Chapter 1

Overview and Purpose of the Caltech Experiments

A plasma is an ionized gas that is thus subject to both fluid-type forces and electromagnetic forces. Of crucial importance for magnetized plasmas is the concept of magnetic helicity, a topological quantity that measures the knottedness or twistedness of the magnetic field. Helicity is approximately conserved throughout plasma evolution and often dissipates more slowly than magnetic energy. A universal relaxation theory, which applies to both astrophysical and laboratory plasmas, dictates that magnetized plasmas will naturally evolve towards an equilibrium state based on the helicity content of the plasma. Both the Caltech Solar Loop Experiment and the Caltech Spheromak Experiment create plasmas with injected helicity so that the evolution towards these relaxed equilibria can be studied. The evolution involves the merging of two or more plasma loops into a single structure, and this thesis studies the merging using two techniques. The first is the design, construction, and utilization of an array of vacuum photodiodes to measure extreme ultraviolet radiation from the Caltech Solar Loop Experiment; the data from the array provides information concerning both radiation losses and magnetic reconnection during the merging. The second is an analytical study of charged particle orbits in magnetic fields aimed at explaining how particle orbits can transition from being localized from one plasma loop to being shared symmetrically among two or more neighboring loops. This study gives insight into how the merging process is initiated.

The remainder of this chapter further develops the motivating ideas behind these studies. The concept of magnetic helicity is explored in more detail along with two additional topics: Woltjer-Taylor states and helicity injection. Woltjer-Taylor states are plasmas with a minimum of magnetic energy given a fixed helicity content, while helicity injection is the process of introducing helicity into a plasma in the first place. The Caltech experiments are grounded in these principles: both utilize novel plasma guns to inject helicity into plasmas and to observe their evolution towards Woltjer-Taylor states. Both projects described in this thesis are motivated by particular observations from these experiments.

Chapter 2 describes the Caltech Solar Loop Experiment and explains how the plasmas are formed and develop helicity. Vacuum photodiodes are then discussed in Chapter 3, which details the construction of the array and noise reduction techniques. Chapter 4 presents the measurements from the array together with other diagnostic data in the study of counter-helicity merging on the Solar Loop Experiment. Strong bursts in extreme ultraviolet radiation are observed; these bursts tend to be localized in space and time and are believed to be related to magnetic reconnection. Chapter 5 presents a new theory of Hamiltonian dynamics that originated from studies of particle trajectories in the Caltech experiments; we find that the action integral for the fastest periodic motion acts as an effective Hamiltonian for the reduced system. This chapter also applies this theorem to charged particle motion in magnetic fields. Chapter 6 presents a model to explain the onset of two plasma loops merging; the model shows how charged particles orbiting two parallel current channels can transition from orbits that remain confined to one current channel to orbits shared symmetrically between both channels. Finally, the appendices contain a description of modifications made to the voltage and current diagnostics for improved measurements, application of Stormer theory to the deflection of charged particles from a diagnostic, a review of action-angle variables and a canonical transformation to the orbit-averaged Hamiltonian system, and additional calculations concerning the application of the new Hamiltonian theory to charged particle motion.

1.1 Magnetic Helicity

Magnetic helicity is a measure of how knotted and twisted the magnetic field of the plasma becomes. Just as a head of hair can get tangled and twisted, so too can the magnetic field lines of a plasma knot and link each other. Magnetic helicity, like magnetic energy, is a conserved quantity for an ideal perfectly conducting plasma; for a realistic plasma with non-zero resistivity, magnetic helicity is often more resilient to resistive decay than magnetic energy. These properties have made helicity a key concept in fusion devices such as tokamaks and spheromaks and astrophysical plasmas such as solar coronal loops.

