EXPLORATORY STUDIES OF OPEN-CHANNEL FLOW

OVER LATERALLY VARYING ROUGHNESS

Thesis by

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ABSTRACT

An exploratory study was made of open-channel flow over beds consisting either entirely or partially of large granular roughness. Steady, uniform flow was established at various depths and velocities over two types of beds, one rough over the entire width of a laboratory flume, the other rough only over half the width and smooth over the other half. Friction factors were determined for these flows, and detailed velocity distributions were measured in three runs.

The friction factors for the entirely rough beds compared closely with those predicted by the Karman-Prandtl equations, and the velocity distributions strongly suggested the existence of secondary circulation of the second kind.

Analysis is offered to show that subdivision of the cross section of a turbulent flow by curves normal to the equal velocity curves does not result in hydraulically independent zones of flow, in that there will be turbulent interchange of the longitudinal component of momentum among such zones; other methods of subdivision are considered and none found to be completely satisfactory.

The customary side-wall correction method is reviewed and found to have no explicit rational basis, and although it is recognized that the method gives reliable results in the situations to which it is usually applied, its application to widely different situations should be undertaken with caution.

Suggestions for needed further research are offered.

ACKNOWLEDGEMENTS

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I. INTRODUCTION

1. General observations

A river is a flow of water and sediment, whose behavior is governed by the water and sediment loads imposed on it and by the nature of the valley in which it flows. The regimes of rivers are continually being altered by man, by the construction or operation of dams, diversion works, or straightening and deepening operations, and the effects of such activity often extend many miles up and down the stream from the actual site of the activity. Hence, a complete assessment of the impact of such man-made changes is important to orderly planning of water resources development and control. However, mechanics of flow in rivers is still not very well understood.

The turbulent flow of water in flumes, lined canals, and other rigid conduits of homogeneous boundary roughness is well understood in its gross aspects, conceptual models of the structure of the turbulence have been fairly successful. Von Karman's similarity hypothesis, originally proposed to apply to the region of flow in the general neighborhood of the boundary, is the basis of workable and successful descriptions of the velocity distribution and over-all resistance to flow found in such channels.

Once consideration is given to channels consisting of loose, movable material (e.g., most natural streams, flowing over sediment), the picture is much less encouraging. For not only must one then investigate the rate of movement of the channel material (i.e., sediment), but one also soon realizes that the resistance-to-flow relation is much more complicated. Where in the former case a friction factor could be related to the stream Reynolds number and to an index of the roughness of the fixed boundary, now the boundary is no longer fixed, and its shape

depends on flow conditions and on the rate of sediment transport. Thus in studying alluvial streams, one seeks relations for both sediment and fluid transport (commonly referred to as "transport" and "resistance-to-flow" or "roughness" relations, respectively).

Of these two functional relations, the transport relation has had much more attention in the past, although there have been several recent efforts to understand the roughness relation. These efforts vary in sophistication and success, and will be only very briefly reviewed here. Each of the principal papers dealing with resistance to flow in alluvial open channels starts by postulating that the boundary shear stress may be considered as the sum of two parts. The first part is that shear which would be felt by a fixed, flat bed of the same texture as that of the actual bed, and is usually referred to as the "grain resistance". The remainder (the "form drag") is considered to arise principally from the effect of bed forms (ripples, dunes, etc.), although being a residual term, it necessarily includes all side effects.* The idea of dividing the shear seems to have been suggested by Meyer-Peter and Müller (1), who chose to express this division in terms of a corresponding division of the energy slope. Einstein (2,3) expressed the division in terms of two hydraulic radii, and he and Barbarossa (4) have published the analysis of some field observations on the relation between a shear parameter and the resistance to flow, in which the effect of the bed forms ("form" or "bar resistance") is expressed in terms embodying the partitioned hydraulic radius. Various workers at the Colorado State University (5, 6, and 7, for example) have written on the subject, depending heavily on dimensional analysis to order and extend the results of their laboratory work. Taylor and Brooks (8) and Taylor (9) have

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*These include effects of channel alignment and changes in shape.

attempted to simplify the means of describing bed configurations, now
largely a subjective process; the former reference is reproduced as an
appendix hereto (pp. 59-69).

All the studies mentioned above have depended either on very
generalized descriptions of the natural channel studied (as in Einstein
and Barbarossa) or on very idealized laboratory models of natural chan-
nels. In the typical laboratory flume, experiments are frequently ar-
ranged so as to have as nearly a two-dimensional situation as possible.
Such simplicity is seldom found in natural streams, it being quite com-
mon to find marked lateral variations of bed configuration and roughness
in such streams, and to find those variations associated with variations
in depth as well. Therefore, the extent to which the laboratory results
apply to the behavior of natural streams is problematical.

In order to explore the influence of lateral variations in bed
roughness, a series of flume experiments was undertaken, wherein the ef-
fect of a large and sudden lateral change in roughness could be studied.
Large roughness elements were selected so as to demonstrate clearly any
peculiarities in this flow situation. A comparison series was under-
taken, to calibrate the roughness elements used; the results of this
series were also intended to serve as a verification of the Karman-
Prandtl equations for rough, two-dimensional channel flow, in a range of
high values of relative roughness.

2. Flow over homogeneous roughness

It is appropriate to review some of the relevant expressions
for resistance to flow in circular pipes and open channels. Since
this creates the need to distinguish among friction factors calculated

for a variety of situations, the following special notation will be established.

For calculated friction factors, we define

Symbol	Description	Defining Equation
f_{so}	Flow in smooth circular pipes	3
f_{ro}	Flow in rough circular pipes	4
f_s	Flow in smooth two-dimensional channels	5
f_r	Flow in rough two-dimensional channels	6

Also, for describing rectangular channels of finite width, and for typographical simplicity, we define

W	Ratio of width, b, to depth, d	$W = b/d$
E	Relative roughness	$E = \epsilon/4r$,

where ϵ is the height of the boundary roughness elements.

Other quantities will be defined as introduced; for completeness and convenience all symbols are defined in the Summary of Notation following the text (see page 70).

The Darcy-Weisbach friction factor, f, is defined as

$$f = \frac{8grS}{u^2},\qquad(1)$$

where g is the acceleration of gravity, r is the hydraulic radius of the pipe or channel, S is its energy gradient, and u is the mean velocity. In terms of the Reynolds number* written

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*The factor four is usually included because the Reynolds number thus defined is more convenient for use with pipe-friction diagrams, the diameter of a circular pipe being four hydraulic radii. While this convention is now generally accepted usage in American hydraulic literature, it was not ever thus: Johnson (10), for example, in 58 pages of tables summarizing significant bed-load experiments, uses r as the characteristic length. Furthermore, Schlichting (11) defines the hydraulic radius as twice the area divided by the wetted perimeter, so that the hydraulic and geometric radii are equal for circular sections.

$$R = \frac{4ru}{\gamma},\qquad(2)$$

ν being the kinematic viscosity of the fluid, the Karman-Prandtl equations for resistance to turbulent flow in a circular pipe may be written

$$1/\sqrt{f_{so}} = 2 \log (R \sqrt{f_{so}}) - 0.8 \quad (\text{smooth pipes}, \frac{u_* \epsilon}{\gamma} < 3)\qquad(3)$$

and

$$1/\sqrt{f_{ro}} = -2 \log E + 0.54, \quad (\text{rough pipes}, \frac{u_* \epsilon}{\gamma} > 70)\qquad(4)$$

where u_*, the shear velocity, is given by $u_* = \sqrt{grS}$.

The form of these equations arises from the assumption of a logarithmic velocity distribution; the constants are empirical and differ but slightly from those calculated on the assumption that the logarithmic velocity distribution holds throughout the main body of the flow. The limiting values of $u_* \epsilon / \nu$ are also empirical.

In the case of two-dimensional open-channel flow, there is no dependable experimental determination of the constants, so the corresponding equations are based on the logarithmic velocity distribution assumption without correction, and are customarily taken to be

$$1/\sqrt{f_s} = 2.03 \log (R \sqrt{f_s}) - 0.47 \quad (\text{smooth channels}, \frac{u_* \epsilon}{\gamma} < 3)\qquad(5)$$

and

$$1/\sqrt{f_r} = -2.03 \log E + 0.91. \quad (\text{rough channels}, \frac{u_* \epsilon}{\gamma} > 70)\qquad(6)$$

Extension of the Karman-Prandtl equations to sections which are neither circular nor two-dimensional rests on the work of Keulegan (12), who concluded that for a first approximation, channels with equal hydraulic radii may be considered equivalent as far as flow relationships are concerned. He found that for more careful work, particularly in the case of polygonal cross sections departing widely from the two-

dimensional, it was necessary to introduce a shape factor, β, into the equations corresponding to 5 and 6 above. The effect of this shape factor on the present work can be seen by using run 36 as an example (see Table 2). In this run, the width/depth ratio and relative roughness were $W = 2.32$ and $E = 0.103$. For this value of W, inclusion of β would increase the constant 0.91 in equation 6 to 1.07; for the observed value of E this change corresponds to a reduction in f from 0.122 to 0.109. For shallower runs (i.e., for larger values of W) the corresponding reduction in f would be less.

Powell (13,14) has reported some experiments in rectangular open channels, on the basis of which he takes issue with Keulegan's analysis and conclusions. In the absence of anything definitive on the question, and in view of the exploratory nature of the work reported herein, no attempt was made to include a shape-factor correction in the subsequent analysis.

In the so-called "transition zone," i.e., for $u_* \epsilon / \nu$ between about 3 and 70, the flow resistance is dependent on both the Reynolds number and the relative roughness, and in particular on the form and distribution of roughness elements. Several empirical curves exist for predicting the friction factor in the transition zone, the best known of which are those of Colebrook (15) and of Nikuradse (16). Colebrook's transition is based on experiments with commercial pipes, in which the major resistance to flow is caused by irregularities, all of which tend to be widely spaced with respect to their size; hence this transition curve may be spoken of as applying to isolated roughness elements.

Nikuradse's results were based on experiments with pipes completely lined with sand grains, and his curve may be spoken of as applying

to close-packed granular roughness elements. Colebrook and White (17) experimented with various combinations of isolated and close-packed granular roughness elements and produced transition curves ranging consistently in character between those of Nikuradse and of Colebrook.

Since the roughness elements in the present work were essentially of the close-packed granular type, the Nikuradse transition curve has been assumed. No opportunity arose to verify this assumption, however, since in all runs the gravel-covered part of the bed acted fully rough in the hydrodynamic sense.

3. Flow over boundaries of laterally varying roughness

In Section 1 above, it was stated that most laboratory experiments on the resistance to flow in open channels has been conducted under carefully two-dimensional conditions, but that these experimental conditions fail in important ways to reflect characteristics of natural streams, in which lateral variation in depth and roughness may be very pronounced.

Two examples of this lateral variation in small streams are shown in Figure 1. The data for the smaller stream, Virgin River near St. George, Utah, were included in a memorandum report (18), and those for the other, Galisteo Creek near Domingo, New Mexico, were furnished by the Albuquerque office, U. S. Geological Survey.

In both streams there is marked lateral variation in depth, and in the Virgin the deeper parts of the cross section were associated with a soft, dune-covered bed. The tendency for dune-covered beds to be softer than flat ones has been noticed before, e.g., by Simons (19). Thus, from comments regarding the softness of the sand bed in the deep parts of Galisteo Creek it would seem likely that there were dunes in the band from about 75 to 85 feet (transverse distance). Although the
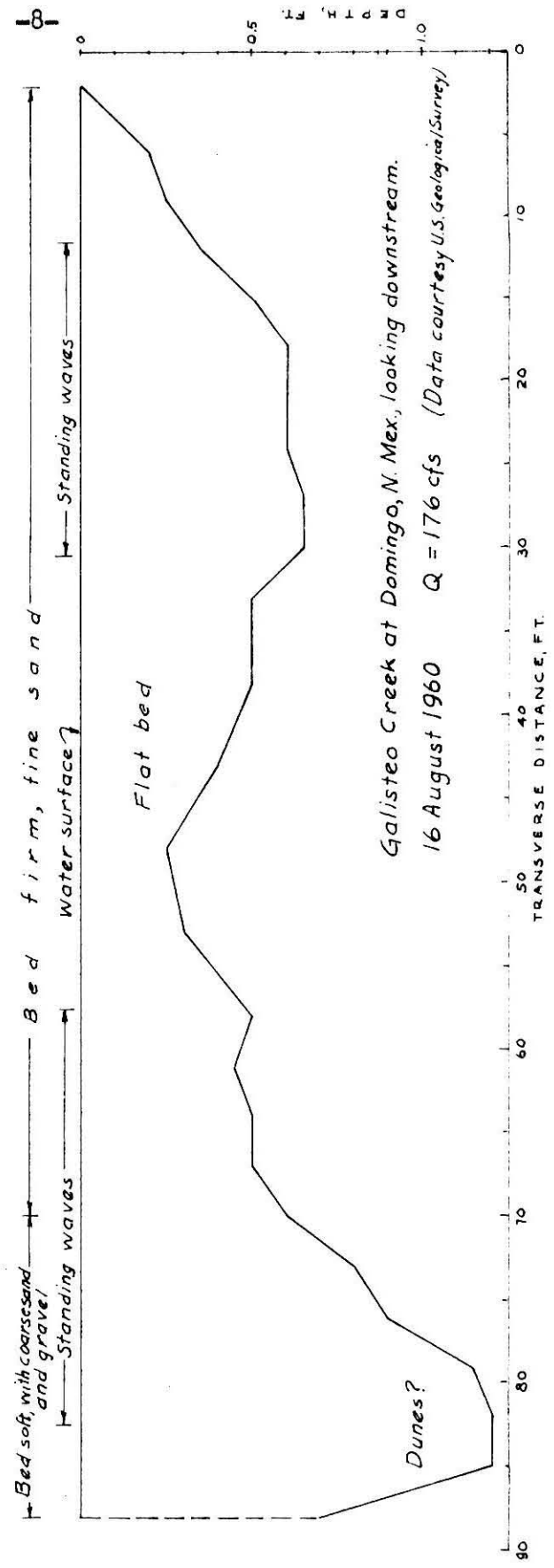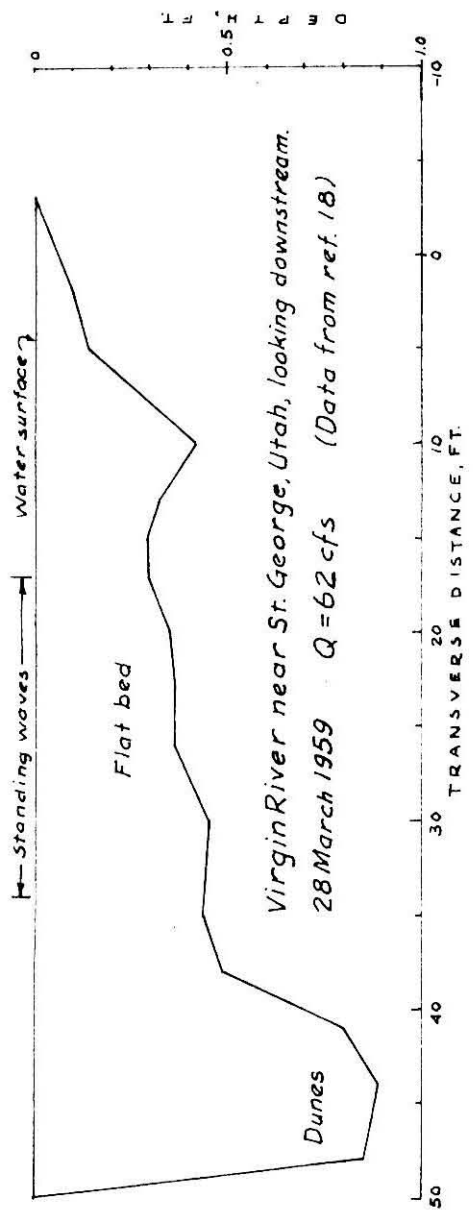
Figure 1. Cross sections of two streams which display lateral variations in depth and bed configuration.

band of standing waves extending to the left of station 70 seems incon-
sistent at first glance, Kennedy (20) has observed standing waves over
a dune-like bed. At any rate, it is clear that in both streams there is
marked lateral variation in flow conditions.

An example of similar variation in a large river is shown in
Figures 2 and 3. Figure 2 shows a cross section of the Missouri River
at Omaha, Nebraska, together with a general map of the river-bed topo-
graphy in the vicinity of the section. From the map and section it is
evident that the left side of the stream is significantly shallower and
smoother than the right half. Figure 3 is a more detailed topographic
map of the right side of the river just above the cross section. This
map shows bed features with amplitudes ranging from three to five feet,
and while their form is irregular, the contours suggest a succession of
dunes or bars.* The maps are based on carefully controlled soundings by
the Missouri River District, U. S. Army Corps of Engineers (21), whose
permission to use the material is appreciated.

It will be noted that the relative difference in depth between
the two types of bed configuration is not much less than in the much
smaller streams shown in Fig. 1. It can be inferred from other reports
that lateral variation in both depth and roughness are fairly common.
Exner (22), for example, reported observations on a sequence of large sand
bars, evenly spaced and on alternate sides of the channel, which were

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*Considering the difference in mapping techniques and in the
ratio of contour interval to dune amplitude, the similarity between
Fig. 3 and Fig. 7 of Thompson (23) is quite striking. Thompson shows
that precision photogrammetric techniques can be used in mapping bed
configurations produced in laboratory flumes, and his Figure 7 is an
example of a bed whose features have an amplitude of about 0.07 feet,
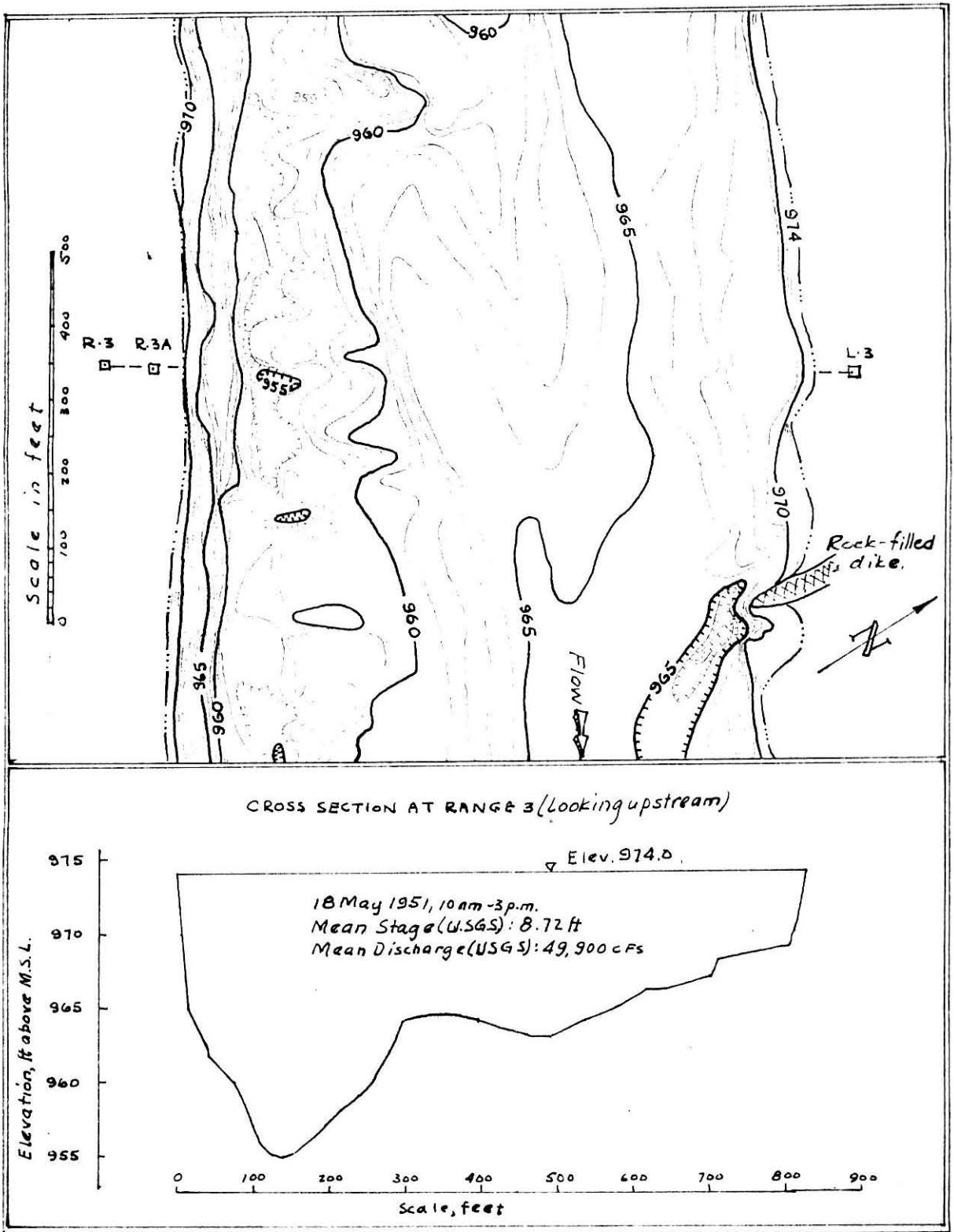shown in a map whose contour interval is 0.01 ft.

