THE ABSORPTION COEFFICIENT OF HARD GAMMA-RAYS

Thesis by

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CONTENTS

SUMMARY

The relation between the frequency of hard radiations and the absorption coefficient is not experimentally well established at the present time. In the following work, the absorption coefficient of γ-rays of ThC" is carefully investigated owing to their homogeneity.

As a result, the observed absorption coefficient of light elements is found to agree fairly well with the theoretical formula of Klein and Nishina. Running from low to high atomic numbers, the absorption per electron is, however, found to increase continuously. This was not noticed before because it was usually covered by the photo-electric absorption of the soft components in a non-homogeneous beam. Furthermore, in some cases emphasis was given to the mass absorption coefficient which is exceptionally high for the hydrogen compounds, consequently the characteristic of the continuous increase of the absorption coefficient per electron was neglected.

In order to interpret this anomalous absorption, several scattering experiments are carried out. The forward scattering can be accounted for by the assumption that all the external electrons behave as if they are free in high frequency radiations around a few x-units. Theoretically the part of the anomalous absorption of heavy elements might be due to the scattering of electrons inside the nucleus or that of tightly bound electrons. The fact that, although such a process does not contribute much scattering in the forward direction its contribution in large scattering angles may become important is also indicated in the experiment.

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I Introduction: The classical theory of the scattering of radiations can only roughly account for the experimental results in the region of relatively long wave-length, like that of soft x-rays. For hard radiations, it departs widely from the observations. A. H. Compton[1] and P. Debye[2], independently, applied the quantum consideration to the problem of scattering. The main conclusions reached then, are the following:

(1) The wave-length of the scattered rays should be increased by an amount $\Delta\lambda$ which is given by

$$\Delta\lambda = \frac{h}{mc}(1-\cos\theta) = 24.24(1-\cos\theta)\times 10^{-11}\ \text{cm} \qquad (1)$$

(2) The intensity of the scattered rays decreases rapidly as the angle of scattering increases. The following formula was proposed by Compton[3]

$$I_\theta = I_0 \cdot \frac{1}{2}\,\frac{1+\cos^2\theta+2\alpha(1+\alpha)(1-\cos\theta)^2}{\{1+\alpha(1-\cos\theta)\}^5} \qquad (2)$$

where I_0 is the scattering intensity at $\theta=0$ and $\alpha = h\nu/mc^2$. The change of wave-length was confirmed very satisfactorily by experiments in the x-ray region. Determinations of the intensity distribution of the scattering with x-rays are very difficult to make. On the other hand, observations of the scattered γ-rays have been made and agree approximately with the theory as shown by the experiments of Compton,[4] Kohlrausch,[5] and Hoffmann,[6] but the quantitative agreement is not conclusive, owing to the lack of a homogeneous beam and the low intensity.

In 1925, Dirac[7] worked out a formula for the scattering intensity by the new quantum theory, which is fairly close to that of Compton. After the development of Dirac's theory of spinning electrons, Klein and Nishina[8] worked out a new scattering formula, which is very much different from those of Compton and Dirac for high frequency radiations, but approaches them for moderate frequency. The scattering intensity due to one electron, according to Klein and Nishima, is given by

$$I_\theta = I_0 \frac{e^4}{2m^2c^4r^2} \frac{1+\cos^2\theta}{[1+\alpha(1-\cos\theta)]^3} \left\{ 1+\alpha^2 \frac{(1-\cos\theta)^2}{(1+\cos^2\theta)[1+\alpha(1+\cos\theta)]} \right\}. \qquad (3)$$

The difference between this formula and that of Dirac is due to the second term in the last factor, which is missing in Dirac's formula. Now, by integrating over all directions and multiplying by the number of free electrons per c.c., we get the scattering coefficient

$$\sigma = \frac{2\pi n e^4}{m^2c^4} \left\{ \frac{1+\alpha}{\alpha^2} \left[\frac{2(1+\alpha)}{1+2\alpha} - \frac{1}{\alpha} \log(1+2\alpha) \right] + \frac{1}{2\alpha} \log(1+2\alpha) - \frac{1+3\alpha}{(1+2\alpha)^2} \right\}. \qquad (4)$$