Magnetic helicity can be defined as the volume integral of a rather unusual quantity [1, ch. 3] [2, ch. 11] [3]:

$$K = \int \mathbf{A} \cdot \mathbf{B} \; d^3r, \tag{1.1}$$

where $\mathbf{A}$ is a vector potential associated with $\mathbf{B}$: $\nabla \times \mathbf{A} = \mathbf{B}$. The above definition must be supplemented by the condition that the magnetic field cannot penetrate the boundary surface of the volume, $\mathbf{B} \cdot d\mathbf{S} = 0$, to ensure K is unchanged by a gauge transformation. We can prove this by making a gauge transformation $\mathbf{A} \to \mathbf{A} + \nabla f$ and evaluating Eq. (1.1):

$$K = \int (\mathbf{A} + \nabla f) \cdot \mathbf{B} \; d^3r = \int \mathbf{A} \cdot \mathbf{B} \; d^3r + \int \nabla \cdot (f\mathbf{B}) \; d^3r \tag{1.2}$$

$$= \int \mathbf{A} \cdot \mathbf{B}\, d^3r + \oint f\mathbf{B} \cdot d\mathbf{S} = \int \mathbf{A} \cdot \mathbf{B}\, d^3r, \tag{1.3}$$

where the surface integral vanishes precisely because of the condition $\mathbf{B} \cdot d\mathbf{S} = 0$. K is thus independent of gauge even though the local helicity "density" $\mathbf{A} \cdot \mathbf{B}$ is gauge-dependent. This suggests that helicity is not a local quantity but rather a quantity associated with an entire field line.

Eq. (1.1) obscures the physical interpretation of helicity as the knotting and twisting of the magnetic field. To make this connection, consider two closed flux tubes that link each other as shown in Fig. 1.1; one can show that Eq. (1.1) evaluates to

$$K = 2\Phi_1\Phi_2, \tag{1.4}$$

where Φ_1 and Φ_2 are the magnetic fluxes of the two flux tubes [2, ch. 11] [4]. If, however, the two flux tubes did not link each other, then the integral of Eq. (1.1) would vanish. This shows the topological nature of helicity. Analogous scenarios exist in fluid mechanics regarding the fluid vorticity [5].

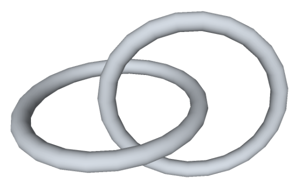

Figure 1.1: Two magnetic flux tubes, or bundles of magnetic field lines, that link each other have non-zero helicity content.

The above example shows that helicity measures the linkage of flux tubes, but helicity is also present when a flux tube is twisted. The flux tube depicted in Fig. 1.2 has a helical magnetic field that can be decomposed into an axial field and an azimuthal field. The azimuthal field clearly wraps around and links the axial field, so the helical magnetic field configuration also has helicity.

Remarkably, magnetic helicity is conserved under ideal magnetohydrodynamic[1] (MHD) conditions. This can be proved by taking a time derivative of Eq. (1.1) [1, sec. 3.7], but an intuitive picture is as follows. In ideal MHD, each magnetic field line retains its identity and cannot break or tear [6]. In this case, the analogy to hair is quite accurate; it is as if the magnetic field lines were actual physical strands that cannot be unlinked without tearing the strands themselves. For instance, under ideal MHD evolution, the two flux tubes in Fig. 1.1 would remain forever linked.

In non-ideal plasmas, electrical resistivity causes diffusion of the magnetic field inside the plasma

[1]Magnetohydrodynamics is one of several possible descriptions of plasma; it treats the plasma as a conducting fluid that carries a current [2, sec. 2.6]. Ideal MHD assumes that the plasma has no resistivity.

