Cooperation in Reciprocity Games and in the Voluntary Contributions Mechanism

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Abstract

Each of the three independent chapters of this dissertation examines or justifies cooperative behavior in one of two specific public goods environments.

The first chapter presents experimental evidence documenting a subject's behavior when faced with simple games that require turn taking for efficiency. Both symmetric and asymmetric games as well as games with explicit punishment actions are studied and compared. The length of the game is a treatment variable; experiments simulating one-shot, finite and infinite repetition games are conducted. Group outcomes are sorted by the player's average payoffs and the importance of focal solution concepts like group welfare, equality, and symmetry are inferred. Individual strategies used in the experiments are also sorted and compared enabling a discussion of endgame effects and conflict within the games.

Standard non-cooperative game theory is not selective enough to discriminate among many of the possible outcomes of the games examined in Chapter One. Relying on focal and axiomatic solution concepts allows discrimination, yet these procedures are inherently ad-hoc. The second chapter examines the outcome to a population game with evolutionary dynamics in

order to theoreticly justify the results of the first chapter in a less ad-hoc manner. In particular, the second chapter applies the Replicator Dynamic. It is shown that under an assumption of limited rationality, specifically limited memory, there is a unique global equilibrium. The unique equilibrium contains a trio of outcomes: non-cooperative Nash play, payoff irrational play, and cooperative turn-taking.

The third chapter presents findings from a second series of experiments, a series designed to study free riding and the voluntary contribution mechanism. In the experimental environment, subjects are randomly assigned constant marginal rates of substitution between the public and the private good. These random assignments are changed each decision period, allowing the measurement of player response functions. These response functions are analogous to the bidding functions obtained in private good, sealed-bid auction experiments. The results are quite different from the results of others in environments with little or no heterogeneity. There is much more free riding, very little evidence of decay across periods, and only sparse evidence of anomalous behavior such as splitting and spite.

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Chapter 1

An Experimental Analysis of Two–Person Reciprocity Games

1.1 Reciprocity Games

As described in Ostrom (1990), the farmers near the city of Valencia, Spain take turns directing water from canals onto their fields. When one farmer has taken all the water he needs, the next farmer, who has been waiting, gets to take all the water he needs. There is obvious temptation for the waiting farmers to try to take water out of turn; Valencia is hot and dry and the crops are in constant danger, especially in drought years. Remarkably enough these turn-taking schemes have survived for centuries.

The purpose of the turn-taking scheme is to insure an efficient, or at least near efficient, use of the water supply. Without the agreement to rotate, the farmers would waste valuable resources fighting amongst themselves over the scarce water. It is possible that farmers closer to the canals, or further upstream, would have an advantage in an unfettered contest for the water. The advantaged farmers might even be better off with free competition than with the turn-taking scheme. However, the disadvantaged farmers might be forced out of business without the turn-taking scheme, and the total amount of crops produced might go down. By following the turn-taking scheme, the farmers avoid these potential problems.

There are other situations in which turn-taking schemes can enable groups of people to exploit a resource to their collective advantage. Two firms, for example, can alternatively offer monopoly price bids in a series of contract auctions. Without the turn-taking scheme, the firms would be forced to offer competitive price bids; the earnings of the auction's winner would be drastically reduced. Similarly, two opposed politicians can alternatively vote against their immediate best interests so that a string of bills, some of which please their constituents, will be assured of passage. If the politicians did not agree on a turn-taking scheme, their votes would cancel out and perhaps no bills would pass.

All these situations can be classified under the rubric of Reciprocity Games. A Reciprocity Game, then, is any noncooperative situation in which some efficient outcomes can only be realized by utilizing nontrivial correlated strategies, or turn-taking. Repeated versions of classical games like the Battle of the Sexes and Chicken are Reciprocity Games, pure coordination games like The Repeated Prisoner's Dilemma are not.

As an example of a Reciprocity Game, consider the repeated, finite action, two player game implied by the stage-game payoff matrix G_1, where

$$G_1 = \begin{bmatrix} (3,3) & (3,7) \\ (7,3) & (4,4) \end{bmatrix}.$$

Label the actions A and B. Let the top and bottom rows represent the payoffs if the row player chooses action A or B, respectively. Let the left and right-hand columns represent the payoffs if the column player chooses action A or B, respectively.

Assuming that both players are rational, or expected utility maximizers, that they are non-altruistic, and that they have complete information about the payoffs and the rationality of the other player, noncooperative game theory offers certain predictions about the player's behavior. The clarity of these predictions depends upon the number of times that the stage-game is repeated.

If the stage-game is not repeated, each player has a dominate strategy, which is to choose action B. Play of this action at every stage is also the unique subgame perfect equilibrium of any finite repetition game. In equilibrium, each player receives a payoff of four in each stage. The equilibrium is efficient only in the non-repeated or one-shot game; in the repeated game, all the efficient outcomes involve alternating between the stage-game payoffs of $(3,7)$ and $(7,3)$. To gain these payoffs, both players must choose their dominated action, and furthermore, the players must coordinate so that they do not choose the dominated action at the same time. Given an even number of stages, the simple alternation scheme of having the players take turns choosing actions A and B leads to an outcome in which each player gets an average stage payoff of five.

If the stage game is repeated an infinite number of times, the folk theorem implies that there are an infinite number of subgame perfect equilibria. Any outcome that has payoffs greater than or equal to four is subgame perfect. In fact, there are an infinite number of efficient subgame perfect equilibria, each one involving some pattern of alternation between $(3,7)$ and $(7,3)$. The multiplicity of equilibria is in itself a problem for the players – which equilibrium should they coordinate on? Axiomatical concepts like symmetry, group welfare, or equality can be used to determine focal points, yet, even with these concepts there need not be a unique equilibrium. The efficient payoffs do share a common trait, however. In the efficient outcomes, the players must resort to a pattern of alternation between the stage-game payoffs of $(3,7)$ and $(7,3)$.

The purpose of this paper, then, is to examine the ability of people to enter into alternation schemes and achieve efficient outcomes to reciprocity games. The games will be studied under three different repetition conditions: one-shot, finite repetition, and infinite repetition. Comparisons will be made between a game that has symmetric payoffs and a game that has asymmetric payoffs. The effects of adding a third action, one intended to be a clear punishment, will also be considered.

1.2 Related Research

The previously mentioned book by Ostrom (1990) is concerned with examining the ability of people to efficiently exploit common pool resources. She reviews several case histories in which groups of people are able to introduce rotation schemes and successfully exploit the resource. Some of her examples have been in place for centuries.

Ostrom *et al.* (1991) have abstracted from these real life examples in an experimental study of the use of a common pool resource. In their study, rotation schemes offer an efficient way to exploit the resource, and, in fact, some of the eight-person groups try to institute these schemes. Ostrom *et al.* find that these schemes fail do to mistrust, mistakes or cheating. The authors find that the efficiency of the use of the resource increases if individuals are allowed to impose fines on one another; however, resource use never reaches optimal levels.

Murninghan *et al.* (1987) studied modified Prisoner's Dilemmas that were in fact Reciprocity Games. They found that in infinite repetition treatments and with the ability to communicate, subjects often resorted to alternation schemes, some sacrificing potential payoffs to do so. Some subjects also attempted *complex alternation* schemes in an effort to generate more equal payoffs.[1] Their treatments are similar to the infinite repetition, symmetric treatment considered here. The main differences between the treatments are that Murninghan *et al.* allow communication, and also the asymmetries in their payoff structure occur on the main diagonal.

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[1]Murninghan *et al.*, p. 17.

Palfrey and Rosenthal (1991a; 1991b) and Cooper *et al.* (1990; 1989; 1987) have studied various public goods and coordination games that with repetition become Reciprocity Games. Cooper *et al.* (1990; 1987) also examined the addition of an action deemed to be a punishment. They found that the availability of the extra action did effect the players choice of strategies.

Selten and Stoecker (1986), in their work on finitely repeated Prisoner's Dilemmas, developed a system of outcome classification that is similar to the strategy classification system used here.[2]

1.3 The Experimental Design

Each of four different payoff treatments will be examined under three different repetition conditions. The four different payoff treatments are: symmetric (G_1), asymmetric (G_2), symmetric with punishment (G_3), and asymmetric with punishment (G_4). Each of these treatments is represented by a payoff matrix in Table 1.1. The different repetition conditions are: one-shot, finite repetition, and infinite repetition.

1.3.1 Equilibria

The equilibria for G_1 have been discussed already, but for completeness, they will also be examined here along with the equilibria in the other three games.

[2]In Selton and Stoecker (1986) either a Cooperative outcome or End-Effect Play occurs if the cooperative alternative in the one-shot game is chosen consecutively for $m > 4$ periods during the supergame. Unlike Selten and Stoecker, this paper examines the sequence of play at the individual level and makes inferences about the types of strategies that each individual plays, either Alternating, or Nash (or Other).

First, in the one-shot conditions of both G_1 and G_2 there is either a unique dominate strategy or dominate solvable Nash equilibria. In G_1 the unique equilibrium is for both players to choose action B, it gives each of them a payoff of four. The outcome will be denoted by the pair $\{B, B\}$ so that each player's move is reflected. In G_2 the unique equilibrium, $\{A, B\}$, is for the row player to choose action A and get a payoff of three, and for the column player to choose action B and get a payoff of seven.

Recall that the games G_3 and G_4 are identical to the games G_1 and G_2, respectively, except that G_3 and G_4 have an additional action available to the players. The action is clearly not a desirable action; if it is played, both players get much worse payoffs. However, the availability of the action means that both G_3 and G_4 have three equilibria instead of only one. They share the equilibria of their counterparts, namely $\{B, B\}$ and $\{A, B\}$, respectively, plus they each have two additional equilibria.

In G_3 the additional equilibria are: $\{(\frac{1}{4}B, \frac{3}{4}C), (\frac{1}{4}B, \frac{3}{4}C)\}$, the fractions representing the weights in a mixed strategy, and $\{C, C\}$. In G_4 the additional equilibria are: $\{(\frac{1}{3}A, \frac{2}{3}C), (\frac{1}{7}A, \frac{6}{7}C)\}$ and $\{C, C\}$. These additional equilibria are dominated, in the sense that both players get higher payoffs, by the $\{B, B\}$ equilibrium in G_3 and the $\{A, B\}$ equilibrium in G_4.

Finite repetition creates no additional equilibria in either G_1 or in G_2. However, in G_3 and in G_4 finite repetition creates many additional equilibria. In fact, due to a finite game folk theorem, any minimax-dominating outcome can be approximated by a subgame perfect equilibrium if the number of stages is large enough.[3] The folk theorem result causes a problem that is very similar to the problem encountered in the infinite repetition games, how do players coordinate on a particular equilibrium when the set of equilibria is very large?

Infinite repetition, in all four games, leads to sets of equilibria that are very large indeed – they are infinite. In fact, the infinite repetition folk theorem says that if the discount rate is low enough, any outcome to a game which results in average stage-game payoffs which are greater than the minimax payoffs is supportable as a subgame perfect equilibrium. [4] Note

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[3]For example, for G_1 repeated $T >= 3$ times,

$$[\{B,A\}_1, \{A,B\}_2, \{B,A\}_3, \ldots, \{A,B\}_{T-1}, \{B,B\}_T]$$

with the threat of playing $\{C,C\}$ for each subsequent stage if there is a defection is subgame perfect. To be more specific, in repeated versions of one-shot games that have multiple Nash equilibria, for any individually rational and feasible outcome u there exists a length T and a subgame perfect equilibrium such that if U is the average stage payoff in the equilibrium,

$$||U - u|| < \varepsilon$$

for any $\varepsilon > 0$. The result holds for two-person games and for n-person games if the dimensionality of the payoff space is equal to the number of players. For details see Benoit and Krishna (1985); p. 919; refer to Theorem 3.7.

[4]The equilibrium payoffs must be such that the following equation holds:

$$\frac{1}{1-\delta} v_i \geq \bar{v}_i + \frac{\delta}{1-\delta} v_i^*$$

$$\frac{1}{1-\delta} v_i^* = \frac{1}{1-\delta}((1-\delta^t)v_{i,min} + \delta^t v_i)$$

where v_i is the average payoff of the equilibrium strategy given no defection, $\bar{v}_i$ is the maximum payoff a player can get by deviating, vi^* is the average payoff of the chosen punishment strategy, and δ is the discount rate. Equation 1 says that the total payoff for playing the equilibrium must be greater than the total payoff for deviating once and then getting the punishment payoff for the rest of the game. For details see Fudenberg and Maskin (1986); pp. 533 - 554; refer to Theorem 1. In the infinite repetition treatments, the discount rate was ten percent.

that the minimax payoffs for G_1 through G_4 are: $(4,4),(3,7),(1\frac{3}{4},1\frac{3}{4})$, and $(1\frac{2}{3},1\frac{3}{4})$. Again, the question is: How do players coordinate on a particular equilibrium when the set of equilibria is very large?

It is possible to pare the sets of equilibrium outcomes down to the manageable level of three or less by applying the axiomatic refinements of Equality, Symmetry, and Welfare Maximization, along with Pareto Optimality. The Equality refinement requires each player to receive the same payoff; the Symmetry refinement requires each player to choose their dominated action the same number of times; and the Welfare Maximization refinement requires the sum of the player's payoffs to be maximized. Pareto Optimality, of course, means that each outcome must be efficient. The equilibria that pass these refinements will be called focal solutions.

Specifically, in G_1 and G_3, the *one to one* alternation scheme leads to average stage payoffs of $(5,5)$ and satisfies all four of these refinements. For the symmetric games, the imposition of the refinements means that the number of focal solutions is the same in the one-shot, finite, and infinite repetition conditions. In each case, there is a unique focal solution.

On the other hand, in G_2 and G_4, a *one to one* alternation scheme satisfies only the Symmetric refinement and leads to average stage payoffs of $(4,5)$. To satisfy the Equality refinement requires a *one to two* alternation scheme. In this scheme the row player chooses action A half as often as the column player chooses action B and players end up with average stage payoffs of $(4\frac{1}{3},4\frac{1}{3})$. Furthermore, to satisfy the Welfare Maximizing refinement leads to play of the $\{A,B\}$ stage game equilibrium and average stage payoffs of $(3,7)$. For the asymmetric games, the imposition of the refinements means that the number of focal solutions is three in the infinite repetition condition and in the finite repetition condition of G_4. The one-shot condition and the finite repetition condition of G_2 have unique focal solutions.

The behavior in the one-shot games should be considered as a calibrating device. The outcomes achieved are worst case outcomes in the sense that there is no chance for the players to use an efficient rotation scheme. Theory predicts that behavior will conform to the Nash Solution, which will be defined as Hypothesis 1.

Although not equilibria in all cases, the following hypotheses will be considered for both the finite and infinite repetition treatments (notice that they do not specify behavior in the earliest stages of the game; they allow a period of time for the players to coordinate):

Hypothesis 1 (Nash Solution) *After a certain period, each player chooses the action which leads to the highest Pareto-Ranked, subgame perfect equilibrium.*

Hypothesis 2 (Alternating Solution) *After a certain period, the outcome to the game will have players alternating between action A and action B such that the realized play will be* $\{\ldots, \{A, B\}, \{B, A\}, \{A, B\}, \ldots\}$.

Hypothesis 3 (Welfare Solution) *After a certain period, the outcome to the game will be such that the sum of the players payoffs is maximized.*

Hypothesis 4 (Equality Solution) *After a certain period, the outcome to the game will maximize the sum of the players payoffs subject to having each player receive the same payoff.*

Hypothesis 1 embodies the predicted outcome in the finite repetition games. The Nash Solution is also an equilibrium in any of the infinite repetition games, although it is not an efficient equilibria in the symmetric cases. Hypothesis 2 embodies the axiomatic refinement of Symmetry, it requires the players to adopt a one to one rotation scheme; Hypothesis 3 embodies the axiomatic refinement of Welfare Maximization; and Hypothesis 4 embodies the axiomatic refinement of Equality. Although not always equilibria, these three solutions are efficient outcomes to the finite repetition games.

1.4 The Experiments

All the experiments were performed in a laboratory at the California Institute of Technology. The experiments were run on a set of computers linked together in a network. The subject pool consisted of students, most of whom were recruited from introductory economics and political science courses. There were nine experimental sessions: one session for each finite and infinite repetition treatment of G_1, G_2, G_3, and G_4; and one session for all the one-shot treatments. The number of subjects in each session varied from ten to fourteen because some recruited subjects did not show up for some experiments.

The following outline describes the order of events that took place in a typical experimental session:

1. Each subject entered the laboratory and sat at the terminal of their choice.

2. The subjects were read a set of directions detailing the rules of the session. The subjects were not shown a payoff matrix, instead each action and payoff was explained to them independently. The subjects were led through two practice periods and then quizzed.[5]

3. In a period, each subject chose either A or B (or C) and was then informed of their payoff and partner's choice. This was repeated under the following conditions:

   (a) In the one-shot treatments, each subject was randomly matched with another at the beginning of each period. The game ended after 15 periods.

   (b) In the finite repetition treatments, each subject played the same person each period. The game ended after 15 periods.

   (c) In the infinite repetition treatments, each subject played the same person every period. After the 15th period, a ten-sided die was rolled so that the subjects could see the result. If a 9 was rolled then the game ended, otherwise the game continued another period after which there was another die roll. The game did not end until a 9 was rolled.

4. At the end of the game, the subjects were randomly matched with a person whom they had not played and another game was started.

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[5]A copy of the directions and quiz used in the one-shot treatment of G_4 is included in the appendix.

5. Each subject in a session played 4 games and was then paid cash for each *point* they earned in the experiment. In the one shot treatments, the order of games was: G_1, G_3, G_2, and G_4. In the finite and infinite repetition treatments, the subjects played the same game four times.

6. The experimental session ended.

In the symmetric treatments, every player faced the same payoffs, so there was no difference between a row and a column player. Hence, in the symmetric treatments, all subjects were treated identically.

On the other hand, in the asymmetric treatments, the labels row and column had meaning, the player unlucky enough to be a row player was at a disadvantage. In order to prevent row players from gambling that they would become column players later in the session, at the beginning of each asymmetric treatment half of the subjects were informed that they would be row players for all four games in the session. In the one-shot session, this division took place before the third game, after all the symmetric games had been played.

Table 1.2 reports the number of subjects and the number of observations, respectively, in each treatment.[6] An observation consists of the outcome of one complete game and two sequences of actions, one for each player involved. The table also shows the dates of each session, the length, the exchange rate, and the order of the one-shot treatments.

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[6]There were 93 subjects total. An effort was made not to have experienced players, however 7 did participate in two sessions. Two participated in 4/20/90 and 5/17/90, one participated in 5/17/90 and 5/18/90, and four participated in 5/11/90 and 5/18/90. These people were never matched with the same person more than once, even across sessions.

1.5 The Results

1.5.1 The One-Shot Treatment

The first step is to examine the players' behavior in the one-shot treatments. The Table 1.3 describes the number of times each possible outcome pair was observed.[7]

In order to determine whether or not an individual's actions changed as s/he gained experience with the game, the data was split into the first eight periods and the last seven periods and then compared using a standard χ^2 test.[8] In no case was there a significant difference between the distribution of actions at the beginning and the distribution of actions at the end. The χ^2s were: 0.3370 for G_1, 0.2983 for G_2 row players, 1.2301 for G_2 column players, 1.6290 for G_3, and 2.5813 for G_4 column players. The column players in G_4 chose action B in every case.

In G_1, fourteen of the 150 observations, or 9.3 percent, assigned payoffs below the minimax to at least one of the players. In G_2, sixteen of the seventy-five row player observations and six of the seventy-five column player observations, 21.3 percent and 8 percent respectively, assigned payoffs below minimax payoffs. Assuming that the true frequency of below individually rational payoffs is the lower end of a 95 percent confidence interval around these observed frequencies would lead to the following percentages: 5.4, 13.4,

[7]In G_1, half of the subjects played A at least once. In G_4, one subject was responsible for all the plays of action C.

[8]χ^2_i, here and elsewhere, is the standard test statistic using Yate's continuity correction. It has a χ^2 distribution with i degrees of freedom. For a complete explanation of this test, see Everitt (1977) pp. 12 - 14.

and 2.8, respectively.

Obviously, there is a substantial minority of players who play non-equilibrium strategies. In an ideal environment, Hypothesis 1, that each player chooses the subgame perfect equilibrium strategy, would be rejected on the basis of even one non-equilibrium play. However, the criteria adopted for this experimental environment allows their rejection only if the upper bound of the 95 percent confidence interval around the observed proportion of plays is less than 0.95. These bounds are displayed in the Table 1.4. Hypothesis 1 must be rejected for G_1, and for the row players in both asymmetric treatments. The fact that not all people always play the unique, subgame perfect equilibrium strategy in one-shot games has been observed many times.[9]

Notice the significant change in the behavior of the column players when comparing G_2 to G_4. In G_2, 8 percent of the actions chosen by the column players violate the Nash Solution, in G_4 no actions chosen violate the Nash Solution. This is an anomaly because behavior does not change for the row player, neither does it change between G_1 and G_3. One explanation for the data is that, because G_2 and G_4 were played in succession by the same players, the column players learned how to play according to Hypothesis 1. Oddly enough, the row players did not share in the revelation.

[9]See Ledyard (1992), Dawes (1980) and Cooper *et al.* (1987; 1990).

1.5.2 The Finite and Infinite Repetition Treatments: Average Payoffs

The outcomes to the finite and infinite repetition treatments are represented by the average payoffs of both players. To allow a period of time for the players to coordinate on a specific outcome, the first four periods are ignored. Also, so that the infinite repetition treatments remain comparable to the finite repetition treatments, the averaging ends with the fifteenth period (the finite repetition treatments were fifteen periods long).

Referring to Figure 1.1, the set of possible outcomes to G_1 if it were infinitely repeated is represented by the triangular figure in both the top and bottom diagrams. Given that a ten period average is used, the possible outcomes are a subset of the triangular set. Actual outcomes to the games are shown by a letter representing one or more observations. The letter is located at the coordinates determined by the average payoffs of the players.

For an outcome to be Pareto Optimal, it must be located on the hypotenuse of the triangular set. The 45° line highlights the outcomes in which the players receive equal payoffs. Every outcome located northeast of the dotted lines payoff dominates the minimax. These minimax dominating outcomes, given a small enough discount rate, are subgame perfect equilibria if the game is infinitely repeated.

In Figure 1.1, the top diagram represents the outcomes of the finite repetition treatment of G_1. The bottom diagram represents the outcomes of the infinite repetition treatment of G_1. Similar figures are constructed for the two treatments of G_2, G_3, and G_4.

Note that in G_1 and G_3 there is no difference between a row and a column player. In order to avoid drawing conclusions from arbitrarily scattered outcomes, all the outcomes are located on or below the 45° line. In G_2 and G_4, there is a difference between a row and a column player.

