PREDICTION OF OFF DESIGN PERFORMANCE OF MULTISTAGE COMPRESSORS

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ABSTRACT

A new method for predicting the off design performance of a multistage compressor has been developed. This method replaces the usual routine numerical integration through the compressor by approximate analytical integration. The overall compressor performance of the compressor is found as a perturbation of the design performance in terms of linear and quadratic expansions in two parameters, the deviation from the design flow rate and the deviation from the design speed. The procedure for this preliminary examination has been simplified by assuming that all the single stage performance characteristics are identical and that the difference between static and stagnation conditions are negligible.

The results of this method are quadratic equations, in terms of the two perturbation parameters, for the temperature ratio, pressure ratio and adiabatic efficiency of the overall compressor. The amount of numerical work in finding the coefficients of these quadratic expansions is not negligible, however the method appears promising. Reasonably accurate results are expected for speeds varying as much as ten per cent from design and flow rates varying as much as four per cent from the design flow.

LIST OF SYMBOLS

A	Flow annulus cross sectional area
c_p	Specific heat at constant pressure
f	Efficiency function $\eta^{\frac{\gamma}{\gamma-1}} - 1$
h	Stagnation enthalpy
m	Mass flow rate
n	Stage number
N	Number of stages
p	Stagnation pressure
R	Gas constant
T	Stagnation temperature
U	Rotor reference speed
V_m	Mean axial gas velocity
α	Design condition constant $(\zeta_0 + 2)$
β	$\dfrac{\Theta \; c_p}{U^2 \; \Upsilon_0}$
γ	Specific heat ratio
γ''	Design condition constant $\dfrac{\Upsilon_0''}{\Upsilon_0} + \dfrac{f_0''}{f_0}$
ζ_0	Design condition constant $f_0 \dfrac{\Upsilon_0'}{\Upsilon_0} \phi_0$
η	Adiabatic efficiency
ξ	Axial "length" variable
ρ	Gas density
Υ	Work coefficient function $\left(\frac{N\Psi}{2}\right)$
ϕ	Flow coefficient
Ψ	Work coefficient
Ψ'	Pressure coefficient

LIST OF FIGURES

I. INTRODUCTION

The usual technique for calculation of multistage compressor performance is numerical. Starting at the entrance with a prescribed flow rate and rotor speed the pressures, temperatures and flow coefficients are determined behind each stage in succession from the known performance characteristics of the individual stages. Although this method can be carried out rather rapidly on machines, it is desirable to have available a method that is more suitable for exploratory investigations of the influence of stage characteristics.

A new method is proposed which replaces the routine numerical integration through the compressor by approximate analytical integration. The overall performance of the compressor can be found as a perturbation of the design performance in terms of linear and quadratic expansions in two parameters, the deviation from the design flow rate and the deviation from the design speed. The amount of numerical work in finding the coefficients in the expansion is not negligible, nevertheless the method appears promising.

The investigation should be considered as preliminary. Certain approximations, such as the assumptions that all single stage performance characteristics are identical and that the difference between stagnation and static conditions are negligible, are not essential but simplify the procedure for the initial examination.

II. DEVELOPMENT OF EQUATIONS

A. Work Coefficient

The performance of multistage compressors is generally presented as plots of temperature and pressure ratios against dimensionless weight flow for given dimensionless compressor speed. The stage performance is most conveniently represented in the dimensionless form as a work coefficient (ψ), pressure coefficient (ψ') and adiabatic efficiency (η) against a flow coefficient (ϕ). These dimensionless quantities are defined as:

$$\psi = \frac{\Delta h_c}{U^2/2} = \frac{c_p \Delta T}{U^2/2}$$

$$\psi' = \frac{\Delta p}{\rho U^2}$$

$$\phi = \frac{V_m}{U} = \frac{\dot{m}}{\rho U A}$$

$$\eta = \frac{\psi'}{\psi}$$

Δh_c is the increase in stagnation enthalpy per stage, c_p the specific heat of the gas at constant pressure, ΔT the increase in stagnation temperature per stage, Δp the increase in stagnation pressure per stage, and ρ the gas density. The compressor rotor reference speed is represented by U and the mean axial gas velocity by V_m. $\dot{m}$ is the mass flow of gas and A the cross sectional area of the flow annulus.

The multistage compressor temperature ratio, pressure ratio, and adiabatic efficiency will be determined from the single stage dimensionless characteristics.

In general it can be said that the work coefficient Ψ is a function of the flow coefficient:

$$\Psi = F(\phi)$$

The temperature rise (ΔT) in a single stage can thus be written as a function of the flow coefficient:

$$\Delta T = \frac{U^2}{2 c_p} \Psi = \frac{U^2}{2 c_p} F(\phi)$$

A multistage compressor will be assumed to be made up of stages which all have the same characteristic. Such a compressor can be considered to be made up of an infinite number of infinitesimal stages such that the temperature can be considered to be a continuous variable along the axis of the compressor. Let the continuous axial variable be ξ, replacing the sequence $\xi_n = \frac{n}{N}$, where n is the stage number from the entrance and N is the total number of stages of the compressor $(0 \leq n \leq N)$. Using the variable ξ the temperature rise can be written in the differential form:

$$\frac{dT}{d\xi} = \frac{U^2}{c_p} \Upsilon(\phi)$$

The compressor then has an overall "length" of unity, and the function $\Upsilon(\phi)$ becomes:

$$\Upsilon(\phi) = N \frac{F(\phi)}{2} = \frac{N\Psi}{2}$$

The adiabatic compressor efficiency is normally defined as:

$$\eta = \frac{T_i - T(o)}{T - T(o)}$$

where T is the actual temperature and T_i is the isentropic final temperature for a given pressure rise. Introducing the isentropic relationship $\frac{T_i}{T(o)} = \left\{\frac{p}{p(o)}\right\}^{\gamma-1/\gamma}$ this becomes:

$$\eta = \frac{\left(P/p(o)\right)^{\frac{\gamma-1}{\gamma}} - 1}{T/T(o) - 1}$$

where $\frac{p}{p(o)}$ and $T/T(o)$ are the total pressure and temperature ratios, $p(o)$ and $T(o)$ are the initial pressure and temperature and γ is the ratio of specific heats of the gas. In a single stage the pressure and temperature increase can be represented by Δp and ΔT such that $p = p(o) + \Delta p$ and $T = T(o) + \Delta T$. The stage efficiency thus becomes:

$$\eta = \frac{\left(1 + \Delta P/p(o)\right)^{\frac{\gamma-1}{\gamma}} - 1}{\frac{\Delta T}{T(o)}}$$

Since Δp is small compared to $p(o)$ the pressure term can be expanded to give:

$$\eta = \frac{\frac{\gamma-1}{\gamma}\left(\Delta P/p(o)\right) + \frac{1}{2}\left(\frac{\gamma-1}{\gamma}\right)\left(-\frac{1}{\gamma}\right)\left(\Delta P/p(o)\right)^2 + \cdots}{\frac{\Delta T}{T}}$$

Therefore in the limiting case, with Δp very small (as in the single stage):

$$\eta = \frac{\frac{\gamma-1}{\gamma} \; \Delta p / p(0)}{\Delta T / T(0)}$$

or

$$\frac{\Delta p}{p(0)} = \eta \left(\frac{\gamma}{\gamma - 1} \right) \frac{\Delta T}{T(0)} \tag{1}$$

This will also be the form of the limiting case as $\Delta p \longrightarrow 0$ so that the differential form can be written as:

$$\frac{1}{p} \frac{dp}{d\xi} = \eta \left(\frac{\gamma}{\gamma - 1} \right) \frac{1}{T} \frac{dT}{d\xi} \tag{1a}$$

The adiabatic efficiency (η) as defined here can be shown to be equivalent to the definition $\eta = \psi'/\psi$ in the limiting case. By the definitions of the work and pressure coefficients:

$$\frac{\psi'}{\psi} = \frac{\frac{\Delta p}{\rho(U^2/2)}}{\frac{c_p \Delta T}{U^2/2}} = \frac{\Delta p}{\rho \, c_p \Delta T}$$

But for Δp small equation (1) applies and:

$$\frac{\psi'}{\psi} = \frac{\eta \left(\frac{\gamma}{\gamma - 1} \right) \frac{p}{T}}{\rho \; c_p}$$

Substituting the thermodynamic relations for a perfect gas:

$$\frac{p}{\rho T} = R \qquad\qquad \frac{R}{c_p} = \frac{\gamma - 1}{\gamma}$$

gives

$$\frac{\psi'}{\psi} = \eta$$

B. Differential Equations for Flow Coefficient

The flow coefficient has been defined as:

$$\phi = \frac{\dot{m}}{\rho u A}$$

Introducing the perfect gas relation,

$$p = \rho R T$$

the flow coefficient becomes:

$$\phi = \frac{\dot{m} R T}{p u A}$$

Taking the logarithm and differentiating with respect to ξ this becomes:

$$\frac{1}{\phi}\frac{d\phi}{d\xi} = \frac{1}{T}\frac{dT}{d\xi} - \frac{1}{p}\frac{dp}{d\xi} - \frac{1}{A}\frac{dA}{d\xi}$$

or

$$\frac{\phi'}{\phi} = \frac{T'}{T} - \frac{p'}{p} - \frac{A'}{A}$$

where the ($'$) denotes differentiation with respect to the variable ξ. Substituting for $\frac{p'}{p}$ from equation (1a) this reduces to:

$$\frac{\phi'}{\phi} = \left\{1 - \eta \frac{\gamma}{\gamma - 1}\right\} \frac{T'}{T} - \frac{A'}{A} \qquad (2)$$

This is an approximate relation since the differences between static and total temperature and pressure have been neglected in the development.