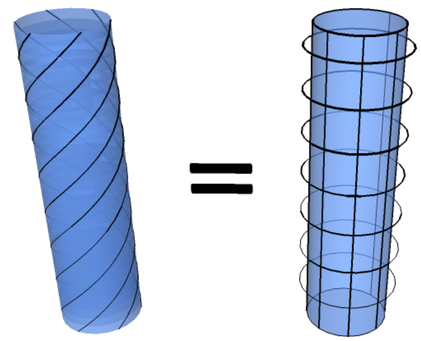

Figure 1.2: A helical flux tube contains helicity and can be thought of as a superposition of an axial field and a toroidal field.

so that field lines do not retain their identity and can meld with each other, but magnetic helicity is remarkably still conserved even under such conditions [1, sec. 3.9 - 3.12]. This process of field lines melding is known as magnetic reconnection [7] and typically involves the annihilation of a component of magnetic field and dissipation of magnetic energy. For instance, the two linked loops can reconnect and form a larger loop as shown in Fig. 1.3. While reconnection typically lowers the magnetic energy of the plasma, the global magnetic helicity is much more nearly conserved. For instance, in Fig. 1.3, helicity has not been destroyed but is rather changed from the linkage of two flux tubes to twist or writhe in the final flux tube. The skeptical reader is referred to Ref. [8], which explains how the above transition of helicity can be seen quite intuitively using simple household items.

1.2 Taylor States and Spheromaks

The idea that magnetic helicity is better conserved than magnetic energy was pushed to an extreme by Taylor, who reasoned that a plasma would continue to undergo magnetic reconnection until it reached a state of minimum magnetic energy given the constraint of constant helicity [6]. Such a variational problem had been proposed by Woltjer [9], who concluded that such plasmas must satisfy

$$\nabla \times \mathbf{B} = \lambda \mathbf{B}, \tag{1.5}$$

where λ is a constant. Since $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$[2], this condition states that the current density is everywhere parallel to the magnetic field. Such states are now known as Woltjer-Taylor states and

[2]In MHD, the displacement current in Ampere's law is quite small and ignored.

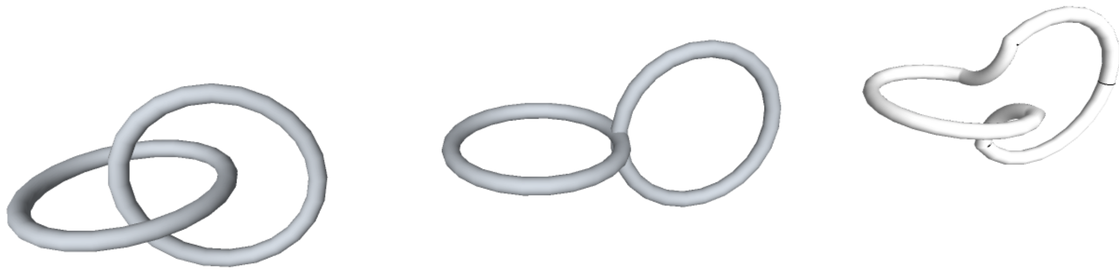

Figure 1.3: With electrical resistivity, magnetic reconnection allows the flux tubes to meld and unlink. However, the helicity lost in the linkage of the flux tubes is now found in the twist of the final flux tube.

are supposed to be equilibrium states as follows. The equation of motion for ideal MHD is

$$\rho \frac{d\mathbf{u}}{dt} = -\nabla P + \mathbf{J} \times \mathbf{B}, \tag{1.6}$$

where ρ is the mass density, $\mathbf{u}$ is the velocity, and P is the pressure. Eq. (1.6) is essentially Newton's second law for a continuous fluid; the lefthand side of Eq. (1.6) is the inertial term, and the righthand side consists of forces acting on the plasma. For many plasmas, the magnitude of the pressure P is much less than the magnetic energy density $B^2/2\mu_0$, and the pressure term in Eq. (1.6) can be ignored. Such plasmas are referred to as low β, where $\beta = 2\mu_0 P/B^2$, the ratio of the thermal energy density to the magnetic energy density. For a low β plasma that is also in equilibrium, the lefthand vanishes as well. Eq. (1.6) is then $0 = \mathbf{J} \times \mathbf{B}$, which is satisfied when $\nabla \times \mathbf{B} = \lambda \mathbf{B}$.

