00725	-0.00592	-0.01004	0.01668	0.03448	-0.00642	0.00931
11.5655	11.5709	11.6200	11.7005	11.6823	11.7603	11.7685	11.8331	11.8361	12.0073	12.0265	12.1401	12.1407	11.9790	11.9855	11.9987	11.9966	12.0258	12.0478	12.0570	12.0818	11.315	11.371	11.547	11.590	11.3133	11.1074
11.5390(9)	11.5408(3)	11.5571(5)	11.5837(7)	11.5777(7)	11.6034(7)	11.6061(9)	11.6273(3)	11.6283(7)	11.6841(7)	11.6903(7)	11.7270(6)	11.7272(7)	11.6749(1)	11.6770(3)	11.6813(3)	11.6806(1)	11.6901(2)	11.6972(3)	11.7002(1)	11.7082(2)	11.455(3)	11.474(3)	11.533(4)	11.547(4)	11.4545(8)	11.3846(8)
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77	77	77	77	77	77	77	77	77	77	77	77	77	78	78	78	78	78	78	78	78	79	79	79	79	80	80

0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	8.223
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000
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0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
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0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000
20	20	20	20	20	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25
8.57	11.53	15.28	24.07	32.47	16.1	17.95	20.65	22.23	22.41	23.02	23.78	25.52	26.82	28.76	29.88	30.91	33.38	2.3	2.49	3.28	4.85	9	6.88	8.03	9.36	10.75
0.01737	0.01105	0.00397	-0.02601	-0.00927	-0.00202	-0.01248	-0.00440	-0.00456	-0.01925	-0.01105	-0.00606	-0.01039	-0.00419	-0.00299	-0.00776	-0.00522	-0.00437	-0.02188	-0.02093	-0.01943	-0.01279	-0.02564	-0.01302	-0.01139	-0.00823	-0.00590
10.8308	10.6762	10.4959	10.1112	9.8398	10.4562	10.3670	10.2656	10.2041	10.1825	10.1676	10.1442	10.0763	10.0363	9.9705	9.9281	9.8966	9.8182	11.1499	11.1391	11.0922	11.0062	10.9281	10.8923	10.8323	10.7665	10.6993
11.2893(7)	11.2353(7)	11.1717(8)	11.0335(8)	10.9339(9)	11.1576(9)	11.1258(8)	11.0894(9)	11.0672(8)	11.0594(8)	11.0540(14)	11.0455(9)	11.0208(9)	11.0062(7)	10.9821(10)	10.9665(9)	10.9549(8)	10.9259(8)	11.3991(8)	11.3954(7)	11.3794(7)	11.3499(8)	11.3230(7)	11.3106(6)	11.2898(7)	11.2669(7)	11.2434(9)
8140	8141	8142	8143	8144	·	·	·	,	,	,	,	,	·	,	,	ı	,	,	ı	ı	,	,	,	,	ı	
80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80

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0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000
25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25
11.44	12.03	13.36	13.89	14.6	15.61	16.74	17.51	19.24	21.58	0.0001	2.3	2.49	3.28	4.85	5.99	6.88	8.05	9.36	10.75	11.8	13	14.03	11.6	12.67	14.9	17.3
06600.0-	-0.00833	-0.00583	-0.01236	-0.01357	-0.00821	-0.01263	-0.01077	-0.01187	0.02529	-0.02383	-0.01771	-0.01983	-0.01990	-0.01715	-0.03624	-0.01797	-0.02314	-0.02236	-0.02273	-0.02069	-0.01755	-0.02429	-0.02719	-0.02983	-0.02309	-0.01844
10.6616	10.6349	10.5750	10.5441	10.511	10.471	10.418	10.387	10.315	10.259	11.5070	11.3762	11.3632	11.3180	11.2338	11.1535	11.1250	11.0598	10.9951	10.9273	10.8796	10.8271	10.7737	10.8824	10.8300	10.7361	10.6372
11.2302(10)	11.2208(8)	11.1997(11)	11.1888(11)	11.177(1)	11.163(2)	11.144(3)	11.133(4)	11.107(3)	11.087(4)	11.5195(8)	11.4757(9)	11.4713(7)	11.4561(8)	11.4276(7)	11.4003(10)	11.3906(9)	11.3683(10)	11.3461(10)	11.3227(8)	11.3062(8)	11.2880(10)	11.2694(10)	11.3072(8)	11.2890(3)	11.2563(7)	11.2216(8)
80	80	80	80	80	80	80	80	80	80	81	81	81	81	81	81	81	81	81	81	81	81	81	81	81	81	81

0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	8.225
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.000	3.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000	3.000
25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25
19.4	21.3	0.0001	2.3	2.49	3.28	4.85	6.17	6.88	8.05	9:36	10.75	12	13.3	14.6	0.0001	2.3	2.49	3.28	4.85	4.97	6.17	6.88	8.05	9:36	10.75	11.8
-0.02522	-0.02106	-0.02467	-0.01540	-0.02044	-0.01770	-0.01741	-0.02512	-0.02019	-0.02217	-0.02506	-0.02862	-0.02382	-0.01859	-0.02362	-0.02993	-0.02393	-0.02707	-0.02719	-0.02240	-0.02588	-0.03160	-0.02625	-0.03077	-0.03172	-0.03709	-0.02962
10.5433	10.4714	11.7706	11.6381	11.6218	11.5785	11.4902	11.4111	11.3786	11.3166	11.2485	11.1781	11.1247	11.0712	11.0091	12.4976	12.3399	12.3239	12.2713	12.1761	12.1653	12.0871	12.0509	11.9799	11.9074	11.8291	11.7834
11.1885(13)	11.1630(13)	11.6068(8)	11.5631(8)	11.5577(8)	11.5433(9)	11.5139(8)	11.4874(9)	11.4765(8)	11.4556(10)	11.4326(10)	11.4087(9)	11.3905(9)	11.3722(10)	11.3509(12)	11.8410(8)	11.7910(7)	11.7859(8)	11.7691(9)	11.7386(9)	11.7351(9)	11.7099(7)	11.6982(8)	11.6752(9)	11.6516(9)	11.6260(8)	11.6110(12)
81	81	81	81	81	81	81	81	81	81	81	81	81	81	81	81	81	81	81	81	81	81	81	81	81	81	81

