Fiscal Policies, Optimal Growth, and Elections under Different Economic Systems

Thesis by
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Dedication

To my parents

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Abstract

Using a general equilibrium approach, I develop a two-period model that provides microeconomic foundations for the relationship among fiscal policies, optimal growth, and elections under two different economic systems: a free economy and a democratic planned economy.

In a free economy (Chapter 2), I assume the government indirectly controls the economy by selecting a fiscal policy, and a firm chooses the growth path. First, I show that fiscal policy determines the endogenous growth of the economy, and fiscal policy is determined by the distribution of income. Second, ceteris paribus, the wealthier are more likely to oppose a larger government and a redistribution-oriented fiscal policy. Third, I show that binary voting procedures always generate the median-income consumer as the majority winner. Fourth, when a private good utility has a constant elasticity of marginal utility of income, then (a) fiscal policy and income distribution have no effects on economic growth; (b) among different distributions of income, the higher the profit share of the decisive consumer (i.e., median-income consumer), the lower the tax rate; (c) under certain conditions, the inverted-U curve relationship between economic development and income inequality (the

Kuznets Curve) does not exist.

In a democratic planned economy (Chapter 3), I assume the government controls the economy by setting wage rates, prices and the growth rate of the economy. First, I show that there exist voting equilibria which are sensitive to agenda setting in most cases. Second, I show that with Cobb-Douglas production technology, decentralization of wage decisions in a democratic planned economy can guarantee a unique political-economic equilibrium and a growth path that is middle-class-oriented. Third, when utility satisfies certain conditions, a democratic planned economy can experience the same growth path and income-distributional neutrality on growth as that of a free economy.

Cross-country and cross-time empirical evidence (Chapter 4) are provided to test theoretical predictions and raise questions for future theoretical explanation. In particular, I find that the growth rate of the population and the ratio of gross private investment to GDP have significantly negative and positive effects on economic growth, respectively.

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Chapter 1

Introduction

1.1 Questions

A comprehensive and systematic understanding of economic growth requires the exploration of both macroeconomic policy and elections simultaneously. This dissertation studies the interrelationship among fiscal policies, optimal growth and political elections under two different economic systems: a free economy and a democratic planned economy. The following standard questions of neoclassical growth models are posed, examined and answered in the dissertation.

For free economies, is it possible to explain observed differences in long-run growth rates without considering exogenous changes in technology or population? Why don't we observe a monotonic relation between income tax rates and economic growth rates as predicted by neoclassical models? Does a political-economic equilibrium exist when candidates compete for office by selecting different fiscal policies? Can the median-income consumer prevail in any binary procedure under the majority rule? What is the predicted relationship of wealth to preferences on tax rates and the size of public good sector? When can we predict fiscal neutrality in economic growth? Does the inverted-U curve relationship between economic development and income inequality (the Kuznets curve) always exist with different utility functions?

In Chapter 3, I study democratic planned economies. Does a political-economic equilibrium exist in a democratic planned economy? What is the effect of decentralizing economic decision-making on electoral outcomes? Can different economic systems experience the same growth path given the same initial economic conditions?

In addition to addressing the above theoretical questions, I present exploratory empirical evidence bearing on the following questions, which are derived from both existing neoclassical growth theory and my theoretical discussions. Do the growth rates across countries converge to steady state? Are the growth rates of population and GDP per capita positively or negatively related? What are the effects of gross private investment, public sector investment, and human capital on economic growth? Does government spending have a negative effect on economic growth? How does income inequality affect economic growth?

1.2 Main Features

This dissertation differs from most of the current literature in the following important ways. First, I incorporate fiscal policies, optimal growth and elections together in a model and am able to systematically study and characterize the political-economic equilibrium.[1] Second, I study two-sector (private good and public good) models[2] where government uses its tax revenue to provide public goods and make private good transfers. Third, a firm (or the economy as a whole) owned by consumers in fixed shares is introduced to decide capital accumulation. This setting enables me to capture the reality that government only indirectly controls the economy by selecting an income tax rate, while at the same time it avoids treating consumers identically as some neoclassical growth models do. Fourth, instead of using Phillips curve or voter myopia, exogenous welfare functions, or cost functions of in-

[1] Chapter 2 of my dissertation is similar to Perotti (1990) in the sense that both papers deal with redistribution, political decisions and economic growth. However, there are some fundamental differences between these two papers: first, Perotti assumes convex costs in collecting taxes and tax revenue is only used in redistribution. I assume no costs in collecting tax and tax revenue can be used in public good production as well as private good transfers (a kind of redistribution). Second, my model deals with the two-sector economy (i.e., private good and public good) instead of the one-sector economy. Third, Perotti assumes linear utility and no explicit production function, while I deal with generally well-defined utility functions and production functions. Finally, some assumptions are made about the distribution of pre-tax incomes in Perotti's paper, for example, there are three groups characterized by different pre-tax incomes, the median voter is in the middle class and the median is initially below the mean, while the distribution of pre-tax income in this paper can be arbitrary.

[2] The two sectors in my context are different from those in the current literature. The two sectors in King and Rebelo (1990) refer to consumption/physical investment and human capital investment. The two sectors in Rebelo (1991) include capital sector and consumption sector. Barro (1990) does employ a similar two-sector model as ours, however the utility function in his model is assumed to be Cobb-Douglas.

flation and/or unemployment for government, I assume that consumers (the firm) maximize their (its) own utilities (profit) subject to technological and budget constraints. The general equilibrium approach provides the microeconomic foundation for this political-economic electoral model as well as the systematic solution. Fifth, I look into the relationships among fiscal policies, economic growth, and elections in a democratic planned economy and compare them with the relationships that exist in a free economy. Finally, in addition to some conventional hypotheses widely studied across countries, I test the hypotheses concerning the effects of public sector investment and income inequality on economic growth.

1.3 Organization

My thesis is organized as follows. Chapter 2, which studies fiscal policies, optimal growth, and elections in a free economy, is divided into the following sections: a review of literature, model, main conclusions, constant elasticity of marginal utility of income, and summary. I study fiscal policies, optimal growth, and elections in a democratic planned economy in Chapter 3. This Chapter includes three sections: model, main conclusions, and summary and system comparison. An empirical study of economic growth is offered in Chapter 4, which consists of five sections: introduction, empirical design, variables and hypotheses, results, summary and data appendix. I conclude in Chapter 5.

Chapter 2

Fiscal Policies, Optimal Growth, and Elections in a Free Economy

2.1 A Review of the Literature

There are four strands of literature relevant to this chapter. First, researchers have constructed a class of endogenous growth models (Solow (1956), Cass (1965), Koopmans (1965), and Uzawa (1965)). These models feature a closed economy with competitive markets, identical rational individuals, and a production technology exhibiting diminishing returns to capital and labor separately and constant returns to both inputs jointly. They have the following properties: (i) the existence of a constant asymptotic growth rate; and (ii) the coincidence of competitive and optimal allocations in the absence of public interventions. As King and Rebelo (1990) put it, the crucial attribute of this class of models is that there is a "core" of capital goods that can be produced without the direct or indirect contribution of nonreproducible factors.[1]

Second, the problem of economic growth and elections has been explored. This line of literature implicitly assumes that a planner can affect the economy by directly choosing capital and/or consumption paths. Beck (1978) politicizes a continuous time, one-sector model of optimal economic growth, where individuals vary only in their rates of time preference. He shows that among a set of optimal plans, the consumption path that is optimal for the voter with the median discount rate is a majority rule core (a political equilibrium). Boylan, Ledyard and McKelvey (1991) study Beck's model in a discrete time setting and prove that there is no majority rule core if nonoptimal plans are feasible. Furthermore, they show that if it is possible for candidates to commit to multi-period plans in a credible fashion, then in general, there will not be a majority rule core. On the other hand, if commitment is impossible, there is a subgame perfect, stationary, symmetric equilibrium to the two-candidate competition game that supports the consumption path that is optimal for the median voter.

The third focus in the literature is the problem of public policy and elections, i.e., the so-called "political business cycle". Underlying the cycle in the original model of Nordhaus (1975) is a Keynesian Phillips curve and voter myopia. MacRae (1977) confirms that if the electorate is myopic, politicians will inflate the economy during election years in order to exploit a Phillips

[1]For a recent survey of this line of literature, see Romer (1989).

curve tradeoff that is more favorable in the short run than in the long run. However, if the voter is rational and votes strategically, vote-loss-minimizing behavior will lead to a long-run inflation-unemployment combination that is a social optimum. Subsequent works study political business cycles without relying on voter myopia. For example, a political budget cycle arises in Rogoff (1990) (see also Rogoff and Sibert (1988)) due to temporary information asymmetries about the incumbent leader's "competence" in administering the public goods production process even if both voters and politicians are rational, utility-maximizing players. By assuming that parties have different exogenous objectives concerning inflation and unemployment, and that voters are rational and forward-looking, Alesina (1987) is able to generate political business cycles due to the fact that the election provides a random shock (see also Alesina, Londregan and Rosenthal (1991)) .[2]

The fourth emphasis has been on the problem of public policy and economic growth. Relying largely on Cobb-Douglas production technology, this strand of literature studies the relationship between public policies and long-term economic growth for a representative consumer without relying on exogenous changes in technology or population. For example, public policy can affect growth rates via the incentives that individuals have to accumulate capital in both its physical and human forms (King and Rebelo (1990)), or via the role of public services as an input to private production and consumption (Barro (1990)). By using the standard convex technology, Jones and Manuelli (1990) are able to show that the long-run growth rate in per

[2]For a review of this strand of literature, see Alesina (1988).

capita consumption depends on the parameters describing tastes, technology, and policies. National growth rates of consumption and output need not converge in a free-trade equilibrium with taxation.

2.2 Model

2.2.1 Basic Assumptions

Consider a free economy with a government, a firm[3] and N consumers. There are two kinds of output, namely a private good X_1 and a public good X_2, and two kinds of input, namely capital K and labor L in the economy. Like Stokey, Lucas and Prescott (1989), I assume P^t to be the price of a unit of output X_1 delivered in period t, and W^t to be the real wage rate in period t. Each consumer i(i$= 1, 2, \ldots, N$) shares a *fixed* part of the firm's profit, $\theta_i \in [0,1]$ and $\sum_{i=1}^{N} \theta_i = 1$. I assume the distribution of θ is common knowledge.

I assume there is an election using some binary procedure under majority rule. Any consumer can be a candidate[4] and compete with other candidate by selecting a fiscal policy in terms of an income tax rate, T, and/or a weight of the public sector, ϕ. Eventually, using a voting procedure to be specified, one of the N consumers is elected and implements the fiscal policy that he

[3]The firm can be thought as an economy. I do not assume multiple firms for simplicity because it is very complicated to compare different profit shares among consumers when there are multiple firms with or without different sizes and products.

[4]I assume no cost for entry because to add a fixed cost does not change my results.

or she prefers.

Assume that each consumer i(i$= 1, 2, \ldots, N$) lives two periods [5] and has a discount factor[6] $\beta \in [0,1]$. In each period i has one unit of time to spend and has a utility function $u(x_{i1}^t, x_2^t)$, where x_{i1}^t and x_2^t are $i's$ consumption levels of the private good X_1 and the public good X_2 in period t respectively.

Assumption 1: $u : R_+^2 \rightarrow R_+^1$ is monotonically increasing, twice continuously differentiable, strictly quasi-concave and additively separable, and satisfies the following Inada conditions:

$$\lim_{x_{i1}^t \downarrow 0} \frac{\partial u(x_{i1}^t, x_2^t)}{\partial x_{i1}^t} = \lim_{x_2^t \downarrow 0} \frac{\partial u(x_{i1}^t, x_2^t)}{\partial x_2^t} = +\infty \text{ and}$$

$$\lim_{x_{i1}^t \uparrow +\infty} \frac{\partial u(x_{i1}^t, x_2^t)}{\partial x_{i1}^t} = \lim_{x_2^t \uparrow +\infty} \frac{\partial u(x_{i1}^t, x_2^t)}{\partial x_2^t} = 0.$$

For simplicity, I henceforth let $u(x_{i1}^t, x_2^t) \equiv \chi(x_{i1}^t) + \xi(x_2^t)$.

Assume that the firm has a production function of the private good given as $f(K^t, L^t)$, where K^t and L^t are the capital input and the labor input in period t, respectively.

Assumption 2: $f : R_+^2 \rightarrow R_+^1$ is monotonically increasing, twice continuously differentiable and strictly quasi-concave, and satisfies $f(0, L) = 0$

[5] It makes no difference for all of my conclusions to assume that each consumer i lives any finite N periods. I will briefly discuss generalization of our conclusion to N-period later.

[6] In fact, our results hold when consumers do not have the same discount factors. However, then, I must assume that all profit shares and discount rates are common knowledge.

($\forall L$) and the following Inada conditions:

$$\lim_{K^t \downarrow 0} \frac{\partial f(K^t, L^t)}{\partial K^t} = \lim_{L^t \downarrow 0} \frac{\partial f(K^t, L^t)}{\partial L^t} = +\infty \quad \text{and}$$

$$\lim_{K^t \uparrow +\infty} \frac{\partial f(K^t, L^t)}{\partial K^t} = \lim_{L^t \uparrow +\infty} \frac{\partial f(K^t, L^t)}{\partial L^t} = 0.$$

In addition, throughout the paper, I also assume: (1) all consumers have the same utility functions;[7] (2) goods will not be wasted; (3) the production technology of the public good is simply one unit of the private good to one unit of the public good.

2.2.2 Notation and Timing of the Model

N is the number of consumers; and t, is a time index, scored 0 or 1; T_j is the tax rate proposed by candidate j ($j = 1, 2, \ldots, N$); ϕ is the proportion of tax revenue used in the production of the public good; $(1 - \phi)$ is the proportion of tax revenue used in the private good transfer; the price of X_1 in period 0 is taken as a numerare, i.e., $P^0 = 1$; $P^1 = P$ is the price of X_1 in period 1; W is the real wage rate; y is the amount of X_1 sold by the firm; K is the aggregate capital input; L is the aggregate labor input; $\pi^0 = y^0 - W^0 L^0$ is the profit of period 0; $\pi^1 = P(y^1 - W^1 L^1)$ is the profit of period 1; θ_i is the fixed share of the profit to consumer i; l_i is the amount of labor supplied by consumer i; β is the discount rate for each consumer; c_i is the pre-transfer consumption

[7]When consumers may have different utility functions, in order to derive the same results, I need to assume all utility functions are common knowledge.

level of X_1 by consumer i; x_{i1} is the total consumption level of X_1 (including transfer from government) by consumer i; x_2 is the consumption level of the public good by each consumer; and U_i is the sum of discounted utility by consumer i. i.e., $U_i = \sum_{t=0}^{1} \beta^t u(x_{i1}^t, x_2^t)$ where

$$\begin{aligned}
x_{i1}^0 &= c_i^0 + \frac{(1-\phi)T}{N}[\sum_{i=1}^{N}(W^0 l_i^0 + \theta_i \pi^0)] \\
x_{i1}^1 &= c_i^1 + \frac{(1-\phi)T}{N}[\sum_{i=1}^{N}(W^1 l_i^1 + \theta_i \frac{\pi^1}{P})] \\
x_2^0 &= \phi T[\sum_{i=1}^{N}(W^0 l_i^0 + \theta_i \pi^0)] \\
x_2^1 &= \phi T[\sum_{i=1}^{N}(W^1 l_i^1 + \theta_i \frac{\pi^1}{P})].
\end{aligned}$$

Each term in brackets is the sum of real wage and profit share across all consumers, i.e., the real gross income, y, in a period. That is, $y = \sum_{i=1}^{N}(W l_i + \theta_i \pi)$.

Following Stokey, Lucas and Prescott (1989), I do not consider leisure in the utility function, and assume candidates as well as consumers maximize their own utilities subject to budget constraints, and the firm maximizes its profit subject to the technical constraint. I constrain the government to run a balanced budget each period and assume that the fiscal policy has two purposes: redistribution of the private good and provision of the public good. In other words, the tax revenue can be used either in the transfer of the private good for the purpose of redistribution[8] and/or the production of

[8]Although I do not explicitly discuss progressive income taxation in the paper, this setting resembles a kind of progressive taxation when T and ϕ are high, and allows far

the public good. The model and its timing[9] are given below (see Figure 2.1):

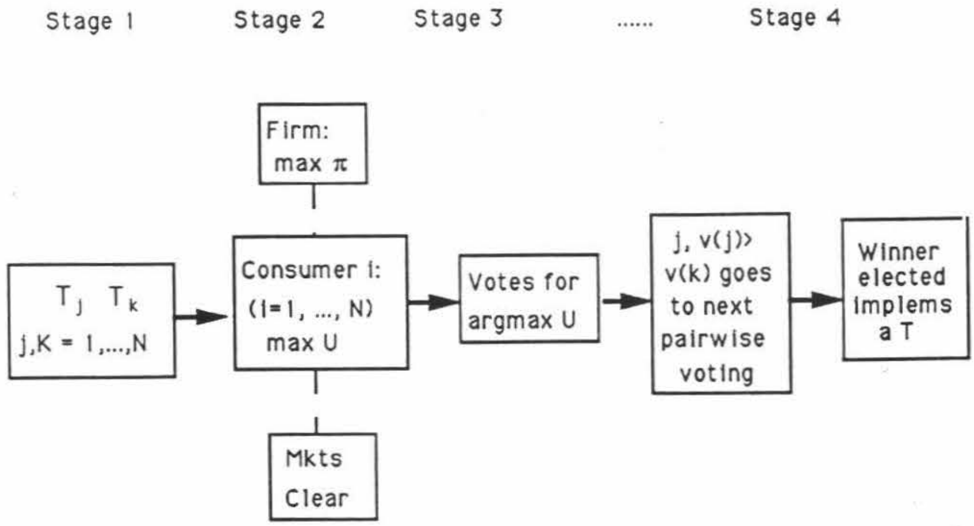


Figure 2.1: Timing for a Free Economy

In stage 0, given the set of consumers and a fixed binary voting procedure, each consumer decides whether he or she will be a candidate. If he or she decides not to be a candidate, his or her opponent automatically wins the binary competition and goes to the next branch of the voting tree. Figure 2.2 gives a voting tree with 4 consumers and a voting procedure in the order of 1, 2, 3 and 4.

In stage 1, In each branch of the voting tree, a candidate j (j =

more flexibility in the tax structure.

[9]The timing of voting on ϕ is similar.

$1, 2, \ldots, N$) proposes a tax rate T_j in order to maximize his or her utility which is given in stage 2.2.

In stage 2.1,[10] for any given P^t and $W^t(t = 0, 1)$, the firm chooses L^t and $K^{t+1}(t = 0, 1)$ to maximize the sum of periodic profits, i.e.,

$$\begin{aligned} \max \pi &= \max \sum_{t=0}^{1} P^t [y^t - W^t L^t] \\ &= \max[(y^0 - W^0 L^0) + P(y^1 - W^1 L^1)] \qquad (2.1) \\ \text{s.t. } y^t &\leq f(K^t, L^t) + (1 - \delta)K^t - K^{t+1} \qquad (2.2) \\ K^1 &\geq 0,\ t = 0, 1, \text{ given } K^0 = \bar{K}. \end{aligned}$$

Where δ is the depreciation rate of the capital.

In stage 2.2, for any given $\phi, T, \theta_i, P^t, W^t$ and $\pi^t(t = 0, 1)$, consumer $i(i = 1, 2, \ldots, N)$ chooses c_i^t, l_i^t to

$$\max U_i = \max \sum_{t=0}^{1} \beta^t u(x_{i1}^t, x_2^t) \qquad (2.3)$$

$$= \max \sum_{t=0}^{1} \beta^t u \left\{ c_i^t + \frac{(1-\phi)T}{P^t N} [\sum_{i=1}^{N} (P^t W^t l_i^t + \theta_i \pi^t)], \frac{\phi T}{P^t} [\sum_{i=1}^{N} (P^t W^t l_i^t + \theta_i \pi^t)] \right\}$$

$$= \max \sum_{t=0}^{1} \beta^t u \left\{ c_i^t + \frac{(1-\phi)T}{N} y^t, \phi T y^t \right\}$$

[10]The following substages 2.1, 2.2 and 2.3 take place at the same time.

$$s.t. \sum_{t=0}^{1} P^t c_i^t \leq \sum_{t=0}^{1} (1-T)[P^t W^t l_i^t + \theta_i \pi^t] \tag{2.4}$$

$$0 \leq l_i^t \leq 1,\ c_i^t \geq 0,\ 0 \leq \theta_i \leq 1,\ \sum_{i=1}^{N} \theta_i = 1, \text{ and } t = 0, 1. \tag{2.5}$$

In stage 2.3, all markets clear, i.e., $\sum_{i=1}^{N} l_i^t = L^t$ and $\sum_{i=1}^{N} c_i^t = (1-T)y^t$. Combining equations (2.1) to (2.5) and market-clearing equations together, each consumer can solve for a competitive equilibrium: $(K^1)^e$, $(W^0)^e$, $(W^1)^e, P^e, (l_i^0)^e$, $(l_i^1)^e$, $(c_i^0)^e$ and $(c_i^1)^e$ $\forall$i$= 1, 2, \ldots, N$.

In stage 3, in each binary competition (i.e., a branch of the binary tree), each consumer $i (i = 1, 2, \ldots, N)$ votes for

$$j = \arg\max \sum_{t=0}^{1} u({x_{i1}^t}^e(T_j), {x_2^t}^e(T_j)),$$

and if indifferent, then splits the vote.

In stage 4, the candidate with the larger number of votes in each binary competition is elected, and implements a tax policy.

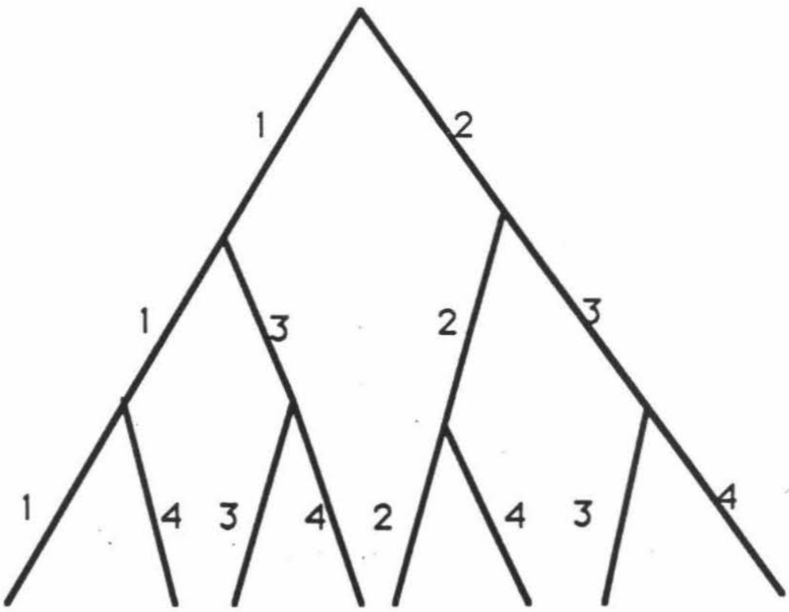


Figure 2.2: A Fixed Binary Procedure When N = 4.

Remark: First, if there is no campaign cost, it is clear that each consumer *is* a potential candidate; however, if there is a positive campaign cost, then only those consumers with expected benefits higher than costs will run a campaign. Second, although I assume there is a fixed binary voting procedure in stage 0, my results hold for any binary voting procedure as we will see later. Third, candidates' promises are not necessarily *binding.* They can do whatever they like if they win office.

Although there are some similarities between the model of Stokey-Lucas

(1989) and mine, such as a finite time horizon and disutility of labor, there are some fundamental differences between these two models: first, I include a public good. Second, the firm owns capital, so capital has only implicit cost, and there is no capital market here. Furthermore because of the ownership of capital, the firm makes profits that are proportionally shared by consumers. Third, by introducing an income tax rate, T, and a weight of the public sector, ϕ, I am able to study the relationship between the income distribution and voting behavior on fiscal policy.

2.2.3 Definitions

First, I define two crucial concepts: a competitive equilibrium and a political-economic equilibrium.

Definition 1: Given T and ϕ, a *competitive equilibrium in a free economy* is a set of prices and wages $\{(P^t, W^t)\}_{t=0}^{1}$, an allocation $\{(L^t, K^{t+1})\}_{t=0}^{1}$ for the firm, and an allocation $\{(c_i^t, l_i^t)\}_{t=0}^{1}$ for consumer $i (i = 1, 2, \ldots, N)$, s.t.,

a. $\{(L^t, K^{t+1})\}_{t=0}^{1}$ solves (2.1) and (2.2) at the stated prices;

b. $\{(c_i^t, l_i^t)\}_{t=0}^{1}$ solves (2.3) to (2.5) at the stated prices;

c. all markets clear, i.e., $\sum_{i=1}^{N} l_i^t = L^t$ and $\sum_{i=1}^{N} c_i^t = (1-T)y^t$, for all $t = 0, 1$.

Definition 2: *A political-economic equilibrium*[11] *in a free economy* is a set containing a consumer (or consumers) $\{j\}_{j=1,2,\ldots,N}$, an income tax rate

[11] A political-economic equilibrium can be similarly defined by substituting T with ϕ when consumers vote on ϕ, or adding ϕ to T when consumers vote on T and ϕ.

$\{T_j\}$ (or a set of T_j), and a competitive equilibrium corresponding to T_j, denoted by $c.e.(T_j)$, such that $\{j, T_j, c.e.(T_j)\}$ solves the model (stages 1 to 4) in 2.2.2.

In addition, I present the following definitions of the Bowen-Lindahl-Samuelson condition, and fiscal and distributional neutrality:[12]

Definition 3: An allocation $(x_{i1}^0, \ldots, x_{iN}^0, x_2^0; x_{i1}^1, \ldots, x_{iN}^1, x_2^1)$ satisfies the Bowen-Lindahl-Samuelson condition if, for each period, the sum over all consumers of the marginal rates of substitution between the public good and the private good is equal to the marginal rate of transformation in production between these two good, i.e., there is (T, ϕ) such that[13]

$$\sum_{i=1}^{N} \frac{u_2^0}{u_1^0} = 1 \text{ and } \sum_{i=1}^{N} \frac{u_2^1}{u_1^1} = 1,$$

where superscripts are time index and subscripts denote partial derivatives with respect to each argument.

Definition 4: *A free economy is fiscally and distributionally neutral* if all macroeconomic variables K^1, W^0, W^1 and P are independent of T, ϕ and θ.

[12]The definition of fiscal neutrality here differs from tax neutralities in Bradford (1980) and Harberger (1980). Bradford defines tax neutrality of investment subsidy as the case when income tax system influences investment only via its effect on savings not via the composition of the capital stock. Tax neutrality in Harberger (1980) requires some social rate of return, such that all independent investment projects meeting the rate will tend to be privately accepted, while no project failing to meet that rate will be privately accepted.

[13]Because of the assumption that each one unit of the private good can produce one unit of the public good, the marginal rate of transformation in production is one.

