THE PERFORMANCE OF JET PROPELLED

SPACE VEHICLES

Thesis by

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SUMMARY

This thesis presents an investigation of the steady state performance of space vehicles partially propelled by jets, the acceleration period required to reach the steady state, the use of jets for the assisted take off of flying boats, and the general economic aspects of jet propulsion applied to aircraft.

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()†		primed quantities are in engineering units, unprimed quantities in f, p, s units
()j		sub j quantities refer to the jet
() _i		sub i quantities refer to the internal combustion engine
() _i		sub i quantities may also be "indicated quantities".
(),		sub W quantities are "weight reduced" quantities
($)_{LD}$		sub LD quantities refer to conditions at maximum lift- drag ratio
Ħ	?	=	aspect ratio
C		=	rate of climb
CD	, c _D	, i	etc. = conventional aerodynamic coefficients
D		=	drag, pounds
Di		=	induced drag
Dp		=	parasite drag
I		=	induced drag correction factor
L		=	lift in pounds
Μ		=	Mach's number
Ρ		=	brake horsepower f.p.s.
P _i		=	power available from the internal combustion engine
Pj		=	power available from the jet engine
Pa		=	power available
$P_{\mathbf{r}}$		=	power required
Py		=	hypothetical power required
Pe		=	excess power
Rv,	8	=	velocity ratio

тj	=	jet thrust in pounds
^T j _c	=	critical jet thrust
^T j _s	=	service jet thrust
Tr _c	=	propeller efficiency ratio
V	=	air speed f.p.s.
٧o	=	design speed
V _M	=	maximum speed
\mathbb{W}_{o}	=	design gross weight pounds
W	=	weight under specified conditions pounds
W_{P}	=	propellant weight in pounds
Ъ	=	wing span, feet
c	=	"effective jet discharge velocity" in f.p.s.
ei or	°j	= specific fuel or propellant consumption
e	=	aerodynamic efficiency
f	=	parasite area
g	=	acceleration of gravity
h	=	altitude at any time
hc	=	critical altitude
lp	=	parasite loading
ls	=	span loading
lt	=	power loading
lj	=	jet loading
m	=	mass flow of propellant per second
q	=	dynamic pressure
t	=	time in seconds

Wh	=	rising speed
ws	=	sinking speed
wy	=	hypothetical sinking speed
ε	=	ratio of weight to standard weight
9	=	propulsive efficiency
θ	=	angle of climb
π	=	3.1416
ρ _o	=	standard atmospheric density, 0.002378
5	=	atmospheric density ratio
λ _s	=	span loading
λ_p	=	parasite loading
λ_t	=	power loading
λ_{j}	=	jet loading
2	=	performance parameter including velocity
¥	=	performance parameter including velocity
Λ_{j}	=	fundamental jet performance parameter

Chapter I

INTRODUCTION

Since the conception of jet propelled vehicles for navigation of space above the earth's surface, men have continuously labored in experimental, mathematical, and imaginative investigation of this mode of transport. In the past years the greatest efforts have been bent toward the development of a successful jet engine. These fruitful experiments indicate the early completion of practical engines, which are reliable, suitable for long term and repeated operation, and utilize a stable propellant.

The application of a successful jet engine to a vehicle designed for space transportation is the next logical step. Many phantasies have been dreamt concerning the phenomenal performances that will result. Experimentation with jet propelled sounding rockets have been undertaken by several individuals. Calculations of specific, predefined, jet propelled flights have been made. It now appears desirable to consider the problem of the behavior of jet propelled space vehicles from a more general point of view, and with practical applications in mind.

This paper investigates the performance of jet propelled space vehicles in various practical applications; proposes a method of performance analysis; attempts to show the effect of variation of the different parameters on performance; presents the solution of several performance problems and discusses the practical and economic aspects of jet propulsion.

THE JET ENGINE

The jet engine is a device for supplying thrust as a result of the combusion of its fuel in a specially designed chamber, and the consequent high velocity discharge of these propellant gases through a nozzle.

The thrust is usually expressed by the formulae:

$$T_{j} = m c \qquad (2.1)$$

where

T_j = thrust in lbs.
m = mass flow of propellant per second
c = "effective discharge velocity" in
 feet per second

From this it is apparent that, for a constant propellant consumption, the thrust varies with the discharge velocity. That is, an efficient jet engine is characterized by a high discharge velocity.

Actually the thrust is not alone a "momentum thrust", as indicated by (2.1), but has a contribution from the pressure drop across the discharge nozzle. This "pressure thrust" is small in comparison with the "momentum thrust". In jet engines, it is convenient to consider that all thrust results from the exhaust velocity, and to use an "effective" value for this velocity to allow for the influence of "pressure thrust".

For all practical purposes the "mass flow" and "effective discharge velocity" may be considered constant over any unadjusted operating period, irrespective of variation in air density or air speed. The thrust is independent of altitude or speed. The flow and discharge velocity are interrelated. Within design limits, they could be controlled by the operator. That is, the thrust of a single engine could be controlled by the operator, but the exercise of such control would modify the engine efficiency as indicated by the change in discharge velocity.

Figs. 2-1 and 2-2 illustrate in two different ways the relation of "propellant mass flow" and "exhaust velocity" to "jet thrust".

For long term operation the weight of a jet engine becomes negligible compared to the weight of propellant involved. In vehicle design this weight is of importance primarily for its moment contribution resulting from probable off CG (center of gravity) location. The great weight addition in many installations will be the weight of pumps and plumbing. Since these items are directly effected by design skill, it is impossible to quote accurate figures for them. However, it may be said that the power requirements for this auxiliary service range upward from 50 HP.





Chapter III

THE JET ENGINE COMPARED WITH THE INTERNAL COMBUSTION

ENGINE WITH PROPELLER

jet The engine is compared with the internal combustion engine — propeller combination as a motor for space vehicles from the following considerations:

- A. Power output available
- B. Specific fuel consumption
- C. Operation at altitude
- D. The launching criterion

A. Power Output Available

The power available from an internal combustion enginepropeller combination is shown as a function of vehicle velocity in Fig. 3-1. The engine brake horsepower is modified by the propellor's efficiency. The power available decreases from a probable maximum of 0.83 BHP at the design vehicle speed, as the speed is varied in either direction from this design value. At high vehicle speeds, the power available may be reduced tremendously by propeller tip losses.