At the compressor design point the flow coefficient, efficiency and temperature rise are assumed to be constants throughout ξ, and equal to the single stage parameters at the design point. Let the subscript ($_o$) denote the design condition. Therefore at design conditions:

$$\phi = \phi_o = \text{constant}$$

$$\eta = \eta_o = \text{constant}$$

$$\left(\frac{dT}{d\xi}\right)_o = T_o' = \frac{U_o^2}{c_p}\tau_o = \text{constant} \tag{3}$$

At the design point equation (3) can easily be integrated to give the temperature at any point in the compressor as:

$$T_o = \frac{U_o^2}{c_p}\tau_o\,\xi + \Theta_o \tag{4}$$

where Θ_o is the inlet temperature.

Dividing equation (3) by (4) gives:

$$\frac{T_o'}{T_o} = \frac{\dfrac{U_o^2\tau_o}{c_p}}{\dfrac{U_o^2\tau_o}{c_p}\xi - \Theta_o} = \frac{1}{\xi + \beta_o} \tag{5}$$

where β_o is defined as:

$$\beta_o = \frac{\Theta_o\ c_p}{U_o^2\ \tau_o}$$

Equation (2) can now be solved to determine the area variation of the compressor:

$$\frac{A'}{A} = -\left\{\eta_o\left(\frac{\gamma}{\gamma-1}\right) - 1\right\}\frac{T_o'}{T_o} - \frac{\phi_o'}{\phi_o}$$

Letting the term $\eta(\frac{\gamma}{\gamma-1}) - 1 = f$ and noting that $\phi_o' = 0$ (since ϕ_o is a constant) and substituting in equation (5):

$$\frac{A'}{A} = -\frac{f_o}{\xi + \beta_o} \tag{6}$$

This area distribution is fixed by the design geometry. Substituting (6) into equation (2) the general differential equation becomes:

$$\frac{\phi'}{\phi} = -f\,\frac{T'}{T} + \frac{f_o}{\xi + \beta_o} \tag{7}$$

To eliminate the temperature (T) equation (7) is solved for T and substituted in the expression for Υ:

$$T = \frac{f\,T'}{\dfrac{f_o}{\xi + \beta_o} - \dfrac{\phi'}{\phi}}$$

$$\Upsilon = \frac{c_p}{U^2}\,T' = \frac{c_p}{U^2}\,\frac{d}{d\xi}\left\{\frac{f\,T'}{\dfrac{f_o}{\xi+\beta_o} - \dfrac{\phi'}{\phi}}\right\} = \frac{d}{d\xi}\left\{\frac{f\,\dfrac{c_p T'}{U^2}}{\dfrac{f_o}{\xi+\beta_o} - \dfrac{\phi'}{\phi}}\right\}$$

$$\Upsilon = \frac{d}{d\xi}\left\{\frac{f\,\Upsilon}{\dfrac{f_o}{\xi+\beta_o} - \dfrac{\phi'}{\phi}}\right\}$$

Performing the differentiations and substituting the relations:

$$\frac{d\Upsilon}{d\xi} = \frac{d\Upsilon}{d\phi}\,\frac{d\phi}{d\xi} = \frac{d\Upsilon}{d\phi}\,\phi'$$

$$\frac{df}{d\xi} = \frac{df}{d\phi}\,\frac{d\phi}{d\xi} = \frac{df}{d\phi}\,\phi'$$

there results the general differential equation in ϕ:

$$f\Upsilon\phi'' + \frac{f_o}{\xi+\beta_o}\left[\left(\Upsilon\frac{df}{d\phi} + f\,\frac{d\Upsilon}{d\phi}\right) + 2\Upsilon\right]\phi' \tag{8}$$

$$- \left[\left(\Upsilon\frac{df}{d\phi} + f\,\frac{d\Upsilon}{d\phi}\right)\phi + \Upsilon(1+f)\right]\frac{\phi'^2}{\phi} + \frac{f_o(f-f_o)\Upsilon\phi}{(\xi+\beta_o)^2} = 0$$

C. Solution for the Flow Coefficient

A series expansion of each of the terms involved in the differential equation has been used to find a solution for the flow coefficient. From the single stage performance it is seen that both the efficiency function f and work coefficient τ are functions of the variable ϕ:

$$f = f(\phi) \qquad\qquad \tau = \tau(\phi)$$

These functions can thus be expressed as a Taylor series expansion about the design point. The stage efficiency at design is assumed to have its maximum value, hence the first derivative of the efficiency function becomes zero $\left(\frac{df}{d\phi}\right)_o = 0$. The Taylor series expansions thus become:

$$f(\phi) = f_o + \frac{1}{2}\left(\frac{d^2 f}{d\phi^2}\right)_o (\phi - \phi_o)^2 + \frac{1}{6}\left(\frac{d^3 f}{d\phi^3}\right)_o (\phi - \phi_o)^3 + \cdots\cdots$$

$$\tau(\phi) = \tau_o + \left(\frac{d\tau}{d\phi}\right)(\phi - \phi_o) + \frac{1}{2}\left(\frac{d^2\tau}{d\phi^2}\right)(\phi - \phi_o)^2 + \frac{1}{6}\left(\frac{d^3\tau}{d\phi^3}\right)(\phi - \phi_o)^3 + \cdots\cdots \qquad (9)$$

The flow coefficient will be represented by a series of the form:

$$\phi = \phi_o + (\phi - \phi_o) = \phi_o + \epsilon\,\phi_1 + \epsilon^2\phi_2 + \epsilon^3\phi_3 + \cdots\cdots \qquad (10)$$

The factor ϵ represents a convenient small parameter to be replaced by unity in the final form. Substitution of the series (9) and (10) into equation (8) results in a series equation in powers of ϵ. Equating terms in like powers of ϵ gives a series of linear differential equations in the flow coefficients ϕ_n (n = 1, 2 . . . . n). Only the first two of these coefficients will be retained. The linear differential equations for these coefficients are:

$$\phi_1'' + \left\{ f_o \frac{\tau_o'}{\tau_o} \phi_o + 2 \right\} \frac{1}{\xi + \beta_o} \phi_1' = 0 \tag{11}$$

$$\phi_2'' + \left\{ f_o \frac{\tau_o'}{\tau_o} \phi_o + 2 \right\} \frac{1}{\xi+\beta_o} \phi_2' = \left\{ \frac{\tau_o'}{\tau_o} \phi_o + \frac{1+f_o}{f_o} \right\} \frac{\phi_1'^2}{\phi_o}$$

$$+ \left[\frac{\tau_o'}{\tau_o} \left\{ \frac{f_o \tau_o'}{\tau_o} \phi_o + 2 \right\} - \frac{f_o \tau_o'}{\tau_o} - f_o \left\{ \frac{\tau_o''}{\tau_o} + \frac{f_o''}{f_o} \right\} \phi_o - \frac{2\tau_o'}{\tau_o} \right] \frac{\phi_1 \phi_1'}{\xi + \beta_o} \tag{12}$$

$$- \frac{1}{2} f_o'' \frac{1}{\xi+\beta_o} \phi_1^2$$

Note that the ($'$) denotes differentiation with respect to the varible ξ when used with the flow function ϕ and denotes differentiation with respect to the variable ϕ when used with f or τ. The constants τ_o, τ_o', τ_o'', f_o, f_o'' and ϕ_o are all known from the single stage characteristic.