2.3 Main Conclusions

2.3.1 Existence of A Competitive Equilibrium

First, I derive the first order conditions for the firm and consumers. For a typical consumer, it is obvious that he or she supplies all of the available factor L, because L causes no disutility to him, i.e., $l_i^0 = l_i^1 = 1, L^0 = L^1 = N$. It is clear that $K^2 = 0$ because period 2 is the end of the world. Since $\lim_{K \to 0} f_1(K^1, L) = +\infty$, the nonnegativity constraint on K^1 in (2.2) is never binding, i.e., $K^1 > 0$. Now let

$$F(K^t) = f(K^t, N) + (1-\delta)K^t,\ t = 0, 1.$$

Because of the assumption that goods will not be wasted, (2.2) becomes

$$\begin{aligned} y^0 &= F(\bar{K}) - K^1 \text{ and} \\ y^1 &= F(K^1). \end{aligned}$$

Thus I have shown that L^0, L^1 and $K^1 > 0$. By Kuhn-Tucker Theorem (see Dixit (1990)), the first order conditions for the firm are:

$$f_2(\bar{K}, N) = W^0 \tag{2.6}$$

$$f_2(K^1, N) = W^1 \tag{2.7}$$

$$P = \frac{1}{F'(K^1)}. \tag{2.8}$$

Let $u(x_{i1}^t, x_2^t) = \chi(x_{i1}^t) + \xi(x_2^t)$. Then the first order conditions for a consumer i ($\forall$i$= 1, 2, \ldots, N$) are:

$$\begin{aligned}
\chi' \left\{ c_i^0 + \frac{(1-\phi)T}{N}[F(\bar{K}) - K^1] \right\} &\leq \lambda_i, \qquad c_i^0 \geq 0 && (2.9) \\
\beta\chi'[c_i^1 + \frac{(1-\phi)T}{N} F(K^1)] &\leq \lambda_i P, \qquad c_i^1 \geq 0 && (2.10) \\
\sum_{t=0}^{1}(1-T)[P^t W^t + \theta_i \pi^t] &\geq \sum_{t=0}^{1} P^t c_i^t, \quad \lambda_i \geq 0 \text{ i.e.,} && (2.11) \\
\sum_{t=0}^{1}(1-T)[P^t y^t + P^t W^t (1-\theta_i N)] &\geq \sum_{t=0}^{1} P^t c_i^t, \quad \lambda_i \geq 0.
\end{aligned}$$

The conditions for market clearing are:

$$\begin{aligned}
\sum_{i=1}^{N} c_i^0 &= (1-T)y^0 \text{ and} && (2.12) \\
\sum_{i=1}^{N} c_i^1 &= (1-T)y^1. && (2.13)
\end{aligned}$$

Proposition 1: Given T and ϕ, there exists a competitive equilibrium.

Proof: Since a consumer cannot do anything about the public good for given T and ϕ and his or her utility is separable in time, the current problem is equivalent to a neoclassical private ownership production economy with one producer, N consumers and two goods, i.e., a present private good and a future private good. Because of the assumptions I make about utility functions and the production function (monotonically increasing, twice continuously differentiable, and strictly quasiconcave), it is easy to verify that all Arrow-Debreu conditions are satisfied, thus there exists a competitive equilibrium (see Debreu (1959), Aliprantis, Brown and Burkinshaw (1989)).

Q.E.D.

Remark: Proposition 1 and the following results can be generalized to the case of any finite N-period game. Roughly speaking, given T and ϕ, the problem given in (2.1) to (2.5) and market clearing conditions for any finite periods t is equivalent to a neoclassical private ownership production economy with one producer, N consumers and t goods (private good in each period corresponds to a good). It can be shown by the Arrow-Debreu Theorem that there is a competitive equilibrium.

Proposition 2: Given the distribution of θ, the wealthier a consumer $i(i = 1, 2, \ldots, N)$ is, the more private good he or she consumes, and the better off he or she is.[14]

Proof: Taking derivatives with respect to θ_i in (2.9) to (2.11), we get

$$\frac{\partial c_i^0}{\partial \theta_i}\chi_0'' - \frac{\partial \lambda_i}{\partial \theta_i} = 0 \qquad (2.14)$$

$$\beta\frac{\partial c_i^1}{\partial \theta_i}\chi_1'' - P\frac{\partial \lambda_i}{\partial \theta_i} = 0 \qquad (2.15)$$

$$(1-T)(\pi^0 + \pi^1) - \sum_{t=0}^{1} P^t \frac{\partial c_i^t}{\partial \theta_i} = 0. \qquad (2.16)$$

[14]Throughout most parts of the paper, I take the distribution of θ as given. So for any given distribution of θ, T and ϕ are endogenously determined, so is the political-economic equilibrium. Therefore, in propositions 2, 3 and 4, which study the relation between wealthiness and the attitude towards fiscal policies, all derivatives with respect to θ or income refer to *the switch of consumers with different profit shares in the same distribution of θ rather than changing of the distribution.* In 4.3, I study the relationship between tax rate and *income distribution* when the elasticity of marginal utility of income is constant.

Substituting (2.14) and (2.15) into (2.16), we have

$$(1-T)(\pi^0+\pi^1)=\left\{\frac{1}{\chi_0''}+\frac{P^2}{\beta\chi_1''}\right\}\frac{\partial\lambda_i}{\partial\theta_i}.$$

Recalling $\chi_0'', \chi_1'' < 0$ (Assumption 1), we get $\frac{\partial\lambda_i}{\partial\theta_i} < 0$. Substituting $\frac{\partial\lambda_i}{\partial\theta_i} < 0$ into (2.14) and (2.15) yields $\frac{\partial c_i^0}{\partial\theta_i}, \frac{\partial c_i^1}{\partial\theta_i} > 0$ and

$$\frac{\partial U}{\partial\theta_i}=\frac{\partial c_i^0}{\partial\theta_i}\chi_0'+\beta\frac{\partial c_i^1}{\partial\theta_i}\chi_1' > 0,$$

where χ_0' and χ_1' are the first derivatives of utility at $t=0$ and $t=1$ with respect to X_1 respectively. **Q.E.D.**

2.3.2 Voting on the Tax Rate

Now in order to derive proposition 3 concerning the relationship between wealthiness and attitude towards the tax rate (or the size of government), I need the following condition:

Condition α: $R_r(x)=\frac{(-x\partial^2U/\partial x^2)}{(\partial U/\partial x)}\le 1\ \forall x\in R$, i.e., a utility function for the private good has an elasticity of marginal utility of income (EMUI) not greater than one. [15]

[15] This condition is generally true for a developed economy with a relatively equal distribution of income, in which marginal utilities will not respond to changes in income dramatically, since consumers are relatively rich and the marginal utility is decreasing. However this condition may not hold for an undeveloped economy.

Proposition 3: If condition α is satisfied, then given ϕ and the distribution of θ, wealthier consumers are more likely to oppose a larger government, i.e.,

$$\frac{\partial(\partial U/\partial T)}{\partial \theta_i} < 0.$$

Proof: Let $I_i = LHS(2.11)$ to be $i's$ income. If one consumer's income differs from the income of other consumers, it only differs on θ_i. There is a positive relation between I_i and θ_i. Hence I need to prove $\frac{\partial(\partial U/\partial T)}{\partial I_i} < 0$.

Recalling $u(x_{i1}^t, x_2^t) = \chi(x_{i1}^t, x_2^t) + \xi(x_2^t)$ and taking the derivative with respect to T in (2.3), we have

$$\begin{aligned}\frac{\partial U}{\partial T} &= \left\{\frac{\partial c_i^0}{\partial T} + \frac{(1-\phi)}{N}[F(\bar{K}) - K^1]\right\}\chi_0' + \phi[F(\bar{K}) - K^1]\xi_0' + \\ &+ \beta\left\{\frac{\partial c_i^1}{\partial T} + \frac{(1-\phi)}{N}F(K^1)\right\}\chi_1' + \beta\phi F(K^1)\xi_1'. \qquad (2.17)\end{aligned}$$

Now taking the derivative with respect to T in (2.11) yields,

$$\sum_{t=0}^{1} P^t \frac{\partial c_i^t}{\partial T} = -\sum_{t=0}^{1}[P^t W^t + \theta_i \pi^t]. \qquad (2.18)$$

Using (2.9), (2.10) and (2.18), (2.17) can be rewritten as

$$\begin{aligned}\frac{\partial U}{\partial T} &= -\lambda_i \sum_{t=0}^{1}[P^t W^t + \theta_i \pi^t] + \frac{(1-\phi)}{N}[F(\bar{K}) - K^1]\chi_0' + \\ &\quad \phi[F(\bar{K}) - K^1]\xi_0' + \beta\frac{(1-\phi)}{N}F(K^1)\chi_1' + \beta\phi F(K^1)\xi_1'.\end{aligned}$$

Recalling $I_i = \sum_{t=0}^{1}(1-T)[P^t W^t + \theta_i \pi^t]$ and taking the derivative with respect to I_i in the above equation gives

$$\frac{\partial}{\partial I_i}(\frac{\partial U}{\partial T}) = -\frac{\lambda_i}{1-T} - \frac{I_i}{1-T}\frac{\partial \lambda_i}{\partial I_i} + \frac{(1-\phi)}{N}[y^0 \frac{\partial c_i^0}{\partial I_i}\chi_0^{''} + Py^1 \frac{\partial c_i^1}{\partial I_i}\chi_1^{''}].$$

Similarly as in Proposition 2, we have $\frac{\partial c_i^0}{\partial I_i}, \frac{\partial c_i^1}{\partial I_i} > 0$. Using the above inequalities, $\lambda_i = \frac{\partial U}{\partial I_i}$ and $R_r(x) \leq 1$ yields

$$\frac{\partial}{\partial I_i}(\frac{\partial U}{\partial T}) = -\frac{\partial U/\partial I_i}{1-T}[1 - R_r(I_i)] + \frac{(1-\phi)}{N}[y^0 \frac{\partial c_i^0}{\partial I_i}\chi_0^{''} + Py^1 \frac{\partial c_i^1}{\partial I_i}\chi_1^{''}] < 0.$$

Q.E.D.

Proposition 3 claims that when the marginal utility of income for the private good is not too elastic,[16] i.e., not too sensitive to a change in income, then given the distribution of income and the percentage of tax revenue used in public good production, the poor would like an expansionary fiscal policy (i.e., a fiscal policy with the higher tax rate) or a larger government. Some direct evidence supports the negative relationship between wealthiness and attitude towards the size of government. For example, Sears and Citrin (1982) study the California tax revolt of 1978 and find that income is linearly related to support for the tax revolt (Tables 5.1 and 5.3, Sears and Citrin (1982)).

Second, it is a "stylized fact" that one of the obvious differences between the major American political parties is found in the social and economic status of their supporters. Republicans are likely to have higher incomes

[16]At this point, it is not clear how far I can weaken condition α in order to derive Proposition 3. However, an example is provided at the end of Subsection 2.4.4 to show that when condition α is violated, counter-intuitive results may follow.

than Democrats. Therefore the attitudes about the size of government of the Democratic and Republican parties to some extent reflect the attitudes of the poor and the rich respectively. Tufte points out that in 1976, the Republican platforms are significantly more concerned of federal spending (22 times vs. 3 times), size and cost of government (11 vs. 2), taxes (45 vs. 37), and private sector (10 vs. 3) than the Democratic platforms (Table 4-1, Tufte (1978)). This conclusion holds for other countries as well. As we see that parties of the Left have traditionally favored a more powerful central government and an expansion of the public economy than parties of the Right. From 1945 to 1969, each additional decade of left-wing government control means an additional 10 percentage point increase in government receipts (Figures 4-4 and 4-5, Tufte (1978)). Kiewiet finds that during 1960-1980, compared to the Republicans, the Democrats were more likely to believe that new or large federal programs were needed; while they were less likely to believe that too much government spending was the nation's worst problem (Figure 6.2, Kiewiet (1983)). Browning states that during 1947-1982, Democratic presidential policy initiatives outnumber the Republican initiatives by a factor of 4 and almost twice as many programs were initiated during periods of Democratic control of both the Presidency and Congress than during all other three combinations of Presidencies and Congress (Table 5-4, Browning (1986)). Similarly, Democratic Presidents and Congresses are far more likely to initiate large programs than Republican Presidents and Congresses (Table 5-5, Browning (1986)).

Lemma 1: Given ϕ and the distribution of θ_i, for any voter i

$(i = 1, \ldots, N)$, $\frac{\partial}{\partial T}(\frac{\partial U}{\partial T}) < 0$.

Proof: Taking the derivative with respect to T in (2.18) yields $\sum_{t=0}^{1} P^t \frac{\partial^2 c_i^t}{\partial T^2} = 0$. Taking the derivative with respect to T in (2.17) gives

$$\begin{aligned}
\frac{\partial}{\partial T}(\frac{\partial U}{\partial T}) &= \left\{\frac{\partial c_i^0}{\partial T} + \frac{(1-\phi)}{N}[F(\bar{K}) - K^1]\right\}^2 \chi_0^{''} + \left\{\phi[F(\bar{K}) - K^1]\right\}^2 \xi_0^{''} \\
&+ \beta[\frac{\partial c_i^1}{\partial T} + \frac{(1-\phi)}{N} F(K^1)]^2 \chi_1^{''} + \beta[\phi F(K^1)]^2 \xi_1^{''} \\
&+ \frac{\partial^2 c_i^0}{\partial T^2}\chi_0^{'} + \frac{\partial^2 c_i^1}{\partial T^2}\beta\chi_1^{'} \\
&= \left\{\frac{\partial c_i^0}{\partial T} + \frac{(1-\phi)}{N}[F(\bar{K}) - K^1]\right\}^2 \chi_0^{''} + \left\{\phi[F(\bar{K}) - K^1]\right\}^2 \xi_0^{''} \\
&+ \beta[\frac{\partial c_i^1}{\partial T} + \frac{(1-\phi)}{N} F(K^1)]^2 \chi_1^{''} + \beta[\phi F(K^1)]^2 \xi_1^{''} \\
&+ \chi_0^{'}[\frac{\partial^2 c_i^0}{\partial T^2} + P\frac{\partial^2 c_i^1}{\partial T^2}] < 0.
\end{aligned}$$

The second equality used $\frac{\beta\chi_1^{'}}{\chi_0^{'}} = P$, which was derived from (2.9) and (2.10),[17] and the last inequality used $\sum_{t=0}^{1} P^t \frac{\partial^2 c_i^t}{\partial T^2} = 0$, and the assumption that all second derivatives are negative. **Q.E.D.**

$\frac{\partial U}{\partial T}$ is shown in a $(T, \frac{\partial U}{\partial T})$ space in Figure 2.3 by Proposition 3 and Lemma 1. I am now able to give the main conclusion of this chapter. The following theorem provides us with the conditions and outcome of a political-economic equilibrium for voting over tax rates in a free economy.

[17] I only consider the interior case here. The boundary cases can be similarly derived.

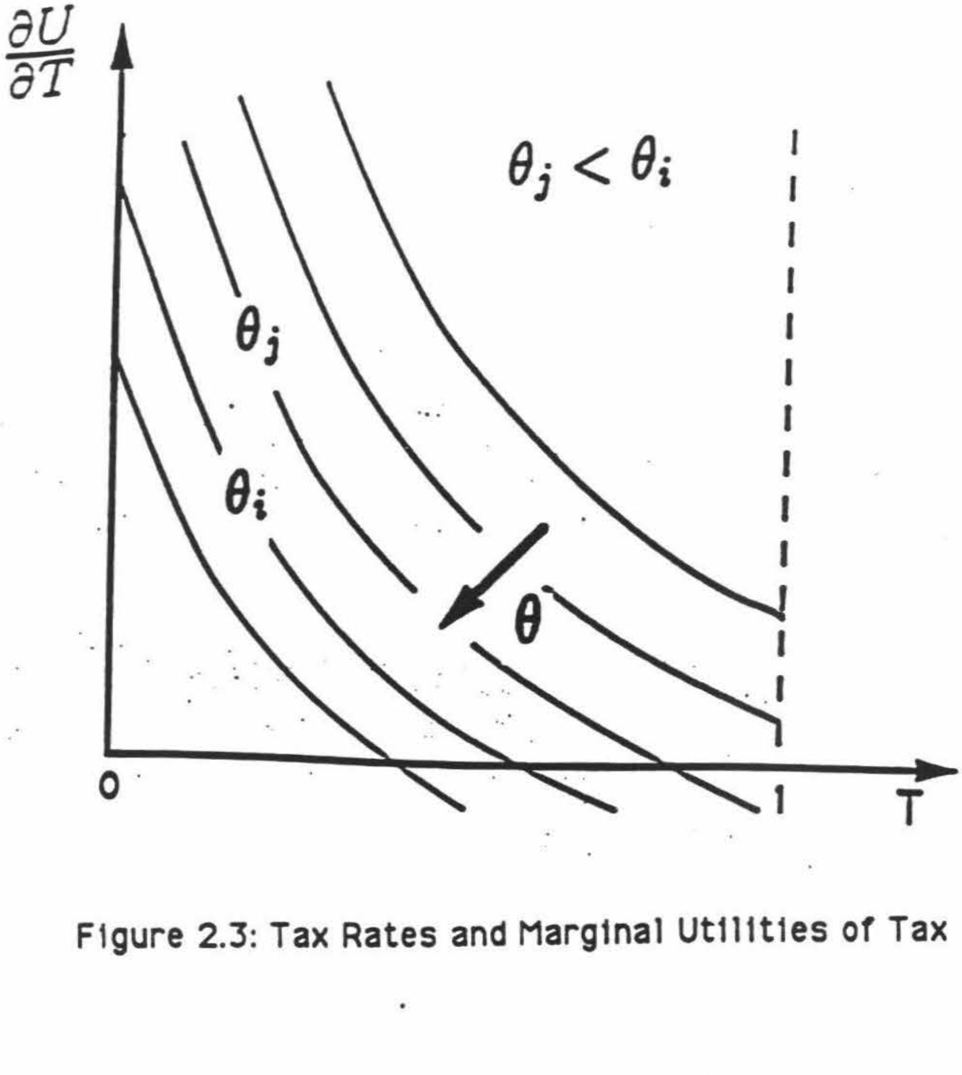


Figure 2.3: Tax Rates and Marginal Utilities of Tax

Theorem 1: (1) Given ϕ and the distribution of θ_i, when condition α is satisfied, the median-income voter, m, will prevail in elections with any binary procedure under majority rule; the political-economic equilibrium is $\{m, T_m^e, c.e.(T_m^e)\}$, where $T_m^e = \arg\max\{U_m^e(1), U_m^e(T^*)\}$, $U_m^e(1)$ and $U_m^e(T^*)$ are utilities for voter

m in competitive equilibrium when $T = 1$ and $T = T^*$ respectively, and T^* satisfies $\frac{\partial U_m}{\partial T}\Big\|_{T=T^*} = 0$.

Proof: (1) **Step 1**, for any voter $i (i = 1, \ldots, N)$, T_i^e is either on the boundary, i.e., 1, [18] or an interior point T^* which satisfies the first order condition: $\frac{\partial U_i}{\partial T}\Big\|_{T=T^*} = 0$.

Step 2, if $\theta_i > \theta_j$, $\forall i, j = (1, \ldots, N)$, then $T_i^e \leq T_j^e$.

(i) If $T_j^e = 1$, it is obvious.

(ii) If $T_i^e = 1$, then according to step 1, it must be the case that $\frac{\partial U_i}{\partial T} > 0$, $\forall T \in [0, 1)$. By Proposition 3, we have $\frac{\partial U_j}{\partial T} > 0$, $\forall T \in [0, 1)$. Therefore $T_j^e = 1$.

(iii) If $T_i^e, T_j^e < 1$. Suppose not, $T_i^e \geq T_j^e$, then using Lemma 1 and Proposition 3 in sequence, we have $0 = \frac{\partial U}{\partial T}\Big\|_{\theta_i, T_i} \leq \frac{\partial U}{\partial T}\Big\|_{\theta_i, T_j} < \frac{\partial U}{\partial T}\Big\|_{\theta_j, T_j} = 0$, which is a contradiction.

Step 3, Step 2 implies that all individual preferences belong to the class of "intermediate preferences" studied by Grandmont (1978). This class has the following useful property: individual preferences are indexed by the parameter θ_i and the distribution of preferences within the group is fully summarized by the distribution of θ_i. As shown by Grandmont (1978), since θ_i is a scaler, preferences are single peaked. Clearly, provided that all tax policy options are compared pairwise, the most preferred tax policy of the median voter is a majority alternative; then according to McKelvey and Niemi (1978), for

[18] $T_i^e = 0$ is impossible because of the Inada condition:

$$\lim_{x_{i2}^t \downarrow 0} u_2(x_{i1}^t, x_2^t) = +\infty.$$

general binary procedures, multistage sophistication will assure the adoption of that alternative.

Step 4, the political-economic equilibrium can be computed by solving the problem (2.6) to (2.13), with T^e maximizing the utility of the median voter. **Q.E.D.**

2.3.3 Voting on the Weight of the Public Sector

The timing of the model and definitions of a competitive equilibrium and a political-economic equilibrium are the same as those in 2.2 and 2.3 except now consumers vote on ϕ instead of voting on T. The following proposition, which is more robust than Proposition 3 in the sense that it can be derived without condition α, studies a case when government controls ϕ and takes T as given.

Proposition 4: Given the tax rate T and the distribution of income, wealthier consumers are more likely to oppose a redistribution-oriented fiscal policy, i.e.,

$$\frac{\partial}{\partial \theta_i}\left(\frac{\partial U}{\partial (1-\phi)}\right) < 0; \; or \; \frac{\partial}{\partial \theta_i}\left(\frac{\partial U}{\partial \phi}\right) > 0.$$

In other words, wealthier consumers would like government to spend more tax revenue on the production of the public good.

Proof: For any given $T \in [0,1]$, $\frac{\partial U}{\partial \phi} = \sum_{t=0}^{1} \beta^t T[F(K^t) - K^{t+1}](\xi_t' - \frac{1}{N}\chi_{t\cdot}')$

Using Assumption 1 and Proposition 2, we have

$$\frac{\partial}{\partial\theta_i}(\frac{\partial U}{\partial\phi}) = -\sum_{t=0}^{1}\beta^t T[F(K^t) - K^{t+1}]\frac{1}{N}\chi_t''\frac{\partial c_i^t}{\partial\theta_i} > 0. \qquad \textbf{Q.E.D.}$$

Proposition 4 states that given the distribution of income and the tax rate, wealthier consumers are more likely to oppose a redistribution-oriented fiscal policy. Some empirical studies confirm this result. Tufte claims that in 1976, compared to the Republican platforms, the Democratic platforms are much more concerned about the distributional issues, such as inequality (30 times vs. 15 times), opportunity (24 vs. 7), and poverty (23 vs. 3) (Table 4-1, Tufte (1978)). He also shows a positive relationship between the high degree of income equalization (measured by the difference between pre-tax and post-tax income for the top 20 percent of households) and the extent of left-wing control of the executive branch in ten industrialized countries (Figure 4-3, Tufte (1978)).

Theorem 2: Given T and the distribution of θ_i, the median-income voter will beat anyone else in elections with any binary procedure under majority rule; the political-economic equilibrium is $\{m, \phi_m^e, c.e.(\phi_m^e)\}$, where $\phi^e = \arg\max\{U_m^e(1), U_m^e(\phi^*)\}$, $U_m^e(1)$ and $U_m^e(\phi^*)$ are utilities for the median voter m in competitive equilibrium when $\phi = 1$ and $\phi = \phi^*$ respectively, and ϕ^* satisfies $\frac{\partial U_m}{\partial\phi}\Big\|_{\phi=\phi^*} = 0$.

Proof: Similar to the proof of Theorem 1.

2.3.4 Voting on the Tax Rate and the Weight of Public Sector

Now consider the case when candidates are allowed to choose ϕ as well as T. Although median-voter-like results follow for both one-dimensional voting cases (i.e., voting on T and ϕ respectively), this may not hold when the voting on T and ϕ is simultaneous and voters are sophisticated. For example, suppose there are three consumers L, M and S in a free economy with $\theta_L > \theta_M > \theta_S$. Then by Theorem 1, given ϕ, $T_L < T_M < T_S$; and by Theorem 2, given T, $1-\phi_L < 1-\phi_M < 1-\phi_S$. Suppose the ideal points for voters L, M and S look like those in Figure 4, then L and S would like to select a point in the shadow area, say N, which is preferred over M for both of them.

What will happen if we focus on sequential voting on T and ϕ (similarly for voting on ϕ and T)? Suppose the timing and definitions of competitive equilibrium and political-economic equilibrium are similar to those in Chapter 2 except now after selecting a majority winner through a binary process in dimension T, consumers vote on ϕ and select a majority winner in dimension ϕ. The rule of the game is as follows: only the majority winners (or winner) on T can be candidates (or a candidate) in the second stage (i.e., voting on ϕ), and the majority winner in the second stage wins the office. This rule precludes coalitional government and guarantees a sole majority winner, thus any fiscal policy will locate on the contract curve of the consumers' ideal points, i.e., any majority winner will implement her ideal T and ϕ. Theorem 3 guarantees the median-income voter as the sole majority winner when the voting is sequential.

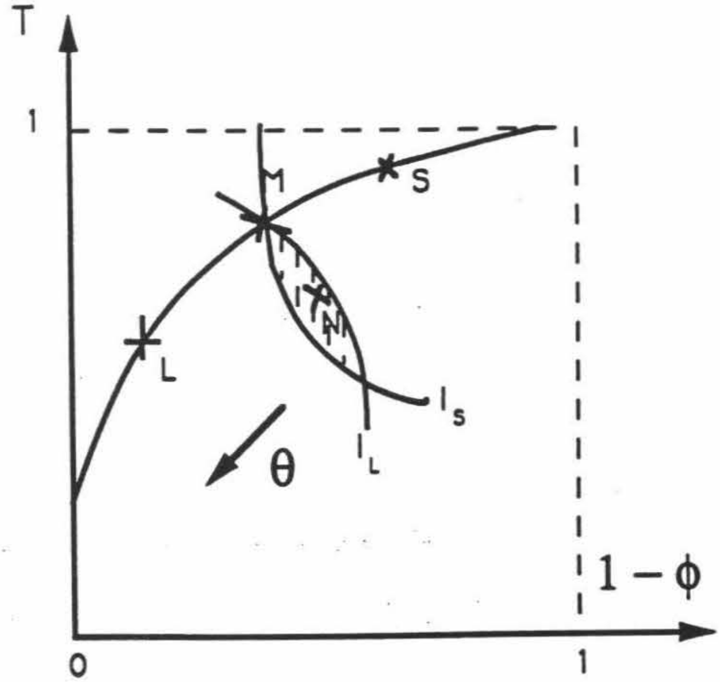


Figure 2.4: Simultaneous Voting Over (T, φ)

Theorem 3: The median-income voter will prevail in any sequential voting over T and ϕ regardless of the agenda (i.e., voting on T and then ϕ, or on ϕ and then T). The political-economic equilibrium is $\{m, T_m^e, \phi_m^e, c.e.(T_m^e, \phi_m^e)\}$, where T_m^e and ϕ_m^e are 1 or the solutions of simultaneous equations $\frac{\partial U_m}{\partial T}\Big\|_{T=T^*} = 0$ and $\frac{\partial U_m}{\partial \phi}\Big\|_{\phi=\phi^*} = 0$.

