Essays on Contests, Coordination Games, and Matching

Thesis by

Mürüvvet İlknur Büyükboyacı Hanay

In Partial Fulfillment of the Requirements

for the Degree of

Doctor of Philosophy

California Institute of Technology

Pasadena, California

2013

(Defended December 3, 2012)

Acknowledgement

I will begin by recognizing my advisor, Leeat Yariv, for her patience, her responsiveness (the quickest replier of an e-mail), her generosity, her toughness (taking the saying “The pain that does not kill you makes you stronger” as a base, she kept the pain level so appropriate that it constantly improved me), and her supportiveness (she helped me keep my faith in myself). I want to express my sincere thankfulness to Thomas Palfrey both for his mentorship and his encouragement. I learned a lot from his knowledge and experience. I would also like to thank Federico Echenique for his guidance and advice. I am also grateful to Jean-Laurent Rosenthal for his caring. He gave a lot of help to all graduate students, including me, through pro-seminars (and by devoting his personal time to discuss the research of many grad students in the department). I cannot find enough words to explain how helpful these seminars and his comments were. I am also grateful to the rest of the department for their feedback during the seminars, especially Marina Agranov, Ben Gillen, and Peter Bossaerts. Last, I thank my master thesis advisor, Ozgur Kibris.

I also thank my friends in the department: Hatice Ozsoy, Julian Romero, Sera Linardi, Gabriel Katz, Andrea Robbett, Ines Levin, Sangmok Lee, Alice Chang, Theresa Gutberlet, Marjan Praljak, Guilherme de Freitas, Luke Boosey, Salvatore Nunnari, Emerson Melo, and Nilanjan Roy. I feel quite privileged to be a part of this close environment at Caltech (between faculty and students, and among the students). I also want to thank Laurel Auchampaugh for her administrative support.

I further thank to my old and current Turkish friends at Caltech, namely, Ozgun Konca, Caglar Tanrikulu, Ersen Bilgin, Grace Song, Omer Durak, Ercan Gurses, Necmiye Ozay and Samet Oymak. I also thank Nida Cakir for sharing the “Ph.D. in Economics” experience with me from the beginning and being such good friends to us (Selim and me) with her husband, Murat Melek. There are also other background supporters of me from old, good friends: Selen Pehlivan, Zeynep Yasemin Teksen, Gozde Yavuz, Ipek Gursel Tapki, Aslihan Dogan, Melek Aslan, Esin Eren, Yasemin Saldanli, and Nur Ayvaz. Thank you all.

Finally, I would like to acknowledge my parents, Abdulkadir Buyukboyaci and Fatime Buyukboyaci and my sister Gulsen Buyukboyaci. The most deserved thank you goes to my husband Mehmet Selîm Hanay from whom I got support in every part of this thesis both emotionally and technically.

Abstract

In this thesis, I use theory and experiments (sometimes only experiments), to investigate the impact of agents' heterogeneity on economic environments such as tournaments, decentralized matching, and coordination games.

The first chapter analyzes a coordination game (a stag-hunt game) in which one of the equilibria gives a higher payoff to the players, but playing the corresponding strategy profile leading to this equilibrium entails strategic risk. In this chapter, I ask whether agents can coordinate on the equilibrium that gives a higher payoff when they are provided information about an opponent's risk aversion. Two key insights result from my analysis. First, a subject's propensity to choose the risky action depends on her opponent's risk attitude. Second, this propensity is independent of the subject's own risk attitude.

The second chapter compares the performance of two tournament designs when contestants are heterogeneous in their abilities. One of the designs is the standard winner-take-all (WTA) tournament, which is common both in the literature and in the real world. The alternative tournament design involves two tournaments with different prizes (parallel tournaments) where individuals choose which tournament to enter before competing. With a simple model and an experiment, I show that when contestants' abilities differ substantially, the designer makes higher profit using parallel tournaments. Nevertheless, when the contestants' abilities are similar, the designer makes higher profit in the WTA tournament.

The third chapter studies a two-period decentralized matching game under complete information with frictions in the form of time discounting. I find that the sub-game perfect Nash equilibrium outcome of this game coincides with a stable outcome for most preference profiles. The selection of which stable outcome emerges depends on the level of frictions: the sub-game

perfect Nash equilibrium outcome of this game yields the firm-optimal stable match (a worker-optimal stable match) when the time discount is sufficiently high or low (intermediate values).

Table of Contents

List of Tables

List of Figures

Chapter 1:

Introduction

In chapter 2, Risk Attitudes and the Stag-Hunt Game, I analyze a coordination game (stag-hunt game). A stag-hunt game (with the risky and safe actions) has two pure Nash equilibria that are Pareto-rankable. The risky action leads either to the Pareto-superior equilibrium (high payoff) or to out of equilibrium (low payoff) depending on the other player's action. Both players may want to end up in the Pareto-superior equilibrium due to its payoff. However, uncertainty about the other player's action may prevent them to take such strategic risk. This paper investigates how information about the risk attitude of an opponent affects a player's action choice in the stag-hunt game with an experiment. In the experiment, I first elicit subjects' risk attitudes. I then allow subjects to play a one-shot stag-hunt game in which subjects are informed about the risk attitudes of their opponents. Finally, I elicit subjects' beliefs about their opponents' action choices. Two key insights result from the analysis. First, a subject's propensity to choose the risky action depends on her opponent's risk attitude. Second, this propensity is independent of the subject's own risk attitude.

Chapter 3, Parallel Tournaments, theoretically and experimentally compares a principal's profits from two tournament designs. The first design is a standard winner-take-all tournament with a single prize. The second design features two parallel tournaments with different prizes where individuals choose which tournament to enter before competing. I develop a simple model that illustrates how the relative performances of these designs change as contestants' abilities differ. The theoretical model shows that the designer's profit is higher (lower) in the parallel tournament when contestants' abilities differ greatly (are similar). I complement these findings with experimental evidence. The experiments show that the parallel tournament is more profitable under high heterogeneity, whereas under low heterogeneity the designer is better off with the single-prize tournament. Further, high-ability agents underparticipate and low-ability agents overparticipate in the high-prize tournament relative to the theoretical prediction.

Chapter 4, Decentralized Matching Market with Time Frictions, studies a two-period decentralized matching game under complete information with time discounting. The players of the game are firms and workers. They calculate their expected gains from their strategies (proposing to a worker or accepting/rejecting an offer) according to cardinal representation of their match utilities. Firms sequentially make directed offers to workers in the game. I show that the subgame perfect Nash equilibrium outcome of this game coincides with a stable outcome for a specific class of preference profiles. For these preference profiles, the selection of which stable outcome emerges depends on the level of frictions: the subgame perfect Nash equilibrium outcome yields the firm-optimal stable matching when the time discount is sufficiently low or high. However, for intermediate time discounts, the subgame perfect Nash equilibrium outcome yields a worker-optimal stable matching.

Chapter 2:

Risk Attitudes and the Stag-Hunt Game

2.1 Introduction

In many social interactions individuals try to learn (or know) other parties' characteristics and accordingly form expectations about their behavior. For instance, an insurance company evaluates how likely its customers are to be involved in an accident with questionnaires; a recruiter tries to foresee whether a job candidate is a good team worker in an interview; a landlord tries to determine whether a potential tenant will pay the rent on time through some clues during a conversation. In all these examples, some additional information decreases the risk of a bad outcome. This paper analyzes the outcome of a stag-hunt game that has a strategic risk, when players are provided information about their opponents' risk attitudes.

In a stag-hunt game, players choose between strategically safe and risky actions (respectively, Action 1 and Action 2 in Table 2.1). Depending on both a player's own and her opponent's action choices, players may end up in the payoff-dominant equilibrium ((Action 2, Action 2)), in the risk-dominant equilibrium ((Action 1, Action 1)), or out of equilibrium ((Action 1, Action 2), (Action 2, Action 1)).

	Action 1	**Action 2**
Action 1	a, a	a, c
Action 2	c, a	b, b

$b > a > c > 0$

Table 2.1: 2 x 2 Stag-Hunt Game

In this paper, I test whether information about their opponents' risk attitudes helps players prevent out of equilibrium outcomes and more importantly, helps them coordinate on the payoff-dominant equilibrium. A player's own risk attitude and information about the other person's risk attitude should affect a player's action (hence the game outcome) for the following reasons: Agents' utility representations differ according to their risk aversion.[1] In the game, risk-averse, risk-neutral, and risk-loving agents expect different payoffs from Action 1 (or Action 2) for the same belief about the other person's action choice. Hence, their optimal action choices may change as a response to their beliefs: It may be optimal for a risk-averse agent to choose Action 1 even when she thinks that her opponent chooses Action 1 with a low probability. On the other hand, it may be optimal for a risk-loving agent to choose Action 1 only when she thinks that her opponent chooses Action 1 with a high probability. This difference stems from the concave (convex) utility function of a risk-averse (risk-loving) agent. Similarly, when a player is informed about how risk averse the other person is, she will form her beliefs accordingly. She will expect a risk-averse opponent to choose Action 1 with a higher (lower) probability and best respond to her belief by choosing Action 1 (Action 2).

To test these predictions, I design an experiment with three stages. In the first stage, I use a common technique (due to Holt and Laury, 2002) to elicit subjects' risk aversion. This technique involves two lotteries: one risky and one safe. The riskiness of the lotteries is determined by the difference between the high and low payoffs. There are ten different situations and individuals are asked at which situation they want to switch from a "safe" lottery to a "risky" lottery (hereafter, the situation at which a subject switches from the "safe" lottery to the "risky" lottery is named as her 'risk threshold'). At a given situation the probability of obtaining the high payoff is identical in both lotteries. These probabilities range from 1/10 to 1 in increments of 1/10 between

[1] Some earlier studies (Bolton 1998; Camerer 1997; Holt and Laury 2002; Goeree, Holt, and Palfrey, 2002) show some deviations from the equilibrium predictions in experimental results when subjects' risk attitudes are not taken into account for the payoffs.

situations. The expected payoff of the risky lottery becomes higher than the expected payoff of the safe lottery after Situation 5. Depending on agents' being risk loving, risk averse, or risk neutral, they can switch from the safe lottery to the risky lottery before, after, or at Situation 5, respectively. In this stage, individuals bear an exogenous risk while choosing a lottery to play because they know with what probability they win the high payoff in a lottery for a particular situation.

In the second stage, subjects play a one-shot stag-hunt game with information about their potential opponents' risk thresholds. A potential opponent could have ten different risk thresholds. Hence, in this stage each subject plays the stag-hunt game against an opponent who has one of these risk thresholds in each time. At the end, subjects are randomly matched with someone in the laboratory. The stated action of each subject conditional on risk threshold of her actual opponent is taken as her action choice for the game. Here, an individual bears an endogenous risk while playing against a specific opponent because she does not know exactly with what probability the other person chooses the safe action in the game.[2] In the last stage, I elicit players' beliefs regarding their actual opponents' action choices by giving information about their actual opponents' risk thresholds.

There are several findings that emerge from my analysis: First, subjects are responsive to information about their opponents' risk attitudes. Indeed, 42% of subjects chose their actions according to their opponents' risk attitudes. Specifically, information about an opponent's risk attitude has a significant and negative effect on a subject's propensity to choose the risky action i.e., the more risk averse an opponent is, the less likely a subject is to choose the risky action, as predicted by the theory.

---

[2] For endogenous risk, a player's uncertainty about her opponent's risk threshold is not taken into account because she chooses her actions as if all cases are real.

Second, although theoretically a risk-averse agent should choose the safe action with a higher probability compared to a risk-loving agent, a player's own risk attitude has no effect on her action choice. In other words, a player who personally takes more risk or less risk is equally likely to choose the risky action or the safe action in the game.[3] Furthermore, due to the first result, one can expect subjects to use the following strategic reasoning while choosing their actions. If a subject thinks that her opponent responds to information that she is given, then she believes that the information about her own risk attitude will increase (if she is risk loving, or decrease if she is risk averse) her opponent's propensity to choose the risky action. Hence, the player best responds to her belief about her opponent's action choice by choosing an action according to her own risk attitude.[4] Nevertheless, subjects may not be sure that all agents would respond to that information, hence they may not choose their actions with such reasoning.

The last finding pertains to the effects of risk attitude information on the equilibrium selection. Giving information about the opponent increased the frequency of risky action choices and likelihood of ending up on the payoff-dominant equilibrium when risk-loving agents chose their actions according to their risk attitudes.[5] However, due to insufficient number of risk-loving agents (most subjects were risk averse), the realized actions of the responsive agents' were the safe action which led either the risk-dominant equilibrium or out of equilibrium at the end.

There are three strands of literature related to this study. The first analyzes the effect of information about an opponent's attributes on a player's action choice in specific strategic

---

[3] Neumann and Vogt (2009) and Al-Ubaydli et al. (2011) test the effect of a subject's own risk attitude on her action choice in different stag-hunt games as well. Similar to my results, they found no effect of a subject's own risk attitude on her action choice.

[4] If they knew that others respond, one would expect a subject to choose her action according to her own risk attitude only if she has the same risk attitude her opponent. If the subjects do not have the same risk attitude (both risk averse, both risk loving, or both risk neutral), they may be confused about choosing their actions according to their opponents' risk attitude or their own risk attitudes.

[5] In Schmidt et al. (2003), with no information about an opponent, subjects chose the safe action with a frequency of 0.60, whereas in my experiment, subjects chose the safe action with a frequency of 0.47 when their potential opponents were risk loving. Such a behavior only increases the coordination on the payoff-dominant equilibrium if the actual opponent is risk loving.

interactions. For instance, in trust games, information about an opponent's race (Glaeser et al., 2000; and Burns, 2012), gender (Scharleman et al., 2001; Chadhuri and Gangadheran, 2002; Croson and Buchon, 1999), and ethnicity (Fershtman and Gneezy, 2001; Bouckaert and Dhaene, 2003) has an effect. In coordination games, information about an opponent's gender (Holm, 2000) and ethnicity (Bogach and Leibbrandt, 2011) has an effect. In bargaining games, information about an opponent's initial endowments (Konow, 2000), gender, income, and status (Holm and Engseld, 2005) has an effect. Nonetheless, theory does not provide any guidance as to the effects of these sorts of attributes on action choices. This paper tests the effect of information about another class of attributes, risk attitudes, which theoretically should affect a player's action choice in a strategic game.

The second related strand of work analyzes the coordination issue in games with Pareto rankable multiple equilibria. Van Huyck et al. (1990, 1991), Cooper et al. (1992), and Straub (1995) show that subjects may not always coordinate on the payoff-dominant equilibrium. There is a fair bit of follow-up work illustrating instruments that allow players to coordinate on the payoff-dominant equilibrium. For instance, repeated interactions (Van Huyck et al., 1990; Schmidt et al., 2003), loss-avoidance (Cachon and Camerer, 1996), and communication (Clark et al., 2001; Cooper et al., 1992; Charness, 2000; Charness and Grosskopf, 2004; and Duffy and Feltovich, 2006) assist in achieving coordination in the lab. Here, I test whether players use information about their opponents to coordinate on either of the equilibria.

The last strand focuses on the relationship between risk and strategic uncertainty. Heinemann et al. (2009) find that subjects who avoid risk also avoid strategic uncertainty, i.e., "subjects have similar perception for exogenous and endogenous risk if both situations are framed in a similar way" (page 6). Contrary to that result, but in line with Neumann and Vogt (2009) and Al-Ubaydli et al. (2011), I find that there is no relationship between subjects' risk attitudes and their action choices in 2 x 2 stag-hunt games. In my experiment, to create similarity while framing exogenous and endogenous risk, I use identical payoffs for the risky action in the game and the risky lottery.[6] Nevertheless, both risk-averse and risk-loving subjects choose the safe action in the game with equal probability. In addition, the current paper studies the effect of information about an opponent's risk attitude on a player's action choice and equilibrium selection, which, to the best of my knowledge, has never been studied.

2.2 Experimental Design

The experiment was conducted at the Social Science Experimental Laboratory (SSEL) at the California Institute of Technology (Caltech). Subjects were recruited by e-mail using the SSEL database, which consists of graduate and undergraduate students at Caltech. Overall, 50 subjects participated in the experiment. There were six sessions and each lasted 45 minutes. Each subject participated in only one session. All sessions were computerized using z-tree (Fischbacher, 2007). Throughout the experiment, payoffs were described in terms of "experimental units" (hereafter, EU). Eight EU corresponded to one dollar. A subject earned $19.05 on average, including a $5 participation fee. The experiment consisted of three stages.[7]

In the first stage, I elicited the risk attitude of each subject by Holt and Laury's (2002) method. According to this method, subjects must choose one of two lotteries available for ten different situations (figure 3 in appendix 1). In Situation 1, the less-risky lottery (Option A) has a higher expected payoff than the more-risky one (Option B). Hence, only very strong risk lovers pick Option B in this situation. Moving further down the table in figure 3, the expected payoff difference between the lotteries in Option A and in Option B decreases and eventually turns negative in Situation 5. In Situation 10, all subjects must choose between a sure payoff of 40 EU (Option A) and a sure payoff of 77 EU (Option B). Since all rational individuals prefer the latter

[6] In Heinemann et al. (2009), individuals stated certainty equivalents for a different game and lottery.
[7] The instructions in a session were read stage by stage. It was ensured that no subject could read the next stage's instruction before the current stage ended to prevent any kind of hedging among different periods' payoffs. Full instructions for the stages and their screenshots can be found in appendix A.

one in the last situation, by then all subjects should have switched from Option A to Option B. In this experiment, a consistent subject should switch from Option A to Option B just once. However, earlier experiments using Holt and Laury's (2002) method showed that some subjects may go back and forth between Option A and Option B. To prevent such behavior in my experiment, I asked subjects when they wanted to switch from Option A to Option B which is named as their *risk thresholds* throughout the paper.

The payoffs for the lottery choices in the experiment were selected so that the risk threshold point would provide an interval estimate of a subject's constant relative risk aversion (CRRA). With these payoffs, it is optimal for a risk-neutral subject to switch from Option A to Option B in Situation 5. Similarly, it is optimal for a risk-averse *(risk-loving)* subject to switch from Option A to Option B after *(before)* Situation 5. The payment for this stage was determined according to a randomly chosen row among these ten situations and the subject's lottery choice in that particular row.

	Action 1	**Action 2**
Action 1	57, 57	57, 2
Action 2	2, 57	77, 77

Table 2.2: The Experimental Stag-Hunt Game

In the second stage, each subject stated her action choice for the game in Table 2.2 contingent on her potential opponent's risk threshold (figure 4 in appendix 1). A potential opponent could have ten different risk thresholds based on her decision in the first stage. I elicit a player's action choice against an opponent with one of these risk thresholds at each time[8] to prevent subjects thinking that they should respond to risk threshold information. At the end of the stage subjects were randomly matched. The stated action of each subject conditional on her actual opponent's risk threshold, was taken as the action she chose for the game and was used to determine her payoff for this stage.

[8] Each player plays the game 10 times.

The monetary payoffs in Table 2.2 were specified as follows: First, to allow subjects to make a connection between the first and second stages of the experiment, similar monetary payoffs were chosen for the risky lottery (Option B) in Stage 1 and for the risky action (Action 2) in Stage 2. In particular, Option B offered 77 EU as the high payoff and 2 EU as the low payoff in Stage 1. Similarly, Action 2 may have resulted in either 77 EU when an opponent chooses Action 2 or 2 EU when an opponent chooses Action 1 in Stage 2. Second, to create a trade-off between the payoff-dominant equilibrium and the risk-dominant equilibrium, the safe action's return was chosen as 57 EU.[9] Schmidt et al. (2003) show that when Selten's (1995) risk measure, $R = \log\left[\frac{(u(Action\ 1, Action\ 1) - u(Action\ 2, Action\ 1))}{(u(Action\ 2, Action\ 2) - u(Action\ 1, Action\ 2))}\right]$, is greater than 0 subjects are more likely to have a conflict between the payoff-dominant equilibrium and the risk-dominant equilibrium in a stag-hunt game.[10] By choosing the safe action's payoff as 57 EU, the risk measure of the game becomes $\log\left(\frac{u(57) - u(2)}{u\ (77) - u(57)}\right)$, which is greater than 0 for any subject.

In the third stage, I elicited subjects' beliefs about their opponents' actions. To elicit their beliefs truthfully regardless of their risk attitudes, I used a method inspired by Becker et al. (1964) and developed by Grether (1981). According to this method, there are ten decisions and two alternatives, Alternative 1 and Alternative 2. Alternative 1 offered 16 EU to subjects only if their opponents had chosen Action 1 in the second stage. Alternative 2 offered subjects lotteries that gave 16 EU with probabilities ranging from 1/10 to 1 in increments of 1/10 between decisions (figure 5 in appendix 1).[11, 12] Each subject was provided information about the risk threshold of her actual opponent in the game and was asked when she wanted to switch from

---

[9] Selten's risk measure is greater than 0 when Action 1's payoff is between 55 and 76. I chose this payoff as 57, closer to the lower bound to have a higher trade-off between risk and payoff, i.e., higher risk measure.
[10] When R is less than 0 subjects are more likely to end up in the payoff-dominant equilibrium.
[11] Belief elicitation was made after the action choice to prevent subjects from thinking that they should act consistently with their beliefs.
[12] I start the probability of the lottery at 1/10 because I can only get interval information for the belief in each decision. Always choosing Alternative 2 implies that the agent believes her opponent will chose the safe action lower than a probability of 1/10.

Alternative 1 to Alternative 2 among ten different decisions. The point she switched at was named as her *belief threshold* throughout the paper. One can interpret the belief threshold as follows: A subject believes that her opponent chose the safe action in the second stage at most with the probability of the lottery that she switched to. For instance, assume that a subject switched from Alternative 1 to Alternative 2 in Decision 5. This implies her belief about her opponent's safe action choice is at most 5/10, and since she did not switch from Alternative 1 to Alternative 2 earlier, her belief is higher than 4/10. Hence, she believes that her opponent chooses the safe action with a probability that is higher than 4/10 and less than 5/10.

There are three theoretical hypotheses that I aimed to test. The first hypothesis is that a subject's own risk threshold negatively affects the likelihood of her risky action choice. This holds for the following reason: The payoffs in the game table are monetary payoffs and when they are converted to CRRA utilities,[13] subjects' best response behavior will be as follows: An extremely risk loving subject (with a risk aversion parameter below -0.49) chooses the safe action if she expects her opponent to choose the safe action with a probability higher than 0.41. However, an extremely risk averse subject (with a risk aversion parameter above 1.37) chooses the safe action if she expects her opponent to choose the safe action with a probability higher than 0.06. As a subject's risk aversion increases, the subject's safe action choice becomes optimal with a lower belief about the other person's safe action choice. In particular, the more risk averse a subject is, the more likely she is to choose the safe action. The second hypothesis is that information about an opponent's risk aversion affects the likelihood of a subject's risky action choice. When a subject is given information about her opponent's risk attitude, she will take into account that information while forming her beliefs about her opponent's action choice. In other words, a player believes that a risk-averse opponent is more likely to choose the safe action;

[13] The CRRA utility of a subject is $u(x) = \frac{x^{1-r}}{1-r}$, where x is monetary payment and r is risk aversion coefficient. For relative risk aversion coefficients, r, I first found the corresponding intervals from the subjects' risk threshold points in Stage 1 then I took the average of the boundaries of these intervals.

hence, with such information about an opponent she is more likely to choose the safe action as a best response. The third hypothesis is that information about an opponent's risk aversion affects players' beliefs and actions in a similar way. In particular, a player's belief about her opponent's safe action would be positively correlated with the subject's own risk aversion and her opponent's risk aversion. This should hold because agents' actions choices are driven by their beliefs.

2.3 Aggregate Risk Distribution and Action Choices

I start by describing the risk-profile distribution of the subjects. Table 2.3 shows the implied risk-aversion intervals by the number of Option A choices in Stage 1 and the risk-profile distributions of the subjects in my experiment and in Holt and Laury's (2002) experiment.[14] I use a two-sample z-test to compare subjects' percentages in each group from both experiments. I find that the percentage differences between the groups in my experiment and Holt and Laury's (2002) experiment are not significant, except for the group that is classified as the "slightly risk averse".

In terms of the action profiles in Stage 2, subjects can be classified into one of five different groups. Subjects with the action profiles Always Action 1, Always Action 2, and Nonmonotonic were not responsive to the information that they were given, i.e., they chose their actions independently of their opponents' risk attitudes. Subjects with the action profile Always Action 1 (Always Action 2) chose the safe (risky) action against potential opponents with any risk preferences. Subjects with the action profile Nonmonotonic chose their actions randomly. Subjects with the action profiles Interior Cutoff and Almost Interior Cutoff were responsive to information provided to them. The first responsive group chose the risky action until a certain risk threshold point (*cutoff point*) of their potential opponents and then shifted to the safe action. The

[14] In Holt and Laury's (2002) treatment, Option A gives \$2 with probability p and \$1.60 with probability $(1-p)$. Option B gives \$3.85 with probability p and \$0.10 with probability $(1-p)$.

second responsive group violated the Interior Cutoff action profile just once.[15] Table 2.4 shows the percentages of subjects using these action profiles.

Number of Option A Choices	Range of Relative Risk Aversion for $U(x) = \frac{x^{1-r}}{1-r}$	Risk Preference Classification	Proportion of Choices	
			Holt-Laury	This paper
0–1	$r < -0.95$	highly risk loving	1%	4%
2	$-0.95 < r < -0.49$	very risk loving	1%	2%
3	$-0.49 < r < -0.15$	risk loving	6%	4%
4	$-0.15 < r < 0.15$	risk neutral	26%	36%
5	$0.15 < r < 0.41$	slightly risk averse	26%	8%
6	$0.41 < r < 0.68$	risk averse	23%	26%
7	$0.68 < r < 0.97$	very risk averse	13%	8%
8	$0.97 < r < 1.37$	highly risk averse	3%	8%
9–10	$r > 1.37$	very highly risk averse	3%	4%

Table 2.3: Risk-Profile Distributions of Subjects in Holt-Laury's and My Experiments

Action profiles	**Percentage of subjects**
Always Action 1	32%
Always Action 2	16%
Interior Cutoff	34%
Almost Interior Cutoff	8%
Nonmonotonic	10%

Table 2.4: Distribution of Subjects' Action Choices

I compared subjects' action choices in my experiment with action choices in a one-shot stag-hunt game experiment where agents knew nothing about their opponents' risk attitudes (The baseline is Schmidt et al., 2003). I chose Schmidt et al. (2003) as a baseline for two reasons: (1) They stated action frequencies for a one-shot stag-hunt game. (2) They used a game with a risk

[15] A subject who uses the Almost Interior Cutoff may use the safe action *once* among the risky actions *before the cutoff* point or she may use the risky action *once* among the safe actions *after the cutoff point*.

measure[16, 17] of log(3), which is very close to the risk measure of the game utilized in the reported experiment, log (2.75). In their experiment, subjects chose the safe action with a frequency of 0.60.[18] In my experiment, subjects chose the safe action with a frequency of 0.47 when their potential opponents were risk loving (having a risk preference smaller than 0.15) and the safe action with a frequency of 0.74 when their potential opponents were risk averse (having a risk preference bigger than 0.15).