Figure 2. Bed Topography, Missouri River near Omaha, Nebraska, 18 May 1951. (Section R-3—L-3 lies approximately at river mile 640.75). After U. S. Army, Corps of Engineers (21).

Figure 3. Detailed Bed Topography, Missouri River near Omaha, Nebraska, 17
May 1951. (Area is near right bank and just upstream from range R-3—L-3;
cf. figure 2.) After U. S. Army, Corps of Engineers (21).

found in a several-kilometer length of the River Mur. His work and some others of interest are reported in Leliavsky (24).

In trying to understand such flow situations and to estimate the resistance to flow which is encountered, the question naturally arises as to the extent to which existing flume data are applicable. Can the cross section of a stream of this type be divided into segments of constant depth and bed configuration, the known results for laboratory models of such segments be applied, and a useful composite result obtained? How should this composite be obtained, and how should it reflect the interactions between adjacent segments of the original flow? The answers to these questions are not clear, but it is quite clear that if workable combining rules can be found, the usefulness of laboratory results for two-dimensional flow might be greatly extended, and the present gap between laboratory and field data reduced materially. Also, as a special case the side-wall correction procedure widely used in the reduction of flume data (see, for example, Johnson (25); cf. note, p.51) could be considered and its validity estimated.

Experiments were undertaken to examine these questions and to estimate their significance. The laboratory work, exploratory in nature, was conducted in a tilting flume 10.5 inches wide, with the gravel covering either the entire bed, or the right or left half. Since variations in depth would have complicated the problem unduly at the present stage, wood inserts were used for the smooth half of the bed to raise it to the same mean level as the gravel bed. The results of those experiments form the basis for this study.

II. EXPERIMENTAL PROCEDURE AND RESULTS

1. Statement of problem

In the interests of simplicity only two bed arrangements were considered. In the first, referred to as the "split" bed, the channel bed was hydrodynamically rough on one side of the centerline and hydrodynamically smooth on the other. In the other, referred to as the "full" bed, the bed was hydrodynamically rough over its entire width. The cross section was rectangular, and the walls were hydrodynamically smooth. All the variables considered significant for the purpose of this investigation are listed below, although for a more careful study others might be added; Figure 4 is a typical cross section and a dimension sketch.
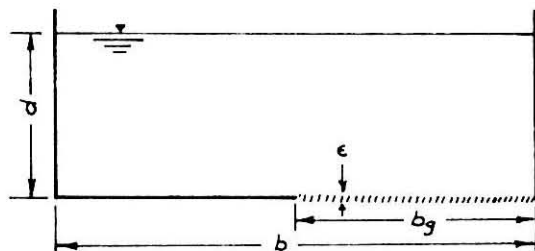
Figure 4

Variables describing channel geometry or fluid properties:

Symbol	Description	Dimensions
b	width of channel	L
b_g	width of rough part of channel	L
ϵ	some measure of roughness height (mean grain diameter, say)	L
ν	kinematic viscosity of fluid	L^2/T
ϱ	density of fluid	FT^2/L^4
g	acceleration of gravity	L/T^2

Variables pertaining principally to the flow:

Q	fluid discharge	L^3/T
u	mean velocity of flow	L/T
d	depth of flow	L
S	energy slope	-
f	Darcy-Weisbach friction factor	-

Of these eleven variables, the first six may be considered as specified for any stream or group of laboratory runs. Of the remainder, only two may be considered as independent variables, and the other three must therefore be considered dependent. For example, in the typical tilting flume experiment Q and d would be known, with V, S, and f to be determined. In a field problem, S and Q might be known, three quantities again remaining to be determined. Considering eleven variables, of which eight are known, three equations are needed. Of these three, two are known, namely the continuity equation,

$$Q = bud, \qquad (7)$$

and the Darcy-Weisbach equation,

$$S = \frac{fu^2}{4r \cdot 2g} . \qquad (8)$$

The latter may be considered a definition of f, in which r, the hydraulic radius, is simply an abbreviation for $bd/(b + 2d)$. The third equation is an as yet unknown flow resistance, or roughness, relation.

Since Q and S are given by eq. 7 and 8, only the other nine variables need be considered in formulating the roughness relation. They may be combined into the following six convenient, independent, dimensionless groups:

$$F = u/\sqrt{gd} \qquad \text{Froude No.}$$

$$R = 4ru/\nu \qquad \text{Reynolds No.} \qquad \Big\} \quad (9)$$

$$\Theta = b_g/(b + 2d) \qquad \text{Roughness distribution ratio}$$

$$f \qquad\qquad \text{Friction Factor}$$
$$E = \in /4r \qquad \text{Relative Roughness} \qquad\qquad\qquad (9)$$
$$W = b/d \qquad\qquad \text{Width-Depth Ratio}$$

Of these F is unimportant so long as there are no wave effects.

Thus, for the present study the problem consists of finding the form of the function φ in

$$f = \varphi (R, \Theta, E, W). \qquad\qquad\qquad (10)$$

In the simple case of flow in a pipe the parameters Θ and W do not appear in the roughness relation.

It should be kept in mind that the foregoing analysis presumes no movement of the bed material. If there is sediment movement, an additional variable must be included to reflect the rate of sediment transport. There will also be variables describing the sediment itself (e.g., its density), which may be considered specified, but the rate of transport will be a variable whose inclusion will make an additional functional relation necessary for the complete solution of the problem. Furthermore, the flow resistance and the transport rate may be interdependent to the extent that it would be useless to consider one without the other. For example, the appearance of ripples or dunes on a previously flat sand bed cannot occur without some transport; however, the appearance of such features significantly alters both the flow resistance and the transport rate, which in turn influence the further development of the bed forms.

2. Objectives

The objectives of the experiments were first to determine how f varies with Θ, the roughness distribution ratio, and second to study the

flow structure as revealed by detailed velocity profiles.

The principal observations made were (1) those necessary to the determination of overall friction factors for runs of both bed types over a range of depths and discharges; and (2) detailed velocity traverses necessary to determine the distribution of flow in the channel, the existence of secondary currents, and the extent to which the assumptions of the standard side-wall correction procedure were realistic in this deliberately extreme situation. The results of these observations are presented in Section 5 and some implications are discussed in subsequent chapters.

3. Laboratory procedure

The laboratory work was done in the Institute's 10.5-inch, tilting, recirculating flume (see Figure 5). This flume is 40 ft long, and has been rather completely described elsewhere, as have the standard experimental techniques used in conjunction with it; see, for example, Vanoni and Brooks (26), Brooks (27), or Kennedy (20). Suffice it to say here that the flume and all appurtenant piping are mounted on a truss supported at two points, one a fixed pivot and the other a pair of jacks; the flow is measured by a venturi meter (6 x 4½ in) in the 6-inch return line; and water surface and other detailed observations are made with respect to rails fixed parallel to the flume, with the flume slope made nearly equal to the energy slope for the flow.

The bed consisted of painted marine plywood for the smooth half of the bottom, and nominal one-inch filter gravel for the rough half, and is illustrated in Figure 6. This gravel was the 3/4- to 1¼-inch fraction of pit-run gravel from a pit located in the alluvial fan of the San Gabriel River, and the stones were nearly all sub-rounded to rounded,
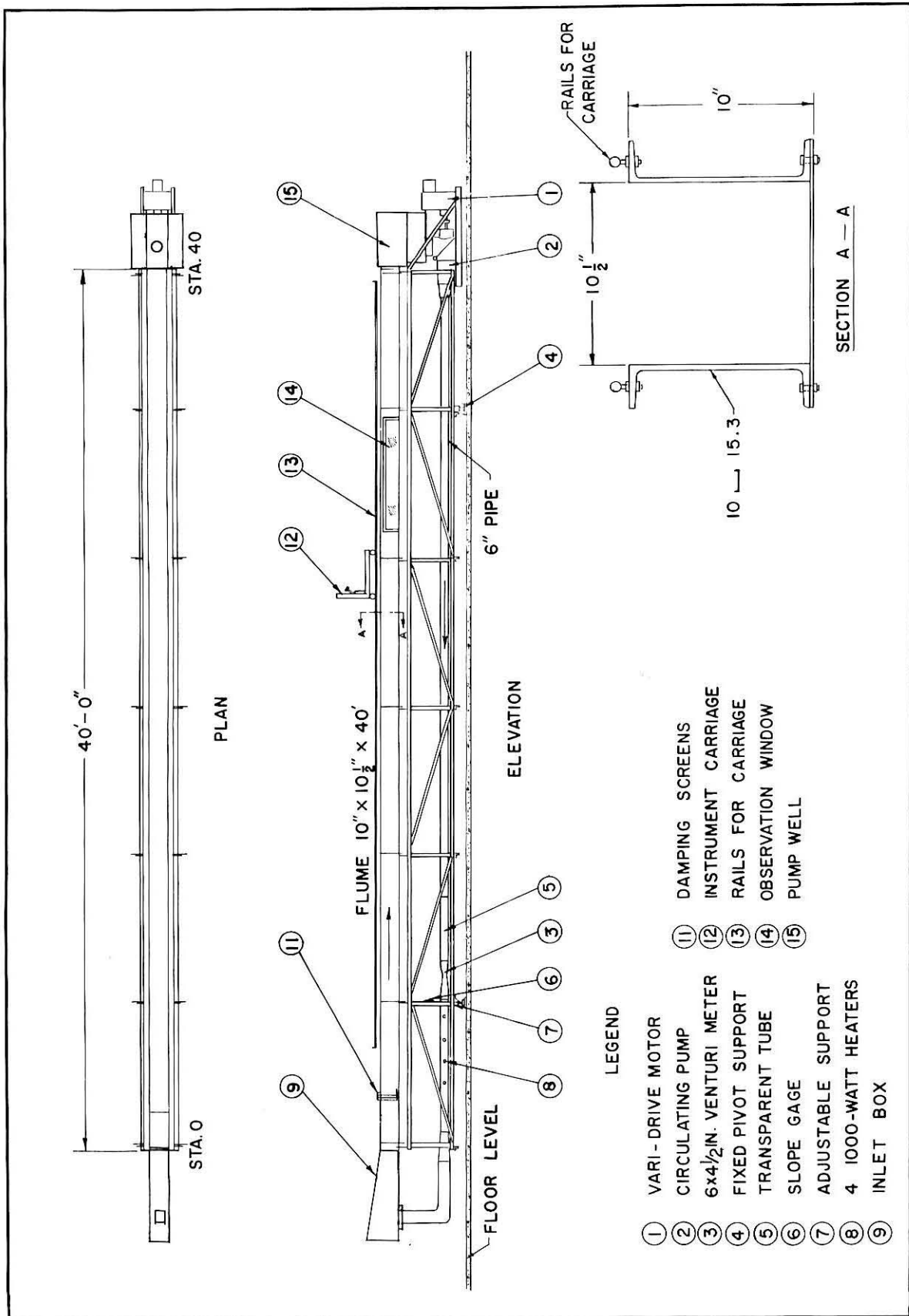
Figure 5. Schematic diagram of the flume
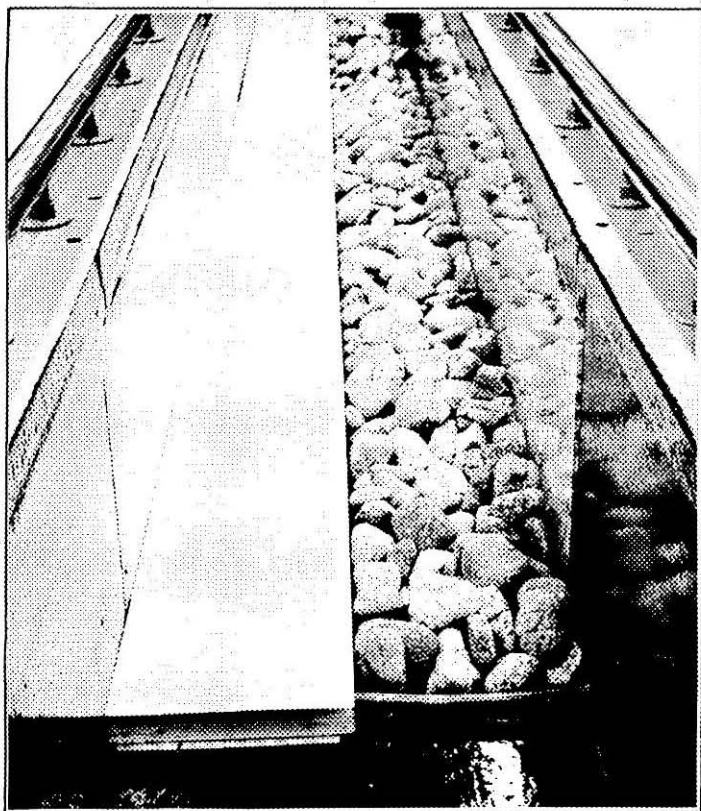
Figure 6. A typical "split" bed. The bed
has been omitted in the foreground to show
details of shimming and bracing.
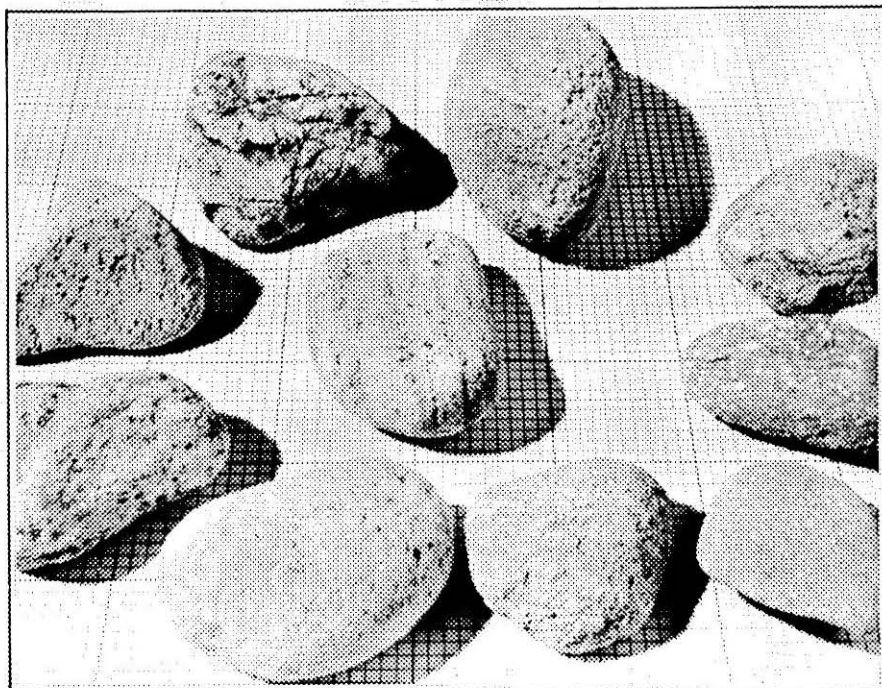
Figure 7. Sample of the gravel used. The back-
ground grid is divided in tenths of inches.

although they tended to be flattened to the extent that the ratios between principal diameters were estimated to be 1:2:4 in many cases. A random specimen of this gravel is illustrated in Figure 7.

These roughness elements were individually hand-placed at first (i.e., for runs 1 - 19.2), in order that a single layer should present a visually uniform character, but the subsequent experiments showed that hand placing did not alter the results enough to justify the effort. (This conclusion was based on consistency shown between runs 21 - 23 and 1 - 9, and was depended upon thereafter in runs 30 -36.) Bed configurations used were 1) rough completely across the section, and 2) one half (the right side in most runs) rough, the other smooth. In the latter, or split bed case, an effort was made to make the effective depth of flow the same on both halves; however, for flow over roughness elements which may be as large as half the unobstructed depth a precise definition of depth is not straightforward.

4. Locating the bottom for the rough bed

In observing flow over a bed of fine sand, the depth of flow is ordinarily very large compared to the grain diameter, and it is unnecessary to consider flow around and between the topmost layer of grains. Furthermore, a visual averaging of the surface gives a sufficiently precise definition of "the surface of the bed". In the present work, however, one grain diameter corresponds to a substantial fraction of the total depth, and "the surface of the bed" is not a sharply defined location. Hence it must be defined arbitrarily.*

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*The size of the roughness elements relative to that of the entire cross section may be seen in Figures 6, 11, and 13.

One approach would be to assume a semi-logarithmic velocity distribution and select that location which makes the straightest plot of velocity vs. log (wall distance). This is neither precise nor productive of consistent results, because the grains are large enough that the effects of individual lee-side eddies distort the profile locally. For a bed of close-packed hemispheres, Einstein and El Samni (28) found that the logarithmic velocity distribution law is followed if distances are measured from a hypothetical wall 0.2 grain diameter down from the tops of the hemispheres, or at $y/r_o = 0.6$. The figure should be different for other shapes and arrays of roughness elements, although they found further, in experimenting on gravel, that the same result could be used provided the 65%-finer grain size is used as the effective diameter of the material.