Proof: It is known that every candidate $j(j = 1, 2, \ldots, N)$ will propose his or her most preferable (T_j, ϕ_j) if he or she comes to office because the campaign promise is not binding, so the political competition over space

(T, ϕ) focuses on the contract curve of the consumers' ideal points. I will prove the median-income consumer m is the Cordocet winner. I only consider the interior equilibrium T and ϕ, i.e., $T,\ \phi < 1$. Corner equilibrium cases can be similarly proved.

First, $\forall$ j ($j = 1, 2, \ldots, N$), assume $\theta_N \geq \theta_j \geq \ldots > \theta_m \geq \ldots \geq \theta_1$ (similarly for $\theta_j < \theta_m$), then $1 > T_m > T_j$ and $1 > \phi_j > \phi_m$. By contradiction, if $1 > T_j \geq T_m$, then because (T_j, ϕ_j) is the most preferable choice for j, we have $(T_j, \phi_j) \succeq_j (T_m, \phi_j)$ with $T_j \geq T_m$, a contradiction to Proposition 3. Similarly $1 > \phi_j > \phi_m$.

Second, following Theorem 2 and Theorem 1 sequentially,

$$(T_m, \phi_m) \succ_{1,\ldots,m} (T_m, \phi_j) \succ_{1,\ldots,m} (T_j, \phi_j),$$

where "$\succ_{1,\ldots,m}$" is the preference relation by at least consumers $1, 2, \ldots, m$. Thus the median-income voter beats any other voter in binary competition.

Q.E.D.

It is interesting to note that theorems 1, 2 and 3 show that a version of the median voter theorem (Downs (1956)) holds in my setting. An economic version of the median voter theorem is the so called Director's Law. Stigler (1970) explains the philosophy of the Director's law as follows: "Government has coercive power, which allows it to engage in acts (above all, the making of resources) which could not be performed by voluntary agreement of the members of a society. *Any portion of the society which can secure control of* *the state's machinery will employ the machinery to improve its own position.* ... **This dominant group will be the middle income classes."** My result is reminiscent of that of Kramer and Snyder (1983), namely, the observed stability and progressivity of income taxation in democratic societies arises from the success of the middle class in minimizing its own tax burden, at the expense of upper- and low-income taxpayers.

Remark: There are two special cases regarding the distribution of income that should be considered: (1) If $\theta_i = \theta$ for all $i (i = 1, 2, \ldots, N)$, then the wealth of the society is uniformly distributed. In this case, anyone is a median-income voter. Since all candidates will choose the same platform, anyone may win the election. It is interesting that in this case, the Bowen-Lindahl-Samuelson condition implies a political-economic equilibrium as we will see in the following Proposition 5. (2) If $\exists j (j = 1, 2, \ldots, N), s.t., \theta_j = 1$, i.e., there exists a consumer j who is the only owner of the firm, then unless j is a dictator or has veto power, anyone other than j can be the winner, and it is obvious that $T^e = 1$. This can be generalized to the conclusion that in an extremely unequal society (i.e., few consumers control the economy), any political election will inevitably result in complete equalization of wealth.

2.3.5 Uniform Distribution of Income

In public economics, the so-called Bowen-Lindahl-Samuelson condition (see Laffont (1988)) is well-known. The following proposition states that the

Bowen-Lindahl-Samuelson condition implies a political-economic equilibrium when the distribution of income is uniform.

Proposition 5: (1) If the distribution of income is uniform, then to satisfy the Bowen-Lindahl-Samuelson condition, i.e., there is (T,ϕ) such that $\sum_{i=1}^{N}\frac{\xi_0'}{\chi_{i0}'}=1$ and $\sum_{i=1}^{N}\frac{\xi_1'}{\chi_{i1}'}=1$ is a sufficient condition for solving the political-economic equilibrium.

(2) When $\chi(.)=\xi(.)=g(.)$ and $\frac{g'(z)}{g'(\hat{z})}=g'(\frac{z}{\hat{z}})$, then the above sufficient condition is necessary too.

Proof: (1) On the one hand, when $\theta_i=1/N$, $c_i^t=\frac{(1-T)}{N}y^t$, then $x_{1t}=\frac{y^t}{N}(1-\phi T)$ and $x_{2t}=\phi T y^t$. Thus $\frac{\partial U}{\partial T}=0$ and $\frac{\partial U}{\partial \phi}=0$ lead to

$$\begin{aligned} 0 &= \phi\left\{-\frac{y^0}{N}\chi_0'+y^0\xi_0'-\beta\frac{y^1}{N}\chi_0'+\beta y^1\xi_1'\right\} \text{ and} \\ 0 &= T\left\{-\frac{y^0}{N}\chi_0'+y^0\xi_0'-\beta\frac{y^1}{N}\chi_0'+\beta y^1\xi_1'\right\}. \end{aligned}$$

$\phi\neq 0$ and $T\neq 0$ because of Inada condition in equilibrium, $\frac{\partial U}{\partial T}=0$ and $\frac{\partial U}{\partial \phi}=0$ is equivalent to the following equation.

$$y^0\chi_0'(\frac{1}{N}-\frac{\xi_0'}{\chi_0'})+\beta y^1\chi_1'(\frac{1}{N}-\frac{\xi_1'}{\chi_1'})=0. \tag{2.19}$$

On the other hand, suppose there is (T,ϕ) satisfying the Bowen-Lindahl-Samuelson condition, then recalling the assumption of the technology of public good production (i.e., one unit of private good produces one unit of public good), we have $\sum_{i=1}^{N}\frac{\xi_0'}{\chi_{i0}'}=1$ and $\sum_{i=1}^{N}\frac{\xi_1'}{\chi_{i1}'}=1$.

Then because $\theta_i=1/N$ $\forall$i$=(1,\dots,N)$, it must be $\frac{\xi_0'}{\chi_{i0}'}=1/N$ and $\frac{\xi_1'}{\chi_{i1}'}=$

$1/N$ for all i. Obviously such (T, ϕ) solves (2.19).

(2) Suppose (T, ϕ) solves the political-economic equilibrium, then when $\chi(.) = \xi(.) = g(.)$ and $\frac{g'(z)}{g'(\bar{z})} = g'(\frac{z}{\bar{z}})$, (2.19) becomes $(y^0\chi_0' + \beta y^1\chi_1')\left\{[\frac{\phi NT}{1-\phi T}] - \frac{1}{N}\right\} = 0$. i.e., $[\frac{\phi NT}{1-\phi T}] = \frac{1}{N}$, which implies $\frac{\xi_0'}{\chi_{i0}'} = 1/N$ and $\frac{\xi_1'}{\chi_{i1}'} = 1/N$ for all i. Summing up these two equations over consumers, we get the Bowen-Lindahl-Samuelson condition. **Q. E. D.**

2.3.6 Tax and Growth Rates

One important focus of the endogenous growth literature deals with the effect of fiscal policy on economic growth. Two questions are considered. Does the tax rate have any effect on the growth rate? If yes, is the growth rate positively or negatively related to the tax rate? Most of the neoclassical growth models agree that national taxation can substantially affect long-run rates of economic growth (King and Rebelo (1990)). With regard to the direction of the effect of taxation on economic growth, most of the theoretical results claim that higher income tax rates translate into lower rates of growth.[19] However, the results of empirical studies are somewhat controversial due to different definitions, interpretations and measurements of government services, and different coverage of countries (Landau (1983), Kormendi and Meguire (1985), Grier and Tolluck (1987), and Barro (1990)).

[19] For key references in this literature, see Becker (1985), Judd (1985), Barro (1990) and Rebelo (1991).

In this subsection, I examine the relationship between the tax and growth rates. The process is intuitive and simple. I derive the partial derivative of K^1 with respect to T by combining the market-clearing conditions, first order conditions and budget constraints. I only consider the case when $c_i^0,\ c_i^1 > 0$ (cases when one of c_i^0 and c_i^1 is binding can be similarly considered).

Taking the derivative with respect to T in $\sum_{i=1}^N c_i^0 = (1-T)y^0$ and recalling $y^0 = F(\bar{K}) - K^1$, I have

$$\sum_{i=1}^{N} \frac{\partial c_i^0}{\partial T} = -[F(\bar{K}) - K^1] - (1-T)\frac{\partial K^1}{\partial T}.$$

Since $c_i^0, c_i^1 > 0$, the first order conditions (2.9) and (2.10) are binding. (2.10)/(2.9) yields

$$\beta F'(K^1)\chi'[c_i^1 + \frac{(1-\phi)T}{N}F(K^1)] = \chi'\left\{c_i^0 + \frac{(1-\phi)T}{N}[F(\bar{K}) - K^1]\right\} \tag{2.20}$$

Substituting (2.6), (2.7) and (2.8) into (2.11) gives

$$\begin{aligned}(1-T)\Bigg\{ f_2(\bar{K},N) + \theta_i[F(\bar{K}) - K^1 - Nf_2(\bar{K},N)] + \frac{f_2(K^1,N)}{F'(K^1)} \\ + \frac{\theta_i}{F'(K^1)}[F(K^1) - Nf_2(K^1,N)]\Bigg\} = c_i^0 + \frac{c_i^1}{F'(K^1)}. \qquad (2.21)\end{aligned}$$

Taking the derivative with respect to T in (2.20), we have

$$\begin{aligned}\beta F'(K^1)\chi_1''[\frac{\partial c_i^1}{\partial T} &+ \frac{(1-\phi)}{N}F(K^1) + \frac{(1-\phi)T}{N}F'(K^1)\frac{\partial K^1}{\partial T}] + \\ \beta F''(K^1)\chi_1'\frac{\partial K^1}{\partial T} &= \chi_0''[\frac{\partial c_i^0}{\partial T} - \frac{(1-\phi)T}{N}\frac{\partial K^1}{\partial T}] +\end{aligned}$$

$$+ \quad + \frac{(1-\phi)}{N}[F(\bar{K}) - K^1]. \tag{2.22}$$

Taking the derivative with respect to T in (2.21) gives

$$\begin{aligned}
- & \left\{ f_2(\bar{K}, N) + \theta_i[F(\bar{K}) - K^1 - N f_2(\bar{K}, N)] + \frac{f_2(K^1, N)}{F'(K^1)} + \right. \\
+ & \left. \frac{\theta_i[F(K^1) - N f_2(K^1, N)]}{F'(K^1)} \right\} + (1-T)\frac{\partial K^1}{\partial T}\left\{ -\theta_i + \right. \\
+ & \theta_i \frac{[F'(K^1) - N f_{21}(K^1, N)]F'(K^1) - F''(K^1)[F(K^1) - N f_2(K^1, N)]}{[F'(K^1)]^2} \\
+ & \left. \frac{f_{21}(K^1, N)F'(K^1) - f_2(K^1, N)F''(K^1)}{[F'(K^1)]^2} \right\} \\
= & \frac{\partial c_i^0}{\partial T} + \frac{\frac{\partial c_i^1}{\partial T} F'(K^1) - c_i^1 F''(K^1)\frac{\partial K^1}{\partial T}}{[F'(K^1)]^2}, \text{ i.e.,}
\end{aligned}$$

$$\begin{aligned}
- & [F'(K^1)]^2 \left\{ f_2(\bar{K}, N) + \theta_i[F(\bar{K}) - K^1 - N f_2(\bar{K}, N)] + \right. \\
+ & \left. \frac{f_2(K^1, N)}{F'(K^1)} + \frac{\theta_i}{F'(K^1)}[F(K^1) - N f_2(K^1, N)] \right\} + \\
+ & (1-T)\frac{\partial K^1}{\partial T}\left\{ -\theta_i[F'(K^1)]^2 + f_{21}(K^1, N)F'(K^1) \right. \\
- & f_2(K^1, N)F''(K^1)] + \theta_i[F'(K^1) - N f_{21}(K^1, N)]F'(K^1) \\
- & \left. \theta_i F''(K^1)[F(K^1) - N f_2(K^1, N)] \right\} + c_i^1 F''(K^1)\frac{\partial K^1}{\partial T} \\
= & [F'(K^1)]^2 \frac{\partial c_i^0}{\partial T} + F'(K^1)\frac{\partial c_i^1}{\partial T}.
\end{aligned} \tag{2.23}$$

Rearranging (2.22) yields

$$\frac{\partial c_i^1}{\partial T} = \frac{1}{\beta F'(K^1)\chi_1''}\left\{ \chi_0''[\frac{\partial c_i^0}{\partial T} + \frac{(1-\phi)}{N}[F(\bar{K}) - K^1] \right.$$

$$
\begin{aligned}
&- \frac{(1-\phi)T}{N}\frac{\partial K^1}{\partial T}] - \beta F''(K^1)\chi_1^{'}\frac{\partial K^1}{\partial T}\bigg\} \\
&- \frac{(1-\phi)}{N}F(K^1) - \frac{(1-\phi)T}{N}F'(K^1)\frac{\partial K^1}{\partial T}. \qquad (2.24)
\end{aligned}
$$

Substituting (2.24) into (2.23), we get

$$
\begin{aligned}
&- [F'(K^1)]^2 \left\{f_2(\bar{K}, N) + \theta_i[F(\bar{K}) - K^1 - Nf_2(\bar{K}, N)] + \right. \\
&+ \left. \frac{f_2(K^1, N)}{F'(K^1)} + \frac{\theta_i}{F'(K^1)}[F(K^1) - Nf_2(K^1, N)]\right\} + \\
&+ (1-T)\frac{\partial K^1}{\partial T}\left\{-\theta_i[F'(K^1)]^2 + f_{21}(K^1, N)F'(K^1)\right. \\
&- f_2(K^1, N)F''(K^1)] + \theta_i[F'(K^1) - Nf_{21}(K^1, N)]F'(K^1) \\
&- \left.\theta_i F''(K^1)[F(K^1) - Nf_2(K^1, N)]\right\} + c_i^1 F''(K^1)\frac{\partial K^1}{\partial T} \\
&= \frac{\chi_0^{''}}{\beta\chi_1^{''}}[\frac{\partial c_i^0}{\partial T} + \frac{(1-\phi)}{N}[F(\bar{K}) - K^1] - \frac{(1-\phi)T}{N}\frac{\partial K^1}{\partial T}] \\
&+ [F'(K^1)]^2\frac{\partial c_i^0}{\partial T} - \frac{\chi_1^{'}F''(K^1)}{\chi_1^{''}}\frac{\partial K^1}{\partial T} \\
&- \frac{(1-\phi)}{N}F(K^1)F'(K^1) - \frac{(1-\phi)T}{N}[F'(K^1)]^2\frac{\partial K^1}{\partial T}. \qquad (2.25)
\end{aligned}
$$

Summing up (2.25) across all $i's$, using (2.13) and (2.19), and simplifying, we have

$$
\begin{aligned}
\frac{\partial K^1}{\partial T}&\left\{(1-\phi T)[F'(K^1)]^2 + \sum_{i=1}^{N}\frac{\chi_0^{''}}{\beta\chi_1^{''}}\frac{(1-\phi)T}{N} + \sum_{i=1}\frac{\chi_1^{'}}{\chi_1^{''}}F''(K^1)\right\} \\
&= \phi F(K^1)F'(K^1) + \sum_{i=1}^{N}\frac{\chi_0^{''}}{\beta\chi_1^{''}}[\frac{\partial c_i^0}{\partial T} + \frac{(1-\phi)}{N}y^0].
\end{aligned}
$$

Since the first and third terms inside the braces of the left hand side of the above equation are positive, and the second term is nonnegative, $\frac{\partial K^1}{\partial T}$ has the same sign as the right hand side. However, the first term on the right hand side is positive, while the second term, which depends on the type of utility functions and the distributions of income, may be positive or negative. Therefore, I have the following conclusion.

Proposition 6: There is not a definite relationship between the tax and growth rates. The relationship between the tax and growth rates generally depends on the type of utility functions and the distribution of income.

I will give two examples with different distributions of wealth in Subsection 2.3.7 to show that when the tax rate increases, the growth rate increases too. In Section 2.4, I study a special type of utility function (namely a utility function with constant elasticity of marginal utility of income[20]) which has the property that the growth rate is not a function of the tax rate (i.e., $\frac{\partial K^1}{\partial T} = 0$) and the distribution of income. *In other words, a free economy with a constant EMUI is fiscally and distributionally neutral.*

2.3.7 Examples

I examine two examples in which utility functions have increasing risk aversion. In Example 1, by varying ϕ (the proportion of tax revenue used in the

[20]From now on, I use EMUI as the abbreviation of elasticity of marginal utility of income.

public good production), I demonstrate that different weights of the public sector generate different growth paths and different welfare levels. Example 2, together with example 1, shows that different distributions of θ lead to different sizes of government and different welfare levels.

Example 1: Let $u(x_1, x_2) = [\log(x_1 + 1)]^{\frac{99}{100}} + 0.20930{x_2}^{1/2}$,

$$f(K, L) = 1.2K^{1/2}L^{1/2} + K,$$

$$\bar{K} = 25,\ L = 9,\ \delta = 0,\ \text{and } \beta = 1,$$

$$\theta_1 = \ldots = \theta_7 = 0 \text{ and } \theta_8 = \theta_9 = 0.5.$$

It is not difficult to check that when $x_1 < 99$,

$$R_r(x_1) = \frac{x_1}{x_1 + 1}\left\{1 + \frac{1}{100}[\log(x_1 + 1)]^{-1}\right\} < 1 \text{ and}$$

$$\frac{\partial R_r(x_1)}{\partial x_1} > 0.$$

Therefore condition α holds, and Proposition 3 and Theorem 1 are both valid. The first order conditions are:

$$W^0 = 0.6\bar{K}^{1/2}L^{-1/2} = 1; \tag{2.26}$$

$$W^1 = 0.6(K^1)^{1/2}L^{-1/2} = 0.2(K^1)^{1/2}; \tag{2.27}$$

$$P = \frac{1}{(2 + 1.8(K^1)^{-1/2})} \tag{2.28}$$

$$y^0 = 68 - K^1 \tag{2.29}$$

$$y^1 = 3.6(K^1)^{1/2} + 2K^1 \tag{2.30}$$

$$
\begin{aligned}
P &= \frac{u_1[c_i^1 + \frac{(1-\phi)T}{9} y^1, \phi T y^1]}{u_1[c_i^0 + \frac{(1-\phi)T}{9} y^0, \phi T y^0]}; && (2.31)\\
c_i^0 + P c_i^1 &= (1-T)\left\{\theta_i[(59 - K^1) + P(1.8(K^1)^{1/2}) + 2PK^1]\right\} \\
&+ (1-T)[1 + 0.2(K^1)^{1/2} P]; && (2.32)\\
\sum_{i=1}^{9} c_i^0 &= (1-T) y^0; && (2.33)\\
\sum_{i=1}^{9} c_i^1 &= (1-T) y^1. && (2.34)
\end{aligned}
$$

Clearly, because of the given distribution of θ, each voter $i (i = 1, \ldots, 7)$ is a median voter and can win the election. In addition, any $i (i = 1, \ldots, 7)$ will propose $T = 1$ during the campaign[21] and implement $T = 1$ when he or she comes into power. Thus (2.31) can be rewritten as:

$$
\frac{(\frac{(1-\phi)T}{9} y^0 + 1)[\log(\frac{(1-\phi)T}{9} y^0 + 1)]^{\frac{1}{100}}}{(\frac{(1-\phi)T}{9} y^1 + 1)[\log(\frac{(1-\phi)T}{9} y^1 + 1)]^{\frac{1}{100}}} = \frac{1}{2 + 1.8(K^1)^{-1/2}}.
$$

Case 1: $\phi = 0.00$ *or* $(1-\phi) = 1.00$. (2.31) becomes

$$
\begin{aligned}
h(K^1) &= (2 + 1.8(K^1)^{-1/2})(8.55556 - 0.11111K^1) \\
&\quad [\log(8.55556 - 0.11111K^1)]^{\frac{1}{100}} - (0.4(K^1)^{1/2} + 0.22222K^1 + 1) \\
&\quad [\log(0.4(K^1)^{1/2} + 0.22222K^1 + 1)]^{\frac{1}{100}}.
\end{aligned}
$$

Since $h'(K^1) < 0$, there is a unique solution for K^1, that is $(K^1)^e = 34.17941$.

[21]It is obvious that given ϕ, $T = 1$ gives i the highest utility from consumption of both the public good and the private good (because he or she enjoys more benefit from the private good transfer than the tax cost he or she pays to the government).

Substituting K^1 into equations (2.26) to (2.30), we have $(W^0)^e = 1.00000$, $(W^1)^e = 1.16926$, $P^e = 0.43330$, $(y^0)^e = 33.82059$ and $(y^1)^e = 89.40556$. And

$$U = u(3.75784, 0) + u(9.93395, 0) = 3.92398.$$

Case 2: $\phi = 0.20$ *or* $(1 - \phi) = 0.80$. Similarly, the unique nonnegative solution is: $(K^1)^e = 34.77731$, $(W^0)^e = 1.00000$, $(W^1)^e = 1.17945$, $P^e = 0.43380$, $(y^0)^e = 33.22269$ and $(y^1)^e = 90.78464$. And $U = u(2.95313, 6.64454) + u(8.06975, 18.15693) = 4.98908$.

Case 3: $\phi = 0.80$ *or* $(1 - \phi) = 0.20$. The unique nonnegative solution is: $(K^1)^e = 43.62335$, $(W^0)^e = 1.00000$, $(W^1)^e = 1.32096$, $P^e = 0.44004$, $(y^0)^e = 24.37665$ and $(y^1)^e = 111.02397$. And $U = u(0.54170, 19.50132) + u(2.46720, 88.81918) = 4.57398$.

Case 4: $\phi = 0.90$ *or* $(1 - \phi) = 0.10$. The unique nonnegative solution is: $(K^1)^e = 54.98424$, $(W^0)^e = 1.00000$, $(W^1)^e = 1.48303$, $P^e = 0.44588$, $(y^0)^e = 13.01576$ and $(y^1)^e = 136.66297$. And $U = u(0.14462, 11.71418) + u(1.51848, 122.99667) = 4.09976$.

Case 5: $\phi = 0.95$ *or* $(1 - \phi) = 0.05$. The unique nonnegative solution is: $(K^1)^e = 67.94598$, $(W^0)^e = 1.00000$, $(W^1)^e = 1.64859$, $P^e = 0.45078$, $(y^0)^e = 0.05402$ and $(y^1)^e = 165.56653$. And $U = u(0.00030, 0.05132) + u(0.91981, 157.28820) = 3.32769$.

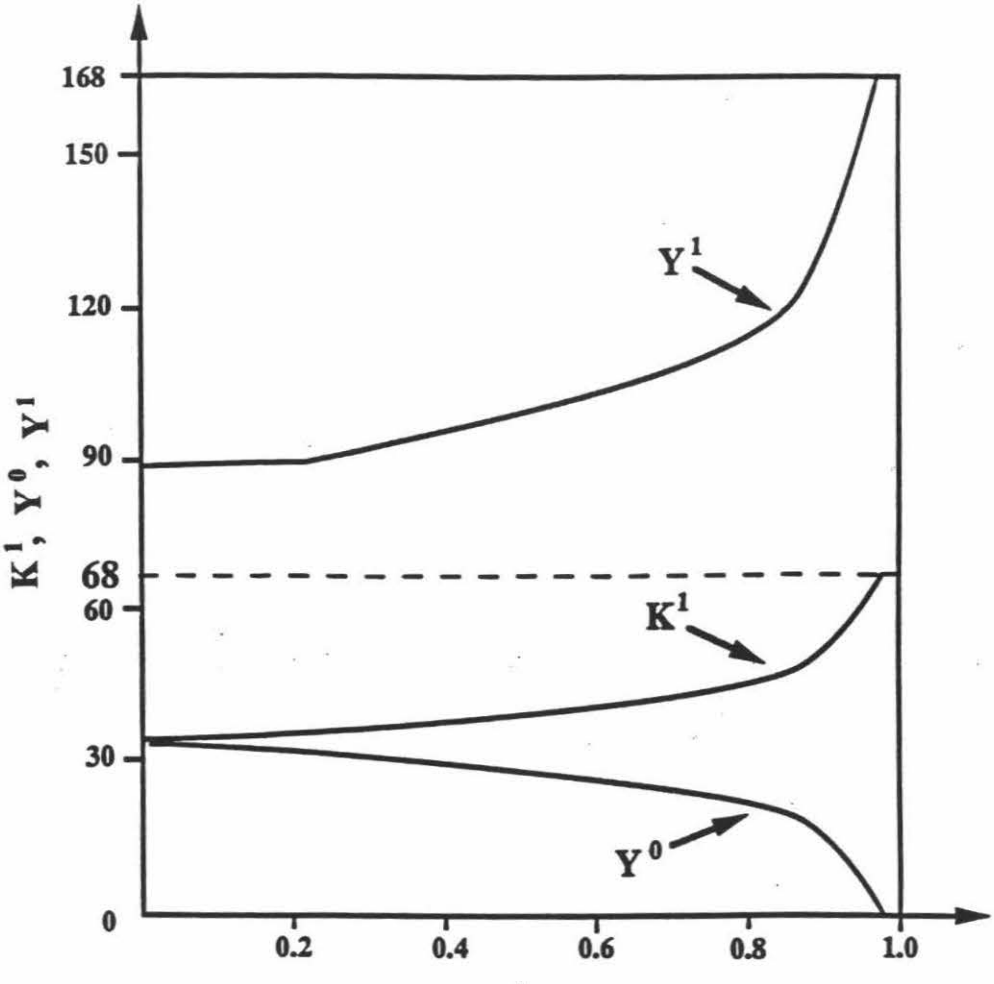


Figure 2.5: Growth Paths for Example 1

So Example 1 presents a situation in which growth becomes more dramatic as the weight of the public sector becomes larger and larger. In other words, consumers save more and more for future consumption when the fiscal policy becomes more and more public good-oriented (see Table 2.1 and Figure 2.5).