The testing of these formulae can be approached either by the measurement of scattering intensity or by the determination of the absorption coefficient. In each method there are special advantages and disadvantages. For example; since the scattered radiations are of different hardness, their effect on the ionization chamber or electroscope are different from that of the primary radiation, or that of radiations scattered at

another angle; on the other hand, the measurement of scattered radiations gives the intensity distribution directly. The absorption measurement has the advantage of dealing with greater intensity, but it only tells the net loss, which might include different causes than true scattering. Better information can only be obtained by the carrying out of both methods. The present work deals mainly with the measurement of the absorption coefficient of hard γ-rays, although several scattering measurements have also been made to supplement it. ThC" is used as the source of γ-rays.

II PREVIOUS WORK: A large number of contributions on the determination of the absorption coefficient of γ-rays can be found in the literature. But different authors usually obtained different results. A good summary was given in Kohlrausch's Handbuch der Experimentalphysik, Vol. 15, Radioaktivitat. The main causes of the previous discrepancies are the following: (1) Most determinations were made on the γ-rays of RaC. Unfortunately, the γ-rays from RaC are very inhomogeneous, as shown in the secondary β-ray spectrum. So it is very difficult to draw any definite conclusions as to the relation between the frequency and the absorption coefficient measured. (2) Errors were often introduced due to the unsuitable experimental arrangement. In most cases the source was not well screened, divergent beams were used, and the absorber was put too close to the measuring apparatus (electroscope or ionization chamber). In

consequence an appreciable amount of scattered radiations was received, and this made the result unreliable. In several experiments like those of Kohlrausch the experimental conditions were quite favorable, but the inhomogeneity of the radiation still made it difficult to interpret.

ThC" is found to give a narrow band of γ-rays of wave-length around 4.7 XU, which is quite intense and far removed from the remaining γ-ray spectrum usually occuring together. Table I shows the composition of the γ-rays of ThC" obtained by Black[9] and re-computed by Bastings.[10]

TABLE I.

λ(XU)	K.V.	$I(\beta)$	$I(\gamma)$
45.1	277	39	2.3
22.5	515	25	9.3
4.7	2649	5.5	88.4

In the above table, K.V. represents the energy of each light quantum in kilovolts, $I(\beta)$ the intensity distbibution on the β-ray spectrum due to different γ-ray components, and $I(\gamma)$ the intensity distribution of the γ-ray components themselves. The requirement of the homogeneity of the rays is therefore satisfied to a certain extent by this element. Investigations of the absorption coefficient of these rays have been carried out by Russell and Soddy,[11] Rutherford and Richardson,[12] almost 20 years ago before the modern technique of the sensitive current measurement was developed. It has

also been measured recently by Bastings. But as all of these observers used fairly divergent beams with the electroscope placed near the absorber, an appreciable amount of the scattered rays entered the electroscope. This tends to lower the apparent value of the absorption coefficient. In fact, in Bastings' experiment this was purposely done in order to obtain the change of this apparent absorption coefficient. It is therefore thought desirable to make measurements on a narrow beam of these rays by the use of modern sensitive apparatus so that the value of the absorption coefficient experimentally determined fits the ordinary definition and thus makes possible a comparison with the theoretical values of Klein and Nishima's or Dirac's formulae. The present work is carried out for various elements to test the variation with the atomic number.

III THE RADIO-ACTIVE SOURCE: *Rd-Th* is used as the radio-active source, and *ThC″* is formed after several transformations. The quantity of *Rd-Th* is separated from *Ms-Th* and found to be quite pure from a measurement of the rate of decay. 88 days after the separation of the *Rd-Th* from *Ms-Th* when the equilibrium state is practically reached, the γ-ray intensity of the source is found to be equivalent to 3.595 *mg Ra* and it decreases to 3.210 *mg Ra* in 115 days. This gives the half period = 705 days.

