Essays on Dynamic Political Economy

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To Pasquale and Maria Luisa

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Abstract

This dissertation comprises three essays that are linked by their focus on dynamic models of political economy, in which farsighted agents interact over an infinite number of periods, and the strategic environment evolves endogenously over time.

In Chapter 2, “Dynamic Legislative Bargaining with Veto Power”, I analyze the consequences of veto power in an infinitely repeated divide-the-dollar bargaining game with an endogenous status quo policy. I show that a Markov equilibrium of this dynamic game exists, and that, irrespective of the discount factor of legislators, their recognition probabilities, and the initial division of the dollar, policy eventually gets arbitrarily close to full appropriation of the dollar by the veto player.

Chapters 3 and 4—coauthored with Thomas Palfrey and Marco Battaglini—study free riding in a dynamic environment where a durable public good provides a stream of benefits over time and agents have opportunities to gradually build the stock. We consider economies with reversibility, where investments can be positive or negative, and economies with irreversibility, where investments are non-negative and the public good can only be reduced by depreciation. In Chapter 3, “The Free Rider Problem: A Dynamic Analysis”, we study and compare the set of Markov equilibria of these models. With reversibility, there is a continuum of equilibrium steady states: the highest equilibrium steady state of the public good is increasing in the group size, and the lowest is decreasing. With irreversibility, the set of equilibrium steady states converges to a unique point as

depreciation converges to zero: the highest steady state possible with reversibility. In Chapter 4, "The Dynamic Free Rider Problem: An Experimental Study", we test the results from this model with controlled laboratory experiments. The comparative static predictions for the treatments are supported by the data: irreversible investment leads to significantly higher public good production than reversible investment.

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Chapter 1

Introduction

The three essays in this thesis are part of a growing literature extending static models of political economy to dynamic environments, in which farsighted agents interact over an infinite number of periods and their strategic incentives evolve endogenously over time. The underlying theme of the thesis is the notion that most public policies and individual decisions have long-lasting consequences, and that political agents with conflicting interests take into account the effect of today's policy on future decisions. To illustrate this notion, consider policies with an endogenous status quo. In most democracies, once enacted, many laws or programs remain in effect until further legislative action is taken. For example, in the U.S., about two-thirds of the federal budget is allocated to mandatory spending, which continues year after year by default. Outside of the fiscal sphere, many other issues, such as immigration, financial regulation, minimum wage, civil liberties, and national security are typically regulated by permanent legislation. Another example is provided by the accumulation of public debt over time: whenever governments borrow money from the capital markets to finance their current expenditure, they increase the level of public debt, affecting the ability to spend of future governments. In all these settings, the *state variable* generates a *dynamic linkage* across periods and present agents with a trade off between optimally responding to the current environment and securing more favorable conditions for the future.

This approach has recently been employed to study a number of different issues: dynamic electoral competition and extensions (or failures) of median voter theorems (Duggan 2000, Banks and Duggan 2008, Forand 2009, Kalandrakis 2009, Van Weelden 2009); dynamic legislative bargaining and the evolution (or unraveling) of compromise (Baron 1996, Dixit et al. 2000, Kalandrakis 2004, Bowen and Zahran 2009, Diermeier and Fong 2011); sequential elections and the formation (or not) of bandwagons (Dekel and Piccione 2000, Callander 2007, Nageeb and Kartik 2010); dynamic institutional choice and the persistence (or reform) of undesirable institutions (Acemoglu and Robinson 2008, Lagunoff 2009, Lizzeri and Persico 2004); dynamic principal-agent models and the electorate ability (or inability) to provide incentives to politicians (Banks and Sundaram 1993, 1998, Schwabe 2009).

In this thesis, I apply this framework to two novel topics: (1) dynamic legislative bargaining with an endogenous status quo alternative and a veto player; and (2) dynamic provision of durable public goods. I find that the dynamic politico-economic equilibrium approach may substantially change the pattern of public policy choices predicted by a static model and thus, provide a new angle to understand empirical evidence which is hard to explain with existing theories. Analytical solutions are provided throughout the thesis, making the underlying mechanism highly transparent.

Chapter 2 is motivated by the observation that many real world voting bodies and assemblies grant one or several of their members the right to block decisions even when a proposal has secured the necessary majority—a veto right. The existence of veto power raises the concern it may grant its holders excessive power: although the formal veto right only grants the power to block undesirable decisions, it could *de facto* allow veto members to impose their ideal decision on the rest of the committee. The existing literature has studied this possibility in static frameworks where the status quo policy is exogenous, or legislation cannot be modified after its initial introduction. These are strong limitations for two reasons. First, in reality, legislators interact over a longer time horizon and many of their decisions are inertial, in the sense that a policy remains in effect only until a new policy is passed. Second, in this dynamic setting, the status quo policy, which determines the bargaining advantage of the veto players, is the product of past decisions rather than being exogenously given. In this paper, I aim to improve our understanding of the consequences of veto power, taking into account the dynamic process by which the status quo policy is generated.

In particular, I use non-cooperative bargaining theory to analyze the consequences of veto power in a setting where legislators interact over an infinite number of periods and, if there is no agreement for a policy change, the last period's decision is implemented. I focus on the case of perfectly opposed preferences (for example, pork barrel policies) and I show that the veto player is eventually able to move the policy arbitrarily close to his ideal point, even when legislators are patient and take into account the future consequences of changing the current status quo policy. However, unless legislators are perfectly impatient, the veto player has to pay his coalition partners a premium in order to pass a proposal that he prefers to the status quo, and it takes an infinite number of bargaining periods to converge to his ideal point. In a companion paper not included in this thesis (Nunnari 2012), I look at a more canonical spatial model with partially aligned preferences (for example, personal income tax rates or mandatory spending). In this case, the veto player cannot always impose his ideal policy, because he often has to compromise with a second de facto veto player, the median legislator. I show that the power of the veto player depends on the initial status quo, and even more so when legislators are patient and forward-looking.

Chapters 3 and 4—coauthored with Thomas Palfrey and Marco Battaglini— are motivated by a salient feature of many public goods provided by governments or groups of individuals: it takes time to accumulate them, and they are durable, depreciating slowly, and projecting their benefits for many years. Although a large literature has studied public good provision in static settings, much less is known about dynamic environments, both theoretically and empirically. How serious is free riding in these cases? To what extent does it depend on the institutions that govern decision making and on the public good production function? In these two papers, we study free riding in a dynamic environment where a durable public good provides a stream of benefits over time and agents have opportunities to gradually build the stock. We consider two cases: economies with reversibility, where investments can be positive or negative; and economies with irreversibility, where investments are non-negative and the public good can only be reduced by depreciation.

In Chapter 3, we study the Markov equilibria of these two models. With reversibility, there is a continuum of equilibrium steady states. while in a static free rider's problem the players' contributions are strategic substitutes, in a dynamic model they may be strategic complements: the highest equilibrium steady state of the public good is increasing in n, and the lowest is decreasing. With irreversibility, the set of equilibrium steady states converges to a unique point as depreciation converges to zero: the highest steady state possible with reversibility. The irreversibility constraint, thus, creates a commitment device and reduces the strategic substitutability of contributions. In both cases, the highest steady state converges to the efficient steady state as agents become increasingly patient.

Chapter 4 tests the results from this model with controlled laboratory experiments, and represents the first experimental study of the dynamic accumulation process of a durable public good. We find investing behavior consistent with the theory, as we observe significantly higher public good levels when investments cannot be reversed. In both treatments, there is overinvestment relative to the equilibrium in the initial stages of the game. This is followed by negative investment approaching the theoretical predictions in reversible investment economies, while the overinvestment decreases but persists in irreversible investment economies. Finally, we propose a novel experimental methodology to test the Markovian assumption using a one-period reduced version of the dynamic game and we conclude that there is evidence of Markovian, forward-looking behavior.

These two papers are part of a broader research agenda in which we explore alternative political mechanisms for the provision of durable public goods and we assess their performance. In a third paper, not included in this thesis (Battaglini, Nunnari, and Palfrey 2012b), we focus on the case of reversible economies and we consider a centralized legislature where, in each period, the representatives of n districts bargain over how to allocate resources between investment in a public good and targeted transfers. We show that the efficiency of the public policy is increasing in the number of votes required to pass a proposal, because a higher majority requirements leads to higher investment in the public good and less private consumption.

Chapter 2

Dynamic Legislative Bargaining with Veto Power

A large number of important voting bodies grant one or several of their members the right to block decisions even when a proposal has secured the necessary majority—a *veto right*. One prominent example is the United Nations Security Council, where a motion is approved only with the affirmative vote of nine members, including the concurring vote of the five permanent members (China, France, Great Britain, Russia, and the U.S.). Another important example is the U.S. President's ability to veto congressional decisions. Additionally, in assemblies with asymmetric voting weights and complex voting procedures, veto power may arise implicitly: this is the case of the U.S. in the International Monetary Fund and the World Bank governance bodies (Leech and Leech 2004).[1]

The existence of veto power raises two concerns. First, the ability of an agent to veto policies increases the possibility of legislative stalemate, or "gridlock". Second, although the formal veto right only grants the power to block undesirable decisions, it could *de facto* allow veto members to

[1] Many other institutions grant veto power to some of their decision makers. For example, some corporate boards of directors grant minority shareholders a "golden share", which confers the privilege to veto any decision. This share is often held by members of the founding family, or governments in order to maintain some control over privatized companies and was widely used in the European privatization wave of the late 90s and early 2000s. For instance, the British government had a golden share in BAA, the UK airports authority; the Spanish government had a golden share in Telefónica; and the German government had a golden share in Volkswagen.

impose their ideal decision on the rest of the committee (Russell 1958, Woods 2000, Blum 2005).[2]

In this paper, I investigate the consequences of veto power in a dynamic bargaining setting where the location of the current *status quo policy* is determined by the policy implemented in the previous period. This is an important feature of many policy domains—for instance, personal income tax rates or entitlement programs—where legislation remains in effect until the legislature passes a new law. In each of an infinite number of periods, one of three legislators, one of whom is a veto player, is randomly recognized to make a proposal on the allocation of a fixed endowment. The proposed allocation is implemented if it receives at least two affirmative votes, including the vote of the veto player. Otherwise, the status quo policy prevails and the resource is allocated as it was in the previous period. In this sense, the status quo policy evolves endogenously.

In this simple setting, I answer two basic questions: To what extent is the veto player able to leverage his veto power into outcomes more favorable to himself? As this leverage is found to be substantial, I then turn to a second question: What are the effects of institutional measures meant to reduce the power of the veto player?

In particular, I fully characterize a Markov Perfect Equilibrium (MPE) and prove it exists for any discount factor, any initial divisions of the resources, and any recognition probabilities.[3] In this MPE, the veto player is eventually able to move the status quo policy arbitrarily close to his ideal point. That is, the veto player is eventually able to fully appropriate all resources, irrespective of the discount factor, the recognition probabilities, and the initial division of the resources.

When agents are impatient, this result comes from the fact that non-veto legislators support any

[2]These concerns were expressed by the delegates of the smaller countries when the founders of the United Nations met in San Francisco in June 1945 (Russell 1958, Bailey 1969), and they have been a crucial point of contention in the ongoing discussion over how to reform the UN Security Council to improve its credibility and reflect the new world order (Fassbender 1998, Weiss 2003, Bourantonis 2005, Blum 2005). A similar debate has recently arisen regarding the IMF's and WB's voting weights determination (Woods 2000, Rapkin and Strand 2006).

[3]The only general existence result for dynamic bargaining games applies to settings with stochastic shocks to preferences and the status quo (Duggan and Kalandrakis 2010). As these features are not present in my model, proving existence is a necessary step of the analysis.

proposal that gives them at least as much as the status quo. Thus, it takes at most two proposals by the veto player to converge to full appropriation of the dollar. When legislators are patient—that is, when they care, even minimally, about the effect of the current policy on future outcomes—the ability of the veto player to change the policy to his advantage remains, but is reduced. This occurs because, when other committee members receive a proposal that increases the veto player's share, they take into account the associated reduction in their future bargaining power and demand a premium to support it. However, unless legislators are perfectly patient, this premium is always smaller than the share of resources not already allocated to the veto player, and the policy displays a *ratchet effect*: with the possible exception of the first period, the share to the veto player will only stay constant (if he is not proposing) or increase (if he is proposing).

The speed of convergence to the veto player's ideal outcome is decreasing in the discount factor of the committee members, as the premium demanded by non-veto legislators increases in their patience. In contrast with the impatient case, when agents are patient, this premium is always positive and, thus, it takes an infinite number of bargaining periods to converge to full appropriation of the dollar by the veto player.

This result suggests that the ability to oppose any decision is indeed a powerful right and guarantees a strong leverage on long run outcomes. Therefore, I analyze potential mechanisms to weaken veto power and find that extreme outcomes are difficult to avoid in the long run. First, I investigate the effect of reducing the agenda setting power of the veto player. As long as the veto player has a positive probability of recognition, he will be able to extract all resources. However, the speed of convergence to this outcome decreases as this probability decreases. Second, adding an additional veto player does not increase the ability of non-veto players to retain a share of the resources in the long run, but it reduces the ability of each veto player to accrue all the resources:

whenever a veto player proposes, he has to share what he extracts from the other legislators with the other veto player. Finally, when the veto right is randomly re-assigned in every period—rather than permanently held by one legislator—the long term outcome is still extreme: policy eventually converges to an absorbing set where all resources go to either the proposer or the veto player.

This paper contributes primarily to the theoretical literature on the consequences of veto power in legislatures. A large number of studies build on models of legislative bargaining à la Baron and Ferejohn (1989) to examine the role of veto power in policy making. Most of these papers model specific environments and focus on the case of the U.S. Presidential veto. These works analyze conditions under which an executive may exercise veto power (Matthews 1989, Groseclose and McCarty 2001, Cameron 2000), evaluate the effect of presidential veto on spending (Primo 2006) or the distribution of pork barrel policies (McCarty 2000a), disentangle the effect of veto and proposal power (McCarty 2000b), and provide a rationale for the emergence of veto points inside Congress (Diermeier and Myerson 1999). More closely related to this paper, Winter (1996) shows that the share of resources to veto players is decreasing in the cost of delaying an agreement, so that the share of resources to non-veto players declines to zero as the cost of delay becomes negligible, that is, as legislators become infinitely patient. Banks and Duggan (2000) derive a similar result in a more general model of collective decision making. A common limitation of this literature, and the main point of departure with my paper, is the focus on static settings: the legislative interaction ceases once the legislature has reached a decision, and policy cannot be modified after its initial introduction. In these frameworks, any conclusion on the effect of veto power on policy outcomes depends heavily on the specific assumptions on the status quo policy (Krehbiel 1998, Tsebelis 2002). In this paper, the status quo policy is not exogenously specified but is rather the product of policy makers' past decisions.

In this sense, the present study belongs to a strand of recent literature on legislative policy making with an endogenous status quo and farsighted players (Baron 1996, Kalandrakis 2004, Bernheim, Rangel, and Rayo 2006).[4] However, with the exception of Duggan, Kalandrakis, and Manjunath (2008), who model the specific institutional details of the American presidential veto and limit their analysis to numerical computations, this literature does not explore the consequences of veto power. The most related work to mine is Kalandrakis (2004) who analyzes a similar environment without a veto player. This institutional variation generates stark differences in strategies and long run outcomes, as the voting behavior of patient players mimics the behavior of impatient ones, and policy quickly converges to an absorbing set where the proposer extracts all resources in all periods.[5] In my setting, the discount factor affects voting strategies and—even if the irreducible absorbing set has the veto player extracting all resources—convergence to absorption typically takes an infinite number of periods, during which the veto player shares resources with one non-veto player.

The paper is organized as follows. Section 2 gives a detailed presentation of the legislative setup and introduces the equilibrium notion. Section 3 outlines the equilibrium analysis and gives the main results. Section 4 investigates the consequences of measures to reduce the power of the veto player. Section 5 concludes with a discussion of the limitations of my approach and of the future directions for the study of veto power in dynamic frameworks.

[4]See also Epple and Riordan (1987), Ingberman (1985), Bowen and Zahran (2009), Dziuda and Loeper (2010), Cho (2005), Gomes and Jehiel (2005), Lagunoff (2008, 2009), Barseghyan, Battaglini, and Coate (2010), Battaglini and Coate (2007, 2008), Baron and Herron (2003), Baron, Diermeier, and Fong (2011),Diermeier and Fong (2011), Duggan and Kalandrakis (2010), Kalandrakis (2009), Penn (2009).

[5]Convergence to this absorbing set is not deterministic, as it depends on the identity of the proposer recognized in each period, but it happens in finite time, in a maximum expected time of 2.5 periods.

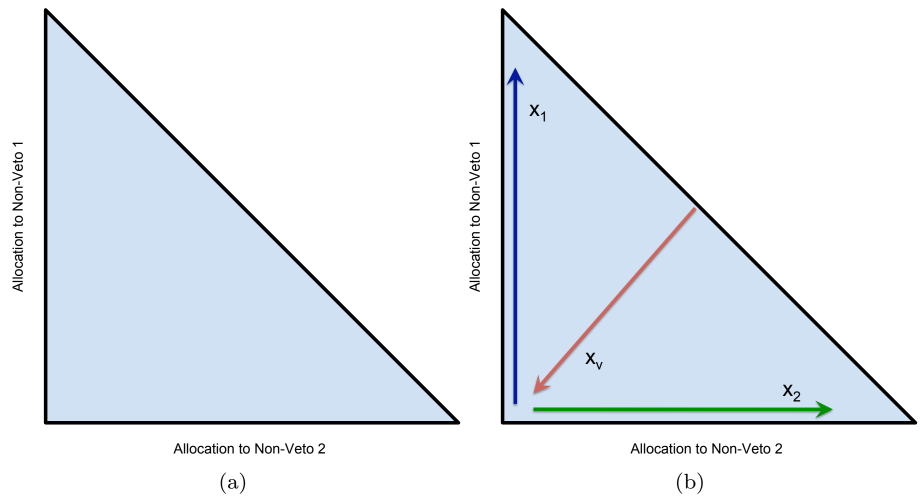


Figure 2.1: The set of possible legislative outcomes in each period, Δ

2.1 Model and Equilibrium Notion

2.1.1 Model

Three agents repeatedly bargain over a legislative outcome $\mathbf{x}^t$ for each $t = 1, 2, \ldots$. One of the three agents is endowed with the power to veto any proposed outcome in every period. I denote the veto player with the subscript v and the two non-veto players with the subscript $j = \{1, 2\}$. The possible outcomes in each period are all possible divisions of a fixed resource (a dollar) among the three players, that is $\mathbf{x}^t$ is a triple $\mathbf{x}^t = (x_v^t, x_1^t, x_2^t)$ with $x_i^t \geq 0$ for all $i = v, 1, 2$ and $\sum_{i \in \{v,1,2\}} x_i = 1$. Thus, the legislative outcome $\mathbf{x}^t$ is an element of the unit simplex in $\mathbb{R}^3_+$, denoted by Δ. Figure 1 represents the set of possible legislative outcomes, $\mathbf{x} \in \Delta$, in $\mathbb{R}^2$. The vertical dimension represents the share to (non-veto) player 1, while the horizontal dimension represents the share to (non-veto) player 2. The remainder is the share that goes to the veto player. Thus, the origin is the point where the veto player gets the entire dollar.

The Bargaining Protocol. At the beginning of each period, one agent is randomly selected to propose a new policy, $\mathbf{z} \in \Delta$. Each agent has the same probability of being recognized as policy proposer, that is $\frac{1}{3}$.[6] This new proposal is voted up or down, without amendments, by the committee. A proposal passes if it gets the support of the veto player and at least one other committee member. If a proposal passes, $\mathbf{x}^t = \mathbf{z}$ is the implemented policy at t. If a proposal is rejected, the policy implemented is the same as it was in the previous period, $\mathbf{x}^t = \mathbf{x}^{t-1}$. Thus, the previous period's decision, $\mathbf{x}^{t-1}$, serves as the *status quo policy* in period, t. The initial status quo $\mathbf{x}^0$ is exogenously specified.

Stage Utilities. Agent i derives stage utility $u_i : \Delta \to \mathbb{R}$, from the implemented policy $\mathbf{x}^t$. I assume players' utilities depend only on their share of the dollar, and that payoffs are linear, so that $u_i(\mathbf{x}) = x_i$. Players discount the future with a common factor $\delta \in [0, 1)$, and their payoff in the game is given by the discounted sum of stage payoffs.

2.1.2 Equilibrium

Strategies. In general, strategies are functions that map histories, that is, vectors that records all proposals as well as all voting decisions that precede an action, to the space of proposals Δ and voting decisions {yes; no}. In what follows, though, I restrict analysis to cases when players condition their behavior only on a summary of the history of the game that accounts for payoff-relevant effects of past behavior (Maskin and Tirole 2001). Specifically, define the state in period t as the previous period's decision $\mathbf{x}^{t-1}$, and denote the state by $\mathbf{s} \in S$, so that we have $\mathbf{s} = \mathbf{x}^{t-1}$ and $S = \Delta$. I restrict attention to Markov strategies such that agents behave identically ex ante, that is, prior to any mixing, in different periods with state $\mathbf{s}$, even if that state arises from different

[6] I will relax this assumption in Section 4.1.

histories.

In general, a *mixed Markov proposal strategy* for legislator i is a function $\mu_i : S \to \mathcal{P}(\Delta)$, where $\mathcal{P}(\Delta)$ denotes the set of Borel probability measures over Δ. For the purposes of this analysis, it is sufficient to assume that for every state $\mathbf{s}$, μ_i has finite support. Thus, the notation $\mu_i[\mathbf{z}|\mathbf{s}]$ represents the probability that legislator i makes the proposal $\mathbf{z}$ when recognized, conditional on the state being $\mathbf{s}$. A *Markov voting strategy* is an *acceptance correspondence* $A_i : S \to \Delta$, where $A_i(\mathbf{s})$ represents the allocations for which i votes *yes* when the state is $\mathbf{s}$. Then, a *Markov strategy* is a mapping $\sigma_i : S \to \mathcal{P}(\Delta) \times 2^{\Delta}$, where for each $\mathbf{s}$, $\sigma_i(\mathbf{s}) = (\mu_i[\cdot|\mathbf{s}], A_i(\mathbf{s}))$.

Continuation Values and Expected Utilities. In this dynamic game, the *expected utility* of agent i from the allocation implemented in period t does not only depend on his stage utility, but also on the discounted expected flow of future stage utilities, given a set of strategies. In order to define properly the *continuation value* of each status quo, I will first introduce the concepts of the *win set* and *transition probabilities.*

For a given set of voting strategies, define the win set of state $\mathbf{s} \in \Delta$, $W(\mathbf{s})$, as the set of all proposals that beat $\mathbf{s}$ by the voting rule described above. In this setting, $W(\mathbf{s})$ is the collection of all proposals $\mathbf{x}$ to which the veto player and at least one non-veto player vote yes. This differs from a simple majority rule, where the win set would be the collection of all proposals $\mathbf{x}$ to which at least two agents, irrespective of their identity, vote yes.

Then, for a triple of Markov strategies $\sigma = (\sigma_v, \sigma_1, \sigma_2)$, we can write the transition probability to decision $\mathbf{x}$ when the state is $\mathbf{s}$, $Q[\mathbf{x}|\mathbf{s}]$ as follows:

$$Q[\mathbf{x}|\mathbf{s}] \equiv I_{W(\mathbf{s})}(\mathbf{x}) \sum_{i=\{v,1,2\}} \frac{1}{3}\mu_i[\mathbf{x}|\mathbf{s}] + I_{\{\mathbf{s}\}}(\mathbf{x}) \sum_{i=\{v,1,2,\}} \frac{1}{3} \sum_{\mathbf{y}:\mu_i[\mathbf{y}|\mathbf{s}]>0} I_{\Delta\setminus W(\mathbf{s})}(\mathbf{y})\mu_i[\mathbf{y}|\mathbf{s}] \tag{2.1}$$

where $I_{W(\mathbf{s})}(\mathbf{x})$ is the indicator function that takes value of 1 when $\mathbf{x}$ is a proposal that beats the status quo and 0 otherwise, $I_{\{\mathbf{s}\}}(\mathbf{x})$ is the indicator function that takes value of 1 when $\mathbf{x} = \mathbf{s}$ and 0 otherwise, and $I_{\Delta \backslash W(\mathbf{s})}(\mathbf{y})$ is the indicator function that takes value of 1 when $\mathbf{y}$ is a proposal that does not beat the status quo and 0 otherwise. The first part of (2.1) reflects the probability of transition to allocations that are proposed by one of the three players and are approved, which is the probability of transition $Q[\mathbf{x}|\mathbf{s}]$ if $\mathbf{x} \neq \mathbf{s}$. The probability of staying in the same state, $Q[\mathbf{x}|\mathbf{s}]$, is given by the probability that $\mathbf{x} = \mathbf{s}$ is proposed (the first term), plus the probability that a proposal $\mathbf{x} \neq \mathbf{s}$ is proposed and rejected by the floor (the second term).

Equipped with this notation, I now define the *continuation value*, $v_i(\mathbf{s})$, of legislator i when the state is $\mathbf{s}$:

$$v_i(\mathbf{s}) = \sum_{\mathbf{x}:Q[\mathbf{x}|\mathbf{s}]>0} \left[u_i(\mathbf{x}) + \delta v_i(\mathbf{x})\right] Q[\mathbf{x}|\mathbf{s}] \tag{2.2}$$

Using (2.2), we can finally write the expected utility of legislator i, $U_i(\mathbf{s})$, as a function of the allocation implemented in period t, $\mathbf{x}^t$:

$$U_i(\mathbf{x}^t) = x_i^t + \delta v_i(\mathbf{x}^t) \tag{2.3}$$

Given that non-veto legislators are otherwise identical, I focus on Markov proposal and voting strategies that are *symmetric* with respect to the two non-veto legislators. A Markov equilibrium is symmetric if it has the following property: for any state $\mathbf{s} \in \Delta$, define $\mathbf{s}^{12}$ by switching s_1 and s_2, that is $\mathbf{s}^{12} = [s_v, s_2, s_1]$. Then an equilibrium is symmetric if $\sigma_1(\mathbf{s}) = \sigma_2(\mathbf{s}^{12})$ for any $\mathbf{s} \in \Delta$.

Equilibrium Notion. We can finally define the equilibrium solution concept as a variant of Markov perfect Nash equilibrium with a standard refinement on voting strategies:

Definition 2.1.1. *A symmetric Markov perfect Nash equilibrium in stage-undominated voting strategies (MPE) is a pair of Markov strategy profiles (symmetric for the two non-veto players),* $\sigma^* = \{\sigma_v^*, \sigma_1^*, \sigma_2^*\}$*, where* $\sigma_v^* = (\mu_v[\cdot|\mathbf{s}],\ A_v^*(\mathbf{s}))$*, and* $\sigma_j^* = \{(\mu_j[\cdot|\mathbf{s}],\ A_j^*(\mathbf{s}))\}_{j=1}^2$*, such that for all* $i = v, 1, 2$ *and all* $\mathbf{s} \in \Delta$*:*

$$\mathbf{y} \in A_i^*(\mathbf{s}) \Longleftrightarrow U_i(\mathbf{y}) \geq U_i(\mathbf{s}) \tag{2.4}$$

$$\mu_i^*[\mathbf{z}|\mathbf{s}] > 0 \Rightarrow \mathbf{z} \in \arg\max\{U_i(\mathbf{x})|\mathbf{x} \in W(\mathbf{s})\} \tag{2.5}$$

An equilibrium, as specified in (2.4), requires that legislators vote yes if and only if their expected utility from the status quo is not larger than the expected utility from the proposal. Such stage undominated voting strategies rule out uninteresting equilibria where voting decisions constitute best responses solely due to the fact that legislators vote unanimously, and thus a single vote cannot change the outcome. The fact that proposers optimize over all feasible proposals, that is over all proposals that would be approved by a winning coalition composed of the veto player and at least one other legislator, is ensured by (2.5).

2.2 Equilibrium Analysis

Proving existence of a symmetric MPE of this dynamic game, and characterizing it, constitutes a challenging problem due to the cardinality of the state space. Thus, I propose natural conditions on strategies, and show that these conditions define an equilibrium. The first condition is that equilibrium proposals involve *minimal winning coalitions* (Riker 1962), such that at most one of the two non-veto players receives a positive fraction of the dollar in each period. Second, the proposer proposes the *acceptable* allocation—that is, an allocation in the win set of the status quo, $\mathbf{x} \in W(\mathbf{s})$—that maximizes his current share of the dollar. Finally, I prove that these strategies, and the associated continuation values, are part of a symmetric MPE that satisfies conditions (2.4) and (2.5).

The remainder of this section describes the dynamics of this equilibrium and explores the mechanisms behind the results. To build intuition, I start from the case where legislators are impatient, $\delta = 0$, and only care about their current allocation, and then move to the general case with patient legislators, $\delta \in (0, 1)$. In both cases, the equilibrium exhibits the two features mentioned above, and legislators' patience changes only the set of allocations they prefer to the status quo.

2.2.1 Impatient Legislators

When legislators are impatient, they value only current allocations. Then, the expected utility agent i derives from an allocation $\mathbf{x}^t \in \Delta$ is:

$$U_i(\mathbf{x}^t) = x_i^t + \delta v_i(\mathbf{x}^t) = x_i^t$$

Therefore, regardless of the other agents' proposal and voting strategies, it is optimal for legislator i to accept any proposal that allocates to him at least as much as the status quo, and to reject everything else.

$$A_i(\mathbf{s}|\delta = 0) = \{\mathbf{x} \in \Delta | x_i \geq s_i\}$$

Confronted with these acceptance sets, the proposer will propose the allocation that gives him the highest share of the dollar among all those that are supported by the veto player, and at least one

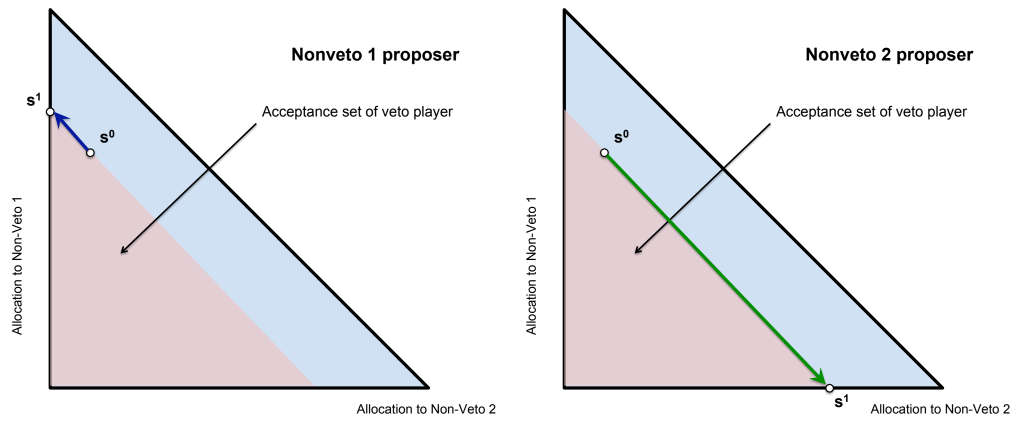


Figure 2.2: Non-veto players' equilibrium proposal strategies for state $\mathbf{s}^0$ and $\delta = 0$

non-veto player.

Figures 2(a) and 2(b) show the acceptance set of the veto player, and the optimal proposal strategy of, respectively, non-veto player 1 and non-veto player 2, when the status quo policy is $\mathbf{s}^0$. Each non-veto proposer simply makes the veto player indifferent between the status quo and his proposal[7] and assigns the remainder to himself, disenfranchising the other non-veto player, whose no vote cannot stop passage.

On the other hand, when the veto player is the proposer, he needs to secure a yes vote from one non-veto player to change the policy. He will, thus, build a coalition with the *poorer* non-veto player—the non-veto player who receives the least in the status quo—giving him as much as he is granted by the status quo. An impatient non-veto player will accept this proposal. This equilibrium proposal strategy is depicted in the left-hand panel of Figure 3. When he is not the proposer, the veto player will oppose any reduction to his allocation. Moreover, whenever he proposes, he will be able to increase his share by exactly the amount held by the non-veto player who receives the

[7]The veto player's indifference curve is defined by the diagonal with slope -1.

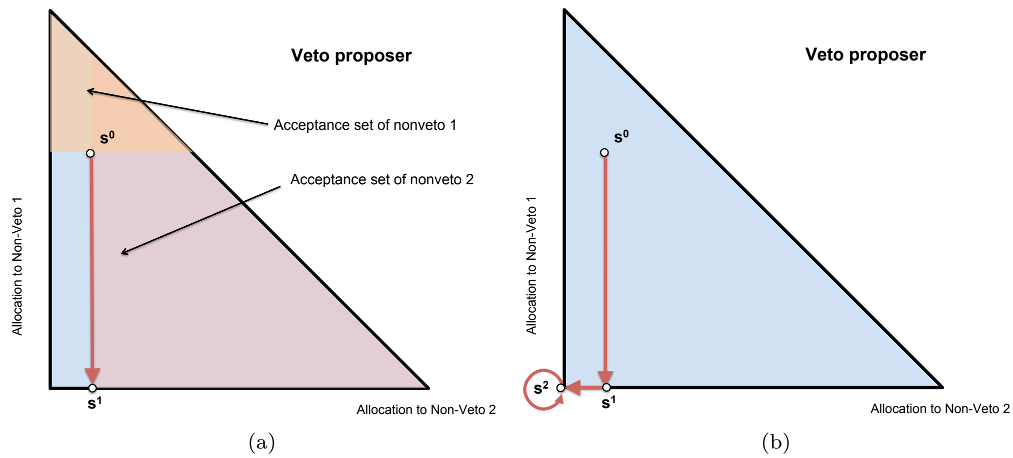


Figure 2.3: Veto player's equilibrium proposal strategy for state $\mathbf{s}^0$ and $\delta = 0$

most in the status quo—the *richer* non-veto player. Given these simple strategies, the equilibrium of the game with $\delta = 0$ has two important features. First, the allocation to the veto player displays a *ratchet effect*: it can only stay constant or increase. Second, the veto player is able to steer the status quo policy to his ideal point in at most two proposals, as he can pass any $\mathbf{x} \in \Delta$ when the poorer non-veto player has zero. Thus, as the veto player can oppose all subsequent changes, he will get the whole dollar in all subsequent periods. The right-hand panel of Figure 3 depicts these transitions when the initial status quo is $\mathbf{s}^0$ and the veto player is randomly assigned to be the proposer in the first two periods.

2.2.2 Patient Legislators

In the more general case, where legislators care about future outcomes, similar results hold. In particular, equilibrium proposals still involve minimal winning coalitions, and the proposer still picks the acceptable allocation that maximizes his current share. However, the acceptance sets of all legislators are now different, and the set of allocations each agent (weakly) prefers to the status quo policy changes with the discount factor, as legislators take into account the impact of the current allocation on future rounds. Not surprisingly, this has important consequences for the dynamics of the game. In the remainder of this section, I first analyze the case when the proposer is the veto player, and then the case when the proposer is a non-veto player.

To help with the exposition, partition the space of possible divisions of the dollar into two subsets, $\overline{\Delta}$, and $\Delta \backslash \overline{\Delta}$. Define $\overline{\Delta} \subset \Delta$ as the set of states $\mathbf{x} \in \Delta$ in which at least one non-veto legislator gets zero:

$$\overline{\Delta} = \{\mathbf{x} \in \Delta \,|x_i = 0 \text{ for some } i = \{1, 2\}\}$$

Note that, if all proposals on the equilibrium path entail minimal winning coalitions, then $\overline{\Delta}$ is an absorbing set, and it is reached in at most one period from any initial status quo allocation. Moreover, define the *demand* of legislator i as the minimum amount he requires to accept a proposal $\mathbf{x} \in \overline{\Delta}$.

Definition 2.2.1. *For a symmetric MPE, non-veto legislator j's demand when the state is $\mathbf{s}$ is the minimum amount $d_j(\mathbf{s}) \in [0, 1]$ such that for a proposal $\mathbf{x} \in \overline{\Delta}$ with $x_j = d_j(\mathbf{s})$, $x_v = 1 - d_j(\mathbf{s})$, we have $U_j(\mathbf{x}) \geq U_j(\mathbf{s})$. Similarly, veto legislator v's demand when the state is $\mathbf{s}$ is the minimum amount $d_v(\mathbf{s}) \in [0, 1]$ such that for a proposal $\mathbf{x} \in \overline{\Delta}$ with $x_v = d_v(\mathbf{s})$, $x_j = 1 - d_v(\mathbf{s})$, for $j = 1, 2$, we have $U_v(\mathbf{x}) \geq U_v(\mathbf{s})$.*

Non-Veto Proposer When a non-veto player is proposing, he needs to secure the vote of the veto player in order to change the current status quo. As a consequence, a proposal that results in a minimal winning coalition assigns a positive share only to the proposer and, if necessary, to the veto player. If the non-veto proposer wants to maximize his current share of the dollar, he will propose the veto player's demand to the veto player, and assign the remainder of the dollar to himself. Therefore, to characterize the equilibrium proposal strategies of a non-veto player, we need to identify the acceptance set of the veto player.

A patient veto player is not indifferent between all states in which he receives the same allocation, and might be better off with allocations that reduce his current share when these decrease his future coalition building costs. This occurs because the future status quo policy affects the future leverage the veto player has when he is the proposer. In this event, he needs to secure the vote of just one non-veto player, and he will, thus, build a coalition with the non-veto player who demands the least, and extract the remainder. As shown below, the demand of each non-veto player is a positive function of what he gets if the policy is unchanged and, therefore, a veto player's coalition building costs when the status quo is $\mathbf{s}$ are a positive function of $\min\{s_1, s_2\}$.

Thus, a veto player prefers an allocation $\mathbf{s}'$ where he gets s'_v and $\min\{s'_1, s'_2\} = s'_{nv}$ to an alternative allocation $\mathbf{s}''$ with $s''_v = s'_v$ but $\min\{s''_1, s''_2\} = s''_{nv} > s'_{nv}$. If the veto player is recognized in the following period, he will be able to increase his share more in the state $\mathbf{s}'$ than in $\mathbf{s}''$.

Figure 4 depicts the acceptance set of a patient veto player for two different values of $\delta > 0$. While an impatient veto player never supports an allocation that reduces his share, a patient one is willing to move from an interior allocation where he gets a higher share, to an allocation towards the edges of the simplex where both he and one non-veto player have a smaller share. In Appendix A, I characterize the amount the veto player demands to accept a proposal that brings the status quo into $\overline{\Delta}$—where one non-veto player gets nothing—as:

$$d_v \quad = \max\{s_v - \tfrac{\delta}{3-2\delta}\underline{s_{nv}}, 0\} \tag{2.6}$$

where $\underline{s_{nv}}$ is the allocation of the poorer non-veto player in the status quo. The reduction accepted by the veto player increases with his discount factor δ and the share to the poorer non-veto player

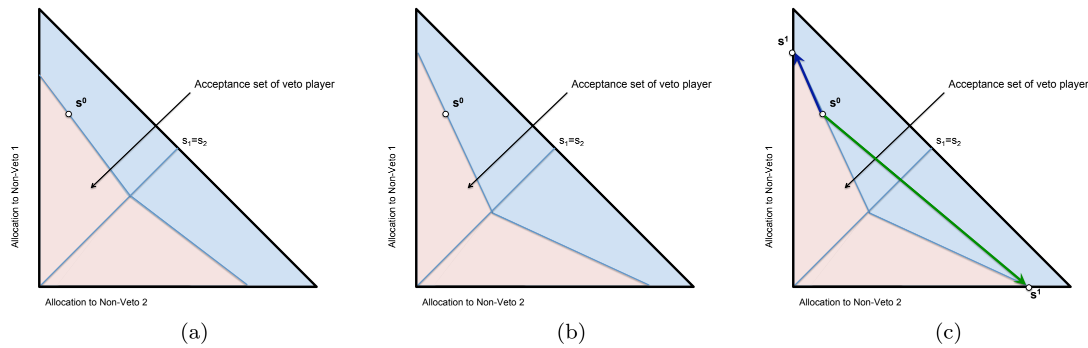


Figure 2.4: Veto's acceptance set and non-veto's proposal strategies for state $\mathbf{s}^0$: (a) $A_v(\mathbf{s}^0)$ when $\delta = \delta_1 > 0$; (b) $A_v(\mathbf{s}^0)$ $\delta = \delta_2 > \delta_1$, (c) equilibrium proposal of non-veto 1 (blue arrow) and non-veto 2 (green arrow)

$\underline{s_{nv}}$. An impatient veto player does not accept any new division of the dollar that gives him less than the status quo. The same is true for a patient veto player when the status quo is in $\overline{\Delta}$ and, thus, $\underline{s_{nv}} = 0$. Note also that the reduction a veto player is willing to accept could be more than what he has in the status quo, in which case his demand is bounded below by 0.

Having identified the acceptance set of the veto player, the non-veto proposer will thus propose the point in the acceptance set of the veto player that maximizes the proposer's stage utility. These are depicted in the right-most panel of Figure 4. A non-veto proposer will completely expropriate the other non-veto player, give the veto player his demand, and allocate the remainder to himself. When the state is in $\overline{\Delta}$, the non-veto proposer can only get $1 - s_v$, but when the state is in $\Delta \backslash \overline{\Delta}$ he can extract an higher amount, namely $1 - d_v$.

Veto Proposer A similar analysis holds for the veto proposer. As mentioned above, when the veto player desires to pass a proposal with a minimal winning coalition, he is not bound to include any specific legislator. Thus, he selects the legislator who accepts the highest increase to the veto player's share—that is, the legislator with the lowest demand—as his coalition partner. With impatient legislators, this is the poorer non-veto player, who accepts any allocation that assigns

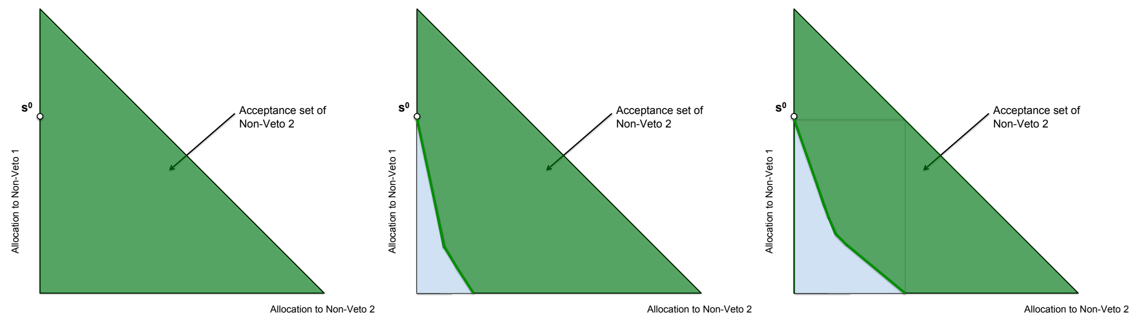


Figure 2.5: Non-veto 2's acceptance sets for state $\mathbf{s}^0$ where $s_1 > s_2$: (a) $A_2(\mathbf{s}^0)$ when $\delta = 0$, (b) $A_2(\mathbf{s}^0)$ when $\delta = \delta_1 > 0$; (c) $A_2(\mathbf{s}^0)$ $\delta = \delta_2 > \delta_1$

him a share greater than or equal to his share in the status quo, regardless of the distribution of the dollar among the other players. However, a patient non-veto player evaluates the impact of the current proposal on his future bargaining power.