The power available at a given altitude from the internal combustion engines is

$$P_i = 9 \times P - Tip$$
 Losses

where

? = propulsive efficiency at low speeds where tip losses are negligible.

P = brake power

P_i = power available from internal combustion engine



Since the thrust is constant, the power available from the jet engine is only a function of vehicle velocity.

$$P_{j} = V T_{j} \tag{3.1}$$

where

P j = power available from jet motor V = vehicle velocity in f.p.s. T j = jet thrust in pounds

Fig. 3-2 depicts this fact. It can be seen, that when plotted against vehicle velocity, jet power available is represented by a straight line emanating from the origin, making an angle with the velocity axis whose tangent equals the jet thrust.

In Fig. 3-3 P₁ and P_j are plotted together. Here it may be seen that from point of power output, the jet is at a disadvantage in the low speed ranges; becomes equal to the internal combustion engine above the design speed; is superior to the internal combustion engine as speeds further increase; and at the very high speeds supplies large powers, while the internal combustion engine supplies negligible power as a result of shock wave losses at the propeller.

B. Specific Fuel Consumption

In the internal combustion motor the specific fuel consumption based on brake horse power is independent of velocity; amounts to about 0.5 lb. per BHP per hour. At the design speed the specific fuel consumption based on propulsive power available is about 0.6 lb. per horsepower available per hour. At zero speed the specific fuel consumption is infinite; dropping rapidly to a minimum of 0.6 at design speed, and then increasing slowly to infinity at the speed at which the entire engine output is absorbed by the shock wave, i.e., about the speed of sound.

In the case of the jet motor the fuel consumption is dependent upon the thrust; whereas the jet power available is a function of velocity. Thus the specific fuel consumption of a jet is infinite at zero velocity and decreases continuously with increasing speed.

The specific fuel consumption of the jet may be determined by writing the equation for jet power.

$$P_{j} = T_{j} V$$

$$P_{j} = m c V$$

$$m = \frac{W_{P}}{gt}$$

$$\frac{W_{P}/t}{P_{j}} = \frac{g}{c V}$$

$$c_{j} = \frac{43.5}{cV'}$$

where m = mass flow of propellant per second

Wp = weight of propellant in pounds

- t = time in seconds
- c = jet velocity in f.p.s.
- c = jet specific fuel consumption in pounds per horsepower per hour
- ci = engine specific fuel consumption in pounds per horsepower available per hour



Fig 3-4

This expression is plotted in Fig. 3-4 for various values of jet velocity.

Fig. 3-5^{*} indicates the jet velocities which may be expected for different thermal efficiencies in the case of gasoline—oxygen propellant. It can be seen that 12,000 f. p.s. corresponds to 100 per cent efficiency. Values between 6,000 and 8,000 f.p.s., at the knee of the curve, correspond to thermal efficiencies which can rationally be expected from a heat engine.

Fig. 3-4 also includes a specific fuel consumption curve, based on power available, for a typical internal combustion engine propeller combination. The dashed line represents the condition if shock waves had not affected propeller efficiency. It is interesting to note, that under this ideal condition, the specific fuel consumptions of jet and internal combustion power plants are equal at vehicle speeds of about 6,000 miles per hour.

In the actual case, it is only near the speed of sound that an intersection occurs between the two specific fuel consumption curves, and this happens at a large value of the ordinate. The behavior of the internal combustion engine curve at Mach's numbers greater than one is unknown, but there is reason to believe that it again drops to finite values in the supersonic region, but has a much greater minimum than exhibited in the subsonic range. The dashed line

* Malina, Dr. Frank--"Characteristics of the Rocket Motor", Journal of the Franklin Institute, Vol. 230, p. 441.



Fig 3-5

indicates the possible behavior of the specific fuel consumption curve above the speed of sound.

From the point of economy in fuel consumption; the jet engine is at a great disadvantage in the subsonic regions, has a rapidly increasing advantage over the internal combustion engine with increasing supersonic speeds. In the case of a slight invasion of the sonic region, the jet fuel consumption quickly becomes equivalent to the internal combustion engine because of the decay of propeller efficiency. Indeed the jet, alone, is capable of supplying power to the vehicle for entering the supersonic region.

C. Operation at Altitude

Above the altitude to which an internal combustion engine is supercharged, its power output drops off with increasing altitude. It becomes zero at approximately 56,000 ft. in the case of geared superchargers, and somewhat higher for turbo superchargers. In addition, the propulsive efficiency decreases with altitude above the design altitude and further decreases the power available.

As may be seen from Equation 3.3, jet power available is independent of altitude.

The jet engine enjoys a tremendous advantage over the internal combustion engine in high altitude applications, and is the only source of power at extremely great heights.

D. The Launching Criterion

In the case of aircraft, the size of the powerplant is usually determined by minimum thrust requirements for take off. Once launched, the aircraft proceeds at a fraction of rated power; frequently, for long range operation, at less than half rated power. In these instances the range is significantly reduced by the excessive powerplant weight carried to meet the take off requirement.

The internal combustion engine propeller combination supplies take off thrust at a motor weight cost of about 600 pounds per thousand pounds of thrust. A jet engine for supplying short term take off thrust could be of the powder type, thus avoiding the heavy auxiliaries of the liquid engine, and providing take off thrust at a motor weight cost of approximately 40 pounds per thousand pounds of thrust. The differences in fuel consumption of the two engines is of no importance from a weight point of view, since this fuel does not contribute to the "in flight" gross weight of the aircraft.

The range of long range aircraft could be greatly benefited by supplying only sufficient conventional power to meet the cruising requirements, and providing the additional short term demands of take off by jet motors.

FUNDAMENTAL EQUATION FOR STEADY STATE AIRCRAFT PERFORMANCE

WITH JET AND INTERNAL COMBUSTION ENGINES

As a result of the high specific fuel consumption of a jet motor at subsonic speeds, it is impossible to utilize it with economy as the primary power source at such speeds. However, this does not preclude consideration of the jet motor as an auxiliary power source for short periods of superperformance.