In applying their results to hemispheres, it may be noted that the hypothetical wall is very nearly that plane at which the volume of those portions of hemispheres above the plane equals the volume of interstices between it and the equatorial plane. The plane that equates these volumes lies at $y/r_o = \pi/3\sqrt{3} = 0.604$. This is not immediately useful for gravel, however, because a uniquely defined equatorial plane does not exist. More useful is the fact that this location of the hypothetical plane is about the same as that of a plane half of whose area lies within the spheres. This plane lies at $y/r_o = \sqrt{1 - \sqrt{3}/\pi} = 0.670$.

This latter criterion, approximate as it is, lends itself to use with a bed of non-spherical grains, and produced about the same results as did the method of straightening out the velocity profile. Furthermore, since it does not involve the measurement of velocity profiles, it can be carried out more easily and accurately, and a rough visual

check is available. The procedure is as follows. Known volumes of water are added to the carefully levelled flume to cover the depth range from zero to complete submergence of the stones, and after each addition, elevations are observed at several points along the flume. Plotting the averages of these elevations against the total water added results in a stage-capacity curve. Then the desired elevation can be read from the curve, remembering that $dQ/dy = A(y)$ where $A(y)$ is the area of surface which is actually water. This process was followed at least once for each bed and therefore the velocity distribution measurements made in runs 22 and 23. The bed elevation for run 36 was found by locating the level required to straighten out the velocity curves on a semilogarithmic graph.

5. Summary of results

Twenty-nine friction factor determinations were made, and in three of these determinations, detailed velocity surveys were also made. The results of the friction factor determinations are presented in Table 1 and the results of the velocity surveys are given in Table 2. In the first group (run numbers below 10) the bed consisted of a single layer of gravel laid over one half and painted 3/4-inch plywood laid over the other half (i.e., a "split"bed). The gravel was placed to present a visually uniform arrangement. Hand placement was at first considered necessary because of the relatively large number of flat stones, which tended to lie flat and thereby produce areas both lower and smoother than the areas occupied by smaller, more nearly spherical stones. The same procedure was followed in placing the bed for the second group of runs, numbered 11 through 19.2, except that here the bed was gravel for the full width. For the third group, runs 21 - 23, the bed was again split, but this time about twice the depth of rock was used. It was

dumped in and generally levelled, the occasional rock which protruded
being moved or replaced. The boards were shimmed up to the elevation of
the hypothetical bed plane (see Section 4, above). In the last group,
runs 30 - 36, the gravel was again only generally levelled, and extended
over the entire width. Detailed velocity surveys were made in runs 22,
23, and 36, at stations which appeared to be free of local flow asym-
metries.

   The coordinate system used in recording data is indicated on each
velocity distribution graph. Elevations were measured from the flume
bottom plate, and transverse distances were measured from an origin
nearly at the center line. Positive distances were toward the right
when looking downstream, and in the split beds the rock was on the right
side.

Table 1

SUMMARY OF DATA: FRICTION FACTOR DETERMINATIONS

Run No	Depth d ft	Mean Velocity u fps	Slope $S \times 10^3$	Temperature T °C	Friction Factor Overall f	Friction Factor Bed f_b	Hydraulic Radius r ft	Relative Roughness $\bar{\varepsilon} = \bar{e}/4r$	Reynolds Number $R = \frac{4ru}{\nu}$
		Group I:	Split bed, one stone thick						
1	.444	.955	.64	19.8	.0396	—	.220	.0946	11,500
2	.440	2.01	2.35	20.6	.0321	—	.219	.0952	165,100
3	.422	3.33	6.70	21.3	.0334	—	.215	.0970	213,000
4	.331	3.16	8.28	22.0	.0402	—	.189	.1104	231,000
5	.330	2.08	3.74	22.2	.0418	—	.188	.1108	152,000
7	.221	2.70	9.33	21.9	.0484	—	.197	.1418	153,200
8	.216	1.38	2.65	22.2	.0514	—	.144	.1448	11,400
9	.218	.805	.97	22.6	.0558	—	.195	.1436	45,800
		Group II:	Bed entirely rough, one stone thick						
11	.447	2.61	6.13	21.3	.0511	.0841	.221	.0942	220,000
12	.446	2.11	3.99	22.1	.0509	.0829	.221	.0942	180,800
13	.456	.997	.96	22.8	.0553	.0896	.223	.0934	87,700
14	.308	2.94	14.59	22.6	.0785	.1195	.182	.1146	210,000
15	.313	1.96	6.24	22.4	.0762	.1152	.183	.1140	140,000
16	.320	.979	1.59	22.2	.0786	.1175	.185	.1128	70,500
17	.147	1.092	6.36	21.8	.1508	.1910	.110	.1896	46,300
18	.156	2.04	18.15	21.4	.1279	.1640	.115	.1808	89,700
19	.156	2.77	32.21	20.9	.1238	.1587	.115	.1808	120,200
19.1	.209	3.52	32.65	20.9	.0960	.1316	.141	.1480	187,700
19.2	.261	3.82	32.42	20.9	.0933	.1367	.163	.1280	235,300
		Group III:	Split bed, several stones thick						
21	.316	2.12	4.15	23.2	.0435	—	.186	.1120	156,600
22	.297	3.04	10.0	23.3	.0493	—	.178	.1172	215,300
23	.154	3.30	25.2	25.6	.0678	—	.114	.1830	157,700
		Group IV:	Bed entirely rough, several stones thick						
30	.127	2.00	19.3	23.6	.1230	.1509	.099	.210	79,400
31	.180	2.08	13.8	23.0	.1043	.1370	.127	.1640	104,600
32	.222	2.27	12.4	22.9	.0911	.1258	.147	.1418	132,000
33	.255	2.45	11.1	24.4	.0767	.1091	.161	.1294	161,300
34	.294	2.55	10.4	23.7	.0725	.1074	.176	.1183	180,500
35	.334	2.61	9.29	23.5	.06665	.1021	.189	.1102	197,400
36	.377	2.64	8.34	22.7	.0620	.0986	.202	.1031	209,700

Table 2

SUMMARY OF DATA: VELOCITY DISTRIBUTIONS

Runs 22 and 23

RUN 23, Sta 27.5 (Cf figure 12)

Transverse Sta, ft	.400	-.200	-.100	-.050	0	+.050	+.200	+.400
Elevation ft	VELOCITIES IN FEET PER SECOND (LEFT SIDE, SMOOTH)					(RIGHT SIDE, ROUGH)		
.230	—	5.34	—	—	—	—	—	—
.220	4.34	5.42	5.14	4.87	4.36	3.94	2.87	2.03
.200	4.37	5.31	5.19	4.90	4.30	3.72	2.76	2.01
.175	—	—	—	—	3.94	3.16	2.47	1.91
.150	4.33	4.71	4.59	4.30	3.36	2.46	2.08	1.74
.125	3.92	4.30	4.00	3.71	2.70	—	—	—

RUN 22, Sta. 24 (Cf figure 11)

Transverse Sta, ft	-.415	-.400	-.250	-.200	-.150	-.100	-.050	0
Elevation ft	(LEFT SIDE, SMOOTH) VELOCITIES IN FEET PER SECOND							
.370	2.81	2.98	3.62	3.76	3.89	3.99	4.00	3.79
.350	2.96	3.13	3.69	3.82	3.91	4.01	4.03	3.87
.300	3.20	3.33	3.86	3.91	3.97	4.04	4.05	3.91
.250	3.21	3.39	3.82	3.90	3.93	3.96	3.91	3.79
.200	3.13	3.26	3.68	3.73	3.69	3.70	3.68	3.36
.150	2.74	2.99	3.41	3.38	3.30	3.27	3.01	2.77
.125	2.47	2.60	2.63	3.09	3.02	2.94	2.66	—

(RIGHT SIDE, ROUGH)	+.100	+.200	+.400	+.415
.370	3.27	2.70	1.88	1.72
.350	3.40	2.85	1.93	1.79
.300	3.54	3.13	1.93	1.81
.250	3.47	3.10	1.78	1.52
.200	2.98	2.45	1.44	1.21
.150	2.21	—	1.08	0.84

Table 2 (continued)
SUMMARY OF DATA: VELOCITY DISTRIBUTIONS
Run 36

| HORIZONTAL TRAVERSES | | | | | VERTICAL TRAVERSES | | | | | |
| Transverse station ft | Elevation, feet | | | | Elevation feet | Transverse station, feet | | | | |
	.493	.343	.243	.193		-.416	-.300	0	+.300	+.399
RIGHT SIDE										
0	3.32	2.87	2.26	1.85	.493	2.34	2.71	3.31	2.84	2.43
.10	3.21	2.88	2.50	1.97	.468	2.45	2.87	___	2.94	2.52
.15	—	3.01	2.39	1.91	.443	2.50	2.97	3.29	3.01	2.54
.20	3.16	3.09	2.58	2.26	.418	2.56	3.06		3.06	2.58
.25	—	3.11	2.71	2.34	.393	2.55	3.15	3.16	3.11	2.56
.30	2.87	3.11	2.79	2.35	.368	2.54	3.18		3.11	2.54
.34	—	2.95	2.71	2.18	.343	2.52	3.21	2.92	3.10	2.49
.38	2.61	2.69	2.47	1.91	.318	2.47	3.14		3.05	2.45
.40	—	2.47	2.21	1.70	.293	2.39	3.10	2.59	3.01	2.41
.41		2.35	2.06	1.48	.268	2.28	2.99		2.93	2.37
.42		2.18	1.83	1.34	.243	2.18	2.89	2.27	2.78	2.21
(Rt wall @ .429 ft)					.218	1.81	2.71		2.64	2.03
LEFT SIDE					.193	1.21	2.47	1.87	2.35	1.64
0	3.31	2.88	2.27	1.76	.183	0.95				1.48
-.10	3.22	2.99	2.42	1.62	.173	0.76	1.90	1.56	2.01	1.32
-.15	—	3.10	2.51	1.81	.163			1.50		
-.20	3.02	3.15	2.60	1.76	.153			1.24		
-.25	—	3.23	2.73	1.81						
-.30	2.80	3.23	2.88	2.37						
-.34	—	3.11	2.88	2.50						
-.38	2.58	2.90	2.63	2.26						
-.40	—	2.74	2.43	1.76						
-.41	2.42	2.63	2.28	1.46						
-.42	2.35	2.49	2.27	1.05						
-.428	2.23	2.33	1.91	0.80						
-.435	2.14	—	—	—						
-.436	—	2.18	—	—						
-.438	—	—	1.70	—						
-.440	—	—	—	0.44						
(Left wall @ -.446 ft.)										

III. DISCUSSION OF RESULTS: FRICTION FACTOR DETERMINATIONS

1. General observations

The values of overall friction factors from Table 1 are plotted in Fig. 8 on an extrapolated Nikuradse pipe-friction diagram, in order to give a clearer idea of the range of Reynolds numbers and friction factors represented by the data. The roughness height, ϵ , was taken as 1 inch for all bed configurations. It will be noted from this figure that all data lie in or very near the range for which the flow is considered fully rough. Thus the Reynolds number effect is minor. As expected, the friction factors were lower in the split bed runs than in the rough bed runs. The observed over-all friction factors for flows over rough and split beds will be denoted f_R and f_S, respectively.

In Figure 9 the Reynolds number effect is neglected, the plot being simply one of friction factor vs. relative roughness. In addition two sets of curves are shown as generally limiting cases. The first of these is based on the Karman Prandtl (equation 6), and the second set is for flow in smooth two-dimensional channels (equation 5), at Reynolds numbers of 40,000, 100,000, and 250,000, which cover the range of values observed in the present investigation. Here, the points for the split beds fall quite nicely between those limiting cases, and tend regularly away from the smooth limiting curves as the relative roughness increases. A constant value of ϵ was assumed since the same roughness elements were used for all runs; hence, increasing the relative roughness means decreasing the depth and therefore increasing θ, the fraction of the perimeter which was rough. Thus the upward drift away from the smooth curves
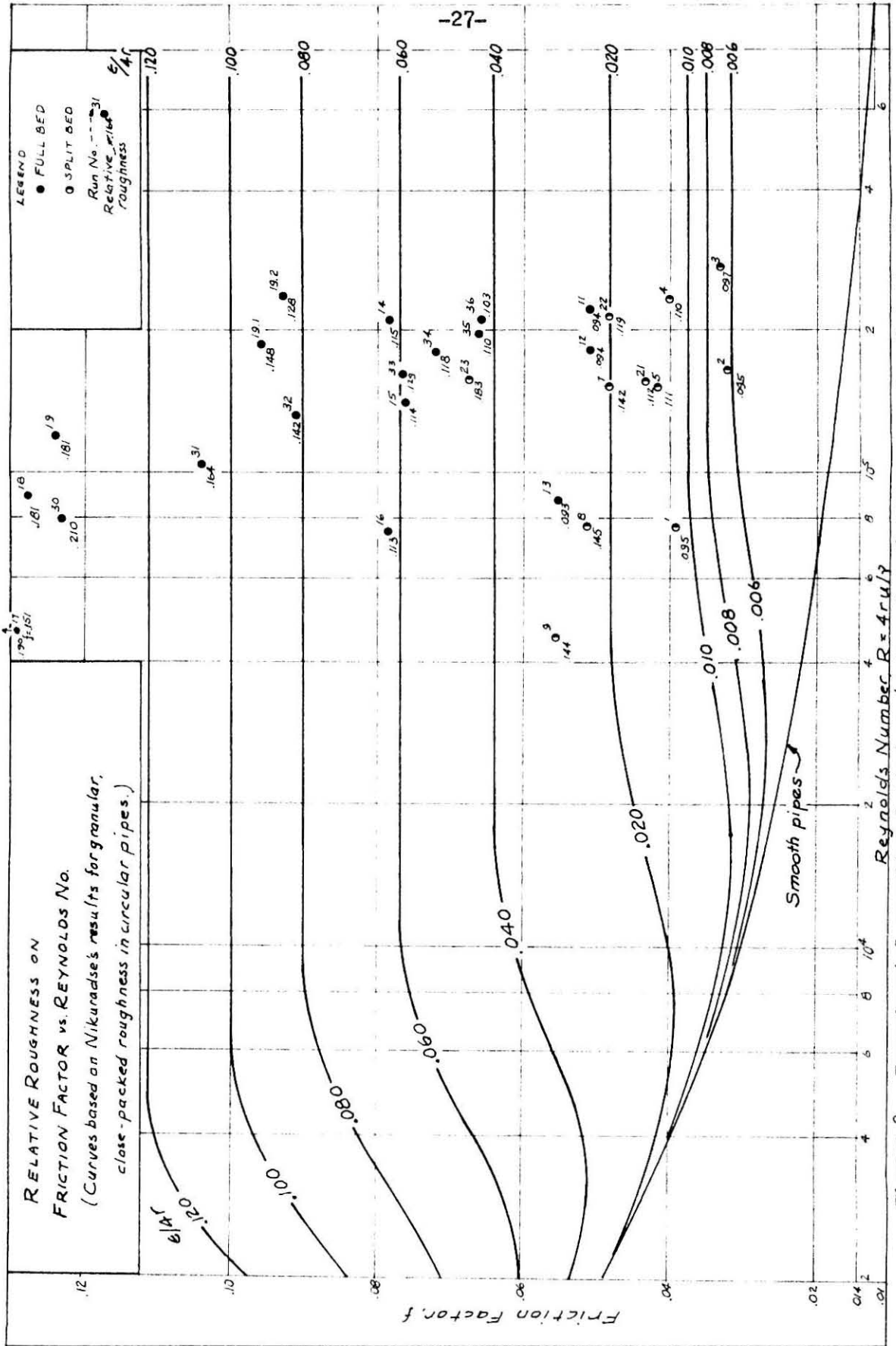
Figure 8. Experimental results plotted on (extrapolated) Nikuradse pipe friction diagram.

Figure 9. Friction factors vs. relative roughness.

is intuitively reasonable.

The points for the rough bed runs falls along a line approximately parallel to that of equation 6, although below it by a distance $\Delta f = 0.06$. While this difference is about constant, it represents a decreasing proportion of the observed friction factor as the relative roughness increases, or, perhaps more significantly, as the hydraulic radius (and therefore the smooth portion of the perimeter, $1 - \Theta$) decreases.

If the customary side-wall correction procedure is applied to the f_R values, as outlined in Chapter V, Section 2, the agreement between them and equation 6 is very much improved, as shown in Figure 10. Observed deviations may be attributable to several things, including the extrapolation of the Karman-Prandtl equation, the doubtful assumptions in the side-wall correction procedure, the very large values of the relative roughness (Nikuradse's largest value was 0.036), and uncertainty as to the location of the "effective" bed, as discussed above. Considering the many uncertainties involved, the agreement is considered very good.

2. Comparison with existing equations

In order to examine the significance of the friction factor determinations, attempts were made to synthesize corresponding values from equations 5 and 6. It is believed that a comparison of the observed and calculated (interpolated) values offers an assessment of the internal consistency of the experimental data and may suggest procedures to be followed when a field problem requires an estimate of the friction factor in a channel of laterally varying roughness. (The question of field procedures is discussed further in Chapter V.)

Two comparison methods were tried.

Under Method A, it was assumed that the friction factor for the flows over the split bed should be a weighted average between (1) the friction factor for flow over a fully rough bed (for the same depth), f_R, and (2) that for flow over a completely smooth bed, f_s, as estimated by equation 5 (for the same hydraulic radius). It was further assumed that the weighting should be in proportion to the rough and smooth areas of bed. Thus the interpolation is between the friction factor for a channel with smooth walls and rough bed and one for a channel with smooth walls and smooth bed. If this interpolated value is denoted f_A, then

$$f_A = \frac{1}{2} (f_R + f_s). \tag{11}$$

The results of interpolation under Method A are given in Table 3 and are quite close to the observed values. This close agreement demonstrates the internal consistency of the data, if Keulegan's assumption is accepted. The method could be used in predicting friction factors, if f_R were estimated by calculating the f for the bed roughness at hand, and using Johnson's side-wall correction procedure in reverse, to add in side-wall effects and thus to obtain a friction factor for a rectangular channel with smooth walls and the desired bed roughness.