Furthermore, I compared my findings with the study of Clark et al. (2001), which tested the effect of communication on subjects' action choices in a stag-hunt game. They used three different games and the closest one of these games to the game in this paper was Game I with a risk measure of log(4). To compare their results with my results, I took only the frequencies in the first round of their 10-round experiment (figure 3 and 4, on page 501 of their paper). In their experiment, without communication subjects chose the safe action with a frequency of 0.70 whereas with communication subjects chose the safe action with a frequency of 0.40.[19] The subjects' safe-action frequency dropped dramatically with communication in Clark et al. (2001). In my experiment depending on an opponent's risk attitude subjects chose the safe action less or more frequently than the baseline. In particular, assume 0.60 as the baseline probability for the safe-action play absent communication.[20] In the experiment the direction of the change was as predicted by the theory: the safe-action frequency dropped from 0.60 to 0.47 when subjects were matched with risk-loving agents and increased from 0.60 to 074 when subjects were matched with risk-averse agents. In terms of magnitudes, the information about an opponent's risk attitude does not affect the safe action frequency as much as direct communication. However, both helped

[16] Game 2 and Game 4 of their experiment has the closest risk measure to the one used in this experiment.
[17] See page 6 for Selten's risk measure formula.
[18] Schmidt et al. (2003) showed frequencies of action choices in one-shot games at the first row of table 3 in page 296. I used Game 2 and Game 4 as baselines, which have frequencies, 0.60 and 0.58 respectively.
[19] The difference in the safe-action frequencies between Schmidt et al. (2003) and without communication treatment of Clark et al. (2001) may stem from the difference in risk measures (log(3) and log(4) respectively) of the games. In particular, the perceived risk in the latter is higher; hence the safe-action frequency is higher.
[20] Baseline is considered as Schmidt et al. (2003).

agents to choose the risky action more often and hence increased the chance of coordinating on the payoff-dominant equilibrium.

To better understand how responsiveness to information about an opponent's risk attitude could affect equilibrium selection, I analyze the responsive groups in more detail. Figure 1 shows the distribution of interior cutoff points for the subjects who used the Interior Cutoff and Almost Interior Cutoff profiles.

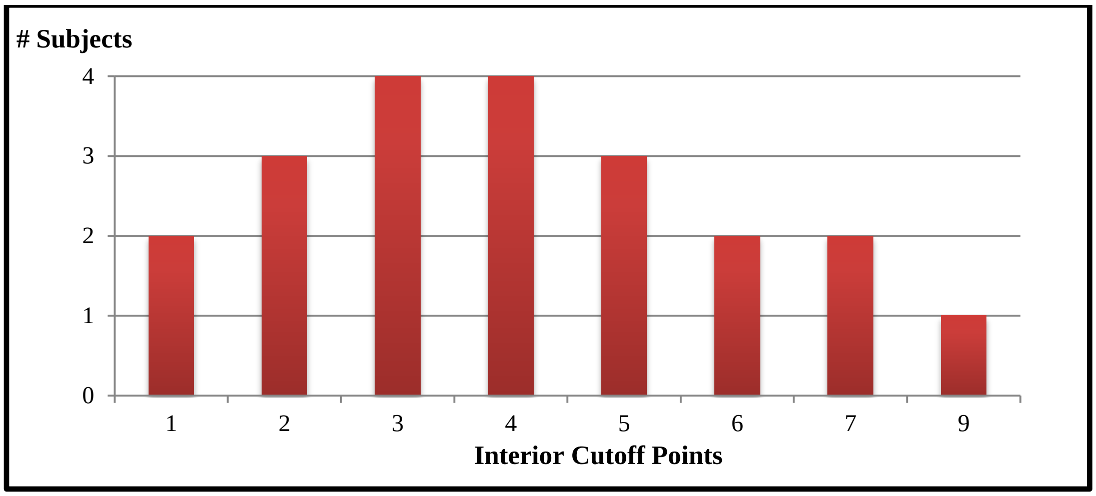


Figure 1: Histogram Illustrating the Distribution of (Responsive) Subjects' Cutoff Points.

On average, subjects' risk preferences could be classified between 0.41 and 0.68, i.e., the subject group was risk averse. However, according to the histogram, only five subjects (out of 42, all subjects except the ones choosing Always Action 2) continued to choose the risky action against potential opponents who had risk preferences above 0.41. Recall that after subjects' action profiles had been elicited they were matched randomly. The actions they chose based on their actual opponents' risk thresholds were taken as their actions for the game. By the distribution in figure 1, one can conclude that subjects' action choices were mostly the safe action when they were matched with their actual opponents in the laboratory. Hence, subjects could either coordinate on the risk-dominant equilibrium or could not coordinate at all.

Higher coordination rate on the payoff-dominant equilibrium in the experiments could have been achieved as risk information is provided, if either of the following conditions held: (1) Most subjects in the subject group continued to choose the risky action when their potential opponents had a risk preference around 0.41 and 0.68 (which is the average risk preference of the subject group). (2) The subject group had a risk preference around -0.49 and 0.15. Only under these conditions would the number of subjects choosing the risky action while playing with their actual opponents increase.

Another reason for ending up out of equilibrium is the difference between chosen action-profiles.[21] Among 25 pairs, 12 could not coordinate (48%), 10 coordinated on the risk-dominant equilibrium (40%), and 3 coordinated on the payoff-dominant equilibrium (12%).

In terms of the pure effects of risk information on selection, the extant literature does not offer assessments of equilibrium frequencies in one-shot stag-hunt games absent any information about subjects' opponents. Since I compared action choices in this paper with action choices in Clark et al. (2001) and Schmidt et al. (2003), I first checked these papers for the frequencies of equilibrium or disequilibrium choices. Both papers have not discussed frequencies of outcomes for one-shot games but Clark et al. (2001) stated these frequencies for all rounds of a repeated stag-hunt game. They found that for all rounds without communication subjects successfully coordinate on the payoff-dominant equilibrium 2% of the time, on the risk-dominant equilibrium 30% of the time, and they could not coordinate 68% of the time. However, with communication subjects successfully coordinate on the payoff-dominant equilibrium 70% of the time, on the risk-dominant equilibrium 13.5% of the time, and they could not coordinate 16.5% of the time.

[21] In three pairs one of the subjects used Always Action 1 and the other used Always Action 2. In three pairs one of the subjects followed Always Action 2 and the other used the Cutoff action profile. In two pairs one of the subjects used Always Action 1 and the other used the Cutoff action profile. In addition to these, there are four more pairs (Always Action 1 and Nonmonotonic, both Nonmonotonic, both Cutoff) with action profiles leading to an out of equilibrium outcome.

In my experiment, subjects ended up playing the payoff-dominant equilibrium 12% of the time, which is less often than in the "with communication" (70%) and more often than in the "without communication" (2%) treatments of Clark et al. (2001). Furthermore, subjects coordinated on the risk-dominant equilibrium 40% of the time which is more often than in the "with communication" (13.5%) and "without communication" (30%) treatments of Clark et al. (2001). Last, they ended up out of equilibrium 48% of the time which is more often than in the "with communication" (16.5%) and less often than in the "without communication" (68%) treatment of Clark et al. (2001). According to these comparisons, subjects coordinate on the payoff-dominant and risk-dominant equilibria in my experiment more often than their "without communication" treatment. Nevertheless, they could not coordinate on the payoff-dominant equilibrium through risk-attitude information as much as they did via direct communication.

2.4 Explaining Action Choices and Beliefs

I start by providing some descriptive results. In Table 2.5, columns correspond to the groups with different action profiles in Stage 2; the Risk row contains the mean and standard deviation of risk thresholds (Stage 1); and the Belief row contains the mean and standard deviation of belief thresholds (Stage 3). The means in all columns of the Risk row are not significantly different from one another, i.e., there is no relationship between the chosen action profile and a subject's own risk aversion.

Thresholds/Groups	Always Action 1 Group	Always Action 2 Group	Interior and Almost Interior Cutoff Group	Nonmonotonic Group
Risk (Stage 1)	6.19 (2.2)	6.25 (1.75)	6 (2.1)	6.2 (1.1)
Belief (Stage 3)	7.19 (2.6)	2.75 (2.49)	5 (2.19)	5.2 (2.05)

Table 2.5: The Mean (Standard Deviation) of Subjects' Risk and Belief Thresholds

The Belief row illustrates the subjects' best-response behavior. The subjects in the Always Action 1 group had significantly higher belief thresholds than the subjects in the Interior and Almost Interior Cutoff group (Wilcoxon rank-sum test, p = 0.0029). In other words, they believe that their opponents chose the safe action with a higher probability than the subjects in the Interior and Almost Interior Cutoff group do. As a response, they chose the safe action more often than the subjects in that group. Similarly, subjects in the Interior and Almost Interior Cutoff group had significantly higher belief thresholds than subjects in the Always Action 2 group (Wilcoxon rank-sum test, p = 0.0265). There is no significant difference between the belief thresholds of the subjects in the Interior and Almost Interior Cutoff group and the subjects in the Nonmonotonic group.

Dependent Variable	Probability of Choosing the Risky Action in the Game		
Model	(1) Probit	(2) Probit	(3) Probit
Condition	All	All	Responsive group
Own Risk Threshold	-0.018 (0.028)	-0.042 (0.034)	-0.051 (0.065)
Opponent's Risk Threshold	-0.043***(0.011)	-0.071**(0.030)	-0.159**(0.069)
Interaction	-	0.005 (0.005)	0.002 (0.009)
# Observations	500	500	210

Robust standard errors are in parenthesis and clustered by subjects

* significant at 10% level; ** significant at 5% level; *** significant at 1% level

Table 2.6: Probit Estimations of Risky Action Choices

Next, I test whether the probability of choosing the risky action is affected by a subject's own risk threshold and her opponent's risk threshold by using various probit models. Table 2.6 contains the results of my estimations. In Model (1) the independent variables are only a subject's own risk attitude and her opponent's risk attitude,[22] whereas in Models (2) and (3), there is an interaction term in addition to the independent variables of Model (1).[23] Furthermore, while Models (1) and (2) utilized all subjects, Model (3) used only the group of subjects that was

[22] An alternative specification using the mean and upper bounds of the risk aversion parameter intervals instead of the risk threshold points generate identical qualitative results.

[23] By interaction term, one can understand the effect of the differences between players' risk aversion parameters on the players' actions, i.e., whether a risk-averse (risk-loving) person is more likely to choose the safe action when she is paired with a risk-averse (risk-loving) person.

responsive to an opponent's risk attitude information, i.e., subjects with Interior Cutoff and Almost Interior Cutoff action profiles.

In Model (1), among the independent variables, I find that only the opponent's risk threshold significantly affects the subject's propensity to choose the risky action. Every additional increase in the opponent's risk threshold decreases the probability of a subject's risky action choice by 0.043. This result supports my prediction regarding the relationship between the probability of the subject choosing the risky action and the risk attitude of an opponent. When I add the interaction term to the regression the result does not change much. In Model (2), the only significant variable is still an opponent's risk threshold, with a marginal effect of -0.071 on the probability of a subject's risky action choice. In Model (3), still only the opponent's risk threshold point significantly affects the probability of the risky action decision, but with a higher magnitude. Every additional increase in an opponent's risk threshold point decreases the probability of the subject choosing the risky action by 0.159. In all regressions, a subject's own risk attitude does not affect the probability of the subject's risky action choice significantly, contrary to what was predicted.

I also look at the relationship between players' beliefs and their risk attitudes. Theoretically players' risk attitudes should affect players' beliefs in the same way they affect players' action choices. Nevertheless, limitation in data volume does not allow me to assess the response of beliefs to information regarding risk attitudes.

Figure 2 reports the relationship between belief thresholds and cutoff points by excluding the subjects with a Nonmonotonic action profile.[24] In the figure, each node corresponds to a belief threshold and a cutoff point for a subject, where a belief threshold gives information about how likely a player thinks her actual opponent will choose Action 1 (i.e., a high belief threshold says

[24] The cutoff point of a subject with Interior Cutoff and Almost Interior Cutoff action profiles ranges from 1 to 10. The cutoff point of a subject with Always Action 1 is considered as zero and the cutoff point of a subject with Always Action 2 is considered as eleven.

that a subject believes that her opponent will choose Action 1 with a high probability). The significant and negative correlation, -0.63, between those two is consistent with best-response behavior. The lower a subject's cutoff point, i.e. the less likely she is to play the risky action, the more she believes her opponent will choose the safe action.

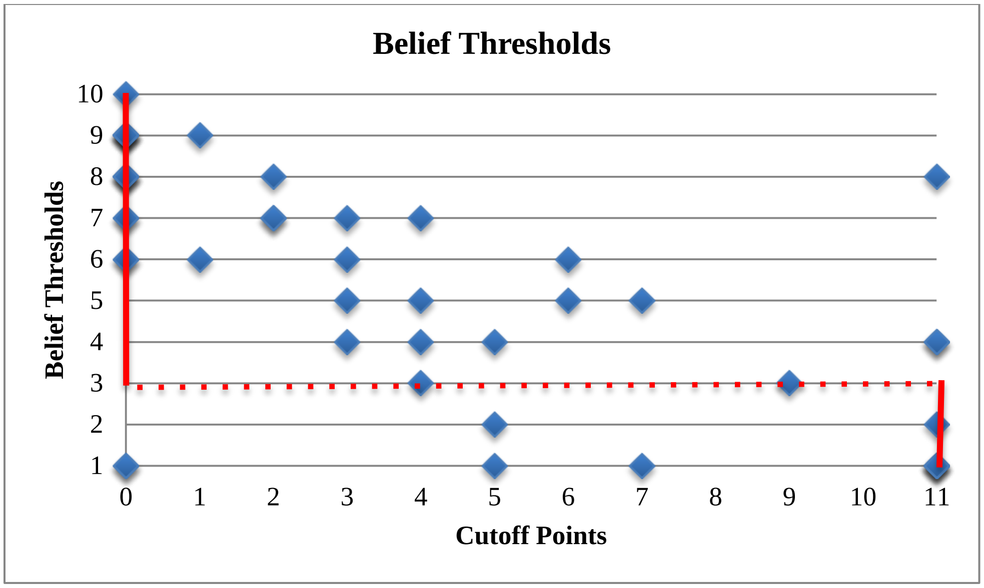


Figure 2: Belief Thresholds and Cutoff Points.

In the figure, the solid red line represents a risk-neutral subject's[25] best-response behavior. A risk-neutral subject chooses the risky action (the safe action) if her belief about her opponent's safe-action choice is below (above) 0.27 (solid red line in figure 2). Although many subjects have belief thresholds above 3, i.e., they believe that their opponents choose the safe-action with a probability higher than 0.30, few agents always chose the safe action (cutoff at zero). Hence, one can conclude from the figure that information about an opponent's risk attitude increased the risky action choice despite the agents' beliefs.

Additionally, I check subjects' best-response behavior with the belief data from Stage 3 and the action data from Stage 2. Since I can only access the intervals of subjects' beliefs by the

[25] Nodes represent agents with various risk attitudes, but I consider a risk-neutral agent's behavior as a representative.

belief thresholds, for the analysis I take the average of the upper and lower bounds of that interval. When subjects are assumed to have linear (CRRA) utility functions, 80% (78%) of them best respond to their beliefs.[26] Costa-Gomes and Weizsacker (2008) studied the best-response behavior of subjects in 3 x 3 games. They found that 50.5% of subjects best respond. Later, Rey-Biel (2009) replicated that experiment with simpler 3 x 3 games (constant-sum and single-digit payoffs). He found that 67.2% of subjects best respond to their beliefs. In particular, the higher percentage of best-response behavior was observed when the game became simpler. In my experiment, the game is 2 x 2 and possibly simpler than the games used in those studies. I observe a higher percentage of subjects best responding than Costa-Gomes and Weizsacker (2008) and Rey-Biel (2009) do.

2.5 Conclusions

This paper is the first study to analyze the effect of information about an opponent's risk attitude on subjects' behavior in stag-hunt games. There are three main messages from the experiment. First, subjects utilize information regarding their opponents' risk attitudes when they choose their actions. Second, players' own risk attitudes have no significant effect on their action choices. Third, such information is insufficient for players to coordinate on the payoff-dominant equilibrium due to insufficient number of risk-loving agents and their action choices that are independent from their risk attitudes.

According to the results, although informing subjects about their opponents' risk attitudes increased the frequency of their risky action choices, the number of agents who coordinated on the payoff-dominant equilibrium remained low. Players' responsiveness to information about their opponents' risk attitude would increase coordination on the payoff-dominant equilibrium

[26] Nine subjects did not best respond under the CRRA utility or the linear utility. Seven of those nine subjects chose their actions independently of her opponent's risk attitude (Always Action 1, Always Action 2, or Nonmonotonic). One subject who best respond under the linear utility but not under the CRRA utility was extremely risk loving.

more if either most agents were risk loving or players continued to choose the risky action even when their potential opponents were slightly risk averse.

2.6 Appendix 1: Instructions and Screenshots in the Stages

Introduction

There will be three stages in this experiment. The instructions for each stage will be read just before the stage starts. In each stage, you will try to maximize your earnings. You will observe your earnings only at the end of the experiment. Throughout the experiment, payoffs are described in terms of "experimental units." Eight experimental units correspond to $1. Your final earnings will be rounded up to a quarter. Besides what you earn during the experiment, you will be paid a $5 show-up fee.

Stage 1

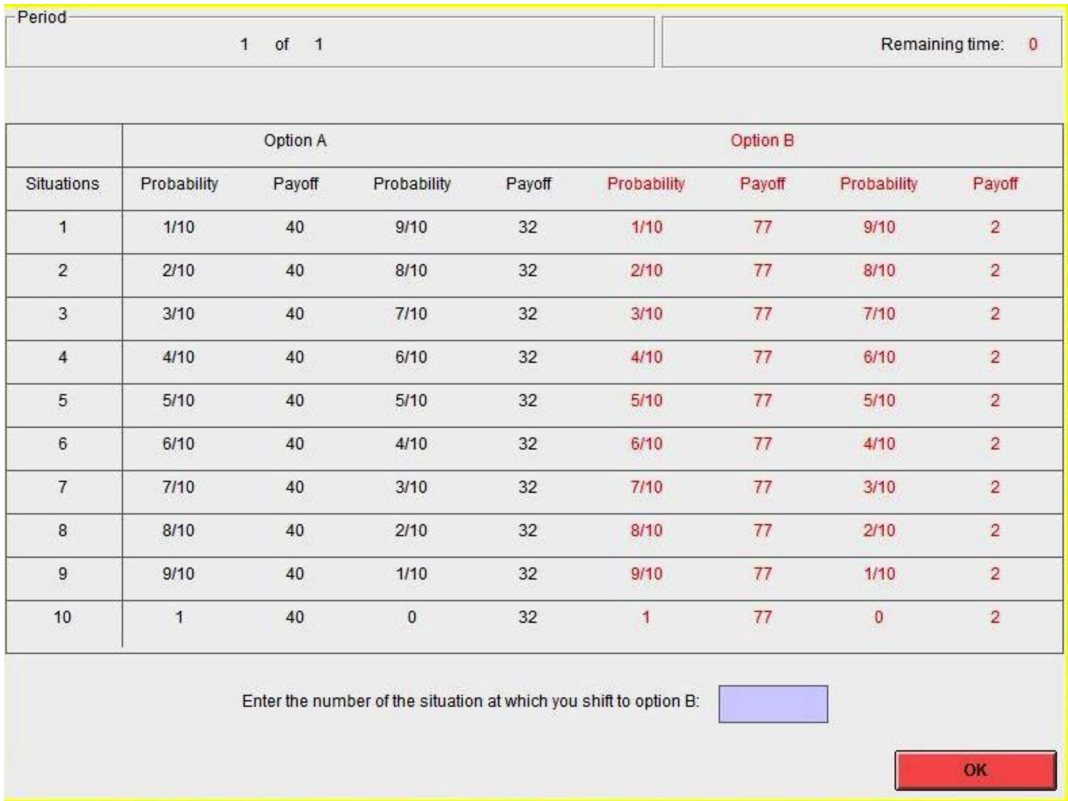
In the first stage of the experiment, you will face 10 different situations. Each situation indicates your chances of winning a certain payoff. For instance, consider **Situation 1**.

In <u>Situation 1</u> Option A offers 40 experimental units with probability 1/10 and 32 experimental units with probability 9/10.

In <u>Situation 1</u> Option B offers 77 experimental units with probability 1/10 and 2 experimental units with probability 9/10.

For **each situation**, you are asked to choose between the two specified options, Option A and Option B, until you choose Option B for some situation. Once you choose Option B, the choices for the following situations can only be Option B.

In the screen, you will notice a box at the bottom of the table. You are asked to enter the number of the situation at which you decide to shift from Option A to Option B in that box.



	Option A				Option B			
Situations	Probability	Payoff	Probability	Payoff	Probability	Payoff	Probability	Payoff
1	1/10	40	9/10	32	1/10	77	9/10	2
2	2/10	40	8/10	32	2/10	77	8/10	2
3	3/10	40	7/10	32	3/10	77	7/10	2
4	4/10	40	6/10	32	4/10	77	6/10	2
5	5/10	40	5/10	32	5/10	77	5/10	2
6	6/10	40	4/10	32	6/10	77	4/10	2
7	7/10	40	3/10	32	7/10	77	3/10	2
8	8/10	40	2/10	32	8/10	77	2/10	2
9	9/10	40	1/10	32	9/10	77	1/10	2
10	1	40	0	32	1	77	0	2

Figure 3: Screenshot of Stage 1.

For instance, if you decide to shift at Situation 3, this implies you chose “Option A” in Situations 1 and 2, and you chose “Option B” in Situation 3 and in all the situations after Situation 3.

Your payment for this stage will be determined randomly (and revealed to you only at the end of the experiment) as follows:

First, we will choose a random number between 1 and 10, which will determine which **situation** will be selected for your payment. Suppose your randomly drawn situation is 7. And suppose you chose Option A in that situation.

Then, we will implement your chosen option in the determined situation. We will again randomize a number between 1 and 10. If it is lower than or equal to 7 (occurring with probability 7/10), you will receive 40 experimental units. Otherwise, if the number is between 8 and 10 (occurring with probability 3/10), you will receive 32 experimental units.

Stage 2

Other person ⇩

You ⇨

	Action 1	Action 2
Action 1	57, 57	57, 2
Action 2	2, 57	77, 77

The table above describes the following game. There are two players in the game: you and the other player. Your actions and payoffs are colored in red; the other player's actions and payoffs are colored in green. You each have two actions: Action 1 and Action 2. In order to play the game, both of you simultaneously choose one of your actions. Your earnings depend both on your action choices and the other person's action choices. If you and the other person both play Action 1, both of you will earn 57 experimental units. If you play Action 1 and the other person plays Action 2, while your payoff from this game will be 57 experimental units, the other person's payoff will be 2 experimental units. If you play Action 2 and the other person plays Action 1, your payoff from this game will be 2 experimental units, while the other person's payoff will be 57 experimental units. If you and the other person both play Action 2, you will each get 77 experimental units.

In this second stage of the experiment, you will be asked to state your action choice conditional on which situation the other person switched at in Stage 1. For instance, one of the questions in Stage 2 is as follows: "Suppose that the other person that you are matched with switched to Option B in **Situation 6**; which action would you like to play, Action 1 or Action 2?". For this question, you are asked to enter **1** for **"Action 1"** choice and **2** for **"Action 2"** choice in the box.

Your payment in this stage will be determined as follows: You will be matched randomly with a person in the experimental group. The action that you stated for the situation at which the other person switched in Stage 1 will be taken as your chosen action for the game. Similarly, the action that the other person stated for the situation at which you switched in Stage 1 will be taken as his/her chosen action for the game. These stated action choices will determine your payments by the game table above.

For example, suppose you switched at Situation 4 and the other person switched at Situation 8 in Stage 1. In Stage 2, when you were asked which action you would choose when the other person switched at Situation 8, you wrote **Action 1**; and when the other person was asked which action he/she would choose when the other person switched at Situation 4, he/she wrote **Action 2**. By these action choices, you earn 57 experimental units and the other person earns 2 experimental units from this stage, according to game table above.

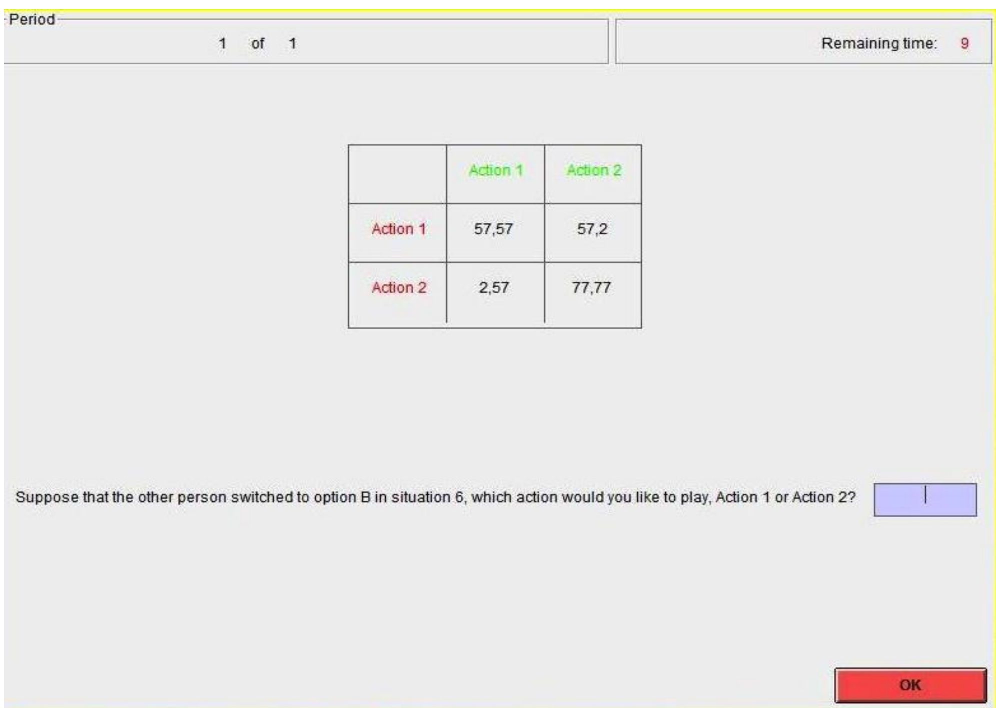


Figure 4: Screenshot of Stage 2.

