# PRODUCTION AND DECAY PROCESSES

# INVOLVING VECTOR MESONS

Thesis by

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# DEDICATION

This work is dedicated to A. (L.) D. Lewis in sincere appreciation for the inspiration which has flowed from many most interesting conversations throughout the course of these investigations.

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ABSTRACT

Many resonances in the scattering of fundamental particles have been recently discovered, which may be interpreted in terms of unstable particles. One of these may be a vector meson coupled to the hypercharge current, whose decay rates are calculated. Predictions are given for the total decay rate, and branching ratios, in terms of $\gamma_h^2/4\pi$, the strength of the hypercharge coupling Another resonance may be correlated with the missing member, $\chi$ , of an octet of pseudoscalar mesons in the "Eightfold Way" of Gell-Mann; a reasonable estimate of the branching ratio $\Gamma(\chi \rightarrow 2\gamma)/\Gamma(\chi \rightarrow \pi^- + \pi^+ + \gamma)$ is obtained. A resonance in the K-$\pi$ system may be Gell-Mann's M-meson; the role of M in the associated production of $\Lambda$'s by pions and in $\Lambda$ production by $\bar{K}$'s is examined. The existence of a new class of asymptotic cross section equalities, which are generalizations of the Pomeranchuk relations, is demonstrated. The pion plus leptons decays of the kaon are considered, and it is shown that they cannot be determined by an intermediate M-meson. The fraction of $\rho$'s, the vector mesons that appear as a resonance in the two-pion system, that decay into four pions is estimated to be less than 1%.

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## 1. INTRODUCTION

In the last two years many resonant configurations of mesons have been discovered, which may be interpreted in terms of unstable mesons. If one examines the mass spectrum of two-pion systems produced by the annihilation of antiprotons, a very prominent peaking is visible at 750 Mev. Since this peak occurs in the $\pi^+\pi^0$ mass spectrum but not in the $\pi^+\pi^+$ spectrum, this resonant configuration is an isotopic triplet. The full width of the peak is near 100 Mev, and at present there are conflicting indications as to the question of fine structure. If we assume that statistical fluctuations account for the observation of multiple peaks within this 100 Mev region, the peak may be thought of as being due to an unstable meson with angular momentum $J = 1$. This vector meson is called the $\rho$.

Also in experiments on the multipion annihilations of protons and antiprotons, the mass spectrum of three-pion systems has been studied. A very sharp peak has been found at 787 Mev. The width of the peak is determined by the experimental resolution; the decay width of the corresponding meson could be of the order of 1 Mev or less. This meson comes in only a neutral variety, and so is an isotopic singlet, $I = 0$. It also is vector, with $J = 1$ and negative parity. The accepted designation for it is $\omega$.

In the reactions $\pi^+ + d \rightarrow p + p + 3\pi$ and $K^- + p \rightarrow \Lambda + 3\pi$ another spike appears at 550 Mev. Recent experiments have shown it to be in the isospin $I = 0$ channel, but its spin and parity are still uncertain. The associated meson was originally called the $\eta$, but if its spin-parity assignment turns out to be $0^-$, it will probably be rechristened as the $\chi$.

In the K-$\pi$ system, which has one unit of strangeness, a 30 Mev wide resonance, called $K^*$, appears at 884 Mev in the reaction $K + N \rightarrow N + K + \pi$. The corresponding meson has $I = 1/2$, and it very likely is vector.

The existence of unstable vector mesons is demanded by two very noteworthy theories of the strong interactions: Sakurai's "Vector Theory of the Strong Interactions"(1) and Gell-Mann's "Eightfold Way"(2,3). By assuming the identity of the observed particles with the mesons of these theories, one is able to make predictions concerning various phenomena in which they are involved. In this thesis, we shall study many production and decay processes from the hopeful point of view that the amplitudes may be dominated by intermediate states of vector mesons.

In Chapter 2, we shall consider the decays of a vector meson coupled to the hypercharge current. Such a meson is present in both the theory of Sakurai and that of Gell-Mann. In Sakurai's scheme it would be identified with the three-pion resonance at 550 Mev, whereas in Gell-Mann's classification it would correspond to the 787 Mev peak. Predictions will be given for the total decay rate, and branching ratios, of such a meson in terms of the strength of the hypercharge coupling.

In Chapter 3 the decays of a neutral pseudoscalar meson of mass 550 Mev are investigated, under the assumption that this meson is even under charge conjugation. A reasonable estimate of the ratio of decay rates $\Gamma(\chi \rightarrow 2\gamma)/\Gamma(\chi \rightarrow \pi^- + \pi^+ + \gamma)$ will be obtained.

We have no theory of strong interactions which we can test rigorously because we do not have the mathematical means of extracting the quantitative dynamical predictions of our theories. We thus must content ourselves, for the time being, with testing our basic ideas in a qualitative, or at best, a semi-quantitative manner, and with correlating the phenomena of particle physics. In the research summarized in this thesis examples of both of these aspects of the investigation of high energy physics will be found.

Today we have a new hypothesis about the semi-quantitative behavior of amplitudes in field theory to check. The Regge pole hypothesis is very attractive for three reasons. First of all, it shows promise of alleviating some of the divergence problems in relativistic quantum mechanics. It also may enable us to group many of the particles into new families. And finally, it indicates a clear way to test whether many of our particles behave as composites or as elementary objects. In Chapter 4, we are concerned primarily with using this hypothesis to elucidate the role of the M-meson in the phenomena of the strong interactions.

Under the hypothesis that the $K$-$\pi$ resonance is vector, we examine the role of the $K^*$ in the associated production of $\Lambda$ by $\pi$ and in $\Lambda$ production by $\bar{K}$. We shall demonstrate the existence of a new symmetry between two reaction amplitudes. This symmetry may be regarded as a generalization of Pomeranchuk's relations and should appear at high energies and low momentum transfers when both amplitudes are dominated by the same pole or pseudo-pole, as is to be expected according to the Regge pole hypothesis. Specifically, we find, in considering the details of the role of the $K^*$ in the processes $\pi + N \rightarrow \Lambda + K$ and $\bar{K} + N + \Lambda + \pi$, that the associated production amplitude in the forward direction (for the K) at high energies is asymptotically equal to the negative of the amplitude characterizing $\Lambda$ production by a $\bar{K}$. The contribution of the dominant pole terms in these amplitudes is constructed for the high energy limit and the energy and momentum transfer dependences are compared for the alternative hypothesis of composite or elementary particle behavior of a pole term. We discuss experiments which are needed to supply data for a test of the Regge pole hyopthesis. The results of these experiments, which are feasible with the new large accelerators, will be most important as guides for the construction of theories of the strong interactions.

A theory based on a vector $K$-$\pi$ resonance with $I = 1/2$ is not capable of accounting for the facts in the decays of a kaon into a pion plus leptons. This negative result will be discussed in Chapter 5, where a more general analysis of the form factors will be proposed.

The meson associated with the two-pion resonance at 750 Mev, the $\rho$, is capable of decaying into four pions. Experiments have been suggested in which these four-pion decays should show up as a peak in the mass spectrum of four-pion systems. An order of magnitude estimate of the branching ratio $\Gamma(\rho \rightarrow 4\pi)/\Gamma(\rho \rightarrow 2\pi)$ is derived in Chapter 6. This branching ratio is estimated to be less than 1%.

## 2. DECAY OF A HYPERCHARGE MESON

### I. Introduction

Three resonances in multipion systems have been discovered, whose existence has been predicted by many theorists. Among the more specific of these predictions are those of Sakurai's vector theory of the strong interactions,[1] Gell-Mann's eightfold way,[2] and similar work by Ne'eman, and Salam and Ward.[3] The two-pion resonance at 750 Mev,[4] the $\rho$, fits the description of a vector meson coupled to the conserved isotopic spin current, which is a common feature of all of these theories. The identification of the three-pion resonances at 787 Mev,[5] the $\omega$, and at 550 Mev,[6] the $\eta$, is still uncertain. Two possibilities are suggested by these theories. Sakurai has proposed[7] that the $\eta$ is a vector meson coupled to the conserved hypercharge current, and that the $\omega$ is a vector meson coupled to the conserved baryon current. On the other hand, the concept of unitary symmetry,[2] which predicts an associated resonance in the p-wave $K\pi$ system that seems to exist also, leads one to conjecture that the $\omega$ is a vector meson coupled to the hypercharge current, and that the $\eta$ is the missing member, the $\chi$, of an octet of pseudoscalar mesons. The other members of that octet would be the three pions and the four kaons.

In this article we shall calculate the decay rates of a vector meson coupled to the hypercharge current. We shall give predictions for the total decay rate, and branching ratios, in terms of one parameter, $\gamma^2/4\pi$, the strength of the hypercharge coupling. If one of the observed $I = 0$ three-pion resonances can be described as a vector meson coupled to a conserved hypercharge current, then one will be able straightforwardly to obtain a good estimate for the strength of the coupling. It must be noted that our approximations and results are based on the hopeful assumption that either there is no vector meson coupled to a baryon current, or the mixing of a baryon meson and a hypercharge meson is not so strong that the sources of both mesons are dominated by the more strongly coupled baryon current. We shall show that even with a Q of 375 Mev, the decay widths to be expected for the triple pion decay of a vector meson with mass 787 Mev are quite small (of the order 1 Mev) with reasonable matrix elements. And finally, we shall show that, due to electromagnetism, the ratio of two pion to three pion decays of a vector meson with zero isotopic spin could easily range from 14% to 56% if the mass is 550 Mev, and could well be as much as about 4% if the mass is 787 Mev.

## II. Neutral Decays

Since the hypercharge meson is characterized by the quantum numbers:

isotopic spin, $I = 0$;

angular momentum, $J = 1$;

parity, $P = -1$;

isoparity, $G = -1$;

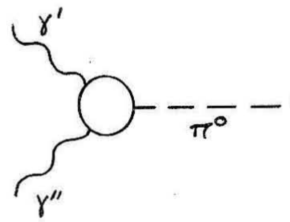
charge conjugation, $C = -1$,

the neutral decays of a hypercharge meson are not allowed by the selection rules valid for the strong interactions. (For convenience in the following discussion, let us introduce the symbol $h$ for the hypercharge meson.) For example, the decays $h \rightarrow n\pi^0$, where n is any integer, and the mode $h \rightarrow \chi + \pi^0$ are forbidden by charge conjugation. All neutral decays, therefore, must be mediated by electromagnetism. The simplest and most probable of these decays is $h \rightarrow \pi^0 + \gamma$ . The second most likely neutral mode should be $h \rightarrow X + \gamma$ , if the $X$ exists with a mass less than that of the $h$ , since the reduction in phase space resulting from additional particles in the final state inhibits greatly modes like $h \rightarrow 2\pi^0 + \gamma$ and $h \rightarrow 3\pi^0 + \gamma$ . The decay $h \rightarrow 2\gamma$ is forbidden by charge conjugation, as is any other neutral mode involving two photons.

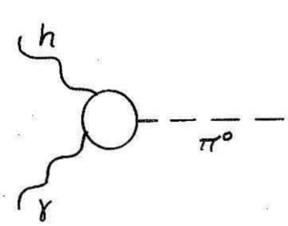
Gell-Mann and Zachariasen[8] showed that it is possible to predict the rate of the $\pi^0 + \gamma$ decay mode of a hypercharge meson in terms of the lifetime of the $\pi^0$. The crucial step in the calculation is to use the facts that the isotopic scalar piece of the electromagnetic current is the hypercharge current, and the isotopic vector piece of the electromagnetic current is the third component of the isotopic spin current. By way of introducing the calculational technique that will be employed in this article, we may paraphrase some of their work.

Since the $h$ is coupled to the isoscalar electromagnetic (EM) current, and the $\rho$ is coupled to the isovector EM current, the following four amplitudes are related.

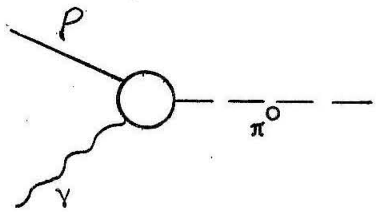
$$f_{\gamma\gamma\pi} F_{\gamma\gamma\pi}(s,t,u)\, \epsilon_{\mu\nu\sigma\tau}\, e'_\mu k'_\nu e''_\alpha k''_\tau =$$



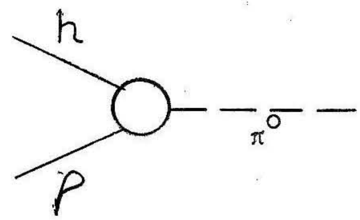

$$f_{h\gamma\pi} F_{h\gamma\pi}(s,t,u)\, \epsilon_{\mu\nu\sigma\tau}\, e^h_\mu k^h_\nu e^\gamma_\sigma k^\gamma_\tau =$$



$$f_{\rho\gamma\pi}\, F_{\rho\gamma\pi}(s,t,u)\ \epsilon_{\mu\nu\sigma\tau}\ e^{\rho}_{\mu} k^{\rho}_{\nu} e^{\gamma}_{\sigma} k^{\gamma}_{\tau} \quad =$$




$$f_{h\rho\pi}\, F_{h\rho\pi}(s,t,u)\ \epsilon_{\mu\nu\sigma\tau}\ e^{h}_{\mu} k^{h}_{\nu} e^{\rho}_{\sigma} k^{\rho}_{\tau} \quad =$$




$e_\mu$ and $k_\mu$ are polarization and momentum four vectors of the particles. We use the metric $(x^1, x^2, x^3, x^4) = (x, y, z, it)$, and $\hbar = c = m_\pi = 1$. In the first amplitude $(k')^2 = -s$, $(k'')^2 = -t$; in the second $(k^h)^2 = -s$, $(k^\gamma)^2 = -t$; in the third $(k^\rho)^2 = -s$, $(k^\gamma)^2 = -t$; in the fourth $(k^h)^2 = -s$, $(k^\rho)^2 = -t$; and in all of them $(k^\pi)^2 = -u$. The constants $f_{\gamma\gamma\pi}$, $f_{h\gamma\pi}$, $f_{\rho\gamma\pi}$, and $f_{h\rho\pi}$ are defined by the normalization $F_{\gamma\gamma\pi}(0, 0, 1) = F_{h\gamma\pi}(0, 0, 1) = F_{\rho\gamma\pi}(0, 0, 1) = F_{h\rho\pi}(0, 0, 1) = 1$.

Strictly speaking, in the theory of the strong interactions diagrams involving $\rho$'s and $h$'s do not exist since these particles are unstable. Such diagrams are always pieces of more complicated diagrams which symbolize interactions involving quite a few particles which are stable with respect to the strong interactions. Many of these multiparticle amplitudes are dominated by resonant terms which are due to poles located on sheets other than the physical sheet. These terms are the same as those associated with unstable particles, and the analysis of the phenomena of particle physics is greatly facilitated by the approximation of working with amplitudes involving unstable particles.