										ber	ntification num	all-Wallace 2003) ider	e (Downs and H	Structure Databas	ogist Crystal S	an Mineral
0.000	3.000	0.000	0.000	0.000	0.000	2.000	0.000	0.000	3.000	25	14	-0.05308	12.2463	11.7611(10)		
0.000	3.000	0.000	0.000	0.000	0.000	2.000	0.000	0.000	3.000	25	13	-0.03963	12.3123	11.7822(10)	·	
0.000	3.000	0.000	0.000	0.000	0.000	2.000	0.000	0.000	3.000	25	11.8	-0.04593	12.3710	11.8009(10)		
0.000	3.000	0.000	0.000	0.000	0.000	2.000	0.000	0.000	3.000	25	10.75	-0.03438	12.4413	11.8232(10)		
0.000	3.000	0.000	0.000	0.000	0.000	2.000	0.000	0.000	3.000	25	9.36	-0.02758	12.5286	11.8508(12)		
0.000	3.000	0.000	0.000	0.000	0.000	2.000	0.000	0.000	3.000	25	8.05	-0.02528	12.6100	11.8764(10)		
0.000	3.000	0.000	0.000	0.000	0.000	2.000	0.000	0.000	3.000	25	6.88	-0.02274	12.6860	11.9002(9)		
0.000	3.000	0.000	0.000	0.000	0.000	2.000	0.000	0.000	3.000	25	5.99	-0.03985	12.7266	11.9129(10)	ı	
0.000	3.000	0.000	0.000	0.000	0.000	2.000	0.000	0.000	3.000	25	4.97	-0.03374	12.8011	11.9361(9)	ı	
0.000	3.000	0.000	0.000	0.000	0.000	2.000	0.000	0.000	3.000	25	4.85	-0.01908	12.8240	11.9432(7)	·	
0.000	3.000	0.000	0.000	0.000	0.000	2.000	0.000	0.000	3.000	25	3.28	-0.01896	12.9348	11.9775(8)	ı	
0.000	3.000	0.000	0.000	0.000	0.000	2.000	0.000	0.000	3.000	25	2.49	-0.02134	12.9906	11.9947(8)		
0.000	3.000	0.000	0.000	0.000	0.000	2.000	0.000	0.000	3.000	25	2.3	-0.01446	13.0117	12.0012(7)	ï	
0.000	3.000	0.000	0.000	0.000	0.000	2.000	0.000	0.000	3.000	25	0.0001	-0.02445	13.1832	12.0537(9)		

# **Supplement 3.** Unconstrained standard deviation of fit parameters

	1σ <sup>*</sup>		
$W_{\sf grs-pyr}$	6.8688	(J/bar/mol)	
$W_{\sf grs-alm}$	0.7088		
W <sub>grs-knr</sub>	39.4974		
$W_{\sf grs-maj}$	1.9888		
$W_{\sf grs-nag}$	11.8925		
W <sub>grs-mmr</sub>	4.3177		
W <sub>grs-sps</sub>	0.7102		
$W_{ m pyr-knr}$	40.8569		
$W_{\sf pyr-maj}$	0.2379		
$W_{\sf pyr-kho}$	5.3615		
W <sub>pyr-mmr</sub>	6.2227		
$W_{alm\text{-}kho}$	25.0387		
$W_{knr-kho}$	8.2109		
$W_{kho}$ -sps	7.8349		
$dW_{ m grs-pyr}$	9.3804		
$dW_{ m grs-alm}$	0.2898		
$dW_{ m grs-nag}$	24.7981		
$dW_{ m pyr-knr}$	88.0329		
$dW_{ m pyr-kho}$	10.3325		
V <sup>grs</sup> o	0.0014		

V <sup>pyr</sup> ₀	0.0016	
$V^{ m alm}_{ m o}$	0.0024	
$V^{knr}{}_{o}$	279.9645	
V <sup>sps</sup> o	0.2312	
*Bootstrap esti	mation of s.d. for eac	h parameter,
varying all othe	r fit parameters	