To simplify the solution of equations (11) and (12) the following notations have been introduced:

$$\zeta_o = f_o \frac{\tau_o'}{\tau_o} \phi_o$$

$$\alpha = \zeta_o + 2$$

$$\gamma'' = \frac{\tau_o''}{\tau_o} + \frac{f_o''}{f_o}$$

The coefficients of the left side of equation (12) become:

$$B = \frac{1}{f_o \phi_o}\left\{ \zeta_o + f_o + 1 \right\} \quad ; \qquad C = \frac{T_o'}{T_o}(\zeta_o - f_o) - f_o \gamma'' \phi_o \, ; \; D = -\frac{1}{2} f_o'' \phi_o$$

Using this notation equations (11) and (12) become:

$$\phi_1'' + \frac{\alpha}{\xi + \beta_o}\phi_1' = 0 \tag{11a}$$

$$\phi_2'' + \frac{\alpha}{\xi + \beta_o}\phi_2' = B\phi_1' + C\frac{\phi_1 \phi_1'}{\xi + \beta_o} + D\frac{\phi_1^2}{(\xi + \beta_o)^2} \tag{12a}$$

Multiplying equation (11a) by the integrating factor $(\xi + \beta_o)^\alpha$

$$(\xi + \beta_o)^\alpha \phi_1'' + \alpha(\xi + \beta_o)^{\alpha - 1}\phi_1' = 0$$

$$\frac{d}{d\xi}\left\{ (\xi + \beta_o)^\alpha \phi_1' \right\} = 0$$

Integrating:

$$(\xi + \beta_o)^\alpha \phi_1' = \text{constant}$$

The constant can be found by introducing the boundary condition at $\xi = 0$; $\phi_1 = \phi_1(o)$ and therefore:

$$\text{Constant} = \beta_o^\alpha \phi_1'(o)$$

Therefore:

$$\phi_1' = \left(\frac{\beta_o}{\xi + \beta_o}\right)^\alpha \phi_1'(o) \tag{11b}$$

Integrating between the limits 0 to ξ:

$$\phi_1 - \phi_1(0) = \int_0^{\xi} \left(\frac{\beta_0}{\xi + \beta_0}\right)^{\alpha} \phi_1'(0)$$

$$\phi_1 = \frac{\beta_0}{\alpha - 1}\left\{1 - \left(\frac{\xi + \beta_0}{\beta_0}\right)^{1-\alpha}\right\} \phi_1'(0) + \phi_1(0)$$

In terms of ζ_0 this becomes:

$$\phi_1 = \phi_1(0) - \frac{\beta_0}{\zeta_0 + 1}\left\{\left(\frac{\beta_0}{\xi + \beta_0}\right)^{\zeta_0 + 1} - 1\right\} \phi_1'(0) \tag{13}$$

Equation 12a is solved in a similar manner. The integrating factor $(\xi + \beta_o)$ is again applied to the left side of the equation and the solutions for ϕ_1' and ϕ_1 form (11b) and (13) substituted into the right side. On integrating once the equation is of the form:

$$\phi_2' = E(\xi - \beta_0)^{-(2\zeta_0 + 3)} + F(\xi + \beta_0)^{-(\zeta_0 + 2)} + G(\xi + \beta_0)^{-1} + H(\xi + \beta_0)^{-(\zeta_0 + 2)} \ln \frac{\xi + \beta_0}{\beta_0}$$

The coefficients E, F, G, and H are a collection of all the constants in the equation not involved in the integration. On integrating the second term of the flow coefficient becomes:

$$\phi_2 = \phi_2(0) - E\frac{\beta_0^{-2(\zeta_0+1)}}{2(\zeta_0+1)}\left\{\left(\frac{\beta_0}{\xi + \beta_0}\right)^{2(\zeta_0+1)} - 1\right\} - F\frac{\beta_0^{-(\zeta_0+1)}}{\zeta_0 + 1}\left\{\left(\frac{\beta_0}{\xi + \beta_0}\right)^{\zeta_0+1} - 1\right\}$$

$$- G \ln\frac{\beta_0}{\xi + \beta_0} + H\frac{\beta_0^{-(\zeta_0+1)}}{\zeta_0 + 1}\left\{\left(\frac{\beta_0}{\xi + \beta_0}\right)^{\zeta_0+1}\left[\ln\frac{\beta_0}{\xi + \beta_0} - \frac{1}{\zeta_0 + 1}\right] + \frac{1}{\zeta_0 + 1}\right\} \tag{14}$$

The coefficients of equation (14) are defined as follows:

$$E = -\left\{B - \frac{C}{\zeta_o+1} + \frac{D}{(\zeta_o+1)^2}\right\} \frac{\beta_o^{2(\zeta_o+2)}}{\zeta_o+1} \quad \phi_1'(0)^2$$

$$F = \left\{B - \frac{C}{\zeta_o+1}\right\} \frac{\beta_o^{\zeta_o+3}}{\zeta_o+1} \phi_1'(0)^2 - \frac{2D}{(\zeta_o+1)^2} \beta_o^{\zeta_o+2} \phi_1'(0)\,\phi_1(0)$$
$$- D \frac{\beta_o^{\zeta_o+1}}{\zeta_o+1} \phi_1^2(0) + \beta_o^{\zeta_o+2} \phi_2'(0) \tag{15}$$

$$G = \frac{D}{(\zeta_o+1)^3} \beta_o^2 \phi_1'(0)^2 + \frac{2D\,\beta_o}{(\zeta_o+1)^2} \phi_1'(0)\,\phi_1(0) + \frac{D}{\zeta_o+1} \phi_1(0)^2$$

$$H = \left\{C - \frac{2D}{\zeta_o+1}\right\} \frac{\beta_o^{\zeta_o+3}}{\zeta_o+1} \phi_1'(0)^2 + \left\{C - \frac{2D}{\zeta_o+1}\right\} \beta_o^{\zeta_o+2} \phi_1'(0)\,\phi_1(0)$$

All terms containing the variable ξ will be designated by lower case b's with subscripts:

$$b_1 = \frac{\beta_o}{2\zeta_o+1}\left\{\left(\frac{\beta_o}{\xi+\beta_o}\right)^{2\zeta_o+1} - 1\right\}$$
$$b_2 = \frac{\beta_o}{\zeta_o}\left\{\left(\frac{\beta_o}{\xi+\beta_o}\right)^{\zeta_o} - 1\right\}$$
$$b_3 = \frac{\beta_o}{\zeta_o+1}\left\{\left(\frac{\beta_o}{\xi+\beta_o}\right)^{\zeta_o+1} - 1\right\}$$
$$b_4 = \frac{\beta_o}{2(\zeta_o+1)}\left\{\left(\frac{\beta_o}{\xi+\beta_o}\right)^{2(\zeta_o+1)} - 1\right\} \tag{16}$$
$$b_5 = \ln \frac{\beta_o}{\xi+\beta_o}$$
$$b_6 = \frac{\beta_o}{\zeta_o}\left(\frac{\beta_o}{\xi+\beta_o}\right)^{\zeta_o}$$

Introducing the coefficients (15) and (16) into equations (13) and (14) the first and second flow coefficient terms become:

$$\phi_1 = \phi_1(0) - b_3 \, \phi_1'(0) \tag{13a}$$

$$\phi_2 = \phi_2(0) + a \, \phi_1'^{\,2}(0) + c \, \phi_1'(0) \, \phi_1(0) + d \, \phi_1^2(0) + e \, \phi_2'(0) \tag{14a}$$

where the coefficients of (14a) are defined as:

$$a = \frac{\beta_0}{\chi_0+1}\left\{ b_4\left(B - \frac{C}{\chi_0+1} - \frac{D}{(\chi_0+1)^2}\right) + b_3\left(-B + \frac{2D}{(\chi_0+1)^2}\right) + b_3 b_5\left(C - \frac{2D}{\chi_0+1}\right) + b_5 \frac{\beta_0}{\chi_0+1}\left(C - \frac{3D}{\chi_0+1}\right)\right\}$$

$$c = b_3\left(-\frac{C}{\chi_0+1} + \frac{4D}{(\chi_0+1)^2}\right) + b_3 b_5\left(C - \frac{2D}{\chi_0+1}\right) + b_5 \frac{\beta_0}{\chi_0+1}\left(C - \frac{4D}{\chi_0+1}\right)$$

$$d = D \, \frac{\beta_0}{\chi_0+1}\left(b_3 - \beta_0 b_5\right) \quad ; \qquad e = -b_3$$

Since a large number of coefficients will be used in the following equations, they will be represented by lower case letters with number subscripts. The letter will be used to indicate coefficients of a given term (a for $\phi_1^2(o)$; c for $\phi_1'(o) \; \phi_1(o)$; d for $\phi_1^2(o)$ and e for either $\phi_1'(o)$ or $\phi_2'(o)$). The number subscript refers to a given equation. The lower case b refers to terms containing the variable ξ.