Stage 3

In the third stage of the experiment, you will face 10 different decisions. For each decision you have 2 different alternatives:

Alternative 1 depends on the other person's action choice in Stage 2 (the action of the person with whom you actually matched in Stage 2) and it is the same under all decisions. In all decisions, Alternative 1 provides 16 experimental units if the other person chose Action 1 in the previous stage and nothing if he/she chose Action 2.

Alternative 2 is always a lottery. For instance, for Decision 1, Alternative 2 provides 16 experimental units with probability 1/10 and nothing with probability 9/10.

For **each decision**, you are asked to choose between the two specified alternatives, Alternative 1 and Alternative 2, until you choose Alternative 2 for some decision. Once you choose Alternative 2, the choices for the following decisions can only be Alternative 2.

On the screen, in addition to the table, you will be given the number of the situation in which the other person with whom you actually played the game in Stage 2 switched in Stage 1. Then you will notice a box at the top of the table. You are asked to enter the number of the decision at which you decide to shift from Alternative 1 to Alternative 2 in that box.

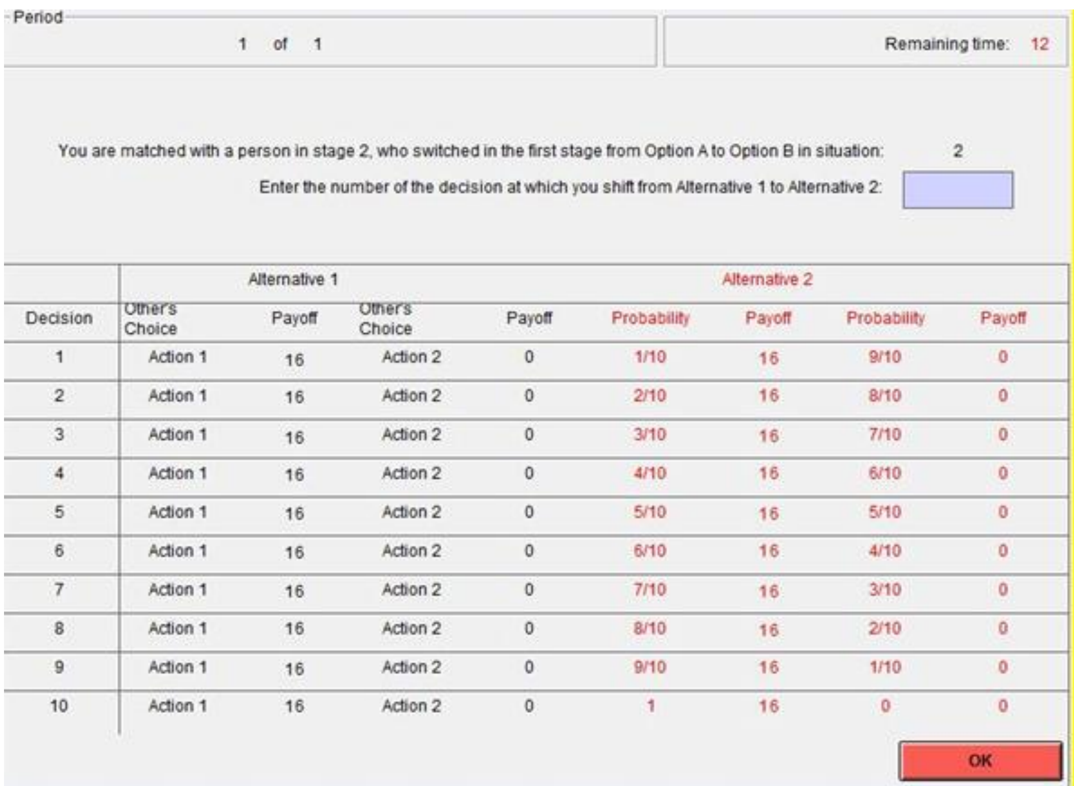
For instance, if you decide to shift at Decision 3, this implies you chose Alternative 1 in Decision 1 and 2, and you chose Alternative 2 in Decision 3 and in all subsequent decisions. In other words, for Decisions 1 and 2, you are choosing the lottery, which gives money if and only if the other person, whom you matched with in the previous stage, chose Action 1. By Decision 3, you are choosing a lottery that pays a certain amount of money with some probability (increasing as the number of the decision increases).

Your payment in this stage will be determined randomly as follows:

First, we will choose a random number between 1 and 10, which will determine which **decision** will be selected for your payment. Suppose your randomly drawn decision is 7.

Then, we will implement your chosen alternative in the determined decision. Suppose you chose Alternative 1 in Decision 7. We will look at the other person's action choice in the previous stage and if it was Action 1, you will earn 16 experimental units; if it was Action 2, you will earn 0.

Or, suppose in this decision, instead of Alternative 1 you chose Alternative 2. Then we will randomize a number between 1 and 10. If it is lower than or equal to 7 (occurring with probability 7/10), you will receive 16 experimental units. Otherwise, if the number is between 8 and 10 (occurring with probability 3/10), you will receive nothing.



	Alternative 1		Alternative 2					
Decision	Other's Choice	Payoff	Other's Choice	Payoff	Probability	Payoff	Probability	Payoff
1	Action 1	16	Action 2	0	1/10	16	9/10	0
2	Action 1	16	Action 2	0	2/10	16	8/10	0
3	Action 1	16	Action 2	0	3/10	16	7/10	0
4	Action 1	16	Action 2	0	4/10	16	6/10	0
5	Action 1	16	Action 2	0	5/10	16	5/10	0
6	Action 1	16	Action 2	0	6/10	16	4/10	0
7	Action 1	16	Action 2	0	7/10	16	3/10	0
8	Action 1	16	Action 2	0	8/10	16	2/10	0
9	Action 1	16	Action 2	0	9/10	16	1/10	0
10	Action 1	16	Action 2	0	1	16	0	0

Figure 5: Screenshot of Stage 3.

Survey

Please briefly write down how you responded to the following experimental tasks:

1. In Stage 2, what affected your action choice? What were you thinking while choosing your action conditional on your opponent's switching point?
2. In Stage 3, what made you to switch from Alternative 1 to Alternative 2 at that specific "decision"?

Chapter 3:

Parallel Tournaments

3.1 Introduction

Schools, sporting events, companies, and many other institutions use tournaments to incentivize agents by giving rewards based on their relative performances. Two features are common. First, agent performance is correlated with underlying ability. Second, ability is heterogeneous. For instance, students may differ in their IQs, which in turn affect their school performance; runners may differ in their physiology, which affects their speed; and firms may have different R&D investment, which affects their ability to innovate.

In this paper, I theoretically and experimentally compare the performance of two tournament designs accounting for heterogeneity in contestants' abilities. One of the designs is the standard winner-take-all (henceforth "single-prize" or "WTA") tournament, which is common in the literature and the real world. In WTA tournaments, only the contestant with the highest performance receives a prize. The other design involves two tournaments with different prizes (henceforth "parallel"), where individuals can choose which tournament to enter before competing (within each tournament, the individual with the highest performance receives a prize). The main conclusion of this paper, both theoretically and experimentally, is that under high heterogeneity the designer is better off using parallel tournaments and under low heterogeneity the designer is better off using a single-prize tournament.[27]

[27] The designer compares the difference between the sum of expected effort, which is observable, and the sum of the expected given prize in two tournament designs.

Consider the following practical example: a school with two types of students, differing in their scholarly abilities, wants to maximize its students' performances in a course with a minimal cost. Each student wants to be the best in the course and hence receive a good reference from the instructor. Suppose the school requires all students to enroll in the same class. If the range of students' abilities is large enough, the best students can obtain good grades without working hard and always obtain better references than the lower-ability students. The lower-ability students may not work hard either, because they anticipate that they will never perform as well as the other students and thus expect to always receive worse references. At the same time, this course may be costly for the school if the school is paying to the teacher hourly. The reason is that, it may be hard for a teacher to teach the same material to high-ability and low-ability students, hence the teacher may need to spend more time at the class. Now, suppose that the school offers two different classes, basic and advanced, and allows students to choose which class to take. The higher-ability students may enroll and compete in the advanced class and try to get the most impressive references. The lower-ability students may opt for the basic class and compete to get good references. A more homogenous division of students in the classes may increase the performances of the high-ability and low-ability students. From the teachers' point of view, this homogeneity may allow them to teach their material more easily; hence they will not spend extra time for teaching. However, having two classes may decrease student performance, because there would be fewer students in each class to compete with; students may not work as hard as they would if they were all together. In this paper, I examine under what ability difference between students the school would offer one or two classes. To answer this question, I develop a simple model with two contestants, which turns out to be enough to explore the key trade-offs.

The model features two agents, who are privately informed about their randomly determined abilities (high or low) and an employer, who only knows the agents' potential abilities. First, the employer decides which tournament to offer: a single-prize tournament or two parallel tournaments with different prizes. In the single-prize tournament, each agent chooses an effort level (low or high) for a given prize. The contestant with the higher effort wins the prize. In the parallel tournaments (basic and advanced courses in the example) with different prizes, each contestant first chooses which tournament to compete in. Afterwards, he learns whether his opponent chose the same tournament or not, and then he chooses an effort level. Again, the contestant with the higher effort wins the chosen tournament's prize. In both tournaments, whether the agent wins the prize or not, he bears the cost of his effort. The designer's profit is the difference between his revenue from the agents' effort and his cost of prizes.

I characterize the unique Perfect Bayesian Nash Equilibrium of each game. I specifically look at equilibrium conditions where agents sort into tournaments according to their abilities, i.e., high-ability agents vie for the high prize and low-ability agents compete for the low prize. To maximize his profit, given the agents' behavior in equilibrium, the designer then offers the smallest prizes that incentivize agents and guarantees them a positive expected utility.[28] Alternatively, the designer can offer a tournament with a single prize to all agents (high and low types). Knowing the agents' total efforts in equilibrium under both tournament designs and the optimal prizes, the designer compares his profit from these tournaments and chooses the one with the higher profit.

I find that when the difference between agents' abilities is large, it is more profitable for the designer to organize parallel tournaments with two different prizes. This is because when bad types' marginal cost of effort is really high, the designer must offer a high prize to ensure a positive expected payoff for low-ability agents in equilibrium of the single-prize tournament. However, it is less costly for the designer to incentivize both types with parallel tournaments where high-ability agents choose the tournament with a high prize and low-ability agents choose

---

[28] The profit is calculated in expected terms. Specifically, with probability 0.5, there will be one high-ability and one low-ability worker and both bonuses will be given. With probability 0.25, both workers will be low ability and only the low bonus will be given. With probability 0.25, both workers will be high ability and only the high bonus will be given.

the tournament with a low prize. Further, I find that when the difference between agents' abilities is small it is more profitable for the designer to organize a single-prize tournament. This stems from the fact that when the cost difference is small the designer can guarantee a positive expected payoff for low-ability agents with a small prize.

Comparing different tournament designs with field data is virtually impossible. The main difficulty is that different principals may choose different tournament structures in different environments (where the distribution of workers' abilities may not be comparable). Hence, I use experiments to test whether the designer chooses the optimal tournament consistent with my theoretical predictions as the contestants' abilities vary. I use a simple structure for the experiment: two institutions (single-prize or parallel tournament) are crossed with two cost differences (large or small) between the types. I vary the cost difference by keeping the high-ability agents' marginal cost level identical and changing low-ability agents' cost levels across the treatments. In additional treatments, I also change (high) tournament's prizes in the parallel and single-prize tournaments.

Three main insights emerge from the experiments. First, the designer obtains a significantly higher profit in the parallel tournament than in the single-prize tournament when types differ greatly from each other. The designer obtains a significantly higher profit in the single-prize tournament than in the parallel tournament when types are similar. These results support the theoretical predictions for the designer's profit.

Second, I find that high-ability agents underparticipate and low-ability agents overparticipate in the high-prize tournament relative to the equilibrium prediction. I conjecture that there are two possible reasons underlying this observation: One is that an agent's belief about winning the high tournament's prize may change as a response to his opponent's ability. A high-ability (low-ability) agent may believe less (more) that he can win the high tournament's prize and hence he may become less (more) competitive, if his opponent has a closer ability (marginal cost of effort) to his. The other possible explanation is that agents' risk aversion may affect their tournament selection behavior. The best response of each agent can change depending on how risk averse or risk loving he is.

Finally, the percentage of subjects choosing the equilibrium effort depends on the number of contestants in the chosen tournament. In particular, when there is just one person in the tournament, equilibrium effort is frequently chosen in all treatments. When there are two people in the tournament, observed equilibrium effort percentages are lower. The marginal effect of being together with an opponent on the observed equilibrium is smaller in the low-prize tournament (-0.14) than in the high-prize tournament (-0.07).

The vast majority of the contest literature focuses on single-prize tournaments[29] (see Falk and Fehr, 2003 and Irlenbusch, 2005 for a literature review), which elicit the highest total effort when contestants are homogenous (Lazear and Rosen, 1981; Rosen, 1986; Prendergast, 1999) but not when the contestants are heterogeneous (Schotter and Weigelt, 1992; Harbring et al., 2007). This paper analyzes whether the tournament-selection decision increases the designer's profit compared to the single-prize tournament (in which there is no selection decision) when the contestants are heterogeneous. In particular, it contributes primarily to the theoretical and experimental literature on tournaments with endogenous entry.

The theoretical literature on tournament selection begins with Lazear and Rosen (1981). In their paper, they consider agents' self-selection into high prize and low-prize tournaments with respect to their abilities, as a solution to the decrease-in-performance problem when agents are heterogeneous. According to their model, an employer cannot obtain agent self-selection into

[29] There are other tournament designs in the literature, such as multistage elimination tournaments (Altman, Folk, and Wibral, 2010; Groh et. al., 2012; Rosen, 1986; and Sheremeta, 2010) and single tournaments with multiple prizes (Moldavunu and Sela, 2001; Muller and Schotter, 2010; and Sheremeta, 2011); however, WTA is the simplest and most common.

tournaments with respect to their abilities, and therefore, he elicits efforts inefficiently. Their model differs from the model used in this paper in many aspects. For example, they define performance as the sum of an agent's effort and luck, as in rank-order tournaments. I define performance solely as the effort, as in all-pay auction models. They assume that the employer gets zero profit from the tournament, but I allow the employer to make a profit.

Later, O'Keeffe et al. (1984) show that an employer can achieve agent self-selection into tournaments with respect to their abilities by changing monitoring precision (derivative of the winning probability in the high-prize and low-prize tournaments) and prize spread with a model based on the rank-order tournament.[30] They assume that the agent believes his opponent in the high-prize contest has high ability and in the low-prize contest has low ability. I demonstrate that without such an assumption it is possible to achieve agent self-selection with appropriate prize levels in an all-pay auction set up.

Last, Leuven et al. (2011) compare total effort elicited in parallel tournaments and single-prize tournaments with a Tullock (1980) tournament when agents are heterogeneous. They show theoretically that single-prize tournaments always deliver higher total effort than parallel tournaments. Here, instead of total effort, I focus on a designer's profit, i.e., the difference between expected total effort and the expected given prize in a deterministic contest. Furthermore, my model differs from theirs in the determination of the WTA prizes: my model uses the optimal prize in the sense that the prize incentivizes both types of agents while maximizing the employer's profit; their model sets the WTA prize simply as the sum of the prizes in the parallel tournament. These key differences in tournament modeling account for the different theoretical expectations between my work and previous literature. Here, I predict theoretically and verify experimentally that the superiority of a tournament mechanism depends on the difference between contestants' abilities.

[30] They change these to guarantee global and local incentives for both types of workers.

There are three groups of experimental studies with endogenous entry in tournaments: One deals with contest entry decisions in which the outside option is a fixed wage[31] (Fullerton et al., 1999; Anderson and Stafford, 2003; Morgan et al., 2012). The second studies contest entry decisions in which the outside option is piece rate (Eriksson et al., 2009; Dohmen and Folk, 2011; Bartling et al., 2009; Balafautas et al., 2012; and Niederle and Vesterlund, 2007), and the third group studies contest entry decisions among alternative contests (Vandegrift, 2007; Cason et al., 2010). Because my study also deals with a contestant's decision between two tournament designs, I focus on the third group. In Vandegrift et al. (2007), subjects choose between piece-rate, single-prize and multiple-prize lottery contests. Holding total payments constant across contests, they find that effort is higher in the single-prize contest than in the multi prize contest. However, entry rates into the single-prize contest as well as subject quality are found to be lower than those into and of the multi prize contest. Cason et al. (2010) test performance and entry decisions under proportional-prize and WTA tournaments when agents' abilities differ. They find that a proportional-prize contest attracts more entrants and generates more aggregate effort than a WTA contest. Here, I compare agents' behavior with two different tournament designs and I find that contestant heterogeneity determines whether the designer benefits more from a single-prize or parallel tournament.

The paper is organized as follows. Section 2 provides the basic theoretical background. Section 3 describes the experimental design. Sections 4 through 6 present aggregate and individual experimental results. Section 7 concludes and discusses the results.

3.2 Theoretical Framework

I consider an environment in which an employer has two workers with different abilities. There are two possible ability levels for each worker. A worker can be of high ability (good type) with probability 0.5 and of low ability (bad type) with probability 0.5. Workers' types are independent

[31] In a contest-entry decision, fixed wage can be thought as zero.

of each other and are determined by the marginal cost of effort. Bad types have a high marginal cost of effort, $c_B \in \mathbb{R}_+$, and good types have a low marginal cost of effort, $c_G \in \mathbb{R}_+$; $c_B > c_G > 0$. Knowing workers' potential types (but not their realized types) the employer offers either a single-prize tournament (section 2.1) or two parallel tournaments with different prizes (section 2.2). In the single-prize tournament, knowing only his own type and the value of the prize, each worker chooses either e_h or e_l, where $e_h > e_l$.[32] Similarly, in the parallel tournaments, knowing only his own type and the values of the prizes, each worker first chooses between tournaments, then chooses either e_h or e_l. In both tournaments, whether a worker i wins the prize or not, he pays his cost of effort, $c_i e_i$, where c_i is the marginal cost of effort (either c_G or c_B) and e_i is the chosen effort (either e_h or e_l).

In the single-prize tournament, the worker exerting the higher effort wins the tournament prize and the worker exerting the lower effort gets nothing.[33] If both choose the same effort level, the prize is divided between two contestants. Here, I assume that workers are risk neutral. In particular, a worker's payoff is the difference between the tournament's prize and his cost of effort if he wins the prize and only the cost of his effort if he does not win the prize. On the other hand, a risk-neutral employer's profit is the difference between the expected elicited total effort and the expected total given prize(s). I assume that effort equals the output of a worker[34] and the price of an output is 1. The employer's revenue is the sum of the workers' output levels. I first look at workers' behavior when the employer chooses to organize a single-prize tournament.

---

[32] Throughout the paper, I limit heterogeneity for the effort levels; instead I use heterogeneity for types, i.e., the marginal cost of efforts.
[33] The payoffs are calculated similarly when two agents choose the same tournament in the parallel-tournament setting.
[34] Hence the set up is similar to all pay auction models.

3.2.1 Single-Prize Tournament

In the single-prize tournament, an employer offers a tournament with a prize K to incentivize each worker. Knowing only his own ability, each worker chooses either e_h or e_l to exert. The strategy of each worker is the probability of choosing e_h, i.e., $p_i \in [0,1]$, where i denotes the worker's type (a good type (G) or a bad type (B)). The expected utilities of a good type from exerting e_h and e_l are as follows:

$$0.5\left(p_G\left(\frac{K}{2}\right)+(1-p_G)(K)\right)+0.5\left(p_B\left(\frac{K}{2}\right)+(1-p_B)(K)\right)-c_G e_h, \text{ and}$$

$$0.5\left(p_G(0)+(1-p_G)\left(\frac{K}{2}\right)\right)+0.5\left(p_B(0)+(1-p_B)\left(\frac{K}{2}\right)\right)-c_G e_l.$$

The expected utility of exerting e_h is greater than the expected utility of exerting e_l for a good type if

$$\left(\frac{K}{2}-c_G(e_h-e_l)\right)\geq 0.$$

There are three potential equilibrium structures to consider: 1) Pooling equilibrium, where either both types exert e_h (appendix 1, Case 1) or both types exert e_l (appendix 1, Case 2); 2) Separating equilibrium, where either good types exert e_h and bad types exert e_l (appendix 1, Case 3) or good types exert e_l and bad types exert e_h; and 3) Semi-pooling equilibrium where good types exert e_h and bad types mix between e_h and e_l (appendix 1, Case 4), or good types mix between e_h and e_l and bad types exert e_l (appendix 1, Case 4), or both types mix between e_h and e_l.[35]

An employer's aim is to maximize his profit by providing a minimal prize and eliciting maximal effort levels from the workers. Hence, knowing the potential marginal cost level of the

[35] The equilibrium structures that are not discussed in appendix 1 are not equilibria due to higher marginal cost levels of bad types.

workers (but not their realized types), and with predetermined e_h and e_l, an employer chooses the lowest K that satisfies the incentive compatibility (IC_B and IC_G) and individual rationality (IR_B and IR_G) conditions for both types. For instance, in order to elicit e_h from both types (appendix 1, Case 1), an employer sets the minimum prize solving the following linear program:

$$\max_{\{K\}}\{2e_h - K\}$$

subject to

i) $\frac{K}{2} - c_B(e_h - e_l) \geq 0,$

ii) $\frac{K}{2} - c_G(e_h - e_l) \geq 0,$

iii) $\frac{K}{2} - c_B e_h \geq 0\ ,$

iv) $\frac{K}{2} - c_G e_h \geq 0.$

The first two conditions state the incentive compatibility and the last two conditions state the individual rationality constraints for bad and good types respectively. Similarly he can set optimal prizes for another equilibrium by using that equilibrium's constraints discussed in appendix 1. To have both types exert e_h in the tournament, there is a constraint over the difference between e_h and e_l as explained in Proposition 1.

Proposition 1 (Constraint on Effort Difference for Pooling on $\boldsymbol{e_h}$): *In the single-prize tournament, if the prize is high enough and $e_h < 3e_l$, both types will exert e_h in equilibrium.*

Proof: See appendix 3.

The condition over the effort levels comes with the incentive compatibility and individual rationality constraints for bad types. Bad types exert e_l and get a positive expected payoff only if the difference between the effort levels is high enough; otherwise they exert e_h. However, when bad types exert e_h, the employer needs to give a high prize (hence he makes less profit) to guarantee positive expected utilities for bad types.[36]

3.2.2 Parallel Tournament

In the parallel tournament, the employer offers two tournaments with different prizes to incentivize each worker. The high-prize tournament gives a high prize, H, to its winner; the low-prize tournament gives a low prize, L, to its winner, i.e., $H > L$. Knowing only his own type and the prizes, each worker is asked to participate in one of two tournaments. Then, tournament choices become public information and each worker chooses to exert e_h or e_l where $e_h > e_l$. If the workers choose different tournaments (hence they are by themselves in a tournament), regardless of the effort they exert, they win the tournament's prize. If both workers choose the same tournament, their payoffs are calculated similarly to the single-prize case.

The strategy of each worker is $(q_i, p_i^H, p_i^L) \in [0,1]$ x $[0,1]$ x $[0,1]$, where i denotes the worker's type (a good type (G) or a bad type (B)); q_i is the probability of the worker i choosing the high-prize tournament; and p_i^H and p_i^L are the probabilities of choosing e_h for a worker i when he is in the high-prize and low-prize tournaments, respectively.[37]

I start my equilibrium analysis by looking at workers' optimal decisions in the subgames, i.e., their effort choices. When a worker is by himself[38] in a given tournament he wins the chosen tournament's prize either he exerts e_h or e_l. To minimize his cost of effort he chooses e_l.

[36] When the difference between the effort levels is really high, the incentive-compatibility conditions to exert a low effort are satisfied, along with the individual-rationality condition for bad types, as opposed to the situation in Proposition 1. In that case, an employer can incentivize both types with a lower prize and my claim about the profit comparison between parallel and single-prize tournaments may not hold.

[37] Since the probability of choosing high effort when the agent by himself in the tournament is always 0, I did not list this among the other strategies.

[38] Workers learn the number of people in a tournament before choosing an effort level and know that they will win the tournament prize when they are alone in a tournament.

When two workers choose the same tournament, there are 81 cases to consider in the subgames in both (the high-prize and low-prize) tournaments for good types and bad types (For four conditions (type x tournament), there are 3 cases to consider with a total of $3^4 = 81$). Similar to the single-prize case, I can eliminate some subgame strategies due to higher marginal cost of effort for bad types (after this elimination the cases to consider become 25) and higher prize in the high tournament (after this elimination the cases to consider become 9). Additionally, I only look at the pure-strategy effort choices to simplify the calculations and have 6 cases to consider given the types' tournament choices.

One can write expected utility of a worker for exerting e_h or e_l in the high-prize and low-prize tournaments similarly to the expected utilities of his effort choices in the single-prize tournament. Workers' optimal effort choices in the tournaments depend on the chosen tournament's prize, effort difference, and their marginal cost of effort. For instance, conditions of a good type exerting e_h in both high-prize and low-prize tournaments are as follows respectively:

$$\left(\frac{H}{2} - c_G(e_h - e_l)\right) \geq 0 \text{ and } \left(\frac{L}{2} - c_G(e_h - e_l)\right) \geq 0.$$

After the optimal effort choices, I consider the conditions for the following equilibrium behavior for tournament selection: 1) Pooling: both types enter the high-prize tournament or both types enter the low-prize tournament. 2) Separating equilibrium: good types enter the high-prize tournament and bad types enter the low-prize tournament or good types enter the low-prize tournament and bad types enter the high-prize tournament. 3) Semi-pooling: good types enter the high-prize tournament and bad types mix between the tournaments, or good types mix between the tournaments and bad types enter the low-prize tournament, or both types mix between the tournaments. The equilibrium structures that are not discussed in appendix 2 are not equilibria due to different prize values in the tournaments. I mostly focused on the pure tournament-selection strategies. Since I have a treatment in the experiment when bad types are mixing, I also look at the conditions in which good types enter the high-prize tournament and bad types mix between the tournaments.