	V <sup>kn</sup> <sub>o</sub> V <sup>sps</sup> <sub>o</sub>	0.9691 0.0007	- 0.6469 0.0073	- 0.9949 0.0007	- 0.0104 0.0056	- 0.0075 0.0073	0.2160 0.0233	- 0.0167 0.9114	0.9845 0.0004	- 0.0133 0.0123	- 0.7761 0.0105	- 0.1519 0.0199	- 0.9863 0.0004	 0.4491 0.0001	- 0.0885 0.0201	- 0.9615 0.0008	 0.1531 0.0091	0.0003 0.0000	0.9982 0.0013	0.9937 0.0023	0.0054 0.0336	0.0190 0.0155	0.0227 0.0063	1.0000 0.0009	0.0009 1.0000
	V <sup>alm</sup> o	- 5 0.0193	9 0.0205	7 0.0227	- 3 0.0352	3 0.0035	5 0.0190	- 5 0.0119	- 5 0.0274	- 3 0.0102	5 0.0338	5 0.0218	9 0.0257	- 1 0.0055	- 9 0.0024	1 0.0274	- 1 0.0077	7 0.0108	- 9 0.0209	7 0.0225	3 0.0065	0.0195	5 1.0000	- 0.0227	5 0.0063
	V <sup>DVr</sup> o	0.0185	0.026	3 0.0207	3 0.0103	- 9 0.0123	- 5 0.0166	t 0.0145	, 0.0226	- t 0.0233	3 0.0356	0.0125	0.023	0.0021	- 5 0.0129	t 0.0251	1 0.0301	l 0.0167	- 3 0.0169	0.0197	0.0093	3 1.0000	0.0195	- t 0.0190	0.0155
	V <sup>ers</sup> o	0.0042	0.000	- 0.0038	- 0.0058	0.0019	- 0.0095	0.0224	0.0067	0.0374	- 0.0083	- 0.0027	0.0059	0.0016	0.0006	- 0.0064	1 0.0381	0.0151	0.0053	0.0050	1.0000	, 0.0093	0.0065	0.0054	- 0.0336
	dW <sub>pyr</sub> . kho	- 0.9636	0.609	0.9970	0.0128	- 0.0066	0.2094	0.0141	0.9760	0.0212	0.772	0.146	066.0	0.4391	0.0802	0.969	0.1534	0.0012	- 0.9891	1.0000	0.0050	0.0197	0.0225	- 0.9937	0.002
	dW <sub>pyr-</sub> knr	0.9730	- 0.6312	- 0.9911	0.0121	0.0081	0.2122	0.0145	0.9746	0.0067	0.7448	- 0.1531	- 0.9758	0.4902	0.0902	- 0.9461	0.1606	0.0004	1.0000	0.9891	0.0053	0.0169	0.0209	0.9982	0.0013
	dW <sub>grs-</sub>	0.000	0.0035	- 0.0005	- 0.0265	- 0.4812	0.1259	0.0008	- 0.005	0.0008	0.0035	0.1461	0.000	0.0021	- 0.0267	- 0.002	0.0103	1.0000	0.000	0.0012	0.0151	0.0167	0.0108	0.0003	0.000
	dW <sub>grs-</sub> alm	- 0.1769	- 0.0110	0.1590	0.0180	0.0080	0.0696	0.0216	0.1258	0.001	- 0.000	0.0967	0.1264	0.2269	0.0100	0.1040	1.0000	- 0.0103	- 0.1606	0.1534	0.0381	0.0301	0.0077	- 0.1531	- 0.0091
	dW <sub>grs-</sub>	- 0.8835	0.6930	0.9617	0.0033	- 0.0041	0.2164	0.0255	0.9743	0.0484	0.8949	0.1360	0.9917	0.2200	0.0773	1.0000	0.1040	0.0002	0.9461	0.9694	0.0064	0.0251	0.0274	0.9615	3000.0
	W <sub>kho</sub> .	- 0.0837	0.1061	0.0841	- 0.1005	0.0100	- 0.1574	- 0.0495	- 0.0867	0.0186	0.0687	- 0.0700	0.0802	0.0410	1.0000	0.0773	0.0100	0.0267	- 0.0902	0.0802	0.006	0.0129	0.0024	0.0885	0.0201
	W <sub>knr-</sub> <sup>kho</sup>	- 0.5960	- 0.0961	0.4589	0.0348	-	0.0479	0.0239	- 0.3200	0.1131	-	0.0882	0.3283	1.0000	0.0410	0.2200	0.2269	0.0021	0.4902	0.4391	0.0016	0.0021	0.0055	0.4491	0.0001
	W <sub>alm</sub> . <sup>kho</sup>	- 0.9366	0.6724	0.9876	0.0057	0.0056	0.2199	0.0230	- 0.9859	0.0326	0.8516	0.1441	1.0000	0.3283	0.0809	0.9917	0.1264	0.0003	0.9758	2066.0	0.0059	0.0239	0.0257	0.9863	0.0004
	W <sub>pyr-</sub>	- 0.1536	0.1088	0.1525	0.0107	0.0898	0.7048	0.0395	0.1454	0.0227	0.1087	1.0000	0.1441	0.0882	0.0700	0.1360	0.0967	0.1461	0.1531	0.1464	0.0027	0.0125	0.0218	0.1519	0.0199
	W <sub>pyr-</sub> <sub>kho</sub>	- 0.6599	0.7922	0.7694	- 0.0230	- 0.0003	- 0.2198	- 0.0533	- 0.8427	- 0.0709	1.0000	0.1087	0.8516	- 0.1593	0.0687	0.8949	-0.0009	0.0035	- 0.7448	0.7723	0.0083	0.0356	0.0338	- 0.7761	0.0105
	W <sub>pyr-</sub>	- 0.0130	- 0.0336	- 0.0116	- 0.0281	- 0.0252	- 0.0386	0.0010	0.0310	1.0000	- 0.0709	0.0227	- 0.0326	0.1131	0.0186	- 0.0484	0.0091	0.0008	0.0067	0.0212	0.0374	0.0233	0.0102	0.0133	- 0.0123
	W <sub>pyr-</sub> <sup>knr</sup>	0.9329	- 0.7005	- 0.9743	- 0.0039	0.0055	0.2217	0.0224	1.0000	0.0310	- 0.8427	- 0.1454	- 0.9859	- 0.3200	- 0.0867	- 0.9743	- 0.1258	0.0005	0.9746	0.9760	0.0067	0.0226	0.0274	0.9845	0.0004
	W <sub>grs-sps</sub>	0.0130	- 0.0450	- 0.0158	- 0.0110	0.0113	- 0.0332	1.0000	0.0224	0.0010	- 0.0533	- 0.0395	- 0.0230	0.0239	- 0.0495	- 0.0259	0.0216	0.0008	0.0145	0.0141	0.0224	0.0145	0.0119	0.0167	- 0.9114
	W <sub>grs-</sub>	0.2106	- 0.2294	- 0.2160	- 0.0639	- 0.1435	1.0000	- 0.0332	0.2217	- 0.0386	- 0.2198	0.7048	- 0.2199	- 0.0479	- 0.1574	- 0.2164	0.0696	0.1259	0.2122	0.2094	0.0095	0.0166	0.0190	0.2160	0.0233
	W <sub>grs-</sub>	0.0091	- 0.0047	- 0.0075	0.1808	1.0000	- 0.1435	0.0113	0.0055	- 0.0252	- 0.0003	- 0.0898	0.0056	- 0.0151	0.0100	- 0.0041	- 0.0080	- 0.4812	0.0081	0.0066	0.0019	0.0123	0.0035	0.0075	0.0073
	W <sub>grs-</sub>	- 0.0116	- 0.0425	0.0126	1.0000	0.1808	- 0.0639	- 0.0110	- 0.0039	- 0.0281	- 0.0230	- 0.0107	0.0057	0.0348	- 0.1005	0.0033	0.0180	- 0.0265	- 0.0121	0.0128	0.0053	0.0103	0.0352	- 0.0104	0.0056
trix	W <sub>grs-knr</sub>	- 0.9733	0.6210	1.0000	0.0126	- 0.0075	- 0.2160	- 0.0158	- 0.9743	- 0.0116	0.7694	0.1525	0.9876	0.4589	0.0841	0.9617	0.1590	0.0005	- 0.9911	0.9970	0.0038	0.0207	0.0227	- 0.9949	0.0007
relation ma	W <sub>grs-</sub>	- 0.5583	1.0000	0.6210	- 0.0425	- 0.0047	- 0.2294	- 0.0450	- 0.7005	- 0.0336	0.7922	0.1088	0.6724	- 0.0961	0.1061	0.6930	- 0.0110	0.0035	- 0.6312	0.6091	0.0006	0.0269	0.0205	- 0.6469	0.0073
nent 4. Cor	W <sub>grs-pyr</sub>	1.0000	- 0.5583	- 0.9733	- 0.0116	0.0091	0.2106	0.0130	0.9329	- 0.0130	- 0.6599	- 0.1536	- 0.9366	- 0.5960	- 0.0837	- 0.8839	- 0.1769	0.0004	0.9730	0.9636	0.0042	0.0185	0.0193	0.9691	0.0007
Suppler		W <sub>grs-pyr</sub>	$W_{\rm grs-alm}$	W <sub>grs-knr</sub>	W <sub>grs-maj</sub>	W <sub>grs-nag</sub>	W <sub>grs-</sub>	W <sub>grs-sps</sub>	W <sub>pyr-knr</sub>	W <sub>pyr-maj</sub>	W <sub>pyr-kho</sub>	mmr	Walm- kho	W <sub>knr-kho</sub>	W <sub>kho-sps</sub>	dWgrs-	alm alm	nag nag	knr knr	kho	V <sup>ers</sup> o	$V^{pyr}$	$V^{\rm alm}_{o}$	$V^{knr}_{o}$	V <sup>sps</sup> o