The bargaining power of a non-veto player decreases with the share held by the veto player in the status quo, s_v. A patient non-veto player values both what he has and the allocation to the veto player, and prefers an allocation $\mathbf{s}' \in \overline{\Delta}$ where he gets $s'_j = 0$ and the veto gets s'_v to an alternative allocation $\mathbf{s}'' \in \overline{\Delta}$ with $s''_j = s'_j$ but $s''_v > s'_v$. The difference between these allocations arises when he is recognized in $t+1$, as he will gain the support of the veto player only for proposals that give him no more than $1 - s_v$. Figure 5 depicts the acceptance set of the poorer non-veto player for a state $\mathbf{s}^0 \in \overline{\Delta}$ and three increasing values of the discount factor.

The veto player's coalition partner now demands a *premium* to vote in favor of an allocation that increases the veto player's share. In other words, the veto player has to compensate his coalition partner with a short term gain in stage utility for the long term loss in future bargaining power.

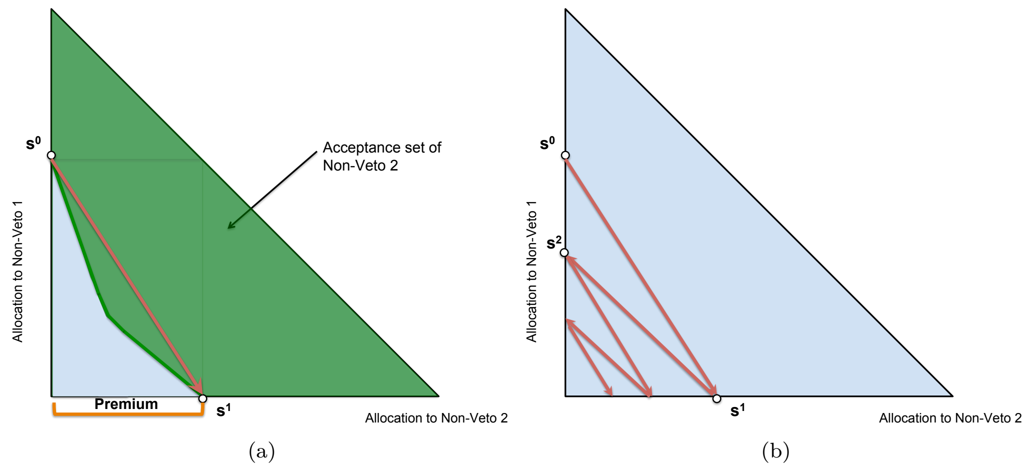


Figure 2.6: Veto's equilibrium proposal strategy for state $\mathbf{s}^0$ and $\delta > 0$

Appendix A shows that the demand of the poorer non-veto player for states $\mathbf{s} \in \overline{\Delta}$ is:

$$d_{nv} = \tfrac{\delta}{3-2\delta}\overline{s_{nv}} \tag{2.7}$$

where $\overline{s_{nv}}$ is the allocation to the richer non-veto player in the status quo. Some properties of d_{nv} are worth noting. First, d_{nv} is smaller than $\overline{s_{nv}}$ for any $\delta \in [0, 1)$. This means that, whenever agents are not perfectly patient ($\delta = 1$), the veto proposer can increase his share, as he can assign himself $1 - d_{nv} > s_v = 1 - \overline{s_{nv}}$. Second, the premium paid by the veto player to his coalition partner is monotonically increasing, and convex, in δ and linearly increasing in $\overline{s_{nv}}$: d_{nv} converges to $\overline{s_{nv}}$ as δ converges to 1, and d_{nv} converges to 0 as δ converges to 0. The fact that the premium is always positive is crucial for the long term dynamics of the game. In particular, this implies that the ratchet effect described above still functions, albeit at a slower rate such that the convergence to the veto player's ideal point happens asymptotically. Figure 6(b) shows how the state would evolve when the veto player always proposes.

One additional equilibrium difference between patient and impatient legislators is that the veto player mixes between coalition partners for some states in the interior of the simplex when the allocations to the two non-veto players are close. This is necessary to guarantee that the proposer's choice of a partner is a best response to what they demand. If the veto player always picked the poorer non-veto player as coalition partner, the poorer player would become the most expensive coalition partner. To see why, note that the demand of a legislator depends both on the current allocation and on the continuation value of the status quo policy. Under pure proposal strategies, the richer non-veto player is sure to be excluded from any future coalition and, when his allocation in the status quo is not much different than the allocation of the poorer non-veto player, this lower continuation value makes him less demanding. In this case, it would not be optimal for the veto player to always propose to the poorer non-veto player.[8]

2.2.3 Results

Proposition 1 provides a summary of the discussion above:

Proposition 1. *For any $\delta \in [0, 1)$ and any initial division of the dollar, $\mathbf{s}^0 \in \Delta$, there exists a symmetric MPE that induces a Markov process over outcomes such that:*

- *For any state $\mathbf{s} \in \Delta \backslash \overline{\Delta}$ there is probability 1 of transition to $\overline{\Delta}$.*
- *$\overline{\Delta}$ is an absorbing set.*
- *All proposals give a positive allocation at most to a minimal winning coalition.*
- *For some $\mathbf{s} \in \Delta \backslash \overline{\Delta}$, the veto proposer mixes between possible coalition partners that have*

[8] Mixed proposal strategies are a common feature of stationary subgame perfect equilibria in models of legislative bargaining à la Baron and Ferejohn (1989), and in Markov perfect equilibria of dynamic legislative bargaining models, for the same reason discussed above. See, for example, Banks and Duggan (2000, 2006), Kalandrakis (2004, 2009), and the discussion in Duggan (2011).

positive and nearly equal allocation under the status quo. For the remaining $\mathbf{s} \in \Delta$*, the veto proposer proposes* d_{nv} *to the poorer non-veto player.*

- *For all* $\mathbf{s} \in \Delta$*, the non-veto proposers proposes* d_v *to the veto player.*

- *For all* $\mathbf{s} \in \overline{\Delta}$*,* $d_v = s_v$ *and* $d_{nv} \geq s_{nv}$*.*

Figure 7 explores the states for which mixing occurs in equilibrium. In regions C and D of Figure 7 the veto player mixes between coalition partners. These regions evolve from left to right as the discount factor grows. In regions B and C of Figure 7 the veto player is willing to accept nothing. Note that mixing occurs when the non-veto players have nearly equal allocations and that the set of status quo policies where mixing occurs grows with δ. This happens because the weight players put in the probability of inclusion in future coalitions diminishes with δ. For $\delta = 0$ coalition building costs are solely determined by status quo allocations, and, thus, there are pure strategy proposals. Regions B and C shrink as δ decreases as well: the lower the discount factor, the lower the benefit the veto player receives from reducing future coalition building costs. For $\delta = 0$, the veto player never accepts anything less than what the status quo grants him, s_v. For status quo allocations in region A of Figure 7, the veto player always includes the poorer non-veto player in his coalition and he always receives a positive allocation when he is not proposing.[9]

The crucial step in the proof of Proposition 1 is verifying the optimality of proposal strategies. While Appendix A contains the details, here I sketch the key passages of the proof. Define the demand of agent i as the amount that makes i indifferent between the status quo $\mathbf{s}$ and a new division $\mathbf{z} \in \overline{\Delta}$, as before. The proof then proceeds in three steps. First, I prove that, for each agent i, $i = v, 1, 2$, $U_i(\mathbf{x})$ is continuous and increasing in x_i for all $\mathbf{x} \in \overline{\Delta}$. This proves that—among

[9] In Appendix A, I give the exact statement of the equilibrium proposal and voting strategies for each region of the simplex, and show that these strategies and the associated value functions constitute part of a symmetric MPE.

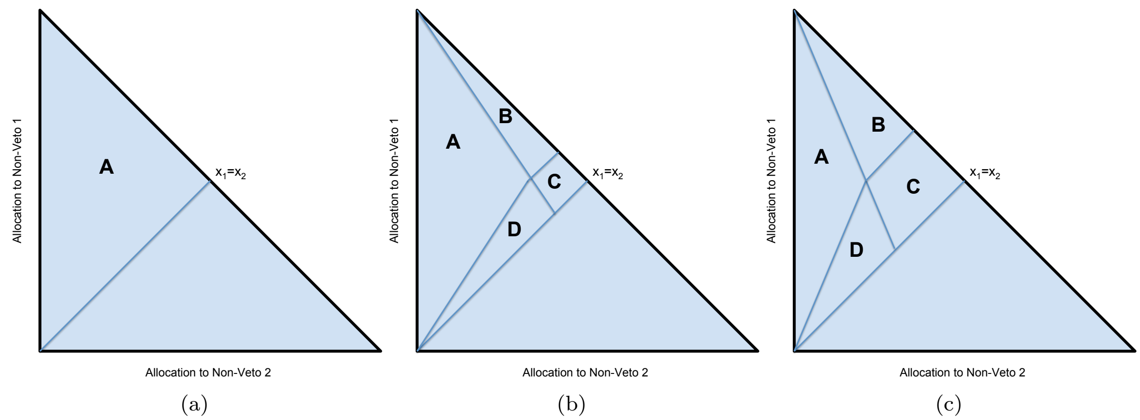


Figure 2.7: Partition of Δ into regions with different equilibrium strategies for allocations where $s_1 \geq s_2$: (a) δ=0; (b) $\delta = \delta_1 > 0$; (c) $\delta = \delta_2 > \delta_1$. In A and B veto proposer builds a mwc with non-veto 2; in C and D veto proposer mixes between coalition partners; in B, and C veto gets 0 when he is not proposing

acceptable allocations in $\overline{\Delta}$— the proposer prefers the one that gives him the highest share of the dollar. Second, I show that the demand of the poorer non-veto player is (weakly) smaller than the demand of the richer non-veto player for any $\mathbf{s} \in \Delta$. This shows that the veto proposer never has an incentive to propose only to the richer non-veto player. Third, I show that the sum of the demands of the veto player and any non-veto player is less than or equal to one for any status quo allocation in Δ. This means that there always exists an acceptable allocation in $\overline{\Delta}$ that guarantees the proposer at least his demand or more. This, together with the monotonicity in the first step, proves that no feasible allocation $\mathbf{x} \in \Delta \backslash \overline{\Delta}$ gives the proposer a higher $U_i(\mathbf{x})$ than his preferred allocation in $\overline{\Delta}$.

I can now state the main result of the paper:

Proposition 2. *There exists a symmetric MPE in which, irrespective of the discount factor and the initial division of the dollar, the status quo policy eventually gets arbitrarily close to the veto player's ideal point, that is,* $\forall\ \varepsilon > 0$*, there exists* T *such that* $\forall\ t \geq T$ *the veto player's allocation in the status quo is greater than or equal to* $1 - \varepsilon$*.*

Proof. The result derives from the features of the MPE characterized in the proof of Proposition 1. In this MPE, once we reach allocations in the absorbing set $\overline{\Delta}$, which happens after at most one period, the veto player is able to increase his share whenever he has the power to propose, and keeps a constant share when not proposing. For any ε and any starting allocation $\mathbf{s}^0$, there exists a number of proposals by the veto player—which depends on δ—such that the veto player's allocation in the status quo will be at least $1 - \varepsilon$ for all subsequent periods. Let this number of proposals be $n^*(\varepsilon, \delta, \mathbf{s}^0)$. Since each player has a positive probability of proposing in each period, the probability that in infinitely many periods the veto player proposes less than $n^*(\varepsilon, \delta, \mathbf{s}^0)$ is zero. □

Proposition 3 addresses the speed of convergence to complete appropriation of the dollar by the veto player.

Proposition 3. *In the symmetric MPE characterized in the proof of Proposition 1, if legislators are impatient, $\delta = 0$, it takes at most two rounds of proposals by the veto player to converge to the irreducible absorbing state where $s_v = 1$. If legislators are patient, $\delta \in (0, 1)$, convergence to this absorbing state does not happen in a finite number of bargaining periods, and the higher the discount factor the slower the convergence.*

Proof. This result follows directly from the equilibrium demand of the poorer non-veto player in the absorbing set $\overline{\Delta}$, $d_{nv}(\mathbf{s}, \delta) = \frac{\delta}{3-2\delta}\overline{s_{nv}}$. When $\delta = 0$, this demand is zero. This means that, when the status quo is in $\overline{\Delta}$—a set that is reached in at most one period—the poorer non-veto supports any proposal by the veto player. The veto player can thus pass his ideal outcome as soon $\mathbf{s} \in \overline{\Delta}$ and he proposes. On the other hand, when $\delta \in (0, 1)$, this is not possible, and the poorer non-veto player always demands a positive share of the dollar to support any allocation that makes the veto player richer. The convergence in this case is only asymptotic as the non-veto player's demand is always positive as long as the allocation to the richer non-veto is positive, that is, as long as the poorer veto player does not have the whole dollar in the status quo.[10] □

Finally, I prove that the equilibrium in Proposition 1 is well behaved, in the sense that proposal strategies are weakly continuous in the status quo, **s**.

Proposition 4. *The continuation value functions, V_i, and the expected utility functions, U_i, induced by the equilibrium in Proposition 1 are continuous.*

In Appendix A, I show that in equilibrium a small change in the status quo implies a small change in proposal strategies and, by extension, to the equilibrium transition probabilities. An immediate implication of the continuity of transition probabilities is the fact that continuation functions and expected utility are continuous.

2.3 Robustness and Extensions

According to the main result, presented in the previous section, the veto player is eventually able to steer the status quo policy arbitrarily close to his ideal point, and fully appropriate all the resources. In this section, I explore three institutional measures that could, in principle, reduce the leverage of the veto player and promote more equitable outcomes: reducing the recognition probability of the veto player, expanding the committee by increasing the number of veto players, and randomly re-assigning veto power in each period. While the first institutional arrangement decreases the agenda setting power of the veto player, the other two introduce competition in the use of veto power.

[10]Notice that when the initial division of the dollar—which is assumed to be exogenous—assigns the whole dollar to the veto player, then the status quo will never be changed and the veto player gets the whole dollar in every period.

2.3.1 Heterogeneous Recognition Probabilities

The previous section assumed that the probabilities of being recognized as proposer are symmetric and history invariant. However, veto players may be outsiders who have lesser ability to set the agenda. For example, the U.S. President has no formal power to propose new legislation and, even if he is able to influence the agenda through like-minded representatives in Congress, his proposal power is lower than any individual member of Congress. In other settings, the veto player has a privileged position to set the agenda, for example, committee chairs in the U.S. Congress. In this section, I relax the assumption of symmetric recognition probabilities, and find that the veto player is still able to eventually appropriate all resources, as long as his recognition probability is positive, and that convergence to this outcome is slower the lower is this probability.

In particular, denote by p_v the probability the veto player is recognized as the proposer in each period, with $p_{nv} = \frac{1-p_v}{2}$ being the probability a non-veto player is recognized. Proposition 5 shows that there exists a MPE equilibrium of this dynamic game that has the same features as the one characterized in the previous section: all proposals entail positive distribution to only a minimal winning coalition and the status quo allocation converges to the ideal point of the veto player as long as $p_v > 0$.

Proposition 5. *With different recognition probabilities of veto and non-veto players, there exists a symmetric MPE in which, irrespective of the initial division of the dollar and the discount factor, the status quo policy eventually gets arbitrarily close to the veto player's ideal point, as long as $p_v > 0$. With the exception of at most the first period, the convergence to the absorbing state is faster the higher is the proposal power of the veto player.*

As in the case with even recognition probabilities, this result hinges on the fact that, once an allocation is in the absorbing set $\overline{\Delta}$—the set of allocations where at least one non-veto gets zero—the veto player is able to increase his share whenever he proposes.

The proposal power of the veto player influences the speed of convergence to his ideal outcome both directly and indirectly. The direct effect is given by the change in the frequency at which the veto player can increase his allocation—which happens only when he proposes.

The indirect effect is given by the change in the amount the veto player can extract from the non-veto players when he proposes. The probability of recognition of the veto player affects the continuation value of the status quo policy for all legislators, and thus it affects how much they demand to support a policy change. In particular, as p_v increases, non-veto players are less likely to be recognized at time $t+1$ and, thus, they are less concerned about their future coalition building costs. This reduces the premium the poorer non-veto player demands from the veto player to support an allocation that increases his share. The proof of Proposition 5 shows that, when $\overline{\Delta}$ is reached, the demand of the poorer non-veto player is

$$d_{nv} = \frac{\delta(1-p_v)}{2-\delta(1+p_v)}\overline{s_{nv}}$$

where $\overline{s_{nv}}$ is the allocation to the richer non-veto player in the status quo. This demand is strictly greater than $\underline{s_{nv}} = 0$ as long as $\overline{s_{nv}} > 0$, $\delta > 0$, and $p_v < 1$. Under these conditions, the poorer non-veto player demands a premium, $d_{nv} \geq \underline{s_{nv}} = 0$, from the veto player. This premium is monotonically decreasing in p_v. Thus, with a higher p_v, the veto player is more likely to increase his share in each period, and he can also extract more from the non-veto players when he is the proposer.

When the initial allocation is in the interior of the simplex, and the proposer in the first period is a non-veto player, p_v has a second, indirect, effect on the demand of the veto player. The continuation value of moving to an allocation in $\overline{\Delta}$ for the veto player increases with p_v as he is more likely to be the proposer in $t+1$ and enjoy the reduction in coalition building costs. This increases his willingness to give up a fraction of his share in order to move into the absorbing set, when a non-veto player is proposing in the initial period.

The proof of Proposition 5 shows that the demand of the veto player is:

$$d_v = \max\{s_v - \frac{2p_v\delta}{2-\delta(1+p_v)}\underline{s_{nv}}, 0\}$$

where s_v is the allocation of the veto player in the status quo, and $\underline{s_{nv}}$ is the allocation of the poorer non-veto player in the status quo. This demand is monotonically decreasing in p_v. That is, the reduction the veto player is willing to accept to move the status quo from $\mathbf{s} \in \Delta \backslash \overline{\Delta}$ into $\overline{\Delta}$ is increasing in his proposal power.

2.3.2 Multiple Veto Players

Next, I consider a committee with more than one veto player and show that the presence of multiple veto players with opposing preferences does not prevent the complete expropriation of the resources initially allocated to non-veto players. However, the presence of other legislators with veto power reduces the amount a veto proposer can extract.

In particular, I study a setting with two veto players and two non-veto players where recognition probabilities are identical and a proposal passes only if it is approved by the two veto players and at least one non-veto player.[11] Proposition 6 shows that this dynamic game has a symmetric MPE—where symmetry applies to legislators of the same type, veto or non-veto—in which the two veto players eventually extract all the surplus regardless of the initial allocation and the discount factor.

[11]Ideally, I would answer the question above studying a game with an arbitrary number of legislators n and veto players $k \leq n$. However, as the dimensionality of the state space increases analytical tractability is quickly lost. Adding one veto player allows me to gain a valuable insight on the issue of multiple veto players but preserves the analytical tractability of the model, even if the set of possible legislative outcomes passes from $\mathbb{R}^2$ to $\mathbb{R}^3$.

Proposition 6. *In the game with two veto players, there exists a symmetric MPE in which, irrespective of the discount factor and the initial division of the dollar, the sum of the allocations to the two veto players eventually gets arbitrarily close to one. In the absorbing state, the share to each veto player is strictly larger than his starting share, unless* $\mathbf{s}^0$ *is an absorbing state.*

As with only one veto player, the result hinges on the fact that a veto proposer can pass an allocation that increases his allocation at the expenses of the richer non-veto player. However, there is an important difference: the veto proposer now has to allocate to both the other veto player and the poorer non-veto player more than what they receive in the status quo, that is, he has to pay both of them a premium in order to increase his allocation.

The other legislators demand this premium because a higher current allocation to one veto player increases his future demand and, thus, decreases the extent to which other legislators can exploit the power to propose in $t+1$, as with only one veto player. The proposing veto player, who builds a minimal winning coalition with the other veto player and the poorer non-veto player, has to share part of the amount he expropriates from the richer non-veto player with his coalition partners, in order to offset this loss and gain their vote.[12] Since each veto player will always be part of a minimal winning coalition, both veto players enjoy a ratchet effect in their allocations, regardless of the identity of the proposer along the equilibrium path.

In the proof of Proposition 6, I show that, once allocations are in $\overline{\Delta}$, the demand of the veto player who is not proposing, denoted by d_v, and the demand of the poorer non-veto player, denoted

[12] Even if I do not formally address the framework with heterogeneous probabilities of recognition, note that, by the same logic explored in Section 4.1, this result would still hold if the two veto players had different probabilities of recognition, as long as both probabilities were strictly positive.

by d_{nv}, are as follows:

$$\begin{aligned} d_v &= s_v + \frac{4\delta(1-\delta)}{16-16\delta+3\delta^2}\overline{s_{nv}} \\ d_{nv} &= \frac{\delta}{4-3\delta}\overline{s_{nv}} \end{aligned}$$

where $\overline{s_{nv}}$ is the allocation to the richer non-veto in the status quo. Some properties of these two demands are worth noting. First, both the non-proposing veto player and the poorer non-veto player asks for a premium, that is, $d_v > s_v$ and $d_{nv} > \underline{s_{nv}} = 0$, as long as $\delta > 0$ and $\overline{s_{nv}} > 0$. Second, out of the two veto players, the one that is proposing will get a greater share of the resources expropriated from the richer non-veto player. Finally, as for the case with one veto player, the premium demanded by coalition partners is increasing in legislators' patience and in the fraction of the dollar in the hands of non-veto players.

2.3.3 Rotating Veto Power

In the basic setting, as well as in the extensions already discussed, veto power is permanently assigned to one or more legislators. This section considers an alternative institutional arrangement where veto power is randomly assigned to a legislator in each period, in a similar—but independent—way as proposal power. In this case, the policy converges in finite time to an absorbing set where, in each period, either the proposer or the veto player get the entire dollar.

This setting is a significant departure from the basic setup, and the existence proof from Proposition 1 does not hold. In this section, I establish a Markov equilibrium of the dynamic game with random veto power, when the space of possible agreements is restricted to minimal-winning coalitions, $\mathbf{x} \in \Delta_2$, that is, the edges of the simplex, where at most two legislators have a positive share.[13] The restriction to minimal winning coalitions simplifies considerably the analysis and it is a sensible conjecture on the properties of equilibria of the unrestricted game, given existing results for similar dynamic bargaining games.[14]

In Appendix A, I prove that a MPE of the restricted game with rotating veto power exists.[15] This equilibrium is summarized by two properties. First, for every status quo, optimal proposals coincide with the feasible allocations that maximize the proposer's share of the surplus. Second, players with zero in the status quo allocation are willing to accept proposals that also allocate them zero, regardless of the identity of the proposer. This second feature is in line with the voting strategies of impatient agents and, contrary to what happens in the setting with permanent veto power, it is preserved when agents are patient. In an equilibrium with these features, the status quo policy converges to an absorbing set where the proposer can allocate the whole dollar to himself, unless he is the only legislator who gets nothing in the status quo, or unless the veto player is another legislator with a positive share in the status quo.

Proposition 7. *In the game with rotating veto power and feasible allocations* $\mathbf{s} \in \Delta_2$*, there exists a symmetric MPE in which, irrespective of the discount factor and the initial division of the dollar, eventually either the proposer or the veto player extract the whole dollar in all periods.*

This result is very intuitive, in light of the features discussed above. With the restriction to minimal-winning coalitions, we have $s_i = 0$ for some i, possibly different across periods, in all periods. Let Δ_1 be the set of allocations where one legislator gets everything, that is, the vertices of the simplex. Now, first consider allocations in which exactly two legislators have a positive share

[13] Note that Δ_2 does not coincide with the partition $\overline{\Delta}$ defined in Section 3, as $\overline{\Delta}$ does not include those allocations where the two non-veto players have a positive allocation and the veto player has zero.

[14] Note that this is is a restriction on the game, and not simply an equilibrium refinement. For dynamic games where minimal winning coalitions arise in equilibrium, see, among others, Kalandrakis (2004, 2009), and Battaglini and Coate (2007, 2008), apart from the results discussed in this paper.

[15] Note that, if minimal winning coalition proposals are optimal also in the unrestricted game where all $\mathbf{s} \in \Delta$ are feasible, this MPE coincides with a MPE of the unrestricted game, at least for all periods $t > 1$.

of the dollar, i.e. $\mathbf{s} \in \Delta_2 \backslash \Delta_1$. If, in equilibrium legislator i with $s_i = 0$ does not object to new divisions of the dollar $\mathbf{z}$ with $z_i = 0$, we have three possibilities:

1. if $j \neq i$ is recognized in period $t+1$ and the veto power is in the hands of either i or j, a coalition of i and j vote for a proposal that allocates the whole dollar to j;
2. if $j \neq i$ is recognized in period $t+1$ and the veto power is in the hands of the third legislator $l \neq j \neq i$, the proposer cannot extract the whole dollar because l will object to it;
3. if i is the proposer, regardless of the identity of the veto player, he will propose the most favorable allocation in Δ_2, as he is not able to allocate the whole dollar to himself.[16]

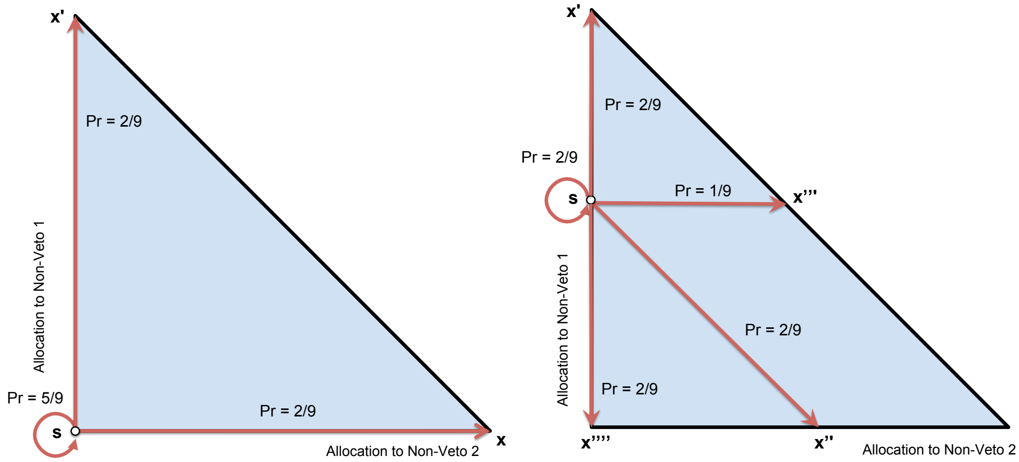
Once we transition to an allocation in Δ_1, where one legislator gets everything, the implemented policy will always be in Δ_1, and it will either be unchanged or move to another vertex of the simplex. Call i the legislator with the whole dollar in $\mathbf{s} \in \Delta_1$. If i has the proposal or veto power, which happens with probability 5/9, the policy does not change. If i has neither power, then the proposer can extract the whole dollar, and will do so. Convergence to the equilibrium absorbing set of policy outcomes is fast, with a maximum expected time before absorption equal to one and a half periods.[17] This implies that—when legislators are patient—permanent veto power promotes less extreme outcomes than rotating veto power. To see why remember that, with permanent veto power, the convergence to the veto player's ideal outcome happens in infinitely many periods, and—along the equilibrium path—the veto has to share the resources with one non-veto player.[18]

Figure 8 represents the transition probabilities for allocations $\mathbf{s}$ in the absorbing set Δ_1, and in the complementary set of minimal winning coalition allocation. To understand the transition

[16]Note that, when the other two legislators have nearly equal allocations, legislator i with $s_i = 0$ mixes between coalition partners. See the proof in Appendix A for details.

[17]Absorption is not deterministic as it depends on the identity of the proposer recognized in each period.

[18]Note that the equilibrium of the game with rotating veto power is similar to the equilibrium of the game without veto power studied by Kalandrakis (2004): it has the same absorbing set—the vertices of the simplex—and similar dynamics, the main difference being a greater status quo inertia with rotating veto power.



(a) (b)

Figure 2.8: Transition probabilities with temporary veto power: (a) $\mathbf{s} \in \Delta_1$; (b) $\mathbf{s} \in \Delta_2 \backslash \Delta_1$

probabilities consider that, ex ante, each legislator has a $1/3$ chance of being selected as the proposer and independent $1/3$ chance of being assigned the power to veto. This means that, ex ante, with probability $1/9$ an agent has both the power to veto and to propose, with probability $2/9$ he has the power to veto but not to propose, with probability $2/9$ he has the power to propose but not to veto, and, finally, with probability $4/9$ he has neither power.

2.4 Discussion

This paper studies the distributive consequences of veto power in a legislative bargaining game with an evolving status quo policy. As the importance of the right to block a decision crucially depends on the status quo, a static analysis cannot draw general conclusions about the effect of veto power on gridlock and policy capture by the veto player. Instead of making ad hoc assumptions on the status quo policy, I study veto power by exploring the inherently dynamic process via which the location of the current status quo is determined. I prove that there exists a Markov Perfect Equilibrium of this dynamic game such that the veto player is eventually able to extract all resources, irrespective of the discount factor, the probability of proposing, and the initial allocation of resources. This result shows that the right to veto is extremely powerful, especially if coupled with proposal power. This is true even when other legislators are patient, and take into account the loss in future bargaining power implied by making concessions to the veto player in the current period.

This paper is the first to derive theoretical predictions on the consequences of veto power in a dynamic setting. While the results certainly add to our understanding of the incentives present in real world legislatures, the setup is intentionally very simple and uses a number of specific assumptions. In the remainder of this section, I discuss some directions for further research.

Extension to General Number of Legislators. I have limited the analysis to legislatures with two non-veto players and, at most, two veto players. It would certainly be interesting to extend the asymptotic result of full appropriation by the veto player(s) to legislatures with an arbitrary number of veto and non-veto legislators. However, the existence proofs for the equilibria proposed in this paper rely on constructing the equilibrium strategies, and the associated continuation values, for any allocation of the dollar, $\mathbf{s} \in \Delta$. It is a very challenging task to extend this existence result and to characterize a Markov equilibrium with a higher number of legislators, as the dimensionality of the state space increases and tractability is quickly lost. Future research could explore the dynamics of a larger legislature using numerical methods, a solution often adopted in the literature on dynamic models with endogenous status quo (Baron and Herron 2003, Penn 2009, Battaglini and Palfrey 2012, Duggan, Kalandrakis, and Manjunath 2008).

Extension to Concave Utilities. The equilibrium I characterize exhibits proposals where at most the members of a minimal winning coalition get a positive share of the dollar. An open question is whether there exist other Markovian equilibria of this game where universal coalitions prevail. One possible avenue for future research is to relax the assumption that legislators' utilities are linear in stage payoffs. It would be interesting to assess whether the equilibrium with minimal winning coalitions is robust to concavity in legislators' utilities, and whether equilibria without minimal winning coalitions may arise when stage payoffs are sufficiently concave. Indeed, Battaglini and Palfrey (2012) have recently explored such equilibria in the context of simple majority without a veto player. Using numerical methods, they find Markov equilibria in which players share the surplus in all periods when stage preferences exhibit sufficient concavity.

Non-Markovian Equilibria. I have focused on Markov perfect equilibria where agents' strategies depend only on the status quo policy. However, this legislative game is an infinite horizon dynamic game with many subgame perfect equilibria, and the Markovian assumption of stationary strategies is very restrictive. As noted in the seminal paper of Baron and Ferejohn (1989), these bargaining games usually have other subgame perfect equilibria that can sustain more equitable outcomes through the use of history-dependent strategies, that is, punishment and rewards for past actions. Bowen and Zahran (2009) explored this avenue without a veto player. They show the existence of non-Markovian equilibria in which players share the surplus as long as the legislators are neither too patient nor too impatient. Interestingly—and related to the previous point—this alternative equilibrium does not survive when players are risk neutral. In Appendix A, I propose strategy profiles for this dynamic game such that the initial allocation is an absorbing state and, thus, there is no convergence to full appropriation by the veto player, as long as the discount factor is high enough, and the two non-veto players receive enough at the beginning of the game.

Extension to Different Policy Domains. This study analyzes a divide-the-dollar game where legislators' preferences are purely conflicting. This is a natural starting point to analyze the consequences of veto power in a dynamic setting as it lays bare the incentives at work. However, there are two important reasons to extend the policy space beyond the pure distributive case. First, many applications, and policy domains, are better modeled with a spatial setting where legislators' preferences are partially aligned. Second, the pure distributive setting leaves little room to ask whether giving a legislator the power to veto is desirable from the societal point of view as, with linear utilities, all outcomes are Pareto-efficient. The welfare consequences, and the normative implications of introducing a veto player can be better analyzed in a setting with less conflicting preferences. One interesting possibility for future research is to analyze the consequences of veto power in a dynamic setting with a unidimensional policy space, and single peaked legislators' preferences over outcomes.[19] An alternative way of exploring a setting with a lower degree of conflict could be to study a different divide-the-dollar game where the dollar can also be allocated to a public good.

Empirical Tests of Theoretical Predictions. The theory provides sharp empirical implications: the ratchet effect for the allocation of the veto player, the monotonic convergence to his ideal point, and the comparative statics on the discount factor, the recognition probabilities, the number of veto players, and the nature of the veto right (permanent vs rotating). One important goal of future research is to assess the empirical validity of these theoretical predictions, in particular with the use of laboratory experiments, which have some important advantages over field data when studying a highly structured dynamic environment such as the one in this paper (Battaglini and Palfrey 2012, Battaglini, Nunnari, and Palfrey 2012b).

[19] A similar setting is studied by Baron (1996) in the context of simple majority without veto power.

Chapter 3

The Free Rider Problem: A Dynamic Analysis

Most free rider problems have a significant dynamic component. Public goods, for example, are often durable: it takes time to accumulate them and they depreciate slowly, projecting their benefits for many years. Similarly, environmental problems depend on variables that slowly evolve over time like capital goods. In all these examples what matters for the agents in the economy is the stock of the individual contributions accumulated over time. Although there is a large literature studying free-rider problems in static environments, much less is known about dynamic environments. A number of important questions still need to be fully answered. What determines the steady states of these problems and their welfare properties? Is the free rider problem better or worse as the number of agents increases? Can we achieve efficient steady states when agents are sufficiently patient?

In this paper, we present a simple model of free riding to address these questions. In the model, n infinitely lived agents allocate their income between private consumption and contributions to a public good in every period. The public good is durable and depreciates at a rate d. We consider two scenarios. First, we study economies with reversibility, in which in every period individual investments can either be positive or negative. Second, we study an economy where the investment is irreversible, so individual investments are non-negative and the public good can only be reduced by depreciation. Although there is a significant literature that has studied free riding in economies with reversibility, in the case of irreversibility progress has been made only in specific environments that do not fit the classical description of large free rider problems.[1] To our knowledge this is the first paper that provides a comparative analysis of Markov equilibria in these environments with and without irreversibility.

We start the analysis by studying the set of equilibria in economies with reversibility. We show that there is a continuum of equilibria, each characterized by a different stable steady state. The set of equilibrium steady states has three notable features. First, it always includes in its interior the level of the public good that would be reached in equilibrium by an agent alone in autarky: the steady state in a community with n agents can be either larger or smaller than when an agent is alone. Second, the upper- and lower-bounds of the set of equilibrium steady states are, respectively, increasing and decreasing in n. This implies that as the number of agents increases, the set of equilibrium steady states expands, and the free rider problem can either improve or worsen with the rise of population. Finally, for any size of population n and any rate of depreciation, the highest (and best) steady state converges to the efficient level as the discount factor converges to one. When agents are sufficiently patient, therefore, the efficient steady state can be achieved with simple Markovian strategies. This is perhaps remarkable since we have a non-cooperative dynamic free riding game, with arbitrarily large numbers of players, and the Markov assumption rules out reward or punishment strategies that are contingent on individual actions or complicated histories, as required in folk-theorem constructions supporting cooperation in repeated games.

In an economy with irreversibility the set of equilibrium steady states is smaller, and contained

[1] A more detailed discussion of the literature is presented at the end of this section.

in the set of equilibrium steady states with reversibility. We show that as the rate of depreciation converges to zero, this set converges to a unique point corresponding to the highest equilibrium steady state that can be supported with irreversibility. An immediate implication is that, as the discount factor converges to one, *all* equilibrium steady states with irreversibility are approximately efficient if the rate of depreciation is small.

The fact that reversibility affects so much the equilibrium set may appear surprising. In a planner's solution the irreversibility constraint is irrelevant: it affects neither the steady state (that is unique), nor the convergence path.[2] The reason why irreversibility is so important in a dynamic free rider game is precisely the fact that investments are inefficiently low. The intuition is as follows. In the equilibria with reversibility, an agent holds back his/her contribution for fear that it will crowd out the contributions of other players, or even be appropriated by other agents in future periods. With irreversibility, however, at some point the equilibrium investment function with reversibility must fall so low that the irreversibility constraint is binding. Even if this happens out of the equilibrium path, this affects the entire equilibrium investment function. In states just below the point in which the constraint is binding, the agents know that the constraint will not allow the other agents to reduce the public good when it passes the threshold. These incentives induce higher investments and a higher value function, with a ripple effect on the entire investment function. This effect induces the agents to cooperate more and results in a unique (high) stable steady state when depreciation is sufficiently small.

From a purely methodological point of view, the paper develops a novel approach to characterize the Markov equilibria that can have more general applicability in the study of stochastic games

[2] On the convergent path the stock of public good is never reduced: it keeps increasing until the steady state is reached, and then it stops; the irreversibility constraint is, thus, never binding on the equilibrium path. This of course is true if the initial state g_0 is smaller than the steady state, an assumption that we will maintain throughout this paper for simplicity of exposition.

with discrete time. The idea is to construct Markov strategies that induce a *weakly* concave value function: the flat regions in the value function allow additional freedom in choosing the players' reaction functions and in sustaining the equilibrium. This approach is essential to prove existence of a Markov equilibrium in the difficult case of an economy with irreversibility.

Our paper is related to three strands of literature. First, it is related to the literature on Markov equilibria in differential public good games initiated by Fershtman and Nitzan (1991).[3] This literature has been the first to propose a framework to study dynamic free rider problems. It differs from our work in two respects. First, it focuses exclusively on the environment of linear quadratic differential games in which preferences are described by specific quadratic functions and strategies are assumed to be linear in the state variable.[4] Second, and most importantly, it restricts the analysis to the case of reversible investments. In our work we consider a standard game with discrete time and general utility functions; we also do not limit the analysis to linear strategies: this, as we will see, will be important to capture the full range of equilibrium phenomena. Finally, we propose a framework that allows the comparison of economies with reversibility and irreversibility.

The second strand of literature to which our paper is related is the research on monotone contribution games (Lockwood and Thomas 2002 and Matthews 2012).[5] These papers assume that players' individual actions can only increase over time. They differ from our work in three important ways. First, the class of games studied in these papers does not include our (standard) free-rider game. Instead, the analysis is focused to environments in which the stage games have

[3]Other significant works in this literature are Dockner and Long (1993), Wirl (1996), Rubio and Casino (2002), Itaya and Shimomura (2001), among others.

[4]Non-linear strategies are discussed in Dockner and Long (1993). Rubio and Casino (2002), however, highlight complications of considering non-linear strategies that arise in this and other related models.

[5]A number of significant papers in the monotone games literature are less directly related. These papers require additional assumptions that make their environments hard to compare to ours. Gale (2001) provides a general framework of monotone games with no discounting, and applies it to a contribution game in which agents care only about the limit contributions as $t \to \infty$. Admati and Perry (1991), Compte and Jehiel (2004) and Matthews and Marx (2000) consider environments in which the benefit of the contribution occurs at the end of the game if a threshold is reached and in which players receive either partial or no benefit from interim contributions. The first two of these papers, moreover, assume that players contribute sequentially, one at a time.

a "prisoner dilemma structure." Both papers assume that keeping the action constant (i.e.„, the most uncooperative action) is a dominant strategy for all players, independently from the level of the action or the level of other players' actions. As shown by Matthews (2012), this assumption is important for the characterization of the equilibria in these papers. In our free-rider environment agents may find it optimal (and indeed do find it optimal) to make a contribution even if the other players choose their minimal contributions.[6] Second, in our model the stock of the public good can either increase or decrease over time. This is obviously true when the investment is reversible, but it is also true with irreversibility because of depreciation. In the literature on monotone games, instead, players' individual contributions (and therefore the aggregate contributions as well) can only increase over time.[7] Third, these papers focus on subgame perfect equilibria supported by trigger strategies, while we focus on Markov equilibria. In environments in which many players interact anonymously as in many natural free rider problems, Markov equilibria seem an important benchmark to understand equilibrium behavior. As we mentioned above, our paper is the first work to study and compare Markov equilibria with and without reversibility.

The final strand of literature is the more recent research on dynamic political economy. This literature studies the Markov equilibria of dynamic accumulation games similar to the game studied here, but with different collective decision processes: free riding with the possibility of writing incomplete contracts (Harstad 2012), noncooperative bargaining (Battaglini and Coate 2007 and Battaglini, Nunnari, and Palfrey 2012b), political agency models (Besley and Persson 2011 and Besley, Ilzetzki and Persson 2011). All of these papers restrict their analysis to environments with reversibility. We are confident that insights developed in our paper on irreversible economies can help understanding public investments even in these alternative models of public decision making

[6]As standard in public good games, we assume the individual benefit for a marginal contribution converges to infinity as the stock of g converges to zero.

[7]Indeed, the game we study is not in the class of monotone games.

in future research.