In a field theory containing isospin and hypercharge mesons whose sources are the renormalized currents $j_\alpha^\rho$ and $j_\alpha^h$, respectively, the relations(8)

$$<m| \, j_\alpha^{\gamma^S} \, |n> = (e/2\gamma_h) \quad (1 - s/m_h^2)^{-1} <m \, | \, j_\alpha^h \, | \, n> \qquad \text{(II.1a)}$$

$$<m| \, j_\alpha^{\gamma^V} \, |n> = (e/2\gamma_\rho) \quad (1 - s/m_\rho^2)^{-1} <m \, | \, j_\alpha^\rho \, | \, n> \qquad \text{(II.1b)}$$

between electromagnetic isospin, and hypercharge current matrix elements are valid in the limit of infinite bare mass for the $h$ and the $\rho$. In these equations, $2\gamma_\rho$ is the universal coupling constant of the $\rho$ to the isospin current when the square of the momentum transfer, $-s$, vanishes, and $\gamma_h$ is the universal coupling constant of the $h$ to the hypercharge current at $s = 0$. In the notation of unitary symmetry(2), $\gamma_h = \sqrt{3} \; \gamma_\omega$. From these equations, it is apparent that the first three amplitudes are related as follows:

$$f_{\gamma\gamma\pi} \, F_{\gamma\gamma\pi}(s, t, u) = (e/2\gamma_\rho) \quad (1 - s/m_\rho^2)^{-1} \, f_{\rho\gamma\pi} \, F_{\rho\gamma\pi}(s, t, u) +$$

$$(e/2\gamma_h) \quad (1 - s/m_h^2)^{-1} \, f_{h\gamma\pi} \, F_{h\gamma\pi}(s, t, u). \qquad \text{(II.2)}$$

Making use of the normalization definitions at $s = t = 0$, and $u = 1$, we find

$$f_{\gamma\gamma\pi} = (e/2\gamma_\rho)\, f_{\rho\gamma\pi} + (e/2\gamma_h)\, f_{h\gamma\pi} \, . \tag{II.3}$$

Furthermore, the second and fourth amplitudes are related by

$$f_{h\gamma\pi}\, F_{h\gamma\pi}(s,\ t,\ u) = (e/2\gamma_\rho)\ (1 - t/m_\rho^2)^{-1}\, f_{h\rho\pi}\, F_{h\rho\pi}(s,\ t,\ u), \tag{II.4}$$

and the third and fourth are also connected,

$$f_{\rho\gamma\pi}\, F_{\rho\gamma\pi}(s,\ t,\ u) = (e/2\gamma_h)\ (1 - t/m_h^2)^{-1}\, f_{h\rho\pi}\, F_{h\rho\pi}(t,\ s,\ u), \tag{II.5}$$

so that

$$f_{h\gamma\pi} = (e/2\gamma_\rho)\ f_{h\rho\pi} \ , \tag{II.6}$$

$$f_{\rho\gamma\pi} = (e/2\gamma_h)\ f_{h\rho\pi} \ , \tag{II.7}$$

and, from Eq. II.3,

$$f_{\gamma\gamma\pi} = (e/\gamma_h)\ f_{h\gamma\pi} = (e/\gamma_\rho)\, f_{\rho\gamma\pi} \ . \tag{II.8}$$

Our equalities hold for vanishing $(k^h)^2$ and $(k^\rho)^2$, but in the calculations the amplitudes on the mass shell are needed. Specifically, in the unstable particle approximation for complicated reactions, quantities such as

$$i\, m_h \Gamma_h \left\{ d\left[\, F_{h\gamma\pi}(s,\ 0,\ 1)\, \right] / ds \right\}_{s\, =\, m_h^2}$$

enter. In the pole approximation, such quantities are unity. Their deviation from unity may be neglected to the same extent that more massive intermediate states may be neglected in a calculation of the dispersion theory. Since we are making use of the latter simplification in this paper, we shall follow a consistent course and employ also the former.

The rate for the two photon decay of the neutral pion is

$$\Gamma(\pi^o \to \gamma + \gamma) = 2\pi \times \frac{1}{2} \sum |T|^2 \; P \; , \tag{II.9}$$

where the relativistic phase space factor, P, for this final state of two massless particles is $(32\pi^2 m_\pi)^{-1}$,

$$\sum |T|^2 = |f_{\gamma\gamma\pi}|^2 \sum \left| \epsilon_{\mu\nu\sigma\tau} e'_\mu \, k'_\nu \, e''_\sigma \, k''_\tau \right|^2 = m_\pi^4/2 \; , \tag{II.10}$$

(the sum being over the polarizations of the gamma rays), and where the factor of 1/2 results from the indistinguishability of the two photons. Putting this together, we obtain

$$\Gamma(\pi^o \to \gamma + \gamma) = |f_{\gamma\gamma\pi}|^2 \, m_\pi^3 \, / \, 64\pi = |f_{\gamma\gamma\pi}|^2 \, / \, 64\pi, \tag{II.11}$$

where we recall the $m_\pi = 1$ with our choice of units.

## III. Decay into $\pi^+ + \pi^- + \pi^0$

The decay of a hypercharge meson into three pions can be treated in an approximation which is the basis of all practical calculations in dispersion theory, namely, that of keeping only those intermediate states with low masses in the dispersion integral. We shall assume the dominance of two pion intermediate states in dispersing the matrix element for $\pi + h \rightarrow \pi + \pi$. In addition, the two pion intermediate states will be treated in the $\rho$-meson pole approximation[8], isotopic spin selection rules together with the generalized Pauli principle allow only states of off angular momentum in the two pion system, and p-wave scattering should be much more important than f-wave scattering at these low energies. Included in the $h$ decay matrix element are three terms, which are obtained from one another by switching the electric charges of the pions, and which may be described graphically by the diagram of Fig. 1. The analytic expression for the matrix element is

$$T = -4\, \gamma_{\pi\pi\rho}\, f_{h\rho\pi}\, \epsilon_{\mu\nu\sigma\tau}\, e^h_\mu k^+_\nu k^o_\sigma k^-_\tau \Big\{ D\left[ (k^- + k^o)^2 \right] + D\left[ (k^o + k^+)^2 \right] + D\left[ (k^+ + k^-)^2 \right] \Big\} \quad \text{(III.1)}$$

where $D(k^2)$ is the $\rho$-meson propagators, $e^h_\mu$ is the polarization four-vector of the $h$, and $k_\nu$ is a pion momentum four-vector. In the pole approximation

$$D(k^2) = (k^2 + m_\rho^2 - i\, m_\rho \Gamma_\rho)^{-1} \quad \text{(III.2)}$$

The expression for the decay rate is complicated by the fact that the square of the matrix element is not a constant and by the necessity

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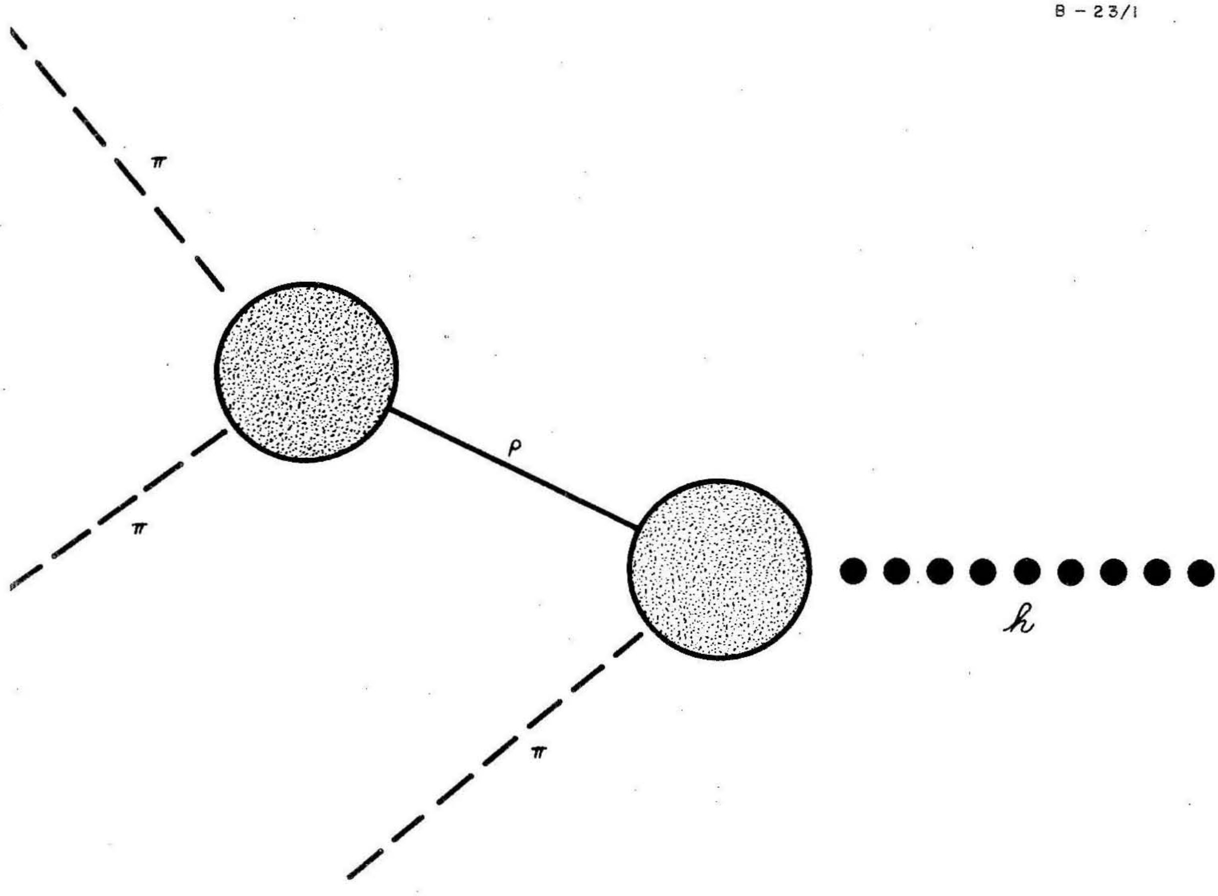



Figure 1. Dominant diagram in $h$ decay

of treating the pion kinematics relativistically. In terms of the pion energies as variables

$$\Sigma \left| \epsilon_{\mu\nu\sigma\tau} e^{h}_{\mu} k^{+}_{\nu} k^{o}_{\sigma} k^{-}_{\tau} \right|^{2} = m_h^2 (\vec{p}^{\,o} \times \vec{p}^{\,-})^2 / 3$$

$$= m_h^2 \left\{ (E^{o2}-1)(E^{-2}-1) - \frac{1}{4}\left[m_h^2 + 1 - 2m_h(E^o+E^-) + 2E^- E^o\right]^2 \right\} / 3, \qquad \text{(III.3)}$$

where $\Sigma$ indicates an average over the initial $h$ polarizations. It has been found that convenient variables are $y = E^o - E^-$, and $x = E^o + E^-$, in terms of which

$$\Gamma(h + 3\pi) = (96\,\pi^3)^{-1}\, \gamma^2_{\pi\pi\rho}\, f^2_{h\rho\pi}\, m_h \iint A\,B\; dx\, dy \quad , \qquad \text{(III.4)}$$

where

$$A = 2m_h\, x(x^2-y^2) - m_h^2\,(5x^2-y^2) - m_\pi^2\,(3x^2+y^2)$$

$$+ 4\, m_h\, x\, (m_h^2 + m_\pi^2) + 4\, m_\pi^4 - (m_h^2 + m_\pi^2)^2 \quad , \qquad \text{(III.5)}$$

$$B = (m_\rho^2 + m_h^2 - m_\pi^2 - 2m_h x)^{-1} + \left[ m_\rho^2 - m_h^2 - m_\pi^2 + m_h(x-y)\right]^{-1}$$

$$+ \left[ m_\rho^2 - m_h^2 - m_\pi^2 + m_h\,(x+y)\right]^{-1} \qquad \text{(III.6)}$$

and the integral extends over the region in which A is positive.

The integral can be evaluated explicitly in the non-relativistic limit. The rate for the three-pion decay may be expressed conveniently in the form

$$\Gamma(h \to \pi^+ + \pi^- + \pi^o) = \frac{(\gamma^2_{\pi\pi\rho}/4\pi)\; f^2_{h\rho\pi}\; m_h\; m_\pi^2\; (m_h - 3m_\pi)^4\; W(m_h)}{12\,\pi\,\sqrt{3}\;\; (m_\rho^2 - 4m_\pi^2)^2} \quad , \qquad \text{(III.7)}$$

Where $W(m_h)$ is a relativistic correction factor; $W(3m_\pi) = 1$. This function has been computed numerically in such a way that we also obtained the spectrum of any one of the pions. If we denote the energy of the $\pi^+$, for example, by E,

$$W(m) = \int_1^{(m^2-3)/2m} S(m,E)\ dE \quad , \tag{III.8}$$

where the spectrum $S(m,E)$ may be expressed as

$$S(m,E) = A(m,E) \left\{ 1 + \frac{(m_\rho^2 - 1 - m^2 + 2mE)}{(m_\rho^2 - 1 - mE)} \frac{3a}{t^3} (t^2 - a^2) \log\left(\frac{a+t}{a-t}\right) + 2at \right. \\ \left. + \frac{(m_\rho^2 - 1 - m^2 + 2mE)^2}{(m_\rho^2 - 1 - mE)^2} \frac{3a}{2t^3} \left[ (t^2 + a^2) \log\left(\frac{t+a}{a-t}\right) - 2at \right] \right\} \tag{III.9}$$

in terms of the functions

$$a = a(m,E) = \sqrt{3}\,(m_\rho^2 - 1 - mE) \,/\, m(m-3) \quad , \tag{III.10}$$

$$t = t(m,E) = \left\{ \frac{6\ m\ (E-1)(E+1)\ \left[(m^2-3)/2m - E\right]}{(m-3)^2\ (m^2 + 1 - 2mE)} \right\}^{\frac{1}{2}} \quad , \tag{III.11}$$

and

$$A(m,E) = \frac{2(m^2 + 1 - 2mE)\ (m_\rho^2 - 4)^2\ t^3}{9\pi\ (m - 3)\ (m_\rho^2 + 2mE - m^2 - 1)^2} \quad . \tag{III.12}$$

The spectrum of the pion in the decay for the case $m_h = 5.7\ m_\pi = 787$ Mev is shown in Fig. 2. It will be noted that the deviations from the statistical spectrum due to pion-pion scattering are very small for this case. This absence of structure in the spectrum may be traced to two factors.

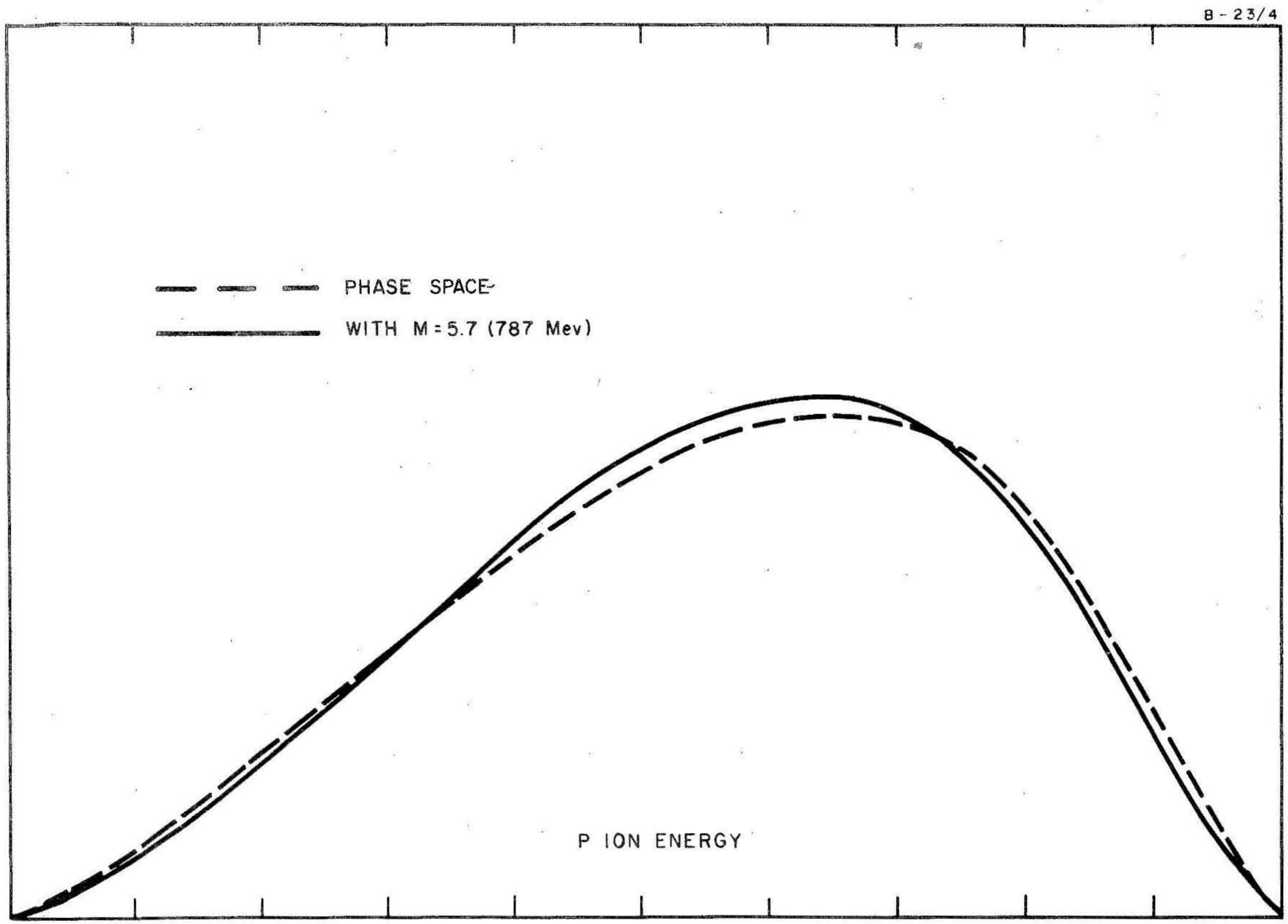



Figure 2. Spectrum of $\pi^+$ in hypercharge meson decay

The maximum energy available to any two pions is well below the mass of the $\rho$, so that only the tail of the resonance is effective. The symmetrization required by the generalized Pauli principle tends to smooth out any departure from uniformity in the matrix element. More specifically, there are three diagrams in the class shown in Fig. 1, and whenever one of them contributes a relatively large term to the decay matrix element, the other two terms are smaller than average.