Again referring to Figure 1.1, specifically to the top diagram which shows the outcomes of the finite repetition treatment, notice that the outcomes occur in two clusters. One cluster is located around the unique one-shot equilibrium or Nash Solution, point $(4, 4)$. The other is located around the focal solution, the outcome that embodies the Alternating Solution, the Equality Solution and the Welfare Maximizing Solution, point $(5, 5)$. The observations are divided roughly between the two clusters. Although the Nash Solution was the most observed with five, fourteen groups were able to improve upon it using some pattern of reciprocation, three actually implemented the focal solution. One player out of the twenty pairs received below minimax payoffs.

The bottom diagram, which shows the outcomes of the infinitely repeated treatment, is in sharp contrast to the top one. Here, twenty-one of twenty-four observations are located at the focal solution. Of the three remaining outcomes, two are located near the Nash Solution, and the last is located at an outcome better than the Nash Solution but not as good as the focal solution. The extension of the time-horizon from finite to infinite draws many outcomes away from the Nash Solution and to the focal solution. People appear to have few problems implementing a rotation scheme and achieving efficient payoffs, approximately 90 percent succeed, if G_1 is infinitely repeated.

Figure 1.2 shows the outcomes of the finite and infinite repetition treatments of G_3. Recall that G_3 is identical to G_1 except that an additional action, a punishment, was added to the action space. Despite the additional strategy, Figure 1.2 closely resembles Figure 1.1. In the top diagram, the finite repetition treatment, thirteen of the twenty outcomes are close to the focal solution. In the bottom diagram, the infinite repetition treatment, nineteen of the twenty-four outcomes are at the focal solution.

The top diagram in Figure 1.3 shows the outcomes of the finite repetition treatments of G_2, the first of the asymmetric games. Seven outcomes were at the Nash and Welfare Maximizing Solutions, point $(7,3)$. One outcome was at the Alternating Solution, point $(5,4)$. No outcomes were at or even near the Equality Solution, point $(4\frac{1}{3}, 4\frac{1}{3})$.[10] More than half of the outcomes, eleven of twenty, have the row player receiving less than minimax payoffs.

The bottom diagram shows the outcomes to the infinite repetition treatment of G_2. Unlike in the symmetric games, there is no improvement in the efficiency of the outcomes as the time horizon gets longer. Roughly the same proportion of outcomes are at the Nash Solution, the Alternating Solution, and the Equality Solution (eight, two, and zero observations out of twenty-four, respectively) as in the finite repetition treatment. Again, half of the outcomes have the row player receiving less than minimax payoffs. If anything, the payoffs in the infinite repetition treatment are worse than the

[10]The Equality Solution requires a *one to two* rotation scheme, *i.e.* row plays A once for each two times that column plays A. This rotation scheme has a three move cycle. What is exhibited in the figures is a ten move average payoff. Even if a *one to two* rotation scheme was implemented, the ten move average would not give equal payoffs. However, any *one to two* rotation scheme would result in payoffs located on the Pareto Frontier and the averaging system used would locate the outcome within 0.2 payoff points of the Equality Solution. No outcomes were within these tolerances.

payoffs in the finite repetition treatment.

Figure 1.4 shows the outcomes to G_4. Recall that G_4 is identical to G_2 except that a punishment action is added. Unlike in the symmetric case, here the presence of the punishment action changes behavior. In the top diagram, the most observed outcome is the Alternating Solution, point $(5, 4)$. This is in contrast to the most observed outcome in the finite repetition treatment of G_2 which was the Nash or Welfare Solution, point $(7, 3)$. However, a substantial number of outcomes are still inefficient outcomes. The bottom diagram has these same features: the most observed point is the Alternating Solution, and many observations are at inefficient outcomes. Again, drawing on the similarity between the top and bottom diagram, infinite repetition did not greatly improve the chances of coordinating on an efficient outcome.

Table 1.5 shows the distribution of outcomes over the focal point solutions. It is clear that infinite repetition makes a difference in the symmetric treatments – it results in a higher percentage of efficient Alternating Solution outcomes. In the asymmetric case, infinite repetition does not seem to make a difference, the distribution over the focal solutions remains similar. However, the addition of a punishment action causes a shift from the Welfare Maximizing Solution to the Alternating Solution. In every asymmetric treatment, a substantial number of outcomes are not efficient.

1.5.3 Comparing Average Payoffs

Table 1.6 shows the average payoffs in the one-shot treatments and in rounds 5 to 15 of the finite and infinite repetition treatments. In the symmetric treatments, the average payoffs rise as the time horizon lengthens. In the one-shot treatment, the average is near the payoff associated with the Nash Solution, which assigns each player four. In the infinite repetition treatments, the average is near the payoff associated with the Alternating Solution, which assigns each player five. There seems to be little lost or gained from the addition of the punishment action.

The asymmetric treatments are much different than the symmetric ones, the longer horizons do not imply more efficient group payoffs. In fact, from the point of view of the column player, the longer time horizon is disastrous – especially when the punishment action is present. The average column player's payoff drops more than 20 percent when moving from the one-shot treatment to either the finite or infinite repetition treatment of G_4. From the group's perspective, this drop in the column player's payoff is not made up for by the small increase in the payoffs of the row player. The average row player only gets around 10 percent more when moving from the one-shot to either repeated treatment of G_4. The finite repetition treatment of G_2 is the only treatment where the players improve upon the payoffs of the one-shot treatment.

1.5.4 The Finite and Infinite Repetition Treatments: The Strategy Space

The following definitions divide the strategy sets associated with each repetition treatment into three disjoint parts:

Definition 1 (Alternating Strategy) *An individual's sequence of play is an Alternating Strategy if, for every period in the sequence, the group's play in the previous period was* $\{A, B\}$ *or* $\{B, A\}$*, then individual's play in this period is B if last period it was A and A if last period it was B.*

Definition 2 (Nash Strategy) *An individual's sequence of play is a Nash Strategy if for every period in the sequence, the individual's play corresponds to the action taken in the highest Pareto ranked, one-shot, subgame perfect equilibrium.*

Definition 3 (Other Strategy) *An individual's sequence of play is an Other Strategy if it is not an Alternating Strategy or a Nash Strategy.*

It is possible to sort every individual's complete sequence of actions into one of the three previous categories. The Alternating Strategy category includes all strategies that try to alternate – dire punishment strategies as well as completely forgiving strategies. The Nash Strategy category includes only the one strategy.[11] The Other Strategy category is a catchall and could contain many things, completely random behavior being one example.

Table 1.7 shows the distribution of strategies for each game's finite repetition treatment. Notice that in the symmetric games G_1 and G_2, the Alternation Strategy is picked most often. Also there is not a significant difference between the distributions, so the punishment action makes little difference.

[11] It is possible to have a sequence of plays defined as both an Alternating and a Nash Strategy. In the symmetric treatments, if both players choose action B in every round, each player's strategy will be put into both categories. Fortunately, no pair of players chooses action B in each round, so the problem does not surface.

In the asymmetric games G_2 and G_4, there is a significant difference between the distribution of strategies with and without the presence of the punishment action. The difference exists for both the row and the column players. The presence of Other Strategies on the part of the row players in G_2 shows that there were attempts at alternation – they do not just play the Nash Strategy. Most of the column players, however, play the Nash Strategy. So, the row players tend to either give up and play the Nash Strategy themselves or they punish their partners with the minimax. Most of them start playing the Nash Strategy.

The proportion of players that play an Alternating Strategy in G_4 is much higher for both types when the the punishment action is present. Note that the players never have to use this action, its presence is enough to cause the shift. A substantial number of players, both row and column, still pick an Other Strategy.

In fact, in each of the finite repetition games, a large number of Other Strategies are chosen. Possible explanations for this is that there is conflict between the players, or that they miscoordinate in the early rounds. In any case, there is uncertainty during the game about which equilibrium strategy, the Alternating Strategy or the Nash Strategy, each player is supposed to use.

Another explanation is that there are end-game effects present. With end-game effects, players who had been choosing their action according to the Alternating Strategy would change to the Nash Strategy before the last period. Unlike in G_1 and G_2, in G_3 and G_4 end-game effects would be consistent with many subgame perfect equilibria.

Table 1.8 reproduces each strategy distribution when the last two periods of play are ignored.[12] There is, in fact, a dramatic end-game effect in both symmetric games; 17.5 percent of the subjects switched from Alternating Strategy to Other Strategy in the last two periods of G_1, 20 percent switched in G_3. The data from the asymmetric games, on the other hand, show positively no evidence of an end-game effect. One must conclude, then, that the Other Strategies present in G_2 and G_4 are due to conflict or miscoordination.

Table 1.9 shows the distribution of strategies for each game's infinite repetition treatment. Notice that in the symmetric games G_1 and G_2, the Alternation Strategy is again picked most often. Also there is not a significant difference between the distributions, so the punishment action makes little difference.

The presence of the punishment action also makes little difference in the asymmetric games, although there is some shift away from the Nash Strategy for the column players. The high number of Other Strategies shows that the conflict and miscoordination present in the finite repetition treatments is still there in the infinite repetition treatments.

The strong difference between the symmetric finite and infinite repetition treatments is not surprising considering the presence of the end-game effects. What is surprising is the strong difference between the finite and infinite repetition treatments of G_2. There was no end-game effect present in the finite treatment of G_2.

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[12]Two was chosen because it is the minimum number of periods that allows both players a chance to defect from the Alternate strategy.

1.6 Conclusions

After considering the evidence presented here, it is not unreasonable to predict that some groups of people, like the aforementioned Valencian farmers, will be able to enter into stable alternation schemes if they are faced with situations similar to Reciprocity Games. The farmers are in a symmetric situation, 80 percent of the farms are less than 1 hectare. The farmers are involved in an infinite repetition conflict, the farms have been there for centuries. Like most of the participants in infinite repetition treatments of G_1 and G_3, the farmers have been able to institute an efficient rotation scheme.

In these experiments, it has been shown that people faced with symmetric Reciprocity Games enact solutions which are progressively more efficient as the time horizon increases from one-shot to finite repetition to infinite repetition. End-game effects have been found in the finite repetition treatments. In symmetric situations, punishment options play very little role.

The ability of groups of people to obtain efficient outcomes if there are large asymmetries between them is much more doubtful. As has been seen, there can be a conflict or miscoordination if the turn-taking and welfare maximizing solutions are different. Although some succeed in instituting one of these two efficient focal outcomes, of those who fail, many get non-individually rational payoffs. Not a single group successfully instituted a *one to two*, or equal payoff, rotation scheme.

Unlike the symmetric games, efficiency in the asymmetric games does not tend to increase as the time horizon lengthens. In fact, due to prolonged conflict or miscoordination, average payoffs in the infinite repetition treatments are below the average payoffs in the one-shot treatments. With finite repetitions, the presence of the punishment action causes an increase in the number of alternation schemes that are successfully implemented or tried, although the number of efficient outcomes does not increase significantly and the average payoffs fall.

Certainly the results of the examination of the asymmetric games highlights problems from a policy standpoint. Common welfare criteria, like the Utilitarian criterion (maximize the sum of the payoffs), the Rawlsian criterion (maximize the minimum payoff), Pareto Optimality, or even simple rationality are not always achievable without intervention. In fact, clearly bad outcomes occur frequently.

And what type of intervention will work? If you care about the sum of the payoffs you may choose to shorten the length of the game. Shortening the length of the game will certainly benefit the group, but the disadvantaged will suffer for it. If you care about equality you may choose to endow people with the ability to punish, or tax, or fine the other participants. Among the efficient outcomes, there will be more egalitarian behavior, but the combined benefits of the group will likely fall on average.

On the other hand, the results of the symmetric games are very encouraging from a policy standpoint. Punishments, taxes or fines are not necessary. Simply increase the time horizon and efficiency rises.

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1.8 Sample Instructions and Quiz

The following is a copy of the instructions given in the one-shot treatments of G_4.

INSTRUCTIONS FOR A DECISION-MAKING EXPERIMENT

This is an experiment in decision making. You will be paid *in cash* at the end of the experiment. The amount of money you earn will depend upon the decisions you make and on the decisions other people make. We request that you do not talk at all or otherwise attempt to communicate with the other subjects except according to the specific rules of the experiment. If you have a question, feel free to raise your hand. One of us will come over to where you are sitting and answer your question in private.

This experiment has 15 separate rounds and then it will end. During each round of the experiment you will be randomly paired with another subject. You will **never** be paired with the same subject for two rounds in a row.

Each round you will be given a token which will be worth either 4 or 2. It will always be worth the same amount. Each round you will be able to use the token in one of three ways: option A, or option B, or option C.

PAYOFFS

The amount of money you earn in a round depends upon which option you pick as well as which option your partner picks. **WHAT HAPPENS**

IN YOUR GROUP HAS NO EFFECT ON THE PAYOFFS TO MEMBERS OF THE OTHER GROUPS AND VICE VERSA. In each round, you have nine possible earnings. These are shown in the following table:

EARNINGS TABLE		
Your Choice	His/Her Choice	Your Earnings
A	A	3 points
A	B	3 points
A	C	1 point
B	A	Your Token Value + 3 points
B	B	Your Token Value
B	C	1 point
C	A	1 point
C	B	1 point
C	C	2 points

To summarize the table:

1 **ROWS 1 to 3:** If you choose option A you will get 3 points if your partner picks either option A or option B. If you choose option A and your partner chooses option C, you will get 1 point.

2 **ROWS 4 to 6:** If you choose option B you will get your token value + 3 points if your partner picks option A, you will get your token value if your partner picks option B, or you will get 1 point if your partner picks option C.

3 **ROWS 7 to 9:** If you choose option C you will get 1 point if your partner picks either option A or option B. If you choose option C and your partner chooses option C, you will get 2 points.

SPECIFIC INSTRUCTIONS:

At the end of the experiment you will be paid 5 cents for every point you have accumulated.

Quiz

The following is a copy of the quiz given in the one-shot treatments of G_4.

QUIZ

id #. ____

1. If my token is worth 4 points, the other player in my group will have a token value equal to:

   i. 4 points.

   ii. 2 points.

   iii. Either 4 or 2 points.

   iv. None of the above.

2. If someone was in my group on round 5 of an experiment, it will be **certain, very likely, impossible** that he or she will be in my group on round 6.

3. If my token value is 2 and I choose option B and my partner chooses option A, how many points will I earn?

4. If I choose option A and my partner chooses option C, how many points will I earn?

5. If at the end of a round I have 2 points, how much am I paid for that round?

1.9 Tables

Table 1.1: The payoff tables for the four different payoff treatments: symmetric (G_1), asymmetric (G_2), symmetric with punishment (G_3), and asymmetric with punishment (G_4).

The Payoff Tables

$$G_1 = \begin{bmatrix} (3,3) & (3,7) \\ (7,3) & (4,4) \end{bmatrix} \qquad G_2 = \begin{bmatrix} (3,3) & (3,7) \\ (5,3) & (2,4) \end{bmatrix}$$

$$G_3 = \begin{bmatrix} (3,3) & (3,7) & (1,1) \\ (7,3) & (4,4) & (1,1) \\ (1,1) & (1,1) & (2,2) \end{bmatrix} \qquad G_4 = \begin{bmatrix} (3,3) & (3,7) & (1,1) \\ (5,3) & (2,4) & (1,1) \\ (1,1) & (1,1) & (2,2) \end{bmatrix}$$

Table 1.2: The date of each experiment along with the number of subjects, the number of observations, the number of periods, the exchange rate, and, if there were different treatments in one session, the order of treatments. O, F, and I stand for one-shot, finite repetition, and infinite repetition, respectively.

Experiments							
game	trtmnt	date	subj.	obs.	length	$\frac{penny}{point}$	order
G_1	O	2/4/91	10	75	1	5	1
	F	1/31/91	10	20	15	4	-
	I	5/18/90	12	24	{61, 37, 17, 29}	4	-
G_3	O	2/4/91	10	75	1	5	2
	F	1/14/91	10	20	15	4	-
	I	5/17/90	12	24	{20, 41, 26, 25}	4	-
G_2	O	2/4/91	10	75	1	5	3
	F	2/1/91	10	20	15	4	-
	I	5/11/90	12	24	{28, 19, 16, 20}	4	-
G_4	O	2/4/91	10	75	1	5	4
	F	2/1/91	14	28	15	4	-
	I	4/20/90	12	24	{16, 29, 21, 24}	4	-

Table 1.3: The distribution of outcomes in the one-shot treatments. The entries in each table represent the number of times each outcome was observed in that treatment. The outcomes that satisfy Hypothesis 4, the Nash Solution, have been underlined. Notice that there are no entries below the diagonal in the symmetric games G_1 and G_3; the symmetric outcomes are classified together. In the asymmetric games, all outcomes are classified separately.

The Distribution of Outcomes
One-Shot Treatments

$$G_1 = \begin{bmatrix} 1 & 12 \\ & \underline{62} \end{bmatrix} \qquad G_2 = \begin{bmatrix} 6 & \underline{53} \\ 0 & 16 \end{bmatrix}$$

$$G_3 = \begin{bmatrix} 1 & 9 & 0 \\ & \underline{63} & 2 \\ & & 0 \end{bmatrix} \qquad G_4 = \begin{bmatrix} 0 & \underline{58} & 0 \\ 0 & 13 & 0 \\ 0 & 4 & 0 \end{bmatrix}$$

Table 1.4: For each **One-Shot** treatment, the breakdown of individual strategy choices between successes and others for the Nash hypothesis is shown. Also shown is the frequency of success and the upper bound of its 95 percent confidence interval. Finally, the distribution of observations under the hypothesis when there is no punishment strategy is compared to the distribution of observations when there is a punishment strategy; a χ^2 statistic is reported.

One-Shot Contingency Table						
Hyp. 1 Nash Solution						
			Row		Column	
	G_1	G_3	G_2	G_4	G_2	G_4
successes	136	137	59	58	69	75
other	14	13	16	17	6	0
freq.	0.9066	0.9133	0.7866	0.7733	0.9200	1.000
high	0.9460	0.9514†	0.8657	0.8541	0.9723†	1.000†
χ_1^2	0.0000		0.0000		4.3403*	

† - significant at $\alpha = 0.05$
* - significant by adopted criteria
high is the upper bound of the 95% c. interval around freq.

Table 1.5: For each finite (F) and infinite (I) repetition treatment, the distribution of outcomes over each focal point solution is shown.

Distribution of Outcomes Over
Focal Point Solution Concepts:

	G_1		G_3		G_2		G_4	
	F	I	F	I	F	I	F	I
Hyp. 2 Alternating	3	21	5	19	1	2	8	7
Hyp. 3 Welfare	*	*	*	*	7	8	3	5
Hyp. 4 Equality	*	*	*	*	0	0	0	0
Hyp. 1 Nash	5	0	1	0	**	**	**	**
Other	12	3	14	5	12	14	17	12

* - Hyp. is the same as Alternating
** - Hyp. is the same as Welfare

Table 1.6: The average payoffs in the one-shot treatment and in rounds 5 – 15 of the finite and infinite repetition treatments.

Average Payoffs						
	One-Shot		Finite		Infinite	
	G_1	G_3	G_1	G_3	G_1	G_3
player	4.147	4.027	4.535	4.585	4.908	4.850
group	8.294	8.054	9.070	9.170	9.816	9.700
	One-Shot		Finite		Infinite	
	G_2	G_4	G_2	G_4	G_2	G_4
row	2.785	2.725	2.955	3.021	2.896	3.029
col	6.040	6.160	6.175	4.757	5.638	4.821
group	8.825	8.885	9.130	7.778	8.534	7.850

Table 1.7: In each **Finite Repetition** treatment, the distribution of strategy choices is shown. The distribution of strategies when there is no punishment strategy is compared to the distribution of strategies when there is a punishment strategy; a χ^2 statistic is reported.

Finite Repetition Contingency Table

			ROW		COL	
	G_1	G_3	G_2	G_4	G_2	G_4
Alt.	21	23	0	11	4	10
Nash	6	4	2	2	15	4
Other	13	13	18	15	1	14
χ^2_2	0.4909		10.2234*		19.4124*	

* - significant at $\alpha = 0.05$

Table 1.8: The different strategy distributions over the focal solutions obtained when all periods are taken into account and also when all but the last two periods are taken into account are displayed for each finite repetition treatment.

Finite Repetition,
Strategy Distributions,
All Periods and All But the Last 2 Periods:

	G_1		G_3	
	all periods	all periods - 2	all periods	all periods - 2
Alt.	21	28	23	30
Nash	6	6	4	5
Other	13	6	13	5

Row Players

	G_2		G_4	
	all periods	all periods - 2	all periods	all periods - 2
Alt.	0	0	11	11
Nash	2	2	2	2
Other	18	18	15	15

Column Players

	G_2		G_4	
	all periods	all periods - 2	all periods	all periods - 2
Alt.	4	4	10	10
Nash	15	15	4	5
Other	1	1	14	13

Table 1.9: In each **Infinite Repetition** treatment, the distribution of strategy choices is shown. The distribution of strategies when there is no punishment strategy is compared to the distribution of strategies when there is a punishment strategy; a χ^2 statistic is reported.