Chapter 3

## NUCLEAR RESONANT INELASTIC X-RAY SCATTERING OF (Mg,Fe)SiO<sub>3</sub> ORTHOENSTATITES

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#### ABSTRACT

We present nuclear resonant inelastic X-ray scattering (NRIXS) measurements on synthetic samples of orthoenstatite-structured (Mg, <sup>57</sup>Fe)SiO<sub>3</sub>, a representative component in Earth's upper mantle. All measurements were performed at ambient conditions. NRIXS spectra were measured for three samples of orthoenstatite containing 20, 13, and 7 mol% FeSiO<sub>3</sub>. The Debye sound velocities were determined from the low-energy region of the partial phonon density of states (PDOS). With known density and bulk modulus, the shear modulus ( $\mu$ ), compressional ( $\nu_P$ ) and shear ( $\nu_S$ ) wave velocities have been computed. The sound velocities obtained from NRIXS are in good agreement with sound velocities obtained using Brillouin spectroscopy and ultrasonic methods for similar compositions. An important advantage of NRIXS is access to additional thermodynamic information, such as the average force constant, mean-square displacement, obtained from the PDOS. We discuss the contribution of the vibrational spectra to these quantities.

#### **INTRODUCTION**

Some of the best-resolved properties throughout Earth's interior are seismologically determined sound velocities, which probe the in situ state of crustal, mantle, and core material with high spatial resolution. Accurate determination of the sound velocities of deep Earth materials is therefore essential for mapping chemical and thermal properties of Earth's interior to seismic observations (e.g., Bass and Anderson 1984; Wagner et al. 2008). Experimental methods to determine the compressional and shear sound velocities of materials include: ultrasonic interferometry (US), impulsively stimulated light scattering (ISLS), Brillouin inelastic light scattering (BS), inelastic neutron scattering (INS), momentum-resolved inelastic X-ray scattering (IXS), and nuclear resonant inelastic X-ray scattering (NRIXS). All of the above-mentioned methods can be applied to single-crystal or polycrystalline specimens. Selective vibrational quantities are obtained for each method. For example, US, BS, and ISLS provide access to the lowenergy (long-wavelength) vibrational states: the sound velocities. The neutron-weighted density of states (DOS) obtained by INS requires larger samples, which in turn strongly limits the highest pressure that can be obtained. IXS provides experimental access to specific phonon branches under extreme pressures (e.g., Antonangeli et al. 2004) and has been successfully combined with theoretical phonon calculations (e.g., Ghose et al. 2006). However, in the case of IXS, one must measure over all momentum-space to determine the DOS, which often requires months of data collection. Under extreme conditions (e.g., confined systems, magnetic fields, high-pressures, high-temperatures), the DOS obtained from NRIXS is much more accessible than other methods and provides access to thermodynamic quantities. The importance in determining accurate thermodynamic quantities from measured vibrational spectra is imperative for accurate modeling of Earth's interior (e.g., Kieffer 1982). Further comparisons and descriptions of these methods can be found in Angel et al. (2009) and references therein. NRIXS requires a sample bearing a nuclear resonant isotope and has been applied to single-crystals or powdered samples as small as 10 mm laterally (and 1 mm thick).