Table 2.1:Growth Paths for Example 1

Case	ϕ \ K^1; y^0; y^1	K^1	Y^0	Y^1
Case 1	0.00	34.179	33.821	89.401
Case 2	0.20	34.777	33.223	90.785
Case 3	0.80	43.623	24.377	111.024
Case 4	0.90	54.984	13.016	136.663
Case 5	0.95	67.946	0.054	165.567

Example 2: Let $\phi = 0.8$ and all assumptions in Example 1 remain valid except the distribution of θ. Now consider the following distribution:

$$\theta_1 = \ldots = \theta_9 = 1/9.$$

Clearly because consumers are identical, they are not lending and borrowing among them. In addition, because the marginal productivity is greater than 1, all leftovers of the first period consumption should be used in production. Thus ∀i$(i = 1, \ldots, 9)$,

$$c_i^0 = \frac{(1-T)y^0}{9} \text{ and } c_i^1 = \frac{(1-T)y^1}{9}.$$

Since in equilibrium $T \neq 1$ due to Inada condition, $c_i^0,\ c_i^1 > 0$, the first order conditions (2.9) and (2.10) are binding. Combining (2.8), (2.9) and (2.10) yields the following equation.

$$\frac{[(\frac{68-K^1}{9})(1-\frac{4T}{5})+1][\log[(\frac{68-K^1}{9})(1-\frac{4T}{5})+1]]^{\frac{1}{100}}}{[(\frac{3.6(K^1)^{1/2}+2K^1}{9})(1-\frac{4T}{5})+1][\log[(\frac{3.6(K^1)^{1/2}+2K^1}{9})(1-\frac{4T}{5})+1]]^{\frac{1}{100}}} = \frac{1}{2+1.8(K^1)^{-1/2}}$$

K^1 and T can be solved by the above equality and equation (2.17), $\frac{\partial U}{\partial T} = 0$, or $T = 1$ if (2.17) has no solution. After simplification, (2.17), $\frac{\partial U}{\partial T} = 0$, becomes

$$0 = \frac{0.09360}{T^{1/2}}[(68-K^1)^{1/2} + (3.6(K^1)^{1/2} + 2K^1)^{1/2}] -$$

$$0.088\left\{\frac{68-K^1}{[(\frac{68-K^1}{9})(1-\frac{4T}{5})+1][\log[(\frac{68-K^1}{9})(1-\frac{4T}{5})+1]]^{\frac{1}{100}}}+\right.$$
$$\left.+\frac{3.6(K^1)^{1/2}+2K^1}{[(\frac{3.6(K^1)^{1/2}+2K^1}{9})(1-\frac{4T}{5})+1][\log[(\frac{3.6(K^1)^{1/2}+2K^1}{9})(1-\frac{4T}{5})+1]]^{\frac{1}{100}}}\right\}.$$

The unique nonnegative solution is: $T^e = 0.5$, $(K^1)^e = 35.77307$, $(W^0)^e = 1.00000$, $(W^1)^e = 1.19621$, $P^e = 0.43460$, $(y^0)^e = 32.22693$ and $(y^1)^e = 93.07800$. And

$$U = u(2.14849, 12.89050) + u(6.20528, 37.23040) = 5.13531,$$

which is greater than 4.57398, the utility in case 3, Example 1 (see Table 2.2 and Figure 2.6).

<u>Table 2.2: Distributions of Income and Growth</u>

<u>when ϕ = 0.80 for Examples 1 and 2.</u>

Example	T, K, y, U / θ	T	K^1	y^0	y^1	U
Example 1	$\theta_1 = \ldots = \theta_7 = 0$ $\theta_8 = \theta_9 = 0.5$	1.00	43.623	24.377	111.024	4.574
Example 2	$\theta_1 = \ldots = \theta_9 = 1/9$	0.50	35.773	32.227	93.078	5.135

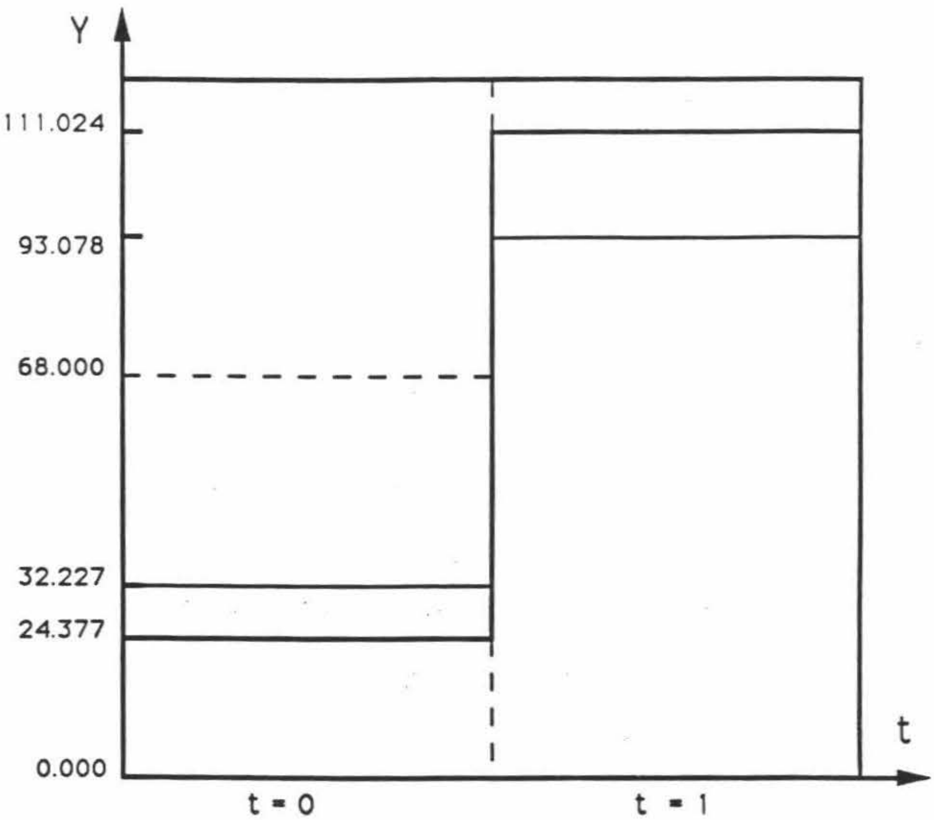
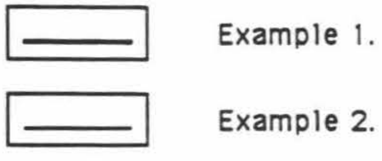


Figure 2.6: Growth Paths for Examples 1 and 2 (ϕ = 0.80).



The fact that $U = 4.57398$, $T = 1$ and $(K^1)^e = 43.62335$ when the distribution of θ is: $\theta_1 = \ldots = \theta_7 = 0$ and $\theta_8 = \theta_9 = 0.5$ (case 3, Example 1) and $U = 5.13532$, $T = 0.5$ and $(K^1)^e = 35.77300$ when the distribution of θ is: $\theta_1 = \ldots = \theta_9 = 1/9$ (Example 2) is in sharp contrast with the statement of neoclassical growth models (Sato (1967), Becker (1985), Judd (1985), Barro (1990) and Rebelo (1991)) which claim that higher income tax rates translate into lower rates of growth.

Remark: First, when the public good and the distribution of income play roles in endogenous growth models, I come to the same conclusion as that of traditional representative-agent-without-public-good models, namely, that national taxation can substantially affect long-run rates of economic growth (see King and Rebelo (1990)). However, my conclusion is that *fiscal policy determines the endogenous growth of the economy, and the distribution of income decides fiscal policy.* [22] *There is not a definitely negative relation between income tax and growth rates unless factors that determine income tax rate are fixed.* While the negative relationship between the income tax rate and the growth rate is too weak to explain the observed wide cross-country differences in growth rates, my model may fill the gap.

Second, by formulating an overlapping generations model that relates equilibrium growth to income inequality and political institutions and assuming that fiscal policy only has a redistributive effect, Persson and Tabellini (1991) conclude that income inequality is harmful for growth. My point is that *pre-tax income equality and post-tax income equality may have different*

[22]In fact, we will see later in Chapter 3, economic system is another determinant of income tax rate.

impacts on economic growth; as the above fact suggests pre-tax inequality may not necessarily be harmful for economic growth although it does jeopardize social welfare.

2.4 Constant Elasticity of Marginal Utility of Income

Proposition 3 and Theorem 1, two of my major conclusions, need a sufficient condition which assumes that utility functions have an elasticy of marginal utility of income (Abbreviated as EMUI) not greater than one. The absolute and relative risk aversion measures were developed by Arrow (1965) and Pratt (1964) respectively. As Arrow (1965) pointed out that the relative risk aversion is the elasticity of the marginal utility of wealth; it is invariant not only with respect to changes in the units of utility but also with respect to changes in the units of wealth.

In this section, I shall focus on utility functions with constant EMUI. The motivation is as follows: first, this kind of utility function is widely used and easy to examine; second, as we will see, this class of utility functions possesses some interesting properties. That is, utility functions in this section satisfy the following condition:

Condition β: A utility function for the private good has a constant EMUI, i.e.,

$$R_r(x) \equiv \frac{(-x\partial^2 U/\partial x^2)}{(\partial U/\partial x)} = \text{constant}.$$

Pratt (1964) claims that any utility function with a constant EMUI can be expressed in terms of a linear transformation of any one of the following three functions:

$$\begin{aligned}
(a)\ u(x) \sim x^{1-c} \quad & \text{if } R_r(x) &=& \; c < 1, \\
(b)\ u(x) \sim \log(x) \quad & \text{if } R_r(x) &=& \; 1, \\
(c)\ u(x) \sim -x^{-(c-1)} \quad & \text{if } R_r(x) &=& \; c > 1.
\end{aligned}$$

Remark: First, any risk-neutral utility function can be approximated by a type-a utility function as c approaches 0. Second, a type-c utility function violates condition α. An example will be given at the end of this chapter to demonstrate a counter-intuitive property, i.e., the wealthiest consumers prefer the highest tax rate. If I exclude type-c utility functions and focus on the first two types, then all results derived in the previous section are valid since now condition α holds.

When a utility function has a constant EMUI, I examine and answer the following three questions: (1) Does fiscal policy have any effect on macroeconomic performance? (2) What is the relationship between the profit share of the decisive consumer and the income tax rate? (3) Can we always predict a

Kuznets curve, i.e., the hypothesis that income inequality first increases and then decreases with development?

2.4.1 Fiscal and Distributional Neutrality

In regard to the first question, whether fiscal policy has effect on macroeconomic performance, I present the following proposition.

Proposition 7: When condition β[23] is satisfied, then (i) a free economy is fiscally and distributionally neutral; (ii) individual consumption levels of the private good depend positively on θ_i and negatively on T. In other words, fiscal policy only has redistributive effects on the economy.

Proof: (i) Because of the additive separability of the utility function (Assumption 1) and irrelevance of the public good to personal wealth, I can simply assume that the utility function of the private good is a linear combination of one of types (a), (b) and (c). First assume that the utility function is in the form of $a + bx^{1-c}, a, b > 0, c < 1$ (I can similarly prove the case with type-b), then $u_1(x_1, x_2) = b(1-c)x^{-c}$, and substituting into (2.20) yields

$$\left\{\frac{c_i^1 + \frac{(1-\phi)T}{N}F(K^1)}{c_i^0 + \frac{(1-\phi)T}{N}[F(\bar{K}) - K^1]}\right\}^{-c} = \frac{1}{\beta F'(K^1)}.\text{ i.e.,}$$

[23] In fact, this condition can be weakened as $\forall x_1,\ \hat{x_1} \in R_+$:

$$\frac{\chi^{'}(x_1)}{\chi^{'}(\hat{x_1})} = \chi^{'}(\frac{x_1}{\hat{x_1}}).$$

Interestingly, most of the noncompound fundamental functions share this property.

$$c_i^0 + \frac{(1-\phi)T}{N}[F(\bar{K}) - K^1] = [\beta F'(K^1)]^{-\frac{1}{c}}[c_i^1 + \frac{(1-\phi)T}{N}F(K^1)] \quad (2.35)$$

Summing up the above equations, and using (2.12) and (2.13), we get

$$(1 - T\phi)[F(\bar{K}) - K^1] = [\beta F'(K^1)]^{-\frac{1}{c}}(1 - T\phi)F(K^1).$$

Since in a political-economic equilibrium, $T \neq 1$ or $\phi \neq 1$, [24]

$$[F(\bar{K}) - K^1] = [\beta F'(K^1)]^{-\frac{1}{c}}F(K^1). \quad (2.36)$$

Thus the solution of K_1, i.e., K_1^e, is independent of T, ϕ, and the distribution of θ_i. Substituting K_1^e into (2.6), (2.7), and (2.8), we can solve for all the other macroeconomic variables, i.e., $(W^0)^e$, $(W^1)^e$ and P^e, which are all independent of T, ϕ and θ_i.

(ii) Comparing (2.35) and (2.36), we have

$$c_i^0 = [\beta F'(K^1)]^{-\frac{1}{c}}c_i^1. \quad (2.37)$$

Substituting the above equality into (2.11),

$$[[\beta F'((K^1)^e)]^{-\frac{1}{c}} + P^e]c_i^1 = (1-T)\left\{(W^0)^e + P^e(W^1)^e + \theta_i[\pi^0 + \pi^1]\right\}.$$

Therefore, c_i^1 depends positively on θ_i and negatively on T. A similar con-

[24]The case when $T = 1$ and $\phi = 1$ cannot occur in a political-economic equilibrium, since no candidate would use up resources in the production of the public good because according to the Inada condition $u_1(0, x_2) = +\infty$ everybody could be better off by slightly reducing T or ϕ.

clusion can be drawn for c_i^0. **Q.E.D.**

2.4.2 Profit Share of the Decisive Consumer and Income Tax Rate

The following proposition studies the relationship between the profit share of the decisive consumer (i.e., the median-income consumer) and the tax rate among different income distributions.

Proposition 8: If Conditions α and β are satisfied, and θ_m^1 and θ_m^2 are the profit shares of the decisive consumers corresponding to two distributions, then $\theta_m^1 > \theta_m^2$ implies $T_m^1 \leq T_m^2$. The inequality is strict if $T_m^1 < 1$.

Proof: **(1) If $\theta_m^2 < \theta_m^1 \leq 1/N$, then $T_m^1 = T_m^2 = 1$.** The proof is trivial since by selecting a proper ϕ, a decisive consumer (i.e., the median-income consumer) with θ_m less than the average θ can always be made better off by increasing the income tax rate.

(2) If $\theta_m^2 \leq 1/N < \theta_m^1$, then $T_m^1 < T_m^2 = 1$. By (1), $T_m^2 = 1$. It is not difficult to prove that complete equalization as a result of $T_m^1 = 1$ is not optimal for the median-income consumer with θ_m^1 larger than the average θ, 1/N.

(3) If $1/N < \theta_m^2 < \theta_m^1$, then $\phi_m^1 = \phi_m^2 = 1$ and $T_m^1 < T_m^2 < 1$. As in (2), $T_m^2 < 1$. It can be proved that any $\phi_m < 1$, i.e., $1 - \phi_m > 0$ is not optimal for the median-income consumer, since the redistributional portion of tax revenue only benefits those consumers with θ lower than the average. Thus, the only thing that needs to be proved is $T_m^1 < T_m^2$. Recalling (2.17) in the proof of Proposition 3, a decisive consumer will select $\phi = 1$ and T such that

$$0 = \frac{\partial U}{\partial T}\Big\|_m = \frac{\partial c_m^0}{\partial T}\chi_0' + \beta\frac{\partial c_m^1}{\partial T}\chi_1' + y^0\xi'(Ty^0) + \beta y^1\xi'(Ty^1).$$

Because of Condition β and Proposition 7, the above equalities for the two decisive consumers only differ in I_m and T. Taking derivative with respect to I_m and using Condition α, similarly as the proof of Proposition 3, we have $\frac{\partial}{\partial I_m}(\frac{\partial U}{\partial T}) < 0$. **Q.E.D.**

Interestingly, if we interpret the size of government as the proportion of tax revenue to aggregate product, namely, the tax rate in this model, then Proposition 8 confirms the conclusion of Meltzer and Richard (1981, 1983) that an increase in mean income[25] relative to the income of the decisive voter increases the size of government.[26]

2.4.3 A Counter-Example to Kuznets Curve

One well-known hypothesis that concerns an inverted-U relationship between per capita income and inequality is the Kuznets curve. Kuznets (1955, 1966,

[25]The different personal incomes come from the different productivities in their paper rather than from the different profit shares as in ours.

[26]For a complete survey of theories of the growth of government, see Lybeck (1988).

pp. 206-217) finds that income inequality first increases then decreases with development.[27] As Perotti (1990) points out, "empirically, this relation seems to be quite robust in cross-section studies, and has been consistently obtained for more than three decades. However, time series studies tend to cast doubts on the shape of the relation." Since technology level[28] is generally positively associated with stage of development, the Kuznets curve can be restated as the hypothesis that income inequality first increases then decreases with technology level. *Can we always predict an inverted U-relation between technology level and tax rate?* The answer is no. The following proposition shows that when the median of the pre-tax income distribution is the same as mean, and the utility function is a linear combination of the same type-a (or type-b) utility functions of the private good and the public good, then in political-economic equilibrium, the tax rate is not a function of technology. In other words, the equilibrium tax rate only depends on the utility function, the weight of the public sector, and (possibly) the size of the population.

Proposition 9: When the median of the pre-tax income distribution is the same as mean, then equilibrium tax rates of types a and b are technology-proof. Specifically, (a) for a type-a utility function, $u(x_1, x_2) = A_1 x_1^{1-c} + A_2 x_2^{1-c} + A_3$, **where** $A_1 > 0$, $A_2 > 0$ **and**

---

[27]For a recent evaluation of the theoretical as well as the empirical work on the Kuznets curve, see Lindert and Williamson (1985) and Perotti (1990).

[28]For example, in an economy with the Cobb-Douglas production function $f(K, L) = AK^{\alpha}L^{\beta}$, higher technology can be interpreted as larger coefficients A, α and β.

A_3 are constants, and $c \in (0,1)$,

$$T = 1 \ or \ \frac{1}{\phi}[1 - \frac{N}{N + (\frac{A_2 N}{A_1})^{\frac{1}{c}}}]. \tag{2.38}$$

(b) For a type-b utility function, $u(x_1, x_2) = A_1 \log x_1 + A_2 \log x_2 + A_3$, where $A_1 > 0$, $A_2 > 0$ and A_3 are constants,

$$T = 1 \ or \ \frac{1}{\phi}[\frac{A_2}{A_1 + A_2}]. \tag{2.39}$$

Proof: (a) According to Theorem 1, the median income consumer will be the majority winner and will eventually implement his or her ideal tax rate. Now, since the median of the pre-tax income distribution is the same as mean, $\theta_m = 1/N$, the utility of the median consumer *is* as follows.

$$\begin{aligned} U_m &= A_1\left\{(1-T)\frac{y^0}{N} + \frac{(1-\phi)}{N}Ty^0\right\}^{1-c} + A_2(\phi T y^0)^{1-c} \\ &+ \beta A_1\left\{(1-T)\frac{y^1}{N} + \frac{(1-\phi)}{N}Ty^1\right\}^{1-c} + \beta A_2(\phi T y^1)^{1-c} + 2A_3. \end{aligned}$$

Recalling $y^0 = F(\bar{K}) - K^1$ and $y^1 = F(K^1)$, and substituting (2.36) into the above equation and simplifying, I have

$$U_m = [A_1(\frac{1-\phi T}{N})^{1-c} + A_2(\phi T)^{1-c}](y^1)^{1-c}\left\{\beta + [\beta F'(K^1)]^{-\frac{1-c}{c}}\right\} + 2A_3.$$

According to Proposition 7, K^1 is not a function of T, so

$$\frac{\partial U_m}{\partial T} = 0 = [-\frac{A_1}{N}(\frac{1-\phi T}{N})^{-c} + A_2(\phi T)^{-c}]\phi(1-c)(y^1)^{1-c}\left\{\beta + [\beta F'(K^1)]^{-\frac{1-c}{c}}\right\},$$

i.e., the marginal benefit of the private transfer equals the marginal benefit of the public good. Since $c < 1$ and $\phi \neq 0$ in equilibrium, the above equation implies

$$\frac{A_1}{N}(\frac{1-\phi T}{N})^{-c} = A_2(\phi T)^{-c}. \tag{2.40}$$

Solving the above equation and using Theorem 1, I get (2.38). Thus an equilibrium tax rate is technology-proof.

(b) (2.39) can be similarly derived. **Q.E.D.**

There are many explanations for the Kuznets curve.[29] The above result provides another reason, namely the particular utility function, for the nonexistence of the Kuznets curve. I show that when the tax rate is flat and the median of the income distribution is the same as mean, then a utility function with constant EMUI implies no Kuznets relation. Since it is unknown under what conditions people have utility functions with EMUI, I do not know the robustness of Proposition 9. However, my conjecture is that different countries may have different utility functions because of different cultures, customs and geographic regions, and it is very likely that different

[29] As Lindert and Williamson (see Figure 1, Lindert and Williamson (1985)) point out, the more robust portion of the Kuznets Curve lies to the right: income inequality falls with the advance of per capita income at the higher levels of development. This may be the results of progressive fiscal policy, union strength, and the rise of the welfare state.

countries may have different relations between income inequality and development. In any case, Proposition 9 does give a theoretical counter-example of the Kuznets curve under certain conditions. Thus it is not surprising at all that some scholars have found evidence which does not supports the Kuznets curve (Saith (1983) and Ram (1988)).

Having (2.38) (or (2.39)), we can undertake a comparative study for interior value of T with respect to ϕ, A_1, A_2, N, and c for a type-a (or ϕ, A_1 and A_2 for a type-b) utility function.

Corollary 1: For a type-a utility function, $\frac{\partial T}{\partial \phi} < 0$, $\frac{\partial T}{\partial A_1} < 0$, $\frac{\partial T}{\partial A_2} > 0$, $\frac{\partial T}{\partial N} > 0$, **and** $\frac{\partial T}{\partial c} >$ *(or* $=$, *or* $<$) 0 **if** $\frac{A_2}{A_1} <$ *(or* $=$, *or* $>$) $\frac{1}{N}$. **And** $\lim_{N\to\infty} \phi T = 1$.

Proof: Taking derivatives of T with respect to ϕ, A_1, A_2, N, and c respectively in (2.38), we have

$$
\begin{aligned}
\frac{\partial T}{\partial \phi} &= -\frac{1}{\phi^2}[1 - \frac{N}{N + (\frac{A_2 N}{A_1})^{\frac{1}{c}}}] < 0; \\
\frac{\partial T}{\partial A_1} &= \frac{-\frac{A_2}{A_1^2}\frac{N^2}{c}(\frac{A_2 N}{A_1})^{\frac{1-c}{c}}}{\phi[N + (\frac{A_2 N}{A_1})^{\frac{1}{c}}]^2} < 0; \\
\frac{\partial T}{\partial A_2} &= \frac{\frac{1}{A_1}\frac{N^2}{c}(\frac{A_2 N}{A_1})^{\frac{1-c}{c}}}{\phi[N + (\frac{A_2 N}{A_1})^{\frac{1}{c}}]^2} > 0; \\
\frac{\partial T}{\partial N} &= \frac{(\frac{1-c}{c})(\frac{A_2 N}{A_1})^{\frac{1}{c}}}{\phi[N + (\frac{A_2 N}{A_1})^{\frac{1}{c}}]^2} > 0; \\
\frac{\partial T}{\partial c} &= \frac{-\frac{N}{c^2}\log(\frac{A_2 N}{A_1})(\frac{A_2 N}{A_1})^{\frac{1}{c}}}{\phi[N + (\frac{A_2 N}{A_1})^{\frac{1}{c}}]^2} > (= \text{ or } <)\ 0 \text{ if } \frac{A_2}{A_1} < (= \text{ or } >)\ \frac{1}{N}.
\end{aligned}
$$

Therefore the tax rate depends positively on A_2 and N and negatively on A_1 and ϕ.

(2.40) can be rewritten as

$$\frac{A_1}{N^{1-c}}(1-\phi T)^{-c} = A_2(\phi T)^{-c}. \tag{2.41}$$

Since $\phi T \leq 1$, $(\phi T)^{-c} \geq 1$; however on the left hand side, when $N \to \infty$, $\frac{A_1}{N^{1-c}} \to 0$, so it must be $(1-\phi T) \to 0$, i.e., $\phi T \to 1$. **Q.E.D.**

Remark: One interesting implication of Corollary 1 is that the equilibrium tax rate increases as the size of the population increases. This might provide another explanation for the widely observed growth of government. The reason behind this result is that when the size of the population increases, the first term in the left hand side of (2.41) decreases, given the other variables, and only a larger T can make the equality (2.41) valid.

Similarly, I present the following corollary concerning the relation between T and A_1, or A_2, or ϕ in (2.39) for a type-b utility function.

Corollary 2: For a type-b utility function, $\frac{\partial T}{\partial \phi} < 0$, $\frac{\partial T}{\partial A_1} < 0$, and $\frac{\partial T}{\partial A_2} > 0$.

Proof: The proof follows from taking derivatives of T with respect to ϕ, A_1 and A_2 respectively in (2.39). As seen, the effects of ϕ, A_1 and A_2 on T have the same signs as those associated with a type-a utility function.

Q.E.D.

2.4.4 Examples

Example 3: All assumptions of Example 1 are valid except that $u(x_1, x_2) = x_1^{1/2} + x_2^{1/2}$ and the distribution of θ is arbitrary. Since $R_r(x) = 1/2 < 1$, conditions α and β hold, Propositions 3 to 8, and Theorems 1, 2 and 3 are all valid. The first order conditions are (2.26) to (2.34). Substituting the utility function and (2.28) to (2.30) into (2.31), we have

$$\frac{\frac{1}{2}[c_i^1 + \frac{(1-\phi)T}{9}(3.6(K^1)^{1/2} + 2K^1)]^{-1/2}}{\frac{1}{2}[c_i^0 + \frac{(1-\phi)T}{9}(68 - K^1)]^{-1/2}} = \frac{1}{2 + 1.8(K^1)^{-1/2}}, \text{ i.e.,}$$

$$\begin{aligned} c_i^0 \; &+ \; \frac{(1-\phi)T}{9}(68 - K^1) = \frac{1}{(2 + 1.8(K^1)^{-1/2})^2}[c_i^1 + \frac{(1-\phi)T}{9}(3.6(K^1)^{1/2} \\ &+ \; 2K^1)]. \end{aligned}$$

Summing up the above equality and using (2.37) yields

$$(1 - T\phi)(68 - K^1) = \frac{(1 - T\phi)}{(2 + 1.8(K^1)^{-1/2})^2}(3.6(K^1)^{1/2} + 2K^1).$$

As argued before, in equilibrium, $T \neq 1$ or $\phi \neq 1$, so we have

$$h(K^1) = 6K^1 + 10.8(K^1)^{1/2} - 268.76 - 489.6(K^1)^{-1/2} - 220.32(K^1)^{-1} = 0.$$

Since $h'(K^1) > 0$ if $K^1 \geq 0$, and

$$h(45.54354) = 0.00007 > 0 \text{ and } h(45.54353) = -0.00001 < 0.$$

Thus there is a unique nonnegative solution for K^1, i.e., $(K^1)^e = 45.54353$, which I substitute it into (2.26) to (2.30), producing $W^{0^e} = 1$, $W^{1^e} = 1.34972$, $P^e = 0.44117$, $y^{0^e} = 22.45647$, $y^{1^e} = 115.38200$, $(\pi^0)^e = 13.45647$, and $\left(\frac{\pi^1}{P}\right)^e = 103.23453$.[30] Comparing (2.40) with (2.41), we get $c_i^0 = P^2 c_i^1 = 0.19463 c_i^1$. Substituting $c_i^0 = 0.19463 c_i^1$ into (2.32), we can solve both c_i^0 and c_i^1, i.e.,

$$c_i^1 = (1-T)(2.50937 + 92.797180\theta_i) \text{ and}$$

$$c_i^0 = (1-T)(0.48840 + 18.061120\theta_i).$$

Therefore we confirm Proposition 7, that fiscal policy has no effect on macroeconomic performance, although it does affect personal consumption.