Infinite Repetition Contingency Table						
			ROW		COL	
	G_1	G_3	G_2	G_4	G_2	G_4
Alt.	42	40	6	6	2	7
Nash	2	1	6	4	12	6
Other	4	7	12	14	10	11
χ_2^2	1.2003		0.5538		4.8254	

* - significant at $\alpha = 0.05$

1.10 Figures

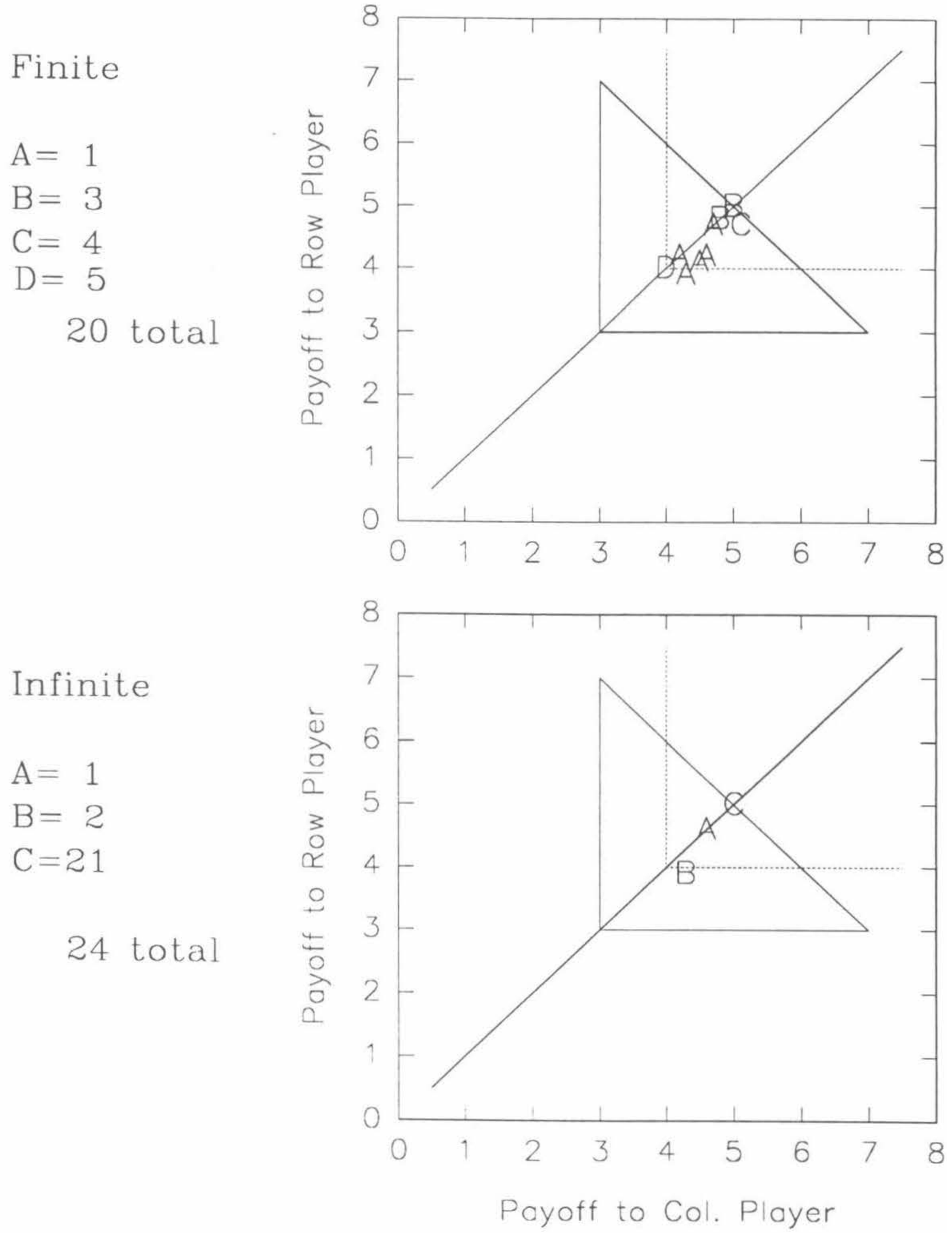

Figure 1.1: The outcomes to the repeated treatments of G_1.

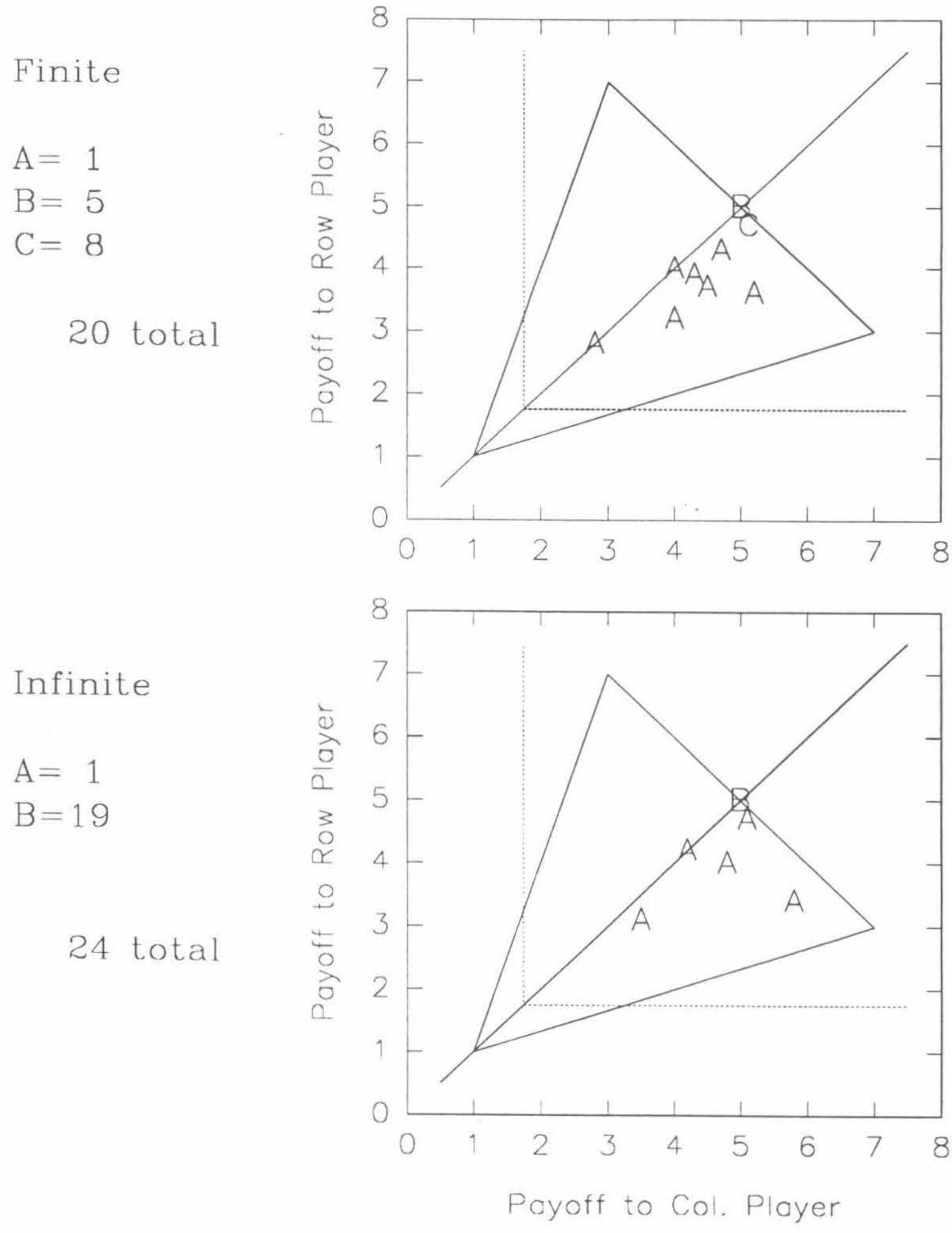

Figure 1.2: The outcomes to the repeated treatments of G_3.

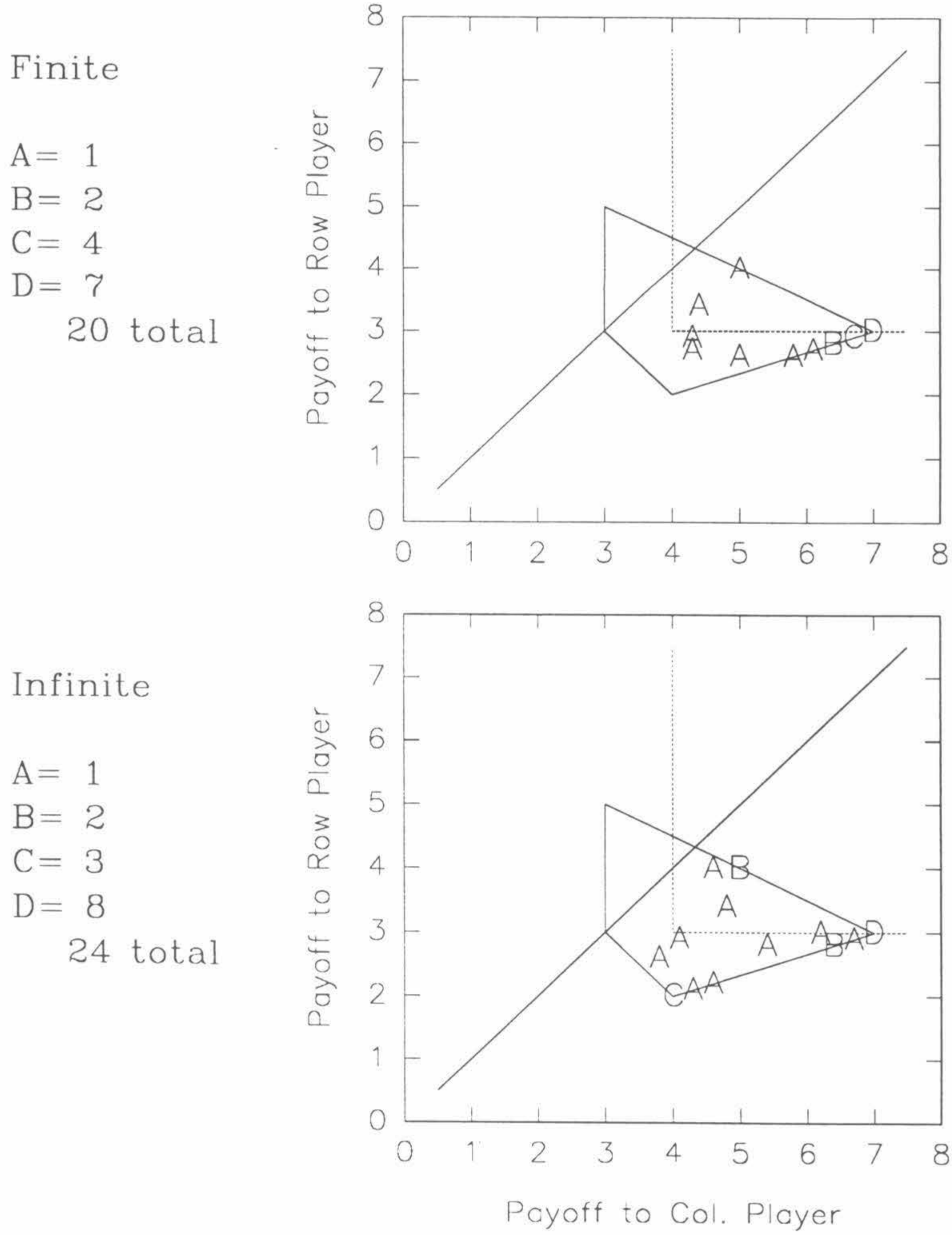

Figure 1.3: The outcomes to the repeated treatments of G_2.

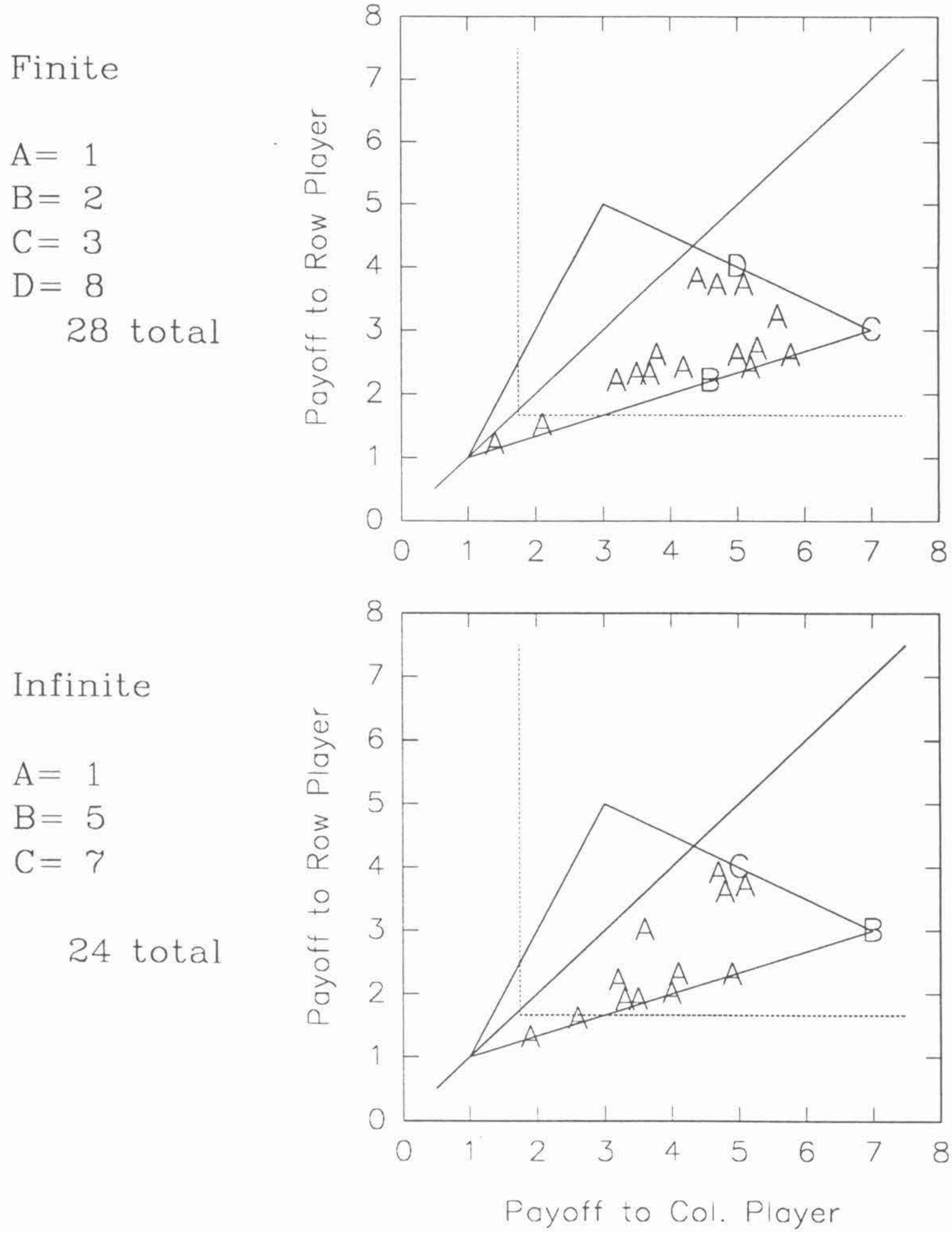

Figure 1.4: The outcomes to the repeated treatments of G_4.

Chapter 2

A Bounded Rationality, Evolutionary Model for Behavior in Two Person Reciprocity Games

2.1 Population Games and the Replicator Dynamic

Due to the Folk Theorem, a generic infinitely repeated game has many equilibria. The multitude of equilibria is a problem for theorists because a justifiable and non-arbitrary method of eliminating the majority of the equilibria has not been found. In the last chapter, experimental evidence was presented that suggested that players in infinitely repeated, symmetric reciprocity games usually succeed in establishing a pattern of alternation. The player's actions have given a clue as to which of the equilibria should remain after elimination. In this chapter a method which eliminates most of the experimentally unobserved equilibria is given.

The method is based on a mathematical model of evolution, developed in biology, called the Replicator Dynamic. The Replicator Dynamic supposes a large population of players, each endowed with a particular strategy. Each player in the population lives (plays a game), creates offspring identical to itself, and then dies. The mixture of player types within the population changes from generation to generation as the population grows and depends upon the success that each player has in creating offspring. In the Replicator Dynamic, each player begets a number of offspring that is proportional to that player's lifetime fitness, or payoff. The result is that later generations have a higher proportion of players endowed with high payoff strategies.

Suppose that the initial population has every possible strategy represented in it. Then, if the Replicator Dynamic is ever in equilibrium, meaning that the mixture of player types remains the same from generation to generation, the strategies that remain in the equilibrium have been justified in a Darwinistic sense.

The problem with the Replicator Dynamic is creating an initial population with every possible strategy in it. In the case of an infinitely repeated game, there are infinitely many possible strategies making it necessary to have a population of infinite size. The analysis of the dynamics on such a population are beyond the state of the art. For any analysis to succeed, there must be only a finite number of possible strategies. It is impossible, then, without further assumptions, to use the Replicator Dynamic as a method of justification. Here, it will be assumed that players have a finite memory. This *bounded rationality constraint* uniformly limits the number of possible strategies.

Application of the model to the infinitely repeated, symmetric reciprocity game succeeds in the sense that there is only one possible equilibrium. The equilibrium encompasses the exchange of favors as well as a behavior associated with short run payoff maximization and a behavior which could be coined as irrational (although none of the players are rational in any sense). All of these behaviors are seen in the experiments reported in the previous chapter. It is not a complete success, however, because the behaviors are not seen in the same ratios and furthermore, the irrational behavior is not an equilibrium in the standard sense.

These types of population games have been studied before, perhaps the best known examples are the papers by Axelrod (1979) and Axelrod and Hamilton (1981) which reported on Repeated Prisoner's Dilemma tournaments. In these tournaments, various people, most of them professional scientists, submitted computer programs which were, in essence, strategies in the repeated Prisoner's Dilemma. Together, the programs made an artificial population which competed by playing a repeated Prisoner's Dilemma in round robin fashion. After competing each strategy was reproduced based on their relative scores, the higher a strategy's score, the higher that strategy's representation in latter generations. They found that the strategy tit-for-tat displaced the other submitted strategies.

A variety of papers focusing on the dynamics of the tournaments followed.

Blad (1986), Hirshleifer and Martinez Coll (1988), Mueller (1987) and Young and Foster (1991) use Replicator Dynamics to justify or determine equilibria in three-strategy Prisoner's Dilemma and perturbed Prisoner's Dilemma games. The strategies considered were: All Defect, All Cooperate, and some variant of tit-for-tat (grim for example). The cooperative outcome was an equilibrium in almost all settings in these works. Smale (1980) applied the Replicator Dynamics to a setting where players only remembered a summary of the past (an average of their past payoffs). He found that if the players play *good* strategies, then cooperation is a globally stable equilibrium.

A different approach was taken by Miller (1989) who used an optimization technique called the Genetic Algorithm. A Genetic Algorithm takes a subset of the possible strategies as a population. The possible strategies, in turn, are determined by the computing power available. The population then evolves much like populations under the Replicator Dynamic. The difference is that each member of the population faces a probability of random mutation (be it gene specific or crossover) before or after the next generation is formed. Miller found that "... cooperative strategies ... tend to proliferate throughout the population under [certain conditions]."[1]

One criticism of these lines of research is that all of them apply their dynamic models to subsets of the possible available strategies. Furthermore, the subsets are determined in relatively arbitrary ways. In the three strategy dynamic models, for example, no reason is given for considering tit-for-tat while not considering, at the same time, the grim strategy.[2] This criticism

[1] Miller (1989), p. 12.

[2] Mueller (1987) attempts an argument by showing that he considers as a third strategy

becomes more powerful when the works of Boyd and Lorberbaum (1987) and Nachbar (1989) are taken into account. Boyd and Lorberbaum showed that, contrary to previous optimistic research, no pure strategy is evolutionarily stable in the infinitely repeated Prisoner's Dilemma. This finding depends upon the fact that all possible strategies have a chance of being played. Nachbar showed that the limit of the Replicator Dynamic in a two-stage Prisoner's Dilemma has everybody defecting (although All Defect is not the only strategy in the limit).

One way to uniformly limit the number of strategies under consideration in an infinitely repeated setting is to apply a bounded rationality constraint. The constraint is a logical one to consider given the comments of Aumann and Sorin (1989) who write:

> The first hint that bounded recall might have something to do with cooperation came in the summer of 1978. Aumann and Kurz, with the help of Jonathan Cave ... worked out a version of the infinitely repeated Prisoner's Dilemma with memory one; this means that each player can base his action only on what his opponent did at the previous stage – he has "forgotten" everything else. This results in an 8×8 bi-matrix game; iterated removal of weakly dominated strategies yields a unique strategy pair, in which both players start by playing "friendly" and continue with "tit-for-tat" thereafter. The outcome is cooperative, both players

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the strategy that in some sense punishes optimally.

always playing "friendly."[3]

However, Aumann and Sorin is not a paper concerned with population dynamics, and so it proceeds down a different path.

The effects of a one period recall will be considered here, only the payoff structure will not be that of a Prisoner's Dilemma. Instead, the analysis will focus on the Reciprocity Game. This paper is, in a sense, an answer to Rapoport (1988) who laments about the "...persistent hegemony of Prisoner's Dilemma ..." and claims that "...it is evident that there is enough to do in this area [of 2×2 games] for an army of investigators."[4]

2.2 The Environment

Let G be a symmetric, two-person, strategic game with finite action spaces $A_i = A_j = \{a, b\}$ and payoff matrix

$$M = \begin{bmatrix} \alpha & \beta \\ \eta & \gamma \end{bmatrix}$$

where the top row and first column correspond with the choice of action a and the bottom row and second column with action b. Let G^{∞} be the supergame made up of an infinite sequence of plays of game G.

A history or memory of length k for player i is defined as $h_i^k \in \Pi_{l=1}^k A_j$. Notice that under this definition, player i only has a memory of the last k

[3]Aumann and Sorin (1989), p. 9.

[4]Rapoport (1988), pp. 400 – 401.

actions of player j; player i does not remember his own actions. Let $s^k \in \mathcal{S}^k : h^k \mapsto A$ (with subscripts suppressed) be a function that maps a player's memory into an action. Call s^k a strategy with a bounded memory of length k and let $S^k \subset S^\infty$ be the set of all k length strategies. Let $S^B \equiv S^1$.

Another way to think of the set S^B is as the set of strategies which can be implemented by a two-state automaton, such automatons are commonly called Moore machines. A Moore machine, here from player i's point of view, consists of a quadruple, $\{H, q_0, f, \lambda\}_i$, where,

1. H is a finite set of histories or states,
2. q_0 is an initial state,
3. $f : H \times A_j \mapsto H$ is a transition function, and
4. $\lambda : H \mapsto A_i$ is a behavior function.

In this particular case, it is convenient to suppress H and f and explicitly enumerate λ. This should cause no confusion because $H = \{a, b\}$ and f maps A_j directly into H, *i.e.* $f(a) = a$ and $f(b) = b$. This convention allows a machine to be written as a triple, for example $\{a, a, a\}$, where the first represents q_0 the initial move of the machine, the second represents $\lambda(a)$, the move that the machine chooses if its opponent chooses action a, and the third represents $\lambda(b)$, the action chosen if its opponent chooses action b. The machine $\{a, a, a\}$ plays action 'a' on the first move, and then plays action 'a' regardless of the action its opponent chooses. There are eight possible two stage machines with these characteristics and they correspond directly with the strategies in the set S^B. Number the eight machines as in Table 2.1.

Suppose that two players, who are limited to choosing strategies in S^B or equivalently to choosing one of the eight machines, meet and play G^∞. Because of the finite strategies, the sequence of play eventually cycles, with the longest cycle being four stages. For example, if player i chooses machine $s_5 = \{a, a, b\}$ and player j chooses machine $s_8 = \{b, b, a\}$, then the sequence of plays will be $\{(a, b), (b, b), (b, a), (a, a), (a, b), (b, b), \ldots\}$, with the first of each pair in the sequence being player i's move. Player i's sequence of payoffs will be $\{\beta, \gamma, \eta, \alpha, \beta, \gamma, \ldots\}$; the payoffs will also cycle. Define the function $\pi : S^B \times S^B \mapsto \Re$ as player i's average cycle payoff. For this example,

$$\pi(s_5, s_8) = \frac{1}{4}(\beta + \gamma + \eta + \alpha).$$

As an alternative example, consider the payoff if player i had chosen s_1 and player j had chosen s_2. In this case, the sequence of play will be $\{(a, b), (a, a), (a, a), \ldots\}$. After the first stage, the machines play (a, a) forever. The average cycle payoff to player i is,

$$\pi(s_1, s_2) = \alpha.$$

The application of the bounded rationality constraint and the particular definition of the payoff functions has transformed the infinitely repeated game G^∞ into a single period game with an 8×8 payoff matrix, Π. Π is shown in Table 2.2

2.3 The Replicator

The following notation is inspired by Taylor and Jonker (1978). Consider a population of N risk-neutral, payoff maximizing players who interact in randomly matched pairs. Let n_i be the number of players who choose strategy i. The population can then be represented as a point $\mathbf{p}$ in the eight-dimensional simplex Δ, with $p_i = n_i/N$ and $\sum_{i=1}^{8} p_i = 1$.