First, I look at the conditions for an equilibrium by which it is possible to create more homogeneous groups for the contests, i.e., conditions for perfect sorting and reverse sorting.

Proposition 3 (Separating Equilibrium for the Tournament Choice):

(i) *Good types enter the high-prize tournament and bad types enter the low-prize tournament only if the ratio of high-prize to low-prize is in the interval* $\frac{4}{3} \leq \frac{H}{L} \leq \frac{3}{2}$.

(ii) *Reverse sorting, where good types enter the low-prize tournament and bad types enter the high-prize tournament, never occurs.*

Proof: See appendix 2, Case 2 for (*i*) and appendix 2, Case 4 for (*ii*).

Good types always enter the high-prize tournament and exert pure equilibrium effort strategies if the prize ratio of the tournaments is high enough (specifically $\frac{H}{L} \geq \frac{4}{3}$). Bad types always enter the low-prize tournament and exert pure equilibrium effort strategies if the prize ratio of the tournaments is low enough (specifically $\frac{H}{L} \leq \frac{3}{2}$). If the prize ratio is below the lower bound, there is not much difference in terms of payoffs between the high- and low-prize tournaments for both types; if the prize ratio is above the upper bound, entering the high-prize tournament becomes more profitable for bad types. When both types are choosing the same effort levels in both tournaments, both types want to be in the high-prize tournament. Thus, there is no perfect sorting for tournament choices under pooling of effort choices. Reverse sorting never occurs because good types always find entering the high-prize tournament more profitable.

Next, I look at the conditions for a pooling equilibrium in which both types enter the high-prize or low-prize tournaments.

Proposition 4 (Pooling Equilibrium for the Tournament Choice):

(i) Pooling where both types enter the high-prize tournament occurs only when the high prize to low prize ratio is high enough.

(ii) Pooling where both types enter the low-prize tournament never occurs.

Proof: See appendix 2, Case 1 for (*i*) and appendix 2, Case 5 for (*ii*).

The threshold for prize ratio that makes bad types enter the high-prize tournament in the equilibrium in *(i)* changes according to bad types' optimal effort choices in the subgames. For instance, when both types are pooling for their effort choices in both tournaments, bad types enter the high-prize tournament if the prize ratio is higher than 2 (Case 1.1, Case 1.6). If bad types exert e_l in the low-prize tournament while exerting e_h in the high-prize tournament, bad types enter the high-prize tournament when the prize ratio is higher than 3 (Case 1.2, Case 1.4). If bad types exert e_l in both tournaments while good types exerting e_h in the high-prize tournament, bad types enter the high-prize tournament when the prize ratio is higher than 4 (Case 1.3, Case 1.5). There is no equilibrium in which both types enter the low-prize tournament because good types always obtain higher gains in the high-prize tournament.

If an employer increases the high tournament's prize, the expected payoff of a bad type by entering the high tournament increases. Hence, he enters the high tournament with a higher probability (appendix 2, Case 3).

After finding equilibrium conditions for the workers, I need to find the minimum prize for the employer to maximize his profit. For instance, for the equilibrium in which types self-select into the tournaments according to their abilities and separate for effort levels, the employer sets the prizes to maximize his profit as follows:

$$\max_{\{H,L\}}\left\{0.25(2e_h - H) + 0.25(2e_l - L) + 0.5\big(2e_l - (H + L)\big)\right\}$$

subject to

i) $\frac{H}{2} - c_G(e_h - e_l) \geq 0,$

ii) $\frac{L}{2} - c_G(e_h - e_l) \geq 0,$

iii) $\frac{H}{2} - c_B(e_h - e_l) \leq 0,$

iv) $\frac{L}{2} - c_B(e_h - e_l) \leq 0,$

v) $0.5\left(\frac{H}{2} - c_G e_h\right) + 0.5(H - c_G e_l) \geq 0.5(L - c_G e_l) + 0.5(L - c_G e_h),$

vi) $0.5(H - c_B e_l) + 0.5(-c_B e_l) \leq 0.5(L - c_B e_l) + 0.5(\frac{L}{2} - c_B e_l),$

vii) $0.5(H - c_G e_l) + 0.5\left(\frac{H}{2} - c_G e_h\right) \geq 0,$

viii) $0.5(L - c_B e_l) + 0.5\left(\frac{L}{2} - c_B e_l\right) \geq 0.$

The first two inequalities state incentive compatibility conditions for good types to exert e_h in the high-prize and low-prize tournaments respectively, the third and fourth inequalities state incentive compatibility conditions for bad types to exert e_l in the high-prize and low-prize tournaments respectively, the fifth and sixth inequalities state incentive compatibility conditions for good types to enter the high-prize and bad types to enter the low-prize tournaments respectively, and the last two conditions state individual rationality conditions for good and bad types when they self-select into the tournaments according to their abilities and choose e_h and e_l respectively, in the chosen tournaments. The employer sets optimal prizes for other potential equilibria as discussed in appendix 2 and implements the equilibrium that maximizes his profit.

3.3 Experimental Design and Procedures

I implemented a 2 x 2 design. Namely, I varied the institution (single-prize or parallel tournament) and the cost difference between the types (large or small). I used "between-" subject design for the institution treatments and "within-" subject design for the cost difference treatments. To vary the cost difference, I kept the good type's marginal cost level (0.4) identical and changed bad types' cost levels across the treatments. To have large (small) cost difference between the types I chose bad types' cost level as 1 (0.5). In additional treatments, I also changed prizes in the parallel (high prize) and single prize-tournaments. In both between- and within-subject treatments, I used the same effort levels: 35 for e_h and 13 for e_l.

	Parallel Tournament				**Single-Prize Tournament**		
(High) Prize	**Optimal**		**Above Optimal**		**Optimal**	**Optimal**	**Below Optimal**
Cost Difference	**Large**	**Small**	**Large**	**Small**	**Large**	**Small**	**Large**
Bad Type's Cost Level	1	0.5	1	0.5	1	0.5	1
Good Type's Cost Level	0.4	0.4	0.4	0.4	0.4	0.4	0.4
(High) Prize	25	25	42	42	70	36	42
Low Prize	18	18	18	18	--	--	--

Table 3.1: Treatments

Now, I will explain what theory predicts about workers' behavior and the employer's profit in these treatments. When the cost difference between the types is "large", good and bad types self-select into the tournaments according to their abilities and separate in effort levels with the optimal high and low prizes as given in the first column of Table 3.1 (The parameters of this treatment satisfy the conditions of the equilibrium corresponding to Case 2.3 in appendix 2). I compare the profits obtained in this treatment with the profits obtained in the "large" cost-difference treatment of the single-prize tournament (last column in Table 3.1).[39] The employer sets a different prize for the single-prize tournament than the sum of prizes in the parallel tournament. Particularly, when the cost difference between the types is large, he uses a greater prize in the single-prize tournament than the sum of high and low prizes in the parallel tournament. This stems from the fact that an employer needs to give a really high prize to motivate both types to exert e_h in the single-prize tournament (The parameters of the treatment in the single-prize tournament with 'large' cost difference satisfy the conditions of the equilibrium corresponding to Case 1 in appendix 1). As shown in the second row of Table 3.2, when the cost difference between

[39] When the prize is optimal, treatment is specified just by the cost difference.

the workers is "large," in equilibrium the employer obtains higher profits in the parallel tournament than in the single-prize tournament.

When the cost difference between the types is "small," good and bad types self-select into the tournaments and separate in effort levels with optimal high and low prizes as given in the second column of Table 3.1 (The parameters of this treatment satisfy the conditions of the equilibrium corresponding to Case 2.2 in appendix 2).[40] I compare the profits obtained in this treatment with the profits obtained in the 'small' cost-difference treatment of the single-prize tournament (the fifth column in Table 3.1). The employer sets a lower prize in the single-prize tournament than the sum of prizes in the parallel tournament when the cost difference between the types is "small." (The parameters of the treatment in the single-prize tournament with "small" cost difference satisfy the conditions of the equilibrium corresponding to Case 1 in appendix 1).[41] As shown in the second row of Table 3.2, when the cost difference between the workers is "small," in equilibrium the employer obtains a lower profit in the parallel tournament than in the single-prize tournament.

I also have treatments by increasing the high prize in the parallel tournament. If the high prize is above optimal and cost difference is "large," good types enter the high-prize tournament and bad types enter the high-prize tournament with 0.5 probability. Good types exert e_h while bad types exerting e_l in both tournaments. (The parameters of this treatment satisfy the conditions

[40] Both in the treatments with "small" and "large" cost differences, the designer can make higher profit if he implements an equilibrium in which both types exert low effort in the low-prize tournament. In this way, both low-prize and high-prize tournaments will be lower and the expected total elicited effort will be the same. However, with these optimal prizes the incentives for both types to choose equilibrium behavior (especially for tournament choice) will be lower. Hence, I chose the prizes in a way that they do not change the main theoretical message of the paper but gives higher incentives to the agents to choose the right tournament.

[41] The same profit can be obtained under parallel tournament if in equilibrium when both types enter the high-prize tournament and choose low effort in the low-prize tournament. The reason is that there will not be a constraint for the low prize, i.e., the employer can set 0 for it. Then parallel tournament will turn out a single-prize tournament.

of the equilibrium corresponding to Case 3.3 in appendix 2).[42] This treatment theoretically generates a lower profit than the “large” cost difference treatment of the parallel tournament with optimal prizes, as shown in the second row of Table 3.2.

If the high prize is above optimal and cost difference is “small,” good types enter the high-prize tournament with probability 1 and bad types enter the high-prize tournament with probability 0.61. Good types exert e_h in both tournaments whereas bad types exert e_l in the low-prize tournament and e_h in the high-prize tournament. (The parameters of this treatment satisfy the conditions of the equilibrium corresponding to Case 3.2 in appendix 2). This treatment generates a higher profit than the “small” cost difference treatment of the parallel tournament with optimal prizes, as shown in the second row of Table 3.2.

I have a treatment with the below optimal prize in the single-prize tournament (the same prize with the high prize of above optimal treatment in the parallel tournament) to test the effect of tournament-selection stage on agents’ effort choices.[43] Such a stage may affect a subject’s belief about the opponent’s type, hence the subject’s effort choice. A summary of all treatments is shown in Table 3.1.

Subjects were recruited by e-mail using the Social Science Experimental Laboratory database, which consists of graduate and undergraduate students at the California Institute of Technology. The experiment was computerized using z-tree (Fischbacher, 2007). In total 88 subjects participated in the experiment. Each subject participated only once, either in a single-prize or in a parallel tournament. In the experiment, there were nine sessions: five sessions of the

[42] The other strategies with mixing for the tournaments are not feasible, i.e., there is no equilibrium when both types are mixing between the tournaments or when good types are mixing between the tournaments, bad types enter the low-prize tournament.
[43] The parameters of the treatment with a “below optimal” prize in the single-prize tournament satisfy the conditions of equilibrium in Case 3, in appendix 1 besides individual rationality constraints for bad types.

single-prize and four sessions of the parallel treatments.[44] Each session consisted of 40 periods, with 10 periods of each treatment.[45] In each period of the parallel (single-prize) sessions, one of four (three) treatments was randomly selected.[46] Subjects were told that in every period the (high) prize, bad type's cost level, or both could change. Throughout the experiment, payoffs were described in terms of "points." Twenty points corresponded to one dollar. Subjects were paid the sum of their periods' earnings at the end of the experiment. Each session lasted about an hour and subjects earned $18.57 on average, including a $5 participation fee.

The experiments replicated the model described in Section 2. In the instructions, a neutral language was used: Match corresponded to the opponent, task corresponded to tournament, decision level corresponded to effort, cost multiplier corresponded to a subject's ability/type, decision cost corresponded to a subject's disutility of effort, and payment corresponded to prize. Now I will describe the single-prize and parallel tournament sessions in more detail.

In the single-prize sessions,[47] at the beginning of each period each subject was assigned a "cost multiplier." Each cost multiplier was randomly drawn from $\{0.4, 1\}$ (in the treatments where the cost difference between the types is large) or from $\{0.4, 0.5\}$ (in the treatments where the cost difference between the types is small) in every period. After subjects were informed only about their cost multipliers, they simultaneously chose between two decision levels: 35 or 13. Subjects were informed that by choosing a decision level they would incur a "decision cost."[48] This decision cost was explained verbally and in the form of a table shown on their screen. After

---

[44] The number of subjects in a session varied from session to session. In particular, in three of the single-prize sessions there were 10 subjects, in one of them there were 6 subjects, and in one of them there were 8 subjects. In three of parallel-tournament sessions, there were 10 subjects, and in one of them there were 14 subjects.

[45] The treatment with a "below optimal" prize in the single-prize tournament in table 3.1 was repeated 20 times in order to make subjects really think about the effort levels. With the parameters used in the other treatments of the single-prize tournament, they may have got bored by always choosing the high effort level.

[46] The order of treatments in parallel tournaments (or in single-prize tournaments) is the same across the sessions. These orders can be found at appendix 6 of this chapter.

[47] The instructions for this session can be found at appendix 5 of this chapter.

[48] Decision cost is the multiplication of cost multiplier and decision level.

each subject had chosen her decision level, the decision levels of the matched two subjects were compared. The player with the higher decision level received a "payment" for that particular period, whereas her partner received no payment. If both match members chose the same decision level, the period payment was divided between the two members. It was also explained and emphasized that decision costs would be subtracted whether or not a subject had won. This implied that subjects could lose money for some periods.

Similarly, in the parallel sessions,[49] subjects were assigned a cost multiplier either from $\{0.4, 1\}$ or from $\{0.4, 0.5\}$ in each period. After subjects were informed only about their cost multipliers, they simultaneously chose between two task choices: Task A or Task B. After the task choice, they were shown whether their opponents chose the same task or not. Then, they were asked to choose either 35 or 13 as their decision levels. While making this choice, they saw the cost table representing potential costs of their decisions. After each subject had chosen her decision level, the decision levels of the matched two subjects who performed the same task were compared. The player with the higher decision level received a "payment" for the task in that particular period; her opponent received no payment. If both subjects in a pair choosing the same task also chose identical decision level, the payment for that task was divided between them. If the subjects in a pair chose different tasks, they received their task payments regardless of the decision level they chose. This was explained during the instructions. Moreover, subjects knew that their decision costs would be subtracted whether or not they won the tournament.

In all sessions, after each period, subjects saw a feedback screen. In the single-prize sessions, subjects were informed whether their decision level was higher or lower than, or coincided with, their opponents' decision levels; whether they won the period payment or not; and what their period payoffs were. Furthermore, the screen reiterated a subject's cost multiplier and decision level. In the parallel sessions, in addition to the feedbacks given in the single-prize

[49] Instructions for this session can be found at appendix 6 of this chapter.

tournaments, subjects were informed about whether they were alone or with their opponents in the task they chose.

3.4 Profit

I start by comparing an employer's profit from an individual in the parallel and single-prize tournaments. In Table 3.2, the first row gives means of profits obtained from a person in the experiment and the second row gives expected profits from a person in equilibrium. In all treatments, each subject plays the same treatment 10 times, resulting in 440 observations. In all statistical tests, standard errors are clustered by subjects within the same treatment.[50]

Tournament	**Parallel**				**Single-prize**	
(High) Prize	**Optimal**		**Above Optimal**		**Optimal**	**Optimal**
Cost Difference	**Large**	**Small**	**Large**	**Small**	**Large**	**Small**
Mean	3.49	4.88	-0.91	5.56	-1.00	15.30
Equilibrium	2.38	2.38	-2.38	3.89	0.00	17.00

Table 3.2: Descriptive Statistics for Profit Levels.

First, I compare the profits obtained in the parallel tournament with the profits obtained in the single-prize tournament when the cost difference is "large." I find that the mean profit (with the described treatment parameters) in the parallel tournament is higher than the mean profit in the single-prize tournament at the 1% significance level (Wilcoxon p-value = 0.0000). Then, I compare the profits obtained in the parallel tournament with the profits obtained in the single-prize tournament when the cost difference is "small." I find that the mean profit (with described treatment parameters) in the parallel tournament is lower than the mean profit in the single-prize tournament at the 1% significance level (Wilcoxon p-value = 0.0000).

Furthermore, I test how an increase in the high tournament's prize in the parallel tournament affects the employer's profit. When the cost difference between the types is 'large', I

[50] Decision levels chosen by the same subject cannot be considered as an independent observation.

find that the mean profit obtained in the treatment with the “optimal” high prize is significantly higher than the mean profit obtained in the treatment with the “above optimal” high prize (Wilcoxon p-value = 0.0003). However, when the cost difference between the types is “small,” I find that the mean profits obtained in the parallel tournament with the “optimal” and “above optimal” high prize treatments are not significantly different (Wilcoxon p-value = 0.48).

These statistical results, which are in line with the theory, tell us that an employer prefers a parallel tournament to a single-prize tournament when the cost difference between the types is large. However, he prefers a single-prize tournament to a parallel tournament when the cost difference between the types is small. In any case (with large or small cost difference between the types), the employer does not prefer increasing the high prize in the parallel tournament because of the following two reasons: 1) When the cost difference between the types is small, the employer always obtains a significantly lower profit in the parallel tournament (either in the optimal prize or above optimal prize treatment) than in the single-prize tournament. 2) When the cost difference between the types is large, the employer obtains a significantly higher profit in the parallel tournament with the ‘optimal’ high prize treatment than with the ‘above optimal’ high prize treatment.

3.5 Tournament and Effort Choices

3.5.1 Aggregate Data

For tournament-selection behavior, I check how frequently subjects choose the high-prize tournament over all periods of a treatment when they are good or bad types.[51] As evident in the first row of Table 3.3 and the equilibrium percentages in parentheses, good types selected the

[51] As evident from table 3.4, experience does not play a significant role in the tournament choice but in the equilibrium effort choice. When I separate the treatments into the groups of ten, I realized that most of the learning occurs in the first two periods. If the aggregate data is calculated by excluding the first two repetition of each treatment, observed equilibrium effort percentages are higher. For instance, by excluding the first two repetition of each treatment observed equilibrium effort increases from 76% to 81% in the OL treatment, from 72% to 79% in the OS treatment, or from 52% to 61% in the AOL treatment.

high-prize tournament significantly less often than predicted by the equilibrium. However, bad types selected the high-prize tournament significantly more often than predicted by the equilibrium in all treatments, except in the AOL treatment of parallel tournament.[52]

The difference between the observed and equilibrium tournament selection behavior may stem from two reasons. The first may be risk aversion. The best response of each agent can change depending on how risk averse or risk loving she is. The second may be that an agent's belief of winning the high tournament's prize changes as the marginal cost difference between the types changes. The closer difference in marginal cost levels of a bad type and a good type, the more a bad type can believe that he can win the high-tournament's prize.[53]

	High Prize	**Optimal**				**Above Optimal**			
	Cost Difference	**Large (OL)**		**Small (OS)**		**Large (AOL)**		**Small (AOS)**	
	Subjects/Types	**Good**	**Bad**	**Good**	**Bad**	**Good**	**Bad**	**Good**	**Bad**
Tournament Selection	All (Equilibrium)	76 (100)	32 (0)	70 (100)	50 (0)	87 (100)	47 (50)	87 (100)	80 (61)
Effort (n = 2)	All	81	76	86	72	93	52	96	93

Table 3.3: Average High-Prize Tournament-Selection and Effort Percentages (%) in Parallel Tournaments (Equilibrium high-prize tournament-selection percentages (%) are given in parentheses)

Then, I check subjects' effort behavior at the sub-games (at the chosen tournament) when they are good or bad types across the treatments.[54] I separately analyze subjects' behavior when they are by themselves or together with their opponents in the tournament. When there is just one person in the tournament, observed equilibrium effort percentages are quite high in all treatments

[52] For significance test, one-sample t-test is used.

[53] In the treatments, I chose the same optimal prizes when the cost difference is large and small, the reader may think that lower prizes would be enough when the cost difference between the types is small. Further, he may think that choosing the prizes that high in the "small" cost difference treatment may lead tournament selection mistakes. However, the reader should note that these same prizes in the treatments lead types to exert different optimal effort choices in the tournaments. If the subgame behavior of both types were the same, when the cost difference was small, the designer would incentivize both types to self-select in to the tournaments with lower low- and high-tournament's prizes.

[54] I check whether the agent's effort decision is according to the subgame equilibrium of that treatment or not, even if the tournament-selection decision is not according to the equilibrium prediction.

and not significantly different from the equilibrium predictions. When there are two people in the tournament, observed equilibrium effort percentages are significantly below the equilibrium predictions as evident in the second row of Table 3.3. The most noticeable observation in the Effort row of Table 3.3 belongs to bad types in the AOL ("above optimal" high prize and "large" cost difference) treatment of parallel tournament. To understand why the percentage in this particular treatment has low equilibrium ratio, I first show the observed equilibrium effort percentages in each tournament separately (appendix 4, Table 3.6). As evident from Table 3.5, bad types' behavior did not accord to the sub-game Nash equilibrium in this treatment when they were in the high-prize tournament (They chose e_h with 0.68 probability and e_l with probability 0.32 even though they should have chosen e_l with probability 1). Then I calculate the expected payoffs with the realized frequencies in the experiment (appendix 4, Table 3.7). One can observe from Table 3.6, the expected payoff difference by exerting e_h and e_l for bad types is low (0.7) in the treatment with the "above optimal" high prize and "large" cost difference. To understand whether there is a correlation between the percentage of observed equilibrium effort and the expected payoff difference between the effort choices (the magnitudes of incentives), I look at the correlation coefficient. I find the correlation to be 0.71, i.e., subjects' response to magnitudes of incentives for effort levels is high.

Similarly, for the single-prize sessions, I look at the observed equilibrium effort percentage of the subjects when they are good or bad types across the treatments. In all treatments of the single-prize sessions, all types exerted equilibrium effort more than 90% of the time, but bad types in the treatment with the 'below optimal' prize exerted equilibrium effort only 52% of the time. To understand why bad types in that treatment were outliers, I follow a similar strategy to the parallel tournament case, i.e., considering the expected payoff for each type (good or bad) from exerting e_h and e_l. The expected payoff difference between the effort choices is the closest to 0 for bad types in the treatment with the "below optimal" prize and it is much higher in other treatments. Again, one can conclude that subjects highly respond to magnitudes of incentives for effort levels in the single-prize tournament.

To understand the effect of the tournament-selection decision prior to the effort choice on subjects' effort choices, I use the same prize in one of the single-prize sessions (the treatment with the "below optimal" prize) and in one of the high-tournaments of the parallel sessions (the treatment with the "above optimal" high prize). I observe high equilibrium effort frequencies for good types in the single-prize and in the parallel tournament treatments (97% and 99%, respectively). However, for bad types, I observe low equilibrium effort frequencies in both treatments (54% and 32%, respectively). The much lower equilibrium behavior for the bad types in the parallel tournament may stem from the fact that a bad type expects e_h from his opponent in the high-prize tournament and as a response he does indeed exert e_h. However, for that treatment, the bad type's best response should be to choose e_l regardless of whether he is in the high-prize or low-prize tournament.

3.5.2 Individual Analysis

To uncover the determinants of tournament selection and to test for learning, I estimate a discrete choice model on each individual's decision to choose high-prize tournament as a function of several explanatory variables. *Low-cost* variable takes the value 1 when the subject is a good type, *Female* variable takes the value 1 when the subject is female, *Experience* variable takes the value 1 for the last 20 periods of a session, *Above-optimal* variable takes the value 1 when the high-tournament's prize is above optimal, *small-cost difference* variable takes the value 1 when the cost difference between good and bad types is small. Table 3.4 corresponds to the marginal effect of the estimation (where standard errors are clustered by subjects).

Condition	Entry into high-prize tournament
Low-cost	0.29*** (0.04)
Female	0.13** (0.06)
Experience	0.01 (0.03)
Above-optimal	0.14*** (0.04)
Small-cost difference	0.07* (0.04)
Above Optimal*Small cost difference	0.13*** (0.04)
Pseduo-Rsquared	0.13
Observations	1760

Table 3.4: Probit Estimation for Entering the High-Prize Tournament

Several insights come out of the tournament-entry estimation. First, and in line with the theoretical predictions, being a good type increases the probability of entry into the high-prize tournament. Second, again in line with the theoretical predictions, if the high tournament's prize is above optimal, the probability of entry into the high-prize tournament increases. Third, subjects' behavior did not differ significantly between the first 20 periods and the last 20 periods. Fourth, small cost-difference between good and bad types increases the probability of entry into the high-prize tournament by 0.07. However, the cost difference between the types should affect subjects' entry decisions only when high tournament's prize is above optimal. Last, being a female increases the probability of entry into the high-prize tournament by 0.13. Different than here, Niederle and Vesterlund (2007) analyzed whether gender affects subjects' choices between tournament and piece rate (where there is no risk), and found that men prefer tournaments more often than women even though there is no significant performance difference between two.[55]

To uncover the determinants of effort behavior in each tournament, I also estimate a discrete choice model on each individual's decision to choose equilibrium effort in the high- and

[55] Niederle and Vesterlund's (2007) tournament environment also differs from mine: In their study, subjects knew their competitors' genders (subjects were in groups of four; two female and two male) whereas in my study, subjects did not know their opponents' genders. In their study, performances were determined via real effort task and subjects only knew their own performance when they made a tournament entry choice. In my study, performances corresponded to pre-determined effort levels. The tournament environment in my study was less uncertain than theirs. In my experiment, each subject not only knew his own ability (which affected his effort choice), but also knew the potential ability of his opponent.

low-prize tournaments as a function of some explanatory variables in addition to listed ones above: *Htn2* variable takes the value 1 when the subject is together with his opponent in the high-prize tournament and *Ltn2* variable takes the value 1 when the subject is together with his opponent in the low-prize tournament. Second and third columns of Table 3.5 correspond to the marginal effects of these estimations respectively (where standard errors are clustered by subjects).

Condition	Equilibrium effort in the HT	Equilibrium effort in the LT
Low-cost	0.08*** (0.02)	0.05*** (0.02)
Female	-0.01 (0.02)	-0.03 (0.02)
Experience	0.08*** (0.02)	0.08*** (0.02)
Above-optimal	0.01 (0.02)	-0.01 (0.02)
Small-cost difference	0.01 (0.02)	-0.00 (0.02)
Above Optimal*Small cost difference	0.07*** (0.02)	0.06*** (0.02)
Htn2	-0.07*** (0.02)	-----
Ltn2	-----	-0.14*** (0.04)
Pseduo-Rsquared	0.07	0.09
Observations	1760	1760

Robust standard errors are in parenthesis and clustered by subjects.