IV THE EXPERIMENTAL ARRANGEMENT: In determining the absorption coefficient, particular care is taken to secure a parallel beam and to avoid the scattered rays from the

container of the source and the absorber. An apparatus similar to that of Kohlrausch is used. The radio-active source is put in the center of a lead cylinder "C" by the use of a lead plug "P" in a special form to cut off any stray rays as shown in Figure 1. The lead cylinder is 32 cm. long and 32 cm. in diameter. It can be rotated about a vertical axis in two ways, first a coarse rotation to any extent, then a slow rotation by screws "S" in an extent of about 20°. The canal on the lead plug is of 5 mm. in diameter at the inner end and 1 cm. in diameter at the outer end, so it allows a conical beam having a semi-angle of approximately 2.5°. Lead filters of different thickness are used to cut off the soft components of the γ-rays. They are represented by "F" in the figure. As measuring apparatus, Professor Millikan's cosmic ray electroscope is used at a distance of 2 meters from the source.

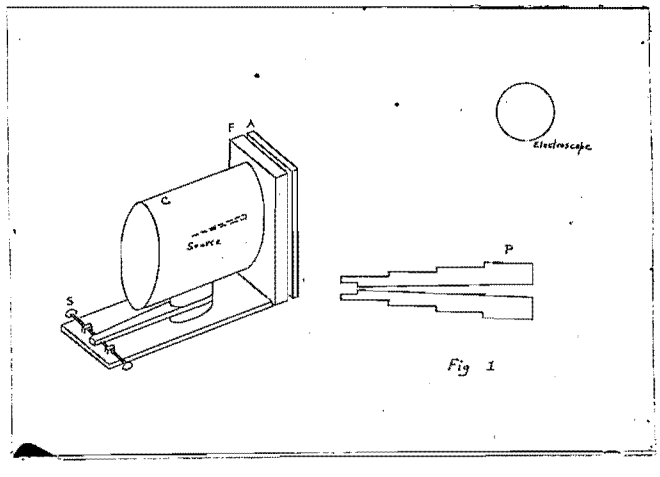

Fig 1

In a later group of experiments an ionization chamber connected to a vacuum electrometer of the type developed by Professor Hoffmann is used at a distance of 1 meter from the source.

The intensity of γ-rays becomes fairly low after being filtered through several cms of lead. So the natural leak of the measuring apparatus becomes very important. As natural leak, we mean the effect of the local radio-activity, the cosmic rays, and the wall effect. The last one is quite serious at first because of its fluctuation. This is eliminated to a great extent by the use of high pressure inside the ionization chamber and also by taking readings of long duration. The cosmic ray electroscope was originally constructed with an air pressure of about 9 atmospheres. In the ionization chamber the air pressure is about 25 atmospheres. The ionization chamber is made of steel and is 16 cm. in length and 10 cm. in diameter with walls of .5 cm. thickness. The sensitivity of the electroscope is about .02 e.s.u./mm; the sensitivity of the Hoffmann electrometer can be pushed very high, but owing to the fluctuation of the wall effect, it is better when used with a low sensitivity.

V PROCEDURE: The absorption coefficient is determined by measuring the ionic current due to γ-rays without and with the absorber. We shall call the readings without the absorber the initial readings, and those with the absorber the final readings. The absorber is put immediately after the

filter, so that it is kept far away from the measuring apparatus, and very little of its scattered rays could enter there. Now, both the initial and final readings are corrected for the natural leak and the scattering (due to the source container and the filter) by subtracting the value of the ionic current observed when the lead cylinder is rotated through 6.5° so that the primary beam just misses the ionization chamber (or the electroscope). Let I and F represent the initial and final ionization currents, let ΔI and ΔF represent the initial and final corrections due to natural leak and scattering, and call the thickness of the absorber d; then the absorption coefficient μ is given by:

$$\mu = \frac{1}{d} \log_e \left(\frac{I - \Delta I}{F - \Delta F} \right). \qquad (5)$$