D. Flow Coefficient Initial Conditions

Equations 13a and 14a can be used to determine the flow coefficient in terms of the initial conditions at $\xi - 0$; ($\phi_1(o)$, $\phi_1'(o)$,

$\phi_2(o)$ and $\phi_2'(o)$). The first stage of the compressor will be assumed to operate with the same characteristic as the single stage, therefore at $\xi = 0$, $\phi(o)$ will be known. Let $\phi(o) = \phi_o + \epsilon\phi_1(o)$ with $\phi_n(o) = 0$ for $n > 1$, thus $\phi_1(o)$ is prescribed and $\phi_2(o) = 0$. For the initial condition at $\xi = 0$ equation (7) becomes:

$$\phi'(o) = \left\{ f(o) \frac{T'(o)}{T(o)} + \frac{f_o}{\beta_o} \right\} \phi(o) \tag{17}$$

By definition:

$$\begin{aligned} T'(o) &= \left(\frac{dT}{d\xi} \right)_{\xi=0} = \frac{U^2}{c_p} \tau(o) \\ T(o) &= \Theta \\ \frac{T'(o)}{T(o)} &= \frac{U^2 \tau(o)}{c_p \Theta} = \frac{U^2 \tau_o}{c_p \Theta} \left(\frac{\tau(o)}{\tau_o} \right) \end{aligned} \tag{18}$$

The term $\dfrac{U^2 \tau_o}{c_p \Theta}$ is similar to the term defined as $1/\beta_o$, therefore let:

$$\beta = \frac{c_p \Theta}{U^2 \tau_o} = \beta_o + \epsilon \beta_1 \tag{19}$$

Substituting from the expansions (9) and (10) gives:

$$\begin{aligned} \phi(o) &= \phi_o + \epsilon \phi_1(o) \qquad \left\{ \phi_n(o) = 0 \text{ for } n > 1 \right\} \\ \tau(o) &= \tau_o + \epsilon \tau_o' \phi_1(o) + \epsilon^2 \frac{\tau_o''}{2} \phi_1(o)^2 \\ f(o) &= f_o + \epsilon^2 \frac{f_o''}{2} \phi_1(o)^2 \\ \phi'(o) &= \epsilon \phi_1'(o) + \epsilon^2 \phi_2'(o) \end{aligned} \tag{20}$$

Substituting (18) and the expansions (19) and (20) into equations (17) results in a series equation in powers of ϵ . Equating terms of like powers of ϵ gives the following expressions for the flow coefficient derivatives at the initial conditions:

$$\phi_1'(0) = \frac{f_0 \phi_0}{\beta_0^2}\beta_1 - \frac{\zeta_0}{\beta_0}\phi_1(0)$$

$$\phi_2'(0) = \left(\frac{\zeta_0 + f_0}{\beta_0^2}\right)\beta_1 \phi_1(0) - \frac{f_0 \phi_0 \beta_1^2}{\beta_0^3} - \frac{f_0}{\beta_0}\left\{\frac{\tau_0'}{\tau_0} + \frac{\gamma'' \phi_0}{2}\right\}\phi_1^2(0) \qquad (21)$$

E. <u>Flow Coefficient - Final Form</u>

Substituting the initial conditions (21) into the expressions for the flow coefficient terms (13a and 14a) results in a quadratic expression for the flow coefficient in terms of the velocity (or speed) perturbation β_1, and the flow coefficient perturbation $\phi_1(o)$ of the form:

$$\phi(\xi) = \phi_0 + \epsilon A_2 \beta_1 + \epsilon^2 A_{22} \beta_1^2 + \epsilon^2 A_{12} \beta_1 \phi_1(0) + \epsilon A_1 \phi_1(0) + \epsilon^2 A_{11} \phi_1(0)^2$$

The upper case letters will be used to represent coefficients of a given equation and the subscripts refer to the terms in the equation. The subscripts 1 and 2 will be used to represent coefficients of the terms $\phi_1(o)$ and β_1 respectively.

Small deviations from the design point are accomplished by letting $\epsilon\,\phi_1(o)$ and $\epsilon\,\beta_1$ be small. This can be accomplished by letting $\epsilon = 1$ and using small perturbation of $\phi_1(o)$ and β_1.

Letting $\epsilon = 1$ the flow coefficient becomes:

$$\phi = \phi_o + A_2 \beta_1 + A_{22} \beta_1^2 + A_{12} \beta_1 \phi_1(o) + A_1 \phi_1(o) + A_{11} \phi_1(o)^2 \qquad (22)$$

The coefficients of equation (22) are expressed in terms of the initial design conditions $(\tau_o, \tau_o', \tau_o'', f_o, f_o'', \phi_o)$ and the location along the length of the compressor ξ. These coefficients are most easily expressed in terms of the coefficients of all the previous equations involved.

Express the initial conditions (21) as:

$$\begin{aligned} \phi_1'(o) &= B_1 \phi_1(o) + B_2 \beta_1 \\ \phi_2'(o) &= B_{11} \phi_1^2(o) + B_{12} \beta_1 \phi_1(o) + B_{22} \beta_1^2 \end{aligned} \qquad (21a)$$

where

$$B_1 = -\frac{\tau_o}{\beta_o} \quad ; \quad B_2 = \frac{f_o \phi_o}{\beta_o^2}$$

$$B_{11} = -\frac{f_o}{\beta}\left\{\frac{\tau_o'}{\tau_o} + \frac{\gamma'' \phi_o}{2}\right\} ; \quad B_{12} = \frac{\tau_o + f_o}{\beta_o} ; \quad B_{22} = -\frac{f_o \phi_o}{\beta_o^3}$$

Substituting into equations (13a) and (14a) the coefficients of equation (22) become:

$$\begin{aligned} A_2 &= -B_2 b_3 \\ A_{22} &= a B_2^2 - b_3 B_{22} \\ A_{12} &= 2 a B_1 B_2 + c B_2 - b_3 B_{12} \\ A_1 &= 1 - b_3 B_1 \\ A_{11} &= a B_1^2 + c B_1 + d - b_3 B_{11} \end{aligned}$$

F. Temperature and Pressure Ratios

The differential equation for the temperature rise through the compressor is integrated to determine the temperature ratios as follows:

$$\frac{dT}{d\xi} = \frac{U^2}{c_p}\,\tau(\phi)$$

$$T(\xi) - T(o) = \frac{U^2}{c_p}\int_0^{\xi} \tau(\phi)\, d\xi$$

Substituting the expansion of $\tau(\phi)$ and dividing by $T(o) = \Theta$ the temperature ratio is:

$$\frac{T(\xi)}{T(o)} = 1 + \frac{U^2}{c_p\Theta}\left\{\xi\tau_0 + \epsilon\tau_0'\int_0^{\xi}\phi_1 d\xi + \epsilon^2\tau_0'\int_0^{\xi}\phi_2\, d\xi + \epsilon^2\frac{\tau_0''}{2}\int_0^{\xi}\phi_1^2 d\xi + \cdots\right\}$$

Let $\epsilon = 1$ and substitute $\frac{U^2\tau_0}{c_p\Theta} = \frac{1}{\beta}$ the temperature ratio becomes:

$$\frac{T(\xi)}{T(o)} = 1 + \frac{1}{\beta}\left\{\xi + \frac{\tau_0'}{\tau_0}\int_0^{\xi}(\phi_1 + \phi_2)d\xi + \frac{\tau_0''}{2\tau_0}\int_0^{\xi}\phi_1^2\, d\xi\right\} \qquad (23)$$

The flow coefficient is defined as:

$$\phi = \frac{V_m}{U} = \frac{\dot m}{\rho U A} = \frac{\dot m R T}{p U A}$$

The flow coefficient ratio at any station in the compressor becomes:

$$\frac{\phi(\xi)}{\phi(o)} = \frac{\dot m R T(\xi)}{\dot m R T(o)} \cdot \frac{p(o)\, U(o)\, A(o)}{p(\xi)\, U(\xi)\, A(\xi)} = \frac{T(\xi)}{T(o)}\,\frac{p(o)}{p(\xi)}\,\frac{A(o)}{A(\xi)}$$

since $\dot m$ and U are constant through the compressor. The pressure ratio is thus given by:

$$\frac{p(\xi)}{p(o)} = \frac{T(\xi)}{T(o)}\,\frac{\phi(o)}{\phi(\xi)}\,\frac{A(o)}{A(\xi)} \qquad (24)$$