Assume that there is exponential growth or decay. Specifically, $\frac{dn_i}{dt} = r_i n_i$, where r_i is the current growth rate for n_i. Growth in the population follows $\frac{dN}{dt} = \bar{r}N$, where $\bar{r}$ is the average growth rate.

By differentiating $p_i = \frac{n_i}{N}$,

$$\begin{aligned} \frac{dp_i}{dt} &= \frac{\frac{dn_i}{dt}}{N} - \frac{n_i \frac{dN}{dt}}{N^2} \\ &= \frac{r_i n_i}{N} - \frac{\bar{r} n_i}{N} \\ &= p_i(r_i - \bar{r}). \end{aligned}$$

Now, assume that the growth rate of players with strategy i is equivalent to the expected payoff, or fitness, of player i. In other words, $F(i|\mathbf{p}) = \sum_{l=1}^{8} p_l \pi(s_i, s_l)$, which is the expected payoff of player i, is equivalent to r_i. Similarly, $F(\mathbf{p}|\mathbf{p}) = \sum_{i=1}^{8} p_i F(i|\mathbf{p})$, which is the expected payoff of a random member of the population, is equivalent to $\bar{r}$. Then by substitution,

$$\frac{dp_i}{dt} = p_i[F(i|\mathbf{p}) - F(\mathbf{p}|\mathbf{p})]. \tag{2.1}$$

Now, $\frac{dp_i}{dt}$ is the instantaneous change in the proportion of players using strategy i. Note that $\frac{dp_i}{dt} > 0$ if and only if $F(i|\mathbf{p}) > F(\mathbf{p}|\mathbf{p})$, and $\frac{dp_i}{dt} < 0$ if and only if $F(i|\mathbf{p}) < F(\mathbf{p}|\mathbf{p})$. Hence, the proportion of players using strategy i rises (or falls) with time only if the expected payoff of strategy i is greater than (or less than) the expected payoff of a random member of the population. If the expected payoff to strategy i is the same as the expected payoff to a random member of the population, then $\frac{dp_i}{dt} = 0$.

Equation 2.1 implies a dynamic in continuous time on the simplex Δ. Given an initial state or initial population in Δ, the dynamic describes a particular trajectory.

Assumption 1 *Every initial population is a point* $\mathbf{p}$ *located in the interior of the simplex* Δ.

The assumption means that every possible strategy has at least some representation in the population.

Definition 1 *An* **equilibrium** *is any population* $\mathbf{p}$ *such that* $\frac{dp_i}{dt} = 0$ *for all* i.

Definition 2 *Given an equilibrium* $\mathbf{p}$, $\mathbf{p}$ *is* **asymptotically stable** *if a trajectory that passes through* $\mathbf{p}'$ *converges to* $\mathbf{p}$ *with time, for all* $\mathbf{p}'$ *in an open neighborhood around* $\mathbf{p}$.

Definition 3 *Given an equilibrium* $\mathbf{p}$, $\mathbf{p}$ *is* **globally asymptotically stable** *if a trajectory that passes through* $\mathbf{p}'$ *converges to* $\mathbf{p}$ *with time, for all* $\mathbf{p}'$ *in the interior of* Δ.

Playing a random member of a population $\mathbf{p}$ is like playing against a mixed strategy $\mathbf{q}$ where $\mathbf{p} = \mathbf{q}$. A particular mixed strategy will be denoted $\mathbf{q}_i$ and will be treated in the obvious way by the function F.

Assumption 2 *The payoff matrix, M, is such that $\eta > \frac{\eta+\beta}{2} > \gamma > max\{\beta, \alpha\}$.*

The assumption means action a is strictly payoff dominated by action b and assures that in what follows $x \in (1/2, 1)$. Furthermore, the assumption defines the properties necessary for the game to be a symmetric Reciprocity Game.

Now, which strategies are of interest in a Reciprocity game? The strategies s_1 and s_2 always play a, which is a dominated strategy in the game G. The strategies s_3 and s_4 always play action b, which is the dominant strategy in the game G. There are three ways alternation can occur: if a player with strategy s_5 meets a player with strategy s_6, or if two players with strategies s_7 or s_8 meet. Only the first of these ways is consistent with the idea of reciprocation.

Define the point $\mathbf{q}^e = \mathbf{p}^e = [0, 0, 0, 0, x, (1-x), 0, 0]$, with x such that the following equality holds:

$$x\alpha + (1-x)\frac{\beta+\eta}{2} = x\frac{\eta+\beta}{2} + (1-x)\gamma.$$

If the population is at point $\mathbf{p}^e$, then the only strategies present are strategies s_5 and s_6.

Lemma 1 *The point $\mathbf{p}^e$ is an equilibrium.*

Proof: This is true since,

$$\begin{aligned} F(5|\mathbf{p}^e) &= x\alpha + (1-x)\frac{\beta+\eta}{2} \\ &= x\frac{\eta+\beta}{2} + (1-x)\gamma \\ &= F(6|\mathbf{p}^e) \\ &= F(\mathbf{p}^e|\mathbf{p}^e) \end{aligned}$$

implies $\frac{dp_i^e}{dt} = 0$ for all i.

□

So, an equilibrium with both s_5 and s_6 players present exists. In this equilibrium, every time an s_5 player meets and s_6 player, there will be Alternation. Of course, meetings between s_5 and s_6 players are not the only types of meetings that occur. When an s_6 player meets another s_6 player, the sequence of play is $\{\ldots, (b, b), \ldots\}$. At each stage, the players myopicly choose the Dominant Strategy Nash equilibrium. It is also an subgame-perfect Nash equilibrium in the game G^∞, although every other equilibrium has Pareto Superior payoffs. The last type of meeting which could occur is between two s_5 players. In this case, the sequence of play is $\{\ldots, (a, a), \ldots\}$ which is not subgame perfect Nash equilibrium play; it will be called Irrational.

Lemma 2 *The equilibrium* $\mathbf{p}^e$ *is locally asymptotically stable.*

Proof: By Lemma 1 $\mathbf{p}^e$ is an equilibrium. In equilibrium, the expected payoff to a random member is $x\frac{\eta+\beta}{2} + (1-x)\gamma$. Suppose point $\mathbf{p}'$ is an element of an open neighborhood around $\mathbf{p}^e$. If the trajectory through $\mathbf{p}'$ converges with time to $\mathbf{p}^e$ then $\mathbf{p}^e$ is locally asymptotically stable.

Two conditions must be meet for the trajectory to converge to $\mathbf{p}^e$. First, no strategy with positive weight at point $\mathbf{p}'$ can have a higher fitness when playing $\mathbf{q}^e$, the equilibrium mixed strategy, than $\mathbf{q}^e$ itself.

Second, if any strategy happens to do equally as well as $\mathbf{q}^e$, then it must be the case that that strategy is in the support of $\mathbf{q}^e$ and $\mathbf{q}^e$ must do better when playing $\mathbf{q}'$ than $\mathbf{q}'$ itself, where $\mathbf{q}'$ is the mixed strategy associated with the point $\mathbf{p}'$.

Formally,

- For all $p_i' > 0$, $F(s_i|\mathbf{p}^e) \leq F(\mathbf{q}^e|\mathbf{p}^e)$ and,
- if $F(s_i|\mathbf{p}^e) = F(\mathbf{q}^e|\mathbf{p}^e)$, then $q_i > 0$ and $F(s_i|\mathbf{p}') < F(\mathbf{q}^e|\mathbf{p}')$.

The fact that these two conditions are sufficient for asymptotic stability is due to Taylor and Jonker (1978).

Note that it is enough to consider only pure strategies with positive weight because any mixed strategy will have a payoff that is a linear combination of the payoffs to pure strategies. If all the pure strategies satisfy the previous two conditions, then any mixed strategy will as well.

The payoffs of all strategies that might have positive weight in a disturbed state against the equilibrium strategy are:

- $F(1, \mathbf{p}^e) = F(2, \mathbf{p}^e) = x\alpha + (1 - x)\alpha = \alpha$
- $F(3, \mathbf{p}^e) = F(4, \mathbf{p}^e) = x\gamma + (1 - x)\gamma = \gamma$

- $F(5, \mathbf{p}^e) = F(6, \mathbf{p}^e) = x\frac{\eta+\beta}{2} + (1-x)\gamma$
- $F(7, \mathbf{p}^e) = F(8, \mathbf{p}^e) = x\frac{\alpha+\beta+\eta+\gamma}{4} + (1-x)\frac{\alpha+\beta+\eta+\gamma}{4} = \frac{\alpha+\beta+\eta+\gamma}{4}$

None of these strategies does better against $\mathbf{q}^e$ than $\mathbf{q}^e$ itself. Both s_5 and s_6 do equally as well, however, so how these strategies do against themselves and how $\mathbf{q}^e$ does against them must also be considered.

- $F(\mathbf{q}^e, 5) = F(\mathbf{q}^e, 6) = x\frac{\eta+\beta}{2} + (1-x)\gamma$
- $F(5, 5) = \alpha$
- $F(6, 6) = \gamma$

Because the payoff to $\mathbf{q}^e$ is higher than the payoff to strategy s_5 when both play against s_5 and because its payoff is higher than the payoff to s_6 when both play s_6, and because no other strategy does as well against it as itself, the equilibrium is locally stable.

□

Hence the equilibrium $\mathbf{p}^e$ is resistant to small shocks or invasion by small numbers of players with a different strategy. But what if there is a large shock or if there are large numbers of players with other strategies in the initial population? To determine what happens in these cases, the following two lemmas will be used.[5]

[5]The two Lemmas show that it is possible to iteratively eliminate strictly dominated strategies in this case. A general theorem encompassing this result can be found in Samuelson and Zhang (1992).

Lemma 3 *Given s_i and s_j such that $F(i,\mathbf{p}) < F(j,\mathbf{p})$ for all $\mathbf{p} \in \Delta$, and any $\mathbf{p}^0$ in the interior of Δ, $\lim_{t\to\infty} p_i = 0$.*

Proof: First, note that because p_i and p_j are in Δ, $\frac{p_i}{p_j}$ is bounded below by 0. Because of this, it is enough to show that $\lim_{t\to\infty} \frac{p_i}{p_j} = 0$. Time is continuous and runs from 0 to ∞. Consider an infinite sequence of points in time $\{t_1, t_2, \ldots, t_n, \ldots\}$, such that $t_n > t_{n-1}$. Given any initial population, it is possible to determine the values of p_i and p_j at any point t_n. Define a second sequence by $T_n = \frac{p_i(t_n)}{p_j(t_n)}$.

Now, to show that the sequence of T_ns is monotonicly decreasing, it is sufficient to show that

$$\frac{\mathrm{d}p_i/p_j}{\mathrm{d}t} = (p_j \frac{dp_i}{dt} - p_i \frac{dp_j}{dt})/p_j^2 < 0$$

which implies that $\frac{\frac{dp_i}{dt}}{p_i} < \frac{\frac{dp_j}{dt}}{p_j}$.

Because $F(i|\mathbf{p}) < F(j|\mathbf{p})$,

$$\begin{aligned} \frac{\frac{dp_i}{dt}}{p_i} &= [F(i|\mathbf{p}) - F(\mathbf{p}|\mathbf{p})] \\ &\text{and} \\ \frac{\frac{dp_j}{dt}}{p_j} &= [F(j|\mathbf{p}) - F(\mathbf{p}|\mathbf{p})] \\ &\Downarrow \\ \frac{\frac{dp_i}{dt}}{p_i} &< \frac{\frac{dp_j}{dt}}{p_j}. \end{aligned}$$

Because the sequence T_n is monotonicly decreasing and bounded below by 0, it must converge, and because it converges, $\lim_{t\to\infty} \frac{p_i}{p_j}$ must also converge.

Suppose the limit converges to a point x greater than zero. Then, at point x, $\frac{\frac{dp_i}{dt}}{p_i} = \frac{\frac{dp_j}{dt}}{p_j}$. This implies that $F(i|\mathbf{p}) = F(j|\mathbf{p})$ which is a contradiction. So, the limit point must be 0.

□

Lemma 4 *Consider population* $\mathbf{p}$ *with* $p_k < \varepsilon$, *strategies* s_l *and* $\mathbf{q}$ *such that* $F(l|i) \leq F(\mathbf{q}|i)$ *for all* $i \neq k$ *and* $F(l|i) < F(\mathbf{q}|i)$ *for at least one* $i \neq k$. *If* ε *is small enough so that* $F(l|\mathbf{p}) < F(\mathbf{q}|\mathbf{p})$ *and if* p_k *is never bigger than* ε, *then for* $\mathbf{p}^0$ *in the interior of* Δ, $\lim_{t\to\infty} p_l = 0$.

Proof: First, note that because p_l and $\mathbf{q}$ are in Δ, $\frac{p_l}{\sum_{i|q_i>0} p_i}$ is bounded below by 0. Because of this, it is enough to show that $\lim_{t\to\infty} \frac{p_l}{\sum_{i|q_i>0} p_i} = 0$. Time is continuous and runs from 0 to ∞. Consider an infinite sequence of points in time $\{t_1, t_2, \ldots, t_n, \ldots\}$, such that $t_n > t_{n-1}$. Given any initial population, it is possible to determine the values of p_l and $\sum_{i|q_i>0} p_i$ at any point t_n. Define a second sequence by $T_n = \frac{p_l(t_n)}{\sum_{i|q_i>0} p_i(t_n)}$.

Now, to show that the sequence of T_ns is monotonicly decreasing, it is sufficient to show that

$$\frac{\mathrm{d}p_l/\sum_{i|q_i>0} p_i}{\mathrm{d}t} = (\sum_{i|q_i>0} p_i \frac{dp_l}{dt} - p_i \sum_{i|q_i>0} \frac{dp_i}{dt})/(\sum_{i|q_i>0} p_i)^2 < 0$$

which implies that $\frac{\frac{dp_l}{dt}}{p_l} < \frac{\sum_{i|q_i>0} \frac{dp_i}{dt}}{\sum_{i|q_i>0} p_i}$.

Because $F(l|\mathbf{p}) < F(\mathbf{q}|\mathbf{p})$,

$$\frac{\frac{dp_l}{dt}}{p_l} = [F(l|\mathbf{p}) - F(\mathbf{p}|\mathbf{p})]$$

and

$$\begin{aligned}
\frac{\sum_{i|q_i>0} \frac{dp_i}{dt}}{\sum_{i|q_i>0} p_i} &= [F(i|\mathbf{p}) - F(\mathbf{p}|\mathbf{p})] \\
&\Downarrow \\
\frac{\frac{dp_l}{dt}}{p_l} &< \frac{\sum_{i|q_i>0} \frac{dp_i}{dt}}{\sum_{i|q_i>0} p_i}.
\end{aligned}$$

Because the sequence T_n is monotonicly decreasing and bounded below by 0, it must converge, and because it converges, $\lim_{t\to\infty} \frac{p_l}{\sum_{i|q_i>0} p_i}$ must also converge.

Suppose the limit converges to a point x greater than zero. Then, at point x, $\frac{\frac{dp_l}{dt}}{p_l} = \frac{\sum_{i|q_i>0} \frac{dp_i}{dt}}{\sum_{i|q_i>0} p_i}$. This implies that $F(l|\mathbf{p}) = F(\mathbf{q}|\mathbf{p})$ which is a contradiction. So, the limit point must be 0.

□

And now, the main result:

Theorem 1 *The equilibrium* $\mathbf{p}^e$ *is globally asymptotically stable.*

Proof: The strategy s_4 strictly dominates the strategies s_1 and s_2, $F(s_4|\mathbf{p}) > F(s_1|\mathbf{p})$ and $F(s_4|\mathbf{p}) > F(s_2|\mathbf{p})$ for all $\mathbf{p}$ in Δ. By Lemma 3, p_1 and p_2 go monotonicly to zero as t goes to infinity. In particular, for any small positive number ε, at some point in time, p_1 and p_2 will both be less then ε.

Now, either s_4 or $\mathbf{q}^e$ strictly dominates s_7 with regard to all strategies except s_1 and s_2. If

$$\frac{\alpha + \beta + \eta + \gamma}{4} - \beta > 0$$

then $\mathbf{q}^e$ is strictly dominant. If the inequality does not hold, then clearly,

$$\gamma - \frac{\alpha + \beta + \eta + \gamma}{4} > 0$$

implying that s_4 is strictly dominant.

Because the inequalities above are strict, it is possible to pick an ε small enough so that if the weights of the strategies s_1 and s_2 are less than ε, either $F(s_4|\mathbf{p}) > F(s_7|\mathbf{p})$ or $F(\mathbf{q}^e|\mathbf{p}) > F(s_4|\mathbf{p})$. Lemma 4 then implies that p_7 goes to zero as t goes to infinity.

Similarly, Lemma 4 can be used to show that p_8 and then p_3 and p_4 go to zero as t goes to infinity; each time $\mathbf{q}^e$ is the mixed strategy needed in the Lemma.

We are now left with only two pure strategies that can have weight greater than ε, s_5 and s_6. Suppose p_5 is very small, then $F(s_5|\mathbf{p}) = (\beta + \eta)/2 + \delta_1$ and $F(s_6|\mathbf{p}) = \gamma + \delta_2$, for some δ_1 and δ_2 small. If p_5 is small, then p_5 will grow with time. What if p_5 is large? Then $F(s_5|\mathbf{p}) = \alpha + \delta_1$ and $F(s_6|\mathbf{p}) = (\beta + \eta)/2 + \delta_2$, for some δ_1 and δ_2 small. If p_5 is large, then p_5 will decay with time.

In any case, the trajectory through any initial point must eventually come within any neighborhood of $\mathbf{p}^e$, and by Lemma 2 converge to $\mathbf{p}^e$.

□

No matter what the initial population is (as long as it is in the interior of Δ), the Replicator Dynamic will converge to an equilibrium with only s_5 and s_6 players. In this equilibrium there will be three types of sequences of play: Alternation, Dominant Strategy Nash, and Irrational.

2.3.1 Other Symmetric Games

What types of problems occur if Assumption 2 is not met? Well, suppose $\alpha > \gamma$, which covers the case of the Prisoner's Dilemma. Then there is no strategy that strictly dominates another. While $\mathbf{p}^e$ is still an equilibrium, it is not a globally asymptotically stable equilibrium. The same result occurs if $\beta > \gamma$, or if $\beta > \eta$; these cases cover the game of Chicken. A numerical example encompassing both of these alternatives will be given later.

Recall Aumann and Sorin's application of a one period recall to the infinitely repeated Prisoner's Dilemma. Aumann and Sorin justify their result by the elimination of strategies based on weak dominance alone. Unfortunately, under the Replicator Dynamic weak dominance alone is not enough to assure that a particular strategy's representation in the population goes to zero. While Nachbar (1988) does prove a theorem which gives positive convergence results in a subset of weakly dominant solvable games, his result cannot be applied in Aumann and Sorin's example. The difficulties encountered in weakly dominant solvable games are covered well in Nachbar (1988) and interested readers are referred there.

2.3.2 Asymmetric Games

There are inherent similarities between the Battle of the Sexes and Reciprocity Games which might lead you to believe that a similar result could be obtained in the Battle of the Sexes. There is a problem, however – the

Battle of the Sexes is an asymmetric game. There are two approaches in the modeling of asymmetric games as population games: analyze two distinct populations, and make the game symmetric through random population assignment.

Analyzing two distinct populations changes the dynamics dramatically. Consider what would happen in the case of the Reciprocity Game. Call the two populations the row population, **rp**, and the column population, **cp**. Let them evolve in the obvious way. Then the populations such that $\mathbf{rp} = \mathbf{cp} = \mathbf{p}^e$ would still be an equilibrium, but instead of being an global attractor, it would be a repeller. Any small deviation leading to a higher number of s_5 row players, for example, will cause the dynamics to flow towards populations consisting entirely of s_5 row players and s_6 column players. There are many other equilibria possible, each depending upon the initial populations. A global result is impossible.

Suppose that the game is made symmetric. The obvious way of accomplishing the task is to randomly choose one of each pair of players to be the row player and to let the other be the column player. A player's payoff would be their average payoff gotten as a row player plus their average payoff gotten as a column player divided by two. Alternatively, each player would face a payoff matrix consisting of cells which were the average payoff across both types given those actions. Specifically, suppose

$$M_c = \begin{bmatrix} \alpha_c & \beta_c \\ \eta_c & \gamma_c \end{bmatrix}$$

and

$$M_r = \begin{bmatrix} \alpha_r & \beta_r \\ \eta_r & \gamma_r \end{bmatrix},$$

where M_c was the payoff matrix faced by column players and M_r was the payoff faced by row players. Then the payoff matrix faced by a player in the version of this game played with random population assignment would be:

$$M = \begin{bmatrix} \frac{\alpha_c+\alpha_r}{2} & \frac{\beta_c+\beta_r}{2} \\ \frac{\eta_c+\eta_r}{2} & \frac{\gamma_c+\gamma_r}{2} \end{bmatrix}.$$

This method is an improvement over the two population method because it does not change the outcome predicted in the Reciprocity Game. In fact, any asymmetric game that meets Assumption 2 after having been made symmetric will meet all the assumptions required by Theorem 1.

Unfortunately, even with random population assignment, the Battle of the Sexes does not meet Assumption 2.

2.4 Examples

Consider the payoff matrix M_1,

$$M_1 = \begin{bmatrix} 3 & 3 \\ 7 & 4 \end{bmatrix}.$$

Then Theorem 1 holds with $x = 1/3$. Figure 2.1 shows a phase portrait for the initial generation that has all strategies with equal representation in the population.

In equilibrium, there are only three possible outcomes to a meeting between two players, call them: Alternation, Dominant Strategy Nash Play, and Irrational Play. Alternation occurs whenever a player with strategy s_5 meets a player with strategy s_6. The sequence of play in this case would be $\{(a,b),(b,a),(a,b),\ldots\}$. Alternation occurs with probability 4/9. Dominant Strategy Nash Play occurs whenever a player with strategy s_6 meets another player with strategy s_6. The sequence of play in this case would be $\{(b,b),(b,b),(b,b),\ldots\}$. Dominant Strategy Nash play occurs with probability 4/9. Irrational Play occurs whenever a player with strategy s_5 meets another player with strategy s_5. The sequence of play in this case would be $\{(a,a),(a,a),(a,a),\ldots\}$. The probability of this outcome is 1/9.