Suppose $\theta_m = 1/9$, then substituting $(K^1)^e$, θ_i, c_i^0 and c_i^1 into our utility function, we have

$$U = u\{2.49516(1-\phi T), 22.45647\phi T\} + u\{12.82022(1-\phi T), 115.38200\phi T\}.$$

After simplification, condition $\frac{\partial U}{\partial \phi} = 0$ becomes $\phi T = 9(1-\phi T)$. It can be verified that this equation also guarantees $\frac{\partial U}{\partial T} = 0$, and $\phi T = 0.9$[31] is the solution which leads to utility $U = 16.31780$ for the median voter regardless of any combination of (T, ϕ) that satisfies $\phi T = 0.9$. Clearly the median voter is much better off now than when there is no government (in that case his or her utility is $U = 5.16014$).

[30] This is an economy with increasing marginal productivity (i.e., $W^1 > W^0$), deflation (i.e., $P < 1$), increasing of profit in real terms (i.e., $(\frac{\pi^1}{P}) > \pi^0$), and increasing of average personal consumption (i.e., $\frac{y^1}{9} > \frac{y^0}{9}$).

[31] If $\phi < 0.9$, then $\phi T < 0.9$ and $\frac{\partial U}{\partial T} > 0$, thus $T = 1$.

Furthermore, if all $\theta_i = 1/9$ and any candidate picks (T, ϕ) such that $\phi T = 0.9$, then for each period $t(t = 0, 1)$, we have $\forall$i$(i = 1, 2, \ldots, N)$,

$$\frac{\partial U/\partial x_2}{\partial U/\partial x_{i1}} = \frac{[22.45647\phi T]^{-1/2}}{[2.49516(1-\phi T)]^{-1/2}} = \frac{1}{3}[\frac{(1-\phi T)}{\phi T}]^{1/2} = \frac{1}{9}.$$

Thus $\sum_{i=1}^{9} \frac{\partial U/\partial x_2}{\partial U/\partial x_{i1}} = 1$. A similar result applies for $t = 1$. Therefore we have verified Proposition 5 because $\sum_{i=1}^{9} \frac{\partial U/\partial x_2}{\partial U/\partial x_{i1}} = 1$.

Example 4: Consider $u(x_1, x_2) = A - x_1^{-1/2} - x_2^{-1/2}$, according to Pratt (1964), $R_r(x) = 1.5$. Assuming all assumptions of example 3 hold and following the same steps as example 3, we find that if $T \neq 1$ or $\phi \neq 1$, then

$$h(K^1) = (68 - K^1)^3 - \frac{\left(3.6(K^1)^{1/2} + 2K^1\right)^3}{\left(2 + 1.8(K^1)^{-1/2}\right)^2} = 0$$

and $h'(K^1) < 0 \; if \; K^1 \geq 0$. As before, we have a unique nonnegative solution: $(K_1)^e = 26.92714$, $(W^0)^e = 1$, $(W^1)^e = 1.03783$, $P^e = 0.42610$, $(y^0)^e = 41.07286$, $(y^1)^e = 72.53517$, $(\pi^0)^e = 32.07286$, and $(\frac{\pi^1}{P})^e = 63.19473$. In addition,

$$c_i^0 = (1 - T)(0.82295 + 33.66637\theta_i) \text{ and}$$

$$c_i^1 = (1 - T)(1.45334 + 59.45496\theta_i).$$

It is not surprising that we have not found any different conclusions regarding Proposition 7 between the above examples 3 and 4 because they share condition β. However, because example 4 violates condition α, I expect it to have a different relationship between the willingness to tax and income. This turns out to be true. Substituting $R_r(x) = 1.5$, $N = 9$ and the solution of all related variables into

$$\frac{\partial}{\partial\theta_i}(\frac{\partial U}{\partial T}) = -\frac{\partial U}{\partial\theta_i}\left\{[\frac{(1-\phi)}{N}(y^0+Py^1)R_r(\theta_i)]/\theta_i + [1-R_r(\theta_i)]\right\}$$

and simplifying, we have

$$\frac{\partial}{\partial\theta_i}(\frac{\partial U}{\partial T}) < 0 \Longleftrightarrow \theta_i < 0.40667(1-\phi) - 0.02444;$$

$$\frac{\partial}{\partial\theta_i}(\frac{\partial U}{\partial T}) = 0 \Longleftrightarrow \theta_i = 0.40667(1-\phi) - 0.02444;$$

$$\frac{\partial}{\partial\theta_i}(\frac{\partial U}{\partial T}) > 0 \Longleftrightarrow \theta_i > 0.40667(1-\phi) - 0.02444.$$

As long as $\phi < 0.93990$ or $(1-\phi) > 0.06010, 0.40667(1-\phi) - 0.02444 = \theta^* > 0$, then if $0 \leq \theta_i < \theta^*$, the willingness to tax decreases as profit share increases; if $\theta_i = \theta^*$, the willingness to tax is a constant; if $\theta^* < \theta_i \leq 1$, the willingness to tax increases as profit share increases. This is counter-intuitive, for instance, let $\phi = 0.5$, then $\theta^* = 0.17890$ (see Figure 2.7).

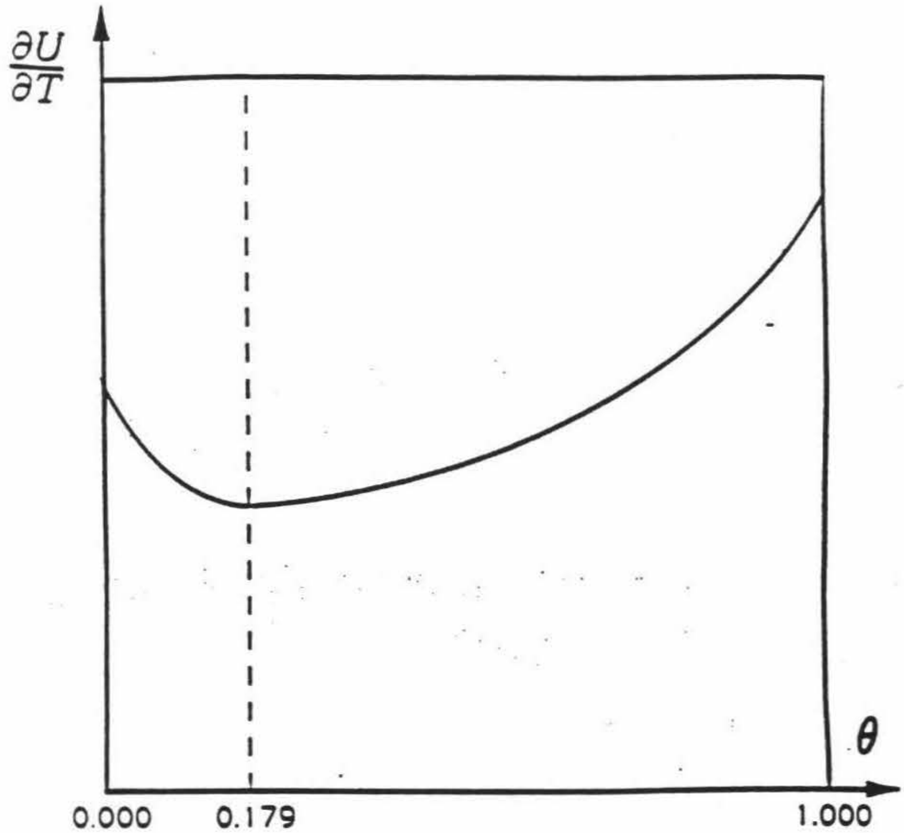


Figure 2.7: Profit Share and the Marginal Utility of Tax for Example 4.

2.5 Summary

Now I am able to answer the questions posed in the Introduction. First, it is possible to explain observed differences in long-run growth rates without relying on exogenous changes in technology or population. Given technology and population, a country's economic growth is determined by its fiscal policy, which is in turn decided by the distribution of income and factors that influence the form of utility functions. Second, a higher tax rate is not definitely associated with a lower rate of economic growth. Third, a political-economic equilibrium does exist when candidates compete for office by selecting different fiscal policies. In equilibrium, the median-income consumer will be the majority winner of the binary electoral process. Fourth, in general, the wealthier consumers are more likely to oppose a higher tax rate or a larger government. Fifth, under constant EMUI, fiscal policy and income distribution have no effects on economic growth. Sixth, an increase in mean income relative to the income of the decisive voter increases the size of government. Seventh, the Kuznets curve is essentially an empirical observation and lacks solid theoretical support. Proposition 9 shows that the income tax rate does not respond to technical change (i.e., economic development).

Chapter 3

Fiscal Policies, Optimal Growth, and Elections in a Democratic Planned Economy

3.1 Model

3.1.1 Basic Assumptions

In this chapter, I study fiscal policies, optimal growth, and elections in a democratic planned economy. The democratic planned economy here is not the traditional authoritarian planned economy since an authoritarian planned economy does not have elections. The democratic planned economy I study in this chapter has the following two fundamental properties. First, there are elections. Second, the elected government directly controls the growth of the economy. Thus a democratic planned economy differs from an authoritarian planned economy in a political respect, i.e., whether there are elections. And a democratic planned economy and a free economy differ in the way the economy is controlled (directly or indirectly).

In reality, two kinds of economies can be fit into my category of democratic planned economies. The first kind of democratic planned economies refers to some Western European countries (such as France and Sweden) where the public sectors are large enough such that governments' decisions on the wages, prices and growth rates of public sectors can substantially influence the wages, prices and growth rates of the economy as a whole. The second kind of democratic planned economies includes most of the Eastern European countries and former Soviet Republics. In those transitional economies, first, the private sector economy has not become a dominant factor in the economy; second, property rights are not well protected; third, market mechanisms are still immature. Thus, elected governments still control wages, prices and economic growth rates.

Consider a planned economy with a government and N consumers.[1] I assume there are two kinds of output, namely a private good X_1 and a public good X_2, and two kinds of input, namely capital K and labor L in the economy. As before, I assume P^t to be the price of a unit of output X_1 delivered in period t, and W^t to be the real wage rate in period t.

Suppose there is a political election using some binary procedure under majority rule. Any consumer can be a candidate and compete with other candidates by selecting a platform of price ratio, wage rates and capital

[1] I neglect firms because the government in a planned economy owns the firms and controls wage rates, prices and capital stocks.

stocks. Eventually, one of the N consumers is elected and implements those macroeconomic variables he proposed during the campaign.

Assume that each consumer $i (i = 1, 2, \ldots, N)$ lives two periods and has a discount rate $\beta \in [0, 1]$. In each period i has one unit of time to spend and has a utility function $u(x_{i1}, x_2)$, where x_{i1} and x_2 are i's consumption levels of the private good X_1 and the public good X_2 in period t respectively. The production technologies of the private good and the public good are assumed to be the same as those in chapter 2 and capital stock in period 0, K^0 is given as $\bar{K}$.

Because government owns the firm, I assume that the government keeps a fixed portion of the profit and this part of profit becomes the government's revenue. As before, government revenue can be used either in the transfer of the private good for the purpose of redistribution and/or the production of the public good.

The profit left over is owned by consumers. Each consumer i shares a fixed $\theta_i \in [0, 1] (\sum_{i=1}^{N} \theta_i = 1)$. All assumptions in Chapter 2 are valid.

3.1.2 Notation and Timing of the Model

N is the number of consumers; and t, which is used as a time index, is 0 or 1; ψ is the fixed share of profit by government; ϕ is the proportion of government revenue used in the public good production; $(1 - \phi)$ is the proportion of government revenue used in the private good transfer; the price of X_1 in period 0 is taken as a numerare, i.e., $P^0 = 1$; $P^1 = P$ is the price of X_1 in period 1; W is the real wage rate; y is the amount of X_1 sold by the firm; K is the aggregate capital input; L is the aggregate labor input; $\pi^0 = y^0 - W^0 L^0$ is the profit of period 0; $\pi^1 = P(y^1 - W^1 L^1)$ is the profit of period 1; θ_i ($\sum_{i=1}^{N} \theta_i = 1$) is consumer i's fixed share of the profit excluding the government's share; l_i is the amount of labor supplied by consumer i; β is the discount factor for each consumer; c_i is the pre-transfer consumption level of X_1 by consumer i; x_{i1} is the total consumption level of X_1 (including transfer from the government) by consumer i; x_2 is the consumption level of the public good by each consumer; and U_i is the sum of discounted utilities by consumer i. I.e.,

$$\begin{aligned}
U_i &= \sum_{t=0}^{1} \beta^t u(x_{i1}^t, x_2^t) \text{ where} \\
x_{i1}^0 &= c_i^0 + \frac{(1-\phi)\psi}{N}[F(\bar{K}) - K^1 - NW^0] \\
x_{i1}^1 &= c_i^1 + \frac{(1-\phi)\psi}{N}[F(K^1) - NW^1] \\
x_2^0 &= \phi\psi[F(\bar{K}) - K^1 - NW^0] \\
x_2^1 &= \phi\psi[F(K^1) - NW^1].
\end{aligned}$$

The model and its timing are given below (see Figure 3.1):

In stage 0, given the set of consumers and a fixed binary voting procedure, each consumer decides whether he or she will be a candidate. If he or she decides not to be a candidate, his or her opponent automatically wins the binary competition and goes to the next branch of the voting tree.

In stage 1, a candidate $j (j = 1, 2, \ldots, N)$ proposes a set of wage rates

W_j^0, W_j^1, price P_j, and capital stocks K_j^1, K_j^2 in order to maximize his or her own utility which is given in stage 2 for any consumer.

In stage 2, for any given W^0, W^1, P, K^1 and K^2 (for simplicity we omit all subscripts later on), a consumer $i (i = 1, 2, \ldots, N)$ chooses c^t, l^t to maximize his or her sum of discounted utilities.

$$\max U_i = \max \sum_{t=0}^{1} \beta^t u(x_{i1}^t, x_2^t) \tag{3.1}$$

$$= \max \sum_{t=0}^{1} \beta^t u \left\{ c_i^t + \frac{(1-\phi)\psi}{N} [F(K^t)\ K^{t+1} - NW^t], \phi\psi[F(K^t) - K^{t+1} - NW^t] \right\}$$

$$\text{s.t. } \sum_{t=0}^{1} P^t c_i^t \leq \sum_{t=0}^{1} P^t \left\{ W^t + \theta_i (1-\psi)[F(K^t) - K^{t+1} - NW^t] \right\}$$

$$0 \leq l^t \leq 1,\ c_i^t \geq 0,\ 0 \leq \theta_i \leq 1,\ \sum_{i=1}^{N} \theta_i = 1,\ \text{and } t = 0, 1.$$

Then each consumer solves c_i^0 and c_i^1 in terms of the proposed macroeconomic variables, i.e., $(c_i^0)^e = c_i^0(W^0, W^1, P, K^1, K^2)$ and

$$(c_i^1)^e = c_i^1(W^0, W^1, P, K^1, K^2).$$

In stage 3, each consumer $i (i = 1, 2, \ldots, N)$ votes for

$$j = \arg\max \sum_{t=0}^{1} u[x_{i1}^t(W^0, W^1, P, K^1), x_2^t(W^0, W^1, P, K^1)]$$

and if indifferent, splits the vote.

In stage 4, the candidate with the larger number of votes in each binary competition is elected and implements the set of macroeconomic variables he or she proposed during the campaign.

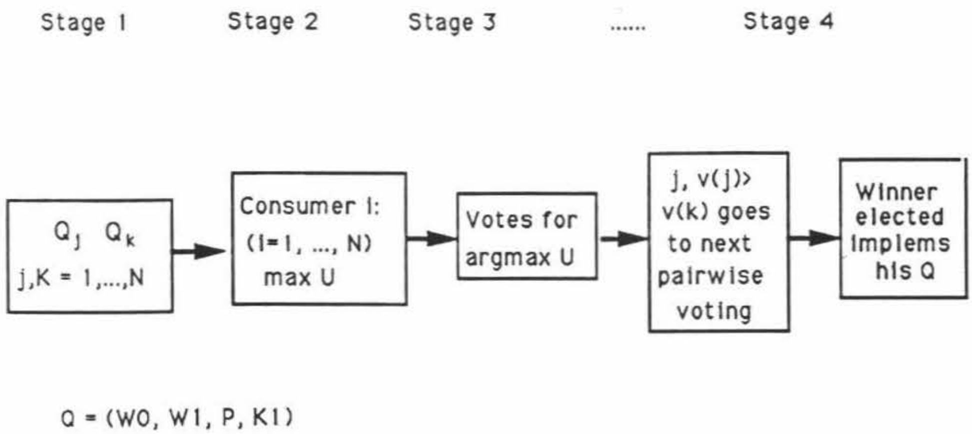


Figure 3.1: Timing for a Democratic Planned Economy

There are some differences between this model and the one in a free economy: first, the government here controls all macroeconomic variables; while the government in a free economy only indirectly controls the economy through tax policy. Second, the government in a free economy uses some of the tax revenue to produce the public good and make private good transfers, while the government in a democratic planned economy sets somewhat lower wage rates to generate profit and uses its fixed share of the profit to produce the public good and make private good transfers.

3.1.3 Definitions

Definition 5: Given W^0, W^1, P and K^1, *a competitive equilibrium in a democratic planned economy* is an allocation $\{(c_i^t, l_i^t)\}_{t=0}^1$ for consumer $i(i = 1, 2, \ldots, N)$, s.t., $\{(c_i^t, l_i^t)\}_{t=0}^1$ solves (3.1) at the stated prices.

Definition 6: *A political-economic equilibrium in a democratic planned economy* is a set of a consumer (or consumers) $\{j\}_{j=1,2,\ldots,N}$, a four-tuple policy instrument (a set of four-tuple policy instruments) (W^0, W^1, P, K^1), and a competitive equilibrium corresponding to W_j^0, W_j^1, P_j and K_j^1, denoted by c.e.$(W_j^0, W_j^1, P_j, K_j^1)$, such that

$$\left\{j; W_j^0, W_j^1, P_j, K_j^1, \text{c.e.}(W_j^0, W_j^1, P_j, K_j^1)\right\}$$

solves the model (stages 1 to 4) in 3.1.2.

Definition 7: *A democratic planned economy is distributionally neutral* if all macroeconomic variables K^1, W^0, W^1 and P are independent of θ.

3.2 Main Conclusions

3.2.1 Existence of a Competitive Equilibrium

For a typical consumer, it is obvious that he or she supplies all available factor L, because L causes no disutility to him or her, i.e., $l^0 = l^1 = 1$, thus $L^0 = L^1 = N$. It is also clear that $y^2 = 0$ because period 2 is the end of the world. As before let $u(x_{i1}^t, x_2^t) = \chi(x_{i1}^t) + \xi(x_2^t)$. For any consumer i, by taking first derivatives with respect to c_i^0, c_i^1, λ_i (Lagrangian multiplier), P, W^0, W^1, K^1, I have the following first order conditions:

$$
\begin{aligned}
\chi_0' &\leq \lambda_i, \qquad c_i^0 \geq 0 && (3.2)\\
\beta\chi_1' &\leq \lambda_i P \quad c_i^1 \geq 0 && (3.3)\\
W^0 &+ \theta_i(1-\psi)[F(\bar{K}) - K^1 - NW^0]\\
&+ P\left\{W^1 + \theta_i(1-\psi)[F(K^1) - NW^1]\right\} \geq c_i^0 + Pc_i^1, \quad \lambda_i \geq 0 && (3.4)
\end{aligned}
$$

$$
\begin{aligned}
\lambda_i c_i^1 \leq \lambda_i \left\{W^1 + \theta_i(1-\psi)[F(K^1) - NW^1]\right\}, \quad P \geq 0 && (3.5)\\
\lambda_i[1 - \theta_i(1-\psi)N] \leq (1-\phi)\psi\chi_0' + \phi\psi N\xi_0', \quad W^0 \geq 0 && (3.6)\\
\lambda_i P[1 - \theta_i(1-\psi)N] \leq \beta[(1-\phi)\psi\chi_1' + \phi\psi N\xi_1'], \quad W^1 \geq 0 && (3.7)
\end{aligned}
$$

$$
\begin{aligned}
&- \frac{(1-\phi)\psi}{N}\chi_0' - \phi\psi\xi_0' + \frac{\beta(1-\phi)\psi}{N}F'(K^1)\chi_1' + \beta\phi\psi F'(K^1)\xi_1'\\
&- \theta_i(1-\psi)\lambda_i + \theta_i(1-\psi)PF'(K^1)\lambda_i \leq 0, \quad K^1 \geq 0. && (3.8)
\end{aligned}
$$

I can now prove the following proposition regarding the optimal problem for each consumer.

Proposition 10: Each consumer i ($\forall$i$= 1, 2, \ldots, N$) has a unique set of

$$\left\{W^0, W^1, P, K^1, c_i^0, c_i^1\right\}$$

which solves his optimal problem.

Proof: Because the marginal utility of income $\lambda_i > 0$, by the Kuhn-Tucker Theorem, (3.4) is binding. Since $\lambda_i > 0$ and $\chi_1^{'} > 0$, by (3.3), $P > 0$, thus (3.5) is binding, i.e.,

$$c_i^1 = W^1 + \theta_i(1 - \psi)[F(K^1) - NW^1]. \tag{3.9}$$

Substituting (3.9) into (3.4), we have

$$c_i^0 = W^0 + \theta_i(1 - \psi)[F(\bar{K}) - K^1 - NW^0]. \tag{3.10}$$

By the Inada condition $\lim_{K \to 0} f_1(K, N) = +\infty$, $K^1 > 0$, thus (3.8) is binding too.

There now remain four inequalities, i.e., (3.2), (3.3), (3.6) and (3.7), that remain undetermined whether they are binding or not. For simplicity, let

$$\begin{aligned} \rho &\equiv \phi\psi \\ \mu &\equiv \theta_i(1 - \psi) + \frac{(1 - \phi)\psi}{N} \end{aligned}$$

$$\tau \equiv 1 - \theta_i(1-\psi)N - (1-\phi)\psi.$$

Case 1: Both (3.6) and (3.7) are not binding. Now by Kuhn-Tucker Theorem, $W^0 = W^1 = 0$.

First, let $\theta_i > 0$. Substituting θ_i into (3.10) and (3.9), I have

$$\begin{aligned} c_i^0 &= \theta_i(1-\psi)F(K^1) > 0 \\ c_i^1 &= \theta_i(1-\psi)[F(\bar{K}) - K^1] > 0. \end{aligned}$$

Since c_i^0 and c_i^1 are both greater than 0 because of $F(K^1) > 0$ and $F(\bar{K}) - K^1 > 0$ (guaranteed by the Inada condition), (3.2) and (3.3) are binding. Using (3.2), (3.3), (3.9) and (3.10) in (3.8) yields

$$\begin{aligned} H(K^1) &\equiv -\mu\chi'\left\{\mu[F(\bar{K}) - K^1]\right\} - \rho\xi'\left\{\rho[F(\bar{K}) - K^1]\right\} \\ &+ \beta\mu\chi'[\mu F(K^1)]F'(K^1) + \beta\rho F'(K^1)\xi'[\rho F(K^1)] = 0. \quad (3.11) \end{aligned}$$

Since $H'(K^1) = \mu^2\chi_0'' + \beta[\mu F'(K^1)]^2\chi_1'' + \rho^2\xi_0'' + \beta[\rho F'(K^1)]^2\xi_1'' + \beta\mu\chi_1'F''(K^1) + \beta\rho\xi_1'F''(K^1) < 0$, and $\lim_{\epsilon\to 0} H(\epsilon) = +\infty$ and $\lim_{\epsilon\to 0} H[F(\bar{K}) - \epsilon] = -\infty$, $\exists$ a unique $K^1 \in (0, F(\bar{K}))$ which solves (3.8). Then by (3.9) and (3.10), I can solve for unique c_i^0 and c_i^1. By (3.2) and (3.3), $\frac{\beta\chi_1'}{\chi_1'} = P$ also gives unique P.

Second, $\theta_i = 0$, then $c_i^0 = c_i^1 = 0$, and (3.8) can be rewritten as

$$\begin{aligned} H(K^1) &\equiv -\frac{(1-\phi)\psi}{N}\chi'\left\{\frac{(1-\phi)\psi}{N}[F(\bar{K}) - K^1]\right\} - \rho\xi'\left\{\rho[F(\bar{K}) - K^1]\right\} \\ &+ \beta\frac{(1-\phi)\psi}{N}\chi'[\frac{(1-\phi)\psi}{N}F(K^1)]F'(K^1) + \beta\rho F'(K^1)\xi'[\rho F(K^1)] = 0. \end{aligned}$$

Similarly as part 1, $H'(K^1) < 0$, K^1 is uniquely determined. By using

$$P = \frac{\beta\chi'[\frac{(1-\phi)\psi}{N}F(K^1)]}{\chi'\left\{\frac{(1-\phi)\psi}{N}[F(\bar{K}) - K^1]\right\}},$$

P can be uniquely solved.

Case 2: (3.6) is binding, while (3.7) is not. Then by the Kuhn-Tucker Theorem, $W^0 > 0$ and $W^1 = 0$. Similarly as case 1, if $\theta_i > 0$ (the case $\theta_i = 0$ can be proved similarly), we have $c_i^0, c_i^1 > 0$. Hence (3.2) and (3.3) are binding. Using (3.2), (3.3), (3.9) and (3.10), (3.6) and (3.8) can be rewritten as

$$\tau\chi'\left\{\tau W^0 + \mu[F(\bar{K}) - K^1]\right\} = \rho N\xi'\left\{\rho[F(\bar{K}) - K^1 - NW^0]\right\}, \quad (3.12)$$

$$\begin{aligned} &- \mu\chi'\left\{\tau W^0 + \mu[F(\bar{K}) - K^1]\right\} - \rho\xi'\left\{\rho[F(\bar{K}) - K^1 - NW^0]\right\} \\ &+ \beta F'(K^1)\mu\chi'[\mu F(K^1)] + \beta\rho F'(K^1)\xi'[\rho F(K^1)] = 0. \quad (3.13) \end{aligned}$$

Using (3.12) in (3.13) yields

$$0 = -\chi'\left\{\tau W^0 + \mu[F(\bar{K}) - K^1]\right\} + \beta F'(K^1)\mu\chi'[\mu F(K^1)] + \beta\rho F'(K^1)\xi'[\rho F(K^1)].$$

Taking the derivative with respect to K^1 in the above equation and solving $\partial W^0/\partial K^1$ yields

$$\frac{\partial W^0}{\partial K^1} = \frac{1}{\tau\chi_0''}\left\{\mu\chi_0'' + \beta\mu\chi_1'F''(K^1) + \beta[\mu F'(K^1)]^2\chi_1''\right.$$

$$+ \quad \beta\rho\xi_1' F''(K^1) + \beta[\rho F'(K^1)]^2 \xi_1'' \Big\} . \tag{3.14}$$

First, K^1 is uniquely determined. Let

$$\begin{aligned} H(K^1) &\equiv LHS(3.12) - RHS(3.12) \\ &= \tau\chi' \left\{ \tau W^0 + \mu[F(\bar{K}) - K^1] \right\} - \rho N\xi' \left\{ \rho[F(\bar{K}) - K^1 - NW^0] \right\}. \end{aligned}$$

Taking the derivative with respect to K^1, using (3.14) and simplifying, I can verify $H'(K^1) < 0$. I am now able to prove the uniqueness of K^1. (i) $H(0) \leq 0$, then combining $H(0) \leq 0$ and $H'(K^1) < 0$ yields $H(K^1) < 0$ for all $K^1 \in (0, F(\bar{K})]$. By the Inada condition, $K^1 \neq 0$, thus $H(K^1) < 0$ for all $K^1 > 0$, which contradicts the assumption that (3.6) is binding. (ii) $H(0) > 0$, then because $\lim_{K^1 \to F(\bar{K}) - NW^0} H(K^1) = -\infty, \forall W^0 > 0$, $\exists$ a unique $K^1 \in (0, F(\bar{K}))$ satisfying $H(K^1) = 0$.