3.1 The Model

Consider an economy with n agents. There are two goods: a private good x and a public good g. The level of consumption of the private good by agent i in period t is x_t^i, the level of the public good in period t is g_t. An allocation is an infinite nonnegative sequence $z = (x_\infty, g_\infty)$ where $x_\infty = (x_1^1, ..., x_1^n, ..., x_t^1, ..., x_t^n, ...)$ and $g_\infty = (g_1, ..., g_t, ...)$. We refer to $z_t = (x_t, g_t)$ as the allocation in period t. The utility U^j of agent j is a function of $z^j = (x_\infty^j, g_\infty)$, where $x_\infty^j = (x_1^j, ..., x_t^j, ...)$. We assume that U^j can be written as:

$$U^j(z^j) = \sum_{t=1}^{\infty} \delta^{t-1} \left[x_t^j + u(g_t) \right],$$

where $u(\cdot)$ is continuously twice differentiable, strictly increasing, and strictly concave on $[0, \infty)$, with $\lim_{g \to 0^+} u'(g) = \infty$ and $\lim_{g \to +\infty} u'(g) = 0$. The future is discounted at a rate δ.

There is a linear technology by which the private good can be used to produce public good, with a marginal rate of transformation $p = 1$. The private consumption good is nondurable, the public good is durable, and the stock of the public good depreciates at a rate $d \in [0, 1]$ between periods. Thus, if the level of public good at time $t - 1$ is g_{t-1} and the total investment in the public good is I_t, then the level of public good at time t will be

$$g_t = (1 - d)g_{t-1} + I_t.$$

We consider two alternative economic environments. In a *Reversible Investment Economy* (RIE) the public policy in period t is required to satisfy three feasibility conditions:

$$\begin{aligned} x_t^j &\geq 0 \ \ \forall j,\ \forall t \\ g_t &\geq 0 \ \ \forall t \\ I_t + \sum_{j=1}^{n} x_t^j &\leq W \ \ \forall t \end{aligned} \tag{3.1}$$

where W is the aggregate per period level of resources in the economy. The first two conditions guarantee that allocations are nonnegative. The third condition is simply the economy's resource constraint. In an *Irreversible Investment Economy* (IIE), the second condition is substituted with:

$$g_t \geq (1-d)g_{t-1} \ \ \forall t \tag{3.2}$$

The RIE corresponds to a situation in which I_t can be negative. The constraint that the state variable g_t is non negative in a RIE is natural when g_t is physical capital and it will maintained throughout the analysis. It should however be noted that it is not relevant for the results.

It is convenient to distinguish the state variable at t, g_{t-1}, from the policy choice g_t and to reformulate the budget condition. If we denote $y_t = (1-d)g_{t-1} + I_t$ as the new level of public good after investing I_t in the current period when the last period's level of the public good is g_{t-1}, then the public policy in period t can be represented by a vector $(y_t, x_t^1, ..., x_t^n)$. Substituting y_t, the budget balance constraint $I_t + \sum_{j=1}^{n} x_t^j \leq W$ can be rewritten as:

$$\sum_{j=1}^{n} x_t^j + [y_t - (1-d)g_{t-1}] \leq W,$$

With this notation, we must have $x_t \geq 0, y_t \geq 0$ in a RIE, and $x_t \geq 0, y_t \geq (1-d)g_{t-1}$ in a IIE.

The initial stock of public good is $g_0 \geq 0$, exogenously given. Public policies are chosen as in the classic free rider problem, modeled by a voluntary contribution game. In period t, each agent j is endowed with $w_t^j = W/n$ units of private good. We assume that each agent has full property rights over a share of the endowment (W/n) and in each period chooses on its own how to allocate its endowment between an individual contribution to the stock of public good (which is shared by all agents) and private consumption, taking as given the strategies of the other agents. The individual contribution by agent j at time t is denoted i_t^j. In a RIE, the level of individual contribution can be negative, with the constraint that $i_t^j \in [-(1-d)g_t/n, W/n]$ $\forall j$, where $i_t^j = W/n - x_t^j$ is the contribution by agent j.[8] In a IIE, an agent's contribution must satisfy $\in [0, W/n]$ $\forall j$. The total economy-wide increase in the stock of the public good in any period is then given by the sum of the agents' individual contributions.

The state variable can have alternative interpretations.

Example 1 (Public capital). It is natural to assume that g is physical public capital. In this case it may seem natural to assume that the environment is irreversible. Once a bridge is constructed, it can not be decomposed and transformed back to consumption. Similarly, a painting donated to a public museum can not typically be withdrawn. The choice of the model to adopt (reversible or irreversible) should depend on the nature of the public good. If the public good is easily divisible and can be easily appropriated (as, for example, wood and other valuable resources from a forest) an agent may choose to appropriate part of the accumulated level. When withdrawals are possible (both because allowed, or because they can not be prevented), then the model may be described as

[8] This constraint guarantees that the sum of reductions in g is never larger than the total stock of public good. The analysis is similar if we allow each player to withdraw up to $(1-d)g$ since no player finds it optimal to reduce g to zero: the marginal utility of g at zero is infinity. In this case, however, we have to assume a rationing rule in case the individuals withdraw more than $(1-d)g$. A simple rationing rule generating identical results is the following. At the beginning of each period player i can claim any amount $\omega_t^i \leq (1-d)g_{t-1}$ from the pool: if $\sum \omega_t^i \leq (1-d)g_{t-1}$, then i receives his demand ω_t^i; if $\sum \omega_t^i > (1-d)g_{t-1}$, then the public good is rationed pro quota, $\omega_t^i = (1-d)\frac{\omega_t^i}{\sum \omega_t^j} g_{t-1}$. The player can then consume x_t^i with $x_t^i \leq W/n + \omega_t^i$ and leave the rest of $W/n + \omega_t^i$ in the public good.

RIE. The ability of the community to prevent agents from "privatizing" (or stealing) the stock of public good is a technological variable that may vary case to case.

Example 2 (Pollution). Suppose the state g is the level of global warming with the convention that the larger is g, the worse is global warming. The utility of an agent now is $u(x, g) = x - c(g)$, where $c(\cdot)$ is increasing, convex and differentiable. It is natural to assume that an agent can either increase or decrease global warming by choosing a "dirty" or a "clean" technology. This environment can be modeled as before if we assume that an increase in the "greenness" of the technology costs, at the margin, a dollar's worth of current consumption. Given this, we have, as before: $g_t = (1-d)g_{t-1} - \sum_j i_t^j$, where now i_t^j stands for the individual contribution to green technology (and it can be positive or negative). In this context it is, therefore, natural to assume the economy is reversible.

Example 3 ("Social Capital" or "Fiscal Capacity") A number of recent works have highlighted the importance of a variety of forms of intangible, or semi-tangible community assets, like social capital (Putnam 2000) or fiscal capacity (Besley and Persson 2011). For the case of social capital, it may seem natural to assume that agents take actions that can be either positive or negative for capital accumulation. Moreover, because social capital is an intangible asset, we may assume it takes values in $g \in (-\infty, +\infty)$. It follows that the accumulation of social capital can probably be modeled as a reversible investment economy as described in Example 1 (where it is assumed that there is a minimum level of capital, zero) or as in Example 2 (where capital is in the real line). The case of fiscal capacity is similar, and indeed Besley and Persson (2011) assume it is reversible. There may be however cases in which even this type of social investment is not reversible, or it is partially reversible. This is probably the case when fiscal capital is embodied in institutions that can not be easily undone. In these cases, an irreversible investment economy can be a more appropriate model.

To study the properties of the dynamic free rider problem described above, we study symmetric Markov perfect equilibria, where all agents use the same strategy, and these strategies are time-independent functions of the state, g. A strategy is a pair $(x(\cdot), i(\cdot))$: where $x(g)$ is an agent's level of consumption and $i(g)$ is an agent's contribution to the stock of public good in state g. Given these strategies, by symmetry, the stock of public good in state g is $y(g) = (1-d)g + ni(g)$. For the remainder of the paper we refer to $y(g)$ as the *investment function*. Associated with any Markov perfect equilibrium of the game is a value function, $v(g)$, which specifies the expected discounted future payoff to an agent when the state is g. An equilibrium is *continuous* if the investment function, $y(g)$, and the value function, $v(g)$, are both continuous in g. In the remaining of the paper we will focus on continuous equilibria. In the following we refer to equilibria with the properties described above simply as equilibria.

The focus on Markov equilibria seems particularly appropriate for this class of dynamic games. Free rider problems are often intended to represent situations in which a large number of agents autonomously and independently contribute to a public good (Olson 1965, Chapter 1.B). In a large economy, it is natural to focus on an equilibrium that is anonymous and independent from the action of any single agent. The Markov perfect equilibrium respects this property, by making strategies contingent only on the payoff relevant economic state. For these reasons, this equilibrium is standard in the applied literature on dynamic public accumulation games.

3.2 The Planner's Problem

As a benchmark with which to compare the equilibrium allocations, we first analyze the sequence of public policies that would be chosen by a benevolent planner who maximizes the sum of utilities of the agents. This is the welfare optimum because the private good enters linearly in each agent's utility function. The planner's solution is extremely simple in the environment described in the previous section: this feature will help highlighting the subtlety of the strategic interaction studied in the next two sections.

Consider first an economy with reversible investment. The planner's problem has a recursive representation in which g is the state variable, and $v_P(g)$, the planner's value function can be represented recursively as:

$$v_P(g) = \max_{y,x} \left\{ \begin{array}{c} \sum_{j=1}^n x^i + nu(y) + \delta v_P(y) \\ s.t \quad \sum_{j=1}^n x^i + y - (1-d)g \leq W, \; x^i \geq 0 \; \forall i, \; y \geq 0 \end{array} \right\} \tag{3.3}$$

By standard methods (see Stokey, Lucas, and Prescott 1989), we can show that a continuous, strictly concave and differentiable $v_P(g)$ that satisfies (3.3) exists and is unique. The optimal policies have an intuitive characterization. When the accumulated level of public good is low, the marginal benefit of increasing y is high, and the planner finds it optimal to spend as much as possible on building the stock of public good: in this region of the state space $y_P(g) = W + (1-d)g$ and $\sum_{j=1}^n x^i = 0$. When g is high, the planner will be able to reach the level of public good $y_P^*(\delta, d, n)$ that solves the planner's unconstrained problem: i.e.„

$$nu'(y_P^*(\delta, d, n)) + \delta v_P'(y_P^*(\delta, d, n)) = 1. \tag{3.4}$$

Applying the envelope theorem, we can show that at the interior solution $y_P^*(\delta, d, n)$ we have

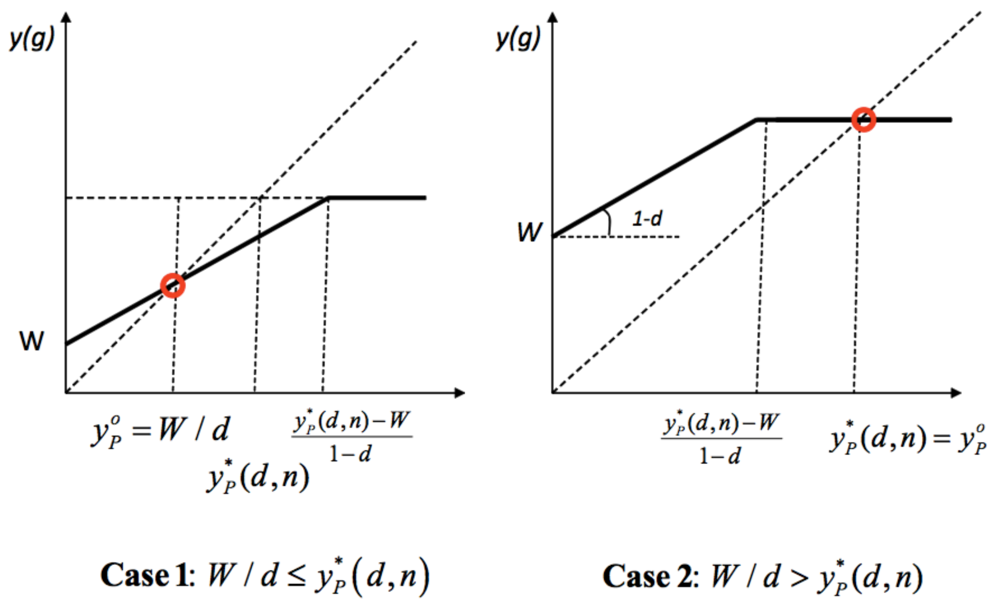


Figure 3.1: The planner's problem

$v_P'(y_P^*(\delta, d, n)) = 1 - d$. From (3.4), we therefore conclude that:

$$y_P^*(\delta, d, n) = [u']^{-1} \left(\frac{1 - \delta(1-d)}{n} \right) \tag{3.5}$$

The investment function, therefore, has the following simple structure. When g is lower than $\frac{y_P^*(\delta,d,n)-W}{1-d}$, $y_P^*(\delta, d, n)$ is not feasible: the planner spends W on the public good so $y_P(g) = (1-d)g + W$. When g is larger or equal than $\frac{y_P^*(\delta,d,n)-W}{1-d}$, instead, the planner does not invest beyond $y_P(g) = y_P^*(\delta, d, n)$. In this case, without loss of generality, we can set $x^i(g) = (W + (1-d)g - y(g))/n \ \forall i$.[9] Summarizing, we have:

$$y_P(g) = \min \{W + (1-d)g, y_P^*(\delta, d, n)\}. \tag{3.6}$$

[9]Indeed, the planner is indifferent regarding the distribution of private consumption.

This investment function implies that the planner's economy converges to one of two possible steady states (see Figure 1). If $W/d \leq y_P^*(\delta, d, n)$, then the rate of depreciation is so high that the planner cannot reach $y_P^*(\delta, d, n)$, (except temporarily if the initial state is sufficiently large). In this case the steady state is $y_P^o = W/d$, and the planner invests all resources in all states on the equilibrium path (Figure 1, Case 1). If $W/d > y_P^*(\delta, d, n)$, $y_P^*(\delta, d, n)$ is sustainable as a steady state. In this case, in the steady state $y_P^o = y_P^*(\delta, d, n)$, and the (per agent) level of private consumption is positive: $x^* = (W + (1-d)g - y)/n > 0$ (Figure 1, Case 2).

An economy in which the planner's optimum can be feasibly sustained as a steady state is the most interesting case. With this in mind we define:

Definition 1. *An economy is said to be regular if* $W/d > y_P^*(\delta, d, n)$.

In the rest of the analysis we focus on regular economies.[10] This is done only for simplicity: extending the results presented below for economies with $W/d \leq y_P^*(\delta, d, n)$ can be done using the same techniques developed in this paper.

The planner's optimum for the IIE case is not very much different. The planner finds it optimal to invest all resources for $g \leq \frac{y_P^*(\delta,d,n)-W}{1-d}$. For $g \in \left(\frac{y_P^*(\delta,d,n)-W}{1-d}, \frac{y_P^*(\delta,d,n)}{1-d}\right)$, the planner finds it optimal to stop investing at $y_P^*(\delta, d, n)$, as before. For $g \geq \frac{y_P^*(\delta,d,n)}{1-d}$, $y_P^*(\delta, d, n)$ is not feasible, so it is optimal to invest 0, and to set $y_P(g) = (1-d)g$. This difference in the investment function for IIE, however, is essentially irrelevant for the optimal path and the steady state of the economy. Starting from any g_0 lower than the steady state y_P^*, levels of g larger or equal than $\frac{y_P^*(\delta,d,n)}{1-d}$ are impossible to reach, and the irreversibility constraint does not affect the optimal investment path.

We conclude this section noting that, both with reversibility and with irreversibility, the planner's solution has a very simple structure: the planner finds it optimal to invest as much as possible

[10] The limit case of $d = 0$ is also included as a regular economy.

in every period until the steady state is reached, and then stop. This has two implications: first, gradualism in investment is never optimal for the planner; second, the irreversibility constraint is irrelevant. As we will see, neither of these two features holds in the free-rider games we study in the next two sections: gradualism is indeed a typical feature of the equilibrium investment function, both with and without irreversibility; and the irreversibility constraint plays an important role in determining the set of equilibrium steady states.

3.3 Reversible Investment Economies

3.3.1 The Equilibrium

We first study equilibrium behavior when the investment in the public good is reversible. Differently from the planner's case, in equilibrium no agent can directly choose the stock of public good y: an agent (say j) chooses only his own level of private consumption x and the level of its own contribution to the stock of public good. The agent realizes that in any period, given g and the other agents' level of private consumption, his/her contribution ultimately determines y. It is therefore as if agent j chooses x and y, subject to three feasibility constraints. The first constraint is a resource constraint that specifies the level of the public good:

$$y = W + (1-d)g - [x + (n-1)x_R(g)]$$

This constraint requires that stock of public good y equals total resources, $W + (1-d)g$, minus the sum of private consumptions, $x + (n-1)x_R(g)$. The function $x_R(g)$ is the *equilibrium* per capita level; naturally, the agent takes the equilibrium level of the other players, $(n-1)x_R(g)$, as given. The second constraint requires that private consumption x is non negative. The third requires total consumption nx to be no larger than total resources $(1-d)g+W$. Agent j's problem can therefore be written as:

$$\max_{y,x}\left\{\begin{array}{c} x+u(y)+\delta v_R(y) \\ s.t\ \ x+y-(1-d)g=W-(n-1)x_R(g) \\ W-(n-1)x_R(g)+(1-d)g-y\geq 0 \\ x\leq(1-d)g/n+W/n \end{array}\right\} \tag{3.7}$$

where $v_R(g)$ is his equilibrium value function.

In a symmetric equilibrium, all agents consume the same fraction of resources, so agent j can assume that in state g the other agents each consume:

$$x_R(g)=\frac{W+(1-d)g-y_R(g)}{n},$$

where $y_R(g)$ is the equilibrium investment function. Substituting the first constraint of (4.2) in the objective function, recognizing that agent j takes the strategies of the other agents as given, and ignoring irrelevant constants, the agent's problem can be written as:

$$\max_{y}\left\{\begin{array}{c} u(y)-y+\delta v_R(y) \\ y\leq\frac{W+(1-d)g}{n}+\frac{n-1}{n}y_R(g),\ y\geq\frac{n-1}{n}y_R(g) \end{array}\right\} \tag{3.8}$$

where it should be noted that agent j takes $y_R(g)$ as given.[11] The objective function shows that an agent has a clear trade off: a dollar in contribution produces an individual marginal benefit $u'(y)+\delta v'_R(y)$; the marginal cost of the contribution is -1, a dollar less in private consumption.[12]

[11]Since $y_R(g)$ is the equilibrium investment function, in a symmetric equilibrium $(n-1)y_R(g)/n$ is the level of investment that agent j expects from all the other agents, and that he/she takes as given in equilibrium.

[12]For simplicity of exposition we assume here that $v_R(g)$ is differentiable. We refer to the proofs in Appendix B

The two constraints define the maximal and minimal feasible level of public good given the other players' investments.

A symmetric Markov equilibrium is therefore fully described in this environment by two functions: an aggregate investment function $y_R(g)$, and an associated value function $v_R(g)$. Two conditions must be satisfied. First, for all values of $g \geq 0$, $y_R(g)$ must solve (4.3) given $v_R(g)$. The second condition for an equilibrium requires that the value function $v_R(g)$ to be consistent with the agents' strategies, and hence consistent with the equilibrium investment function, $y_R(g)$. Each agent receives the same benefit for the expected investment in the public good, and consumes the same share of the remaining resources, $\left(W + (1-d)g - y_R(g)\right)/n$. This implies:

$$v_R(g) = \frac{W + (1-d)g - y_R(g)}{n} + u\left(y_R(g)\right) + \delta v_R(y_R(g)) \tag{3.9}$$

We can therefore define:

Definition 2. *An equilibrium in a Reversible Investment Economy is a pair of functions,* $y_R(\cdot)$ *and* $v_R(\cdot)$*, such that for all* $g \geq 0$*,* $y_R(g)$ *solves (4.3) given the value function* $v_R(\cdot)$*; and for all* $g \geq 0$*,* $v_R(g)$ *solves (3.9) given* $y_R(\cdot)$*.*

For a given value function, if an equilibrium exists, the problem faced by an agent looks similar to the problem of the planner, but with two important differences. First, in the objective function the agent does not internalize the effect of the public good on the other agents. This is the classic free rider problem, present in static models as well: it induces a suboptimal investment in g. The second difference with respect to the planner's problem is that the agent takes the contributions of the other agents as given. The incentives to invest depend on the agent's expectations about the other agents' current and future contributions, which are captured implicitly by the investment

for the details.

function $y_R(g)$. This radically changes the nature of the equilibria. Thus an agent may be willing to invest more or less today, depending on the exact shape of the investment function, which depends on how other agents plan to invest in the future at different levels of g. The relevant question is: Does this make the static free rider problem worse or better in a dynamic environment?

3.3.2 Characterization

To characterize the properties of equilibrium behavior, we first study a particular class of equilibria, the class of *weakly concave equilibria*. An equilibrium is said to be weakly concave if $v(y;g)$ is weakly concave on y for any state g, where $v(y;g)$ is the expected value of investing up to a level of public good, y:

$$v(y\,|g\,) = \frac{W + (1-d)g - y}{n} + u(y) + \delta v(y)$$

We show that this class of equilibria is nonempty and characterize its key properties. We then prove that there is no loss of generality in focusing on this particular class in order to study the set of equilibrium steady states. We therefore use the class of weakly concave equilibria as a tool to gain insight on the more general equilibrium properties of the game.

In a weakly concave equilibrium, the agent's problem (4.3) is a standard concave programming problem similar to (3.3). Because the objective function may have a flat region, however, the investment function typically takes a more general form than the planner's solution (3.6). Figure 2 represents a typical equilibrium. The equilibrium investment function will generally take the following form:

$$y_R(g) = \begin{cases} \min\left\{W + (1-d)g, y\left(g^2\right)\right\} & g < g^2 \\ y\left(g\right) & g \in \left[g^2, g^3\right] \\ y\left(g^3\right) & g > g^3 \end{cases} \tag{3.10}$$

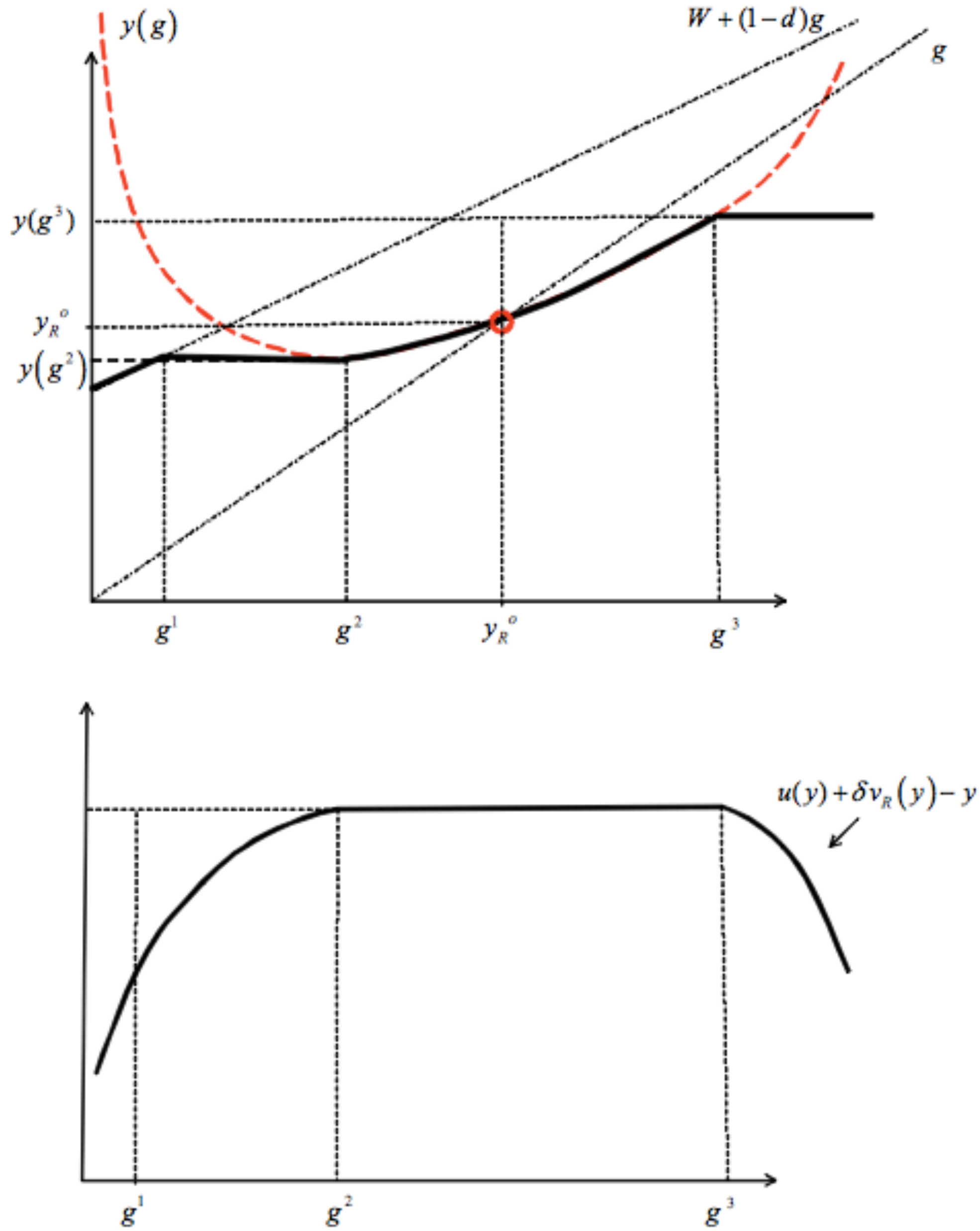


Figure 3.2: The equilibrium in an economy with reversibility

where g^2, g^3 are two critical levels of g, and $y(g)$ is a non decreasing function with values in $[g, W + (1-d)g]$. To see why $y_R(g)$ may take the form of (3.10), consider Figure 2. The top panel of the figure illustrates a canonical equilibrium investment function. The steady state is labeled y_R^o in the figure, the point at which the (bold) investment function intersects the (dotted) diagonal. The bottom panel of the figure graphs the corresponding objective function, $u(y) - y + \delta v_R(y)$. For $g < g^2$, the objective function of (4.3) is strictly increasing in y: either resources are sufficient to reach the level that maximizes the unconstrained objective function and so $y(g) \in \left[g^2, g^3\right]$ (in Figure 2, $y(g) = y\left(g^2\right)$ in $g^1 \leq g \leq g^2$); or it is optimal to invest all resources (in Figure 2, $y(g) = W + (1-d)g$ in $g \leq g^1$).[13] For $g > g^3$, the objective function is decreasing: the investment level is so high that the agents do not wish to increase g over $y(g^3)$. For intermediate levels of $g \in \left[g^2, g^3\right]$, an interior level of investment $y \in \left(g^2, g^3\right)$ is chosen. This is possible because the objective function is flat in this region: an agent is indifferent between any $y \in \left[g^2, g^3\right]$. The key observation here is that since the objective function has a flat region, the agents find it optimal to choose an *increasing* investment function in $\left[g^2, g^3\right]$: a weakly concave objective function, therefore, gives us more freedom in choosing the equilibrium investment function and even a higher level of investment.

The open questions are whether the flat region in Figure 2 is a general equilibrium phenomenon or just an intellectual curiosity, and what degrees of freedom we have in choosing investment functions that are consistent with an equilibrium. For an investment curve as in Figure **??** to be an equilibrium, agents must be indifferent between investing and consuming for all states in $\left[g^2, g^3\right]$. If this condition does not hold, the agents do not find it optimal to choose an interior

[13] In Figure 2 it is assumed that we have $W + (1-d)g > g^2$ for for $g \geq g^1$, so the agent can afford to choose a level of y that maximizes the objective function (i.e.„ $y \in \left[g^2, g^3\right]$) if and only if $g \geq g^1$.

level $y(g)$. The marginal utility of investments is zero if and only if:

$$u'(g) + \delta v'(g) - 1 = 0 \ \forall g \in \left[g^2, g^3\right] \tag{3.11}$$

Since the expected value function is (3.9), we have:

$$v'(g) = \frac{1 - d - y'(g)}{n} + u'(y(g))y'(g) + \delta v'(y(g))y'(g) \tag{3.12}$$

Substituting this formula in (3.11), we see that the investment function $y(g)$ must solve the following differential equation:

$$\frac{1 - u'(g)}{\delta} = \frac{1 - d - y'(g)}{n} + u'(y(g))y'(g) + \delta v'(y(g))y'(g) \tag{3.13}$$

This condition is useful only if we eliminate the last (endogenous) term: $\delta v'(y(g))y'(g)$. To see why this is possible, note that $y(g)$ is in $\left[g^2, g^3\right]$ for any $g \in \left(g^2, g^3\right)$ in the example of Figure 2. In this case, (3.11) implies $\delta v'(y(g)) = 1 - u'(y(g))$. Substituting this condition in (3.13) we obtain the following necessary condition:

$$y'(g) = \frac{1 - d - \frac{n\left(1-u'(g)\right)}{\delta}}{1 - n} \tag{3.14}$$

Condition (4.6) shows that there is a unique way to specify the shape of the investment function that is consistent with a "flat" objective function in equilibrium. This necessary condition, however, leaves considerable freedom to construct multiple equilibria: (4.6) defines a simple differential equation with a solution $y(g)$ unique up to a constant. To have a steady state at y_R^o we need a second condition: $y(y_R^o) = y_R^o$. This equality provides the initial condition for (4.6), and so uniquely defines $y(g|y_R^o)$ in $\left[g^2, g^3\right]$ (see the dashed curve in Figure 2).

Proposition 1, presented below, shows that the degrees of freedom allowed by (4.6) are sufficient to characterize all the stable steady states we can have in equilibrium, weakly concave or not. A steady state y_R^o is said to be stable if there is a neighborhood $N_\varepsilon(y_R^o)$ of y_R^o such that for any $N_{\varepsilon'}(y_R^o) \subseteq N_\varepsilon(y_R^o)$, $g \in N_{\varepsilon'}(y_R^o)$ implies $y_R(g) \in N_{\varepsilon'}(y_R^o)$. Intuitively, starting in a neighborhood of a stable steady state, g remains in a neighborhood of a stable steady state for all following periods.[14] Define the two thresholds:

$$y_R^*(\delta, d, n) = [u']^{-1}\left(1 - \delta(1-d)/n\right), \text{ and } y_R^{**}(\delta, d, n) = [u']^{-1}\left(1 - \delta\left(1 - d/n\right)\right) \tag{3.15}$$

We say that an equilibrium steady state y_R^o is supported by a concave equilibrium if there is a concave equilibrium $y_R(g), v_R(g)$ such that $y_R(y_R^o) = y_R^o$. An equilibrium is monotonic if the investment function, $y(g)$, is non decreasing in g. The following Proposition shows that the set of equilibrium steady states of monotonic equilibria can be easily characterized in closed form. The details about the equilibrium strategies are in Appendix B.

Proposition 1. *A public good level y_R^o is a stable steady state of a monotonic equilibrium in a RIE if and only if $y_R^o \in [y_R^*(\delta, d, n), y_R^{**}(\delta, d, n)]$. Each y_R^o in this set is supported by a concave equilibrium with investment function as illustrated in Figure* 2.

We may obtain an intuitive explanation of why the steady state must be in $[y_R^*(\delta, d, n), y_R^{**}(\delta, d, n)]$ by making three observations. First, an equilibrium steady state must be in the interior of the feasibility region, that is $y_R^o \in (0, W/d)$:[15] intuitively, $y_R^o > 0$, since at 0 the marginal utility of the

[14] An alternative stability concept that has been used in the literature is achievability (Matthews 2012). A steady state is achievable if it is the limit of an equilibrium path. Our concept of stability is weaker: this allows us to have a stronger characterization of the equilibrium set in our environment. It is easy to see that if an equilibrium is not stable, then it is not reachable in a monotonic Markov equilibrium. On the other hand, all steady states characterized in Proposition 1 are achievable (as in the equilibrium illustrated by Figure 2).

[15] The feasibility set is given by $y \geq 0$ and $y \leq W + (1-d)g$, so a steady state must satisfy $y_R^o \geq 0$ and

public good is infinite; and $y_R^o < W/d$ since even in the planner's solution we have this property. Second, in a stable steady state we must have $y_R'(y_R^o) \in (0,1)$.[16] The highest and the lowest steady states, moreover, correspond to the equilibria with the highest and, respectively, the lowest $y_R'(g)$: so $y_R'(y_R^{**}(\delta, d, n)) = 1$ and, respectively , $y'(y_R^*(\delta, d, n)) = 0$. Third, since the solution is interior and the agents can choose the investment they like in a neighborhood of y_R^o, $y_R(g)$ can have positive slope at y_R^o only if the agents's objective function is flat in the neighborhood (otherwise the agents would choose the same optimum point irrespective of g). By the argument presented above, this implies (4.6). Using (4.6) and $y_R'(y_R^o) \leq 1$, we obtain the upper bound, $y_R^{**}(\delta, d, n)$; similarly, using (4.6) and $y_R'(y_R^o) \geq 0$, we obtain the lower bound, $y_R^*(\delta, d, n)$. Proposition 1 formalizes this argument, and it uses the construction described above to prove that $y_R^o \in [y_R^*(\delta, d, n), y_R^{**}(\delta, d, n)]$ is sufficient as well as necessary for y_R^o to be a stable steady state.

In the working paper version of the paper (Battaglini, Nunnari and Palfrey 2012a) we show that non-monotonic equilibria always exist in reversible economies. When we consider non-monotonic equilibria the maximal steady state remains the same as in Proposition 1, the minimal however can be lower than $y_R^*(\delta, d, n)$.[17] The consideration of non-monotonic equilibria, however, is not particularly relevant for the comparison of irreversible versus reversible economies since except when depreciation is high, all the steady states that can be achieved in irreversible economies can be achieved with monotonic equilibria as well. To keep the presentation focused in this paper, the analysis of non-monotonic equilibria is left in the working paper version (Battaglini, Nunnari and Palfrey 2012a).

$y_R^o \leq W + (1-d)y_R^o$. The second inequality implies $y_R^o \leq W/d$.

[16]Here we are assuming that $y_R(g)$ is differentiable for the sake of the argument. Details are provided in Appendix B.

[17]In Battaglini. Nunnari, and Palfrey (2012a) we show that the lowest possible steady state with non monotonic strategies is $[u']^{-1}(1 + \delta(n + d - 2)/n) < y_R^*(\delta, d, n)$.

3.3.3 Efficiency

Proposition 1 shows that, as in the static model, an equilibrium allocation is always inefficient, even in the best equilibrium: $y_P^*(\delta, d, n) > y_R^{**}(\delta, d, n)$ for any $n > 1$ and $\delta < 1$. Let $\overline{y}(\delta, d)$ be the steady state that would be achieved by an agent alone in autarky: $y_P^*(\delta, d, 1) = [u']^{-1}(1 - \delta(1 - d))$. We can make three observation regarding the magnitude of the inefficiency.

Corollary 1. *In a RIE we have:*

- *For any $n > 1$ we have $\overline{y}(\delta, d) \in (y_R^*(\delta, d, n), y_R^{**}(\delta, d, n))$;*
- *The highest equilibrium steady state increases in n; the smallest steady state decreases in n. As $n \to \infty$, $y_R^*(\delta, d, n) \to [u']^{-1}(1)$ and $y_R^{**}(\delta, d, n) \to [u']^{-1}(1 - \delta)$;*
- *For any n and d, $|y_R^{**}(\delta, d, n) - y_P^*(\delta, d, n)| \to 0$ as $\delta \to 1$.*

The first point in Corollary 1 shows that the accumulated level of g in a community with n players may be *either* higher *or* lower than the level that an agent alone in autarky would accumulate. This is in contrast to the static case (when $\delta = 0$), where the level of accumulation is independent of n. The second point shows that, in terms of the steady state level of g, the common pool problem may become better or worse as the size of the community increases. The multiplicity of equilibria, moreover, is not an artifact of the assumption of a finite population. Finally, the last point highlights the fact that the best equilibrium steady state converges to the efficient level as $\delta \to 1$. What is remarkable in this result is the fact that the efficient steady state can be achieved with an extremely simple equilibrium (Markov) in which agents focus exclusively on the state g.

To interpret Proposition 1 it is useful to start from the special case in which $\delta = 0$ and so the free rider problem is static. In this case there is a unique equilibrium “steady state” at

$y_R^o = [u']^{-1}(1)$, independent of n. In addition, the agents' actions are pure strategic substitutes. If agent j is forced to invest $1/n + \Delta$, then all the other agents find it optimal to reduce their investment exactly by $\Delta/(n-1)$. Previous research on dynamic public good games has stressed this aspect of the games. Fershtman and Nitzan (1991), in particular, show an equilibrium in which the substitutability effect is so strong that the steady state is even lower than the equilibrium of a static game. In general, however, the strategic interaction in dynamic games is much richer. In the working paper version of this work, we formally prove that all the equilibria in Proposition 1 with steady state lower than $\overline{y}(\delta, d)$ the players' contributions are strategic substitutes. Indeed, in these equilibria the steady state is lower that the steady state the players would achieve in autarky. But in the equilibria with steady states larger than $\overline{y}(\delta, d)$ the players' contributions must be strategic complements on the equilibrium path. Strategic complementarity is necessary in these equilibria because an agent is willing to keep investing until $y_R^o > \overline{y}(\delta, d)$ only if he expects the other agents to react to his investment by increasing their own investments. This complementarity allows the agents to mitigate the free rider problem and partially "internalize" the public good externality. In these equilibria, the agents accumulate more than what would be reached by an agent in perfect autarky. As Corollary 1 proves, this complementarity effect may be extremely powerful, allowing to achieve an efficient steady as $\delta \to 1$ with simple Markov strategies.

3.4 Irreversible Economies

We now turn to irreversible investment economies. When the agents cannot directly reduce the stock of the public good, the optimization problem of an agent can be written like (4.2), but with an additional constraint: the individual level of investment cannot be negative; the only way to reduce the stock of g, is to wait for the work of depreciation. Following similar steps as before, we can write the maximization problem faced by an agent as:

$$\max_{y} \left\{ \begin{array}{c} u(y) - y + \delta v_{IR}(y) \\ y \leq \frac{W+(1-d)g}{n} + \frac{n-1}{n} y_{IR}(g), \; y \geq \frac{(1-d)g}{n} + \frac{n-1}{n} y_{IR}(g) \end{array} \right\} \tag{3.16}$$

where the only difference with respect to (4.3) is the second constraint. To interpret it, note that it can be written as $y \geq (1-d)g + \frac{n-1}{n}\left[y_{IR}(g) - (1-d)g\right]$: the new level of public good cannot be lower than $(1-d)g$ plus the investments from all the other agents (in a symmetric equilibrium, an individual investment is $\left[y_{IR}(g) - (1-d)g\right]/n$).

As in the reversible case, a continuous symmetric Markov equilibrium is fully described in this environment by two functions: an aggregate investment function $y_{IR}(g)$, and an associated value function $v_{IR}(g)$. The aggregate investment function $y_{IR}(g)$ must solve (4.5) given $v_{IR}(g)$. The value function $v_{IR}(g)$ must be consistent with the agents' strategies. Similarly, as in the reversible case, we must have:

$$v_{IR}(g) = \frac{W + (1-d)g - y_{IR}(g)}{n} + u\left(y_{IR}(g)\right) + \delta v_{IR}(y_{IR}(g)) \tag{3.17}$$

We can therefore define:

Definition 3. *An equilibrium in a Irreversible Investment Economy is a pair of functions, $y_{IR}(\cdot)$ and $v_{IR}(\cdot)$, such that for all $g \geq 0$, $y_{IR}(g)$ solves (4.5) given the value function $v_{IR}(\cdot)$, and for all $g \geq 0$, $v_{IR}(g)$ solves (3.17) given $y_{IR}(g)$.*

As pointed out in Section 3, when public investments are efficient, irreversibility is irrelevant for the equilibrium allocation. The investment path chosen by the planner is unaffected because the planner's choice is *time consistent*: he never finds it optimal to increase g if he plans to reduce

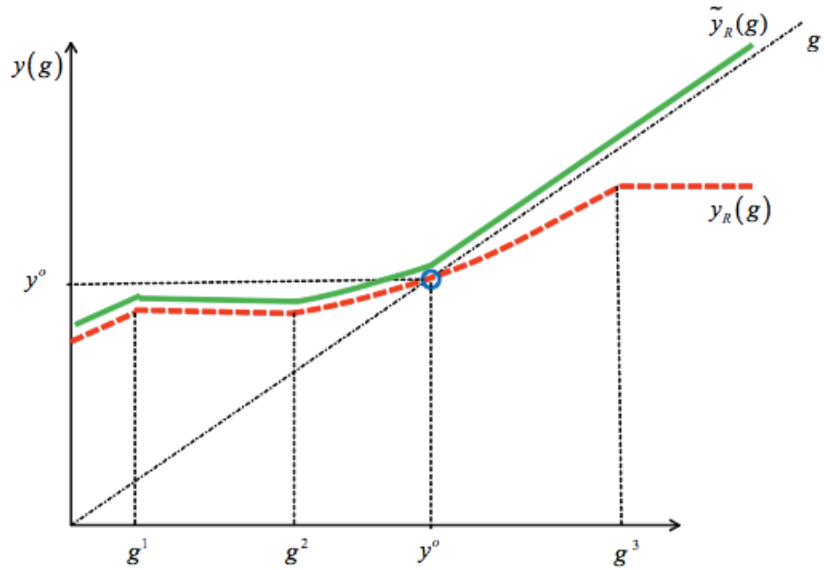


Figure 3.3: The irreversibility constraint and the reversible equilibrium

it later. In the monotone equilibria characterized in the previous section, the investment function may be inefficient, but it is weakly increasing in the state. Agents invest until they reach a steady state, and then they stop. It may seem intuitive, therefore, that irreversibility is irrelevant in this case too. In this section we show that, to the contrary, irreversibility changes the equilibrium set: it induces the agents to significantly increase their investment and it leads to significantly higher steady states when depreciation is small.