As an example of what happens if the payoff matrix is not constructed with the correct inequalities, consider the payoff matrix M_2,

$$M_2 = \begin{bmatrix} 4 & 3 \\ 7 & 2 \end{bmatrix}.$$

In this case, Theorem 1 does not hold. Figure 2.2 shows a phase portrait for the initial generation that has all strategies with equal representation in the population. Figure 2.3 shows a phase portrait for a different initial generation. Notice that the equilibria are different for these two initial generations.

2.5 Conclusion

It has been shown that a large class of two-player, bi-matrix games, both symmetric and asymmetric, have a unique equilibrium when they are modeled as population games containing players with bounded recall. The class is the set of all games which meet Assumption 2. In the unique equilibrium, both trading favors and short term maximization occur. A third irrational outcome also occurs. Normative justification for all three of these behaviors can be obtained from the Darwinistic maxim claiming that only the fittest should survive.

2.6 References

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2.7 Tables

s_1 :	$\{a, a, a\}$	s_2 :	$\{b, a, a\}$
s_3 :	$\{a, b, b\}$	s_4 :	$\{b, b, b\}$
s_5 :	$\{a, a, b\}$	s_6 :	$\{b, a, b\}$
s_7 :	$\{a, b, a\}$	s_8 :	$\{b, b, a\}$

Table 2.1: The eight machines or strategies contained in S^B.

$$\Pi = \begin{bmatrix} \alpha & \alpha & \beta & \beta & \alpha & \alpha & \beta & \beta \\ \alpha & \alpha & \beta & \beta & \alpha & \alpha & \beta & \beta \\ \eta & \eta & \gamma & \gamma & \gamma & \gamma & \eta & \eta \\ \eta & \eta & \gamma & \gamma & \gamma & \gamma & \eta & \eta \\ \alpha & \alpha & \gamma & \gamma & \alpha & \frac{\beta+\eta}{2} & \frac{\alpha+\beta+\eta+\gamma}{4} & \frac{\alpha+\beta+\eta+\gamma}{4} \\ \alpha & \alpha & \gamma & \gamma & \frac{\eta+\beta}{2} & \gamma & \frac{\alpha+\beta+\eta+\gamma}{4} & \frac{\alpha+\beta+\eta+\gamma}{4} \\ \eta & \eta & \beta & \beta & \frac{\alpha+\beta+\eta+\gamma}{4} & \frac{\alpha+\beta+\eta+\gamma}{4} & \frac{\alpha+\gamma}{2} & \beta \\ \eta & \eta & \beta & \beta & \frac{\alpha+\beta+\eta+\gamma}{4} & \frac{\alpha+\beta+\eta+\gamma}{4} & \eta & \frac{\gamma+\alpha}{2} \end{bmatrix}$$

Table 2.2: The environment Π.

2.8 Figures

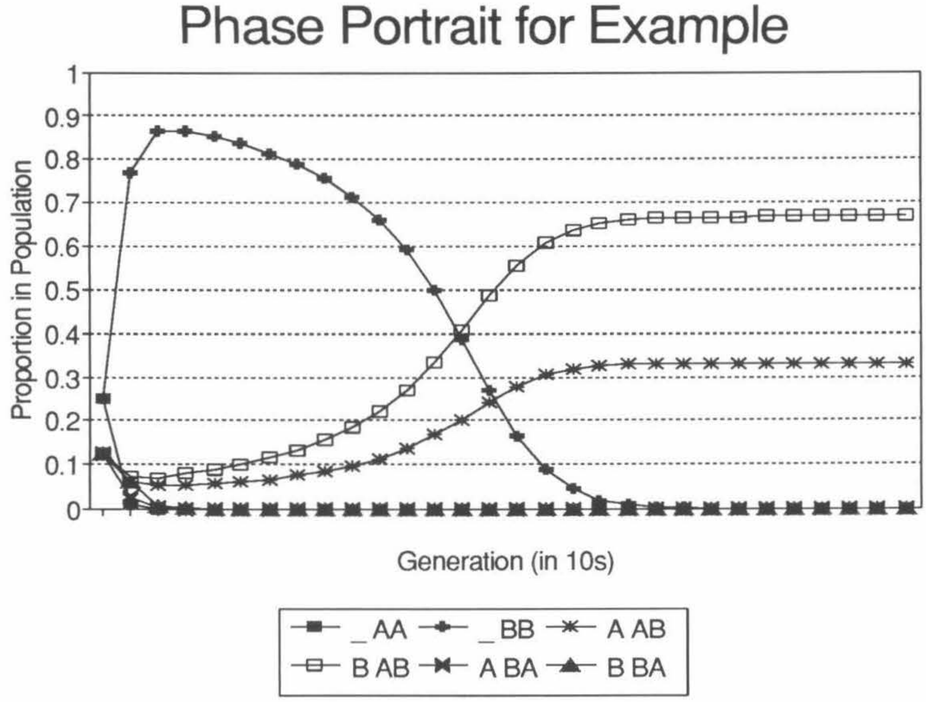

Figure 2.1: The phase portrait for payoff matrix M_1. The term _ AA stands for the sum of the representation of s_1 and s_2, _ BB is similar. The initial generation has all strategies equally represented in the population.

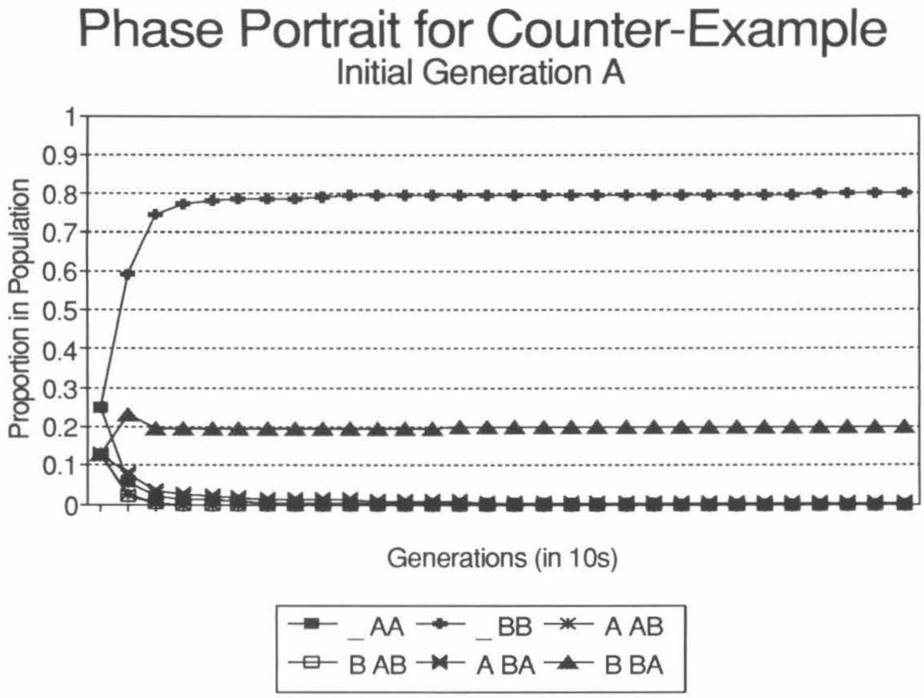

Figure 2.2: The phase portrait for payoff matrix M_2. The term _ AA stands for the sum of the representation of s_1 and s_2, _ BB is similar. The initial generation has all strategies equally represented in the population.

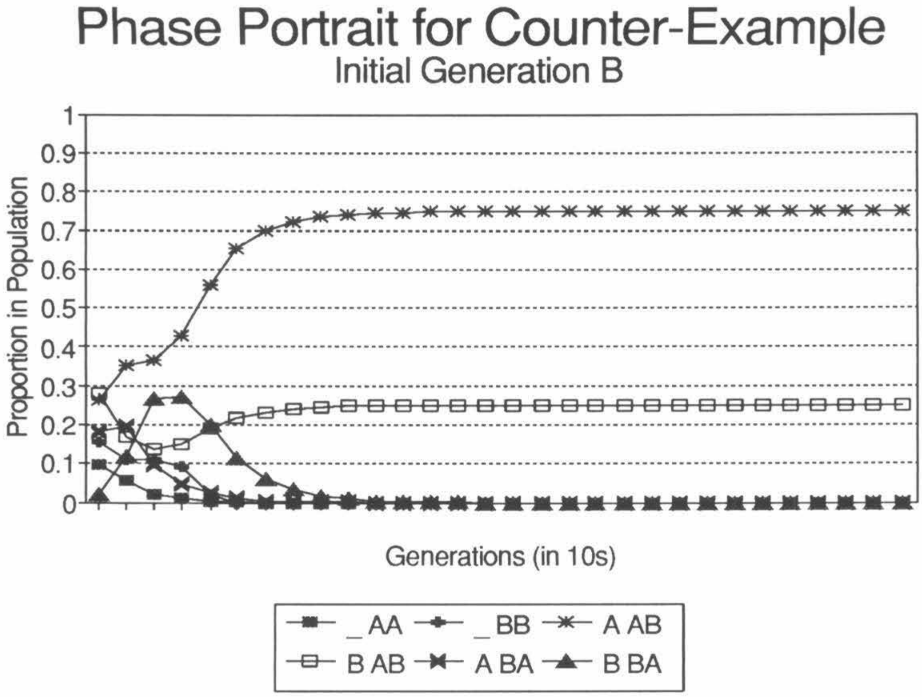

Figure 2.3: The phase portrait for payoff matrix M_2. The term _ AA stands for the sum of the representation of s_1 and s_2, _ BB is similar. The initial generation is $\mathbf{p}^0 = [0.048, 0.048, 0.078, 0.078, \ 0.264, 0.282, 0.182, 0.020]$.

Chapter 3

Anomalous Behavior in Linear Public Goods Experiments: How Much and Why?

3.1 Introduction

[1] There is a growing body of data obtained from experiments on voluntary contributions in linear public goods environments with a single public good and a single private good. Many features of the data have been difficult to explain; for example, subjects violate dominant strategies on a regular basis. They give away money, apparently just to be nice (Isaac and Walker [1984, and elsewhere]); at least as often, they seem to give away money just to be mean (Saijo and Yamaguchi [1992]). Furthermore, individual behavior

[1] This chapter contains work that is joint with Thomas Palfrey.

over time exhibits erratic patterns, it alternates back and forth between extreme generosity and extreme selfishness. Ledyard's (1992) excellent survey documents these and several other anomalies.

These anomalies might be cause for alarm as they signal trouble for any but the most schizophrenic models of behavior. However, the range of enviornments for which these experimental results have been reported is very narrow, and the designs employed make it difficult if not impossible to identify decision rules at an individual level. The point of this paper is to broaden the playing field in a natural direction, using a design that permits estimation of individual behavior. By changing both the information structure and the distribution of preferences, this design also provides a robustness check on the anomalous findings of past experiments.

We offer the following thought experiment in the context of a well-studied private goods allocation mechanism, the second-price auction, in hopes that it will help the reader understand some of our concerns about design, and to foreshadow what follows.

A Thought Experiment:

> Imagine conducting a second-price sealed bid auction experiment with four players, where each is told to bid for an object that is worth exactly \$1.58 to him. After careful explanation of the rules, ten identical, sealed bid, second-price auctions are then conducted in sequence. Bids are required to be greater than or equal to 0 and less than \$1.58 and ties are broken randomly. After each auction, subjects are told the winning bid and the second

> highest bid. When the tenth auction is over, everyone is paid by the experimenter and thanked for showing up.
>
> What do you think the distribution of bids will be, and how will this distribution change from period to period? How would you plan to bid in such an auction?

The first observation to be made about the thought experiment is that it shares some of the traits of many voluntary contribution, public goods experiments that have been reported in the literature. In the most common voluntary contribution, public goods experiment, like in the thought experiment, there are a number of identical players. Also, the players are asked to make a decision about buying a good and they are given personal incentives not to buy it, or at least to spend as little as possible on it. Much of what is known about free riding is based on experiments with this type of design.

The second observation to be made is that little can be learned about the general bidding behavior of the participants. In the auction, each player attaches the same value to the good in each of the ten auctions. Furthermore, every other player also attaches this same value to the good. The measurement of a general bidding function is practically impossible; the best one can do is estimate behavior at a particular point.

It would be possible, by running a number of experiments and varying the value of the good, to construct something that looked like a bidding function. However, that function would depend upon the fact that every player attaches the same value to the auctioned good. This function would only measure how an individual's choice behavior changes when their own value *and the joint distribution of all bidders' values* change simultaneously. The estimated function would have other limitations as well—to obtain the data required, an individual would have to participate in a large number of 10-auction sequences. The amount of play necessary might lead to a confounding of the effects of bidding behavior and of experience, unless a large number of experiments were conducted.

A final observation is that, in spite of the fact that there is a dominant strategy equilibrium where each bids \$1.57, one can, for a variety of reasons, imagine players bidding differently. In fact, it is difficult to guess what might actually happen, especially if the players are inexperienced.[2]

It should be no surprise to learn that auction experiments are not usually conducted like the thought experiment. Auction experiments have focused exclusively on different environments, environments in which players have diverse preferences and diverse information. These are the environments in which auctions most naturally occur. What is surprising is that voluntary contribution experiments have, for the most part, not shared this focus.[3]

This paper, and the experimental design it employs, is motivated by our

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[2]One might also notice that the thought experiment is a repeated game not a one-shot game. We do not address this potential complication until later in the paper.

[3]There are a few exceptions, notably Fisher et al. 1991 and Isaac et al. 1985, both of which consider environments with two types. The former provides subjects with identical information about other subjects' preferences as in parallel homogeneous preference experiments. The latter has several other different features, including nonlinearities, and does not conduct any baseline experiments with homogeneous preference. Brookshire et al. (1991), Smith (1980), and Marwell and Ames (1980) also have conducted experiments with heterogeneous preferences, but these are not comparable for other reasons. None of these experiments varied individual subject preferences across decisions, nor did they provide explicit information about the distribution of preferences in the population. Palfrey and Rosenthal (1991) use an environment similar to the one explained here, but the public good technology is step-level, not linear.

reflections about the thought experiment, and by a view that much can be gained by shifting the research agenda in the direction of this different class of environments. One benefit is simply better measurement: response (*bidding*) functions can be estimated at the individual level. Also, we can check for the robustness of existing results to environments that include features, such as heterogeneity of preferences, that are endemic to natural settings. In what follows, we report results from our experiments that study this kind of environment, and we contrast these results with previous findings.

3.2 Background

This paper investigates contribution behavior under the Voluntary Contribution Mechanism in simple linear public good environments where all players have dominant strategies. The typical environment consists of N individuals, each endowed with X_i discrete units of a private good. The marginal rate of transformation between the public good, y, and the private good is one-for-one, and individual utility functions are of the form: $U(y, x_i) = Vy + r_i x_i$. We refer to V as the *value of the public good*, and it is normalized to be the same for all individuals.

The Voluntary Contribution Mechanism defines a simple game, in which each individual simultaneously decides how much public good (between 0 and X_i) to produce on his own. Total public good production in the economy is the sum of all private production of the public good. Payoff functions are then defined from the final allocation and the utility functions in the obvious way. This game is repeated several times.

As pointed out in Section 1, much of what we think we know about behavior in this game is based on experiments in which X_i and r_i are the same across individuals and repetitions and $r_i/V > 1$. This paper concentrates on a group size of four.

Several findings have emerged from these other investigations: (1) nearly all players in this game violate their one-shot dominant strategy, with many contributing upwards of half their endowment, even when r_i/V is three or more; (2) there is a strong negative relationship between the marginal rate of substitution r_i/V and the rate at which violations are observed; (3) roughly half the aggregate private endowment is contributed by inexperienced subjects on the first play of the game; (4) violations of dominant strategies diminish with repetition and with experience (playing a second sequence of games with a new group); (5) violations of dominant strategies to contribute ($r_i/V_i < 1$, Saijo and Yamaguchi [1992]) appear to be even more prevalent than violations of dominant strategies to free ride.

3.3 Our Design and Procedures

Our experiment looks at the above findings more closely by studying environments with both non-degenerate distributions of r_i/V, and with private information. These innovations are introduced to overcome the limitations of past designs, limitations suggested by the thought experiment. The innovations permit us to measure responsiveness to r_i/V, via response or bidding functions, at both the individual level and the aggregate level, and to measure a baseline of deviant or erroneous behavior due to nuisance factors, such as boredom or confusion.

There are a number of specific features of our design that enable us to address other issues that are relevant to understanding other commonly observed patterns of behavior. These features are listed below. A sample copy of the instructions is in the Appendix.

1. In all our environments, subjects receive r_i's that are randomly assigned according to a uniform distribution between 1 and 20. We sometimes refer to these as *token values.* Each time a subject is to make a new decision, he is independently and randomly assigned a new r_i for that decision. Subjects do not know the other subjects' assignments of r_j's, but the distribution is publicly announced at the beginning. The value of V is also announced at the beginning.

   Therefore, the data contain multiple observations of the choice behavior of each individual, observations at different levels of r_i/V, and permits the estimation of response functions at both the individual and aggregate levels.

2. We vary the distribution of marginal rates of substitution, (r_i/V), by shifting V. We look at the four different distributions given by $V \in \{3, 6, 10, 15\}$. One of the distributions, $V = 3$, has the feature that group efficiency is *not* maximized when all subjects contribute in every round. In that condition, on average, forty percent of the time subjects are assigned a token value that is worth more than four times the individual marginal value of the public good. In these cases, contribution *reduces* group efficiency.

3. We vary the endowment. In one condition, everyone is endowed with one indivisible unit of the private good. In the other condition everyone is endowed with nine discrete units.

4. Each subject makes a sequence of ten decisions in a fixed group with three other players. This allows a direct comparison to some past experiments, notably those reported in the Isaac and Walker studies.

5. Each subject participates in a total of four sequences, each time with a different group of subjects. The first two sequences have the same parameters; the last two sequences have the same parameters (but different from the first two). This allows us to identify experience effects. All four sequences occur in a single session that lasts approximately $1\frac{1}{2}$ hours. Each session includes sixteen subjects.

6. All sessions were conducted at the Caltech Laboratory for Experimental Economics and Political Science, using a collection of PC's that are linked together in a network.

7. Each subject was paid cash, based on a session-specific exchange rate, for each point they earned in the session. The exchange rate was picked so that the sum of equilibrium payoffs was approximately the same across sessions.

[Table 1 here]

3.4 Response Functions and Background Noise

We focus mainly on two aspects of the data. The first has to do with attempting to identify what we call *errors* or background noise—behavior that is grossly inconsistent with standard theory. Second, we attempt to measure response functions, which are the analog to bidding functions in auctions. The functions answer the question: How do contribution decisions depend on the marginal rate of substitution? We measure errors and response functions at both the aggregate and individual levels, using nonparametric and parametric models of the error structure.

It is useful to think of our analysis in the context of a random utility model, of the sort found in Maddala (1983), McFadden (1982), and elsewhere, for the analysis of data with limited dependent variables. For example, in the condition where subjects have a single indivisible unit of the private good, they face a simple binary decision. We model the statistical structure of residuals by assuming that utility functions have a random component that is not observed. For lack of a better name, we call this the *altruism* (or *warm glow*) term. Depending upon the value of the altruism term, subjects may receive some additional utility from contributing a unit of their endowment, over and above the utility induced by the payment method used in the experiment.

Theoretically, an optimal response function for an individual with an additive warm glow term, ε_i, is to contribute X_i if $r_i/V < 1 + \varepsilon_i$, and to contribute 0 if $r_i/V > 1 + \varepsilon_i$. Any behavior is optimal when $r_i/V = 1 + \varepsilon_i$. This is what we call a *cutpoint strategy* (Palfrey and Rosenthal [1988]). In fact, this optimal strategy is a one-shot dominant strategy for any values of ε_i, r_i, V, and X_i.

If the value of ε_i is stochastic, and varies according to some assumed distribution, an estimated response function gives the probability of contribution as a function of other controlled variables, such as experience, etc. In addition, the response function gives us indirectly an estimate of "background noise." We look at the effect of the following variables on response functions:

- The induced marginal rate of substitution (r_i/V).
- Experience.
- Endowment (divisible or indivisible – i.e. one or nine units).
- The value of the public good (V).
- Repetition (Is there a decay over the ten rounds of play?).

3.5 Analysis of the data

3.5.1 Some baselines

We present three different baseline error rates. This gives a rough calibration of a lower bound on the amount of *background noise*[4] in the experiment. By

[4]Contemporaneous work by Andreoni (1992) is also pursuing this issue.

this, we mean the percent of observed decisions that appear incongruous with nearly any currently accepted theory of rational decisionmaking. We also make an attempt to compare our baseline with baselines observed elsewhere, to the extent possible.

Splitting

By splitting, we mean that a subject contributes some fraction of his endowment, but not all of it. Because of the linear structure of the environment, such behavior is not rational even if a subject has a warm glow term added to his marginal rate of substitution. While it might be possible to think up models where such behavior is rational, such explanations would likely be quite contrived. Tables 2, 3, and 4 present the splitting data from our experiments. Recall that in half of our experiments, subjects were not capable of splitting, since they had only a binary choice. Thus, the data in this table is based on only half the sample. One can see two striking features. First, splitting is more prominent among inexperienced subjects and in the early periods of each 10-period game. Second, splitting almost never occurs when subjects have $r_i/V < 1$. In other words, almost all splitting can be accounted for by subjects who have a dominant strategy to free ride.

[Table 2, Table 3, and Table 4 here]

These findings contrast sharply with those of Issac and Walker. They observe splitting well over half of the time in their data and, for their marginal rate of substitution, or MRS, of 1.33 experiments, there is very little decay of splitting over the course of the ten periods.

[Table 5 here]

Spiteful behavior

Many have speculated that subjects violate their dominant strategy to free ride because of some form of altruism, or alternatively, because their utility function depends on group payoffs in a positive way. If this is the main driving force behind the past findings, then we should see very little free riding when subjects have $r_i/V < 1$. Based on this scenario, violations of dominant strategies to contribute can reasonably be attributed to effectively random behavior. This gives us a second kind of baseline error rate. In our experiments, four percent of the decisions violate the dominant strategy to contribute when $r_i/V < 1$. This number is remarkably stable across periods and across the experience treatment (see Table 6).

[Table 6 here]

Sacrificial behavior

In one of our designs, $V = 3$, the group optimum is not obtained by everyone contributing for every possible r_i they might draw. In particular, the group payoff is maximized if subjects contribute if and only if $r_i \leq 4V = 12$. A subject who contributes when $r_i > 12$ sacrifices more than the entire group benefits. It is hard to imagine any except the most fervent altruists contributing under these circumstances. The frequency of this type of contribution also provides, in a slightly different way, a lower bound on the amount of "crazy" or random behavior. As Table 7 shows, this kind of behavior is approximately as common as spiteful behavior, but virtually disappears with experience (1 observation out of 129).