It should be noted that the condition $\mathbf{J} \times \mathbf{B} = 0$ is satisfied more generally by a field

$$\nabla \times \mathbf{B} = \lambda(\mathbf{r})\mathbf{B}, \tag{1.7}$$

where λ is now not constant but is a function of position. Plasmas that satisfy Eq. (1.7) are known as force-free plasmas. Clearly, all Woltjer-Taylor states are force-free, but the converse is not true. Force-free plasmas may be in equilibrium, but they generally contain more magnetic energy than the Woltjer-Taylor states with the same helicity, since the latter minimize energy by definition. Therefore, while force-free states may be in equilibrium, the equilibrium is unstable as the plasma can further shed magnetic energy via reconnection.

A plasma, in the absence of outside constraints, naturally tends towards a Woltjer-Taylor state determined solely by the magnetic field boundary conditions and the amount of helicity in the plasma. This is a powerful concept with applications to both laboratory devices and astrophysics. Using this idea, Taylor explains why laboratory devices such as a reverse-field pinch always settle down to the same state despite variations of experimental parameters [6]. Allowing the plasma to reach a Woltjer-Taylor state in a simply connected volume gives the configuration known as a spheromak, a magnetic-confinement concept aimed at achieving nuclear fusion [2, 10]. Astrophysical plasmas, including solar coronal loops, are also frequently assumed to be in Woltjer-Taylor states [11, 12, 13, 14, 15].

1.3 Helicity Injection

Magnetic helicity is an important concept but is only well-defined when the magnetic field lines do not penetrate the bounding volume. In situations such as the one depicted in Fig. 1.4, helicity is not well-defined because the volume of interest does not enclose all field lines. However, in such situations, it is possible to define a new but related quantity, called relative helicity, that is gauge-independent [1, sec. 3.5] [3, 16]. Relative helicity is essentially the difference in helicity of the magnetic field from a reference field; it resolves the gauge ambiguity while preserving the essence of helicity.

In such situations, it is possible to *inject* relative helicity into the system by applying a voltage between the points at which the magnetic field enters and leaves the volume [1, sec. 3.7]. The rate for helicity injection is, ignoring dissipation,

$$\frac{dK}{dt} = 2V\Phi, \tag{1.8}$$

where V is the voltage difference and Φ is the magnetic flux penetrating the boundary. Helicity injection can be understood intuitively as follows. Suppose that in Fig. 1.4 the field lines inside the flux tube point axially along the tube with no wrapping or twisting around the tube. A voltage applied between the points where the field lines penetrate the boundary drives currents along these field lines, and these currents generate their own field that wraps around the original field lines according to the righthand rule. The superposition of the original field, which is directed along the flux tube, and the field generated by the currents, which wraps around the flux tube, is a helical field which twists in proportion to the current driven. As twisted field lines contain helicity, we see that the applied voltage is indeed increasing the helicity content of the plasma. Indeed, this is precisely what happens in the Caltech experiments and will be discussed further in Sec. 2.1.3

The concept of helicity injection may seem incompatible with Woltjer-Taylor states because the former implies a time-varying helicity content whereas the latter describes an equilibrium. Nonetheless, it is often assumed in many spheromak experiments that the rate of helicity injection is slow

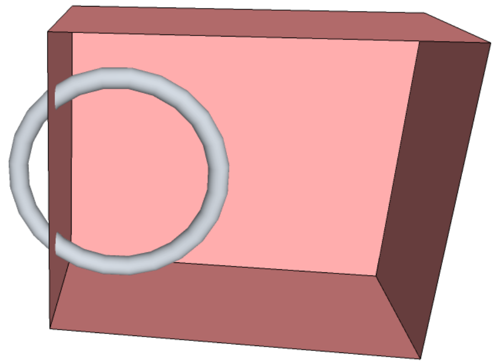

Figure 1.4: If the magnetic field penetrates the boundary, depicted by the red box, then the usual definition of helicity becomes gauge-dependent. In such cases, it is possible to define a relative helicity that is gauge-independent, and it is possible to inject relative helicity into the volume by applying a potential difference between the points where the magnetic field enters and leaves the volume.

enough so that the plasma settles down to the Woltjer-Taylor state determined by the instantaneous value of the plasma gun current. This scenario is called a “driven spheromak” [1, ch. 11].