For the numerical work, the mass of the pion was taken to be 138 Mev which is the average mass of the three pions in this decay mode. In this unit, the mass of the $\omega$ is 5.70 and that of the $\eta$ is 3.99. The spectrum and the correction factor, W(m), were computed by an IBM 7090 for the mass values: m = 3.10, 3.99, 5.00, 5.70, 6.20. The results are given in Tables 1-6. In Fig. 2 the comparison of the spectrum and the statistical prediction has been shown for the $\omega$ mass; the differences between the two, if the hypercharge meson is taken to be the $\eta$, are much smaller.

The ratio of the $\pi^0 + \gamma$ and the $\pi^+ + \pi^- + \pi^0$ decay rates of the $h$ meson is independent of the hypercharge coupling constant, $\gamma_h$:

$$R = \frac{\Gamma(h \rightarrow \pi^0 + \gamma)}{\Gamma(h \rightarrow \pi^+ + \pi^- + \pi^0)} = \frac{\sqrt{3}\ \alpha\ (m_h^2 - 1)^2 (m_\rho^2 - 4)^2}{32\ (\gamma_\rho^2/4\pi)\ (\gamma_{\pi\pi\rho}^2/4\pi)\ m_h^4\ (m_h - 3)^4\, W(m_h)} . \tag{III.13}$$

Table 1: Relativistic Correction Factor, $W(m)$

| $m$ | $W(m)$ |
|---|---|
| 3.10 | 1.033 |
| 3.99 | 1.430 |
| 5.00 | 2.286 |
| 5.70 | 3.558 |
| 6.20 | 5.687 |

Table 2: Pion Spectrum in $h \rightarrow \pi^+ + \pi^- + \pi^0$ , m = 3.10

E = 1 + X(m-3)(m+1)/200m , S(m,E) = 20 m P / (m-3)(m+1) .

| X | P | X | P | X | P | X | P | X | P |
|---|---|---|---|---|---|---|---|---|---|
| 1 | .004 | 21 | .303 | 41 | .544 | 61 | .541 | 81 | .287 |
| 2 | .012 | 22 | .319 | 42 | .550 | 62 | .534 | 82 | .270 |
| 3 | .022 | 23 | .335 | 43 | .556 | 63 | .526 | 83 | .253 |
| 4 | .033 | 24 | .350 | 44 | .561 | 64 | .517 | 84 | .235 |
| 5 | .046 | 25 | .365 | 45 | .565 | 65 | .508 | 85 | .217 |
| 6 | .059 | 26 | .380 | 46 | .569 | 66 | .498 | 86 | .200 |
| 7 | .073 | 27 | .395 | 47 | .572 | 67 | .488 | 87 | .182 |
| 8 | .088 | 28 | .409 | 48 | .574 | 68 | .476 | 88 | .164 |
| 9 | .104 | 29 | .422 | 49 | .576 | 69 | .465 | 89 | .147 |
| 10 | .120 | 30 | .435 | 50 | .577 | 70 | .453 | 90 | .130 |
| 11 | .136 | 31 | .448 | 51 | .577 | 71 | .440 | 91 | .113 |
| 12 | .153 | 32 | .460 | 52 | .576 | 72 | .427 | 92 | .096 |
| 13 | .169 | 33 | .472 | 53 | .575 | 73 | .413 | 93 | .080 |
| 14 | .186 | 34 | .483 | 54 | .573 | 74 | .399 | 94 | .065 |
| 15 | .203 | 35 | .493 | 55 | .571 | 75 | .384 | 95 | .050 |
| 16 | .220 | 36 | .503 | 56 | .567 | 76 | .369 | 96 | .036 |
| 17 | .237 | 37 | .513 | 57 | .564 | 77 | .353 | 97 | .024 |
| 18 | .254 | 38 | .521 | 58 | .559 | 78 | .337 | 98 | .013 |
| 19 | .270 | 39 | .529 | 59 | .554 | 79 | .321 | 99 | .005 |
| 20 | .287 | 40 | .537 | 60 | .548 | 80 | .304 | 100 | .000 |

Table 3: Pion Spectrum in $h \rightarrow \pi^+ + \pi^- + \pi^0$ , m = 3.99

$E = 1 + X(m-3)(m+1)/200m$ , $S(m,E) = 20 \; m P / (m-3)(m+1)$ .

| X | P | X | P | X | P | X | P | X | P |
|---|---|---|---|---|---|---|---|---|---|
| 1 | .005 | 21 | .360 | 41 | .735 | 61 | .840 | 81 | .518 |
| 2 | .013 | 22 | .382 | 42 | .749 | 62 | .834 | 82 | .491 |
| 3 | .023 | 23 | .403 | 43 | .762 | 63 | .828 | 83 | .463 |
| 4 | .036 | 24 | .424 | 44 | .774 | 64 | .820 | 84 | .434 |
| 5 | .050 | 25 | .445 | 45 | .785 | 65 | .811 | 85 | .405 |
| 6 | .064 | 26 | .466 | 46 | .795 | 66 | .801 | 86 | .375 |
| 7 | .080 | 27 | .487 | 47 | .805 | 67 | .790 | 87 | .344 |
| 8 | .097 | 28 | .508 | 48 | .814 | 68 | .778 | 88 | .314 |
| 9 | .115 | 29 | .528 | 49 | .821 | 69 | .765 | 89 | .283 |
| 10 | .113 | 30 | .548 | 50 | .828 | 70 | .750 | 90 | .251 |
| 11 | .152 | 31 | .567 | 51 | .834 | 71 | .735 | 91 | .220 |
| 12 | .172 | 32 | .586 | 52 | .839 | 72 | .718 | 92 | .189 |
| 13 | .191 | 33 | .605 | 53 | .844 | 73 | .700 | 93 | .159 |
| 14 | .212 | 34 | .623 | 54 | .847 | 74 | .681 | 94 | .130 |
| 15 | .232 | 35 | .641 | 55 | .849 | 75 | .661 | 95 | .101 |
| 16 | .253 | 36 | .658 | 56 | .850 | 76 | .639 | 96 | .074 |
| 17 | .274 | 37 | .675 | 57 | .850 | 77 | .617 | 97 | .049 |
| 18 | .296 | 38 | .691 | 58 | .849 | 78 | .594 | 98 | .028 |
| 19 | .317 | 39 | .706 | 59 | .847 | 79 | .569 | 99 | .010 |
| 20 | .339 | 40 | .721 | 60 | .844 | 80 | .544 | 100 | .000 |

Table 4: Pion Spectrum in $h \longrightarrow \pi^+ + \pi^- + \pi^0$ , m = 5.00

$E = 1 + X(m-3)(m+1)/200m$ ,  $S(m,E) = 20\ m\ P\ /\ (m-3)(m+1)$ .

| X | P | X | P | X | P | X | P | X | P |
|---|---|---|---|---|---|---|---|---|---|
| 1 | .006 | 21 | .515 | 41 | 1.126 | 61 | 1.425 | 81 | 1.013 |
| 2 | .018 | 22 | .547 | 42 | 1.152 | 62 | 1.425 | 82 | .968 |
| 3 | .032 | 23 | .579 | 43 | 1.176 | 63 | 1.423 | 83 | .920 |
| 4 | .049 | 24 | .612 | 44 | 1.200 | 64 | 1.419 | 84 | .871 |
| 5 | .068 | 25 | .644 | 45 | 1.223 | 65 | 1.412 | 85 | .819 |
| 6 | .089 | 26 | .676 | 46 | 1.245 | 66 | 1.404 | 86 | .765 |
| 7 | .112 | 27 | .708 | 47 | 1.266 | 67 | 1.394 | 87 | .709 |
| 8 | .135 | 28 | .740 | 48 | 1.286 | 68 | 1.382 | 88 | .651 |
| 9 | .160 | 29 | .772 | 49 | 1.305 | 69 | 1.367 | 89 | .592 |
| 10 | .186 | 30 | .804 | 50 | 1.323 | 70 | 1.351 | 90 | .532 |
| 11 | .213 | 31 | .836 | 51 | 1.340 | 71 | 1.332 | 91 | .470 |
| 12 | .240 | 32 | .867 | 52 | 1.354 | 72 | 1.311 | 92 | .408 |
| 13 | .269 | 33 | .898 | 53 | 1.368 | 73 | 1.287 | 93 | .346 |
| 14 | .298 | 34 | .928 | 54 | 1.381 | 74 | 1.261 | 94 | .285 |
| 15 | .328 | 35 | .958 | 55 | 1.392 | 75 | 1.233 | 95 | .224 |
| 16 | .358 | 36 | .987 | 56 | 1.401 | 76 | 1.202 | 96 | .166 |
| 17 | .389 | 37 | 1.016 | 57 | 1.410 | 77 | 1.169 | 97 | .112 |
| 18 | .420 | 38 | 1.045 | 58 | 1.416 | 78 | 1.134 | 98 | .063 |
| 19 | .451 | 39 | 1.073 | 59 | 1.421 | 79 | 1.096 | 99 | .023 |
| 20 | .483 | 40 | 1.100 | 60 | 1.424 | 80 | 1.056 | 100 | .000 |

Table 5: Pion Spectrum in $h \rightarrow \pi^+ + \pi^- + \pi^0$ , m = 5.70

$E = 1 + X(m-3)(m+1)/200m$ , $S(m,E) = 20 \ m \ P \ / \ (m-3)(m+1)$ .

| X | P | X | P | X | P | X | P | X | P |
|---|---|---|---|---|---|---|---|---|---|
| 1 | .012 | 21 | .789 | 41 | 1.661 | 61 | 2.221 | 81 | 1.784 |
| 2 | .033 | 22 | .834 | 42 | 1.700 | 62 | 2.231 | 82 | 1.717 |
| 3 | .059 | 23 | .880 | 43 | 1.738 | 63 | 2.238 | 83 | 1.644 |
| 4 | .088 | 24 | .925 | 44 | 1.776 | 64 | 2.243 | 84 | 1.567 |
| 5 | .121 | 25 | .970 | 45 | 1.812 | 65 | 2.245 | 85 | 1.484 |
| 6 | .155 | 26 | 1.015 | 46 | 1.848 | 66 | 2.245 | 86 | 1.396 |
| 7 | .192 | 27 | 1.060 | 47 | 1.883 | 67 | 2.241 | 87 | 1.303 |
| 8 | .230 | 28 | 1.105 | 48 | 1.916 | 68 | 2.234 | 88 | 1.205 |
| 9 | .269 | 29 | 1.150 | 49 | 1.949 | 69 | 2.223 | 89 | 1.103 |
| 10 | .309 | 30 | 1.194 | 50 | 1.980 | 70 | 2.209 | 90 | .997 |
| 11 | .350 | 31 | 1.239 | 51 | 2.010 | 71 | 2.192 | 91 | .887 |
| 12 | .392 | 32 | 1.283 | 52 | 2.039 | 72 | 2.171 | 92 | .775 |
| 13 | .435 | 33 | 1.327 | 53 | 2.066 | 73 | 2.146 | 93 | .661 |
| 14 | .478 | 34 | 1.370 | 54 | 2.092 | 74 | 2.116 | 94 | .546 |
| 15 | .522 | 35 | 1.413 | 55 | 2.116 | 75 | 2.083 | 95 | .433 |
| 16 | .566 | 36 | 1.456 | 56 | 2.139 | 76 | 2.045 | 96 | .322 |
| 17 | .610 | 37 | 1.498 | 57 | 2.159 | 77 | 2.002 | 97 | .218 |
| 18 | .654 | 38 | 1.539 | 58 | 2.178 | 78 | 1.955 | 98 | .123 |
| 19 | .699 | 39 | 1.580 | 59 | 2.194 | 79 | 1.903 | 99 | .045 |
| 20 | .744 | 40 | 1.621 | 50 | 2.209 | 80 | 1.846 | 100 | .000 |

Table 6: Pion Spectrum $h \rightarrow \pi^{+} + \pi^{-} + \pi^{0}$ , m = 6.20

E = 1 + X(m-3)(m+1)/200m , S(m,E) = 20 m P / (m-3)(m+1) .

| X | P | X | P | X | P | X | P | X | P |
|---|---|---|---|---|---|---|---|---|---|
| 1 | .052 | 21 | 1.346 | 41 | 2.381 | 61 | 3.337 | 81 | 3.262 |
| 2 | .129 | 22 | 1.399 | 42 | 2.433 | 62 | 3.374 | 82 | 3.175 |
| 3 | .211 | 23 | 1.451 | 43 | 2.484 | 63 | 3.409 | 83 | 3.074 |
| 4 | .293 | 24 | 1.503 | 44 | 2.536 | 64 | 3.441 | 84 | 2.960 |
| 5 | .373 | 25 | 1.554 | 45 | 2.587 | 65 | 3.471 | 85 | 2.830 |
| 6 | .449 | 26 | 1.606 | 46 | 2.638 | 66 | 3.498 | 86 | 2.686 |
| 7 | .523 | 27 | 1.657 | 47 | 2.689 | 67 | 3.522 | 87 | 2.527 |
| 8 | .593 | 28 | 1.709 | 48 | 2.740 | 68 | 3.543 | 88 | 2.352 |
| 9 | .660 | 29 | 1.760 | 49 | 2.790 | 69 | 3.560 | 89 | 2.164 |
| 10 | .725 | 30 | 1.812 | 50 | 2.840 | 70 | 3.572 | 90 | 1.962 |
| 11 | .788 | 31 | 1.863 | 51 | 2.890 | 71 | 3.580 | 91 | 1.748 |
| 12 | .849 | 32 | 1.915 | 52 | 2.939 | 72 | 3.582 | 92 | 1.526 |
| 13 | .908 | 33 | 1.966 | 53 | 2.987 | 73 | 3.578 | 93 | 1.297 |
| 14 | .966 | 34 | 2.018 | 54 | 3.034 | 74 | 3.569 | 94 | 1.066 |
| 15 | 1.023 | 35 | 2.070 | 55 | 3.081 | 75 | 3.552 | 95 | .837 |
| 16 | 1.078 | 36 | 2.112 | 56 | 3.127 | 76 | 3.527 | 96 | .618 |
| 17 | 1.133 | 37 | 2.173 | 57 | 3.172 | 77 | 3.494 | 97 | .412 |
| 18 | 1.187 | 38 | 2.225 | 58 | 3.215 | 78 | 3.452 | 98 | .230 |
| 19 | 1.241 | 39 | 2.277 | 59 | 3.257 | 79 | 3.400 | 99 | .083 |
| 20 | 1.294 | 40 | 2.329 | 60 | 3.298 | 80 | 3.337 | 100 | .000 |

## IV. Decays into Pairs of Charged Particles

As is evident from Eq. II.2, the coupling of the photon to the $\rho$ and to the $h$ in the sense of dispersion theory must be $em_\rho^2/2\gamma_\rho$ and $em_h^2/2\gamma_h$, respectively. An estimate of the rates for the decay of the hypercharge meson into pairs of charged particles may be obtained by assuming the dominance of the single-photon intermediate state in the imaginary part of the $e\bar{e}$, $\bar{\mu}\mu$, and $\pi^-\pi^+$ vertices. For the matrix elements involving lepton pairs, such an approximation leads to the following expressions

$$T = i\,(e^2/2\gamma_h)\; e_\nu^h\; \bar{u}\,\gamma_\nu\, v \quad , \tag{IV.1}$$

where $e_\nu^h$ is the polarization four-vector of the hypercharge meson, u is a Dirac spinor for the lepton, and v is a Dirac spinor for the anti-lepton. A straightforward calculation of the decay rate from this matrix element results in

$$\Gamma(h \to e + \bar{e}) = \alpha^2\, m_h\, (1 - 4m_e^2/m_h^2)^{1/2}\, (1 + 2m_e^2/m_h^2)/\, 12(\gamma_h^2/4\pi) \;; \tag{IV.2}$$

the corresponding expression for $\Gamma(h \to \mu + \bar{\mu})$ is obtained by replacing $m_e$ by $m_\mu$.