In Method B the attempt was made to synthesize friction factors for runs with both the full rough bed and the split bed, by combining the friction factors given by equations 5 and 6 in the same proportion in which the total perimeter was composed of rough or smooth portions. That is, since Θ is the ratio of the width of rock bed to the total perimeter, the interpolated friction factor by Method B, denoted by f_B, is given by

$$f_B = \Theta f_r + (1 - \Theta) f_s. \tag{12}$$

The results of this scheme of interpolation are given in Table 4
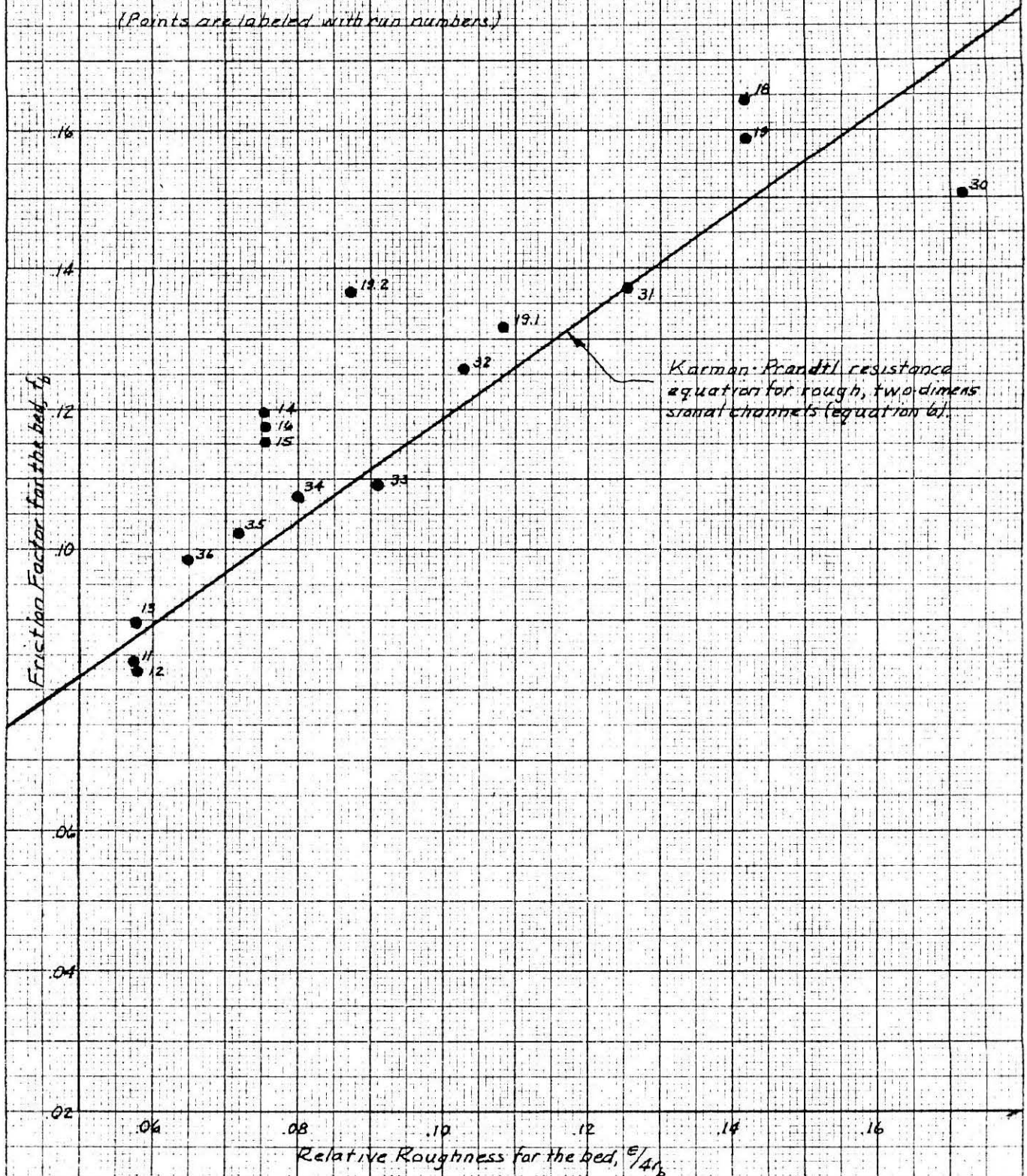
Figure 10. Friction factors vs. relative roughness (after side-wall correction).

Table 3. Friction factors calculated by Method A.

Run No.	F r i c t i o n f a c t o r s				$\dfrac{f_A}{f_S}$
	Observed for rough beds *	Calculated (eq. 5)	Calculated (eq. 11)	Observed	
	f_R	f_s	f_S	f_S	
1	.0536	.0170	.0353	.0396	0.892
2	.0540	.0146	.0343	.0321	1.069
3	.0555	.0133	.0344	.0334	1.030
4	.0666	.0137	.0402	.0402	1.000
5	.0670	.0149	.0420	.0418	1.005
7	.0928	.0149	.0539	.0484	1.113
8	.0952	.0170	.0563	.0514	1.095
9	.0942	.0190	.0566	.0558	1.013
21	.0679	.0148	.0414	.0435	0.953
22	.0722	.0139	.0431	.0493	0.872
23	.1269	.0148	.0709	.0678	1.046

Mean: 1.008

Mean of deviations from unity: .059

* Taken from line drawn in figure 9, equation for which is
$f_R = 0.83E - 0.025$.

Table 4. Friction factors calculated by Method B.

Run No.	1 - θ	Friction Factors				Ratio
		Calculated (eq. 6) f_r	Calculated (eq. 5) f_s	Calculated (eq. 12) f_B	Observed f_R or f_S	$\dfrac{f_B}{f_R \text{ or } f_S}$

Split beds

1	.754	.1157	.0170	.0413	.0396	1.043
2	.753	.1162	.0146	.0397	.0321	1.237
3	.748	.1175	.0133	.0395	.0334	1.183
4	.718	.1278	.0137	.0459	.0402	1.143
5	.717	.1281	.0149	.0470	.0418	1.124
7	.670	.1507	.0149	.0597	.0484	1.233
8	.668	.1328	.0170	.0554	.0514	1.077
9	.669	.1519	.0190	.0629	.0558	1.127
21	.689	.1290	.0148	.0504	.0435	1.159
22	.704	.1330	.0139	.0492	.0493	0.998
23	.634	.1800	.0148	.0753	.0678	1.110

Mean: 1.130

Mean of deviations from unity: .130

Full rough beds

11	.505	.1152	.0138	.0640	.0511	1.253
12	.504	.1152	.0144	.0644	.0509	1.266
13	.510	.1148	.0166	.0647	.0553	1.169
14	.413	.1309	.0140	.0826	.0785	1.052
15	.417	.1305	.0151	.0824	.0762	1.081
16	.422	.1297	.0174	.0822	.0786	1.047
17	.251	.1847	.0190	.1430	.1508	0.948
18	.262	.1785	.0165	.1361	.1279	1.064
19	.262	.1785	.0159	.1360	.1238	1.098
19.1	.323	.1550	.0143	.1095	.0960	1.140
19.2	.373	.1407	.0137	.0932	.0933	0.999
30	.225	.1994	.0169	.1585	.1230	1.289
31	.291	.1667	.0160	.1229	.1043	1.178
32	.336	.1507	.0153	.1051	.0911	1.154
33	.368	.1417	.0157	.0952	.0767	1.242
34	.402	.1337	.0144	.0856	.0725	1.181
35	.433	.1278	.0141	.0785	.0665	1.181
36	.462	.1223	.0140	.0723	.0620	1.167

Mean: 1.087

Mean of deviations from unity: .145

from which it will be noted that the mean of the ratio of f_B to the observed friction factor is 1.101 for all 29 runs, being higher for the split-bed runs than for the full bed runs. Thus Method B produces results in greater disagreement with the observed ones than do the other schemes. This indicates that the effect of the smooth side walls does not go as the proportion they bear to the total wetted perimeters; comparison with the results of Method A show that the side-wall correction procedure seems to reflect the side-wall effect more accurately. Method B relies on equation 6, as would the suggested use of Method A. While this reliance on equation 6 requires using it in the range of very large values of relative roughness, this seems to be justified by the agreement noted above (Cf. Figure 10) between it and the experimental results for the full bed runs, if the side-wall correction method of Johnson is applied to the latter.

IV. DISCUSSION OF RESULTS: VELOCITY DISTRIBUTIONS

1. General observations

The velocity distributions in runs 22, 23, and 36 (Table 2) are plotted in Figures 11, 12, and 13, respectively. It is apparent in all three figures that the thread of maximum downstream velocity is located significantly below the surface. Also, in run 36 the displacement of equal-velocity curves (isotachs) toward the corners is very pronounced. There could be a similar such displacement in the other runs, but the velocity traverses are not well enough spaced to show whether this occurred or not. These features are thought to be the result of a pronounced system of secondary circulation, and will be discussed further in the next section.

In runs 22 and 23, the effect of the split bed in the velocity distribution is qualitatively what one would expect. The mean velocity of the flow over the smooth bed is quite a bit higher than that over the rough bed, and the shallower the flow is, the more nearly is the core of high velocity centered over the smooth bed. Also, the fact that the isotachs are fairly steeply sloping in the region above the break in bed roughness confirms what would be suspected intuitively, that there is considerable longitudinal shear on the vertical plane between the smooth and rough halves of the channel. It is not so obvious, however, why the maximum velocity in a given vertical section should be farther below the surface over the smooth side of the bed than it is over the rough side. As will be discussed in the next section, this is consistent with Prandtl's "secondary circulation of the second kind".

2. Evidence of secondary circulation

Prandtl (29), in discussing the velocity distributions obtained

TRANSVERSE DISTANCE, FEET
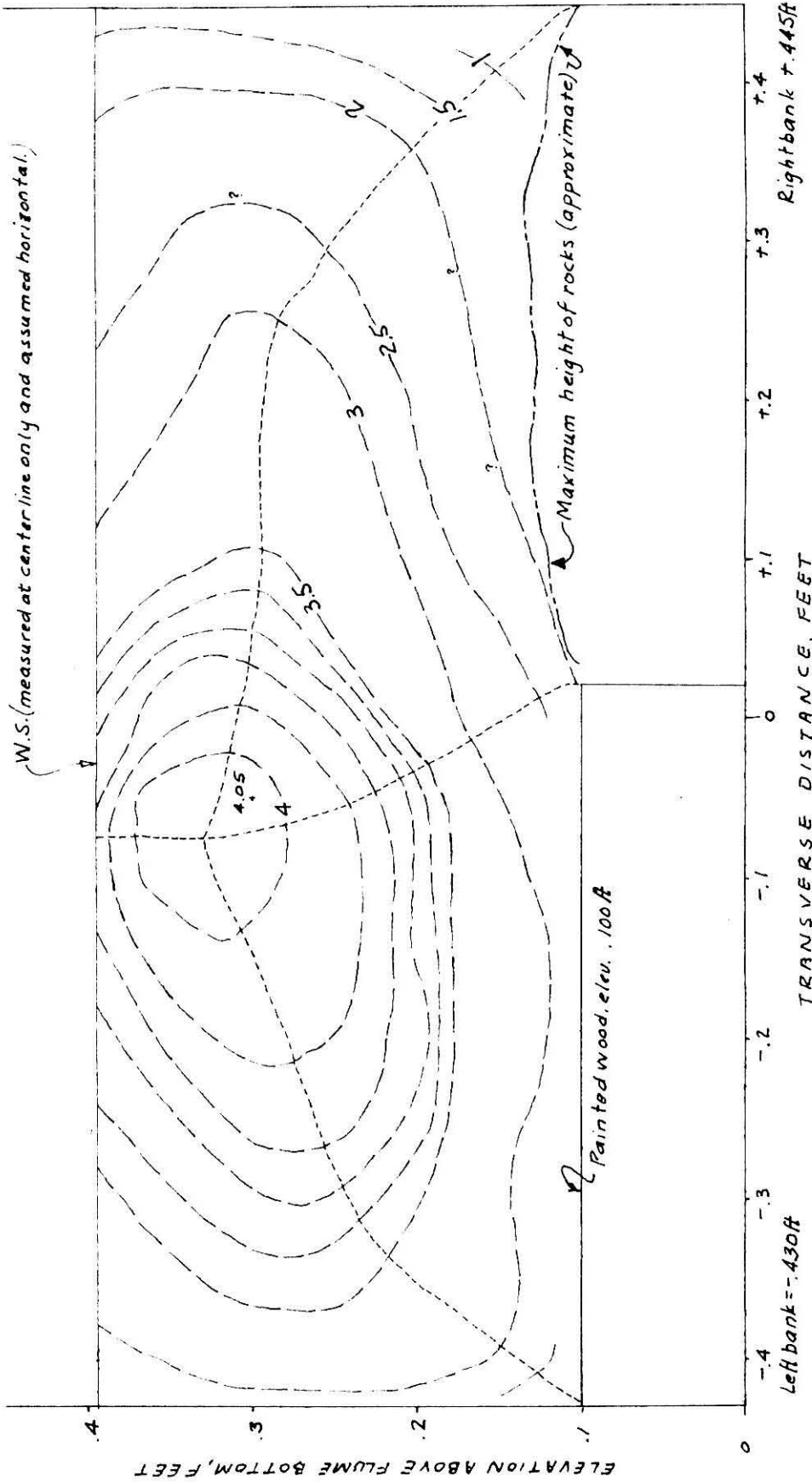
Figure 11. Velocity distribution, run 22, station 24. Velocities in feet per second; scale, 1 inch in figure equals 0.1 foot in flume. Zone boundaries for Leighly method indicated ———. Looking downstream.
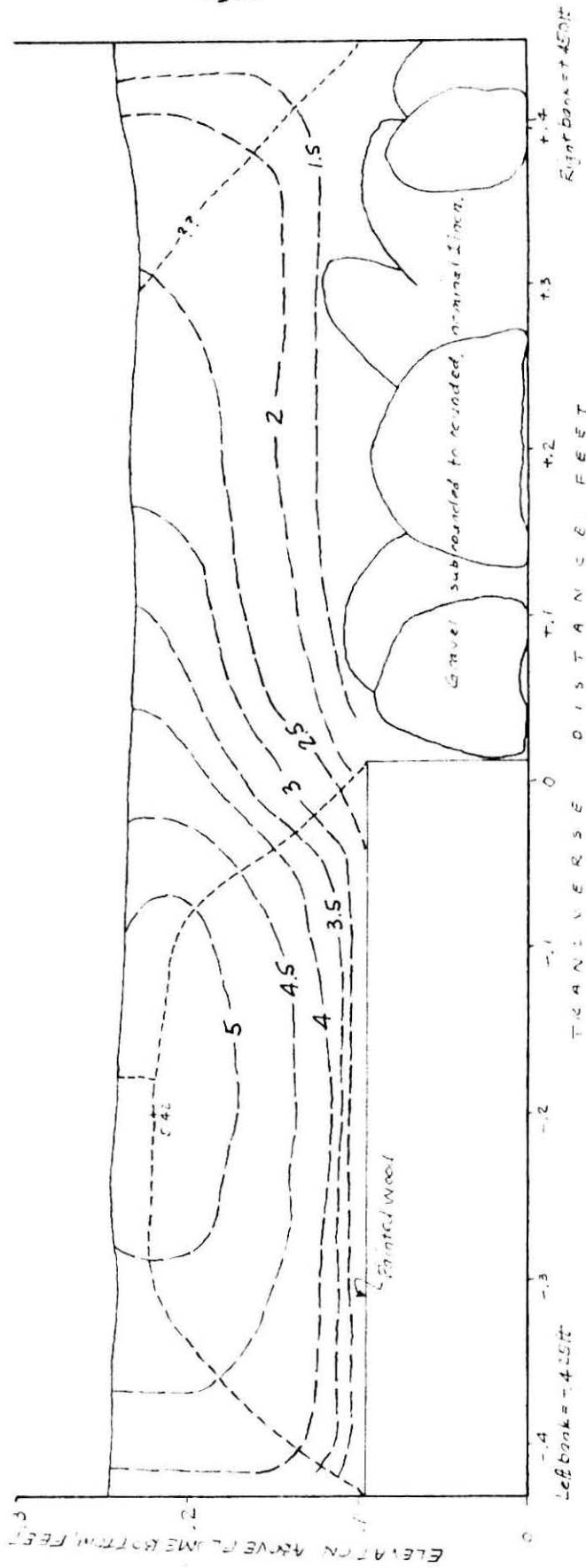
Figure 12. Velocity distribution, run 23, station 27.5. Velocities in feet per second; scale 1 inch in figure equals 0.1 foot in flume. Zone boundaries for Leighly method indicated ———. Looking downstream.
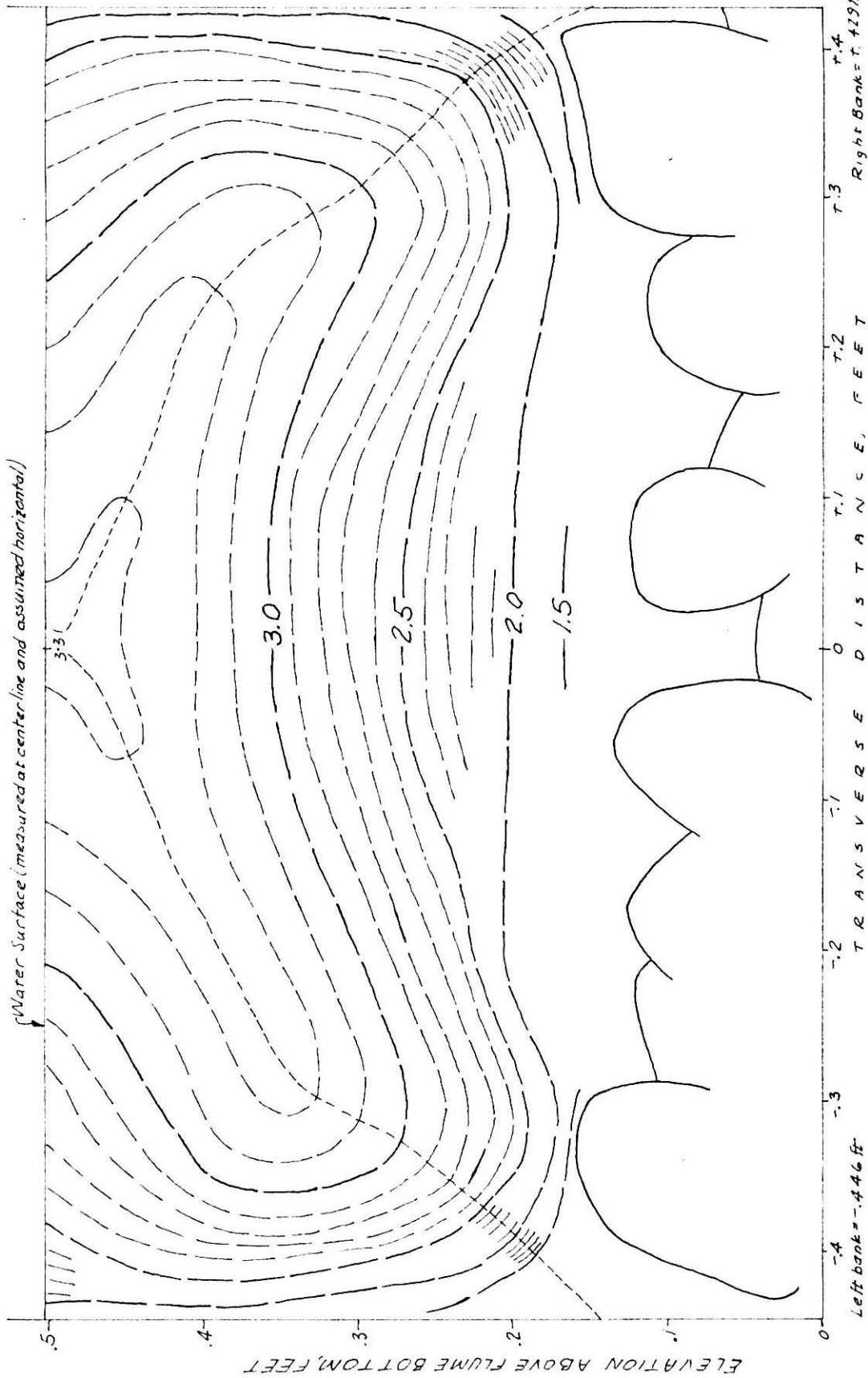
Figure 13. Velocity distribution, run 36, station 8.0. Velocities in feet per second, scale, 1 inch in figure equals 0.1 feet in flume. Zone boundaries for Leighly method indicated ———. Looking downstream.