To illustrate the impact of irreversibility on equilibrium behavior, suppose for simplicity that $d = 0$ and consider Figure 3, where the red dashed line represents some arbitrary monotone equilibrium with steady state y^o in the model with reversibility. Next, suppose we ignore the irreversibility constraint where it is not binding, so we keep the same investment function for $g \leq y^o$ where $y_R(g) \geq g$ and then set the investment function equal to g when $y_R(g) < g$. This gives us the modified investment function $\widetilde{y}_R(g)$, represented by the green solid line. This investment function

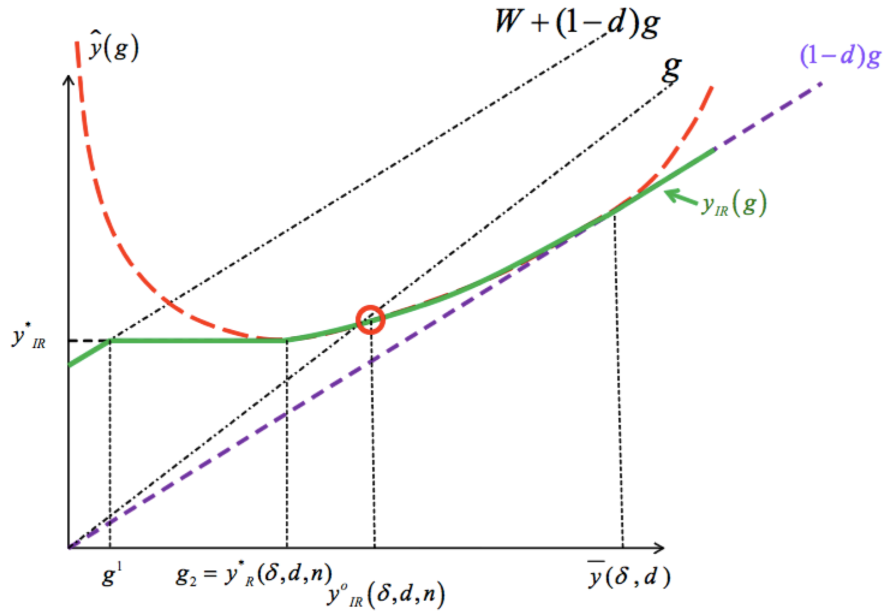


Figure 3.4: The irreversible equilibrium as $d \to 0$

induces essentially the same allocation: the same steady state y^o and the same convergent path for any initial $g_0 \leq y^o$. Unfortunately, $\widetilde{y}_R(g)$ is no longer an equilibrium. On the left of y^o the objective function, $u(y) - y + \delta v_{IR}(y)$, is flat. On the right of y^o, the objective function would remain flat if the investment were the red dashed line as with reversibility; with irreversibility, however, the constraint $y \geq g$ forces the investment to increase at a faster rate than $y_R(g)$. Because $y_R(g)$ is ex ante suboptimal, the "forced" increase in investment makes the objective function increase on the right of y^o. But then choosing y^o would no longer be optimal in state y^o, so it cannot be a steady state.[18]

[18]This problem does not arise with the planner's solution because the planner's solution is time consistent. After the planner's steady state y^*_P is reached the planner would keep g at y^*_P. If the planner's is forced to increase y on the right of y^*_P, we would have a kink at y^*_P, but it would be a "downward" kink. Such a kink makes the objective function fall at a faster rate on the right of the steady state, so it preserves concavity and it does not disturb the optimal solution. The kink is "upward" in the equilibrium with irreversibility because the steady state is not optimal, so the irreversibility constraint, $y \geq g$, increases expected welfare. This creates a sort of "commitment device" for the future; the agents know that g can not be reduced by the others (or their future selves).

Does an equilibrium exist? What does it look like? Let $\widehat{y}(g)$ be the unique solution of (4.6) that is tangent to the line $y = (1-d)g$ (see Figure 4 for an example). As it can be easily verified from (4.6), the point at which $\widehat{y}(g)$ is tangent to $y = (1-d)g$ is $\overline{y}(\delta, d)$.[19] Define $y^o_{IR}(\delta, d, n)$ as the fixed point of this function:[20]

$$\widehat{y}(y^o_{IR}(\delta, d, n)) = y^o_{IR}(\delta, d, n). \tag{3.18}$$

The following Proposition states the existence result. In Appendix B we provide a detailed description of the equilibrium strategies.

Proposition 2. *In any IIE there is a monotonic equilibrium with an investment function as illustrated in Figure 4 and steady state* $y^o_{IR}(\delta, d, n)$ *as defined in (3.18). This equilibrium is weakly concave.*

Proposition 4 establishes that the dynamic free rider game with irreversibility admits an equilibrium with standard concavity properties. Figure 4 shows the investment function associated with the equilibrium. In equilibrium the investment function merges smoothly with the irreversibility constraint: at the point of the merger (i.e.,, $\overline{y}(\delta, d)$, where the constraint becomes binding), the investment function has slope $1-d$. This feature is essential to avoid the problems discussed above.

Proposition 4 does not establish that the there is a unique equilibrium steady state. The following result establishes bounds for the set of stable steady states in a IIE and shows that when depreciation is not too high all stable steady states must be close precisely to $y^o_{IR}(\delta, d, n)$:

Proposition 3. *There is lower bound* $y^*_{IR}(\delta, d, n) \geq y^*_R(\delta, d, n)$ *such that* y_{IR} *is a stable steady state of a monotonic equilibrium only if* $y_{IR} \in [y^*_{IR}(\delta, d, n), y^{**}_R(\delta, d, n)]$*. Moreover, as* $d \to 0$ $y^*_{IR}(\delta, d, n)$,

[19]Formally, $\widehat{y}(g)$ is the solution of (4.6) with the initial condition $\widehat{y}(\overline{y}(\delta, d)) = (1-d)\overline{y}(\delta, d)$.

[20]Note that $y^o_{IR}(\delta, d, n)$ is a function of δ, d and n since $\widehat{y}(g)$ depends on these variables.

$y_R^{**}(\delta, d, n)$ and $y_{IR}^o(\delta, d, n)$ *all converge to* $[u']^{-1}(1-\delta)$.

There is an intuitive explanation for Proposition 5. Because of decreasing returns, the investment in g declines over time, so the constraint $y \geq (1-d)g$ must be binding when g is high enough. When this happens the agents are forced to keep the investment higher than what they would like. Since the equilibrium is inefficiently low (because the agents do not fully internalize the social benefit of g), the constraint $y \geq (1-d)g$ increases expected welfare in these states. The states where the constraint $y \geq (1-d)g$ is binding are typically out of equilibrium (that is on the right of the steady state): in the equilibrium illustrated in Figure 4, for example, the constraint is binding for $g > y_{IR}^o(\delta, d, n)$. The irreversibility constraint, however, has a ripple effect on the entire investment function. In a left neighborhood of $\overline{y}(\delta, d)$, the constraint is not binding; still, the agents expect that the other agents will preserve their investment, so the strategic substitutability will not be too strong. Steady states lower than $y_{IR}^o(\delta, d)$ can occur with reversibility because the agents expect high levels of "strategic substitutability." Proposition 5 shows that when d is sufficiently low, the irreversibility constraint makes these expectations impossible in equilibrium, inducing an equilibrium steady state close to the maximal steady state of the reversible case, $y_R^{**}(\delta, d, n)$. Thus, as $d \to 0$, there is a unique equilibrium steady state in the irreversible case, i.e.,„, $y_{IR}^o = y_R^{**}(\delta, d, n)$.

An immediate implication of Propositions 4 and 5 is the following result:

Corollary 2. *As* $\delta \to 1$ *the highest stable steady state in a IIE converges to the efficient level. As* $\delta \to 1$ *and* $d \to 0$, *every stable steady state in a IIE converges to the efficient level.*

Results proving the efficiency of the best steady state in monotone games as $\delta \to 1$ have been previously presented in the literature by Lockwood and Thomas (2002) and more recently by Matthews (2012). We have already explained in Section 1.1. that these results do not apply to our environment because they rely on assumptions that are not verified in our environment. We emphasize here three additional novel aspects of Corollary 2. First, the result shows that the community can achieve efficiency using a very simple equilibrium (Markov) that requires minimal coordination among the players (in previous results the efficient steady states are supported by subgame perfect equilibria where behavior depends on the entire history of the game). Second, we do not need $d = 0$ to have the result: when d is small, all equilibria must be approximately efficient. Finally, here irreversibility is not necessary for efficiency, the same equilibrium exists in a RIE: irreversibility only guarantees its uniqueness as $d \to 0$.

Propositions 1 and 2 show that both with reversibility and with irreversibility the agents' contributions are gradual and aggregate investment is inefficiently slow: indeed, as it can immediately be seen from the equilibria represented in Figures 2 and 5, the steady state is typically reached only in the limit.[21] This is a purely strategic phenomenon since, as we have discussed in Section 3, a planner never finds gradualism in investment optimal. Previous to our work, a number of authors have highlighted how gradualism is a necessary feature of contribution games with irreversibility and no depreciation (see, in particular, Lockwood and Thomas 2002 and Matthews 2012). We are, however, not aware of previous results that have shown that gradualism is a feature of Markov equilibria in irreversible economies with depreciation, or in economies with reversibility.

3.5 Conclusions

In this paper we have studied a simple model of free riding in which n infinitely lived agents choose between private consumption and contributions to a durable public good. We have considered two possible cases: economies with reversible investments, in which in every period individual investments can either be positive or negative; and economies with irreversible investments, in which

[21] As it can be formally proven, of the equilibria constructed in Proposition 1, the steady state is reached in finite time on in the equilibrium correspondent to the minimal steady state, $y_R^*(\delta, d, n)$.

the public good can only be reduced by depreciation. For both cases we have characterized the set of steady states that can be supported by symmetric Markov equilibria in continuous strategies.

We have highlighted three main results. First, we have shown that economies with reversible investments have typically a continuum of equilibria. In the best equilibrium the steady state is higher in a community with n agents than in autarky, and it is increasing in n; in the worst equilibrium, the steady state is lower in autarky, and it decreases in n. While in a static free rider's problem the players' contributions are strategic substitutes, in a dynamic model they may be strategic complements. Second, we have shown that in economies with irreversible investments, the set of equilibrium steady states is much smaller: indeed, as depreciation converges to zero, the set of equilibrium steady states converges to the best equilibrium that can be reached in economies with reversible investments. Irreversibility, therefore, helps the agents removing the coordination problem that plagues most of the equilibria in the reversible case, and so it necessarily induces higher investment. Third, as agents become increasingly patient, the best steady state in both economies with reversibility and irreversibility converges to the efficient level. As patience increases and depreciation decreases, all equilibrium steady states in an irreversible economy converge to the efficient level.

Although in this paper we have focused on a free rider problem in which agents act independently and there is no institution to coordinate their actions, the approach we have developed to characterize the Markov equilibria has a wider applicability and can be used to study dynamic games in other environments as well. In future work, it would be interesting to investigate economies with irreversible investments when public decisions are taken by legislative bargaining or other types of centralized political processes.

Chapter 4

The Dynamic Free Rider Problem: An Experimental Study

Most public goods are durable, and hence dynamic in nature. It takes time to accumulate them, and they depreciate slowly, projecting their benefits for many years. Prominent examples are public infrastructure, environmental protection, and social capital. Although a large literature has studied public good provision in static models both theoretically and empirically, much less is known about dynamic environments and a number of important questions remain still unanswered.

First of all: How serious is free riding in the provision of durable public goods? Laboratory experiments have shown that theoretical predictions tend to overestimate the seriousness of free riding in static environments.[1] In dynamic environments, we have both the familiar free rider problem present in static public good provision, and a new *dynamic free rider* phenomenon that further erodes incentives for efficient provision: an increase in current investment by one agent triggers a reduction in future investment by all agents. On the other hand, these free rider problems will be severe only if agents coordinate on stationary equilibria where strategies depend only on the

[1] See Ledyard (1995) for a survey. The failure of theoretical predictions seems more serious in cases where the equilibrium level of investment is zero. In experiments where the equilibrium level of investment is positive, the results are mixed, and sometimes very close to equilibrium or even underprovision. See, for example, Palfrey and Prisbrey (1996, 1997), Palfrey and Rosenthal (1991), Holt and Laury (2008).

accumulated level of the public good. The infinite horizon public good game we study has a plethora of nonstationary equilibria that provide strategic opportunities to endogenously support cooperative outcomes using carrot-and-stick strategies. In principle, this could completely overcome both the static and the dynamic free rider problems. Thus, it is an open empirical question whether or not the free rider problem is exacerbated or ameliorated in the case of dynamic provision of durable public goods, as opposed to one-shot public goods problems.

While a handful of studies have recently provided some data on sequential mechanisms for one-shot provision of a discrete public good (Dorsey 1992; Duffy, Ochs, and Vesterlund 2007; Choi, Gale and Kariv 2008; Diev and Hichri 2008; Noussair and Soo 2008; Cho, Gale, Kariv, and Palfrey 2011), or static public goods experiments motivated by intertemporal public good allocations (notably common pool resource problems[2]), to date there have been no laboratory studies of free riding in a truly dynamic environment such as the one presented here, where a durable public good provides a stream of benefits over time and players have opportunities to gradually build the stock. One of the central contributions of this paper is to break out of this static laboratory paradigm for studying free riding and public good provision, and provide some initial empirical findings about dynamic voluntary contribution behavior with durable public goods. To our knowledge, this is the first experimental study of the dynamic accumulation process of a durable public good.

A second set of questions centers around some new issues that arise in the case of durable public good provision that are not present in static or one-shot public good allocation problems. These questions relate to how the free rider problem depends on the production technology of the durable public good. Again, here little work has been done except for static environments, partly because the variations on the production technology are rather limited (for example, continuously increasing production levels vs. thresholds). In dynamic environments, where the public good is durable, at

[2]See Ostrom (1999) for a survey.

least three new additional dimensions of technology play an important role. We call these three dimensions *time-to-build*, *depreciation,* and *reversibility*.

Time-to-build reflects the obvious fact that public infrastructure projects cannot be feasibly developed overnight, but take years of investment to achieve full potential. Rome was not built in a day, nor was the U.S. railroad or highway system or the great underground urban transportation systems of the world. Depreciation is an important and realistic dimension because most public goods are not perfectly durable. Bridges, roads, and aircraft carriers require maintenance and repair. Decisions to invest today in a durable public good must take into account a willingness to invest in maintenance in the future. The greater the stock of the public good that exists today, the higher the maintenance costs in the future. Reversibility of public investment is an especially complex dimension, and reflects the extent to which today's investments in the public good can be converted back to private consumption at a future date. Most investments are partially reversible, and the degree (or cost) of reversibility varies widely. For example, the art collection at the Louvre, which took centuries to accumulate, could be sold off to private collectors and the proceeds distributed as transfer payments to the citizens of France. Cobblestone roads have been dug up and the stones used to build private dwellings. Military vehicles and aircraft can be privatized and converted to civilian use. Publicly owned open space, even with conservation easements, are routinely converted to the private development of shopping malls and new residential communities.

These three dimensions of the dynamic production technology do not only determine the extent to which the long run policy will reflect the welfare of the citizens, but also affect the timing of investment and the extent to which current decisions internalize benefits that will accrue in future period. In a word, they affect how "shortsighted" is the dynamic investment in the durable public good.

Battaglini, Nunnari, and Palfrey (2012a) develop a theoretical approach to explore these questions. The time-to-build dimension is captured by two variables: the per-period *endowment*, which determines the maximum rate of investment in the durable public good, and the *discount factor*, which measures the cost associated with investment lags. Depreciation is captured by the *depreciation rate* which specifies how fast the stock of the public good deteriorates over time. Reversibility is addressed by comparing two extreme cases: the case of *full reversibility*, in which any or all of the stock of the public good can be converted to private consumption instantaneously; and the *irreversible* case, in which none of the public good can ever be converted into private consumption.

In this work, we make a first attempt to answer the questions raised above empirically, by testing the results from this model in a laboratory experiment. Experimental analysis is particularly important when studying a highly structured dynamic environment that cannot be easily replaced by field data; this is because strategic behavior can be observed only if there is a precise measurement of the *state variable* and the actions space available to the players.

The economy we study has n individuals. In each period, each individual is endowed with w units of input that can be allocated between personal consumption and investment in the public good. Utility is linear in consumption of the private good and concave in the accumulated stock of the durable public good. To keep the experiment simple, there is no depreciation, so at time t the stock of the public good is simply the sum of individual investments across all periods up to time t. Total payoffs for a player in the game is the discounted sum of utility over an infinite horizon of the game, where the discount factor is δ. We characterize the efficient accumulation path as a function of w, n, and δ.[3]

We solve for the unique symmetric concave Markov perfect equilibrium of the game under

[3] Battaglini, Nunnari, and Palfrey (2012a) also characterize the efficient path and the equilibrium accumulation paths for arbitrary depreciation rates, $d \in [0, 1]$.

two different assumptions about reversibility: full reversibility and irreversibility. We prove that investment is always higher in the irreversible case, and this theoretical property of our model is the basis for the main theoretical treatment in our experiment: reversibility vs. irreversibility. We also have a secondary treatment dimension, which is the number of individuals in the game: we compare $n = 3$ and $n = 5$. Thus, the experiment has four different treatments depending on n and whether investments are reversible. The Markov equilibrium of our model provides clear qualitative predictions about the difference in investment across the four treatments. In addition to the comparative static predictions of the model, the equilibrium generates quantitative predictions about the steady state levels of public good that can be supported, and also how the dynamics of investment evolves across time and as a function of the current stock of the public good.

In the experiments, the comparative static predictions for the treatments are supported by the data: our main finding is that irreversible investment leads to significantly higher public good production than reversible investment. We do, however, observe some differences between the finer details of the theoretical predictions and the data, mainly with respect to the path of convergence to the steady state. In equilibrium, convergence should be monotonic. That is, the stock of public good should gradually increase over time until the steady state is reached after which investment is zero. Instead, there is a tendency for initial overinvestment in the early periods, compared to the equilibrium investment levels. In the treatment with reversibility, this is followed by a significant reversal, i.e., *negative investment,* with the stock of public good gradually declining in the direction of the equilibrium steady state. After several periods of play, the stock of the public good is very close to the Markov equilibrium of the game. When disinvestment is not feasible, investment steadily decreases but the initial overinvestment cannot be corrected and the long run level of the public good remains above the equilibrium steady state.

Finally, we construct a novel test for the Markovian restriction, designing a one-period experiment where subjects payoffs form the public good are given by the equilibrium value function of the unique concave Markov perfect equilibrium of the game with reversible investment. In this reduced-form version of the game, the incentives to invest in the public good are the same as in the fully dynamic game (under the assumption that subjects condition their strategies only on the public good stock), but there is no possibility to sustain a higher public good outcome through the nonstationary strategies that can arise in a repeated game. We observe no systematic difference in investment levels between this dynamic (yet not repeated) reduced form game and the fully dynamic game. We conclude that observed behavior in the dynamic treatments is well approximated by the predictions of a purely forward looking Markov equilibrium, rather than by an equilibrium in which agents look back at the past to punish uncooperative behavior (or reciprocate cooperative behavior) by other members of the group.

4.1 The Model

Here we describe a simplified version of the model in Battaglini, Nunnari, and Palfrey (2012a), which we will use in our experimental design. Consider an economy with n agents. There are two goods: a private good x and a public good g. The level of consumption of the private good by agent i in period t is x_t^i, the level of the public good in period t is g_t. We refer to $z_t = (x_t, g_t)$ as the allocation in period t. The utility U^j of agent j is a function of $z^j = (x_\infty^j, g_\infty)$, where $x_\infty^j = (x_1^j, ..., x_t^j, ...)$, and and $g_\infty = (g_1, ..., g_t, ...)$. We assume that the future is discounted at a rate δ and that U^j can be written as:

$$U^j(z^j) = \sum_{t=1}^{\infty} \delta^{t-1} \left[x_t^j + 2\sqrt{g_t} \right]$$

There is a linear technology by which the private good can be used to produce public good, with a marginal rate of transformation $p = 1$. The private consumption good is nondurable, the public good is durable and does not depreciate between periods. Thus, if the level of public good at time $t-1$ is g_{t-1} and the total investment in the public good is I_t, then the level of public good at time t will be

$$g_t = g_{t-1} + I_t.$$

It is convenient to distinguish the state variable at t, g_{t-1}, from the policy choice g_t. In the remainder, we denote $y_t = g_{t-1} + I_t$ as the new level of public good after an investment I_t when the last period's level of the public good is g_{t-1}. The initial stock of public good is $g_0 \geq 0$, exogenously given. Public policies are chosen as in the classic free rider problem. In each period, each agent j is endowed with $w = W/n$ units of private good and chooses on its own how to allocate its endowment between an individual investment in the public good (which is shared by all agents) and private consumption, taking as given the strategies of the other agents. The key difference with respect to the static free rider problem is that the public good can be accumulated over time. The level of the state variable g, therefore, creates a dynamic linkage across policy making periods.

We consider two alternative economic environments. In a *Reversible Investment Economy* (RIE), the level of individual investment can be negative, with the constraint that $i_t^j \in [-g_t/n, W/n]$ $\forall j$, where $i_t^j = W/n - x_t^j$ is the investment by agent j.[4] In an *Irreversible Investment Economy*

[4]This constraint guarantees that (out of equilibrium) the sum of reductions in g can not be larger than the stock of g. The analysis would be similar if we allow each agent to withdraw up to g. In this case, however, we would have to assume a rationing rule in case the individuals withdraw more than g.

(IIE), an agent's investment cannot be negative and must satisfy $i_t^j \in [0, W/n]\ \forall j$.

The RIE corresponds to a situation in which the public investment can be scaled back in the future at no cost. An example can be an art collection, land for common use, the level of global warming, or less tangible investments like "social capital". The IIE corresponds to situations in which once the investment is done it cannot be undone. This seems the appropriate case for investments in public infrastructure (say a bridge or a road). In this environment, private consumption cannot be negative and the total economy-wide investment in the public good in any period is given by the sum of the agent investments.

4.1.1 The Planner's Solution

As a benchmark with which to compare the equilibrium allocations, we first analyze the sequence of public policies that would be chosen by a benevolent planner who maximizes the sum of utilities of the agents. This is the welfare optimum because the private good enters linearly in each agent's utility function.

Denote the planner's policy as $y_P(g)$ and consider first an economy with reversible investment. As shown by Battaglini, Nunnari, and Palfrey (2012a), the objective function of the planner's is continuous, strictly concave and differentiable and a solution of its maximization problem exists and is unique. The optimal policies have an intuitive characterization. When the accumulated level of public good is low, the marginal benefit of investing in g is high, and the planner finds it optimal to invest as much as possible: in this case $y_P(g) = W + g$ and $\sum_{j=1}^{n} x^i = 0$. When g is high, the planner will be able to reach the level of public good $y_P^*(n)$ that solves the planner's unconstrained problem:

$$y_P^*(n) = \left(\frac{n}{1-\delta}\right)^2 \tag{4.1}$$

The investment function, therefore, has the following simple structure. For $g < y_P^*(n) - W$, $y_P^*(n)$ is not feasible: the planner invests everything and $y_P(g) = g + W$. For $g \geq y_P^*(n) - W$, instead, investment stops at $y_P(g) = y_P^*(n)$. This investment function implies that the planner's economy converges to the steady state $y_P^o = y_P^*(n)$. In this steady state, without loss of generality, we can set $x^i(g) = (W + g - y(g))/n \; \forall i$.[5]

The planner's optimum for the IIE case is not very much different. The planner finds it optimal to invest all resources for $g \leq y_P^*(n) - W$. For $g \in (y_P^*(n) - W, y_P^*(n))$, the planner finds it optimal to stop investing at $y_P^*(n)$, as before. For $g \geq y_P^*(n)$, $y_P^*(n)$ is not feasible, so it is optimal to invest 0, and to set $y_P(g) = g$ This difference in the investment function for IIE, however, is essentially irrelevant for the optimal path and the steady state of the economy. Starting from any g_0 lower than the steady state y_P^*, levels of g larger or equal than $y_P^*(n)$ are impossible to reach, and the irreversibility constraint does not affect the optimal investment path.

4.1.2 Reversible Investment Economies

We first study equilibrium behavior when the investment in the public good is reversible. We focus on continuous, symmetric Markov-perfect equilibria, where all agents use the same strategy, and these strategies are time-independent functions of the state, g. A strategy is a pair $(x(\cdot), i(\cdot))$: where $x(g)$ is an agent's level of consumption and $i(g)$ is an agent's level of investment in the public good in state g. Associated with any equilibrium is a value function $v_R(g)$ which specifies the expected discounted future payoff to a legislator when the state is g. The optimization problem for agent j if the current level of public good is g, the agent's value function is $v_R(g)$, and other agents'

[5]Indeed, the planner is indifferent regarding the distribution of private consumption.

investment strategies are given by $x_R(g)$, can be represented as:

$$\max_{y,x}\left\{\begin{array}{c} x+2\sqrt{g}+\delta v_R(y) \\ s.t \quad x+y-g=W-(n-1)x_R(g) \\ W-(n-1)x_R(g)+g-y\geq 0 \\ x\leq g/n+W/n \end{array}\right\} \tag{4.2}$$

Contrary to the planner, agent j cannot choose y directly: it chooses only its level of private consumption and the level of its own contribution to the public investment. The agent, however realizes that given the other agents' level of private consumption $(n-1)x_R(g)$, his/her investment ultimately determines y. It is therefore *as if* agent j chooses x and y, provided that he satisfies the feasibility constraints. The first constraint is the resource constraint: it requires that total resources, $W+g$, are equal to the sum of private consumption, $(n-1)x_R(g)+x$, plus the public investment y. The second constraint requires that private consumption x is non negative. The third constraint requires that no agent can reduce y by more than his share g/n.

In a symmetric equilibrium, all agents consume the same fraction of resources, so agent j takes as given that in state g the other agents each consume:

$$x_R(g)=\frac{W+g-y_R(g)}{n},$$

where $y_R(g)$ is the equilibrium level of investment in state g. Substituting the first constraint of (4.2) in the objective function, recognizing that agent j takes the strategies of the other agents as given, and ignoring irrelevant constants, the agent's problem can be written as:

$$\max_{y} \left\{ \begin{array}{c} 2\sqrt{y} - y + \delta v_R(y) \\ \\ y \leq \frac{W+g}{n} + \frac{n-1}{n} y_R(g),\ y \geq \frac{n-1}{n} y_R(g) \end{array} \right\} \tag{4.3}$$

where it should be noted that agent j takes $y_R(g)$ as given.[6] The objective function shows that an agent has a clear trade off: a dollar in investment produces a marginal benefit $\frac{1}{\sqrt{g}} + \delta v'_R(y)$, the marginal cost is -1, a dollar less in private consumption.[7] The first constraint shows that at the maximum the agent can increase the investment of the other players (i.e., $\frac{n-1}{n} y_R(g)$) by $\frac{W+g}{n}$. The second constraint makes clear that at most the agent can consume his endowment W/n and his share of g, g/n.

We restrict attention to equilibria in which the objective function in (4.3) is strictly concave, and we refer to these equilibria as *concave equilibria*. Depending on the state g, the solution of (4.3) falls in one of two cases: the first case corresponds to the situation where the first constraint in (4.3) is binding, so all resources are devoted to investment in the public good. In this case, $x_R(g) = 0$, $y_R(g) = W + g$, and investment by each agent is $i_R(g) = \frac{W}{n}$. In the second case, private consumption is positive, that is, $x_R(g) > 0$, and the solution is characterized by a unique public good level y_R^* satisfying the first order equation:

$$\frac{1}{\sqrt{y_R^*}} + \delta v'_R(y_R^*) = 1 \tag{4.4}$$

In this second case, the investment by each agent is equal to $i_R(g) = \frac{1}{n}\left[y_R^* - g\right]$ and per capita private consumption is $x_R(g) = \frac{W+g-y_R^*}{n} > 0$. The first case is possible only if and only if $W \leq$

[6]Since $y_R(g)$ is the equilibrium level of investment, in a symmetric equilibrium $(n-1)y_R(g)/n$ is the level of investment that agent j expects from all the other agents, and that he/she takes as given in equilibrium.

[7]For simplicity of exposition we assume here that $v_R(g)$ is differentiable. This is essentially without loss of generality and we refer to the proofs in Battaglini, Nunnari, and Palfrey (2012a) for the details.

$y_R^* - g_R$, that is, if g is below some threshold g_R defined by: $g_R = \max\{y_R^* - W, 0\}$. We summarize this in the following proposition, which also proves the existence of an equilibrium and its uniqueness when $v_R(g)$ is strictly concave:

Proposition 1. *In the game with reversible investment, a concave equilibrium exists and it is unique. In this equilibrium, public investment is:* $y_R(g) = \min\{W + g, y_R^*\}$ *where* $y_R^*(n) = \left(\frac{n}{n-\delta}\right)^2 < y_P^*(n)$.

Proof. See Appendix C.

The public good function $y_R(g)$ is qualitatively similar to the corresponding planner's function $y_P(g)$. The main difference is that $y_R^* < y_P^*$ and $g_R < g_P$, so public good provision is typically smaller (and always smaller in the steady state). This is a dynamic version of the usual free rider problem associated with public good provision: each agent invests less than is socially optimal because he/she fails to fully internalize all legislators' utilities. Part of the free rider problem can be seen from (4.4): in choosing investment, legislators count only their marginal benefit, $u'(y) + \delta v'_R(y)$, rather than $nu'(y) + \delta n v'_R(y)$, but all the marginal costs (-1). In this dynamic model, however, there is an additional effect that reduces incentives to invest, called *dynamic free riding.* To see this, consider the value function for $g > g_R$ (where we have an interior solution):

$$\begin{aligned} v_R(g) &= W - (n-1)x_R(g) - (y_R^* - g) + 2\sqrt{y_R^*} + \delta v_R(y_R^*) \\ &= \frac{W - (y_R^* - g)}{n} + 2\sqrt{y_R^*} + \delta v_R(y_R^*) \end{aligned}$$

where the last equation follows by the fact that in a symmetric equilibrium: $x_R(g) = W - (n - 1)x_R(g) - (y_R^* - g)$. A marginal increase in g has two effects. An immediate effect, corresponding to the increase in resources available in the following period: g. But there is also a delayed effect on next period's investment: the increase in g triggers a reduction in the future investment of all the other agents through an increase in $x_R(g)$: for any level of $g > g_R$, $y_R(g)$ will be kept at y_R^*. In a symmetric equilibrium, if agent j increases the investment by 1 dollar, he will trigger a reduction in future investment by all agents by $1/n$ dollars; the net value of the increase in g for j will be only δ/n.

4.1.3 Irreversible Investment Economies

When the stock of the public good cannot be reduced, the optimization problem of an agent can be written like (4.2), but with an additional constraint: the individual level of investment cannot be negative or, in other words, each agent's private consumption cannot exceed his endowment, $x_i(g) \leq W/n$. Following similar steps as before, we can write the maximization problem faced by an agent as:

$$\max_y \left\{ \begin{array}{c} 2\sqrt{y} - y + \delta v_{IR}(y) \\ y \leq \frac{W+g}{n} + \frac{n-1}{n} y_{IR}(g), \; y \geq \frac{g}{n} + \frac{n-1}{n} y_{IR}(g) \end{array} \right\} \tag{4.5}$$

where the only difference with respect to (4.3) is the second constraint. To interpret it, note that it can be written as $y \geq g + \frac{n-1}{n} [y_{IR}(g) - g]$: the new level of public good cannot be lower than g plus the investments from all the other agents.

As pointed out in Section 2.1, when public investments are efficient, irreversibility is irrelevant for the equilibrium allocation. The investment path chosen by the planner is unaffected because the planner's choice is *time consistent*: he never finds it optimal to increase g if he plans to reduce it later. In the concave equilibrium characterized in the previous section, the investment function may be inefficient, but it is weakly increasing in the state. Agents invest until they reach a steady state, and then they stop. It may seem intuitive, therefore, that irreversibility is irrelevant in this case too. To the contrary, irreversibility destroys the concave equilibrium we characterized for reversible investment economies and induces the agents to significantly increase their investment, leading to a significantly higher unique steady state. Intuitively, the reason is that the agents no longer have to worry about the dynamic free rider problem: the irreversibility constraint creates a "commitment device" for the future; the agents know that g can not be reduced by the others (or their future selves).

Proposition 2. *In an economy with irreversible investment, there is a unique weakly concave equilibrium with associated steady state* $y^*_{IR}(n) = \left(\frac{1}{1-\delta}\right)^2$. *This steady state level is strictly greater than* $y^*_R(n)$ *and strictly smaller than* $y^*_P(n)$ *for any* $n > 1$ *and any* $\delta \in [0, 1)$.

Proposition 2 follows directly as a special case of Propositions 4 and 5 in Battaglini, Nunnari, and Palfrey (2012a); it establishes that the dynamic free rider game with irreversibility admits an equilibrium with standard concavity properties and that this game has a unique steady state. In this steady state, the public good stock is strictly smaller than the one accumulated by a benevolent planner, but strictly higher than the one accumulated in the unique concave equilibrium of RIE. Notice that this steady state, $y^*_{IR}(n) = \left(\frac{1}{1-\delta}\right)^2$, is the same level that an agent alone would accumulate and it is independent of n.

There is an intuitive explanation for Proposition 2. Because of decreasing returns, the investment in g declines over time, and so the constraint $y \geq g$ is binding in any equilibrium when g is high enough. When this happens, the agents are forced to keep the investment higher than what they would like. Since the equilibrium is inefficiently low (because the agents do not fully internalize the social benefit of g), the constraint $y \geq g$ increases expected welfare in these states. The states where the constraint $y \geq g$ is binding are typically out of equilibrium, that is, on the right of the steady state. The irreversibility constraint, however, has a ripple effect on the entire investment function. In a left neighborhood of y^*_{IR}, the constraint is not binding; still, the agents expect that the other agents will preserve their investment, so the strategic substitutability will not be too strong. A steady state lower than y^*_{IR} occurs with reversibility because the agents expect high levels of "strategic substitutability". However, the construction of such equilibrium relies on the existence of states where agents can invest negative amounts.

The investment function in the unique weakly concave equilibrium from Proposition 2 is different than the one for the reversible investment case, where the agents would either find it optimal to invest everything, or just enough to maintain the steady state. In particular, Battaglini, Nunnari, and Palfrey (2012a) show that there is a region of the state space in which the investment function $y(g)$ must solve the differential equation:

$$y'(g) = \frac{1 - \frac{n\left(1-\frac{1}{\sqrt{g}}\right)}{\delta}}{1-n} \tag{4.6}$$

The previous expression defines a simple differential equation with a solution $y(g)$, unique up to a constant. The unique weakly concave equilibrium from Proposition 2 has the following investment function:

$$y_{IR}(g) = \begin{cases} \min\left\{W+g, \widehat{y}\left(g_{IR}\right)\right\} & g \leq g_{IR} \\ \widehat{y}\left(g\right) & g_{IR} < g \leq \left(\frac{1}{1-\delta}\right)^2 \\ g & g \geq \left(\frac{1}{1-\delta}\right)^2 \end{cases} \tag{4.7}$$

where $g_{IR} = \max\left\{\min_{g\geq 0}\left\{\widehat{y}\left(g\right) \leq W+g\right\}, y^*_R(n)\right\},$ *and* $\widehat{y}\left(g\right)$ is the the unique solution of (4.6) with initial condition $\widehat{y}\left(\left(\frac{1}{1-\delta}\right)^2\right) = \left(\frac{1}{1-\delta}\right)^2$.

4.1.4 Cooperation Using Non-Stationary Strategies

We have restricted our attention to symmetric Markov perfect equilibria. However, the voluntary contribution game we study is an infinite horizon dynamic game with many subgame perfect equilibria. The Markovian assumption of stationary strategies is very restrictive and it is possible that some other equilibria can sustain more efficient outcomes through the use of history-dependent strategies (punishment and rewards for past actions). As we show below, in economies with reversible investment, the optimal solution can indeed be supported as the outcome of a subgame perfect equilibrium:

Proposition 3. *There is a $\widehat{\delta}_R \in [0, 1)$ such that, for $\delta > \widehat{\delta}_R$, the efficient investment path characterized by the optimal solution is a Subgame Perfect Nash Equilibrium of the voluntary contribution game with reversible investment.*[8]

In Appendix C, we derive nonstationary strategies for the voluntary contribution game with reversible investment whose outcome is the efficient level of public good (the optimal solution), and show that these strategies are a subgame perfect Nash equilibrium.[9]

The strategy for each agent is to allocate the optimal level of investment to public good production, $(y_P^*(g) - g)/n$, and to consume the remainder. A deviation from this investment behavior by any agent is punished by reversion to the unique concave Markov perfect equilibrium characterized in Section 2.2. This is a simple strategy that involves the harshest (individually rational) punishment for deviation from cooperation: whenever $g > y_R^*$ and a deviation is observed, the public good

[8] Looking ahead to the next section, with the parameters of the experiments, the threshold $\widehat{\delta}_R$ defined in Proposition 3 is equal to 0.80 for n=3, and 0.86 for n=5. We use $\delta = 0.75$ in all the experimental sessions, which means that the nonstationary strategies we propose cannot support the efficient steady state (144 in n=3 and 400 in n=5). With n=3, the highest steady state sustainable with these nonstationary strategies is 130, while with n=5, it is 333.

[9] Our goal is to show that the optimal solution is the outcome of some subgame perfect Nash equilibria of the game. We do not claim that the strategies proposed in the proof of Proposition 3 are the best punishment schemes, and there may be different nonstationary strategies that work for lower δ.

will revert to y_R^* and it will stay at this level for all future periods.

When investment is irreversible, the efficient outcome cannot be sustained with strategies similar to the ones proposed above for environments with reversible investment. Matthews (2012) shows that, with discounting, no subgame perfect equilibrium of a general family of dynamic contribution games is efficient, in the sense of supporting the optimal public good stock in each period. In particular, this result applies to our environment, which gives the following Proposition as corollary.

Proposition 4 *There is no $\widehat{\delta}_{IR} \in [0,1)$ such that, for $\delta > \widehat{\delta}_{IR}$ the optimal investment strategies are a Subgame Perfect Nash Equilibrium of the voluntary contribution game with irreversible investment.*

The intuition behind Proposition 4 is that the potential for punishment is significantly dampened by the irreversibility constraint. Whenever $g > y_{IR}^*$ and a deviation is observed, agents cannot disinvest down to y_{IR}^* and the harshest punishment is characterized by no investment and a constant stock in all periods following the first deviation. For any $\delta < 1$ and any steady state level $y^* > y_{IR}^*$, there exist a $g < y^*$ such that an agent prefers to deviate and consume his whole budget, rather than contributing his share to increase the stock according to the optimal investment path.

It is interesting to note that the "best" subgame perfect equilibrium (that is, the Pareto superior equilibrium from the point of view of the agents) is more efficient in RIE than in IIE. This is in contrast with the predictions of the unique concave Markov perfect equilibria analyzed above and suggests that whether we observe higher investment in RIE or in IIE crucially depends on the focal equilibrium.

4.2 Experimental Design

The experiments were all conducted at the Social Science Experimental Laboratory (SSEL) using students from the California Institute of Technology. Subjects were recruited from a pool of volunteer subjects, maintained by SSEL. Eight sessions were run, using a total of 105 subjects. No subject participated in more than one session. Half of the sessions were for Reversible Investment Economies and half for Irreversible Investment Economies. Half were conducted using 3 person committees, and half with 5 person committees. In all sessions the discount factor was $\delta = 0.75$, and the current-round payoff from the public good was proportional to the square root of the stock at the end of that period, that is, $u(g) = 2\sqrt{(g)}$. In the 3 person committees, we used the parameters $W = 15$, while in the 5 person committees $W = 20$. Payoffs were renormalized so subjects could invest fractional amounts.[10] Table 1 summarizes the theoretical properties of the equilibrium for the four treatments. It is useful to emphasize that, as proven in the previous sections, given these parameters the steady state is uniquely defined both for the RIE and IIE game and for all treatments: so the theoretical predictions of the convergence value of g is independent of the choice

[10]We do this in order to reduce the coarseness of the strategy space and allow subjects to make budget decisions in line with the symmetric Markov perfect equilibrium in pure strategies. This is particularly important for the RIE where the steady state level of the public good is 1.77 for n=3 and 1.38 for n=5, and the equilibrium level of individual investment is, respectively, 0.59 and 0.28 in the first period and 0 in all following periods.

of equilibrium.

Treatment	**n**	**W**	$\mathbf{y}^*_R$	$\mathbf{g}_{IR}$	$\widehat{y}(g)$	$\mathbf{y}^*_{IR}$	$\mathbf{y}^*_P$
RIE	3	15	1.77				
RIE	5	20	1.38				
IIE	3	15		1.77	$8 + 1.5g - 4\sqrt{g}$	16	
IIE	5	20		1.38	$6.67 + 1.42g - 3.33\sqrt{g}$	16	
Planner	3 15						144
Planner	5 20						400

Table 1: Experimental parameters and equilibrium investment functions

Discounted payoffs were induced by a random termination rule by rolling a die after each period in front of the room, with the outcome determining whether the game continued to another period (with probability .75) or was terminated (with probability .25). The $n = 5$ sessions were conducted with 15 subjects, divided into 3 groups of 5 members each. The $n = 3$ sessions were conducted with 12 subjects, divided into 4 groups of 3 members each.[11] Groups stayed the same throughout the periods of a given match, and subjects were randomly rematched into groups between matches. A match consisted of one multiround play of the game which continued until one of the die rolls eventually ended the match. As a result, different matches lasted for different lengths (that is, for

[11] One of the $N = 3$ sessions used 9 subjects.

a different number of periods). Table 2 summarizes the design.

Treatment	**n**	**# Groups**	**# Subjects**
RIE	3	70	21
RIE	5	60	30
IIE	3	80	24
IIE	5	60	30

Table 2: Experimental design

Before the first match, instructions[12] were read aloud, followed by a practice match and a comprehension quiz to verify that subjects understood the details of the environment including how to compute payoffs. The current period's payoffs from the public good stock (called *project size* in the experiment) was displayed graphically, with stock of public good on the horizontal axis and the payoff on the vertical axis. Subjects could click anywhere on the curve and the payoff for that level of public good appeared on the screen.

At the end of the last match each subject was paid privately in cash the sum of his or her earnings over all matches plus a showup fee of $10. Earnings ranged from approximately $20 to $50, with sessions lasting between one and two hours. There was considerable range in the earnings and length across sessions because of the random stopping rule.

[12] Sample instructions are reported in Appendix D.

4.3 Experimental Results

4.3.1 Public Good Outcomes

We start the analysis of the experimental results by looking at the long-run stock of public good by treatment. We consider as the *long-run* stock of public good, the stock reached by a group after 10 periods of play.[13] Table 3 compares the theoretical and observed levels of public good by treatment. In order to aggregate across groups, we use the median level of the public good from all groups in a given treatment at period 10 ($\mathbf{y}^{10}_{mdn}$). Similar results hold if we use the mean or other measures of central tendency.[14] We compare this to the stock predicted by the Markov perfect equilibrium of the game after 10 periods (y^{10}_{MP}), and to the stock accumulated in the optimal solution after 10 periods (y^{10}_{P}).