Such an approximation would not be very good for the two-pion mode, because there we obviously have an additional physical effect to consider. The $\pi^-\pi^+$ pair will be in a state with the quantum numbers $J^\pi = 1^-$. In this configuration there is a resonance in pion-pion scattering due to the $\rho$-meson, and so final state interactions will be important in the decay $h \to \pi^- + \pi^+$. The effect of these may be included by putting in the pion electromagnetic form factor, $F_{\pi\pi\gamma}(q^2)$, which, near the two pion resonance, takes the form

$$F_{\pi\pi\gamma}(q^2) = m_\rho^2 \, (\gamma_{\pi\pi\rho}/\gamma_\rho) \, (q^2 + m_\rho^2 - i m_\rho \Gamma_\rho) \, . \tag{IV.3}$$

In the decay matrix element, the form factor is to be evaluated at $q^2 = -m_h^2$, and thus if $m_h$ is close to $m_\rho$, a very strong enhancement of the decay rate will result. This effect has recently been discussed qualitatively by Fubini,(9) Glashow,(10) and Nambu and Sakurai,(11) but we are able to give a semi-quantitative estimate of the magnitude of the branching ratio $\Gamma(h \rightarrow \pi^- + \pi^+) \, / \, \Gamma(h \rightarrow \pi^- + \pi^+ + \pi^0)$ because our calculations give a prediction for the latter rate. The decay rate of the $h$-meson into a pair of charged pions is given, in our approximation, by

$$\Gamma(h \rightarrow \pi^- + \pi^+) = \frac{\alpha^2 \; m_h \; (1 - 4/m_h^2)^{3/2} \; m_\rho^4 \; (\gamma_{\pi\pi\rho}/\gamma_\rho)^2}{(\gamma_h^2/4\pi) \; 48 \; [(m_\rho^2 - m_h^2)^2 + m_\rho^2 \Gamma_\rho^2]} \, . \tag{IV.4}$$

## V. Conclusions

On the basis of Eqs. II.15, III.13, IV.2, and IV.4, the partial widths for the decay of a hypercharge meson are determined in terms of the three parameters characterizing the strengths of vector meson couplings; $(\gamma_\rho^2/4\pi)$, $(\gamma^2_{\pi\pi\rho}/4\pi)$, and $(\gamma_h^2/4\pi)$. One of these is known experimentally; using 100 Mev for the width of the two pion resonance, one finds that

$$\gamma^2_{\pi\pi\rho}/4\pi = 1/2 .$$

Next we note that the ratio $\gamma_\rho/\gamma_{\pi\pi\rho}$ is the zero momentum form factor of the $\rho$ meson,[8] and can be related to the strength of the $\rho$ meson "pole" term in the pion electromagnetic form factor. This can be measured in experiments with colliding beams of electrons and positrons, which are now under consideration at many laboratories. But for the present, we shall have to be content with the observation that this ratio is likely to be quite close to unity, since we know that the analogous zero momentum nucleon $-\rho$ form factor departs from unity by not more than about 40%.[8,12] The only parameter that is really unknown is, thus, the hypercharge coupling strength $\gamma_h^2/4\pi$. This may be determined from any one of the partial widths, and then be used to predict the branching ratios for all the modes. For illustrative purposes, however, we shall display the decay rates for two reasonable values of the hypercharge coupling strength: $\gamma_h^2/4\pi = 1.5$, as would be suggested by unitary symmetry, and $\gamma_h^2/4\pi = 3$; these are given in Tables 7 and 8. The tables were constructed using $\Gamma_{\pi^0\to\gamma+\gamma} = 3$ ev, and $\gamma^2_{\pi\pi\rho}/4\pi = \gamma_\rho^2/4\pi = 1/2$.

Several points appear to be worth noting concerning these results.

(1) The charged particle-pair decays of the $\omega$ and the $\eta$ should be

Table 7: Typical decay rates of a hypercharge meson of mass 550 Mev

| | $(\gamma_h^2/4\pi) = 1.5$ | $(\gamma_h^2/4\pi) = 3$ |
|---|---|---|
| $h \rightarrow e + \bar{e}$ | 1.6 kev | 0.81 kev |
| $h \rightarrow \mu + \bar{\mu}$ | 1.6 | 0.81 |
| $h \rightarrow \pi^- + \pi^+$ | 1.1 | 0.55 |
| $h \rightarrow \pi^0 + \gamma$ | 21.5 | 43.0 |
| $h \rightarrow \pi^- + \pi^+ + \pi^0$ | 2.0 | 4.0 |
| $\Gamma_h$ | 27.8 | 49.2 |

Table 8: Typical decay rates of a hypercharge meson of mass 787 Mev

| | $(\gamma_h^2/4\pi) = 1.5$ | $(\gamma_h^2/4\pi = 3$ |
|---|---|---|
| $h \to e + \bar{e}$ | 2.3 kev | 1.2 kev |
| $h \to \mu + \bar{\mu}$ | 2.3 | 1.2 |
| $h \to \pi^- + \pi^+$ | 17 | 8.5 |
| $h \to \pi^0 + \gamma$ | 69 | 138 |
| $h \to \pi^- + \pi^+ + \pi^0$ | 394 | 788 |
| $\Gamma_h$ | 485 | 937 |

searched for, since it is quite conceivable that up to 15% of the $\eta$'s or up to 5% of the $\omega$'s decay in this manner. (2) The ratio of neutral decays to charged decays is supposed to be less than 30% for the $\omega$, and in the range $3 \to 7$ for the $\eta$. If either the $\omega$ or the $\eta$ were to be identified with a hypercharge meson, this piece of data would be consistent with our prediction. But perhaps it would be more reasonable to wait for additional statistics at this stage of the game. (It may be of interest to point out here that if the $\eta$ turns out to be pseudoscalar, the $\chi$, one expects the branching ratio $\Gamma_{\chi \to \gamma+\gamma} / \Gamma_{\chi \to \pi^-+\pi^++\gamma} = 3.6$.) (3) Sakurai has recently claimed that the reported narrow width of the $\omega$, $\Gamma_\omega < 30$ Mev,[5] is quite a mystery,[13] and gives an explanation of this fact on the basis of R invariance. Coleman and Glashow, however, showed[14] that R invariance would lead to a vanishing magnetic moment for the neutron and no mass difference between $\Sigma^-$ and $\Sigma^+$. They considered these unacceptable results to be sufficient grounds for abandoning R invariance. Our calculations indicate that a very tiny width for the $\omega$ is to be expected with reasonable coupling strengths, and hence it does not seem that a selection rule based on some invariance principle is needed to explain the small width. (4) Electromagnetism causes a mixing of an $I = 1$ vector meson and a hypercharge vector meson with $I = 0$, so that the correct decaying states are given by[10] $\tilde{h} = (1-a^2)^{1/2} h + a\rho^o$ and $\tilde{\rho}^o = (1-a^2)^{1/2} \rho^o - a h$. It is easy to see that $a^2$ must be given by $\Gamma_{h \to \pi^++\pi^-} / \Gamma_\rho$, and thus we find that $a = .0117$ for $m_h = 787$ Mev, and $a = .0033$ for $m_h = 550$ Mev.

Some of the results of this article were reported in a letter to the Physical Review.[15] Other communications on these decay modes have also appeared. Unfortunately, there appear to be serious errors in the Feinberg calculations[16], so that most of his conclusions are not valid. Brown and Singer[17] have quoted estimates for the ratio $R = \Gamma(h \rightarrow \pi^{0} + \gamma) / \Gamma(h \rightarrow \pi^{-} + \pi^{+} + \pi^{0})$ which are very much smaller than our estimates; their subsequent conclusion that the $\eta$ must be pseudoscalar is unreliable from a theoretical standpoint.

## 3. DECAY OF THE $\chi$-MESON

Recent experimental evidence on the decay of the neutral meson at 550 Mev[6] into three pions suggests that for this object the most likely quantum numbers are (spin 0, parity -, G = +1), $0^{-+}$, although statistical limitations and uncertainties in subtracting background do not permit the assignment $J^{\pi G} = 1^{--}$ to be ruled out.[18] Since it is known that this neutral meson has isospin I = 0,[19] if it were pseudoscalar with G = +1, it would fit the description of the missing member, called $\chi$, of an octet of pseudoscalar mesons in the "Eightfold Way" of Gell-Mann.[2] The three pion decay modes of the $\chi$ are forbidden by the conservation of G, and occur only through the intermediary of virtual photons. Accordingly it is very difficult to estimate their rates; perhaps the only statement one may trust is that the ratio of the rates of the two decays into three pions, $(3\pi^0)/(\pi^+ + \pi^- + \pi^0)$, must be less than 3/2 since charge conjugation invariance requires the final three pion state to have I = 1. It is possible, however, to obtain a reasonable estimate of the branching ratio $\Gamma(\chi \rightarrow 2\gamma)/\Gamma(\chi \rightarrow \pi^+ + \pi^- + \gamma)$ by assuming that the $\pi^+ + \pi^- + \gamma$ matrix element is dominated by the $\rho$-meson intermediate state, in the sense of dispersion theory, and by using the eightfold way to estimate the ratio of the $\chi\gamma\gamma$ to the $\chi\rho\gamma$ vertex. A ratio[18] of neutral to charged decay modes of the 550 Mev meson near 3 implies that the two photon decay will be the dominant decay mode if the meson is pseudoscalar, since the only other two body neutral decays, $\chi \longrightarrow \pi^0 + \gamma$ and $\chi \longrightarrow 2\,\pi^0$, are strictly forbidden by angular momentum and parity selection rules, and decays into more particles are highly inhibited by phase space factors. Thus essentially we will be able to give an estimate of the fraction of $\chi$'s that decay into $\pi^+ + \pi^- + \gamma$, a number which may be compared with experiment.

First, let us set up the expression for the rate of the decay $\chi \to 2\gamma$. We define the constant $f_{\gamma\gamma\chi}$ such that the T-matrix element for this decay has the form

$$T = f_{\gamma\gamma\chi} \epsilon_{\nu\mu\sigma\tau} \, e'_{\nu} \, k'_{\mu} \, \epsilon''_{\sigma} k''_{\tau} \,, \tag{1}$$

where $e_{\nu}$ and $k_{\nu}$ are polarization and momentum four vectors of the gamma rays. This form is uniquely determined by the pseudoscalar nature of the $\chi$. In terms of this decay constant, the decay rate is found to be

$$\Gamma(\chi \to 2\gamma) = |f_{\gamma\gamma\chi}|^2 \; m_{\chi}^3/64\pi \quad . \tag{2}$$

The phase space integral for the mode $\chi \to \pi^+ + \pi^- + \gamma$ is considerably more complicated, and we shall outline the calculation in more detail. The matrix element is taken to be that resulting from keeping only a $\rho$-meson intermediate state, which is

$$T = 2\gamma_{\pi\pi\rho} \, \epsilon_{\mu\nu\alpha\tau} (p^+ - p^-)_{\mu} (p^+ + p^-)_{\nu} \, e_{\alpha}^{\gamma} \, k_{\tau}^{\gamma} \, f_{\rho\gamma\chi} \Big/ \left[ (p^+ + p^-)^2 + m_{\rho}^2 \right] , \tag{3}$$

where $\gamma_{\pi\pi\rho}$ is the $\rho\pi\pi$ coupling constant, and $f_{\rho\gamma\chi}$ is a number characterizing the $\rho\gamma\chi$ vertex. The geometric factors in the latter vertex are identical to those in the $\gamma\gamma\chi$ vertex. Putting in the density of states, we find for the decay rate

$$\Gamma = (4m_\chi/\pi)(\gamma^2_{\pi\pi\rho}/4\pi)(f^2_{\rho\gamma\chi}/4\pi) \iint dE^+ dE^- \frac{(E^{+2}-m_\pi^2)(E^{-2}-m_\pi^2)(1-\cos^2\theta)}{\left[(p^+ + p^-)^2 + m_\rho^2\right]^2} , \tag{4}$$

where $\cos\theta = \left[(m_\chi - E^- - E^+)^2 + 2m_\pi^2 - E^{+2} - E^{-2}\right] / \left[4(E^{+2}-m_\pi^2)(E^{-2}-m_\pi^2)\right]^{\frac{1}{2}}$ .

(5)

This integral may be simplified by changing variables to $y = E^+ - E^-$ and $x = 2m_\chi(m_\chi - E^+ - E^-)/(m_\chi^2 - 4m_\pi^2)$; the integral over y may be done analytically, but the other must be done numerically. The formula for the rate is

$$\Gamma(\chi \to \pi^+ + \pi^- + \gamma) = \frac{(\gamma^2_{\pi\pi\rho}/4\pi)(f^2_{\rho\gamma\chi}/4\pi)\,(m_\chi^2 - 4m_\pi^2)^5\, U}{24\,\pi\, m_\chi^3\, (m_\rho^2 - m_\chi^2)^2} \tag{6}$$

where

$$U = \int_0^1 x^3 (1-x)^{3/2} \left[m_\chi^2/(m_\chi^2 - 4m_\pi^2) - x\right]^{-\frac{1}{2}} \left[1 + x(m_\chi^2 - 4m_\pi^2)/(m_\rho^2 - m_\chi^2)\right]^{-2} dx$$

$$= .0143 = 1/70 . \tag{7}$$

In order to complete the calculation, we must be able to estimate $f_{\rho\gamma\chi}/f_{\gamma\gamma\chi}$. That we shall do by dispersing the vertices, assuming the dominance of the $\rho$-meson and $\omega$-meson intermediate states, and using the unitary symmetry(2) value for the ratio of the $\rho\rho\chi$ and the $\omega\omega\chi$ vertices. The calculation is most conveniently summarized by diagrams; see Fig. 3 and 4. Analytically, we have

$$f_{\gamma\gamma\chi} \approx (e/2\gamma_\rho)^2 f_{\rho\rho\chi} + \tfrac{1}{3}(e/2\gamma_\omega)^2 f_{\omega\omega\chi} , \tag{8}$$

and $f_{\rho\gamma\chi} \approx (e/2\gamma_\rho)\, f_{\rho\rho\chi}$ , (9)

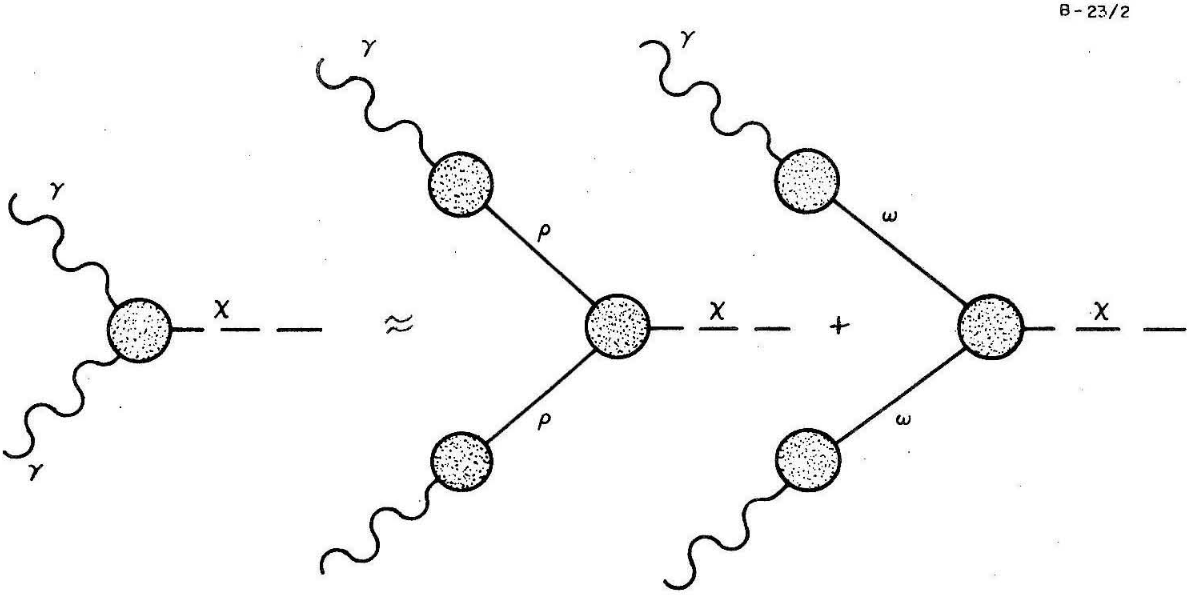



Figure 3. Dominant diagrams for the $\gamma\gamma\chi$ vertex

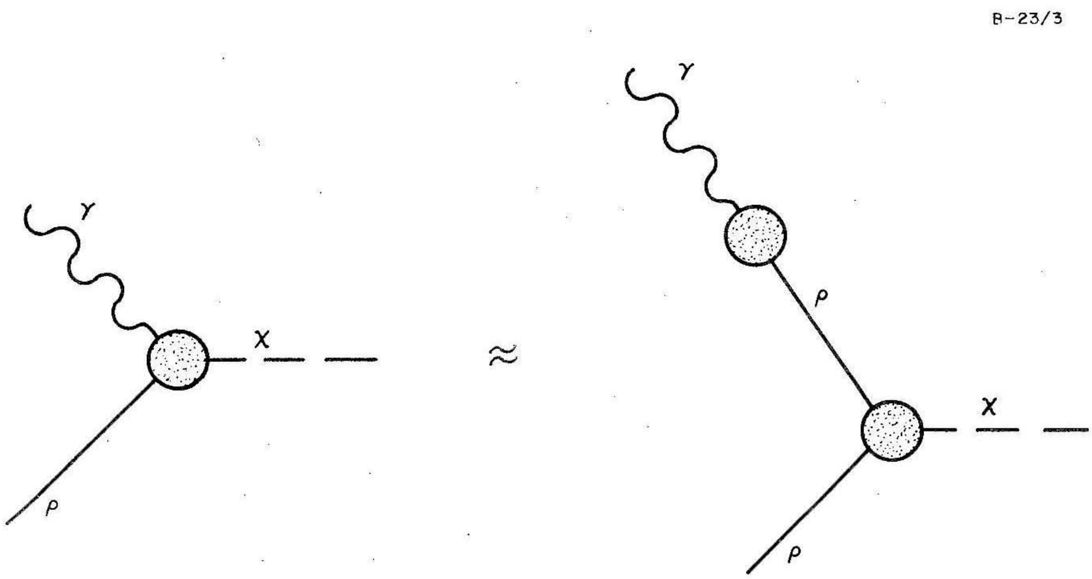



Figure 4. Dominant diagram in the $\rho\chi\gamma$ vertex

since the transformation of the $\rho$-meson and the $\omega$-meson into photons in the sense of dispersion theory have the amplitudes[8] $(em_\rho^2/2\gamma_\rho)$ and $(em_\omega^2/2\sqrt{3}\gamma_\omega)$, respectively. In the limit of unitary symmetry, $\gamma_\omega = \gamma_\rho$, and $f_{\rho\rho\chi} = -f_{\omega\omega\chi}$. Thus we obtain $3\gamma_\rho/e$ as an estimate for the desired ratio.