Accordingly the governing performance equation is developed for an aircraft powered with conventional internal combustion engine and constant speed propeller plus a jet supplying constant thrust. Gross weight is considered constant. The notation follows that of Oswald in NACA TR #408.

A. Determination of the Performance Equation

From the sketch of the equilibrium condition, Fig. 4-1, forces along the aerodynamic axis and perpendicular to the flight path can be equated

$$L = W \cos \theta$$

$$T_{i} + T_{i} = D_{b} + D_{i} + W \sin \theta$$
(4.1)

Multiplying 4.1 (b) by flight velocity to obtain a power equation

$$T_{j} V + T_{i} V = D_{p} V + D_{i} V + V W \sin \theta \qquad (4.21)$$

$$P_{i} + P_{i} = V (D_{p} + D_{i}) + W C \qquad (4.22)$$

or



Fig. 4-1

$$C_{\rm D} = C_{\rm Dp} + C_{\rm Di} = \frac{f}{s} + \frac{C_{\rm L}^2}{\pi R} e$$

$$C_{\rm L} = \frac{L}{q S} \qquad \text{and from (4.1a)}$$

$$C_{\rm L} = \frac{W \cos \theta}{q S}$$

$$C_{\rm D} = \frac{f}{q} + \frac{W^2 \cos^2 \theta}{\pi S b^2 R}$$

so

$$C_{\rm D} = \frac{1}{S} + \frac{\pi q S b^2 e}{\pi q S b^2 e}$$

$$D = fq + \frac{W^2 \cos^2 \theta}{\pi q b^2 e}$$
(4.3)

Substituting (4.3) in (4.22),

$$P_{j} + P_{i} = V fq + \frac{V W^{2} \cos^{2} \theta}{\pi q b^{2} e} + W C$$
 (4.4)

This may be written,

$$P_{a} = P_{y} + P_{e} \qquad (4.50)$$

where

P_a = total power available
P_e = excess power; power available for climb
P_y = hypothetical power required; power required
to overcome the drag along the flight path

Rearranging terms and dividing by W

$$C = \frac{dh}{dt} = \frac{P_a}{W} - \frac{P_y}{W}$$
(4.51)

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$$\frac{dh}{dt} = w_h - w_y \tag{4.52}$$

where

w_h = rising speed

 W_{τ} = hypothetical sinking speed

$$W_{h} = \frac{P ?}{W} + \frac{T j V}{W}$$

$$W_{y} = \frac{2 W \cos^{2} \theta}{\pi \rho_{o} \sigma V b^{2} e} + \frac{f \rho_{o} \sigma V}{2W}$$

$$(4.53)$$

Substituting in (4.52)

$$\frac{dh}{dt} = \frac{P}{W} + \frac{T_j V}{W} - \frac{2W \cos^2 \theta}{\pi \rho_0 \sigma V b^2 e} - \frac{f \rho_0 \sigma V^3}{2W}$$
(4.6)

This is the fundamental performance equation for an aircraft propelled by a combination of jet and internal combustion engines.

Attention is invited to the "hypothetical sinking speed", w_y , which equals the "sinking speed, w_s , if the square of the cosine of the angle of climb is put equal to 1. Similarly the "hypothetical power required" is different from "the power required". The hypothetical power required is the power necessary to overcome the drag along the flight path. At low speeds and large angles of climb P_y may be materially smaller than P_R , because of the decrease in induced drag corresponding to the difference between 1 and cos ² 6.

or

The performance parameters are defined as follows:

Span loading
$$\equiv \lambda_s = \frac{2W}{\rho_o. \pi b^2 e}$$

Parasite loading
$$= \lambda_p = \frac{2W}{\rho_0 f}$$

Thrust loading
$$\Xi \lambda_t = \frac{W}{9 P}$$
 (4.7)

Jet loading
$$\equiv \lambda_j = \frac{W}{T_j}$$

These parameters are substituted in Equation 4.6 to obtain another form of the fundamental performance equation.

$$\frac{dh}{dt} = \frac{1}{\lambda_t} + \frac{V}{\lambda_j} - \frac{\lambda_s \cos^2 \theta}{V \sigma} - \frac{\sigma V^3}{\lambda_p}$$
(4.71)

Likewise it can be seen that

$$w_{h} = \frac{1}{\lambda_{t}} + \frac{V}{\lambda_{j}}$$

$$w_{y} = \frac{\lambda_{s} \cos^{2} \theta}{V \sigma} + \frac{\sigma V^{3}}{\lambda_{p}}$$

$$w_{y} = \frac{\lambda_{s}}{V \sigma} \left(1 - \frac{C^{2}}{V^{2}}\right) + \frac{\sigma V^{3}}{\lambda_{p}}$$
(4.72)

or

.

C. The Equation in Engineering Parameters

The lambda parameters are defined in the foot-poundsecond units which are unwieldy in engineering use. Corresponding parameters utilizing engineering units are defined below.

Span loading
$$\equiv l_s = \frac{W}{eb}$$

Parasite loading $\equiv l_p = \frac{W}{f}$
Thrust loading $\equiv l_t = \frac{W}{P!}$
(4.8)

Jet loading
$$\equiv l_j = \frac{W}{T_j}$$

These parameters are introduced in Equation 4.6 to obtain another form of the fundamental performance equation.

$$l_{t} \frac{dh}{dt} = 550 + 1.47 \ V' \frac{l_{t}}{l_{j}} - 183 \frac{\cos^{2} \theta}{\sigma \ V'} \ l_{s} l_{t} \neq 0.00375 \ \sigma \ V'^{3} \left(\frac{l_{t}}{l_{p}}\right) \ (4.81)$$

Likewise

$$w_{h} = \frac{550}{l_{t}} + \frac{1.47 \text{ V'}}{l_{j}}$$

(4.82)

$$w_{y} = 183 \frac{\cos^{2} \theta}{\sigma V^{1}} l_{s} + 0.00375 \frac{\sigma V^{13}}{l_{p}}$$

D. Concluding Remarks on the Performance Equation

The direct analytical solution of the fundamental performance equation for the general case is impossible. Exhaustive efforts were made to rearrange or re-express the terms of the equation so that they could be combined in a parameter which did not include velocity, and which would assist in a simple solution. It is believed there is no such simple and useful parameter, similar to in the conventional Rockefeller analysis, which can be formed from the terms of the equation.