In the experiment with the electrometer, two sets of readings are taken for each substance, each set consists of about 10 initial readings and 10 final readings and 5 initial correction readings and 5 final correction readings. The ratio of the initial reading to the final reading from two sets differ by less than .6%. The consistancy of the correction readings is of about the same order. The probable error of the absorption coefficient computed from their mean is about 2%. As an illustration, the readings for the determination of the absorption coefficient in water of the γ-rays of ThC" after being filtered through 6.6 cm. lead is given below. In this case

the time intervals for the scale reading of the electrometer to change by 12 cms. are first determined. The initial and final time intervals are indicated as tI and tF in Table II, those for initial and final corrections are indicated as t'I and t'F. Since $I=K/t$ and only the ratio of I is used, we set $K=10^3$ arbitrarily.

TABLE II

(All readings are given in seconds)

First Set		Second Set		First Set		Second Set	
tI	tF	tI	tF	t'I	t'F	t'I	t'F
217.4	262.8	219.2	265.1	633.0	644.0	637.0	657.5
217.2	262.1	217.4	267.7	638.5	648.0	633.5	650.0
212.7	271.0	214.0	265.1	630.0	647.5	640.0	643.5
216.4	268.7	211.1	261.7	644.0	637.0	638.0	655.0
214.5	266.0	211.7	262.7	643.0	646.5	636.0	639.0
217.2	267.4	218.2	267.5	637.7	644.6	636.9	649.0
215.2	266.7	213.7	266.9	t'F/t'I = 1.011,			1.019
213.6	264.9	213.8	268.2				
220.5	261.2	219.4	260.5				
217.5	270.6	219.9	266.9				
216.2	266.1	215.8	265.2				
tF/tI = 1.231,			1.229				

$\therefore (t_I)_{mean} = 216.0''$, $(t_F)_{mean} = 265.7''$,

$(t'_I)_{mean} = 637.3''$, $(t'_F)_{mean} = 646.8''$.

$I = (10^3/t) = 4.630$, $F = 3.763$,

$\Delta I = 1.569$, $\Delta F = 1.546$,

$I - \Delta I = 3.061$, $F - \Delta F = 2.217$,

$\frac{I - \Delta I}{F - \Delta F} = 1.381$, $d = 7.42$ cm, $\therefore \mu_{H_2O} = .0435$

The precision of the measurements by the electroscope is of about the same order.

VI RESULTS OF THE ABSORPTION COEFFICIENT: The absorption coefficient μ of lead for γ-rays of ThC" is first investigated when they are filtered through different thicknesses of lead. The purpose is to get some idea of the homogeneity of the rays. This part of the work is done with the electroscope, and the result is given in Table III, where the first column gives the thickness of the lead filter and the last column gives the corresponding value of μ, which is obtained when a separate lead absorber is introduced in addition to the lead filter. The lead absorber is .682 cm. thick. The notations used in Table III have been explained already.

It is also to be noted that different parts of the scale in the electroscope are used for different sets of readings, so the values of the current in different rows of Table III are not comparable.

TABLE III

μ of Pb after passing through different Pb filters

Thickness of Pb Filter cm.	I	ΔI	F	ΔF	μ
.0	4.072	.106	2.280	.103	.88
1.36	2.053	.132	1.439	.130	.563
2.72	1.138	.152	.8446	.1483	.510
4.08	.8511	.1818	.6579	.1805	.496
5.44	.5214	.1783	.4223	.1762	.487
6.80	.3511	.1746	.3021	.1746	.477

In determining the absorption coefficients of various elements, the γ-rays are filtered through 6.8 cms. of lead in order to get the hardest component as free from the soft components as possible. The same lead absorber of thickness .682 cms. is used here, the thickness of other absorbers is approximately equivalent to that of lead so that the values of μ of different elements correspond to γ-rays of the same hardness. This part of the work is done both with the electroscope and the electrometer. The result determined by the electroscope is given in Table IV, and that by the electrometer is given in Table V.

