Chapter 3

MIP Flow and Structure Behavior

3.1 Model Verification

Please see appendix 1.

3.2 Model Validation

We perform the same computation for different mesh and time step refinements keeping constant the product $C = t_s * N$ where t_s is the time step and N the total number of elements. We perform the mesh and time step independence tests for the model excited at one of the highest excitation frequency (f=11 Hz) so that the accuracy of the simulation for lower frequencies will be ensured. Our model has a total of 10,500 elements (6,000 fluid elements and 4,500 solid elements) and we use 1,000 time step per each excitation period (C=0.9545). We compare our model to 4 other cases (see table 2), and each computation runs until periodicity in the flow is achieved.

We define the instantaneous error in mesh refinement to be the mean error in axial velocity relative to the finest mesh at a specific point (y_o, z_o) (14).

$$Error_{(y_{o},z_{o})} = mean_{t \in [t_{o},t_{o}+T]} \frac{|v_{z}^{test}(t,y_{o},z_{o}) - v_{z}^{finest}(t,y_{o},z_{o})|}{|v_{z}^{finest}(t,y_{o},z_{o})|}$$
(14)

The time average of the error calculated for the point belonging to the axis of symmetry at the exit of the pump decreases with mesh refinement and our model has an average relative difference with the finest mesh possible $Error_{(0,L)}$ of about 3%.

 Table 2. Mesh and time steps refinements test cases and associated error with respect to the finest mesh.

Case #	Number of elements	Time steps per period	Mean Error
1	13,650	1,300	0
2	12,600	1,200	0.0069
3	10,500	1,000	0.0315
4	8,400	800	0.0565
5	5,200	500	0.0583

3.3 Identification of the natural frequencies of the

system

A free vibration test is performed. The model is impulsively actuated and the pincher is held to resting position until every motion in the fluid and solid domains disappears. The triangular impulse duration is 1.66 e-2 s, corresponding to 200 time steps of 8.3 e-5 s each. The time step length t_s =8.3 e-5 s corresponds to the smallest time step duration used throughout the computations (*f*=12 Hz).

Because the model is fairly complex, the spectral analysis of the impulse response is carried on for different parameters extracted from the flow (pressure, axial velocity) and the structure (radial displacement), and at different points throughout the model. Each of these observables {point, parameter} has an associated Power Spectrum Density (PSD) that exhibits several frequencies.



Figure 8. (Top) Impulse response: exit flow rate variation in time under triangular impulse excitation. (Bottom) The associated Power Spectrum Density (PSD). The Fourier transform was calculated using 4,096 points, and a time resolution of 4 e-4 s.

However, throughout the model the different observables' PSD contains the same frequencies, but expressed at different strengths (amplitude of the PSD). We chose to present the exit flow rate variation in time and its associated PSD (figure 8) because in addition to exhibiting all the natural frequencies present in the system, it is a relevant observable for the system *pump*. We identify $f_n=11$ Hz as a natural frequency and f=22 Hz, f=33 Hz, f=44 Hz, f=55 Hz as harmonics. Additional natural frequencies are f=41 Hz, f=49 Hz and f=59 Hz. $f_d=33$ Hz is the dominant frequency of the spectrum. We choose to study the system around the natural frequency $f_n=11$ Hz because the dominant frequency is its harmonic.

3.4 Pulse velocity

The pressure wave speed was calculated using a single pressure step at one extremity of the tube of magnitude. The pressure is modeled as a normal traction force of magnitude 1e+4 dyn/cm². The time step resolution for the computation is t_s =9.0909 e-5 s. The pressure wave speed is estimated to 172.7 cm.s⁻¹, based on the time needed for the pressure step to propagate along the model (*L*=15.2 cm) at rest. This velocity is closed to the value $c_0 = 155.6$ cm.s⁻¹ found using the Moens-Korteweg formula⁴⁹ (15) (derived for inviscid flow in a thin walled elastic tube that possesses some material compressibility):

$$c_0 = \sqrt{\frac{Eh}{2\rho a \left(1 - \nu^2\right)}},\tag{15}$$

where *E* is the stiffness of the gelatin layer (E_{gel} =5e+4 dyn/cm²), *h* the thickness of the gelatin layer (h_{gel} =0.405 cm), ρ the density of the gelatin layer (ρ_f =1 g/cm³) and *a* the fluid domain radius (R_f =0.55 cm) and ν the Poisson's ratio of the gelatin (v_{gel} =0.3).