by Nikuradse (30) in experiments with turbulent flow in straight channels of non-circular cross section, took note of two characteristics of those distributions which are also present in the results of run 36 of the present study. They are the displacement of isotachs toward corners of the cross section, and the occurrence of the thread of maximum velocity, in open channel flows, at a point below the free surface. Prandtl's explanation is that somehow there is a pattern of secondary circulation ("of the second kind", since the more easily explained secondary flows arising from bends in the channel had already been discussed) which proceeds from zones of high velocity outward toward the corners, somewhat in the manner indicated in Figure 14. This flow, it is argued, tends to
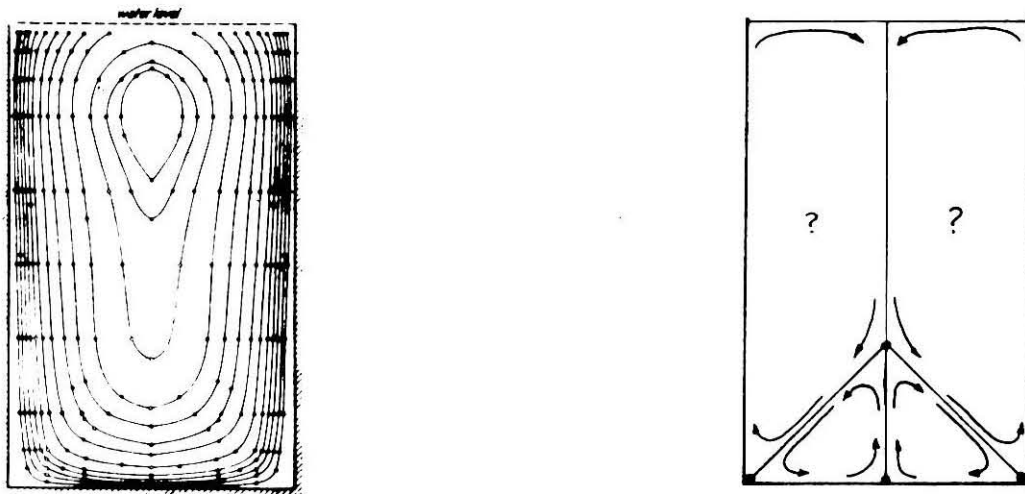
Figure 14. An example of secondary circulation of the second kind. After Prandtl (29).

carry fast-moving water into the areas near the corners, and to bring slow-moving water out toward the center from a neighboring section of the perimeter. The uppermost cells of this circulation would tend to bring slower-moving water in from the walls toward the center, and thus to reduce the velocity at the center of the free surface to a value slightly less than that at points just below.

Very few laboratory studies of this type of flow have been re-
ported, Nikuradse's paper still being the principal reference. Two
theoretical studies are of interest, however, those of Howarth (32) and
Einstein and Li (32). Howarth deduced the condition for existence of
this type of flow in pipes from the modified vorticity transfer model
for turbulent flow, using Goldstein's assumed form for the mixture length
tensor. He concluded that "secondary motion arises if the mixture length
is not constant on the curves along which $\left|\mathrm{grad}\ \overline{u}\right|$ is constant, $\overline{u}$ being
the mean velocity parallel to the pipe axis. Einstein and Li proceed
from the Navier-Stokes equations as applied to uniform flow, and write
an expression for the time derivative of the downstream component of
vorticity. This expression is in terms of those Reynolds stresses which
do not contain downstream velocity fluctuations. They argue that the
vanishing of this expression is equivalent to the absence of a spontaneous
occurrence of secondary flow. Asknowledging that this is a rather weak
approach to the problem, they then consider conditions under which their
function is non-vanishing. Among other things, they conclude that
"secondary flows do not develop spontaneously in a laminar, straight,
uniform flow. In turbulent flows secondary currents . . . may be expected
to occur particularly near the frictional boundary where the lines of
constant velocity are not parallel."

In speculating about the pattern of secondary circulation, one
is led to the consideration of individual helices, or cells of circula-
tion of a particular sense (i.e., clockwise, or counterclockwise). These
cells correspond to areas of the cross section throughout which the
vorticity does not change sign, and are therefore bounded by lines
along which the vorticity vanishes. These lines are separation stream-
lines with respect to the transverse components of such helical flow

(i.e., for the yz-plane, if x is taken in the downstream direction), and contain all the "stagnation points".* Some idea could be gained of the nature of the circulation pattern if it were possible to state general properties of these lines.

It has been observed that the secondary flow moves toward corners whose walls were equally rough, and one would expect that in the immediate neighborhood of such a corner the direction of flow would be along the bisector of the corner angle. This much can be argued from considerations of symmetry about such a bisector; this same symmetry requires that the vorticity must be zero along the bisector in the neighborhood of the corner. Hence, the bisector is a separation stream-line, and the corner is a stagnation point. (This is clearly indicated in reference 12, particularly in the closed triangular pipe illustrated in figure 20.15.)

The observation of Einstein and Li that their expression may not vanish "if the flow pattern changes as one follows the boundary in the cross section" suggests that there would be a circulation across any point of roughness change not at a corner. This follows because of the difference in velocity profiles to be expected over the boundary seg-ments of differing roughness. Thus, in the split bed situation, the corner between wall and smooth bed would be a stagnation point, whereas the midpoint of the bed would not.

At present, little can be said beyond these suggestions, because

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*Reference to Figure 14 may clarify this line of argument: that figure illustrates four cells, adjacent ones being of opposite sign or sense. These cells are divided by separation streamlines, and the points indicated by large dots are stagnation points, with respect to transverse and vertical (i.e., y- and z-) components of velocity.

not enough is known of the spatial variations of the mixture length (for Howarth's theory) or of the Reynolds stresses within the flows ordinarily encountered in open channels. On the basis of what has been suggested, however, a few qualitative comments may be made concerning the possible nature of the secondary circulation in runs 22 and 36. Run 23 is believed to be too shallow to allow the development of a secondary circulation with large enough or strong enough cells to make any obvious sort of modification of the isotachs.

In run 36, the isotachs are drawn much more markedly into the corners than in Nikuradse's cases (Cf. ref. 11, figs. 20.13 and 20.16), from which it seems that the strength of the secondary circulation is considerably greater in the present case. It is suggested that this greater strength can be associated with the great difference in roughness on the two walls making up the corner. This suggestion is consistent with the velocity distribution in run 22, if it is assumed that the depression of the thread of maximum velocity in a vertical is also a measure of the strength of the secondary circulation, because it appears that the level of maximum velocity does drop somewhat as one moves toward the right wall (from transverse station 0.35 ft to about 0.70 ft). In conclusion, only tentative suggestions can be offered as to the pattern of secondary circulation in the observed runs, but it seems very likely that a secondary circulation "of the second kind" did in fact exist.

3. Estimation of boundary shear stress distribution

Several methods are available for determining the intensity of shear stress on portions of the fixed boundary of a fluid flow. Direct methods include direct measurement of the force on a free-floating element of the wall or measurement of the movement of a spring-loaded element

of wall (33). Methods have been used which depend on measuring the rate
of heat or mass transfer from a wall element (34,35). While each of
these methods presents considerable difficulties in technique, it can
be used in an empirical way so as not to depend on assumptions regarding
the nature of the flow or of the velocity profile. Methods which involve
assumptions regarding the nature of the velocity profile include the use
of Preston or Stanton tubes, or simply of careful determination of the
velocity profile near the wall (36).

No systematic program of boundary shear stress observation was
included in the present study, although any extension of this work should
certainly include such a program. In its absence, an estimate of shear
stress distribution was attempted by using the velocity data given in
Table 2, as follows.

The logarithmic velocity distribution law may be written as

$$u = m \log (y/y_o) \tag{14}$$

where

$$m = 2.303 \ u_*/\kappa \ = \frac{2.303}{\kappa} \sqrt{\frac{\tau_o}{\rho}}, \tag{15}$$

and κ is von Karman's constant.* Thus, if velocity profiles are ob-
served in the vicinity of the bed or wall, and these profiles are closely
enough spaced about the perimeter, the distribution of m can be deter-
mined. Integrating m^2 over the perimeter will then allow the mean value
of κ to be determined, since $\overline{\tau_o}$ is known.

Thus the distribution of m^2 about the perimeter gives a measure

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*It is well known that κ is not constant, depending in some un-
known way on characteristics of the flow. It can therefore be thought
of as defined locally along the boundary (except possibly in a corner)
if τ_o is similarly defined.

of τ_o/κ^2 and if κ^2 is assumed constant, m^2 gives a measure of shear stress distribution. Unfortunately, the data of Table 2 are not adequate to define the variation of m^2 along the perimeter, although m^2 did seem to vary rather widely and to show higher values near the center of the bed in the split-bed cases. How much of this is variation in local shear stress, how much is variation in κ, and how much is just experimental error, is not known. The method is indirect at best, and depends on having very carefully measured velocity profiles near the bed. For future work, a more direct method would be desirable, so that local variations in τ_o can be separated from local variations in κ.

V. DISCUSSION OF RESULTS: HYDRAULIC SUBDIVISION OF THE CROSS SECTION

There are several situations of flow over laterally varying rough-
ness in which this variation is taken into account. In attempting to ac-
count for this variation it is customary to divide the cross sections of
flow and to consider the resulting parts hydraulically independent. This
is true in calculating the velocities and discharge of a river in flood,
where the flow along a vegetated flood plain would usually be treated
separately from that within the channel proper. Such a division is in-
corporated in the side-wall correction procedure widely used to reduce
the results of observations in laboratory flumes to the values which would
be obtained in the corresponding two-dimensional case. It is appropriate
to review some of the hydraulic subdivision schemes which have been pro-
posed, in particular of the side-wall correction procedures of Johnson
(10), in order to determine the usefulness of such a process in predicting
the resistance to flow in channels similar to that of the split bed runs
of the present study.

1. Basic considerations

The argument supporting most methods of hydraulic subdivision goes
as follows: steady, uniform flow can be partitioned in such a way that
each zone of flow is associated with a segment of perimeter of constant
roughness, and that each of these zones is hydraulically independent of
the others. Each such zone may then be analyzed alone, and the customary
equations applied. In the case of side-wall corrections, for example,
the resulting values of f, r, etc. for the zone associated with the bed
are supposedly equivalent to those for an infinitely wide channel of the
same bed material.

In the past it has been tacitly assumed that the dividing surfaces

should be everywhere normal to curves of equal mean downstream velocity (i.e., they should be parallel to grad $\bar{u}$). Indeed, this assumption was stated by Leighly (37), who proceeded on the basis of it to find the distribution of the boundary shear stress and of the eddy diffusivity coefficient for several streams. Accordingly, the velocity surveys for runs 22, 23, and 36 were used to divide the flow and the pertinent hydraulic parameters were calculated.* The partition is indicated by dotted lines in Figures 11, 12, and 13, and the results of the calculations are given in Table 5.

In none of the three runs is there good agreement between the values calculated by the Leighly assumptions and those predicted by equation 5 for the smooth parts of the perimeter. In fact, the calculated friction factor for each wall agrees much more closely with that for the adjacent bed section than it does with the identical opposite side-wall. Thus it is obvious that the Leighly method of partitioning the flow is incorrect, in that the zones it produces are not independent. Hence, more careful consideration of the partition idea is in order.

Consider a turbulent flow which is statistically steady and uniform, and at any point within it choose local coordinates so that the x-axis is downstream, the y-axis is parallel to grad $\bar{u}$, and positive toward increasing velocities (i.e., in general not normal to the bed). The z-axis will then lie parallel to the isotachs (Fig. 15). As is customary, the x-, y-, and z-components of velocity are represented by u, v, and w, respectively. Each one (as well as the pressure, p) may be

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*It should be noted that this differs from the usual side-wall correction procedure, for which detailed velocity traverses are rarely available. Instead, the wall roughness is assumed and the partitioning of the cross sectional area is computed. See Section 2 below.
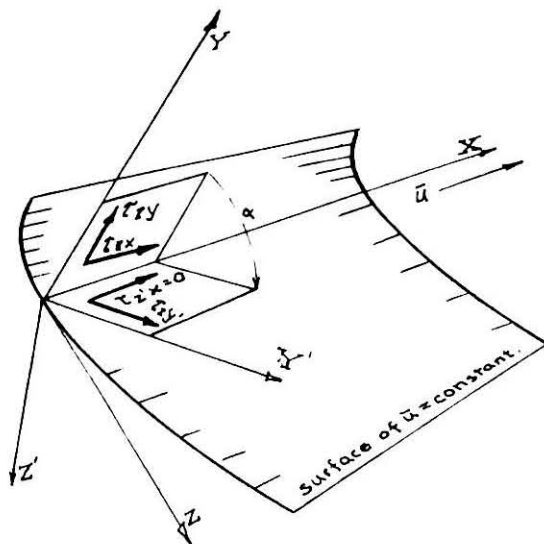
Figure 15. Local coordinate system.
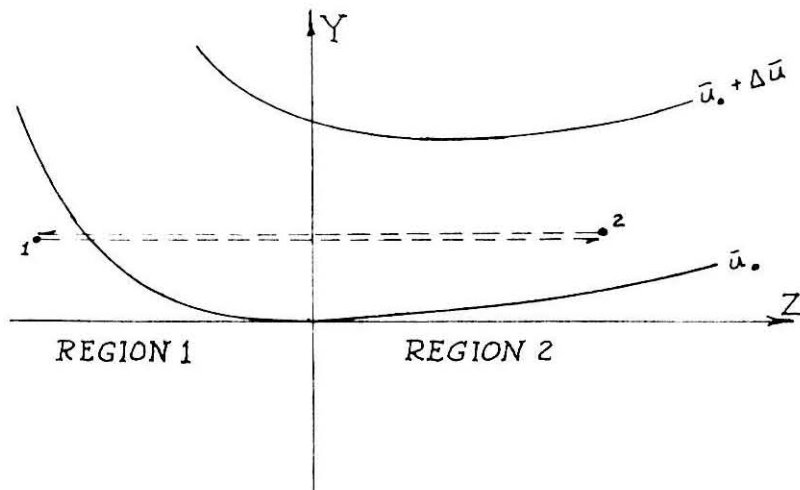
Figure 16. Momentum interchange with
asymmetric isotachs.

Table 5

RESULTS OF PARTITIONING RUNS 22, 23, & 36 BY LEIGHLY METHOD

Section or Zone	Material	Estimated Roughness Height ϵ_i, ft	Area of Associated Flow Zone A_i, ft²	Mean Velocity u_i, fps	Hydraulic Radius r_i, ft	Relative Roughness E_i	Reynolds Number R_i	Friction Factor f_i calc by defin.	f_i predic. eq. (5) or (6)	$\dfrac{f_i\ calc}{f_i\ pred}$
			RUN 22:	$T = 23.3°C$	$S = 1.00\%$		(SPLIT BED)			
LEFT WALL	Glass	10^{-6}	.0527	3.38	.1766	1.4×10^{-6}	238,000	.0400	.0137	2.92
LEFT ½ BED	Painted Wood	10^{-4}	.0608	3.07	.1353	1.8×10^{-4}	167,000	.0371	.0146	2.54
RIGHT ½ BED	Gravel	.0833	.0776	2.36	.1826	.1140	173,000	.0848	.1298 †	0.65
RIGHT WALL	Glass	10^{-6}	.0681	2.61	.2337	1.1×10^{-6}	245,000	.0887	.0136	6.52
ENTIRE CHANNEL	—	—	.2592	2.80	.1771	—	215,300	.0587	—	—
			RUN 23:	$T = 25.6°C$,	$S = 2.52\%$		(SPLIT BED)			
LEFT WALL	Glass	10^{-6}	.0125	4.08	.0833	3.0×10^{-6}	130,000	.0324	.0153	2.12
LEFT ½ BED	Painted Wood	10^{-4}	.0417	4.14	.0942	2.7×10^{-4}	149,000	.0355	.0149	2.38
RIGHT ½ BED	Gravel	.0833	.0595	2.59	.1372	.1518	135,000	.1324	.1567 †	0.85
RIGHT WALL	Glass	10^{-6}	.0102	1.64	.0723	3.5×10^{-6}	49,700	.1740	.0187	9.31
ENTIRE CHANNEL	—	—	.1239	3.18	.1063	—	151,700	.0685	—	—
			RUN 36:	$T = 22.7°C$,	$S = 0.834\%$		(FULL ROUGH BED)			
LEFT WALL	Painted Steel	10^{-4}	.0663	2.74	.1761	1.4×10^{-4}	189,800	.0503	.0143	3.52
BOTTOM	Rock	.0833	.2062	2.25	.236	.0884	272,000	.1000	.1104 †	0.91
RIGHT WALL	Painted Steel	10^{-4}	.0575	2.90	.1525	1.6×10^{-4}	174,300	.0389	.0145	2.68
ENTIRE CHANNEL	—	—	.3300	2.54	.202	—	192,300	.0672	—	—
			RUN 36 Partitioned by Johnson-Brooks side-wall correction procedure							
LEFT WALL	Painted Steel	10^{-4}	.025	2.64	.0664	3.8×10^{-6}	68,900	.0206	—	—
BOTTOM	Gravel	.0833	.280	2.64	.321	.0649	333,000	.0986	.0928	1.060
RIGHT WALL	Painted Steel	10^{-4}	.025	2.64	.0664	3.8×10^{-6}	68,900	.0206	—	—
ENTIRE CHANNEL	—		.330	2.64	.202	—	209,700	.0620	—	—

† Hydrodynamically rough, equation 6; all others are from equation 5.

NOTE: Discharge data for Leighly method are from velocity distributions and differ slightly from those in table 1 and those in Johnson-Brooks example.

divided into a mean portion (overscored) and a fluctuating portion (primed);
e.g., $u = \bar{u} + u'$, and $\overline{u'} = 0$.

For the steady flow situation, the Reynolds equation for the x
component becomes

$$\varrho\left(\bar{u}\frac{\partial\bar{u}}{\partial x} + \bar{v}\frac{\partial\bar{u}}{\partial y} + \bar{w}\frac{\partial\bar{u}}{\partial z}\right) = -\varrho g S - \frac{\partial\bar{p}}{\partial x} + \frac{\partial}{\partial x}\left[\mu\frac{\partial\bar{u}}{\partial x} - \varrho\,\overline{u'^2}\right] + \frac{\partial}{\partial y}\left[\mu\frac{\partial\bar{u}}{\partial y} - \varrho\,\overline{u'v'}\right]$$

$$\tag{15}$$

$$+ \frac{\partial}{\partial z}\left[\mu\frac{\partial\bar{u}}{\partial z} - \varrho\,\overline{u'w'}\right] \; ,$$

where the first term in each bracket on the right hand side is a true
mean stress term, the last term (the so-called Reynolds stress) being a
measure of momentum advected by turbulent velocity fluctuations. To
speak of $\varrho\,\overline{u'v'}$ as a stress is not strictly correct, but it is established
usage and is satisfactory so long as the distinction between true and
Reynolds stresses is kept in mind. I.e., we write

$$\tau_{ji} = \mu\frac{\partial u_i}{\partial x_j} - \varrho\,\overline{u_i'u_j'} \; , \tag{16}$$

where τ_{ji} is the stress in the i direction on the plane normal to the j
axis.