NRIXS is a high-resolution X-ray spectroscopic method that provides direct access to the partial phonon density of states (PDOS) of the nuclear resonant isotope, <sup>57</sup>Fe in this case. That is, all lattice vibrations involving <sup>57</sup>Fe-nuclei contribute to the measured PDOS, and one may obtain averaged thermodynamic quantities related to the <sup>57</sup>Feparticipating nuclei, including: vibrational specific heat per atom at constant volume  $(c_{\rm V})$ , vibrational entropy per atom ( $S_{vib}$ ), Lamb-Mössbauer factor ( $f_{LM}$ ), mean force constant (D), vibrational kinetic energy  $(E_K)$ , and vibrational kinetic energy at 0 K  $(E_Z)$  (Sturhahn 2004). From the kinetic energy of the <sup>57</sup>Fe-nuclei, one can calculate the  $\beta$ -factor, which relates the equilibrium iron isotope fractionation between two substances (e.g., Polyakov et al. 2007). With known density, the Debye sound velocity is obtained from the lowenergy portion of the PDOS. If the bulk modulus of the material is known, the compressional and shear wave velocities and shear modulus can be computed. NRIXS studies related to geophysical applications have primarily been conducted on high symmetry and/or iron-rich materials (Giefers et al. 2000; Mao et al. 2001; Struzhkin et al. 2001; Hu et al. 2003; Mao et al. 2004; Lin et al. 2005; Lin et al. 2006; Gao et al. 2008),

but far fewer measurements have been conducted on low symmetry phases (Mao et al. 2006; Gao et al. 2008). We discuss the relevance of symmetry to the PDOS below.

In this contribution, we present NRIXS measurements for three powdered synthetic samples of orthorhombic-structured (Mg,<sup>57</sup>Fe)SiO<sub>3</sub> orthoenstatite containing representative upper mantle iron concentrations of 20, 13, and 7 mole percent FeSiO<sub>3</sub>. We show that the sound velocities of orthoenstatite determined from NRIXS are in good agreement with previous ultrasonic (Kumazawa 1969; Frisillo and Barsch 1972; Webb and Jackson 1993; Flesch et al. 1998; Kung et al. 2004) and Brillouin scattering studies (Weidner et al. 1978; Bass and Weidner 1984; Duffy and Vaughan 1988; Jackson et al. 1999, 2007) on similar compositions. Analysis of the raw NRIXS data and interpretation of results were completed as part of the author's first-year graduate work and were published in Jackson et al. (2009). These sections have been extracted from that work and are presented here.

#### SAMPLE DESCRIPTION

Three powdered samples of (Mg,<sup>57</sup>Fe)SiO<sub>3</sub> orthoenstatite containing 20, 13, and 7 mole percent FeSiO<sub>3</sub> were prepared for the NRIXS measurements. The <sup>57</sup>Fe-enriched polycrystalline samples were synthesized from oxides in a piston-cylinder apparatus at the Geophysical Laboratory of the Carnegie Institution of Washington. The starting material consisted of 95% enriched <sup>57</sup>Fe<sub>2</sub>O<sub>3</sub>, SiO<sub>2</sub>, and MgO. The <sup>57</sup>Fe<sub>2</sub>O<sub>3</sub> was reduced to FeO in a gas-mixing furnace. SiO<sub>2</sub> and MgO were furnace-fired in an effort to dehydrate the starting materials. Synthesis conditions in the piston cylinder were 1.5 GPa and 1000 °C for a duration of 48 hours. Verification of the structure (space group: *Pbca*) and chemistry of the samples were obtained via powder X-ray diffraction (XRD) and electron microprobe analysis (EMPA), respectively. (Mg,<sup>57</sup>Fe)SiO<sub>3</sub> was identified as the only phase present at the resolution of the above-mentioned techniques (XRD: 5 volume % and EMPA: 5 μm).

#### NRIXS EXPERIMENTS

The NRIXS experiments were performed at sector 3 ID-B of the Advanced Photon Source (APS) at Argonne National Laboratory under ambient conditions. The energy bandwidth of the incident X-rays determines the resolution of the phonon spectra of the samples. The X-rays were prepared with bandwidths of 1 meV using a multiple-crystal Bragg reflection monochromator (Toellner 2000). We used a Kirkpatrick-Baez mirror system to obtain a focal spot size of  $6 \times 6 \mu m^2$  at the full width at half maximum (Zhao et al. 2004). The X-ray flux in this spot was  $8 \times 10^8$  ph/s, and the resulting spectral flux density was  $2 \times 10^{16}$  ph/s/eV/mm<sup>2</sup> (Sturhahn 2004). The storage ring was operated in low-emittance top-up mode with 24 bunches that were separated by 153 ns. The ~2 mm thick samples were mounted in air on holders for the NRIXS measurements. For each spectrum, the monochromator was tuned from -80 to +100 meV (in 0.25 meV step size with 5 s collection time per energy point) around the nuclear resonance energy of <sup>57</sup>Fe. 14.4125 keV. The radiation emitted from the samples was observed with two avalanche photodiode detectors. One detector was placed close to the sample (~2 mm away) to collect the incoherent inelastic scattered photons, and the other detector was placed

downstream (100 cm) in the forward scattering direction, in order to obtain the resolution function independently (Fig. 1). High counting rates were achieved due to the thickness and enrichment of the samples. Therefore, one spectrum per composition was collected. The raw NRIXS spectra are shown in Fig. 2.