Treatment	**n**	$\mathbf{y}^{10}_{mdn}$	y^{10}_{MP}	y^{10}_{P}
Reversible Investment (RIE)	3	4	1.38	144
Reversible Investment (RIE)	5	7.21	1.77	200
Irreversible Investment (IIE)	3	71.88	10.91	144
Irreversible Investment (IIE)	5	91.75	10.57	200

Table 3: Long-Run Stock of Public Good, Theory vs. Results by Treatment

How do groups get to these stocks of public good? Figure 1 gives us a richer picture, showing the time series of the stock of public good by treatment.[15] The horizontal axis is the time period and the vertical axis is the stock of the public good. As in Table 3, we use the median level of the public good from all groups in a given treatment. Superimposed on the graphs are the theoretical

[13]In the experiment, the length of a match is stochastic and determined by the roll of a die. No match lasted longer than 17 periods and we have very few observations for periods 11-17.

[14]The statistical tests in the remainder of this section compare average stocks between different treatments using t-tests and their underlying distributions using Wilcoxon-Mann-Whitney tests.

[15]These and subsequent figures show data from the first ten periods. Data from later periods (11 for IIE with $n = 5$, 11-13 for RIE with $n = 5$ and $n = 3$, and 11-17 for IIE with $n = 7$) are excluded from the graphs because there were so few observations. The data from later periods are included in all the statistical analyses.

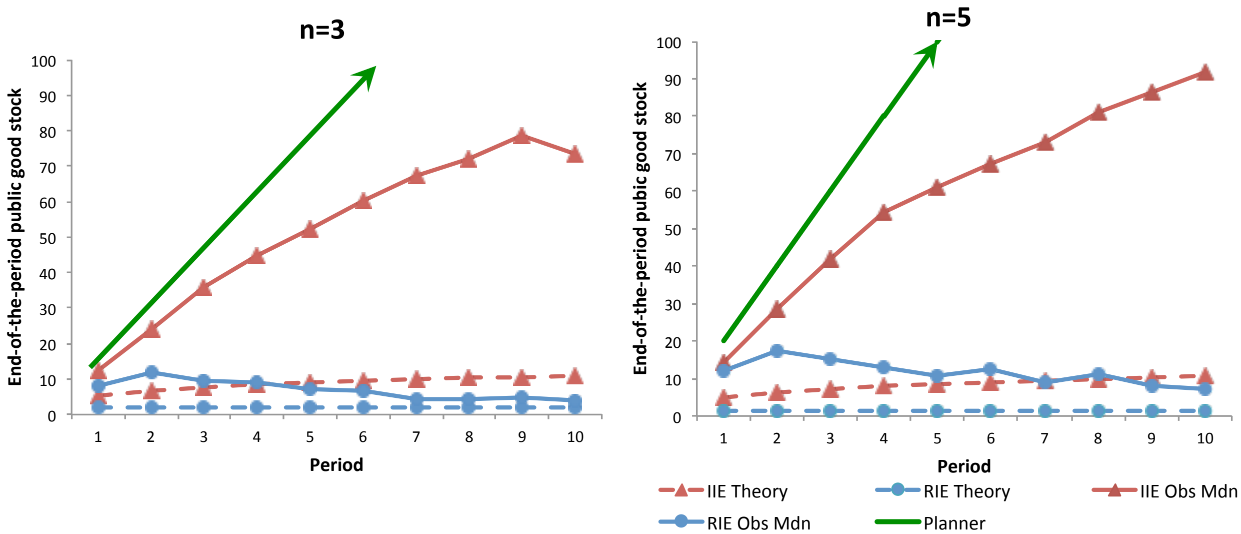


Figure 4.1: Median time paths of the stock of g, RIE vs. IIE

time paths (represented with solid lines), corresponding to the Markov perfect equilibria and to the optimal solution.

Table 3 and Figure 1 exhibit several systematic regularities, which we discuss below in comparison with the theoretical time paths.

FINDING 1. Irreversible investment leads to higher public good production than reversible investment. According to t-tests and Wilcoxon-Mann-Whitney tests[16], the average stock of public good is significantly lower in RIE than in IIE in every single period for both group sizes. This difference is statistically significant at the 1% level (p¡0.01) for periods 1-10 for both group sizes. Not only are the differences statistically significant, but they are large in magnitude. The median stock of public good is around four times greater in the IIE treatment than in the RIE

[16]The null hypothesis of a t-test is that the averages in the two samples are the same. The null hypothesis of a Wilcoxon-Mann-Whitney test is that the underlying distributions of the two samples are the same. We are treating as unit of observation a single group.

treatment, averaged across all periods for which we have data (38.75 in IIE vs. 10.75 in RIE for $n = 5$ and 44.25 in IIE vs. 7.25 in RIE for $n = 3$). The differences between the two economies are relatively small in the initial period, but they increase sharply as more periods are played. By period 10, the differences are very large (71.88 vs. 4 for $n = 3$, and 91.75 vs. 7.21 for $n = 5$).

FINDING 2. Both reversible and irreversible investment lead to significantly inefficient long-run public good levels. The optimal steady state is y*=400 for $n = 5$ and y*=144 for $n = 3$, and the optimal investment policy is the fastest approach: invest W in every period until y* is achieved. After 10 periods, the median stock of public good achieved with the optimal investment trajectory is 200 for $n = 5$ and 144 for $n = 3$. In the experiments, the median stock of public good levels out at about 7 ($n = 5$) or 4 ($n = 3$) under reversible investment economies, while it keeps growing, but at an inefficiently slow pace, under irreversible investment. The median stock averages 9.29 in periods 7-10 in RIE with $n = 5$, 4.34 in periods 7-10 in RIE with $n = 3$, 83.26 in periods 7-10 in IIE with $n = 5$, and 72.70 in periods 7-10 in IIE with $n = 3$. In all treatments the average stock of public good in the last periods (rounds 8 on) is significantly smaller than the level predicted by the optimal solution (the level attainable investing W each period) according to the results of a t-test on the equality of means (p¡0.01).

FINDING 3. Public good accumulation is higher in five members groups than in three member groups. This difference, however, is statistically significant only in the initial periods. For the same accumulation mechanism (reversible or irreversible investment), the average and median stock of public good is higher with five members groups than with three members groups in every single period. However, this difference is small in magnitude (especially for the earlier periods and for the reversible investment games) and, according to t-tests[17], statistically

[17] Similar results are obtained using Wilcoxon-Mann-Whitney tests.

significant at conventional levels (p¡0.05) only for the first two periods in RIE, and the first four periods in IIE. This is in line with the Markovian equilibria discussed in the previous section, which predict small differences between the two group sizes. In RIE, the stock is predicted to converge quickly to similar steady state levels (1.77 and 1.38). In IIE, while the steady state levels are predicted to be exactly the same (16), the equilibrium investment trajectory is somewhat slower with larger groups. However, the differences induced by the different group sizes are small, with the predicted stock after 10 periods equal to 10.26 with five members groups and 10.64 with three members groups.

Because of the possibility of nonstationary equilibria it is natural to expect a fair amount of variation across groups. Figure 1, by showing the median time path of the stock of public good, mask some of this heterogeneity. Do some groups reach full efficiency? Are some groups at or below the equilibrium? We turn next to these questions.

Figure 2 illustrates the variation across groups by representing, for each period, the first, second and third quartile of investment levels for RIE with $n = 5$ (panel (a)), IIE with $n = 5$ (panel (b)), RIE with $n = 3$ (panel (c)), and IIE with $n = 3$ (panel (d)) games. The continuous line represents the median, while the dashed lines represent the range interquartile.

There was remarkable consistency across groups, especially considering this was a complicated infinitely repeated game with many non-Markov equilibria.With irreversible investment, many groups invested significantly more heavily than predicted by the Markov perfect equilibrium, but this was not enough to achieve efficient levels of the public good in the long-run, as nearly always such cooperation fell apart in later periods. The most efficient group in IIE with $n = 5$ invested 98% of W in the first period and W in periods 2-6, resulting in a public good level of 119.5. In the remaining four periods, group investment slowed down because of the contagious defection of some

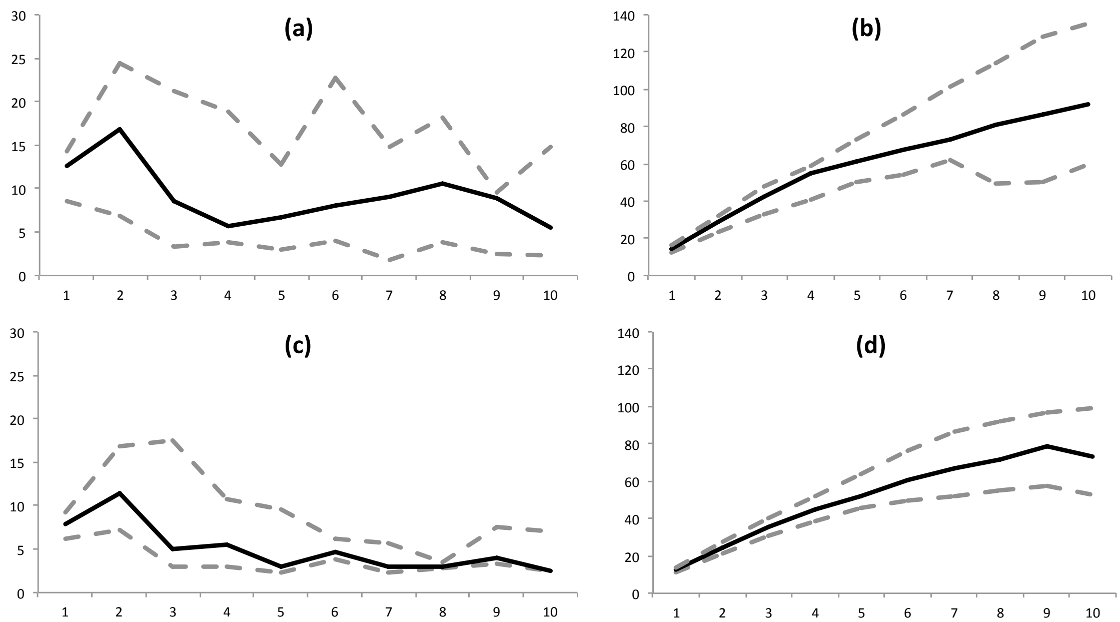


Figure 4.2: Quartiles of time paths of the stock of g, all treatments

of his members: in period 7, one member invests zero; in period 8, two members invest zero, and in periods 9-10, three members invest zero (with average investment in the last three periods at 49% of W and final stock at period 10 of 157.8). Even this very successful group, started consuming resources for private investment well short of the efficient level (400). In RIE with $n = 5$, the most efficient committee in the early periods invested W in each of the first 2 periods, resulting in a public good level of 40. In the following period, one member disinvested the maximum allowed (that is, 1/5 of the stock). The same investment behavior was followed by two members in the fourth period and, finally, in the fifth and final period, every member disinvested his share of the public good, bringing the stock to zero. We observe similar patters for the most efficient groups with $n = 3$.

These findings are perhaps surprising since, from Proposition 3, we know that, for the parameters of the experiment, almost efficient levels of the public good can be supported as the outcome of the RIE game using nonstationary strategies.[18] In the IIE games, on the other hand, the optimal solution cannot be supported by any subgame perfect equilibrium with nonstationary strategies when there is discounting. This is in stark contrast with the unique Markov perfect equilibria derived in Sections 2.2 and 2.3, which predict the opposite comparative static: the long run level of the public good is predicted to be 10 times as large with irreversible investment than with reversible investment.

Figure 1 and 2, therefore, make clear that the predictions of the Markov perfect equilibrium are substantially more accurate than the prediction of the "best" subgame perfect equilibrium (that is, the Pareto superior equilibrium from the point of view of the agents). This observation may undermine the rationale for using the "best equilibrium" as a solution concept.

4.3.2 Investing Behavior

So far, we have presented results for the public good stock accumulated by each group. In this section, we analyze the data at a finer level, using the investment decisions of each single individual in each group.

How much do individual agents invest in the public good? Figure 3 shows the time series of the median investment in the public good by treatment. The horizontal axis is the time period and the vertical axis is the investment in the public good. The maximum amount each agent can allocate to investment is the same in each period, and it is given by w/n, which is equal to 5 for the three-members groups and to 4 for the five-members groups. The minimum amount each agent can invest is always zero in the irreversible investment treatment, but it depends on the stock at the

[18] With the parameters of the experiment, the public good stock sustainable with the nonstationary strategies proposed in the proof of Proposition 3 is 141 (vs. an efficient level of 150) for three members groups, and 351 (vs. an efficient level of 400) for five members groups.

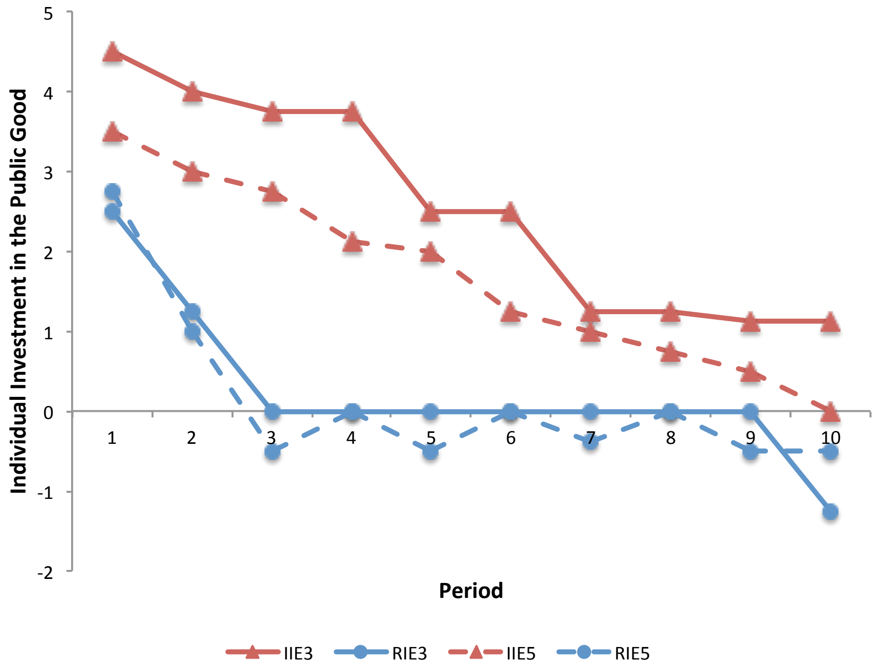


Figure 4.3: Median time paths of individual investment

beginning of the period in the reversible investment treatment (since each agent can disinvest up to g/n units of the public good). For each period, we use the median level of individual investment from all subjects in a given treatment. Similar results hold if we use the mean or other measures of central tendency.

Figure 3 shows a series of interesting patterns. First, the median individual investment is always higher with irreversible investment than with reversible investment in periods 1-10. Second, the level of investment is decreasing, with median investment converging quickly to values around zero for the reversible investment economies and steadily decreasing towards zero for the irreversible economies.

How do these levels of individual investment compare to the theoretical predictions? The median

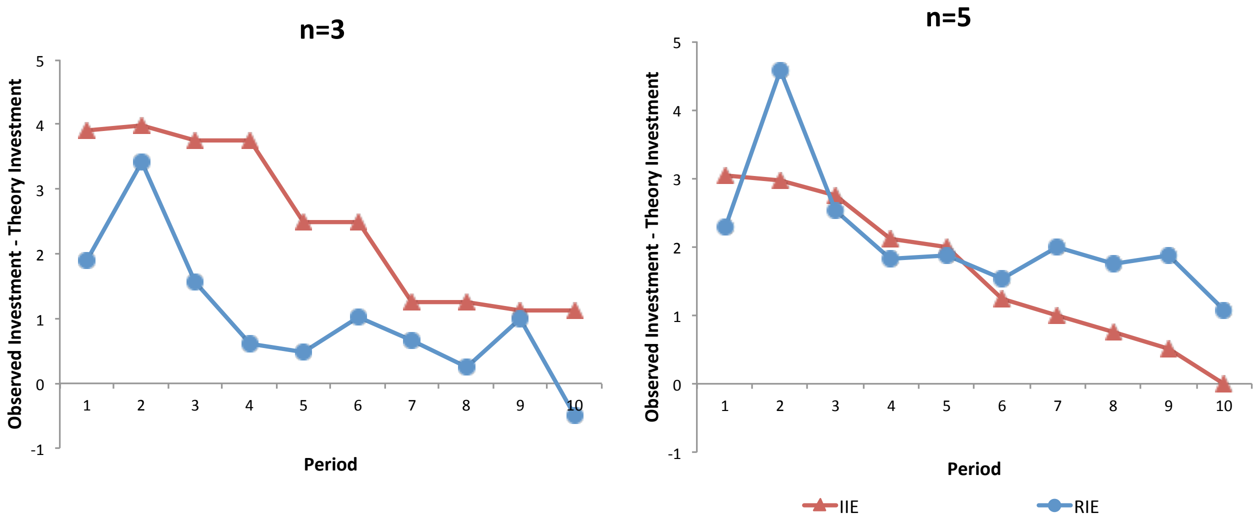


Figure 4.4: Time paths of median difference between observed and theoretical investment

time paths from Figure 3 are qualitatively in line with the predicted time paths: with reversible investment, the theory predicts positive investment only in the first period (when the equilibrium steady state is reached) and zero investment from the second period on; with irreversible investment, the theory predicts positive investment in each period, but at a monotonically decreasing pace (with convergence to the equilibrium steady state only asymptotically). There are, however, some differences between the finer details of the theoretical predictions and the data. We observe overinvestment in the early periods: while individual investment is predicted to be less than 1 unit in the first period for all treatments, we observe medians between 2.5 and 4.5. In the reversible economies, this overinvestment is corrected in the later periods: the median investment falls sharply to zero and a large fraction of individuals disinvests, with higher early overinvestment followed by higher disinvestment. We discuss these observations in detail below.

The game we study is a dynamic game with an evolving state variable. In this game, the strategic incentives in each period are possibly different and determined by the level of the state variable and by subjects' expectations on other subjects' behavior. It follows that, to better compare the observed level of investment with the theoretical predictions, we need to take into account the state variable faced by each agent when making an allocation decision, that is, the stock of the public good at the beginning of a period. For each subject in each period, we calculate the difference between his observed behavior and the investment predicted by the theory given the public good stock in his group in that period. Figure 4 shows the time series of the median of this difference. This series starts out significantly above zero for all treatments but decreases as more periods of the same match are played, suggesting that subjects' decisions respond to the evolution of the state variable, with their investment behavior closely matching the predictions of the unique concave Markovian equilibrium for later periods. Notice that this pattern leads to public good outcomes that are in line with the equilibrium steady states for reversible economies, but not for irreversible economies: in the former, subjects can correct the initial overinvestment with negative investment, while in the latter the equilibrium investment for any level above the steady state (16) is bound to be zero and the initial overinvestment persists. We summarize these findings below.

FINDING 4. In both treatments, there is overinvestment relative to the equilibrium in the early periods. This is followed by negative investment approaching the theoretical predictions in RIE, while the overinvestment decreases but persists in IIE. In RIE, the median investment in the first two periods are (7.88=0.53W, 4.13=0.28W) for three members groups, and (12.63=0.63W, 4.88=0.24W) for five members groups. As a result, the median public good stock by the end of period 2 equals, respectively, 11.38, and 16.75. This compares with equilibrium investment policies in the first two periods equal to (1.77, 0) for $n = 3$, and (1.38, 0) for $n = 5$, and a predicted stock equal to, respectively, 1.38 and 1.77. In IIE, on the other hand, the median investment in the first two periods are (12.5=0.63W, 11.63=0.58W) for three members groups, and (14.03=0.71W, 14.03=0.71W) for five members groups. As a result, the median public good stock by the end of period 2 equals, respectively, 24.25, and 28.5. This compares with equilibrium investment policies in the first two periods equal to (1.77, 3.56) for $n = 3$ and (1.38, 3.34) for $n = 5$, and a predicted stock equal, respectively, to 5.33 and for 4.72. Thus, in all treatments, committees overshoot the equilibrium in early periods by a factor of ten (R3 and R5), five (IR3), and six (IR5).[19]

In RIE, this overshooting is largely corrected in later periods via disinvestment. When investment is reversible, convergence of the public good stock is close to equilibrium, with the difference between the median public good levels and the equilibrium public good levels in the last 4 periods of data measuring less than 3 units of the public good for three members groups (4.34 vs. 1.77) and less than 4 for five members groups (5.50 vs 1.38). With irreversible investment, investment remains positive but is monotonically decreasing with periods of play (in the same match): the median investment in periods 7-10 is 3.8 (=0.25W) with $n = 3$ and 6.63 (=0.33W), with the minimum median investment reached in period 10 (4.75=0.24W). Given the public good stock by the end of period 2 is already above the predicted stead state level (16), the positive - albeit slower - investment flow in the following periods brings the long-run level of public good to be six (91.75 vs. 16 for IR5) and five times (73.34 vs. 16 for IR3) larger than predicted.

We now turn to a descriptive analysis of the individual allocations between investment in the public good and current consumption good, as a function of period of play (within a match) and accumulation mechanism.

We break down the investment decisions into 3 canonical types: (1) *Positive Investment*; (2) *Zero Investment*; and, for RIE, (3) *Negative Investment*. The first category is further broken down

[19] The difference between the average investment in the early periods and the predicted investment in these same periods is statistically significant at the 1% level for all treatments.

by whether investment in the public good accounts for the whole budget, most of the budget (but less than W), or a minority of the budget. Similarly, the third category is further broken down by whether the negative investment is the maximum allowed by the mechanism ($= g/n$) or a smaller amount.

	RIE 3					IIE 3				
Investment Type	**ALL**	**R1**	**R2-4**	**R5-7**	**R8-10**	**ALL**	**R1**	**R2-4**	**R5-7**	**R8-10**
INV > 0	67.0	100	57.8	44.7	39.2	82.8	100	96.4	77.9	59.5
* $I = W$	5.1	12.4	2.6	-	-	27.7	47.9	35.7	21.4	9.5
* $I \in (.5W, W)$	16.2	31.4	13.9	3.5	2.0	24.2	33.3	34.4	20.3	12.3
* $I \in (0, .5W]$	45.7	56.2	41.3	41.2	37.3	30.9	18.8	26.4	36.3	37.7
INV $= 0$	6.7	-	5.9	14.9	17.7	17.2	-	3.6	22.1	40.5
INV < 0	26.4	-	36.3	40.4	43.1	-	-	-	-	-
* $I \in (0, -g/n)$	21.6	-	36.3	40.4	43.1	-	-	-	-	-
* $I = -g/n$	4.8	-	6.9	6.1	11.8	-	-	-	-	-

Table 4: Individual Investment Types, $n = 3$, # Observations: 705 for RIE, 1716 for IIE

	RIE 5					IIE 5				
Investment Type	**ALL**	**R1**	**R2-4**	**R5-7**	**R8-10**	**ALL**	**R1**	**R2-4**	**R5-7**	**R8-10**
INV > 0	51.5	82.3	45.3	35.0	35.8	83.1	96	89.8	76.7	60.9
* $I = W$	26.4	47.7	23.8	14.2	15.8	33.1	46.3	37.5	25.0	18.2
* $I \in (.5W, W)$	3.5	4.7	4.6	1.3	0.8	16.5	19.0	20.0	13.3	9.3
* $I \in (0, .5W]$	21.5	30.0	17.0	19.6	19.2	33.5	30.7	32.4	38.3	33.3
INV $= 0$	12.9	17.7	8.8	13.3	14.2	17.0	4.0	10.2	23.3	39.1
INV < 0	35.7	-	45.9	51.7	50.0	-	-	-	-	-
* $I \in (0, -g/n)$	29.0	-	37.9	39.6	40.8	-	-	-	-	-
* $I = -g/n$	6.7	-	8.0	12.1	9.2	-	-	-	-	-

Table 5: Individual Investment Types, $n = 5$, # Observations: 1230 for RIE, 1740 for IIE

Tables 4 and 5 show the breakdown of investment decisions for the four treatments. In each table, the first column lists the various investment types. The second (for RIE) and sixth (for IIE) columns lists the proportion of each allocation type when we lump together all observations. Columns 3 through 5 (for RIE), and 7 through 9 (for IIE) show how these proportions evolve within a match.

FINDING 5. In all treatments, most allocations involve positive investment in the public good. The proportion of decisions that belong to this category decreases with the period of play (within a single match). In RIE, a significant proportion of allocations involve negative investment. The proportion of decisions that belong to this category increases with the period of play (within a single match). In all treatments, most allocations had a positive amount of investment. In RIE3 and RIE5 this investment type accounts, respectively, for 67%, and 51.5% of all decisions; in IIE3 and IIE5, this type accounts, respectively, for 82.8% and 83.1%. Allocations with zero or negative investment occurred 33.1% of the time in RIE3 groups, 48.6% of the time in RIE5 groups, but only 17.2% of the time in IIE3 groups and 17.0% of the time in IIE5 groups. The difference is mostly due to the negative investment allocations (which are not allowed in IIE). In contrast to the data, the Markov perfect equilibrium allocations should have been concentrated in the category "$I = 0$" for RIE and "$I \in (0, .5W)$" for IIE. However, notice that, in IIE, "$I \in (0, .5W)$" is the most common proposal type and accounts for around 1/3 of all allocations.

4.3.3 The Effect of Experience

Within the same match, subjects' investing behavior gets closer to the predictions as more periods are played. It is therefore natural to ask whether we observe a similar pattern across matches. Do subjects choose allocations closer to the predictions of the Markov equilibria when they are more experienced? Or do they still overinvest in early periods and reduce investment in later periods, even after many matches of the same (multi-period) game?

To answer these questions, we regress the amount invested on the investment predicted by the unique Markov equilibrium (given the public good stock at the beginning of the period), and the number of periods played in the match. We add to these independent variables, the interaction between the two, to test whether there is any significant difference in the correlation between observed and predicted investment as experience grows. Table 6 shows OLS estimates for all treatments. An observation is a single subject's allocation decision in a single period.[20]

	(1)	(2)	(3)	(4)
Treatment	*RIE, 3*	*RIE, 5*	*IIE, 3*	*IIE, 5*
Predicted Investment	0.38* (0.16)	0.58** (0.14)	0.04 (0.03)	0.03** (0.01)
Match #	-0.04 (0.03)	-0.03 (0.03)	0.14** (0.03)	0.05* (0.02)
Match #*Pred Inv	0.01 (0.03)	-0.03 (0.02)	0.01 (0.01)	-0.01* (0.01)
Constant	1.71** (0.33)	1.47** (0.25)	1.90** (0.22)	1.90** (0.22)
R-squared	0.1128	0.1004	0.0953	0.0059
Observations	705	1230	1716	1740

Table 6: OLS estimates. Dependent variable: Observed investment. Standard errors clustered by subject in parentheses; * significant at 5% level; ** significant at 1% level

FINDING 6. There is no evidence of an impact of experience, in any of the treatments. The estimates from Table 6 suggest that the investment decisions are not affected by

[20] We cluster standard error by subjects to take into account possible correlations among decisions taken by the same individuals. We also tried the standard error correction by clustering by groups, and Tobit estimates to take into account the upper bound on contributions given by w/n. The results are unchanged.

experience, at least not in the sense of playing closer to the theoretical predictions. In RIE, as subjects play more matches within the same session their endowment allocation is not significantly altered. In IIE, as subjects play more matches within the same session, they slightly increase their investment in the public good. This change however does not increase the correlation between the observed investment and the investment predicted by the unique Markov equilibrium of the game.

4.3.4 Test for Markovian Behavior

The final questions we attempt to address are: To what extent are the models we use adequate to study this problem? What equilibrium concepts should be used? This is a particularly important question since, depending on the equilibrium concept, we can have very different predictions for the same model. While it is difficult to identify the equilibrium adopted by players, the analysis of public good outcomes and investing behavior provides some interesting insights. As discussed above, we observe a consistent pattern of behavior across groups, despite the fact that we have multiplicity of potential equilibria; the investing behavior is correlated to the evolution of the stock in a way predicted by the theory; and, at least for RIE, the long term public good outcomes are close to the equilibrium steady states.

To further pursue this question, we construct a more direct test of the Markovian restriction, that is, of the assumption that players are forward-looking and condition their strategy only on the stock of the public good at the beginning of the period, irrespective of the histories. In particular, we conduct a one-period version of the reversible investment game, where the payoffs from the public good stock are complemented by the equilibrium value functions of the unique concave Markov perfect equilibrium of the game. In each one-period game, agent j receives the following payoff:

$$U^j(x^j, y) = x^j + 2\sqrt{y} + \delta v_R(y),$$

where x^j is the private consumption of agent j, y is the end-of-period public good stock, and $\delta v_R(y)$ is the discounted equilibrium value function from the dynamic game with reversible investment. In each experimental session, subjects play for 40 matches. Contrary to the dynamic game, the length of each match is known and equal to one period. At the end of each one-period match, subjects are reshuffled into new groups and the public good stock starts out at a (potentially different) exogenous level. We use eight different g_0, to elicit an investment strategy (as a function of the state variable) comparable to the one observed in the fully dynamic game.[21] Table 7 below summarizes the experimental design.

n	**W**	δ	**# Groups**	**# Subj**	**# Matches**	g_0
3	15	0.75	80	24	40	0, 1.25, 2.5, 3.75, 5, 6.25, 7.5, 8.75
5	20	0.75	60	30	40	0, 1.25, 2.5, 3.75, 5, 6.25, 7.5, 8.75

Table 7: Experimental design, one-period reduced form treatments.

In the one-period reduced form treatments, the unique equilibrium of the game prescribes the same investment level predicted for the fully dynamic game under the Markovian assumption that subjects condition their strategies only on the public good stock. While there is no other equilibrium

[21] The beginning-of-period public good stocks we use in this one-period reduced form treatment are 0, 1.25, 2.5, 3.75, 5, 6.25, 7.5, 8.75. In each experimental session, each of these beginning-of-period stock is used in five different matches, in random order, for a total of forty matches. The range $[0 - 8.75]$ covers around 75% of observations in the dynamic game with three-members groups and around 55% of observations in the dynamic game with five-members groups.

in this one-period game, in the fully dynamic game there is a plethora of different subgame perfect equilibria that can sustain higher level of investment with nonstationary strategies. Therefore, if we observe an identical behavior in the two treatments, we can consider this as evidence of Markovian strategies in the fully dynamic game. On the other hand, we can attribute any difference in behavior to the nonstationary strategies that can arise in a repeated game.

Figure 5 illustrates the median individual investment as a function of the initial stock for the one-period reduced form games described above and for the fully dynamic games.[22]

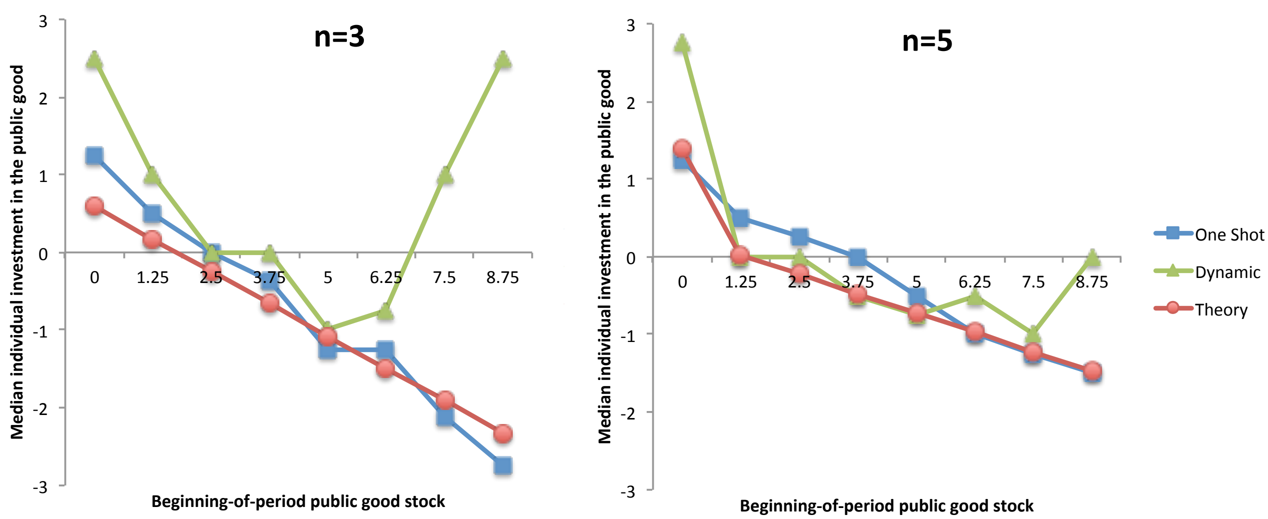


Figure 4.5: Investment as a function of beginning-of-round stocks, reduced form vs. dynamic

FINDING 7. In RIE, there is evidence of Markovian, forward-looking behavior. For three-members groups, investment is significantly higher in the dynamic treatment for initial

[22]Since the beginning-of-round stock in the dynamic games is endogenous and does not necessarily match the values used in the one-round games, for these games we use a weighted median of investment levels for all periods-groups that started with a public good stock in a 6 experimental units interval around the starting size used in the one-round games. For example, the median investment corresponding to a beginning-of-round stock of 5 is computed as the weighted median investment from all periods-groups starting at a stock between 4.25 and 5.75. This allows us to have a comparable number of observations between one-round and dynamic games. The results are the same when we use intervals of 8 or 10 experimental units.

stocks of 0, 6.25, 7.5, and 8.25, and statistically indistinguishable for the remaining initial stocks. For five-members groups, investment is significantly higher in the dynamic treatment for initial stocks of 0, 6.25, and 8.75; it is significantly higher in the reduced form treatment for initial stocks of 1.25, 2.5, 3.75, and 5; and it is statistically indistinguishable for $g_0 = 3.75$. While there is some significant difference, these differences are small in magnitude (with the exception of initial stocks greater than 5 for $n = 3$), and we cannot conclude that investment is higher in the fully dynamic game than in the reduced form game (as a consequence of nonstationary strategies). Regarding the high investment in the dynamic treatment for three-member groups and stocks greater than 5, this is due to a few groups who invested significantly more heavily than predicted by the Markov perfect equilibrium, but this only happened rarely and most of the observations from the dynamic treatment (where the initial public good stock is endogenous) have a beginning-of-period stock smaller than 6.25.[23]

We, thus, conclude that observed behavior is well approximated by the predictions of a purely forward looking Markov equilibrium, rather than by an equilibrium in which agents look back at the past to punish uncooperative behavior (or reciprocate cooperative behavior) by other members of the group.

4.4 Discussion and Conclusions

This paper investigated the dynamic accumulation process of a durable public good in a voluntary contribution setup. Despite the fact that most, if not all, public goods are durable and have an

[23]Since the beginning-of-period stock in the dynamic treatment is endogenous we have a reduced number of observations for these high values: we use only 10 groups to compute the weighted median investment for a starting stock of 8. The remaining 60 groups never accumulated these levels of public good. The beginning-of-period stock is smaller than 6.25 in 60% of observations (regardless of period number). The average beginning-of-period public good stock in rounds 8-10 (that is, the long run level of public good) is 4.6.

important dynamic component, very little is known on this subject, both from a theoretical and empirical point of view. We attempt to provide some initial empirical findings about voluntary contribution behavior with durable public goods.

We have considered two possible cases: economies with reversible investments (RIE), in which in every period individual investments can either be positive or negative; and economies with irreversible investments (IIE), in which the public good cannot be reduced. Reversibility is an important feature of many public goods problems (for example, common pool problems), which is completely missed by static analysis. We also have a secondary treatment dimension: we compare three-members and five-members groups. For all treatments, we have characterized the steady states and the accumulation paths that can be supported by the optimal solution and by the unique symmetric concave Markov equilibrium.

We have highlighted three main results. First, in line with the comparative static predictions, irreversible investment leads to significantly higher public good production than reversible investment. With reversibility, the dynamic dimension exacerbates the free rider problem present in static public good provision: if an agent contributes above the equilibrium levels, not only this reduces the future contributions by all agents, but it triggers negative investment by other agents that transform part of the public good stock in private consumption. On the other hand, the irreversibility constraint creates a *commitment device* and reduces the strategic substitutability of contributions.

Second, we have shown that, in both treatments, there is overinvestment in the early periods, compared to the equilibrium investment levels. In the treatment with reversibility, this is followed by a significant reversal, with the stock of public good gradually declining in the direction of the equilibrium steady state. When disinvestment is not feasible, investment steadily decreases but the initial overinvestment cannot be corrected and the long run level of the public good remains above the equilibrium steady state.

Finally, we have proposed a novel experimental methodology to test the assumption that subjects' strategies in this complex infinite-horizon game depend only on the state variable, that is, the accumulated level of the public good. We have shown that, for the reversible investment treatment, there is evidence of Markovian, forward-looking behavior.

This is the first experimental study of the dynamic accumulation process of a durable public good. Our design was intentionally very simple and used a limited set of treatments. As a consequence, there are many possible directions for the next steps in this research. The theory has interesting comparative static predictions about the effect of other parameters that we have not explored in this work, such as: the discount factor; the depreciation level; preferences; and endowments. For example, a higher discount factor increases both the optimal steady state and the equilibrium steady state of the durable public good for all values of n and for both reversible and irreversible economies. For similar reasons, positive depreciation in the public good technology leads to a decrease in the steady state of the Markov equilibrium studied here. Among these extensions, it would be particularly interesting to run experiments that allow a closer comparison with the results from the static literature. This can be done in a number of different ways: for example, experiments with a finite and known horizon of one period (that is, $\delta = 0$), or experiments with full depreciation of the stock at the end of each period and an infinite horizon (that is, $\delta > 0$).

Moreover, our model and experimental design does not consider different rules for negative investment (for example, allowing subjects to disinvest unilaterally up to the whole stock and adopting a rationing rule to keep a nonnegative level of public good), or the effect of a completion benefit at a specified accumulation threshold. We have also limited the analysis to voluntary contribution mechanisms that turn out to be highly inefficient, both in theory and in practice. Battaglini, Nunnari, and Palfrey (2012b) study how centralized mechanisms fare in providing durable public goods and show that efficiency increases with the majority rule required to approve an allocation decision. It would be interesting to consider different decentralized mechanisms and explore which ones are more efficient for the provision of durable public goods.

Finally, it would be interesting to complement our novel direct test of the Markovian assumption running the reduced form experiments for the irreversible investment economies, or applying this innovative experimental methodology to different infinite horizon games to explore the relative importance of forward-looking behavior and history-dependent strategies in different contexts.