* significant at 10% level; ** significant at 5% level; *** significant at 1% level

Table 3.5: Probit Estimation for Choosing Equilibrium Effort in the High (Low)-Prize Tournaments

There are several results emerging from equilibrium effort estimations in both tournaments. First, in both tournaments, being a good type increases the probability of choosing equilibrium effort slightly. Second, there is no effect of gender on equilibrium effort. Third, among the treatments only the treatment with above optimal prize and small cost difference increases the equilibrium effort behavior slightly in both tournaments. Fourth, in both tournaments, being together with the opponent decreases the probability of exerting equilibrium effort. Last, in contrast to learning behavior in tournament selection, subjects learn to exert equilibrium effort in both tournaments by time.[56]

3.6 Conclusions

I report results from experiments considering two different tournament designs when workers are heterogeneous: a tournament with a single prize and two parallel tournaments with different prizes. With respect to outcomes, there are three main insights. First, as predicted by the theory, when types differ greatly from each other, the parallel tournament generates higher profits to an employer than the single-prize tournament. When types are similar, the single-prize tournament generates higher profits. Moreover, by increasing the high prize in the parallel-tournament case, an employer cannot increase his profit. Second, good types always under participate and bad types over participate in the high-prize tournament. Furthermore, for the tournament-selection decision, I find that females enter the high-prize tournament significantly more often than males. Third, subjects' equilibrium effort behavior is related to the number of contestants in the tournament, they exert equilibrium effort behavior more frequently when there are fewer subjects in a tournament.

These results are important from a tournament-design perspective. In the real world, there is often substantial heterogeneity among workers; hence a tournament should be designed considering this fact. Additionally, according to my experimental observations, it may not be possible to obtain self-selection into tournaments with respect to ability only, other characteristics, such as risk aversion, beliefs, etc., may affect the tournament selection dynamics

[56] El-Gamal, McKelvey, and Palfrey (1993) compared two learning models in their paper: one is based on reputation building and the other concerns population learning. Learning by reputation building can be achieved by playing with a specific opponent over a sequence of moves, then using observations about the opponent's earlier play to make inferences about how the opponent is likely to play in subsequent moves. In population learning situations, each subject plays similar games against a sequence of opponents then predicts how a randomly selected opponent is likely to act by learned population parameters. My experiment is classified under the second model because subjects were re-matched in every period and did not play the same game over a period of time (the parameters also changed in every period).

of workers. Hence, an employer needs to understand the underlying reasons for workers' tournament choices to better organize tournaments to increase his profits.

As I stated earlier, the employer experiences a trade-off by organizing parallel tournaments as an alternative to single-prize tournaments because by parallel tournaments an employer can increase the homogeneity of contestants in each group, and hence increase his profit. At the same time, by these tournaments, the number of contestants in each contest decreases and the elicited total effort decreases. My experiment was designed using the simplest model, with only two person, two types, and two effort levels, as a first step toward understanding this trade-off, and it succeeds in uncovering the basic dynamics of this trade-off.

3.7 Appendix 1: Single-Prize Tournament

Case 1: To have both (good and bad) types pool for e_h, the expected utility of exerting e_h should be higher than the expected utility of exerting e_l, i.e.,

$$\left(\frac{K}{2} - c_G(e_h - e_l)\right) \geq 0\,, \qquad IC_G$$

$$\left(\frac{K}{2} - c_B(e_h - e_l)\right) \geq 0, \qquad IC_B$$

$$\left(\frac{K}{2} - c_G e_h\right) \geq 0, \qquad IR_G$$

$$\left(\frac{K}{2} - c_B e_h\right) \geq 0. \qquad IR_B$$

Case 2: To have both types pool for e_l, the expected utility of exerting e_l should be higher than the expected utility of exerting e_h for both types.

$$\left(\frac{K}{2} - c_G(e_h - e_l)\right) \leq 0, \qquad IC_G$$

$$\left(\frac{K}{2} - c_B(e_h - e_l)\right) \leq 0, \qquad IC_B$$

$$\left(\frac{K}{2} - c_G e_l\right) \geq 0, \qquad IR_G$$

$$\left(\frac{K}{2} - c_B e_l\right) \geq 0. \qquad IR_B$$

Case 3: To have a separating equilibrium in which good types exert e_h and bad types should exert e_l, the expected utility of exerting e_h should be higher than the expected utility of exerting e_l for good types and the expected utility of exerting e_l is higher than the expected utility of exerting e_h for bad types, i.e.,

$$\left(\frac{K}{2} - c_G(e_h - e_l)\right) \geq 0, \qquad IC_G$$

$$\left(\frac{K}{2} - c_B(e_h - e_l)\right) \leq 0, \qquad IC_B$$

$$0.5\left(\frac{K}{2}-c_G e_h\right)+0.5(K-c_G e_h)\geq 0, \qquad IR_G$$

$$0.5\left(\frac{K}{2}-c_B e_l\right)+0.5(-c_B e_l)\geq 0. \qquad IR_B$$

Case 4: The necessary condition for a semi-pooling equilibrium in which good types exert e_h but bad types mix between e_h and e_l (bad types choose e_h with probability p_B) are:

$$\left(\frac{K}{2}-c_G(e_h-e_l)\right)\geq 0, \qquad IC_G$$

$$\left(\frac{K}{2}-c_B(e_h-e_l)\right)=0, \qquad IC_B$$

$$0.5\left(\frac{K}{2}-c_G e_h\right)+0.5\left(p_B\left(\frac{K}{2}-c_G e_h\right)+(1-p_B)(K-c_G e_h)\right)\geq 0, \qquad IR_G$$

$$0.5\left(\frac{K}{2}-c_B e_h\right)+0.5\left(p_B\left(\frac{K}{2}-c_B e_h\right)+(1-p_B)(-c_B e_h)\right)\geq 0. \qquad IR_B$$

The necessary conditions for a semi-pooling equilibrium in which good types mix between e_h and e_l (good types choose e_h with probability p_B) and bad types exert e_l are:

$$\left(\frac{K}{2}-c_G(e_h-e_l)\right)=0, \qquad IC_G$$

$$\left(\frac{K}{2}-c_B(e_h-e_l)\right)\leq 0, \qquad IC_B$$

$$0.5\left(p_B\left(\frac{K}{2}-c_G e_h\right)+(1-p_B)(K-c_G e_h)\right)+0.5(K-c_G e_h)\geq 0, \qquad IR_G$$

$$0.5\left(p_B(-c_B e_l)+(1-p_B)\left(\frac{K}{2}-c_B e_l\right)\right)+0.5(\frac{K}{2}-c_B e_l)\geq 0. \qquad IR_B$$

3.8 Appendix 2: Parallel Tournaments

Case 1: Pooling for both types to enter the high-prize tournament, $q_G=q_B=1$.

Case 1.1: Both types pool for effort level e_h in both tournaments:

$$p_G^L=1, p_B^L=1, p_G^H=1, p_B^H=1.$$

To have both types exert effort levels with the above probabilities, the following constraint should hold on the prize:

$$\frac{L}{2} - c_B(e_h - e_l) \geq 0, \qquad \frac{H}{2} - c_B(e_h - e_l) \geq 0, \qquad IC_B$$

$$\frac{L}{2} - c_G(e_h - e_l) \geq 0, \qquad \frac{H}{2} - c_G(e_h - e_l) \geq 0, \qquad IC_G$$

To have both types participate in the high-prize tournament, the following constraint should hold on the prize:

$$\frac{H}{2} - L - c_B(e_h - e_l) \geq 0, \qquad IC_B$$

$$\frac{H}{2} - L - c_G(e_h - e_l) \geq 0. \qquad IC_G$$

To have both types follow the above strategies and gain a positive expected utility, the following constraint should hold on the prize:

$$\frac{H}{2} - c_B e_h \geq 0, \qquad IR_B$$

$$\frac{H}{2} - c_G e_h \geq 0. \qquad IR_G$$

Case 1.2: Types separate for effort levels in the low-prize tournament; types pool for effort level e_h in the high-prize tournament:

$$p_G^L = 1, p_B^L = 0, p_G^H = 1, p_B^H = 1.$$

To have both types exert effort levels with the above probabilities, the following constraint should hold on the prize:

$$\frac{L}{2} - c_B(e_h - e_l) \leq 0, \qquad \frac{H}{2} - c_B(e_h - e_l) \geq 0, \qquad IC_B$$

$$\frac{L}{2} - c_G(e_h - e_l) \geq 0, \qquad \frac{H}{2} - c_G(e_h - e_l) \geq 0. \qquad IC_G$$

Conditions of selecting the high-prize tournament for both types are the same as in Case 1.1 because on the equilibrium path both types choose the same strategies. Similarly, individual rationality conditions will be same as in Case 1.1 because both types select the high-prize tournament and exert e_h.

Case 1.3: Types separate for effort levels in the low-prize and high-prize tournaments:

$$p_G^L = 1, p_B^L = 0, p_G^H = 1, p_B^H = 0.$$

To have both types exert effort levels with the above probabilities, the following constraint should hold on the prize:

$$\frac{L}{2} - c_B(e_h - e_l) \leq 0, \qquad \frac{H}{2} - c_B(e_h - e_l) \leq 0, \qquad IC_B$$

$$\frac{L}{2} - c_G(e_h - e_l) \geq 0, \qquad \frac{H}{2} - c_G(e_h - e_l) \geq 0. \qquad IC_G$$

To have both types select the high-prize tournament with the strategies above, the following constraint should hold on the prize:

$$-L + \frac{H}{2} - L \geq 0, \qquad IC_B$$

$$\frac{H}{2} - L - c_G(e_h - e_l) + H - L - c_G(e_h - e_l) \geq 0. \qquad IC_G$$

To have both types follow the above strategies and gain a positive expected utility, the following constraint should hold on the prize:

$$0.5\left(\frac{H}{2} - c_B e_l\right) + 0.5(-c_B e_l) \geq 0, \qquad IR_B$$

$$0.5\left(\frac{H}{2} - c_G e_h\right) + 0.5(H - c_G e_h) \geq 0. \qquad IR_G$$

Case 1.4: Types pool for effort level e_l in the low-prize tournament; types pool for effort level e_h in the high-prize tournament:

$$p_G^L = 0, p_B^L = 0, p_G^H = 1, p_B^H = 1.$$

To have both types exert effort levels with the above probabilities, the following constraint should hold on the prize:

$$\frac{L}{2} - c_B(e_h - e_l) \leq 0, \qquad \frac{H}{2} - c_B(e_h - e_l) \geq 0, \qquad IC_B$$

$$\frac{L}{2} - c_G(e_h - e_l) \leq 0, \qquad \frac{H}{2} - c_G(e_h - e_l) \geq 0. \qquad IC_G$$

Conditions for selecting the high-prize tournament for both types are the same as in Case 1.1 because on the equilibrium path both types choose the same strategies. For a similar reason, individual rationality conditions will be same as in Case 1.1.

Case 1.5: Types pool for effort level e_l in the low-prize tournament; types separate for effort level in the high-prize tournament:

$$p_G^L = 0, p_B^L = 0, p_G^H = 1, p_B^H = 0.$$

To have both types exert effort levels with the above probabilities, the following constraint should hold on the prize:

$$\frac{L}{2} - c_B(e_h - e_l) \leq 0, \qquad \frac{H}{2} - c_B(e_h - e_l) \leq 0, \qquad IC_B$$

$$\frac{L}{2} - c_G(e_h - e_l) \leq 0, \qquad \frac{H}{2} - c_G(e_h - e_l) \geq 0. \qquad IC_G$$

Conditions for selecting the high-prize tournament for both types are the same as in Case 1.3 because on the equilibrium path both types choose the same strategies. For a similar reason, individual rationality conditions will be same as in Case 1.3.

Case 1.6: Types pool for effort level e_l in the low-prize and high-prize tournaments:

$$p_G^L = 0, p_B^L = 0, p_G^H = 0, p_B^H = 0.$$

To have both types exert effort levels with the above probabilities, the following constraint should hold on the prize:

$$\frac{L}{2} - c_B(e_h - e_l) \leq 0, \qquad \frac{H}{2} - c_B(e_h - e_l) \leq 0, \qquad IC_B$$

$$\frac{L}{2} - c_G(e_h - e_l) \leq 0, \qquad \frac{H}{2} - c_G(e_h - e_l) \leq 0. \qquad IC_G$$

To have both types select the high-prize tournament with the strategies above after the tournament choice, the following constraint should hold on the prize:

$$\frac{H}{2} - L \geq 0, \qquad IC_B$$

$$\frac{H}{2} - L \geq 0. \qquad IC_G$$

To have both types follow the above strategies and gain a positive expected utility, the following constraint should hold on the prize:

$$\left(\frac{H}{2} - c_B e_l\right) \geq 0, \qquad IR_B$$

$$\left(\frac{H}{2} - c_G e_l\right) \geq 0. \qquad IR_G$$

Case 2: Separating equilibrium: Good types enter the high-prize tournament; bad types enter the low-prize tournament, $q_G = 1$ and $q_B = 0$.

Case 2.1: Both types pool for effort level e_h in both tournaments:

$$p_G^L = 1, p_B^L = 1, p_G^H = 1, p_B^H = 1.$$

For types' optimal effort choice, the same conditions (for effort levels) as in Case 1.1 should be satisfied. To have good types participate in the high-prize tournament and bad types participate in the low-prize tournament, the following conditions should hold:

$$H - \frac{L}{2} + c_B(e_h - e_l) + \frac{H}{2} - L - c_B(e_h - e_l) \leq 0, \qquad IC_B$$

$$H - \frac{L}{2} + c_G(e_h - e_l) + \frac{H}{2} - L - c_G(e_h - e_l) \geq 0. \qquad IC_G$$

There is no equilibrium like this because bad types always want to participate in the high-prize tournament with these equilibrium effort choices.

Case 2.2: Types separate for effort levels in the low-prize tournament; both types pool for effort level e_h in the high-prize tournament:

$$p_G^L = 1, p_B^L = 0, p_G^H = 1, p_B^H = 1.$$

For types' optimal effort choice, the same conditions (for effort levels) as in Case 1.2 should be satisfied. To have types separate into the tournaments, the following conditions should hold:

$$\frac{H}{2} - L - c_B(e_h - e_l) + H - \frac{L}{2} \leq 0, \qquad IC_B$$

$$\frac{H}{2} - L - c_G(e_h - e_l) + H - L + c_G(e_h - e_l) \geq 0. \qquad IC_G$$

To have both types follow the above strategies and gain a positive expected utility, the following conditions should hold:

$$0.5(L - c_B e_l) + 0.5\left(\frac{L}{2} - c_B e_l\right) \geq 0, \qquad IR_B$$

$$0.5(H - c_G e_l) + 0.5\left(\frac{H}{2} - c_G e_h\right) \geq 0. \qquad IR_G$$

Case 2.3: Types separate for effort levels in the low-prize tournament; types separate for effort levels in the high-prize tournament:

$$p_G^L = 1, p_B^L = 0, p_G^H = 1, p_B^H = 0.$$

The same conditions (for effort levels) as in Case 1.3 should be satisfied for the effort levels here. To have types separate into the tournaments, the following conditions should hold:

$$-L + H - \frac{L}{2} \leq 0, \qquad IC_B$$

$$\frac{H}{2} - L - c_G(e_h - e_l) + H - L + c_G(e_h - e_l) \geq 0. \qquad IC_G$$

Individual rationality conditions for both types in equilibrium satisfy the conditions in Case 2.2 because types self-select into the tournaments; on the equilibrium path types exert the same effort levels in the tournaments.

Case 2.4: Types pool for effort level e_l in the low-prize tournament; types pool for effort level e_h in the high-prize tournament:

$$p_G^L = 0, p_B^L = 0, p_G^H = 1, p_B^H = 1.$$

The same conditions (for effort levels) as in Case 1.4 should be satisfied for the effort levels here. To have types separate into the tournaments, and have both types follow the above strategies and gain a positive expected utility, the following conditions should hold:

$$\frac{H}{2} - L - c_B(e_h - e_l) + H - \frac{L}{2} \leq 0, \qquad IC_B$$

$$\frac{H}{2} - L - c_G(e_h - e_l) + H - \frac{L}{2} \geq 0, \qquad IC_G$$

$$0.5\left(\frac{L}{2} - c_B e_l\right) + 0.5(L - c_B e_l) \geq 0, \qquad IR_B$$

$$0.5\left(\frac{H}{2} - c_G e_h\right) + 0.5(H - c_G e_l) \geq 0. \qquad IR_G$$

Case 2.5: Types pool for effort level e_l in the low-prize tournament; types separate for effort level in the high-prize tournament:

$$p_G^L = 0, p_B^L = 0, p_G^H = 1, p_B^H = 0.$$

The same conditions (for effort levels) as in Case 1.5 should be satisfied for the effort levels here. To have types separate into the tournaments, and have both types follow the above strategies and gain a positive expected utility, the following conditions should hold:

$$-L + H - \frac{L}{2} \leq 0, \qquad IC_B$$

$$\frac{H}{2} - L - c_G(e_h - e_l) + H - \frac{L}{2} \geq 0, \qquad IC_G$$

$$0.5\left(\frac{L}{2} - c_B e_l\right) + 0.5(L - c_B e_l) \geq 0, \qquad IR_B$$

$$0.5\left(\frac{H}{2} - c_G e_h\right) + 0.5(H - c_G e_l) \geq 0. \qquad IR_G$$

Case 2.6: Types pool for effort level e_l in the low-prize tournament; types pool for effort level e_l in the high-prize tournament.

$$p_G^L = 0, p_B^L = 0, p_G^H = 0, p_B^H = 0.$$

The same conditions (for effort levels) as in Case 1.6 should be satisfied for the effort levels here. To have types separate into the tournaments, the following conditions should hold:

$$\frac{H}{2} - L + H - \frac{L}{2} \leq 0, \qquad IC_B$$

$$\frac{H}{2} - L \geq 0. \qquad IC_G$$

There is no equilibrium like this because bad types always want to participate in the high-prize tournament with these equilibrium effort choices.

Corollary: According to the incentive compatibility conditions for effort levels and tournament selection, the prize ratio should be set in the following interval $\frac{4}{3} \leq \frac{H}{L} \leq \frac{3}{2}$ to guarantee types self-select into the different tournaments.

Case 3: Semi-pooling equilibrium: Good types enter the high-prize tournament, $\boldsymbol{q_G = 1}$, and bad types mix between tournaments, $\boldsymbol{q_B \in (0, 1)}$.

Case 3.1: Both types pool for effort level e_h in both tournaments:

$$p_G^L = 1, p_B^L = 1, p_G^H = 1, p_B^H = 1.$$

The same conditions as in Case 1.1 should be satisfied for the effort levels here. To have good types enter the high-prize tournament and bad types mix between the tournaments, and have both types follow the above strategies (for effort choice and tournament selection) and gain a positive expected utility, the following conditions should hold:

$$0.5(1 - q_B)\left(H - \frac{L}{2} + c_B(e_h - e_l)\right) + 0.5(1 + q_B)\left(\frac{H}{2} - L - c_B(e_h - e_l)\right) = 0, \qquad IC_B$$

$$0.5(1 - q_B)\left(H - \frac{L}{2} + c_G(e_h - e_l)\right) + 0.5(1 + q_B)\left(\frac{H}{2} - L - c_G(e_h - e_l)\right) \geq 0, \qquad IC_G$$

$$0.5\left(\frac{H}{2} - c_B e_h\right) + 0.5\left((1 - q_B)(H - c_B e_l) + q_B\left(\frac{H}{2} - c_B e_h\right)\right) \geq 0, \qquad IR_B$$

$$0.5\left(\frac{H}{2} - c_G e_h\right) + 0.5\left((1 - q_B)(H - c_G e_l) + q_B\left(\frac{H}{2} - c_G e_h\right)\right) \geq 0. \qquad IR_G$$

Case 3.2: Types separate for effort level in the low-prize tournament; both types pool for effort level e_h in the high-prize tournament:

$$p_G^L = 1, p_B^L = 0, p_G^H = 1, p_B^H = 1.$$

The same conditions as in Case 1.2 should be satisfied for the effort levels. To have high types enter the high-prize tournament and low types mix between the tournaments, the following conditions should hold:

$$0.5\left(\frac{H}{2} - L - c_B(e_h - e_l)\right) + 0.5\left[(1 - q_B)\left(H - \frac{L}{2}\right) + 0.5q_B\left(\frac{H}{2} - L - c_G(e_h - e_l)\right)\right] = 0, \qquad IC_B$$

$$0.5(1 - q_B)\left(H - \frac{L}{2} + c_G(e_h - e_l)\right) + 0.5(1 + q_B)\left(\frac{H}{2} - L - c_G(e_h - e_l)\right) \geq 0. \qquad IC_G$$

Individual rationality conditions will be same as in Case 3.1; when bad types are in the high-prize tournament they still exert e_h.

Case 3.3: Types separate for effort levels in the low-prize tournament; types separate for effort levels in the high-prize tournament:

$$p_G^L = 1, p_B^L = 0, p_G^H = 1, p_B^H = 0.$$

The same conditions as in Case 1.3 should be satisfied for the effort levels here. To have good types enter the high-prize tournament and bad types to mix between the tournaments, and have both types follow the above strategies (for effort choice and tournament selection) and gain a positive expected utility, the following conditions should hold:

$$0.5(-L) + 0.5\left[(1 - q_B)\left(H - \frac{L}{2}\right) + q_B\left(\frac{H}{2} - L\right)\right] = 0, \qquad IC_B$$

$$0.5\left(\frac{H}{2} - L - c_G(e_h - e_l)\right) + 0.5\left[(1 - q_B)(H - L + c_G(e_h - e_l)) + q_B\left(H - L - c_G(e_h - e_l)\right)\right] \geq 0, \qquad IC_G$$

$$0.5(-c_B e_l) + 0.5\left((1 - q_B)(H - c_B e_l) + q_B\left(\frac{H}{2} - c_B e_l\right)\right) \geq 0, \qquad IR_B$$

$$0.5\left(\frac{H}{2} - c_G e_h\right) + 0.5\big((1 - q_B)(H - c_G e_l) + q_B(H - c_G e_h)\big) \geq 0. \qquad IR_G$$

Case 3.4: Types pool for effort level e_l in the low-prize tournament; types pool for effort level e_h in the high-prize tournament.

$$p_G^L = 0, p_B^L = 0, p_G^H = 1, p_B^H = 1.$$

The same conditions as in Case 1.4 should be satisfied for the effort levels here. To have good types enter the high-prize tournament and bad types mix between the tournaments, and have both types follow the above strategies (for effort choice and tournament selection) and gain a positive expected utility the following inequalities should hold.

$$0.5\left(\frac{H}{2} - L - c_B(e_h - e_l)\right) + 0.5\left((1 - q_B)\left(H - \frac{L}{2}\right) + q_B\left(\frac{H}{2} - L - c_B(e_h - e_l)\right)\right) = 0, \qquad IC_B$$

$$0.5\left(\frac{H}{2} - L - c_G(e_h - e_l)\right) + 0.5\left((1 - q_B)\left(H - \frac{L}{2}\right) + q_B\left(\frac{H}{2} - L - c_G(e_h - e_l)\right)\right) \geq 0, \qquad IC_G$$

$$0.5\left(\frac{H}{2} - c_B e_h\right) + 0.5\left((1 - q_B)(H - c_B e_l) + q_B\left(\frac{H}{2} - c_B e_h\right)\right) \geq 0, \qquad IR_B$$

$$0.5\left(\frac{H}{2} - c_G e_h\right) + 0.5\left((1 - q_B)(H - c_G e_l) + q_B\left(\frac{H}{2} - c_G e_h\right)\right) \geq 0, \qquad IR_G$$

Case 3.5: Types pool for effort level e_l in the low-prize tournament; types separate for effort level in the high-prize tournament:

$$p_G^L = 0, p_B^L = 0, p_G^H = 1, p_B^H = 0.$$

The same conditions as in Case 1.5 should be satisfied for the effort levels here. To have good types enter the high-prize tournament and bad types mix between the tournaments, and have both types follow the above strategies (for effort choice and tournament selection) and gain a positive expected utility, the following conditions should hold:

$$0.5(-L) + 0.5\left[(1-q_B)\left(H-\frac{L}{2}\right) + q_B\left(\frac{H}{2}-L\right)\right] = 0, \qquad IC_B$$

$$0.5\left(\frac{H}{2}-L-c_G(e_h-e_l)\right) + 0.5\left[(1-q_L)\left(H-\frac{L}{2}\right) + q_L\left(H-L-c_G(e_h-e_l)\right)\right] \geq 0, \qquad IC_G$$

$$0.5(-c_Be_l) + 0.5\left((1-q_L)(H-c_Be_l) + q_L\left(\frac{H}{2}-c_Be_l\right)\right) \geq 0, \qquad IR_B$$

$$0.5\left(\frac{H}{2}-c_Ge_h\right) + 0.5\left((1-q_L)(H-c_Ge_l) + q_L(H-c_Ge_h)\right) \geq 0. \qquad IR_G$$

Case 3.6: Types pool for effort level e_l in the low-prize and high-prize tournaments:

$$p_G^L = 0, p_B^L = 0, p_G^H = 0, p_B^H = 0.$$

The same conditions as in Case 1.6 should be satisfied for the effort levels. To have good types enter the high-prize tournament and bad types to mix between the tournaments, and have both types follow the above strategies (for effort choice and tournament selection) and gain a positive expected utility, the following conditions should hold:

$$0.5\left(\frac{H}{2}-L\right) + 0.5\left[(1-q_B)\left(H-\frac{L}{2}\right) + q_B\left(\frac{H}{2}-L\right)\right] = 0, \qquad IC_B$$

$$0.5\left(\frac{H}{2}-L\right) + 0.5\left[(1-q_B)\left(H-\frac{L}{2}\right) + q_B\left(\frac{H}{2}-L\right)\right] \geq 0, \qquad IC_G$$

$$0.5(-c_Be_l) + 0.5\left((1-q_B)(H-c_Be_l) + q_B\left(\frac{H}{2}-c_Be_l\right)\right) \geq 0, \qquad IR_B$$

$$0.5\left(\frac{H}{2}-c_Ge_h\right) + 0.5\left((1-q_B)(H-c_Ge_l) + q_B(H-c_Ge_h)\right) \geq 0. \qquad IR_G$$

Case 4: Good types enter the low-prize tournament; bad types enter the high-prize tournament, $q_G = 0$, and $q_B = 1$.