1.4 The Caltech Experiments

Woltjer-Taylor states depend solely on the magnetic boundary conditions and helicity content with no dependence on the plasma density, temperature, or flow. Plasmas starting with varying densities, flows, etc., may, by Taylor’s theory, evolve to the same equilibrium state as determined by the total helicity content. However, it is not clear exactly how the plasma evolves towards this equilibrium: the plasma may evolve through a sequence of Woltjer-Taylor states throughout its entire lifetime as in the driven spheromak picture, or it may be out of a force-free state in its initial stages.

The Caltech experiments allow research into plasma formation and the steps needed to reach a Woltjer-Taylor state. Both Caltech experiments use planar magnetized plasma guns to create plasma and inject helicity into them. These planar guns, in contrast to the typical coaxial design, allow direct observation of the plasma in its infantile stages. Experiments at Caltech have shown that modeling plasma evolution as a sequence of Woltjer-Taylor states is too simplistic and that plasma flows and pressures *are* important [17].

The two topics of this thesis, an array of vacuum photodiodes and a Hamiltonian picture for particle orbits, evolved out of helicity studies done on both the Solar Loop Experiment and the Spheromak Experiment. We describe each in turn.

1.4.1 The Caltech Solar Loop Experiment, Bright Spots, and Vacuum Photodiodes

The Caltech Solar Loop Experiment creates plasma arcs resembling solar coronal loops on the sun's surface [18, 19, 20] as shown in Fig. 1.5. While there is an obvious discrepancy in size, temperature, density, and field strength between a laboratory plasma and the solar corona, the underlying physics is the same [21, sec. 1.3.1] [18]. Indeed, the solar corona is often modeled as a force-free or Woltjer-Taylor state [11, 12, 14, 15], so studies of near force-free plasma loops in the laboratory are expected to give insight into solar phenomena, particularly the impulsive eruption of coronal loops [22].

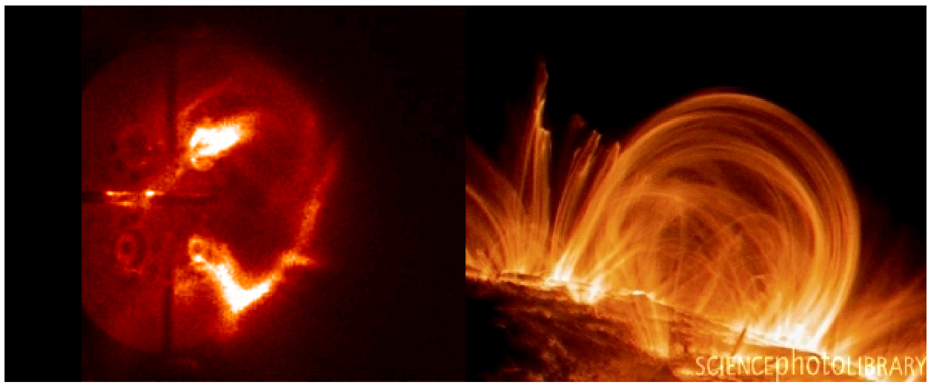

Figure 1.5: The plasma loops created by the Caltech Solar Loop Experiment are designed to resemble plasma structures on the surface of the sun. The image on the right is from the TRACE satellite.