3.5 Flow rate variation in time

Instantaneous flow rate $Q(t, z_o)$ at a cross section located at $z = z_o$ of the tube is, by convention, positive when flow is exiting the pump (flow in Z direction), and is expressed as:

$$Q(t,z_o) = 2\pi \int_{0}^{R_f(t)} v_z(y(t,z_o),z_o)y(t,z_o)dy(t,z_o), \qquad (16)$$

where v_z is the axial velocity, y is the radial position, R_f is the fluid domain radius and z_o the longitudinal position of the considered cross-section and t the time. For each excitation frequency, we compute the cross sectional flow at the pump extremity distant to the actuator Q(t, L). Exit flow history plots show a transient phase where the flow is building up before reaching a steady state of periodic oscillations and constant mean value (figure 9).

3.6 Mean exit flow rate and frequency

For the various frequencies of excitation, the mean exit flow rate (\overline{Q}) is calculated by averaging at steady state conditions, the instantaneous exit flow rate Q(t,L) over one excitation period.



Figure 9. Typical exit flow rate history plot. Excitation frequency is f=11.5 Hz. Periodicity is achieved after 15 pinching cycles and mean flow at steady state is 45.7 cc/s. The solid line is a filtered curve of the flow rate using a moving average window of one cycle.

The mean exit flow rate is nonlinearly dependent on frequency as expected for an IP. In addition, it exhibits a zone of negative flow for frequencies below 9 Hz (figure 10). Maximum positive flow reaches 86.87 cc/s when the pump is excited at 10.1 Hz. Therefore, for the system *pump* f_{res} =10.1 Hz will be referred it as the resonant frequency of the system. Flow resonance has been also observed in single layer impedance pumps.^{5,29,42}



Figure 10. Mean exit flow rate (\overline{Q}) as a function of the excitation frequency (*f*).

3.7 Reynolds number and Womersley number

The Reynolds number in a steady flow is defined as the

$$R_e = \frac{\overline{u} \, d}{\nu},\tag{17}$$

where \overline{u} is a characteristic velocity, d is a characteristic length, and v the kinematic viscosity of the fluid. For the fluid-filled elastic tube problem \overline{u} is defined as the mean axial velocity \overline{v}_z , d as the fluid domain radius R_f , and v as the ratio of the dynamic viscosity of water over the density of water $\frac{\mu_f}{\rho_f}$. The Reynolds number can be expressed

as a function of the mean exit flow:

$$R_e = \frac{4\overline{Q}}{\pi d\nu}.$$
(18)

For the different frequencies of excitation, the mean exit flow ranges from -78cc/s to +86cc/s leading to mean Reynolds number up to $R_e^{mean} = 9,959$. The instantaneous Reynolds number based on the maximum axial velocity for each frequencies of excitation ranges between 2,000 and 20,000.

The Womersley number (α) is defined as the ratio of the inertial forces to the viscous forces for pulsatile flows and can be seen as the equivalent of the Reynolds number but for pulsatile flow. It is expressed as:⁵⁰

$$\alpha = R_f \sqrt{\frac{\omega}{\nu}}, \qquad \qquad \omega = 2\pi f, \quad (19)$$

where R_f is the fluid domain radius, ω is a characteristic frequency in radians per second of the oscillatory motion, and ν the kinematic viscosity of the fluid. For the fluid-filled elastic tube problem ω is expressed in terms of the frequency of excitation of the system f, and ν is the ratio of the dynamic viscosity of water over the density of water $\frac{\mu_f}{\rho_f}$. For the frequency range 7.2Hz to 12.2Hz studied, the Womersley number spans 36.9 to 48.1.