[Table 7 here]

3.5.2 Estimation of response functions from aggregate data

A Simple Model

We measure response functions as the probability of contribution as a function of the marginal rate of substitution or MRS. First, consider the following family of theories, a family that includes both the dominant strategy (game) theory and the altruism theories based on an additive warm glow altruism term. Each member of this family is characterized by an error rate, ε, and a threshold, M. An (ε, M) theory states that "Individuals contribute to a public good if and only if the marginal rate of substitution (token value divided by public good value plus warm glow) is less than or equal to M. However, they make errors at a rate of ε."

If $M = 1$, then this is just the dominant strategy theory, modified appropriately to account for the possibility of error. If $M > 1$ this indicates some degree of altruism, everyone is altruistic. If $M < 1$, this indicates negative altruism. According to our data, what is the best theory in this family? Using the criterion of maximum likelihood, the answer is the M^* that produces the fewest classification errors in the data, together with ε^* equal to whatever the classification error generated by M^* is. This is not only easy to calculate, it is also easy to illustrate graphically. Figure 1 displays the answer: In our data, the best theory is $M = 1.1$. It results in only 12.5 percent ($\varepsilon*$) classification errors and is very close to the selfish cutpoint equal to 1.0. Figures 2 and 3 break this analysis down across the various levels of the V-treatment and the two levels of the endowment treatment.

Probit Analysis

An alternative, more familiar way to estimate response functions is by Probit analysis. In effect, the Probit analysis fits curves through the raw data shown in Figures 4-7. In this analysis, we assume that an altruism term, ε_{it}, is a Normally distributed random term added to an individual's MRS that it is independently distributed across individuals and across decisions.

The impact of experience, endowment and other experimental treatments are easily assessed by introducing dummy variables. The simplest probit model, with only a constant term and r_i/V, or MRS, entering on the right hand side gives us an estimate of the average altruism term, which we denote by $\bar{\varepsilon}$, and its standard deviation σ_ε.

We consider five Probit Models which are built by recursively adding independent variables to the basic model. Note that an observation in these models is a decision involving a single token. In order to maintain equal representation between the conditions with an endowment of one and those with an endowment of nine, an investment decision in the endowment of one conditions is given the same weight as nine similar investment decisions in the endowment of nine conditions.

The intercept coefficients in a Probit model represent changes in $\bar{\varepsilon}/\sigma_\varepsilon$ and the slope coefficients represent changes in $-1/\sigma_\varepsilon$. The estimated mean, $\bar{\varepsilon}$, is equal to minus the slope coefficient divided by the intercept coefficient. It follows that a negative change in the already negative slope coefficient leads to a decrease in $\bar{\varepsilon}$, holding everything else constant. This decrease is implied by the decrease in variance due to the more negative slope coefficient. If everything is to stay the same, $\bar{\varepsilon}$ must also decrease. The decrease in variance also makes the slope of the curve steeper.

From each Probit Model, we can obtain a response function $\mathcal{P}(\cdot)$, which returns the probability that a subject invests in the public good. The six variables in the other models are: *exper.s*, a slope dummy for subjects with experience; *exper*, a constant dummy for subjects with experience; *endow.s*, a slope dummy for treatments with an endowment of nine; *endow*, a constant dummy for subjects with an endowment of nine; V, the marginal return from the public good; and *period* which ranges from 1 to 10. Coefficients, t-statistics, log likelihoods, and the percentages correctly predicted for each model are given in Table 8.

[Table 8 here]

Turning to specific models, even the simple model $\mathcal{P}_1$, in which a player's investment decision depends only upon MRS, is able to correctly predict 83.064 percent of the observations.

In model $\mathcal{P}_2$, the slope coefficient for the experience variable, *exper.s*, is negative which means the response curve for experienced subjects is steeper than the response curve for inexperienced subjects. The coefficient for the intercept variable for experience, *exper*, is positive. This tends to offset the change in $\bar{\varepsilon}$ implied by the reduced variance, however, the total change in $\bar{\varepsilon}$ is still negative.

A player's cutpoint is the point at which he is indifferent between investing in the public good and investing in the private good, the point where $\mathcal{P}_i = 1/2$. For inexperienced subjects, the estimated cutpoint is 1.641, and for experienced subjects, it is 1.399. This finding reinforces the findings of Isaac and Walker. Experienced subjects are more consistent with the dominant strategy model than inexperienced subjects. In this case, the effect is even significant. Of independent interest is that experienced subjects' response functions are *steeper*, indicating less random behavior.

Probit model $\mathcal{P}_3$, shows a minor effect of the addition of a pair of endowment variables, both equal 1 if the endowment is nine tokens and 0 if the endowment is one token. In this case, the slope shift is positive and the intercept shift is negative. The consequence is that the response function for subjects in the high endowment condition is flatter than the response function for subjects in the low endowment condition. The negative intercept is enough to counteract the higher variance, however, and the high endowment means are less than the low endowment means. The magnitudes of these coefficients are much smaller than those associated with the experience effect and the effect of the endowment change is similarly smaller.[5] The actual differences are shown in Figure 8.

[5]The magnitudes are comparable because the variables, both dummies, are of the same scale, namely 0 or 1.

The variable V, which is added in model $\mathcal{P}_4$, measures the marginal valuation of the public good. One interpretation (since we have controlled for MRS) is that its coefficient tells us what happens to a subject's behavior as the payoffs rise. Although the effect is very small, we find that a player's response function becomes steeper, and the average deviation becomes smaller. A similarly small result holds when the period of the decision is taken into account. Holding everything else constant, a player is less likely to contribute in later periods than in earlier periods.

Quite clearly, the major effects are due to MRS and experience. While the endowment condition has some effect, it is not as important. The effects due to the size of the payoffs and to the period of the decision pale in comparison.

3.5.3 Response Functions and Errors: Individual Level Analysis

The analysis in the previous section assumes that individuals are identical. In fact, there are indications of heterogeneity in our data. Similar indications have also been noted in past work. This section offers a simple approach to look at differences between individuals, based on minimization of classification errors (as in section 5.2.1). We do two things. First, we break down that analysis by individual, and obtain a distribution of classification minimizing cutpoints for individuals. This allows us to identify the fraction of subjects who behave consistently with the Nash equilibrium, subjects we call *Nash players*. Second, from these estimated individual cutpoints, we can obtain a distribution of the error rates across individuals. This gives us a way to identify what fraction of subjects are behaving consistently with *some* cutpoint model.

We define a *Nash Player* as a player who is rational and non-altruistic.[6] That is $\bar{\varepsilon}_i = 0$. With this in mind, consider Tables 7 and 8 which report, by subject, the raw number of classification errors for each of the twenty possible cutpoints. These cutpoints correspond to the possible token values. They are the only applicable cutpoints, because they relate directly to every possible realization of r_i.

[Tables 7 and 8 here]

Each possible cutpoint is given a score based on how well it represents that subject's decisions in the experiment. The score is simply the number of times a violation would have occurred if that was the actual cutpoint rule the subject used.[7] More specifically, we hypothesize that a particular player is using a cupoint that corresponds to token value x (we consider every possible x in turn). Hypothetically, each time that player receives a token value r_i, he compares it to x and then spends only if $r_i < x$. A classification error occurs if one of the two following events occurs: $r_i < x$ and the player does not spend, or $r_i > x$ and the player does spend. The lower the cutpoint's score, the better it represents that person's decisions. In these two tables we report the data from one of the $\{6, 1\}$ treatments and one of the $\{6, 9\}$

[6]Because our estimation allows for errors, a Nash Player may be different than a player who *perfectly* follows the decision rule implied by the self-interested model. The difference is that a Nash Player is allowed to make mistakes.

[7]When a particular rule was imprecise, *i.e.*, when the player was indifferent, it was assumed that no errors were made.

treatments.[8]

The first thing to notice is that the minimum error cutpoint is not always unique. When forced to estimate a unique cutpoint, we select the one closest to 1, which is Nash play. In Table 9, subjects $\{4, 6, 10, 14, 15, 16\}$ are classified as Nash players, as are subjects $\{3, 4, 5, 6, 8, 10, 12, 13, 15\}$ in Table 10. A second thing to notice is that not every subject has the same estimated cutpoint. In Table 9, for example, subject #2 has an estimated cutpoint of 2.17 (corresponding to a token value of 13) while subject #16 has an estimated cutpoint of 1.0 (corresponding to a token value of 6). Another observation is that, for some subjects, the minimum number of errors is strictly greater than zero.

Pooling across all experiments, we find that 144/256, or 56 percent of the observations are Nash players. The entire distribution of cutpoints is illustrated in Figure 9. On the x-axis is the difference between the estimated cutpoint and the value of the public good in token value units. For example, subject #1 from Table 10 would be included in the "3" category in this figure, since his estimated cutpoint is 9 and the value of the public good is 6. An x-value of 0 in this figure corresponds to Nash play. This figure can also be broken down by experience, and doing so illustrates the effect of experience on inducing Nash (non-altruistic) play. This is shown in Figure 10.

Finally, we define *consistent players* as players that can be perfectly classified, so that they never make an error at their estimated cutpoint. Pooling across all experiments, we find 178/256, or 70 percent consistent players. The

[8]These two tables are meant to be representative.

percentages of experienced and inexperienced consistent players are 75 and 64 respectively. Figure 11 displays the distribution of error rates, measured as the proportion of an individual's decisions that are inconsistent with his estimated cutpoint. Comparing to the earlier baselines, these error rates are again mostly in a range of five percent or below.

3.5.4 Comparison to Previous Results

There are a few simple comparisons between our data and the data from four person experiments conducted by Isaac and Walker. Recall that, in Issac and Walker's experiments, all subjects have identical marginal rates of substitution, equal to either 1.33 or 3.33 (which they refer to as High MPCR and Low MPCR). Their experiments also used a ten-period repetition design.

The most notable difference between their data and ours is in the frequency with which we observed *consistent Nash* play. This occurs when a subject, for an entire ten-period repetition, makes no decision that is inconsistent with dominant strategy Nash equilibrium. In terms of Figures 9 and 11, these subjects are in the 0-categories in *both* figures. We observe this 118 out of 256 observations, or 45 percent of the time. Isaac and Walker observe this 7 out of 76 observations, or 9 percent of the time. Thus we find five times as much consistent Nash play. Large differences also occur in the frequency of splitting, as pointed out earlier (Tables 2–5).

A second comparison is to look at the decisions made by our subjects when they had MRS = 1.33 and MRS = 3.33. The comparison is given in Table 11.

[Table 11 here]

Again, the same kind of pattern emerges. We find lower contribution rates. In fact, our contribution rate for MRS = 3.33 is roughly the same magnitude as the background noise measured in our baselines.

A third comparison is what we call repetition effects and what has been referred to elsewhere as *decay* — it is typical in these experiments to see less contribution in later periods than in early periods. In fact, in comparable experiments, contribution rates in early periods have ranged from two to four times as much as contribution rates in later periods. We measure an effect in our data (recall the Probit analysis), but we find the magnitude of the decay to be very small. It is true that there is more free riding in later periods, but this is attributable to a decrease in subject errors, or an increase in their consistency, not to a change in their decision rule. This fact is also reflected in the decline of splitting behavior documented earlier.

Andreoni (1988) conducted experiments similar to those of Isaac and Walker and observed magnitudes of contribution, free riding, and decay that by interpolation are roughly the same as those found in the data generated by Isaac and Walker. Those experiments used five person groups and MRS = 2. Instructions were somewhat different and some new treatments were explored. Andreoni's results are similar to those of Issac and Walker, and differences between our data and his are likewise similar to the differences between our data and Issac and Walker's.

Our findings also contrast sharply with the highly anomalous behavior in the experiments done by Saijo and Yamaguchi. They conducted homogeneous preference experiments with MRS = .7 and MRS = 1.42. Like Andreoni, they observe magnitudes of free riding, and decay for their experiments with an MRS = 1.42 that are roughly the same as those in Isaac and Walker's data. Saijo and Yamaguchi and Issac and Walker also observe similar split rates. The splitting rates observed in *both* of Saijo and Yamaguchi's treatments are 55 percent. They get as much splitting when subjects have a dominant strategy to contribute, as when subjects have a dominant strategy to free ride! Our findings are *dramatically* different.

Saijo and Yamaguchi observe aggregate contribution rates that are different from ours and also from Isaac and Walker's. For the 1.42 treatment, they observe 27 percent contribution, which is quite a bit less contribution than that seen in Isaac and Walker's data for MRS = 1.33. Our closest observations to MRS = 1.42 are at MRS = 1.5 and MRS = 1.4. We observed contribution rates of .27 and .36, respectively for those two values of MRS.

In their MRS = .7 treatment, Saijo and Yamaguchi see a contribution rate of 58 percent! Recall that our observed contribution rate was so close to 1 (.96) for this range of MRS, that we used this as one of our baselines for the rate of background noise! We have no satisfactory explanation for this enormous difference between their results and ours. However, we do note that those experiments were conducted somewhat differently in a number of ways, which may partially account for the differences in data.

Saijo and Yamaguchi employed seven member groups instead of four member groups, they conducted the experiments manually instead of through a computer network, and they used different instruction methods. In fact, they used two instruction sets as a treatment, and found significant differences due to that treatment. Also, they required subjects to make each decision within 20 seconds, and they used a different subject pool. Saijo and Yamaguchi suggest that the differences may be attributable to cultural differences between Japan and the U.S. We are skeptical of that explanation, but have no better one to offer.

3.6 Interpreting the Results

The main differences between our findings and previous findings can be summarized by the following observations:

1. We observe less splitting.
2. We do not observe significant decay.
3. We observe lower contribution rates.
4. We observe more Nash behavior.
5. We observe essentially no spiteful behavior.

The findings that replicate from past experiments with comparable group sizes are that experience leads to lower contribution rates, and contribution rates are declining in the marginal rate of substitution (marginal valuation of the private good).

Explanations for the differences that we observe are either methodological or environmental in nature. Possible methodological explanations abound: we utilize slightly different experimental procedures, or our instructions and computer screens are different, we employ a different subject pool, *etc.* On the environmental side, our experiments utilize a different economic environment, by which we mean the information structure and the profile of preferences in the group are different. In particular, as emphasized in the introduction, the information structure and profile of preferences correspond almost exactly to the standard environment used for auction experiments. In each period, preferences in the group are randomly and independently drawn from a known distribution of marginal rates of substitution, thereby inducing heterogeneity across individuals. This contrasts sharply with environments that have been explored in earlier investigations of the voluntary contributions mechanism.

To try to assess the relative importance of the methodological and environmental explanations, we have subsequently tried to replicate Issac and Walker's findings using our procedures and subject pool and their homogeneous environment. Specifically, we conducted an additional experimental session where every subject had a publicly announced marginal rate of substitution equal to 3.33, and every subject was endowed with multiple units of the private good.

Figure 12 compares the results of this session with the data from Issac and Walker. There is very little difference. The main features of the data replicate: there are very high contribution rates early on, and these rates decay significantly. In this extra session, we also observed similar splitting rates and amounts of Nash behavior. Based on this data, we dismiss the possibility that differences in our experimental procedures or subject pool are responsible for the differences in our results.

Thus we are left only with environmental explanations. This leads us to conclude that the findings from earlier experiments, experiments that utilized homogeneous environments, are not robust to public goods environments which exhibit variation in preferences, even if we limit attention only to linear public goods environments. This is a significant finding, even more so if one suspects, as we do, that heterogeneous preferences are a factor in most natural settings. There is an interesting question left open, namely "Why does heterogeneity lead to such different results?"

It is possible that, with homogeneous preferences, it is easier for a group to achieve a cooperative solution of the sort suggested by repeated game arguments. For example, if subjects adopt the type of strategies that reciprocate generous behavior by others, or believe that others adopt these strategies (see Kreps, Milgrom, Roberts, and Wilson [1985]), then some of the patterns of behavior that have been noticed in the homogeneous preference experiments, decay and pulsing, for example, can be rationalized.

In our design, since preferences are private information, the ability to signal one's generosity to other players is interfered with.[9] If one is observed to contribute, other subjects cannot tell if you are being generous, or simply acting selfishly.

To identify the effects of the private information in our experiments, we conducted two revealed-information sessions (with $V = 6$ and $X = 9$) where all token value draws were revealed to everyone in the group. In the first of

[9]Actually, in most of the homogeneous design experiments, homogeneity is not publicly announced. However, experiments by Isaac and Walker (1990) find that common knowledge of the homogeneity has no effect on behavior. They conjecture that subjects infer from the wording in the instructions that other subjects have similar payoff tables.

these sessions, token values were revealed after the decisions were made. In the second, token values were revealed before the decisions were made. In both cases, the signal interference problem is eliminated, which, if the above explanation is correct, should lead to greater contribution and less free riding.

The pooled results for the revealed information sessions are displayed in Figure 13, which compares the empirical response function with the data from all the other heterogeneous preference experiments (those with no revealed information).[10] There is very little difference. In fact, if anything, revealed information seems to lead to even more free riding behavior, which is contrary to the reputation hypothesis.

This leaves us without a complete explanation for why we observe such different results in our environment. At this point, we simply do not know. A number of other possible explanations can be imagined. Perhaps it was important (because of faster learning, less boredom, or something else) that subjects in our design are assigned a new MRS for each decision. This sort of explanation unfortunately seems to be currently beyond the reach of existing theoretical models of behavior in these kinds of games. On the other hand, the findings here are suggestive of possible new directions for theoretical work, as well as some directions for new experimental designs.

[10]There is no significant difference between the two revealed information sessions, so pooling the data is reasonable.

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3.8 Sample Instructions

Decision-Making Experiment

This is an experiment in decision making. You will be paid IN CASH at the end of the experiment. The amount of money you earn will depend upon the decisions you make and on the decisions other people make. It is important that you do not talk at all or otherwise attempt to communicate with the other subjects except according to the specific rules of the experiment. If you have a question, feel free to raise your hand. One of us will come over to where you are sitting and answer your question in private.

This session you are participating in is broken down into a sequence of four separate experiments. Each experiment will last 10 rounds. At the end of the last experiment, you will be paid the total amount you have accumulated during the course of all 4 experiments. Everyone will be paid in private and you are under no obligation to tell others how much you earned. Your earnings are given in FRANCS. At the end of the last experiment, you will be paid 11 cents for every 100 FRANCS you have accumulated during the course of all 4 experiments.

In each experiment you will be divided into 4 groups of 4 persons each. Those groups will stay the same for all 10 rounds of the experiment. After each of the 10 round experiments, everyone will be regrouped into 4 entirely new groups. Therefore, whenever we change groups, the other people in your group will be completely different from the last group you were in. You will not be told the identity of the other members in your group. Since we will be running 4 experiments tonight, you will be assigned 4 different groupings, one for each 10 round experiment.

RULES FOR EXPERIMENT #1

Each round of the experiment you will have 9 tokens. You must choose how many of these tokens you wish to keep and how many tokens you wish to spend. The amount of money you earn in a round depends on how many tokens you keep, how many tokens you spend, and how many tokens are spent by others in your group. Each round, you will be told how many FRANCS each token is worth if you keep it. This amount, called your TOKEN VALUE, will change from round to round and will vary from person to person randomly. To be more specific, in each round, this amount is equally likely to be anywhere from 1 to 20 FRANCS. There is absolutely no systematic or intentional pattern to your token values or the token values of anyone else. The determination of token values across rounds and across people is entirely random. Therefore, everyone in your group will generally have different token values. Furthermore, these token values will change from round to round in a random way. You will be informed PRIVATELY what your new token value is at the beginning of each round and you are not permitted to tell anyone what this amount is.

After being told your token value, you must wait at least 10 seconds before making your decision of how many tokens to spend and how many to keep. Your keyboard will be frozen for this period of time. When everyone has made a decision, you are told how many tokens were spent in your group and what your earnings were for that round. This will continue for 10 rounds. Following each round you begin with 9 new tokens and are randomly assigned a new token value between 1 and 20 FRANCS.

PAYOFFS

You will receive 6 FRANCS times the total number of tokens spent in your group. In addition, you will also receive your token value times the number of tokens you keep. Notice that this means every time anyone in your group spends a token, everyone in the group (including the spender) gets an additional 6 FRANCS, but the spender foregoes his or her token value for that token. WHAT HAPPENS IN YOUR GROUP HAS NO EFFECT ON THE PAYOFFS TO MEMBERS OF THE OTHER GROUPS AND VICE VERSA. Therefore, in each round, you have the following possible earnings, as shown in the table:

[HAND OUT EARNINGS TABLE. ALSO WRITE ON BOARD]

Suppose everyone else in your group spends 13 tokens in all and you spend 4 tokens and your token value was 12. You would earn 24 + 78 + 60 = 162 FRANCS. If you had spent 3 tokens you would have earned 18 + 78 + 72 = 168 FRANCS. If you had spent 5 tokens you would have earned 30 + 78 + 48 = 156 FRANCS.

ADDITIONAL PROCEDURES:

1. Are there any questions? [ANSWER QUESTIONS]

2. Hand out quiz.

3. Correct quiz answers and read them aloud.

4. Answer any additional questions.

5. Two practice rounds – Tell them not to press any keys unless you tell them to. In round 1 have all even ID#'s spend and odd keep. In round 2 do it the other way. Go over screen display and history. Tell subjects to refrain from pressing keys for no reason.

Specific instructions for Experiment 2:

Experiment 2 is the same as experiment 1 except you now have been regrouped with a completely different set of people.

Specific instructions for Experiment 3:

Experiment 3 is the same as experments 1 and 2 except now everyone in a group receives 10 FRANCS times the number of spenders in the group. Again, in addition, nonspenders also receive their token values. Remember that everyone has been reassigned to a group with a new set of people. Here is your new payoff table:
[HAND OUT NEW EARNINGS TABLE, AND COLLECT OLD ONE. CHANGE BOARD. EXPLAIN.]

Suppose everyone else in your group spends 13 tokens in all and you spend 4 tokens and your token value was 12. You would earn 40 + 130 + 60 = 230 FRANCS. If you had spent 3 tokens you would have earned 30 + 130 + 72

= 232 FRANCS. If you had spent 5 tokens you would have earned 50 + 130 + 48 = 228 FRANCS.

Specific instructions for Experiment 4:

Experiment 4 is the same as experiment 3 except you have been regrouped again.