Second, W^0, W^1, P, c_i^0, c_i^1 are all uniquely determined for any given K^1. Clearly $W^1 = 0$ is unique. If I can prove that W^0 is unique, then c_i^0 and c_i^1 are unique too because of (3.9) and (3.10). Then by (3.2) and (3.3), P is unique too. Now let $G(W^0) = LHS(3.12) - RHS(3.12)$. Then it is easy to check $G'(W^0) < 0$. (i) If $G(0) \leq 0$, then combining $G'(W^0) < 0$ and $W^0 > 0$, we have $G(W^0) < 0$, which contradicts the assumption that (3.12) is binding. (ii) If $G(0) > 0$, then by $\lim_{W^0 \to \frac{F(\bar{K}) - K^1}{N}} G(W^0) = -\infty$ and $G'(W^0) < 0$, $\exists$ a unique $W^0 \in (0, \frac{F(\bar{K}) - K^1}{N})$ such that (3.12) holds.

Case 3: (3.6) is not binding, while (3.7) is binding. The unique existence of $\{W^0, W^1, P, K^1, c_i^0, c_i^1\}$ can be similarly proved as Case 2.

Case 4: Both (3.6) and (3.7) are binding. Then by the Kuhn-Tucker

Theorem, $W^0, W^1 > 0$, by (3.9) and (3.10), c_i^0, $c_i^1 > 0$, thus both (3.2) and (3.3) are binding. Substituting (3.6) and (3.7) into (3.8), I have

$$
\begin{aligned}
& - \lambda_i \left\{ \frac{(1-\phi)\psi}{N} + [\frac{1}{N} - \theta_i(1-\psi) - \frac{(1-\phi)\psi}{N}] + \theta_i(1-\psi) \right\} [1 - PF'(K^1)] \\
& = -\frac{\lambda_i}{N}[1 - PF'(K^1)] = 0.
\end{aligned}
$$

Recalling $\lambda_i = \chi_0' > 0$, I have $P = \frac{1}{F'(K^1)}$. Using (3.2), (3.3), (3.9) and (3.10), we can rewrite (3.6), (3.7) and (3.8) as follows.

$$
\begin{aligned}
\tau\chi' \left\{\tau W^0 + \mu[F(\bar{K}) - K^1]\right\} &= \rho N\xi' \left\{\rho[F(\bar{K}) - K^1 - NW^0]\right\} && (3.15) \\
\tau\chi'[\tau W^1 + \mu F(K^1)] &= \rho N\xi' \left\{\rho[F(K^1) - NW^1]\right\} && (3.16) \\
\chi' \left\{\tau W^0 + \mu[F(\bar{K}) - K^1]\right\} &= \beta F'(K^1)\chi'[\tau W^1 + \mu F(K^1)]. && (3.17)
\end{aligned}
$$

Taking derivatives with respect to K^1 in (3.15) and (3.16) gives

$$
\begin{aligned}
\frac{\partial W^0}{\partial K^1} &= \frac{\tau\mu\chi_0'' - \rho^2 N\xi_0''}{\tau^2\chi_0'' + (\rho N)^2\xi_0''} && (3.18) \\
\frac{\partial W^1}{\partial K^1} &= \frac{(\rho^2 N\xi_1'' - \tau\mu\chi_1'')F'(K^1)}{\tau^2\chi_1'' + (\rho N)^2\xi_1''}. && (3.19)
\end{aligned}
$$

First, K^1 is uniquely determined. Let $H(K^1) \equiv RHS(3.17) - LHS(3.17) = 0$. Taking the derivative with respect to K^1, using (3.18) and (3.19) and simplifying, I have $H'(K^1) < 0$.

Using (3.15) and (3.16), $H(K^1)$ can be rewritten as

$$
H(K^1)\frac{\tau}{\rho N} = \beta F'(K^1)\xi' \left\{\rho[F(K^1) - NW^1]\right\} - \xi' \left\{\rho[F(\bar{K}) - K^1 - NW^0]\right\}.
$$

Because $H'(K^1) < 0$, and $\lim_{\epsilon \to 0} H(\epsilon) = +\infty$ and $\lim_{\epsilon \to 0} H[F(\bar{K}) - \epsilon] = -\infty$, $\exists$ a unique $K^1 \in (0, F(\bar{K}))$ satisfying (3.17).

Second, $W^0, W^1, P, c_i^0, c_i^1$ are all uniquely determined for any given K^1. Let $G(W^0) = LHS(3.15) - RHS(3.15) = 0$, then the uniqueness of W^0 can be similarly proved as the second part in Case 2. Similarly, W^1 is unique. So are c_i^0, c_i^1 and P. **Q.E.D.**

Corollary 1: If $\psi > 0$ and i ($\forall$i$= 1, \ldots, N$) has a large profit share such that

$$\theta_i \geq \frac{1 - (1 - \phi)\psi}{(1 - \psi)N},$$

then he or she will propose $W^0 = W^1 = 0$.

Proof: Since $\theta_i > 0$, then by (3.9) and (3.10), $c_i^0 > 0$ and $c_i^1 > 0$. Therefore (3.2) and (3.3) are binding. Substituting (3.2) into (3.9), I have

$$\lambda_i[1 - \theta_i(1 - \psi)N - (1 - \phi)\psi]\chi_0' \leq \phi\psi\xi_0'.$$

If $\theta_i \geq \frac{1-(1-\phi)\psi}{(1-\psi)N}$, since $\lambda_i \geq 0$, $\lambda_i[1 - \theta_i(1 - \psi)N - (1 - \phi)\psi] \leq 0$, however $\psi > 0$, and $\xi_t' > 0$, therefore inequality (3.6) is never binding, thus $W^0 = 0$ by the Kuhn-Tucker Theorem. Similarly, $W^1 = 0$. **Q.E.D.**

3.2.2 Existence of Voting Equilibrium

One might suspect that as a multi-dimensional problem (in terms of W^0, W^1,P, K^1), the election in a democratic planned economy will lead us to nowhere, i.e., no political-economic equilibria or infinity of equilibria. Fortunately I can prove strict quasi-concavity (Proposition 10 below) of U on W^0, W^1, P, K^1, thus according to Kramer (1972), there exists a voting equilibrium $X^0 \in R^4_+$, such that none of the individual components of any feasible change (i.e., change X^0 only by one dimension each time) from X^0 will be preferred to the status quo by any decisive coalition.

Proposition 11: (a) The utility function in (3.1) is strictly quasiconcave in W^0, W^1, P, K^1.

(b) There exists a voting equilibrium X^0 which will defeat any feasible change in only one dimension.

Proof: (a) Taking derivatives with respect to W^0, W^1, P, K^1 in (3.1) gives

$$\begin{aligned}
\frac{\partial U}{\partial W^0} &= \lambda_i[1-\theta_i(1-\psi)N] - (1-\phi)\psi\chi_0' - \phi\psi N\xi_0' \\
\frac{\partial U}{\partial W^1} &= \lambda_i P[1-\theta_i(1-\psi)N] - \beta(1-\phi)\psi\chi_1' - \beta\phi\psi N\xi_1' \\
\frac{\partial U}{\partial P} &= \lambda_i W^1 + \lambda_i\theta_i(1-\psi)[F(K^1) - NW^1] - \lambda_i c_i^1 \\
\frac{\partial U}{\partial K^1} &= \lambda_i\theta_i(1-\psi)PF'(K^1) - \lambda_i\theta_i(1-\psi) - \frac{(1-\phi)\psi}{N}\chi_0' \\
&- \phi\psi\xi_0' + \frac{\beta(1-\phi)\psi}{N}F'(K^1)\chi_1' + \beta\phi\psi F'(K^1)\xi_1'.
\end{aligned}$$

The second derivatives are

$$\begin{aligned}
\frac{\partial^2 U}{\partial (W^0)^2} &= [(1-\phi)\psi]^2\chi_0'' + (\phi\psi N)^2\xi_0'' \\
\frac{\partial^2 U}{\partial W^0\partial W^1} &= 0 \\
\frac{\partial^2 U}{\partial W^0\partial P} &= 0
\end{aligned}$$

$$
\begin{aligned}
\frac{\partial^2 U}{\partial W^0 \partial K^1} &= \frac{[(1-\phi)\psi]^2}{N}\chi_0'' + (\phi\psi)^2 N \xi_0'' \\
\frac{\partial^2 U}{\partial W^1 \partial W^0} &= 0 \\
\frac{\partial^2 U}{\partial (W^1)^2} &= \beta[(1-\phi)\psi]^2 \chi_1'' + \beta(\phi\psi N)^2 \xi_1'' \\
\frac{\partial^2 U}{\partial W^1 \partial P} &= \lambda_i[1-\theta_i(1-\psi)N] \\
\frac{\partial^2 U}{\partial W^1 \partial K^1} &= -\beta\frac{[(1-\phi)\psi]^2}{N} F'(K^1)\chi_1'' - \beta(\phi\psi)^2 N F'(K^1)\xi_1'' \\
\frac{\partial^2 U}{\partial P \partial W^0} &= 0 \\
\frac{\partial^2 U}{\partial P \partial W^1} &= \lambda_i[1-\theta_i(1-\psi)N] \\
\frac{\partial^2 U}{\partial P^2} &= 0 \\
\frac{\partial^2 U}{\partial P \partial K^1} &= \lambda_i\theta_i(1-\psi)F'(K^1) \\
\frac{\partial^2 U}{\partial K^1 \partial W^0} &= \frac{[(1-\phi)\psi]^2}{N}\chi_0'' + (\phi\psi)^2 N \xi_0'' \\
\frac{\partial^2 U}{\partial K^1 \partial W^1} &= -\beta\frac{[(1-\phi)\psi]^2}{N} F'(K^1)\chi_1'' - \beta(\phi\psi)^2 N F'(K^1)\xi_1'' \\
\frac{\partial^2 U}{\partial K^1 \partial P} &= \lambda_i\theta_i(1-\psi)F'(K^1) \\
\frac{\partial^2 U}{\partial (K^1)^2} &= \lambda_i\theta_i(1-\psi)PF''(K^1) + \frac{[(1-\phi)\psi]^2}{N^2}\chi_0'' + (\phi\psi)^2\xi_0'' \\
&+ \beta\frac{[(1-\phi)\psi F'(K^1)]^2}{N^2}\chi_1'' + \beta\frac{(1-\phi)\psi}{N}F''(K^1)\chi_1' \\
&+ \beta[\phi\psi F'(K^1)]^2\xi_1'' + \beta\phi\psi F''(K^1)\xi_1'.
\end{aligned}
$$

After simplification, the determinant of the second order derivative is

$$
|D| = -[[(1-\phi)\psi]^2\chi_0'' + (\phi\psi N)^2\xi_0'']\left\{\lambda_i{}^2[1-\theta_i(1-\psi)N]^2 F''(K^1)\right.
$$

$$[\lambda_i\theta_i(1-\psi)P + \frac{\beta(1-\phi)\psi}{N}\chi_1' + \beta\phi\psi\xi_1']$$
$$+ \quad \beta[\frac{\psi\lambda_i F'(K^1)}{N}]^2 [(1-\phi)^2\chi_1'' + (\phi N)^2\xi_1''] \Bigg\} < 0.$$

Thus the utility function is strictly quasiconcave in W^0, W^1, P, K^1 because it is negative definite (see Diewert, Avriel and Zang (1981)).

(b) By Kramer (1972), there exists a voting equilibrium

$$X^0 = \left\{(W^0)^e, (W^1)^e, P^e, (K^1)^e\right\}$$

such that none of the individual components of any feasible change from X^0 will be preferred by any decisive coalition to the status quo. **Q.E.D.**

3.2.3 Uniform Distribution of Income

Similarly to Proposition 5, when the distribution of wealth is uniform, I can prove that satisfying the Bowen-Lindahl-Samuelson condition is a necessary and sufficient condition for achieving a political-economic equilibrium.

Proposition 12: If $\psi > 0$, $\phi > 0$,[2] the distribution of wealth is uniform, and W^0, W^1, $c_i^0, c_i^1 > 0$, then to satisfy the Bowen-Lindahl-Samuelson condition is a necessary and sufficient condition for a political-economic equilibrium.

[2]Any economy with $\psi = 0$ or/and $\phi = 0$ produces no public good and is not socially efficient because of the Inada condition.

Proof: (Necessity) Suppose there is a set of (W^0, W^1, P, K^1) which solves a political-economic equilibrium. Since $W^0, W^1 > 0$, by the Kuhn-Tucker Theorem, (3.6) and (3.7) must both be binding. Applying the result of case 4, the proof of Proposition 10, (3.15), (3.16) and (3.17) are all satisfied. Now with $\psi > 0$, $\phi > 0$ and $\theta_i = 1/N$, (3.16) can be rewritten as

$$\frac{\xi_0'}{\chi_0'} = \frac{\tau}{\rho N} = \frac{1-\theta_i(1-\psi)N-(1-\phi)\psi}{\phi\psi N} = \frac{1}{N}.$$

Thus $\sum_{i=1}^{N} \frac{\xi_0'}{\chi_0'} = 1$. Similarly, by (3.9), $\sum_{i=1}^{N} \frac{\xi_1'}{\chi_1'} = 1$.

(Sufficiency) First, $c_i^0, c_i^1 > 0$ corresponds to Case 4, the proof of Proposition 10. As known, a political-economic equilibrium is characterized by equations (3.15), (3.16), (3.17) and $P = \frac{1}{F'(K^1)}$ and its corresponding economic equilibrium. By Proposition 10, for any given (W^0, W^1, P, K^1), its economic equilibrium always uniquely exists. Thus the only thing that needs to be proved is to show there is (W^0, W^1, P, K^1) such that (3.15), (3.16), (3.17) and $P = \frac{1}{F'(K^1)}$.

Second, since all $\theta_i = \frac{1}{N}$, to satisfy the Bowen-Lindahl-Samuelson condition implies

$$\chi' \left\{\tau W^0 + \mu[F(\bar{K}) - K^1]\right\} = N\xi' \left\{\rho[F(\bar{K}) - K^1 - NW^0]\right\} \qquad (3.20)$$

$$\chi'[\tau W^1 + \mu F(K^1)] = N\xi' \left\{\rho[F(K^1) - NW^1]\right\}. \qquad (3.21)$$

These two equations are equivalent to (3.15) and (3.16) when $\theta_i = 1/N$ $\forall$i$= 1, \ldots, N$.

Third, substituting $\theta_i = 1/N$, (3.20) and (3.21) into (3.8), and simplifying, I have

$$\chi_0' = \beta F'(K^1)\chi_1'. \tag{3.22}$$

This is exactly (3.17). Recalling $c_i^0, c_i^1 > 0$ and applying the Kuhn-Tucker Theorem, (3.2) and (3.3) are binding. Thus $\beta\frac{\chi_0'}{\chi_1'} = P$. Comparing the above equation with (3.22), we have $P = \frac{1}{F'(K^1)}$. **Q.E.D.**

3.2.4 Limitation of Electoral Outcomes

Proposition 11 states there exists a voting equilibrium which will defeat any feasible change in only one dimension in a democratic planned economy. Thus only *sequential* voting over (W^0, W^1, P, K^1) guarantees such a voting equilibrium. One limitation is that such a voting equilibrium (W^0, W^1, P, K^1) may not produce *only one type* of consumer (i.e., consumers with different profit shares can win in different dimensions),[3] then unless a coalitional government is allowed,[4] *the electoral outcomes in a democratic planned economy are sensitive to agenda setting.* Thus if every voting order is possible, then generically there is not a unique majority winner like the median-income

[3]One extreme exception is when someone is the median voter in all four dimensions. A somewhat general case is when the utility of the private good is the same as that of the public good and has a constant EMUI, then there is a unique majority winner, namely, the median-income voter. See Proposition 13).

[4]If coalitional governments are allowed, then one equilibrium occurs when all winners in four dimensions form the government. There might be other equilibria when some of the dimensional winners form a coalition.

consumer in a free economy (see Example 5).

Example 5: Assumptions in Example 1 remain the same except that (a) this is a planned economy in which candidates choose W^0, W^1, P and K^1; (b) $(1-\phi) = 0.5$ and $\psi = 0.2$; (c) the distribution of θ is as follows: 0.00, 0.00, 0.00, 0.05, 0.15, 0.15, 0.20, 0.20, 0.25. I index consumers and types from lowest θ to highest. For example, the voter with $\theta = 0.05$ is voter 4 and is a type-2 voter.

(i) For voters 1 to 4, they use simultaneous equations (3.15), (3.16), (3.17) and $P = \frac{1}{F'(K^1)}$ (see case 4, the proof of proposition 10). Voters 1, 2 and 3 will propose $(W^0, W^1, P, K^1) = (3.37842, 8.96354, 0.43431, 35.41122)$; voter 4 proposes $(2.93177, 7.14919, 0.43483, 36.05735)$. For $i = 5, \ldots, 9$, since $\theta_i \geq \frac{1-(1-\phi)\psi}{(1-\psi)N} = \frac{1}{8} = 0.125$, by Corollary 1, I have $W^0 = W^1 = 0$, and K^1 can be determined by (3.11) and P can be solved by $P = \frac{\beta \chi_1'}{\chi_0'}$ (see case 1, the proof of Proposition 10). Voters 5 and 6 will propose $(0, 0, 0.39739, 35.59468)$; voters 7 and 8 propose $(0, 0, 0.39643, 35.09448)$; voter 9 proposes $(0, 0, 0.39586, 34.78101)$.

(ii) First, consider the case when everyone votes sincerely and the voting order is as follows: first vote on W^0, W^1 (or W^1, W^0), then on P, K^1 (or K^1, P). It can be verified that the first stage eliminates voters 1 to 4, stage 3 yields one of voters 5 and 6 as the majority winner. Next consider another voting order: first vote on K^1, P (or P, K^1), then on W^0, W^1 (W^1, W^0). Then three types of consumers can be the majority winners when everyone votes on K^1 conditional on the expectation of other voters' positions in (W^0, W^1, P). As shown in Table 3.1 and Figure 3.2, K^1 *(or P) is now no longer monotone in* θ, and if all voters 1, 2 and 3 are expected to be type-1 voters, then either the type-3 voters (when both of them are correctly or incorrectly expected) or the type-2 voter (when one of the type-3 voters is thought to be a voter with $\theta = 0.01888$) will be the majority winner. However, if all voters 1, 2 and 3 are expected to be voters with $\theta = 0.16543$, then one of them will be the winner. Clearly the last case corresponds to the situation when extremists take over the government and implement a non-middle-class-oriented fiscal policy.

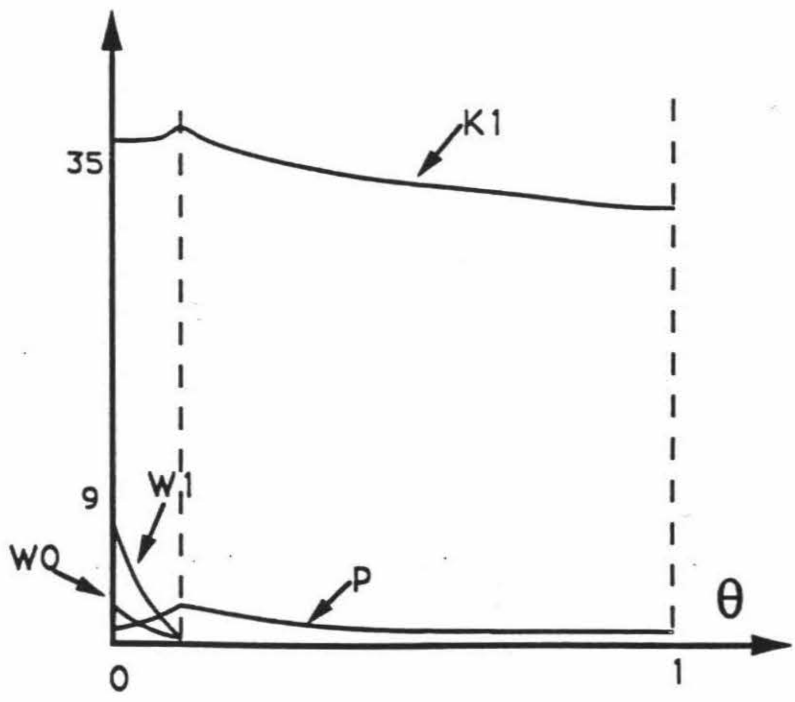


Figure 3.2: Policy Instruments for Example 5

Table 3.1: Policy Instruments for Example 5

θ	W^0	W^1	K^1	P
0.00000	3.37842	8.96354	35.41122	0.43431
0.01888	3.27216	8.55640	35.59468	0.43446
0.05000	2.93177	7.14919	36.05735	0.43483
0.12058	0.00000	0.00000	36.05735	0.39835
0.15000	0.00000	0.00000	35.59468	0.39739
0.16543	0.00000	0.00000	35.41122	0.39703
0.20000	0.00000	0.00000	35.09448	0.39643
0.25000	0.00000	0.00000	34.78101	0.39586

3.2.5 Constant Elasticity of Marginal Utility of Income

As shown in Chapter 2, when utility from the private good has a constant EMUI, then we have three interesting properties. First, fiscal policy and income distribution have no effect on economic growth. Second, among different distributions of wealth, the higher the profit share of the decisive consumer (i.e., median-income consumer), the lower the tax rate. Third, when the median of the pre-tax income distribution is the mean, then I have provided a counter-example of the Kuznets curve. Can we get any interesting results in a democratic planned economy when utility of the private good has a constant EMUI? For instance, can we avoid agenda-dependent equilibrium? Fortunately, when consumers are not very rich such that everyone prefers some non-zero wage rates, this class of utility functions completely solves the multi-dimensionality problem encountered in the last section.

Proposition 13: When nobody prefers zero wage rates, and χ and ξ have constant EMUIs and $\chi = a\xi + b$ $(a > 0)$, then

(a) each consumer will propose the same K^1 and P that are determined independently of $\theta_i (i = 1, \ldots, N)$; $W_i^1 = d(K^1)W_i^0$, that is, every consumer will propose a pair of wage rates which are linearly proportional by a variable d which is a function of K^1;

(b) the median-income consumer will become the majority winner,

(c) the democratic planned economy generates the same growth path as the free economy.

Proof: (a) The fact that each consumer i proposes a pair of non-zero wage rates corresponds to case 4, the proof of Proposition 10. Therefore, equations (3.15) to (3.17) must be satisfied. Let us assume $\chi(x) = ax^{1-c} + b$, where $a, b > 0$ and $c < 1$ (the other two cases can be similarly proved). Then (3.17) can be rewritten as

$$[\frac{\tau W^0 + \mu[F(\bar{K}) - K^1]}{\tau W^1 + \mu F(K^1)}]^{-c} = \beta F'(K^1),$$

Recalling $\mu = \theta_i(1-\psi) + \frac{(1-\phi)\psi}{N}$ and $\tau = 1 - \theta_i(1-\psi)N - (1-\phi)\psi$,

$$\begin{aligned} W^0 &+ [\theta_i(1-\psi) + \frac{(1-\phi)\psi}{N}][F(\bar{K}) - K^1 - NW^0] \\ &= [\beta F'(K^1)]^{-1/c}\Big\{W^1 + [\theta_i(1-\psi) + \frac{(1-\phi)\psi}{N}] \\ &\quad [F(K^1) - NW^0]\Big\}. \end{aligned} \tag{3.23}$$

Substituting (3.15) and (3.16) into (3.17) yields

$$[\rho(F(\bar{K}) - K^1 - NW^0)]^{-c} = \beta F'(K^1)[\rho(F(K^1 - NW^1)]^{-c}.$$

So, I have

$$[F(\bar{K}) - K^1 - NW^0] = [\beta F'(K^1)]^{-1/c}[(F(K^1) - NW^1)]. \tag{3.24}$$

Substituting (3.24) into (3.23) yields $W^0 = [\beta F'(K^1)]^{-1/c} W^1$. Thus for any given K^1, W^1 is linearly related to W^0. Using the equality in (3.24), we have

$$F(\bar{K}) - K^1 = [\beta F'(K^1)]^{-1/c} F(K^1). \tag{3.25}$$

K^1 can be solved independently of θ_i, so can P since $P = \frac{1}{\beta F'(K^1)}$ in this case.

(b) Part (a) shows that policy parameters among consumers differ only in W^0 and W^1. Moreover because W^0 and W^1 are linearly related for the given K^1, *the four-dimensional voting problem is in fact only one dimensional*, namely, a straight line in (W^0, W^1) space. (3.16) can be rewritten as

$$H(W^1, \theta_i) \equiv \tau\chi'[\tau W^1 + \mu F(K^1)] - \rho N\xi'\left\{\rho[F(K^1) - NW^1]\right\} = 0.$$

Now taking total derivatives with respect to W^1 and θ_i in (3.16), and recalling

$$\mu = \theta_i(1-\psi) + \frac{(1-\phi)\psi}{N} \text{ and}$$

$$\tau = 1 - \theta_i(1-\psi)N - (1-\phi)\psi.$$

I have

$$\begin{aligned} d(W^1, \theta_i) &= (\tau^2\chi_1'' + (\rho N)^2\xi_1'')dW^1 + \\ &+ \left\{-(1-\psi)N\chi_1' + \tau(1-\psi)[F(K^1) - K^1]\chi_1''\right\} d\theta_i = 0. \end{aligned}$$

So $\frac{\partial W^1}{\partial \theta_i} < 0$. Similarly as the proof of Theorem 1, the median-income consumer will be the majority winner.

(c) The proof follows by comparing (2.36) and (3.25). **Q.E.D.**

The example below illustrates what Proposition 13 has shown.