Our final result for the branching fraction comes out

$$\Gamma(\chi \rightarrow \pi^+ + \pi^- + \gamma)/\Gamma(\chi \rightarrow 2\gamma) = (9/8)\,(\gamma^2_{\rho\pi\pi}/4\pi)\,(\gamma_\rho^2/4\pi) \quad , \quad (10)$$

using unitary symmetry to estimate the relative importance of the $\rho$-meson and the $\omega$-meson intermediate states. $(\gamma^2_{\rho\pi\pi}/4\pi) = \frac{1}{2}$ for a $\rho$-meson decay width of 100 Mev, and $\gamma_\rho^2/4\pi$ should be about the same. The actual estimate of .28 for the branching ratio is not in violent disagreement with the preliminary data of Bastien, et al.[18]

A calculation such as the one described here may not necessarily be expected to be quantitatively valid. Its purpose is to provide, at worst, the correct order of magnitude of the quantity under consideration, and hopefully it should be considerably better. The important result of this calculation, therefore, is that because of an inhibiting phase space factor, the mode $\pi^+ + \pi^- + \gamma$ is expected to be somewhat rarer than the $2\gamma$ decay mode of a $\chi$, even though the rate of the latter involves the electromagnetic fine structure constant to one higher power.

# 4. THE M-MESON AND ITS EFFECT ON $\Lambda$-K ASSOCIATED PRODUCTION AND $\bar{K}$-N SCATTERING AT HIGH ENERGIES

## I. Introduction

A resonance has been found in the K-$\pi$ system which is most likely to have angular momentum $J = 1$. It is quite possible that this resonance belongs to the octet of vector mesons predicted by Gell-Mann and Ne'eman; adopting the notation of Gell-Mann, we shall call it the M-meson, or simply the M. There are two important general questions to be raised regarding this object: (1) How strongly is it coupled to the other particles? (2) Does the assumption that the M contribution dominates a given amplitude enable us to understand any important features of reactions in which it is exchanged?

Both questions will be considered in this paper. In Part II, the strength of the coupling of the M to the K-$\pi$ system will be related to the width of the resonance. The same coupling constant is involved in the production of the M in the reaction $K^- + p \rightarrow M^- + p$, which we shall also investigate. In Part III, the contribution of the M to the associated production amplitude will be studied. We shall treat the M according to the Regge pole hypothesis there, and shall discuss how experiments at beam energies within the range of existing accelerators can be used to decide whether the M behaves as predicted by the Regge pole hypothesis. The crossed hyperon production reaction, $\bar{K}^o + p \rightarrow \Lambda + \pi^+$, will be examined in the same spirit in Part IV. Finally, the existence of a new class of symmetries in asymptotic amplitudes, which are generalized Pomeranchuk relations, will be illustrated in Part V.

II. Properties of the M-Meson

There is at 884 Mev an object which appears as an $I = 1/2$ resonance[20] in the $K\pi$ system. Assuming it to be a vector particle, we define the coupling constant $\gamma_{MK\pi}$ so that the matrix element for the decay $M^+ \rightarrow K^+ + \pi^o$ is

$$T = \gamma_{MK\pi} \; e^M \cdot (p_\pi - p_K) \;, \tag{II:1}$$

where $e^M$ is the polarization four-vector of the M, and $p_\pi$, $p_K$ are the four-momenta of the decay products. The rate for the decay is

$$\Gamma(M^+ \longrightarrow K^+ + \pi^o) = \gamma^2_{MK\pi} \; k^3/6\pi m^2_M \;, \tag{II:2}$$

where

$$4 \; m^2_M \; k^2 = \left[ m^2_M - (m_K - m_\pi)^2 \right] \left[ m^2_M - (m_K + m_\pi)^2 \right]. \tag{II:3}$$

Since the M has $I = 1/2$, the charged M decays more often into a charged pion and neutral kaon; the branching ratio is two. Neglecting other decay modes, which certainly have much smaller widths, the decay rate for the M-meson is

$$\Gamma_M = (\gamma^2_{MK\pi}/4\pi) \;\; 58 \text{ Mev} \;\; . \tag{II:4}$$

The width of the M is quoted to be 30 Mev[21], so that

$$\gamma^2_{MK\pi}/4\pi = \;.51 \;\; . \tag{II:5}$$

According to the unitary symmetry scheme[2], this number should be comparable to the coupling of the $\rho$-meson to the two pion system, which is $\gamma^2_{\rho\pi\pi}/4\pi \; = \; .45$ if we assume 90 Mev for the $\rho$-width.[22]

The coupling constant $\gamma_{MK\pi}$ enters also in the pion pole approximation to the M production amplitude in the reaction $K + N \rightarrow M + N$. One finds

$$\gamma^2_{MK\pi}/4\pi = \lim_{t \rightarrow m^2_\pi} \left\{ 4\,(g^2_{\pi NN}/4\pi)^{-1} \frac{(p_K/p_M)\, s\, (t - m^2_\pi)^2\, m^2_M}{(-t)\left[(m^2_M + m^2_K - t)\, 4m^2_M\, m^2_K\right]} \frac{d\sigma}{d\Omega} \right\} , \tag{II:6}$$

where s is the square of the total energy in the center-of-mass system, and

$$4\, s\, p^2_M = \left[s - (m_N - m_M)^2\right]\left[s - (m_N + m_M)^2\right] , \tag{II:7}$$

$$4\, s\, p^2_K = \left[s - (m_N - m_K)^2\right]\left[s - (m_N + m_K)^2\right] , \tag{II:8}$$

$$2\sqrt{s}\, E_M = s + m^2_M - m^2_N , \tag{II:9}$$

$$2\sqrt{s}\, E_K = s + m^2_K - m^2_N , \tag{II:10}$$

$$t = (E_M - E_K)^2 - p^2_M - p^2_K + 2\, p_M p_K \cos\Theta . \tag{II:11}$$

Angular distributions for this reaction are not yet available, so that the coupling constant cannot be determined by this extrapolation procedure. However, Baqi-Beg and DeCelles[23] and Chan[24] have proposed that existing experimental data on the total production cross section be fitted in the pion pole approximation. M. Alston, _et al._[20] state that at $s = 3.48$ Gev$^2$ the total cross section for $M^-$ production is $1.4 \pm 0.3$ mb. If it is assumed that the pion pole dominates the amplitude, this leads to a value of .21 $\pm$ 20% for $\gamma^2_{MK\pi}/4\pi$, which is not in agreement with the value obtained from the M width. Theoretically, however, we have no reason to expect the pion pole to dominate the total cross section at such low energies, and one suspects strongly that any agreement would be fortuitous. That it indeed must be so has recently been demonstrated by a measurement of the total cross section for $M^o$ production by the Alston group[25]. Their value of 0.7 mb is 1/8 of what should be expected if the pion pole dominates. It is thus apparent that angular distributions at considerably higher energies are needed to test the correlation expected between M production and its decay width.

## III. Associated Production

In considering the amplitude for associated production by pions, we shall treat the reaction

$$\pi^- + p \rightarrow \Lambda + K^o .$$

All other amplitudes can be obtained from it, since when $\Lambda$'s are produced, the reaction is in a pure $I = 1/2$ state. The amplitude contains only two independent functions of the relativistic invariants, and can be written as

$$T = \bar{u}_\Lambda \left\{ A(s,t) - i\, B(s,t)\, (\not{q} + \not{r})/2 \right\} u_p , \qquad \text{(III:1)}$$

since the relative (K$\Lambda$N) parity is almost certainly negative. In our work we designate the four-momentum of the N, $\Lambda$, $\pi$, K by p, p', q, r, respectively, and we adhere to the convention that

$$\begin{aligned} s &= -(p+q)^2 , \\ t &= -(p'-p)^2 , \\ u &= -(r-p)^2 . \end{aligned} \qquad \text{(III:2)}$$

The subsidiary condition

$$s + t + u = m_N^2 + m_\Lambda^2 + m_\pi^2 + m_K^2 \qquad \text{(III:3)}$$

expresses the well-known fact that there are only two relativistically invariant variables in the problem.

Let us proceed by analyzing the t-exchange channel. In this channel only a system with unit hypercharge, zero baryonic charge, and $I = 1/2$ can be exchanged. By developing the $\pi + \bar{K} \longrightarrow \Lambda + \bar{N}$ amplitude in partial waves, as is done in Appendix A, it can be shown that the exchange of a state of spin J, which must necessarily have the parity $(-)^J$, gives the following expressions for the invariant functions A and B:

$$A(s,t) \rightarrow c_J^{(1)}(t) \quad s^J \quad , \tag{III.4}$$

$$B(s,t) \rightarrow J \, c_J^{(2)} \, (t) \, s^{J-1} \quad . \tag{III:5}$$

Only the leading term at high energies has been retained.

If the Regge pole hypothesis[26] is correct, then at high energies in the forward direction, i.e., $s \rightarrow \infty$, and $t$ small, the functions A and B will be dominated by a term associated with the exchange of a vector meson with one unit of hypercharge: the M-meson, which is presumably the $K$-$\pi$ resonance ($K^*$) at 884 Mev. The asymptotic form of these functions at high energies will be:

$$A(s,t) \underset{s\rightarrow\infty}{\rightarrow} \frac{1 - e^{-i\pi\alpha_M(t)}}{2 \sin \pi\alpha_M(t)} \left\{ \frac{2\,s}{(m_N+m_\Lambda)\,(m_\pi+m_K)} \right\}^{\alpha_M(t)} \times$$

$$(m_\pi+m_K)\; b^{(1)}_{\Lambda NMK\pi}\,(t) \quad , \tag{III:6}$$

$$B(s,t) \underset{s\rightarrow\infty}{\rightarrow} \frac{1 - e^{-i\pi\alpha_M(t)}}{2 \sin \pi\alpha_M(t)} \quad 2\,\alpha_M(t) \quad \times$$

$$\left\{ \frac{2\,s}{(m_N+m_\Lambda)\,(m_\pi+m_K)} \right\}^{\alpha_M(t) - 1} b^{(2)}_{\Lambda NMK\pi}\,(t) \; . \tag{III:7}$$

The signature of the Regge trajectory is negative, since the resonance has $J = 1$, and the two functions, $b(t)$, are independent. The fact that there are two is a reflection of the two possible spin states, $S = 0$ or $S = 1$, for the $\Lambda\overline{N}$ system with a given total angular momentum J.

On comparing these asymptotic expressions with those resulting from the exchange of the M-meson in the pole approximation, which are derived in Appendix B, one can identify various quantities at $t = m_M^2$. First of all, since the resonance occurs in p-wave K-$\pi$ scattering, we have $\mathrm{Re}\ \alpha_M(m_M^2) = 1$. The width of the resonance is proportional to $\mathrm{Im}\ \alpha_M(m_M^2) = I_M$, and inversely proportional to the slope of the Regge trajectory, $\epsilon_M = \mathrm{Re}\ (d\alpha/dt)_{t=m_M^2}$ :

$$\Gamma_M = I_M/m_M\epsilon_M \quad . \tag{III:8}$$

And finally, (leaving off some of the subscripts where their omission results in no ambiguity), we have:

$$b^{(1)}(m_M^2)/\pi\epsilon_M = -\sqrt{6}\ \gamma_{MK\pi}\ (m_\Lambda + m_N)\ \mu_{\Lambda NM} \quad , \tag{III:9}$$

and

$$b^{(2)}(m_M^2)/\pi\epsilon_M = \sqrt{6}\ \gamma_{MK\pi}\ \left[\gamma_{\Lambda NM} + \mu'_{\Lambda NM}(m_\Lambda - m_N) + \mu_{\Lambda NM}(m_\Lambda + m_N)\right] , \tag{III:10}$$

where $\gamma_{MK\pi}$ and $\gamma_{\Lambda NM}$ are the coupling constants of the M to the K$\pi$ and the $\Lambda$N currents, respectively, $\mu_{\Lambda NM}$ is the anomalous magnetic moment in the $\Lambda$NM vertex, and $\mu'_{\Lambda NM}$ is an "anomalous" anomalous moment term in that vertex. The last term is unfamiliar because in many reactions it can be eliminated on some symmetry consideration; in electrodynamics, the conservation of the current requires its absence.

To calculate the cross sections and polarizations, it is convenient to write T in a reduced form which is subsequently sandwiched between two component spinors. Defining in such a manner functions T' and T" such that $T \rightarrow T' + i\, T'' \; \vec{\sigma}.\vec{q}\times\vec{r}/qr$, one finds

$$\{(E_\Lambda+m_\Lambda)(E_N+m_N)\}^{1/2} \; T' = (E_\Lambda+m_\Lambda)(E_N+m_N)\left[A + B(E_\pi+E_K)/2\right]$$
$$+ B\left[(E_\Lambda+m_\Lambda)q^2 + (E_N+m_N)r^2\right]/2 \qquad \text{(III:11)}$$
$$+ q\, r \cos\theta\left[-A + \tfrac{1}{2}B(2\sqrt{s} + m_\Lambda+m_N)\right].$$

$$\left[(E_\Lambda+m_\Lambda)(E_N+m_N)\right]^{1/2} T'' = q\, r \sin\theta\left[A - \tfrac{1}{2}B(2\sqrt{s} + m_\Lambda + m_N)\right]. \qquad \text{(III:12)}$$

In these reduced expressions, E refers to the energy of the particle and q,r the magnitude of the three-momentum in the center-of-mass system. The $\Lambda$'s produced will be partially polarized in the $\vec{q} \times \vec{p}' = -\vec{q} \times \vec{r}$ direction; the degree of polarization is P, which is easily shown to be

$$P = \frac{2\, \mathrm{Im}\, T'^{*}\, T''\, \sin\theta}{|T'|^2 + |T''|^2 \sin^2\theta}. \qquad \text{(III:13)}$$

The cross section for associated production is

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\,\frac{r}{q}\left\{|T'|^2 + |T''|^2 \sin^2\theta\right\}, \qquad \text{(III:14)}$$

where $(r/q)^2 = \left[s - (m_\Lambda-m_K)^2\right]\left[s - (m_\Lambda+m_K)^2\right]/\left[s - (m_N-m_\pi)^2\right]\left[s - (m_N+m_\pi)^2\right]$.