Appendices

Appendix A

Proofs of Chapter 1

Proof of Proposition 1

The results of Proposition 1 follow from the existence of a symmetric MPE with the following minimal winning coalition proposal strategies for all $\mathbf{s} \in \boldsymbol{\Delta}$, where $s_1 \geq s_2$:

- Case A $\left(s_1 \leq 1 - \frac{3-\delta}{3-2\delta} s_2,\ s_1 \geq \frac{3-\delta}{3-2\delta} s_2\right)$:

$$\begin{aligned} \mathbf{x}^v &= [1-d_2, 0, d_2], \mathbf{x}^1 = [d_v, 1-d_v, 0], \mathbf{x}^2 = [d_v, 0, 1-d_v] \\ d_v &= s_v - \frac{\delta s_2}{3-2\delta} \\ d_2 &= \frac{\delta}{3-2\delta} s_1 + \frac{(3-\delta)}{(3-2\delta)} s_2 \end{aligned}$$

- Case B $\left(\ s_1 > 1 - \frac{3-\delta}{3-2\delta} s_2,\ s_1 \geq \frac{27-27\delta+3\delta^2+\delta^3}{(3-2\delta)(3-\delta)^2} s_2 + \frac{\delta^2}{(3-\delta)^2}\right)$:

$$\begin{aligned} \mathbf{x}^v &= [1-d_2, 0, d_2], \mathbf{x}^1 = [d_v, 1-d_v, 0], \mathbf{x}^2 = [d_v, 0, 1-d_v] \\ d_v &= 0 \\ d_2 &= \frac{9-12\delta+3\delta^2}{(3-2\delta)^2} s_2 + \frac{\delta}{(3-2\delta)} \end{aligned}$$

- Case C $\left(s_1 > 1 - \frac{3-\delta}{3-2\delta} s_2,\ s_1 < \frac{27-27\delta+3\delta^2+\delta^3}{(3-2\delta)(3-\delta)^2} s_2 + \frac{\delta^2}{(3-\delta)^2} \right)$:

$$
\begin{aligned}
\mathbf{x}^v &= \begin{cases} [1-d_2, d_2, 0] & \text{w/ Pr} = \mu_v^C \\ [1-d_2, 0, d_2] & \text{w/ Pr} = 1-\mu_v^C \end{cases}, \mathbf{x}^1 = [d_v, 1-d_v, 0], \mathbf{x}^2 = [d_v, 0, 1-d_v] \\
d_v &= 0 \\
d_2 &= \frac{9-12\delta+3\delta^2}{(3-2\delta)^2} s_2 + \frac{\delta}{(3-2\delta)} \\
\mu_v^C &= \frac{(-27+36\delta-15\delta^2+2\delta^3)s_1}{2\delta((9-12\delta+3\delta^2)s_2-2\delta^2+3\delta)} + \frac{(27-27\delta+3\delta^2+\delta^3)s_2+3\delta^2-2\delta^3}{2\delta((9-12\delta+3\delta^2)s_2-2\delta^2+3\delta)}
\end{aligned}
$$

- Case D $s\left(_1 \leq 1 - \frac{3-\delta}{3-2\delta} s_2,\ s_1 < \frac{3-\delta}{3-2\delta} s_2 \right)$:

$$
\begin{aligned}
\mathbf{x}^v &= \begin{cases} [1-d_2, d_2, 0] & \text{w/ Pr} = \mu_v^D \\ [1-d_2, 0, d_2] & \text{w/ Pr} = 1-\mu_v^D \end{cases}, \mathbf{x}^1 = [d_v, 1-d_v, 0], \mathbf{x}^2 = [d_v, 0, 1-d_v] \\
d_v &= s_v - \frac{\delta s_2}{3-2\delta} \\
d_2 &= \frac{\delta}{3-2\delta} s_1 + \frac{(3-\delta)}{(3-2\delta)} s_2 \\
\mu_v^D &= \frac{3}{2} \frac{(-3+2\delta)s_1 + (3-\delta)s_2}{\delta(\delta s_1 + (3-\delta)s_2)}
\end{aligned}
$$

It is tedious but straightforward to check that, if players play the proposal strategies in cases A-D and these proposals pass, their continuation values are as follows:

- Case A

$$
\begin{aligned}
v_v(\mathbf{s}) &= \frac{1}{1-\delta} - \frac{2-\delta}{(3-\delta)(1-\delta)} s_1 - \frac{1}{(1-\delta)} s_2 && \text{(A.1)} \\
v_1(\mathbf{s}) &= \frac{(3-3d+\delta^2)}{(3-\delta)^2(1-\delta)} s_1 + \frac{(3-\delta)}{(3-\delta)^2(1-\delta)} s_2 && \text{(A.2)} \\
v_2(\mathbf{s}) &= \frac{(3-2\delta)}{(3-\delta)^2(1-\delta)} s_1 + \frac{(6-5\delta+\delta^2)}{(3-\delta)^2(1-\delta)} s_2 && \text{(A.3)}
\end{aligned}
$$

- Case B

$$\begin{aligned}
v_v(\mathbf{s}) &= \frac{1}{(1-\delta)(3-\delta)} - \frac{(3-4\delta+\delta^2)}{(3-2\delta)(1-\delta)(3-\delta)}s_2 \\
v_1(\mathbf{s}) &= \frac{(3\delta-4\delta^2+\delta^3)}{(3-\delta)^2(1-\delta)(3-2\delta)}s_2 + \frac{(9-15\delta+9\delta^2-2\delta^3)}{(3-\delta)^2(1-\delta)(3-2\delta)} \\
v_2(\mathbf{s}) &= \frac{(3-2\delta)}{(3-\delta)^2(1-\delta)} + \frac{(3-4\delta+\delta^2)}{(3-\delta)^2(1-\delta)}s_2
\end{aligned}$$

- Case C

$$\begin{aligned}
v_v(\mathbf{s}) &= \frac{1}{(1-\delta)(3-\delta)} - \frac{(3-4\delta+\delta^2)}{(3-2\delta)(1-\delta)(3-\delta)}s_2 \\
v_1(\mathbf{s}) &= \frac{(-9+18\delta-11\delta^2+2\delta^3)}{2\delta(3-\delta)(1-\delta)(3-2\delta)}s_1 + \frac{(9-15\delta+7\delta^2-\delta^3)}{2\delta(3-\delta)(1-\delta)(3-2\delta)}s_2 + \frac{6\delta-7\delta^2+2\delta^3}{2\delta(3-\delta)(1-\delta)(3-2\delta)} \\
v_2(\mathbf{s}) &= \frac{(9-18\delta+11\delta^2-2\delta^3)}{2\delta(3-\delta)(1-\delta)(3-2\delta)}s_1 + \frac{(-9+21\delta-15\delta^2+3\delta^3)}{2\delta(3-\delta)(1-\delta)(3-2\delta)}s_2 + \frac{6\delta-7\delta^2+2\delta^3}{2\delta(3-\delta)(1-\delta)(3-2\delta)}
\end{aligned}$$

- Case D

$$\begin{aligned}
v_v(\mathbf{s}) &= \frac{1}{1-\delta} - \frac{2-\delta}{(3-\delta)(1-\delta)}s_1 - \frac{1}{(1-\partial)}s_2 \\
v_1(\mathbf{s}) &= \frac{(-3+6\delta-2\delta^2)}{2\delta(3-\delta)(1-\delta)}s_1 + \frac{1}{2\delta(1-\delta)}s_2 \\
v_2(\mathbf{s}) &= \frac{(3-2\delta)}{2\delta(3-\delta)(1-\delta)}s_1 + \frac{(-3+7d-2\delta^2)}{2\delta(1-\delta)(3-\delta)}s_2
\end{aligned}$$

On the basis of these continuation values, we obtain players' expected utility functions, $U_i(\mathbf{x}) = x_i + \delta V_i(\mathbf{x})$. The reported demands are in accordance with Definition 2. In particular, d_i, $i = 1, 2$

and d_v can be easily derived from the following equations:

$$\begin{aligned} s_i + \delta V_i(\mathbf{s}) &= d_i + \delta V_i([1-d_i, d_i, 0]) \\ s_v + \delta V_v(\mathbf{s}) &= d_v + \delta V_v([d_v, 1-d_v, 0]) \end{aligned}$$

The demands for non-veto player 1 are never part of a proposed allocation and have therefore been omitted in the statement of the equilibrium proposal strategies above but we will use them in the remainder of the proof. In cases C and D, the mixing of the veto player is such that $d_1 = d_2$. In the other two cases, d_1 is as follows:

- Case A $\left(s_1 \leq 1 - \frac{3-\delta}{3-2\delta}s_2,\ s_1 \geq \frac{3-\delta}{3-2\delta}s_2\right)$:

$$d_1 = \frac{(4\delta^2 - 12\delta + 9)}{(3-2\delta)^2}s_1 + \frac{(3\delta - \delta^2)}{(3-2\delta)^2}s_2$$

- Case B $\left(\ s_1 > 1 - \frac{3-\delta}{3-2\delta}s_2,\ s_1 \geq \frac{27-27\delta+3\delta^2+\delta^3}{(3-2\delta)(3-\delta)^2}s_2 + \frac{\delta^2}{(3-\delta)^2}\right)$:

$$d_1 = \frac{(27 - 63\delta + 51\delta^2 - 17\delta^3 + 2\delta^4)}{(3-2\delta)^3}s_1 + \frac{(3\delta^2 - 4\delta^3 + \delta^4)}{(3-2\delta)^3}s_2 + \frac{9\delta - 15\delta^2 + 9\delta^3 - 2\delta^4}{(3-2\delta)^3}$$

Furthermore, all reported non-degenerate mixing probabilities are well defined. On the basis of the expected utility functions, U_i, we can then construct equilibrium voting strategies, $A_i^*(\mathbf{s}) = \{\mathbf{x} | U_i(\mathbf{x}) \geq U_i(\mathbf{s})\}$, $i = v, 1, 2$, for all $\mathbf{s} \in \Delta$. These voting strategies obviously satisfy equilibrium condition (2.4). Then, to prove Proposition 1 it suffices to verify equilibrium condition (2.5). To do so, we make use of five lemmas. We seek to establish an equilibrium with proposals that allocate a positive amount to at most one non-veto player. Lemma 1 shows that the expected utility function for these proposals satisfies the following continuity and monotonicity properties. Lemma 2 proves that minimal winning coalition proposals are optimal among the set of feasible proposals in $\overline{\Delta}$. Lemma 3 establishes that the equilibrium demands of the veto player and one non-veto player sum to less than unity and that the demands of the two non-veto players are (weakly) ordered in accordance to the ordering of allocations under the state $\mathbf{s}$. Lemma 4 then establishes that the proposal strategies for legislators $i = v, 1, 2$ in Proposition 1 maximize $U_i(\mathbf{x})$ over all $\mathbf{x} \in W(\mathbf{s}) \cap \overline{\Delta}$; these proposals would then maximize $U_i(\mathbf{x})$ over all $\mathbf{x} \in W(\mathbf{s})$ if there is no $\mathbf{x} \in W(\mathbf{s}) \cap \Delta/\overline{\Delta}$ that accrues i higher utility. We establish that this is indeed the case in Lemma 5.

Lemma 1. *Consider a symmetric Markov Perfect strategy profile with expected utility $U_i(\mathbf{s})$, $\mathbf{s} \in \overline{\Delta}$, determined by the continuation values in equations (A.1)-(A.3). Then, for all $\mathbf{x} = (x, 1 - x, 0) \in \Delta$ (a) $U_i(\mathbf{x})$, $i = v, 1, 2$ is continuous and differentiable with respect to x, (b) $U_v(\mathbf{x})$ increases with x, while $U_1(\mathbf{s})$ and $U_2(\mathbf{s})$ does not increase with x.*

Proof. An allocation $\mathbf{x} = (x, 1 - x, 0) \in \Delta$ belongs to case A in Proposition 2. Therefore we can write $U_i(\mathbf{x}) = x_i + \delta V_i(\mathbf{x})$ as follows:

$$U_v(\mathbf{x}) = x + \frac{\delta}{1-\delta} - \frac{\delta(2-\delta)}{(3-\delta)(1-\delta)}(1-x) \tag{A.4}$$

$$U_1(\mathbf{x}) = 1 - x + \delta\frac{(3-3d+\delta^2)}{(3-\delta)^2(1-\delta)}(1-x) \tag{A.5}$$

$$U_2(\mathbf{x}) = \delta\frac{(3-2\delta)}{(3-\delta)^2(1-\delta)}(1-x) \tag{A.6}$$

$U_i(\mathbf{x})$ is linear and continuous in x for $i = v, 1$, establishing part (a) of the Lemma. Regarding part (b):

$$\begin{aligned}
\frac{\partial U_v(\mathbf{x})}{\partial x} &= 1 + \frac{\delta(2-\delta)}{(3-\delta)(1-\delta)} > 0 \\
\frac{\partial U_1(\mathbf{x})}{\partial x} &= -\left(1 + \delta\frac{(3-3d+\delta^2)}{(3-\delta)^2(1-\delta)}\right) < 0 \\
\frac{\partial U_2(\mathbf{x})}{\partial x} &= -\delta\frac{(3-2\delta)}{(3-\delta)^2(1-\delta)} < 0
\end{aligned}$$

$\frac{\partial U_v(\mathbf{x})}{\partial x} > 0$ for any $\delta \in [0,1)$, since both the numerator and the denominator of $\frac{\delta(2-\delta)}{(3-\delta)(1-\delta)}$ are positive for any $\delta \in [0,1)$; $\frac{\partial U_1(\mathbf{x})}{\partial x} < 0$ for any $\delta \in [0,1)$, since both the numerator and the denominator of $\frac{(3-3d+\delta^2)}{(3-\delta)^2(1-\delta)}$ are positive for any $\delta \in [0,1)$; and $\frac{\partial U_2(\mathbf{x})}{\partial x} < 0$ for any $\delta \in [0,1)$, since both the numerator and the denominator of $\frac{(3-2\delta)}{(3-\delta)^2(1-\delta)}$ are positive for any $\delta \in [0,1)$. □

By the definition of demands and the monotonicity established in part (b) of Lemma 1 we immediately deduce:

Lemma 2. *Consider a symmetric Markov Perfect strategy profile with expected utility, $U_i(\mathbf{x})$, for $\mathbf{x} \in \overline{\Delta}$, $i = v, 1, 2$, given by (A.4)-(A.6). Every minimal winning coalition proposal of the veto player $x(v, i, d_i(\mathbf{s}))$, $i = \{1, 2\}$ is such that $x(v, i, d_i(\mathbf{s})) \in \arg\max\{U_v(\mathbf{x}) | \mathbf{x} \in \overline{\Delta}, U_i(\mathbf{x}) \geq U_i(\mathbf{s})\}$; similarly, every minimal winning coalition proposal of a non-veto player $x(i, v, d_v(\mathbf{s}))$, $i = \{1, 2\}$ is such that $x(i, v, d_i(\mathbf{s})) \in \arg\max\{U_i(\mathbf{x}) | \mathbf{x} \in \overline{\Delta}, U_v(\mathbf{x}) \geq U_v(\mathbf{s})\}$.*

Lemma 3. *For all $\mathbf{s} \in \Delta$, the demands reported in Proposition 1 are such that (a) $s_i \geq s_j \Rightarrow d_i \geq d_j$, $i.j = 1, 2$, and (b) $d_i + d_v \leq 1$, $i = 1, 2$.*

Proof. Part (a). Since we focus on the half of the simplex in which $s_1 \geq s_2$, we want to prove that $d_1 \geq d_2$. In cases C and D the mixed strategy of the veto player is such that $d_1 = d_2$, so we focus on cases A and B.

- Case A:

$$\begin{aligned}
\frac{(4\delta^2-12\delta+9)}{(3-2\delta)^2}s_1+\frac{(3\delta-\delta^2)}{(3-2\delta)^2}s_2 &\geq \frac{\delta}{3-2\delta}s_1+\frac{(3-\delta)}{(3-2\delta)}s_2\\
s_1 &\geq \frac{3-\delta}{3-2\delta}s_2
\end{aligned}$$

- Case B:

$$\begin{aligned}
&\frac{(27-63\delta+51\delta^2-17\delta^3+2\delta^4)}{(3-2\delta)^3}s_1+\frac{(3\delta^2-4\delta^3+\delta^4)}{(3-2\delta)^3}s_2+\frac{9\delta-15\delta^2+9\delta^3-2\delta^4}{(3-2\delta)^3}\\
&\geq \frac{9-12\delta+3\delta^2}{(3-2\delta)^2}s_2+\frac{\delta}{(3-2\delta)}\\
s_1 &\geq \frac{27-27\delta+3\delta^2+\delta^3}{(3-2\delta)(3-\delta)^2}s_2+\frac{\delta^2}{(3-\delta)^2}
\end{aligned}$$

Part (b). Since we focus on the half of the simplex in which $s_1 \geq s_2$, by part (a) of the same Lemma, it is enough to prove that $d_1 + d_v \leq 1$.

- Case A:

$$\begin{aligned}
s_v-\frac{\delta s_2}{(3-2\delta)}+\frac{(4\delta^2-12\delta+9)}{(3-2\delta)^2}s_1+\frac{(3\delta-\delta^2)}{(3-2\delta)^2}s_2 &\leq 1\\
s_v+s_1+\frac{\delta^2}{(3-2\delta)^2}s_2 &\leq 1
\end{aligned}$$

which holds for any $\delta \in [0,1)$, because $s_v + s_1 + s_2 = 1$ and $\frac{\delta^2}{(3-2\delta)^2} \in [0,1)$. To see this notice that $\frac{\delta^2}{(3-2\delta)^2}$ is monotonically increasing in δ and is equal to 1 when $\delta = 1$.

- Case B:

$$\frac{(27-63\delta+51\delta^2-17\delta^3+2\delta^4)s_1}{(3-2\delta)^3}+\frac{(3\delta^2-4\delta^3+\delta^4)s_2}{(3-2\delta)^3}+\frac{9\delta-15\delta^2+9\delta^3-2\delta^4}{(3-2\delta)^3}\leq 1$$

Notice that $\frac{(27-63\delta+51\delta^2-17\delta^3+2\delta^4)}{(3-2\delta)^3} \geq \frac{(3\delta^2-4\delta^3+\delta^4)}{(3-2\delta)^3}$ for any $\delta \in [0,1)$, so the LHS has an upper bound when $s_1 = 1$ and $s_2 = 0$. Therefore, we can prove the following inequality:

$$\begin{aligned} \frac{(27-63\delta+51\delta^2-17\delta^3+2\delta^4)}{(3-2\delta)^3} + \frac{9\delta-15\delta^2+9\delta^3-2\delta^4}{(3-2\delta)^3} &\leq 1 \\ \frac{(3-2\delta)^3}{(3-2\delta)^3} &\leq 1 \end{aligned}$$

- Case C:

$$\begin{aligned} \frac{9-12\delta+3\delta^2}{(3-2\delta)^2} s_2 + \frac{\delta}{(3-2\delta)} &\leq 1 \\ s_2 &\leq \frac{3-2\delta}{3-\delta} \end{aligned}$$

which holds for any $\delta \in [0,1)$, since $s_v + s_1 + s_2 = 1$ and $\frac{3-2\delta}{3-\delta} \leq 1$ for any $\delta \in [0,1)$. To see this notice that $\frac{3-2\delta}{3-\delta}$ is monotonically decreasing in δ and it is equal to 1 when $\delta = 0$.

- Case D:

$$\begin{aligned} s_v - \frac{\delta s_2}{3-2\delta} + \frac{\delta}{3-2\delta} s_1 + \frac{(3-\delta)}{(3-2\delta)} s_2 &\leq 1 \\ s_v + s_2 + \frac{\delta}{3-2\delta} s_1 &\leq 1 \end{aligned}$$

which holds for any $\delta \in [0,1)$ because $s_v + s_1 + s_2 = 1$ and $\frac{\delta}{3-2\delta} \in [0,1)$. To see this notice that $\frac{\delta}{3-2\delta}$ is monotonically increasing in δ and is equal to 1 when $\delta = 1$. □

We now show that equilibrium proposals are optimal over feasible alternatives in $\overline{\Delta}$.

Lemma 4. *$\mu_i[\mathbf{z}|\mathbf{s}] > 0 \Rightarrow \mathbf{z} \in \arg\max\{U_i(\mathbf{x})|\mathbf{x} \in W(s) \cap \overline{\Delta}\}$, for all $\mathbf{z}, \mathbf{s} \in \Delta$.*

Proof. All equilibrium proposals take the form of minimal winning coalition proposals: $\mathbf{x}(v, j, d_j(\mathbf{x}))$

when the veto player is proposing and $\mathbf{x}(j, v, d_v(\mathbf{x}))$ when a non-veto player is proposing. Also, whenever $\mu_v[\mathbf{x}(v, 1, d_1)|\mathbf{s}] > 0$ and $\mu_v[\mathbf{x}(v, 2, d_2)|\mathbf{s}] > 0$, we have $d_1 = d_2$ so that $U_v(\mathbf{x}(v, 1, d_1)) = U_v(\mathbf{x}(v, 2, d_2))$. Thus, in view of Lemma 2 it suffices to show that if $\mu_i[\mathbf{x}(i, j, dj)|\mathbf{s}] = 1$, then $U_i(\mathbf{x}(i, j, d_j)) = U_i(\mathbf{x}(i, h, d_h))$, $h \neq i, j$, i.e. proposer i has no incentive to coalesce with player h instead of j. This is immediate for a non-veto player, since only coalescing with the veto player guarantees the possibility to change the state. To show that - for the veto player - if $\mu_v[\mathbf{x}(v, j, dj)|\mathbf{s}] = 1$, then $U_v(\mathbf{x}(v, j, d_j)) = U_v(\mathbf{x}(v, h, d_h))$, $j \neq h$, it suffices to show $d_h \geq d_j$ by part (b) of Lemma 1. In Proposition 1 we have $s_1 \geq s_2$, (by part (a) of Lemma 3) $d_1 \geq d_2$, and when $d_1 \neq d_2$, we have $\mu_v[\mathbf{x}(v, 1, d_1)|\mathbf{s}] = 0$ which gives the desired result. □

We conclude the proof by showing that optimum proposal strategies cannot belong in $\Delta/\overline{\Delta}$. In particular, we show that if an alternative in $\Delta/\overline{\Delta}$ beats the status quo by majority rule, then for any player i we can find another alternative in $\overline{\Delta}$ that is also majority preferred to the status quo and improves i's utility.

Lemma 5. *Assume $\mathbf{x} \in W(\mathbf{s}) \cap \Delta/\overline{\Delta}$; then for any $i = v, 1, 2$ there exists $\mathbf{y} \in W(\mathbf{s}) \cap \overline{\Delta}$ such that $U_i(\mathbf{y}) \geq U_i(\mathbf{s})$.*

Proof. Consider first the veto player, $i = v$. Let $\mathbf{x} \in W(\mathbf{s}) \cap \Delta/\overline{\Delta}$. Consider first the case $\mathbf{x} \in A_v^*(\mathbf{s})$. Then, $\mathbf{x}$ is weakly preferred to s by v and at least one i, $i = 1, 2$. Now set $\mathbf{y} = \mathbf{x}(v, j, d_j(\mathbf{x}))$, where $d_j(\mathbf{x})$ is the applicable demand from Proposition 1. We have $U_j(\mathbf{x}(v, j, d_j(\mathbf{x}))) \geq U_j(\mathbf{x})$, by the definition of demand. From part (b) of Lemma 3 have $d_v(\mathbf{x}) + d_j(\mathbf{x}) \leq 1$ and as a result $x_v(v, j, d_j(\mathbf{x})) = 1 - d_j(\mathbf{x}) \geq d_v(\mathbf{x})$; hence, $U_v(\mathbf{x}(v, j, d_j(\mathbf{x}))) \geq U_v(\mathbf{x})$, which follows from the weak monotonicity in part (b) of Lemma 1. Thus, $\mathbf{y} = \mathbf{x}(v, j, d_j(\mathbf{x})) \in W(\mathbf{s})$ (because is supported by v and j), and we have completed the proof for this case. Now consider the case $\mathbf{x} \notin A_v^*(\mathbf{s})$, i.e. $U_v(\mathbf{s}) > U_v(\mathbf{x})$. Part (a) of Lemma 3 ensures that $d_v(\mathbf{s}) + d_j(\mathbf{s}) \leq 1$, hence proposal $\mathbf{y} = \mathbf{x}(v, j, d_j(\mathbf{s}))$ has

$x_v(v, j, d_j(\mathbf{s})) = 1 - d_j(\mathbf{s}) \geq d_v(\mathbf{s})$. Then $U_v(\mathbf{y}) \geq U_v(\mathbf{s}) > U_v(\mathbf{x})$, and $\mathbf{y} \in W(\mathbf{s}) \cap \overline{\Delta}$.

Now consider a non veto player, $i = 1, 2$. Let $\mathbf{x} \in W(\mathbf{s}) \cap \Delta/\overline{\Delta}$. Consider first the case $\mathbf{x} \in A_i^*(\mathbf{s})$. Then, $\mathbf{x}$ is weakly preferred to s by v and (at least) i. Now set $\mathbf{y} = \mathbf{x}(i, v, d_v(\mathbf{x}))$, where $d_v(\mathbf{x})$ is the applicable demand from Proposition 1. We have $U_v(\mathbf{x}(i, v, d_v(\mathbf{x}))) \geq U_v(\mathbf{x})$, by the definition of demand. From part (b) of Lemma 3 have $d_v(\mathbf{x}) + d_i(\mathbf{x}) \leq 1$ and as a result $x_i(i, v, d_v(\mathbf{x})) = 1 - d_v(\mathbf{x}) \geq d_i(\mathbf{x})$; hence, $U_i(\mathbf{x}(i, v, d_v(\mathbf{x}))) \geq U_i(\mathbf{x})$, which follows from the weak monotonicity in part (b) of Lemma 1. Thus, $\mathbf{y} = \mathbf{x}(i, v, d_v(\mathbf{x})) \in W(\mathbf{s}) \cap \overline{\Delta}$ (because is supported by v and i), and we have completed the proof for this case. Finally, consider the case $\mathbf{x} \notin A_i^*(\mathbf{s})$, i.e. $U_i(\mathbf{s}) > U_i(\mathbf{x})$. Part (a) of Lemma 3 ensures that $d_v(\mathbf{s}) + d_i(\mathbf{s}) \leq 1$, hence proposal $\mathbf{y} = \mathbf{x}(i, v, d_v(\mathbf{s}))$ has $x_i(i, v, d_v(\mathbf{s})) = 1 - d_v(\mathbf{s}) \geq d_i(\mathbf{s})$. Then $U_i(\mathbf{y}) \geq U_i(\mathbf{s}) > U_i(\mathbf{x})$, and $\mathbf{y} \in W(\mathbf{s}) \cap \overline{\Delta}$, which completes the proof. □

As a result of Lemmas 4 and 5, equilibrium proposals are optima over the entire range of feasible alternatives. It then follows that proposal strategies in Cases A-D of Proposition 2 satisfy the equilibrium condition (2.5) which completes the proof. ■

Proof of Proposition 4

The result of Proposition 4 follows once we establish that the proposal strategies in the equilibrium from Proposition 1 are weakly continuous in the status quo $\mathbf{s}$, i.e. that in equilibrium a small change in the status quo implies a small change in proposal strategies and, by extension, to the equilibrium transition probabilities.

Lemma 6. *The equilibrium proposal strategies μ_i^* in the proof of Proposition 1 are such that for every $\mathbf{s} \in \Delta$ and every sequence $\mathbf{s}_n \in \Delta$ with $\mathbf{s}_n \to \mathbf{s}$, $\mu_i^*[\cdot|\mathbf{s}_n]$ converges weakly to $\mu_i^*[\cdot|\mathbf{s}]$.*

Proof: The equilibrium is such that $\mu_i^*[\cdot|\mathbf{s}]$ $i = 1, 2$ has mass on only one point $\mathbf{x}(i, v, d_v(\mathbf{s}))$

and that $\mu_v^*[\cdot|\mathbf{s}]$ has mass on at most two points $\mathbf{x}(v,1,d_1(\mathbf{s}))$, and $\mathbf{x}(v,2,d_2(\mathbf{s}))$. It suffices to show that these proposals (when played with positive probability) and associated mixing probabilities are continuous in s (see Kalandrakis (2004) and Billingsley (1999)). Continuity holds in the interior of Cases A-D in Proposition 1, so it remains to check the boundaries of these cases. In order to distinguish the various applicable functional forms we shall write d_i^w and $\mu_v^w[\cdot|\mathbf{s}]$ where $w = \{A,B,C,D\}$ identifies the case for which the respective functional form applies.

- Boundary of Cases A and B: at the boundary (as in the interior of the two cases) we have $\mu_v^A[\mathbf{x}(v,1,d_2)|\mathbf{s}] = \mu_v^B[\mathbf{x}(v,1,d_2)|\mathbf{s}] = 0$; at the boundary we have $s_1 = 1 - \frac{3-\delta}{3-2\delta}s_2$, then:

$$\begin{aligned} d_v^A &= d_v^B = 0 \\ d_1^A &= d_1^B = 1 - \frac{9-12\delta-3\delta^2}{(3-2\delta)^2}s_2 \\ d_2^A &= d_2^B = \frac{9-12\delta-3\delta^2}{(3-2\delta)^2}s_2 + \frac{\delta}{(3-2\delta)} \end{aligned}$$

- Boundary of Cases B and C: at the boundary we have $s_1 = \frac{27-27\delta+3\delta^2+\delta^3}{(3-2\delta)(3-\delta)^2}s_2 + \frac{\delta^2}{(3-\delta)^2}$; then:

$$\begin{aligned} \mu_v^B[\mathbf{x}(v,1,d_2)|\mathbf{s}] &= \mu_v^C[\mathbf{x}(v,1,d_2)|\mathbf{s}] = 0 \\ d_v^B &= d_v^C = 0 \\ d_1^B &= d_1^C = \frac{9-12\delta+3\delta^2}{(3-2\delta)^2}s_2 + \frac{\delta}{(3-2\delta)} \\ d_2^B &= d_2^C = \frac{9-12\delta-3\delta^2}{(3-2\delta)^2}s_2 + \frac{\delta}{(3-2\delta)} \end{aligned}$$

- Boundary of Cases C and D: at the boundary we have $s_1 = 1 - \frac{3-\delta}{3-2\delta}s_2$; then:

$$\begin{aligned}
\mu_v^C[\mathbf{x}(v,1,d_2)|\mathbf{s}] &= \mu_v^D[\mathbf{x}(v,1,d_2)|\mathbf{s}] = \frac{3}{2}\frac{(-3+2\delta)((-2+2s_2)\delta+3-6s_2)}{\delta((3s_2-2)\delta^2+(-12s_2+3)\delta+9s_2)} \\
d_v^C &= d_v^D = 0 \\
d_1^C &= d_1^D = d_2^C = d_2^D = \frac{9-12\delta-3\delta^2}{(3-2\delta)^2}s_2 + \frac{\delta}{(3-2\delta)}
\end{aligned}$$

- Boundary of Cases D and A: at the boundary we have $s_1 = \frac{3-\delta}{3-2\delta}s_2$; then:

$$\begin{aligned}
\mu_v^D[\mathbf{x}(v,1,d_2)|\mathbf{s}] &= \mu_v^A[\mathbf{x}(v,1,d_2)|\mathbf{s}] = 0 \\
d_v^D &= d_v^A = s_v - \frac{\delta s_2}{3-2\delta} \\
d_1^D &= d_1^A = d_2^D = d_2^A = \frac{(3-\delta)^2}{(3-2\delta)^2}s_2
\end{aligned}$$

■

Proof of Proposition 5

As before, we focus on the allocations in which $s_1 \geq s_2$. The other cases are symmetric. Consider the following equilibrium proposal strategies (all supported by a minimal winning coalition) and demands (as defined in the proof of Proposition 1):

- CASE A: $s_1 \geq 1 - \frac{2-\delta(1-p_v)}{2-\delta(1+p_v)}s_2$; $s_1 \geq \frac{2p_v\delta^2+10\delta-8-3\delta^2-2p_v\delta+p_v^2\delta^2}{(2-\delta(1+p_v))(1-p_v)\delta}s_2 + \frac{4-4p_v\delta-4\delta+\delta^2+p_v^2\delta^2+2p_v\delta^2}{(2-\delta(1+p_v))(1-p_v)\delta}$

$$\begin{aligned}
\mathbf{x}^v &= [1-d_2^A, 0, d_2^A], \mathbf{x}^1 = [d_v^A, 1-d_v^A, 0], \mathbf{x}^2 = [d_v^A, 0, 1-d_v^A] \\
d_v^A &= s_v - \frac{2p_v\delta}{2-(1+p_v)\delta}s_2 \\
d_2^A &= \frac{\delta(1-p_v)}{2-\delta(1+p_v)}s_1 + \frac{2-\delta(1-p_v)}{2-\delta(1+p_v)}s_2 \\
d_1^A &= \frac{-4p_v\delta+4+2p_v\delta^2-4\delta+p_v^2\delta^2+\delta^2}{(2-\delta(1+p_v))^2}s_1 + \frac{-p_v^2\delta^2-\delta^2-2p_v\delta+2\delta+2p_v\delta^2}{(2-\delta(1+p_v))^2}s_2
\end{aligned}$$

- CASE B:

$s_1 < 1 - \frac{2-\delta(1-p_v)}{2-\delta;(1+p_v)} s_2; s_1 \geq \frac{-2\delta^3 p_v^2 + p_v^3\delta^3 + p_v\delta^3 + \delta^2 + p_v^2\delta^2 - 2p_v\delta^2 - 4\delta + 4}{(p_v\delta-\delta+2)(p_v\delta-1)(-2+p_v+p_v\delta)} s_2 + \frac{-p_v\delta^3 - 2p_v^2\delta^2 + p_v^3\delta^3 + 2p_v\delta^2}{(p_v\delta-\delta+2)(p_v\delta-1)(-2+p_v+p_v\delta)}$

$$
\begin{aligned}
\mathbf{x}^v &= [1-d_2^B, 0, d_2^B], \mathbf{x}^1 = [d_v^B, 1-d_v^B, 0], \mathbf{x}^2 = [d_v^B, 0, 1-d_v^B] \\
d_v^B &= 0 \\
d_2^B &= \frac{-2p_v\delta^2 + 2\delta^2 + 2p_v\delta - 6\delta + 4}{(2-\delta(1+p_v))^2} s_2 + \frac{p_v^2\delta^2 - \delta^2 - 2p_v\delta + 2\delta}{(2-\delta(1+p_v))^2} \\
d_1^B &= \frac{-16\delta + 10\delta^2 + 2p_v\delta^4 + 2p_v^3\delta^3 - 10p_v\delta^3 - 2p_v^2\delta^2}{(2-\delta(1+p_v))^3} s_1 + ... \\
&+ \frac{16p_v\delta^2 + 2\delta^3 p_v^2 + 8 - 2\delta^3 - 2p_v^3\delta^4 - 8p_v\delta}{(2-\delta(1+p_v))^3} s_1 + ... \\
&+ \frac{2p_v\delta^2 - 4p_v^2\delta^2 - 6p_v\delta^3 - 4p_v^2\delta^4 + 4p_v\delta^2 + 8p_v^2\delta^3 - 2p_v^3\delta^3 - 2p_v^3\delta^4}{(2-\delta(1+p_v))^3} s_2 + ... \\
&+ \frac{4\delta - 4p_v\delta - 2p_v\delta^4 - 4\delta^2 - 5\delta^3 p_v^2 + 7p_v\delta^3 - 3p_v^3\delta^3 + \delta^3 + 8p_v^2\delta^2 - 4p_v\delta^2 + 2p_v^3\delta^4}{(2-\delta(1+p_v))^3}
\end{aligned}
$$

- CASE C:

$s_1 < 1 - \frac{2-\delta(1-p_v)}{2-\delta;(1+p_v)} s_2; s_1 < \frac{-2\delta^3 p_v^2 + p_v^3\delta^3 + p_v\delta^3 + \delta^2 + p_v^2\delta^2 - 2p_v\delta^2 - 4\delta + 4}{(p_v\delta-\delta+2)(p_v\delta-1)(-2+p_v+p_v\delta)} s_2 + \frac{-p_v\delta^3 - 2p_v^2\delta^2 + p_v^3\delta^3 + 2p_v\delta^2}{(p_v\delta-\delta+2)(p_v\delta-1)(-2+p_v+p_v\delta)}$

$$
\begin{aligned}
\mathbf{x}^v &= \begin{cases} [1-d_2^C, d_2^C, 0] & \text{w/ Pr} = \mu_v^C \\ [1-d_2^C, 0, d_2^C] & \text{w/ Pr} = 1-\mu_v^C \end{cases}, \mathbf{x}^1 = [d_v^C, 1-d_v^C, 0], \mathbf{x}^2 = [d_v^C, 0, 1-d_v^C] \\
d_v^C &= 0 \\
d_1^C &= d_2^C = \frac{-2p_v\delta^2 + 2\delta^2 + 2p_v\delta - 6\delta + 4}{(2-\delta(1+p_v))^2} s_2 + \frac{p_v^2\delta^2 - \delta^2 - 2p_v\delta + 2\delta}{(2-\delta(1+p_v))^2} \\
\mu_v^C &= \frac{(-3p_v^2\delta^2 + 3p_v\delta^2 - 3\delta - 3p_v\delta + 6)s_1}{2(-2+\delta+p_v\delta)\delta d_2^C} + \frac{(3p_v^2\delta^2 - 3p_v\delta^2 + 3\delta + 3p_v\delta - 6)s_2}{2(-2+\delta+p_v\delta)\delta d_2^C} + \ldots \\
&+ \frac{\delta^2 d_2^C p_v + \delta^2 d_2^C - 2\delta d_2^C}{2(-2+\delta+p_v\delta)\delta d_2^C}
\end{aligned}
$$

- CASE D: $s_1 \geq 1 - \frac{2-\delta(1-p_v)}{2-\delta(1+p_v)} s_2$; $s_1 < \frac{2p_v\delta^2+10\delta-8-3\delta^2-2p_v\delta+p_v^2\delta^2}{(2-\delta(1+p_v))(1-p_v)\delta} s_2 + \frac{4-4p_v\delta-4\delta+\delta^2+p_v^2\delta^2+2p_v\delta^2}{(2-\delta(1+p_v))(1-p_v)\delta}$

$$\begin{aligned}
\mathbf{x}^v &= \begin{cases} [1-d_2^D, d_2^D, 0] & \text{w/ } \Pr = \mu_v^D \\ [1-d_2^D, 0, d_2^D] & \text{w/ } \Pr = 1-\mu_v^D \end{cases}, \mathbf{x}^1 = [d_v^D, 1-d_v^D, 0], \mathbf{x}^2 = [d_v^D, 0, 1-d_v^D] \\
d_v^D &= s_v - \frac{2p_v\delta}{2-(1+p_v)\delta} s_2 \\
d_1^D &= d_2^D = \frac{\delta(1-p_v)}{2-\delta(1+p_v)} s_1 + \frac{2-\delta(1-p_v)}{2-\delta(1+p_v)} s_2 \\
\mu_v^D &= \frac{(-3p_v^2\delta^2+3p_v\delta^2-3\delta-3p_v\delta+6)s_1}{2(-2+\delta+p_v\delta)\delta d_2^D} + \frac{(3p_v^2\delta^2-3p_v\delta^2+3\delta+3p_v\delta-6)s_2}{2(-2+\delta+p_v\delta)\delta d_2^D} + \ldots \\
&\quad + \frac{\delta^2 d_2^D p_v + \delta^2 d_2^D - 2\delta d_2^D}{2(-2+\delta+p_v\delta)\delta d_2^D}
\end{aligned}$$

where μ_v^C and μ_v^D are the probabilities that the veto player coalesces with non-veto 1 in cases C, and D respectively. These are well defined probability in [0,1] such that $d_1^C = d_2^C$ and $d_1^D = d_2^D$, or such that $s_1 + \delta v_1(\mathbf{s}, \mu_v, d_2) = s_2 + \delta v_2(\mathbf{s}, \mu_v, d_2)$.

It is tedious but straightforward to show that these equilibrium strategies and the associated value functions are part of a symmetric MPE, using the same strategy employed in the proof of Proposition 1. ■

Proof of Proposition 6

In this case an allocation is $\mathbf{s} = [s_{v1}, s_{v2}, s_1, s_2]$, where s_{vi}, $i = 1, 2$, denote the share to a veto player and s_j, $j = 1, 2$, denote the share to a non-veto player. In the remainder of the proof, we focus on the allocations in which $s_1 \geq s_2$ and $s_{v1} \geq s_{v2}$. The other cases are symmetric. The equilibrium I characterize is similar to the one from Proposition 1 and the steps behind the proof are the same. In particular, we partition the state space into regions where the veto proposer mixes or not between coalition partners and regions where the "demand" of a veto player to the proposal of a non-veto (as defined in the proof of Proposition 1) is bounded at zero. Since there are two veto players we have 6 regions, 3 where the veto proposers do not mix and three where they do (in order to keep the demand of the two non-veto players equal). The three regions with no mixing are characterized by A) $d_{v1} \geq d_{v2} > 0$; B) $d_{v1} > 0$ and $d_{v2} = 0$; and C) $d_{v1} = d_{v2} = 0$. In these regions, a veto proposer coalesces with non-veto player 2 with probability 1. The remaining three regions are analogous with the difference that the veto proposer coalesces with non-veto player 1 with probability $\mu \in [0, 1]$.