3.9 Tables

Endowment \ V	3	6	10	15
1 token	2	2	2	2
9 tokens	2	2	2	2

Table 3.1: Each cell has two 10-period sequences of a cohort with sixteen subjects divided into four groups. The first sequence is called "inexperienced"; the second is called "experienced." Groups were shuffled between sequences.

	early	late
inexp.	.22 (320)	.11 (320)
exp.	.12 (320)	.04 (320)

Table 3.2: Analysis of Splits. All data with endowment nine.

	early	late
inexp.	.36 (182)	.19 (176)
exp.	.21 (180)	.07 (170)

Table 3.3: Analysis of Splits. Endowment = 9, MRS > 1.

	early	late
inexp.	.029 (138)	.021 (144)
exp.	.021 (140)	.0067 (150)

Table 3.4: Analysis of Splits. Endowment = 9, MRS $\leq$ 1.

	MRS = 1.33	MRS = 3.33
periods 1-5	.56 (120)	.60 (260)
periods 6-10	.56 (120)	.40 (260)

Table 3.5: Splitting behavior in the Isaac and Walker data.

	early	late
inexp.	.03 (262)	.04 (285)
exp.	.04 (263)	.04 (288)

Table 3.6: Spiteful behavior. Free-riding rates for subjects with MRS < 1 (Dominant Strategy to Contribute)

	early	late
inexp.	.08 (63)	.04 (65)
exp.	0 (65)	.002 (64)

Table 3.7: Sacrifical behavior. Contribution Rates for Subjects with MRS > 4

Probit Models					
	1	2	3	4	5
ones	1.778	1.504	1.612	1.801	1.850
	(85.301)	(57.596)	(45.538)	(34.252)	(32.222)
MRS	-1.156	-0.916	-0.973	-1.013	-1.015
	(-86.358)	(-58.866)	(-44.078)	(-42.878)	(-42.896)
exper.s		-0.861	-0.858	-0.867	-0.868
		(-25.252)	(-25.084)	(-25.235)	(-25.233)
exper		0.983	0.980	0.992	0.994
		(20.013)	(19.919)	(20.075)	(20.089)
endow.s			0.104	0.108	0.107
			(3.742)	(3.888)	(3.856)
endow			-0.199	-0.207	-0.205
			(-4.618)	(-4.761)	(-4.730)
V				-0.015	-0.015
				(-4.923)	(-4.993)
period					-0.008
					(-2.146)
lg lkhd	-8912.7	-8522.7	-8511.9	-8499.7	-8497.4
% pred.	83.064	83.160	83.238	83.429	83.607

Table 3.8: In each Probit Model, the dependent variable is the investment decision. Equal weight has been given to both the one token treatment and to the nine token treatment. Under each coefficient is the asymptotic t-statistic. The log likelihood and the percentage correctly predicted are also given for each model.

Token Value (Cutpoint)

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	1	5	5	5	3	3	3	3	3	2	2	0	0	0	0	1	1	2	4	4	5
	2	3	2	2	2	1	1	1	1	1	1	1	1	0	0	0	0	0	2	4	5
S	3	5	4	4	3	3	3	3	3	2	2	1	1	1	2	3	3	3	3	3	4
u	4	2	1	1	1	0	0	0	0	0	0	0	1	3	3	3	4	5	5	5	5
b	5	4	3	2	2	2	2	1	1	0	0	0	0	1	4	4	4	4	5	5	5
j	6	2	2	2	2	1	0	0	0	1	2	2	4	4	4	5	5	7	7	7	7
e	7	3	3	2	2	1	1	1	0	0	2	2	2	2	2	3	5	5	5	5	7
c	8	4	4	3	3	3	3	1	0	0	1	2	2	2	2	2	3	3	4	6	6
t	9	5	3	3	3	3	3	3	3	3	3	3	3	1	1	1	1	4	4	4	4
	10	3	3	2	2	1	0	0	0	1	1	1	1	3	5	5	5	7	7	7	7
#	11	3	3	3	2	1	1	1	0	0	0	0	3	4	4	4	5	5	5	7	7
	12	5	5	4	4	4	4	3	2	1	0	1	1	1	2	3	4	4	4	4	4
	13	6	6	5	4	4	3	3	3	0	0	0	0	1	1	1	2	2	2	2	2
	14	2	1	1	1	0	0	0	1	3	4	4	5	6	6	6	7	7	7	7	8
	15	2	2	1	1	0	0	0	1	1	1	1	1	2	4	5	6	6	6	6	7
	16	3	2	1	1	1	1	1	2	2	2	2	3	4	4	4	5	6	6	7	7

Table 3.9: The raw number of classification errors for the first repetition of treatment $\{6, 1\}$

Token Value (Cutpoint)

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	1	23	23	23	5	5	5	5	5	3	10	8	24	24	23	31	31	40	58	58	67
	2	22	15	17	17	12	16	16	16	16	16	16	16	11	15	15	15	10	23	41	50
S	3	54	45	45	27	18	18	18	18	18	27	27	36	36	36	36	36	36	36	36	36
u	4	18	9	9	9	0	0	0	0	0	0	0	9	27	27	27	36	45	45	45	45
b	5	18	9	0	0	0	0	0	9	9	18	18	18	27	54	54	54	54	63	63	63
j	6	18	18	18	18	9	0	0	0	9	18	18	36	36	36	45	45	63	63	63	63
e	7	31	31	22	22	13	13	13	10	10	22	22	22	22	19	24	41	41	41	41	59
c	8	11	11	2	2	2	2	2	20	29	37	45	45	45	45	45	54	54	62	79	79
t	9	30	12	12	12	12	12	12	12	12	12	12	12	9	24	24	19	41	39	46	53
	10	27	27	18	18	9	0	0	0	9	9	9	9	27	45	45	45	63	63	63	63
#	11	18	18	18	18	18	18	18	14	19	19	14	36	45	45	45	54	54	54	72	72
	12	9	9	0	0	0	0	0	9	18	27	45	45	45	54	63	72	72	72	72	72
	13	19	19	10	1	1	1	10	10	9	35	35	35	44	44	44	53	53	53	53	53
	14	44	35	35	35	26	26	17	9	19	28	20	21	30	30	29	37	37	37	37	46
	15	22	22	13	13	4	4	4	13	13	13	13	13	20	34	41	50	50	50	50	59
	16	30	12	12	22	12	12	19	26	26	26	21	23	30	30	28	34	42	42	51	51

Table 3.10: The raw number of classification errors for the first repetition of treatment $\{6, 9\}$

	IW data	Our data
MRS = 1.33	.50 (240)	.37 (90)
MRS = 3.33	.20 (520)	.05 (56)

Table 3.11: Contribution rates. Comparison to IW data, when MRS = 1.33 and MRS = 3.33

3.10 Figures

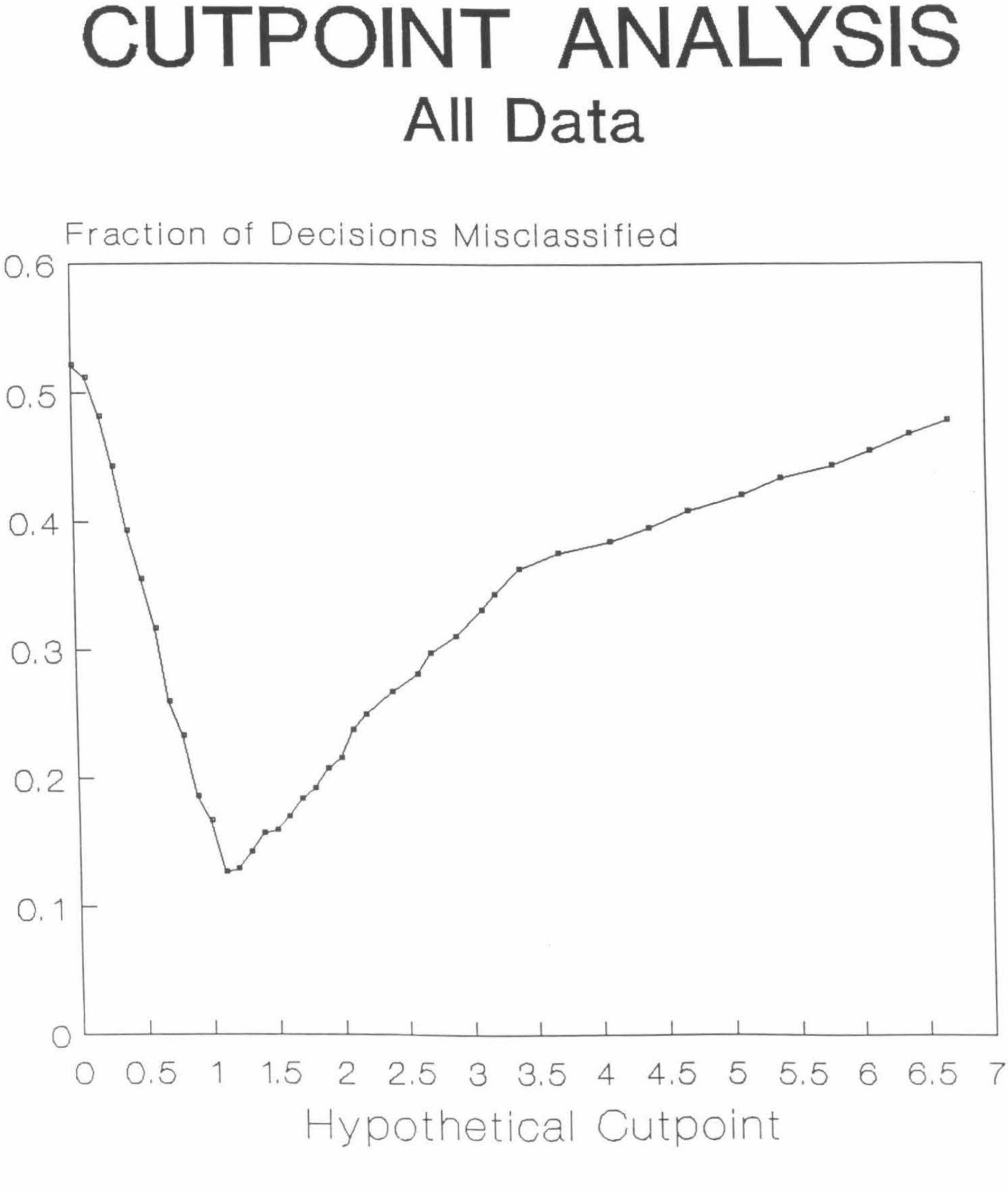

Figure 3.1: Cutpoint analysis: aggregate level

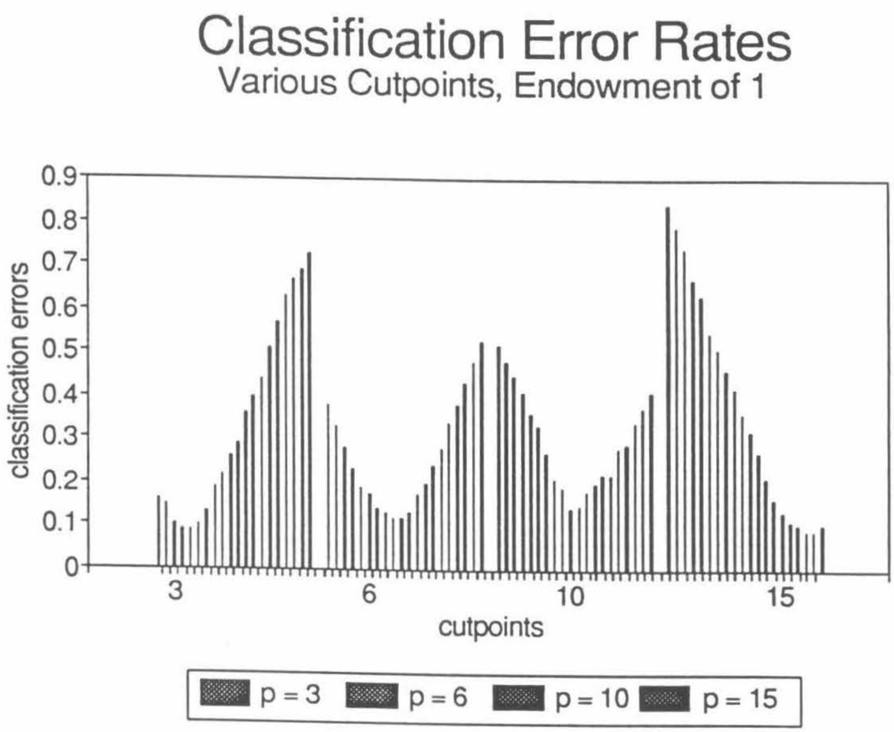

Figure 3.2: Classification errors aggregated over all subjects shown for all treatments with an endowment condition of one.

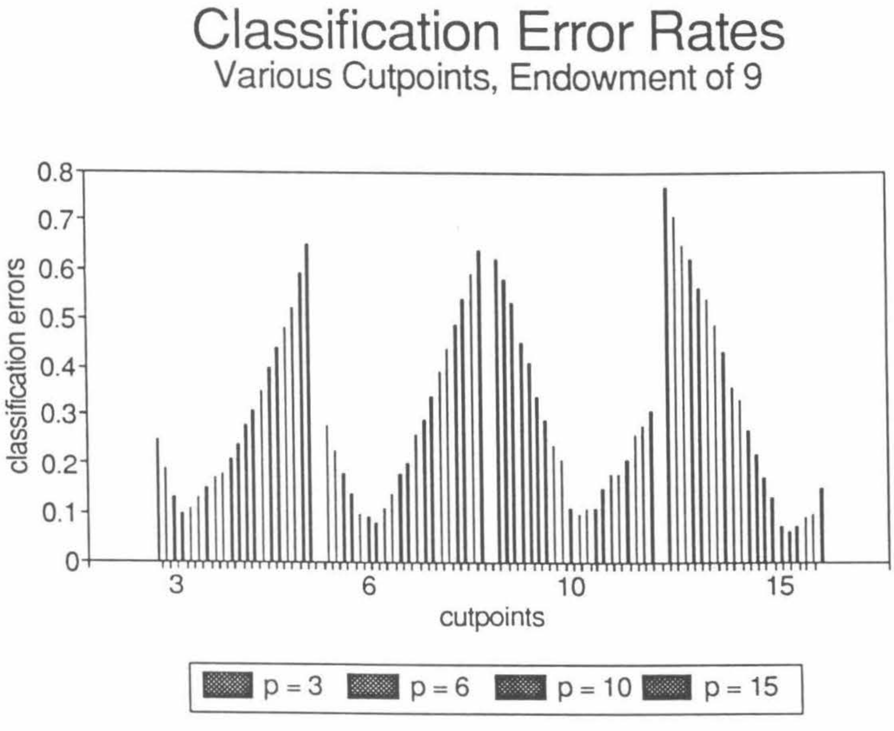

Figure 3.3: Classification errors aggregated over all subjects shown for all treatments with an endowment condition of nine.

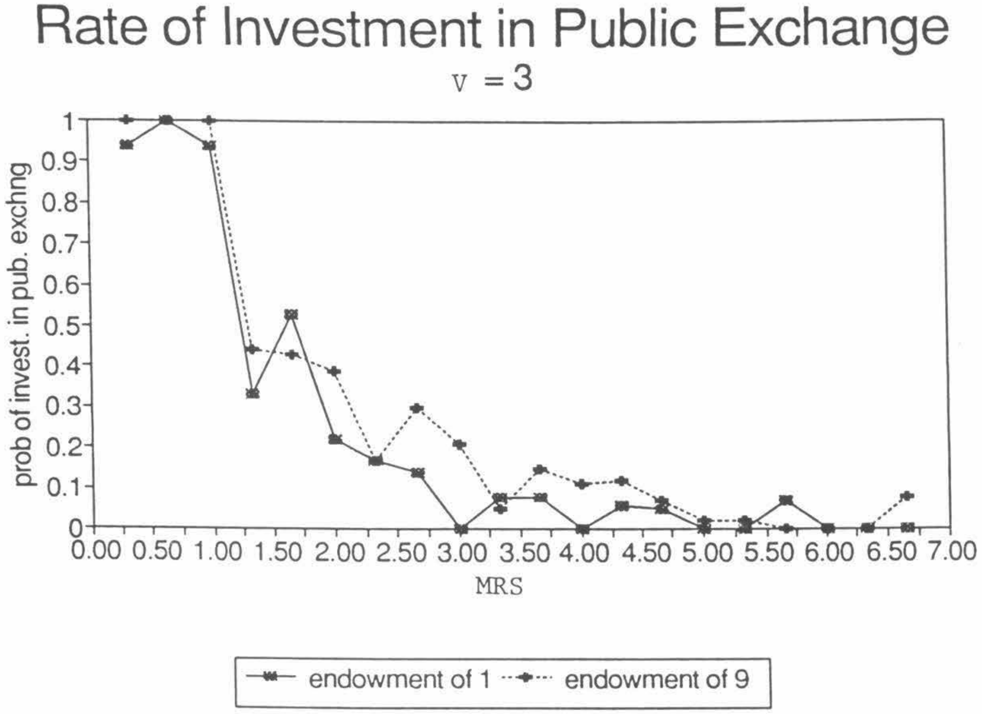

Figure 3.4: The aggregate percentage of tokens invested in the public exchange *vs.* the marginal rate of substitution, plotted for both the endowment of one and the endowment of nine conditions. $V = 3$.

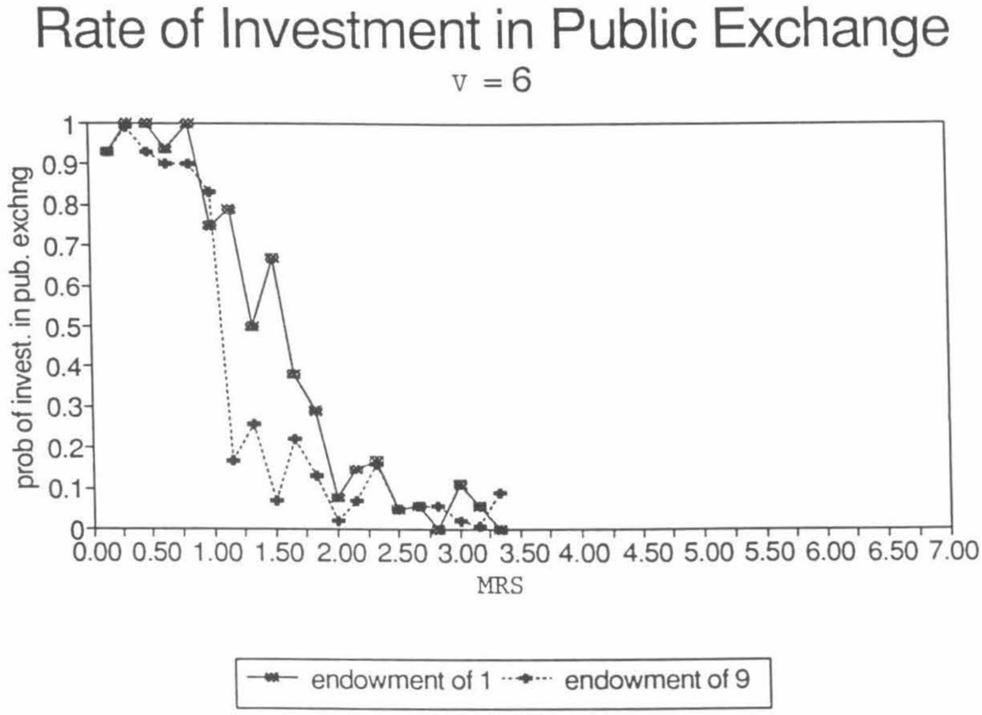

Figure 3.5: The aggregate percentage of tokens invested in the public exchange *vs.* the marginal rate of substitution, plotted for both the endowment of one and the endowment of nine conditions. $V = 6$.

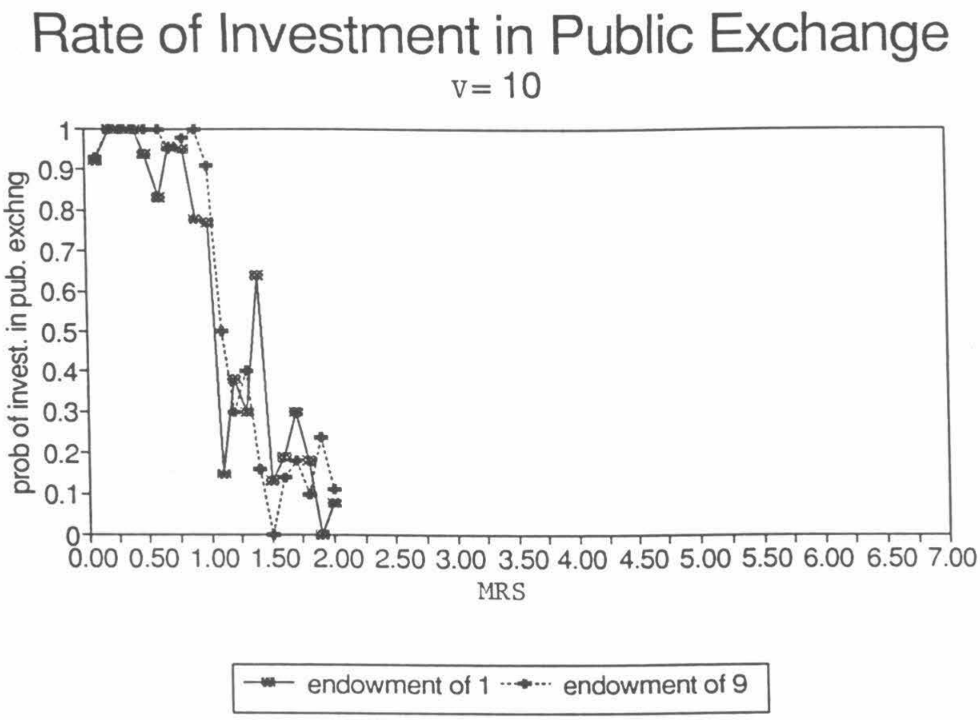

Figure 3.6: The aggregate percentage of tokens invested in the public exchange *vs.* the marginal rate of substitution, plotted for both the endowment of one and the endowment of nine conditions. $V = 10$.