Several performance analyses were undertaken, and their solution accomplished by graphical means. As a result of this graphical study, it became apparent that several of the performance factors could be analytically approximated if a few assumptions were made.

In the ensuing chapters the method of graphical performance analysis and the analytic approximations are discussed.

Chapter V

GRAPHICAL PERFORMANCE ANALYSIS

A. Nature of the Power Diagrams

In a graphical performance analysis it is convenient to use power rather than force diagrams. In the analysis of the performance of conventional aircraft, the power required to steadily propel the aircraft is plotted as a function of speed. Similarly the power available from the power plant is plotted on the same coordinates. The speed at which the power available and power required curves intersect is the maximum speed. The speed at which these two curves have their maximum separation is the speed for best climb. The maximum separation is the maximum excess power which, if divided by the aircraft weight, will yield the maximum rate of climb.

A similar analysis is made for the jet-internal combustion engine combination. However it is somewhat simpler, in this case, to use indicated quantities for the speed-power coordinates. In this way one power required curve suffices for all altitudes. Jet power curves likewise do not change with altitude. However, the power available from the internal combustion engine is a function of altitude, and a different curve has to be plotted for each altitude considered.

Fig. 5-1 shows a typical indicated power required curve. The indicated power available curve is the sum of jet and internal combustion powers. It is somewhat simpler to plot on the indicated coordinates the function "power required minus jet power". One curve represents this function for all altitudes for any jet thrust considered. The relation of this curve to the indicated internal combustion power curves for different altitudes is used for performance prediction. Fig. 5-2 depicts this method.

In the case of the power required curve plotted on indicated coordinates, a straight line can be drawn from the origin tangent to the curve. The tangency occurs at the indicated speed for maximum lift to drag ratio. This is the speed at which greatest range and endurance can be achieved using the jet alone. If the tangent of the angle which this line makes with the abscissa is equal to the jet thrust, then the line represents the jet power at all altitudes. Furthermore this tangent line represents the minimum jet thrust which will just keep the aircraft in flight at any altitude with no power contribution from the internal combustion engine. This minimum thrust is called the critical jet thrust, T_{jc} . An aircraft equipped with a jet motor supplying a thrust exceeding this critical value has an unlimited ceiling. These facts are illustrated in Fig. 5-3.

It is desirable to define a jet thrust which by itself results in some finite rate of climb. Accordingly, "service jet thrust" is the jet thrust required to produce an indicated rate of climb of 1000 f.p.m. at any altitude, i.e.



climb = 1000/ σ ft./min. The service jet thrust may be graphically determined in the following manner:

 From the aircraft weight compute the indicated excess power, required to produce an indicated climb of 1000 f.p.m.

$$P_{e} = 1000 W$$

2. Add this value to the power required curve, i.e., plot $(P_e + P_r) \sqrt{\sigma}$

3. From the origin draw a straight line tangent to the new curve.

4. The tangent of the angle which this new line makes with the abscissa is the service jet thrust, T_{is}.

This construction is illustrated in Fig. 5-3.

Another form of possible graphical analysis is illustrated in Fig. 5-4. The coordinates are indicated air speed and indicated vertical speed, w. In this method the power required curve is replaced by a curve representing the indicated hypothetical sinking speed of the aircraft, $w_y \sqrt{\sigma}$ (See Chapter IV). The indicated rising speed, $w_h \sqrt{\sigma}$ is the sum of the rising speeds due to the internal combustion engine, $w_i \sqrt{\sigma}$; and the jet, $w_j \sqrt{\sigma}$. It is plotted on the same coordinates. The intersection of the two curves occurs at maximum indicated speed, the greatest separation of the curve is the maximum indicated rate of climb. A straight line drawn from the origin represents the indicated jet



rising speed, where the tangent of the angle which the line makes with the abscissa equals the reciprocal of the jet loading, $1/l_j$.

B. Power Required

The determination of the indicated power required curve is the first step in graphical analysis. The expression for the "hypothetical power required" is taken from Chapter IV.

$$P_{y} = \frac{2W^{2} \cos^{2} \theta}{\pi \rho_{o} \sigma V b^{2} e} + \frac{1}{2} f \rho_{o} \sigma V^{3}$$
(5.1)

Multiplying by σ and substituting I for $\cos^2 \theta$

$$P_{y} \sqrt{\sigma} = \frac{I}{\sqrt{\sigma}} \left(\frac{2 W^{2}}{\pi \rho_{o} b^{2} e} \right) + \frac{1}{2} (\sqrt{\sigma})^{3} f \rho_{o} \quad (5.11)$$

which in engineering units is

$$P_{y}' = \frac{I}{\nabla \sqrt{\sigma}} \left(\frac{0.332 \text{ W}^{2}}{b^{2} \text{ e}} \right) + 6.82 \text{ f} \left(\frac{\nabla \sqrt{\sigma}}{100} \right)^{3} \quad (5.12)$$

When I equals one, $P_y = P_r$. In graphing this expression it is desirable to plot a solid line for I = 1, and dashed lines at the lower speed ranges for arbitrary values of I, say 0.9, 0.8 and 0.7. The power diagram constructed for the analysis of the "Composite Pursuit" illustrates this method in Fig. 5-5.

C. Power Available

The power available is the sum of jet and internal combustion engine powers

$$P_a = P_j + P_i \tag{5.2}$$

$$P_a = V T_j + ? P$$
 (5.21)

$$P_{a} \overline{\sigma} = V \overline{\sigma} T_{j} + 9 P \overline{\sigma}$$
 (5.22)

$$P_{a}' \sqrt{\sigma} = 0.00267 \ V' \sqrt{\sigma} T_{j} + 9 P' \sqrt{\sigma} (5.23)$$

Because the first term of this expression is linear, and independent of altitude on the indicated coordinates, it may be convenient to plot it separately. In the second term both **?** and P' are functions of the altitude so that this term is different for each altitude investigated.