3.8 Wall motion

Each layer of the tube has a distinct thickness and distinct material properties which influences the speed, damping and amplitude of the traveling elastic waves. The thickness and softness of the gelatin layer are used to amplify wave motion, while the stiffness of the external layer is used to limit outward radial motion. The concept of multilayer pumping relies on large amplitude wave motion at the fluid-gelatin interface combined with a very limited motion of the external surface of the pump. The maximum wall deflection in the stiffer layer outer surface is found to range from 0.37% to 6.10% from resting position, while the gelatin inner surface deflects from 24% to 32% from resting position, depending on the frequency of excitation. At resonance f_{res} =10.1 Hz, gelatin stretch is particularly important (figure 11) and plays a role in the pumping performance.



Figure 11. Gelatin maximum positive radial strain in time and space as a function of the frequency of excitation (*f*).

3.9 Wave interaction in a multilayer impedance pump

Upon compression elastic waves are created in both layers of the tube. They travel along the length of the tube and reflect at the tube extremities. The constructive wave mechanism occurring in the tube's walls of a MIP (mainly in the gelatin layer) is similar to the one described by Avrahami and Gharib⁵ for a SLIP. When the pump is excited at resonance a strong wave interaction occurs toward the pump extremity distant from the pincher. This interaction creates a suction zone where fluid fills quickly the newly created cavity. As this cavity travel downstream toward the tube's extremity a strong pressure gradient is created between the cavity and the extremity of the tube. A net exiting flow is created by inertia. More specifically, the wave mechanism over a period of time T is as follows (figure 12 and appendix 2).

The cycle begins with the elastic tube at resting state (t=0*T). Upon compression (t=0.16*T), two primary positive elastic waves are created on each side of the pincher. The positive elastic wave close to the short side of the tube reflects into a negative wave (t=0.26*T). At t=0.36*T the pinching action is over and the result is the creation of a pressure gradient associated with a reflected negative elastic wave (z=4.8 cm) and a primary positive elastic wave (z=7 cm) traveling toward the exit of the tube. While traveling, the positive wave (t=0.45*T, z=10 cm) steepens due to the influence of the nearby forward negative wave (t=0.45*T, z=7 cm). Small amplitude secondary waves (t=0.45*T, z=[0,3] cm) that have being created from the release action of the pincher and have reflected on the short side of the pincher, are now traveling toward the exit of the tube. The forward positive wave reaches the tube's extremity (t=0.57*T), and reflects in a negative wave traveling now toward the pincher (t=0.67*T). At that instant, a strong wave interaction occurs between this reflected wave and the still-forward-traveling negative wave creating a large suction zone (t=0.67*T). Fluid fills quickly the newly created opening, and a strong pressure gradient is present between the cavity and the extremity of the tube as the cavity travels further downstream. Fluid is washed out by inertia and exits the pump (t=0.74*T). The suction zone reflects at the tube extremity

(*t*=0.8**T*), squeezing the fluid out of the pump (*t*=0.9**T*). Motion in the tube damps and a new cycle is about to begin (*t*=1**T*).











Figure 12. Illustration of the propagating waves in the multilayer impedance pump.
Example for *f*=10.1 Hz. Selected frames at time *t* as a fraction of the period time *T*. (Top)
Outline of the model. Walls position against longitudinal axis. (Middle) Corresponding snapshots of the axial velocity fluid field. (Bottom) Axial pressure longitudinal

distribution.

3.10 Velocity profiles

For the MIP excited at 10.1Hz, we select a specific time (t=8.77129 s) and plot the velocity profiles for discrete cross sections along the tube.



Figure 13 (a)





Figure 13 (b)



Figure 13 (c)

Figure 13. Velocity profiles. (a) Instantaneous axial velocity field and velocity profiles at 11 cross sections along the tube. (b) Enlarged view of the different velocity profiles. (c) Velocity profile at the exit of the pump (#11) for selected times over a period of time.

3.11 Wall position, axial velocity and axial pressure

longitudinal distribution

For each frequency in the positive flow domain, the wall displacement, axial pressure longitudinal distribution and axial velocity longitudinal distribution is plotted over a period of time once periodicity in the flow is achieved.









Figure 14. Wall displacement, axial pressure longitudinal distribution and axial velocity longitudinal distribution over a period of time once periodicity in the flow is achieved.