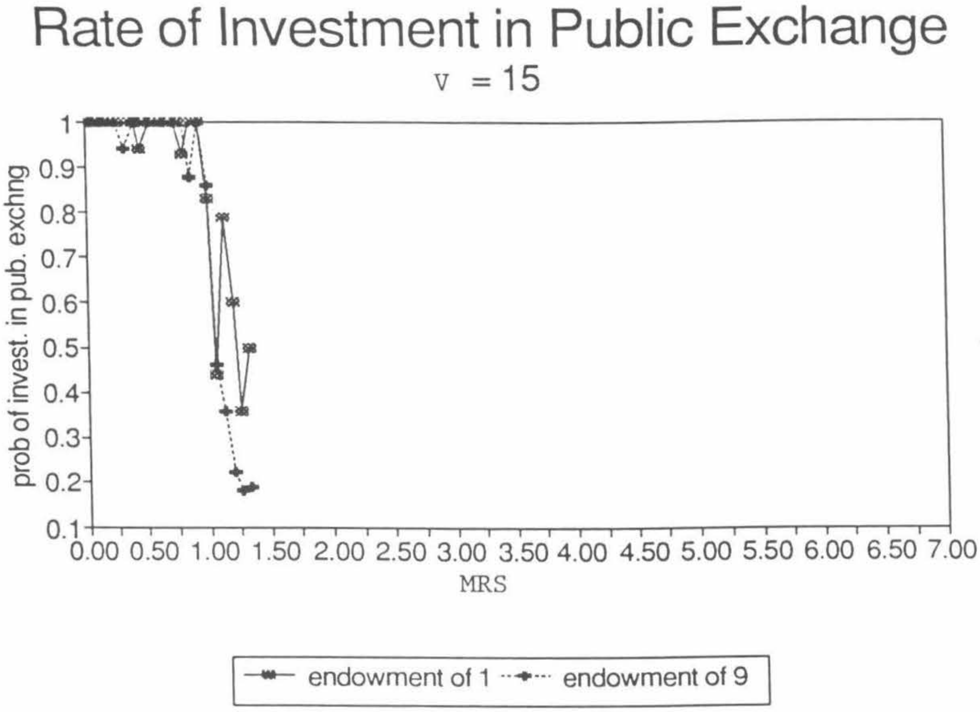

Figure 3.7: The aggregate percentage of tokens invested in the public exchange *vs.* the marginal rate of substitution, plotted for both the endowment of one and the endowment of nine conditions. $V = 10$.

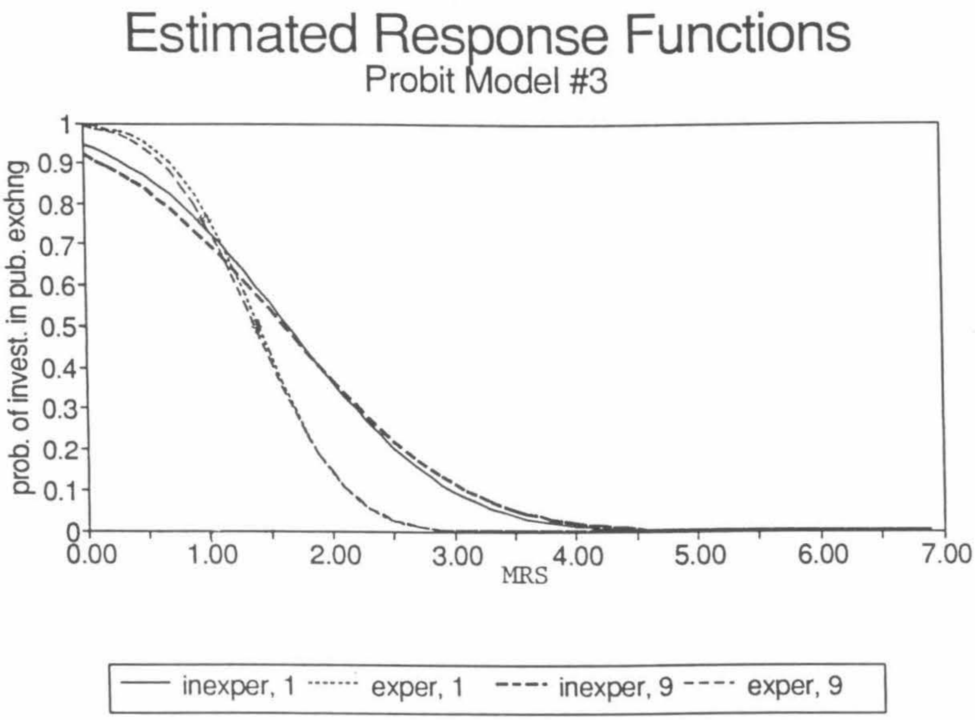

Figure 3.8: The different response functions generated by Probit Model No. 3.

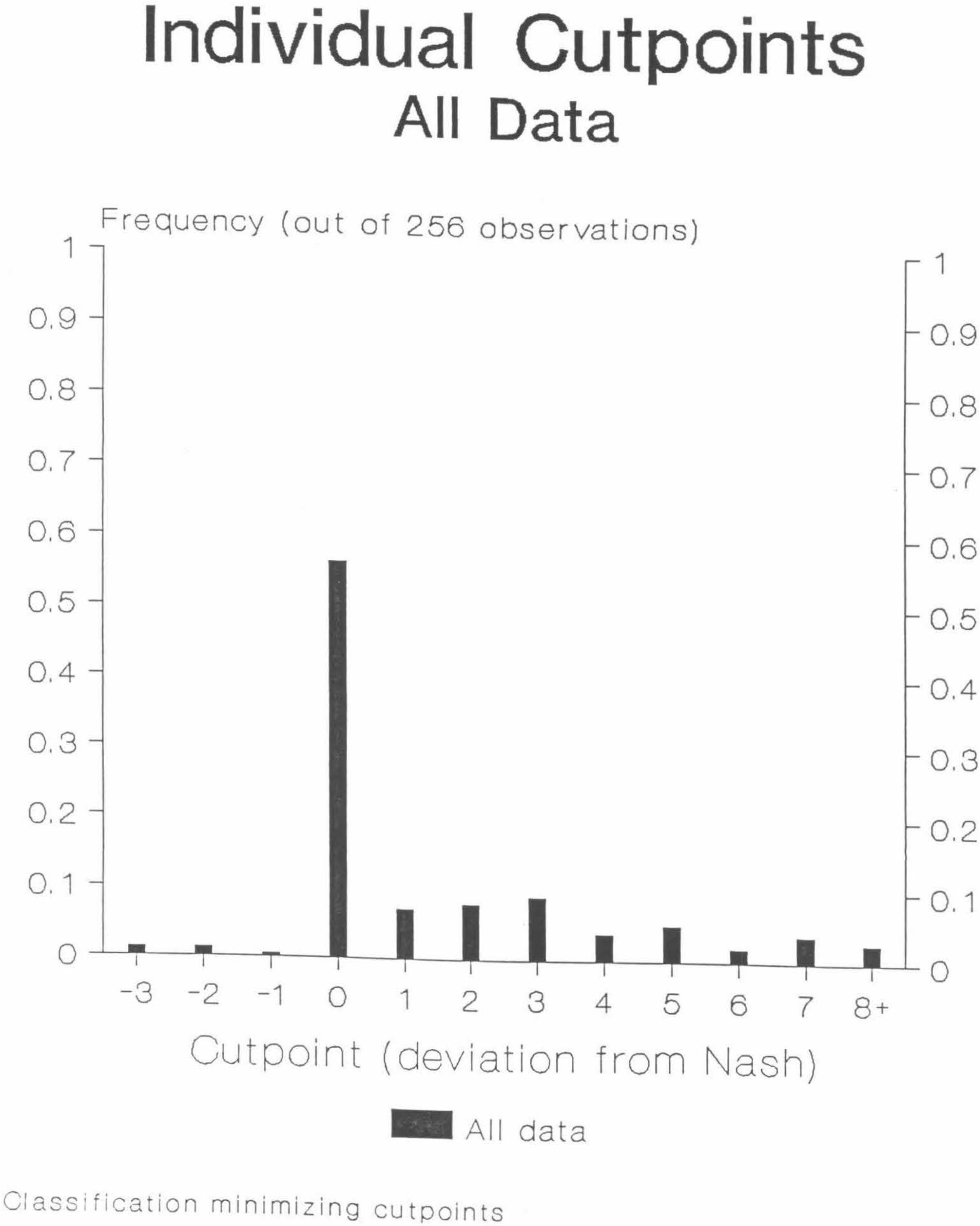

Figure 3.9: Estimated cutpoints measured as deviation from Nash play (in token value units). All data.

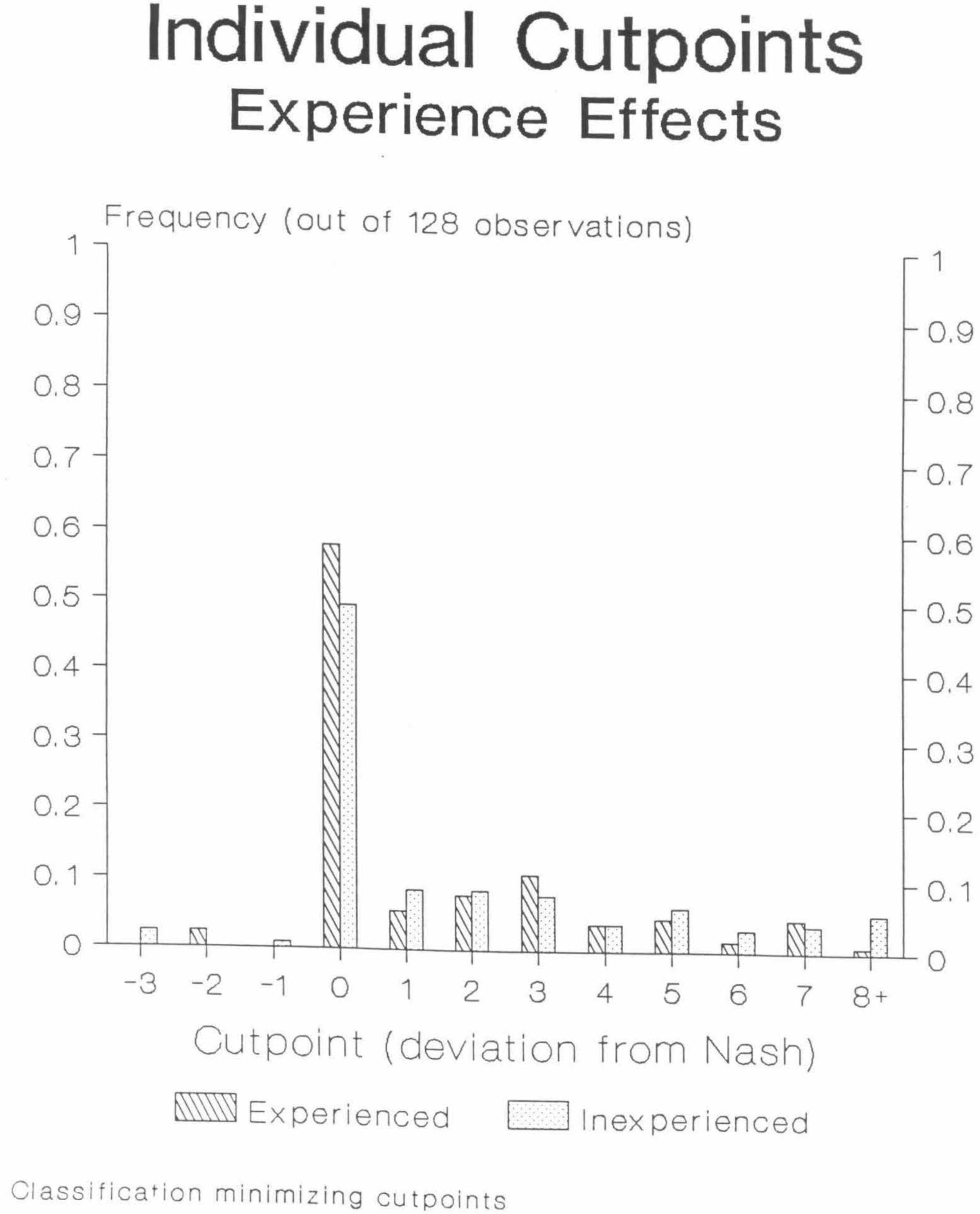

Figure 3.10: Estimated cutpoints measured as deviation from Nash play (in token value units). Experience effects.

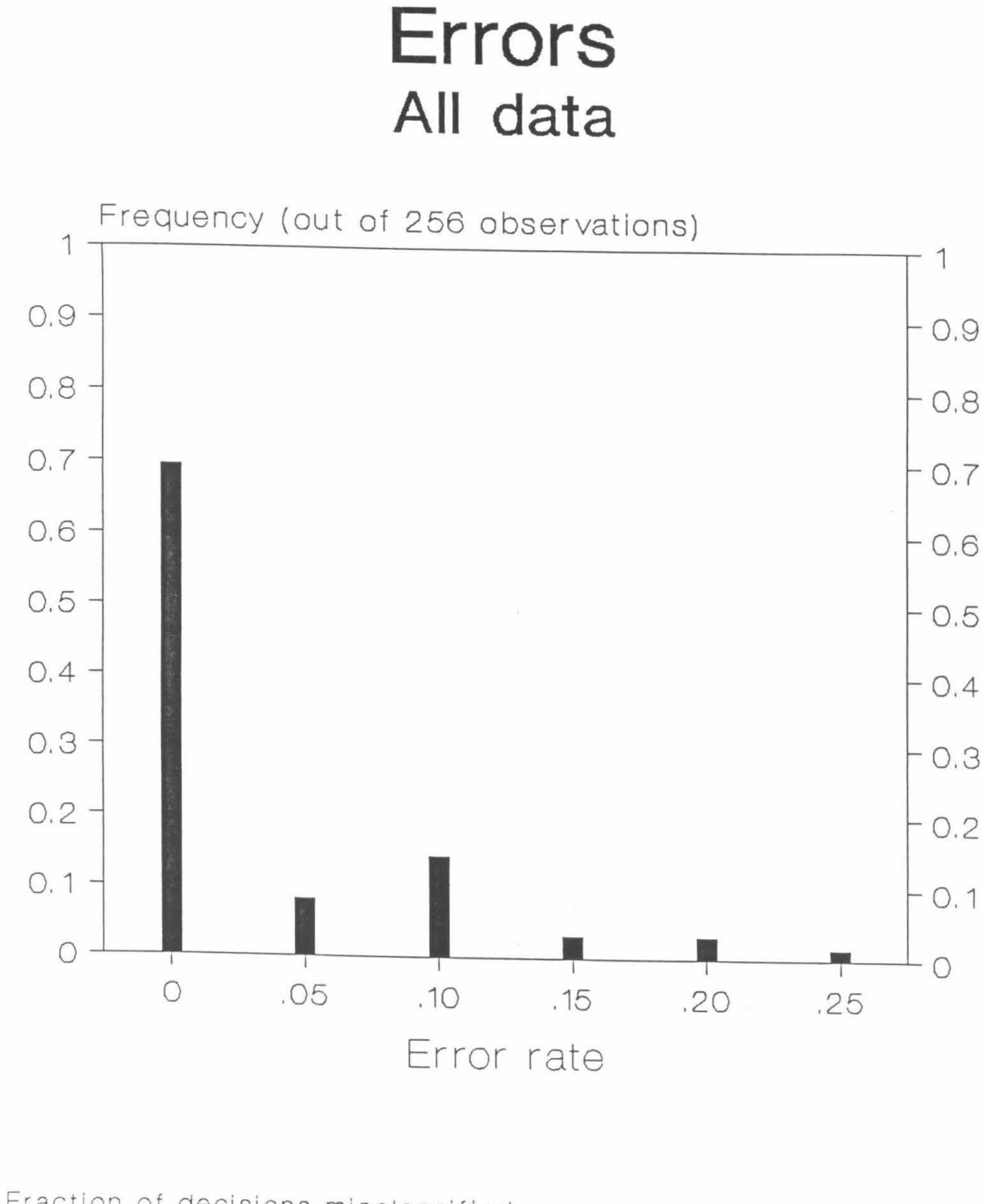

Figure 3.11: Classification errors.

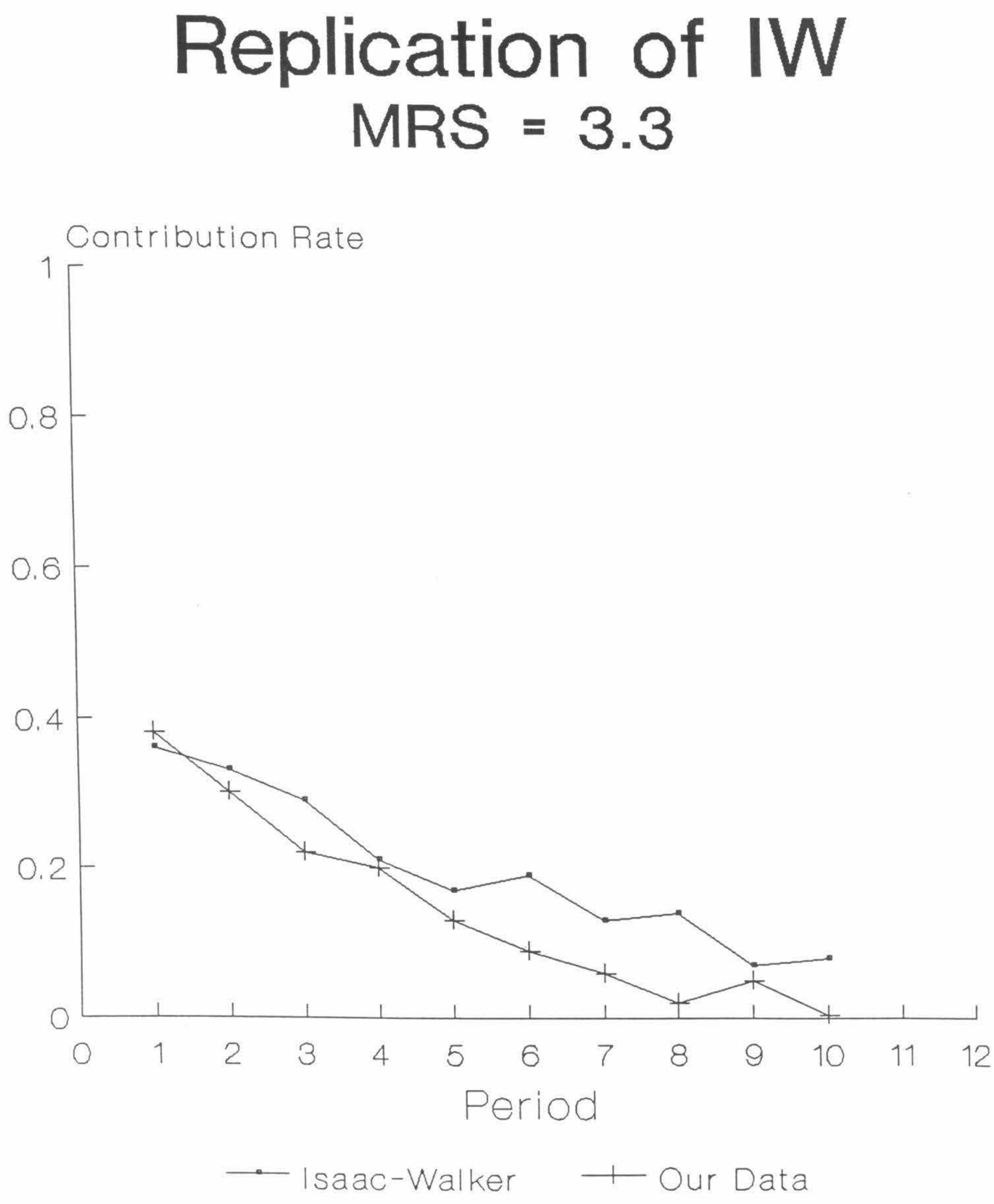

Figure 3.12: Replication of homogeneous preference experiments with $V = 6, r = 20, X = 9$ (MRS= 3.3).

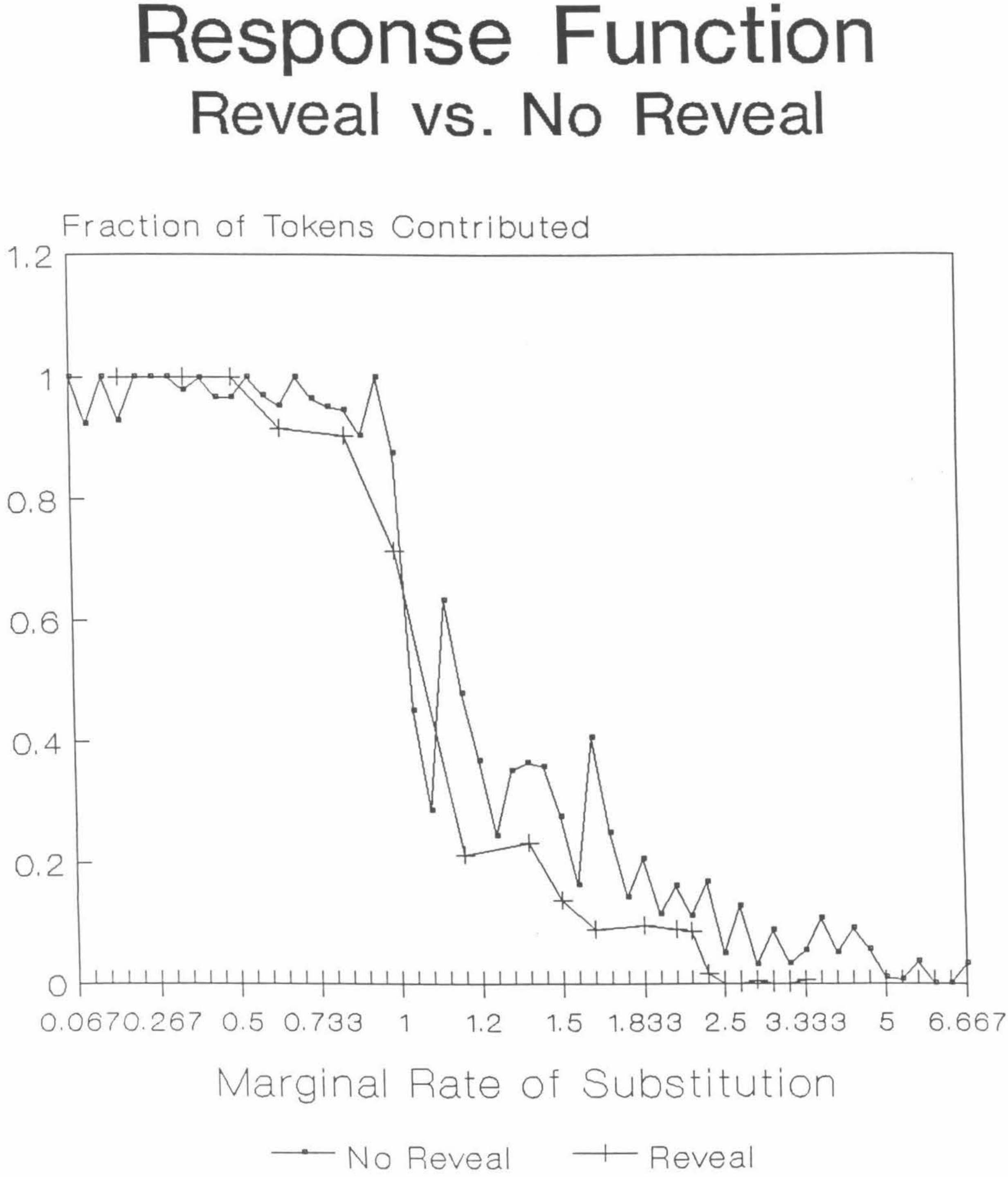

Figure 3.13: Empirical response function with (reveal) and without (no reveal) publicly reported token values.