In Figs. 5-5 and 5-7 the engine power available for seven different altitudes has been graphed, and jet power available for two (and four) different thrusts has been graphically added to it. This results in over twenty power available curves used in the analysis.

The jet power curves are simply constructed by drawing a straight line from the origin through a point whose coordinates satisfy Equation 3.1. For instance the ordinate of this line at 200 mph indicated velocity is

$$P_j' \sqrt{\sigma} = 0.534 T_j$$
 (5.31)


Fig 5-5

+3

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.

and the thrust of any jet, a point on whose power line has coordinates 200 mph indicated and $P_j' = 75$, is

$$T_j = 1.874 P_j' \sqrt{\sigma}$$
 (5.32)

The determination of the second term of Equation 5.23 is more difficult. The engine power available depends on the supercharging and general characteristics of the engine and propeller. In the investigations made in this paper, engines were considered to have geared superchargers. Rated BHP was assumed constant from sea level to critical altitude, and was considered to decrease thereafter in accordance with the function $\left(\frac{\sigma - 0.117}{0.883}\right)x$ fictitious horsepower, where sigma is the atmospheric density ratio.

To simplify the numerous calculations, the following assumptions were made regarding the propeller:

 Chosen for maximum speed, no jet thrust, at critical altitude.

2. No variation of propulsive efficiency with altitude.

3. The reduction of propeller efficiencies, T_{r_c} , at speeds different from the design speed is presumed to conform with the function graphed in Fig. 5-8 where R_{V_c} is the ratio of the indicated velocity involved, to the design indicated velocity.

This figure was determined by calculating the efficiencies for several differently chosen propellers in accordance with NACA TR594. The function shown in the figure fitted approximately all the calculations made. It is believed to fairly well represent the variation in propulsive efficiency with indicated speed for the designs considered.

D. Performance Diagram

The power diagram used in the graphical analysis of a pursuit aircraft is shown in Figs. 5-5 and 5-7. Section A describes the method used to obtain top speed, climbing speed, maximum rate of climb which correspond to the different power available curves, i.e., the different altitudes and jet thrusts. At altitudes above 56,000 ft. the engine power is zero and the entire power available is supplied by the jet. Since the indicated jet power curve does not vary with altitude, the indicated power available curve above 56,000 ft. is the same at all increasing altitudes. That is the indicated top speed, the indicated speed for best climb, and the indicated maximum climb remain constant. It is only necessary to apply the density ratio, σ , to get the true value of these quantities at the different altitudes.

The performance diagram is illustrated in Fig. 5-10. The data compiled from the power diagrams are used to construct this figure. Against the altitude ordinate are plotted, for the different jet thrusts considered, rate of climb, time to climb, best climbing speed, top speed. The ceiling is the value of the ordinate where it is intersected by the rate of climb curve; and also where the speed for best climb and top speed intersect. Note that for jet thrusts above the critical value the ceiling is unlimited. The dashed





Fig 5-10

curve labeled T_{jc} shows the only speed at which level flight can be maintained with minimum jet thrust. This is the most economic operating condition under jet power. Level speeds less than this curve can be maintained only by the use of a greater jet thrust, because of the increase in induced drag. If this curve were continued to lower altitudes, it would be tangent to the speed envelopes corresponding to other jet thrusts at their ceiling. This fact is of assistance in constructing the performance curve.

The time to climb curves result from the graphical evaluation at different altitudes of the integral

$$t = \int_{0}^{h} \frac{1}{C} dh$$

and is illustrated in Fig. 5-11.

The long dashed lines represent respectively speeds corresponding to Mach's number 0.7 and 0.8. The decrease in speed with altitude to the tropopause is corroborated by measurements. The parabolic increase in the stratosphere is indicated by indirect observations of the molecular temperature. Since shock wave formation results from an approach to Mach's number of one, the useful portion of the performance diagram must be to the left of a line representing a Mach's number somewhat less than one. The shape of such curves much above sixty thousand feet is unsubstantiated, and indeed the importance and relation of Mach's number to drag at such low densities and high temperature is unknown.



The results of the analysis of the P-40 airplane with jet thrusts of 400 and 800 lbs. are shown in the performance diagram, Fig. 5-12.

E. Effect of the Jet on Performance

The effect of the jet on the performance of two aircraft is illustrated in Fig. 5-13. The solid lines refer to the P-40; the dashed lines to a "Composite Pursuit". The physical characteristics of these airplanes are:

> P-40 Composite Gross Weight 6,769 lb. 10,500 lb. 37.3 ft. Wing Span 40 ft. (assumed) 4.33 ft: 4.58 ft? Parasite Area 1,090 HP 1,900 HP BHP Critical Altitude 12,000 ft. 22,000 ft.

In the figure, the ordinate is jet thrust; the abscissa is the item of performance considered. The curves show the effect of jet thrust on top speed, ceiling, time to climb, and rate of climb. There is also a curve associated with the same ordinate showing rate of propellant consumption for different jet velocities.

SYNOPSIS OF THE EFFECT OF JET THRUST

Ceiling:

The response of ceiling to jet thrust is nearly linear, but somewhat greater at the increased thrusts. The degree of response is dependent on the ratio of span loading (W/b^2e) to parasite loading (W/f) of the aircraft. The fact that



Fig 5-12



Fia 5-13

the ceiling becomes unlimited at fairly low values of jet thrust, known as critical jet thrust, is one of the most interesting results of this investigation.

Climb:

The response of climb to jet thrust is approximately linear and independent of altitude in the troposphere, amounting to about 250 FPM/100 lbs. In the stratosphere the response increases with altitude, and at 60,000 ft. amounts to about 600 FPM/100 lbs.

Likewise the effect of the jet on time to climb is greatly increased at high altitudes.

The effect of the jet at altitude is much greater than at sea level in all cases.

Maximum Speed:

The maximum speed is increasingly effected at increased altitudes; from 8 mph/100 lbs. thrust to 30 mph/100 lbs. thrust at the tropopause. The response to increasing thrust is about linear up to 800 lb. where it begins to drop off. In the two cases studied, the response curves for the critical altitude and above, very nearly intersect at a point. The ordinate of these points is a thrust equal to about sixtenths of the critical thrust for each case. With such a thrust the aircraft would have approximately a constant top speed at all altitudes up to 56,000 ft., and an increasing maximum speed thereafter.