Example 6: All assumptions in Example 3 are valid except that $u(x_1, x_2) = {x_1}^{0.5} + 0.112{x_2}^{0.5}$, $\phi = \frac{6}{7}$, and $\psi = 0.7$. Suppose we have five types of consumers, that is, consumers with profit shares 0, 1/18, 1/9, 1/6 and 2/9. Then since the largest $\theta = 2/9$ is less than 1/3, beyond which a consumer will choose zero wage rates, all consumers choose n0n-zero wage rates. The solutions are given in Table 3.2. Compared to Example 3, Proposition 13 is true.

It can be verified for different distributions of θ that the median voter always prevails. For example, if the distribution is given as: 0, 0, 1/18, 1/18, 1/9, 1/6, 1/6, 2/9, 2/9, then the majority winner is the type-3 ($\theta = 1/9$) consumer. If the distribution is: 0, 0, 0, 0, 1/6, 1/6, 2/9, 2/9, 2/9, then the winner will be one of the type-4 ($\theta = 1/6$) consumer. If the distribution is given as: 0, 0, 1/18, 1/18, 1/18, 1/6, 2/9, 2/9, 2/9, then one of the three type-2 voters is the majority winner.

Table 3.2: Policy Instruments for Example 6

θ	W^0	W^1	K^1	P
2/9	0.96313	4.94857	45.54354	0.44117
1/6	1.76971	9.09285	45.54354	0.44117
1/9	2.07330	10.65268	45.54354	0.44117
1/18	2.21958	11.40427	45.54354	0.44117
0.00	2.30111	11.82316	45.54354	0.44117

3.2.6 Controlling Only Growth and Inflation

Economic systems described by the political economy model in which the government controls all macroeconomic instruments no longer exist in Eastern Europe and the former Soviet Union, let alone the democratic planned economies in Western Europe. In this subsection, I consider a more *relaxed* model in which government only controls economic growth and inflation, i.e., K^1 and P. All assumptions and timing remain the same except that now *the government only controls K^1 and P, and W^0 and W^1 are determined by marginal productivities.*[5] Can this setup, which has only two parameters that voters vote on, solve the generic problem of agenda-dependent political-economic equilibrium encountered in a complete democratic planned economy (see Subsection 3.2.4)? This turns out to be true when the production technology is Cobb-Douglas.

Proposition 14: In a democratic planned economy with Cobb-Douglas production technology, when government only controls capital stock and inflation, then any sequential voting over (K^1, P) yields the median-K^1 voter as the majority winner.

Proof: First, suppose the production function is

$$f(K, L) = AK^a L^b,$$

where $A > 0$, $0 < a$, and $b < 1$. Similarly as before, $L = N$ because labor causes no disutility for consumers. Since $F(K^1) = f(K^1, N) + (1 - \delta)K^1$, it is easy to check

$$f''_{21}(K^1, N) = Aab(K^1)^{a-1} N^{b-1} > 0 \text{ and} \tag{3.26}$$

[5] As seen in the model of a free economy, the fact wage rates are equal to marginal productivities (equations (2.6) and (2.7)) results from maximization of the firm.

$$
\begin{aligned}
F'(K^1) - Nf''_{21}(K^1,N) &\geq f'_1(K^1,N) - Nf''_{21}(K^1,N) \\
&= Aa(1-b)(K^1)^{a-1}N^b > 0. \qquad (3.27)
\end{aligned}
$$

Second, because the marginal utility of income $\lambda_i > 0$, by the Kuhn-Tucker Theorem, (3.4) is binding. Since $\lambda_i > 0$ and $\chi'_1 > 0$, by (3.3), $P > 0$, thus (3.5) is binding. (3.4) and (3.5) imply (3.9) and (3.10), i.e., $c_i^1 = W^1 + \theta_i(1-\psi)[F(K^1) - NW^1]$. Substituting (3.9) into (3.4), I have $c_i^0 = W^0 + \theta_i(1-\psi)[F(\bar{K}) - K^1 - NW^0]$.

By the Inada condition $\lim_{K\to 0} f_1(K,N) = +\infty$, $K^1 > 0$, thus (3.8) is binding. (3.2) and (3.3) are binding too because $c_i^0, c_i^1 > 0$. Letting $H(K^1) = LHS(3.8) - RHS(3.8) \equiv 0$, $\mu \equiv \frac{(1-\phi)\psi}{N} + \theta_i(1-\psi)$, substituting (3.2), (3.3), (3.9) and (3.10) into $H(K^1) = 0$, and taking derivative with respect to K^1, we have

$$
\begin{aligned}
\frac{\partial H(K^1)}{\partial K^1} &= \mu^2\chi''_0 + \beta\mu\chi'_1 F''(\phi\psi)^2\xi''_0 + \beta\phi\psi\xi'_1 F'' + \\
&+ \beta[\frac{(1-\phi)\psi}{N} + \theta_i(1-\psi)]\left\{f''_{21} + \mu[F' - Nf''_{21}]\right\} F'\chi''_1 + \\
&+ \beta\phi\psi\left\{\phi\psi[F' - Nf''_{21}]\right\} F'\xi''_1 < 0.
\end{aligned}
$$

The above inequality holds because of (3.26) and (3.27). Thus for each consumer i, there is a unique optimal K^1.

Third, (3.2) and (3.3) yield

$$
P = \frac{\beta\chi'\left\{f'_2(K^1,N) + \mu[F(K^1) - Nf'_2(K^1,N)]\right\}}{\chi'\left\{f_2(\bar{K},N) + \mu[F(\bar{K}) - K^1 - Nf'_2(\bar{K},N)]\right\}}.
$$

Now taking the derivative of P with respect with K^1 in the above equality yields

$$\frac{\partial P}{\partial K^1} = \beta \frac{[f_{21}'' + \mu(F' - f_{21}'')]\chi_0'\chi_1'' + \mu\chi_1'\chi_0''}{(\chi_0')^2} < 0.$$

Therefore P is a decreasing function of K^1.

Fourth, as shown in part three, individual preferences are indexed by the parameter K^1, thus by Grandmont (1978), the median-K^1 voter will beat any rival in the final stage (i.e., voting on P) because everyone is sincere. By backward induction, this median-K^1 voter will defeat any opponent in stage 1 too. **Q. E. D.**

3.3 Summary and System Comparison

Let me answer the questions raised at the beginning of the Introduction. First, when voting is sequential on the macroeconomic instruments, there exist political-economic equilibria. However, those equilibria are generally agenda-dependent. Thus if a coalitional government is not allowed, then certain voting procedures may cause political chaos and social instability when extremists take power and implement their favorable growth plans.[6] Second, with a Cobb-Douglas production technology, decentralization of wage

[6]To some extent, an authoritarian planned economy can be thought as a special case of a "democratic" planned economy where communist elites are agenda setters. They set certain voting procedures to eliminate their opponents.

decisions in a democratic planned economy can guarantee a unique political-economic equilibrium and a growth path that is middle-class-oriented. Third, when consumers have the same constant EMUI utility functions for the private good and the public good, the economic growth path in a democratic planned economy is the same as that in a free economy. The growth path is distributionally neutral too.

By comparing results of a free economy with those of a democratic planned economy, I can reach the following three conclusions concerning the differences between the two economic systems:

First, when the distribution of wealth is uniform, then regardless of economic systems, the Bowen-Lindahl-Samuelson condition always implies a political-economic equilibrium.

Second, when consumers have the same constant EMUI utility functions for both the private good and the public good, then the economic growth path is fiscally and distributionally neutral in a free economy or distributionally neutral in a democratic planned economy. In addition, the growth paths of these two systems are the same.

Third, a political-economic equilibrium in which the median (θ) voter prevails always exists in a free economy; while there are generally multiple equilibria which depend on the agenda setting in a democratic planned economy. In this regard, *fiscal policies in a free economy are more middle-class oriented than those in a democratic planned economy.*

Chapter 4

An Empirical Study of Economic Growth

4.1 Introduction

Many economists have examined the relationship between economic growth and various macroeconomic variables, such as income equality (Kuznets (1955), Ahluwalia (1976), Saith (1983), Lindert and Williamson (1985), Ram (1988), and Persson and Tabellini (1991)), government spending (Landau (1983), Kormendi and Meguire (1985), Grier and Tullock (1987), Barth and Bradley (1987), and Barro (1991), (1990)), and the initial economic conditions (Kormendi and Meguire (1985), and Barro (1991)).

This chapter presents exploratory empirical evidence bearing on a set of hypotheses which concern the effects of economic determinants on economic growth. These hypotheses are derived from both neoclassical growth models and the theoretical analysis in chapter 2 and tested across a sample of fifty-two countries. Among the hypotheses investigated are the effects of (i) the initial economic conditions, (ii) the growth rate of the labor force, (iii) the ratio of private capital investment to GDP, (iv) the weight of public sector, (v) human capital, (vi) government spending, (vii) geographical regions, and (viii) income inequality on economic growth.[1]

My empirical study differs from other similar studies in the following respects: First, both private investment and public sector investment are included in my study. According to theoretical discussions in Chapters 2 and 3, the weight of public sector as well as private investment may affect economic growth, so my setup could provide additional explanatory factors for economic growth. Second, I use the Gini coefficient to measure income inequality and study the effects of income inequality on economic growth. [2] However, because of limitations of the *World Development Report* (abbreviated as WDR) data, the Gini coefficients are not available on annual bases. Third, I study economic growth across time and countries as well as across countries. Although this setup may decrease the fit of our empirical models, we avoid loss of efficiency of data encountered when pooling data across time (see eg., Johnston (1960)).[3]

[1]There is much literature concerning the opposite problem, namely the effects of economic growth on income distribution. This literature is inspired by Kuznets' inverted U-curve hypothesis: in the early stages of growth, inequality increases, then stabilizes and finally in the later stages of growth it declines. For a recent evaluation of the theoretical as well as empirical work on the Kuznets curve, see Lindert and Williamson (1985).

[2]Persson and Tabellini (1991) examine the same problem by using the share in personal income of the top 20 % of the population, INCSH, as the measurement of inequality. They find the correlation coefficient between the Gini-coefficient and INCSH is close to 0.8.

[3]Persson and Tabellini (1991) do consider the variation of regressions for economic

The rest of this chapter is divided into three parts: empirical design and hypotheses, results, and summary. There is also a short appendix giving descriptions and definitions of the variables used in the paper.

4.2 Empirical Design, Variables, and Hypotheses

4.2.1 Empirical Design

The data for this study come from a sample of fifty-two countries from the *World Development Report 1991: Supplementary Data.* A list of countries under study is given in Table 4.1.[4] The countries were chosen because they had continuous annual series for GDP per capita, total population, the ratio of investment to GDP (private and public), average years of education, and the ratio of government spending to GDP over the period from 1970 to 1986.

growth across time. They add a set of period dummies (which cover 15 to 20 years) to their cross-country regressions, and find the time dummies add considerable explanatory power.

[4]As seen in Table 4.1, the list of countries under study does not include any European or North American countries besides Turkey.

Table 4.1: List of Countries Under Study

1. Argentina
2. Burundi*
3. Benin*
4. Bolivia*
5. Brazil
6. Central African Rep*
7. Chile
8. Côte d'Ivoire
9. Cameroon*
10. Congo*
11. Columbia
12. Costa Rica
13. Algeria*
14. Egypt
15. Ethiopia*
16. Guatemala
17. Hong Kong
18. Burkina Faso*
19. Indonesia
20. India
21. Israel#
22. Jamaica
23. Kenya
24. South Korea
25. Liberia*
26. Sri Lanka
27. Morocco
28. Madagascar*
29. Mexico
30. Mauritania*
31. Mauritius
32. Malawi
33. Malaysia
34. Nigeria*
35. Pakistan
36. Panama
37. Peru
38. Philippines
39. Rwanda*
40. Sudan
41. Senegal*
42. Singapore
43. El Salvador
44. Syria*
45. Togo*
46. Thailand
47. Turkey
48. Tanzania
49. Venezuela
50. Zaire
51. Zambia
52. Zimbabwe*

*: Missing data of income distribution.

#: Missing data of PERT and SERT.

I measure secular economic growth by using the average annual rate of growth of GDP per capita (MGROW). I model MGROW as a function of other variables drawn from the previous sample period that, according to various hypotheses, should affect secular economic growth. I use the following simple cross-sectional model:

$$MGROW_i^{t+1} = \alpha + X_i^t\beta + \epsilon_i,$$

where $MGROW_i^{t+1}$ is the mean growth of GDP per capita over the 1971-1987 period, X_i^t is a vector of initial economic variables, dummy variables, and the means of explanatory variables (such as the size of labor force, private capital investment, the weight of public sector, technical skill or production efficiency, income tax rate, and income distribution) at over the 1970-1986 period, β is a coefficient vector associated with X_i, and ϵ_i is error term.

4.2.2 Variables

1. MGROW, the average annual growth rate of GDP per capita, is the dependent variable in this analysis. I calculate the mean of per capita GDP for fifty-two countries under study across the time span 1970-87 and the average annual growth rate of GDP per capita across the time span 1971-87. [5] Although pooling the sample across time may increase the R-square in my

[5] All other variables are calculated in the same way except that the time span is 1970-86.

study, it might jeopardize efficiency. So I would like to look at all data across time as well as across countries. Over the 1971-1987 period the mean value of MGROW is 1.403 %, and it ranges from -2.599% (for Liberia) to 7.242 % (for South Korea). Summary statistics for all variables are given in Table 4.2.

2. GDPINI, the level of GDP per capita in 1969, represents the initial economic condition. MGDP is the average annual GDP per capita from 1970 to 1986. First, I use the level of GDP per capita in 1969 instead of 1970, in which all independent variables begin. Second, since there is a high correlation between GDPINI and MGDP (R^2 = 0.88, SE = 0.06411, and t = 19.37713), I concentrate on GDPINI only. The mean of GDPINI is 922.596, and it ranges from 103 (for Ethiopia) to 4819 (for Venezuela).

3. POPINI is the total population in 1969, while MRPOP is the average annual growth rate of population. Since there is a high correlation between total population (POP4) and total labor force, interpolated (LABOR4) (R^2 = 0.99739, SE = 0.00282, and t = 138.339), I focus on total population in this study. The variable POPINI has a mean of 27,443,500, and it ranges from 829,000 (for Mauritius) to 547,569,000 (for India). And the variable MRPOP has a mean of 2.596 (%), and it ranges from 1.380 (%) (for Mauritius) to 4.025 (%) (for Cote d'Ivoire).

4. MPRI is the average annual ratio of gross private investment to GDP and represents the size of investment in the private sector. MPUB, the average annual ratio of public sector investment to GDP, is used to characterize

the weight of public sector.[6] MPRI has a mean of 13.208 (%), and it ranges from 2.913 (%) (for Burundi) to 31.525 (%) (for Singapore). And MPUB has a mean of 9.097 (%), and it ranges from 2.610 (%) (for El Salvador) to 30.092 (%) (for Algeria).

5. MEDU is the mean of the *estimated* average years of education of the population of working age (15 to 64), MPERT is the average total gross primary enrollment rate (percent), and MSERT is the average total gross secondary enrollment rate (percent). Because WDR data lacks direct measurements, such as the ratio of teachers to students, for technical skills, I use MEDU, or MPERT and MSERT, indexes of investment in human capital, as approximations. Since it is unclear which of MEDU, MPERT and MSERT better measures technical skill, I will use them as independent variables separately (MEDU in Table 4.3, and MPERT and MSERT in Table 4.5). My conjecture is that among MEDU, MPERT and MSERT, MSERT, the average total gross secondary enrollment should be the best measurement for quality as well as quantity of education. MEDU has a mean of 4.154, and it ranges from 0.415 (for Burkina Faso) to 8.613 (for Sri Lanka). MPERT has a mean of 83.480, and it ranges from 17.875 (for Mauritius) to 130.000 (for

[6]MDR points out, for developing countries, private sector investment data is usually not compiled as part of national accounts. It must be determined as the *residual* between a measure of total investment for an economy and that of the consolidated public sector. Thus, INVGPR4 and INVPUB4 add up to total domestic investment as a share of GDP. The public sector investment is defined as capital expenditure of the consolidated general government plus that of public corporate entities. The assumption was made that all *inventories (stocks) were held in the private sector.* The often "spotty" nature – both in definition and in availability – of existing time series data on public sector capital expenditure for these countries leads to a chance of potentially significant error in the estimates.

Congo). And MSERT has a mean of 24.092, and it ranges from 1.750 (for Rwanda) to 59.500 (for Congo).

6. MGCON is the average annual share of government consumption in GDP (GCONX2) and serves an our measure of government spending. Since WDR does not measure income tax rate directly, I use MGCON as the approximation of income tax rate in the paper. The mean value of MGCON is 14.269 (%), and it ranges from 7.189 (%) (for Hong Kong) to 36.494 (%) (for Israel).

7. AFRICA is the dummy variable for Africa, and LATAMER is the dummy variable for Latin America. Among my sample of 52 countries, there are 26 African nations, 13 Latin American nations, and 13 other nations (which include 12 Asian nations and 1 European nation).

8. GINI, the Gini coefficient, is the ratio of the area between the cumulative percentage share of household income and the equality line (45-degree-line) to the area under the equality line in Figure 4.1. It measures income inequality. The higher the Gini coefficient, the less equalized the income distribution. Since WDR data provides the percentage share of household income for five quintile groups of households and the top 10 % households, the Gini coefficient can be calculated according to the following formula.

$$\mathrm{GINI} = 1 - 0.001[18 * q1 + 14 * q2 + 10 * q3 + 6 * q4 + 3 * q5 - 2 * q6],$$

where q1, q2, q3, q4, q5 and q6 are the percentage shares of household income by the lowest 20 percent of households, the second quintile household, the third quintile household, the fourth quintile household, the highest 20 percent of household and the highest 10 percent of households respectively (see the Appendix). GINI has a mean of 0.442, and it ranges from 0.282 (for Morocco) to 0.582 (for Brazil).[7]

Table 4.2 Summary Statistics
(Cross-Country)

VARIABLES	#OBS	MEAN	STD.DEV	MIN	MAX
MGROW	52	0.01403	0.02261	-0.02599	0.07242
GDPINI	52	922.596	928.677	103.000	4819.00
POPINI	52	$2.74*10^{7}$	$7.73*10^{7}$	$8.29*10^{5}$	$5.48*10^{8}$
MRPOP	52	0.02596	0.00617	0.01380	0.04025
MPRI	52	0.13208	0.05168	0.02913	0.31525
MPUB	52	0.09097	0.04278	0.02610	0.30092
MEDU	52	4.15385	2.12669	0.41500	8.61294
MPERT	51	83.4799	28.2998	17.8750	130.000
MSERT	51	24.0915	16.2823	1.75000	59.5000
MGCON	52	0.14269	0.05353	0.07189	0.36494
GINI	33	0.44226	0.07092	0.28180	0.58190

[7]One interesting scenario is that as GDP grows, GINI increases first and then decreases, since

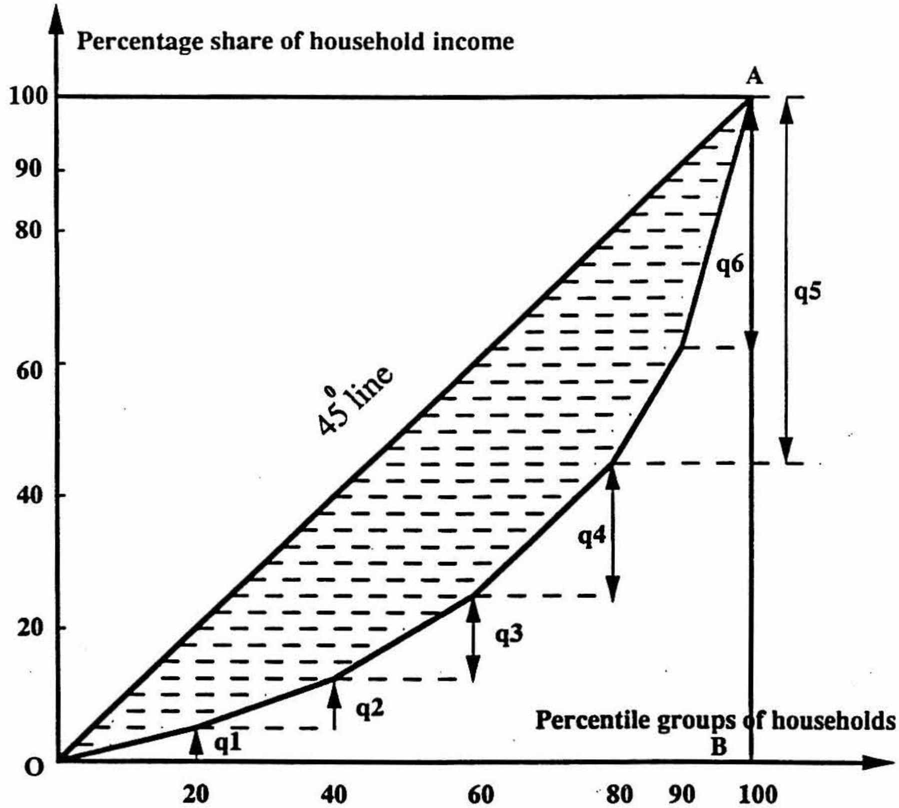
$$\text{GINI} = 0.37756 + 9.5510^{-5} MGDP - 1.8710^{-8} MGDP^2,$$

and the t-values for MGDP and $MGDP^2$ are 3.34314 and -3.56335, $R^2 = 0.29918$, and $SE = 0.06132$.

The regression equation is expressed as:

$$\begin{aligned} MGROW &= \alpha + \beta_1 GDPINI + \beta_2 POPINI + \beta_3 MRPOP + \beta_4 MPRI \\ &+ \beta_5 MPUB + \beta_6 MEDU + \beta_7 MGCON + \beta_8 AFRICA \\ &+ \beta_9 LATAMER + \beta_{10} GINI + \epsilon. \end{aligned} \tag{4.1}$$

Remark: Since there are only 33 countries which have at least one data of the percentage share of household income for five quintile groups of households and the top 10 % household and those datum are not collected on annual bases, any results in my paper concerning income distribution are very preliminary.



q1, q2, q3, q4, q5 and q6: Percentage share of household income by the lowest 20%, the second quintile, the third quintile, the fourth quintile, the highest 20% and the highest 10% of households respectively.

$$\text{Gini Coefficient} = \frac{\text{Shaded Area}}{\text{Area (OAB)}}.$$

Figure 4.1: Gini Coefficient.

4.2.3 Hypotheses

I focus on the following hypotheses concerning the effects of economic determinants on growth. These hypotheses are derived from either neoclassical growth theory or the theoretical analyses in Chapter 2.

(i) Hypothesis concerning the effects of initial economic conditions.

In neo-classical growth models with diminishing returns to capital, a country's per capita growth rate tends to be inversely related to its initial level of income per person (Solow (1956), Cass (1965), and Koopmans (1965)). This convergence hypothesis seems to be inconsistent with cross-country evidence, which indicates that per capita growth rates for about 100 countries in the post-World War II period are uncorrelated with the starting level of per capita product (Barro (1991)). However, Barro shows if one holds constant measures of initial human capital – measured by primary and secondary school-enrollment rates – there is evidence that countries with lower per capita product tend to grow faster (see also Mankiw, Romer, and Weil (1990)).

(ii) Hypothesis concerning the effects of the growth rate of labor force.

Under standard neoclassical growth theory, the steady state growth rate should equal the growth rate of the labor force plus the growth rate of exogenous technological change, implying a positive relation between the mean population growth rate and the mean annual economic growth. Kormendi and Meguire (1985) find a significantly positive effect of the mean population growth rate on the mean annual rate of growth of aggregate real gross domestic or national product. Since I examine the growth rate of *GDP per* *capita* instead of the growth rate of real GDP, it is less obvious whether the positive relation should still hold.

(iii) Hypothesis concerning the effects of the ratio of private investment to GDP.

Neoclassical growth theory predicts a positive relation between the steady state growth rate and the growth rate of exogenous technological change. Since, generally speaking the growth rate of exogenous technological change is positively related to the size of private investment, I expect a positive relation between the mean ratio of gross private investment to GDP and the mean annual growth rate of GDP per capita.

(iv) Hypothesis concerning the effects of the weight of the public sector.

There is no existing neoclassical growth theory regarding the effects of the weight of public sector. My theory does not predict a definite relation between the weight of public sector and economic growth, since the weight of public sector could be an endogenous variable which is determined by the distribution of income (see Subsection 2.2.3 and 2.2.4).[8]

(v) Hypothesis concerning the effects of the quantity of human capital per person.

Neoclassical growth models point out that increases in the quantity of human capital per person tend to lead to high rates of investment in human and physical capital, and hence, to higher per capita growth (Becker and Murphy (1990)) (See also Lucas (1988) and Rebelo (1991)). Barro (1991)

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[8]Example 1v
in Chapter 2 does show that when the weight of the public sector increases, economic growth rate also increases. However this result relies on some critical assumptions, such as the substitution rate of public good and private good is 1, and the discount rate is 1.

demonstrates that given the level of initial per capita GDP, the growth rate is substantially positively related to the starting amount of human capital.

(vi) Hypothesis concerning the effects of government spending.

"Supply side" theories hypothesize that the taxes necessary to support government spending distort incentives, generally reduce efficient resource allocation, and hence reduce the level of output. This prediction has its empirical support from Landau (1983), Grier and Tullock (1987) and Barro (1991). However, Kormendi and Meguire (1985) do find an insignificantly positive effect of the growth of government spending as a proportion of output on growth. Barth and Bradley (1987) also show an insignificantly positive effect of the share of government investment in GDP on growth.

(vii) Hypothesis concerning geographical regions.

As known in Chapter 2, different economic growth patterns may result from different forms of utility functions, which may be due to different cultures, customs and geographical regions. Since there are no other variables in WDR data that measure cultures and customs, AFRICA and LATAMER are included to capture some of the difference. I do not have any theoretical prediction concerning the effects of geographical regions on economic growth. However, Barro (1991) finds significantly negative coefficients on these two dummies.

(viii) Hypothesis concerning income inequality.

As discussed in Chapter 2, the distribution of income can indirectly affect economic growth by deciding a fiscal policy through political election. However, the direction of effect is not determined by my model. Persson and Tabellini (1991) demonstrate that income inequality is harmful for economic growth, because it leads to policies that do not protect property rights and do not allow full private appropriation of returns from investment. They also find statistically strong support for the negative effect of income inequality on economic growth (see also Murphy, Shleifer and Vishny (1989), and Greenwood and Jovanovich (1990)).

4.3 Results

4.3.1 Cross-Sectional Results

The cross-section regression results are presented in Tables 4.3 to 4.5. Tables 4.3 and 4.5 report the results of estimating cross-section regressions in the form of (4.1) when the Gini coefficient is excluded, where the independent variables are discussed in the last subsection. The difference between Tables 4.3 and 4.5 is that the variable (variables) relating to human capital is MEDU in Table 4.3, and are MPERT and MSERT in Table 4.5.