#### DETERMINATION OF THE PDOS AND SOUND VELOCITIES

The NRIXS method directly provides the Fourier-transformed self-intermediate scattering function,  $S(\mathbf{k}, E) = \frac{1}{2\pi\hbar} \int \langle e^{i\mathbf{k}\mathbf{r}(t)} e^{-i\mathbf{k}\mathbf{r}(0)} \rangle e^{iEt/\hbar} dt$ , where  $\hbar$  is Plank's constant,  $\mathbf{k}$ is the wave vector of the X-rays incident on the sample, and r(t) is the displacement operator of the resonant nucleus (Sturhahn 2004). The quasi-harmonic model of lattice vibrations is then used to extract the partial (due to information about motions of the resonant nuclei only) and projected (due to a potential angular dependence on k) phonon DOS from S(k,E) (Sturhahn et al. 1998; Sturhahn 2000) for each composition (Fig. 3). The dependence of  $S(\mathbf{k}, E)$  on the direction of the incident X-rays is implicitly contained in  $S(\mathbf{k}, E)$  and is expressed via the directional dependence of the phonon DOS. The description of the anisotropy of the PDOS is given by a symmetric second-rank tensor (e.g., Sturhahn and Kohn 1999), whereas the elastic anisotropy requires a symmetric fourth-rank tensor. For a sample characterized with symmetry lower than cubic, direct inversion of the measured NRIXS spectrum provides a reliable value of the averaged PDOS, if the Lamb-Mössbauer factor remains high (see Sturhahn and Jackson 2007, for a detailed discussion). In the case of orthoenstatite, the Lamb-Mössbauer factor remains high for all three compositions,  $\sim 0.7$  (see following section).

As explained in previous reports, the energy of an acoustic mode  $a(E_a)$  with a small wave number q (long wavelength) that propagates in the direction  $\mathbf{q}$  is given by  $E_a = \hbar q v_a(\mathbf{q})$ , where  $v_a$  is the sound velocity of mode a. The number of phonon states  $(N_a)$  in momentum space is then  $dN_a = V \int k_a^2 dk_a d\Omega_q$ , where  $k_a = E_a/(\hbar v)$ , V is a normalization volume, and the integration is performed over all directions  $\mathbf{q}$ , symbolized by " $d\Omega_q$ ". The linear phonon dispersion leads to a Debye-like phonon DOS:

$$D(E) = \frac{m}{2\pi^2 \hbar^3 \rho} \frac{1}{v_D^3} E^2,$$
 (1)

with 
$$\frac{1}{v_D^3} = \frac{1}{3} \sum_a \int \frac{1}{v_a^3(\mathbf{q})} \frac{d\Omega_q}{4\pi}$$
, (2)

where  $v_D$  is the Debye sound velocity,  $\rho$  is the density of the material, and *m* is the mass of the nuclear resonant isotope, <sup>57</sup>Fe in this case. This relationship is exact for sufficiently small energies (long phonon wavelengths).

The quantitative description of the low-energy region of the phonon DOS provides the Debye sound velocity,  $v_{\rm D}$ . However, the derivation of the Debye sound velocity relies on a linear dispersion that will only be accurate within a limited energy range. Obtaining high-resolution data at low energies is crucial for studies like the present case, where iron is the heavy element in a relatively light matrix of Mg, Si, and O, thereby providing only a small window of accessible low-energy phonon-energies to evaluate the sound velocities. A systematic evaluation of the errors resulting in the selective energy interval  $(E_1, E_2)$  of the low-energy portion of the PDOS is therefore necessary in the present case, where  $E_1$  and  $E_2$  represent the low- and high-energy cutoffs in the PDOS. The energy interval  $(E_1, E_2)$  for each PDOS is as follows: En80 (2.8, 10.8 meV), En87: (3.8, 12.8 meV), and En93 (6.8, 12 meV). The ranges were selected so that the elastic contribution was less than 10% over the selected energy range, therefore ensuring that the influence of elastic scattering is small compared to the total scattering. Using these selective energy intervals and an improved empirical relation for the dispersion of the acoustic phonons at low-energy (Sturhahn and Jackson 2007), the Debye sound velocity has been determined from the PDOS of each composition. We show a representative fit procedure in Fig. 4 for En80.

For an isotropic solid, the relationship of the Debye sound velocity to the compressional,  $v_P$ , and shear,  $v_S$ , velocities is determined from Eqs. 1 and 2 (Sturhahn and Jackson 2007):

$$\left(\frac{3}{v_D^3}\right) = \left(\frac{1}{v_P^3}\right) + \left(\frac{2}{v_S^3}\right).$$
(3)

One can see from the above relationship that the Debye sound velocity is heavily weighted toward the shear sound velocity. With known density ( $\rho$ ) and adiabatic bulk modulus ( $K_S$ ), the isotropic  $v_P$  and  $v_S$  and shear modulus ( $\mu$ ) follow the additional relationships:

$$\frac{K_s}{\rho} = v_{\phi}^2 = v_P^2 - \frac{4}{3}v_s^2, \tag{4}$$

$$\frac{\mu}{\rho} = v_s^2 \,. \tag{5}$$

Combining Eqs. 3 and 4, one obtains the approximate general solutions for  $v_S$  and  $v_P$  (within 0.1% error, see Sturhahn and Jackson 2007):

$$v_S = 0.952 v_D - 0.041 v_\phi \tag{6}$$

and

$$v_P = 0.908v_{\phi} + 0.297v_D + (0.243v_D^2/v_{\phi}).$$
<sup>(7)</sup>

The density of our samples was determined from their measured volumes and chemistry and corrected for their natural iron-enrichment. The  $K_S$  values for this study were determined from a linear regression of the Brillouin scattering results for the Mg-(Weidner et al. 1978; Jackson et al. 1999, 2007) and Fe- (Bass and Weidner 1984) end members. In Figs. 5 and 6, we plot our determined sound velocities and elasticity of (Mg,Fe)SiO<sub>3</sub> orthoenstatites from NRIXS (Table 1) along with results from previous ultrasonic and Brillouin scattering measurements. In the cases where the full elastic tensor was available, the  $v_D$  values were determined from the Christoffel equation and Eqs. 1 and 2. If only the isotropic values of  $v_S$  and  $v_P$  were given, Eq. 3 was used to determine  $v_D$ . We find good agreement with the results from NRIXS in comparison with Brillouin (Weidner et al. 1978; Bass and Weidner 1984; Duffy and Vaughan 1988; Jackson et al. 1999, 2007) and ultrasonic measurements (Kumazawa 1969; Frisillo and