TABLE IV

μ of Various Substances by Electroscope

Substance	I	ΔI	F	ΔF	μ
H_2O	.3502	.1701	.2853	.1696	.0438
Al	.3870	.1705	.2821	.1699	.1029
Cu	.3359	.1763	.2789	.1759	.338
Zn	.3509	.1771	.2991	.1768	.275
Sn	.3355	.1762	.2879	.1759	.280
Pb	.3511	.1746	.3012	.1746	.477

TABLE V

μ of Various Substances by Electrometer

Substance	I	ΔI	F	ΔF	μ
H_2O	4.630	1.569	3.763	1.546	.0435
Al	4.876	1.618	4.100	1.592	.1023
Cu	4.697	1.574	3.557	1.534	.335
Zn	4.926	1.645	3.923	1.612	.274
Sn	4.946	1.653	3.939	1.618	.278
Pb	4.906	1.660	3.962	1.616	.478

VII DISCUSSION OF THE RESULTS OF THE ABSORPTION COEFFICIENT:

In order to compare the experimental results with the theory and to have a general survey of the situation of the problem, the above results together with some derived constants are again

listed in Table VI. Here ρ represents the density of each substance, Z the number of the external electrons in one atom, σ the theoretical value of the absorption coefficient as computed from equation (4); i.e., from Klein and Nishima's formula for $\lambda = 4.7 \times U$, μ, the absorption coefficient determined by the use of the electroscope, μ_2 that by the electrometer, and μ_m the mean experimental value, μ/ρ the mass absorption coefficient. The last row μ_e denotes the absorption coefficient per electron which is obtained by dividing the absorption coefficient μ by the number of external electrons per c.c. of each substance. $N = 6.06 \times 10^{23}$.

TABLE VI

Comparison of μ Observed and μ Calculated

	H_2O	Al	Cu	Zn	Sn	Pb
ρ	1.00	2.68	8.90	7.17	7.29	11.36
Z	8+1+1	13	29	30	50	82
σ	.0419	.0973	.306	.248	.231	.338
μ_1	.0438	.1029	.338	.275	.280	.477
μ_2	.0435	.1023	.335	.274	.278	.478
μ_m	.0437	.1026	.337	.275	.279	.478
μ/ρ	.0437	.0383	.0378	.0383	.0383	.0420
$\mu_e \times N$	.0787	.0794	.0829	.0834	.0909	.1062

The above values of $\mu_e \times N$ are plotted against Z in Fig. 2, where the value of water is plotted on $Z = 8$, this being of course arbitrary.

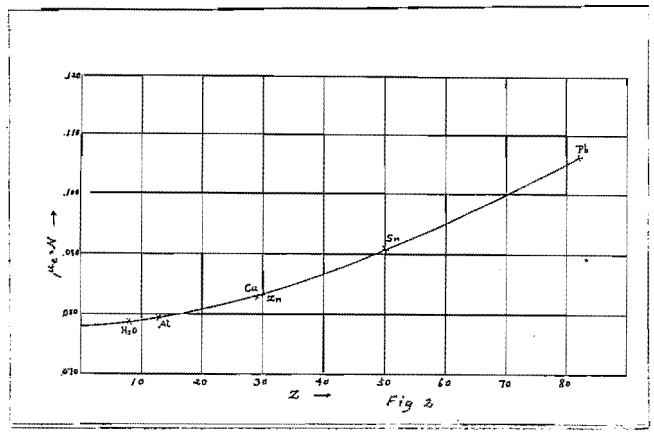

Fig 2

Now, in the derivation of Klein and Nishima's formula, all the external electrons are assumed to be free, the same is also assumed in deriving Compton's formula and Dirac's formula. Thus, theoretically, the value μ_a would be constant, while those given in Table VI increase with the atomic number. This increase of μ_a might be explained by the following alternatives. (1) A part of the scattering may be due to electrons inside the nucleus as suggested by Professor Millikan and Professor Bowen. (2) The scattering of a tightly bound electron of the atoms of high atomic number may be greater than that of a loosely bound electron. (3) There may still be some true absorption due to the photo-electric effect, which should,