F. Discussion

The preceeding analysis presumes that jet power is available for periods sufficient to reach the steady state conditions which are predicted. This implies the use of a long running jet, which must therefore be of the liquid type. For short term jet operation the acceleration period discussed in Chapter VII is of interest.

The economies are of course not comparable with those of the conventional power plant. But for certain short term super-performance requirements, the high rate of propellant consumption may be excusable. This is a problem in aerial tactics, the answer to which can be based on the performance predictions made in this paper.



Fig 6-1

Chapter VI

APPROXIMATE ANALYTIC PERFORMANCE PREDICTION

A. Critical Jet Thrust

The critical jet thrust corresponds to a jet power curve tangent to the power required curve as illustrated in Fig. 6-1. The point of tangency occurs at the speed for maximum lift-drag ratio. The analytic expression for critical jet thrust may be obtained by writing the expression for sinking speed (see Chapter IV).

$$w_{g} = \frac{\lambda_{g}}{\sigma_{V}} + \frac{\sigma V^{3}}{\lambda_{p}}$$
(6.11)

$$w_{s} = \frac{DV}{W}$$
(6.12)

 $\frac{D}{L} = \frac{\lambda_{s}}{\sigma V^{2}} + \frac{\sigma V^{2}}{\lambda_{p}}$ (6.13)

$$\frac{d \mathbf{E}}{d\mathbf{V}} = -\frac{2 \lambda_{s}}{\sigma \, \mathbf{V}^{3}} + \frac{2 \sigma \, \mathbf{V}}{\lambda_{p}} \tag{6.14}$$

Equating the differential to zero to determine the condition for maximum lift to drag ratio,

$$V_{LD} \sqrt{\sigma} = (\lambda_s \lambda_p)^{\frac{1}{4}}$$

$$V'_{LD} \sqrt{\sigma} = 14.9 (l_s l_p)^{\frac{1}{4}}$$
(6.15)

Substituting this value of indicated velocity in Equation 6.13

$$\begin{pmatrix} D \\ L \end{pmatrix}_{LD} = \frac{\lambda_{\rm g}}{(\lambda_{\rm g} \ \lambda_{\rm p})^{\frac{1}{2}}} + \frac{(\lambda_{\rm g} \ \lambda_{\rm p})^{\frac{1}{2}}}{\lambda_{\rm p}}$$
(6.16)

From which

$$D_{LD} = 2W \left(\frac{\lambda_{s}}{\lambda_{p}}\right)^{\frac{1}{2}}$$
(6.17)

It is apparent from Fig. 6-1 that

$$D_{LD} = T_{j_{c}}$$

$$T_{j_{c}} = 2W \left(\frac{\lambda_{s}}{\lambda_{p}}\right)^{\frac{1}{2}}$$

$$T_{j_{c}} = 1.13 W \left(\frac{1_{s}}{1_{p}}\right)^{\frac{1}{2}}$$

$$T_{j_{c}} = 1.13 \frac{W}{b} \left(\frac{f}{e}\right)^{\frac{1}{2}}$$
(6.18)

The expression for the critical jet thrust and for the indicated velocity for maximum lift-drag ratio (Equation 6.15) are extremely useful in constructing a performance diagram.

B. Maximum Speed

Approximate maximum speed analysis can be made by ignoring the induced drag. This results in negligibly optimistic predictions except in the vicinity Δf the maximum ceiling where the induced drag term is predominant.

Then for level flight:

$$V = V_{M}$$
 $\frac{dh}{dt} = 0$ $\theta = 0$ $\lambda_{s} = 0$

and Equation 4.71 becomes

$$\frac{\nabla_{M}^{3}}{\lambda_{p}} = \frac{1}{\lambda_{t}} + \frac{\nabla_{M}}{\lambda_{j}}$$
(6.21)

$$\mathbb{V}_{M}^{\mathbf{3}} \left[\frac{\sigma \lambda_{t}}{\lambda_{p}} \right]^{\mathbf{1}_{\mathbf{3}}} = 1 + \mathbb{V}_{M} \left[\frac{\lambda_{t}}{\lambda_{j}} \right]^{\mathbf{1}_{\mathbf{3}}}$$
(6.22)

Define:

Then 6.22 becomes

$$\int \int (6.24)$$

$$\Lambda_{j} = \frac{(\lambda_{t}^{2} \lambda_{p})^{\frac{1}{3}}}{\lambda_{j} \sigma^{\frac{1}{3}}}$$
(6.25)

$$\Lambda_{j}' = \frac{(l_{t}^{2} l_{p}/\sigma)^{1/3}}{l_{j}} = 7.11 \Lambda_{j}$$
 (6.26)

$$\Lambda_{j} = 7.11 \quad \frac{\Lambda^{3} - 1}{-\Lambda}$$
 (6.27)

From Equation 6.23a

$$V_{\rm M}' = 52.73 \, \left(\frac{l_{\rm p}}{\sigma \, l_{\rm t}} \right)^{1/3} \tag{6.28}$$

To determine the maximum velocity, compute Λ'_j from Equation 6.26. Use it in Equation 6.27 to determine $-\Lambda$. This solution has been accomplished graphically in Fig. 6-2. Use $-\Lambda$ in Equation 6.28 to determine maximum speed.

The simultaneous solution of these three equations has been accomplished in Fig. 6-3. This graph is entered with Λ'_j and $l_p/\sigma l_t$ to determine the maximum velocity in miles per hour.

The approximate analytic solution for climb will be completed and supplied as an insert to this report at an early date.





THE ACCELERATION PERIOD

The preceeding analysis deals only with the steady state condition. The time required to reach this limiting condition from any flight condition is of definite importance.

In the case of "jet on" climb from "jet off" level flight at or above climbing speed, the maximum climb can be reached at once. The acceleration period is negligible. This results from the fact that the aircraft speed does not have to be increased. The only acceleration involved is angular acceleration resulting from actuation of the controls in assuming the climbing attitude.