This may be rewritten in terms of the functions A and B, in which case it becomes

$$\frac{d\sigma}{d\Omega} = \left\{16\pi\left[s - (m_N-m_\pi)^2\right]\left[s - (m_N+m_\pi)^2\right]\right\}^{-1} \tfrac{1}{2}\sum|T|^2,$$

where

$$\frac{1}{2}\sum|T|^2 = |A|^2\left[(m_N+m_\Lambda)^2 - t\right] + \text{Re } A^*B\left[(m_\Lambda+m_N)(2s+t-m_\Lambda^2-m_N^2)\right.$$

$$\left. - 2m_N m_K^2 - 2m_\Lambda m_\pi^2\right] + |B|^2\left[s^2 - s(m_\Lambda^2+m_N^2+m_K^2+m_\pi^2-t)\right. \qquad \text{(III:15)}$$

$$\left. - \frac{1}{4}t(m_\Lambda+m_N)^2 + \frac{1}{4}(m_\Lambda^2+m_N^2)^2 + m_K^2 m_\pi^2 + m_\Lambda m_N(m_K^2+m_\Lambda^2)\right] \quad .$$

In units of the mass of the charged pion,

$$16\,\pi\,(s - 32.7)\,(s - 59.7)\,\frac{d\sigma}{dt} =$$

$$|A|^2\,(217 - t) + \text{Re } A^*B\,(29.4\,s + 14.7\,t - 1797)$$

$$+ |B|^2\,(s^2 - 122.9\,s + st - 54.2\,t + 3737) \quad . \qquad \text{(III:16)}$$

At high energies the cross section in the backward $(\Lambda)$ direction will approach

$$\frac{d\sigma}{dt} \to \frac{1}{16\pi}\left|\frac{1 - e^{-i\pi\alpha_M(t)}}{2\sin\pi\alpha_M(t)}\right|^2 \left\{\frac{2\,s}{(m_\Lambda+m_N)(m_K+m_\pi)}\right\}^{2\alpha_M(t) - 2} \times$$

$$\left\{\left|b^{(1)}(t) + \alpha_M(t)\,b^{(2)}(t)\right|^2 - \frac{t}{(m_\Lambda+m_N)^2}\left|b^{(1)}(t)\right|^2\right\} \quad . \qquad \text{(III:17)}$$

We may recall that at large s in the center-of-mass system,

$$t = m_\Lambda^2 + m_N^2 - 2E_\Lambda E_N + 2p'p\cos\theta \to -\frac{1}{2}(s - m_\Lambda^2 - m_N^2 - m_K^2 - m_\pi^2)(1 - \cos\theta) \qquad \text{(III:18)}$$

or, in units of the $\pi^-$ mass,

$$t \to -\frac{1}{2}\,(s - 122.9)(1 - \cos\theta) \quad .$$

Data on this reaction at high energies are not yet available. The best one has at this time are those of Eisler, *et. al.*[27] at the pion lab momentum of 1.43 Gev/c, which corresponds to $s = 3.58 \text{ Gev}^2 = 184\, m_{\pi^-}^2$. This is certainly not a large enough energy to suggest that the Regge pole on the M trajectory must dominate the associated production amplitude; at ten times this energy, which is now possible with the CERN and Brookhaven machines, we would expect the dominance of this Regge pole in the forward ($K^o$) direction. The $\Lambda$ is backward peaked even at these low energies; however, the degree of peaking appears to be too small to fit the prediction of a dominant Regge pole. This latter statement is made assuming that the M-meson behaves as a composite particle with $\epsilon_M$ of the order 1 $(\text{Gev})^{-2}$. If the M contributes in the fashion of an elementary particle,[7] the trajectory degenerates to a point, and the amplitude will not drop off exponentially in the momentum transfer.

It is very important that the angular distribution at small angles and high energies be measured in order to determine the character of the M-pole. The formulae we have derived will be useful in analyzing such experiments.

## IV. Hyperon Production in $\bar{K}N$ Scattering

As a specific case of the I = 1 reaction $\bar{K} + N \rightarrow \Lambda + \pi$, let us consider $\bar{K}^0 + p \rightarrow \Lambda + \pi^-$, which corresponds to the u-channel of the associated production reaction studied in the preceding section. If q and r again denote the pion and kaon four momentum, respectively, then the amplitude for this $\bar{K}N$ inelastic scattering process is given by:

$$T = \bar{u}_\Lambda \left\{ A(s,t) + \frac{1}{2} i\, B(s,t)\, (\not{q} + \not{r}) \right\} u_p \,, \qquad \text{(IV:1)}$$

where

$$\begin{aligned} s &= -(p-q)^2 = u_u \\ t &= -(p'-p)^2 = t_u \\ u &= -(p+r)^2 = s_u \,, \end{aligned} \qquad \text{(IV:2)}$$

and the functions A and B are analytic continuations of those in the preceding section.

According to the Regge hypothesis, at high energies in the forward direction, ($s_u \longrightarrow \infty$, $u_u \longrightarrow -\infty$, $t_u$ small), the functions A and B are dominated again by the pole associated with the M-meson. In fact, all our results on the asymptotic form of the functions, and cross sections for the reaction $\pi + N \longrightarrow \Lambda + K$ apply also to this inelastic $\bar{K}N$ scattering process. In particular, as $u = s_u \rightarrow \infty$, for small t, the following asymptotic relations will be valid:

$$A(s,t) \underset{u\rightarrow\infty}{\rightarrow} \frac{1 - e^{-i\pi\alpha_M(t)}}{2 \sin \pi\alpha_M(t)} \left\{ \frac{2u}{(m_\Lambda + m_N)(m_K + m_\pi)} \right\}^{\alpha_M(t)} (m_K + m_\pi)\, b^{(1)\prime}_{\Lambda N M K \pi}(t) \,,$$

$$\text{(IV:3)}$$

$$B(s,t) \underset{u\to\infty}{\rightarrow} \frac{1 - e^{i\pi\alpha_M(t)}}{2 \sin \pi\alpha_M(t)} \left\{\frac{2u}{(m_\Lambda+m_N)(m_K+m_\pi)}\right\}^{\alpha_M(t)-1} \times$$

$$2\,\alpha_M(t)\; b^{(2)'}_{\Lambda NMK\pi}(t) \quad . \tag{IV:4}$$

But, we can go further than this. The functions b and b' are characteristic of the cross channel, the t channel, which is the same for both the associated production and the $\bar{K}N$ reactions. Therefore, the functions b and b' are essentially one and the same, provided only that we put in the angular functions in a consistent fashion. This latter requirement is easily fulfilled just by continuing to write $x_t = \cos\theta_t$ as $(s + 2E_N E_\pi - m_N^2 - m_\pi^2)/(2q_t p_t)$. On going from the s channel to the u channel, therefore, in the asymptotic region the only change is that of the sign of $x_t$. But such an interchange gives back the same amplitude except for the factor $(^-)^\sigma$, where $\sigma$ is the signature of the Regge pole. Accordingly, we see that

$$b^{(1)'}_{\Lambda NMK\pi}(t) = - b^{(1)}_{\Lambda NMK\pi}(t) \quad , \tag{IV:5}$$

$$b^{(2)'}_{\Lambda NMK\pi}(t) = + b^{(2)}_{\Lambda NMK\pi}(t) \quad . \tag{IV:6}$$

(The sign change coming from $(\not{q} + \not{p})$ on going from the s to the u channel is responsible for the apparent asymmetry between Eq. IV:5 and Eq. IV:6.) At a given center-of-mass energy in the asymptotic region and at a given small momentum transfer, the amplitudes for $\pi + N \rightarrow \Lambda + K$ and $\bar{K} + N \rightarrow \Lambda + \pi$ are related by a minus sign, and thus the differential cross sections, polarizations, etc., will be the same for the two processes.

The changes in the cross section formulae are very slight and may be obtained by the interchanges: $m_K \leftrightarrow m_\pi$ , $s \leftrightarrow s_u = u$ . For example, from Eq. III:15 and Eq. III:16 we get the differential cross section for $\overline{K}^0 + p \longrightarrow \Lambda + \pi^+$:

$$\frac{d\sigma}{dt} = \frac{|A|^2 (217 - t) - \mathrm{Re}\, A^*B\,(29.4\, s_u + 14.7\, t - 1827) + |B|^2 \left[s_u^2 - s_u(122.9 - t) - 54.2\, t + 3737\right]}{16\,\pi \left[s - 10.0\right]\left[s - 106.0\right]} , \qquad \text{(IV:7)}$$

in units of $m_{\pi^-}^2$.

Data on this reaction at high energies are not yet available. The CERN and Brookhaven machines do produce meson beams with energies in the 10-15 Gev regions; however, experiments with these beams so far have not been designed to measure the two body inelastic processes. It is essential that such experiments be undertaken because of the greater simplicity in the analysis of these reactions.

## V. Generalization of the Pomeranchuk Relations

In the course of this work we have found a new set of relationships between asymptotic cross sections(28), which may be regarded as generalizations of the Pomeranchuk relations. Our basic result, that Regge pole dominance implies that the two asymptotic amplitudes in the s and u channels are equal to each other for small values of t (except possibly for a sign), is quite general for the case of scalar particles. In our problem we saw that going from one channel to the other in the asymptotic region amounts to changing the sign of cos $\theta_t$. This change of sign results in the factor $\sigma$, which is the orbital parity, or "signature", of the Regge pole in the t channel.

In the diagrammatic representation of amplitudes two channels of a scattering process are related to each other by the reversal of two external lines. If the lines to be reversed involve scalar bosons, the effect is unambiguous and simple. We are dealing with a three point vertex representing the coupling of two spinless particles to an intermediate boson of spin J, which we take to be integral for the purpose of formulating the rule. Such an intermediate boson may be represented by a tensor field of rank J, which is symmetric and divergenceless in all indices, and traceless in any pair. The vertex must also be a completely symmetric tensor of rank J constructed from the four-momenta $r_\mu$ and $r'_\mu$ of the two bosons. It is more convenient, however, to consider the linear combinations, $\Sigma_\mu = r_\mu + r'_\mu$ and $q_\mu = r_\mu - r'_\mu$, which have simple transformation properties under line reversal when the energies are high enough that any mass differences may be neglected, namely: $\Sigma_\mu \longrightarrow - \Sigma_\mu$ ,

$q_\mu \longrightarrow + q_\mu$ . We also note that the tensor must be constructed solely from $\Sigma_\mu$; the other vector $q_\mu$ is ineffective because it gives zero for the residue of the pole when it is dotted into the propagator of rank 2J representing the intermediate state of spin J. Since the tensor is thus the direct product of J $\Sigma_\mu$'s, under line reversal we get the factor $(-)^J$, which is the signature of the intermediate state.

In the case of KN-scattering, our result is in agreement with that of Ferrari et al.[29], and in contradiction to that of Lee[30].

For the reversal of baryon lines we must consider as well the transformation of the Dirac matrices. This transformation is the same as for particle-antiparticle conjugation, under which;

$$1 \rightarrow 1,\ \gamma_5 \rightarrow \gamma_5,\ \gamma_\mu \gamma_5 \rightarrow \gamma_\mu \gamma_5,\ \gamma_\mu \rightarrow -\gamma_\mu,\ \sigma_{\mu\nu} \rightarrow -\sigma_{\mu\nu} \ .$$

A $\sigma_{\mu\nu}$ term will always appear here in the combination $\sigma_{\mu\nu} q_\nu$ which is a vector and is odd under line reversal. It is apparent that our signature rule does not hold in complete generality because of the peculiar transformation properties of $\gamma_\mu \gamma_5$. In vertex tensors of rank J involving the pseudovector Dirac matrices, the operation of line reversal results in the factor $-(-)^J$, the negative of the signature. Our simple signature rule remains valid, however, whenever (signature)(parity) = +1. This is the case for the exchange of the K*, the $\rho$, and the $\omega$.

In some cases, the existence of particular symmetries among the baryons provides an alternate rule. These symmetries obviously must be such as to prohibit the mixing of the two pseudovector forms $\Sigma_\mu \gamma_5$ and $\gamma_\mu \gamma_5$ . Charge conjugation C and more generally the isoparity operation G yield the desired selection rules when the object being exchanged has a definite value of C and/or G. Reversal of baryon lines in the same isotopic multiplet introduce the factor $(-)^I G$, which is C for the neutral objects.

We must stress that the result of line reversal is a function of the type of couplings. Only when the particles being reversed belong to the same isotopic multiplet does the factor reduce to $(-)^I G$ or C. It is easy to imagine possible couplings where this last rule would fail. As an example, one may consider the coupling of $\pi$ and $X$ to a fictitious particle with quantum numbers $J^P$, $I^G = 1^-, 1^-$ .

It is interesting to note that our above result when applied to the relation between that part of the interaction between N and $\bar{N}$ due to the exchange of pions and the corresponding part in the NN interaction yields a conclusion differing from that usually quoted(31).

Apparently, no symmetry exists if the exchanged object has half-integral spin.

We may close this article by listing some quadruplets of asymptotic amplitudes which should be equal, except for the "signature" factor, on the basis of the Regge pole hypothesis.

| POLE | ASYMPTOTIC AMPLITUDES | |
|---|---|---|
| M | $T(\pi + N \rightarrow \Lambda + K)$ | (1a) |
| | $- T(\bar{K} + N \rightarrow \Lambda + \pi)$ | (1b) |
| | $- T(\pi + \bar{\Lambda} \rightarrow \bar{N} + K)$ | (1c) |
| | $T(\bar{K} + \bar{\Lambda} \rightarrow \bar{N} + \pi)$ | (1d) |
| M | $T(\pi + N \rightarrow \Sigma + K)$ | (2a) |
| | $- T(\bar{K} + N \rightarrow \Sigma + \pi)$ | (2b) |
| | $- T(\pi + \bar{\Sigma} \rightarrow \bar{N} + K)$ | (2c) |
| | $T(\bar{K} + \bar{\Sigma} \rightarrow \bar{N} + \pi)$ | (2d) |

| POLE | ASYMPTOTIC AMPLITUDES | |
|---|---|---|
| M | $T(\bar{N} + N \to \Lambda + \bar{\Lambda})$ | (3a) |
| | $- T(\Lambda + N \to \Lambda + N)$ | (3b) |
| | $- T(\bar{N} + \bar{\Lambda} \to \bar{N} + \bar{\Lambda})$ | (3c) |
| | $T(\Lambda + \bar{\Lambda} \to \bar{N} + N)$ | (3d) |
| M | $T(\bar{N} + N \to \Sigma + \bar{\Sigma})$ | (4a) |
| | $- T(\Sigma + N \to \Sigma + N)$ | (4b) |
| | $- T(\bar{N} + \bar{\Sigma} \to \bar{N} + \bar{\Sigma})$ | (4c) |
| | $T(\Sigma + \bar{\Sigma} \to \bar{N} + N)$ | (4d) |
| $\rho$ | $T(\pi^{0} + n \to \pi^{-} + p)$ | (5a) |
| | $- T(\pi^{+} + n \to \pi^{0} + p)$ | (5b) |
| | $- T(\pi^{0} + \bar{p} \to \pi^{-} + \bar{n})$ | (5c) |
| | $T(\pi^{+} + \bar{p} \to \pi^{0} + \bar{n})$ | (5d) |
| $\rho$ | $T(p + n \to p + n)$ | (6a) |
| | $- T(\bar{n} + n \to p + \bar{p})$ | (6b) |
| | $- T(p + \bar{p} \to \bar{n} + n)$ | (6c) |
| | $T(\bar{n} + \bar{p} \to \bar{n} + \bar{p})$ | (6d) |

We note that our result about the asymptotic equality of (5a) and (5b) is actually quite weak, since by charge independence we know that the two amplitudes are negatives of each other at any energy and angle. Similarly for (5c) and (5d). Also G conjugation is sufficient to guarantee strict equality between (5a) and (5c ), (5b) and (5d), (6a) and (6d), and (6b) and (6c). We may also remark that the Pomeranchuk relations hold when the amplitudes are dominated by the Pomeranchon pole[32, 26], for which C = + 1.

## 5. THE DECAYS OF A K INTO A PION PLUS LEPTONS

Bernstein, Fubini, Gell-Mann, and Thirring[33] and, independently, Kuang-Chao[34] have presented a reasonable explanation of the remarkable formula[35] relating the axial vector coupling constant in nuclear beta-decay to the decay rate of the charged pion. The Goldberger-Treiman relation may be understood to be a consequence of the dominance, at low $q^2$, of the pion pole in the dispersion relation for the matrix elements of the divergence of the axial current in beta-decay. The divergence is assumed to be a highly non-singular operator; that is, its matrix elements obey unsubtracted dispersion relations or, equivalently, vanish at infinite $q^2$.