however, be very small theoretically. Further discussion regarding these points is reserved to the last section after some consideration of the scattering intensity. It is only to be remarked here that this continuous increase of the value μ_e for hard radiations with respect to Z was not noticed before because of the following reasons: (1) Most previous works were done with non-homogeneous radiations, so that this effect is covered by the photo-electric absorption of the soft components which is increased as the third power of Z . (2) This effect was indeed present in the early works on ThC", but at that time emphasis was given to the mass absorption coefficient, while hydrogen contains one electron with one mass, the hydrogen compounds give high values of μ/ρ , so the character of the continuous increase of μ_e with respect to Z was obscured.

VIII MEASUREMENT OF THE SCATTERED RADIATIONS: Owing to the low intensity of the scattered rays, we can not here filter the primary beam with much lead. At present, it is only filtered through 2.7 cm. of lead. The scattering substance is set across the primary beam and is just large enough to cover the whole beam. The intensity of the scattered rays is measured by the ionization chamber at a distance of about 20 or 30 cm. from the scattering substance. The Al-block used for observing the scattering in the forward direction is of 1 inch thickness and about 25 inches square. It is set perpendicular to the primary beam. For observing the scattering

at 90°, the Al-block is set symmetrically with respect to the primary and scattered rays (i.e., its front surface makes angles of 45° with both directions) so as to make the computation simple. In this case the Al-block is therefore wider in the horizontal dimension in order to cover the whole beam. Lead blocks of similar area are also used, of course the thickness of the Pb-blocks is made approximately equivalent in mass per unit area to that of Al-blocks. In determining the scattering intensity, two sets of readings of the ionization current are taken, one is taken without the scattering block and the other by introducing it in the primary beam. The scattering in the forward direction can be measured quite well, but that at 90° is already very small. The object of this measurement is to compare the scattering of the light element with that of the heavy element rather than determining the absolute intensity. The result is given in Table VII. In this table, θ represents the mean value of the angle of scattering, γ the distance between the center of the ionization chamber and that of the scattering block, t represents the time interval for a given displacement of the electrometer needle, I the current in arbitrary unit or $I = 1000/t$, ΔI the current due to scattered rays entering the ionization chamber. In part A, ΔI_1 gives the result of one group of readings, ΔI_2 that of another group by using a different sensitivity of the electrometer. The thicknesses of the scattering blocks are represented by d_{Al} and d_{Pb}.

TABLE VII

A. Scattering in the Forward Direction:

$\theta = 22.5^\circ$, $r = 31.5$ cm, $d_{Al} = 2.56$ cm, $d_{Pb} = .522$ cm.

(1) Readings of the Time Intervals of Group 1.

Without Scattering	With Al-Scattering	Without Scattering	With Pb-Scattering
1568"	1443"	1583"	1494"
1580	1437	1573	1478
1570	1438	1563	1484
1582	1467	1566	1490
1565	1451	1574	1482
1582	1460	1556	1486
1566	1434	1584	1482
1588	1447	1574	1479
1554	1447	1581	1486
1571	1445	1572	1485
1573	1447	1573	1485

(2) Result of Group 1 and Group 2.

	t_1	I_1	t_2	I_2
Without Scattering	1573"	.6357	5078"	.1969
With Al-Scattering	1447	.6911	4683	.2135
Without Scattering	1573	.6357	5071	.1962
With Pb-Scattering	1485	.6734	4792	.2087

	ΔI_{Al}	ΔI_{Pb}	$\Delta I_{Al}/\Delta I_{Pb}$
Group 1.	.0554	.0377	1.47
Group 2.	.0166	.0115	1.44