In the case of acceleration to a "jet on" top speed, the case is somewhat different. The acceleration time is considerable and is expressed by the integral

$$t = \frac{W}{g} \int \frac{V_1}{V_2} \frac{V}{P_e} dV$$
 (7.1)

$$F = m a$$

$$\frac{P_e}{V} = \frac{W}{g} \frac{dv}{dt}$$
or
$$dt = \frac{WV}{P_e g} dv$$

*

where Pe is a function of velocity



Fig 7-1



The evaluation of this integral has been performed graphically in the case of the composite pursuit in Fig. 7-1. It is convenient to use indicated excess power and indicated velocities obtained directly from the power diagram in this calculation. From an inspection of Equation 7.1 it can be seen that such a procedure will yield a time multiplied by $\sqrt{\sigma}$, or an "indicated time". This value must of course be divided by the $\sqrt{\sigma}$.

The results of two such solutions are shown in Figs. 7-2 and 7-3. Fig. 7-2 shows the time to accelerate between different speeds at a great many different jet thrusts for the P-40 at 15,000 ft. Fig. 7-3 shows the acceleration time for the composite pursuit with different thrusts at various altitudes. The general result is that it requires about 30 or 40 sec. to accelerate from conventional top speed to the vicinity of jet top speed for jet installations that might logically be considered. A desirable pilot technique would be a short dive to assist in reducing this acceleration period.



Fig 7-3

Chapter VIII

ASSISTED TAKE OFF TO INCREASE THE RANGE OF FLYING BOATS

A. Discussion

Detailed studies have been made of the assisted launching of aircraft on wheels by the use of jets. The investigations have included the launching of land planes from conventional runways as well as the launching of flying boats from rolling carriages. The results have shown that such an application of jets will yield a substantial increase in gross weight that can be launched in a given distance, or a small decrease in the launching distance for the normal gross weight.

In these studies the launching distance is a vital factor, limited by the size of operating fields, or by the length of launching track which could be installed. As a required result the jet thrust is very high. The weight of the jet motor, accessories, fuel storage, and mounting becomes so very large as to logically require the detachment of the jet apparatus from the aircraft after launching.

The disadvantages of such an application are threefold:

1. The propellant consumption is so very great that the cost of this item alone in the assisted launching of a medium bomber is of considerable importance.

2. The apparatus for assisted take off does not remain with the aircraft; and in the case of operations to advanced bases, this fact places an additional load on the logistic problem.

3. In the case of flying boats, the carriage and track installation is not feasible at advanced bases which may be only casually used. The boats are therefore deprived of their additional gross weight (increased range) at precisely the time it may be most critically needed.

It is proposed that the most advantageous application of the jet to the assisted take off problem is the launching of overloaded flying boats from the water utilizing powder jets built integral with the hull.

In such an application the length of take off run is of secondary importance. Accordingly the jet power can be conserved until the acceleration under engine power virtually ceases. Applied at such a time, the required duration of jet operation is reduced. This type of installation has the tremendous advantage that the aircraft is at all times equipped to accomplish an overload take off. No involved carriage, track, accessory jet motor, or catapult, need be made available. The only requirement for the aircraft to use any suitable water surface as a base, is that there be available a fuel cache or service which includes a supply of jet propellant.

B. Best Jet Thrust for Maximum Overload of PBY

The increase in aircraft weight resulting from the jet installation is the motor and mounting weights. For powder jets these weights can be considered approximately proportional to the weight of the propellant burned. The actual propellant weight is of no concern since it is not aboard the



Fig 8-1

aircraft in flight. This section deals with the investigation of the best jet thrust to use in order to keep the installation weight to a minimum.

The investigation is made for PBY aircraft with two 1200 BHP engines. The thrust and drag data for this airplane was kindly furnished by the Consolidated Aircraft Co.

A study of the take off resistance curves for this airplane at different gross weights shows that they can be closely approximated by a single resistance curve plotted to weight reduced coordinates. These weight reduced parameters are defined in Fig. 8-1.

The following more or less arbitrary criterion is established for the study:

1. Take off accelerations less than 0.027 gravity are unsatisfactory, i.e., when this value is reached on take off, jet thrust is applied and constantly maintained until this acceleration can again be supplied by engines alone for the remainder of the take off.

2. Wind calm. Wind assisting take off of hydroplanes has the general effect of a reduction in gross weight.

The dashed line in Fig. 8-1 shows the thrust required to accomplish this minimum acceleration.

Since it is understood that 35,000 lb. is the maximum gross weight to which a PBY can be loaded without change in step position, this is the weight chosen for first consideration. In Fig. 8-1 the thrust available line corresponding to 35,000 lb. gross weight crosses the desired thrust line initially at $V_W = 23.5$ MPH and finally $V_W = 68.0$ MPH. Between these speeds the acceleration is assisted by jets. Different values of constant thrust were considered to be applied for this period and the total propellant consumption computed for each case. This was done by determining the time for acceleration by a graphical solution of the integral

$$t = \frac{W}{g \sqrt{\epsilon}} \int_{V_1}^{V_2} \frac{dV_W}{T_{e_W}}$$
(8.1)

and multiplying the time by the rate of propellant consumption taken from Fig. 2-2. A jet velocity of 8,000 f.p.s. was assumed.

The plot of weight of propellant required to accomplish this take off versus jet thrust used is shown in Fig. 8-2. It may be seen that the least thrust which will satisfactorily accomplish take off will be the most economical. Operation at the minimum of the curve is not practical because the small excess thrust corresponding to this condition provides insufficient margin for variation in pilot technique and consequent increase in take off resistance.

The minimum satisfactory thrust of 1170 lb. requires a jet operation period of 48 seconds. This is a long term for powder jets, and the practical solution of the problem leads to the consideration of two sets of jets fired in sequence. Further consideration of the thrust-resistance curves of Fig. 8-1 shows that after passing the first resistance hump there is a period during which the thrust available closely approximates the desired thrust. The first set of jets could be designed to carry through the first hump, after which acceleration would continue under engine power until negligible increase in speed was experienced by the pilot. The ignition of the second set of jets would then be accomplished for acceleration thru the second hump. By this technique the total period of jet operations could be shortened to about 33 seconds. Furthermore acceleration through the first hump would only require a thrust of 575 lb. as against 1170 lb. for the second. This technique would result in a considerable saving in jet weight at the expense of an increase in take off distance.

