Chapter 2

The Multilayer Impedance Pump Model

2.1 Physical model

The MIP was a fluid-filled elastic tube with an excitation zone located asymmetrically with respect to the length of the pump. The pump had an aspect and a layered wall structure similar to the embryonic heart. The fluid domain accounts for only 35% of the total volume occupied by the pump. The layered walls of the elastic tube were made of a thick gelatin layer for about 80% of the elastic tube volume, and a thin stiffer layer for the remaining 20%, following the heart tube geometry (figure 5). Each layer constituting the tube walls was made of an isotropic linear elastic material. The material properties of each layer have been chosen so that a large enough stiffness ratio between the elastic layers enables the combined effect of wave amplification through the gelatin and the prevention of outward motion at the external layer. In addition, following the embryonic heart structure, the gelatin-like layer has been given some compressibility (ν_{gel}=0.3), while the stiffer layer was relatively incompressible (ν_{sl}=0.49) (table 1).

The periodic excitation consisted of imposed radial displacements $y(t,z)$ on a section of the outer surface of the tube (1). The pump was actuated for 20% of the period time T. The tube's external radius was maintained to original position during the remaining 80% of the period time. During actuation, the elastic tube was compressed following a sinusoidal time function $g(t)$ that depended on the frequency of excitation f (2). The amplitude A of the compression was set to 10% of the pump external radius, so that to model the displacement resulting from the myocites' contractions. The spatial repartition of the compression zone followed a quadratic spatial function *s(z)* to simulate a physical pincher (3):

$$y(t,z) = g(t) * s(z), \qquad (t,z) \in [0,T] * [a_l, a_l + a_w], \quad (1)$$

$$g(t) = A * \sin(5\pi\ f\ t) * Heaviside(\frac{T}{5} - t), \qquad (t) \in [0,T], \quad (2)$$

$$s(z) = 1 - (\frac{1}{13} z - \frac{14}{13})^4, \qquad (z) \in [a_l, a_l + a_w]. \quad (3)$$

The impedance mismatch was achieved by fixing the tube's extremities, ensuring total reflection of the elastic waves. The fluid filling the tube was water.

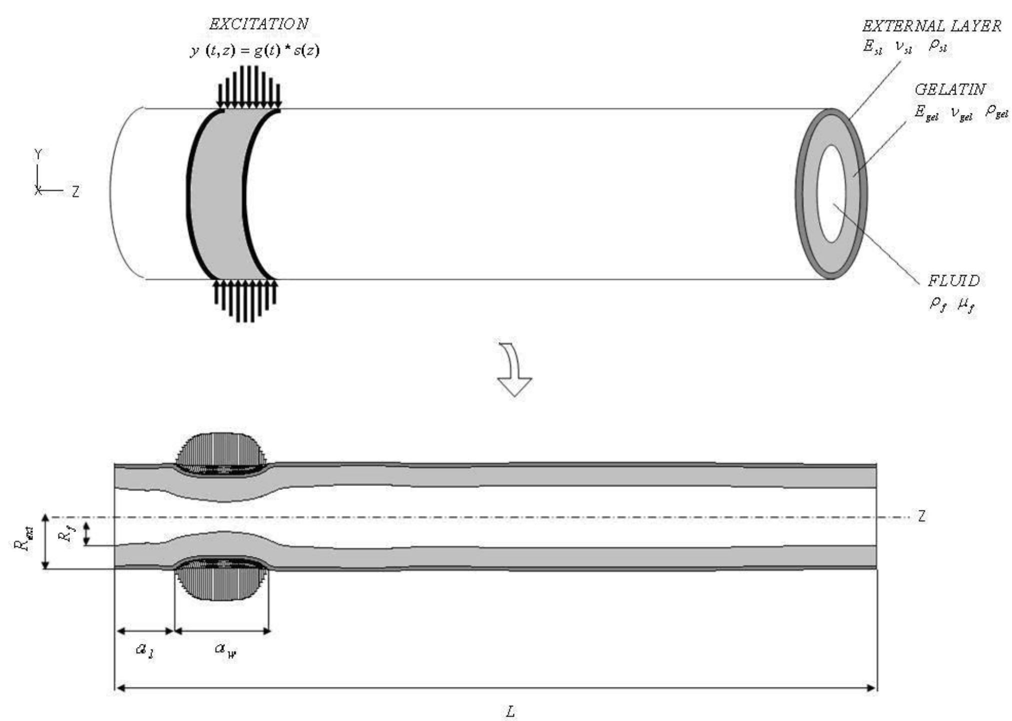

Figure 5. (Top) 3D view and (Bottom) 2D view in longitudinal cross section of the physical model of the MIP.