B. Scattering at 90°

$\theta = 90°$, $r_1 = 31.5$ cm, $r_2 = 22.0$ cm, $d_{Al} = 2.56$ cm, $d_{Pb} = .536$ cm

	t_1	I_1	t_2	I_2
Without Scattering	4915"	.2035	4247"	.2355
With Al-Scattering	4876	.2051	4195	.2384
Without Scattering	4888	.2046	4247	.2355
With Pb-Scattering	4859	.2058	4204	.2379

	ΔI_{Al}	ΔI_{Pb}	$\Delta I_{Al}/\Delta I_{Pb}$
Group 1 with dist. r_1	.0016	.0012	1.3
Group 2 with dist. r_2	.0029	.0024	1.2

In the above table, the result of Group 2, part A can be compared with that of Group 1, Part B if we divide the value ΔI of the latter by 1.2. This reduces the two current values to the same arbitrary unit.

The absorption coefficient of Al, for γ-rays of ThC" filtered through different thicknesses of lead, is given in Table VIII for later use in the computation of scattering.

TABLE VIII

μ of Al after Different Pb-filters

Thickness of Pb-filter	μ
.0 cm	.150
1.36	.118
2.72	.109
6.80	.1026

IX COMPARISON OF THE EXPERIMENTAL RESULT OF SCATTERING WITH THE THEORY: Let μ be the absorption coefficient of the primary rays, μ' that of the scattered rays making an angle θ with respect to the primary rays. Let $\Delta\sigma$ be the fraction of energy scattered by 1 c.c. of the substance in the solid angle $\Delta\omega$, which is occupied by the ionization chamber. Referring to Fig. 3a, the amount of scattered energy entering the ionization chamber is then given by

$$E = A\int_0^d I_0\, e^{-\mu x}\,\Delta\sigma\left\{e^{-\frac{\mu'(d-x)}{\cos\theta}}\right\}dx, \qquad (6)$$

where A represents the cross-section of the primary beam.

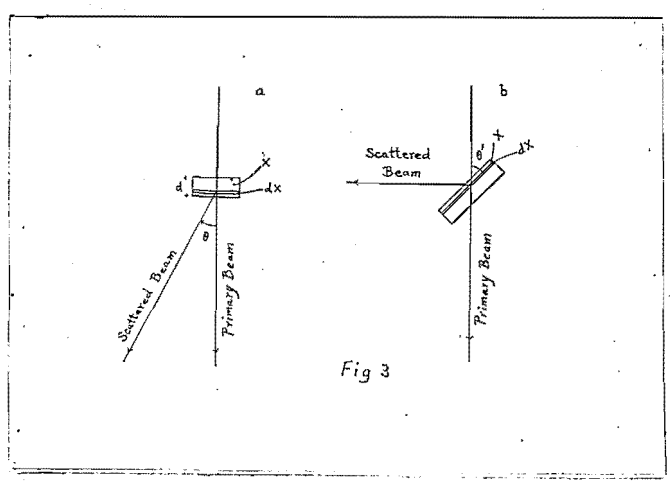

Fig 3

Now, in the above formula, μ' is only used for correction. For the present purpose, it is sufficient to estimate μ' by the following approximate method. The value μ for Al can be found in Table VIII, it is equal to .109 after the γ-rays are filtered through 2.7 cm. of lead. From Table VI,

we know that the absorption coefficient of light elements is fairly close to the theoretical value given by Klein and Nishima's formula. So we can calculate back from μ_{Al} to get the mean value of λ of the primary beam. This gives

$$\lambda_m = 5.7 \times U$$

By the use of Compton's formula (1), we can calculate the wave-length λ' of the scattered rays at any angle θ. From λ' we can again calculate μ' with the Klein and Nishima's formula. The value μ of Pb is given in Table III. μ'_{Pb} is calculated in another approximate method. From Table III and VIII we can calculate the ratio μ_{Pb}/μ_{Al} for different values of μ_{Al}. By an interpolation, we can find μ'_{Pb}. With this process we get, for $\theta = 22.5°$,