Case 4.1: Both types pool for effort level e_H in both tournaments:

$$p_G^L = 1, p_B^L = 1, p_G^H = 1, p_B^H = 1.$$

The same conditions as in Case 1.1 should be satisfied for the effort levels here. The following conditions should hold to have good types choose the low-prize tournament and bad types choose the high-prize tournament:

$$H - \frac{L}{2} + c_B(e_h - e_l) + \frac{H}{2} - L - c_B(e_h - e_l) \geq 0, \qquad IC_B$$

$$H - \frac{L}{2} + c_G(e_h - e_l) + \frac{H}{2} - L - c_G(e_h - e_l) \leq 0. \qquad IC_G$$

However, this cannot hold because $H > L > 0$; good types want to enter the high-prize tournament as well.

Case 4.2: Types separate for effort level in the low-prize tournament; types pool for effort level e_h in the high-prize tournament:

$$p_G^L = 1, p_B^L = 0, p_G^H = 1, p_B^H = 1.$$

The same conditions as in Case 1.2 should be satisfied for the effort levels here. The following conditions should hold to have good types choose the low-prize tournament and bad types choose the high-prize tournament:

$$H + \frac{H}{2} - L - c_B(e_h - e_l) \geq 0, \qquad IC_B$$

$$H - \frac{L}{2} + c_G(e_h - e_l) + \frac{H}{2} - L - c_G(e_h - e_l) \leq 0. \qquad IC_G$$

However, this cannot hold because $H > L > 0$; good types want to enter the high-prize tournament as well.

Case 4.3: Types separate for effort level in the low-prize and high-prize tournaments:

$$p_G^L = 1, p_B^L = 0, p_G^H = 1, p_B^H = 0.$$

The same conditions as in Case 1.3 should be satisfied for the effort levels here. The following conditions should hold to have good types choose the low-prize tournament and bad types choose the high-prize tournament:

$$H + \frac{H}{2} - L \geq 0, \qquad IC_B$$

$$H - \frac{L}{2} + c_G(e_h - e_l) + H - L - c_G(e_h - e_l) \leq 0. \qquad IC_G$$

However, this cannot hold because $H > L > 0$; good types want to enter the high-prize tournament as well.

Case 4.4: Types pool for effort level e_l in the low-prize tournament; types pool for effort level e_h in the high-prize tournament:

$$p_G^L = 0, p_B^L = 0, p_G^H = 1, p_B^H = 1.$$

The same conditions as in Case 1.4 should be satisfied for the effort levels here. The following conditions should hold to have good types choose the low-prize tournament and bad types choose the high-prize tournament:

$$H - \frac{L}{2} + \frac{H}{2} - L - c_B(e_h - e_l) \geq 0, \qquad IC_B$$

$$H - \frac{L}{2} + \frac{H}{2} - L - c_G(e_h - e_l) \leq 0. \qquad IC_G$$

However, these conditions cannot hold together because if bad types enter the high-prize tournament, good types should also enter because $c_B > c_G$.

Case 4.5: Types pool for effort level e_l in the low-prize tournament; types separate for effort level in the high-prize tournament.

$$p_G^L = 0, p_B^L = 0, p_G^H = 1, p_B^H = 0.$$

The same conditions as in Case 1.5 should be satisfied for the effort levels here. The following conditions should hold to have good types choose the low-prize tournament and bad types choose the high-prize tournament:

$$H - \frac{L}{2} + \frac{H}{2} - L \geq 0, \qquad IC_B$$

$$H - \frac{L}{2} + H - L - c_G(e_h - e_l) \leq 0. \qquad IC_G$$

However, high types' incentive compatibility condition in the tournament choice contradicts with their incentive compatibility condition in the effort choice.

Case 4.6: Types pool for effort level e_l in the low-prize and high-prize tournaments:

$$p_G^L = 0, p_B^L = 0, p_G^H = 0, p_B^H = 0.$$

The same conditions as in Case 1.6 should be satisfied for the effort levels here. The following conditions should hold to have good types choose the low-prize tournament and bad types choose the high-prize tournament:

$$H - \frac{L}{2} + \frac{H}{2} - L \geq 0, \qquad IC_B$$

$$H - \frac{L}{2} + \frac{H}{2} - L \leq 0. \qquad IC_G$$

Contradiction, no equilibrium exists with these strategies for tournament selection and effort choices.

Case 5: Both types participate in the low-prize tournament, $q_G = 0$, and $q_B = 0$.

Case 5.1: Both types pool for effort level e_h in both tournaments:

$$p_G^L = 1, p_B^L = 1, p_G^H = 1, p_B^H = 1.$$

The same conditions as in Case 1.1 should be satisfied for the effort levels here. The following conditions should hold to have both types choose the low-prize tournament:

$$H - \frac{L}{2} + c_B(e_h - e_l) \leq 0, \qquad IC_B$$

$$H - \frac{L}{2} + c_G(e_h - e_l) \leq 0. \qquad IC_G$$

However, this cannot hold because $H > L > 0, c_B, c_G > 0, e_h - e_l > 0$.

Case 5.2: Types separate for effort level in the low-prize tournament; types pool for effort level e_h in the high-prize tournament:

$$p_G^L = 1, p_B^L = 0, p_G^H = 1, p_B^H = 1.$$

The same conditions as in Case 1.2 should be satisfied for the effort levels here. The following conditions should hold to have both types choose the low-prize tournament:

$$H + H - \frac{L}{2} \leq 0, \qquad IC_B$$

$$H - \frac{L}{2} + c_G(e_h - e_l) + H - L + c_G(e_h - e_l) \leq 0. \qquad IC_G$$

However, this cannot hold because $H > L > 0$.

Case 5.3: Types separate for effort level in the low-prize and high-prize tournaments:

$$p_G^L = 1, p_B^L = 0, p_G^H = 1, p_B^H = 0.$$

The same conditions as in Case 1.3 should be satisfied for the effort levels here. The following conditions should hold to have both types choose the low-prize tournament:

$$H + H - \frac{L}{2} \leq 0, \qquad IC_B$$

$$H - \frac{L}{2} + c_G(e_h - e_l) + H - L + c_G(e_h - e_l) \leq 0. \qquad IC_G$$

However, this cannot hold because $H > L > 0$.

Case 5.4: Types pool for effort level e_l in the low-prize tournament; types pool for effort level e_h in the high-prize tournament:

$$p_G^L = 0, p_B^L = 0, p_G^H = 1, p_B^H = 1.$$

The same conditions as in Case 1.4 should be satisfied for the effort levels here. The following conditions should hold to have both types choose the low-prize tournament:

$$H - \frac{L}{2} \leq 0, \qquad IC_B$$

$$H - \frac{L}{2} \leq 0. \qquad IC_G$$

However, this cannot hold because $H > L > 0$.

Case 5.5: Types pool for effort level e_l in the low-prize tournament; types separate for effort level in the high-prize tournament.

$$p_G^L = 0, p_B^L = 0, p_G^H = 1, p_B^H = 0.$$

The same conditions as in Case 1.5 should be satisfied for the effort levels here. The following conditions should hold to have both types choose the low-prize tournament:

$$H - \frac{L}{2} \leq 0, \qquad IC_B$$

$$H - \frac{L}{2} \leq 0. \qquad IC_G$$

However, this cannot hold because $H > L > 0$.

Case 5.6: Types pool for effort level e_l in the low-prize and high-prize tournaments:

$$p_G^L = 0, p_B^L = 0, p_G^H = 0, p_B^H = 0.$$

The same conditions as in Case 1.5 should be satisfied for the effort levels here. The following conditions should hold to have both types choose the low-prize tournament:

$$H - \frac{L}{2} \leq 0, \qquad IC_B$$

$$H - \frac{L}{2} \leq 0. \qquad IC_G$$

However, this cannot hold because $H > L > 0$.

3.9 Appendix 3: Proof of Proposition 1

Suppose, bad types choose e_l when $e_h < 3e_l$ in the single-prize tournament. So, the incentive compatibility condition for bad types is satisfied if the following condition holds:

$$\frac{K}{2} - c_B(e_h - e_l) \leq 0.$$

The individual rationality condition for bad types is satisfied when they exert e_l if the following condition holds:

$$0.5\left(\frac{K}{2} - c_B e_l\right) + 0.5(-c_B e_l) \geq 0,$$

which implies that the principal needs to set K such that

$$2c_B(e_h - e_l) \geq K \geq 4c_B e_l.$$

This contradicts with the initial condition for the effort levels. Hence, if $e_h < 3e_l$, bad types exert e_h in the tournament as well.

3.10 Appendix 4:

High Prize	Cost Difference	Types	High-Prize Tournament e_h	High-Prize Tournament e_l	Low-Prize Tournament e_h	Low-Prize Tournament e_l
Optimal	Large	Good	85 (100)	15 (0)	65 (100)	35 (0)
	Large	Bad	30 (0)	70 (100)	14 (0)	86 (100)
	Small	Good	93 (100)	7 (0)	64 (100)	36 (0)
	Small	Bad	77 (100)	23 (0)	38 (0)	62 (100)
Above Optimal	Large	Good	97 (100)	3 (0)	46 (100)	54 (0)
	Large	Bad	68 (0)	32 (100)	18 (0)	82 (100)
	Small	Good	98 (100)	2 (0)	40 (100)	60 (0)
	Small	Bad	94 (100)	6 (0)	33 (0)	67 (100)

Table 3.6: Observed effort percentages (%) in the tournaments
(Equilibrium percentages are in parentheses)

High Prize		Optimal				Above Optimal			
Cost Difference		Large		Small		Large		Small	
	Types	Good	Bad	Good	Bad	Good	Bad	Good	Bad
High-Prize Tournament (HT)	High Effort	10.41** (9.15)**	-4.52 (-5.25)	8.05** (9.15)**	5.43** (6.75)**	17.99** (19.7)**	0.95 (2)	12.60** (12.81)**	9.47** (9.74)**
	Low Effort	8.41 (7.3)	0.61** (-0.5)**	5.83 (7.3)	4.53 (6)	9.45 (10.55)	1.65** (2.75)**	2.42 (2.99)	1.12 (1.69)
Payoff Difference due to Efforts in HT		2.00 (1.85)	-5.13 (-4.75)	2.22 (1.85)	0.90 (0.75)	8.54 (9.15)	-0.70 (-0.75)	10.19 (9.82)	8.35 (8.05)
Low-Prize Tournament (LT)	High Effort	7.62* (8.4)*	-6.25 (-6)	7.56* (8.4)*	5.38 (6)	9.51* (10.6)*	-2.25 (-0.5)	10.86* (11.08)*	9.15 (9.36)
	Low Effort	7.53 (8.3)	-0.27* (0.5)*	7.48 (8.3)	6.18* (7)*	9.45 (10.55)	1.65* (2.75)*	10.78 (11.05)	9.48* (9.74)*
Payoff Difference due to Efforts in LT		0.09 (0.1)	-5.98 (-6.5)	0.08 (0.1)	-0.80 (-1)	0.06 (0.05)	-3.90 (-3.25)	0.07 (0.03)	-0.33 (-0.38)
Payoff Difference between the tournaments (HT-LT)		2.79 (0.75)	-0.88 (-1)	0.26 (0.75)	0.98 (-0.25)	8.48 (9.1)	0.00 (0)	1.75 (1.73)	-0.01 (0)

Table 3.7: Expected payoffs with experimental distributions

(Equilibrium predictions in parentheses)

3.11 Appendix 5: Experimental Instructions for the Single Prize Tournaments

Welcome

- Welcome to the Lab, and thank you for participating in today's experiment.
- It is very important that you do not touch the computer until you are instructed to do so. And when you are told to use the computer, use it only as instructed. In particular, do not attempt to browse the web or do other things unrelated to the experiment.
- Place all of your personal belongings away, so we can have your complete attention.

1

The Experiment

- The experiment you will be participating in today is an experiment in decision making. At the end of the experiment you will be paid for your participation in cash. Each of you may earn different amounts. The amount you earn depends on your decisions, chance, and the decisions of other participants.
- Please DO NOT socialize or talk during the experiment.
- We will go over the instructions slowly. It is important you follow.
- When you have a question please raise your hand and one of us will come to you to answer it in PRIVATE.

2

Matching

- The experiment will consist of 40 periods.
- You will be matched with another subject randomly in every period.
- The identity of your match will not be revealed to you.

3

Cost Multiplier

- At the beginning of the game, you and your match will be given a cost multiplier.
  - This cost multiplier will be either:

    low cost multiplier, **0.4** with **½** probability, or

    high cost multiplier,**1.0** with **½** probability.
  - You can have same or different cost multiplier with your match.
- Then you will be informed about your cost multiplier. You will not, however, be informed about your match's cost multiplier.

4

Decision Level Choice

You will be asked to choose a decision level: 35 or 13.

5

Decision Level Choice

You will be asked to choose a decision level: 35 or 13.

Cost Multiplier	Decision Level	Total Cost
0.4	35	14
	13	5.2
1	35	35
	13	13

6

Timeline-Summary

- You will get matched.
- You and your match will learn your own cost multipliers. (but not your matches, vice versa)
- You will choose a decision level.
- You will learn your period payoff.

7

Payoff Calculation-I

	Decision Level	Payment
	Higher	25 points
	Coincides	25/2 points
	Lower	0

8

Payoff Calculation-II

- You will have to subtract your total cost in all cases.
- To summarize:

Payoff = Payment – Total Cost.

- Your payoff will be converted into dollars at the rate of 20 Points = $1.

Payoff Calculation-III

	Decision Level	Payment	Payoff
	Higher	25 points	25 points- **Total Cost**
	Coincides	25/2 points	25/2 points-**Total Cost**
	Lower	0 points	0 points – **Total Cost**

Continuing Periods I

- This procedure described for the 1st period will be repeated for 40 periods with some modifications.
- The cost multipliers of you and your match will be redrawn in every period:

low cost multiplier = **0.4** with **½** probability

high cost multiplier = **1.0** with **½** probability.

OR IN SOME PERIODS…

low cost multiplier = **0.4** with **½** probability

high cost multiplier = **0.5** with **½** probability.

11

Continuing Periods II

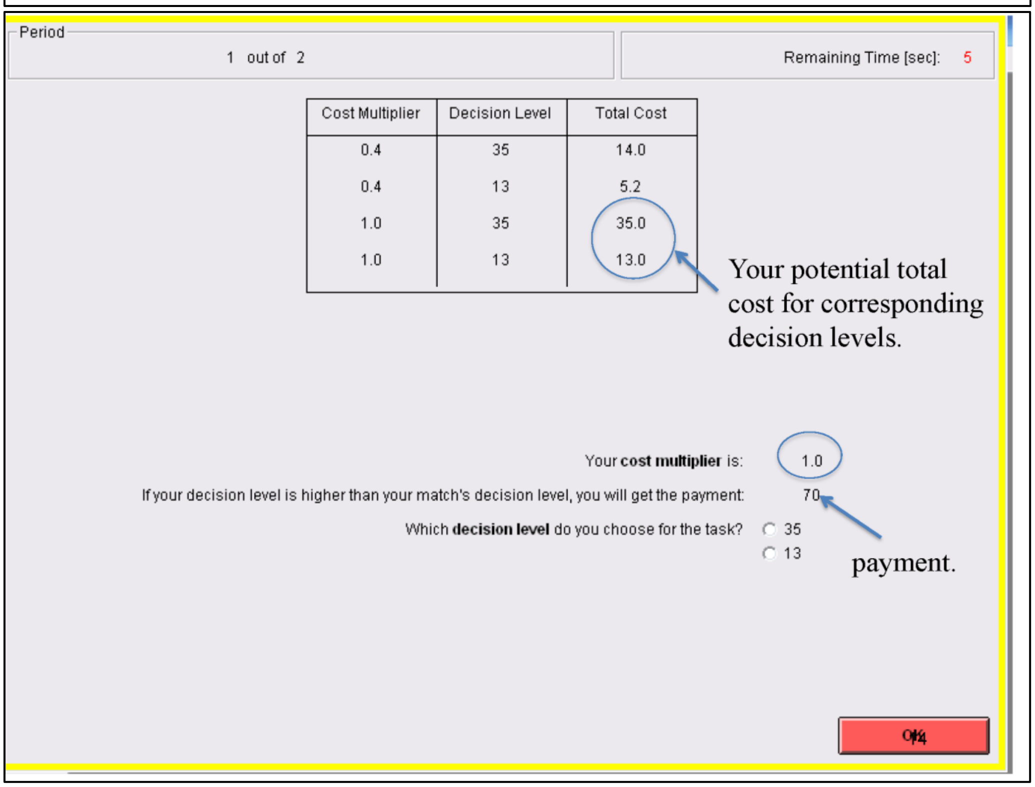
- Additionally, there can be three possible cases for the payment for each period.:
  - The payment can be **36.**
  - The payment can be **42.**
  - The payment can be **70.**
- You will learn the payment for that period before you make your period decision.
- After each period you will be informed of
  - your cost multiplier,
  - which decision level you choose,
  - whether your decision level is higher than your match
  - the payment you receive.

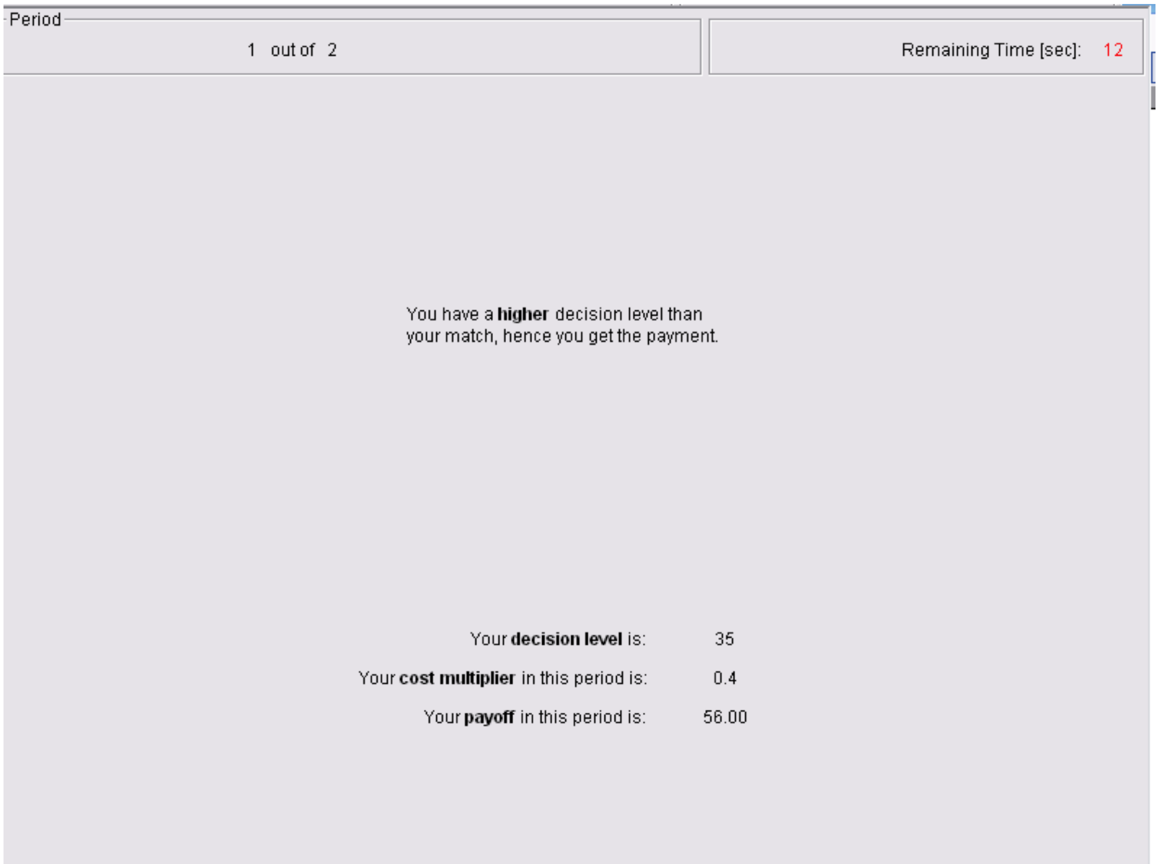
12

Continuing Periods III

- In every period, after you get matched with a different subject, then you choose a decision level: 35 or 13.
- The decision level that you choose will be compared to your match's decision level, and your earnings will be calculated for the period.
- At the end of 40 periods, your final payoff in the experiment will be the sum of your individual earnings in each period.

13




Period
1 out of 2
Remaining Time [sec]: 12
You have a higher decision level than your match, hence you get the payment.
Your decision level is: 35
Your cost multiplier in this period is: 0.4
Your payoff in this period is: 56.00

3.12 Appendix 6: Experimental Instructions for the Parallel Tournaments

Welcome

- Welcome to the Lab, and thank you for participating in today's experiment.
- It is very important that you do not touch the computer until you are instructed to do so. And when you are told to use the computer, use it only as instructed. In particular, do not attempt to browse the web or do other things unrelated to the experiment.
- Place all of your personal belongings away, so we can have your complete attention.

1

The Experiment

- The experiment you will be participating in today is an experiment in decision making. At the end of the experiment you will be paid for your participation in cash. Each of you may earn different amounts. The amount you earn depends on your decisions, chance, and the decisions of other participants.
- Please DO NOT socialize or talk during the experiment.
- We will go over the instructions slowly. It is important you follow.
- When you have a question please raise your hand and one of us will come to you to answer it in PRIVATE.

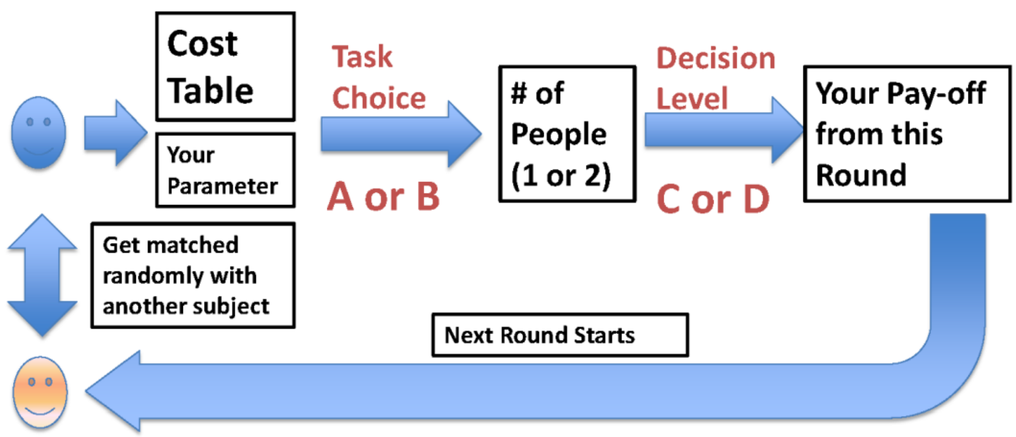
2

Matching

- The experiment will consist of 40 periods.
- You will be matched randomly with another subject in every period.
- The identity of your match will not be revealed to you.

3

Experiment Overview



4

Cost Multiplier

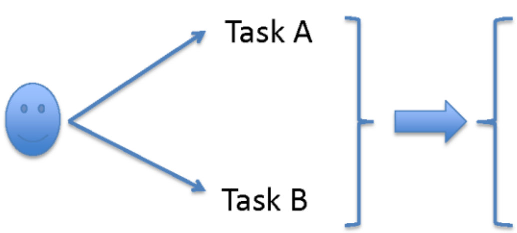
- At the beginning of the game, you and your match will be given a **cost multiplier.**
  - This cost multiplier will be either:

    low cost multiplier, **0.4** with **½** probability, or

    high cost multiplier, **1.0** with **½** probability.
  - You may have same or different cost multiplier with your match.
- Then you will be informed about your cost multiplier. You will not, however, be informed about your match's cost multiplier.

5

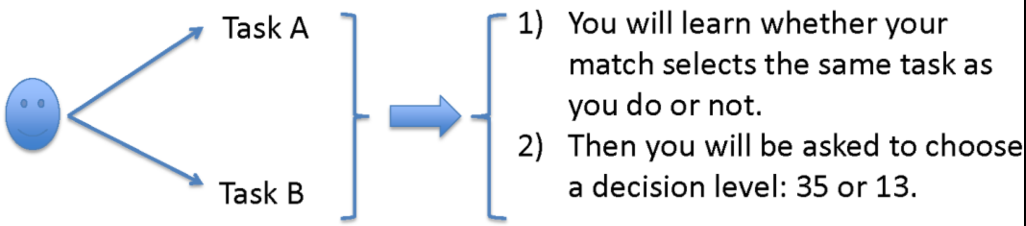
Task and Decision Level Choice



1) You will learn whether your match selects the same task as you do or not.
2) Then you will be asked to choose a decision level: 35 or 13.

6

Task and Decision Level Choice



Cost Multiplier	Decision Level	Total Cost
0.4	35	14
	13	5.2
1	35	35
	13	13

7

Timeline-Summary

- You will get matched randomly at the beginning of each period.
- You and your match will learn your own cost multipliers (but not your match's, vice versa).
- Each of you will select Task A or Task B.
- You will learn either you are alone or with your match, in the task you select.
- You will choose a decision level.
- You will learn your period payoff.

8

Payoff Calculation-I

Task A

Alone or with a match	Decision Level	Payment
	---------	25 points
	Higher	25 points
	Coincides	25/2 points
	Lower	0

Task B

Alone or with a match	Decision Level	Payment
	----------	18 points
	Higher	18 points
	Coincides	18/2 points
	Lower	0

9

Payoff Calculation-II

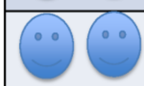
- You will have to subtract your total cost in all cases.
- To summarize:

  Payoff = Payment – Total Cost.
- Your payoff will be converted into dollars at the rate of 20 Points = $1.

10

Payoff Calculation-III

Task A

Alone or with a match	Decision Level	Payment	Payoff
	---------	25 points	25 points – **Total Cost**
	Higher	25 points	25 points – **Total Cost**
	Coincides	25/2 points	25/2 points – **Total Cost**
	Lower	0	0 – **Total Cost**

11

Continuing Periods I

- This procedure described for the 1st round will be repeated for 40 periods with some modifications.
- The cost multipliers of you and your match will be redrawn in every period:

low cost multiplier = **0.4** with **½** probability

high cost multiplier = **1.0** with **½** probability.

OR IN SOME PERIODS…

low cost multiplier = **0.4** with **½** probability

high cost multiplier = **0.5** with **½** probability.