Table 1. Physical parameters of the MIP.

Physical parameter	Symbol	Value
Length of the pump	L	15.2 cm
External radius of the pump	R_{ext}	1.03 cm
Fluid domain radius	R_f	0.55 cm
Gelatin thickness	h_{gel}	0.405cm
Stiffer layer thickness	h_{sl}	0.075 cm
Actuator location with respect to the tube's nearest extremity	a_l	1.2 cm
Actuator width	a_w	1.8 cm
Gelatin stiffness	E_{gel}	5 e+ 4 dyn/cm^2
Stiffer layer stiffness	E_{sl}	1 e+ 7 dyn/cm^2
Gelatin Poisson's ratio	ν_{gel}	0.3
Stiffer layer Poisson's ratio	ν_{sl}	0.49
Gelatin density	ρ_{gel}	1 g/cm^3
Stiffer layer density	ρ_{sl}	1 g/cm^3
Fluid viscosity	μ_f	0.01 g/cm s
Fluid density	ρ_f	1 g/cm^3
Excitation amplitude	A	0.1 cm
Frequency	f	7 Hz to12.2 Hz

2.2 Mathematical model

The fluid motion was derived by the conservative Navier-Stokes equations using the Arbitrary Lagrange Eulerian formulation:

$$\nabla \cdot \mathbf{v} = 0, \tag{4}$$

$$\rho_f \left(\frac{\partial \mathbf{v}}{\partial t} + \left(\mathbf{v} - \mathbf{v}_g \right) \cdot \nabla \mathbf{v} \right) + \nabla \cdot \boldsymbol{\tau}_\mathbf{f} = 0\,, \tag{5}$$

where $\boldsymbol{\tau}_\mathbf{f}$ is the stress tensor, $\mathbf{v}$ is the flow velocity vector and $\mathbf{v}_\mathbf{g}$ is the local coordinate velocity vector, ρ_f is the density of the fluid and t is the time.

The fluid is Newtonian, incompressible and viscous, and its state of stress $\boldsymbol{\tau}_\mathbf{f}$ follows:

$$\boldsymbol{\tau}_\mathbf{f} = -P\,\mathbf{I} + \mu_f (\nabla \mathbf{v} + \nabla \mathbf{v}^T), \tag{6}$$

where P is the static pressure and μ_f is the dynamic viscosity.

The dynamics of each layer of the flexible wall were calculated using the balance of momentum equation in Lagrangian form (7) and the constitutive relation for a linear isotropic elastic material (8):

$$\nabla \boldsymbol{\tau}_s + \mathbf{b}^\mathbf{f} = \rho\, \ddot{\mathbf{u}}\,, \tag{7}$$

$$\boldsymbol{\tau}_\mathbf{s} = \lambda Tr(\boldsymbol{\varepsilon}_\mathbf{s})\mathbf{I} + 2\mu \boldsymbol{\varepsilon}_\mathbf{s}\,, \tag{8}$$

where $\boldsymbol{\tau}_\mathbf{s}$ is the Cauchy stress tensor, $\boldsymbol{\varepsilon}_\mathbf{s}$ the strain tensor, $\mathbf{b}^\mathbf{f}$ the body forces vector per unit volume, $\ddot{\mathbf{u}}$ the acceleration vector ρ the density, and λ and μ the Lamé constants of the considered structural domain.