The success of such a hypothesis invites its further use in attempts to correlate the phenomena of the weak interactions. Extensions to other processes, such as $\Sigma \rightarrow \Lambda + \ell + \nu$ and $\Lambda \rightarrow p + \ell + \bar{\nu}$, have been proposed[33,36], but unfortunately most of them are dependent on the many parameters which are quite difficult to determine. However, the pion plus leptons decay modes of the kaon do offer the possibility of making definite predictions based on a straightforward extension of the hypothesis stated above. It is possible to specify the branching ratio of muons to electrons and the pion spectrum in these decays. This fact was noted by Bernstein and Weinberg in their discussion[37] of the possible existence of a scalar resonance in the K-$\pi$ system.

A resonance has been found[20] in this system, the $K^*$ at 884 Mev, so that the idea of treating the K-$\pi$ intermediate states in the particle approximation now has considerable merit. However, this particle is most likely to be vector, in which case it fits the description of Gell-Mann's M-meson.[2] It would thus seem to be of interest to carry out the calculation of the features of the decay of the K into $\pi + \ell + \nu$ for the case of the vector $K^*$, hereafter called the M. Such an investigation was undertaken, and it was found that the hypothesis is incompatible with experiment. Specifically, the result that the form factor in the electron decay vanishes at the maximum pion energy is almost certainly ruled out by the data of Brown et al.[38] on the $K^+_{e3}$ decay interaction and by the data of Luers et al.[39] on $K_2^o$ decays. The other result that the branching ratio of muons to electrons should be 3:5 also disagrees with experiment; Roe et al.[40] give the value $1.0 \pm 0.2$ for the ratio in $K^+$ decays, and Luers et al.[39] quote $0.79 \pm 0.19$ in the case of the $K_2^o$.

Very recently, Ely et al.[41] have reported the breakdown of the $\Delta S = \Delta Q$ rule and, consequently, the $\Delta I = 1/2$ rule in the pion plus electron decay of the neutral kaon. The absence of a $\Delta I = 1/2$ rule, of course, has the very important consequence that it renders intolerable any analysis based on the dominance of the M pole for both the $K^+$ and the $K_2^o$. The decays of both particles must be treated separately, and terms other than those related to the M must be prominent in the form factors.

In view of these facts, we are forced to abandon any hope of using a generalized Goldberger-Treiman relation in this problem. We have analyzed these decays in terms of much more general formulae for the form factors, which contain three parameters. It is our hope in so doing that the parameters we define will serve as convenient links between future experimental and theoretical studies on these decays.

Since the K is almost certainly pseudoscalar, only the vector current of the weak interactions contributes to the non-leptonic half of the matrix element which, written in invariant form, involves two form factors. To fix the notation, let us write the T-matrix element as

$$T = (G/\sqrt{2}) \left\langle \pi \mid V_\alpha \mid K \right\rangle \bar{u}_\ell \gamma_\alpha (1 + \gamma_5) u_\nu \tag{1}$$

with

$$(G/\sqrt{2}) \left\langle \pi \mid V_\alpha \mid K \right\rangle = (-i/2) \left[ F_1(q^2) (P^\pi_\alpha + P^K_\alpha) + F_3(q^2) (P^\pi_\alpha - P^K_\alpha) \right] \tag{2}$$

where $P^K$ and $P^\pi$ are the four-momenta of the K and $\pi$, and $q = P^\pi - P^K$. In the rest frame of the kaon, $q^2 = -m_K^2 - m_\pi^2 + 2m_K E_\pi$. The decay rate may be written conveniently as an integral over the pion energy spectrum;

$$\Gamma(K \rightarrow \pi + \ell + \nu) = (16 \pi^3 m_K^2)^{-1} \int_{-(m_K - m_\pi)^2}^{-m_\ell^2} p_\pi(q^2) J(q^2)\, dq^2 \ , \tag{3}$$

where $p_\pi$ is the magnitude of the pion momentum in the rest frame of the kaon,

$$2 m_K p_\pi(q^2) = \left\{ (m_K^2 - m_\pi^2 - q^2)^2 + 4 m_K^2 q^2 \right\}^{1/2} \ , \tag{4}$$

and

$$24 (1 + m_\ell^2 / q^2)^{-2} J(t) =$$

$$F_1^2 \left\{ 2 \left[ (m_K^2 - m_\pi^2)^2 + q^2 (2m_K^2 + 2m_\pi^2 + q^2) - m_\ell^2 (2m_K^2 + 2m_\pi^2 + q^2) \right.\right.$$

$$\left.\left. + 4(m_K^2 - m_\pi^2)^2 / q^2 \right] \right\} - 6 F_1 F_3 m_\ell^2 (m_K^2 - m_\pi^2) - 3 m_\ell^2 q^2 F_3^2 \ . \tag{5}$$

We note that in electron decays, because of the presence of the factor $m_e^2$, only the form factor denoted by $F_1$ is of interest. (There is very little likelihood that the form factor $F_1$ would be so small, relative to $F_3$, that the latter could be significant in the electron decays.)

We suggest that the data on the spectrum and the rates be analyzed in terms of the following representations for the form factors:

$$F_1(q^2) = A + C\, m_M^2/(q^2+m_M^2) \quad , \tag{6}$$

$$F_3(q^2) = B + D\, m_M^2/(q^2+m_M^2) \quad . \tag{7}$$

Using the tables of Brene, et al.[42] we have carried out the calculation of the twelve numerical integrals occurring in the formulae for the decay rates. In terms of the mass of the charged pion,

$$\Gamma(K \rightarrow \pi + e + \nu) = (48\, \pi^3\, m_K)^{-1}\ (\, 285\ A^2 + 595\ AC + 311\ C^2\, ), \tag{8}$$

$$\begin{aligned}\Gamma(K \rightarrow \pi + \mu + \nu) = (48\, \pi^3\, m_K)^{-1}\ (\, & 179.5\ A^2 + 390\ AC + 210\ C^2 \\ & - 40.9\ CD - 37.3\ AD - 37.3\ BC - 34.0\ AB \\ & + 6.21\ D^2 + 11.1\ BD + 5.00\ B^2\, )\quad .\end{aligned} \tag{9}$$

Because of the low values of $q^2$ which occur in this decay ($q^2$ ranges from -6.6 to 0), the form factors are quite well represented by the constant plus linear term in the power series expansion of Eqs. 6 and 7:

$$F_1(q^2) \approx A + C - C\, q^2/m_M^2 \quad , \tag{10}$$

$$F_3(q^2) \approx B + D - D\, q^2/m_M^2 \quad . \tag{11}$$

In the vector pole approximation the coefficients C and D are not independent. If the resonance behavior is not to appear in the s-wave K-$\pi$ system,

$$D = C\,(m_K^2 - m_\pi^2)/m_M^2 = 0.291\ C \quad . \tag{12}$$

The parameter C is proportional to the coupling constant of the M to the K-$\pi$ current and to the constant in the weak coupling vertex of the M and the leptons. The first constant is related simply to the width of the M,

$$\Gamma_M = (\gamma_{MK\pi}^2/16\pi m_M^5)\left\{\left[m_M^2 - (m_K - m_\pi)^2\right]\left[m_M^2 - (m_K + m_\pi)^2\right]\right\}^{3/2} \quad , \tag{13}$$

but we can see no clear way to determine the second constant. For reference, though, we record the fact that

$$C = 2\ f\ \gamma_{MK\pi} \tag{14}$$

where $+i\ J_{-\alpha} M_\alpha^- f$ is the effective weak Lagrangian for the vertex coupling the M with the leptons.

The s-wave piece of the K$\pi$ matrix element is proportional to the divergence, $D(q^2)$, of the matrix element.

$$D(q^2) = (m_K^2 - m_\pi^2)\,F_1(q^2) + q^2\,F_3(q^2) \quad . \tag{15}$$

In our representation, restricted by Eq. 12, the divergence is

$$D(q^2) = (m_K^2 - m_\pi^2)\,(A + C) + q^2\,B \quad . \tag{16}$$

Now, since it is likely that there is not a resonance in the s-wave K-$\pi$ system, one might be tempted to assume that the divergence is almost a constant. If one sets B = 0, the ratio of the muon rate to the electron rate and the spectra in these decays are determined in terms of one constant, $x = C/A$. The ratio of the rates is

$$\Gamma(K \rightarrow \pi + \mu + \nu)/\Gamma(K \rightarrow \pi + e + \nu) = (180 + 379\,x + 198\,x^2)/(285 + 595\,x + 311\,x^2) \tag{17}$$

This ratio is $0.63 \pm 0.03$ for all reasonable values of x. (Near $x = -1$ the right side of Eq. 17 is indeterminate since it approaches 0/0. But there is no question that the ratio of the rates remains approximately 3:5.) Such a ratio is incompatible with the data[40] on the $K^+$, but falls within the error bars in the case[39] of the $K_2^o$.

Theoretically, however, we do not have a good reason for the vanishing of B, and so we propose that the experimental data be analyzed so as to provide the values of A, B, and C. Future theoretical studies should be directed towards expressing these parameters in terms of measurable quantities in other strong and weak interaction processes.

Let us conclude by showing how the existing experimental data may be analyzed to provide estimates of A, B, and C. If the constant terms dominate the form factors, the ratio of the rates and the spectra are determined by the parameter

$$\xi - F_3/F_1 = -(B + .291\,C)/(A + C) \tag{18}$$

This case was considered by Brene *et al.*[42] who found that

$$\Gamma(K \rightarrow \pi + \mu + \nu)\ /\Gamma(K \rightarrow \pi + e + \nu) = 0.65 + .124\,\xi + .0190\,\xi^2\ . \tag{19}$$

For the $K^+$, where this ratio is $1.0 + 0.2$, the corresponding value of $\xi$ is either $-8.7 \mp 1.0$ or $+2.1 \pm 1.0$. Very recent experimental results[43] on the muon spectrum show that

$$\xi = -8.7 \pm 1.0 \quad . \tag{20}$$

The determination of the pion spectrum in the decays giving electrons provides a convenient way to measure the ratio $C/A$. Early results[38] show that

$$-2 < C/(A+C) < 10 \tag{21}$$

with 95% confidence. For the neutral kaon, $\xi$ is either $-7.5 \mp 1.4$ or $+1.0 \pm 1.4$, and the data of Luers *et al.*[39] indicate that

$$0 < C/(A+C) < 8 \tag{22}$$

(It may be of interest, with respect to this last piece of data, to point out that the effect of an intermediate boson in the theory of the weak interactions is indistinguishable from the effect of the form factors.) Finally, we recall that when the ratios are accurately fixed, the magnitude of the parameters A, B, and C may be found from one of the decay rates.

## 6. FOUR-PION DECAYS OF THE $\rho$-MESON

The $\rho$-meson appears strikingly as a peak in the mass spectrum of two-pion systems which are created in the annihilation of antiprotons. The average multiplicity of pions in these annihilations is about five, so that these events may be analyzed to determine whether there are peaks in the mass spectra of three and four-pion configurations. Because its G-parity is +, the $\rho$-meson cannot decay strongly into three pions, but the four-pion decays are allowed by the strong interactions. Accordingly, one expects a peak in the four-pion mass spectrum located at about 750 Mev. It is the purpose of this article to present an estimate of the branching ratio $\Gamma(\rho \to 4\pi)/\Gamma(\rho \to 2\pi)$ so that some idea of the prominence of the four-pion peak may be obtained.

Four models have been considered in order to estimate the relevant matrix element. They may be described simply by stating the composition of the intermediate states included in each model. They are: (1) a neutral vector meson of mass 550 Mev; (2) a neutral vector meson with mass 787 Mev; (3) two $\rho$-mesons; (4) a single pion.

If the neutral object at 550 Mev has the quantum numbers of our hyper-charge meson, i.e. $J^P \; I^G = 1^- \; 0^-$, the decay of the $\rho$-meson into four pions will be dominated by the two step process.

$$\rho \to \pi + h \to 4\pi \qquad (1)$$

The decay rate $\Gamma(\rho \to 4\pi)$ will be quite rapid due to the fact that it is governed by a two-body phase space factor. Explicitly,

$$\Gamma(\rho \rightarrow 4\pi) = \Gamma(\rho \rightarrow h + \pi)\ \Gamma(h \rightarrow 3\pi)/\Gamma_h \quad , \tag{2}$$

where the branching fraction $\Gamma(h \rightarrow 3\pi)/\Gamma_h$ would be expected to be close to 1/4 from the experimental data(19). The matrix element for the decay $\rho \rightarrow h + \pi$ is

$$T = f_{h\rho\pi}\ F_{h\rho\pi}(m_h^2,\ m_\rho^2,\ m_\pi^2)\ \epsilon_{\mu\nu\sigma\tau} e_\mu^h k_\nu^h e_\sigma^\rho k_\tau^\rho \ , \tag{3}$$

where the symbols are the same as those employed in Part II of Chapter 2. We shall again approximate the form factor for the vertex by unity, and make use of Eqs. II.6 and II.8 of Chapter 2 in order to relate the parameter $f_{h\rho\pi}$ to the parameter $f_{\gamma\gamma\pi}$ which enters into the formula for the life-time of the neutral pion:

$$f_{h\rho\pi} = (2\gamma_h \gamma_\rho / e^2)\ f_{\gamma\gamma\pi} \quad ; \tag{4}$$

$$\Gamma(\pi^0 \rightarrow 2\gamma) = f_{\gamma\gamma\pi}^2\ m_\pi^3 \ / \ 64\pi \ . \tag{5}$$

The rate for the strong decay is

$$\Gamma(\rho \rightarrow h + \pi) = f_{h\rho\pi}^2\ k^3 \ / \ 12\pi \quad , \tag{6}$$

where

$$4\ m_\rho^2\ k^2 = \left[m_\rho^2 - (m_h - m_\pi)^3\right]\left[m^2 - (m_h + m_\pi)^2\right] . \tag{7}$$

Using 3 ev for the $\pi^0$ decay rate and $3\gamma_\rho^2/4\pi = \gamma_h^2/4\pi = 3/2$, we obtain

$$\Gamma(\rho \rightarrow h + \pi) = 0.8 \text{ Mev.} \tag{8}$$

On the basis of this model, the branching fraction $\Gamma(\rho \rightarrow 4\pi)/\Gamma(\rho \rightarrow 2\pi)$ would be expected to be of the order of 0.2%, since the width of the $\rho$ is about 100 Mev.

If the 550 object does not have quantum numbers which allow the strong decay of a $\rho$ into it plus a pion, then one must consider more complicated models. One that immediately springs to mind is very similar to the first, except that in this case the hypercharge meson is to be identified with the vector meson at 787 Mev, the $\omega$. There is no resonance term in the physical decay region, so that we must compute the four-body phase space factor. This factor is quite small and the matrix element in the model with the $\omega$ intermediate state is not large. The fraction of $\rho$'s that decay into four pions is less than $10^{-7}$, which is so small that the details of this calculation do not appear to be of great interest.

Since the $\rho$-meson carries isotopic spin, there must be a trilinear coupling of the $\rho$'s if the $\rho$ is to be identified with a meson that couples to the isospin current. The magnitude of that coupling can be estimated from the $\rho$-width, and an estimate for the decay of the $\rho$ into four pions may be obtained by keeping only the intermediate state of two $\rho$-mesons. The spinology is somewhat complicated, and the four-body phase space integral necessitates the numerical evaluation of a five-dimensional integral. An exact computation, therefore, would not be justified unless we felt that the contribution of this intermediate state dominated the matrix element. It is thus important to be able to make estimates of the order of magnitude of the decay rates.