12

Continuing Periods II

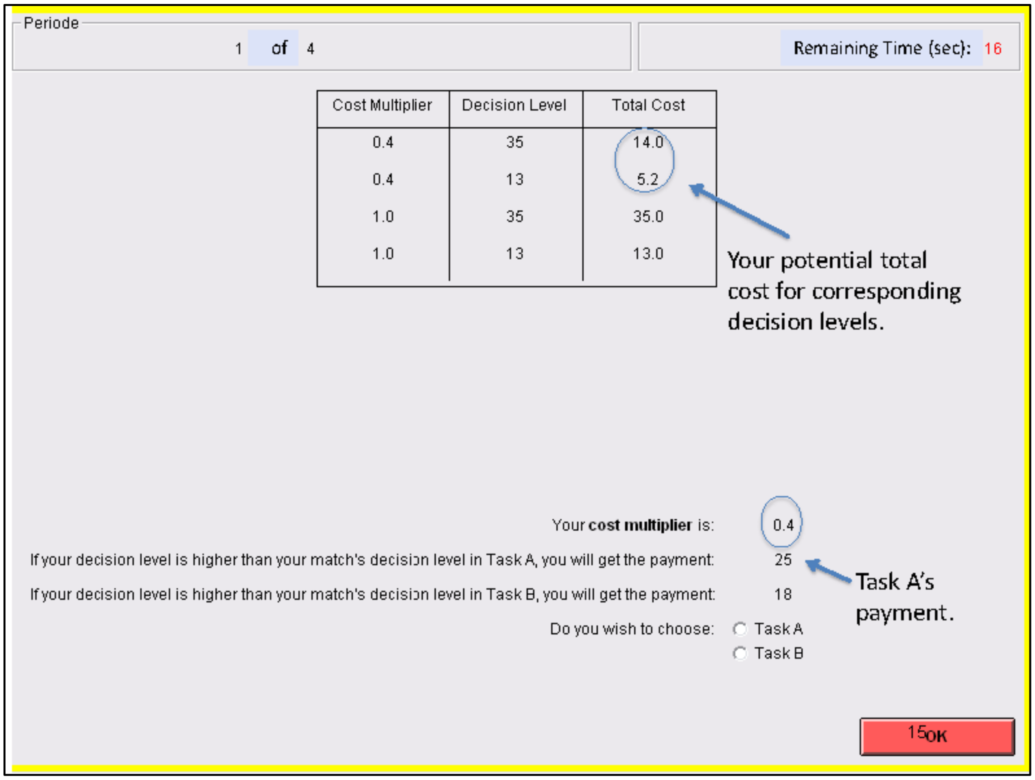
- Additionally, there can be two different cases in terms of Task A's payment.
  - Task A's payment is **25**.
  - Task A's payment is **42**.
- After each period you will be informed of…
  - your cost multiplier,
  - which task you select,
  - which decision level you choose,
  - whether your decision level is higher than your match
  - the payment you receive.

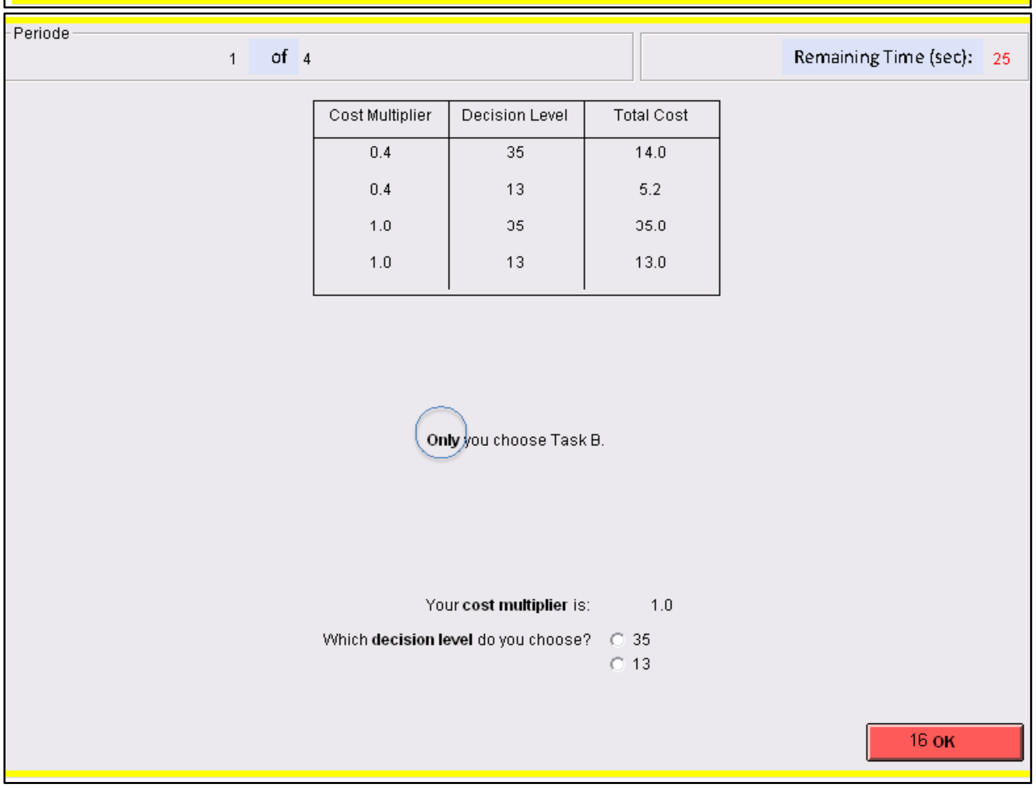
13

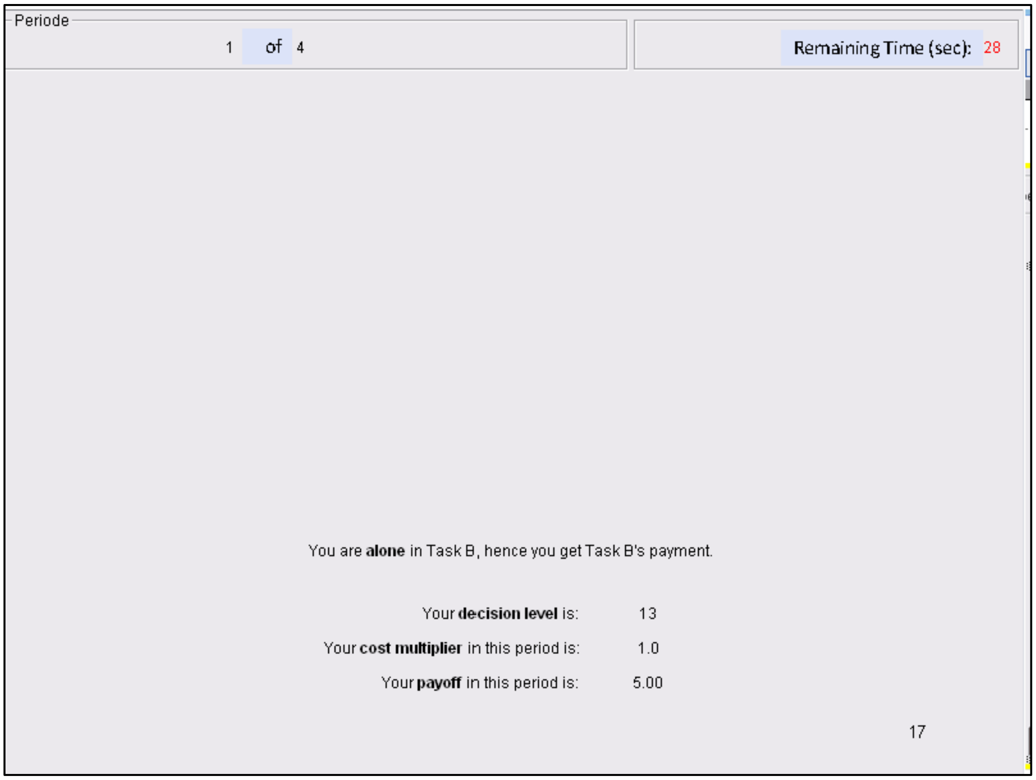
Continuing Periods III

- In every period, after you get matched with a different subject, you will select Task A or Task B.
- You will learn whether your match selected the same Task as you do or not; then you choose a decision level: 35 or 13.
- The decision level that you choose will be compared to the decision level of your match; and your earnings will be calculated for the period.
- At the end of 40 periods, your final payoff in the experiment will be the sum of your individual earnings in each period.

14


Periode
1 of 4
Remaining Time (sec): 16
Cost Multiplier | Decision Level | Total Cost
0.4 | 35 | 14.0
0.4 | 13 | 5.2
1.0 | 35 | 35.0
1.0 | 13 | 13.0
Your potential total cost for corresponding decision levels.
Your cost multiplier is: 0.4
If your decision level is higher than your match's decision level in Task A, you will get the payment: 25
If your decision level is higher than your match's decision level in Task B, you will get the payment: 18
Task A's payment.
Do you wish to choose: Task A Task B
15 OK


Periode
1 of 4
Remaining Time (sec): 25
Cost Multiplier | Decision Level | Total Cost
0.4 | 35 | 14.0
0.4 | 13 | 5.2
1.0 | 35 | 35.0
1.0 | 13 | 13.0
Only you choose Task B.
Your cost multiplier is: 1.0
Which decision level do you choose? 35 13
16 OK


Periode
1 of 4
Remaining Time (sec): 28
You are **alone** in Task B, hence you get Task B's payment.
Your **decision level** is: 13
Your **cost multiplier** in this period is: 1.0
Your **payoff** in this period is: 5.00
17

3.13 Appendix 7: Treatment Orders in the Single-Prize and Parallel Tournaments

Periods 1-10	OL	BOL	OS	BOL	BOL	OS	BOL	OL	OS	BOL
Periods 11-20	OL	BOL	BOL	OL	BOL	OS	OL	BOL	OS	BOL
Periods 21-30	BOL	OS	BOL	OL	OS	BOL	OL	BOL	BOL	OL
Periods 31-40	BOL	OS	OL	BOL	OS	BOL	BOL	OS	BOL	OL
BOL: Below-optimal prize Large cost-difference				OL: Optimal prize Large cost-difference				OS: Optimal prize Small cost-difference		

Table 3-8: Treatment Orders in the Single-Prize Tournaments

Periods 1-10	OL	OS	AOS	AOL	AOL	AOS	OS	OL	AOS	AOL
Periods 11-20	OL	OS	OS	OL	AOL	AOS	OL	OS	AOS	AOL
Periods 21-30	AOL	AOS	OS	OL	AOS	AOL	OL	OS	OS	OL
Periods 31-40	AOL	AOS	OL	OS	AOS	AOL	AOL	AOS	OS	OL
AOL: Above-Optimal Prize Large cost-difference			AOS: Above-Optimal Prize Small cost-difference			OL: Optimal Prize Large cost-difference			OL: Optimal Prize Small cost-difference	

Table 3-9: Treatment Orders in the Parallel Tournament

Chapter 4:

Decentralized Matching Market with Time Frictions[57]

4.1 Introduction

There are many real-life examples in which two-sided matching markets are centralized, such as the medical residency match, school allocations in New York City and Boston, and so on. In these markets, an agent from each side of the market submits a rank-ordered preference list to a central clearinghouse, which then produces a matching. The literature has mostly focused on centralized matching markets;[58] however, in markets such as college admissions in the US, the market for law clerks, junior economists, etc., matching proceeds in a decentralized manner. This paper studies a two-period decentralized game in which agents care about matching with their partners sooner (e.g., an unemployed person wants to find a job as quickly as possible and a firm looks for an employee for a particular job with some urgency). I study whether this game reaches a stable matching[59] as an equilibrium, and if so, whether the selected stable outcome changes for different levels of time discounting.

The success of centralized matching markets (their continued use in a market) is associated with whether they produce a stable outcome (Roth, 1991; Niederle and Roth, 2009). Hence, most centralized matching mechanisms implement a stable matching. If there are multiple

---

[57] I thank Julian Romero for the simulations in this chapter.

[58] See Niederle and Yariv (2009), Pais (2008), Haeringer and Wooders (2010) and references therein for exceptions.

[59] A stable match is a pairing of workers and firms in which no firm (worker) who is matched to a worker (firm) prefers to be alone, and no firm and worker pair prefer to jointly deviate by matching to one another.

stable matchings for the reported preferences (which is quite likely for large number of market participants[60]), the centralized matching algorithm chooses one of them to implement. For example, Gale-Shapley deferred acceptance algorithm (Gale and Shapley, 1962) results in either the firm-optimal stable matching or the worker-optimal stable matching, depending on who proposes first in the algorithm. However, a decentralized game might not necessarily implement only one stable matching. In this paper, I identify conditions under which different stable matchings[61] are selected as the outcome of a (noncooperative) decentralized game.

I study a simple model of a decentralized matching game in a market that is composed of firms, workers, and their preferences (firms' preferences are over workers and workers' preferences are over firms). I assume that these preferences are strict and are represented by cardinal utilities. Each firm can hire at most one worker and each worker can work for at most one firm. This one-to-one matching process in the market is modeled as an extensive form game where firms, named by index numbers,[62] sequentially propose to workers according to the ordering given by their index numbers. Those who receive a proposal immediately respond by accepting or rejecting the offer. If a worker gets a better offer in the same period, he can accept it even if he accepted another offer earlier, i.e., workers can hold offers. All agents know the others' preferences and observe all past realized actions. I assume that all firms (workers) prefer being matched with a worker (firm) to remaining unmatched. There is no commitment between the firms and workers; firms can propose to another worker in the second period even if they were matched with a worker in the first period or workers can accept another offer even if they accepted an offer earlier. Each firm can make only one offer per period. If all agents get matched in the first period, the game ends.

---

[60] Pittel (1989) shows that the number of stable matchings tends to increase as the number of market participants increases.

[61] I consider environments with multiple stable matchings.

[62] For instance, the name of a firm with the index number 1 is f_1.

When there is at least one firm and one worker that did not get matched in the first period a similar procedure occurs in the second period. In this case, if a firm gets rejected in the first period, it cannot make a proposal to the same worker again in the second period. All agents (even if they got matched in the first period) stay in the market until the game ends. Agents' utilities are affected by which period and whom they get matched with in each period. All agents get a period utility from their first-period matches and a discounted lifetime utility from their second-period matches. Firms and workers share a common discount factor, and receive their match utilities at the end of each period.

Under complete information, for specific preference profiles[63] I find that the subgame perfect Nash equilibrium (SPNE) of the game coincides with a stable matching for any time discount, either the firm optimal or the worker-optimal stable matching. Which stable matching coincides with the SPNE depends on the time discount.

When the time discount is low, I find that the unique SPNE outcome of the game yields the firm-optimal stable matching. This is because since firms know that workers will accept any offer in the first period[64] they will make offers to the best-achievable workers for them, i.e., to their firm-optimal matches. When the time discount is sufficiently high, I find that the SPNE outcome of the game yields the firm-optimal stable matching. This is because with a high time discount, one of the firms may profit delaying its matching to the second period.[65] By knowing the workers will accept any proposal in the last period, all firms propose to the best possible matches for them, to their firm-optimal matches, in that period.

---

[63] In these preference profiles, there is no conflict of interest for the first-ranked workers among the firms and there is no conflict of interest for the first-ranked firms among the workers. The threshold for a worker to reject an offer is smaller then the threshold of a firm to delay the matching to the second period.
[64] Even if a rejection of an offer in the first period brings the best-achievable match to a worker in the second period, it is not profitable for the worker to reject any offer for a low time discount.
[65] The model does not allow firms to just wait for the second period without making an offer.

When the time discount is intermediate the unique SPNE outcome of the game yields a worker-optimal stable matching. In this case, firms know that they will be rejected if they make offers to their firm-optimal stable matches in the first period. Moreover, it is not profitable for any firm to delay its matching to the second period and propose to its favorite worker then (unlike in the case of a very large time discount). According to the preference profiles that I consider in this paper, a firm ranks its match under firm-optimal stable matching and worker-optimal stable matching consecutively. Hence, for intermediate discount factors, firms immediately make offers to their worker-optimal stable matches in the first period and get accepted.

In the literature, there are only a few papers on decentralized matching markets (Roth and Xing, 1997; Blum et al., 1997; Alcalde et al., 1998; Pais, 2008; Diamantoudi et al., 2006; Niederle and Yariv, 2009; and Haeringer and Wooders, 2010). Among those studies, my work is most related to the last two.

Niederle and Yariv (2009) show that when agents have aligned preferences and the time discount is high enough, the unique stable matching outcome coincides with the Bayesian Nash equilibrium outcome of the game with iterated elimination of weakly dominated strategies. They work with aligned preferences to guarantee a unique stable matching and hence to eliminate coordination problems due to multiple stable matchings. Here, I also study a decentralized game with time discounting. In contrast, I consider preference profiles that guarantee multiple stable matching. Since I work on a sequential game with perfect information in this paper, I could overcome problems of coordination on specific stable matches for some preference profiles.

Haeringer and Wooders (2010) consider the case of complete information and restrict firms' strategies such that they cannot make offers to workers who had rejected them previously, as in our case. In their game, firms can only make exploding offers, i.e., offers must be accepted or rejected right away and the decisions are irreversible. The game ends when all firms exit the market. The authors show that there is a unique SPNE that coincides with the worker-optimal matching. In my model, agents can hold offers, i.e., decisions are reversible. Additionally, my game has two periods and assumes a time discount. I show that, depending on the time discount and preference profile, the SPNE coincides with either the firm-optimal or worker-optimal stable matching.

Last, Echenique and Yariv (2012) study a decentralized matching market in an experimental environment. Their experiment allows nonbinding offers under complete information without friction. According to their experimental results, most outcomes coincide with stable matchings; which stable matching gets selected depends on the cardinal representation of ordinal preferences. In contrast, I consider time frictions. Similar to their result, I find that the selected stable matching (with preferences that allow multiple stable matchings) depends on the time discount and the cardinal utility representation of the firm-optimal and worker-optimal stable matches.

This chapter is organized as follows: section 2 provides the marriage model and section 3 describes the decentralized matching game. Section 4, 5, and 6 present the results. Section 7 discusses the results and concludes.

4.2 The Marriage Model

I consider a finite set of workers $W = \{w_1, w_2, \ldots, w_n\}$ and a finite set of firms $F = \{f_1, f_2, \ldots, f_n\}$, where the cardinality of these sets is the same, i.e., $|F| = |W| = n$. Each worker $w \in W$ has a strict, complete, transitive, and asymmetric preference relation P_w over $F \cup \{w\}$. Similarly, each firm $f \in F$ has a strict, complete, transitive, and asymmetric preference relation P_f over $W \cup \{f\}$. The preferences of firms and workers, $P = P_f \cup P_w$, are represented by the utility function $U = (u_f(w), u_w(f))$, where the first component represents the utility of a firm from being matched with a worker or itself (unmatched), i.e., $u_f(w): W \cup \{f\} \rightarrow \mathbb{R}$, and the second component represents the utility of a worker from being matched with a firm or himself, i.e., $u_w(f): F \cup \{w\} \rightarrow \mathbb{R}$. The utility representation of firms and workers is common knowledge to all agents. The final utility obtained by a worker (or a firm) from the matching process depends on the matches that they have in each period and on the time discount factor, $\delta \in (0,1)$. Each (firm's) worker's utility from the game is the sums of the first period match utility and the discounted second period match utility for their remaining lives.

I assume that match utilities are strictly positive, i.e., $u_w(f) > 0$ for all $w \in W$ and $u_f(w) > 0$ for all $f \in F$. The utilities of unmatched firms and workers are normalized to 0 and represented as $u_f(f) = 0$ and $u_w(w) = 0$, respectively. Hence, all agents prefer being matched to being unmatched, for all $w \in W, u_w(f) > u_w(w) = 0$, and for all $f \in F, u_f(w) > u_f(f) = 0$.

For fixed sets of firms F and workers W, a market is denoted by (F, W, U).

A match is a function $\mu: F \cup W \rightarrow F \cup W$, such that for all $f \in F, \mu(f) \in W \cup \{f\}$, and for all $w \in W$, $\mu(w) \in F \cup \{w\}$. Moreover, the matching function is one-to-one; i.e., if $(f, w) \in F \times W$, then $\mu(w) = f$ if and only if $\mu(f) = w$. A **blocking pair** in a match is formed if there exists a pair $(f, w) \in F \times W$, where f and w would be happier if they were matched to each other than they are with their current matches under μ; i.e., $u_f(w) > u_f(\mu(f))$ and $u_w(f) > u_w(\mu(w))$. In this case, a pair $(f, w) \in F \times W$ blocks matching. A match is **stable** if it does not have any blocking pairs. The most preferred stable matching by firms (workers), the firm-optimal (worker-optimal) stable matching μ^F (μ^W) is the least preferred stable matching by workers (firms).

4.3 The Decentralized Game

For a given market (F, W, U), I consider the following two-period decentralized matching game. The market is common knowledge to all agents and all agents are unmatched at the beginning of the game.[66] In both periods (t = 1, 2) firms are given the opportunity to make offers according to their index numbers and workers who get offers immediately respond to them. There are $2n$ stages in a period, where $2n$ is the total number of firms and workers in the market. In the first stage of the first period, the firm indexed by 1, f_1, proposes to a worker.[67] In the second stage, the worker receiving the offer, immediately replies to the firm either by accepting or rejecting it, or she does nothing. In the third stage, by observing f_1's offer to a worker and the worker's response, the firm indexed by 2, f_2, makes an offer to a worker (it can be to any worker in the market). In the fourth stage, the worker, who receives that offer, responds and so on. The game continues by giving firms the opportunity to make offers, according to their index numbers. The first period ends when all firms in the market make their offers to workers and all workers who have received an offer have replied (either accepted or rejected the offers). If all firms and workers are matched in the first period the game ends, otherwise the second period starts.

All matched and unmatched agents stay in the market for the second period and the second period proceeds similarly. In particular, firms make offers in the order of their indices to workers who did not reject them in the first period after observing the first period's matches, proposals, and rejections/acceptances. Then workers who obtain the offers reply back immediately by observing all past realized actions. Matchings are not binding for both parties; any worker or firm can break up with his (its) match at any time. For each period, an agent receives utility from the match she has at the end of a period. An agent receives his match utility from the first period and a discounted lifetime utility from the second period.

Formally, the described extensive form decentralized matching game is an array $G = (F \cup W, \mathcal{H}, S, (u_v^t)_{t=1}^2)$ where $\mathcal{H}$ is the set of all nonterminal histories (stage) with a generic element $h \in \mathcal{H}$, S is the strategy space, $S = \prod_{v \in F \cup W} S_v$ where $S_v = \prod_{h \in \mathcal{H}} S_v^h$ for every $v \in F \cup$

[66] The initial matching for the first period is taken as empty, but the initial matching for the second period is taken as the matching that is formed in the first period.
[67] This order affects the game outcome for some preference profiles.

W and $(u_v^t)_{t=1}^2$ is the utility of each agent $v \in F \cup W$ from his matches in the first and second periods as a result of his strategy. Let $S^h = \prod_{v \in F \cup W} S_v^h$ be the set of strategies available to player v after observing the history h. After observing history h only one of the players is active at history (stage) $h+1$, i.e., due to the structure of the game $S^h = S_v^h$. For instance, in the first stage of the first period, the firm f_1 proposes to a worker ($s_{f_1}^1$) after observing initial history $\emptyset$; in the second stage the proposed worker (w_i where $i = 1, \dots, n$) accepts or rejects the offer ($s_{w_i}^2$); in the third stage, the firm f_2 proposes to a worker ($s_{f_2}^3$) after observing $h^3 = \left(\emptyset, s_{f_1}^1, s_{w_i}^2\right)$ and so on. In particular, the history at stage r is an ordered collection of strategies before stage r, $(\emptyset, s^1, \dots, s^{r-1}) = h^r$. If $h^{r+1} = (h^r, s^r)$ then history h^{r+1} proceeds history h^r. If there is no history following some $h \in \mathcal{H}$, it is called a terminal history. The set of all terminal histories is denoted by Z. For every $h \in \mathcal{H} \backslash Z$ there is a subgame $G(h) = (F \cup W, \mathcal{H}(h), S(h), (u_v^t)_{t=1}^2)$ where h is the initial history, $\mathcal{H}(h) = \{h' \in \mathcal{H} | h' \text{ proceeds } h\}$ and $S(h) = \prod_{h' \in \mathcal{H}(h)} S^{h'}$. Given history h and the strategy s there are two resulting matching outcomes (one for each period), respectively $\mu^1[s|h]$ for the first period match and $\mu^2[s|h]$ for the second period match. A player $v \in F \cup W$ obtains the utility, u_v, by playing s_v as a strategy when the initial history is h (in this case the resulting matchings are $\mu^1[s_v|h]$ and $\mu^2[s_v|h]$) and time discount $\delta \in (0,1)$:

$$u_v(s_v|h) = u_v(\mu^1[s_v|h]\,) + \frac{\delta}{1-\delta} u_v(\mu^2[s_v|h]).$$

For each period, agents receive utilities from the matches that they have at the end of a period. They get their match utility from the first period and discounted lifetime match utility from the second period.[68]

[68] The first period match can be thought as a date, in men-women matching or temporary firm-worker matching etc. For most agents there is a trial-error period until they find their final (life time) matches.

The equilibrium concept that I consider in this paper is the subgame perfect Nash equilibrium. A strategy profile s^* is a SPNE if there is no strategy s such that $u_v(s_v, s_{-v}|h) \geq u_v(s_v^*, s_{-v}|h)$ for each player $v \in F \cup W$, and for each history $h \in \mathcal{H}$.

4.4 Preferences

I focus on a specific class of preference profiles that are as follows for firms and workers:

$f_i: w_i \succ w_{i+1} \succ \cdots \succ w_{i+n-1\,(mod\,n)}$ for all $i \in \{1,2,\ldots,n\}$

$w_j: f_{j-1} \succ f_{j-2} \succ \cdots \succ f_{j-n\,(mod\,n)}$ for all $j \in \{1,2,\ldots,n\}$

There are two stable matchings, the firm-optimal stable matching $\mu^F(f_i) = w_i$ and the worker-optimal stable matching $\mu^W(f_i) = w_{i+1(mod\,n)}$. According to these preference profiles, there is no conflict of interest among firms, there is no conflict of interest among workers but there is a conflict of interest between firms and workers.[69]

Proposition 1: *The decentralized matching game with the described market has a subgame perfect Nash equilibrium that coincides with one of the stable matches.*

Proof: There are three cases in here. First, if the time discount is small, none of the workers finds profitable to reject any offer. By knowing that firms sequentially propose to the best achievable workers for themselves that are their firm-optimal stable matches. Hence with the described preferences, all firms propose to their firm-optimal stable matches (who are ranked first by the firms) and get accepted in the first period.

[69] The statements of this paper can be extended for the preference profiles in which at least one of the workers finds profitable to reject a proposal (i.e. ranks his firm-optimal match really lower than his worker-optimal stable match) but all firms rank their worker-optimal stable matches as the second (just after their firm-optimal matches).