For uniform flow we have

$$\frac{\partial u_i}{\partial x} = 0 \quad \text{and} \quad \frac{\partial}{\partial x}\left(\overline{u_i'u_j'}\right) = 0$$

and because the z-axis parallels the isotachs, $\partial\bar{u}/\partial z = 0$. Using these
relations, equation 15 becomes

$$\varrho\left(\bar{v}\frac{\partial\bar{u}}{\partial y}\right) = -\varrho g S - \frac{\partial\bar{p}}{\partial x} + \frac{\partial}{\partial y}\left[\mu\frac{\partial\bar{u}}{\partial y} - \varrho\,\overline{u'v'}\right] + \frac{\partial}{\partial z}\left[-\varrho\,\overline{u'w'}\right] \; , \tag{17}$$

where now

$$\tau_{xx} = 0 \; , \tag{18}$$

$$\tau_{yx} = \mu\frac{\partial\bar{u}}{\partial y} - \varrho\,\overline{u'v'} \; , \tag{19}$$

$$\tau_{zx} = -\varrho\,\overline{u'w'} \; . \tag{20}$$

Now if the z-plane is considered to divide the flow in the neighborhood of the origin into two parts, the resistance offered one part by the other is measured by τ_{zx}, which does not in general vanish under the assumptions merely of steady uniform flow. (If the flow is laminar, both τ_{zx} and τ_{zy} vanish, so that surfaces normal to the isotachs will be surfaces of zero shear, both longitudinal and transverse.)

If the dividing planar element is rotated through an angle α about the x-axis, and y'- and z'-axes are defined as shown in Figure 15, then the x-component of shear on the element, $\tau_{z'x}$, will vanish if tan α $= -\tau_{zx}/\tau_{yx}$, whereas in general the y'-component, $\tau_{z'y'}$, will not. Thus it cannot be assumed that both components vanish simultaneously.

We have thus shown that with each point within the flow there can be associated a planar element of such orientation that the longitudinal shear stress on it vanishes. In the absence of internal singularities in this set of oriented planar elements, it is therefore possible (conceptually, at least) to start at any point and construct a single continuous surface which is everywhere tangent to the planar elements, and which, therefore, will be a surface of zero longitudinal shear. Hence, if the starting points selected are points on the boundary at which the roughness changes (e.g., from the corners, and in the split-bed runs, from the point where the rocks meet the board), the flow will have been divided into zones between which there is no net interchange of momentum in the longitudinal direction. Such a division is always possible, but as shown above, these surfaces will not in general fall normal to the equal velocity surfaces.

To visualize a situation in which τ_{zx} on an element normal to the isotachs does not vanish, consider first a simple situation in which it does, namely that of two-dimensional flow. Here the isotachs are

straight, and the momentum interchange across any z-plane can be seen to be zero by symmetry. That is, the parcels of fluid carried across the plane in one direction have the same momentum, on the average, as those carried in the opposite direction. The same can be said in the case of a circular pipe, in which case the z-plane at any point will be a radial plane, and while individual parcels may now have different velocities $\bar{u} + u'$ from having been carried different distances to cross the z-plane, symmetry still prevails so that the net transport of momentum is seen to be zero.

If, however, the isotachs are not symmetrical with respect to the z-axis in the neighborhood of a particular point (Figs. 15 and 16), there will be a systematic bias in the distribution of u'-values for the parcels being carried across the z-plane. In Figure 16, parcels such as that labelled "2" will have higher velocities than their counterparts (labelled "1" in the figure), so there will be a net turbulent transport of momentum. In the figure, the flow in region 2 will experience a drag from the flow in region 1. The magnitude of this effect cannot be estimated, but if the mixing length concept has any physical reality, the fact that calculated values of the mixing length are of the order of tenths of feet rather than thousandths, say, means that even modest asymmetry of the isotachs may make the momentum flux significant.

If there is a secondary circulation such that $\bar{w} \neq 0$, the above argument is not affected. This can be seen by noticing that only the turbulent transport of momentum, $-\rho \overline{u'w'}$ is involved in the Reynolds stress, and observing that $\overline{u'w'} = \overline{u'w} - \overline{u'\bar{w}} = \overline{u'w}$, regardless of the value of $\bar{w}$.

Consideration of Figures 11 - 13 will show that asymmetry of

the isotachs could occur over much of the cross section, including the regions in which the zero-shear surfaces might be expected to fall. Thus it is quite reasonable that partition of the flow along lines normal to isovels should have produced such inconsistent results as Table 5 showed.

There still remain some physical objections to the partition concept, however, arising from the fact demonstrated above, that it is not necessarily true that because the longitudinal shear stress ($\tau_{z'x}$) has vanished, the transverse shear stress ($\tau_{z'y'}$) must have vanished also. Thus, for example, the flow in a zone associated with a side-wall properly separated by a surface of zero longitudinal shear, is still influenced by the slower flow over the rough bottom, not directly by a longitudinal drag, but by the other component of shear on the dividing surface. The nature of this influence is not clear, but it seems likely that it might significantly modify the velocity distributions and turbulence properties of the zones, so as to make questionable the use of equations developed for quite a different flow situation (the Karman-Prandtl equations, for example). An additional point of concern is that these zones, particularly those associated with side-walls, may be more nearly triangular than rectangular, and as Keulegan (12) has pointed out, the shape of the flow section must be accounted for, although it would be quite difficult to decide on an appropriate value of his shape factor, β, for a narrow triangular section with only one wall!

2. Procedure

Thus far the general rationale of hydraulic subdivision of flow has been considered. However, the side-wall correction method as actually practiced departs somewhat from the rationale, so a detailed examination of the procedure is in order. For simplicity, consider steady, uniform

flow in a rectangular open channel whose bed is of one roughness and whose sides are both of another roughness. Of the physical quantities associated with the overall flow, those which are common to all zones are ρ, ν, g, and S, while those already known separately for the zones are P and ϵ. Therefore, the quantities to be partitioned may be taken as u, A, f, and r. Because of the symmetry about the vertical center line of the channel, the two wall sections will be considered together. A subscript w will apply to those quantities pertaining to the zones associated with the walls, subscript b to those pertaining to the zones associated with the bed, and no subscript to overall values.

Considering overall, wall-associated, and bed-associated quantities, there are twelve variables, four for each zone. Four of these, usually the overall quantities, are known. The following eight relations[*] can be written among the eight remaining variables

Geometric continuity: $A_w + A_b = A$ (21a)

Flow continuity: $A_w u_w + A_b u_b = Au$ (21b)

Definition: $r_w = A_w/P_w$ (21c)

Definition: $r_b = A_b/P_b$ (21d)

Definition: $f_b = 8gr_b S/u_b^2$ (21e)

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[*]Various authors differ in details of their procedure. Einstein (2) uses Manning's equation to define both the bed and bank roughness, and estimates the bank roughness n_w independently. Johnson (25) prefers the Karman-Prandtl resistance equation for the bed, because it is better able to account for the effects of temperature, and the roughness coefficient is dimensionless. The wall roughness, f_w, is usually estimated in his method, too; it need not be, however. Brooks (27, 38) has developed a modification which permits direct solutions for the situation in which the walls may be assumed to be hydrodynamically smooth, and he also considers (38, p. 236 ff) the effect of assuming some value for u_w/u other than unity.

Definition: $$f_w = 8gr_w S/u_w^2 \qquad (21f)$$

Empirical relation: $$f_w = \varphi_1(r_w, u_w; \nu, \epsilon_w) \qquad (21g)$$

Empirical relation: $$f_b = \varphi_2(r_b, u_b; \nu, \epsilon_b) \qquad (21h)$$

However, since the goal of most series of experiments is to investigate the functional relation which describes the behavior of f_b, it is begging the question to assume one. Hence, we are short one equation, and the assumption usually made to meet this deficiency is that $u_w = u_b$. While this is not strictly justifiable, Brooks (38) showed that assuming $u_w/u = 0.9$ gave results only a few percent different than did assuming $u = u_w$. The results of the velocity survey for run 36 were used to calculate the actual values of u_w/u for various arbitrarily chosen wall-associated zones. The zones measured are indicated in Figure 17, and the results are as follows:

Division Number	A_w/A	u_w/u
1	0.0245	0.868
2	0.0735	0.996
3	0.147	1.097
L4	0.196	1.244

Here, area L4 is a triangle chosen to approximate the left zone defined by the Leighly method.

Thus for small values of A_w/A, the assumption of $u_w = u$ is not a bad one. When A_w/A is large, as would be the case when wall and bed roughnesses are nearly the same, the similarity in roughness itself would tend to result in u_w/u not greatly different from unity.

It is interesting to note that nowhere in the side-wall correction procedure (as contrasted with the rationale) is the location of

the dividing surfaces specified, much less that they be required to be surfaces of zero shear. Indeed, if f_b and f_w are eliminated among equations 21 e, f, g, and h, and if equations 21 c and d are used to eliminate r_w and r_b, the resulting four equations in A_w, A_b, u_w, and u_b will be seen to reflect nothing of the internal nature of the flow, and the solution is seen to depend, for its recognition of physical reality, only on the particular resistance functions which are chosen. Since it is debatable whether these functions can realistically describe the behavior of flow in a channel with rigid boundaries but drastically non-regular shape, it is difficult to see that the procedure has any physical significance at all except in an approximate way. Thus the excellent results of its application to the data of the present study (see Chapter III, especially Figure 10), must for the time being be considered fortuitous, and use of the procedure in situations very different from those in which it is previously known to work should be undertaken with caution.
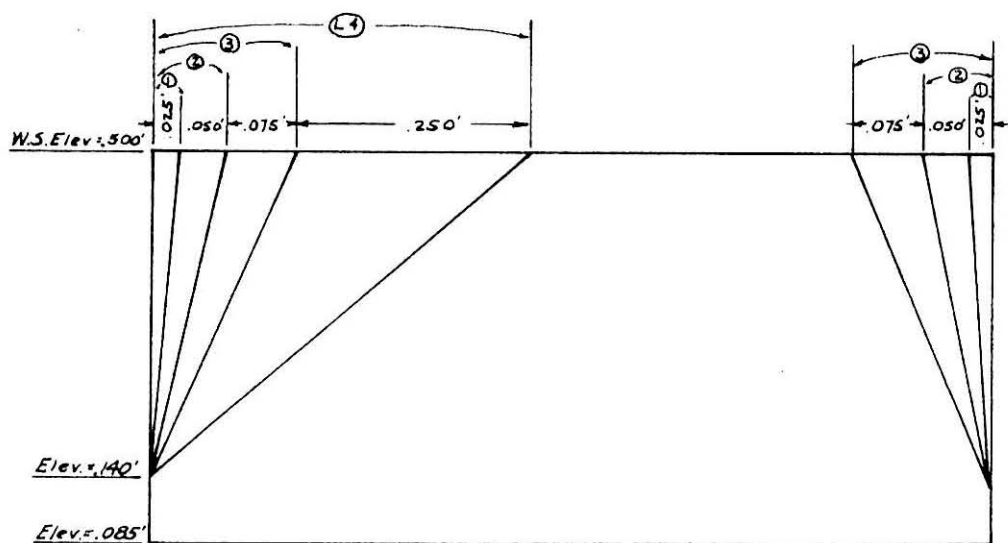
Figure 17. Arbitrarily chosen wall-associated zone, run 36.

VI. SUMMARY AND CONCLUSIONS; SUGGESTIONS FOR FUTURE WORK

1. Summary

Steady, uniform, open-channel flow was established at various depths and velocities over two types of beds, one rough over the entire width of a laboratory flume, the other rough only over half the width and smooth over the other half. Friction factors were determined for these flows, and detailed velocity distributions were measured in three runs. The friction factors were compared with those predicted by the Karman-Prandtl equations, and the velocity traverses were used to investigate the existence of secondary circulations and to assess the validity of methods of subdividing such a flow into hydrodynamically independent parts.

2. Conclusions

The following conclusions may be stated:

A. The bed friction factors determined for flow over the rough bed are consistent with the Karman-Prandtl equation for turbulent flow in rough, two-dimensional open channels, if the side-wall correction procedure of Johnson is applied, this consistency having been observed for relative roughnesses ranging from 0.09 to 0.18.

B. The overall friction factors found for flow over a bed rough on only one half varied consistently with relative roughness or hydraulic radius. They can be estimated within about six percent by weighting equally the values for the rough bed runs and for smooth, two-dimensional channels of the same hydraulic radius. They can be estimated only to within 30 percent of the true value by weighting the values for smooth and rough two-dimensional channels of the same hydraulic radius, if the latter values are taken from the Karman-Prandtl equations and if the

weighting is in proportion to the roughness distribution over the wetted perimeter. Values estimated by this latter method averaged about 10 percent high.

C. In considering methods of subdividing stream cross sections into hydrodynamically independent zones, it was concluded that surfaces of subdivision which lie normal to surfaces of equal mean downstream velocity are generally not surfaces of zero longitudinal shear stress (where the term shear stress is taken to include the advected turbulent momentum described as Reynolds stresses). Surfaces of zero longitudinal shear do exist; however, they do not necessarily divide the flow into independent parts because the transverse shear stress will not in general vanish on these surfaces. Therefore, subdivision of a turbulent flow into hydrodynamically independent zones is not in general possible, because the turbulence generated at the bed is undoubtedly diffused throughout the channel. Thus, while the customary side-wall correction procedure seems to be adequate for eliminating the effect of smooth side walls on the flow over a rough bed, this adequacy must be considered fortuitous, as there is no firm rational basis for the individual steps of the procedure.

D. The detailed velocity profiles for three runs indicate the existence of strong secondary circulation of the general pattern described by Prandtl as being "of the second kind".

3. Suggestions for future work

It was noted previously (p. 31) that the results of the present study could probably be extended to split beds of different roughness. The procedure mentioned was to determine the friction factor for a two-dimensional channel of the new roughness, to change it to one for a

channel of the required shape by using the side-wall correction procedure
in reverse, and then to take the average of the friction factor thus
modified and the friction factor for a smooth, two-dimensional channel
of the same hydraulic radius. It remains to be seen, however, whether
such a procedure would be effective for beds of other lateral distribu-
tions of roughness, and the assumption that it would be effective neces-
sarily relies on an assumption of the validity of the side-wall correction
procedure. Thus it is apparent that not only does considerably more
work need to be done with different bed roughness sizes and distributions
and with a wider range of width/depth ratios, but also a closer look must
be taken at the internal nature of the flow. For example, more thorough
and more precise velocity distribution measurements are needed, and the
velocity measurements should be extended to include determinations of
direction of flow, so as to shed more light on the secondary circulation
patterns. These patterns are more than a curiosity, because it seems
likely that they play some part in the interaction of different portions
of the flow, even if not in the advection of turbulent momentum.

Of a more fundamental nature are observations (1) of the spatial
variation of the turbulent velocity fluctuation correlations (or of the
Reynolds stresses) or of the mixing length; (2) of the nature of the
boundary layer in a wide channel in the neighborhood of a break in
boundary roughness; (3) of the effect on a boundary layer of turbulent
fluctuations arising from a nearby wall of different roughness (as in a
closed wide channel with different roughnesses on opposite walls).

In addition to all the above, it should be remembered that the
laterally varying roughness found in nature is interrelated with the
transport of sediment. Once the questions mentioned above are extended

to flows transporting sediment, they become very much more complicated. This is particularly true of those questions relating to local properties of the flow.

Particular projects which should be undertaken next might include (1) determinations of velocity and Reynolds stress distributions in a wind tunnel approximating a two-dimensional channel, in which one or both walls were divided into respect to roughness or in which one wall were smooth and the other rough; (2) determination of the velocity vector distribution throughout an open-channel flow over laterally varying roughness; and (3) determination of the distribution of boundary shear stress about the perimeter.

APPENDIX A

Discussion of

"Resistance to Flow in Alluvial Channels"

by D. B. Simons and E. V. Richardson (7)

R. H. Taylor, Jr., and N. H. Brooks (8)

The notation used in the attached reprint follows that of Simons and Richardson and differs in some respects from that used in the rest of this thesis. The symbols which are used differently in the reprint and which are not defined in its text are as follows:

Symbol in Appendix A	Description	Symbol in text	Dimensions
d	fall diameter of sediment particle	--	L/T
D	depth of flow	d	L
C	Chezy coefficient (in $V = C\sqrt{rS}$)	--	$L^{1/2}/T$
F_r	Froude number	F	--
Re	Reynolds number	R	--
V	Mean downstream velocity	u	L/T
w	Settling velocity of sediment particle	--	L/T

R. HUGH TAYLOR, JR.,[44] A. M. ASCE, and NORMAN H. BROOKS,[45] M. ASCE.—The authors are to be commended for their comprehensive experimen-

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[45] Assoc. Prof. of Civ. Engrg., W. M. Keck Lab. of Hydr. and Water Resources, Calif. Inst. of Tech., Pasadena, Calif.

tal program on sediment transport at the Colorado State University under sponsorship of the U. S. Geological Survey. Their experiments are valuable because they have been conducted in an 8-ft wide flume which is larger than any tilting flume used heretofore in sediment transport studies. The data which they are collecting, not only for the 0.45-mm sand reported on in the paper, but for all the sizes used in their program, will be of lasting value to the profession. For the record the writers ask in what form the complete record of their experiments will finally be published.

In the analysis and interpretation of the experimental results there are a number of points which warrant discussion. Under the section "Theory of Flow in Alluvial Channels," the authors have not actually presented a theory but only some dimensional analysis, leading to Eq. 6. To apply dimensional analysis sensibility to the roughness problem requires selection of the pertinent variables; but furthermore one must be careful to assume the correct number of functional relationships. In other words there may be more than just one dependent variable, as several relations may have to be satisfied simultaneously.

For example, the writers believe that for natural rivers the bed-material sediment discharge (Q_s) should be included as another independent variable, because field evidence shows that river behavior changes significantly with a change in Q_s supplied to a reach. The transport relation is strongly interlocked with the flow relation, and neither one can be properly considered without the other. Thus in Eq. 1 variables such as B, V, and S cannot be considered to vary at will as independent variables, but are constrained by other unmentioned relations such as a transport equation. Hence a simple listing of the variables is not as important as a careful consideration of the constraints, which at the present state of knowledge are none too clear.

However, one redundancy is apparent in Eq. 6. Because the drag coefficient for the particles C_D is the same as for spheres (because the fall diameter d is used for the characteristic length), C_D is a function of the particle Reynolds number, namely

$$C_D = f\left(\frac{w\,d}{\nu}\right) \dots\dots\dots\dots\dots (22)$$

in which f is the experimentally determined curve given in any elementary textbook. Rewriting,

$$\frac{w\,d}{\nu} = \frac{w}{V}\,\frac{d}{D}\,\frac{V\,D}{\nu}$$

$$= \left(\frac{w}{V}\right)\left(\frac{d}{D}\right)\left(R_e\right) \dots\dots\dots\dots\dots (23)$$

The last three parameters in parenthesis are given in Eq. 6, so C_D, a known function of the preceding product, is superfluous; that is, C_D is already a dependent variable by another relation not expressed.