At the fluid-structure interface the fluid is fully coupled to the gelatin. The fundamental conditions applied to the fluid-structure interface are displacement compatibility and traction equilibrium between the two surfaces:

$$\mathbf{d} = \mathbf{u}, \tag{9}$$

$$\mathbf{n} \cdot \boldsymbol{\tau}_{\mathbf{f}} = \mathbf{n} \cdot \boldsymbol{\tau}_{\mathbf{s}}, \tag{10}$$

where $\mathbf{d}$ and $\mathbf{u}$ are the fluid and solid displacement vectors respectively, and $\mathbf{n}$ is the unit normal.

To ensure total wave reflection, fixed ends in both layers are modeled by imposing zero displacements in all directions and at all time at the two tube extremities:

$$\mathbf{u} = \mathbf{0} \qquad \text{at } z=0 \text{ and } z=L. \tag{11}$$

The no-slip condition ($\mathbf{n} \times \dot{\mathbf{u}} = \mathbf{n} \times \mathbf{v}$) is applied at the fluid-structure interface, and the tube lies in a stress-free and pressure-free environment (figure 3):

$$\mathbf{n} \cdot \boldsymbol{\tau}_{\mathbf{s}} = 0 \qquad \text{on the lateral surface of the tube,} \tag{12}$$

$$P = 0 \qquad \text{at } z = 0 \text{ and } z = L. \tag{13}$$

Initial conditions are resting state: zero pressure and zero velocity in the fluid, no stress or strain in the structure.

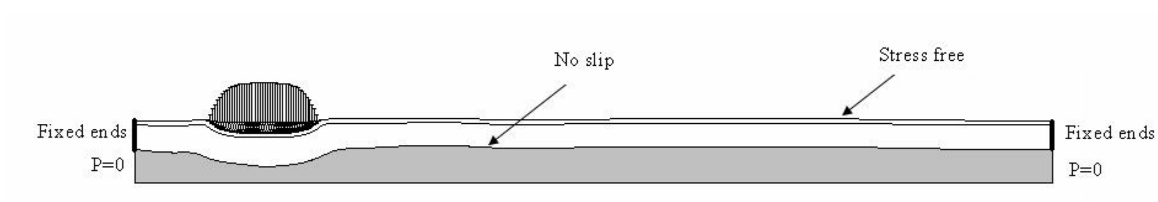

Figure 6. 2D axisymmetric longitudinal outline of the MIP model with excitation and boundary conditions (the shaded region represents the fluid domain).

2.3 Numerical model

The finite elements method was used to discretize both the fluid and structure domains, and the fully coupled problem was solved using the commercial package ADINA (ADINA R&D, MA).

The fluid and the solid domain were meshed using 4-noded axisymetric elements. The solid mesh was refined at the pinching zone. A total of 10,500 elements were used, 6,000 for the fluid and 4,500 for the solid (figure 7). An embedded actuation pincher was modeled by imposing radial displacements on a series of nodes corresponding to the pincher location, at the outer surface of the tube. The solid part is solved using the small strain, small deformation hypothesis, and the flow is assumed to be laminar. A constant number of 1,000 time steps per pinching cycle are used to march throughout the transient simulations.

The time integration scheme is implicit Euler backward (α=1), which is first-order accurate in time. The equations of motion are integrated by using the implicit damped Newmark scheme (δ=0.5, α =0.25), and the full Newton Method was used for the non linear equations. The fluid and solid are 2-ways direct fully coupled, and the fluid mesh is updated at each time step using Arbitrary Lagrange Eulerian formulation. All computations are starting from resting state and are carried on until periodicity in the fluid motion is achieved (mean exit flow is constant within 1% for at least 5 periods).

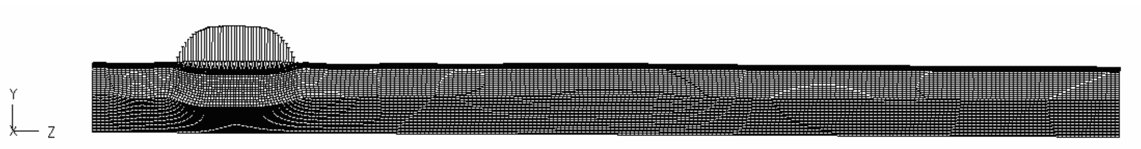

Figure 7. 2D axisymmetric longitudinal view of the mesh.