It has been proposed that the sudden eruption of solar prominences might be triggered by the interaction of two magnetic structures [15], and this possibility was explored on the Caltech Solar Loop Experiment by producing two loops side-by-side [20]. The loops carry parallel currents and hence attract each other and merge together. Previous studies of two merging plasmas suggest that the final state after the merging is determined by the initial helicity content in accordance with Taylor's theorem [23, 24, 25, 26]. Similarly, the Caltech Solar Loop Experiment produces two loops with either the same helicity, called co-helicity, or opposite helicities, called counter-helicity, and the difference in helicity content results in different plasma behavior. In co-helicity experiments, one of the two loops tends to expand sooner and faster than the other. In counter-helicity experiments, a bright spot appears at the loop apex, and, around the same time, a burst of soft x-rays registers on a set of x-ray diodes. Fig. 1.6 shows an example of both the bright spot and the x-ray burst, and the possible correlation of these two events forms the original motivation for the array of vacuum photodiodes, as will be discussed.

The reason for both the bright spot and the x-ray bursts is believed to be the extra amount of magnetic reconnection that occurs for counter-helicity merging. Magnetic reconnection is present in both co- and counter-helicity merging because the azimuthal field, the field that is generated

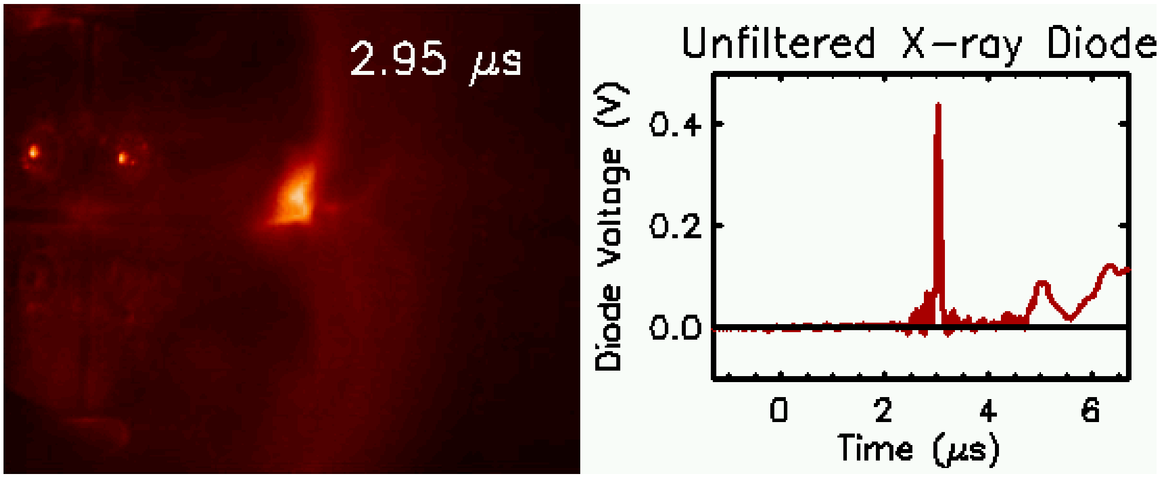

Figure 1.6: In counter-helicity merging, a bright spot forms in the central regions. At the same time, a burst of soft x-rays registers on the x-ray diodes. The array of vacuum photodiodes was constructed to study these phenomena further.

by the plasma current and that wraps around the plasma loop, is the same for both experiments and is annihilated as the loops merge; see Fig. 1.7. There is additional magnetic reconnection in counter-helicity experiments because the axial fields also annihilate, as shown in Fig. 1.7. This additional reconnection is not present in co-helicity experiments because, in that case, the axial fields are parallel, not anti-parallel, and do not annihilate. Magnetic reconnection involves a decrease in magnetic energy that is transferred to the plasma particles [7], and both the bright spot and the x-ray bursts are thought to be energized by the additional amount of reconnection, and hence energy released, in counter-helicity experiments. The bright spot is believed to be caused by the formation of a strong current sheet between the two loops as they merge; the current sheet deposits thermal energy into the plasma by Ohmic dissipation, and this thermal energy is then lost to radiation [27, sec. 1.2]. The enhanced x-ray emission might be due to a population of energetic electrons that are accelerated by the reconnection process [28, 29]; these energetic electrons would emit x-rays through bremsstrahlung. The correlation between x-rays and MHD activity such as sawtooth oscillations has been known for some time [30], but recently these x-rays have been ascribed to electrons directly energized by the magnetic reconnection associated with the MHD activity [31, 32].