Finally, dimensional analysis should give dimensionless combinations which are convenient. Since the paper is on resistance to flow, a dimensionless friction factor such as the Darcy-Weisbach f should be used instead of slope S in Eq. 6.

Furthermore, in the tabulation of experimental values (Table 1) it would be most helpful if the authors would give the computed roughness coefficients, and $C/\sqrt{g}$-values, upon which the subsequent discussion in the text is based. The arrangement of data in Table 1 according to increasing slope values is not a sequence with increasing roughness or shear because of the variable depth.

Bed Forms.—In the list of forms of bed roughness, "Plant bed and water surface" has been placed under the "Rapid flow regime, $F_r > 1$." While this may be true for their flume experiments (although none in the category are cited in Table 1) it should be made very clear that $F_r > 1$ is not a necessary condition for existence of a plane or flat bed. When there is no strong coupling between the bed and water surface profiles (as there is for standing waves or antidunes), the Froude number is a minor or irrelevant parameter because of the absence of a local gravitational effect on the flow pattern. For the plane bed condition it is only necessary to have velocities near the bed of such magnitude that a dune, if one were to form, would be "sheared" off. The stability of the sand surface depends on the magnitude of the local fluid forces relative to the resistance of particles to motion, and should not be sensitive to depth. Since the Froude number does depend on depth, and does not depend on the sand characteristics, it is an illogical parameter for describing bed forms which are not coupled with surface waves.

This brings us to the crucial weakness of the authors' entire method of description of the various bed forms in alluvial channels under "Observed Flow Phenomena." In attempting to make their description appear more generalized they have used the dimensionless quantities d/σ and F_r as criteria for separation of the various types of bed forms (for example, "transition" occurs when "$d/\sigma' > 2$ and $0.6 < F_r < 1.0$"). The citing of dimensionless values has merit, not a priori, but only if they are meaningful over a significant range of conditions. Because there is no physical reasoning to support the use of these parameters, and the cited values are easily proved to be lacking in generality, their use obscures the solution to the problem rather than clarifying it.

As indicated previously the Froude number has relevance only for the standing wave and antidune cases. It means very little to say that ripples change to dunes at $F_r = 0.28$; the authors would make their point more clearly by stating that dunes begin at a mean velocity of about 1.40 fps for their experiments. With change to another flow system of a different depth but with the same sand, the velocity at the point where ripples become dunes is more likely to be about the same actual value cited above than a new value computed from $F_r = 0.28$.

Consider now the parameter d/σ', the ratio of the grain size to the so-called thickness of the laminar sublayer. When the bed is covered with ripples, dunes, or other form resistance, σ' lacks physical interpretation, but still can be computed by the relation

$$\sigma' = \frac{11.6\,\nu}{V_*} \quad \dots \dots \dots \dots \dots \dots \quad (24)$$

Consequently

$$d/\sigma' = \frac{dV_*}{11.6\,\nu} \quad \dots \dots \dots \dots \dots \dots \quad (25)$$

Now for example the authors state that motion begins when d/σ' was between 0.48 and 0.53. By Shields' graph[46] the initiation of motion could begin at values of this parameter ranging from say 0.2 to 50 or more depending on the properties of the particles and of the fluid. (By comparison $\tau_0/\Delta\gamma_s d$ ranges from 0.035 to 0.07 as a function of d/σ'). Whether the boundary is hydrodynamically smooth or rough (as determined by the value of d/σ') is irrelevant; the question involves the balance of forces on the particles.

46 "Sediment Transportation," by C. B. Brown, Engineering Hydraulics, ed. H. Rouse, Wiley, New York, 1950, p. 790.

The features which the authors call ripples have been called dunes in the past by the writers in describing laboratory experiments at the California Institute of Technology[47,48] (CIT). The larger features called dunes by the authors have not been observed at CIT, because of depths limited to 0.5 ft or less, and widths of 0.875 ft or 2.79 ft respectively in two flumes. They may be related, however, to what has been described as sand waves, which were features with long flat tops, and not three-dimensional in plan as shown by Fig. 4. Apparently there is no essential difference between ripples and dunes other than size, but it is curious that there is no gradual gradation from ripples to dunes. Still much larger dune-type features have been observed in rivers (as by Carey and Keller[49] on the Mississippi River). Perhaps for a given sand only certain ranges of dune (or ripple) heights and wave lengths are possible, with appearance of the large set(s) suppressed by inadequate depth or width. If this is so, it makes the roughness analysis and modeling of alluvial channels a very troublesome problem, which cannot be resolved by laboratory work alone.

The general description of antidunes and standing waves agrees well with recent research at CIT by John F. Kennedy[50,51] on the mechanics of these waves, sponsored by the Agricultural Research Service. It may be noted that antidune behavior is considerably influenced by the grain size, as well as the velocity. For example, Kennedy found that for 0.23 mm sand, the friction factor f consistently rose as F_r increased, due to heavy wave breaking, but that for 0.55 mm sand, f decreased as V increased because the amplitude of antidunes decreased, and wave breaking was less important than for the more easily transported 0.23 mm sand. Also for 0.55 mm sand, a flat bed regime was found following antidunes with increasing velocity (at $F_r \approx 2$); but for 0.23 mm sand this regime was not found.

In Fig. 8 there are a number of runs listed as "Standing Waves" (Runs 28, 31, 33, 34, 35, 37, 38, 39, 41), but in Table 1 these are all listed as "Transition." Clarification of the bed configuration is needed for these runs.

Analysis of Roughness by Authors.—At the end of the section "Analysis of Data," the authors present Fig. 11 (Eq. 9) which, it is claimed, can be used to compute $C/\sqrt{g}$ (or simply $\sqrt{8/f}$). Since $C/\sqrt{g}$ does not appear explicitly, one may instead seek S by trial and error, given depth, velocity, and sand and water properties. It must be noted that this curve is by no means general, but applies only for the particular sand and flume used by the author.

To see how well Fig. 11 allows the slope to be determined, trial-and-error solutions for S were attempted for several of the reported runs, assuming everything known by S. Fig. 33 shows the curve of Fig. 11, together with two other curves, each of which is obtained by assuming arbitrary values of S while using the depths and velocities reported for the runs indicated (runs 12 and 40).

47 "Mechanics of Streams with Movable Beds of Fine Sand," by N. H. Brooks, Transactions, ASCE, Vol. 123, 1958, pp. 526-594.

48 "Laboratory Studies of the Roughness and Suspended Load of Alluvial Streams," by V. A. Vanoni and N. H. Brooks, Sedimentation Laboratory, Calif. Inst. of Tech., Pasadena, Calif., Report E-68, 1957.

49 "Systematic Changes in the Beds of Alluvial Rivers," by W. C. Carey and M. D. Keller, Proceedings, ASCE, Vol. 83, No. HY 4, August, 1957.

50 "Stationary Waves and Antidunes in Alluvial Channels," by J. F. Kennedy, thesis presented to the Calif. Inst. of Tech., Pasadena, Calif., in 1960, in partial fulfilment for the degree of Doctor of Philosophy.

51 "Study of Relations between Transportation of Sediment and Hydraulic Characteristics of Streams," by J. F. Kennedy, Keck Lab. of Hydr. and Water Resources, Calif. Inst. of Tech., Pasadena, Calif., Report KHWR-6, in press.

The assumed values range from 0.1 to 10 times the observed slope. For definitive trial-and-error solutions, of course, these curves should have well defined intersections with the given function. Unfortunately this is not the case.
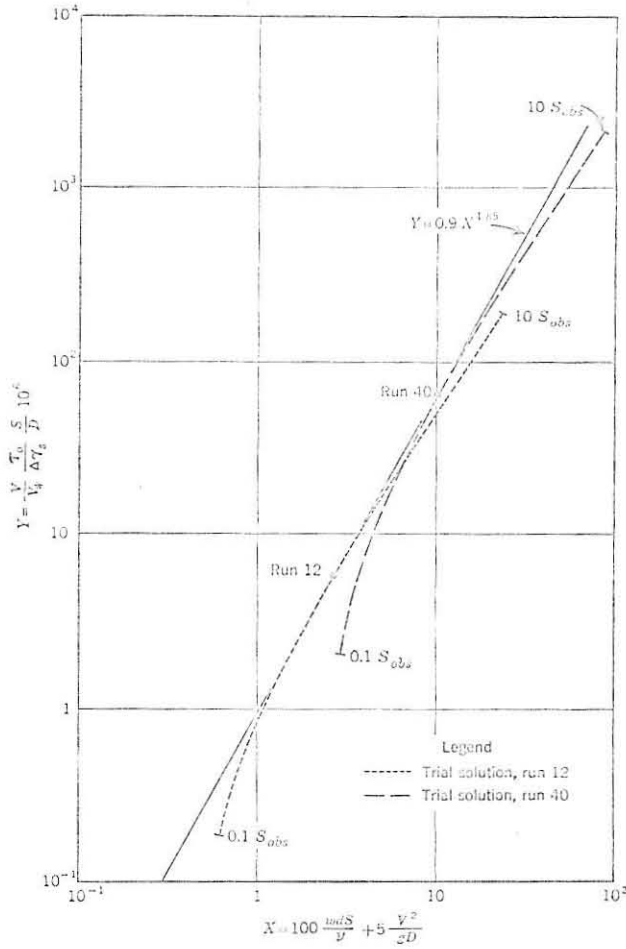

FIG. 33.—AUTHORS' FIG. 11, SHOWING IMPOSSIBILITY
OF TRIAL-AND-ERROR SOLUTIONS FOR
SLOPE

The trial curves shown represent a variation of a hundredfold in assumed S-values and still lie within the belt of scattering of the data points upon which the function is based. Several other runs were tried, with similar results. It must therefore be concluded that Fig. 11 is of no help in the estimation of slope

(or friction factor). The indiscriminate juggling of relationships may thus, as in this case, lead to relations more nearly approaching disguised mathematical identities than physical laws.

That the same conclusion holds for Fig. 10 is evident when it is noted that each of the variables contains slope to the one-half power. Thus the "trial" curves are straight lines which slope upward to the right at 45° and which like-wise have rather unsatisfactory intersections with the zone within which the data lie.

An Alternate Analysis of Roughness. — In clear-water flow in pipes or in open channels with rigid boundaries, the slope of the energy grade line may be de-termined by means of the Darcy-Weisbach friction factor, f, which is a well-established function of the Reynolds number and of the relative roughness (and is usually expressed in a pipe friction diagram). If the flow is in an alluvial channel, however, the problem is complicated by the presence of additional causes of energy dissipation, such as the turbulence in the wakes of dunes and ripples on the moveable bed. Hence Einstein and others have chosen to appro-tion the total boundary shear stress τ_0 into a part, τ'_0 due to grain roughness and the additional part, τ''_0, due to form drag, so that $\tau_0 = \tau'_0 + \tau''_0$. Since $\tau_0 = \gamma\, r\, S$, this partition requires some partition either of the hydraulic radius or of the slope (or of both). If the hydraulic radius is partitioned, following Einstein and Barbarossa,[52] the results are two virtual channels of the same slope and flow velocity but of different geometries, giving rise to a certain amount of intuitional difficulty. If, on the other hand, the slope is partitioned, as Meyer-Peter and Müller[53] proposed, one can write $S = S' + S''$, and visu-alize S' as that energy slope necessary for the given flow to overcome the re-sistance due to the grain resistance on a flat bed, where S'' would be the ad-ditional energy slope necessary to enable the flow to overcome the form drag caused by the bed configuration.

This being the case, the ratio of the true slope to that corresponding to the grain drag only (S/S') would be a simple measure of the energy dissipation arising out of the alluvial nature of the stream, as related to that expected in clear-water flow over a flat bed. Furthermore, to the extent that most of the additional dissipation arises from form drag, S/S' could be a useful descrip-tion of the bed configuration itself. Since the Darcy-Weisbach friction factor, f, for open channel flow is defined as

$$f = \frac{8\, g\, r\, S}{V^2} \quad\dots\dots\dots\dots\dots\dots\dots (26)$$

in which r is the hydraulic radius, and V is the mean channel velocity, then a corresponding friction factor, related to the grain resistance only, may be de-fined as

$$f' = \frac{8\, g\, r\, S'}{V^2} \quad\dots\dots\dots\dots\dots\dots (27)$$

Hence,

$$\frac{S}{S'} = \frac{f}{f'} \quad\dots\dots\dots\dots\dots\dots (28)$$

and thus the ratio f/f' has the same physical significance as S/S'.

52 "River Channel Roughness," by H. A. Einstein and N. L. Barbarossa, Transactions, ASCE, Vol. 117, 1952, p. 1121-1146.

53 "Formulas for Bed-Load Transport," by E. Meyer-Peter, and R. Müller, Inter-natl. Assoc. for Hydr. Research, Second Meeting, Stockholm, Sweden, 1948, pp. 39-65.

According to the definition of S', f' is simply the friction factor for clear flow over a flat bed with grain roughness only. Hence a reliable value of f' may readily be found from the Moody pipe-friction diagram, using 4 r in place of the pipe diameter, and taking the equivalent sand roughness ϵ to be the mean grain size of the bed material. (Some investigators say that the equivalent roughness is bigger than the mean size (for example, the 65% -size is advocated by Einstein[52]); this may well be true, and further research is needed on this point, but it is of minor importance here). Namely,

$$f' = \phi\left(R_e, \frac{\epsilon}{4\,r}\right) \quad \dots\dots\dots\dots\dots\dots (29)$$

Thus to compute the value of the ratio f/f' from measured data one has only to divide the f of Eq. 26 by the f' from the pipe friction diagram.

TABLE 4.—FRICTION FACTORS AND f/f' FOR AUTHORS' DATA

Run No. (1)	f (2)	f' (3)	f/f' (4)	Form of Bed Roughness[a] (5)	Run No. (6)	f (7)	f' (8)	f/f' (9)	Form of Bed Roughness[a] (10)
14	.0308	.0208	1.48	Plane	20	.0927	.0193	4.80	Dunes
17	.0514	.0189	2.70	Ripples	23	.0537	.0187	2.87	Dunes
16	.0471	.0196	2.40	Ripples	24	.1288	.0192	6.71	Dunes
13	.0367	.0235	1.54	Plane	40	.0474	.0177	2.68	Dunes
15	.0630	.0195	3.23	Ripples	39	.0204	.0190	1.07	Transition
18	.0684	.0210	3.24	Ripples	26	.0101	.0211	0.48	Transition
2	.0435	.0190	2.29	Ripples	28	.0274	.0207	1.32	Transition
3	.0528	.0187	2.82	Ripples	29	.0740	.0223	3.32	Transition
9	.0657	.0210	3.12	Ripples	31	.0244	.0201	1.21	Transition
1	.0479	.0191	2.50	Ripples	27	.0384	.0213	1.80	Transition
5	.0435	.0191	2.28	Ripples	36	.0508	.0252	2.02	Transition
11	.0830	.0237	3.50	Ripples	41	.0222	.0190	1.17	Transition
4	.0420	.0192	2.19	Dunes	30	.0517	.0226	2.33	Transition
8	.0802	.0212	3.78	Ripples	35	.0380	.0230	1.65	Transition
7	.0585	.0192	3.04	Dunes	34	.0267	.0220	1.21	Transition
10	.1223	.0240	5.11	Ripples	33	.0187	.0222	0.84	Transition
6	.0816	.0216	3.77	Ripples	35	.0244	.0194	1.26	Transition
12	.1018	.0243	4.20	Ripples	37	.0201	.0200	1.00	Transition
19	.0624	.0215	2.91	Dunes	32	.0228	.0206	1.11	Antidune
21	.0907	.0178	5.09	Dunes	45	.0926	.0220	4.21	Antidune
22	.0886	.0177	5.01	Dunes	44	.0267	.0217	1.22	Antidune
25	.0850	.0211	4.00	Dunes	42	.0256	.0215	1.19	Antidune
					43	.0266	.0198	1.34	Antidune

[a] From Table 1.

Ideally one would expect f/f' to be near unity when the stream bed is flat, and much higher when the bed is dune-covered. In the situation where suspended load dampens the turbulence, one could even expect values of f/f' to be less than one.[48] These expectations are very clearly demonstrated by the authors' data. Table 4 shows the values of f, f', and f/f' computed from their data (Table 1). It will be seen that a value of f/f' = 2 very neatly separates the runs with bed forms (for example, dunes or ripples) from those either without bed forms or with only the gradual sinusoidal variations of the type associated with standing waves or antidunes.