Consider the following equilibrium proposal strategies (all supported by a minimal winning coalition) and demands (as defined in the proof of Proposition 1):

- CASE A: $s_{v1} \geq s_{v2} \geq \frac{\delta(\delta^3-36\delta^2+72\delta-32)}{2(3\delta^2-10\delta+8)(\delta^2+6\delta-8)} s_2; s_1 \geq \frac{2(4-5\delta+\delta^2)}{\delta^2-10\delta+8} s_2$

$$
\begin{aligned}
\mathbf{x}^{v1} &= [1 - d_{v2}^A - d_2^A, d_{v2}^A, 0, d_2^A], \mathbf{x}^{v2} = [d_{v1}^A, 1 - d_{v1}^A - d_2^A, 0, d_2^A] \\
\mathbf{x}^{1} &= [d_{v1}^A, d_{v2}^A, 1 - d_{v1}^A - d_{v2}^A, 0], \mathbf{x}^{2} = [d_{v1}^A, d_{v2}^A, 0, 1 - d_{v1}^A - d_{v2}^A] \\
d_{v1}^A(v) &= s_{v1} + \frac{4\delta(1-\delta)}{16 - 16\delta + 3\delta^2} s_1 + \frac{(3\delta^5 - 9\delta^4 + 72\delta^3 - 248\delta^2 + 320\delta - 128)\delta}{(-4+3\delta)(\delta-4)(\delta^2+6\delta-8)(\delta-2)} s_2 \\
d_{v1}^A(nv) &= s_{v1} - \frac{\delta(\delta^3 - 36\delta^2 + 72\delta - 32)}{2(3\delta^2 - 10\delta + 8)(\delta^2 + 6\delta - 8)} s_2 \\
d_{v2}^A(v) &= s_{v2} + \frac{4\delta(1-\delta)}{16 - 16\delta + 3\delta^2} s_1 + \frac{(3\delta^5 - 9\delta^4 + 72\delta^3 - 248\delta^2 + 320\delta - 128)\delta}{(-4+3\delta)(\delta-4)(\delta^2+6\delta-8)(\delta-2)} s_2 \\
d_{v2}^A(nv) &= s_{v2} - \frac{\delta(\delta^3 - 36\delta^2 + 72\delta - 32)}{2(3\delta^2 - 10\delta + 8)(\delta^2 + 6\delta - 8)} s_2 \\
d_2^A &= \frac{\delta}{4 - 3\delta} s_1 + \frac{4\delta^3 + 48\delta - 18\delta^2 - 32}{3\delta^3 + 14\delta^2 - 48\delta + 32} s_2 \\
d_1^A &= -\frac{\delta^7 - 62\delta^5 + 5\delta^6 - 72\delta^4 + 736\delta^3 + 3072\delta - 3264\delta^2 - 1024}{(32 - 56\delta + 32\delta^2 - 7\delta^3 + \delta^4)(-4+3\delta)(\delta^2+6\delta-8)} s_1 + ... \\
&\quad - \frac{4\delta^7 - 752\delta^4 - 58\delta^6 + 330\delta^5 - 256\delta^2 + 736\delta^3}{(32 - 56\delta + 32\delta^2 - 7\delta^3 + \delta^4)(-4+3\delta)(\delta^2+6\delta-8)} s_2
\end{aligned}
$$

- CASE B: $s_{v1} \geq \frac{\delta(\delta^3-36\delta^2+72\delta-32)}{2(3\delta^2-10\delta+8)(\delta^2+6\delta-8)} s_2 \geq s_{v2}; s_1 \geq \frac{2(4-5\delta+\delta^2)}{\delta^2-10\delta+8} s_2$

$$\begin{aligned}
\mathbf{x}^{v1} &= [1-d^B_{v2}-d^B_2, d^B_{v2}, 0, d^B_2], \mathbf{x}^{v2} = [d^B_{v1}, 1-d^B_{v1}-d^B_2, 0, d^B_2] \\
\mathbf{x}^{1} &= [d^B_{v1}, d^B_{v2}, 1-d^B_{v1}-d^B_{v2}, 0], \mathbf{x}^{2} = [d^B_{v1}, d^B_{v2}, 0, 1-d^B_{v1}-d^B_{v2}] \\
d^B_{v1}(v) &= s_{v1} + \frac{\delta(18\delta^6-428\delta^4+816\delta^3+384\delta^2-1792\delta+1024)}{4(-4+3\delta)^2(\delta+4)(\delta-2)} s_1 + ... \\
&+ \frac{\delta(21\delta^6-80\delta^5-108\delta^4+1296\delta^3-3136\delta^2+3072\delta-1024)}{4(-4+3\delta)^2(\delta+4)(\delta-2)} s_2 + ... \\
&+ \frac{\delta(18\delta^6-284\delta^4+864\delta^3-1088\delta^2+512\delta)}{4(-4+3\delta)^2(\delta+4)(\delta-2)} s_{v1} + \dots \\
&+ \frac{\delta(284\delta^4+1088\delta^2-512\delta-18\delta^6-864\delta^3)}{4(-4+3\delta)^2(\delta+4)(\delta-2)} \\
d^B_{v1}(nv) &= s_{v1} - \frac{\delta(-22\delta^2-32+48\delta+3\delta^3)}{(-4+3\delta)^2(\delta+4)(\delta-2)} s_1 - \frac{(-8\delta+2\delta^2+2\delta^3)\delta}{(-4+3\delta)^2(\delta+4)(\delta-2)} s_2 + ... \\
&- \frac{(-22\delta^2-32+48\delta+3\delta^3)\delta}{(-4+3\delta)^2(\delta+4)(\delta-2)} s_{v1} - \frac{(32-48\delta-3\delta^3+22\delta^2)\delta}{(-4+3\delta)^2(\delta+4)(\delta-2)} \\
d^B_{v2}(v) &= s_{v2} + \frac{\delta(18\delta^6-428\delta^4+816\delta^3+384\delta^2-1792\delta+1024)}{4(-4+3\delta)^2(\delta+4)(\delta-2)} s_1 + ... \\
& \frac{\delta(21\delta^6-80\delta^5-108\delta^4+1296\delta^3-3136\delta^2+3072\delta-1024)}{4(-4+3\delta)^2(\delta+4)(\delta-2)} s_2 + ... \\
&+ \frac{\delta(18\delta^6-284\delta^4+864\delta^3-1088\delta^2+512\delta)}{4(-4+3\delta)^2(\delta+4)(\delta-2)} s_{v2} + \dots \\
&+ \frac{\delta(284\delta^4+1088\delta^2-512\delta-18\delta^6-864\delta^3)}{4(-4+3\delta)^2(\delta+4)(\delta-2)} \\
d^B_{v2}(nv) &= 0 \\
d^B_2 &= -\frac{16\delta^2-5\delta^4+6\delta^5-16\delta^3}{(-4+3\delta)^2(\delta+4)} s_1 - \frac{-576\delta+400\delta^2+256-25\delta^4+7\delta^5-64\delta^3}{(-4+3\delta)^2(\delta+4)} s_s + ... \\
& \frac{-14\delta^4+6\delta^5-28\delta^3+96\delta^2-64\delta}{(-4+3\delta)^2(\delta+4)} s_{v1} - \frac{-6\delta^5+64\delta+14\delta^4-96\delta^2+28\delta^3}{(-4+3\delta)^2(\delta+4)} \\
d^B_1 &= -\frac{44544\delta^2+8320\delta^4+784\delta^5+8192-30720\delta}{(32-56\delta+32\delta^2-7\delta^3+\delta^4)(\delta+4)(-4+3\delta)^3} s_1 + ... \\
&- \frac{-30336\delta^3-47\delta^8+6\delta^9-1016\delta^6+265\delta^7}{(32-56\delta+32\delta^2-7\delta^3+\delta^4)(\delta+4)(-4+3\delta)^3} s_1 + ... \\
&- \frac{-2048\delta+8192\delta^2+10624\delta^4-4160\delta^5-13184\delta^3-74\delta^8+7\delta^9+412\delta^6+227\delta^7}{(32-56\delta+32\delta^2-7\delta^3+\delta^4)(\delta+4)(-4+3\delta)^3} s_2 + ... \\
&- \frac{-492\delta^6+4640\delta^4-656\delta^5-8320\delta^3+6656\delta^2-2048\delta-56\delta^8+6\delta^9+262\delta^7}{(32-56\delta+32\delta^2-7\delta^3+\delta^4)(\delta+4)(-4+3\delta)^3} s_{v1} + ... \\
&- \frac{-6\delta^9+2048\delta+8320\delta^3+492\delta^6-6656\delta^2-4640\delta^4-262\delta^7+656\delta^5+56\delta^8}{(32-56\delta+32\delta^2-7\delta^3+\delta^4)(\delta+4)(-4+3\delta)^3}
\end{aligned}$$

- CASE C: $\frac{\delta(\delta^3-36\delta^2+72\delta-32)}{2(3\delta^2-10\delta+8)(\delta^2+6\delta-8)}s_2 \geq s_{v1} \geq s_{v2}; s_1 \geq \frac{2(4-5\delta+\delta^2)}{\delta^2-10\delta+8}s_2$

$$
\begin{aligned}
\mathbf{x}^{v1} &= [1-d_{v2}^C-d_2^C, d_{v2}^C, 0, d_2^C], \mathbf{x}^{v2} = [d_{v1}^C, 1-d_{v1}^C-d_2^C, 0, d_2^C] \\
\mathbf{x}^{1} &= [d_{v1}^C, d_{v2}^C, 1-d_{v1}^C-d_{v2}^C, 0], \mathbf{x}^{2} = [d_{v1}^C, d_{v2}^C, 0, 1-d_{v1}^C-d_{v2}^C] \\
d_{v1}^C(v) &= s_{v1}\left(1+\frac{\delta(-36\delta^3+240\delta^2-448\delta+256)}{8(-4+3\delta)^2(\delta-4)}\right)+\frac{\delta(9\delta^4-54\delta^3+8\delta^2+160\delta-128)}{8(-4+3\delta)^2(\delta-4)}s_1+... \\
&\quad \frac{\delta(12\delta^4-134\delta^3+368\delta^2-384\delta+128)}{8(-4+3\delta)^2(\delta-4)}s_2+\frac{\delta(-9\delta^4-104\delta^2+54\delta^3+64\delta)}{8(-4+3\delta)^2(\delta-4)} \\
d_{v1}^C(nv) &= 0 \\
d_{v2}^C(v) &= s_{v2}\left(1+\frac{\delta(-36\delta^3+240\delta^2-448\delta+256)}{8(-4+3\delta)^2(\delta-4)}\right)+\frac{\delta(9\delta^4-54\delta^3+8\delta^2+160\delta-128)}{8(-4+3\delta)^2(\delta-4)}s_1+... \\
&\quad \frac{\delta(12\delta^4-134\delta^3+368\delta^2-384\delta+128)}{8(-4+3\delta)^2(\delta-4)}s_2+\frac{\delta(-9\delta^4-104\delta^2+54\delta^3+64\delta)}{8(-4+3\delta)^2(\delta-4)} \\
d_{v2}^C(nv) &= 0 \\
d_2^C &= -\frac{3\delta^3-4\delta^2}{2(-4+3\delta)^2}s_1-\frac{4\delta^3-28\delta^2+56\delta-32}{2(-4+3\delta)^2}s_2-\frac{-3\delta^3+10\delta^2-8\delta}{2(-4+3\delta)^2} \\
d_1^C &= -\frac{3\delta^7-1024-4160\delta^2+2544\delta^3-824\delta^4+3328\delta-25\delta^6+160\delta^5}{2(-4+3\delta)^2(32-56\delta+32\delta^2-7\delta^3+\delta^4)}s_1+... \\
&\quad -\frac{4\delta^7-56\delta^6+308\delta^5+256\delta-896\delta^2+1232\delta^3-848\delta^4}{2(-4+3\delta)(32-56\delta+32\delta^2-7\delta^3+\delta^4)^2}s_2+... \\
&\quad -\frac{-256\delta+768\delta^2+544\delta^4-174\delta^5+31\delta^6-3\delta^7-912\delta^3}{2(-4+3\delta)^2(32-56\delta+32\delta^2-7\delta^3+\delta^4)}
\end{aligned}
$$

- CASE D: $s_{v1} \geq s_{v2} \geq \frac{\delta(\delta^3-36\delta^2+72\delta-32)}{2(3\delta^2-10\delta+8)(\delta^2+6\delta-8)}s_2; s_1 < \frac{2(4-5\delta+\delta^2)}{\delta^2-10\delta+8}s_2$

$$\begin{aligned}
\mathbf{x}^{v1} &= \begin{cases} [1-d_2^D-d_{v2}^D, d_{v2}^D, d_2^D, 0] & \text{w/ Pr} = \mu_v^D \\ [1-d_2^D-d_{v2}^D, d_{v2}^D, 0, d_2^D] & \text{w/ Pr} = 1-\mu_v^D \end{cases} \\
\mathbf{x}^{v2} &= \begin{cases} [d_{v1}^D, 1-d_2^D-d_{v1}^D, d_2^D, 0] & \text{w/ Pr} = \mu_v^D \\ [d_{v1}^D, 1-d_2^D-d_{v1}^D, 0, d_2^D] & \text{w/ Pr} = 1-\mu_v^D \end{cases} \\
\mathbf{x}^1 &= [d_{v1}^D, d_{v2}^D, 1-d_{v1}^D-d_{v2}^D, 0], \mathbf{x}^2 = [d_{v1}^D, d_{v2}^D, 0, 1-d_{v1}^D-d_{v2}^D] \\
d_{v1}^D &= d_{v1}^A \\
d_{v2}^D &= d_{v2}^A \\
d_1^D &= d_2^D = d_2^A
\end{aligned}$$

- CASE E: $s_{v1} \geq \frac{\delta(\delta^3-36\delta^2+72\delta-32)}{2(3\delta^2-10\delta+8)(\delta^2+6\delta-8)}s_2 \geq s_{v2}; s_1 < \frac{2(4-5\delta+\delta^2)}{\delta^2-10\delta+8}s_2$

$$\begin{aligned}
\mathbf{x}^{v1} &= \begin{cases} [1-d_2^C-d_{v2}^C, d_{v2}^C, d_2^C, 0] & \text{w/ Pr} = \mu_v^E \\ [1-d_2^C-d_{v2}^C, d_{v2}^C, 0, d_2^C] & \text{w/ Pr} = 1-\mu_v^E \end{cases} \\
\mathbf{x}^{v2} &= \begin{cases} [d_{v1}^C, 1-d_2^C-d_{v1}^C, d_2^C, 0] & \text{w/ Pr} = \mu_v^E \\ [d_{v1}^C, 1-d_2^C-d_{v1}^C, 0, d_2^C] & \text{w/ Pr} = 1-\mu_v^E \end{cases} \\
\mathbf{x}^1 &= [d_{v1}^A, d_{v2}^A, 1-d_{v1}^A-d_{v2}^A, 0], \mathbf{x}^2 = [d_{v1}^A, d_{v2}^A, 0, 1-d_{v1}^A-d_{v2}^A] \\
d_{v1}^E &= d_{v1}^B \\
d_{v2}^E &= d_{v2}^B \\
d_1^C &= d_2^C = d_2^B
\end{aligned}$$

- CASE F: $\frac{\delta(\delta^3-36\delta^2+72\delta-32)}{2(3\delta^2-10\delta+8)(\delta^2+6\delta-8)}s_2 \geq s_{v1} \geq s_{v2}; s_1 < \frac{2(4-5\delta+\delta^2)}{\delta^2-10\delta+8}s_2$

$$\mathbf{x}^{v1} = \begin{cases} [1-d_2^F - d_{v2}^F, d_{v2}^F, d_2^F, 0] & \text{w/ Pr} = \mu_v^F \\ [1-d_2^F - d_{v2}^F, d_{v2}^F, 0, d_2^F] & \text{w/ Pr} = 1-\mu_v^F \end{cases}$$

$$\mathbf{x}^{v2} = \begin{cases} [d_{v1}^F, 1-d_2^F - d_{v1}^F, d_2^F, 0] & \text{w/ Pr} = \mu_v^F \\ [d_{v1}^F, 1-d_2^F - d_{v1}^F, 0, d_2^F] & \text{w/ Pr} = 1-\mu_v^F \end{cases}$$

$$\mathbf{x}^1 = [d_{v1}^F, d_{v2}^F, 1-d_{v1}^F - d_{v2}^F, 0]$$

$$\mathbf{x}^2 = [d_{v1}^F, d_{v2}^F, 0, 1-d_{v1}^F - d_{v2}^F]$$

$$d_{v1}^F = d_{v1}^C$$

$$d_{v2}^F = d_{v2}^C$$

$$d_1^F = d_2^F = d_2^C$$

where μ_v^J,is the probability that a veto proposer coalesces with non-veto 1 in case J, $d_{vi}^J(v)$ is the demand of veto player i when the proposer is the other veto in case J, and $d_{vi}^J(nv)$ is the demand of veto player i when the proposer is a non-veto in case J. Notice that μ_v^J are well defined probability in [0,1] such that $d_1^i = d_2^i$, $i = D, E, F$, or such that $s_1+\delta v_1(\mathbf{s}, \mu_v, d_2) = s_2+\delta v_2(\mathbf{s}, \mu_v, d_2)$ .It is tedious but straightforward to show that these equilibrium strategies and the associated value functions are part of a symmetric MPE, using the same strategy employed in the proof of Proposition 1. In particular, the crucial steps will be 1) showing that in the absorbing set where one non-veto player receives zero, the expected utility of all agents are weakly increasing in their current allocation (the main passage in proving that the proposed proposals are optimal among all minimal winning coalition proposal), and 2) showing that the sum of the demands of a minimal winning coalition is always weakly smaller than 1 (meaning that there always exists a minimal winning coalition proposal that makes the proposer at least as well off as he is in a status quo where everyone has a positive share). ■

Proof of Proposition 7

By assumption, we are restricting the set of possible legislative outcomes to allocations on the edges of the simplex, i.e. to $\mathbf{s} \in \Delta_2$. I focus on allocations where $s_1 \geq s_2 \geq s_3 = 0$ (the other cases being symmetric). Since the endowment is 1 and $s_3 = 0$, we can reduce the problem to one dimension replacing $s_2 = 1 - s_1$ and focusing on allocations where $s_1 \geq 1/2$. Consider the proposal and voting strategies that would be part of an equilibrium with perfectly impatient agents: each agent, when proposing, tries to maximize his current allocation (i.e. he proposes the "acceptable" allocation, $\mathbf{x} \in W(\mathbf{s})$, that give him the greatest share) and each agent votes yes to any proposal that gives him as much as he gets in the status quo. I want to show that these strategies and the associated value functions are part of an equilibrium even when agents are patient. First of all, consider the allocations in the absorbing set $\mathbf{s} \in \Delta_1$ where one agent gets the whole dollar. Denote with $\overline{V}_0$ the continuation value from an allocation $\mathbf{s} \in \Delta_1$ where the agent gets nothing, and $\overline{V}_1$ the continuation value from an allocation $\mathbf{s} \in \Delta_1$ where the agent gets the whole dollar. We can derive $\overline{V}_0$ and $\overline{V}_1$, using the transition probabilities discussed in Section 5.3:

$$
\begin{aligned}
\overline{V}_1 &= \frac{5}{9}(1 + \delta\overline{V}_1) + \frac{4}{9}(0 + \delta\overline{V}_0) \\
\overline{V}_0 &= \frac{7}{9}(0 + \delta\overline{V}_0) + \frac{2}{9}(1 + \delta\overline{V}_1) \\
&\implies \overline{V}_1 = \frac{5 - 3\delta}{3(3 - 4\delta + \delta^2)} \\
&\implies \overline{V}_0 = \frac{2}{3(3 - 4\delta + \delta^2)}
\end{aligned}
$$

Using these continuation values, the probability of being selected as veto and as proposer, and the conjectured strategies discussed in Section 5.3, we can derive the continuation values for any allocation $\mathbf{s} \in \Delta_2$. It is tedious but straightforward to verify that the value functions are as follows:

- CASE A: $s_1 \geq \frac{134\delta^4+7371\delta^2-1665\delta^3+6561-12393\delta}{27(\delta^2+3-4\delta)(8\delta^2-99\delta+162)}$

$$\begin{aligned}
v_1^A(\mathbf{s}) &= -\frac{-4374\delta+2790\delta^2+39\delta^4+2187-642\delta^3}{3(\delta-1)(\delta-3)(13\delta^3-207\delta^2+729\delta-729)}s_1+\ldots \\
&\quad -\frac{1458-100\delta^3-1620\delta+630\delta^2}{3(\delta-1)(\delta-3)(13\delta^3-207\delta^2+729\delta-729)} \\
v_2^A(\mathbf{s}) &= \frac{39\delta^4-696\delta^3-5508\delta+2916+3249\delta^2}{3(\delta-1)(\delta-3)(13\delta^3-207\delta^2+729\delta-729)}s_1+\ldots \\
&\quad \frac{-39\delta^4-4374+7047\delta+743\delta^3-3753\delta^2}{3(\delta-1)(\delta-3)(13\delta^3-207\delta^2+729\delta-729)} \\
v_3^A(\mathbf{s}) &= \frac{-108\delta^2+81\delta+27\delta^3}{3(\delta-1)(\delta-3)(13\delta^3-207\delta^2+729\delta-729)}s_1+\ldots \\
&\quad +\frac{-85\delta^3+243\delta+243\delta^2-729}{3(\delta-1)(\delta-3)(13\delta^3-207\delta^2+729\delta-729)} \\
d_1^A &= \frac{35\delta^4-1998\delta^3+8424\delta^2-13122\delta+6561}{81(\delta^2+3-4\delta)(\delta^2-15\delta+27)}s_1+\frac{-53\delta^4+126\delta^3-81\delta^2}{81(\delta^2+3-4\delta)(\delta^2-15\delta+27)} \\
d_2^A &= s_2 \\
d_3^A &= 0
\end{aligned}$$

- CASE B: $s_1 < \frac{134\delta^4+7371\delta^2-1665\delta^3+6561-12393\delta}{27(\delta^2+3-4\delta)(8\delta^2-99\delta+162)}$

$$\begin{aligned}
v_1^B(\mathbf{s}) &= -\frac{24\delta^4-393\delta^3+1746\delta^2-2835\delta+1458}{3(\delta-1)(\delta-3)\delta(8\delta^2-99\delta+162)}s_1-\frac{-10\delta^3-207\delta^2-729+810\delta}{3(\delta-1)(\delta-3)\delta(8\delta^2-99\delta+162)} \\
v_2^B(\mathbf{s}) &= \frac{24\delta^4-393\delta^3+1746\delta^2-2835\delta+1458}{3(\delta-1)(\delta-3)\delta(8\delta^2-99\delta+162)}s_1+\frac{-24\delta^4+403\delta^3-1539\delta^2+2025\delta-729}{3(\delta-1)(\delta-3)\delta(8\delta^2-99\delta+162)} \\
v_3^B(\mathbf{s}) &= -\frac{2(7\delta^2-27)}{(\delta-1)(\delta-3)\delta(8\delta^2-99\delta+162)} \\
\mu_3^B &= -\frac{216\delta^4-3537\delta^3+15714\delta^2-25515\delta+13122}{2(52\delta^3+207\delta^2-972\delta+729)\delta}s_1+\ldots \\
&\quad -\frac{-134\delta^4+1665\delta^3-7371\delta^2+12393\delta-6561}{2(52\delta^3+207\delta^2-972\delta+729)\delta} \\
d_1^B &= d_2^B = s_2 \\
d_3^B &= 0
\end{aligned}$$

where μ_3^B is the probability that legislator 3 chooses legislator 1 as coalition partner when he is both the proposer and the veto player. The difference between Case A and Case B lies in whether the legislator who receives zero in the status quo mixes between coalition partners or not (when he is both the proposer and the veto player). As discussed in Section 5.3, when the other two legislators have similar allocations, coalescing always with the "poorer" one would not constitute an equilibrium because, for some states, the "richer" legislator would be "cheaper". When legislator 3 uses pure strategies and always coalesces with legislator 2, legislator 1 demands more than legislator 2 as long as $s_2 \leq d_1 + \delta v_2(1 - d_1)$ (or $s_1 + \delta v_1 = s_2 + \delta v_2$). This gives us the boundary between the two cases, $s_1 \geq \frac{134\delta^4 + 7371\delta^2 - 1665\delta^3 + 6561 - 12393\delta}{27(\delta^2 + 3 - 4\delta)(8\delta^2 - 99\delta + 162)}$.

Lemma 7. Consider a symmetric Markov Perfect strategy profile with expected utility $U_i(\mathbf{s})$, $\mathbf{s} \in \Delta_2$, determined by the continuation values above. Then, for all $\mathbf{x} = (x, 1 - x, 0) \in \Delta_2$, $U_1(\mathbf{x})$ does not decrease with x, while $U_2(\mathbf{x})$ does not increase with x.

Proof. Denote $\widehat{x} = \frac{134\delta^4 + 7371\delta^2 - 1665\delta^3 + 6561 - 12393\delta}{27(\delta^2 + 3 - 4\delta)(8\delta^2 - 99\delta + 162)}$. Then we have:

$$U_1(x, 1-x, 0) = \begin{cases} 1 + \delta \frac{5 - 3\delta}{3(3 - 4\delta + \delta^2)} & \text{if } x = 1 \\ x + \delta v_1^A(s_1 = x) & \text{if } x \in (\widehat{x}, 1) \\ x + \delta v_1^B(s_1 = x) & \text{if } x \in (1/2, \widehat{x}) \\ x + \delta v_2^B(s_1 = 1 - x) & \text{if } x \in (1 - \widehat{x}, 1/2) \\ x + \delta v_2^A(s_1 = 1 - x) & \text{if } x \in (0, 1 - \widehat{x}) \\ \delta \frac{2}{3(3 - 4\delta + \delta^2)} & \text{if } x = 0 \end{cases}$$

Notice that we have $\frac{\partial U_1(\mathbf{x})}{x} > 0$ for all pieces of the function and for any $\delta \in [0, 1)$. Symmetry completes the proof for $U_2(\mathbf{x})$. $\square$

The optimality of the conjectured proposal and voting strategies for states $\mathbf{s} \in \Delta_2$ follows from the monotonicity established in Lemma 6. ■

Non-Markov Equilibria

I propose strategy profiles such that the initial allocation can be supported as the outcome of a Subgame Perfect Nash Equilibrium (SPNE) and, thus, there is no convergence to full expropriation by the veto player. This SPNE exists as long as the discount factor is high enough and the two non-veto players receive enough. In particular, I want to prove that:

Proposition 8. *For any* $\mathbf{s} \in \Delta$ *such that* $\min_{j=1,2} s_j \geq 1/4$ *, there is a* $\overline{\delta}(\mathbf{s})$ *such that for* $\delta > \overline{\delta}(\mathbf{s})$ *the initial division of the dollar can be supported as the outcome of a Subgame Perfect Nash Equilibrium of the game.*

The idea behind the proof is the following: if a non-veto player accepts a proposal that expropriates the other non-veto player, we switch to a punishment phase in which we reverse to the MPE characterized above. The discount factor needed to support this outcome depends on the share granted to the two non-veto legislators at the beginning of the game: the lower the allocation an agent receives in the initial status quo, the more profitable a deviation.

Proof. To support the initial allocation $\mathbf{s^0}$ as the outcome of a Subgame Perfect Nash Equilibrium, employ the following strategy configuration:

1. whenever a member is recognized, he proposes the status quo allocation $\mathbf{s^0}$ and everyone supports it;

2. if a proposer deviates by proposing $\mathbf{z} \neq \mathbf{s^0}$, every non-veto player j votes against the proposal;

3. if a non-veto player j deviates by voting contrary to the strategies above, from the following

period on we reverse to the MPE equilibrium proposal and voting strategies characterized in Section 3.

The strategies for the punishment phase are clearly a SPNE as shown in the proof of Proposition 1 (MPE being one of the many SPNEs of this game). We need to show that, under certain conditions on $\mathbf{s}^0$ and δ, the non-veto players have no profitable deviation from the equilibrium strategy on the equilibrium path. The payoff to a non-veto player if she follows the equilibrium strategy is:

$$V^j_{EQ}(\mathbf{s}) \quad = \quad \frac{s_j}{1-\delta}$$

The payoff to deviating and proposing or voting in favor an allocation $\mathbf{z} \neq \mathbf{s^0}$ is given by:

$$V^j_{DEV}(\mathbf{x}) \quad = \quad x_j + \delta v^j_{MPE}(\mathbf{x})$$

where $v^j_{MPE}(\mathbf{x})$ is the value function from the MPE characterized in the proof of Proposition 1. The most profitable deviation when proposing is a proposal that assigns the whole dollar to oneself (if this is in the acceptance set of the veto player). Similarly, the most profitable deviation when voting is to accept a veto player's proposal that assigns the whole dollar to oneself. In both cases the expected utility from the deviation is as follows (assuming the deviator is agent 2):

$$V^j_{DEV}(0,1,0) \quad = \quad 1 + \delta\frac{3-3\delta+\delta^2}{(3-\delta)^2(1-\delta)}$$

When is the payoff from the equilibrium strategies higher than the payoff from the most profitable deviation?

$$\begin{aligned} \frac{s_j}{1-\delta} &\geq 1+\delta\frac{3-3\delta+\delta^2}{(3-\delta)^2(1-\delta)} \\ s_j &\geq \frac{(3-2\delta)^2}{(3-\delta)^2} \end{aligned}$$

Since this condition has to hold for both non-veto players, we conclude that an equilibrium where the initial status quo is never changed can be supported by a SPNE if the following condition holds:

$$\min_{i=1,2} s_i^0 \geq \frac{(3-2\delta)^2}{(3-\delta)^2}$$

The right-hand side is a linear and decreasing function of δ, and it is equal to 1 when $\delta = 0$ and to 1/4 when $\delta = 1$. This means that there exists a discount factor for which the proposed strategies can support the initial status quo allocation forever, only as long both non-veto player have at least 1/4 of the dollar each at the beginning of the game. ■

Appendix B

Proofs of Chapter 2

Proof of Proposition 1

Let $y_R^*(\delta, d, n)$ and $y_R^{**}(\delta, d, n)$ be defined by (3.15). Since we are in a regular economy, we have $W/d > y_R^{**}(\delta, d, n)$. We first prove here that for any $y^o \in [y_R^*(\delta, d, n), y_R^{**}(\delta, d, n)]$, there is Markov equilibrium with steady state equal to y^o. Each y^o is supported by a concave equilibrium with investment function $y_R(g\,|y^o\,)$ described by (3.10), where

$$g^2 = \max\left\{\min_{g\geq 0}\left\{g\,|y\left(g\,|y^o\,\right) \leq W + (1-d)g\,\right\}, y_R^*(\delta, d, n)\right\}, \tag{B.1}$$

g_3 is defined by $y\left(g^3\,|y^o\,\right) = y_R^{**}(d, n)$, and $y(g) = y\left(g\,|y^o\,\right)$ is the the unique solution of (4.6) with initial condition $y\left(y_R^o\,|y_R^o\,\right) = y^o$. This proves the "sufficiency" part of the statement. Then we prove that the steady state must be in $[y_R^*(\delta, d, n), y_R^{**}(\delta, d, n)]$. This proves the "necessity" part of the statement.

Sufficiency

To construct the equilibrium we proceed in 3 steps.

Step 1. We first construct the strategies for a generic y^o and prove their key properties. Let $y\left(g\,|y^o\,\right)$ be the solution of the differential equation when we require the initial condition: $y\left(y^o\,|y^o\,\right) = y^o$, for $y^o \in [y^*_R(\delta, d, n), y^{**}_R(\delta, d, n)]$. Let $g^2(y)$ be defined by (B.1). This, essentially, is the largest point between the point at which $y\left(g\,|y^o\,\right)$ crosses from below $W + (1-d)g$, and $y^*_R(\delta, d, n)$. Let $g^3(y^o)$ be defined by $y\left(g^3(y^o)\,|y^o\,\right) = y^{**}_R(\delta, d, n)$.

Lemma A.1. *$y'\left(g\,|y^o\,\right) \in (0,1)$ in $\left[g^2(y^o), y^{**}_R(\delta, d, n)\right]$ and $y''\left(g\,|y^o\,\right) \geq 0$.*

Proof. From (4.6), $y'\left(g\,|y^o\,\right) \geq 0$ for $g \geq y^*_R(\delta, d, n)$, *and* $y'\left(g\,|y^o\,\right) \leq 1$ for $g \leq y^{**}_R(\delta, d, n)$. Since $y''\left(g\,|y^o\,\right) = \frac{n}{1-n}\left[\frac{u''(g)}{\delta}\right]$, $y''(g) > 0$. ■

Lemma A.2. *For any $y^o \in [y^*_R(\delta, d, n), y^{**}_R(\delta, d, n)]$, $g^3(y^o) \geq y^{**}_R(\delta, d, n)$.*

Proof. Note that $y\left(y^{**}_R(\delta, d, n)\,|y^o\,\right)$ is increasing in y^o. Moreover $y\left(y^{**}_R(\delta, d, n)\,|y^{**}_R(\delta, d, n)\,\right) = y^{**}_R(\delta, d, n)$. So $y\left(y^{**}_R(\delta, d, n)\,|y^o\,\right) < y^{**}_R(\delta, d, n)$ for $y^o < y^{**}_R(\delta, d, n)$. It follows that $g^3(y^o) \geq y^{**}_R(\delta, d, n)$ for any $y^o \leq y^{**}_R(\delta, d, n)$. ■

We have:

Lemma A.3. *$y\left(g\,|y^o\,\right) \in (0, W + (1-d)g)$ in $\left(g^2(y^o), g^3(y^o)\right)$.*

Proof. First note that $y\left(g^2(y^o)|y^o\right) \leq W + (1-d)g^2(y^o)$. Since $y'\left(g\,|y^o\,\right) < 1$ for $g < y^{**}_R(\delta, d, n)$ we must have $y\left(g\,|y^o\,\right) < W + (1-d)g$ for $g \in \left(g^2(y^o), y^{**}_R(\delta, d, n)\right)$. For $g > y^{**}_R(\delta, d, n)$, we have $W + (1-d)g > W + (1-d)y^{**}_R(\delta, d, n)$. Since $y\left(g\,|y^o\,\right) < y^{**}_R(\delta, d, n)$ in $\left(g^2(y^o), g^3(y^o)\right)$, We have $y\left(g\,|y^o\,\right) < y^{**}_R(\delta, d, n) < W + (1-d)y^{**}_R(\delta, d, n) < W + (1-d)g$ in $\left[y^{**}_R(\delta, d, n), g^3(y^o)\right)$ as well. Similarly, since $y'\left(g\,|y^o\,\right) \geq 0$ for $g > g^2(y^o)$ and $y\left(g^2(y^o)|y^o\right) \geq 0$, we must have $y\left(g\,|y^o\,\right) > 0$ for $g > g^2(y^o)$. Note that $y\left(g^2(y^o)\,|y^o\,\right) \geq 0$ since $y'\left(g\,|y^o\,\right) \in (0, 1-d)$ in $[y^*_R(\delta, d, n), y^o]$ implies that $y\left(g\,|y^o\,\right) > g$ for all $g \in [y^*_R(\delta, d, n), y^o]$. ■

For any $y^o \in [y_R^*(\delta,d,n), y_R^{**}(\delta,d,n)]$, we now define the investment function:

$$y_R(g\,|y^o\,) = \begin{cases} \min\left\{W + (1-d)g, y\left(g^2(y^o)\,|y^o\,\right)\right\} & g \leq g^2(y^o) \\ y\left(g\,|y^o\,\right) & g^2(y^o) < g \leq g^3(y^o) \\ y_R^{**}(\delta,d,n) & g \geq g^3(y^o) \end{cases}$$

For future reference, define $g^1(y^o) = \max\left\{0, \left(y\left(g^2(y)\,|y^o\,\right) - W\right)/(1-d)\right\}$. This is the point at which $W + (1-d)g^2(y^o) = y\left(g^2(y^o)\,|y^o\,\right)$, if positive. Clearly, we have $g^1(y^o) \in \left[0, g^2(y^o)\right]$. We have:

Lemma A.4. *For any* $y^o \in [y_R^*(\delta,d,n), y_R^{**}(\delta,d,n)]$, $y\left(g\,|y^o\,\right) \in \left[g^2(y^o), g^3(y^o)\right]$ *for* $g \in \left[g^2(y^o), g^3(y^o)\right]$.

Proof. Since $y\left(g\,|y^o\,\right)$ it is monotonic non-decreasing in $g \in \left[g^2(y^o), g^3(y^o)\right]$,

$$y\left(g\,|y^o\,\right) \in \left[y\left(g^2(y^o)\,|y^o\,\right), y\left(g^3(y^o)\,|y^o\,\right)\right] \quad \forall g \in \left[g^2(y^o), g^3(y^o)\right].$$

Since $y\left(g\,|y^o\,\right)$ has slope lower than one in $\left[g^2(y^o), g^3(y^o)\right]$ and $y\left(y^o\,|y^o\,\right) = y^o$ for $y^o \geq g^2(y^o)$, we must have $y\left(g^2(y^o)\,|y^o\,\right) \geq g^2(y^o)$, so $y\left(g\,|y^o\,\right) \geq g^2(y^o)$ *for* $g \in \left[g^2(y^o), g^3(y^o)\right]$. Similarly, $y\left(g^3(y^o)\,|y^o\,\right) \leq g^3(y^o)$, so $y\left(g\,|y^o\,\right) \leq g^3(y^o)$ *for* $g \in \left[g^2(y^o), g^3(y^o)\right]$. ∎

Step 2. We now construct the value functions corresponding to each steady state $y^o \in [y_R^*(\delta,d,n), y_R^{**}(\delta,d,n)]$. For $g \in \left[g^2(y^o), g^3(y^o)\right]$ define the value function recursively as

$$v(g|y^o) = \frac{W + (1-d)g - y\left(g\,|y^o\,\right)}{n} + u(y\left(g\,|y^o\,\right)) + \delta v(y\left(g\,|y^o\,\right)). \tag{B.2}$$

By Theorem 3.3 in Stokey, Lucas, and Prescott (1989), the right hand side of (B.2) is a contraction: it defines a unique, continuous and differentiable value function $v_0(g\,|y^o\,)$ for this interval of g.

(differentiability follows from the differentiability of $y\left(g\left|y^o\right.\right)$). We have

Lemma A.5. For any $y^o \in [y_R^*(\delta,d,n), y_R^{**}(\delta,d,n)]$ and any $g \in \left[g^2(y^o), g^3(y^o)\right]$, $u'(g) + \delta v_0'(g; y^o) = 1$.

Proof. Note that by Lemma A.4, for $g \in \left[g^2(y^o), g^3(y^o)\right]$, we have $y\left(g\left|y^o\right.\right) \in \left[g^2(y^o), g^3(y^o)\right]$. From (4.6) we can write (for simplicity we write $y'\left(g\left|y^o\right.\right) = y'(g)$):

$$\frac{1-u'(g)}{\delta} = \frac{1-y'(g)}{n} + u'(y(g))y'(g) + \left[1-u'(y(g))\right]y'(g)$$

for any $g \in \left[g^2(y^o), g^3(y^o)\right]$. But then using (4.6) again allows to substitute $1-u'(y(g))$ to obtain:

$$\begin{aligned}\frac{1-u'(g)}{\delta} &= \frac{1-y'(g)}{n} + u'(y(g))y'(g) \\ &\quad +\delta\left[\frac{1-y'(y(g))}{n} + u'(y^2(g))y'((y(g)) + \left[1-u'(y^2(g))\right]y'(y(g))\right]y'(g)\end{aligned}$$

where $y^0(g) = g$, $y^1(g) = y(g)$, $y^m(g) = y(y^{m-1}(g))$, and $\left[y'\right]^0(g) = 1$, $\left[y'\right]^1(g) = y'(g)$, and $\left[y'\right]^m(g) = y'(\left[y'\right]^{m-1}(g))$. Iterating we have:

$$\begin{aligned}\frac{1-u'(g)}{\delta} &= \lim_{n\to\infty}\sum\nolimits_{j=0}^{n}\delta^j\left[\frac{1-y'(y^j(g\left|y^o\right.)\left|y^o\right.)}{n} + u'(y^{j+1}(g))y'(y^j(g\left|y^o\right.)\left|y^o\right.)\right]_{i=0}^{j}\left[y'\right]^i(y^{i-1}(g)) \\ &= v'(g\left|y^o\right.)\end{aligned}$$

This implies $u'(g) + \delta v_0'(g; y^o) = 1$. ■

In the rest of the state space we define the value function recursively. In $\left[g^1(y^o), g^2(y^o)\right]$, if $g^1(y^o) < g^2(y^o)$, the value function is defined as:

$$v_0(g\left|y^o\right.) = \frac{W + (1-d)g - y\left(g^2(y^o)\left|y^o\right.\right)}{n} + u(y\left(g^2(y^o)\left|y^o\right.\right)) + \delta v_0(y\left(g^2(y^o)\left|y^o\right.\right)) \qquad \text{(B.3)}$$

for $y\left(g^2(y^o)\,|y^o\right) \in \left[g^2(y^o), g^3(y^o)\right]$.

Lemma A.6. *For any* $g \in \left[g^1(y^o), g^3(y^o)\right]$*,* $u(g) + \delta v(g\,|y^o\,)$ *is concave and has slope larger or equal than* 1*.*

Proof. If $g^1(y^o) = g^2(y^o)$, the result follows from the previous lemma. Assume therefore, $g^1(y^o) < g^2(y^o)$. In this case $g^2(y^o) = y_R^*(\delta, d, n)$. For any $g \in \left[g^1(y^o), g^2(y^o)\right]$, $y(g; y^o) = y\left(y_R^*(\delta, d, n)\,|y^o\,\right)$. So we have $v_0'(g\,|y^o\,) = (1-d)/n$ implying: $u'(g) + \delta v_0'(g\,|y^o\,) = u'(g) + \delta(1-d)/n > 1$ since $g \le g^2(y^o) = y_R^*(\delta, d, n)$. The statement then follows from this fact and Lemma A.5. ■

Consider $g < g^1(y^o)$. In $\left[g_{-1}, g^1(y^o)\right]$ the value function is defined as:

$$v_{-1}(g\,|y^o\,) = u(W + (1-d)g) + \delta v_0(W + (1-d)g\,|y^o\,)$$

where $g_{-1} = \max\left\{0, \frac{g^1(y^o)-W}{1-d}\right\}$. Assume that we have defined the value function in $g \in \left[g_{-t}, g_{-(t-1)}\right]$ as v_{-t}, for all t such that $g_{-(t-1)} > 0$. Then we can define $v_{-(t+1)}$ as:

$$v_{-(t+1)}(g\,|y^o\,) = u(W + (1-d)g) + \delta v_{-t}(W + (1-d)g\,|y^o\,),$$

in $\left[g_{-(t+1)}, g_{-t}\right]$ with $g_{-(t+1)} = \frac{g_{-t}-W}{1-d}$.

Lemma A.7. *For any* $g \in [0, g^3(y^o)]$*,* $u(g) + \delta v(g\,|y^o\,)$ *is concave and it has slope greater than or equal than* 1*.*

Proof. We prove this by induction on t. Consider now the interval $\left[\frac{g^1(y^o)-W}{1-d}, g^1(y^o)\right]$. In this range we have

$$v_{-1}'(g\,|y^o\,) = \left[u'(W + (1-d)g) + \delta v_0'(W + (1-d)g\,|y^o\,)\right](1-d) \ge 1-d$$

since $W+(1-d)g \in \left[g^1(y^o), g^3(y^o)\right]$. It follows that for $g \in \left[\frac{g^1(y^o)-W}{1-d}, g^1(y^o)\right]$:

$$u'(g) + \delta v'_{-1}(g\,|y^o\,) \geq u'(g) + \delta(1-d) > 1 \tag{B.4}$$

Where the last inequality follows from the fact that $g \leq g^2(y^o) < y_R^{**}(\delta, d, n)$. Note, moreover, that the right and left derivative of $v(g\,|y^o\,)$ at $g^1(y^o)$ are the same. To see this note that by the argument above, the left derivative is $(1-d)/n$; by Lemma A.5, however, the right derivative is $(1-u'(y_R^*(\delta, d, n)))\,/\delta = (1-d)/n$ as well. We conclude that $u'(g) + \delta v'_{-1}(g\,|y^o\,)$ is concave, it has derivative larger than 1. Assume that we have shown that for $g \in \left[g_{-t}, g^3(y^o)\right]$, $u(g) + \delta v_{-t}(g\,|y^o\,)$ is concave and $u'(g) + \delta v'_{-t}(g\,|y^o\,) > 1$. Consider in $g \in \left[g_{-(t+1)}, g_{-t}\right]$. We have:

$$v'_{-(t+1)}(g\,|y^o\,) = \left[u'(W+(1-d)g) + \delta v'_{-t}(W+(1-d)g\,|y^o\,)\right](1-d) \geq 1-d$$

since $W+(1-d)g \geq \left[g_{-t}, g^3(y^o)\right]$. So $u'(g) + \delta v'_{-(t+1)}(g\,|y^o\,) \geq u'(g) + \delta(1-d) \geq 1$. By the same argument as above, moreover, v is concave at g_{-t}. We conclude that for any $g \leq g^1$, $u(g) + \delta v(g\,|y^o\,)$ is concave and it has slope larger than 1. ■

We can define the value function for $g \geq g^3(y^o)$ as:

$$v_1(g\,|y^o\,) = \frac{W+(1-d)g - y_R^{**}(\delta, d, n)}{n} + u(y_R^{**}(\delta, d, n)) + \delta v_0(y_R^{**}(\delta, d, n)\,|y^o\,)$$

since, by Lemma A.2, $g^3(y^o) \geq y_R^{**}(\delta, d, n)$.