This pair of phenomena, bright spots and x-ray bursts, motivate the vacuum photodiode array described in Chapter 3. The bright spot forms consistently from shot to shot, but the x-ray bursts are rather fickle. Indeed the large variations in the x-ray signals are documented by Hansen et. al., who quoted x-ray signals, averaged over a number of shots, of 176.3 ± 100.6 mV [20]. The possible explanation for such large variations are numerous. The x-ray burst might not happen every shot, or perhaps it emits in variable directions. The x-ray diodes have a single line of sight to the plasma, and their alignment is quite sensitive to small changes in its inclination angle, so perhaps the x-ray bursts

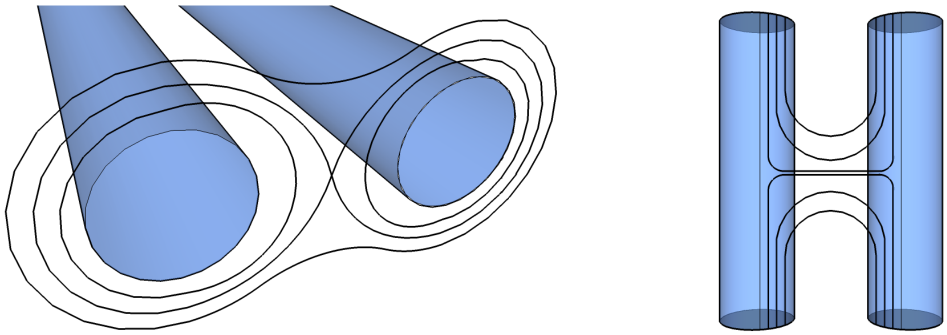

Figure 1.7: Counter-helicity merging involves two types of magnetic reconnection: (a) the azimuthal component, generated by the plasma current, and (b) the axial component, generated by the bias field coils. Co-helicity merging only involves the first type.

occur consistently but are not observed consistently by the diodes. To address these possibilities, an array of radiation detectors was needed to provide spatial as well as temporal resolution, and vacuum photodiodes were selected based on their low material cost and potential to scale into an array. The design of the array is discussed in Chapter 3, while Chapter 4 presents the experimental results.

1.4.2 Spheromaks, Spider Legs, and Particle Orbits

The Spheromak Experiment produces spheromaks using a novel plasma gun in which the electrodes are co-planar and concentric as opposed to being coaxial cylinders. The planar electrodes allow direct observation of the plasma during its initial stages when, as shown in Fig. 1.8, the plasma consists of eight loops, or “spider legs.” This eightfold symmetry occurs because the gas for the plasma is injected through eight pairs of gas inlets; the eightfold symmetry is in contrast to the initial magnetic field, which is generated by a coil of wire behind the electrodes and is entirely axisymmetric. The Woltjer-Taylor state for the magnetic boundary conditions is hence axisymmetric, and, indeed, the spider legs quickly expand and merge to form an axisymmetric plasma, as shown in Fig. 1.8. The exact mechanism of how the plasma transitions from the eightfold symmetry of the initial gas distribution to the axisymmetry of the magnetic field boundary conditions is not clear, and an explanation is sought by studying the motion of single charged particles in magnetic fields that model the Spheromak Experiment.