The decay rate may be written as

$$\Gamma = 2\pi \, S \left< |T|^2 \right> P \quad , \tag{9}$$

where S is a factor depending on the statistics of the particles, $\left<|T|^2\right>$ is an appropriate average of the square of the matrix element, and P is the relativistic phase space factor. P may be evaluated simply in the non-relativistic limit by a technique which we shall illustrate for our case. Using non-relativistic kinematics, we write

$$P = (2\pi)^{-9} \, 2m_\pi \, I \, / \, 32 \, m_\rho m_\pi^4 \quad , \tag{10}$$

where

$$I = \int d^3k_1 \, d^3k_2 \, d^3k_4 \, d^3k_5 \, \delta^3(\textstyle\sum k) \, \delta(\textstyle\sum k^2 - Q) \ , \tag{11}$$

and

$$Q = 2 \, m_\pi \, (m_\rho - 4m_\pi). \tag{12}$$

Dimensionally, $I = A \, Q^{7/2}$, and by using the relation

$$\delta^3 \, (\textstyle\sum k) = (2\pi)^{-3} \displaystyle\int e^{i \sum kx} \, d^3x \tag{13}$$

we find

$$\int I \, e^{-Q} \, dQ = (7/2)! \ \ A = (2\pi)^{-3} \int 4\pi x^2 \, dx \left\{ \int e^{ikx/\sqrt{3}} \, e^{-k^2} \, dk \right\}^{12} \tag{14}$$

Thus we see that

$$A = 2 \, \pi^4 / \, 105 \quad . \tag{15}$$

For the decays of a $\rho^0$ into $\pi^+ + \pi^- + 2\,\pi^0$, the statistical factor $S = 1/2$, and

$$\Gamma(\rho^{\circ} \to \pi^+ + \pi^- + 2\pi^{\circ}) = \frac{(2m_{\pi})^{\frac{1}{2}} (m_{\rho} - 4m_{\pi})^{7/2} \langle |T|^2 \rangle}{512 \cdot 105 \; \pi^4 \; m_{\rho}} \quad . \tag{16}$$

Using the trilinear $\rho$-coupling with the constant $\gamma_{\rho}$,[2] we obtain the following expression for the matrix element.

$$T = 32 \, \gamma^2_{\pi\pi\rho} \gamma_{\rho} \, B \left[ (k^{\circ\prime} + k^-)^2 + m^2_{\rho} \right]^{-1} \left[ (k^{\circ\prime\prime} + k^+)^2 + m^2_{\rho} \right]^{-1} \tag{17}$$

$$+ \, k^{\circ\prime} \longleftrightarrow k^{\circ\prime\prime} \, ,$$

where

$$B = k^{\rho} \cdot (k^{\circ\prime} - k^-) \, e^{\rho} \cdot (k^{\circ\prime\prime} - k^+) - k^{\rho} \cdot (k^{\circ\prime\prime} - k^+) \, e^{\rho} \cdot (k^{\circ\prime} - k^-) \tag{18}$$

$$+ \, e^{\rho} \cdot (k^{\circ\prime} + k^-) \, (k^{\circ\prime\prime} - k^+) \cdot (k^{\circ\prime} - k^-) \quad .$$

In the above formulae, $e^{\rho}$ is the polarization four-vector of the $\rho^{\circ}$, and $k^{\rho}$, $k^+$, $k^-$, $k^{\circ\prime}$, $k^{\circ\prime\prime}$ are the four-momenta of the particles. We estimate $\langle |T|^2 \rangle$ to be less than $64/m^2_{\pi}$, and thus

$$\Gamma(\rho^{\circ} \to \pi^+ + \pi^- + 2\pi^{\circ}) < 2 \text{ kev}, \; \Gamma(\rho \to 4\pi)/\Gamma(\rho \to 2\pi) < 2 \cdot 10^{-5}, \tag{19}$$

according to this model.

The branching ratios are very small in the last two models because the matrix element depends on a high number of pion momenta, i.e. because most of the pions are in p-states. We have begun to investigate a model in which most of the pions are in s-states. It is clear now that this model will provide the largest estimate of the branching ratio because of the very large enhancement of pion-pion scattering near threshold[44]. A preliminary estimate using the results of Schnitzer[45] for the s-wave pion-pion scattering length gives a branching ratio $\Gamma(\rho \to 4\pi)/\Gamma(\rho \to 2\pi)$

of not more than 1%. We feel that this estimate is so large that a detailed calculation is desirable. Such a calculation is in progress.

## APPENDIX A

Partial Wave Decomposition for $\pi + \bar{K} \longrightarrow \Lambda + N$

In the center-of-mass system, choose coordinates such that

$$
\begin{aligned}
\bar{p} &= -p = (0, 0, -p_t, iE_N) \, , \\
p' &= (0, 0, p_t, iE_\Lambda) \, , \\
q &= (-q_t \sin\theta_t, \, 0, \, -q_t \cos\theta_t, \, iE_\pi) \, , \\
\bar{r} &= -r = (q_t \sin\theta_t, \, 0, \, q_t \cos\theta_t, \, iE_K) \, .
\end{aligned}
\tag{A:1}
$$

In terms of relativistic invariants, the momenta and energies are given by

$$4t \, p_t^2 = \left[t - (m_\Lambda - m_N)^2\right] \left[t - (m_\Lambda m_N)^2\right] \, , \tag{A:2}$$

$$4t \, q_t^2 = \left[t - (m_K - m_\pi)^2\right] \left[t - (m_K + m_\pi)^2\right] \, , \tag{A:3}$$

$$2\sqrt{t} \; E_N = t + m_N^2 - m_\Lambda^2 \, , \tag{A:4}$$

$$2\sqrt{t} \; E_\Lambda = t + m_\Lambda^2 - m_N^2 \, , \tag{A:5}$$

$$2\sqrt{t} \; E_K = t + m_K^2 - m_\pi^2 \, , \tag{A:6}$$

$$2\sqrt{t} \; E_\pi = t + m_\pi^2 - m_K^2 \, , \tag{A:7}$$

and

$$x_t = \cos\theta_t = (s + 2E_N E_\pi - m_N^2 - m_\pi^2)/(2q_t p_t) \, . \tag{A:8}$$

The first step is to evaluate the helicity amplitudes in terms of the functions A and B appearing in Eq. III:1 . The computation is straightforward, and so only the results will be given here. If the helicity states are denoted by $(\lambda\bar{\lambda})$, the amplitudes are

$$T(++) = T(--) = \left[(E_\Lambda+m_\Lambda)(E_N+M_N)\right]^{-\frac{1}{2}} \Big\{-A\ p_t\ (E_\Lambda+E_N+m_\Lambda+m_N) \tag{A:9}$$

$$+ B\ q_t\ (E_N+m_N)(E_\Lambda+m_\Lambda-E_N+m_N)\cos\theta_t + \tfrac{1}{2}B\ p_t(E_K-E_\pi)(E_\Lambda+m_\Lambda-E_N-m_N)\Big\},$$

$$T(+-) = -T(-+) = -\ B\ q_t\left[(E_N+m_N)/(E_\Lambda+m_\Lambda)\right]^{\frac{1}{2}}(E_\Lambda+E_N+m_\Lambda-m_N)\sin\theta_t\ . \tag{A:10}$$

Secondly, for a partial wave with angular momentum J, and parity $(-)^J$, we must determine the form of the helicity amplitudes. The following helicity combinations are eigenstates of S, and $S_z$ with our sign conventions:

| $(S,S_z)$ | $(\Lambda,\bar{N})$ | $(\lambda\bar{\lambda})$ |
|---|---|---|
| $(0,0)$ | $2^{-\frac{1}{2}}\left[(\uparrow\downarrow)-(\downarrow\uparrow)\right]$ | $2^{-\frac{1}{2}}\left[(++)+(--)\right]$ |
| $(1,1)$ | $(\uparrow\uparrow)$ | $-(+-)$ |
| $(1,0)$ | $2^{-\frac{1}{2}}\left[(\uparrow\downarrow)+(\downarrow\uparrow)\right]$ | $2^{-\frac{1}{2}}\left[(++)-(--)\right]$ |
| $(1,-1)$ | $(\downarrow\downarrow)$ | $(-+)$ |

The projection of a partial wave amplitude onto a given helicity state is

$$\langle(\lambda\bar{\lambda})\mid J,0\rangle = \sum_{S_z}\sum_{S}\sum_{\ell}\sum_{m}\langle(\lambda\bar{\lambda})|S,S_z;\ell,m\rangle\langle S,S_z;\ell,m|J,0;\ell;S\rangle,$$

where

$$\langle(\lambda\bar{\lambda})|S,S_z;\ell,m\rangle = \langle(\lambda\bar{\lambda})|S,S_z\rangle\ Y_\ell^m(\theta,\phi)\quad,$$

and $\langle S,S_z;\ell,m|J,0;\ell;S\rangle$ are the Clebsch-Gordan vector coupling coefficients. In general, for a given J, there are four elements of the T-matrix, corresponding to: $S=0,\ \ell=J$; and $S=1,\ \ell=J,\ J+1, J-1$. (For J=0, of course, there are but three.) If we choose to label them in the following way

$$\mathcal{T}(J,0,t) = \left[(2J+1)/8\pi\right]^{\frac{1}{2}} < \ell=J,S=0\,||T(t)||\,J > \quad ,$$

$$\mathcal{T}(J,1,t) = \left[(2J+1)/8\pi J(J+1)\right]^{\frac{1}{2}} < \ell=J,S=1\,||T(t)||\,J > ,$$

$$\mathcal{T}(J,1+,t) = \left[(J+1)/8\pi\right]^{\frac{1}{2}} < \ell=J+1,S=1\,||T(t)||\,J > \quad ,$$

and $$\mathcal{T}(J,1-,t) = \left[J/8\pi\right]^{\frac{1}{2}} < \ell=J-1,S=1\,||T(t)||\,J > \quad ,$$

the helicity amplitudes can be written in the rather simple form:

$$T(++) = \mathcal{T}(J,0,t)\,P_J(x_t) - \mathcal{T}(J,1+,t)\,P_{J+1}(x_t) + \mathcal{T}(J,1-,t)\,P_{J-1}(x_t), \tag{A.11}$$

$$T(--) = \mathcal{T}(J,0,t)\,P_J(x_t) + \mathcal{T}(J,1+,t)\,P_{J+1}(x_t) - \mathcal{T}(J,1-,t)\,P_{J-1}(x_t)\ , \tag{A.12}$$

$$T(+-)/\sin\theta_t = \mathcal{T}(J,1,t)\,P'_J(x_t) - \mathcal{T}(J,1+,t)\,P'_{J+1}(x_t)/(J+1) - \mathcal{T}(J,1-,t)\,P'_{J-1}(x_t)/J \quad , \tag{A.13}$$

$$T(-+)/\sin\theta_t = -\,\mathcal{T}(J,1,t)\,P'_J(x_t) - \mathcal{T}(J,1+,t)\,P'_{J+1}(x_t)/(J+1) - \mathcal{T}(J,1-,t)\,P'_{J-1}(x_t)/J \quad . \tag{A.14}$$

Only two of these amplitudes, $\mathcal{T}(J,S,t)$, occur if parity is conserved. In our case, $\mathcal{T}(J,1\pm,t) = 0$, since the parity of the system is $(-)^J$.

From these formulae, we can read off the form of the functions A and B resulting from the exchange in the t-channel of a pure J state with parity $(-)^J$:

$$B(s,t) = -\,\mathcal{T}(J,1,t)\,P'_J\left[x_t(s,t)\right] \Big/ \left\{ q_t(E_\Lambda+E_N+m_\Lambda-m_N)\left[(E_N+m_N)/(E_\Lambda+m_\Lambda)\right]^{\frac{1}{2}} \right\} \tag{A.15}$$

$$A(s,t) = -\mathcal{T}(J,0,t)\ P_J(x_t)\ \left[(E_\Lambda+m_\Lambda)(E_N+m_N)\right]^{\frac{1}{2}} / \left\{p_t(E_\Lambda+E_N+m_\Lambda+m_N)\right\}$$

$$-\mathcal{T}(J,1,t)\ P_J'(x_t)\ \left[(E_\Lambda+m_\Lambda)/E_N+m_N)\right]^{\frac{1}{2}} (E_\Lambda+E_N+m_\Lambda+m_N)^{-1}$$

$$(E_\Lambda+E_N+m_\Lambda-m_N)^{-1} \left\{(E_K-E_\pi)(E_\Lambda+m_\Lambda-E_N-m_N)/q_t \right. \tag{A:16}$$

$$\left. - x_t\ (E_N+m_N)(E_\Lambda+m_\Lambda-E_N+m_N)/p_t\right\} ,$$

where we recall that as $s \to \infty$, $x_t \to s/2q_t p_t$. We may put these formulae into a convenient relativisitic form by defining

$$F_J^{(1)}(t) = -\mathcal{T}(J,0,t)\ \left[(E_\Lambda+m_\Lambda)(E_N+m_N)\right]^{\frac{1}{2}} / \left\{(2q_t p_t)^J p_t(E_\Lambda+E_N+m_\Lambda+m_N)\right\} , \tag{A:17}$$

and
$$F_J^{(2)}(t) = -\mathcal{T}(J,1,t)\ \left[(E_\Lambda+m_\Lambda)/(E_N+m_N)\right]^{\frac{1}{2}} / \left\{(2q_t p_t)^{J-1}\ q_t\ (E_\Lambda+E_N+m_\Lambda-m_N)\right\} \tag{A:18}$$

in terms of which we have our final result for the functions A and B resulting from a pure J state:

$$B(s,t) = F_J^{(2)}(t)\ \left\{(2p_t q_t)^{J-1}\ P_J'\left[(s+2E_N E_\pi-m_N^2-m_\pi^2)/2p_t q_t\right]\right\} , \tag{A:19}$$

$$A(s,t) = F_J^{(1)}(t)\ \left\{(2p_t q_t)^J\ P_J\left[(s+2E_N E_\pi-m_N^2-m_\pi^2)/2p_t q_t\right]\right. ,$$

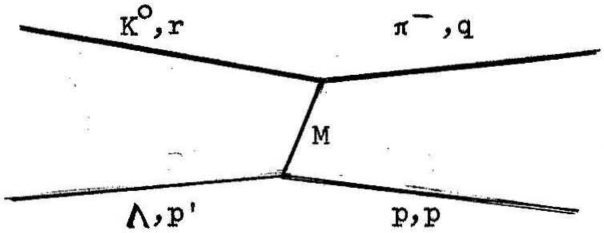
$$+ F_J^{(2)}(t)\ \left\{(2q_t p_t)^{J-1}\ P_J'\right\}\ \left\{(m_\Lambda+m_N)(s+2E_N E_\pi-m_N^2-m_\pi^2)/\left[t-(m_\Lambda+m_N)^2\right]\right.$$

$$\left. + (m_\Lambda-m_N)(m_K^2-m_\pi^2)/2t\right\} . \tag{A:20}$$

APPENDIX B

Contribution of the M Pole to the Amplitudes in $\pi^- + p \rightarrow \Lambda + K^o$

From this diagram




using the Feynman rules as in perturbation theory, we can compute the "pole" in the amplitude at $t = m_M^2$ due to the exchange of the M-meson. (I write "pole", since this pole lies off the physical sheet because of the instability of the M.) Near the pole, the amplitude for associated production is given by

$$(-i)\ (m_M^2 - t)^{-1}\ \sqrt{6}\ \gamma_{MK\pi}\ (q+r)_\alpha\ \left[\delta_{\alpha\beta} + (r-q)_\alpha (r-q)_\beta / m_M^2\right]\ \bar{u}_\Lambda\ X_\beta u_p \quad , \qquad (B:1)$$

where $$X_\beta = \gamma_{\Lambda NM}\ \gamma_\beta - \mu_{\Lambda NM}\ \sigma_{\beta\nu}(p'-p)_\nu - \mu'_{\Lambda NM}\ \sigma_{\beta\nu}(p'+p)_\nu . \qquad (B:2)$$

$\mu_{\Lambda NM}$ is the anomalous magnetic moment term in the coupling of the M to $\Lambda N$. $\mu'_{\Lambda NM}$ is an additional term which is seldom encountered since this second type of tensor coupling is ruled out in electrodynamics and in some other theories by a certain class of symmetries having to do with the existence of mirror diagrams.

By using various formulae for the spinor matrix elements, one can show that the pole contributions to the functions A and B are:

$$B(s,t) = \frac{-\sqrt{6}}{t - m_M^2}\ \gamma_{MK\pi}\ 2\left\{\gamma_{\Lambda NM} + \mu_{\Lambda NM}(m_\Lambda + m_N) + \mu'_{\Lambda NM}(m_\Lambda - m_N)\right\} \quad ; \qquad (B:3)$$

$$A(s,t) = \frac{-\sqrt{6}}{t-m_M^2}\, \gamma_{MK\pi} \left\{ (m_\Lambda - m_N)(m_K^2 - m_\pi^2)\left[\gamma_{\Lambda NM} + \mu'_{\Lambda NM}(m_\Lambda - m_N)\right]/m_M^2 \right. \tag{B:4}$$

$$\left. + \mu_{\Lambda NM}(m_\Lambda^2 + m_N^2 + m_K^2 + m_\pi^2 - 2s - t) - \mu'_{\Lambda NM}(t - m_M^2)(m_K^2 - m_\pi^2)/m_M^2 \right\} .$$

We note that $\mu'_{\Lambda NM}$ contributes a singular term to the amplitude only in the combination $\gamma_{\Lambda NM} + \mu'_{\Lambda NM}(m_\Lambda - m_N)$, and can be eliminated from consideration near the pole by redefining $\gamma_{\Lambda NM}$ to be

$$\gamma'_{\Lambda NM} = \gamma_{\Lambda NM} + \mu'_{\Lambda NM}(m_\Lambda - m_N) \quad . \tag{B:5}$$

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