The temperature ratio is given by equation (23) and the flow coefficients by equations (20) and (22). The area ratio is fixed by the geometry at the design condition given by equation (6) as:

$$\frac{1}{A}\,\frac{dA}{d\xi} = -\left(\frac{f_o}{\xi+\beta_o}\right)$$

Integrating gives:

$$\int_o^{\xi}\frac{dA}{A} = -f_o\int_o^{\xi}\frac{d\xi}{\xi+\beta_o}$$

$$\ln\frac{A(\xi)}{A(o)} = f_o\ \ln\frac{\beta_o}{\xi+\beta_o}$$

$$\frac{A(o)}{A(\xi)} = \left(\frac{\xi+\beta_o}{\beta_o}\right)^{f_o}$$

The integrals in equation (23) are found by direct integration of ϕ_1, ϕ_1^2 and ϕ_2 as given by equations (13) and (14). The results are again expressed in terms of the initial conditions and coefficients involving the design conditions and axial position (ξ) through the compressor. Thus:

$$\int_o^{\xi}\phi_1\,d\xi = \xi\,\phi_1(o) + e_1\,\phi_1'(o)$$

$$\int_o^{\xi}\phi_1^2\,d\xi = a_2\,\phi_1'(o)^2 + c_2\,\phi_1'(o)\,\phi_1(o) + d_2\,\phi_1(o)^2$$

$$\int_o^{\xi}\phi_2\,d\xi = a_3\,\phi_1'(o)^2 + c_3\,\phi_1'(o)\,\phi_1(o) + d_3\,\phi_1(o)^2 + e_3\,\phi_2'(o)$$

In terms of the coefficients of equation (12) and the ξ variable coefficients (16) the coefficients of these equations are given by:

$$e_1 = \frac{\beta_o}{\chi_o+1}\left(\xi + b_2\right)$$

$$a_2 = \left(\frac{\beta_o}{\chi_o+1}\right)^2\left(\xi + 2b_2 - b_1\right) \;; \quad c_2 = \frac{2\beta_o}{\chi_o+1}\left(\xi + b_2\right) \;; \; d_2 = \xi$$

$$a_3 = \left(\frac{\beta_o}{\chi_o+1}\right)^2\left\{ \frac{\xi}{2}\left(B + \frac{C-2D}{\chi_o+1} - \frac{5D}{(\chi_o+1)^2}\right) - \frac{b_1}{2}\left(B - \frac{C}{\chi_o+1} + \frac{D}{(\chi_o+1)^2}\right) \right.$$
$$\left. + b_2\left(B + \frac{C}{\chi_o} - \frac{2D(2\chi_o+1)}{\chi_o(\chi_o+1)^2}\right) - b_5\left[b_6\left(C - \frac{2D}{\chi_o+1}\right) + (\xi+\beta_o)\frac{D}{\chi_o+1}\right]\right\}$$

$$c_3 = \frac{\beta_o}{\chi_o+1}\left\{\xi\left(\frac{C-2D}{\chi_o+1} - \frac{4D}{(\chi_o+1)^2}\right) + b_2\left[\frac{C(2\chi_o+1)}{\chi_o(\chi_o+1)} - \frac{6D}{(\chi_o+1)^2} - \frac{2D}{\chi_o(\chi_o+1)^2}\right]\right.$$
$$\left. - b_5\left[b_6\left(C - \frac{2D}{\chi_o+1}\right) + (\xi+\beta_o)\frac{2D}{\chi_o+1}\right]\right\}$$

$$d_3 = -\frac{D}{\chi_o+1}\left\{\xi + \frac{\xi+b_2}{\chi_o+1} + (\xi+\beta_o)\,b_5\right\}$$

$$e_3 = e_1$$

Substituting these integrals into equation (23) and introducing the initial conditions as given by equations (21a) the temperature ratio also reduces to a quadratic expression in the velocity perturbation β_1 and the initial flow coefficient perturbation $\phi_1(o)$. This equation is of the form:

$$\frac{T(\xi)}{T(o)} = \frac{1}{\beta_o}\left\{(\xi+\beta_o) + D_2\beta_1 + D_{22}\beta_1^2 + D_{12}\beta_1\phi_1(o) + D_1\phi_1(o) + D_{11}\phi_1^2(o)\right\} \quad (23a)$$

The coefficients of equation (23a) are given by:

$$D_2 = \frac{\tau_o'}{\tau_o} B_2 e_1 - \frac{\xi}{\beta_o}$$

$$D_{22} = \frac{\tau_o'}{\tau_o}\left(a_3 B_2^2 + 2e_1 B_{22}\right) + \frac{\tau_o''}{2\tau_o}\left(a_2 B_2^2\right) + \frac{\xi}{\beta_o^2}$$

$$D_1 = \frac{\tau_o'}{\tau_o}\left(B_1 e_1 + \xi\right)$$

$$D_{12} = \frac{\tau_o'}{\tau_o}\left(2a_3 B_1 B_2 + B_2 c_3 + B_{12} e\right) + \frac{\tau_o''}{2\tau_o}\left(2a_2 B_1 B_2 + B_2 c_2\right) - \frac{D_1}{\beta_o}$$

$$D_{11} = \frac{\tau_o'}{\tau_o}\left(B_1^2 a_3 + B_1 c_3 + d_3 + B_{11} e_1\right) + \frac{\tau_o''}{2\tau_o}\left(a_2 B_1^2 + B_1 c_2 + \xi\right)$$

The maximum value of the temperature ratio at a given speed (β_1 = constant) is found by setting the derivative of the temperature ratio with respect to $\phi_1(o)$ equal to zero:

$$\frac{d}{d\phi_1(o)}\left\{\frac{T(\xi)}{T(o)}\right\} = \frac{1}{\beta_o}\left\{D_{12}\beta_1 + D_1 + 2D_{11}\phi_1(o)\right\} = 0$$

The value of the flow coefficient for the maximum temperature ratio is thus:

$$\phi_1(o)_{max} = -\frac{D_{12}\beta_1 + D_1}{2D_{11}}$$

The second derivative corresponds to the curvature of the temperature ratio curve and is:

$$\frac{d^2}{d\,\phi_1(o)^2}\left\{\frac{T(\xi)}{T(o)}\right\} = \frac{2D_{11}}{\beta_o}$$

and is independent of β_1. The temperature ratio curves can thus be completely determined by the coefficients of equation (23a).

A similar parabolic curve can be found for the pressure ratio by substituting equations (20), (22), (23a) and (25) into equation (24), and dropping all the terms of β_1 and $\phi_1(o)$ higher than the second order. The same procedure can be used to express the adiabatic efficiency η by substituting the equations of pressure and temperature ratio in the equation:

$$\eta = \frac{\left(\frac{p(\xi)}{p(o)}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\frac{T(\xi)}{T(o)} - 1}$$

The compressor efficiency (η_c) is determined by setting $\xi = 1$. The second order equations for the pressure ratio and efficiency thus reduce to the following form;

$$\frac{p(\xi)}{p(o)} = \left(\frac{\xi+\beta_o}{\beta_o}\right)^{f_o+1}\left\{1 + E_2\,\beta_1 + E_{22}\,\beta_1^2 + E_{12}\beta_1\phi_1(o) + E_1\,\phi_1(o) + E_{11}\,\phi_1^2(o)\right\} \quad (25)$$

$$\eta = \eta_o\left\{\,1 + C_2\,\beta_1 + C_{22}\,\beta_1^2 + C_{12}\beta_1\phi_1(o) + C_1\,\phi_1(o) + C_{11}\,\phi_1^2(o)\right\} \quad (26)$$