In the case of the PBY the second hump is the critical one. In fact, at 35,000 pounds gross weight, the first hump could be crossed with very meagre accelerations on engine thrust alone; with the jet becoming absolutely essential at the second hump. However this type of marginal take off has so long been the bane of overload operation that it would seem desirable to provide a positive excess acceleration through out the run.

C. Best Jet Thrust for Overload Critical at Second Hump

At a gross weight of 32,300 lb. the thrust does not drop below desirable thrust until the second hump. An analysis of this condition shows a minimum satisfactory thrust of 635



lb. must be applied between $V'_W = 52$ MPH and $V'_W = 67$ MPH which for this thrust requires **24** seconds. The propellant consumption for this case was studied at different jet thrusts and the results plotted in Fig. 8.2. Again it is seen that the minimum thrust which will supply satisfactory accelerations is the most economical.

It is interesting to consider the weight of propellant required for the jet to accelerate the flying boat between two speeds in the take off run, assuming that the internal combustion engines just supply a thrust equal to the resistance.

From Newton's Law

$$T_{j} = -\frac{W}{g} a \qquad (8.2)$$

$$T_{j} = c \frac{W_{P}}{gt}$$
(8.3)

Combining the two equations

$$W_{\rm P} = \frac{\rm at \ W}{\rm c} \tag{8.4}$$

or

$$W_{\rm P} = -\frac{W}{c} (V_2 - V_1)$$
 (8.5)

For this hypothetical case it can be seen that the propellant weight is not a function of the time, jet thrust, or acceleration.

D. Design Consideration

From a general consideration of the problem it would seem desirable to have the jet thrust simply adjustable to meet the demands of various effective gross weights, that is weights as effected by take off wind. This is most easily accomplished by dividing the jet thrust between several small jet motors, each utilizing a standard size powder cartridge. The pilot could then determine the number of jets to use on each hump from his loading condition and wind.

The ignition timing of the jet is a matter which can eventually yield additional economy. After the pilot has actuated the jet thrust, the individual jets need not be simultaneously ignited but could be automatically timed to ignite in succession at appropriate intervals. Thus the peak thrust would occur at approximately peak resistance. The thrust would taper in either direction from the peak with a corresponding economy in propellant consumption (also in jet motor weight).

The location of the jets is a matter of prime importance for balance, utility, and structural reasons. Installation in the boat step has been suggested. This conception finds the flying boat peculiarly adaptable to jet auxiliary power. The step forms a natural and ample exit for the jets. This underwater region is at low pressure during take off and therefore conducive to proper jet operation. Jets installed in the step could be easily arranged so their centerlines passed near the CG, thus contributing negligible moments to the aircraft. Structurally the application of thrust at the step is a logical thing, requiring the transfer of thrust forces only very short distances to the regions of application of resistance forces.

Chapter IX

DISCUSSION AND CONCLUSION

A. General Design Problems

The installation of jet motors in aircraft is confronted with four problems. They are, structural, balance, jet exhaust location, and considerations of accidents.

The structural problem is one which will yield to normal attack.

The problem of balance is best handled by striving not to introduce eccentric forces. When this is impossible, moment arms should be kept small and should be in the plane in which the aircraft has the greatest margin of control. Land planes with tricycle landing gear are particularly adaptable to an integral jet installation in the after belly. In such a location the jet center lines may be easily installed to pass near the CG. Detachable jet motors will undoubtedly introduce some eccentric thrust because of their probable attachment in the vicinity of the main landing gear.

The jet exhaust may be considered to present a fire hazard for a distance of about five feet thru a cone whose apex angle is about twenty five degrees.

The most serious accident possibility which must be considered is the failure of an exhaust plate and nozzle. The high velocity discharge of this mass rearward would result in a serious forward reaction of the jet motor. The use of several small units, rather than one large jet would tend
to minimize this danger, by making it only necessary to consider the reaction of a much smaller unit. The use of guide bolts with heads, whereby the runaway exhaust plate could couple some of its energy back into neutralizing the motor reaction, may be a partial solution.

The development of the liquid motor with liquid cooling promises to supply a source of jet thrust for long term operation. This will be the motor used in supersonic flight, and may perhaps find an application in subsonic superperformance designs. The liquid motor presents the installation problems previously described and in addition calls for the following special services:

- 1. Cooling system
- 2. Suitable propellant storage
- 3. Propellant pumping system

The first two may be supplied by conventional methods. The propellant pumping, if handled in a direct manner, calls for considerable power. A solution has been suggested for this problem consisting of placing the propellant storage tanks under the jet combustion chamber pressure. The only auxiliary power then required is that needed to overcome the drop across the injection orifice.

B. Recommendations

The jet powder motor has reached the development stage where its operation in aircraft should be investigated. The hand in hand solution of further problems of the motor, and problems of installation and operation will most rapidly result in its early practical application.

The problem of an efficient, stable, fuel for the liquid motor is one calling for considerable investigation. The ultimate propellant would be a liquid under atmospheric pressures and temperatures, and should be chemically stable. It should contain a minimum of dormant elements, that is should consist primarily of oxygen and uncombined hydrocarbons. The nearest approach to this ideal propellant is liquid oxygen and gasoline. The pressure required to maintain the oxygen in liquid state is, of course, the disqualifying factor in this combination. Other fuel combinations are under consideration whose physical state are adaptable to aircraft storage facilities. They all suffer the disadvantage of containing large percentages of dormant elements, which radically reduce the jet velocity. A number of suggestions have been made, including cellulose solutions, and solutions of ozone and an unsaturated hydrocarbon. Most of these suggestions involve combinations which primarily are dangerously unstable. A qualified investigation in this field might yield the ideal liquid propellant.

The development of the liquid motor will supply motive power for supersonic flight. A much increased supply of information regarding aerodynamic phenomenae in this range is sorely needed. In particular, investigations should be conducted to determine:

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1. Propeller efficiencies and proper design at supersonic speeds and very low densities.

2. The behavior of airfoils and simple shapes at supersonic speeds and very low densities.

3. The temperature lapse rate in the higher strato-