$$\lambda' = 7.5 \times U; \quad \mu'_{Al} = .127, \quad \mu'_{Pb} = .64$$

For $\theta = 90°$ we can not interpolate the value of μ_{Pb}/μ_{Al} in our previous result. In this case μ'_{Al} is calculated as before, μ'_{Pb} is calculated from the following semi-empirical formulae[3]

$$\mu_e = \tau_e + \sigma_e, \quad \tau_e = 2.24 \times 10^{-2} N^3 \lambda^3, \qquad (7)$$

where σ_e is given by Klein and Nishina's formula, τ_e is the photo-electric absorption per electron. In this way we get, for $\theta = 90°$,

$$\lambda' = 30 \times U, \quad \mu'_{Al} = .24, \quad \mu'_{Pb} = 1.74.$$

Referring to Fig. 3b, the amount of scattered energy entering the ionization chamber is given by

$$E = A \int_0^d I_0 e^{-\frac{\mu x}{\cos\theta'}} \Delta\sigma \left\{ e^{-\frac{\mu' x}{\cos\theta'}} \right\} \frac{dx}{\cos\theta'}, \qquad (8)$$

where θ' is the angle between the primary beam and the surface of the scattering block and is also equal to the angle between the scattered beam and the surface. In the present case $\theta = 90°$ $\theta' = 45°$.

The above computation of μ' is certainly very rough, but the values found are sufficiently close for the purpose of correcting terms.

By the use of the above values μ_{Al}, μ_{Pb}, μ'_{Al} μ'_{Pb} and d_{Al}, d_{Pb} we can calculate the ratio E_{Al} / E_{Pb} from equation (6) and (8) if we assume a certain relation between $\Delta \sigma_{Al}$ and $\Delta \sigma_{Pb}$. Since E is proportional to ΔI, this ratio can be compared with the experimental ratio $\Delta I_{Al} / \Delta I_{Pb}$.

We are going to test different assumptions, which can be put over $\Delta \sigma_{Al}$ and $\Delta \sigma_{Pb}$. First we might assume that the absorption of different elements is all due to scattering and they scatter in the same way with respect to space distribution. In this case we should have

$$\frac{\Delta \sigma_{Al}}{\Delta \sigma_{Pb}} = \frac{\mu_{Al}}{\mu_{Pb}}.$$

Carrying out the computation, we get

$$\left(\frac{E_{Al}}{E_{Pb}}\right)_{\theta = 22.5°} = 1.05, \qquad \left(\frac{E_{Al}}{E_{Pb}}\right)_{90°} = 1.20.$$

Compare these theoretical values with the experimental values of $(\Delta I_{Al} / \Delta I_{Pb})$, we see that the theoretical value for the forward scattering is much too low, in which case the difference certainly lies outside the limit of the experimental error.

We might assume that the scattering intensity is approximately proportional to the number of external electrons, but a part of the absorption is due to some other causes such as the nuclear scattering which contributes quite appreciably to the absorption but very little to the scattering intensity, at least in the forward direction. This anomalous absorption can also be traced to the scattering of tightly bound electrons. Based on this assumption, we get

$$\left(\frac{E_{Al}}{E_{Pb}}\right)_{22.5^\circ} = 1.42, \qquad \left(\frac{E_{Al}}{E_{Pb}}\right)_{90^\circ} = 1.62.$$

We see that the forward scattering can be accounted by this assumption very satisfactorily, while the departure of the values at 90° is also to be expected. For the part of the anomalous absorption might contribute some scattering, which is not appreciable in the forward direction but becomes important when the normal scattering intensity decreases as the angle of scattering increases. However, the scattering intensity at 90° is so low that the probable error is very high and we could not give too much emphasis on this point.

To sum up, we can say that the forward scattering is predicted fairly well by the Klein and Nishina's theory regarding the external electrons as free for high frequencies. But an anomalous absorption is present which increases as the atomic number Z increases, and this anomalous absorption does not give much scattered rays in the forward direction as the normal scattering process, it might, however, become important as the angle of scattering increases.

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