Lemma A.8. *For any $g \geq 0$, $u(g) + \delta v(g\,|y^o\,)$ is concave and it has slope less than or equal than 1.*

Proof. For $g > g^3(y^o)$, $v'(g\,|y^o\,) = (1-d)\,/n$. Since, by Lemma A.2, $g \geq y_R^{**}(\delta, d, n) \geq y_R^*(\delta, d, n)$, we have $u'(g) + \delta v'(g\,|y^o\,) < 1$. Previous lemmas imply $u(g) + \delta v(g|y^o)$ is concave and has slope greater than or equal than 1 for $g \leq g^3(y^o)$. This establishes the result. ■

Step 3. Define

$$x(g\,|y^o\,) = \frac{W + (1-d)g - y(g\,|y^o\,)}{n}, \text{ and } i(g\,|y^o\,) = \frac{y(g\,|y^o\,) - (1-d)g}{n}$$

as the levels of per capita private consumption and investment, respectively. Note that by construction, $x(g\,|y^o\,) \in [0, W/n]$. We now establish that $y(g\,|y^o\,)$, $x(g\,|y^o\,)$ and the associated value function $v\,(g\,|y^o\,)$ defined in the previous steps constitute an equilibrium. We first show that given $y(g\,|y^o\,)$, $v\,(g\,|y^o\,)$ describes the expected continuation value to an agent, starting at state g. Since $y(g\,|y^o\,) \in \left[g^2(y^o), g^3(y^o)\right]$ for $g \in \left[g^2(y^o), g^3(y^o)\right]$, $v\,(g\,|y^o\,)$ must be described by (B.2) for $g \in \left[g^2(y^o), g^3(y^o)\right]$. By construction, moreover, $v(g\,|y^o\,)$ is the expected continuation value to an agent in all states $g \geq g^3(y^o)$, and $g \leq g^2(y^o)$. We now show that $y(g\,|y^o\,)$ is an optimal reaction function given $v(g\,|y^o\,)$. An agent solves the problem (4.3), where $y_R(g) = y(g\,|y^o\,)$. Note that $y(g\,|y^o\,)$ satisfies the constraints of this problem if $y(g\,|y^o\,) \leq \frac{W+(1-d)g}{n} + \frac{n-1}{n} y(g\,|y^o\,)$, so if $y(g\,|y^o\,) \leq W + (1-d)g$; and if $y(g\,|y^o\,) \geq \frac{n-1}{n} y(g\,|y^o\,)$, so if $y(g\,|y^o\,) \geq 0$. Both conditions are automatically satisfied by construction. If $g < g^1(y^o)$, we have $u'(y) + \delta v'(y) \geq 1$ for all $y \in [0, W + (1-d)g]$, so $y(g\,|y^o\,) = W + (1-d)g$ is optimal. If $g \geq g^1(y^o)$, then $y(g\,|y^o\,)$ is an unconstrained optimum, so again it is an optimal reaction function.

Necessity

We now prove that any stable steady state of an equilibrium must be in $[y^*_R(\delta, d, n), y^{**}_R(\delta, d, n)]$. We proceed in two steps.

Step 1. We first prove that $y_R^o \leq y_R^{**}(\delta, d, n)$. Suppose to the contrary that there is stable steady state at $y_R^o > y_R^{**}(\delta, d, n)$. We must have $y_R^o \in (y_R^{**}(\delta, d, n), W/d]$, since it is not feasible for a steady state to be larger than W/d. Consider a left neighborhood of y_R^o, $N_\varepsilon(y_R^o) = (y_R^o - \varepsilon, y_R^o)$. The value function can be written in $g \in N_\varepsilon(y_R^o)$ as:

$$\begin{aligned} v_R(g) &= \frac{W + (1-d)g - y_R(g)}{n} + u(y_R(g)) + \delta v_R(y_R(g)) \\ &= u(y_R(g)) + \delta v_R(y_R(g)) - y_R(g) + \frac{W + (1-d)g}{n} + (1 - 1/n)\, y_R(g) \end{aligned} \tag{B.5}$$

In $N_\varepsilon(y_R^o)$ the constraint $y \geq \frac{n-1}{n} y_R(g)$ cannot be binding, else we would have $y_R(g) = (1 - 1/n)y_R(g)$, so $y_R(g) = 0$: but this is not possible in a neighborhood of $y_R^o > 0$. We consider two cases.

Case 1. Suppose first that $y_R^o < W/d$. We must therefore have that $y_R(g) < W + (1-d)g$ in $N_\varepsilon(y_R^o)$, so the constraint $y \leq \frac{W+(1-d)g}{n} + \frac{n-1}{n} y$ is not binding. The solution is in the interior of the constraint set of (4.3), and the objective function $u(y_R(g)) + \delta v_R(y_R(g)) - y_R(g)$ is constant for $g \in N_\varepsilon(y_R^o)$.

Lemma A.9. *There is a neighborhood $N_\varepsilon(y_R^o)$ in which $y_R(g)$ is strictly increasing.*

Proof. Suppose to the contrary that, for any $N_\varepsilon(y_R^o)$, there is an interval in $N_\varepsilon(y_R^o)$ in which $y_R(g)$ is constant. Using the expression for $v_R(g)$ presented above, we must have $v_R'(g) = (1-d)/n$ for any g in this interval. Since $N_\varepsilon(y_R^o)$ is arbitrary, then we must have a sequence $g^m \to y_R^o$ such that $v_R'(g^m) = (1-d)/n\ \forall m$. We can therefore write:

$$\begin{aligned} v_R^-(y_R^o) &= \lim_{\Delta \to 0} \frac{v_R(y_R^o) - v_R(y_R^o - \Delta)}{\Delta} = \lim_{\Delta \to 0} \lim_{m \to \infty} \frac{v_R(g^m) - v_R(g^m - \Delta)}{\Delta} \\ &= \lim_{m \to \infty} \lim_{\Delta \to 0} \frac{v_R(g^m) - v_R(g^m - \Delta)}{\Delta} = \frac{1-d}{n} \end{aligned}$$

where $v_R^-(y_R^o)$ is the left derivative of $v_R(g)$ at y_R^o, and the second equality follows from the continuity of $v_R(g)$. Consider now a marginal reduction of g at y_R^o. The change in utility is (as $\Delta \to 0$):

$$\begin{aligned}\Delta U\left(y_R^o\right) &= u(y_R^o - \Delta) - u(y_R^o) + \delta\left[v_R(y_R^o - \Delta) - v_R(y_R^o)\right] + \Delta \\ &= \left[1 - \left(u'(y_R^o) + \delta\frac{1-d}{n}\right)\right]\Delta\end{aligned}$$

In order to have $\Delta U\left(y_R^o\right) \leq 0$, we must have $u'(y_R^o)+\delta(1-d)/n \geq 1$. This implies $y_R^o \leq y_R^*(\delta, d, n) < y_R^{**}(\delta, d, n)$, a contradiction. Therefore, if there is stable steady state at $y_R^o > y_R^{**}(\delta, d, n)$, then $y_R(g)$ is strictly increasing in a neighborhood $N_\varepsilon(y_R^o)$. ■

Lemma A.9 implies that there is a neighborhood $N_\varepsilon(y_R^o)$ in which $u(g) + \delta v_R(g) - g$ is constant. Since y_R^o is a stable steady state and $y_R(g)$ is strictly increasing. Moreover, for any open left neighborhood $N_{\varepsilon'}(y_R^o) = (y_R^o - \varepsilon', y_R^o) \subset N_\varepsilon(y_R^o)$, $g \in N_{\varepsilon'}(y_R^o)$ implies $y_R(g) \in N_{\varepsilon'}(y_R^o))$. These observations imply:

Lemma A.10. *There is a neighborhood $N_\varepsilon(y_R^o)$ in which*

$$y_R'(g) = \frac{n}{n-1}\left(\frac{1-u'(g)}{\delta} - \frac{1-d}{n}\right) \tag{B.6}$$

Proof. There is a $N_\varepsilon(y_R^o)$ and a constant K such that $\delta v_R\left(g\right) = K + g - u(g)$ for $g \in N_\varepsilon(y_R^o))$. Hence $v_R\left(g\right)$ is differentiable in $N_\varepsilon(y_R^o)$. Moreover, $y_R(g) \in N_\varepsilon(y_R^o)$ for all $g \in N_\varepsilon(y_R^o)$. Hence $u(y_R(g)) + \delta v\left(y_R(g)\right) - y_R(g)$ is constant in $g \in N_\varepsilon(y_R^o)$ as well. These observations and the definition of $v_R(g)$ imply that $v_R'(g) = \frac{1-d}{n} + \left(1 - \frac{1}{n}\right) y_R'(g)$ in $N_\varepsilon(y_R^o)$ (where $y_R(g)$ must be differentiable otherwise $v_R(g)$ would not be differentiable). Given that $u'(g) + \delta v_R'(g) = 1$ in

$g \in N_\varepsilon(y_R^o)$, we must have:

$$u'(g) + \delta v_R'(g) = u'(g) + \delta \left[\frac{1-d}{n} + \left(1 - \frac{1}{n}\right) y_R'(g) \right] = 1$$

which implies (B.6) for any $g \in N_\varepsilon(y_R^o)$. ■

Let g^m be a sequence in $N_\varepsilon(y_R^o)$ such that $g^m \to y_R^o$. We must have

$$\begin{aligned} y_R^-(y_R^o) &= \lim_{\Delta \to 0} \frac{y_R(y_R^o) - y_R(y_R^o - \Delta)}{\Delta} = \lim_{\Delta \to 0} \lim_{m \to \infty} \frac{y_R(g^m) - y_R(g^m - \Delta)}{\Delta} \\ &= \lim_{m \to \infty} \lim_{\Delta \to 0} \frac{y_R(g^m) - y_R(g^m - \Delta)}{\Delta} = \frac{n}{n-1} \left(\frac{1 - u'(y_R^o)}{\delta} - \frac{1-d}{n} \right) \end{aligned} \tag{B.7}$$

where $y_R^-(y_R^o)$ is the left derivative of $y_R(y_R^o)$, and the second equality follows from continuity. Consider a state $(y_R^o - \Delta)$. For y_R^o to be stable we need that for any small Δ:

$$y_R(y_R^o - \Delta) \geq y_R^o - \Delta = y_R(y_R^o) + (y_R^o - \Delta) - y_R^o$$

where the equality follows from the fact that $y_R(y_R^o) = y_R^o$. As $\Delta \to 0$, this implies $y_R^-(y_R^o) \leq 1$ in $N_\varepsilon(y_R^o)$. By (B.7), we must therefore have:

$$\frac{n}{n-1} \left(\frac{1 - u'(y_R^o)}{\delta} - \frac{1-d}{n} \right) \leq 1$$

This implies: $y_R^o \leq y_R^{**}(\delta, d, n)$, a contradiction.

Case 2. Assume now that $y_R^o = W/d$ and it is a strict local maximum of the objective function $u(y) + \delta v_R(y) - y$. In this case in a left neighborhood $N_\varepsilon(y_R^o)$, we have that the upperbound $y \leq \frac{W+(1-d)g}{n} + \frac{n-1}{n} y_R(g)$ is binding: implying $y_R(g) = W + (1-d)g$ in $N_\varepsilon(y_R^o)$. We must therefore have a sequence of points $g^m \to y_R^o$ such that $g^m = y_R(g^{m-1})$ and $y_R(g^m) = W + (1-d)g^m$ $\forall m$. Given this, we can write:

$$\begin{aligned} v_R(g^m) &= u(g^{m+1}) + \delta v_R(g^{m+1}) = u(g^{m+1}) + \delta \left[u(g^{m+2}) + \delta v_R(g^{m+2}) \right] \\ &= \sum\nolimits_{j=0}^{\infty} \delta^j u(W + (1-d)g^{m+j}) \end{aligned}$$

note that since $g^{m+1} = W + (1-d)g^m$, the derivative of g^{m+1} with respect to g^m is $\left[g^{m+1}\right]' = (1-d)$. By an inductive argument, it is easy to see that $\left[g^{m+j}\right]' = (1-d)^j$. So $v_R(g^m)$ is differentiable and:

$$\delta v_R'(g^m) = \sum\nolimits_{j=0}^{\infty} \left[\delta(1-d)\right]^{j+1} u'(W + (1-d)g^{m+j}).$$

Since $u'(g^m) + \delta v_R'(g^m) \geq 1$, we have:

$$u'(g^m) + \sum\nolimits_{j=0}^{\infty} \left[\delta(1-d)\right]^{j+1} u'(W + (1-d)g^{m+j}) \geq 1$$

for all m. Consider the limit as $m \to \infty$. Since $u'(g)$ is continuous and $g^m \to y_R^o$, we have:

$$\begin{aligned} 1 &\leq \lim_{m\to\infty} \left[u'(g^m) + \sum\nolimits_{j=0}^{\infty} \left[\delta(1-d)\right]^{j+1} u'(W + (1-d)g^{m+j}) \right] \\ &= u'(y_R^o) + \sum\nolimits_{j=0}^{\infty} \left[\delta(1-d)\right]^{j+1} u'(y_R^o) = \frac{u'(y_R^o)}{1 - \delta(1-d)} \end{aligned}$$

This implies $y_R^o \leq \left[u'\right]^{-1} (1 - \delta(1-d)) < y_R^{**}(\delta, d, n)$, a contradiction.

Case 3. Assume now that $y_R^o = W/d$, but it is not a strict maximum of $u(y) + \delta v_R(y) - y$ in any left neighborhood. It must be that $u(y) + \delta v_R(y) - y$ is constant in some left neighborhood $N_\varepsilon(y_R^o)$. If this were not the case, then in any left neighborhood we would have an interval in which

$y_R(g)$ is constant, but this is impossible by Lemma A.9. But then if $u(y) + \delta v_R(y) - y$ is constant in some $N_\varepsilon(y_R^o)$, the same argument as in Case 1 of Step 1 implies a contradiction.

Step 2. We now prove that $y_R^o \geq y_R^*(\delta, d, n)$. Assume there is stable steady state at $y_R^o < y_R^*(\delta, d, n)$. Since $\lim_{g\to 0} u'(g) = \infty$, $y_R^o > 0$. There is therefore a neighborhood $N_\varepsilon(y_R^o) = (y_R^o, y_R^o + \varepsilon)$ in which $y_R(g)$ satisfies all the constraints of (4.3) and it maximizes $u(y) + \delta v_R(y) - y$ We conclude that the objective function $u(y_R(g)) + \delta v_R(y_R(g)) - y_R(g)$ is constant in $N_\varepsilon(y_R^o)$. By the same argument as in Lemma A.9 it follows that there is a neighborhood $N_\varepsilon(y_R^o)$ in which $y_R(g)$ is strictly increasing. Since y_R^o is a stable steady state and $y_R(g)$ is strictly increasing in $N_\varepsilon(y_R^o)$, there is a neighborhood $N_\varepsilon(y_R^o)$ of y_R^o such that for any open right neighborhood $N_{\varepsilon'}(y_R^o) = (y_R^o, y_R^o + \varepsilon') \subset N_\varepsilon(y_R^o)$, $g \in N_{\varepsilon'}(y_R^o))$ implies $y_R(g) \in N_{\varepsilon'}(y_R^o)$. By the same argument as in Lemma A.10, it follows that there is a $N_{\varepsilon'}(y_R^o)$ in which $y_R'(g)$ is given by (B.6). Equation (B.6), however, implies that $y_R'(g) \geq 0$ only for states $g \geq y_R^*(\delta, d, n)$. This implies that $y_R(g)$ is non-monotonic, a contradiction. ■

Proof of Proposition 2

Since we are in a regular economy, we have $W/d > y_R^{**}(\delta, d, n)$. We construct here a concave and monotonic equilibrium with steady state is $y_{IR}^o(d, n)$ as defined in (3.18). We proceed in two steps.

Step 1. We first construct the strategies. Remember that $\overline{y}(\delta, d) \equiv y_P^*(\delta, d, 1) = [u']^{-1}(1 - \delta(1 - d))$. This is the point at which the solution of the differential equation (4.6) has slope $(1 - d)$. Define g_{IR}^2 as:

$$g_{IR}^2 = \max\left\{\min_{g\geq 0}\{g\,|\widehat{y}(g) \leq W + (1-d)g\}, y_R^*(\delta, d, n)\right\}. \qquad \text{(B.8)}$$

The investment function is defined as:

$$y_{IR}(g) = \begin{cases} \min\left\{W + (1-d)g, \widehat{y}\left(g^2_{IR}\right)\right\} & g \leq g^2_{IR} \\ \widehat{y}\left(g\right) & g^2_{IR} < g \leq \overline{y}(\delta, d) \\ (1-d)g & g \geq \overline{y}(\delta, d) \end{cases}$$

Using the same argument as in the proof of Proposition 1, we can prove that $y_{IR}(g)$ is continuous and almost everywhere differentiable with right and left derivative at any point, and $y_{IR}(g) \in [(1-d)g, W + (1-d)g]$ for any g. Finally, it is easy to see that $y_{IR}(g)$ has a unique fixed-point y^o_{IR} such that $y_{IR}(y^o_{IR}) = y^o_{IR} \in \left[g^2_{IR}, \overline{y}(\delta, d)\right]$.

Step 2. We now construct the value function $v_{IR}(g)$ associated to $y_{IR}(g)$, and prove that $y_{IR}(g), v_{IR}(g)$ is an equilibrium. For $g \leq \overline{y}(\delta, d)$, we define the value function exactly as in Step 2 of Section 7.1.1. For $g \geq \overline{y}(\delta, d)$, note that $y_{IR}(g) < g$, so we can define the value function recursively as:

$$v_{IR}(g) = \frac{W}{n} + u((1-d)g) + \delta v_{IR}((1-d)g). \tag{B.9}$$

The value function defined above is continuous in g. Using the same argument as in Step 2 of Section 7.1.1 we can show that $u(g) + \delta v(g; y^o_{IR}) - y$ is weakly concave in g for $g \leq \overline{y}(\delta, d)$; it is strictly increasing in $\left[0, g^2_{IR}\right]$, and flat in $\left[g^2_{IR}, \overline{y}(\delta, d)\right]$. Consider now states $g > \overline{y}(\delta, d)$. Let $g^4 = \frac{\overline{y}(\delta,d)}{1-d}$. In $\left[\overline{y}(\delta, d), g^4\right]$, we must have $(1-d)g \in \left[g^2_{IR}, \overline{y}(\delta, d)\right]$. Note that $u'(g) + \delta v'_{IR}(g) = 1$ for $g \in \left[g^2_{IR}, \overline{y}(\delta, d)\right]$, so by (B.9) we have

$$v'_{IR}(g) = (1-d)\left[u'((1-d)g) + \delta v'_{IR}((1-d)g)\right] = 1-d$$

for $g \in \left[\overline{y}(\delta, d), g^4\right]$. This fact implies that $u'(g) + \delta v'_{IR}(g) = u'(g) + \delta\,(1-d)$ for any $g \in \left[\overline{y}(\delta, d), g^4\right]$, and hence it is concave in this interval. It follows that $v_{IR}(g)$ is concave in $g \leq g^4$ because $u'(g) + \delta v'_{IR}(g) \leq 1$ for any $g \in \left[\overline{y}(\delta, d), g^4\right]$. Using a similar approach we can prove that $v_{IR}(g)$ is concave for all g, and we have $u'(g) + \delta v'_{IR}(g) \leq 1$ for $g \geq \overline{y}(\delta, d)$. To prove that $y_{IR}(g), v_{IR}(g)$ is an equilibrium, we proceed exactly as in Step 3 of Section 7.1.1 to establish that $y_{IR}(g)$ is optimal given $v_{IR}(g)$, and that $v_{IR}(g)$ satisfied (3.17) given $y_{IR}(g)$.■

Proof of Proposition 3

We proceed in 2 steps.

Step 1. The same argument used in Step 1 of Section 7.1.2 shows that no equilibrium stable steady state can be greater than $y^{**}_R(\delta, d, n) = [u']^{-1}(1 - \delta(1 - d/n))$. The same argument used in Step 2 in Section 7.1.2 we can show that no equilibrium can be less than $y^*_R(\delta, d, n)$, so $y^*_{IR}(\delta, d, n) \geq y^*_R(\delta, d, n)$.

Step 2. Consider a sequence $d^m \to 0$. For each d^m there is at least an associated equilibrium $y_m(g)$, $v_m(g)$ with steady state y^o_m. If follows trivially that $\lim_{m\to\infty} y^{**}_R(d^m, n) = [u']^{-1}(1 - \delta) = \overline{y}(0)$.

What remains to be shown is that $\lim_{m\to\infty} y^*_{IR}(d^m, n) = [u']^{-1}(1 - \delta) = \overline{y}(0)$. Let Γ_m be the set of equilibrium steady states when the rate of depreciation is d^m. We now show by contradiction that for any $\xi > 0$, there is a $\widetilde{m}$ such that for $m > \widetilde{m}$, $\inf_y \Gamma_m \geq \overline{y}(0) - \xi$. Since $\inf_y \Gamma_m \leq y^{**}_R(\delta, d, n)$, this will immediately imply that $y^*_{IR}(\delta, d, n) \to \overline{y}(0)$. Suppose to the contrary there is a sequence of steady states y^0_m, with associated equilibrium investment and value functions $y_m(g)$, $v_m(g)$, and an $\xi > 0$ such that $y^0_m < \overline{y}(0) - \xi$ for any arbitrarily large m. Define $y^0_m(g) = y_m(g)$, and $y^j_m(g) = y_m(y^{j-1}_m(g))$ and consider a marginal deviation from the steady state from y^0_m to $y^0_m + \Delta$. By the irreversibility constraint we have $y_m(g) \geq (1 - d^m)\, g$. Using this property and the fact that

y_m^0 is a steady state, so $y_m^j(y_m^0) = y_m^0$, we have:

$$y_m(y_m^0 + \Delta) - y_m(y_m^0) \geq (1 - d^m)\left(y_m^0 + \Delta\right) - y_m^0 = (1 - d^m)\,\Delta - d^m y_m^0$$

This implies that, as $m \to \infty$, for any given Δ:

$$\frac{y_m(y_m^0 + \Delta) - y_m^0}{\Delta} \geq 1 + o_1\left(d^m\right)$$

where $o_1\left(d^m\right) \to 0$ as $m \to 0$. We now show with an inductive argument that a similar property holds for all iterations $y_m^j(y_m^0)$. Assume we have shown that:

$$\frac{y_m^{j-1}(y_m^0 + \Delta) - y_m^0}{\Delta} \geq 1 + o_{j-1}\left(d^m\right)$$

where $o_{j-1}\left(d^m\right) \to 0$ as $m \to 0$. We must have:

$$y_m(y_m^{j-1}(y_m^0 + \Delta)) - y_m^j(y_m^0) \geq (1 - d^m)\, y_m^{j-1}(y_m^0 + \Delta) - y_m^0$$

We therefore have:

$$y_m(y_m^{j-1}(y_m^0 + \Delta)) - y_m^0 \geq y_m^{j-1}(y_m^0 + \Delta) - y_m^0 - d^m y_m^{j-1}(y_m^0 + \Delta)$$

so we have:

$$\begin{aligned} \frac{y_m(y_m^{j-1}(y_m^0 + \Delta)) - y_m^0}{\Delta} &\geq \frac{y_m^{j-1}(y_m^0 + \Delta) - y_m^0}{\Delta} - \frac{d^m y_m^{j-1}(y_m^0 + \Delta)}{\Delta} \\ &\geq 1 + o_j\left(d^m\right) \qquad \text{(B.10)} \end{aligned}$$

where $o_j(d^m) = o_{j-1}(d^m) - \frac{d^m y_m^{j-1}(y_m^0+\Delta)}{\Delta}$, so $o_j(d^m) \to 0$ as $m \to 0$.

We can write the value function after the deviation to $y_m^0 + \Delta$ as:

$$V(y_m^0+\Delta) = \sum_{j=0}^{\infty} \delta^{j-1} \left[\frac{W + (1-d^m)\, y_m^{j-1}(y_m^0+\Delta) - y_m^j(y_m^0+\Delta)}{n} + u(y_m^j(y_m^0+\Delta)) \right]$$

For any given function $f(x)$, define $\Delta f(x) = f(x+\Delta) - f(x)$. We can write:

$$\begin{aligned} \Delta V(y_m^0)/\Delta &= \sum_{j=0}^{\infty} \delta^{j-1} \left[\begin{array}{c} \frac{(1-d^m)\Delta y_m^{j-1}(y_m^0)/\Delta - \Delta y_m^j(y_m^0)/\Delta}{n} \\ + \left[u'(y_m^0) + o(\Delta)\right] \Delta y_m^j(y_m^0)/\Delta \end{array} \right] \\ &\geq \sum_{j=0}^{\infty} \delta^{j-1} \left[\begin{array}{c} \frac{(1-d^m)(1+o_{j-1}(d^m)) - (1+o_j(d^m))}{n} \\ + \left[u'(y_m^0) + o(\Delta)\right] (1+o_j(d^m)) \end{array} \right] \end{aligned} \tag{B.11}$$

where $o(\Delta) \to 0$ as $\Delta \to 0$. In the first equality we use the fact that if we choose Δ small, since $y_m(g)$ is continuous, $\Delta y_m^j(y_m^0)$ is small as well. This implies that

$$\left(u(y_m^j(y_m^0+\Delta)) - u(y_m^j(y_m^0))\right) / \left[y_m^j(y_m^0+\Delta) - y_m^j(y_m^0)\right]$$

converges to $u'(y_m^j(y_m^0))$ as $\Delta \to 0$. The inequality in B.11 follows from (B.10). Given Δ, as $m \to \infty$, we therefore have $\lim_{m\to\infty} \Delta V(y_m^0)/\Delta \geq \frac{u'(y_m^0)+o(\Delta)}{1-\delta}$. We conclude that for any $\varepsilon > 0$, there must be a Δ_ε such that for any $\Delta \in (0, \Delta_\varepsilon)$ there is a m_Δ guaranteeing that $\Delta V(y_m^0)/\Delta \geq \frac{u'(y_m^0)}{1-\delta} - \varepsilon$ for $m > m_\Delta$. After a marginal deviation to $y_m^0 + \Delta$, therefore, the change in agent's objective function is:

$$u'(y_m^0) + \delta \Delta V(y_m^0)/\Delta - 1 \geq \frac{u'(y_m^0)}{1-\delta} - \delta\varepsilon - 1$$

for m sufficiently large. A necessary condition for the un-profitability of a deviation from y_m^0 to

$y_m^0 + \Delta$ is therefore:

$$y_m^0 \geq [u']^{-1} (1 - \delta + \delta\varepsilon (1 - \delta)) . \tag{B.12}$$

Since ε can be taken to be arbitrarily small, for an arbitrarily large m, (B.12) implies $y_m^0 \geq \overline{y}(0) - \xi/2$, which contradicts $y_m^0 < \overline{y}(0) - \xi$. We conclude that $y_{IR}^*(\delta, d, n) \to \overline{y}(0)$ as $d \to 0$. ■

Appendix C

Proofs of Chapter 3

Proof of Proposition 1

The fact that a concave equilibrium has the property stated in the proposition follows from the discussion in the text. Here we prove existence and uniqueness.

Existence. Let $y_R^* = \left[u^{-1}\right]'(1-\frac{\delta}{n})$, and $g_R^1 = \max\{0, y_R^* - W\}$. For any $g > g_R^1$ define a value function $v_R^1(g) = \frac{W-(y_R^*-g)}{n} + u\left(y_R^*\right)$. Note that this function is continuous, non decreasing, concave, and differentiable with respect to g, with $\frac{\partial}{\partial g}v_R^1(g) = \frac{1}{n}$. Let $g_R^2 = \max\left\{0, g_R^1 - W\right\}$, and define:

$$v_R^2(g) = \begin{cases} v_R^1(g) & g \geq g_R^1 \\ u(g+W) + \delta v_R^1\left(g+W\right) & g \in \left[g_R^2, g_R^1\right) \end{cases}$$

Note that $v_R(g)$ is continuous and differentiable in $g \geq g_R^2$, except at most at g_R^1 To see that it is also concave in this interval, note that it is concave for $g \geq g_R^1$. Moreover, for any $g \in \left[g_R^2, g_R^1\right)$ and $g' \geq g_R^1$ we have:

$$\begin{aligned} \frac{\partial}{\partial g}v_R^2(g) &= u'(g+W) + \delta v_R^{1\prime}(g+W) \\ &> u'(y_R^*) + \delta v_R^{1\prime}(y_R^*) = 1 > \frac{1}{n} = \frac{\partial}{\partial g}v_R^2(g') \end{aligned}$$

The first inequality derives from $y_R^* > g + W$ (which is true, by definition of g_R^1 and g_R^2, for all $g \in [g_R^2, g_R^1))$, and concavity of $u(g)$. So $v_R^2(g)$ is concave in $g \geq g_R^2$. Assume that for all $g \geq g_R^n$, with $g_R^n \geq 0$ and either $g_R^n < g_R^2$ or $g_R^n = 0$, we have defined a value function $v_R^n(g)$ that is concave and continuous, and that is differentiable in $g > g_R^1$. Define $g_R^{n+1} = \max\{0, g_R^n - W\}$, and

$$v_R^{n+1}(g) = \begin{cases} v_R^n(g) & g \geq g_R^n \\ u(g+W) + \delta v_R^n(g+W) & g \in [g_R^{n+1}, g_R^n) \end{cases}$$

We can easily show that this function is concave, continuous in $g \geq g_R^{n+1}$, and differentiable for $g > g_R^1$. Moreover, either $g_R^{n+1} = 0$ or $g_R^{n+1} < g_R^n$. We can therefore define inductively a value function $v_R(g)$ for any $g \geq 0$ that is continuous and concave, and that is differentiable at least for $g > g_R^1$ and so, in particular, at y_R^*. Define now the following strategies:

$$y_R(g) = \min\{W + g, y_R^*\}, \text{ and } x_A(g) = \frac{W + g - y_R(g)}{n}. \tag{C.1}$$

We will argue that $v_R(g), y_R(g), x_A(g)$ is an equilibrium. To see this note that by construction, if the agent uses strategies $y_R(g), x_A(g)$, then $v_R(g)$ describe the expected continuation value function of an agent. To see that $y_R(g), x_A(g)$, are optimal given $v_R(g)$ note that for $g \geq g_R^1$, $\left\{y_R^*, \frac{W+g-y_R^*}{n}\right\}$ maximizes (4.2) when all the constraints except the second are considered; and for $g \geq g_R^1$, $W + g > y_R^*$, so the second constraint is satisfied as well. For $g < g_R^1$, we must have $y_R(g) = W + g$, $x_A(g) = 0$. We conclude that $y_R(g)$, $x_A(g)$ is an optimal reaction function given $v_R(g)$. ■

Uniqueness. In the steady state we must have $y(y_R^*) = y_R^*$ and $x(y_R^*) > 0$. The steady state cannot be lower than y_R^*. In this case, $W + g = g$: but this implies $W = 0$, a contradiction. Since $y(g)$ is constant for $g \geq \max\{y_R^* - W, 0\}$, it is straightforward to show that the derivative of the value function in this region is $v'(g) = \frac{1}{n}$. Using the first order condition we must have $u'(y_R^*) + \delta v_R'(y_R^*) = 1$, so:

$$y_R^*(n) = [u']^{-1}\left(1 - \delta\frac{1}{n}\right) \tag{C.2}$$

for such an equilibrium to exist we need that $y_R^* > g_R$, that is always true when the public good stock does not depreciate. We conclude that, when the public good stock does not depreciate, y_R^* is given by (C.2), and the equilibrium steady state is unique. ■

Proof of Proposition 3

The efficient outcome (the social planner solution characterized in Section 2.1) can be sustained in the voluntary contribution game with reversible investment, when agents use nonstationary strategies entailing reversal to the unique concave Markov equilibrium characterized in Section 2.2. To show this, we construct strategies whose outcome is the efficient level of public good and we show that there is no profitable deviation from the equilibrium path. The symmetric strategy for each committee member is to invest $i_P^*(g) = \min\left\{\frac{W}{n}, \frac{y_P^*-g}{n}\right\}$ if $g_t = y^*(g_{t-1})$ (i.e. if the observed level of the public good at the beginning of the period is consistent with equilibrium strategies, or, in other words, it is the efficient level of public good given the stock of g at the beginning of the previous period) and to invest $i_R^*(g) = \min\left\{\frac{W}{n}, \frac{y_R^*-g}{n}\right\}$ where $y_R^* < y_P^*$ (i.e. the investment associated with the Markov equilibrium characterized in Proposition 1) if $g_t \neq y^*(g_{t-1})$ (i.e. if a deviation from equilibrium has occurred in the previous period). To prove that this strategy profile is an equilibrium we show that agents have no profitable deviation.

An agent's payoff if she follows the equilibrium strategy is:

$$\frac{W}{n} - i_P^*(g) + 2\sqrt{g + ni_P^*(g)} + \delta V_{EQ}(g + ni_P^*(g))$$

An agent's payoff if she deviates (according to her most profitable deviation) is:

$$\frac{W}{n}+\frac{g}{n}+2\sqrt{g-\frac{g}{n}+(n-1)i_P^*(g)}+\delta V_{DEV}\left(g-\frac{g}{n}+(n-1)i_P^*(g)\right)$$

An agent's most profitable deviation is to invest $-g/n$ (i.e. to subtract from the public good her share and to consume it). The gains from this deviation are greater the closer g is to y_P^*. Therefore, we will check whether an agent has an incentive to deviate when $g \in [g_P, y_P^*]$, or whether:

$$\frac{W}{n}-\frac{y_P^*-g}{n}+2\sqrt{y_P^*}+\delta V_{EQ}(y_P^*) \geq \frac{W}{n}+\frac{g}{n}+2\sqrt{g-\frac{g}{n}+(n-1)\frac{y_P^*-g}{n}}+\delta V_{DEV}\left(g-\frac{g}{n}+(n-1)\frac{y_P^*-g}{n}\right)$$

where:

$$V_{EQ}(y_P^*) = \frac{1}{1-\delta}\left[\frac{W}{n}+2\sqrt{y_P^*}\right]$$

and:

$$\begin{aligned} V_{DEV}\left(\frac{n-1}{n}y_P^*\right) &= \frac{W}{n}-\frac{y_R^*-\frac{n-1}{n}y_P^*}{n}+2\sqrt{y_R^*}+\delta V_{DEV}\left(y_R^*\right) \\ &= \frac{W}{n}-\frac{y_R^*-\frac{n-1}{n}y_P^*}{n}++2\sqrt{y_R^*}+\frac{\delta}{1-\delta}\left(\frac{W}{n}+2\sqrt{y_R^*}\right) \end{aligned}$$

After we plug in $V_{EQ}(y_P^*)$ and $V_{DEV}\left(\frac{n-1}{n}y_P^*\right)$, the inequality above becomes:

$$\begin{aligned} \frac{W}{n}-\frac{y_P^*-g}{n}+2\sqrt{y_P^*}+\frac{\delta}{1-\delta}\left[\frac{W}{n}+2\sqrt{y_P^*}\right] &\geq \frac{W}{n}+\frac{g}{n}+2\sqrt{\frac{n-1}{n}y_P^*}+\delta \\ & \left[\frac{W}{n}-\frac{y_R^*-\left(\frac{n-1}{n}y_P^*\right)}{n}+2\sqrt{y_R^*}+\frac{\delta}{1-\delta}\left(\frac{W}{n}+2\sqrt{y_R^*}\right)\right] \\ \frac{1}{1-\delta}\left[2\sqrt{y_P^*}-\delta 2\sqrt{y_R^*}\right]-\frac{\delta}{n}\left[\frac{(n-1)}{n}y_P^*-y_R^*\right] &\geq 2\sqrt{\frac{n-1}{n}y_P^*}+\frac{y_P^*}{n} \end{aligned}$$

Replacing y_P^* and y_R^* (who both depend on δ), the inequality we want to prove becomes:

$$\frac{1}{1-\delta}\left[\frac{2n}{1-\delta}-\frac{\delta 2n}{n-\delta}\right]-\frac{\delta(n-1)}{(1-\delta)^2}+\frac{\delta}{n}\left(\frac{n}{n-\delta}\right)^2 \quad \geq \quad \sqrt{\frac{n-1}{n}}n^2+\frac{n}{(1-\delta)}$$

Multiplying both sides by $(1-\delta)^2$ and rearranging, we have:

$$n-(n-1)\delta \geq \frac{(1-\delta)^2\delta 2n}{n-\delta}+\frac{\delta}{n}\left(\frac{n}{n-\delta}\right)^2(1-\delta)^2+\sqrt{\frac{n-1}{n}}n^2(1-\delta)$$

There is $\widehat{\delta}_R$ such that $\forall\delta > \widehat{\delta}_R$ the inequality above holds. To see this note that as δ approaches 1 the RHS approaches zero, while the LHS is positive for any $\delta \in [0,1]$. ■

Using the parameters and the utility function of our experiments, $\widehat{\delta}_R = 0.80$ for $n = 3$ and $\widehat{\delta}_R = 0.86$ for $n = 5$. We use $\delta = 0.75$, which means that, in the experimental setting, the efficient level of the public good cannot be sustained in equilibrium. However, it can be shown that nonstationary strategies of the type proposed above can sustain an almost efficient level of the public good, y^*. In this case, the inequality we want to prove is:

$$\frac{1}{1-0.75}\left[2\sqrt{y^*}-0.752\sqrt{y_R^*}\right]-\frac{0.75}{n}\left[\frac{(n-1)}{n}(y^*)-y_R^*\right] \geq 2\sqrt{\frac{n-1}{n}(y^*)}+\frac{y^*}{n}$$

This inequality holds for $y^* = 130$ in the treatment with 3 agents (where $y_P^* = 144$) and for for $y^* = 333$ in the treatment with 5 agents (where $y_P^* = 400$).

Appendix D

Experimental Instructions

INSTRUCTIONS FOR RIE5 TREATMENT

Thank you for agreeing to participate in this experiment. During the experiment we require your complete, undistracted attention and ask that you follow instructions carefully. Please turn off your cell phones. Do not open other applications on your computer, chat with other students, or engage in other distracting activities, such as reading books, doing homework, etc. You will be paid for your participation in cash, at the end of the experiment. Different participants may earn different amounts. What you earn depends partly on your decisions, partly on the decisions of others, and partly on chance. It is important that you not talk or in any way try to communicate with other participants during the experiments.

Following the instructions, there will be a practice session and a short comprehension quiz. All questions on the quiz must be answered correctly before continuing to the paid session. At the end you will be paid in private and you are under no obligation to tell others how much you earned. Your earnings are denominated in FRANCS which will be converted to dollars at the rate of 75 FRANCS to a DOLLAR.

This is an experiment in group decision making. The experiment will take place over a sequence of 10 matches. We begin the match by dividing you into THREE groups of five members each. Each of you is assigned to exactly one of these groups. In each match each member of your group will make investment decisions.

In each round, each member of your group has a budget of 16 francs. Each member must individually decide how to divide his or her budget into private investment and project investment, in integer amounts. The private investment always has to be greater than or equal than 0. The project investment can be either positive, or zero, or negative. Any amount you allocate to private investment goes directly to your earnings for this round. The project investment produces earnings for all group members in the following way.

[SHOW SLIDE]

The project earnings in a round depend on the size of the project at the end of that round. Specifically, each committee member earns an amount in francs proportional to the square root of the size of the project at the end of the round (precisely equal to 4*sqrt(project size)). Thus, for example, if the size of the project at the end of the round equals 9, then each member earns exactly 4*sqrt(9)=12 additional francs in that round. If the size is equal to 36, each member earns exactly 4*sqrt(36)=24 additional francs in that round. In your display, earnings are always rounded to two decimal places. So, for example if the project size at the end of a round equals 5, each member earns 4*sqrt(5)=8.94 francs from the project in that round.

The second important fact about the project is that it is durable. That is, project investment in a round increases or decreases the size not just for that round, but also for all future rounds. The size of your groupÕs project starts at 0 in the first round of the match. At the end of the first round it is equal to the sum of your group membersÕ project investment in that round. This amount gets carried over to the second round. Whenever the size of the project is greater than 0, you can propose a negative project investment. However, in this case, the proposed negative investment cannot exceed one fifth of the size of the project at the beginning of the round (in other words, you can dispose only of your share of the project).At the end of the second round, the size of the project equals to the combined amount invested in the project in rounds 1 and 2 by all members of your group, and so forth. So, every round project investment changes the size of the project for the current round and all future rounds of the match.

The total number of rounds in a match will depend on the rolling of a fair 8-sided die. When the first round ends, we roll it to decide whether to move on to the second round. If the die comes up a 1 or a 2 we do not go on to round 2, and the match is over. Otherwise, we continue to the next round. We continue to more rounds, until a 1 or a 2 is rolled at the end of a round and the match ends. At the end of each round your earnings for that round are computed by adding the project earnings to your private investment. For example, if your private investment is 20 and the end-of-round project size is 9, then your earnings for that round equal 20 + 4*sqrt(9) = 20+12 = 32. Your earnings for the match equal the sum of the earnings in all rounds of that match.

After the first match ends, we move to match 2. In this new match, you are reshuffled randomly into THREE new groups of five members each. The project size in your new group again starts out at 0. The match then proceeds the same way as match 1. After match 10, the experiment is over. Your total earnings for the experiment are the sum of your earnings over all rounds and all matches.

We will now go through one practice match very slowly. During the practice match, please do not hit any keys until I tell you, and when you are prompted by the computer to enter information, please wait for me to tell you exactly what to enter. You are not paid for this practice match.

[AUTHENTICATE CLIENTS]

Please double click on the icon on your desktop that says BP2. When the computer prompts you for your name, type your First and Last name. Then click SUBMIT and wait for further instructions. You now see the first screen of the experiment on your computer. It should look similar to this screen.

[SHOW SLIDE]

At the top left of the screen, you see your subject ID. In the top right you can see that you have been assigned by the computer to a group of FIVE subjects, and assigned a group member number: 1, 2, 3, 4, or 5. This group assignment and your member number stays the same for all rounds of this match, but will change across matches. It is very important that you take careful note of your group member number.

As a visual aid, there is a graph on the left that shows exactly how project earnings will depend on project size. The current size of the project is marked with a large dot at the origin. If each member of your group decides to invest nothing this period, then this will be the size that determines your project earnings at the end of the round. You can use your mouse to move the curser along the curve to figure out what your earnings will be for different levels of project investment. Also, if you type an amount in the Project Investment box, the computer will compute and display the corresponding project earnings for you just below the box. Take a minute to practice using your curser to move along the curve, and typing in different possible investment levels. But do not hit the confirm button yet.

At this time, go ahead and type in any investment decision you wish and hit the confirm button. You are not paid for this practice match so it does not matter what you enter.

[SHOW SLIDE]

This screen now summarizes the outcome of the round. Here you see your committee member number, and the end of round project size. The investment decisions of each member are displayed in a table. Below the table are displayed your earnings for the round, given the outcome. This marks the end of the round. The table with columns in the bottom of your screen is the History panel and summarizes all of this important information.

We now roll an eight-sided die to decide whether to move on to round 2. If the die comes up a 1 or a 2, we do not go on to round 2, and the match is over. If the die comes up 3 through 8, we continue to a second round of the match. [Roll die and do second round unless it comes up a 1 or 2. Next say "the die roll was X, so we will continue to the next round". If X=(1 or 2) say "if this was a real match, there would be no second round. That would be the end of the match. However, we want to go through one more practice round to make sure you are familiar with the computer interface."]

[SHOW SLIDE]

In this second round, you keep the same committee member number as in the first round, and the members of your committee all stay the same. Notice that the project investment from round 1 carries over, so the round 2 beginning project size equals the project size at the end of round 1. In this second round please follow the same instructions of the first round. You can go ahead now.

Since this is a practice match, we will not roll a die after the second round, and the practice match will end. During the paid matches, each match will continue until the die comes up a 1 or a 2.

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