The coefficients of these equations are most easily expressed in terms of the coefficients of the previous equations and are:

$$E_2 = \frac{D_2}{\xi+\beta_0} - \frac{A_2}{\phi_0} \quad ; \quad E_1 = \frac{1-A_1}{\phi_0} + \frac{D_1}{\xi+\beta_0}$$

$$E_2 = \left(\frac{A_2}{\phi_0}\right)^2 - \frac{A_{22}}{\phi_0} - \frac{A_2 D_2}{\phi_0(\xi+\beta_0)} + \frac{D_{22}}{\xi+\beta_0}$$

$$E_{12} = \frac{2A_1A_2 - A_2}{\phi_0^2} - \frac{A_{12}}{\phi_0} + \frac{1}{\xi+\beta_0}\left\{D_{12} + \frac{D_2 - A_1D_1 - A_2D_1}{\phi_0}\right\}$$

$$E_{11} = \left(\frac{A_1}{\phi_0}\right)^2 - \frac{A_1}{\phi_0^2} - \frac{A_{11}}{\phi_0} + \frac{1}{\xi+\beta_0}\left\{D_{11} + \frac{D_1 - A_1D_1}{\phi_0}\right\}$$

$$C_2 = \frac{K}{K-1}\left(\frac{\gamma-1}{\gamma}\right)E_2 - D_2 \quad ; \quad C_1 = \frac{K}{K-1}\left(\frac{\gamma-1}{\gamma}\right)E_1 - D_1$$

$$C_{12} = \frac{K}{K-1}\,\frac{\gamma-1}{\gamma}\left\{E_{12} - E_2D_1 - E_1D_2 - \frac{E_1E_2}{\gamma}\right\} + 2D_1D_2 - D_{12}$$

$$C_{22} = \frac{K}{K-1}\,\frac{\gamma-1}{\gamma}\left\{E_{22} - E_2D_2 - \frac{E_2^2}{2\gamma}\right\} + D_2^2 - D_{22}$$

$$C_{11} = \frac{K}{K-1}\,\frac{\gamma-1}{\gamma}\left\{E_{11} - E_1D_1 - \frac{E_1^2}{2\gamma}\right\} + D_1^2 - D_{11}$$

$$K = \left(\frac{\xi+\beta_0}{\beta_0}\right)^{\frac{\gamma-1}{\gamma}(1+f_0)} \quad ; \quad \eta_0 = \beta_0(K-1)$$

The coefficient C_1 can be assumed to be zero since the maximum efficiency at design speed is known to occur at the single stage design flow rate. Setting the derivative of the efficiency with respect to the initial flow coefficient equal to zero:

$$\frac{d\eta}{d\phi_1(0)} = \eta_0\left\{C_{12}\beta_1 + C_1 + 2C_{11}\phi_1(0)\right\} = 0$$

the maximum efficiency is found to occur at an initial flow coefficient perturbation of:

$$\phi_1(0) = -\left\{\frac{C_1 + C_{12}\beta_1}{2C_{11}}\right\}$$

Since the maximum adiabatic efficiency at design speed $(\beta_1 = 0)$ must occur very close to the design flow, C_1 should be very small.

The speed function (β_1) for maximum efficiency is found by substituting the value of the flow coefficient perturbation $(\phi_1(o))$ for maximum efficiency into equation (26) which becomes:

$$\eta = \eta_o\left[1 + \left(C_2 - \frac{C_1 C_{12}}{C_{11}}\right)\beta_1 + \left(C_{22} - \frac{C_{12}^2}{4C_{11}}\right)\beta_1^2\right.$$

Setting the derivative with respect to β_1 equal to zero the maximum efficiency is found to occur at a speed function of:

$$\beta_1 = \frac{C_1 C_{12} - 2C_{11}C_2}{4C_{11}C_{22} - C_{12}^2}$$

III. APPLICATION

The performance of a ten stage compressor has been computed to illustrate the use of the method. The single stage characteristics for the research compressor as described in reference 1 have been used in this analysis. The single stage characteristics for this compressor are shown in Figure 1 in terms of the parameters τ and f. The values of the design point constants for this example are; $\tau_o = 1.95$, $\tau_o' = -7.25$, $\tau_o'' = -37.5$, $f_o = 2.01$, $f_o'' = -118$ and $\phi_o = 0.44$. The reference rotor speed for this example is $U_o = 1000$ fps. The gas is air at standard temperature and pressure with $\Theta_o = 520^oR$, $C_p = .24$ Btu/lb^{o}R and $\gamma = 1.4$. The value of β_o and ζ_o then become; $\beta_o = 1.603$ and $\zeta_o = -3.288$. The broken line curves in Figure 1 are the approximate curves representing the quadratic expansions found by dropping all terms higher than the second order in equations (9). This approximation is seen to be reasonably accurate for flow coefficients from 0.40 to 0.50.

The overall compressor performance is found by letting $N = 10$ and $\xi = 1.0$ in equations (23a), (25) and (26). The speed variable is computed from equation (19) which can be simplified to give:

$$\beta_1 = -\beta_o + \frac{c_p\Theta}{U^2\tau_o} = -\beta_o + \left(\frac{U_o}{U}\right)^2 \frac{c_p\Theta}{U_o^2\tau_o} = \beta_o\left[\left(\frac{U_o}{U}\right)^2 - 1\right]$$

The resulting temperature ratio curves for the ten stage compressor are shown in Figure 2 as a function of the inlet flow coefficient. The pressure ratio and efficiency curves are shown in Figure 3. The maximum efficiency is found to occur at $\beta_1 = 0.363$ which corresponds to $U/U_o = 0.903$.

The operating point of each stage is found by varying ξ in ten equal increments and solving equation (22) for the flow coefficient. The operating points at design speed for the first, second, fifth and last stage of the ten stage compressor are shown in Figure 4. The stage flow coefficient (ϕ) is plotted as a function of the compressor inlet flow coefficient (ϕ (o)). The stage efficiency (η), work coefficient (Ψ) and pressure coefficient (Ψ') are shown as functions of the stage flow coefficient (ϕ). These curves indicate the range in which the quadratic approximation is reasonably accurate. The quadratic expansion of τ and f are shown in Figure 1 to be reasonably accurate for stage flow coefficients between 0.40 and 0.50. The tenth stage is seen to be outside this range at design speed for compressor inlet flow coefficients greater than 0.455 and less than 0.400.

The last stage flow coefficient will have the greatest variation from the compressor inlet flow coefficient for off design speed conditions. The operating points for the tenth stage at varying speeds are shown in Figure 5.

The operating range of the tenth stage indicates that the method should be reasonably accurate for a fairly wide range of compressor inlet flow coefficients at design speed. For varying speeds reasonable accuracy can be expected at speeds varying up to ten per cent off design with a similar flow coefficient deviation.

CONCLUDING REMARKS

It has been shown that the off design characteristics of a multi-stage compressor can be predicted by a method using approximate analytical integration. The method can be expected to give reasonably accurate results at design speed for compressor inlet flow coefficients varying as much as four per cent from the design condition. At off design speeds the method should be accurate for speeds varying up to ten per cent from design with similar flow coefficient variation for each speed. A comparison with actual test results has not been made since none could be found for a compressor with all stages having identical characteristics.

This method should be extended to include varying stage characteristics. This could be done by assuming a given single stage characteristic over a section of the compressor and different single stage characteristics over other sections. The approximate integrations could then be carried out over the various sections using the appropriate stage constants for that section.

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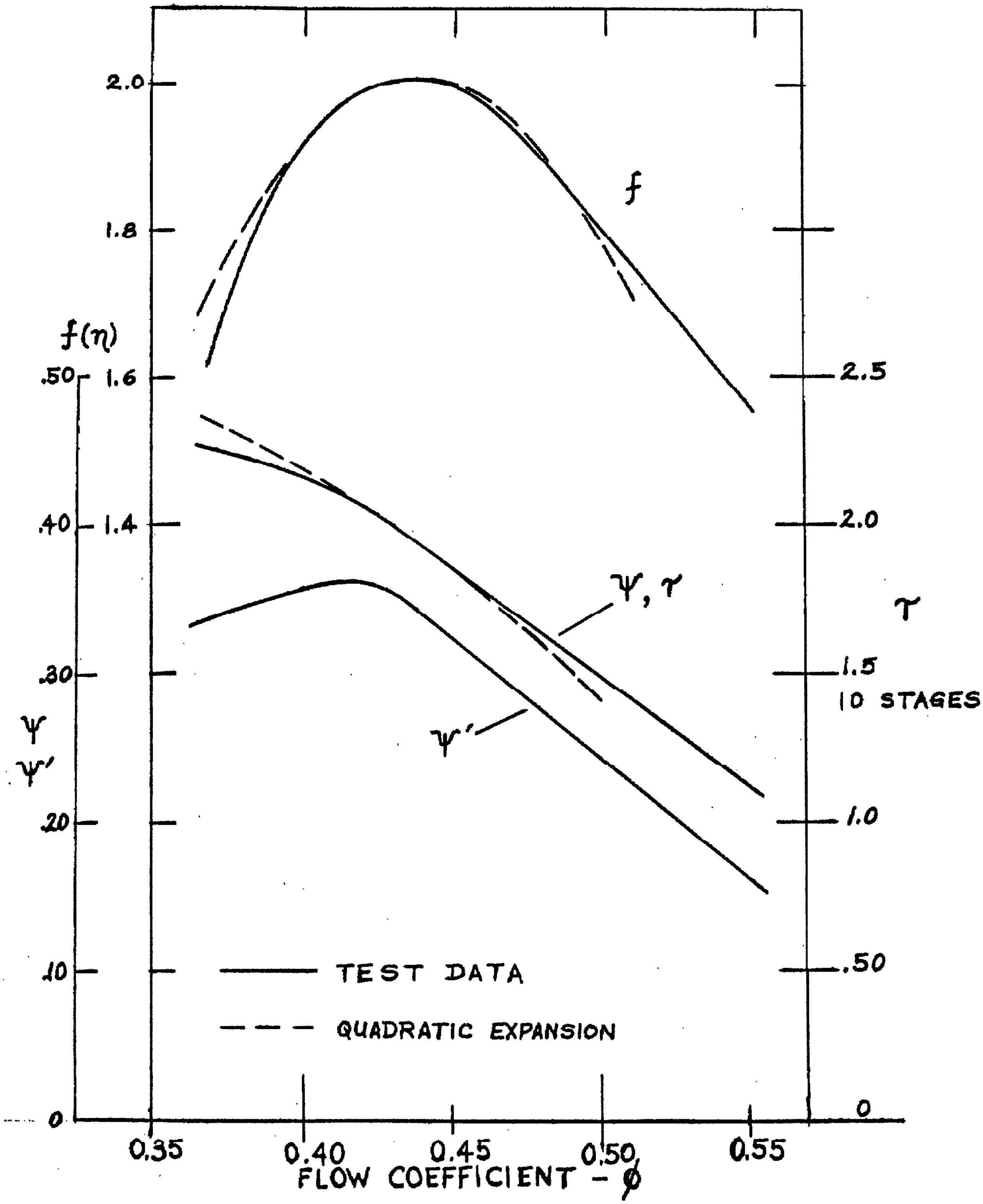