Barsch 1972; Webb and Jackson 1993; Flesch et al. 1998; Kung et al. 2004) on similar compositions. Within the experimental uncertainties of our NRIXS data, all three ironbearing compositions exhibit the same Debye sound velocity. Reports from ultrasonic measurements on similar compositions also appear independent of iron content for some elastic parameters (Kumazawa 1969; Frisillo and Barsch 1972; Webb and Jackson 1993; Kung et al. 2004). In Fig. 6, we also include the bulk modulus obtained on a suite of synthetic (Mg,Fe)SiO<sub>3</sub> samples determined from single-crystal X-ray diffraction measurements (Hugh-Jones and Angel 1994, 1997), using the isothermal to adiabatic bulk modulus conversion of  $K_{0S} = K_{0T}(1 + \alpha \gamma T)$  at 300 K, where  $\alpha = 3.2 \times 10^{-5} \text{ K}^{-1}$  and  $\gamma = 1.009$  (Angel and Jackson 2002). Natural orthopyroxenes (OPX) are known to contain minor amounts of Al and Ca, in addition to iron. The substitution of small amounts of Ca into the M2 octahedral site in OPX does not significantly affect its elasticity (Nestola et al. 2006; Perrillat et al. 2007). However, either the coupled substitution of Al into the tetrahedral and octahedral sites or simply the substitution of Al into the tetrahedral site of OPX appears to stiffen the bulk modulus of OPX, in comparison to the Mg end-member (Chai et al. 1997). The  $v_{\rm D}$  values calculated from the measured elastic tensors of the (Al,Fe)- (Chai et al. 1997) and Ca-bearing (Perrillat et al. 2007) orthopyroxenes are 5.37 km/s and 5.31 km/s, respectively, compared to a  $v_{\rm D}$  values of 5.40 km/s for MgSiO<sub>3</sub> (Jackson et al. 2007) and 5.20 km/s for (Mg<sub>0.8</sub>Fe<sub>0.2</sub>)SiO<sub>3</sub> (Frisillo and Barsch 1972) orthoenstatites.

#### THERMODYNAMIC PARAMETERS EXTRACTED FROM THE PDOS

The PDOS provides access to several thermodynamic quantities of the material (Sturhahn 2004). For the orthoenstatite samples measured, we determined the following from the PDOS:  $c_V$ , vibrational specific heat per atom at constant volume (k<sub>B</sub>/atom);  $S_{vib}$ , the vibrational entropy per atom (k<sub>B</sub>/atom);  $f_{LM}$ , the average Lamb-Mössbauer factor; D (N/m), the mean force constant;  $E_K$ , the vibrational kinetic energy;  $E_Z$ , the vibrational

kinetic energy at 0 K of the <sup>57</sup>Fe nucleus (Table 2; Fig. 7). These are thermodynamic parameters of the <sup>57</sup>Fe-participating vibrations. It is interesting to note that although the sound velocities of orthoenstatite do not show an obvious trend within the low ironconcentration region of the solid-solution, quantities derived from the whole spectrum (the PDOS) do show trends. The shape of the PDOS indeed changes as a function of iron content (Fig. 3). This can be understood quantitatively by analyzing the contributions of the vibrational spectrum to the individual thermodynamic quantities. The energy dependence of the PDOS, g(E), of the contribution to the mean force constant (*D*) is proportional to  $g(E) * E^2$  (Fig. 8). The higher energy regions (E > 10 meV) of the spectra differ significantly as a function of iron content.

In Fig. 9 we plot the energy dependence of the vibrational contributions to the vibrational specific heat per atom at constant volume ( $c_V$ ) for En80:

$$c_V = 3k_B \int E^2 [2k_B T \sinh(E/2k_B T)]^{-2} g(E) dE.$$
(8)

One can see that all energies are essentially weighted equally at 300 K, which results in minimal effects from  $c_V$  to the shapes of the PDOS. The differences will decrease even further at higher temperatures. This effect is the same for En87 and En93. The relationship between the PDOS, g(E), and the Lamb-Mössbauer factor ( $f_{LM}$ ) (Chumakov and Sturhahn 1999) is as follows:

$$-\ln f_{LM} = E_R \int \left(\frac{1+e^{-\beta E}}{1-e^{-\beta E}}\right) \frac{g(E)}{E} dE = k^2 \langle x^2 \rangle, \tag{9}$$

where *k* is the incident wave vector and  $\langle x^2 \rangle$  is the mean-square-displacement of the <sup>57</sup>Fe atoms. For vibrational entropy, the relationship is as follows (Sturhahn 2004):

$$S_{vib} = 3k_B \int \left\{ \frac{\beta E}{2} \left( \frac{e^{\beta E} + 1}{e^{\beta E} - 1} \right) - \ln \left[ e^{\frac{\beta E}{2}} - e^{-\frac{\beta E}{2}} \right] \right\} g(E) dE \,.$$
(10)

### **CONCLUDING REMARKS**

We have shown that the sound velocities of a suite of orthoenstatite samples measured by NRIXS are in good agreement with sound velocities obtained using ultrasonic and Brillouin scattering methods. More importantly, the lattice vibrations pertaining to the <sup>57</sup>Fe-nuclei have been measured. The quantities derived from NRIXS measurements of the PDOS for orthoenstatites offer unique insights into the behavior of iron in materials. The nature of the PDOS obtained from NRIXS involving only <sup>57</sup>Fe participating vibrations suggests that these measurements should be complemented with the total phonon density of states to obtain representative thermodynamic behavior of the entire sample.