Second, if the time discount is larger, firms may want to delay their matchings to the second period and propose to the best achievable worker for themselves by then. Again since it is the last period, all workers will accept the offers. By knowing that all firms propose to their firm-optimal stable matches in the second period. In the first period, all firms know that if they propose to their first ranked worker in their lists, they will be rejected. Hence, except the last firm (it is disadvantageous to propose the last) all firms propose to their worker-optimal stable matches in the first period. The last firm observes others' offers and responses then it proposes to a worker who will reject him for sure in the first period. Then all firms propose to their firm-optimal matches in the second period.

Third, if the time discount is intermediate, none of the firms finds profitable to be unmatched in the first period and to delay its matching to the second period. Further, if they propose to their firm-optimal stable matches in the first period, their proposals will be rejected. Hence, they propose to the next best achievable workers, their worker-optimal stable matches and get accepted in the first period.

4.5 Small Time Discount

I analyze the game outcome for three groups of time discounts. First, suppose the time discount is so small (hence, the discounted gain from the second period is small) that none of the workers profits from being unmatched in the first period but being matched with their worker-optimal matches in the second period. In this case, all workers accept any offer in the first period. The following proposition summarizes the result in this condition.

Proposition 2: *When* $\delta < \min_{w_j} \frac{u_{w_j}\left(\mu^F(w_j)\right)}{u_{w_j}\left(\mu^W(w_j)\right)}$ *for all* $w_j \epsilon \{w_1, w_2, \dots, w_n\}$*, the unique SPNE of the decentralized matching game for the described market coincides with the firm-optimal stable matching.*[70]

Proof*:* Since δ is so small, a worker w_j would accept his first period offer immediately even if the rejection of a first period's offer brings him his worker-optimal stable match in the second period, $\left(1 + \frac{\delta}{1-\delta}\right) * u_{w_j}\left(\mu^F(w_j)\right) > \frac{\delta}{1-\delta} * u_{w_j}\left(\mu^W(w_j)\right)$. As a result, all firms make offers to their firm optimal stable matches and get accepted in the first period.

Table 4.1 in the Appendix summarizes the frequency of game outcomes (for a particular time discount) coinciding with stable or unstable matchings for a market with three firms and three workers. This frequency is taken over all preferences that guarantee multiple stable matchings for this market. As one can see from that Table 4.1, for small δ the SPNE corresponds to the firm-optimal stable matching in most preference profiles (with a frequency of %99.7).

4.6 Intermediate Time Discounts

Second, I analyze the game outcome for intermediate time discounts. The lower bound for the time discount is determined by a worker's willingness to reject an offer (he may reject if he prefers being unmatched in the first period and being matched with a better firm in the second period to being matched with his firm-optimal stable match for the rest of his life). The upper bound for the time discount is determined by a firm's willingness to delay the matching to the second period (it may prefer delaying its matching, if the firm expects its favorite worker to reject it in the first period). The following proposition summarizes the result with intermediate time discount.

---

[70] If the game were simultaneous, this theorem would satisfy for all preferences. In this case, the best a firm would achieve its firm-optimal match.

Proposition 3: *When* $\min_{w_j}\left\{\frac{u_{w_j}\left(\mu^F(w_j)\right)}{u_{w_j}\left(\mu^W(w_j)\right)}\right\} < \delta < min_{f_i}\left\{\frac{u_{f_i}\left(\mu^W(f_i)\right)}{u_{f_i}\left(\mu^F(f_i)\right)}\right\}$ *for all* $w_j \epsilon \{w_1, \dots, w_n\}$*, and* $f_i \epsilon \{f_1, \dots, f_n\}$ *the unique SPNE of the decentralized matching game for the described market coincides with the worker-optimal stable matching.*[71]

Proof: If $\min_{w_j}\left\{\frac{u_{w_j}\left(\mu^F(w_j)\right)}{u_{w_i}\left(\mu^W(w_j)\right)}\right\} < \delta$ it means that there exists one worker who thinks that it is better for him not to accept an offer from his firm-optimal stable match in the first period but to accept an offer from his worker-optimal stable match in the second period. Hence all firms know that when they make offers to their firm-optimal matches, one of the workers can reject such an offer and lead its matching to the second period. By rejecting the offer from his firm-optimal match, this worker guarantees an offer from a better firm because rejected firms cannot propose to the same worker in the second period (their firm-optimal matches) and the second-best workers for the firms are their matches under μ^W by construction. In particular, if all workers accept the offers from their firm-optimal matches, the game will end and the resulting match will be the firm-optimal stable match. When a worker rejects the offer from his firm-optimal match he can be unmatched in the first period but he guarantees an offer from his worker-optimal match in the second period. Hence, if $\frac{\delta}{1-\delta}u_{w_j}\left(\mu^W(w_j)\right) > \frac{1}{1-\delta}u_{w_j}\left(\mu^F(w_j)\right)$, then it is profitable to reject the offer and get an offer from his worker-optimal stable match in the second period.

From the proposer side, firms know that if they make offers to their firm-optimal matches and get rejected in the first period, they cannot make offer(s) to the same worker(s) in the second period. Instead, in order to obtain their firm-optimal matches in the second period, one of the firms may prefer to make an offer to the same worker with another firm and to be unmatched

[71]The likelihood of having the preferences leading to the outcome in Proposition 3, among the preferences that guarantee multiple-stable matching in a 3x3 market is at most 0.23. If this condition does not hold, i.e., $\min_{f_i}\left\{\frac{u_{f_i}\left(\mu^W(f_i)\right)}{u_{f_i}\left(\mu^F(f_i)\right)}\right\} \geq \min_{w_j}\left\{\frac{u_{w_j}\left(\mu^F(w_j)\right)}{u_{w_j}\left(\mu^W(w_j)\right)}\right\}$ for all $f_i \epsilon \{f_1, \dots, f_n\}$ and $w_j \epsilon \{w_1, \dots, w_n\}$ then the SPNE outcome would coincide with firm-optimal stable match for all time discount.

(i.e., to make an offer to a worker who is preferred less than its worker-optimal match) in the first period. However, since $\delta < \min_{f_i}\left\{\frac{u_{f_i}(\mu^W(f_i))}{u_{f_i}(\mu^F(f_i))}\right\}$ for all firms, none of the firms has an incentive to do so. If δ is in the specified interval, all firms will propose to their matches under μ^W and get accepted in the first period.

Example 2 (SPNE for intermediate time discounts): Consider a market with three firms $F = \{f_1, f_2, f_3\}$, three workers $W = \{w_1, w_2, w_3\}$, and the following preference profiles. Suppose the utilities corresponding to these preference profiles are such that all firms (workers) get 9 as utility from their first-ranked workers (firms), 6 from their second-ranked workers (firms), and 3 from their last-ranked workers (firms).

f_1	f_2	f_3	w_1	w_2	w_3
$\langle w_1 \rangle$	$\langle w_2 \rangle$	$\langle w_3 \rangle$	$[f_3]$	$[f_1]$	$[f_2]$
$[w_2]$	$[w_3]$	$[w_1]$	f_2	f_3	f_1
w_3	w_1	w_2	$\langle f_1 \rangle$	$\langle f_2 \rangle$	$\langle f_3 \rangle$

Under these preferences, there are two stable matchings: in one stable matching (firm-optimal one, marked by $\langle . \rangle$), f_1, f_2, and f_3 are matched with w_1, w_2, and w_3, respectively; in the second stable matching (worker-optimal one, marked by $[.]$), f_1, f_2, and f_3 are matched with w_2, w_3, and w_1 respectively.

If $\min_{w_j}\left\{\frac{u_{w_j}(\mu^F(w_j))}{u_{w_j}(\mu^W(w_j))}\right\} = \frac{3}{9} < \delta < \frac{6}{9} = min_{f_i}\left\{\frac{u_{f_i}(\mu^W(f_i))}{u_{f_i}(\mu^F(f_i))}\right\}$, f_1 will make an offer to w_2; f_2 will make an offer to w_3; f_3 will make an offer to w_1 and all offers will be accepted in the first period in the SPNE. Suppose that f_1 made an offer to w_1(its firm-optimal match), then w_1 would reject it and f_1 would be unmatched in the first period. In the second period, it cannot offer to w_1 (because it got rejected in the first period) again, hence it can make an offer to its second-best worker (worker-optimal match) in the second period. Similar situations may occur for the other firms too.

One of the firms can make an offer to a worker who is less preferred than its worker-optimal match to be unmatched (at worst scenario) in the first period and makes an offer to its firm-optimal match in the second period (to delay the matching). However, since $\delta < \frac{6}{9}$, none of the firms has an incentive to delay its match to the second period. Hence, in equilibrium all firms make offers to their worker-optimal matches and all workers accept these offers in the first period.

4.7 Large Time Discounts

Last, I analyze the game outcome for large time discounts. The lower bound for the time discount is determined by a firm's willingness to delay the matching to the second period. For larger time discounts firms may prefer being matched with their favorite one in the second period to being matched with a less preferred worker for the rest of their lives in the first period. The following proposition summarizes the result in this condition.

Proposition 4: *When* $\delta > min_{f_i}\left\{\frac{u_{f_i}\left(\mu^W(f_i)\right)}{u_{f_i}\left(\mu^F(f_i)\right)}\right\}$ *for all* $f_i \epsilon \{f_1, \dots, f_n\}$ *the unique SPNE of the decentralized matching game for the described market coincides with the firm-optimal stable matching.*

Proof: If $\delta > \min_{f_i} \frac{u_{f_i}\left(\mu^W(f_i)\right)}{u_{f_i}\left(\mu^F(f_i)\right)}$ for all $f_i \epsilon \{f_1, \dots, f_n\}$ then all firms have an incentive to delay the game to the second period and to make offers to their firm-optimal matches in the second period.

First suppose, one of the firms makes an offer to its firm-optimal match in the first period, then it is certain that it will be rejected (due to first part of Proposition 3) and furthermore, it cannot make an offer to the same worker again in the second period. Now suppose, it makes an offer to the second-best worker (which is its match under μ^W according to the considered preference profiles) in its preference list then it will get accepted immediately. If all firms make offers to their second-best workers they will get accepted and the game will end in the first period. However, any of the firms prefer to be unmatched in the first period and get matched with its firm-optimal match in the second period if $\delta u_{f_i}(\mu^F(f_i)) > u_{f_i}(\mu^W(f_i))\ \forall f_i \in \{f_1, \dots, f_n\}$. Since, the firms propose sequentially and history is observable, there won't be a coordination problem about the firm who will lead the delay. One of the firms will make an offer to the same worker with another firm and get unmatched in the first period then in the second period all firms will make offers to their firm-optimal matches. Particularly, in the SPNE, all firms but the last one will make an offer to the second-best workers in their preference lists and will be accepted immediately. The last firm will make an offer to a worker who also got an offer from his worker-optimal match in the same period and hence the last firm will be rejected in the first period. An unmatched firm and worker pair will lead a matching in the second period where all firms will make offers to best workers and will be accepted (resulting in the firm-optimal stable match).

Example 3 (SPNE for large time discounts): Consider the same market as in the Example 2. If $\delta > \frac{u_{f_i}(\mu^W(f_i))}{u_{f_i}(\mu^F(f_i))}\ \forall f_i \in \{f_1, f_2, f_3\}$, all firms prefer to be unmatched in the first period but to be matched with their favorite workers in the second period. Further, it is enough for one of them to be unmatched in the first period to delay the matching to the second period. If the game were simultaneous this would have led a coordination problem. In the SPNE of the example, first two firms can be matched with their worker-optimal matches in the first period, the last firm will propose to the same worker with one of the first two and will be rejected. This will lead the matching to the second period. In the second period, all firms will propose to their firm-optimal matches and they will be accepted.

As one can see from that Table 4.1 in Appendix, for large δ the SPNE corresponds to the firm-optimal stable matching in most preference profiles (with a frequency over %95).

4.8 Conclusions and Extensions

This paper studies a (non-cooperative) decentralized matching game in an environment with two stable matchings. Particularly, it analyzes whether the SPNE outcome of this game coincides with a stable matching and if it does, whether the coinciding stable outcome changes with the time discount.[72] I find that for specific preference profiles, the SPNE always coincides with a stable matching. Further, for small time discounts, the SPNE coincides with the firm-optimal stable matching. If the time discount is intermediate, so that workers do not mind matching in the second period whereas firms want to match in the first period, the SPNE coincides with the worker-optimal stable matching. Last, if the time discount is large enough, all firms prefer not to be matched in the first period but to be matched with their favorite worker in the second period the SPNE coincides with the firm-optimal stable matching.

Some limitations of this model are the following. First, it assumes that firms propose according to an order.[73] Such an ordering may lead equilibrium outcomes that are unstable for some preference profiles. Particularly it may give an advantage (disadvantage) to the late (early)-moving firms. Second I assume a particular class of preferences. Third, I assume that preference profiles and history are observable in the game. However, it is clear that the firm offering first has

[72] According to Gale-Shapley proposing side always has the advantage, but in our decentralized game depending on the time discount and utility ratios between the firm-optimal and the woman-optimal stable matches, both proposing and proposed parties can have advantage. Proposed party has an option to reject and obtain a better match in the last period and proposing party has option to delay an offer if it is profitable.

[73] Pais (2008) explained such an order could be explained through speed of each firm's proposing devices: email, phone, mail.

a disadvantage for some preference profiles. Hence such a firm may prefer not to tell its preferences truthfully. Fourth, I assume that the game ends in two periods. However, this is not too realistic. Although in most matching occasions, matching parties look for their lifelong matches for a finite period (the matches before their final matches are their trials), they may not be able to find exactly in two periods. Further, this search period may change from person to person.

4.9 Appendix 1: Frequency of Stable Matches as a Response to Time Discount

Consider a market with three firms $F = \{f_1, f_2, f_3\}$, three workers $W = \{w_1, w_2, w_3\}$, and the utilities in which all firms (workers) get 9 from being matched with their first-ranked workers (firms), 6 from being matched with their second-ranked workers (firms), and 3 from being matched with their last-ranked workers (firms).

	First period SPNE outcomes			Second period SPNE outcomes		
δ	μ^F	μ^W	Unstable	μ^F	μ^W	Unstable
0.01	0.997	0	0.003	0	0	0
0.15	0.997	0	0.003	0	0	0
0.33	0.997	0	0.003	0	0	0
0.34	0.498	0.231	0.032	0.236	0	0.004
0.4	0.498	0.231	0.032	0.236	0	0.004
0.45	0.498	0.231	0.032	0.236	0	0.004
0.49	0.498	0.231	0.032	0.236	0	0.004
0.5	0.579	0.114	0.014	0.289	0	0.005
0.51	0.173	0.101	0.003	0.699	0	0.024
0.55	0.173	0.101	0.003	0.699	0	0.024
0.66	0.173	0.101	0.003	0.703	0	0.02
0.67	0.027	0	0	0.955	0	0.018
0.7	0.027	0	0	0.955	0	0.018
0.75	0.025	0	0	0.948	0	0.027
0.8	0.025	0	0	0.932	0	0.043
0.99	0.025	0	0	0.932	0	0.043

Table 4.1: The Frequency of Game Outcomes

Bibliography

Al-Ubaydli, O., G. Jones, and J. Weel (2011), "Patience, cognitive skill and coordination in the repeated stag-hunt," Working Paper, George Mason University.

Alcalde J., D. Perez-Castrillo, and A. Romero-Medina (1998), "Hiring procedures to implement stable allocations," *Journal of Economic Theory,* 82: 469–480.

Altmann, S., A. Falk, and M. Wibral (2008), "Promotions and incentives: The case of multi- stage elimination tournaments," IZA Discussion Paper No. 3835.

Anderson, L. A. and S. L. Stafford (2003), "An experimental analysis of rent seeking under varying competitive conditions," *Public Choice,* 115, 199–216.

Balafoutas, L., R. Kerschbamer, and M. Sutter (2012), "Distributional preferences and competitive behavior," *Journal of Economic Behavior and Organization*, 83, 125–135.

Bartling, B., E. Fehr, M. A. Marechal, and D. Schunk (2009), "Egalitarianism and competitiveness," *American Economic Review,* 99, 93–98.

Becker, G., M. DeGroot, and J. Marschak (1964), "Measuring utility by a single-response sequential method," *Behavioral Science*, 9, 226–236.

Blum Y., A. E. Roth, U.G. Rothblum (1997), "Vacancy chains and equilibration in senior-level labor markets," *Journal of Economic Theory*, 76: 362–411.

Bogach, O. and A. Leibbrandt (2011), "An experimental study of the relevance and scope of culture as a focal point," Working Papers 201104, University of Hawai at Manoa.

Bolton, G. E. (1998), "Bargaining and Dilemma Games: From Laboratory Data Towards Theoretical Synthesis," *Experimental Economics,* 1, 257–281.

Bouckaert, J. and G. Dhaene (2003), "Inter-ethnic trust and reciprocity: Results from an experiment with small businessmen," *European Journal of Political Economy,* 4, 869–886.

Burns, J. (2012), "Race, diversity and pro-social behavior in a segmented society," *Journal of Economic Behavior and Organization,* 81, 366–378.

Cachon, G. P. and C. F. Camerer (1996), "Loss-avoidance and forward induction in experimental coordination games," *Quarterly Journal of Economics,* 111, 165–194.

Camerer, C. F. (1997), "Progress in behavioral game theory," *Journal of Economic Perspectives,* 11, 167-188.

Cason, T., W. Masters, and R. Sheremeta (2010), "Entry into winner-take-all and proportional prize contest: An experimental study," *Journal of Public Economics*, 94, 604–611.

Chadhuri, A., and L. Gangadheran (2002), "An experimental analysis of trust and trustworthiness," *Southern Economic Journal,* 73, 959–985.

Charness, G. (2000), "Self-serving cheap talk: A test of Aumann's conjecture," *Games and Economic Behavior,* 33, 177–194.

Charness, G. and B. Grosskopf (2004), "What makes cheap talk effective? Experimental evidence," *Economic Letters*, 83, 383–389.

Clark, K.S., S. Kay, and M. Sefton (2001), "When are Nash equilibria self enforcing? An experimental analysis," *International Journal of Game Theory*, 29, 495–515.

Cooper, R., D. De Jong, R. Forsythe, and T. Ross (1992), "Communication in coordination games," *Quarterly Journal of Economics*, 107, 739–771.

Costa-Gomes, M. and G. Weizsacker (2008), "Stated beliefs and play in normal form games," *Review of Economic Studies*, 75(3), 729–762.

Croson, R. and N. Buchan (1999), “Gender and culture: international experimental evidence from trust games,” *American Economic Review*, 89 (2), 386–391.

Diamantoudi E., E. Miyagawa, and L. Xue (2006), “Decentralized matching: the role of commitment,” Mimeo.

Dohmen, T. and A. Falk (2011), “Performance pay and multidimensional sorting: Productivity, preferences, and gender,” *American Economic Review*, 101, 556–90.

Duffy, J. and N. Feltovich (2006), “Words, deeds, and lies: Strategic behavior in games with multiple signals,” *Review of Economics Studies*, 73, 669–688.

Echenique, F. and L. Yariv (2012), “An experimental study of decentralized matching”, Working Paper.

El-Gamal, M., R.D. McKelvey, and T. R. Palfrey (1993), “A bayesian sequential experimental study of learning in games,” Journal of American Statistical Association, 88, 428–435.

Epstein, G. S. and S. Nitzan (2002), “Asymmetry and corrective public policy in contests,” *Public Choice*, 113, 231–240.

Eriksson, T., S. Teyssier, M. C. Villeval (2009), “Self-selection and the efficiency of tournaments,” *Economic Inquiry*, 47, 530–548.

Falk, A. and E. Fehr (2003), “Why labour market experiments?,” *Labour Economics*, l0, 399–406.

Fershtman, C. and U. Gneezy (2001), “Discrimination in a segmented society: an experimental approach,” *Quarterly Journal of Economics*, 115, 351–377.

Fischbacher, U. (2007), “Z-Tree: Zurich Toolbox for Ready-Made Economic Experiments,” *Experimental Economics*, 10 (2), 171–178.

Fullerton, R., B. G. Linster, M. McKee, and S. Slate (1999), "An experimental investigation of research tournaments," *Economic Inquiry*, 37, 624–636.

Gale D. and S. L. Shapley (1962), "College admissions and the stability of marriage," *American Mathematical Monthly*, 69, 9–15.

Glaeser, E., D. Laibson, J. Scheinkman, and C. Soutter (2000), "Measuring trust," *Quarterly Journal of Economics*, 115 (3), 811–846.

Goeree, J. K., A. H. Holt, and T. R. Palfrey (2002), "Quantal response equilibrium and overbidding in private-value auctions," *Journal of Economic Theory*, 104(1), 247–272.

Grether, D. M. (1981), "Financial incentive effects and individual decision making," Social Science Working Paper 401, California Institute of Technology.

Groh, C., B. Moldovanu, A. Sela, and U. Sunde (2012): "Optimal seedings in elimination tournaments," *Economic Theory*, 49(1), 59–80.

Haeringer G. and M. Wooders (2010), "Decentralized job matching," *International Journal of Game Theory*, DOI: 10.1007/s00182-009-0218-x.

Harbring, C., B. Irlenbusch, M. Kräkel, and R. Selten (2007): "Sabotage in asymmetric contests— an experimental analysis," *International Journal of the Economics of Business*, 14, 201–223.

Heinemann, F., R. Nagel, and P. Ockenfels (2009), "Measuring Strategic Uncertainty in Coordination Games," *Review of Economic Studies*, 76(1), 181–221.

Holm, H. (2000), "Gender based focal points," *Games and Economic Behavior*, 32, 292–314.

Holm, H. J., and P. Engseld (2005), "Choosing bargaining partners: An experimental study on the impact of information about income, status and gender," *Experimental Economics*, 8(3), 183–216.

Holt, C., and S. Laury (2002), "Risk aversion and incentive effects," *American Economic Review*, 92, 1644–1655.

Irlenbusch, B. (2006): "Experimental perspectives on incentives in organisations," *Central European Journal of Operations Research,* 14, 1–24.

Konow, J. (2000), "Conditional versus unconditional altruism: Theory and evidence. Working paper, Loyola Marymount University.

Lazear, E.P., S. Rosen (1981): "Rank-order tournaments as optimum labor contracts," *Journal of Political Economy,* 89, 841–864.

Leuven, E., H. Oosterbeek, and B. van der Klaauw (2011): "Splitting tournaments," 8016, C.E.P.R. Discussion Papers.

Moldovanu, B., A. Sela (2001): "The optimal allocation of prizes in contests," *The American Economic Review,* 91, 542–558.

Morgan, J., H. Orzen, and M. Sefton (2012): "Endogenous entry in contests," *Economic Theory*, 51, 435–463.

Muller, W., and A. Schotter (2010): "Workaholics and dropouts in organizations," *Journal of the European Economic Association,* 4, 717–743.

Neumann, T. and B. Vogt (2009), "Do players' beliefs or risk attitudes determine the equilibrium selections in 2x2 Coordination Games," Mimeo.

Niederle, M. and L. Vesterlund (2007): "Do women shy away from competition? Do men compete too much?," *Quarterly Journal of Economics*, 122, 1067–1101.

Niederle M. and A.E. Roth (2009), "Market culture: How rules governing exploding offers affect market performance," *American Economic Journal: Microeconomics,* 2(1), 199–219.

Niederle M. and L. Yariv (2009), "Decentralized matching with aligned preferences," Mimeo

O'Keeffe, M., W. K. Viscusi, and R. J. Zeckhauser (1984): "Economic contests: Comparative reward schemes," *Journal of Labor Economics,* 2, 27–56.

Pais J. (2008), "Incentives in decentralized random matching markets," *Games and Economic Behavior,* 64: 632–649.

Pittel B. (1989), "The average number of stable matchings," *SIAM Journal on Discrete Mathematics*, 2(4), 530–549.

Prendergast, C. (1999): "The provision of incentives in firms," *Journal of Economic Literature*, 37(1), 7–63.

Rey-Biel, P. (2009), "Equilibrium play and best response to (stated) beliefs in normal form games," *Games and Economic Behavior,* 65, 572–585.

Rosen, S. (1986): "Prizes and incentives in elimination tournaments," *American Economic Review*, 76(4), 701–715.

Roth A. E. (1991), "A natural experiment in the organization of entry level labor markets: Regional markets for new physicians and surgeons in the U.K.," *American Economic Review*, 81, 415–440.

Roth A.E., X. Xing (1994): "Jumping the gun: Imperfections and institutions related to the timing of market transactions", *American Economic Review*, 84, 992–1044.

Roth A.E., X. Xing (1997), "Turnaround time and bottlenecks in market clearing: decentralized matching in the market for clinical psychologists," *Journal of Political Economy*, 105: 284–329.

Scharleman, J., C. Eckel, A. Kacelnik, and R. Wilson (2001), "The value of smile: Game theory with a Human face," *Journal of Economic Psychology*, 22, 617–640.

Schmidt, D., R. Shupp, J.M. Walker, and E. Ostrom, (2003), “Playing safe in coordination games: The roles of risk dominance, payoff dominance, and history of play,” *Games and Economic Behavior,* 42, 281–299.

Schotter, A., K. Weigelt (1992): “Asymmetric tournaments, equal opportunity laws, and affirmative action: some experimental results,” *Quarterly Journal of Economics,* 107, 511–539.

Selten, R. (1995), “An axiomatic theory of a risk dominance measure for bipolar games with linear incentives,” *Games and Economic Behavior,* 8, 213–263.

Sheremeta, R. M. (2010): “Experimental comparison of multi-stage and one-stage contests,” *Games and Economic Behavior*, 68, 731–747.

Sheremeta, R.M. (2011): “Contest design: An experimental investigation,” *Economic Inquiry*, 2, 573–590.

Straub, P.G. (1995), “Risk dominance and coordination failures in static games,” *Quarterly Review of Economics and Finance,* 35, 339–363.

Tullock, G. (1980): “Towards a theory of the rent-seeking society,” Edward Elgar Publishing, 153–179.

Van Huyck, J. B., R. C. Battalio, and R. O. Beil (1990), “Tacit coordination games, strategic uncertainty, and coordination failure,” *American Economic Review*, 80, 234–248.

Van Huyck, J. B., R. C. Battalio, and R. O. Beil (1991), “Strategic uncertainty, equilibrium selection, and coordination failure in average opinion games,” *Quarterly Journal of Economics,* 106, 885–911.

Vandegrift, D., A. Yavas, and P. Brown (2007): “Incentive effects and overcrowding in tournaments: An experimental analysis,” *Experimental Economics*, 10, 345–368.