3.12 Mechanical work done by the elastic tube

An energy balance on the fluid domain inside the long portion of the elastic tube past the pincher allows us to compute the mechanical work of the elastic tube done on the fluid. Because the energy balance is made for the portion of the tube devoid of active compression (i.e. pincher), this calculation aims to focus on the energetic role of the elastic tube itself in pumping.

We use a fixed control volume (CV) delimited by the "input" and "output" cross sections, the axis of symmetry and the fluid-structure interface. The "input" cross section is located downstream next to the pinching zone at z=4.56 cm, and the "output" upstream, just before the exit of the pump at z=13.68 cm, away from the exit enough to avoid the

results to be biased by the "too-close" compression zone and the zero pressure boundary condition, respectively (figure 15).

In the absence of added heat, the conservation of energy principle applied to the system (fluid inside the control volume) states that the time rate of change of the system total energy (E) is balanced by the time rate of change of the work done to the system (W).⁶⁰

$$\frac{DE}{Dt} = \frac{DW}{Dt}$$
(20)



Figure 15. (Top) Input and Output cross sections defining the portion of the tube for which mechanical work is calculated. (Bottom) Control volume and fluid energy balance.A fixed control volume (solid line box) enclosing the wall (dashed line) is used in order to consider the wall motion as a shaft work. Pumping work defined as shaft work minus

the losses is balanced by the differential of energy between the output (E_{out}) and input

$$(E_{in})$$
 of the system.

In the absence of gravitational forces, the work done to the system, i.e. the work done to the fluid domain, is decomposed into the work done by the environment on the fluid (involving fluid pressure and viscous terms) and the mechanical power done on the fluid and due to wall motion or shaft work (W_{mech}) .

$$W = W_{mech} + W_{press} + W_{viscous}$$
(21)

On the other hand, the fluid's total energy per unit mass (e) is decomposed to its internal and kinetic energies since gravitational forces are omitted.

$$e = e_{\text{internal}} + e_{kinetic} \tag{22}$$

Internal energy depends on temperature only, and is part of the fluid losses by internal friction (\dot{E}_{loss}). Using Reynolds transport theorem, the material derivative of the fluid's total energy (*E*) becomes:

$$\frac{DE}{Dt} = \frac{d}{dt} \iiint \rho e \, dV + \iint \rho e \, \mathbf{v} \cdot \mathbf{n} dS$$

$$= \frac{d}{dt} \iiint \rho \left(e_{\text{int ernal}} + \frac{\mathbf{v}^2}{2} \right) dV + \iint \rho \left(e_{\text{int ernal}} + \frac{\mathbf{v}^2}{2} \right) \mathbf{v} \cdot \mathbf{n} dS$$

$$= \dot{E}_{loss} + \frac{d}{dt} \iiint \rho \frac{\mathbf{v}^2}{2} \, dV + \iint \rho \frac{\mathbf{v}^2}{2} \, \mathbf{v} \cdot \mathbf{n} dS,$$
(23)

where dS is a surface differential element and dV a volume differential element of the CV.

The volume integral represents the kinetic power of the fluid inside the CV. Because of steady state periodic conditions, its contribution to the energy balance will be zero after integration over a time period. The surface integral represents the flux of kinetic energy at the CV boundaries, and comprises the input and output cross sections only since no fluid crosses the top part of the CV and $\mathbf{v} \cdot \mathbf{n} = 0$ on the bottom part of the CV (axis of symmetry).

On the other hand, the rate at which the environment does work on the fluid is decomposed into fluid pressure and viscous stress components. Integration along the surfaces of the CV is nonzero at the input and output cross sections only, and viscous stress or shear contribution on the two cross sections is small enough to be neglected. The pressure power becomes:

$$\dot{W}_{press} = \iint_{1\cup 2} P \,\mathbf{v} \cdot \mathbf{n} dS \,. \tag{24}$$

Therefore the balance of rate of change of energy is as follow:

$$\dot{E}_{loss} + \iint_{1\cup 2} \rho \frac{\mathbf{v}^2}{2} \mathbf{v} \cdot \mathbf{n} dS = \dot{W}_{mech} + \iint_{1\cup 2} P \mathbf{v} \cdot \mathbf{n} dS , \qquad (25)$$

$$\dot{W}_{mech} - \dot{E}_{loss} = \iint_{1\cup 2} \left(\rho \frac{\mathbf{v}^2}{2} + P \right) \mathbf{v} \cdot \mathbf{n} dS = \dot{E}_{out} - \dot{E}_{in} \,. \tag{26}$$

