Appendix A

Derivation of the mineral physics derivatives

We use the same notation as presented in Chapter 2. Subscripts pv and ppv denote the perovskite (Pv) and post-perovskite (pPv) phase, respectively.

$$\delta vp = \frac{1}{2}\left(\frac{\delta K + 4R_1\delta G/3}{1+4R_1/3} - \delta\rho\right) \tag{A.1}$$

$$\delta vs = \frac{1}{2}(\delta G - \delta\rho). \tag{A.2}$$

For Pv:

$$\delta K_{pv} = \frac{\partial \ln K}{\partial T} dT \tag{A.3}$$

$$\delta G_{pv} = \frac{\partial \ln G}{\partial T} dT \tag{A.4}$$

$$\delta\rho_{pv} = -\bar{\rho}\bar{\alpha}\alpha_0 dT\Delta T \tag{A.5}$$

where dT is a (non-dimensional) temperature perturbation from a reference state.

For pPv:

$$\delta K_{ppv} = \delta K_{pv} + \frac{\partial \ln K}{\partial \Gamma} \tag{A.6}$$

$$\delta G_{ppv} = \delta G_{pv} + \frac{\partial \ln G}{\partial \Gamma} \tag{A.7}$$

$$\delta\rho_{ppv} = \delta\rho_{pv} + \frac{Rb}{Ra}\alpha_0\Delta T \tag{A.8}$$

$$\delta vp_{ppv} = \frac{1}{2}\left(\frac{\delta K_{pv} + 4R_1\delta G_{pv}/3}{1+4R_1/3} - \delta\rho_{pv} + \frac{\frac{\partial \ln K}{\partial \Gamma} + 4R_1\frac{\partial \ln G}{\partial \Gamma}/3}{1+4R_1/3} - \frac{Rb}{Ra}\alpha_0\Delta T\right) \tag{A.9}$$

$$\delta vs_{ppv} = 1/2\left(\delta G_{pv} - \delta\rho_{pv} + \frac{\partial \ln G}{\partial \Gamma} - \frac{Rb}{Ra}\alpha_0\Delta T\right). \tag{A.10}$$

Considering the fractional increase in the S- and P-wave velocities across the Pv-pPv phase transition:

$$\delta vp_{ppv} = \delta vp_{pv} + \delta vp^{\Gamma} \tag{A.11}$$

$$\delta vs_{ppv} = \delta vs_{pv} + \delta vs^{\Gamma} \tag{A.12}$$

where δvs^{Γ} and δvp^{Γ} are the fractional perturbations to the S- and P-wave velocity, respectively, due to the Pv-pPv phase transition. By substitution:

$$\frac{\partial \ln G}{\partial \Gamma} = 2\delta vs^{\Gamma} + \frac{Rb}{Ra}\alpha_0 \Delta T \tag{A.13}$$

$$\frac{\partial \ln K}{\partial \Gamma} = \left(2\delta vp^{\Gamma} + \frac{Rb}{Ra}\alpha_0 \Delta T\right)(1 + 4R_1/3) - 4R_1/3\frac{\partial \ln G}{\partial \Gamma}. \tag{A.14}$$