There are two advantages which may be realized in the use of the f/f' parameter. The first is in the matter of terminology, and the second is in the problem of predicting roughness.
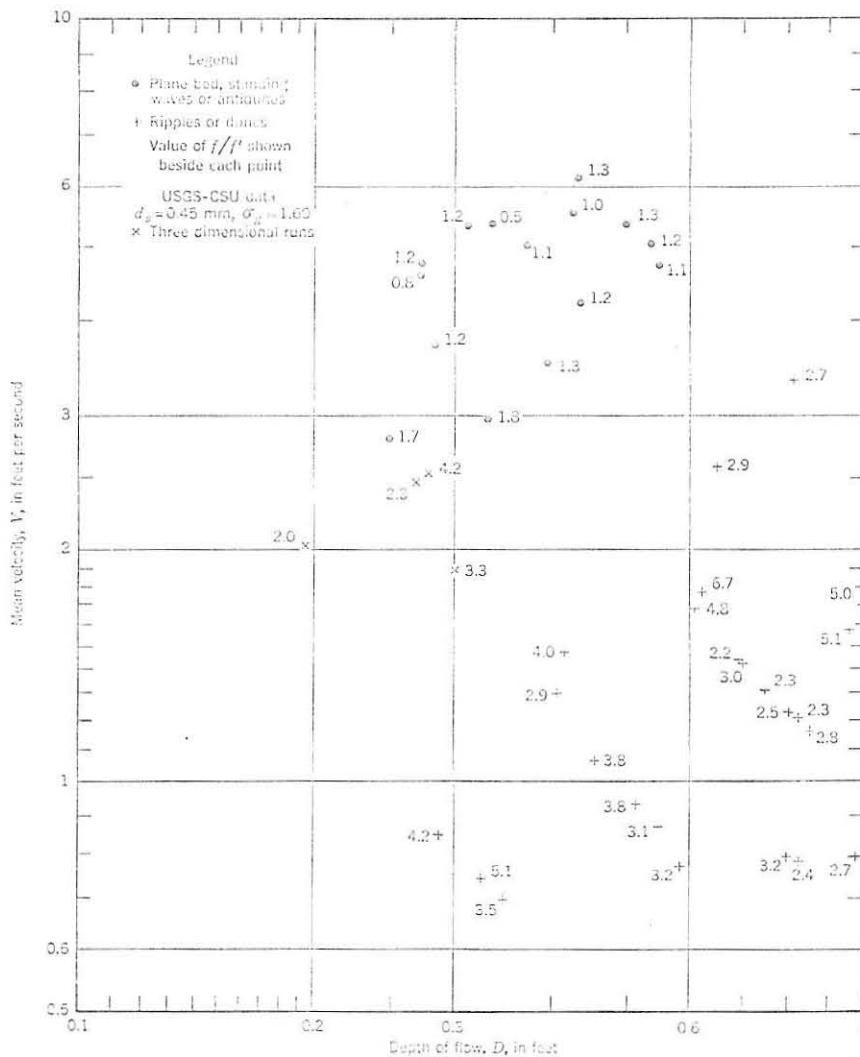

FIG. 34.—VARIATION OF f/f' WITH VELOCITY AND DEPTH FOR 0.45 mm SAND IN CSU FLUME ACCORDING TO AUTH-ORS' DATA

The present situation with respect to terminology for description of bed configurations is unsatisfactory. A list of all the terms which have been applied in

the literature is a long one, and in current use one can find a laboratory call-
ing a ripple the same form that another calls a dune, while the latter uses the
term "dune" to refer to something else again. Rather than depending solely on
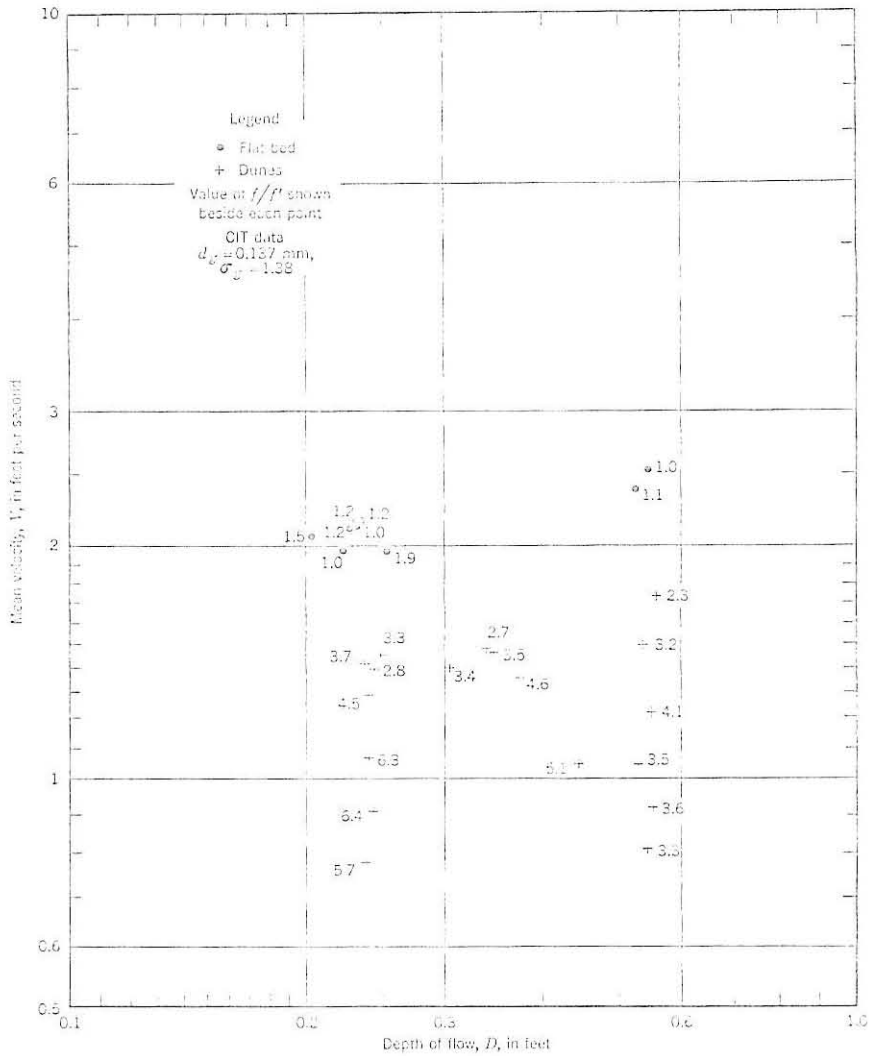

FIG. 35.—VARIATION OF f/f' WITH VELOCITY AND DEPTH FOR
0.14 mm SAND IN CIT FLUME

an agreement on terminology as a necessary step in the quantitative analysis
of roughness, it would seem that a more fruitful approach would be to use a
numerical quantity which can be objectively determined. In such a case the

use of curves for alluvial open-channel discharge (Liu and Hwang,[54] for example) would not require both a prediction of the bed forms to be expected and also a knowledge of what particular term had been used to describe them.

TABLE 5.—DATA FROM RUNS ON CALIFORNIA INSTITUTE
OF TECHNOLOGY SAND NO. 4

Run No.	Velocity V fps	Depth D ft	f	f'	f/f'	Bed Configuration
(a) Vanoni and Brooks[b,c]						
2-9	0.77	0.238	.124	.0216	5.74	Dunes
2-3	0.90	0.243	.133	.0208	6.39	Dunes
2-8	1.07	0.240	.129	.0205	6.29	Dunes
2-1	1.28	0.240	.090	.0201	4.48	Dunes
2-7	1.40	0.237	.074	.0201	3.68	Dunes
2-6	1.44	0.249	.064	.0196	3.27	Dunes
2-17D[a]	1.39	0.302	.067	.0196	3.42	Dunes
2-17F[a]	2.07	0.203	.029	.0193	1.51	Flat
2-2	2.13	0.203	.0235	.0192	1.22	Flat
2-12	0.80	0.541	.051	.0193	3.84	Dunes
2-5	1.04	0.528	.063	.0178	3.54	Dunes
2-10	1.22	0.540	.072	.0175	4.12	Dunes
2-11	1.49	0.536	.055	.0170	3.24	Dunes
2-13D[a]	1.72	0.353	.038	.0169	2.25	Dunes
2-16F[a]	2.39	0.524	.0186	.0168	1.10	Flat
2-4	2.53	0.544	.0170	.0164	1.04	Flat
(b) Nomicos[c]						
H-2	2.13	0.233	.0215	.0207	1.04	Flat
H-3	1.97	0.223	.022	.0211	1.04	Flat
H-7a	1.38	0.243	.059	.0213	2.77	Dunes
(c) Kennedy[d]						
3-1	0.91	0.550	.068	.0187	3.63	Dunes
3-4	1.14	0.441	.096	.0187	5.13	Dunes
3-2	1.35	0.373	.086	.0189	4.55	Dunes
3-7	1.45	0.345	.067	.0190	3.51	Dunes
3-5D[a]	1.47	0.340	.052	.0192	2.71	Dunes
3-5F[a]	1.97	0.254	.036	.0192	1.87	Flat
3-6	2.15	0.233	.023	.0193	1.19	Flat
3-6A	2.13	0.235	.023	.0193	1.19	Flat
3-6B	2.21	0.228	.023	.0193	1.19	Flat

[a] D = dune section in run with long sand wave. F = flat section in run with long sand wave. No side-wall correction made.

[b] "Mechanics of Streams with Movable Beds of Fine Sand," by N. H. Brooks, _Transactions,_ ASCE, Vol. 123, 1958, pp. 526-594.

[c] "Laboratory Studies of the Roughness and Suspended Load of Alluvial Streams," by V. A. Vanoni and N. H. Brooks, Sedimentation Laboratory, Calif. Inst. of Tech., Pasadena, Calif., Report E-68, 1957.

[d] "Study of Relations between Transportation of Sediment and Hydraulic Characteristics of Streams," by J. F. Kennedy, Keck Laboratory of Hydraulics and Water Resources, Calif. Inst. of Tech., Pasadena, Calif., Report KHWR-6, in press.

Because the principal reason for wanting to know what bed form will be found under given conditions is so that the friction factor can be estimated, it is use-

54 "Discharge Formula for Straight Alluvial Channels," by H.-K. Liu and S.-Y. Hwang, _Proceedings,_ ASCE, Vol. 85, No. HY 10, October, 1959, pp. 65-98.

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ful to have a means of estimating the friction factor directly. In connect with a roughness relation, moreover, it would seem unnecessary to distinguish between those features the authors call "ripples" and those they call "dunes." (While these situations are clearly different with respect to transport, they are similar as far as roughness is concerned). It is therefore suggested that an objective, numerical quantity is a necessary part of the description of a bed configuration, and that the value of f/f' is a measure suitable to this purpose. It is hardly necessary to say that such a value in itself is an incomplete description, in the same way that the verbal description, in itself, is also incomplete.

The second advantage which the use of f/f' may offer is in predicting the roughness which may be expected for given flow conditions. A number of interesting ideas are suggested by Figs. 34 and 35, which show f/f' as a function of depth and velocity. Fig. 34 uses data for the authors' experiments, $d = 0.45$ mm, (runs 13 and 14 have been omitted because there is no transport and the flat bed was artificially prepared). Fig. 35 uses data from experiments (2, 3, 6) made at the California Institute of Technology on a finer sand (geometric mean, $d_g = 0.137$ mm). (Tables 4 and 5 respectively.) First, it will be noted that the variation of f/f' seems fairly regular, especially when it is remembered that as a percentage of the actual friction factor a difference between f/f' values of 2 and 3 is much greater than between f/f' values of 6 and 7. Second, it is interesting to note that the separation between dune-type bed configurations and flat configurations occurs at about the same f/f' value in both sands.

The directness and simplicity with which f may be estimated for a given depth and velocity may be illustrated by the following example. Suppose it were desired to estimate the friction factor for a flow over the authors' sand if the depth were 0.8 ft and the velocity were 2.5 fps. From Fig. 34, f/f' would be about 3, and a rough (dune or ripple) bed would be expected. Taking the mean diameter for the effective roughness height and assuming $\nu = 10^{-5}$ sq ft per sec, the relative roughness, $\epsilon/(4 \, r)$, is .00055, and $R_e = 4 \, r \, V/\nu = 940,000$; for these values the pipe friction diagram gives $f' = .05$, approximately. The slope can be determined by substituting directly into the definition of f, Eq. 26; in this example, the result is about $S = .0015$.

Further research on the roughness problem is urgently needed. It may be hoped that continued flume studies like the authors' and more field investigations will make a comprehensive and logical roughness analysis possible in the future.

ERRATA:

Thesis page	Line	For	read
59	1	A.M. ASCE	M. ASCE
60	13	sensibility	sensibly
61	1	"Plant	"Plane
61	19	d/σ	d/σ'
61	38	known by	known but
64	16	appro-	appor-

APPENDIX B

Summary of Notation

In the following summary, the page number refers to the page on which each symbol is first used or defined.

Symbol	Description	Page
A	Area of cross section or of a zone thereof	51
$A(y)$	Area of surface which is actually water	21
b	Width of channel	13
b_g	Width of rough part of channel	13
d	Depth of flow	14
E	Relative roughness	4
f	Darcy-Weisbach friction factor	4
f_A	Friction factor estimated by Method A	31
f_B	Friction factor estimated by Method B	31
ff_r	Friction factor for flow in rough two-dimensional channels	4
f_R	Observed friction factor for rough bed	26
f_{ro}	Friction factor for flow in rough circular sections	4
f_s	Friction factor for flow in smooth two-dimensional channels	4
f_S	Observed friction factor for split bed	26
f_{so}	Friction factor for flow in smooth circular sections	4
F	Froude No.	14
g	Acceleration of gravity	4
m	Slope of semilogarithmic velocity distribution	42
P	Perimeter	51
p	Pressure	47
Q	Fluid discharge	14

Subscripts and other supplementary symbols systematically used

b	Quantities pertaining to zones associated with the bed (A, P, f, u)	

w   Quantities pertaining to zones associated with the
walls (A, P, f, u)

——   Mean value (of velocity, u, v, or w; or of pressure, p)

'   Fluctuating value (of velocity, u, v, or w)

REFERENCES

1. Meyer-Peter, E. and Müller, R., "Formulas for Bed-Load Transport", Internatl. Assn. for Hydr. Res., Proc. 2nd Meeting, Stockholm, 1948, p. 39 - 65.

2. Einstein, H. A., "Formulas for the Transportation of Bed Load", Trans. Amer. Soc. Civil Engrs., v. 107, 1942, p. 561 - 577.

3. Einstein, H. A., "The Bed-Load Function for Sediment Transportation in Open Channel Flows", U. S. Dept. of Agriculture, Soil Conservation Service, Tech. Bul. No. 1026, Sept. 1950, 71 p.

4. Einstein, H. A. and Barbarossa, N. L., "River Channel Roughness", Trans. Amer. Soc. Civil Engrs., v. 117, 1952, p. 1121 - 1146.

5. Ali, S. M. and Albertson, M. L., "Some Aspects of Roughness in Alluvial Channels", Dept. of Civil Eng., Colorado A. and M. College, Fort Collins, Colorado, Aug. 1953, 9 p.

6. Liu, H.-K. and Hwang, S.-Y., "Discharge Formula for Straight Alluvial Channels", Proc. Amer. Soc. Civil Engrs., v. 85, n. HY 10, October 1959, p. 65 - 98.

7. Albertson, M. L., Simons, D. B., and Richardson, E. V., "Discussion of 'Mechanics of Sediment Ripple Formation' by H.-K. Liu", Proc. Amer. Soc. Civil Engrs., v. 84, paper 1558, p. 23 - 32. See also: Simons, D. B. and Richardson, E. V., "Resistance to Flow in Alluvial Channels", Proc. Amer. Soc. Civil Engrs., v. 86, n. HY 5, p. 73 - 99.

8. Taylor, R. H., Jr., and Brooks, N. H., "Discussion of 'Resistance to Flow in Alluvial Channels', by D. B. Simons and E. V. Richardson", Proc. Amer. Soc. Civil Engrs., v. 86, n. HY 1, January 1961, p. 246 - 256.

9. Taylor, R. H., Jr., "Resistance to Flow in Alluvial Channels: The Relative Friction Factor"; U. S. Geological Survey, 1961 (in process).

10. Johnson, J. W., "Laboratory Investigations on Bed-Load Transportation and Bed Roughness", U. S. Dept. of Agriculture, Soil Conservation Svc. Publ. SCS-TP-50, 116 p.

11. Schlichting, Hermann, "Boundary Layer Theory", New York, McGraw-Hill, 1960, 647 p.

12. Keulegan, G. H., "Laws of Turbulent Flow in Open Channels", J. Research Nat. Bureau Stds., v. 21, 1938, p. 707 - 741.

13. Powell, R. W., "Resistance to Flow in Rough Channels", Trans. Amer. Geophys. Union, v. 31, 1950, p. 575 - 582.

14. Powell, R. W., "Flow in a Channel of Definite Roughness", Trans. Amer. Soc. Civil Engrs., v. 111, 1946, p. 531 - 566.

15. Colebrook, C. F., "Turbulent Flow in Pipes, with Particular Reference to the Transition Region between the Smooth and Rough Pipe Laws", Proc. Inst. Civil Engrs., v. 11, 1933, p. 133 - 156.

16. Nikuradse, Johann, "Strömungsgesetze in rauhen Rohren", Forschungs-Arb. Ing.-Wesen, n. 361, Berlin, 1933.

17. Colebrook, C. F. and White, C. M., "Experiments with Fluid Friction in Roughened Pipes", Proc. Royal Soc. (London), v. 161, 1937, p. 367 - 378.

18. Taylor, R. H., Jr., "Notes on Flow and Roughness Observations in Two Alluvial Streams", U. S. Geol. Survey unpublished memorandum, 1960, 13 p.

19. Simons, D. B., "Theory and Design of Stable Channels in Alluvial Materials", Colorado State University, Report CER57DBS17, 1957, 394 p.

20. Kennedy, J. F., "Stationary Waves and Antidunes in Alluvial Channels", California Institute of Technology, W. M. Keck Laboratory of Hydraulics and Water Resources, Report KH-R-2, 1961, 146 p.

21. Missouri River Division, U. S. Corps of Engineers, "Sediment Transportation Characteristics Study", First Interim Report, Appendix D, Topographic Maps, Omaha, Nebraska, Feb. 1952 (unpublished).

22. Exner, Felix, "Über die Wechselwirkung zwischen Wasser and Geschiebe in Flüssen", Akad. d. Wiss. (Wien), Abt IIA, v. 134, 1925, p. 199 - 210.

23. Thompson, M. M., "Photogrammetric Mapping of Sand Beds in a Hydraulic Test Flume", Photogrammetric Engineering, June 1958, p. 468 - 475.

24. Leliavsky, Serge, "An Introduction to Fluvial Hydraulics", London, Constable, 1955, 257 p.

25. Johnson, J. W., "The Importance of Side-Wall Friction in Bed-Load Investigations", Civil Eng., v. 12, n. 6, 1942, p. 329 - 331.

26. Vanoni, V. A. and Brooks, N. H., "Laboratory Studies of the Roughness and Suspended Load of Alluvial Streams", California Institute of Technology, Sedimentation Laboratory Report E-68, 1957, 121 p.

27. Brooks, N. H., "Mechanics of Streams with Movable Beds of Fine Sand", Trans. Amer. Soc. Civil Engrs., v. 123, 1958, p. 526 - 594.

28. Einstein, H. A. and El-Samni, E. A., "Hydrodynamic Forces on a Rough Wall", Rev. Mod. Phys., v. 21, 1949, p. 520 - 524.

29. Prandtl, Ludwig, "Essentials of Fluid Dynamics", New York, Hafner, 1952(?), 452 p.

30. Nikuradse, J., "Untersuchungen über die Geschwindigkeits-verteilung in turbulenten Strömungen", Verein Deutscher Ingenieure Forschungsheft 281, Berlin, 1926.

31. Howarth, L., "Concerning Secondary Flow in Straight Pipes", Proc. Camb. Phil. Soc., v. 34, 1938, p. 335 - 344.

32. Einstein, H. A. and Li, Huon, "Secondary Currents in Straight Channels", Trans. Amer. Geophys. Union, v. 39, 1958, p. 1085 - 1088.

33. Liepmann, H. W. and Dhawan, S., "Direct Measurement of Local Skin Friction in Low-Speed and High-Speed Flow", Proc. 1st U. S. Natl. Congr. Appl. Mech., 1952, p. 869.

34. Ludwieg, H., "Ein Gerät zur Messung der Wandschubspannung turbulenter Reibungsschichten", Ing.-Arch., v. 17, 1949, p. 207-218.

35. Murphy, J. S. and Smith, A. M. O., "Measurement of Wall Shearing Stress in the Boundary Layer by Means of an Evaporating Liquid Film", J. appl. Phys., v. 27, 1956, p. 1097 - 1103.

36. Hsu, E. Y., "The Measurement of Local Turbulent Skin Friction by Means of Surface Pitot Tubes", U. S. Navy Dept., Taylor Model Basin Res. and Devel. Rpt. 957, 1955, 18 p.

37. Leighly, J. B., "Toward a Theory of the Morphologic Signifi-cance of Turbulence in the Flow of Water in Streams", Univ. of California (Berkeley) Publs. in Geogr., v. 6, 1932, p. 1 - 22.

38. Brooks, N. H., "Laboratory Studies of the Mechanics of Streams Flowing over a Movable Bed of Fine Sand", California Institute of Technology, Thesis, 1954, 248 p.