Fig. 1 Single Stage Flow Characteristics (Ref.1, Fig. 81).

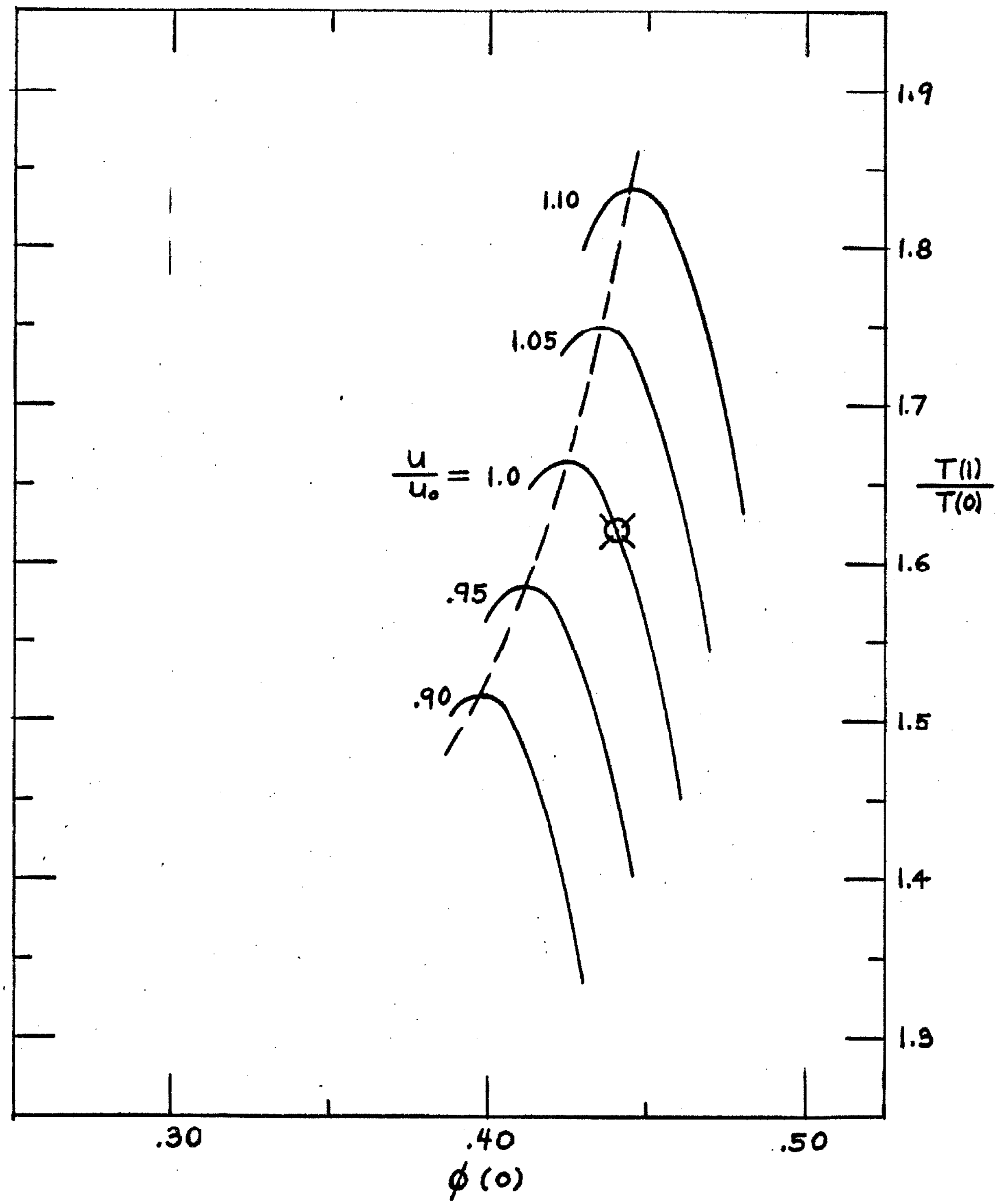


Fig. 2 Ten Stage Compressor - Overall Temperature Ratio.

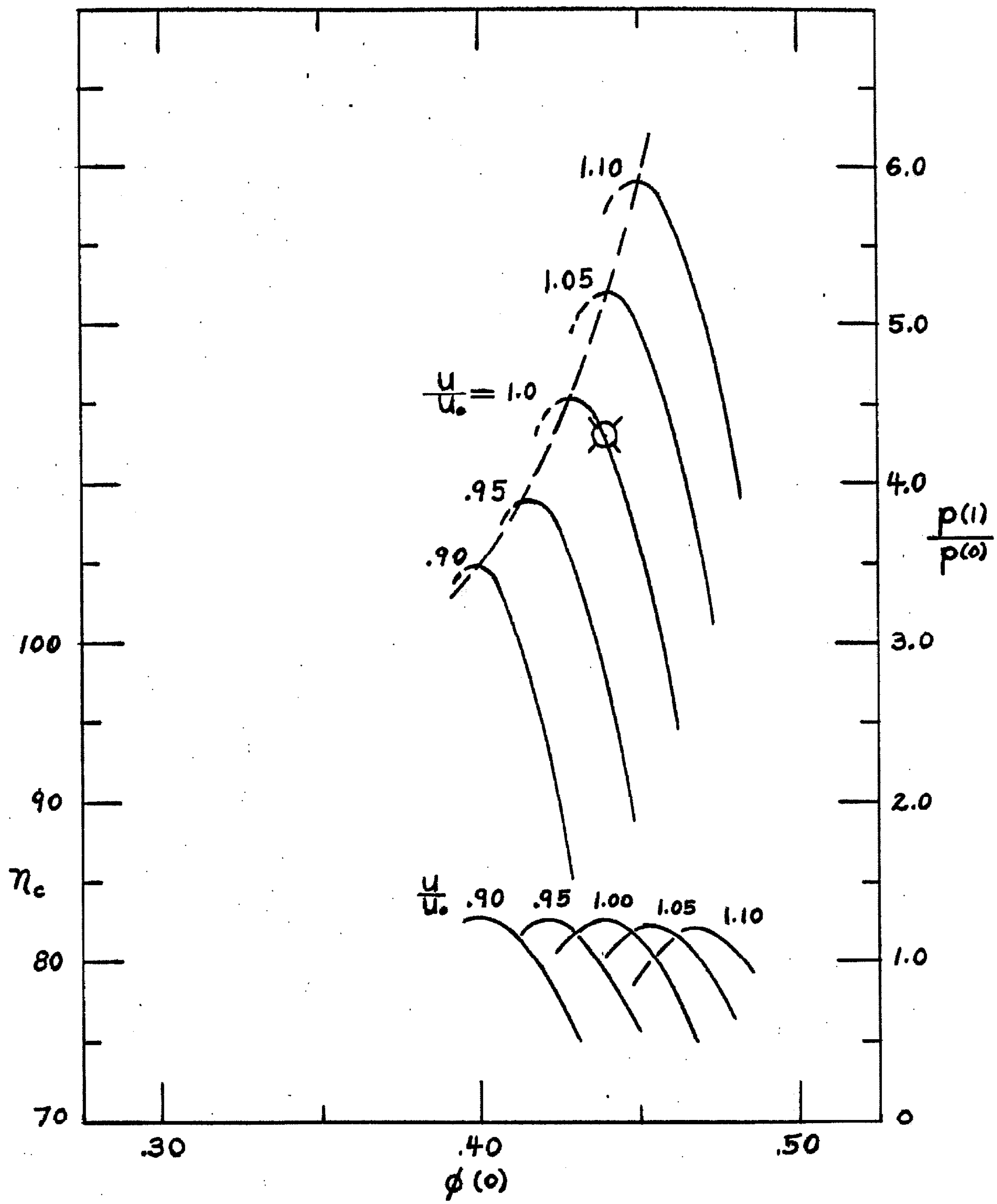


Fig. 3 Ten Stage Compressor - Overall Pressure Ratio and Adiabatic Efficiency.