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## TABLES

**Table 1.** Debye sound velocities and other elastic parameters of orthoenstatites determined from the low-energy region of the PDOS.

	Density	$v_D$	$v_S$	$v_P$	$K_{0S}^{*}$	μ
	(g/cc)	(km/s)	(km/s)	(km/s)	(GPa)	(GPa)
En93	3.26(1)	5.12(20)	4.62(2)	7.83(20)	107.2	70(6)
En87	3.31(1)	5.11(5)	4.62(5)	7.79(10)	106.8	71(2)
Em80	3.36(1)	5.08(7)	4.59(7)	7.23(11)	106.4	71(2)

Notes: (\*) fixed values obtained from a linear regression of previous Brillouin scattering results for MgSiO<sub>3</sub> and FeSiO<sub>3</sub> orthopyroxene as a function of density. A density correction (<1%) to account for natural iron-enrichment was applied to our data. Values in parentheses represent the error in the last significant digit(s).

		C	f	מ	E	E
_	$(k_{\rm B}/atom)$	$S_{vib}$ (k <sub>B</sub> /atom)	JLM	D (N/m)	(meV/atom)	(meV/atom)
En93	2.74(2)	3.61(2)	0.723(3)	195(5)	6.38(8)	14.7(1)
En87	2.75(1)	3.60(1)	0.730(2)	170(3)	6.15(6)	14.01(7)
En80	2.77(2)	3.71(2)	0.709(3)	165(5)	6.11(8)	15.1(1)

**Table 2.** The <sup>57</sup>Fe weighted thermodynamic parameters of orthoenstatites determined from the PDOS.

Notes:  $c_V$  - specific heat per atom at constant volume;  $S_{vib}$  - the vibrational entropy per atom;  $f_{LM}$  - the average Lamb-Mössbauer factor; D - the mean force constant;  $E_Z$ ,  $E_K$  - vibrational kinetic energy of the nucleus at 0 K and room-temperature, respectively.

## **FIGURE CAPTIONS**

**Fig. 1** Typical experimental set-up for NRIXS experiments at third generation synchrotron sources. For high-pressure experiments, focusing mirrors are placed after the monochromator and a diamond anvil cell contains the sample. The "SMS" detector is placed in the forward scattering direction and hence, measures the resolution function (RF) of the NRIXS spectrum independently (modified from Sturhahn and Jackson 2007).

**Fig. 2** Raw NRIXS energy spectra, S(E), of (Mg,<sup>57</sup>Fe)SiO<sub>3</sub> orthoenstatites. The elastic peak has not been removed. Data for En87 and En93 are shifted vertically. The errors in S(E) for En87 and En93 are similar to those of En80, but are not plotted for clarity.

**Fig. 3** Partial phonon density of states of iron in (Mg, <sup>57</sup>Fe)SiO<sub>3</sub> orthoenstatites. The PDOS for En87 and En93 are shifted vertically by 100 ev<sup>-1</sup> and 200 ev<sup>-1</sup>, respectively.

Fig. 4 Debye velocity ( $v_D$ ) determination for En80 using an improved method for extracting the sound velocity (see text). The fit was performed on the data (open circles) starting from 2.8 meV ( $E_1$ ) and ending at 10.8 meV ( $E_2$ ) (solid line), then extrapolated to E = 0 to determine  $v_D$  (dashed line). This fit produced a  $\chi^2$  of 0.33.

**Fig. 5** Debye ( $v_D$ , circles), compressional ( $v_P$ , diamonds), and shear ( $v_S$ , squares) velocities from this experiment (open symbols) along with previous measurements of the enstatite-ferrosilite solid solution series (filled symbols). The dashed lines represent a

linear regression of the enstatite and ferrosilite end-members from Brillouin scattering. Sources of data: 1 = Weidner et al. 1978; 2 = Flesch et al. 1998; 3 = Jackson et al. 1999; 4 = Kung et al. 2004; 5 = Jackson et al. 2007; 6 = Duffy and Vaughan 1988; 7 =Kumazawa 1969; 8 = Frisillo and Barsch 1972; 9 = Webb and Jackson 1993; 10 = Bass and Weidner 1984. A density correction (<1%) to account for natural iron-enrichment was applied to our data. In the cases where the full elastic tensor was available, the  $v_D$ values were determined from the Christoffel equation and Eqs. 1 and 2.

**Fig. 6** Adiabatic bulk and shear elastic moduli determined from this experiment (triangles) along with previous results (squares) for the orthoenstatite-orthoferrosilite solid solution series. Note that  $K_S$  was fixed in our study (see text). Symbols and reference numbers have the same meaning as in Fig. 5. Additionally,  $11 = (Mg,Fe)SiO_3$  orthoenstatite (Hugh-Jones and Angel 1997);  $12 = MgSiO_3$  orthoenstatite (Hugh-Jones and Angel 1997).

**Fig. 7** Selected thermodynamic parameters extracted from the PDOS of the orthoenstatites measured with NRIXS in this study: (a)  $c_V$ : vibrational specific heat per atom at constant volume (k<sub>B</sub>/atom), (b)  $S_{vib}$ : the vibrational entropy per atom (k<sub>B</sub>/atom), (c)  $f_{LM}$ : the Lamb-Mössbauer factor, and (d) the mean force constant, D (N/m).

**Fig. 8** The energy dependence of the vibrational contributions to the mean force constant (*D*). One can see that the higher energy regions of the spectra differ significantly as a function of iron content.

**Fig. 9** The energy dependence of the vibrational contributions to the vibrational specific heat at constant volume ( $c_V$ ), where  $k_B$  is the Boltzman factor and T = 300 K for En80. One can see that all energies are essentially weighted equal at this temperature, which results in minimal effects on the shape of the PDOS. This effect is the same for the two other compositions.

## FIGURES

Fig. 1



Fig. 2







Fig. 4



Fig. 5



Fig. 6



Fig. 7



Fig. 8



Fig. 9