The result of the estimating cross-sectional regression in the form of (4.1) is given in Regression 1, Table 4.3. The regression as a whole explains around 52 percent of the variation in measured economic growth. The estimated coefficient on GDPINI is negative as expected from neoclassical growth theory, and thus the convergence hypothesis seems to lack empirical support. However, this negative relation is not as strong as those of Kormendi and Meguire (1985), and Barro (1991). Since a one percent increase in initial GDP per capital is associated with 0.0055 to 0.0092 percent decrease in the growth rate of GDP per capita in their reports, the negative effect here is almost negligible. Thus there is no strong indication of convergence in growth rates in my study. I believe that difference in samples may have caused the difference of explanatory power of GDPINI on MGROW between my study and their. The WDR data covers either developing countries or those fast growing developed countries, such as Taiwan, Korea and Hong Kong. Adding many maturely developed countries, which should be more likely to have achieved the full capacity of growth, may intensify the negative effect of GDPINI on MGROW.

I find that the average annual growth rate of GDP per capita is negatively, although not significantly, related to the initial population and the average growth rate of population. A one percent increase in the growth rate of the population is associated with a 0.85465 percent decrease in the annual growth rate of GDP per capita. This finding suggests that if we examine the mean annual rate of growth of GDP per capita instead of the mean annual rate of growth of aggregate gross domestic products, then the effect of population growth changes sign.

The coefficient on MPRI is positive and significant as expected. A one percent increase in the annual ratio of gross private investment to GDP is associated with a 0.17197 percent increase in the annual growth rate of GDP per capita. Singapore, Hong Kong and South Korea have the three highest values for the annual ratio of gross private investment to GDP, which raised the estimated growth rates by 5.42% for Singapore, 4.03% for Hong Kong, and 3.97% for South Korea. This strongly positive relation between the annual ratio of gross private investment to GDP and economic growth has been widely noted. It is commonly known that Japan, West Germany and the four Little Dragons (i.e., Hong Kong, Singapore, South Korea and Taiwan) have relatively high savings rates which enable them to use a big part of GDP in private investment. We therefore accept the hypothesis that private investment has a strong positive effect on economic growth.

A positive although not significant effect of MPUB is found on economic growth. A one percent increase in the ratio of public sector investment to GDP is associated with a 0.07949 increase in the annual growth rate of GDP per capita.

The negative coefficient on MEDU is contrary to expectations, although it is insignificant and quite small.[9] My first conjecture is that the estimated average years of education of the population of the working age group (15 to 64) does not fully measure the quality of education of the labor force. For example, Sri Lanka, Jamaica, Malaysia, Philippines and Mauritius rank the top five, while Hong Kong ranks number 20 in MEDU among 52 countries under study. This quite contradicts with the common belief that Hong Kong, Singapore and South Korea all have very high quality labor forces. I will consider the other measurements of education, namely the average total gross primary enrollment rate (percent), MPERT, and the average total gross secondary enrollment rate (percent), MSERT in Table 4.5.

As expected from neoclassical growth theory, the estimated coefficient on MGCON is negative although insignificant. A one percent increase in

[9]As Persson and Tabellini (1991) show, SCHOOL, an index for average skills, has the positive effect on economic growth, but is never statistically significant.

the average annual share of government consumption in GDP is associated with a 0.05796 percent decrease in the average annual growth rate of GDP per capita. This is consistent with the results of Landau (1983), Grier and Tullock (1987), and Barro (1991). [10] Kormendi and Meguire (1985) find an insignificantly positive effect of government spending on economic growth.

With regard to geographical dummy variables, I find both the estimated coefficients on AFRICA and LATAMER are significantly negative. An African country and a Latin American country would be associated with a 0.02042 and 0.02475 percent decrease in the annual growth rate of GDP per capita respectively. This finding is consistent with that of Barro (1991).

[10]Barro (1991) measures government consumption by subtracting estimates of the ratio of nominal government spending on education and defense to nominal GDP from the Summer-Heston (1988) figures on the ratio of real government consumption purchases to real GDP. He argues that expenditures on education and defense are more like public investment than public consumption; in particular, these expenditures are likely to affect private-sector productivity or property rights, which matter for private investment.

Table 4.3: Regressions for MGROW

(Cross-Country-I)

	(1)	(2)	(3)	(4)
no. obs.	52	52	33	33
const.	0.03582	0.03242	0.04163	0.03158
	(1.85162)	(2.16920)	(1.64890)	(1.40274)
GDPINI	$-2.33*10^{-6}$		$-2.51*10^{-6}$	
	(-0.67824)		(-0.79909)	
POPINI	$-2.36*10^{-11}$		$-2.41*10^{-11}$	
	(-0.67562)		(-0.85198)	
MRPOP	-0.85465@	-0.83504@	-1.97085#	-2.15240#
	(-1.84621)	(-1.93144)	(-3.31589)	(-3.80941)
MPRI	0.17197#	0.14162#	0.20417#	0.20418#
	(2.84709)	(2.73878)	(3.17055)	(3.83648)
MPUB	0.07949		-0.08340	
	(1.28435)		(-0.86755)	
MEDU	$-5.02*10^{-4}$		$-4.79*10^{-3}$#	$-5.11*10^{-3}$#
	(-0.29490)		(-2.24999)	(-2.59047)
MGCON	-0.05796		-0.02945	
	(-1.14388)		(-0.56202)	
GINI			0.11617#	0.11777#
			(2.62578)	(2.85664)
AFRICA	-0.02042#	-0.01804#	-0.01587@	-0.01484#
	(-2.38148)	(-2.69946)	(-1.90470)	(-2.12640)
LATAMER	-0.02475#	-0.02560#	-0.03197#	-0.03137#
	(-3.27087)	(-3.74807)	(-4.20870)	(-4.87569)
R^2	0.52241	0.48240	0.76207	0.72383
SER	0.01722	0.01694	0.01327	0.01315

Note: OLS regressions; t-values in brackets; # & @: Significant at 0.025 & 0.05 level resp.

Table 4.4 Marginal Contribution to R^2

(Cross-Country)

Regressor	Marginal Contribution to R^2
GDPINI	0.523%
POPINI	0.519%
MRPOP	3.876%
MPRI	9.218%
MPUB	1.876%
MEDU	0.099%
MGCON	1.488%
AFRICA	6.450%
LATAMER	12.166%
TOTAL	36.215%

Note: marginal contribution to R^2 is defined as the difference between the R^2 from Regression 1 in Table 4.3 and the R^2 from estimating Regression 1 with each variable deleted in turn. The "total" R^2 of 0.36215 is below 0.52241, the R^2 in Reg. 1 because of some intercorrelation among the regressors.

Table 4.4 gives the marginal contribution to R^2 in Regression 1. As seen, MPRI, AFRICA and LATAMER explain 27.834 (%) out of 36.215 (%), the total marginal contribution to R^2 across all 9 variables. Then I regress MGROW against MPRI, AFRICA and LATAMER, and report the result in Column 2, Table 4.3. As seen, these three variables explain 48.636 percent of the variation in measured economic growth (89.398 % of 54.404 % explained by Regression 1)! Thus the mean annual ratio of gross private investment to GDP and geographical dummies are extremely critical in explaining cross-section economic growth.

Regression 3 provides similar results to regression 1 except now the Gini coefficient is included in the explanatory vector, thus only 33 out of the 52 countries are available now. The regression as a whole explains around 76 percent of the variation in measured economic growth. In sharp contrast with conventional belief, the coefficient of GINI has a significantly positive sign. A one percent increase in GINI is associated with a 0.11617 percent increase in the annual growth rate of GDP per capita. I then regress MGROW on only GINI, but find a negative, although insignificantly, correlation between MGROW and GINI ($MGROW = 0.04021 - 0.04517GINI$, with $R^2 = 0.02016$, $SE = 0.02269$, and $t = -0.79869$). Thus controlling other variables makes the sign of the coefficient of GINI change from negative to positive.[11] Except MPUB, all other coefficients affect economic growth in the same direction they do in Regression 1. Moreover, the negative effect of MEDU on MGROW becomes significant.

Based on Regression 3, Regression 4 examines MGROW by only those variables that have significant effects on MGROW (i.e., MRPOP, MPRI, MEDU, GINI, AFRICA and LATAMER) and explains around 72 percent of the variation in measured economic growth.

The second set of cross-sectional regressions in Table 4.5 contains the same set of variables discussed in Table 4.3 with the exception of the variable

[11] It is unclear whether this positive relation results from the spotty data of GINI or the fact that my sample covers primarily non-democratic developing countries. Those countries may not face the same "growth trap" as Western developed countries when income inequality is large or becomes so profound that it discourages further accumulation and growth, see Persson and Tabellini (1991).

concerning human capital. Regression 5 uses MPERT and MSERT instead of MEDU and explains around 54 percent of the variation in measured economic growth. All estimated coefficients in Regression 5 have the same signs and levels of significance as those of Regression 1 except that first, MPERT and MSERT have positive although insignificant effects on MGROW; second, the effect of AFRICA on MGROW is no longer significant.

Regression 6 includes those variables with high t-statistics in Regression 5, namely MPRI, MSERT, AFRICA and LATAMER and explains 49 percent of the variation in measured economic growth. Now we see a significantly (at 0.05 level, one-tail) positive effect of MSERT on MGROW and a significantly negative effect of AFRICA on MGROW.

Regression 7 includes GINI and explains around 72 percent of the variation in measured economic growth. As in Regression 3, GINI affects economic growth positively, although insignificantly. All estimated coefficients have the same signs and levels of significance as those of Regression 5 except that the effects of MPUB and MPERT become negative, although insignificanly.

Regression 8 includes only MPRI, MSERT, MGCON, GINI and LATAMER as explanatory variables, and explains 65 percent of the variation in measured economic growth. As seen, MPRI and MSERT have significantly positive effects on MGROW, while MGCON and LATAMER have significantly negative effects on MGROW.

Table 4.5: Regressions for MGROW

(Cross-Country-II)

	(5)	(6)	(7)	(8)
no. obs.	51	51	32	32
const.	0.02674	0.00652	0.01362	-0.00352
	(1.45572)	(0.64142)	(0.48312)	(-0.16067)
GDPINI	$-4.53*10^{-6}$		$-3.57*10^{-6}$	
	(-1.07885)		(-0.81093)	
POPINI	$-1.88*10^{-11}$		$-7.52*10^{-12}$	
	(-0.54482)		(-0.23696)	
MRPOP	-0.59691		-0.87137	
	(-1.23388)		(-1.23959)	
MPRI	0.13370#	0.10559@	0.17732#	0.15145#
	(2.02512)	(1.74115)	(2.18763)	(2.25545)
MPUB	0.07060		-0.06195	
	(1.01735)		(-0.41205)	
MPERT	$7.81*10^{-7}$		$-1.06*10^{-4}$	
	(0.00577)		(-0.43605)	
MSERT	$3.33*10^{-4}$	$3.72*10^{-4}$@	$2.60*10^{-4}$	$4.43*10^{-4}$#
	(1.13355)	(1.75259)	(0.76841)	(2.08102)
MGCON	-0.09243		-0.11304	-0.15867#
	(-1.33458)		(-0.97271)	(-2.24786)
GINI			0.08626	0.04018
			(1.47095)	(0.94382)
AFRICA	-0.01369	-0.01774#	-0.00449	
	(-1.62768)	(-2.47854)	(-0.40441)	
LATAMER	-0.02066#	-0.02513#	-0.02343#	-0.02211#
	(-2.33198)	(-3.58993)	(-2.18148)	(-3.63142)
R^2	0.54404	0.48636	0.71894	0.64858
SER	0.01720	0.01703	0.01512	0.01483

Note: OLS regressions; t-values in brackets; # & @: Significant at 0.025 & 0.05 level resp.

4.3.2 Cross-Sectional and Cross-Time Results

As argued before, aggregation of data across time may increase R^2, while decreasing the efficiency of the regression (i.e., lower t-values). In this part of Chapter 4, I examine economic growth across both countries and time. The variables are quite similar to those in last subsection except that first, there are not variables concerning initial conditions, such as GDPINI and POPINI; second, I do not consider GINI since the data is too spotty. Summary statistics are given in Table 4.6 (for definitions of all variables, see Appendix). The regressional equation can be written as

$$\begin{aligned} MGROW &= \alpha + \beta_1 RPOP + \beta_2 IVPRI + \beta_3 IVPUB + \beta_4 EDU + \\ &+ \beta_5 GCON + \beta_6 AFRICA + \beta_7 LATAMER + \epsilon. \end{aligned} \tag{4.2}$$

Next, I report the results of estimating cross-sectional and cross-time regressions in the form of (4.3). Regression 9 uses EDU as the index of human capital, and has 860 observations. Regression 10 uses PERT and SERT instead, and has 808 observations. Regression 11 examines GROW excluding both RPOP and education variables. Compared to the results of the cross-sectional regressions in Tables 4.3 and 4.5, as expected, the results of estimating cross-section and cross-time regressions in Table 4.7 reduce their explanatory power, while increasing the levels of significance of their estimated coefficients. Regressions 9, 10 and 11 have very similar patterns. First, they explain around only 11.3 to 11.5 percent of the variation in measured economic growth. Second, the higher the growth rate of the population, the lower the annual growth rate of GDP per capita. This negative effect is statistically significant now. Third, the annual ratio of gross private investment to GDP has a significantly positive effect on the annual growth rate of GDP per capita. Fourth, the annual growth rate of GDP per capita is significantly negatively related to the annual share of government consumption in GDP. Fifth, both dummies AFRICA and LATAMER significantly negatively affect GROW.

Table 4.6 Summary Statistics

(Cross-Country, Cross-Time)

VARIABLES	#OBS	MEAN	STD.DEV	MIN	MAX
GROW	860	0.01421	0.04997	-0.15504	0.22354
RPOP	860	0.02605	0.00721	0.00510	0.08310
IVPRI	860	0.13320	0.06491	0.00510	0.37080
IVPUB	860	0.09079	0.05268	0.00870	0.41320
EDU	860	4.25235	2.20064	0.21000	10.1000
PERT	808	82.9167	29.0885	12.0000	143.100
SERT	808	22.7797	16.2361	1.00000	77.0000
GCON	860	0.14189	0.05917	0.04800	0.50990

Signs of effects for most of the variables remain unchanged between cross-sectional, cross time and cross-sectional regressions. The coefficients of variables in (9) remain the same as those in (1), and most of the coefficients in (10) have the same signs as those in (5) except PERT.

I also presents the marginal contribution to R^2 for all explanatory variables in Table 4.8. In fact, when regressing GROW against IVPRI only, I have $R^2 = 0.06416$ with $t = 7.66935$ and $SE = 0.04837$. While all independent variables explain 11.523 % of economic growth, the annual ratio of gross private investment to GDP itself explains 6.416 % of economic growth for cross-country and cross-time regressions.

Table 4.7: Regressions for GROW

(Cross-Country and Cross-Time)

	(9)	(10)	(11)
no. obs.	860	808	860
const.	0.03797 (3.62990)	0.03917 (3.48586)	0.03082 (3.54367)
RPOP	-0.48009@ (-1.90991)	-0.59428# (-2.17042)	-0.39746 (-1.63333)
IVPRI	0.15616# (5.76780)	0.13267# (4.56339)	0.14885# (5.61482)
IVPUB	0.01000 (0.31179)	0.00148 (0.04404)	
EDU	-0.00134 (-1.35329)		
PERT		$-1.46*10^{-5}$ (-0.18139)	
SERT		$5.63*10^{-7}$ (0.00337)	
GCON	-0.06963# (-2.45996)	-0.06861@ (-1.80394)	-0.07294# (-2.61283)
AFRICA	-0.02202# (-4.40669)	-0.01884# (-3.56065)	-0.01876# (-4.28138)
LATAMER	-0.02603# (-5.65836)	-0.02710# (-5.45402)	-0.02584# (-5.64317)
R^2	0.11523	0.11365	0.11332
SER	0.04719	0.04693	0.04719

Note: OLS regressions; t-values in brackets; # & @: Significant at 0.025 & 0.05 level.

Table 4.8 Marginal Contribution to R^2

(Cross-Country, Cross-Time)

Regressor	Marginal Contribution to R^2
RPOP	0.378%
IVPRI	3.454%
IVPUB	0.010%
EDU	0.190%
GCON	0.628%
AFRICA	2.016%
LATAMER	3.324%
TOTAL	10.000%

Note: marginal contribution to R^2 is defined as the difference between the R^2 from Regression 9 in Table 4.7 and the R^2 from estimating Regression 9 with each variable deleted in turn. The "total" R^2 of 0.10000 is below 0.11523, the R^2 in Reg. 9 because of some intercorrelation among the regressors.

4.4 Summary

We are now in a better position to address the questions posed in Section 1.1. First, there is no strong indication of convergence in growth rates. Second, cross-country regressions show that larger initial population level is associated with lower growth rate of the economy, although these negative effects are frequently statistically insignificant. Third, there is a strong negative correlation between the growth rates of population and GDP per capita. Fourth, the ratio of gross private investment to GDP has a strong positive effect on economic growth. Fifth, the ratio of public sector investment generally has an insignificant effect on economic growth. Sixth, the effect of education on growth is mixed and needs further exploration. Seventh, government spending has a negative effect on economic growth. This negative effect is significant across countries and time. Eighth, although income inequality negatively, not significantly, affects economic growth, my study shows that income inequality actually enhances economic growth, not always significantly, after controlling other variables for those non-democratic countries in the sample. Lastly, geographical variables explain a large part of economic growth. Specifically, African and Latin American countries experienced much lower economic growth during the period 1971-1987.

4.5 Data Appendix

4.5.1 Descriptions of Data Used

The variables used in Chapter 4 are listed below. They are generally annual time-series ordered by country and year (see *World Development Report 1991*: Supplementary Data). Asterisk (*) denotes a variable reported in five-year intervals. For each series, the maximum possible time coverage and the number of countries with at least one observation are included in parentheses.

1. GDPKD: GDP at constant 1980 prices, U.S. dollars. (1960-89: 86)

2. POP4: Total population. (1960-89: 91)

3. GDPCAP4: Gross Domestic Product (GDP) per capita = GDPKD/POP4. (1960-87: 58)

4. INVGPR4: Ratio of gross private investment to GDP. (1970-88: 94)

5. INVPUB4: Ratio of public sector investment to GDP. (1970-88: 98)

6. EDT: estimated average years of education of the population of working age group (15 to 64). Based on UNESCO data on enrollment rates for the period 1960-88, and on mortality and birth statistics. (1960-86: 68)

7. PERT3*: Gross primary enrollment rate, total (percent). (1960-85: 91)

8. SERT3*: Gross secondary enrollment rate, total (percent). (1960-85: 92)

9. GCONX2: Share of Government consumption in GDP. (1960-89: 85)

10. YDISTN: Income distribution, which is in terms of percentage share of household income, by percentile groups of households, is not included in the World Development Report data set. I enter the data according to tables "Income Distribution" in *World Development Report* (1980-91).

4.5.2 Definitions of Variables

Since most of our variables discussed above cover the period 1970-86 (there is missing data occasionally), we focus our empirical analysis on this 17-year span. 52 countries under study are listed in Table 4.1. All means are cross time.

MGDP = Mean(GDPCAP4).

GROW = GDPCAP4/GDPCAP4[-1] - 1.

MGROW = Mean(GROW).

GDPINI = GDPCAP4 in 1969.

POP = POP4.

RPOP = POP4/POP4[-1] - 1.

MRPOP = Mean(RPOP).

POPINI = POP4 in 1969.

IVPRI = INVGRI4.

MPRI = Mean(INVGRI4).

IVPUB = INVPUB4.

MPUB = Mean(INVPUB4).

EDU = EDT.

MEDT = Mean(EDT).

PERT = PERT3.

MPERT = Mean(PERT3).

SERT = SERT3.

MSERT = Mean(SERT3).

GCON = GCONX2.

MGCON = Mean(GCONX2).

GINI, the Gini coefficient $= 1 - 0.001[18 * q1 + 14 * q2 + 10 * q3 + 6 * q4 + 3 * q5 - 2 * q6]$ (mathematical account is given in next part), where q1, q2, q3, q4, q5 and q6 are the percentage share of household income by the lowest 20 percent of households, the second quintile household, the third quintile household, the fourth quintile household, the highest 20 percent of household and the highest 10 percent of households respectively.

AFRICA: Dummy variable for Africa.

Latamer: Dummy variable for Latin America.

4.5.3 Derivation of Gini Coefficient

As seen in Figure 4.1, the Gini coefficient is equal to the ratio of shaded area over Area(OAB). The shaded area equals Area(OAB) subtracts the area under the curve OA. Now, the area under the curve OA, A($\underline{OA}$), can be calculated as follows.

$$
\begin{aligned}
A(\underline{OA}) &= 0.5 * 20 * q1 + 0.5 * 20 * [q1 + (q1 + q2)] \\
&+ 0.5 * 20 * [(q1 + q2) + (q1 + q2 + q3)] \\
&+ 0.5 * 20 * [(q1 + q2 + q3) + (q1 + q2 + q3 + q4)] \\
&+ 0.5 * 10 * [(q1 + q2 + q3 + q4) + (q1 + q2 + q3 + q4 + q5 - q6)] \\
&+ 0.5 * 10 * [(q1 + q2 + q3 + q4 + q5 - q6) + (q1 + q2 + q3 + q4 + q5)]
\end{aligned}
$$

$$= 0.5 * 10 * [18 * q1 + 14 * q2 + 10 * q3 + 6 * q4 + 3 * q5 - 2 * q6].$$

Thus

$$\begin{aligned}
\text{Gini} &= \frac{A(OAB) - A(\underline{OA})}{A(OAB)} \\
&= 1 - \frac{0.5 * 10 * [18 * q1 + 14 * q2 + 10 * q3 + 6 * q4 + 3 * q5 - 2 * q6]}{0.5 * 100 * 100} \\
&= 1 - 0.001[18 * q1 + 14 * q2 + 10 * q3 + 6 * q4 + 3 * q5 - 2 * q6].
\end{aligned}$$

Chapter 5

Conclusions

5.1 Conclusions

The dissertation study the relationship among fiscal policies, optimal growth, and elections under two different economic systems: a free economy and a democratic planned economy.

In a free economy (Chapter 2), I assume the government indirectly controls the economy by selecting a fiscal policy, and a firm chooses the growth. First, I show that the distribution of income and the form of the utility function determine a fiscal policy (in terms of an income tax rate and a weight of the public sector) that, together with the utility function, decides economic growth. Compared to neoclassical growth models which conclude that higher income tax rates translate into lower rates of growth, my model takes fiscal policy as endogenous and provides another explanation for the widely observed cross-country differences in growth rates. In other words, economic growth in a specific nation may have something to do with its income distribution and the form of utility functions, which may in turn be a result of its culture, customs and geographic region. Therefore, to undertake a cross-country study of economic growth and fiscal policy without paying any attention to specific characteristics of a country may generate misleading and unreliable results. For instance, I provide examples using different income distributions (examples 1 and 2 in Chapter 2) to show that a higher income tax rate can be associated with a higher growth rate.

Second, given the distribution of income and the weight of the public sector ϕ, when the EMUI of the private good utility function is not greater than one, then the wealthier are more likely to oppose a larger government. Given the tax rate, the wealthier are more likely to oppose a redistribution-oriented fiscal policy (i.e., smaller ϕ). I have shown that the median-income voter will prevail in any sequential pairwise voting over T and ϕ (regardless of agenda setting) under majority rule.

Third, when a private good utility function has a constant EMUI, then fiscal policy and income distribution have no effects on economic growth (i.e., fiscal and distributional neutrality). I also find the following two interesting properties. One property concerns the profit share of the decisive consumer and the tax rate. Among different distributions of income, the higher the profit share of the decisive consumer (i.e., median-income consumer), the lower the tax rate. This conclusion confirms the statement of Meltzer and Richard (1981, 1983) that an increase in mean income relative to the income of the decisive voter increases the size of government. The other property is related to the Kuznets curve (the hypothesis that income inequality first increases and then decreases with development). I have shown that when the median of the income distribution (pre-tax) is the same as mean, then the tax rate is technology-proof, i.e., post-tax inequality has nothing to do with economic development. One interesting case is when the utility function is $u(x, y) = A_1 x^c + A_2 y^c + A_3$, where $c \in (0, 1)$, and A_1, $A_2 > 0$ and A_3 are constants. Then, the size of government increases as the size of the population increases. This may provide another explanation for the growth of government.

In a democratic planned economy (Chapter 3), I assume the government controls the economy by setting wage rates, prices and the growth rate of the economy. I show there exist political-economic equilibria in a democratic planned economy; however, these equilibria are generally agenda-dependent. I further show that with Cobb-Douglas production technology, decentralization of wage decision in a democratic planned economy can guarantee a unique political-economic equilibrium and a growth path that is middle-class-oriented. Moreover, under certain conditions, the economic growth path in a democratic planned economy could be the same as that in a free economy.

In addition to theoretical work, I present an empirical study of economic growth (Chapter 4) and show that there is a strong negative correlation between the growth rates of population and GDP per capita. Private investment has a strong positive effect on economic growth. After controlling other variables in my sample, I find that income inequality actually enhances economic growth, although not always significantly.

5.2 Directions for Future Work

This is a first attempt to bring together fiscal policies, economic growth and elections in order to explain economic growth more realistically and systematically. There are some theoretical limitations in this paper. First, throughout the dissertation, I assume that the population is politically homogeneous in the sense that there are no political parties or interest groups pursuing their own goals. Second, I assume consumers have full information about all profit shares θ_i and are sophisticated enough to calculate their payoffs through competitive equilibrium solution for any given fiscal policy. Third, I use the model of Stokey, Lucas and Prescott (1989) which does not consider leisure in the utility function and avoids dealing with problems concerning labor supply and unemployment. This is certainly not realistic.

There are quite a few directions for future research. First, can we predict political business cycles by introducing a cost of entry, interest groups (for example, a candidate has to serve for an interest group or party in order to get funding or win the primary) and technology shock? Second, how will coalitional governments change the electoral outcomes in a democratic planned economy? Third, if coalitional governments can somehow solve the agenda-dependence problem of the electoral outcome, is there any empirical evidence which suggests that coalitional governments are more likely to occur in a democratic planned economy than a free economy? Fourth, does there exist a subgame perfect equilibrium in a democratic planned economy when we request consumers to vote on W^0, W^1, P, and K^1 simultaneously (i.e., candidates' ideal points) instead of voting on them sequentially?

In addition, some of my empirical results pose a challenge for theoretical explanations. First, why does higher education not always lead to higher economic growth? Second, what are the political and economic reasons for the lower growth rate of the economy in Africa and Latin America? Third, does income inequality really jeopardize economic growth?

Future extensions of the empirical analysis include: first, more countries, particularly those in Europe and North America, need to be studied in order to find out whether some of my empirical findings can be generalized to democratic developed countries. Second, the fact that there are few planned economies included in the sample under study prevents me from comparing economic growth between free economies and planned economies, this problem can be solved when more data of planned economies becomes available. Third, a more systematic data set on income distribution should be included in the empirical study in order to obtain reliable results concerning the effects of income inequality on economic growth.

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