Single particle motion provides a rather detailed perspective of plasma by following one particle through the electromagnetic field of the plasma. Such an approach is quite different from but complementary to a fluid-like description such as MHD. A robust approximation scheme to the motion of a charged particle through rather arbitrary magnetic fields was developed by Alfvén [33]. In a uniform magnetic field, particle motion in the plane normal to **B** is perfectly circular and is referred to as Larmor motion. Alfvén studied particle motion when the strength and direction of

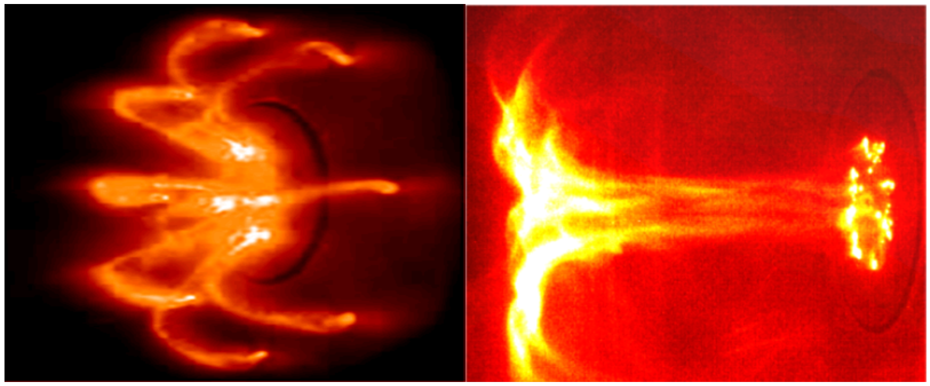

Figure 1.8: The spheromak starts out as eight discrete spider legs but eventually merges into an axisymmetric configuration.

the magnetic field varied and concluded that the particle still executes Larmor-like motion except that the center of the circle slowly drifts over the course of many orbits. A set of equations, known as guiding center theory [2, ch. 3], describes the average position of the particle without providing the details of the Larmor motion. These equations provide a substantial simplification of the exact equations of motion and hold quite generally as long as the magnetic field does not change much over the course of a Larmor orbit. These equations can be derived by averaging Newton's law:

$$m\frac{d\mathbf{v}}{dt} = q\mathbf{E}(\mathbf{r},t) + q\mathbf{v}\times\mathbf{B}(\mathbf{r},t). \tag{1.9}$$

To study particle motion in the Spheromak experiment, we employ a Hamiltonian approach. Hamiltonian mechanics is a formulation of mechanics in which a single function, the Hamiltonian, generates all equations of motion. Hamiltonian mechanics is entirely equivalent to using Newton's law but often leads to different insights. In this case, the Hamiltonian approach reveals a surprising connection between the guiding center equations and another aspect of charged particle motion, adiabatic invariants [2, sec. 3.3] [34, sec. 49], which are quantities that are approximately conserved over long periods of time even when other quantities, such as energy, are not. For instance, the first adiabatic invariant μ [33, sec. 2.3] is

$$\mu = \frac{(1/2)mv_L^2}{B}, \tag{1.10}$$

where v_L is the Larmor velocity. Up to three adiabatic invariants can be found for charged particle motion [35]. The Hamiltonian theory presented in this thesis states that the adiabatic invariant associated with the fastest periodic motion acts as an effective Hamiltonian for the reduced or orbit-averaged system. For charged particle motion, this means that μ acts as a Hamiltonian for the guiding center motion. The strength of the new Hamiltonian theory presented here, however, is that it applies more generally than charged particle motion, and, in this way, concepts from guiding center theory are generalized to a broader class of systems.

Chapter 5 describes the new Hamiltonian theory in its full generality and also applies it to charged particle motion in electromagnetic fields. Chapter 6 returns to the original problem of explaining the merging of the spider legs. There, a model magnetic field for the experiment is introduced that supports two classes of trajectories, those shared symmetrically between two plasma loops and those confined to a single loop. A mechanism for transitions between the two classes is then proposed. Although the model greatly simplifies the complexity of the actual experiment, it provides a good starting point for future investigation.