We define the pumping power (\dot{W}_{pump}) as the mechanical power done by the moving wall (\dot{W}_{mech}) minus losses (\dot{E}_{loss}) :

$$\dot{W}_{pump} = \dot{W}_{mech} - \dot{E}_{loss}, \qquad (27)$$

Finally the pumping work is found by integrating equation (26) over a period of time *T*:

$$W_{pump} = \int_{t}^{t+T} \left[\iint_{1\cup 2} \left(\rho \frac{\mathbf{v}^2}{2} + P \right) \mathbf{v} \cdot \mathbf{n} dS \right] dt \,.$$
(28)

We found a nonlinear relationship of the pumping work and the frequency of excitation (figure 16), reaching maximum around the resonant frequency. Significant positive work occurs for frequencies ranging from 10Hz to 10.5Hz, meaning that the elastic tube does work on the fluid. It is of particular interest since the considered portion of the tube actually does not contain active components (such as pincher). This implies that the elastic tube does not act as a resistor, but contribute to pumping by transmitting energy to the fluid. At resonance is the transfer of energy from the elastic tube to the flow maximized.



Figure 16. Pumping work of the elastic tube $(W_{pump}, 1 \text{ Erg}=1\text{e-7 J})$ and frequency of

excitation (f).

3.13 Pumping efficiency

The efficiency (ε) of a pump is the ratio of the useable work over the work dispensed to actuate the pump. In the case of the MIP, it corresponds to the work produced by the pump (W_{pump}) over the work dispensed at the actuation zone (W_{actua}):

$$\varepsilon = \frac{W_{pump}}{W_{actua}}.$$
(29)

The work done by the actuation zone is the result of the work done by each of the nodes that undergoes the prescribed compressive displacement. Nodal work is calculated using the nodal reaction force integrated over the nodal radial displacement (motions in the other directions are constrained). The model being axisymmetric, the work for the whole pinching section is recovered by multiplying by 2π :

$$W_{actua} = 2\pi \sum_{pinching nodes} \int_{t}^{t+T} \left(\int R_{y}^{node} dy(t,z) \right) dt .$$
(30)

The work produced by the pump (W_{pump}) represents the increase of energy of the fluid between the entrance and exit of the pump (31). It is composed of pressure and kinematic terms. Because we imposed a zero pressure boundary condition at the two extremities of the tube (entrance z=0 and exit z=L), the pressure term in equation (31) is null, which would lead to a wrong estimation of the work produced by the pump.

$$W_{pump} = \int_{t}^{t+T} \left[\iint_{entrance \cup exit} \left(\rho \frac{\mathbf{v}^2}{2} + P \right) \mathbf{v} \cdot \mathbf{n} dS \right] dt$$
(31)

We propose to evaluate the pressure term using an equivalent model. The MIP can be seen as a tube producing a mean flow, and one can consider as an equivalent model a similar Poiseuille tube driven by a pressure gradient (figure 17). The pressure at one extremity being zero, one can calculate, knowing the geometry and the mean flow, the pressure at the other extremity of the Poiseuille tube (32).⁶¹

$$P(z=0) = \frac{8\mu L}{\pi R_f^4} \overline{Q}$$
(32)

The Poiseuille pressure represents the pressure that would be produced by the MIP for the same resulting mean flow \overline{Q} . Consequently, the work produced by the pump is evaluated by plugging the Poiseuille pressure (32) into equation (31).

The efficiency is computed for frequencies of excitation ranging from 9Hz to 12 Hz, where flow is exiting the pump in the positive direction (figure 18). We found that the efficiency ranges between 5% and 15%, except when excited at resonance, where the pump exhibits a clear peak at almost 35%. This confirms the role of resonance where the MIP reaches its maximum pumping efficiency.



Figure 17. Equivalent model: Poiseuille flow driven by a pressure gradient.



Figure 18. Efficiency (ε) of the MIP and frequency of excitation (*f*).