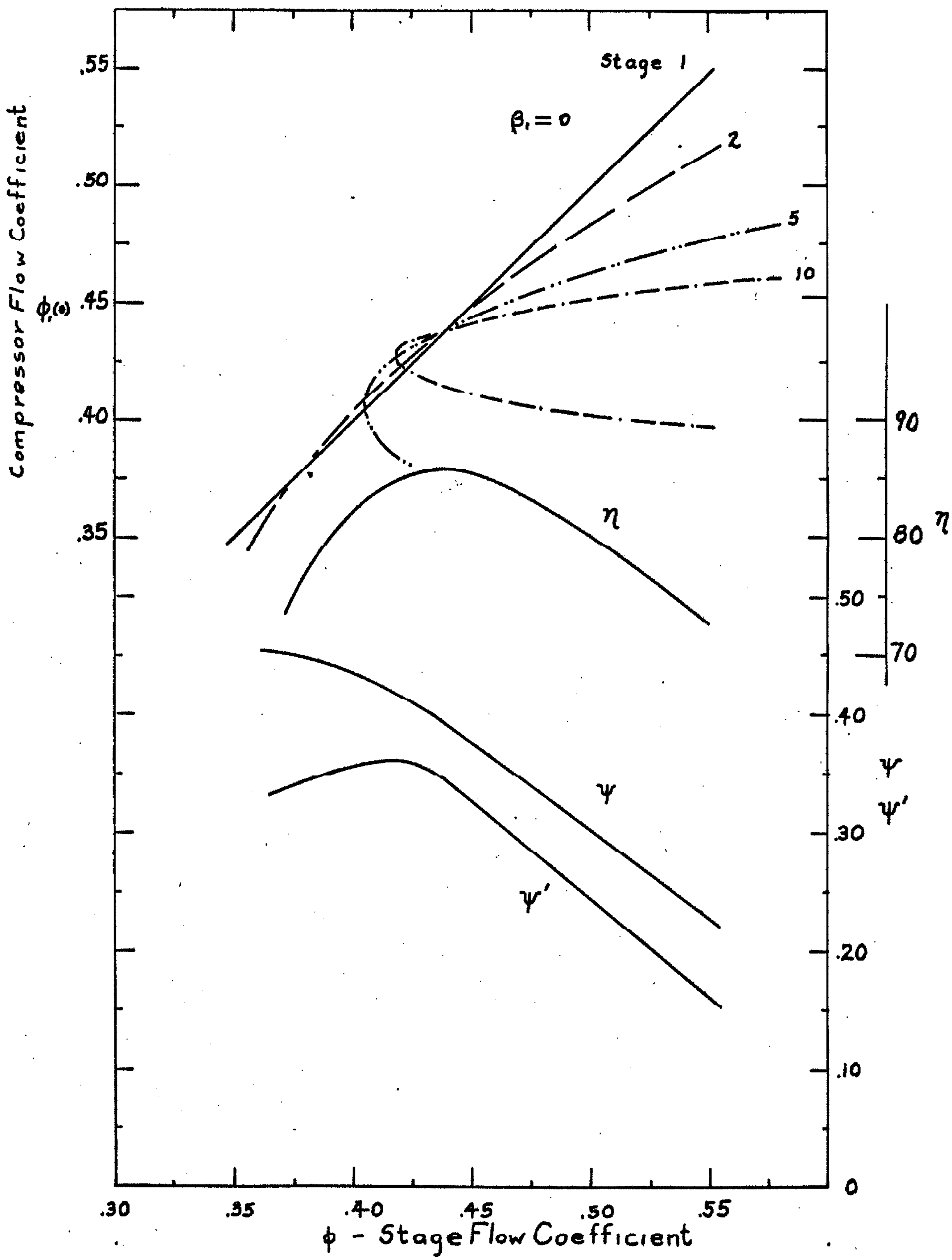
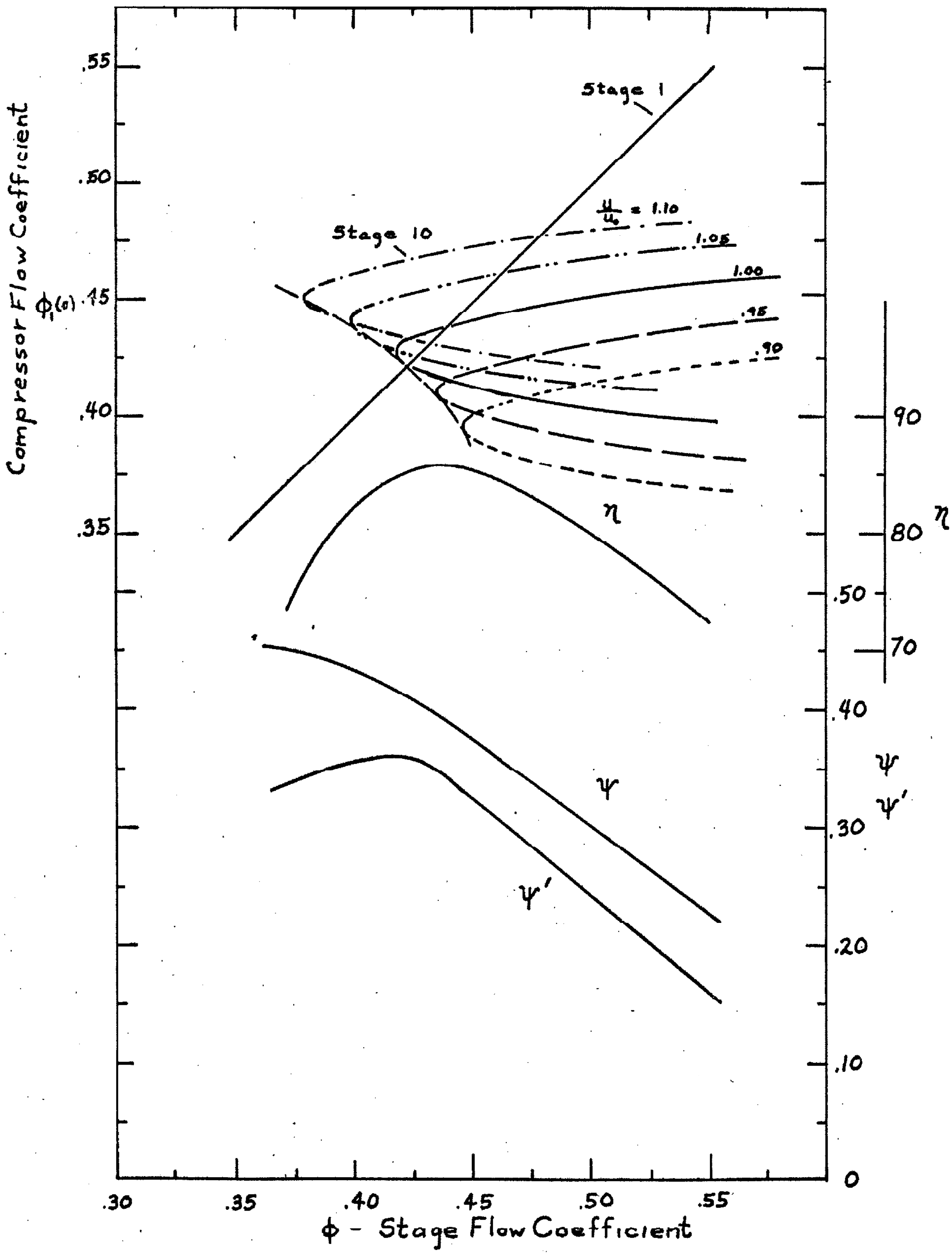


Fig. 4 Stage Operating Points at Design Speed ($\beta_1 = 0$)



Fif. 5 Tenth Stage Operating Points at Off Design Speeds.